Design of a Cylindrical Cavity Resonator for Measurements of Electrical Properties of Dielectric Materials

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ABSTRACT

In microwave communications, the main aspects for affecting the dielectric losses in the materials are relating to the dielectric properties and the radiation frequencies. Normally, the different dielectric materials will lead to the different losses and reflections for microwave frequencies. To evaluate the dielectric properties from the different materials plays an essential role in the microwave engineering.

There are many approaches can be used to measure the dielectric materials, e.g. capacitor methods, transmission line methods, cavity resonator methods, open cavity methods and so on. The cavity resonator method is one of the most popular ways for measuring the dielectric materials. In this thesis, some of the techniques will be reviewed, and the $\text{TM}_{010}$ mode cylindrical cavity resonator with perturbation technique will be used for determining the dielectric properties. The design and measurements will be presented in both simulations and practice. With 1.2GHz cavity resonator, in the simulations, the dielectric permittivity for Teflon is measured as $2.09 - 0.0023i$ and $2.12 - 0.0116$ in copper cavity and ferromagnetic cavity. Finally the sample is measured as $3.83 - 0.12i$ in practice.

Key words: Dielectric constant measurement, Cavity resonator, Dielectric property, $\text{TM}_{010}$ mode, Coupling device, Perturbation technique.
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Abbreviations

DC – Direct Current
TE – Transverse Electric
TM – Transverse magnetic
MUT – Material Under Test
HFSS – High Frequency Structure Simulator
VNA – Vector Network Analyzer
HL – High Loss
LL – Low Loss
IEEE – Institution of Electrical and Electronics Engineers
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ABSTRACT

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I. Introduction

1.1 Background

The measurements for permittivity have been introduced and applied since many years ago. The earlier concept of the permittivity measurements is based on DC electrical resistance to determine grain moisture content. With the development of microwave engineering, the measurement methods are enhancing, there are many new methods have been studied and used in the measurement region, e.g. waveguide methods, cavity resonator methods, open resonator methods, free space methods and so on. However, the methods shown different benefits and defects, the designer has to understand the features of them and find out the property one to estimate the dielectric materials.

In this thesis work, one of the methods will be applied for determining the dielectric sample from Radarbolaget. The characteristic of the sample is semi-solid, which is a kind of sand. According to the futures of the sample, the cavity resonator has ability to measure it.

In Radarbolaget, the purpose of the dielectric sample is for constructing the wall of the building. Radarbolaget uses radar to detect the objects from the inner of the building. For the case of the radio frequencies coming from the outside and transmitting to the inner of the building, the wall yields some losses and reflections to the radio signals. As we known, the affections to the radio frequencies due to the dielectric materials are different. To determine the dielectric properties from the sample can help us to know how many losses and reflections are produced, and finally, the accuracy of the results from the detected objects will be raised.

1.2 Review of the Literatures

The dielectric and magnetic measurements base on radio and microwave methods have been studied in depth and the measurements for dielectric materials are studied in different aspects. The literature by Mohammed Nurul Afsar [2] covers the most of the techniques used to measure dielectric properties of materials over frequency range 1MHz to 1500MHz. A literature by Bussey [3] introduces the methods for radio and microwave measurements of dielectric and
magnetic properties of materials. A literature by Redheffer [4] covers the most of microwave arrangements that may be used. In this thesis, some of the methods will be looked.

In literature [5] and [6], the authors introduced that the parallel plate capacitor is used as the test cell for measuring the dielectric samples. The complex dielectric permittivity is obtained by measuring the change of capacitance and of conductance due to the device with and without specimen, the fringing field affections are solved by mathematical corrections. Figure 1.1 shows the measurement by using capacitor method.

![Parallel Plate Capacitor Measurement Method](image)

**Figure 1.1:** Parallel Plate Capacitor Measurement Method

Transmission line method is simple and conveniently, it does not need the particular device in the measurements. An advantage of this method is that the method is suitable for the broadband frequencies, but the arrangement of the sample is somewhat complex, it has to be made into a slab or annular geometry, the sample with the transmission line are illustrated in Figure 1.2. The coaxial lines and waveguide are normally used to measure samples. [7] used the transmission-reflection and short-circuit line method measured the dielectric materials and presented the uncertainty analysis for this method. For the granular and liquid materials, and [8] has shown the measurement. [9] introduced the materials are measured inside a partially filled waveguide.

![Transmission Line Measurement Method](image)
Figure 1.2: Transmission Line Methods;

(a) Sample inside the Waveguide; (b) Sample inside the Coaxial Line

The microwave cavity resonators are popularly used to measure the dielectric constant and permeability measurement area, Figure 1.3. It is very conveniently for dielectric measurements and loss tangents over a wide range. Cook [10] obtained the best uncertainties for permittivity and loss, which shows approximately ±0.2 percent in a permittivity of 2 and ±3 percent in 100 \( \mu \text{rad} \) at 95 percent confidence level.

Perturbation technique is a simple and conveniently method for measurement, literature [1] and [12] introduced the mathematical model and principle of the perturbation theory for resonator cavities, [12], [13], [14], [15], [16] employed the cavity perturbation method and provided the theoretical analysis for different cavity resonators, in which [12] measured the dielectric materials for solid and liquids, the performance by using cavity resonator for the measurements shown accurate results, [13] measured the dielectric material by using the rectangular cavity resonator at the TE\(_{10P}\) mode, [14] analyzed the error of air gap which is happened due to the sample length less than the cavity length, and [15], [16] measured the dielectric constant in a cylindrical cavity resonator at the TM\(_{0n0}\) mode,

Figure 1.3: The Dielectric Material Measured by Cylindrical Cavity Resonator at the TM\(_{0n0}\) Mode
There are some literatures [17, 18, 19] using open resonators to measure the dielectric properties. This method have been used in many years, basically, the open resonators can be defined as two types, hemispherical and spherical, Figure 1.4 (a), (b). Dudorov and his co-workers [17] used both hemispherical and spherical resonator to measure the thin film materials, [18] measured the non-planar dielectric object by using the open resonator and [19] used the open resonator to measure the materials at 100GHz, the uncertainty for measurement is 0.02% to 0.04% for $\varepsilon_r \geq 2$ and $6 - 40 \times 10^{-6}$ for $\tan \delta(10^{-4} \leq \tan \delta \leq 10^{-3})$. The operation frequencies for open resonator methods are commonly used in the millimeter region (30~200 GHz), however, it also can be used in low frequency region if the sample size is large diameters. There is also the other application [20] included in the magnetic materials measurement for open resonators.

**Figure 1.4: Open Resonators Measuring Dielectric Materials;**

(a) Hemispherical Mirror; (b) Spherical Mirror

Free space methods use two antennas for transmitting and receiving, the MUT is placed between these two antennas, the sample usually fixed on a slab, see Figure 1.5 (a). For this method, the materials are not required to specify the geometry, therefore this method can be used to measure the materials at high temperature. For determine the dielectric properties, the attenuation and phase shift are evaluated. Literature [21] is using the free space method to measure the intrinsic material properties. On the other hand, the reflection measurements are also possible for free space method [22], see Figure 1.5 (b).

Table 1.1 lists some general comparisons for the above methods.
Figure 1.5: Free Space Method;

(a) Transmission Measurement; (b) Reflection Measurement
Table 1.1: The Simple Comparisons for the Different Measurement Methods

<table>
<thead>
<tr>
<th></th>
<th>Accuracy for Low Loss (LL) and High Loss (HL) materials</th>
<th>Frequency band</th>
<th>Sample’s type</th>
<th>Preparation for sample</th>
<th>Valid parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Capacitor method</strong></td>
<td>High (Both)</td>
<td>Single</td>
<td>Thin films</td>
<td>Difficult</td>
<td>Permittivity</td>
</tr>
<tr>
<td><strong>Transmission line methods</strong></td>
<td>Moderate (Both)</td>
<td>Broadband</td>
<td>Solid, Sand, Liquids</td>
<td>Difficult</td>
<td>Permittivity and permeability</td>
</tr>
<tr>
<td><strong>Cavity resonator methods</strong></td>
<td>High (LL) Low (HL)</td>
<td>Single</td>
<td>Solid, Sand, Liquids</td>
<td>Difficult</td>
<td>Permittivity and permeability</td>
</tr>
<tr>
<td><strong>Open resonator methods</strong></td>
<td>High (LL) Low (HL)</td>
<td>Single</td>
<td>Thin films</td>
<td>Easy</td>
<td>Permittivity and permeability</td>
</tr>
<tr>
<td><strong>Free space methods</strong></td>
<td>Moderate (Both)</td>
<td>Banded</td>
<td>Flat materials</td>
<td>Easy</td>
<td>Permittivity and permeability</td>
</tr>
</tbody>
</table>

1.3 Problem Definitions

There are many parameters decide the accuracy of the results for the dielectric materials measurements. Q-value is one of the most important factors for estimating the quality of the cavity resonator, as high Q-value as high accuracy and narrow bandwidth. The effects for the Q-value can be decided by many conditions, e.g. the metallic material for building cavity resonator, the filled material inside cavity, the coupling device and the transverse modes. However these conditions can be fixed during designing. On the other hand, the effects from the coupled external circuit also needs to be taken into account, when the external circuit connected to the cavity resonator, the measured Q-value will no longer be the original Q-value ($Q_0$), which will be
changed to the loaded Q-value (QL). Therefore, the accuracy of the measured results for the dielectric samples is depending on the accuracy of the Q-values.

To achieve the matching during the sample placing inside the cavity resonator i.e. critical coupled (g=1), the cavity resonator must be over coupled to the external circuit, which means the Q-value (QL) of the cavity resonator must be higher than the Q-value (Qe) of the external circuit, briefly, the resistance (R) of the cavity resonator must be higher than the resistance (Z0) of the external circuit so that the device is able to achieve over coupled. The dielectric sample also needs to be fabricated to a property size so that when the sample is fully placed in the cavity resonator the critical coupled occurring.

1.4 Objectives

The main objective for the thesis work is to find the dielectric properties of the materials in the specified frequency. For the microwave cavity resonator design, the aspects of the cavity physical size, coupling loop, Q-values, and the coupling factor have to be taken into account. The thesis work can be separated to the following missions:

- Simulate the copper cylindrical cavity resonator and measure the dielectric material
- Simulate the ferromagnetic cylindrical cavity resonator and measure the dielectric material
- Design of the cylindrical cavity resonator in practice and measure the dielectric sample
- Compare and analyze the simulated results in simulations
- Analyze the measured results in practical
II. Theoretical Analysis

2.1 Cylindrical Cavity Resonator

Generally, the cavity resonator can be constructed from circular waveguide shorted at both ends or built by a cylindrical metal box, i.e. cylindrical cavity resonator [23]. The basic concept of the circular waveguide and the cylindrical cavity resonator are similar. An illustration for the cylindrical cavity resonator is given in Figure 2.1. Inside the cavity resonator, the electric and magnetic fields exist, the total energies of the electric and magnetic field are stored within the cavity, and the power can be dissipated in the metal wall of the cavity resonator as well as the filled dielectric material. Beside, the filled dielectric material will affect the resonant frequency and Q-value. The transverse modes used in the cavity resonators are the TE and TM mode, which will provide the different dimensions, resonant frequencies and Q-values.

For the use of the cavity resonator, it has to be excited via the other coupling device. The energies are transmitted from the external equipment to the cavity via the coupling device. for the TE and TM mode, the coupling devices will be different, section 2.2 will focus on the excitation for the cavity with the different coupling devices.

![Figure 2.1: Cylindrical Cavity Resonator](image)
2.1.1 Propagation Constant

The propagation constant of the TM<sub>nm</sub> mode is described in (2-1), the values of <i>p</i><sub>nm</sub> for the TM<sub>nm</sub> mode has listed in Table 2.1

\[ \beta_{nm} = \sqrt{k^2 - \left( \frac{p_{nm}}{a} \right)^2} \]  \hspace{1cm} (2-1)

where \( k \) is the wavenumber of the resonant wave

\[ a \] is the radius of the cylindrical cavity resonator

<table>
<thead>
<tr>
<th>n</th>
<th>( P_{n1} )</th>
<th>( P_{n2} )</th>
<th>( P_{n3} )</th>
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<tr>
<td>0</td>
<td>2.405</td>
<td>5.520</td>
<td>8.654</td>
</tr>
<tr>
<td>1</td>
<td>3.832</td>
<td>7.016</td>
<td>10.174</td>
</tr>
<tr>
<td>2</td>
<td>5.135</td>
<td>8.417</td>
<td>11.620</td>
</tr>
</tbody>
</table>

2.1.2 Resonant Frequency

The resonant frequency operates in the cavity resonator, which has the same properties as the frequency operate in the waveguide. The resonant frequencies have to higher than the cut off frequency when it is operating. The dominate mode for the TM<sub>nm</sub> mode is the TM<sub>010</sub> mode, there are also other modes resonant inside the cavity at higher frequencies, Figure 2.2 illustrates the other transverse modes and resonant frequencies. Basically, the resonant frequency is related to the dimension of the cavity and the filling materials. Equation (2-2) gives the derivation of the resonant frequency

\[ f_{nm} = \frac{c}{2\pi \sqrt{\mu \varepsilon_r}} \sqrt{\left( \frac{p_{nm}}{a} \right)^2 + \left( \frac{\ell \pi}{d} \right)^2} \]  \hspace{1cm} (2-2)
where \( f_{\text{nnl}} \) is the operation frequency of the cylindrical cavity resonator

\[ c \] is the speed of light

\( \mu_r \) is the permittivity of the filled material inside the cylindrical cavity resonator

\( \varepsilon_r \) is the permeability of the filled material inside the cylindrical cavity resonator

\( d \) is the height of the cylindrical cavity resonator

![Figure 2.2: Resonant Mode Chart for a Cylindrical Cavity Resonator](image)

**2.1.3 TM\textsubscript{010} Mode**

As described in section 2.1.2, the TM\textsubscript{010} mode is the dominate mode, with \( p_{01} = 2.405 \). Figure 2.3 (a) shows that the electric and magnetic fields of the TM\textsubscript{010} mode within the cylindrical cavity resonator, in which the magnetic field is parallel to the cavity bottom and perpendicular to the electric field. The density of magnetic field is increasing from the cavity center to the boundary, and the electric field is opposition. Figure 2.3 (b) shows the relationship between them, the dashed line is the magnetic fields and the solid line is the electric fields. The equations for the TM\textsubscript{010} mode are described in formula (2.3), the derivations and detail parts have been discussed in literature [23] and [24].

\[
E_r = -j\beta K_0 J_0(K_r r)(B^+ e^{-j\beta z} - B^- e^{j\beta z})
\]  

(2-3a)
\[ E_z = K_z J_0(K_z r)(B^+ e^{-j\beta z} - B^- e^{j\beta z}) \]  
\[ H_\phi = j\omega \varepsilon_0 K_z J_0(K_z r)(B^+ e^{-j\beta z} - B^- e^{j\beta z}) \]

where \( B^+ = B^- = B \), so we obtain the field components \( E_z \) and \( H_\phi \) of the TM\(_{010}\) as follow,

\[ E_z = 2B\beta_0^2 J_0(\beta_0 r) \]  
\[ H_\phi = 2jB \frac{\beta_0^2}{\eta} J_1(\beta_0 r) \]

where \( \eta \) is the intrinsic impedance of the material filling the cavity resonator (\( \eta = 377 \) for air)

![Image](image-url)

(a) Electric and Magnetic Fields for TM\(_{010}\) Mode  
(b) Electric and Magnetic Fields as a Function for a TM\(_{010}\) Mode in a Cylindrical Cavity Resonator

2.1.4 Quality Factor

Quality factor (Q-value) is an essential parameter for estimating the quality of cavity resonators [23]. The high Q-value indicates the high accuracy and the narrow bandwidth of the cavity resonator. The unloaded Q-value (\( Q_0 \)) means the Q-value for the cavity resonator without connection of the external circuit, see equation (2-6), which is relevant to the conductivity of the metal material as well as the filling materials within the cavity. For an air filled cylindrical cavity
resonator, Figure 2.4 shows the normalized $Q_0$ due to conductor loss for various resonant modes. Equation (2-7) gives an unloaded $Q$-value with air filled cavity for the $TM_{010}$ mode

$$Q_0 = \left( \frac{1}{Q_c} + \frac{1}{Q_d} \right)^{-1}$$

(2-6)

where $Q_c$ is the $Q$-value of the metal conducting wall

$Q_d$ is the $Q$-value of the filled material

$$Q_0 = Q_c = \frac{2V}{S \sqrt{\frac{2}{\omega \mu \sigma}}}$$

(2-7)

where $V$ is the volume of the cavity resonator

$S$ is the surface area of the cavity resonator

$\sigma$ is the conductivity of the metal wall

In practice, however, the cavity resonator has to be connected to an external circuit which has a $Q$-value ($Q_e$). The external circuit will always have the effect of lowering the overall of the system. The expressions for the unloaded $Q$ ($Q_0$), the external $Q$ ($Q_e$) and the loaded $Q$ ($Q_L$) are given in (2-8), (2-9), (2-10) respectively

$$\frac{1}{Q_L} = \frac{1}{Q_0} + \frac{1}{Q_e}$$

(2-8)

$$Q_0 = \frac{R}{\omega_0 L_c}$$

(2-9)

$$Q_e = \frac{R_t}{\omega_0 L_{eq}}$$

(2-10)
2.1.5 The Equivalent Circuit

The cylindrical cavity resonator can be equal to a parallel RLC resonant circuit, which is illustrated in Figure 2.5. The input impedance of the resonator is represented as, [23]

\[ Z_{in} = \left( \frac{1}{R} + \frac{1}{j\omega L} + j\omega C \right)^{-1} \]  

(2-11)

After rearrange formula (2-11), the input impedance is

\[ Z_{in} = \frac{R}{1 + jR \left( \omega C - \frac{1}{\omega L} \right)} \]  

(2-12)

However, the resonant frequency \( \omega_0 = \frac{1}{\sqrt{LC}} \Rightarrow C = \frac{1}{\omega_0^2 L} \), then with (2-12) the input impedance can be obtained

\[ Z_{in} = \frac{R}{1 + j \frac{R}{L} \left( \frac{\omega}{\omega_0} - \frac{1}{\omega} \right)} \]  

(2-13)

where \( \left( \frac{\omega}{\omega_0} - \frac{1}{\omega} \right) = \frac{\omega^2 - \omega_0^2}{\omega_0 \omega} = \frac{1}{\omega} \left( \omega + \omega_0 \right) \left( \omega - \omega_0 \right) \frac{\omega^2}{\omega_0^2} \)
When \( \omega \approx \omega_0 \), we have \( \omega + \omega_0 = 2\omega_0 \), it can be derived in (2-14)

\[
\frac{\omega}{\omega_0^2} - \frac{1}{\omega} \approx \frac{1}{\omega_0} \frac{2\omega_0(\omega - \omega_0)}{\omega_0^2} \approx \frac{\omega - \omega_0}{\omega_0} \frac{2}{\omega_0}
\]

(2-14)

![Resonator diagram](image)

**Figure 2.5:** The Resonator with an External Circuit

Let \( \Delta = \frac{\omega - \omega_0}{\omega_0} \), with (2-13) and (2-14), the input impedance is

\[
Z_{in} = \frac{R}{1 + jQ_o 2\Delta}
\]

(2-15)

where \( Q_o = \left( \frac{R}{\omega_0 L} \right) \)

For the parallel resonant circuit, the coupling factor is given by

\[
g = \frac{R}{Z_0}
\]

(2-16)

\( g < 1 \), the resonator is said to be under coupled

\( g = 1 \), the resonator is said to be critical coupled

\( g > 1 \), the resonator is said to be over coupled

With (2-15) and (2-16), the input impedance of parallel resonant circuit is derived

\[
Z_{in} = \frac{gZ_0}{1 + jQ_o 2\Delta}
\]

(2-17)
Finally the reflection coefficient can be obtained as

$$\Gamma = \frac{Z_{in} - Z_0}{Z_{in} + Z_0} = \frac{gZ_0}{1 + jQ_0 \Delta} + \frac{Z_0}{gZ_0 + jQ_0 \Delta} = \frac{g - 1 - j2Q_0 \Delta}{g + 1 + j2Q_0 \Delta} \tag{2-18}$$

In the reflection coefficient measurement, the examples for smith chart and the reflection coefficient are shown in Figure 2.6 and 2.7 respectively. In Figure 2.6, the smith chart shows three situations for coupling factor, in which the coupling factor is derived by (2-16). When the cavity resonator (R) critical coupled to the transmission line (Z_0), the imaginary part is equal to zero. The real reflection coefficient can be expressed as (2-19)

$$\Gamma_{\text{real}} = \frac{g - 1}{g + 1} \tag{2-19}$$

The unloaded Q (Q_0) is expressed in (2-20), which can be derived by rearranging (2-9), (2-10) and inserting into (2-16). The loaded Q (Q_L) can be derived by inserting (2-20) into (2-8), the final expression is given in (2-21)

$$Q_0 = g \times Q_e \tag{2-20}$$

$$Q_L = \frac{Q_0}{1 + g} \tag{2-21}$$

The loaded Q-value can be also calculated from the reflection coefficient measurement curve (Figure 2.7) by

$$Q_L = \frac{f_0}{\Delta f} \tag{2-22}$$

where \( f_0 \) is the resonant frequency

\( \Delta f \) is the bandwidth of the reflection curve
2.2 Coupling Devices

The coupling devices are used for transferring the energy from the feed line to the resonator. The coupling methods have been discussed in literature [25-27]. If the method of the reflection coefficient is used, only one coupling device is needed, and two in the case of method of transmission coefficient. In this thesis design, the coupling loop is used to excite the cavity resonator, the principles and mathematical equations for it are introduced in section 2.2.3. There are also some other methods can be used to excite the waveguide resonator or cavity resonator,
such as coupling holes and coupling probes. The section 2.2.1 and 2.2.2 will introduce the basic principles for them.

### 2.2.1 Coupling Holes

Coupling holes are designed in a waveguide which is used as the feeding transmission line, the aperture, Figure 2.8, is the natural coupling device. Depending on the location of the aperture in the waveguide, the tangential magnetic field or normal electric field will penetrate the aperture and couple to the resonance mode. The strength of the coupling (the electric or magnetic dipole moment) is proportional to the third power of the radius of the aperture. The coupling depends, of course, on the location of the aperture with respect to the field of the resonance mode and the direction of the field lines in the case of magnetic coupling [26].

![Coupling Hole Diagram](image)

**Figure 2.8:** The Coupling Hole Coupled to a Cavity Resonator

### 2.2.2 Coupling Probe

Coupling probe is formed by extending the feeding coaxial cable with a small distance into the cavity resonator, which is shown in Figure 2.9, the length of the probe is small compare to the wavelength, normally it is equal to quarter wave length, and the input impedance is therefore nearly equivalent to that of an open circuit. The current in the probe is small, but the voltage creates an electric field between the probe and the adjacent wall of the resonator. The field radiates energy into the resonator like a small monopole antenna. The probe couples to the electric field that is perpendicular to the wall at the location of the probe. The coupling is stronger the closer to a field maximum the probe is located. Coupling to unwanted modes can be
avoided by locating at least one of the probes in a place, where the electric field of such a mode is zero [26].

![Coupling Probe Coupled to a Cavity Resonator](image)

**Figure 2.9:** The Coupling Probe Coupled to a Cavity Resonator

### 2.2.3 Coupling Loop

Coupling loop is formed by extending the feeding coaxial cable with a distance into the cavity resonator and bent and grounded on the cavity wall, the center of the loop is located midway between the top and the bottom walls of the cavity, Figure 2.10 shows the position of the coupling loop inside the cavity resonator. The size of the loop is small compare to the wavelength, therefore the voltage is nearly zero but the current is large, and the input impedance is nearly equivalent to that of a short circuit. The current generates a magnetic field that radiates like a magnetic dipole tangential to the wall. The dipole moment is proportional to the loop area. The radiation couples to the magnetic field of a resonance mode that is tangential to the wall and perpendicular to the plane of the loop. Therefore the orientation of the loop is also important.

As the TM_{010} mode cavity resonator coupling with the external circuit, the cavity resonator and the coupling loop will be a parallel resonant circuit, which has shown in Figure 2.5. Due to the cavity resonator coupled to the external coupling loop, the coupling factor (g) is related to the surface resistivity of the cavity resonator and the resistance of the coupling loop, the principle for the coupling loop coupled to the cavity resonator are broad introduced in the literates [24], [25]. The accurate size of the coupling loop can be calculated by formula (2-23),

\[ g = \frac{Z_0}{Z_L} \]
\[ R_c = \frac{(\omega \mu A)^2}{2\pi a(h + a)R_s} \]  \hspace{1cm} (2-23)

where  
\( R_c \) is the resistance for the cavity resonator

\( a \) is the radius of cavity resonator

\( h \) is the height of the cavity resonator

\( R_s \) is the surface resistivity of the cavity resonator

\( \mu \) is the permeability of the cavity resonator

\( A \) is the area of the coupling loop enclose to the cavity

Rearrange the formula (2-23), the radius of the coupling loop can be obtained in (2-24)

\[ r = \sqrt{\frac{2\sqrt{R_c 2\pi a(h + a)R_s}}{\omega \mu \pi}} \]  \hspace{1cm} (2-24)

**Figure 2.10**: The Coupling Loop Coupled to the Cylindrical Cavity Resonator

### 2.3 The Properties of Dielectric Materials

The complex permittivity \( \varepsilon' \) and tangent loss \( \tan \delta \) are two important parameters for dielectric materials, in which \( \varepsilon' \) indicates the real part and \( \varepsilon'' \) indicates the imaginary part. The real part \( \varepsilon' \) is the dielectric constant (relative to air), it is related to the stored energy within the medium,
and the imaginary part $\varepsilon''$ is related to the dissipation (or loss) of energy within the medium. The equations for them are given in the below, [23]

$$
\varepsilon' = \varepsilon' - j\varepsilon'' \tag{2-25}
$$

$$
\tan \delta = \frac{\varepsilon''}{\varepsilon'} \tag{2-26}
$$

### 2.4 Cavity Perturbations

In practical applications [23], the cavity resonators are normally modified by making small changes of the cavity volume, or introduce a small dielectric sample. To achieve that, the cavity resonators can be tuned with a small screw that enters the cavity volume, or by changing the size of the cavity resonator. The other way involves the measurement of the dielectric constant by determine the shift of the resonant frequency when a small dielectric sample is inserted into the cavity. The perturbation method assumes that the actual fields of a cavity with a small shape or material perturbation are not greatly different from those of the unperturbed cavity.

For the material filling a part of the cavity resonator, the permittivity is expressed as $\varepsilon$ and the permeability is expressed as $\mu$, $\overrightarrow{E}_0$ and $\overrightarrow{H}_0$ represents the electric field and magnetic field for the original cavity, $\overrightarrow{E}$ and $\overrightarrow{H}$ represents the fields of the perturbed cavity, the Maxwell’s curl equations can be written as

$$
\nabla \times \overrightarrow{E}_0 = j\omega_0 \mu \overrightarrow{H}_0 \tag{2-27a}
$$

$$
\nabla \times \overrightarrow{H}_0 = j\omega_0 \varepsilon \overrightarrow{E}_0 \tag{2-27b}
$$

$$
\nabla \times \overrightarrow{E} = -j\omega(\mu + \Delta\mu) \overrightarrow{H} \tag{2-28a}
$$

$$
\nabla \times \overrightarrow{H} = j\omega(\varepsilon + \Delta\varepsilon) \overrightarrow{E} \tag{2-28b}
$$

where $\omega_0$ and $\omega$ are the resonant frequency of the original cavity and the perturbed cavity

$\Delta\varepsilon$ and $\Delta\mu$ are the cavity perturbed by a change in permittivity and permeability
Multiply the conjugate of (2-27a) by \( \overline{H} \) and multiply (2-27b) by \( \overline{E_0} \) to get

\[
\overline{H} \cdot \nabla \times \overline{E_0} = j\omega_0\mu \overline{H} \cdot \overline{H_0} \quad (2-29a)
\]

\[
\overline{E_0} \cdot \nabla \times \overline{H} = j\omega (\varepsilon + \Delta \varepsilon) \overline{E_0} \cdot \overline{E} \quad (2-29b)
\]

Subtracting these two equations and using vector identity that \( \nabla \cdot (A \times B) = B \cdot \nabla \times A - A \cdot \nabla \times B \) gives

\[
\nabla \cdot (\overline{E_0} \times \overline{H}) = j\omega_0\mu \overline{H} \cdot \overline{H_0} - j\omega (\varepsilon + \Delta \varepsilon) \overline{E_0} \cdot \overline{E} \quad (2-30)
\]

Similarly, we multiply the conjugate of (2-27b) by \( \overline{E} \) and multiply (2-28a) by \( \overline{H_0} \) to get

\[
\overline{E} \cdot \nabla \times \overline{H_0} = -j\omega_0 \overline{E_0} \cdot \overline{E} \quad (2-31a)
\]

\[
\overline{H_0} \cdot \nabla \times \overline{E} = -j\omega (\mu + \Delta \mu) \overline{E_0} \cdot \overline{H} \quad (2-31b)
\]

Subtracting these two equations and using vector identity gives

\[
\nabla \cdot (\overline{E} \times \overline{H_0}) = -j\omega (\mu + \Delta \mu) \overline{E_0} \cdot \overline{H} + j\omega_0 \varepsilon \overline{E_0} \cdot \overline{E} \quad (2-32)
\]

Now add (2-30) and (2-32), integrate over the volume \( V_0 \), and use the divergence theorem to obtain

\[
\int_{V_0} \nabla \cdot (\overline{E_0} \times \overline{H} + \overline{E} \times \overline{H_0}) \, dv = \left[ \int_{V_0} \overline{E_0} \times \overline{H} + \overline{E} \times \overline{H_0} \right] \cdot d\vec{S} = 0
\]

\[
= j\int_{V_0} \{[\omega_0\varepsilon - \omega (\varepsilon + \Delta \varepsilon)] \overline{E_0} \cdot \overline{E} + [\omega_0\mu - \omega (\mu + \Delta \mu)] \overline{H_0} \cdot \overline{H} \} \, dv
\]

where the surface integral is zero because \( \hat{n} \times \overline{E} = 0 \) on \( S_0 \). Rewriting gives

\[
\frac{\omega - \omega_0}{\omega} = \frac{-\int_{V_0} (\Delta \varepsilon \overline{E} \cdot \overline{E_0} + \Delta \mu \overline{H} \cdot \overline{H_0}) \, dv}{\int_{V_0} (\varepsilon \overline{E} \cdot \overline{E_0} + \mu \overline{H} \cdot \overline{H_0}) \, dv}
\]

(2-34)
This is an exact equation for the change in resonant frequency due to material perturbations, but is not in a very usable form since we generally do not know $\overline{E}$ and $\overline{H}$, the exact fields in the perturbed cavity. But, if we assume that $\Delta \varepsilon$ and $\Delta \mu$ are small, then we can approximate the perturbed fields $\overline{E}$, $\overline{H}$ by the original fields $\overline{E}_0$, $\overline{H}_0$, and $\omega$ in the denominator of (2-34) by $\omega_0$, to give the fractional change in resonant frequency as

$$\frac{\omega - \omega_0}{\omega_0} = \frac{-\int_{V_c} (\Delta \varepsilon |E_0|^2 + \Delta \mu |H_0|^2) dv}{\int_{V_s} (\varepsilon |E_0|^2 + \mu |H_0|^2) dv}$$

(2-35)

This result shows that any increase in $\varepsilon$ or $\mu$ at any point in the cavity will decrease the resonant frequency, the term (2-35) can be related to the stored electric and magnetic energies in the original and perturbed cavities, so that the decrease in resonant frequency can be related to the increase in stored energy of the perturbed cavity.

For the case of the rod MUT inserted into a TM$_{010}$ mode cylindrical cavity resonator, von Hippel has derived the equations for calculating the complex dielectric properties of the dielectric sample, the detail parts and the derivations are discussed in the literature [1]. The real and imaginary part of the permittivity is determined by the frequency shift and the in Q-value changes, the equations are given in (2-36) and (2-37)

$$\varepsilon' = 1 + 0.539 \times \frac{V_s (f_e - f_s)}{V_c f_o}$$

(2-36)

where $\varepsilon'$ is the dielectric constant

$V_c$ is the volume of the cavity resonator

$V_s$ is the volume of the under test dielectric sample

$f_o$ is the original frequency of the cavity resonator

$f_s$ is the shifted frequency
\[
\varepsilon' = 0.269 \times \frac{V_s}{V_r} \times \left( \frac{1}{Q_{LS}} - \frac{1}{Q_{L0}} \right)
\]  

(2-37)

where \( Q_{LS} \) is the Q-value for the sample inside the cavity resonator

\( Q_{L0} \) is the Q-value for the empty cavity resonator loaded with coupling loop

The MUT should be placed in the center of the cavity resonator, because the center position is the strongest electric fields at, and the height of the sample is equal to the height of the cavity resonator. For preventing the perturbation too much strong, the sample should be enough small compare to the cavity resonator, otherwise the electric and magnetic fields will be distortion. Basically, the volume of the sample will be built equal to or less than 1/1000 of the volume of the cavity.
III. Measurement of the Dielectric Properties

3.1 Experiment Setup

The cavity resonator measurement working with perturbation technique have to be performed by using the Vector Network Analyzer (VNA), the VNA have the function for measuring the transmission and reflection of the radio frequencies, the experiment can be realized by measuring of the reflection coefficient (S11) from cylindrical cavity resonator, the measured data will be updated to a computer to calculate the dielectric properties and loss factor, the Figure 3.1 shows the experiment setup in the experiment, Figure 3.2 shows the block diagram.

Figure 3.1: The Cavity Perturbation Method’s Setup

Figure 3.2: Block Diagram of the Experiment Setup


3.2 Procedure

3.2.1 The Design of the Cylindrical Cavity Resonator

The TM_{010} mode cylindrical cavity resonators are both designed in the HFSS and practice. Generally the copper or brass can be taken for the fabrication of cavity. Other materials like aluminum can also be used. If the cavity resonator is big and weighty, the low density metal or alloy can be considered to use. Some designers coat the silver on the inner surface of cavity to reduce skin effect. In the simulations, the cavity metal wall is selected to use copper. In the practice, the cookie box can be chose to make the cavity. There are different benefits and defects for copper cavity and cookie box. Table 2.2 illustrates the basic comparisons for them.

<table>
<thead>
<tr>
<th>Comparisons</th>
<th>Benefits</th>
<th>Defects</th>
</tr>
</thead>
<tbody>
<tr>
<td>Copper</td>
<td>High Q-value; Low losses; High accuracy</td>
<td>High Cost; Long produce period</td>
</tr>
<tr>
<td>Cookie box</td>
<td>Low cost; Without manufacture; Can achieve the goals</td>
<td>Low Q-value; High losses; Low accuracy</td>
</tr>
</tbody>
</table>

The cavity resonators are designed at 1.2GHz, the radius can be calculated from formula (3-1). For the TM_{010} mode, the height of the cavity is not limited, because the $\ell$ is equal to 0. Figure 3.3 shows the copper cavity resonator with and without sample in the HFSS simulations. The radius of the cavity resonator is 95 mm, the height is 65 mm, the coupling loop is designed on the middle of the metal wall, and we can see it in the figure. The radius of coupling loop in the simulations is 5 mm and the radius of the coupling loop in the practice is 25 mm. Figure 3.4 shows the simulations of the electric and magnetic fields inside the TM_{010} mode empty cavity resonator.

The Q-value of the air filling TM_{010} mode cavity resonators is decided with the following parameters: The intrinsic impedance of the air ($\eta$), the dimension of the cavity (h and a), the
permeability ($\mu$) and the conductivity ($\sigma$) of the cavity metal. Formula (3-2) can be used to calculate the Q-value [24].

$$a = \frac{P_{\text{sum}}}{\sqrt{\left(\frac{2\pi f \sqrt{\mu \varepsilon_r}}{c}\right)^2 - \left(\frac{\ell \pi}{d}\right)^2}}$$  \hspace{1cm} (3-1)

$$Q_{LO} = \frac{\eta}{\omega \mu} \frac{P_{01}}{\sqrt{\frac{2\sigma}{\omega \mu}} \left(1 + \frac{a}{h}\right)}$$  \hspace{1cm} (3-2)

**Figure 3.3:** The Copper Cylindrical Cavity Resonator in the Simulations

(a) Empty Cavity; (b) Cavity with Sample

**Figure 3.4:** The Fields inside Cavity; (a) The Magnetic Field; (b) The Electric Field
A picture for the cylindrical cavity resonator is shown in the Figure 3.5. A magnet can be used to determine the material of the metal wall for the cavity, therefore, we can conclude that the material belongs to ferromagnetic materials, e.g. Iron, ferrite. There are two holes made on box at top and bottom sides, the radius of the holes are manufactured very close to the radius of the sample for preventing the microwave leakage. The cookies box’s cover is soldered with the box wall, otherwise there will be producing a capacitor between the cover and the box wall.

![Figure 3.5: The Cavity Resonator Made by Cookies Box](image)

### 3.2.2 Preparations of the Dielectric Sample

The state of dielectric sample from Radarbolaget likes the sand. It is illustrated in Figure 3.6 (a). To measure the semi-solid or liquids samples in cavity resonator, a good method is to fill the sample inside a tube. Figure 3.6 (b) shows the small tubes for filling the samples. The diameters of them are 2.8 mm and 4.8 mm, the 2.8 mm tube is used to fill the reference material, i.e. water, and the 4.8 mm tube is used to fill the dielectric sample from Radarbolaget.
There are two necessary situations need to be considered. The first, in the practical experiment, the diameter of the tubes need to be measured in average value, because the tubes is not the perfect rod, the average measurement will provide the accurate results for the radius of the tubes, and the empty tubes have to be inserted into the cavity before measuring the sample, this is for examining the frequency shift from tube. Basically, the tube will not provide too much perturbations, but to enhancing the accuracy of the measurement, the frequency shift from that needs to be taken into account.
3.2.3 Experiment Processes

According to the perturbation technique and the reflection coefficient measurement, the VNA has to be calibrated for one port state in the specified frequency range. The MUT is introduced into a cavity resonator and the permittivity of the sample is evaluated by comparing the shifted resonant frequency and the original resonant frequency. The real part of the permittivity can be calculated from (2-36). According to (2-37), the imaginary part is decided by the Q-values, the Q-value ($Q_o$) for unloaded empty cavity resonator and the Q-value ($Q_L$) for the empty cavity loaded with the coupling loop have been described in section 2.1.4 in formulas (2-7) and (2-8). However the Q-value can be also determined from bandwidth, see section 2.1.5, and Figure 2.7. Hence, the measurement steps can be summarized in the below:

1. Calibrate the VNA for one port state in the specified range.

2. Measure the cavity resonator with the empty tube, record the resonant frequency ($f_0$) and the bandwidth.

3. Measure the cavity resonator with the filled tube, record the shifted resonant frequency ($f_s$) and the bandwidth.

4. Compute the permittivity and tangent loss from the measured data
IV. Measurement Results

In this section, the cylindrical cavity resonator will be examined for the determinations of the dielectric properties and tangent loss of the sample. The MUT is decided to use Teflon, because Teflon belongs to the low loss dielectric materials, the cavity resonator method is in good performance for the measurement of low loss materials. In the practice, the semi-solid sample from Radarbolaget will be measured and the reference sample is decided to use water. The measured results from both of them will be presented. Finally, the dielectric properties from simulations and practices will be calculated and compared and the change of the stored energy due to sample will be estimated.

4.1 The Simulated Results

4.1.1 The Simulations of Copper Cylindrical Cavity Resonator

The first, the cylindrical cavity resonator will be simulated in the HFSS. We measure the resonant frequency \(f_0\) from the empty cavity resonator, and then measure the shifted resonant frequency \(f_s\) from the filled cavity resonator.

The copper cavity and the ferromagnetic material cavity will be both simulated in HFSS. The results from them will be compared and discussed. The simulations for both kinds of cavity can show the uncertainty and accuracy difference between them.

In the copper cavity resonator, the diameter of the Teflon is 6mm, which is placed in the center of the cavity resonator, because the electric field is maximum in the center of the TM\(_{010}\) mode. Figure 4.1 (a) gives the measured reflection coefficients from the empty cavity and the filled cavity, the dashed curve indicates the original frequency and the solid curve indicates the shifted frequency. Figure 4.1 (b) gives the smith charts for the cases of the empty and the filled cavity resonator.
Figure 4.1: (a) The Simulated Resonant Frequencies (Dashed: Original; Solid: Teflon);
(b) The Smith Charts in Simulations (Dashed: Original; Solid: Teflon)

According to the measured parameters, the Q-values, the complex dielectric properties and the tangent loss are calculated by using formula (2-22), (2-36), (2-37), and (3-2). Table 4.1 lists the calculated results from the HFSS simulations for Teflon.
Table 4.1: The Simulated Results for Copper Cavity with Teflon in HFSS

<table>
<thead>
<tr>
<th>Cavity resonator (copper)</th>
<th>Theoretical empty cavity</th>
<th>Simulated empty cavity</th>
<th>Simulated for filling Teflon (d=6mm)</th>
<th>Errors in percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>f (GHz)</td>
<td>1.2087</td>
<td>1.22469</td>
<td>1.22213</td>
<td>/</td>
</tr>
<tr>
<td>Q-value</td>
<td>9488</td>
<td>9420</td>
<td>8729</td>
<td>0.717%</td>
</tr>
<tr>
<td>Coupling factor (g)</td>
<td>1.15</td>
<td>1.15</td>
<td>1</td>
<td>/</td>
</tr>
<tr>
<td>$\varepsilon'$</td>
<td>/</td>
<td>/</td>
<td>2.09</td>
<td>0.476%</td>
</tr>
<tr>
<td>$\varepsilon''$</td>
<td>/</td>
<td>/</td>
<td>0.0023</td>
<td>/</td>
</tr>
<tr>
<td>Tangent loss</td>
<td>/</td>
<td>/</td>
<td>0.0011</td>
<td>/</td>
</tr>
</tbody>
</table>

(Note: The errors for Q-values are compared between theoretical and simulated empty cavities)

4.1.2 The Simulations of Ferromagnetic Cylindrical Cavity Resonator

In the simulations of ferromagnetic material cavity resonator, the diameter of Teflon is 1cm. The measured reflection coefficient and smith chart are given in Figure 4.2, and the simulated results are listed in Table 4.2.
**Figure 4.2:** (a) The Simulated Resonant Frequencies (Dashed: Original; Solid: Teflon);

(b) The Smith Charts in Simulations (Dashed: Original; Solid: Teflon)

**Table 4.2:** The Simulated Results for Ferromagnetic Cavity with Teflon in HFSS

<table>
<thead>
<tr>
<th>Cavity resonator (Ferromagnetic)</th>
<th>Theoretical empty cavity</th>
<th>Simulated empty cavity</th>
<th>Simulated for filling Teflon (d=10mm)</th>
<th>Errors in percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>f (GHz)</td>
<td>1.23140</td>
<td>1.22426</td>
<td>1.22213</td>
<td>/</td>
</tr>
<tr>
<td>Q-value</td>
<td>193</td>
<td>194</td>
<td>190</td>
<td>0.518%</td>
</tr>
<tr>
<td>Coupling factor (g)</td>
<td>1.16</td>
<td>1.16</td>
<td>1.15</td>
<td>/</td>
</tr>
<tr>
<td>$\varepsilon'$</td>
<td>/</td>
<td>/</td>
<td>2.12</td>
<td>0.952%</td>
</tr>
<tr>
<td>$\varepsilon^*$</td>
<td>/</td>
<td>/</td>
<td>0.0116</td>
<td>/</td>
</tr>
<tr>
<td>Tangent loss</td>
<td>/</td>
<td>/</td>
<td>0.0055</td>
<td>/</td>
</tr>
</tbody>
</table>

(Note: The errors for Q-values are compared between theoretical and simulated empty cavities)
4.1.3 The Stored Energy due to Sample

As we described, in the TM_{010} mode, the strength of the electric field is maximum in the cavity center but the magnetic field is zero, and the sample have to be placed in the center during measurements. Hence, the sample can be considered in the place where the electric field is maximum but the magnetic is zero. Finally, with the computations of the stored energy, the magnetic field can be neglect and only the electric field needs to be taken into computations.

The energy density of electromagnetic field can be calculated by (4-1), and the total energy stored in the sample can be calculated by integral the volume of the sample with energy density

\[ w = \frac{1}{2}(\varepsilon_0 E^2 + \frac{1}{\mu_0} B^2) \]  

(4-1)

where \( \varepsilon_0 \) is the permittivity of free space

\( E \) is the electric field

\( \mu_0 \) is the permeability of free space

\( B \) is the magnetic field

When the Teflon is inserted into the cavity, the actual permittivity in the center place will be \( \varepsilon \), which can be calculated by

\[ \varepsilon = \varepsilon_{Teflon} \times \varepsilon_0 \]  

(4-2)

In HFSS simulations, the maximum electric field in the cavity center can be found, see Figure 4.3. The estimations of the stored and changed energy due to sample are given in the Table 4.3.
Figure 4.3: The Electric Fields Found in the Different Cavity Resonators in HFSS Simulations. 
(a) The Empty Copper Cavity; (b) The Copper Cavity with Teflon; 
(c) The Empty Ferromagnetic Cavity; (d) The Ferromagnetic Cavity with Teflon

Table 4.3: The Stored and Changed Energy duo to Sample

<table>
<thead>
<tr>
<th>Cavity Configuration</th>
<th>E(V/m)</th>
<th>Stored (J)</th>
<th>Changed (J)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Empty copper cavity</td>
<td>$9.9292 \times 10^{-1}$</td>
<td>$8.0213 \times 10^{-7}$</td>
<td>$1.2649 \times 10^{-8}$</td>
</tr>
<tr>
<td>Copper cavity with d=6mm Teflon</td>
<td>$9.8506 \times 10^{-1}$</td>
<td>$7.8948 \times 10^{-7}$</td>
<td></td>
</tr>
<tr>
<td>Empty ferromagnetic cavity</td>
<td>$9.9162 \times 10^{-1}$</td>
<td>$2.2223 \times 10^{-6}$</td>
<td>$8.4632 \times 10^{-9}$</td>
</tr>
<tr>
<td>Ferromagnetic cavity with d=10mm Teflon</td>
<td>$9.8973 \times 10^{-1}$</td>
<td>$2.2138 \times 10^{-6}$</td>
<td></td>
</tr>
</tbody>
</table>
4.2 The Practical Results of Cookie Box

The practical cylindrical cavity resonator is manufactured by a cookie box, which has been shown in the Figure 3.5. The cavity metal is concluded in the magnetic materials, the popular materials for making the cookie box are illustrated in the Table 4.4, and the relative permeability of the different materials are listed [28].

<table>
<thead>
<tr>
<th>Magnetic materials</th>
<th>Relative permeability (μ )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cast iron</td>
<td>200~400</td>
</tr>
<tr>
<td>Ferrite</td>
<td>More than 640</td>
</tr>
</tbody>
</table>

To observe the inner of the cookie box, we can find that there are some other materials have been plated on the surface of iron. We conclude that the pure cast iron is not possible for making the cookie box, and therefore, the Ferrite is taken for the theoretical computations. We use the value $\mu = 640$ for permeability. According to the Q-value calculation of empty cavity by formula (3-2), $Q_{LO}$ can be obtained as 330 for the cookie box under the unloaded condition. However, this $Q_{LO}$ cannot be the accurate Q-value for the cavity. It can be reference Q-value during the measurement.

The reference sample (water) will be measured for examining the practical cavity resonator. We fill the water inside a 2.8 mm diameter tube, and the semi-solid sample will be filled into a 4.8 mm diameter tube. Before to measure the water and sample, the tube is first evaluated, then we can see how much frequency shift due to that. This will enhance the accuracy of the results. The reflection coefficients of the empty cavity and the empty cavity with tube are given in Figure 4.4 (a). The Figure 4.4 (b) shows the responses in smith chart.
From Figure 4.4 (a), we can see the frequency shift due to the tube is weak, the affections from the tube is very small, which indicates that the tube can be used for filling the samples. After examine of the tube, we measure the reference sample (water), see Figure 4.5. This step will help us to know if the cavity resonator has ability to measure the dielectric materials and how much of
the errors will be produced. Because the water is filled inside the tube, the frequency curve of the tube will be used to instead of the original frequency, i.e. $f_0_{\text{tube}}$ will indicate the “$f_0_{\text{Empty}}$”.

![Diagram](image1)

**Figure 4.5:** (a) The Measured Resonant Frequencies (Dashed: Original; Solid: Water); (b) The Measured S11 in Smith Chart (Dashed: Original; Solid: Water)

The measured results of the sample from Radarbolaget are given in the Figure 4.6, and the results of the above measurements are listed in Table 4.5.
Figure 4.6: (a) The Measured $S_{11}$ for Determine Dielectric Constant of Sample

(Dashed: Original; Solid: Sample);

(b) The Measured $S_{11}$ in Smith Chart (Dashed: Original; Solid: Sample)
Table 4.5: The Measured Results from the Practice

<table>
<thead>
<tr>
<th>Cavity Resonator (Cookie box)</th>
<th>Theoretical empty cavity</th>
<th>Measured empty cavity</th>
<th>Measured for tube</th>
<th>Measured for filling water (d=2.8 mm)</th>
<th>Simulated for filling sample (d=4.8 mm)</th>
<th>Errors in percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>f (GHz)</td>
<td>1.2087</td>
<td>1.2054</td>
<td>1.2051</td>
<td>1.16655</td>
<td>1.20105</td>
<td>/</td>
</tr>
<tr>
<td>Q-value</td>
<td>137</td>
<td>79</td>
<td>80</td>
<td>61</td>
<td>78</td>
<td>42.33%</td>
</tr>
<tr>
<td>Coupling factor (g)</td>
<td>1.47</td>
<td>1.45</td>
<td>1.47</td>
<td>0.6185</td>
<td>1.328</td>
<td>/</td>
</tr>
<tr>
<td>( \varepsilon' )</td>
<td>/</td>
<td>/</td>
<td>/</td>
<td>80.4</td>
<td>3.83</td>
<td>1.14%</td>
</tr>
<tr>
<td>( \varepsilon'' )</td>
<td>/</td>
<td>/</td>
<td>/</td>
<td>4.8</td>
<td>0.12</td>
<td>6.67%</td>
</tr>
<tr>
<td>Tangent loss</td>
<td>/</td>
<td>/</td>
<td>/</td>
<td>0.06</td>
<td>0.03</td>
<td>7.25%</td>
</tr>
</tbody>
</table>

(Note: The errors for Q-values are compared between theoretical and measured empty cavities; The errors for \( \varepsilon' \), \( \varepsilon'' \) and Tangent loss are compared between standard water and measured water)

Table 4.6: The Standard Dielectric Properties for Water at 1.2GHz

<table>
<thead>
<tr>
<th>Standard dielectric properties (Water)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \varepsilon' )</td>
</tr>
<tr>
<td>( \varepsilon'' )</td>
</tr>
<tr>
<td>tan ( \delta )</td>
</tr>
</tbody>
</table>

The copper and ferromagnetic cavity resonators were simulated in HFSS, and the measured results for both of them have been shown. The error in dielectric constant in copper cavity is 0.476% and in ferromagnetic cavity is 0.952%. These results shown the copper cavity resonator has better reliability and accuracy than the ferromagnetic cavity resonator.
In the practice, the cavity resonator have been designed by a cookie box, the reasons for using cookie box are low cost and saving time. The measured results shown the cookie box has high reliability in dielectric constant but poor in loss factor and tangent loss. The errors of them are 1.14%, 6.67% and 7.25% respectively. Finally, the results of dielectric sample from Radarbolaget are 3.83-0.12i, and the tangent loss is 0.03. (Table 4.6 shown the standard dielectric properties for water at 1.2 GHz, which can be compared with the measured results from water)
V. Discussions/Conclusions

According to the above works, the cavity resonators measurements by using one port reflection coefficient were performed in HFSS and practice. The results in simulations for the copper cavity and the ferromagnetic cavity are presented.

In the simulations, the cylindrical cavity resonators were designed at 1.2 GHz by copper and ferromagnetic material. The Q-values between the theoretical and simulated cavities were compared in Table 4.1 and 4.2, which shows the good matching in the theoretical and simulation. The dielectric constant measurement for Teflon in copper cavity shown better accuracy than that in ferromagnetic cavity, but the loss factors were obtained in big difference between two cavities. Consider the reason of the errors, the big difference in Q-values between the copper cavity and ferromagnetic cavity may be the possible reason, because the loss factor is mainly decided by the change of Q-values.

In the practice, the cavity resonator made by cookie box at 1.2GHz has been manufactured and performed. To reduce the cost and manufacture time, the cavity resonator was build by a cookie box. Due to that, there are some uncertainty conditions with the cookie box, such as the cavity materials, insertion holes, Q-values. These uncertainties yield errors when we conclude the results in Q-values and the permittivity. However, for estimating the real part of dielectric permittivity, the uncertainties can be neglected, because the frequency shift is the mainly parameter in the measurement. Compare with the previous workers [13], [29], the errors in dielectric constant are acceptable, but the errors in loss factor and tangent loss were somewhat big. The possible reasons for the errors in the practice due to manufactures and operations are analyzed as following.

1. The sample is not paced in the center position where the electric field is maximum, the small shift of the position will result in the error.
2. The cookie box is not stable, which is easily effected with the shape.
3. The two holes on the top and bottom may result in the energy loss.
4. The tubes are not perfect rod.
5. The radius of the tubes is measured by slider caliper, which is accurate measurement for the radius, however, that is still too hard to measure the exact radius for the tubes.

6. The low Q-value.

There are some approaches can be considered to improve the accuracy in the measurements, such as building a known material cavity, use more tubes during measurements, analyze the sample holes and estimate the uncertainties due to the holes.
VI. The Suggested Future Work

For the different frequency band, the dielectric materials will provide different characteristics. To fully understand the sample from Radarbolaget, the best method is to establish the more cavity resonators in different frequency range for measuring the dielectric constant. This will give the positive affections for understanding the properties of the sample.
References


