How useful are intraday data in Risk Management?

An application of high frequency stock returns of three Nordic Banks to the VaR and ES calculation

Master’s Thesis in Financial Mathematics

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Abstract

The work is focused on the Value at Risk and the Expected Shortfall calculation. We assume the returns to be based on two pillars - the white noise and the stochastic volatility. We assume that the white noise follows the NIG distribution and the volatility is modeled using the nGARCH, NIG-GARCH, tGARCH and the non-parametric method. We apply the models into the stocks of three Banks of the Nordic market. We consider the daily and the intraday returns with the frequencies 5, 10, 20 and 30 minutes.

We calculate the one step ahead VaR and ES for the daily and the intraday data. We use the Kupiec test and the Markov test to assess the correctness of the models. We also provide a new concept of improving the daily VaR calculation by using the high frequency returns.

The results show that the intraday data can be used to the one step ahead VaR and the ES calculation. The comparison of the VaR for the end of the following trading day calculated on the basis of the daily returns and the one computed using the high frequency returns shows that using the intraday data can improve the VaR outcomes.
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Chapter 1

Introduction

In the beginning of the nineteen seventies the new era for the international financial markets began. From that moment on the markets experience an unprecedented level of volatility and, as a consequence, the risk. Therefore the accurate risk estimation became a crucial matter, not only for banks but also for individual traders.

Over the last twenty years the Value at Risk (VaR) has become a tool used most frequently in the risk management. The Value at Risk is a measure of market risk. It allows to define the upper boundary of the expected losses that a market participant may be exposed to during a given time period and with a certain probability.

After the introduction of the second Basel Accord in 2004, BASEL II, which allows banks to use their own Value at Risk model to regulate the capital requirements, the Value at Risk method became even more significant. The procedure is to model the stock price behavior and afterwards to predict the future losses. The capital requirements are then calculated. The amount of the requirements has to be high enough to satisfy the regulators but on the other hand as low as possible to retain the investment capacity.

Empirical studies show that the behavior of the stock returns does not follow the normal distribution. In reality they are leptokurtic and often skewed. Therefore in stock returns modeling more flexible distributions than the normal distribution should be used.

There were many models introduced over the years. Eberlein, Kallsen and Kristen (2002) proved that models based on the hyperbolic distributions provide a good reflection of the dynamics of the returns. Moreover they showed
that the GARCH-M models are successful in volatility forecasts. The meaning of GARCH-based models is also stressed in Wilhelmsson (2007). Even though in his work he created a complicated model involving the time varying variance, skewness and kurtosis, the much simpler NIG-GARCH model performed equally good. Giot (2000) shows that the tGARCH model, a GARCH model with the Student’s t as a conditional distribution, produces good results. In our work we concentrate on the GARCH models as they are commonly used and are proved to produce reliable results. Even restricted to GARCH class, there exist an enormous number of different models. The study of Hansen and Lunde (2005) examined about 330 different GARCH-type models and yet their investigation was not even close of being exhaustive.

In our work we focus on the stock returns of three Nordic banks - the Swedbank, the SHB and the SEB. We use four models: nGARCH, NIG-GARCH, tGARCH and the non-parametric model to predict the volatility. Based on the results we calculate the Value at Risk and the Expected Shortfall. We work both with the daily and the intraday data with the frequencies of 5, 10, 20 and 30 minutes.

Intraday Value at Risk although not used in connection to BASEL II, can still be a very useful tool for market participants involved in frequent trading. The BASEL II considers only the daily price changes. In the long time horizon trading, predicting the volatility using the daily price quotations can be sufficient. We cannot neglect however the impact of the stock price behavior observed within a day on the accurate volatility estimation. High intraday price oscillations occur often and considering only the price taken at the end of the trading day may not be enough for an active trader.

Giot (2000) performed studies on the intraday data for three stocks traded on the New York Stock Exchange with 15 and 30 minutes quotations. We aim to obtain a broader spectrum of information by concerning a greater variety of the data. Investigating the impact on Value at Risk by usage of the intraday data is justified based on the results of McMillan, Speight and Evans (2007). They proved that indeed the high frequency returns provide a better volatility forecasts than the predictions based on the daily price changes. Based on their work we also abandon the idea of performing the deseasonalization of the returns, as suggests Giot (2005). McMillan, Speight and Evans (2007) found out that deseasonalized returns provide worse forecasts than the raw or standardized data.
Our main research problem is to answer the question whether the intraday data are useful in Risk Management. Therefore, in addition to calculating the Value at Risk and the Expected Shortfall one step ahead, we also perform an investigation whether the fact of gathering the high frequency data can improve in the daily Value at Risk estimation. The basic concept is to compute the Value at Risk for the end of a trading day using the intraday returns up to the end of the previous day. This involves making the volatility predictions for a multiple-step ahead. We compare the generated results with the Value at Risk for the same trading days but obtained using the daily returns.

As far as we are concerned there has been no investigation concerning the high frequency data for the Nordic market. To the best of our knowledge no other work investigated the usage of the intraday data to the daily Value at Risk calculation.

The thesis is organized as follows. In Chapter 2 we introduce the model of the financial returns with all the necessary features. We describe the volatility estimation techniques with the GARCH and non-parametric models. We also describe all probability distributions that are used in this work. In Chapter 3 we define the Value at Risk and the Expected Shortfall. We discuss the usefulness of these two measures and point out the differences between them. Chapter 4 contains the methodology of backtesting procedures of the Value at Risk. In Chapter 5 we describe the intraday and the daily returns of the SEB, the SHB and the Swedbank. The results of our work are presented in Chapter 6. Chapter 7 contains the conclusions. In the Appendix we enclose the program code we wrote and used in all the computations.
Chapter 2

Financial returns modeling

To model the financial returns a proper mathematical tools and methods need to be taken.

2.1 General model - notation and assumptions

For all the models we will use the following notation. The price process is denoted by \( P = \{P_t\}_{t \geq 0} \). The return process \( R = \{R_t\}_{t \geq 0} \) is given by the equation

\[
R_t = \log \left( \frac{P_t}{P_0} \right).
\]

Adopting the assumption of Eberlein, Kallsen and Kristen (2003) we claim that the increments of the returns \( \Delta R_t = R_t - R_{t-1} \Rightarrow \Delta R_t = \log \left( \frac{P_t}{P_{t-1}} \right) \) take the form

\[
\Delta R_t = \sigma_t \Delta L_t,
\]

where \( \sigma \) denotes the volatility process and \( (\Delta L_t)_{t=1,2,...} \) denotes white noise.

Definition 1 (White noise) We call a sequence \( \epsilon = (\epsilon_n) \) white noise (in the wide sense) if \( E\epsilon_n = 0, \ E\epsilon_n^2 < \infty \) and

\[
E\epsilon_k\epsilon_n = 0, \quad \forall k \neq n.
\]

Having made the above assumption, Eberlein, Kallsen and Kristen (2003) considered the logarithmic squared returns of the Dow Jones Industrial Average from May 26, 1896 to January 4, 2001

\[
\log(\Delta R_t)^2 = \log(\sigma_t^2) + \log(\Delta L_t)^2.
\]
Then they took the logarithmic squared returns minus estimated logarithmic squared volatility and they compared the result with simulated residuals in the case of normal daily returns. The outcome showed that the stock return data were heavier tailed in comparison to the normal distributed ones. Furthermore they observed clustering of extremely high values which was not present in the simulated sample. That stands that the normal distribution is not adequate to describing white noise. Eberlein, Kallsen and Kristen (2003) suggested the generalized hyperbolic distribution which is a special class of distributions tailor-made in the way that fits the financial returns well.

In our work we use the subclass of the generalized hyperbolic distributions - the Normal Inverse Gaussian (NIG) distribution. The main motivation of using it is the closure under the convolution which is a convenient property to work with.

2.1.1 Levy processes

It has been proved that the GH distribution are infinitely divisible and therefore imply a Levy process, see Barndorff-Nielsen and Halgreen (1977). Thus the NIG distribution also implicate a Levy process as the member of the GH distributions family.

Shiryaev (1999) defines the Levy process in the following way:

**Definition 2 (Levy process)** A stochastic process $X$ is called the Levy process if it satisfies the following properties:

1. $P(X_0 = 0) = 1$,

2. The property of independent increments, i.e.
   For each $n \geq 1$ and each collection $t_0, t_1, \ldots$ such that $0 \leq t_0 < t_1 < \ldots < t_n$, the variables $X_{t_0}, X_{t_1} - X_{t_0}, \ldots, X_{t_n} - X_{t_{n-1}}$ are independent.

3. The property of stationary increments, it means:
   For all $s \geq 0$ and $t \geq 0$,
   $$X_{t+s} - X_s \overset{d}{=} X_t - H_0$$

4. The property of stochastic continuity
   For all $t \geq$ and $\epsilon > 0$,
   $$\lim_{s \to t} P(|X_s - X_t| > \epsilon) = 0$$

5. $t \to X_t$ is almost surely right continuous with left limits.
2.2 Probability distributions

2.2.1 Generalized Hyperbolic distributions

The Generalized Hyperbolic distributions (GH) were introduced into finance by Brandorff-Nielsen (1977). The subclasses of the GH distributions were proved to give almost exact fits to the financial log returns. All subclasses of the GH distributions can be derived from the GH distribution itself.

Definition 3 (General Hyperbolic distribution) The one-dimensional GH distribution is defined by the density function given by

\[
f(x; \lambda, \alpha, \beta, \delta, \mu)_{GH} = a(\lambda, \alpha, \beta, \delta, \mu)(\delta^2 + (x - \mu)^2)^{(\lambda - \frac{1}{2})/2} \times K_{\lambda - \frac{1}{2}}(\alpha \sqrt{\delta^2 + (x - \mu)^2}) \exp\{-\beta(x - \mu)\},
\]

where

\[
a(\lambda, \alpha, \beta, \delta, \mu) = \left(\frac{\alpha^2 - \beta^2}{\sqrt{2\pi \alpha^{\lambda - \frac{1}{2}}} \delta^\lambda K_{\lambda}(\delta \sqrt{\alpha^2 - \beta^2})}\right),
\]

and \(K_{\lambda}(x)\) denotes the modified Bessel function of the third kind and order \(\lambda\) as

\[
K_{\lambda}(x) = \frac{1}{2} \int_0^\infty y^{\lambda-1} \exp\{-\frac{x}{2}(y + y^{-1})\} dy, \quad x > 0.
\]

The parameter restrictions are

\[
\mu \in \mathbb{R},
\]

\[
\delta \geq 0, |\beta| < \alpha \quad \text{if} \quad \delta > 0,
\]

\[
\delta > 0, |\beta| < \alpha \quad \text{if} \quad \delta = 0,
\]

\[
\delta > 0, |\beta| \leq \alpha \quad \text{if} \quad \delta < 0.
\]

The parameters \(\mu\) and \(\delta\) describe the location and the scale. \(\beta\) stands for the skewness and \(\alpha\) determines the shape in the way that the decrease in \(\delta \sqrt{\alpha^2 - \beta^2}\) reflects the increase in the kurtosis. If \(\beta = 0\) then the distribution is symmetric. Positive or negative value of \(\beta\) characterize the type of the skewness. \(\lambda\) defines the subclass of the GH distributions. It is also related to the tail flatness.

Furthermore the moment generating function \(M(u)_{GH}\) and the characteristic function \(\varphi(u)_{GH}\) are given by the formulas

\[
M(u)_{GH} = e^{\mu u} \left(\frac{\alpha^2 - \beta^2}{\alpha^2 - (\beta + u)^2}\right)^{\lambda/2} K_{\lambda}(\delta \sqrt{\alpha^2 - (\beta + u)^2}) / K_{\lambda}(\delta \sqrt{\alpha^2 - \beta^2}), \quad |\beta + u| < \alpha.
\]

\[
\varphi(u)_{GH} = M(iu)_{GH}.
\]
2.2.2 The NIG distribution

According to Barndorff-Nielsen (1997), the NIG distribution as a subclass of the GH distributions is obtained by taking $\lambda = -\frac{1}{2}$, i.e.

$$NIG(\alpha, \beta, \delta, \mu) \sim GH(-\frac{1}{2}, \alpha, \beta, \delta, \mu).$$

The density function of a $NIG(\alpha, \beta, \delta, \mu)$ distribution is given by

$$f_{NIG}(x; \alpha, \beta, \delta, \mu) = \frac{\alpha \delta}{\pi} \frac{K_1(\alpha \delta \sqrt{1 + (\frac{x - \mu}{\delta})^2})}{\sqrt{\delta^2 + (x - \mu)^2}} e^{\delta \sqrt{\alpha^2 - \beta^2} + \beta (x - \mu)},$$

where $0 \leq |\beta| < \alpha$ and $\delta > 0$.

**Lemma 1** The moment generating function of a random variable following the NIG$(\alpha, \beta, \delta, \mu)$ distribution is

$$M(u) = \exp(\mu u) \frac{\exp(\delta \sqrt{\alpha^2 - \beta^2})}{\exp(\delta \sqrt{\alpha^2 - (\beta + u)^2})}.$$

**Proof:**
For any density function $f(x)$ it holds

$$\int_{-\infty}^{\infty} f(x) dx = 1.$$

For the NIG density function

$$1 = \frac{\delta \alpha}{\pi} \exp(\delta \gamma - \beta \mu) \int_{-\infty}^{\infty} \frac{K_1(\alpha \sqrt{\delta^2 + (x - \mu)^2}) e^{\beta x}}{\sqrt{\delta^2 + (x - \mu)^2}} dx,$$

where $\gamma = \sqrt{\alpha^2 - \beta^2}$.

We should notice that

$$\left(\int_{-\infty}^{\infty} \frac{K_1(\alpha \sqrt{\delta^2 + (x - \mu)^2}) e^{\beta x}}{\sqrt{\delta^2 + (x - \mu)^2}} dx\right)^{-1} = \frac{\delta \alpha}{\pi} \exp(\delta \sqrt{\alpha^2 - \beta^2} - \beta \mu).$$

Thus we have the moment generating function

$$M(u) = E[e^{\alpha X}]$$
\[ \int_{-\infty}^{\infty} K_1\left(\frac{\alpha \sqrt{\beta^2 + (x - \mu)^2}}{\sqrt{\beta^2 + (x - \mu)^2}}\right)e^{(\beta+u)x} dx \]

\[ = \frac{\delta \alpha}{\pi} \exp\left(\delta \sqrt{\alpha^2 - \beta^2} - \beta \mu\right) \int_{-\infty}^{\infty} K_1\left(\frac{\alpha \sqrt{\beta^2 + (x - \mu)^2}}{\sqrt{\beta^2 + (x - \mu)^2}}\right)e^{(\beta+u)x} dx \]

\[ = \frac{\delta \alpha}{\pi} \exp\left(\delta \sqrt{\alpha^2 - \beta^2} - \beta \mu\right) \frac{\pi}{\delta \alpha} \exp\left(-\delta \sqrt{\alpha^2 - (\beta + u)^2 + (\beta + u)\mu}\right) \]

\[ = \exp(\mu u) \exp\left(\delta \sqrt{\alpha^2 - \beta^2}\right) \exp\left(-\delta \sqrt{\alpha^2 - (\beta + u)^2}\right). \]

The four first moments (mean, variance, skewness and kurtosis) of the random variable \(X\) that follows the NIG distribution are presented below

\[ E[X] = \mu + \frac{\beta \delta}{\sqrt{\alpha^2 - \beta^2}}, \]

\[ V[X] = \frac{\delta \alpha^2}{(\alpha^2 - \beta^2)^{\frac{3}{2}}}, \]

\[ S[X] = \frac{3\beta}{\alpha \sqrt{\delta(\alpha^2 - \beta^2)^{\frac{1}{2}}}}, \]

\[ K[X] = \frac{3(1 + 4(\frac{\delta}{\alpha})^2)}{\delta \sqrt{\alpha^2 - \beta^2}}. \]

The NIG distribution is the only subclass of the GH distributions with the property of closure under convolution, see Theorem 1 point 2.

**Theorem 1** The properties of NIG distribution:

1. For \(X \sim NIG(\alpha, \beta, \delta, \mu)\) and constant \(k\) we have \(kX \sim NIG(\frac{\alpha}{k}, \frac{\beta}{k}, k\delta, k\mu)\);

2. If \(X \sim NIG(\alpha, \beta, \delta_1, \mu_1)\) and \(Y \sim NIG(\alpha, \beta, \delta_2, \mu_2)\) and \(X \perp Y\) then \(X + Y \sim NIG(\alpha, \beta, \delta_1 + \delta_2, \mu_1 + \mu_2)\).

**Proof**

1. We apply the moment generating function to \(cX\). For the moment generating function we have

\[ M_{cX}(t) = M_X(ct) = Ee^{ctX} \]
Thus
\[ M_{cX}(t) = \exp(\mu c t) \frac{\exp(\delta \sqrt{\alpha^2 - \beta^2})}{\exp(\delta \sqrt{\alpha^2 - (\beta + ct)^2})} = \exp(\mu c t) \frac{\exp(c \delta \sqrt{(\frac{\alpha}{c})^2 - (\frac{\beta}{c})^2})}{\exp(c \delta \sqrt{(\frac{\alpha}{c})^2 - (\frac{\beta}{c} + t)^2})}, \]

which is the moment generating function of \( \text{NIG}\left(\frac{\alpha}{c}, \frac{\beta}{c}, c\delta, c\mu\right) \).

2. A property of moment generating function for independent \( X \) and \( Y \) states
\[ M_{X+Y}(t) = M_X(t) M_Y(t). \]
For \( X \sim \text{NIG}(\alpha, \beta, \mu_1, \delta_1) \) and \( Y \sim \text{NIG}(\alpha, \beta, \mu_2, \delta_2) \) we obtain
\[ M_{X+Y}(t) = \exp(\mu_1 t) \frac{\exp(\delta_1 \sqrt{\alpha^2 - \beta^2})}{\exp(\delta_1 \sqrt{\alpha^2 - (\beta + t)^2})} \frac{\exp(\mu_2 t)}{\exp(\delta_2 \sqrt{\alpha^2 - (\beta + t)^2})} = \exp((\mu_1 + \mu_2) t) \frac{\exp((\delta_1 + \delta_2) \sqrt{\alpha^2 - \beta^2})}{\exp((\delta_1 + \delta_2) \sqrt{\alpha^2 - (\beta + t)^2})}, \]

which is the moment generating function of \( \text{NIG}(\alpha, \beta, \mu_1 + \mu_2, \delta_1 + \delta_2) \).

\[ \square \]

In Figures 2.1 to 2.4 we present the effects of manipulations on the parameters of the \( \text{NIG} \) distribution. Figure 2.1 illustrates how the \( \alpha \) parameter influences the shape of the density with the other parameters remaining unchanged. Figure 2.2 presents the changes made by the \( \beta \) parameter and Figure 2.3 by \( \delta \) parameter. The last Figure, 2.4, pictures the simultaneous changes of more than one parameter.
Figure 2.1: $NIG(\alpha, 0, 1, 0)$ for $\alpha = 0.2, 1, 3$ respectively.

Figure 2.2: $NIG(1, \beta, 1, 0)$ for $\beta = 0, 1.8, -1.5$ respectively.

Figure 2.3: $NIG(1, 0, \delta, 0)$ for $\delta = 3, 1, 0.6$ respectively.

Figure 2.4: The $NIG$ densities for various parameter sets, $NIG(0.7, -0.5, 0.5, 0)$, $NIG(1, 0, 0.8, -2)$, $NIG(2, 1.8, 1, 0)$ respectively.
2.2.3 Student’s t distribution

The Student’s t distribution is another case of the GH distributions denoted by $GH(-\frac{\nu}{2}, 0, 0, \sqrt{\nu}, \mu)$, where $\nu$ denotes the degrees of freedom. The probability density function of the Student’s t distribution is given as follows

$$f_{\text{Stud}}(x; \nu, \mu) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)\sqrt{\pi\nu}} \left(1 + \frac{(x - \mu)^2}{\nu}\right)^{-\frac{\nu+1}{2}},$$

where $\Gamma$ is a gamma function, i.e. for $z \in \mathbb{R}$ we have $\Gamma(z) = \int_0^{\infty} t^{z-1}e^{-t}dt$ and for $n \in \mathbb{N}$ the formula changes to $\Gamma(n) = (n-1)!$.

The degrees of freedom and the kurtosis are strongly related. The parameter $\nu$ can be computed from the kurtosis by the following expression

$$k = \frac{6}{\nu - 4} + 3 \quad \forall \quad \nu > 4.$$

The above equality is proved below. The kurtosis is given by

$$k = \frac{\mu_4}{\sigma^4},$$

where $\mu_4$ is the fourth central moment and $\sigma$ is the standard deviation. The second and fourth central moment functions are defined as

$$\mu_2 = E[(x - \mu)^2] = \frac{\nu}{\nu - 2},$$

$$\mu_4 = E[(x - \mu)^4] = \frac{3\nu^2}{(\nu - 2)(\nu - 4)}.$$

Now when we notice that $k = \frac{\mu_4}{\sigma^4} = \frac{\mu_4}{\mu_2^2}$ we immediately obtain the above dependence.

The higher the degree of freedom the less peaked the probability density function is, i.e. the more similar it is to the normal distribution. This property is illustrated in Figure 2.5.

2.3 A volatility estimation

The volatility is a crucial factor in an accurate risk estimation. Even though the famous Black-Scholes model makes an assumption about constant volatility, the authors - Black and Scholes (1973) realized from the very beginning that a not varying volatility is only a convenient simplification which is far from being realistic. Indeed the empirical studies of the price returns have proved the existence of random volatility fluctuations around a mean level.
Definition 4 (Stochastic volatility) A process in which the return fluctuation includes an unobservable component which cannot be predicted using current available information is called a stochastic volatility process.

Changes in volatility over time have been investigated with many approaches. There are two main classes of the volatility prediction models:

- Parametric models,
- Nonparametric models.

The first group assumes that the returns for which the volatility we want to estimate follows some kind of distribution. The distribution always depends on one or more parameters, and here the name comes from. The nonparametric models make no assumption about distribution of the returns.

In our work we focus on three parametric models - three types of the Generalized Autoregressive Conditional Heteroscedastic (GARCH) models, and on the non-parametric model.

2.3.1 GARCH

The GARCH model is a generalization of the ARCH model developed by Engle (1982). The GARCH model was proposed in Bollerslev (1986).

Definition 5 (GARCH(p,q) process) Let \( (\epsilon_t) \) be a real-valued and discrete time stochastic process, and \( \mathcal{F}_t \) be the information set (\( \sigma \)-field) of all information up to time \( t \). We consider \( (\epsilon_t) \) of the form

\[
\begin{align*}
\epsilon_t &= \sigma_t z_t, \\
E[z_t] &= 0, \\
Var[z_t] &= 1,
\end{align*}
\]
where $\sigma_t$ is a positive, time-varying and measurable function of the time $t-1$ information set, $\mathcal{F}_{t-1}$. We call $(\epsilon_t)$ a GARCH($p,q$) process if

$$
\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \epsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2, \tag{2.9}
$$

where

- $\alpha_0 > 0$, $\alpha_i \geq 0$ for $i \in \{1, \ldots, q\}$,
- $\beta_j \geq 0$ for $j \in \{0, \ldots, p\}$,
- $\sum_{i=1}^{\max(p,q)} \alpha_i + \beta_i < 1$.

The first two conditions guarantee a nonnegative variance whereas the last one ensures the stationarity of the process (the unconditional variance is finite).

Notice that if we take $p = 0$ in (2.9) we reduce the model to an ARCH($q$).

Among all of the GARCH-type models, the GARCH(1, 1) is the most frequently used for modeling the financial time series. Lower order GARCH models often perform significantly better than higher order ones (Hansen and Lunde, 2005). The GARCH(1, 1) provides the main dynamic characteristics of the returns. In comparison with the traditional ARCH modeling it provides a better forecasts of the volatility.

We use GARCH-M process, that is GARCH(1, 1) process to model a mean-adjusted return series $(\epsilon_t)$. Thus in our case $(\epsilon_t)$ have the form

$$
\epsilon_t = (R_t - \mu) = \left( \log \frac{P_t}{P_{t-1}} - \mu \right),
$$

that is

$$
R_t = \mu + \epsilon_t,
$$

where $R_t$ is the observed logarithmic return. Moreover, we make the assumption that the errors, $z_t$, in (2.6), (2.7), (2.8) follow a GH distribution, i.e.

$$
\epsilon_t = \sigma_t z_t, \quad z_t \sim GH(\lambda; \alpha, \beta, \delta, \mu), \quad z_t \text{ i.i.d.},
$$

$$
E[z_t] = 0, \quad Var[z_t] = 1.
$$
The conditional variance $\sigma_t^2$ changes over time as it follows the \textit{GARCH}(1, 1) process

$$\sigma_t^2 = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2,$$

with $\alpha_0 > 0, \alpha_1 \geq 0, \beta_1 \geq 0, \alpha_1 + \beta_1 < 1$.

The restriction $\alpha_1 + \beta_1 < 1$ is sufficient for a wide-sense stationarity, see Bollerslev (1986).

To make the n-steps-ahead prediction of the volatility in time $t$ we proceed as follows.
Let $\sigma_{t+n}$ denotes a forecast for the $n$ future steps made in the moment $t$. Then

$$\sigma_{t+1}^2 = \alpha_0 + \alpha_1 \epsilon_t^2 + \beta_1 \sigma_t^2,$$

$$\sigma_{t+2}^2 = \alpha_0 + (\alpha_1 + \beta_1) \sigma_{t+1}^2,$$

$$\vdots$$

$$\sigma_{t+n}^2 = \alpha_0 + (\alpha_1 + \beta_1) \sigma_{t+(n-1)}^2 = \frac{\alpha_0 [1 - (\alpha_1 + \beta_1)^{n-1}]}{1 - \alpha_1 - \beta_1} + (\alpha_1 + \beta_1)^{n-1} \sigma_{t+1}^2.$$

The parameters of the model are estimated in every step. There are many methods allowing to do so, e.g.

- Maximum Likelihood method,
- Generalized Method of Moments,
- Expectation Maximization method.

Below we present the concepts of these techniques.

\textbf{The Maximum likelihood method (ML)}

The Maximum Likelihood estimation was introduced by Sir Ronald A. Fisher about the year 1920. The idea of the method is to maximize the likelihood functions. Given a sample of random variables $X_1, \ldots, X_n$ distributed with the density function $f(x; \Theta)$ where $\Theta$ denotes a vector the model parameter ($\theta_1, \ldots, \theta_k$), then the likelihood function of the random sample takes the form

$$L(\Theta; X_1, \ldots, X_n) = f(X_1, \ldots, X_n; \Theta),$$
where \( f \) denotes the joint probability density function. If \( X_1, \ldots, X_n \) are i.i.d. random variables, then the likelihood function can be reduced to

\[
L(\Theta; X_1, \ldots, X_n) = f(X_1; \Theta) \times \cdots f(X_n; \Theta) = \Pi_{i=1}^n f(X_i; \Theta),
\]

which denotes the product of 1-dimensional density functions.

The ML estimate is the parameter vector \( \hat{\Theta} = (\hat{\theta}_1, \ldots, \hat{\theta}_k) \) that maximizes the likelihood function.

The analytical approach of finding \( \hat{\Theta} \) is to take the derivative of \( L \) and assume that it is equal to zero, i.e.

\[
\frac{\partial}{\partial \Theta} L(\Theta) = 0.
\]

Note that a solution to the likelihood equation may not necessarily be the maximum, it can be as well minimum or a stationary point. It should be investigated whether the computed vector of parameters indeed maximizes the function.

Another important thing to investigate whether the maximum is global or local one. Every local extrema gives the zero value of the derivatives of \( L \), but we are interested only in the global maximum.

It is often more convenient to make the optimization of the logarithm of the likelihood function, \( \ln(L(\Theta)) \). This is motivated because the maximum of \( L(\Theta) \) and \( \ln(L(\Theta)) \) occurs for the same \( \Theta \). In the sequent we will use the logarithm of the maximum likelihood function for our calculations

\[
\frac{\partial}{\partial \Theta} \log L(\Theta) = 0, \quad \text{where} \quad \log L(\Theta) = \sum_{i=1}^n \log f(X_i; \Theta).
\]

To estimate the parameters for the models that follow different distributions of course we will need different likelihood functions.

The likelihood function for a normal distribution with an unknown mean \( \mu \), an unknown variance \( \nu \) and if the size of the observed sample \( (x_i) \) is equal to \( n \), is given by the expression

\[
L(\Theta; x) = \Pi_{i=1}^n \frac{1}{\sqrt{2\pi\nu}} e^{\frac{(x_i-\mu)^2}{2\nu}} = \left( \frac{1}{2\pi\nu} \right)^\frac{n}{2} e^{-\frac{1}{2\nu} \sum_{i=1}^n (x_i-\mu)^2}.
\]
Since we have the observed log-returns $R = (R_1, \ldots, R_n)$, the formula changes to
\[
\log (L(\Theta; R)) = -\frac{n}{2} \log(2\pi\nu) - \frac{1}{2\nu} \sum_{i=1}^{n} (R_i - \mu)^2.
\]

By differentiating the above equation partially with respect to $\mu$ and $\nu$ and setting the results equal to 0 we obtain
\[
\hat{\mu} = \bar{R} = \frac{1}{n} \sum_{i=1}^{n} R_i,
\]
and
\[
\hat{\nu} = \frac{1}{n} \sum_{i=1}^{n} (R_i - \bar{R})^2.
\]

For the $GH$ distributions with the parameter set $\Theta = (\lambda, \alpha, \beta, \delta, \mu)$ the log-likelihood function is given by
\[
\log (L(\Theta; R)) = \log a(\lambda, \alpha, \beta, \delta, \mu) + \frac{\lambda - 1}{2} \sum_{i=1}^{n} \log (\delta^2 + (R_i - \mu)^2)
+ \sum_{i=1}^{n} \log K_{\lambda-1/2} \left( \alpha\sqrt{\delta^2 + (R_i - \mu)^2} \right) + \sum_{i=1}^{n} \beta(R_i - \mu),
\]
where $a(\lambda, \alpha, \beta, \delta, \mu)$ is given by [2.4] and $K$ by [2.5].

In our work we consider the $NIG$ and the Student’s $t$ distributions which are the special cases of the $GH(\lambda, \alpha, \beta, \delta, \mu)$ distribution with the parameters $GH(-\frac{1}{2}, \alpha, \beta, \delta, \mu)$ and $GH(-\frac{1}{2}, 0, 0, \sqrt{\nu}, \mu)$ respectively, where $\nu$ denotes the degrees of freedom.

**The Generalized Method of Moments**

Another way of computing the set of parameters $\Theta = (\theta_1, \ldots, \theta_k)$ is to make use of the knowledge about the moments of the underlying model. The GMM is the generalization of the Method of Moments introduced by Karl Pearson in 1894. The MM is one of the oldest estimation methods. The basic requirement is that having $k$ parameters to estimate we have to know at least the $k$ first moments $M_1, \ldots, M_k$. Then given the random sample $(X_1, \ldots, X_n)$ we compute the sample moments $\bar{X}_1, \ldots, \bar{X}_k$ in the way $\bar{X}_i = \frac{1}{n} \sum_{j=1}^{n} X_j^i$. From the above steps and by equating the distribution moments with the
sampled moments, i.e. \( M_i(\Theta) = \hat{M}_i \), we obtain \( k \) equations with \( k \) unknown parameters and therefrom we can get \( \hat{\Theta} \). In the GMM which was formalized by Hansen in 1982 there can exist some moment conditions that are derived from the assumptions of the econometric model.

### The Expectation Maximization method

The Expectation Maximization (EM) algorithm is the approach of finding maximum likelihood estimates based on an incomplete data set. The algorithm is basically an iterative scheme that enables to split a complicated problem into a sequence of simpler ones. The estimating procedure looks as follows: the initial estimates for the parameters are obtained either by previous knowledge of the data or by a random guess. Then during the iterative process the new estimates are computed and the process continuous until estimates converge, see Figure 2.3.1.

In our work we use the Maximum Likelihood estimation. The reason for this is that the statistical software that we use gives us a convenient numerical treatment for the ML technique.

### 2.3.2 Nonparametric model - a historical volatility with the moving-window length calculated by cross-validation

Lets remind that from the equation (2.2) we look for the signal \( \log \sigma_t^2 \) which is perturbed by the noise \( \log(\Delta L_t)^2 \). We make an assumption that the volatility \( \sigma \) follows some arbitrary stochastic process. We also suppose that \( \sigma \) changes over time but with a lower frequency than the frequency of the returns. In other words, we assume that the volatility changes over time period that is longer than the sampling interval.

To calculate the volatility we proceed in the way adopted by Eberlein, Kallsen and Kristen (2003), i.e. we use a moving average

\[
\log \hat{\sigma}_t^2 = \frac{1}{k} \sum_{i=0}^{k-1} \log(\Delta R_{i-1})^2,
\]
EM algorithm

Input:
Distribution of data set; unknown parameters $\Theta = (\theta_1, \ldots, \theta_k)$,
Incomplete data set $X$,
Convergence threshold $\tau$.

Output:
Parameter estimations $\hat{\Theta} = (\hat{\theta}_1, \ldots, \hat{\theta}_k)$,

Initialize $\hat{\Theta}^0$,
Start with $n = 0$.

Repeat:
Having $n - 1$ we compute $\hat{\Theta}^n$ using the function $Q$ defined as

$$Q(\Theta, \hat{\Theta}^{n-1}) = E[\log p(X, Y; \Theta)|X, \hat{\Theta}^{n-1}],$$

so that $\Theta$ maximizes $Q(\Theta, \hat{\Theta}^{n-1})$
which means $\Theta^n = \arg\max_{\Theta} Q(\Theta, \hat{\Theta}^{n-1})$

until $|Q(\Theta, \hat{\Theta}^n) - Q(\Theta, \hat{\Theta}^{n-1})| < \tau$.

Result: $\hat{\Theta} = \hat{\Theta}^n$,

where $Y$ is the completion of incomplete set $X$ and $p$ is the joint density function.

Figure 2.6: The EM algorithm, step-by-step.

where $k$ denotes the appropriate smoothing parameter (the estimated time interval during which the volatility process changes).

To find the parameter $k$ we use the Cross Validation technique, i.e. we choose $k$ that minimizes

$$CV(k) = \frac{1}{T} \sum_{t=1}^{T} \left( \log(\Delta R_t)^2 - (\log \hat{\sigma}_t^2)^{-t} \right)^2,$$

where $T$ denotes the total size of the empirical sample and

$$\left( \log \hat{\sigma}_t^2 \right)^{-t} = \frac{1}{k} \sum_{i=1}^{k} \log(\Delta R_{t-i})^2$$
is the moving average that calculates information in the moment \( t \). Therefore \( CV \) is in fact a sum of errors.

As we do not assume that the returns follow any distribution the most obvious way to make the prediction for volatility in time \( t+1 \) is to take

\[
\sigma_{t+1} = \hat{\sigma}_t.
\]
Chapter 3
Risk Management, 
VaR and ES methods

It is a well know fact that the financial returns are volatile. We are not certain about the exact level of the future profit or loss. There is a proportional relationship between the volatility and the risk. The more volatile the market is, the more risky it is considered to be. The Value at Risk and the Expected Shortfall are the attempts to predict the future losses and therefore to manage the risk.

3.1 Value at Risk

The Value at Risk (VaR) denotes the maximum loss that can be incurred over the certain period $t$ with a given probability $\alpha$, the confidence level. Let us consider the logarithmic returns in the form $r_t = \log(\frac{P_t}{P_{t-1}})$, where $P_t$ denotes the price in the time moment ‘$t$’. We formulate the definition in the terms of the probability in a similar way as in Acerbi and Tasche (2001).

Definition 6 (Value at Risk)

$$VaR_\alpha(r) = VaR(\alpha, r)$$
$$= -sup \{x \in \mathbb{R} : P(r \leq x) \leq \alpha\}$$
$$= -inf \{x \in \mathbb{R} : P(r \leq x) \geq \alpha\}$$

There are three common ways of obtaining $VaR$ value

- Historical approach,
• Monte Carlo Simulation,
• Variance covariance approach.

3.1.1 The Historical simulation method

The historical simulation method is the easiest way of computing the VaR. It is a nonparametric method, which means it has no assumption about the distribution of the returns. On the one hand this is an advantage because of its simplicity but on the other hand it is a serious weakness. The basic idea of the historical simulation is to take the empirical financial returns from a given time period and put them in order from the worst to the best, basically in the form of a histogram. We assume that the general behavior of the returns will not change in the future. Thus from the histogram we can take the worst $\alpha \times 100\%$ of the returns and say that with the probability $(1 - \alpha)$ the future return will be greater than those lowest $\alpha \times 100\%$ of the returns.

3.1.2 The Monte Carlo simulation

The Monte Carlo simulation assumes that the returns follow some distribution. The specific type of the distribution is not proposed. The idea behind this approach is to generate series of the financial returns based on the chosen distribution. Subsequently we take the $\alpha \times 100\%$ of the worst returns and say that the future return should be greater with the probability $(1 - \alpha)$. The method is fairly simple nevertheless the more returns we generate the better and more reliable the results are, but it requires more time and demands proper computer resources. The method is illustrated in Figure 3.1.

3.1.3 The Variance covariance method

The third method - the variance covariance approach is also a parametric method. The classical approach assumes normally distributed returns. We calculate the parameters of the distribution using the historical data. In this particular case we have the mean and the standard deviation only. Then the concept is similar to the historical simulation method but instead of looking at the actual data we look at the distribution function. The VaR is defined as the $\alpha$ quantile of the cumulative distribution function $F$ of the returns $R_t = \log\left(\frac{P_t}{P_{t-1}}\right)$, i.e.

$$VaR_\alpha(r) = VaR(\alpha, r) = F^{-1}(\alpha | \Omega_t),$$
Example 1

Figure 3.1: The histogram representing 100 financial returns generated by a Monte Carlo procedure. The generated returns are put into the 'buckets' representing the return rates (in percent). In this example we have 19 cases of 0% return, 18 cases of 5% loss and so on. If we have the confidence level $\alpha = 5\%$ in 95% of the cases the future loss should not be equal or greater than 15%.

where $\Omega_t$ represents the information up to time $t$. The $\alpha$-quantile of the random variable $X$ is

$$q_{\alpha}(X) = \sup \{x \in \mathbb{R} : P(X \leq x) \leq \alpha \}.$$

Therefore given the random variable $R_t$ describing the financial returns we have

$$-q_{\alpha}(R_t) = \text{VaR}_{\alpha}(R_t)$$

and

$$\alpha = \int_{-\infty}^{q_{\alpha}} f_{R_t}(s)ds,$$

where $f_{R_t}$ is the density function of the returns.

In some cases it is more convenient to consider the losses instead of the financial returns. To derive the loss in the moment $t$ we simply take

$$l_t = \max(0, -R_t).$$
The standard approach in the variance covariance method is to use the Gaussian distribution. In our work we consider also two other distributions. The first is the Normal Inverse Gaussian (NIG) distribution and the second one is the Student’s t distribution. This choice is motivated by the fact that the empirical returns do not tend to follow the normal distribution as they have heavy tails.

3.1.4 Is the VaR a good measure of risk?

The Value at Risk is a convenient method for the institutions, however the construction and usage of the model used in estimation of the VaR method can cause some problems.

The advantages of the VaR are

- Universality - the same concept of a risk measurement can be used in many types of risk, e.g. both the operational and the credit risk. The methods used are obviously different, but the risk is expressed in the similar way. This fact introduces the benchmark in the domain of the risk control.

- The VaR can be also used to different types of assets, i.e. it can be applied to interest rates, foreign exchanges, credits or commodity
about the two above points we must notice that the VaR has the property of the invariance of the strong law. This means that the distributions of two random variables X and Y do not have to be identical if we want to say that \( \text{VaR}_\alpha(X) = \text{VaR}_\alpha(Y) \). In this way two random variables, one distributed with heavy tails and second one light tailed, may have the same \( \text{VaR}_\alpha \) value. This is of course a serious drawback of the VaR method. It is important to recognize that on the one hand the universal character of the VaR seems to be an advantage, but on the other hand it is a flaw.

- The VaR express the risk in the easy to interpret and convenient to monitor way - it simply calculates the worst case scenario and presents the result as a single value.

- It is the model that was sanctioned by the Basel Committee in 1996 for computing the market risk capital requirements, therefore it has become the measure used commonly in financial markets.

However the VaR possesses many strengths it also suffers from serious drawbacks. The Value at Risk is of course the risk measure in the sense of definition, Acerbi and Tasche (2001).

**Definition 7 (Risk measure)** Every function mapping random variables to the real number is the risk measure. Consider the probability space \((\Omega, \mathcal{F}, P)\) and the non-empty set \(V\) such that \(V \in \mathcal{F}\). Then any mapping \(\rho : V \to \mathbb{R} \cup \{\infty\}\) is called a risk measure.

However one of the biggest shortcomings of VaR is the fact that it is not a coherent risk measure.

**Definition 8 (Coherent risk measure)** A risk measure \(\rho\) such that \(\rho : V \to \mathbb{R} \cup \{\infty\}\) is called coherent if it fulfills the following properties:

- **Monotonicity:** \(X, Y \in V, X \leq Y \Rightarrow \rho(X) \leq \rho(Y)\)
- **Positive homogeneity:** \(X \in V, h > 0, hX \in V \Rightarrow \rho(hX) = h\rho(X)\)
- **Translation invariance:** \(X \in V, a \in \mathbb{R}, X + a \in V \Rightarrow \rho(X + a) = \rho(X) - a\)
- **Subadditivity:** \(X, Y \in V, X + Y \in V \Rightarrow \rho(X + Y) \leq \rho(X) + \rho(Y)\).
All of the above properties are easily justified from the logical and economical point of view. Monotonicity denotes that the higher the returns we want to achieve the higher risk we must accept. Homogeneity shows that the larger position we take the greater risk we take. Translation invariance stands that the more we invest in risk-free asset the lower the overall portfolio risk we have. Finally the subadditivity shows the positive effects of portfolio diversification on the risk reduction.

The Value at Risk fails to fulfill the last property. That means that the \( VaR \) does not consider the effects of a portfolio diversification. Another shortcoming of the \( VaR \) is the fact that it does not take into account the fact how much the loss exceeds the estimated \( VaR \) level.

Under these circumstances the other risk measures have been searched for. The alternative risk measure that can be a remedy for some of the shortcomings of the \( VaR \) is the Expected Shortfall (or the Expected Tail Loss or the Conditional Value at Risk or the Tail Value at Risk).

### 3.2 The Expected Shortfall

The Expected Shortfall (\( ES \)) is a coherent risk measure while \( VaR \) does not fulfill the subadditive property. According to Acerbi and Tasche (2001), the axiom of coherence is necessary to be taken into account if we want to consider the complex reality, it cannot be omitted. Quoting these authors, measuring risk with the measure that does not fulfill the axioms of coherency (such like the \( VaR \)) is like measuring the temperature using the barometer. Those authors go a step further saying that the risk measure that is not coherent should not be called the risk measure at all.

So while the \( VaR \) measure answers the question "what is the minimum loss incurred in the \( \alpha \% \) of the worst cases of the portfolio" the \( ES \) searches for the solution of the problem "what is the expected loss incurred in the \( \alpha \% \) of the worst cases of the portfolio". To give the proper definition we refer to Tasche (2002).

**Definition 9 (Expected shortfall)** Let \( (\Omega, \mathcal{F}, P) \), \( \alpha \in (0, 1] \) be fixed and \( X \) be a continuous random variable. Additionally let \( E[L = \max(0, -X)] < \infty \) and \( q_\alpha(X) = \inf \{x \in \mathbb{R} : P(X \leq x) \geq \alpha \} \). Then the value

\[
ES_\alpha(X) = -E[X|X < q_\alpha] = -E[X|X < -VaR_\alpha(X)]
\]
\[
= -\frac{1}{\alpha} \int_{-\infty}^{-VaR_{\alpha}} xf(x)dx \\
= -\frac{1}{\alpha} \int_{0}^{\alpha} F^{-1}(t)dt \\
= -\frac{1}{\alpha} \int_{0}^{\alpha} q_u du
\]

is called the Expected Shortfall \( ES_{\alpha} \) at the level \( \alpha \) of \( X \).

The Expected Shortfall is the smallest coherent measure that dominates the \( VaR \). It is essential to realize that the \( ES \) in the case of the general distributions may violate the subadditivity property. However when restricted to the continuous distributions it is always a coherent risk measure. It preserves the primary values of the \( VaR \) and at the same time it takes into consideration the extreme losses.
Chapter 4

Backtesting

When a financial institution has computed its 1% \( VaR \) in order to determine the capital requirements, it is important to examine whether the accepted \( VaR \) model is accurate, i.e. whether it represents a precise measure of the actual risk level that the institution is subjected to. To verify this we have to make use of statistical tests.

To examine the backtests for the \( VaR \) we firstly introduce so called hit sequence, i.e. the sequence of the \( VaR_\alpha \) violations, defined as

\[
I_t(\alpha) = \begin{cases} 
1, & \text{if } l_t > VaR_\alpha(R_t), \\
0, & \text{else.} 
\end{cases}
\]

This sequence contains only the information about whether the \( VaR \) violation occurred but gives no information about the magnitude of the existing exception.

Following the terminology of Christoffersen (1998) an accurate \( VaR \) model must fulfill two properties

1. **The Unconditional Coverage Property**
   It measures the ability of the model of producing the correct number of violations. In terms of the hit sequence we should have \( P(I_t(\alpha) = 1) = \alpha \). It is important to notice that an incorrect \( VaR \) model is not only the one in which the exceptions occur more frequently than \( \alpha \times 100\% \) over time. When too few violations occur it is a sign that the model is too conservative. It is also a bad feature as it causes the financial institution to keep too high capital reserves.

2. **The Independence property**
   Exceptions from the correct \( VaR \) models should be independently dis-
tributed over time. Any two factors of the hit sequence \( I_{t+j}(\alpha), I_{t+k}(\alpha) \) have to be independent from each other. Intuitively this means that the \( VaR \) sequence \( \{ \ldots, I_{t-1}(\alpha), I_t(\alpha) \} \) must not bring any information about whether the additional exception, \( I_{t+1}(\alpha) = 1 \), will occur. Clustering of the exceptions is a signal that the independence property may be violated. For example given 1\% \( VaR \) the probability of occurring the exception from the \( VaR \) level is 1\%. If there constantly occurs the situation in which one violation is immediately followed by another the actual probability of observing the second exception is 100\%, not 1\% like it should be. Therefore the 1\% \( VaR \) level should be higher.

Although the second property is not required by BASEL II, which is only concerned with correct unconditional coverage, it is important to pay attention to the independence requirement.

A variety of tests to examine whether the hit sequence satisfies one or both of the properties have been proposed over the years. We use the Kupiec test for the unconditional coverage and the Markov test for the independence.

### 4.1 The Unconditional Coverage and the Kupiec test

The Kupiec test was proposed in Kupiec (1995). Kupiec proposed the so-called proportion of failures (POF) test to check whether the total number of violations in the hit sequence differs considerably from \( \alpha_0 \times 100\% \) of the whole sample. If we have \( T \) observations the Kupiec test statistic looks like

\[
POF = 2 \log \left( \left( \frac{1 - \hat{\alpha}}{1 - \alpha_0} \right)^{T-I(\alpha)} \left( \frac{\hat{\alpha}}{\alpha_0} \right)^{I(\alpha)} \right),
\]

\[
\hat{\alpha} = \frac{1}{T} I(\alpha),
\]

\[
I(\alpha) = \sum_{t=1}^{T} I_t(\alpha).
\]

The \( POF = LR_{uc} \) is a statistic used in testing the hypothesis

\[
H_0^{uc} : \quad \alpha = \alpha_0,
\]

versus the alternative hypothesis

\[
H_1^{uc} : \quad \alpha \neq \alpha_0.
\]
where $\alpha$ is the failure rate (estimated by $\hat{\alpha}$).

The Kupiec test is asymptotically distributed as a chi-square distribution with the degree of freedom equal to one. It is convenient to consider the p-value. We can obtain this value from the formula

$$p-value(LR_{uc}) = 1 - \int_{-\infty}^{LR_{uc}} f_{\chi^2(1)}(x)dx.$$

### 4.2 The Independence test and the Markov test

The Markov test was introduced by Christoffersen (1998). It checks whether the VaR violation depends on the past violations. If the hit sequence fulfills this property then the chance that the violation occurrences will not increase under the influence of the past violations.

The test involves creating a $2 \times 2$ probability matrix that records the exceptions from the VaR with respect to the preceding day, i.e.

$$\Pi = \begin{bmatrix} 1 - \pi_{01} & \pi_{01} \\ 1 - \pi_{11} & \pi_{11} \end{bmatrix},$$

where $\pi_{ij}$ denotes the probability that the hit sequence will have $j$ value at the time $t$ if it had $i$ value in the moment $t - 1$.

We test two hypotheses

$$H_{0}^{ind} : \pi_{01} = \pi_{11}$$
$$H_{1}^{ind} : \pi_{01} \neq \pi_{11}$$

The test statistic is given by

$$LR_{ind} = 2 \left( \log \left( (1 - \hat{\pi}_{01})T_{0} - T_{01} \hat{\pi}_{01}(1 - \hat{\pi}_{11})T_{1} - T_{11} \hat{\pi}_{11} \right) - \log \left( \hat{\pi}_{1}T_{1}(1 - \hat{\pi}_{1})T_{-T_{1}} \right) \right),$$

where $T$ is the total number of observations, $T_{i}$ is the quantity of $i$-values and $T_{ij}$ represents the amount of observations valued $i$, when the previous observation had the value $j$

$$\hat{\pi}_{i} = \frac{T_{i}}{T},$$
$$\hat{\pi}_{ij} = \frac{T_{ij}}{T_{i}}.$$
Chapter 4. Backtesting

The Markov test is also follows the chi-square distribution with the degree of freedom equal to one. We compute the p-value for the test statistic from

\[ p - value(LR_{ind}) = 1 - \int_{-\infty}^{LR_{ind}} f_{\chi^2(1)}(x)dx. \]

Those two tests can combined into the one that checks the correct conditional coverage, i.e. whether the VaR model fulfills both unconditional coverage and independence property. It means that

\[ I_t \sim i.i.d. \ Bernoulli \ distribution \ with \ parameter \ \alpha. \]

The test statistic simply looks like

\[ LR_{cc} = LR_{uc} + LR_{ind}. \]

This test follows asymptotically the \( \chi^2(2) \) distribution.
Chapter 5

Data sets

In our study we work with the stock prices of three Nordic banks listed on the Stockholm Stock Exchange - Skandinaviska Enskilda Banken (SEB) Svenska Handelsbanken (SHB) and Swedbank (SWED). The data were obtained from the SIX Telekurs. We consider the daily returns taken from the period from March 21st, 2007 to March 19th, 2010. That gives the overall number of 751 observations. We also use the intraday data with the frequencies 5, 10, 20 and 30 minutes. The high frequency data spread over the period between March 23rd, 2009 to June 6th, 2009. That period covers 51 trading days which is equal to 5202, 2602, 1327 and 868 price quotations for 5, 10, 20 and 30 minutes respectively.

We conducted a preliminary recognition to obtain some information about what tools we should or should not use. To do so we transformed the stock prices into the stock returns. The results are presented below.

5.1 The Intraday data

5.1.1 The stock prices of SEB

In Figure 5.1 - Figure 5.4 we present the SEB stock prices. Although in the later investigation we consider the stock returns, in our opinion it is better firstly to look how does the price behave.

The general pattern is the same for each frequency. The higher the frequency the thicker the plot appears to be. That is due to the higher concentration of the time moments when the stock price is recorded.
Figure 5.1: The SEB stock prices for 5 minutes frequency of quotation in the period from March 23rd, 2009 to June 6th, 2009.

Figure 5.2: The SEB stock prices for 10 minutes frequency of quotation in the period from March 23rd, 2009 to June 6th, 2009.

It might seem that taking the higher frequencies is pointless. However we must remember that we are interested in the financial returns. And these small oscillations of stock prices result in the more significant differences in the returns series, see Figure 5.5 - Figure 5.8.

In Figure 5.9 we present the histograms of the returns. The dashed line denotes the normal probability density curve fitted to the data. It is worth noticing that the more frequent the price quotations are, the more often the
Figure 5.3: The SEB stock prices for 20 minutes frequency of quotation in the period from March 23rd, 2009 to June 6th, 2009.

Figure 5.4: The SEB stock prices for 30 minutes frequency of quotation in the period from March 23rd, 2009 to June 6th, 2009.
Figure 5.5: The SEB stock returns for 5 minutes frequency of quotation in the period from March 23rd, 2009 to June 6th, 2009.

Figure 5.6: The SEB stock returns for 10 minutes frequency of quotation in the period from March 23rd, 2009 to June 6th, 2009.

Figure 5.7: The SEB stock returns for 20 minutes frequency of quotation in the period from March 23rd, 2009 to June 6th, 2009.
zeros occur. This means that in many cases the price did not change during the measurement period. It is also immediately clear that the returns do not follow the normal distribution. We investigate this fact more precisely in Chapter 6.
5.1.2 SHB

In Figure 5.10 we show only the plot of the stock prices for the SHB 20 minutes data. The plots for 5, 10 and 30 minutes look similar, as it is shown for the SEB stock prices in Figure 5.1 - Figure 5.4. In Figure 5.11 we present the corresponding returns.
Figure 5.10: The SHB stock prices for 20 minutes frequency of quotation in the period from March 23rd, 2009 to June 6th, 2009.

Figure 5.11: The SHB stock returns for 20 minutes frequency of quotation in the period from March 23rd, 2009 to June 6th, 2009.

Figure 5.12 presents four histograms. We can observe that the quantity of zeros is considerably higher than in the SEB case. The conclusion may be the fact that the SHB stocks are not so popular among traders as the SEB stocks. Indeed we found out that the total volume of the SHB shares is often about eight times less than the SEB stocks volume.
Chapter 5. Data sets

Figure 5.12: Histograms of the returns of SHB in the period from March 23rd, 2009 to June 6th, 2009.

5.1.3 The Swedbank stocks

For the Swedbank we illustrate the stock prices and the stock returns for the 20 minute frequency, see Figures 5.13 and 5.14. The movement of the stock prices seems to be less rapid in comparison to the SEB and the SHB stock prices. From the plot of the returns, Figure 5.14, we observe that indeed the returns behave in the quite regular way except from the single extreme loss which occurred on April 23, 2009 (which corresponds to the index number 547).
Intraday data in Risk Management

Figure 5.13: The Swedbank stock prices for 20 minutes frequency of quotation in the period from March 23rd, 2009 to June 6th, 2009.

Figure 5.14: The Swedbank stock returns for 20 minutes frequency of quotation in the period from March 23rd, 2009 to June 6th, 2009.

From the histograms (Figure 5.15) we can see that the number of zeros is considerably less than in the case of SHB stock and close to the amount of zeros in the SEB stock.
Figure 5.15: Histograms of the returns of the Swedbank in the period from March 23rd, 2009 to June 6th, 2009.

5.2 The Daily data

In this section we analyze the daily stock returns for the SEB, the SHB and the Swedbank stocks. We perform this analysis in another section than the intraday data due to the fact that in the case of the daily returns the overall time horizon of the analyzed returns is much longer than the time horizon of the intraday returns. In Figure 5.16 and Figure 5.17 we present the stock prices and in Figure 5.18 we illustrate the corresponding returns for the three considered bank stocks.
Figure 5.16: Plots of the stock prices. The top panel: SEB stock, the bottom panel: SHB stock.
We observe that the general trends of the stock prices of all the three banks were going down from the beginning of the considered period approximately to the middle of March, 2009. We can suspect that this behavior was caused by the financial crisis. In March, 2009 the banks began to recover and the trend started to go up. The SHB stocks prices increased most dynamically. From Figure 5.18 we see that from the approximately 300th record the fluctuations in the returns considerably increased but lasted for a different periods of time for the considered banks.

From the enclosed histograms (Figures 5.19 to Figure 5.21) we can observe that the returns do not follow the normal distribution. Indeed this observation is supported by the Shapiro-Wilk normality test. The p-values for the returns of each of the banks were considerably lower than 0.01. The exact results are presented in Table 5.1.

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Table 5.1: The Shapiro-Wilk test of the normal distribution for the daily returns of the SEB, the SHB and the Swedbank stocks.
Figure 5.18: Plots of the stock returns. The top panel: the SEB stock, the middle panel: the SHB stock, the bottom panel: the Swedbank stock.
Figure 5.19: The histogram of the daily returns of the SEB stock.

Figure 5.20: The histogram of the daily returns of the SHB stock.

Figure 5.21: The histogram of the daily returns of the Swedbank stock.
Chapter 6

Results

In this chapter we provide the results of our investigation. We start with the outcomes of the application of different volatility estimation models to the \textit{VaR} and the \textit{ES} estimation for the daily returns. The considered models are the three GARCH-based models, i.e. the normal GARCH (nGARCH), the Student’s t GARCH (tGARCH) and the Normal Inverse Gaussian GARCH (NIG-GARCH). We also use the non-parametric volatility model. The non-parametric volatility is computed using the moving window. The length of the window is calculated using the Cross Validation method. Therefore the length differs for the different data sets. The GARCH parameters estimation is based on the constant period of 500 data points which corresponds approximately to two years of daily data. The motivation to use such a time interval is described in the work of Eberlein, Kallsen, Kristen (2003). We calculate the VaR level one step ahead, i.e. for the next day. The purpose is to see how the particular model deals with the volatility estimation and consequently with the proper \textit{VaR} and the proper \textit{ES} level adjustment in the way that is required by the BASEL.

In the next step we perform the similar investigation in application to the high frequency data. We find out how taking the more frequent returns influences the behavior of the models in the one step ahead estimation. The volatility estimated by the GARCH models is again based on the last 500 records and the non-parametric volatility prediction is based on length which is the outcome of the Cross Validation algorithm.

The final part presents the results of the usage of the high frequency data for the daily \textit{VaR} and \textit{ES} estimation. The idea is to model the volatility for the next day basing on the last 500 data points, see Figure 6.1. Of course the period equal to the 500 data points differs for the different frequencies.
For 5 minute returns 102 data points corresponds to one trading day. For 10 minute data one trading day is equal to 51 data points, for 20 minute frequency it is 26 data points and for 30 minutes 17 data points.

Figure 6.1: The scheme of the volatility estimation based on the 500 latest data points.

All the calculations are computed using the R Software, the language and the environment for statistical computing and graphics.

6.1 The VaR and ES calculation

From the first 500 points we estimate the parameters necessary to make the volatility predictions in the case of the parametric models. To make the parameters estimation we use the Maximum Likelihood method. Having computed the parameters we make the volatility forecast and therefrom we calculate the VaR and the ES values.

For the non-parametric model we calculate the volatility in the straightforward way using the moving average window of the length calculated by the Cross Validation technique. The window is calculated on the first 500 data points, so the length may oscillate between 5 and 500. In the non-parametric method the time of the occurrence of the first volatility forecast is calculated in the way that it coincides with the first occurrence of the parametric models result. So the first VaR and ES values are calculated for the 501 return.

We calculate the VaR and the ES for the 1% and 5% confidence levels. The relation between the 1% VaR and the 5% VaR is showed in Figure 6.2. The 1% VaR plot is always above the 5% VaR. That is due to the fact that the 5% VaR assumes the losses to occur with the probability of 5% in every step. The same for the 1% VaR. Therefore the 5% VaR is more tolerant.
The both lines are similar but not identical. The same relation occurs for the ES method.

Figure 6.2: The example of the general relationship between the 1% VaR and the 5% VaR level.

6.1.1 The VaR and ES calculation for the daily returns

Firstly we look at the distribution of the first 500 data points on which we base the VaR and the ES calculations. From the left side of Figure 6.3 which presents the Quantile-Quantile (Q-Q) plots, we see that the empirical returns fit into the NIG distribution well. On the right side of Figure 6.3 we observe that the fit to the normal distribution is not good. Therefore the assumption about white noise, see the Definition 2.1 following the NIG distribution is proved to be adequate.

Figure 6.4 to Figure 6.11 display the VaR and the ES plots for different daily stock returns. Taking into account the transparency of the plots it is illegible to present on a single chart more than two results. Therefore we provide just a sample of diagrams to give an idea of the behavior of the models. Note that the black vertical lines on the Figure 6.4 to Figure 6.11 represent the losses. The upper plot represents the ES which always dominates the VaR - the lower plot. In Table 6.1 we present number of exceptions from the VaR and the ES for the considered models.
Figure 6.3: The Q-Q plots of the SEB stock (top), the SHB stock (middle) and the Swedbank stock (bottom) for the NIG (left) and the Normal distribution (right).
Figure 6.4: $VaR$ and $ES$ for the SHB stock for the 1% confidence level. Calculations are made using the nGARCH model.

Figure 6.5: $VaR$ and $ES$ for the SHB stock for the 5% confidence level. Calculations are made using the nGARCH model.
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Figure 6.6: \( VaR \) and \( ES \) for the SEB stock for the 1\% confidence level. Calculations are made using the tGARCH model.

Figure 6.7: \( VaR \) and \( ES \) for the SEB stock for the 5\% confidence level. Calculations are made using the tGARCH model.
Figure 6.8: *VaR* and *ES* for the Swedbank stock for the 1% confidence level. Calculations are made using the NIG-GARCH model.

Figure 6.9: *VaR* and *ES* for the Swedbank stock for the 5% confidence level. Calculations are made using the NIG-GARCH model.
Figure 6.10: \( \text{VaR} \) and \( \text{ES} \) for the Swedbank stock for the 1% confidence level. Calculations are made using the non-parametric model.

Figure 6.11: \( \text{VaR} \) and \( \text{ES} \) for the Swedbank stock for the 5% confidence level. Calculations are made using the non-parametric model.
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<table>
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<th>ES 1%</th>
<th>ES 5%</th>
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Table 6.1: The number of exceptions from the VaR and ES for the 250 daily returns.

From Figure 6.4 to Figure 6.11 and from Table 6.1 we cannot say much about the quality of the models. The results from the table do not differ much. The reason is that for the daily data we consider the one year period only, that is 250 data points. That number is too small to base conclusions upon it. We also cannot make any deductions based on the raw data, we need some statistical tests. We use them in the later part of our investigation.

**6.1.2 VaR and ES calculation for the intraday returns**

In Figure 6.13 we show an example of the Q-Q plot for the NIG distribution of the SHB 20 minutes stock returns. The fit into the NIG distribution is very accurate. An interesting fact to notice is the shape of the Q-Q for the 5 minutes frequency, see 6.12. In the fit there emerged the "steps". It can be so due to the fact that the higher frequency we consider, the more returns of the same value or the returns equal to zero exist. In the lower frequencies these "steps" disappear. The interpretation may be that the frequency equal to 5 minutes may contain the irrelevant information that can perturb our estimations.
Figure 6.12: The Q-Q plot of the SHB 5 minutes stock for the NIG distribution.

Figure 6.13: The Q-Q plot of the SHB 20 minutes stock for the NIG distribution.
In Figures 6.14 - Figure 6.17 we give an example of the usage of all four models in calculating the 1% \textit{VaR} and the 1% \textit{ES} for the SEB 20 minutes returns. Again if we wanted to include all the graphical results, the number of the images in this section would exceed 100. Therefore we choose the SEB 20 minutes results as they represent the main features of the models.

Figure 6.14: The 1% \textit{VaR} and \textit{ES} levels for the SEB 20 minutes data calculated using the nGARCH model.

Figure 6.15: The 1% \textit{VaR} and \textit{ES} levels for the SEB 20 minutes data calculated using the NIG-GARCH model.
Figure 6.16: The 1% VaR and ES levels for the SEB 20 minutes data calculated using the tGARCH model.

Figure 6.17: The 1% VaR and ES levels for the SEB 20 minutes data calculated using the non-parametric model.

Figure 6.14 illustrates that the normal distribution is not sensitive to the oscillations of the returns. The large losses are symbolized by the high spikes. Notice however that the changes in the VaR or the ES levels are not proportional to the magnitude of the losses.
A more sensitive, in comparison to the nGARCH, is the NIG-GARCH model, see Figure 6.15. In this case the VaR and ES plots are more ragged. The dependence between the occurrence of a loss and the change in VaR and ES is clearer.

The model based on the Student’s t distribution, see Figure 6.16, seems to be even more sensitive distribution than the NIG. Although both plots look similar, see Figure 6.16 and Figure 6.15, changes in the VaR and the ES for the Student’s t are more rapid and bigger than in the NIG case.

The non-parametric model, Figure 6.17, results in a very chaotic VaR and ES estimation. The values are often too high to fit in the plot area. This model is likely to overestimate the risk and to be unstable because of the unexpected spikes.

In Table 6.2 - Table 6.5 we show the number of exceptions from the VaR and ES levels. We can observe that the outcomes are similar for the nGARCH, NIG-GARCH and the tGARCH. The non-parametric model results are not consistent. They suggest that the model is unpredictable in its behavior. However without the proper statistical treatment we cannot draw any relevant conclusions. The statistical treatment is presented in the later part of the work.
### Chapter 6. Results

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Table 6.2: The number of exceptions from the $VaR$ and $ES$ for 4701 of 5 minute returns in the period from March 23rd, 2009 to June 6th, 2009.

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Table 6.3: The number of exceptions from the $VaR$ and $ES$ for 2100 of 10 minute returns in the period from March 23rd, 2009 to June 6th, 2009.
### Table 6.4: The number of exceptions from the VaR and ES for 825 of 20 minute returns in the period from March 23rd, 2009 to June 6th, 2009.

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### Table 6.5: The number of exceptions from the VaR and ES for 366 of 30 minute returns in the period from March 23rd, 2009 to June 6th, 2009.

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<tr>
<td>non-parametric (window length 14)</td>
<td>8</td>
<td>25</td>
<td>2</td>
<td>15</td>
</tr>
</tbody>
</table>
6.2 Results of the Backtesting

In Table 6.6 - Table 6.9 we present the statistical results for all the returns that we use, grouped with respect to different volatility models. Each Table contains the value of the test statistic for the unconditional coverage, $LR_{uc}$, and the test statistic for the independence property, $LR_{ind}$, with the corresponding p-values. The cells with the p-value that is smaller than 0.05 are marked with a gray background. If additionally the value is less than 0.01 it is written in bold. This denotation refers to the level of significance. It is important to make a distinction between the significance level and the confidence level. The confidence level, $\alpha$, occurs in the VaR and the ES methods. The level of significance depends on an arbitrary choice of a statistician and is used to determine whether we do or do not reject the null hypothesis. These two values are not correlated.

To our surprise, in the model that we consider - the model where the perturbations follow the NIG distribution and the volatility is estimated by the GARCH method with different conditional distributions or by the non-parametric method, the nGARCH appears to produce the best VaR estimations. With respect to the unconditional coverage the nGARCH model can be rejected only once, for the 5% VaR for the daily SHB returns, and only if we assume the significance level equal to 5%. It can be however rejected with respect to the independence property 5 times. Independence property is not the subject of concern of the BASEL II, nevertheless it is an essential feature of the proper VaR measure, therefore we investigate it as well. Note that majority of rejections of the tGARCH and the NIG-GARCH in our work would be a result of the violations of the independence property.

All the GARCH models perform good for the 20 and 30 minutes data. Only the non-parametric model produces the results that cannot be accepted from the statistical point of view.

The unexpectedly good behavior of the normal based GARCH and worst than expected behavior of the tGARCH and NIG-GARCH in some sense may be justified. Consider Figure 6.14 - Figure 6.15. The nGARCH model tend to overestimate the risk and is not so sensitive enough to the losses and their magnitude. The NIG-GARCH and tGARCH are more sensitive and reflect the fluctuations in losses in a more precise way. Of course the tests show that these models are not perfect and should be improved. However when adopting the nGARCH method, in the sense of BASEL II, we may incur the bigger capital requirements.
We decide not to introduce the backtesting of the ES in our work. This method is not used by the BASEL II, however we think that in the future it will become more and more popular. It is also an important method because of its mathematical consistence. Therefore we calculate the ES values and compare them to the VaR but we do not include the backtest to keep the work more transparent.

<table>
<thead>
<tr>
<th></th>
<th>VaR 1%</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$LR_{uc}$</td>
<td>p-val</td>
<td>$LR_{ind}$</td>
<td>p-val</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SEB</td>
<td>0.356</td>
<td>0.551</td>
<td>3.415</td>
<td>0.065</td>
<td>0.0170</td>
<td>0.896</td>
<td>5.101</td>
<td>0.239</td>
<td></td>
</tr>
<tr>
<td>SHB</td>
<td>2.323</td>
<td>0.127</td>
<td>4.452</td>
<td>0.035</td>
<td>2.916</td>
<td>0.088</td>
<td>0.0199</td>
<td>0.888</td>
<td></td>
</tr>
<tr>
<td>Swed</td>
<td>0.356</td>
<td>0.551</td>
<td>3.415</td>
<td>0.065</td>
<td>2.043</td>
<td>0.153</td>
<td>3.757</td>
<td>0.053</td>
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</table>

<table>
<thead>
<tr>
<th></th>
<th>VaR 5%</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$LR_{uc}$</td>
<td>p-val</td>
<td>$LR_{ind}$</td>
<td>p-val</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SEB</td>
<td>1.588</td>
<td>0.208</td>
<td>12.762</td>
<td>&lt;0.001</td>
<td>0</td>
<td>1</td>
<td>9.861</td>
<td>0.002</td>
<td></td>
</tr>
<tr>
<td>SHB</td>
<td>0.199</td>
<td>0.656</td>
<td>9.775</td>
<td>0.002</td>
<td>0.089</td>
<td>0.765</td>
<td>6.339</td>
<td>0.012</td>
<td></td>
</tr>
<tr>
<td>Swed</td>
<td>0.725</td>
<td>0.394</td>
<td>1.060</td>
<td>0.303</td>
<td>0.040</td>
<td>0.842</td>
<td>0.059</td>
<td>0.807</td>
<td></td>
</tr>
</tbody>
</table>

Table 6.6: The behavior of the nGARCH model under the different time frequencies.
Table 6.7: The behavior of the NIG-GARCH model under the different time frequencies.

<table>
<thead>
<tr>
<th></th>
<th>VaR 1%</th>
<th></th>
<th></th>
<th>VaR 5%</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$LR_{uc}$</td>
<td>p-val</td>
<td>$LR_{ind}$</td>
<td>p-val</td>
<td>$LR_{uc}$</td>
<td>p-val</td>
</tr>
<tr>
<td>5 minute frequency</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SEB</td>
<td>9.873</td>
<td>0.002</td>
<td>0.718</td>
<td>0.397</td>
<td>0.070</td>
<td>0.792</td>
</tr>
<tr>
<td>SHB</td>
<td>0.740</td>
<td>0.390</td>
<td>0.231</td>
<td>0.631</td>
<td>14.465</td>
<td>&lt;0.001</td>
</tr>
<tr>
<td>Swed</td>
<td>1.112</td>
<td>0.292</td>
<td>0.687</td>
<td>0.407</td>
<td>4.661</td>
<td>0.031</td>
</tr>
<tr>
<td>10 minute frequency</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SEB</td>
<td>0.047</td>
<td>0.828</td>
<td>9.587</td>
<td>0.002</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>SHB</td>
<td>2.752</td>
<td>0.097</td>
<td>9.273</td>
<td>0.002</td>
<td>5.844</td>
<td>0.016</td>
</tr>
<tr>
<td>Swed</td>
<td>0.414</td>
<td>0.520</td>
<td>9.557</td>
<td>0.002</td>
<td>0.159</td>
<td>0.691</td>
</tr>
<tr>
<td>20 minute frequency</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SEB</td>
<td>0.067</td>
<td>0.796</td>
<td>0.199</td>
<td>0.656</td>
<td>1.423</td>
<td>0.233</td>
</tr>
<tr>
<td>SHB</td>
<td>0.202</td>
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<td>0.120</td>
<td>0.729</td>
<td>1.048</td>
<td>0.306</td>
</tr>
<tr>
<td>Swed</td>
<td>0.008</td>
<td>0.930</td>
<td>0.157</td>
<td>0.692</td>
<td>0.040</td>
<td>0.841</td>
</tr>
<tr>
<td>30 minute frequency</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SEB</td>
<td>0.910</td>
<td>0.340</td>
<td>0.022</td>
<td>0.882</td>
<td>1.790</td>
<td>0.181</td>
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<tr>
<td>SHB</td>
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<td>0.860</td>
<td>0.088</td>
<td>0.766</td>
<td>0.028</td>
<td>0.867</td>
</tr>
<tr>
<td>Swed</td>
<td>0.031</td>
<td>0.860</td>
<td>0.088</td>
<td>0.766</td>
<td>0.742</td>
<td>0.390</td>
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<td>daily frequency</td>
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<tr>
<td>SEB</td>
<td>1.177</td>
<td>0.278</td>
<td>0.008</td>
<td>0.929</td>
<td>1.944</td>
<td>0.163</td>
</tr>
<tr>
<td>SHB</td>
<td>1.176</td>
<td>0.278</td>
<td>0.008</td>
<td>0.929</td>
<td>6.071</td>
<td>0.014</td>
</tr>
<tr>
<td>Swed</td>
<td>0.108</td>
<td>0.742</td>
<td>0.032</td>
<td>0.857</td>
<td>3.009</td>
<td>0.083</td>
</tr>
</tbody>
</table>
Table 6.8: The behavior of the tGARCH model under the different time frequencies.
### Table 6.9: The behavior of the non-parametric model under the different time frequencies.

<table>
<thead>
<tr>
<th></th>
<th>VaR 1%</th>
<th></th>
<th>VaR 5%</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$LR_{uc}$</td>
<td>p-val</td>
<td>$LR_{ind}$</td>
<td>p-val</td>
</tr>
<tr>
<td>5 minute frequency</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SEB</td>
<td>0.741</td>
<td>0.390</td>
<td>8.859</td>
<td><strong>0.003</strong></td>
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<tr>
<td>SHB</td>
<td>5.573</td>
<td>0.018</td>
<td>18.153</td>
<td><strong>&lt;0.001</strong></td>
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<tr>
<td>Swed</td>
<td>6.174</td>
<td>0.013</td>
<td>19.954</td>
<td><strong>&lt;0.001</strong></td>
</tr>
<tr>
<td>10 minute frequency</td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>SEB</td>
<td>0.199</td>
<td>0.656</td>
<td>11.480</td>
<td><strong>&lt;0.001</strong></td>
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<tr>
<td>SHB</td>
<td>0.199</td>
<td>0.656</td>
<td>9.775</td>
<td><strong>0.002</strong></td>
</tr>
<tr>
<td>Swed</td>
<td>0.187</td>
<td>0.666</td>
<td>1.311</td>
<td>0.252</td>
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<td>20 minute frequency</td>
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</tr>
<tr>
<td>SEB</td>
<td>0.067</td>
<td>0.796</td>
<td>0.199</td>
<td>0.656</td>
</tr>
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<td>SHB</td>
<td>0.685</td>
<td>0.408</td>
<td>0.088</td>
<td>0.767</td>
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<td>Swed</td>
<td>0.067</td>
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<td>3.029</td>
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<td>30 minute frequency</td>
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<td>SEB</td>
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<td>0.031</td>
<td>0.860</td>
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<td>0.766</td>
</tr>
<tr>
<td>Swed</td>
<td>3.884</td>
<td>0.049</td>
<td>7.033</td>
<td><strong>0.008</strong></td>
</tr>
<tr>
<td>daily frequency</td>
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</tr>
<tr>
<td>SEB</td>
<td>1.177</td>
<td>0.278</td>
<td>0.008</td>
<td>0.929</td>
</tr>
<tr>
<td>SHB</td>
<td>1.177</td>
<td>0.278</td>
<td>0.008</td>
<td>0.929</td>
</tr>
<tr>
<td>Swed</td>
<td>1.177</td>
<td>0.278</td>
<td>0.008</td>
<td>0.929</td>
</tr>
</tbody>
</table>
6.3 Application of the high frequency returns to the daily VaR calculation

In this section we present the results of our attempt of using the intraday data for the daily prediction of the VaR for the end of the following trading day. The graphical illustration of the concept is showed in Figure 6.1. We compare the daily VaR predictions based on the daily returns with the predictions based on 5, 10, 20 and 30 minute returns. Some of our results are illustrated in Figure 6.18 - Figure 6.23. Because of the multitude of the results and keeping in mind the transparency of our work we present only the representative set of the plots.

Considering the length of the intraday returns series we are able to compare the daily VaR predictions on a short time periods only. For 5 minute returns we have 46 time points in which we compare the simulated results, for 10 minutes it is 41 points, for 20 minutes - 31 points and for 30 minutes - 21 points.

We divide this section into two subsections. In the first part we consider the VaR computation based on the non-parametric volatility estimation. In the second part we consider the GARCH-based volatility.

6.3.1 The VaR based on the non-parametric model

In Figure 6.18 we present the behavior of the daily VaR computed on the basis of the high frequency data in comparison with the VaR calculated from the daily returns. As we can see the VaR based on the intraday data behaves in a very unstable way. It is difficult to assess the model on so few data points. Nevertheless we consider the non-parametric model to be too unpredictable for our purposes. Therefore we focus on the GARCH models.
6.3.2 VaR based on the GARCH models

Figure 6.19 presents two VaR lines. One line is computed using the daily data and the other one using the 5 minutes data. We should realize that it means the first line is calculated on the two previous years and the second one on the last 5 days. We do not consider the GARCH models in separation because of a short time horizon and relatively similar results. The both lines behave in a similar way. In some cases the VaR based on the 5 minute returns behaves even better than the one based on an daily data, trying to omit high losses, while the daily based VaR pays no attention to them.

Figure 6.20 presents the VaR calculation based on 5 minutes data that is less accurate than the values from Figure 6.19. The estimation based on the high frequency data produces two high peaks that greatly overestimate the risk.
Figure 6.19: The comparison of the $VaR$ levels computed using the daily and the 5 minutes returns. Left column presents 1% $VaR$ and right column presents 5% $VaR$ for the SEB stock (top), the SHB stock (middle) and the Swedbank stock (bottom).

In Figure 6.21, Figure 6.22 and Figure 6.23 we present the results of using the frequency equal to 10, 20 and 30 minutes respectively. From these plots we conclude that using the high frequency data to the daily $VaR$ predictions
is not a senseless idea. The daily predictions based on the intraday data may result in a better reaction to the rapid or unpredictable changes in the returns. However a further studies should be performed.

Figure 6.20: The example of the comparison of the \textit{VaR} levels computed using the daily and the SHB 5 minute returns.
Figure 6.21: The comparison of the VaR levels computed using the daily and the 10 minute returns for the SEB (top left), the SHB (top right) and the Swedbank (bottom).
Figure 6.22: The comparison of the VaR levels computed using the daily and the 20 minute returns for the SEB (top left), the SHB (top right) and the Swedbank (bottom).
Figure 6.23: The comparison of the VaR levels computed using the daily and the 30 minute returns for the SEB (top left), the SHB (top right) and the Swedbank (bottom).
Chapter 7

Conclusions

This work is concentrated on the usage of the high frequency data in the Risk Management. We have analyzed the daily and the intraday stock returns of three Nordic Banks: SEB, SHB and Swedbank. The intraday data involved the stock prices with the frequencies of the quotations equal to 5, 10, 20 and 30 minutes.

We adopted the returns model introduced in Eberlein, Kallsen and Kristen (2003), i.e. we assume that the increments of the returns can be presented as two processes, the volatility process and the white noise. Additionally we made an assumption that the white noise follows the Normal Inverse Gaussian distribution. We have justified this choice. The volatility process has been estimated using four different methods. Three of them are GARCH-based methods, i.e. the GARCH with the condition distribution assumed to be normal, NIG and Student’s t. The fourth technique was the non-parametric method with the moving average window with the length calculated using the Cross Validation. Firstly we checked the behavior of the models using the daily returns and than we applied them to the intraday returns.

The main idea of this work was to investigate how useful are the intraday data in the Value at Risk and the Expected Shortfall calculation. To do so we performed two types of research. The first one was simply the calculation of the VaR and the ES one step ahead. The second idea was to try to calculate the VaR one day ahead - for the end of the following trading day, using the high frequency returns. The VaR obtained in this way was compared to the VaR calculated from the daily returns. As far as we are concerned there has been no research involving the usage of the high frequency data on the Nordic market done before. It is also important to stress that we were unable to find any work at all that considered improving the one day ahead
VaR calculation by using the intraday data.

The results of the VaR and the ES calculation were assessed on the basis of the statistical tests. Those were the Kupiec test for the unconditional coverage and the test for the independence property - the Markov test. For the daily returns there were a very small differences between the three GARCH-based models. The non-parametric model proved to be unstable and chaotic. This fact was also observed in the intraday data. For the high frequency returns the model that managed to produce the best VaR and ES calculations turned out to be nGARCH. However this model has been showed to overestimate the risk. That is a major disadvantage both for long and short term investors. It may result in too high capital requirements and therefore a lower investment capacity. The NIG-GARCH and the tGARCH reflect the behavior of the losses in a precise way, and still satisfy the unconditional coverage and the independence property in most of the cases. The disadvantage of these tests is the fact that they do not take into account the size of the difference between the VaR level and the losses.

Our attempt of calculating the one day ahead VaR based on the intraday data provided interesting results. It turned out that the VaR computed on the basis of the 500 daily returns and 500 intraday returns can be similar. We have shown that in some cases the VaR based on the high frequency returns can perform even better than the one computed in the standard way. We were unable to perform a more precise investigation due to the not sufficient number of the data points.

The short time horizon of the high frequency data is the biggest drawback of our research. Such data are not publicly available. There are companies who collect the high frequency data but non of them were eager to share them with us for free. Because of the cost of such data we were unable to obtain an access to a wider range of the stock returns. To give a more reliable results further investigations should be conducted.
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Appendix

Programme code in R

One step ahead VaR and ES calculations using daily stock price of SEB.

```r
### Loading libraries
library(timeSeries)
library(fCalendar)
library(fGarch)
library(ghyp)

### Loading data from file
seb <- readSeries("SEB.csv", header=FALSE)

### Exporting plot of price movement to file
png(filename = "seb.png", width = 410, height = 220, units = "px", pointsize = 10, bg = "white", res = NA, restoreConsole = TRUE)
plot(seb, type="l", xlab="Index", ylab="Stock price SEB daily")
dev.off()

### Creating vector of numeric returns
rseb <- returns(seb[,1])
ret <- as.numeric(rseb)

### Exporting plot of returns to file
png(filename = "rseb.png", width = 410, height = 220, units = "px", pointsize = 10, bg = "white", res = NA, restoreConsole = TRUE)
plot(ret, type="l", xlab="Index", ylab="Returns SEB daily")
dev.off()

### Normal QQ plot from first 500 returns
png(filename="qseb.png")
qqnorm(ret[1:500], main="Normal Q-Q PLOT SEB daily")
qqline(ret[1:500], col="green")
```
dev.off()

### Vector of losses

```r
losses <- c(length(ret), NA)
for (i in 1:length(ret)) {
  if (ret[i] < 0)
    losses[i] <- -ret[i]
  else losses[i] <- 0
}
```

### BACKTESTING

#### Unconditional Coverage

```r
LR_uc <- function (hits, it, alpha) {
  it = it
  T = length(hits)
  2 * log(((1-it/T)/(1-alpha))^(T-it)*((it/T)/alpha)^it);
}
```

#### Independence Property

```r
LR_ind <- function (hits, it) {
  T_01 = 0
  T_11 = 0
  for (i in 2:length(hits)) {
    if (hits[i-1] == 0 && hits[i] == 1) T_01 = T_01 + 1
    if (hits[i-1] == 1 && hits[i] == 1) T_11 = T_11 + 1
  }
  T = length(hits)
  pi_1 = it/T
  pi_0 = (T-it)/T
  pi_11 = T_11/it
  pi_01 = T_01/(T-it)
  2*(log((1-pi_01)^(T-it-T_01)*pi_01^T_01*(1-pi_11)^(it-T_11)*pi_11^T_11) - log(pi_1^it*(1-pi_1)^(T-it)))
}
```

#### p-value

```r
p_val <- function (LR) {
  print(1-pchisq(LR, 1))
}
```
```r
### GARCH(1,1) volatility
gf <- garchFit(~garch(1,1), ret[1:500], trace=FALSE)
vol <- gf@sigma.t

### Standardized returns
sret <- ret[1:500]/vol

### Fit of 500 standardized returns into NIG distribution
NIGfit <- fit.NIGuv(sret, silent=T)

### QQ-plot with NIG distribution
png(filename="qnigseb.png")
qqghyp(NIGfit, gaussian=FALSE, main="NIG Q-Q Plot SEB daily")
dev.off()

### Vectors for VaR estimations for 1% and 5%
var1 <- c(length(ret), NA)
var5 <- c(length(ret), NA)
var1[1:500] <- 0
var5[1:500] <- 0

### Vectors for ES estimations for 1% and 5%
etl1 <- c(length(ret), NA)
etl5 <- c(length(ret), NA)
etl1[1:500] <- 0
etl5[1:500] <- 0

### Hit sequence
hit1 <- c((length(ret)-500), NA)
hit5 <- c((length(ret)-500), NA)

### Counters of VaR exception
it1 <- 0
it5 <- 0

### Counters of ES exception
es1 <- 0
es5 <- 0

### VaR for 1% and 5% level of confidence. Expected Shortfall
### for 1% and 5%
for(i in 501:(length(ret))){
```
vol[i]<-sqrt(gf@fit$matcoef[2,1]+ gf@fit$matcoef[3,1]*(
ret[i-1]-gf@fit$matcoef[1,1])^2+ gf@fit$matcoef[4,1]*gf@sigma.t[500]^2);

fitgh<-transform(NIGfit,0,vol[i])

l1<-qghyp(0.01,fitgh)
l5<-qghyp(0.05,fitgh)
var1[i]=-l1
var5[i]=-l5

etl1[i]=-ESghyp(0.01,fitgh)
etl5[i]=-ESghyp(0.05,fitgh)

gf1<-tryCatch(garchFit(~ garch(1,1),ret[(i-499):i],
trace=FALSE),
error = function(e) gf)
gf<-gf1

sret<-ret[(i-499):i]/gf@sigma.t;
NIGfit<-fit.NIGuv(sret,silent=T);

if(is.finite(var1[i])){
  if(var1[i]<losses[i]){
    it1=it1+1
    hit1[i-500]=1
    except1[i]=losses[i]-var1[i]
  }
  else hit1[i-500]=0
}
else hit1[i-500]=0

if(is.finite(var5[i])){
  if(var5[i]<losses[i]){
    it5=it5+1
    hit5[i-500]=1
    except5[i]=losses[i]-var5[i]
  }
  else hit5[i-500]=0
}
else hit5[i-500]=0

if(is.finite(etl1[i])){
  if(etl1[i]<losses[i]){
    es1=es1+1
  }
}
if(is.finite(etl5[i])){
    if(etl5[i]<losses[i])
        es5=es5+1
}

### Exporting plot of losses, 1% VaR and 1% ES to a file
a<-1.25*max(losses[501:length(losses)])
png(filename = "seb_n1.png", width = 400, height = 220, units = "px", pointsize = 10, bg = "white", res = NA, restoreConsole = TRUE)
plot(losses[501:length(losses)],type="h",xlim=c(0,1.05*(length(losses)-500)),ylim=c(0,a),ylab="Losses; 1% VaR and 1% ES")
lines(etl1[501:length(losses)],t="l",col="red")
lines(var1[501:length(losses)],t="l",col="green")
lines(losses[501:length(losses)],t="h",col="black")
legend(0.87*length(losses[501:length(losses)]), 1.03*a, c("VaR", "ES"), lwd=2,col=c("green","red"),cex=0.9)
dev.off()

### Exporting plot of losses, 5% VaR and 5% ES to a file
a<-1.25*max(losses[501:length(losses)])
png(filename = "seb_n5.png", width = 400, height = 220, units = "px", pointsize = 10, bg = "white", res = NA, restoreConsole = TRUE)
plot(losses[501:length(losses)],type="h",xlim=c(0,1.05*(length(losses)-500)),ylim=c(0,a),ylab="Losses; 5% VaR and 5% ES")
lines(etl5[501:length(losses)],t="l",col="red")
lines(var5[501:length(losses)],t="l",col="green")
lines(losses[501:length(losses)],t="h",col="black")
legend(0.87*length(losses[501:length(losses)]), 1.03*a, c("VaR", "ES"), lwd=2,col=c("green","red"),cex=0.9)
dev.off()

### Backtesting

# VAR 1% Unconditional Coverage
LR_uc(hit1,iti1,0.01)
p_val(LR_uc(hit1,iti1,0.01))

# VAR 1% Independence property
LR_ind(hit1,iti1)
p_val(LR_ind(hit1,iti1))
# VAR 5% Unconditional Coverage
LR_uc(hit5,it5,0.05)
p_val(LR_uc(hit5,it5,0.05))

# VAR 5% Independence property
LR_ind(hit5,it5)
p_val(LR_ind(hit5,it5))

### Saving the results into the unique variables
losses_seb <- losses
vol_seb_n <- vol
var1_seb_n <- var1
var5_seb_n <- var5
etl1_seb_n <- etl1
etl5_seb_n <- etl5
hit1_seb_n <- hit1
hit5_seb_n <- hit5
it1_seb_n <- it1
it5_seb_n <- it5
es1_seb_n <- es1
es5_seb_n <- es5

# GARCH(1,1) volatility
gf <- garchFit(~garch(1,1),ret[1:500],cond.dist="std",trace=FALSE)
vol <- gf@sigma.t

### Standardized returns
sret <- ret[1:500]/vol

### Fit of 500 standardized returns into NIG distribution
NIGfit <- fit.NIGuv(sret,silent=T)

### QQ-plot with NIG distribution
png(filename="qnigseb.png")
qqghyp(NIGfit, gaussian=FALSE, main="NIG Q-Q Plot SEB daily")
dev.off()

### Vectors for VaR estimations for 1% and 5%
var1 <- c(length(ret),NA)
var5 <- c(length(ret),NA)
var1[1:500] <- 0
var5[1:500] <- 0
### Vectors for ES estimations for 1% and 5%

```r
etl1 <- c(length(ret), NA)
etl5 <- c(length(ret), NA)
etl1[1:500] <- 0
etl5[1:500] <- 0
```

### Hit sequence

```r
hit1 <- c((length(ret) - 500), NA)
hit5 <- c((length(ret) - 500), NA)
```

### Counters of VaR exception

```r
it1 <- 0
it5 <- 0
```

### Counters of ES exception

```r
es1 <- 0
es5 <- 0
```

### VaR for 1% and 5% level of confidence. Expected Shortfall for 1% and 5%

```r
for (i in 501:(length(ret))){
  vol[i] <- sqrt(fit@fit$matcoef[2,1] + fit@fit$matcoef[3,1]*((ret[i-1] - fit@fit$matcoef[1,1])^2 + fit@fit$matcoef[4,1]*fit@sigma.t[500]^2);

  fitgh <- transform(NIGfit, 0, vol[i])

  l1 <- qghyp(0.01, fitgh)
l5 <- qghyp(0.05, fitgh)
var1[i] <- -l1
var5[i] <- -l5
etl1[i] <- -ESghyp(0.01, fitgh)
etl5[i] <- -ESghyp(0.05, fitgh)

  gf1 <- tryCatch(garchFit(~garch(1,1), ret[(i-499):i],
                           cond.dist = "std", trace = FALSE),
                           error = function(e) gf)
gf <- gf1
sret <- ret[(i-499):i]/fit@sigma.t;
NIGfit <- fit.NIGuv(sret, silent = T);
```

```r
if(is.finite(var1[i])){
  if(var1[i]<losses[i]){
    it1 = it1+1
  }
}87
```
```r
hit1[i-500]=1
except1[i]=losses[i]-var1[i]
}
else hit1[i-500]=0
}
else hit1[i-500]=0

if(is.finite(var5[i])){
  if(var5[i]<losses[i]){  
    it5=it5+1
    hit5[i-500]=1
    except5[i]=losses[i]-var5[i]
  }
  else hit5[i-500]=0
}
else hit5[i-500]=0

if(is.finite(etl1[i])){
  if(etl1[i]<losses[i]){  
    es1=es1+1
  }
}

if(is.finite(etl5[i])){
  if(etl5[i]<losses[i]){  
    es5=es5+1
  }
}

### Exporting plot of losses, 1% VaR and 1% ES to a file
a<-1.25*max(losses[501:length(losses)])
png(filename = "seb_t1.png", width = 400, height = 220, units = "px", pointsize = 10, bg = "white", res = NA, restoreConsole = TRUE)
plot(losses[501:length(losses)],type="h",xlim=c(0,1.05*(length(losses)-500)),ylim=c(0,a),ylab="Losses; 1% VaR and 1% ES")
lines(etl1[501:length(losses)],t="l",col="red")
lines(var1[501:length(losses)],t="l",col="green")
lines(losses[501:length(losses)],t="h",col="black")
legend(0.87*length(losses[501:length(losses)]), 1.03*a, c("VaR", "ES"), lwd=2,col=c("green","red"),cex=0.9)
dev.off()

### Exporting plot of losses, 5% VaR and 5% ES to a file
a<-1.25*max(losses[501:length(losses)])
```
```r
png(filename = "seb_t5.png", width = 400, height = 220, units = "px", pointsize = 10, bg = "white", res = NA, restoreConsole = TRUE)
plot(losses[501:length(losses)], type="h", xlim=c(0,1.05*(length(losses)-500)), ylim=c(0,a), ylab="Losses; 5% VaR and 5% ES")
lines(etl5[501:length(losses)], t="l", col="red")
lines(var5[501:length(losses)], t="l", col="green")
lines(losses[501:length(losses)], t="h", col="black")
legend(0.87*length(losses[501:length(losses)]), 1.03*a, c("VaR", "ES"), lwd=2, col=c("green", "red"), cex=0.9)
dev.off()

### Backtesting

# VAR 1% Unconditional Coverage
LR_uc(hit1, it1, 0.01)
p_val(LR_uc(hit1, it1, 0.01))

# VAR 1% Independence property
LR_ind(hit1, it1)
p_val(LR_ind(hit1, it1))

# VAR 5% Unconditional Coverage
LR_uc(hit5, it5, 0.05)
p_val(LR_uc(hit5, it5, 0.05))

# VAR 5% Independence property
LR_ind(hit5, it5)
p_val(LR_ind(hit5, it5))

### Saving the results into the unique variables
vol_seb_t<-vol
var1_seb_t<-var1
var5_seb_t<-var5
etl1_seb_t<-etl1
etl5_seb_t<-etl5
hit1_seb_t<-hit1
hit5_seb_t<-hit5
it1_seb_t<-it1
it5_seb_t<-it5
es1_seb_t<-es1
es5_seb_t<-es5
```

---

# NIG–GARCH

---
### GARCH(1,1) volatility

gf <- garchFit(~garch(1,1), ret[1:500], cond.dist="snig", trace = FALSE)
vol <- gf@sigma.t

### Standardized returns
sret <- ret[1:500]/vol

### Fit of 500 standardized returns into NIG distribution
NIGfit <- fit.NIGuv(sret, silent=T)

### QQ-plot with NIG distribution
png(filename="qnigseb.png")
qqghyp(NIGfit, gaussian=FALSE, main="NIG Q-Q Plot SEB daily")
dev.off()

### Vectors for VaR estimations for 1% and 5%
var1 <- c(length(ret), NA)
var5 <- c(length(ret), NA)
var1[1:500] <- 0
var5[1:500] <- 0

### Vectors for ES estimations for 1% and 5%
etl1 <- c(length(ret), NA)
etl5 <- c(length(ret), NA)
etl1[1:500] <- 0
etl5[1:500] <- 0

### Hit sequence
hit1 <- c((length(ret)-500), NA)
hit5 <- c((length(ret)-500), NA)

### Counters of VaR exception
it1 <- 0
it5 <- 0

### Counters of ES exception
es1 <- 0
es5 <- 0

### VaR for 1% and 5% level of confidence. Expected Shortfall for 1% and 5%
for(i in 501:(length(ret))){
    vol[i] <- sqrt(gf@fit$matcoef[2,1]+ gf@fit$matcoef[3,1]*(
        ret[i-1]-gf@fit$matcoef[1,1])^2+ gf@fit$matcoef[4,1]
        *gf@sigma.t[500]^2);
    fitgh <- transform(NIGfit, 0, vol[i])
}
l1 <- qghyp(0.01, fitgh)
l5 <- qghyp(0.05, fitgh)
var1[i] = - l1
var5[i] = - l5
etl1[i] = - ESghyp(0.01, fitgh)
etl5[i] = - ESghyp(0.05, fitgh)

gf1 <- tryCatch(garchFit(~ garch(1,1), ret[(i-499):i],
cond.dist = "snig", trace = FALSE),
error = function(e) gf)
gf <- gf1
sret <- ret[(i-499):i]/gf@sigma.t;
NIGfit <- fit.NIGuv(sret, silent = T);

if(is.finite(var1[i])){
  if(var1[i] < losses[i]){
    it1 = it1 + 1
    hit1[i-500] = 1
    except1[i] = losses[i] - var1[i]
  }
  else hit1[i-500] = 0
}
else hit1[i-500] = 0

if(is.finite(var5[i])){
  if(var5[i] < losses[i]){
    it5 = it5 + 1
    hit5[i-500] = 1
    except5[i] = losses[i] - var5[i]
  }
  else hit5[i-500] = 0
}
else hit5[i-500] = 0

if(is.finite(etl1[i])){
  if(etl1[i] < losses[i]){
    es1 = es1 + 1
  }
}
else

if(is.finite(etl5[i])){
  if(etl5[i] < losses[i]){
    es5 = es5 + 1
  }
}
### Exporting plot of losses, 1% VaR and 1% ES to a file

```R
a <- 1.25 * max(losses[501:length(losses)])
png(filename = "seb_nig1.png", width = 400, height = 220, units = "px", pointsize = 10, bg = "white", res = NA, restoreConsole = TRUE)
plot(losses[501:length(losses)], type = "h", xlim = c(0, 1.05 * (length(losses) - 500)), ylim = c(0, a), ylab = "Losses; 1% VaR and 1% ES")
lines(etl1[501:length(losses)], t="l", col="red")
lines(var1[501:length(losses)], t="l", col="green")
lines(losses[501:length(losses)], t="h", col="black")
legend(0.87 * length(losses[501:length(losses)]), 1.03 * a, c("VaR", "ES"), lwd = 2, col = c("green", "red"), cex = 0.9)
dev.off()
```

### Exporting plot of losses, 5% VaR and 5% ES to a file

```R
a <- 1.25 * max(losses[501:length(losses)])
png(filename = "seb_nig5.png", width = 400, height = 220, units = "px", pointsize = 10, bg = "white", res = NA, restoreConsole = TRUE)
plot(losses[501:length(losses)], type = "h", xlim = c(0, 1.05 * (length(losses) - 500)), ylim = c(0, a), ylab = "Losses; 5% VaR and 5% ES")
lines(etl5[501:length(losses)], t="l", col="red")
lines(var5[501:length(losses)], t="l", col="green")
lines(losses[501:length(losses)], t="h", col="black")
legend(0.87 * length(losses[501:length(losses)]), 1.03 * a, c("VaR", "ES"), lwd = 2, col = c("green", "red"), cex = 0.9)
dev.off()
```

### Backtesting

#### VAR 1% Unconditional Coverage

```R
LR_uc(hit1, it1, 0.01)
p_val(LR_uc(hit1, it1, 0.01))
```

#### VAR 1% Independence property

```R
LR_ind(hit1, it1)
p_val(LR_ind(hit1, it1))
```

#### VAR 5% Unconditional Coverage

```R
LR_uc(hit5, it5, 0.05)
p_val(LR_uc(hit5, it5, 0.05))
```

#### VAR 5% Independence property
LR_ind(hit5, it5)
p_val(LR_ind(hit5, it5))

### Saving the results into the unique variables
vol_seb_nig <- vol
var1_seb_nig <- var1
var5_seb_nig <- var5
etl1_seb_nig <- etl1
etl5_seb_nig <- etl5
hit1_seb_nig <- hit1
hit5_seb_nig <- hit5
it1_seb_nig <- it1
it5_seb_nig <- it5
es1_seb_nig <- es1
es5_seb_nig <- es5

###############################
# NON-PARAMETRIC #
###############################

### Cross Validation Method for an optimal window

temp <- ret[1:500]

for (i in 1:500) {
  if (temp[i] == 0) temp[i] = 1e-10
}
temp <- temp^2

summ <- function(vec, k, t) {
  s = 0
  for (i in 1:k) {
    if (t > i) s = s + log(vec[t-i])
  }
  summ = s / k
}

CV <- function(vec, k, T) {
  s = 0
  for (t in 1:T) {
    s = s + (log(vec[t]) - summ(vec, k, t))^2
  }
  CV = s / T
}

T = 500
k = 5
win = k
\[ \text{min} = \text{CV}(\text{temp}, k, T) \]
\[ k = 6 \]
while \((k \leq 500)\) {
    \[ \text{aa} = \text{CV}(\text{temp}, k, T) \]
    if \((\text{aa} < \text{min})\) {
        \[ \text{min} = \text{aa} \]
        \[ \text{win} = k \]
    }
    \[ k = k + 1 \]
}
\[ \text{win} \]

### Non-parametric Volatility Estimation

\[
\text{logvol} \leftarrow \text{function}(\text{vec}, k, t) \{
\text{s} = 0 
\text{tmp} \leftarrow \text{vec}[1: t] 
\text{tmp} \leftarrow \log(\text{tmp}^2) 
\text{for } (i \text{ in } 0:(k-1)) \{
    \text{if} (\text{is.finite} (\text{tmp}[t-i])) \text{ s} = \text{s} + \text{tmp}[t-i] 
\} 
\text{logvol} = \text{s} / k 
\}
\]

\[ \text{vol} \leftarrow \text{c}(\text{length} (\text{ret}), \text{NA}) \]
\[ \text{vol}[1: \text{win}] = 0 \]

\[
\text{for } (i \text{ in } (\text{win} + 1) : \text{length} (\text{ret})) \{
\text{t} = i - 1 
\text{tmp} = \text{logvol} (\text{ret}, \text{win}, t) 
\text{vol}[i] = \sqrt{\exp(\text{tmp})} 
\}
\]
\[ \text{vol}[1: \text{win}] \leftarrow \text{mean}(\text{vol}[\text{win} + 1: 500]) \]

### VaR calculation

### Vectors for VaR estimations for 1% and 5%
\[ \text{var1} \leftarrow \text{c}(\text{length} (\text{ret}), \text{NA}) \]
\[ \text{var5} \leftarrow \text{c}(\text{length} (\text{ret}), \text{NA}) \]
\[ \text{var1}[1: 500] <\text{-} 0 \]
\[ \text{var5}[1: 500] <\text{-} 0 \]

### Vectors for ES estimations for 1% and 5%
\[ \text{etl1} \leftarrow \text{c}(\text{length} (\text{ret}), \text{NA}) \]
\[ \text{etl15} \leftarrow \text{c}(\text{length} (\text{ret}), \text{NA}) \]
\[ \text{etl1}[1: 500] <\text{-} 0 \]
\[ \text{etl15}[1: 500] <\text{-} 0 \]
## Hit sequence

hit1 <- c((length(ret) - 500), NA)
hit5 <- c((length(ret) - 500), NA)

## Counters of VaR exception

it1 <- 0
it5 <- 0

## Counters of ES exception

es1 <- 0
es5 <- 0

## Declaration of a vector of VaR exceptions

except1 <- c(length(ret), NA)
except1[1:length(ret)] <- 0
except5 <- c(length(ret), NA)
except5[1:length(ret)] <- 0

### VaR

for (i in 501:(length(ret))){
    sret <- (ret[(i - 500):(i - 1)]) / (vol[(i - 500):(i - 1)]);
    NIGfit <- fit.NIGuv(sret, silent = T);

    fitgh <- transform(NIGfit, 0, vol[i])

    l1 <- qghyp(0.01, fitgh)
    l5 <- qghyp(0.05, fitgh)
    var1[i] = -l1
    var5[i] = -l5
    if (is.na(var1[i])) var1[i] <- mean(var1[500:i - 1])
    if (is.na(var5[i])) var5[i] <- mean(var5[500:i - 1])

    if (is.finite(var1[i])){
        if (var[i] < losses[i]){
            it1 = it1 + 1
            hit1[i - 500] = 1
            except1[i] = losses[i] - var1[i]
        }
    } else hit1[i - 500] = 0

    if (is.finite(var5[i])){
        if (var[i] < losses[i]){
            it5 = it5 + 1
            hit5[i - 500] = 1
            except5[i] = losses[i] - var5[i]
        }
    } else hit5[i - 500] = 0
}
except5[i]=losses[i]-var5[i]
}
else hit5[i-500]=0
else hit5[i-500]=0

etl1[i]<-ESghyp(0.01,fitgh)
etl5[i]<-ESghyp(0.05,fitgh)

if(is.finite(etl1[i])){
  if(etl1[i]<losses[i]){
    es1=es1+1
  }
}

if(is.finite(etl5[i])){
  if(etl5[i]<losses[i]){
    es5=es5+1
  }
}

# # # Exporting plot of losses, 1% VaR and 1% ES to a file
a<-1.25*max(losses[501:length(losses)])
png(filename = "seb_np1.png", width = 400, height = 220,
    units = "px", pointsize = 10, bg = "white", res = NA,
    restoreConsole = TRUE)
plot(losses[501:length(losses)],type="h",xlim=c(0,1.05*(
    length(losses)-500)),ylim=c(0,a),ylab="Losses; 1% VaR and
1% ES")
lines(etl1[501:length(losses)],t="l",col="red")
lines(var1[501:length(losses)],t="l",col="green")
lines(losses[501:length(losses)],t="h",col="black")
legend(0.87*length(losses[501:length(losses)]), 1.03*a, c("VaR", "ES"), lwd=2,col=c("green","red"),cex=0.9)
dev.off()

# # # Exporting plot of losses, 5% VaR and 5% ES to a file
a<-1.25*max(losses[501:length(losses)])
png(filename = "seb_np5.png", width = 400, height = 220,
    units = "px", pointsize = 10, bg = "white", res = NA,
    restoreConsole = TRUE)
plot(losses[501:length(losses)],type="h",xlim=c(0,1.05*(
    length(losses)-500)),ylim=c(0,a),ylab="Losses; 5% VaR and
5% ES")
lines(etl5[501:length(losses)],t="l",col="red")
lines(var5[501:length(losses)],t="l",col="green")
lines(losses[501:length(losses)],t="h",col="black")
legend(0.87*length(losses[501:length(losses)]), 1.03*a, c("VaR", "ES"), lwd=2,col=c("green","red"),cex=0.9)
de.v.off()

### Backtesting

# VAR 1% UC
LR_uc(hit1,it1,0.01)
p.val(LR_uc(hit1,it1,0.01))

# VAR 1% IND
LR_ind(hit1,it1)
p.val(LR_ind(hit1,it1))

# VAR 5% UC
LR_uc(hit5,it5,0.05)
p.val(LR_uc(hit5,it5,0.05))

# VAR 5% IND
LR_ind(hit5,it5)
p.val(LR_ind(hit5,it5))

### Saving the results into the unique variables

win.seb<-win
vol.seb.np<-vol
var1.seb.np<-var1
var5.seb.np<-var5
etl1.seb.np<-etl1
etl5.seb.np<-etl5
hit1.seb.np<-hit1
hit5.seb.np<-hit5
it1.seb.np<-it1
it5.seb.np<-it5
es1.seb.np<-es1
es5.seb.np<-es5

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One day ahead VaR calculations using 5 minutes stock returns of SEB.

```r
### Loading data from file
seb5 <- readSeries("SEB5.csv", header = FALSE)

### Creating vector of numeric returns
rseb5 <- returns(seb5[,1])
ret <- as.numeric(rseb5)

### Loading daily losses
losses <- losses_seb[506:551]

########################################################################
# nGARCH #
########################################################################

### Vector of the Volatility
vol <- c(46, NA);
vol[1:46] <- 0

### Vectors for VaR estimations for 1% and 5%
var1 <- c(46, NA)
var5 <- c(46, NA)
var1[1:46] <- 0
var5[1:46] <- 0

### GARCH(1,1) volatility and standardized returns
gf <- garchFit(~garch(1,1), ret[10:509], trace = FALSE)
sret <- ret[10:509]/gf@sigma.t

### Fit of 500 standardized returns into NIG distribution
NIGfit <- fit.NIGuv(sret, silent = T)

### VaR calculation
k = 1
for (i in seq(611, length(ret), 102)) {
  sigmsq <- gf@fit$matcoef[2,1] + gf@fit$matcoef[3,1]*(ret[i-102] - gf@fit$matcoef[1,1])^2 + gf@fit$matcoef[4,1]*(gf@sigma.t[500])^2;
  tmp = gf@fit$matcoef[2,1]*(1-(gf@fit$matcoef[3,1]+gf@fit$matcoef[4,1])^101)/(1-gf@fit$matcoef[3,1]-gf@fit$matcoef[4,1]) + (gf@fit$matcoef[3,1]+gf@fit$matcoef[4,1])^101*
sigmsq
  vol[k] = sqrt(102*tmp)
}
```
fitgh <- transform(NIGfit,0,vol[k])

l1 <- qghyp(0.01, fitgh)
l5 <- qghyp(0.05, fitgh)
var1[k] = -l1
var5[k] = -l5
if(is.na(var1[k])) var1[k] = mean(var1[1:k-1])
if(is.na(var5[k])) var5[k] = mean(var5[1:k-1])

gf1 <- tryCatch(garchFit(~garch(1,1), ret[(i-499):i],
                       trace=FALSE),
                error = function(e) gf)
gf <- gf1
sret <- ret[(i-499):i]/gf@sigma.t;
NIGfit <- fit.NIGuv(sret, silent=T);

k=k+1

### Exporting plot of daily losses, 1% VaR for daily and 5 minute returns
a <- 2.5 * max(losses)
ng(filename = "in_seb5_n1.png", width = 350, height = 270,
    units = "px", pointsize = 10, bg = "white", res = NA,
    restoreConsole = TRUE)
plot(losses,type="h",xlim=c(0,1.05*length(losses)),ylim=c(0,a)
     ,ylab="Losses; daily and 5 minutes 1% VaR")
lines(var1_seb_n[506:551],t="1",col="red")
lines(var1[1:46],t="1",col="green")
lines(losses,t="h",col="black")
legend(0.74*length(losses), 1.03*a, c("daily VaR", "5 min VaR "),
       lwd=2,col=c("red","green"),cex=0.9)
dev.off()

### Exporting plot of daily losses, 5% VaR for daily and 5 minute returns
a <- 2.5 * max(losses)
png(filename = "in_seb5_n5.png", width = 350, height = 270,
    units = "px", pointsize = 10, bg = "white", res = NA,
    restoreConsole = TRUE)
plot(losses,type="h",xlim=c(0,1.05*length(losses)),ylim=c(0,a)
     ,ylab="Losses; daily and 5 minutes 5% VaR")
lines(var5_seb_n[506:551],t="1",col="red")
lines(var5[1:46],t="1",col="green")
lines(losses,t="h",col="black")
legend(0.74*length(losses), 1.03*a, c("daily VaR", "5 min VaR ")
       , lwd=2,col=c("red","green"),cex=0.9)
dev.off()
### Vector of the Volatility

```r
vol <- c(46, NA);
vol[1:46] <- 0
```

### Vectors for VaR estimations for 1% and 5%

```r
var1 <- c(46, NA)
var5 <- c(46, NA)
var1[1:46] <- 0
var5[1:46] <- 0
```

### GARCH(1,1) volatility and standardized returns

```r
gf <- garchFit(~garch(1,1), ret[10:509], cond.dist = "std", trace = FALSE)
sret <- ret[10:509]/gf@sigma.t
```

### Fit of 500 standardized returns into NIG distribution

```r
NIGfit <- fit.NIGuv(sret, silent = T)
```

### VaR calculation

```r
for (i in seq(611, length(ret), 102)) {
  sigmsq <- gf@fit$matcoef[2,1] + gf@fit$matcoef[3,1]*(ret[i-102] - gf@fit$matcoef[1,1])**2 + gf@fit$matcoef[4,1]*(gf@sigma.t[500])**2;
  tmp <- gf@fit$matcoef[2,1]*(1-(gf@fit$matcoef[3,1]+gf@fit$matcoef[4,1])**101)/((1-gf@fit$matcoef[3,1]-gf@fit$matcoef[4,1])**101)*sigmsq
  vol[k] = sqrt(102*tmp)
  fitgh <- transform(NIGfit, 0, vol[k])
  l1 <- qghyp(0.01, fitgh)
  l5 <- qghyp(0.05, fitgh)
  var1[k] <- -l1
  var5[k] <- -l5
  if(is.na(var1[k])) var1[k] <- mean(var1[1:k-1])
  if(is.na(var5[k])) var5[k] <- mean(var5[1:k-1])
}
```
gf1 <- tryCatch(garchFit(~garch(1,1), ret[(i-499):i],
    cond.dist="std", trace=FALSE),
    error = function(e) gf)
gf <- gf1

sret <- ret[(i-499):i] / gf@sigma.t;
NIGfit <- fit.NIGuv(sret, silent=T);

k = k + 1

### Exporting plot of daily losses, 1% VaR for daily and 5 minute returns

a <- 2.5 * max(losses)
ng(filename = "in_seb5_t1.png", width = 350, height = 270,
    units = "px", pointsize = 10, bg = "white", res = NA,
    restoreConsole = TRUE)
plot(losses, type = "h", xlim = c(0, 1.05 * length(losses)), ylim = c(0, a),
    ylab = "Losses; daily and 5 minutes 1% VaR")
lines(var1_seb_t[506:551], t="l", col="red")
lines(var1[1:46], t="l", col="green")
lines(losses, t="h", col="black")
legend(0.74 * length(losses), 1.03 * a, c("daily VaR", "5 min VaR"),
    lwd=2, col=c("red", "green"), cex=0.9)
dev.off()

### Exporting plot of daily losses, 5% VaR for daily and 5 minute returns

a <- 2.5 * max(losses)
png(filename = "in_seb5_t5.png", width = 350, height = 270,
    units = "px", pointsize = 10, bg = "white", res = NA,
    restoreConsole = TRUE)
plot(losses, type = "h", xlim = c(0, 1.05 * length(losses)), ylim = c(0, a),
    ylab = "Losses; daily and 5 minutes 5% VaR")
lines(var5_seb_t[506:551], t="l", col="red")
lines(var5[1:46], t="l", col="green")
lines(losses, t="h", col="black")
legend(0.74 * length(losses), 1.03 * a, c("daily VaR", "5 min VaR"),
    lwd=2, col=c("red", "green"), cex=0.9)
dev.off()

# Vector of the Volatility
vol <- c(46, NA);
vol[1:46]<-0

### Vectors for VaR estimations for 1% and 5%
var1<-c(46,NA)
var5<-c(46,NA)
var1[1:46]<-0
var5[1:46]<-0

### GARCH(1,1) volatility and standardized returns
gf<-garchFit(~garch(1,1),ret[10:509], cond.dist="snig",trace=FALSE)
sret<-ret[10:509]/gf@sigma.t

### Fit of 500 standardized returns into NIG distribution
NIGfit<-fit.NIGuv(sret,silent=T)

### VaR calculation
k=1
for(i in seq(611,length(ret),102)){
  sigmsq<-gf$fit$matcoef[2,1]+gf$fit$matcoef[3,1]*(ret[i-102]-gf$fit$matcoef[1,1])**2+gf$fit$matcoef[4,1]*(gf@sigma.t[500])**2;
  tmp=gf$fit$matcoef[2,1]*(1-(gf$fit$matcoef[3,1]+gf$fit$matcoef[4,1])**101)/(1-gf$fit$matcoef[3,1]-gf$fit$matcoef[4,1])+(gf$fit$matcoef[3,1]+gf$fit$matcoef[4,1])**101*sigmsq;
  vol[k]=sqrt(102*tmp)
  fitgh<-transform(NIGfit,0,vol[k])
  l1<-qghyp(0.01,fitgh)
l5<-qghyp(0.05,fitgh)
  var1[k]=-11
  var5[k]=-15
  if(is.na(var1[k]))var1[k]=mean(var1[1:k-1])
  if(is.na(var5[k]))var5[k]=mean(var5[1:k-1])
  gf1<-tryCatch(garchFit(~garch(1,1),ret[(i-499):i], cond.dist="snig",trace=FALSE),
    error = function(e) gf)
  gf<-gf1
}
sret<-ret[(i-499):i]/gf@sigma.t;
NIGfit<-fit.NIGuv(sret,silent=T);
k=k+1
Exporting plot of daily losses, 1% VaR for daily and 5 minute returns

```r
a <- 2.5 * max(losses)

# Exporting plot of daily losses, 1% VaR for daily and 5 minute returns
png(filename = "in_seb5_nig1.png", width = 350, height = 270,
     units = "px", pointsize = 10, bg = "white", res = NA,
     restoreConsole = TRUE)
plot(losses, type = "h", xlim = c(0, 1.05 * length(losses)), ylim = c(0, a),
     ylab = "Losses; daily and 5 minutes 1% VaR")
lines(var1_seb_nig[506:551], t = "l", col = "red")
lines(var1[1:46], t = "l", col = "green")
lines(losses, t = "h", col = "black")
legend(0.74 * length(losses), 1.03 * a, c("daily VaR", "5 min VaR"),
       lwd = 2, col = c("red", "green"), cex = 0.9)
dev.off()
```

Exporting plot of daily losses, 5% VaR for daily and 5 minute returns

```r
a <- 2.5 * max(losses)

# Exporting plot of daily losses, 5% VaR for daily and 5 minute returns
png(filename = "in_seb5_nig5.png", width = 350, height = 270,
     units = "px", pointsize = 10, bg = "white", res = NA,
     restoreConsole = TRUE)
plot(losses, type = "h", xlim = c(0, 1.05 * length(losses)), ylim = c(0, a),
     ylab = "Losses; daily and 5 minutes 5% VaR")
lines(var5_seb_nig[506:551], t = "l", col = "red")
lines(var5[1:46], t = "l", col = "green")
lines(losses, t = "h", col = "black")
legend(0.74 * length(losses), 1.03 * a, c("daily VaR", "5 min VaR"),
       lwd = 2, col = c("red", "green"), cex = 0.9)
dev.off()
```

Loading the window length for 5 minutes returns

```r
win <- win_seb5
```

Vectors for VaR estimations for 1% and 5%

```r
var1 <- c(46, NA)
var5 <- c(46, NA)
var1[1:46] <- 0
var5[1:46] <- 0
```

### VaR

```r
k = 1
for(i in seq(510, length(ret), 102)) {
```
sret <- (ret[(i-500):(i-1)]/vol[(i-500):(i-1)]);
NIGfit <- fit.NIGsv(sret, silent=T);

fitgh <- transform(NIGfit, 0, sqrt(102)*vol[i])

l1 <- qghyp(0.01, fitgh)
l5 <- qghyp(0.05, fitgh)
var1[k] <- l1
var5[k] <- l5
if(is.na(var1[k])) var1[k] <- mean(var1[500:k-1])
if(is.na(var5[k])) var5[k] <- mean(var5[500:k-1])

k = k + 1

### Exporting plot of daily losses, 1% VaR for daily and 5 minute returns
a <- 2.5*max(losses)
gg(filename = "in_seb5_np1.png", width = 350, height = 270,
units = "px", pointsize = 10, bg = "white", res = NA,
restoreConsole = TRUE)
plot(losses, type = "h", xlim = c(0, 1.05*length(losses)), ylim = c(0, a)
, ylab = "Losses; daily and 5 minutes 1% VaR")
lines(var1_seb_nig[506:551], t = "l", col = "red")
lines(var1[1:46], t = "l", col = "green")
lines(losses, t = "h", col = "black")
legend(0.74*length(losses), 1.03*a, c("daily VaR", "5 min VaR"),
lwd = 2, col = c("red", "green"), cex = 0.9)
dev.off()

### Exporting plot of daily losses, 5% VaR for daily and 5 minute returns
a <- 2.5*max(losses)
gg(filename = "in_seb5_np5.png", width = 350, height = 270,
units = "px", pointsize = 10, bg = "white", res = NA,
restoreConsole = TRUE)
plot(losses, type = "h", xlim = c(0, 1.05*length(losses)), ylim = c(0, a)
, ylab = "Losses; daily and 5 minutes 5% VaR")
lines(var5_seb_nig[506:551], t = "l", col = "red")
lines(var5[1:46], t = "l", col = "green")
lines(losses, t = "h", col = "black")
legend(0.74*length(losses), 1.03*a, c("daily VaR", "5 min VaR"),
lwd = 2, col = c("red", "green"), cex = 0.9)
dev.off()
One day ahead VaR calculations using 10 minutes stock returns of SEB.

```r
### Loading data from file
seb10 <- readSeries("SEB10.csv", header=FALSE)

### Creating vector of numeric returns
rseb10 <- returns(seb10[,1])
ret <- as.numeric(rseb10)

### Loading daily losses
losses <- losses_seb[511:551]

####################################################################
### nGARCH ####################################################################
####################################################################

### Vector of the Volatility
vol <- c(41, NA);
vol[1:41] <- 0

### Vectors for VaR estimations for 1% and 5%
var1 <- c(41, NA)
var5 <- c(41, NA)
var1[1:41] <- 0
var5[1:41] <- 0

### GARCH(1,1) volatility and standardized returns
gf <- garchFit(~ garch(1,1), ret[10:509], trace=FALSE)
sret <- ret[10:509]/gf@sigma.t

### Fit of 500 standardized returns into NIG distribution
NIGfit <- fit.NIGuv(sret, silent=T)

### VaR calculation
k=1
for(i in seq(560, length(ret), 51)){
sigmsq <- gf$fit$matcoef[2,1]+ gf$fit$matcoef[3,1]*(ret[i-51] -
gf$fit$matcoef[1,1])^2+ gf$fit$matcoef[4,1]*(gf@sigma.t[500])^2;

tmp=gf$fit$matcoef[2,1]*(1-(gf$fit$matcoef[3,1]+gf$fit$matcoef[4,1])^50)/(1-gf$fit$matcoef[3,1]-gf$fit$matcoef[4,1])+(gf$fit$matcoef[3,1]+gf$fit$matcoef[4,1])^50*
sigmsq
vol[k]=sqrt(51*tmp)
}
```

105
fitgh <- transform(NIGfit, 0, vol[k])

l1 <- qghyp(0.01, fitgh)
l5 <- qghyp(0.05, fitgh)
var1[k] = -l1
var5[k] = -l5
if(is.na(var1[k])) var1[k] = mean(var1[1:k-1])
if(is.na(var5[k])) var5[k] = mean(var5[1:k-1])

gf1 <- tryCatch(garchFit(~garch(1,1), ret[(i-499):i],
                             trace = FALSE),
                             error = function(e) gf)
gf <- gf1

sret <- ret[(i-499):i] / gf@sigma.t;
NIGfit <- fit.NIGuv(sret, silent = T);

k = k + 1

### Exporting plot of daily losses, 1% VaR for daily and 10 minute returns
a <- 2.5 * max(losses)
ng(filename = "in_seb10_n1.png", width = 350, height = 270,
     units = "px", pointsize = 10, bg = "white", res = NA,
     restoreConsole = TRUE)
plot(losses, type = "h", xlim = c(0, 1.05 * length(losses)), ylim = c(0, a),
     ylab = "Losses; daily and 10 minutes 1% VaR")
lines(var1_seb_n[511:551], t = "l", col = "red")
lines(var1[1:41], t = "l", col = "green")
lines(losses, t = "h", col = "black")
legend(0.74 * length(losses), 1.03 * a, c("daily VaR", "10 min VaR"),
       lwd = 2, col = c("red", "green"), cex = 0.9)
dev.off()

### Exporting plot of daily losses, 5% VaR for daily and 10 minute returns
a <- 2.5 * max(losses)
png(filename = "in_seb10_n5.png", width = 350, height = 270,
     units = "px", pointsize = 10, bg = "white", res = NA,
     restoreConsole = TRUE)
plot(losses, type = "h", xlim = c(0, 1.05 * length(losses)), ylim = c(0, a),
     ylab = "Losses; daily and 10 minutes 5% VaR")
lines(var5_seb_n[511:551], t = "l", col = "red")
lines(var5[1:41], t = "l", col = "green")
lines(losses, t = "h", col = "black")
legend(0.74 * length(losses), 1.03 * a, c("daily VaR", "10 min VaR"),
       lwd = 2, col = c("red", "green"), cex = 0.9)
dev.off()
VaR

dev.off()

### Vector of the Volatility
vol<-c(41,NA);
vol[1:41]<-0

### Vectors for VaR estimations for 1% and 5%
var1<-c(41,NA)
var5<-c(41,NA)
var1[1:41]<-0
var5[1:41]<-0

### GARCH(1,1) volatility and standardized returns
gf<-garchFit(~garch(1,1),ret[10:509],cond.dist="std",trace=FALSE)
sret<-ret[10:509]/gf@sigma.t

### Fit of 500 standardized returns into NIG distribution
NIGfit<-fit.NIGuv(sret,silent=T)

### VaR calculation
k=1
for(i in seq(560,length(ret),51)){
sigmsq<-gf@fit$matcoef[2,1]+gf@fit$matcoef[3,1]*(ret[i-51]-
gf@fit$matcoef[1,1])^2+gf@fit$matcoef[4,1]*(gf@sigma.t[500])^2;
tmp=gf@fit$matcoef[2,1]*(1-(gf@fit$matcoef[3,1]+gf@fit$
matcoef[4,1])^-50)/(1-gf@fit$matcoef[3,1]-gf@fit$matcoef
[4,1])*(gf@fit$matcoef[3,1]+gf@fit$matcoef[4,1])^-50*
sigmsq
vol[k]=sqrt(51*tmp)
fitgh<-transform(NIGfit,0,vol[k])
ll<-qghyp(0.01,fitgh)
l5<-qghyp(0.05,fitgh)
var1[k]=-ll
var5[k]=-l5
if(is.na(var1[k]))var1[k]=mean(var1[1:k-1])
if(is.na(var5[k]))var5[k]=mean(var5[1:k-1])
gf1 <- tryCatch(garchFit(~garch(1,1), ret[((i-499):i),
cond.dist="std", trace=FALSE),
error = function(e) gf)
gf <- gf1

sret <- ret[((i-499):i)]/gf@sigma.t;
NIGfit <- fit.NIGuv(sret, silent=T);

k <- k+1

### Exporting plot of daily losses, 1% VaR for daily and 10 minute returns
a <- 2.5 * max(losses)
png(filename = "in_seb10_t1.png", width = 350, height = 270,
 units = "px", pointsize = 10, bg = "white", res = NA,
 restoreConsole = TRUE)
plot(losses, type="h", xlim=c(0,1.05*length(losses)), ylim=c(0,a
),ylab="Losses; daily and 10 minutes 1% VaR")
lines(var1_seb_t[511:551], t="l", col="red")
lines(var1[1:41], t="l", col="green")
lines(losses, t="h", col="black")
legend(0.74*length(losses), 1.03*a, c("daily VaR", "10 min
 VaR"), lwd=2, col=c("red", "green"), cex=0.9)
dev.off()

### Exporting plot of daily losses, 5% VaR for daily and 10 minute returns
a <- 2.5 * max(losses)
png(filename = "in_seb10_t5.png", width = 350, height = 270,
 units = "px", pointsize = 10, bg = "white", res = NA,
 restoreConsole = TRUE)
plot(losses, type="h", xlim=c(0,1.05*length(losses)), ylim=c(0,a
 ),ylab="Losses; daily and 10 minutes 5% VaR")
lines(var5_seb_t[511:551], t="l", col="red")
lines(var5[1:41], t="l", col="green")
lines(losses, t="h", col="black")
legend(0.74*length(losses), 1.03*a, c("daily VaR", "10 min
 VaR"), lwd=2, col=c("red", "green"), cex=0.9)
dev.off()

###############################################################################
### NIG−GARCH
###############################################################################

### Vector of the Volatility
vol <- c(41, NA)
### Vectors for VaR estimations for 1% and 5%

```r
var1 <- c(41, NA)
var5 <- c(41, NA)
var1[1:41] <- 0
var5[1:41] <- 0
```

### GARCH(1,1) volatility and standardized returns

```r
gf <- garchFit(~garch(1,1), ret[10:509], cond.dist = "snig", trace = FALSE)
sret <- ret[10:509]/gf@sigma.t
```

### Fit of 500 standardized returns into NIG distribution

```r
NIGfit <- fit.NIGuv(sret, silent = T)
```

### VaR calculation

```r
k = 1
for (i in seq(560, length(ret), 51)){
sigmsq <- gf@fit$matcoef[2,1] + gf@fit$matcoef[3,1] * (ret[i-51] -
gf@fit$matcoef[1,1])^2 + gf@fit$matcoef[4,1] * (gf@sigma.t[500])^2;

tmp = gf@fit$matcoef[2,1] * (1 - (gf@fit$matcoef[3,1] + gf@fit$matcoef[4,1])^50) / (1 - gf@fit$matcoef[3,1] - gf@fit$matcoef[4,1])^50 *
sigmsq

vol[k] = sqrt(51 * tmp)
fitgh <- transform(NIGfit, 0, vol[k])

l1 <- qghyp(0.01, fitgh)
l5 <- qghyp(0.05, fitgh)
var1[k] = -l1
var5[k] = -l5
if (is.na(var1[k])) var1[k] = mean(var1[1:k-1])
if (is.na(var5[k])) var5[k] = mean(var5[1:k-1])

gf1 <- tryCatch(garchFit(~garch(1,1), ret[(i-499):i],
cond.dist = "snig", trace = FALSE),
error = function(e) gf)
gf <- gf1

sret <- ret[(i-499):i]/gf@sigma.t;
NIGfit <- fit.NIGuv(sret, silent = T);
k = k + 1
```
### Exporting plot of daily losses, 1% VaR for daily and 10 minute returns

```r
a <- 2.5 * max(losses)
```

```r
png(filename = "in_seb10_nig1.png", width = 350, height = 270,
    units = "px", pointsize = 10, bg = "white", res = NA,
    restoreConsole = TRUE)
plot(losses, type = "h", xlim = c(0, 1.05 * length(losses)), ylim = c(0, a
    ), ylab = "Losses; daily and 10 minutes 1% VaR")
lines(var1_seb_nig[511:551], t = "l", col = "red")
lines(var1[1:41], t = "l", col = "green")
lines(losses, t = "h", col = "black")
legend(0.74 * length(losses), 1.03 * a, c("daily VaR", "10 min
    VaR"), lwd = 2, col = c("red", "green"), cex = 0.9)
dev.off()
```

### Exporting plot of daily losses, 5% VaR for daily and 10 minute returns

```r
a <- 2.5 * max(losses)
```

```r
png(filename = "in_seb10_nig5.png", width = 350, height =
    270, units = "px", pointsize = 10, bg = "white", res = NA,
    restoreConsole = TRUE)
plot(losses, type = "h", xlim = c(0, 1.05 * length(losses)), ylim = c(0, a
    ), ylab = "Losses; daily and 10 minutes 5% VaR")
lines(var5_seb_nig[511:551], t = "l", col = "red")
lines(var5[1:41], t = "l", col = "green")
lines(losses, t = "h", col = "black")
legend(0.74 * length(losses), 1.03 * a, c("daily VaR", "10 min
    VaR"), lwd = 2, col = c("red", "green"), cex = 0.9)
dev.off()
```

### Loading the window length for 10 minutes returns

```r
win <- win_seb10
```

### Vectors for VaR estimations for 1% and 5%

```r
var1 <- c(41, NA)
var5 <- c(41, NA)
var1[1:41] <- 0
var5[1:41] <- 0
```

### VaR

```r
k = 1
for (i in seq(510, length(ret), 51)) {
```
sret <- (ret[(i-500):(i-1)]/vol[(i-500):(i-1)]);
NIGfit <- fit.NIGuv(sret, silent=T);
fitgh <- transform(NIGfit, 0, sqrt(51) * vol[i])

l1 <- qghyp(0.01, fitgh)
l5 <- qghyp(0.05, fitgh)
var1[k] <- l1
var5[k] <- l5
if(is.na(var1[k])) var1[k] <- mean(var1[500:k-1])
if(is.na(var5[k])) var5[k] <- mean(var5[500:k-1])
k <- k+1

### Exporting plot of daily losses, 1% VaR for daily and 10 minute returns
a <- 2.5 * max(losses)
ng(filename = "in_seb10_np1.png", width = 350, height = 270, units = "px", pointsize = 10, bg = "white", res = NA, restoreConsole = TRUE)
plot(losses, type="h", xlim=c(0, 1.05 * length(losses)), ylim=c(0, a), ylab="Losses; daily and 10 minutes 1% VaR")
lines(var1_seb_nig[511:551], t="l", col="red")
lines(var1[1:41], t="l", col="green")
lines(losses, t="h", col="black")
legend(0.74 * length(losses), 1.03 * a, c("daily VaR", "10 min VaR"), lwd=2, col=c("red","green"), cex=0.9)
dev.off()

### Exporting plot of daily losses, 5% VaR for daily and 10 minute returns
a <- 2.5 * max(losses)
png(filename = "in_seb10_np5.png", width = 350, height = 270, units = "px", pointsize = 10, bg = "white", res = NA, restoreConsole = TRUE)
plot(losses, type="h", xlim=c(0, 1.05 * length(losses)), ylim=c(0, a), ylab="Losses; daily and 10 minutes 5% VaR")
lines(var5_seb_nig[511:551], t="l", col="red")
lines(var5[1:41], t="l", col="green")
lines(losses, t="h", col="black")
legend(0.74 * length(losses), 1.03 * a, c("daily VaR", "10 min VaR"), lwd=2, col=c("red","green"), cex=0.9)
dev.off()
One day ahead VaR calculations using 20 minutes stock returns of SEB.

```r
### Loading data from file
seb20 <- readSeries("SEB20.csv", header=FALSE)

### Creating vector of numeric returns
rseb20 <- returns(seb20[,1])
ret <- as.numeric(rseb20)

### Loading daily losses
losses <- losses_seb[521:551]

# GARCH

### Vector of the Volatility
vol <- c(31, NA);
vol[1:31] <- 0

### Vectors for VaR estimations for 1% and 5%
var1 <- c(31, NA)
var5 <- c(31, NA)
var1[1:31] <- 0
var5[1:31] <- 0

### GARCH(1,1) volatility and standardized returns
gf <- garchFit(~ garch(1,1), ret[20:519], trace=FALSE)
sret <- ret[20:519] / gf@sigma.t

### Fit of 500 standardized returns into NIG distribution
NIGfit <- fit.NIGuv(sret, silent=T)

### VaR calculation
k=1
for(i in seq(542, length(ret), 26)){
  sigmsq <- gf@fit$matcoef[2,1] + gf@fit$matcoef[3,1] * (ret[i-26] -
  gf@fit$matcoef[1,1])^2 + gf@fit$matcoef[4,1] * (gf@sigma.t
  [500])^2;
  tmp = gf@fit$matcoef[2,1] * (1-(gf@fit$matcoef[3,1]+gf@fit$
  matcoef[4,1])^25) / (1-gf@fit$matcoef[3,1]-gf@fit$matcoef
  [4,1]) + (gf@fit$matcoef[3,1]+gf@fit$matcoef[4,1])^25 *
  sigmsq
  vol[k] = sqrt(26*tmp)
}
```

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fitgh <- transform(NIGfit, 0, vol[k])

l1 <- qghyp(0.01, fitgh)
l5 <- qghyp(0.05, fitgh)
var1[k] = -l1
var5[k] = -l5
if(is.na(var1[k])) var1[k] = mean(var1[1:k-1])
if(is.na(var5[k])) var5[k] = mean(var5[1:k-1])

gf1 <- tryCatch(garchFit(~garch(1, 1), ret[(i-499):i],
  trace = FALSE),
  error = function(e) gf)
gf <- gf1
sret <- ret[(i-499):i]/gf@sigma.t;
NIGfit <- fit.NIGuv(sret, silent = T);

k = k + 1}

### Exporting plot of daily losses, 1% VaR for daily and 20 minute returns
a <- 2.5 * max(losses)
ng(filename = "in_seb20_n1.png", width = 350, height = 270,
  units = "px", pointsize = 10, bg = "white", res = NA,
  restoreConsole = TRUE)
plot(losses, type = "h", xlim = c(0, 1.05 * length(losses)), ylim = c(0, a),
  ylab = "Losses; daily and 20 minutes 1% VaR")
lines(var1_seb_n[521:551], t = "l", col = "red")
lines(var1[1:31], t = "l", col = "green")
lines(losses, t = "h", col = "black")
legend(0.74 * length(losses), 1.03 * a, c("daily VaR", "20 min VaR"),
  lwd = 2, col = c("red", "green"), cex = 0.9)
dev.off()

### Exporting plot of daily losses, 5% VaR for daily and 20 minute returns
a <- 2.5 * max(losses)
png(filename = "in_seb20_n5.png", width = 350, height = 270,
  units = "px", pointsize = 10, bg = "white", res = NA,
  restoreConsole = TRUE)
plot(losses, type = "h", xlim = c(0, 1.05 * length(losses)), ylim = c(0, a),
  ylab = "Losses; daily and 20 minutes 5% VaR")
lines(var5_seb_n[521:551], t = "l", col = "red")
lines(var5[1:31], t = "l", col = "green")
lines(losses, t = "h", col = "black")
legend(0.74 * length(losses), 1.03 * a, c("daily VaR", "20 min VaR"),
  lwd = 2, col = c("red", "green"), cex = 0.9)
dev.off()
VaR"), lwd=2, col=c("red", "green"), cex=0.9)
dev.off()

### Vector of the Volatility
vol <- c(31, NA);
vol[1:31] <- 0

### Vectors for VaR estimations for 1% and 5%
var1 <- c(31, NA)
var5 <- c(31, NA)
var1[1:31] <- 0
var5[1:31] <- 0

### GARCH(1,1) volatility and standardized returns
gf <- garchFit(~garch(1,1), ret[20:519], cond.dist="std", trace = FALSE)
sret <- ret[20:519]/gf@sigma.t

### Fit of 500 standardized returns into NIG distribution
NIGfit <- fit.NIGuv(sret, silent=T)

### VaR calculation
k=1
for(i in seq(542, length(ret), 26)){
  sigmsq <- gf$fit$matcoef[2,1] + gf$fit$matcoef[3,1]* (ret[i-26] -
  gf$fit$matcoef[1,1])^2 + gf$fit$matcoef[4,1]* (gf@sigma.t
  [500])^2;
  tmp = gf$fit$matcoef[2,1]* (1 - (gf$fit$matcoef[3,1] + gf$fit$
    matcoef[4,1])^25) / (1 - gf$fit$matcoef[3,1] - gf$fit$matcoef
    [4,1]) + (gf$fit$matcoef[3,1] + gf$fit$matcoef[4,1])^25 *
  sigmsq
  vol[k] = sqrt(26 * tmp)
  fitgh <- transform(NIGfit, 0, vol[k])
  l1 <- qghyp(0.01, fitgh)
  l5 <- qghyp(0.05, fitgh)
  var1[k] <- -l1
  var5[k] <- -l5
  if(is.na(var1[k])) var1[k] = mean(var1[1:k-1])
  if(is.na(var5[k])) var5[k] = mean(var5[1:k-1])

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gf1 <- tryCatch(garchFit(~garch(1,1), ret[(i-499):i],
    cond.dist="std", trace=FALSE),
    error = function(e) gf)
gf <- gf1

sret <- ret[(i-499):i]/gf@sigma.t;
NIGfit <- fit.NIGuv(sret, silent=T);

k = k + 1

### Exporting plot of daily losses, 1% VaR for daily and 20 minute returns

a <- 2.5 * max(losses)

ng(filename = "in_seb20_t1.png", width = 350, height = 270,
    units = "px", pointsize = 10, bg = "white", res = NA,
    restoreConsole = TRUE)

plot(losses, type="h", xlim = c(0, 1.05 * length(losses)), ylim = c(0, a),
    ylab = "Losses; daily and 20 minutes 1% VaR")

lines(var1_seb_t[521:551], t="l", col="red")
lines(var1[1:31], t="l", col="green")
lines(losses, t="h", col="black")

legend(0.74 * length(losses), 1.03 * a, c("daily VaR", "20 min VaR"),
    lwd = 2, col = c("red", "green"), cex = 0.9)

dev.off()

### Exporting plot of daily losses, 5% VaR for daily and 20 minute returns

a <- 2.5 * max(losses)

png(filename = "in_seb20_t5.png", width = 350, height = 270,
    units = "px", pointsize = 10, bg = "white", res = NA,
    restoreConsole = TRUE)

plot(losses, type="h", xlim = c(0, 1.05 * length(losses)), ylim = c(0, a),
    ylab = "Losses; daily and 20 minutes 5% VaR")

lines(var5_seb_t[521:551], t="l", col="red")
lines(var5[1:31], t="l", col="green")
lines(losses, t="h", col="black")

legend(0.74 * length(losses), 1.03 * a, c("daily VaR", "20 min VaR"),
    lwd = 2, col = c("red", "green"), cex = 0.9)

dev.off()

####################################################################

### NIG-GARCH

####################################################################

### Vector of the Volatility

vol <- c(31, NA);
vol[1:31]<-0

### Vectors for VaR estimations for 1% and 5%
var1<-c(31,NA)
var5<-c(31,NA)
var1[1:31]<-0
var5[1:31]<-0

### GARCH(1,1) volatility and standardized returns
gf<-garchFit(~garch(1,1),ret[20:519],cond.dist="snig",trace=FALSE)
sret<-ret[20:519]/gf@sigma.t

### Fit of 500 standardized returns into NIG distribution
NIGfit<-fit.NIGuv(sret,silent=T)

### VaR calculation
k=1
for(i in seq(542,length(ret),26)){
sigma<-(gf$fit$matcoef[2,1]+gf$fit$matcoef[3,1]*(ret[i-26]-
gf$fit$matcoef[1,1])^2+gf$fit$matcoef[4,1]*(gf@sigma.t[500])^2;
tmp=gf$fit$matcoef[2,1]*(1-(gf$fit$matcoef[3,1]+gf$fit$matcoef[4,1])^25)/(1-gf$fit$matcoef[3,1]-gf$fit$matcoef [4,1])+(gf$fit$matcoef[3,1]+gf$fit$matcoef[4,1])^25*sigma
vol[k]=sqrt(26*tmp)
fitgh<-transform(NIGfit,0,vol[k])
}

l1<-qghyp(0.01,fitgh)
l5<-qghyp(0.05,fitgh)
var1[k]<-l1
var5[k]<-l5
if(is.na(var1[k]))var1[k]=mean(var1[1:k-1])
if(is.na(var5[k]))var5[k]=mean(var5[1:k-1])

gf1<-tryCatch(garchFit(~garch(1,1),ret[((i-499):i)],
    cond.dist="snig",trace=FALSE),
    error = function(e) gf)
gf<-gf1
sret<-ret[((i-499):i)/gf@sigma.t;
NIGfit<-fit.NIGuv(sret,silent=T);

k=k+1
### Exporting plot of daily losses, 1% VaR for daily and 20 minute returns

```r
a <- 2.5 * max(losses)
png(filename = "in_seb20_nig1.png", width = 350, height = 270,
    units = "px", pointsize = 10, bg = "white", res = NA,
    restoreConsole = TRUE)
plot(losses, type = "h", xlim = c(0, 1.05 * length(losses)), ylim = c(0, a),
    ylab = "Losses; daily and 20 minutes 1% VaR")
lines(var1_seb_nig[521:551], t = "l", col = "red")
lines(var1[1:31], t = "l", col = "green")
lines(losses, t = "h", col = "black")
legend(0.74 * length(losses), 1.03 * a, c("daily VaR", "20 min VaR"),
    lwd = 2, col = c("red", "green"), cex = 0.9)
dev.off()
```

### Exporting plot of daily losses, 5% VaR for daily and 20 minute returns

```r
a <- 2.5 * max(losses)
png(filename = "in_seb20_nig5.png", width = 350, height = 270,
    units = "px", pointsize = 10, bg = "white", res = NA,
    restoreConsole = TRUE)
plot(losses, type = "h", xlim = c(0, 1.05 * length(losses)), ylim = c(0, a),
    ylab = "Losses; daily and 20 minutes 5% VaR")
lines(var5_seb_nig[521:551], t = "l", col = "red")
lines(var5[1:31], t = "l", col = "green")
lines(losses, t = "h", col = "black")
legend(0.74 * length(losses), 1.03 * a, c("daily VaR", "20 min VaR"),
    lwd = 2, col = c("red", "green"), cex = 0.9)
dev.off()
```

### Loading the window length for 20 minutes returns

```r
win <- win_seb20
```

### Vectors for VaR estimations for 1% and 5%

```r
var1 <- c(31, NA)
var5 <- c(31, NA)
var1[1:31] <- 0
var5[1:31] <- 0
```

### VaR

```r
k = 1
for (i in seq(520, length(ret), 26)) {
```
sret <- (ret[(i-500):(i-1)]/vol[(i-500):(i-1)]);
NIGfit <- fit.NIGuv(sret, silent=T);

fitgh <- transform(NIGfit, 0, sqrt(26)*vol[i])
l1 <- qghyp(0.01, fitgh)
l5 <- qghyp(0.05, fitgh)
var1[k] <- l1
var5[k] <- l5
if(is.na(var1[k])) var1[k] <- mean(var1[500:k-1])
if(is.na(var5[k])) var5[k] <- mean(var5[500:k-1])

k=k+1
}

### Exporting plot of daily losses, 1% VaR for daily and 20 minute returns
a <- 2.5*max(losses)
png(filename = "in_seb20_np1.png", width = 350, height = 270, units = "px", pointsize = 10, bg = "white", res = NA, restoreConsole = TRUE)
plot(losses, type = "h", xlim = c(0, 1.05*length(losses)), ylim = c(0, a), ylab = "Losses; daily and 20 minutes 1% VaR")
lines(var1_seb_np[521:551], t="l", col="red")
lines(var1, t="l", col="green")
lines(losses, t="h", col="black")
legend(0.74*length(losses), 1.03*a, c("daily VaR", "20 min VaR"), lwd=2, col=c("red", "green"), cex=0.9)
dev.off()

### Exporting plot of daily losses, 5% VaR for daily and 20 minute returns
a <- 2.5*max(losses)
png(filename = "in_seb20_np5.png", width = 350, height = 270, units = "px", pointsize = 10, bg = "white", res = NA, restoreConsole = TRUE)
plot(losses, type = "h", xlim = c(0, 1.05*length(losses)), ylim = c(0, a), ylab = "Losses; daily and 20 minutes 5% VaR")
lines(var5_seb_np[521:551], t="l", col="red")
lines(var5, t="l", col="green")
lines(losses, t="h", col="black")
legend(0.74*length(losses), 1.03*a, c("daily VaR", "20 min VaR"), lwd=2, col=c("red", "green"), cex=0.9)
dev.off()
One day ahead VaR calculations using 30 minutes stock returns of SEB.

```r
### Loading data from file
seb30 <- readSeries("SEB30.csv", header=FALSE)

### Creating vector of numeric returns
rseb30 <- returns(seb30[,1])
ret <- as.numeric(rseb30)

### Loading daily losses
losses <- losses_seb[531:551]

# Vector of the Volatility
vol <- c(21, NA);
vol[1:21] <- 0

### Vectors for VaR estimations for 1% and 5%
var1 <- c(21, NA)
var5 <- c(21, NA)
var1[1:21] <- 0
var5[1:21] <- 0

### GARCH(1,1) volatility and standardized returns
gf <- garchFit(~garch(1,1), ret[10:509], trace=FALSE)
sret <- ret[10:509]/gf@sigma.t

### Fit of 500 standardized returns into NIG distribution
NIGfit <- fit.NIGuv(sret, silent=TRUE)

### VaR calculation
k=1
for(i in seq(526, length(ret), 17)){
sigmsq <- gf@fit$matcoef[2,1]+gf@fit$matcoef[3,1]*(ret[i-17]-
gf@fit$matcoef[1,1])^2+gf@fit$matcoef[4,1]*(gf@sigma.t[500])^2;

tmp = gf@fit$matcoef[2,1]*(1-(gf@fit$matcoef[3,1]+gf@fit$matcoef[4,1])^16)/(1-gf@fit$matcoef[3,1]-gf@fit$matcoef[4,1])+(gf@fit$matcoef[3,1]+gf@fit$matcoef[4,1])^16*
sigmsq

vol[k] = sqrt(17*tmp)
}
```
fitgh <- transform(NIGfit, 0, vol[k])

l1 <- qghyp(0.01, fitgh)
l5 <- qghyp(0.05, fitgh)
var1[k] <- -l1
var5[k] <- -l5
if(is.na(var1[k])) var1[k] <- mean(var1[1:k-1])
if(is.na(var5[k])) var5[k] <- mean(var5[1:k-1])

gf1 <- tryCatch(garchFit(~garch(1,1), ret[(i-499):i],
                           trace = FALSE),
                           error = function(e) gf)
gf <- gf1

sret <- ret[(i-499):i]/gf@sigma.t;
NIGfit <- fit.NIGuv(sret, silent=T);

k <- k+1
}

### Exporting plot of daily losses, 1% VaR for daily and 30 minute returns
a <- 2.5 * max(losses)
ng(filename = "in_seb30_n1.png", width = 350, height = 270,
    units = "px", pointsize = 10, bg = "white", res = NA,
    restoreConsole = TRUE)
plot(losses, type="h", xlim=c(0, 1.05*length(losses)), ylim=c(0, a),
     ylab="Losses; daily and 30 minutes 1% VaR")
lines(var1_seb_n[531:551], t="l", col="red")
lines(var1[1:21], t="l", col="green")
lines(losses, t="h", col="black")
legend(0.74*length(losses), 1.03*a, c("daily VaR", "30 min VaR"),
       lwd=2, col=c("red", "green"), cex=0.9)
dev.off()

### Exporting plot of daily losses, 5% VaR for daily and 30 minute returns
a <- 2.5 * max(losses)
png(filename = "in_seb30_n5.png", width = 350, height = 270,
    units = "px", pointsize = 10, bg = "white", res = NA,
    restoreConsole = TRUE)
plot(losses, type="h", xlim=c(0, 1.05*length(losses)), ylim=c(0, a),
     ylab="Losses; daily and 30 minutes 5% VaR")
lines(var5_seb_n[531:551], t="l", col="red")
lines(var5[1:21], t="l", col="green")
lines(losses, t="h", col="black")
legend(0.74*length(losses), 1.03*a, c("daily VaR", "30 min VaR"),
       lwd=2, col=c("red", "green"), cex=0.9)
dev.off()
VaR")}, lwd=2,col=c("red","green"),cex=0.9)
dev.off()

# GARCH

### Vector of the Volatility
vol<-c(21,NA);
vol[1:21]<-0

### Vectors for VaR estimations for 1% and 5%
var1<-c(21,NA)
var5<-c(21,NA)
var1[1:21]<-0
var5[1:21]<-0

### GARCH(1,1) volatility and standardized returns
gf<-garchFit(~garch(1,1),ret[10:509],cond.dist="std",trace=FALSE)
sret<-ret[10:509]/gf@sigma.t

### Fit of 500 standardized returns into NIG distribution
NIGfit<-fit.NIGuv(sret,silent=T)

### VaR calculation
k=1
for(i in seq(526,length(ret),17)){
  sigmsq<-gf@fit$matcoef[2,1]+gf@fit$matcoef[3,1]*(ret[i-17]-
    gf@fit$matcoef[1,1])^2+gf@fit$matcoef[4,1]*(gf@sigma.t[500])^2;
  tmp=gf@fit$matcoef[2,1]*(1-(gf@fit$matcoef[3,1]+gf@fit$
    matcoef[4,1])^16)/(1-gf@fit$matcoef[3,1]-gf@fit$matcoef
    [4,1])+(gf@fit$matcoef[3,1]+gf@fit$matcoef[4,1])^16*
    sigmsq
  vol[k]=sqrt(17*tmp)
  fitgh<-transform(NIGfit,0,vol[k])
  l1<-qghyp(0.01,fitgh)
  l5<-qghyp(0.05,fitgh)
  var1[k]=-l1
  var5[k]=-l5
  if(is.na(var1[k]))var1[k]=mean(var1[1:k-1])
  if(is.na(var5[k]))var5[k]=mean(var5[1:k-1])

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gf1 <- tryCatch (garchFit(~garch(1,1),ret[(i-499):i],
cond.dist="std",trace=FALSE),
error = function(e) gf)
gf<-gf1
sret<-ret[(i-499):i]/gf@sigma.t;
NIGfit<-fit.NIGuv(sret,silent=T);

t=1
k=k+1
}

### Exporting plot of daily losses, 1% VaR for daily and 30 minute returns
a<-2.5*max(losses)
gg(filename = "in_seb30_t1.png", width = 350, height = 270,
units = "px", pointsize = 10, bg = "white", res = NA,
restoreConsole = TRUE)
plot(losses,type="h",xlim=c(0,1.05*length(losses)),ylim=c(0,a
),ylab="Losses; daily and 30 minutes 1% VaR")
lines(var1_seb_t[531:551],t="l",col="red")
lines(var1[1:21],t="l",col="green")
lines(losses,t="h",col="black")
legend(0.74*length(losses), 1.03*a, c("daily VaR", "30 min
VaR"), lwd=2,col=c("red","green"),cex=0.9)
dev.off()

### Exporting plot of daily losses, 5% VaR for daily and 30 minute returns
a<-2.5*max(losses)
gg(filename = "in_seb30_t5.png", width = 350, height = 270,
units = "px", pointsize = 10, bg = "white", res = NA,
restoreConsole = TRUE)
plot(losses,type="h",xlim=c(0,1.05*length(losses)),ylim=c(0,a
),ylab="Losses; daily and 30 minutes 5% VaR")
lines(var5_seb_t[531:551],t="l",col="red")
lines(var5[1:21],t="l",col="green")
lines(losses,t="h",col="black")
legend(0.74*length(losses), 1.03*a, c("daily VaR", "30 min
VaR"), lwd=2,col=c("red","green"),cex=0.9)
dev.off()

# Vector of the Volatility
vol<-c(21,NA);
### Vectors for VaR estimations for 1% and 5%

```r
var1 <- c(21, NA)
var5 <- c(21, NA)
var1[1:21] <- 0
var5[1:21] <- 0
```

### GARCH(1,1) volatility and standardized returns

```r
gf <- garchFit(~garch(1,1), ret[10:509], cond.dist="snig", trace = FALSE)
sret <- ret[10:509]/gf@sigma.t
```

### Fit of 500 standardized returns into NIG distribution

```r
NIGfit <- fit.NIGuv(sret, silent = T)
```

### VaR calculation

```r
k = 1
for (i in seq(526, length(ret), 17)) {
  sigmsq <- gf@fit$matcoef[2,1] + gf@fit$matcoef[3,1]*(ret[i-17] -
    gf@fit$matcoef[1,1])^2 + gf@fit$matcoef[4,1]*(gf@sigma.t[500])^2;
  tmp = gf@fit$matcoef[2,1]*(1-(gf@fit$matcoef[3,1]+gf@fit$matcoef
    [4,1])^16)/(1-gf@fit$matcoef[3,1]-gf@fit$matcoef
    [4,1])*(gf@fit$matcoef[3,1]+gf@fit$matcoef[4,1])^16*
    sigmsq
  vol[k] = sqrt(17*tmp)
  fitgh <- transform(NIGfit, 0, vol[k])
  l1 <- qghyp(0.01, fitgh)
  l5 <- qghyp(0.05, fitgh)
  var1[k] = -l1
  var5[k] = -l5
  if(is.na(var1[k])) var1[k] = mean(var1[1:k-1])
  if(is.na(var5[k])) var5[k] = mean(var5[1:k-1])
}
gf1 <- tryCatch(garchFit(~garch(1,1), ret[(i-499):i],
    cond.dist="snig", trace = FALSE),
    error = function(e) gf)
gf <- gf1
sret <- ret[(i-499):i]/gf@sigma.t;
NIGfit <- fit.NIGuv(sret, silent = T);
k = k + 1
```
Exporting plot of daily losses, 1% VaR for daily and 30 minute returns

```r
a <- -2.5 * max(losses)
png(filename = "in_seb30_nig1.png", width = 350, height = 270,
    units = "px", pointsize = 10, bg = "white", res = NA,
    restoreConsole = TRUE)
plot(losses, type = "h", xlim = c(0, 1.05 * length(losses)), ylim = c(0, a ),
    ylab = "Losses; daily and 30 minutes 1% VaR")
lines(var1_seb_nig[531:551], t = "l", col = "red")
lines(var1[1:21], t = "l", col = "green")
lines(losses, t = "h", col = "black")
legend(0.74 * length(losses), 1.03 * a, c("daily VaR", "30 min VaR"),
    lwd = 2, col = c("red", "green"), cex = 0.9)
dev.off()
```

Exporting plot of daily losses, 5% VaR for daily and 30 minute returns

```r
a <- -2.5 * max(losses)
png(filename = "in_seb30_nig5.png", width = 350, height = 270,
    units = "px", pointsize = 10, bg = "white", res = NA,
    restoreConsole = TRUE)
plot(losses, type = "h", xlim = c(0, 1.05 * length(losses)), ylim = c(0, a ),
    ylab = "Losses; daily and 30 minutes 5% VaR")
lines(var5_seb_nig[531:551], t = "l", col = "red")
lines(var5[1:21], t = "l", col = "green")
lines(losses, t = "h", col = "black")
legend(0.74 * length(losses), 1.03 * a, c("daily VaR", "30 min VaR"),
    lwd = 2, col = c("red", "green"), cex = 0.9)
dev.off()
```

### NON-PARAMETRIC

Loading the window length for 30 minutes returns

```r
win <- win_seb30
```

Vectors for VaR estimations for 1% and 5%

```r
var1 <- c(21, NA)
var5 <- c(21, NA)
var1[1:21] <- 0
var5[1:21] <- 0
```

### VaR

```r
k = 1
for(i in seq(510, length(ret), 17)) {
```
sret<-(ret[(i-500):(i-1)])/(vol[(i-500):(i-1)]);
NIGfit<-fit.NIGuv(sret,silent=T);

fitgh<-transform(NIGfit,0,sqrt(17)*vol[i])

l1<-qghyp(0.01,fitgh)
l5<-qghyp(0.05,fitgh)
var1[k]=-l1
var5[k]=-l5
if(is.na(var1[k])) var1[k]<-mean(var1[500:k-1])
if(is.na(var5[k])) var5[k]<-mean(var5[500:k-1])
k=k+1

### Exporting plot of daily losses, 1% VaR for daily and 30 minute returns
a<-2.5*max(losses)
png(filename = "in_seb30_np1.png", width = 350, height = 270,
    units = "px", pointsize = 10, bg = "white", res = NA,
    restoreConsole = TRUE)
plot(losses,type="h",xlim=c(0,1.05*length(losses)),ylim=c(0,a
    ),ylab="Losses; daily and 30 minutes 1% VaR")
lines(var1_seb_np[531:551],t="l",col="red")
lines(var1,t="l",col="green")
lines(losses,t="h",col="black")
legend(0.74*length(losses), 1.03*a, c("daily VaR", "30 min
    VaR"), lwd=2,col=c("red","green"),cex=0.9)
dev.off()

### Exporting plot of daily losses, 5% VaR for daily and 30 minute returns
a<-2.5*max(losses)
png(filename = "in_seb30_np5.png", width = 350, height = 270,
    units = "px", pointsize = 10, bg = "white", res = NA,
    restoreConsole = TRUE)
plot(losses,type="h",xlim=c(0,1.05*length(losses)),ylim=c(0,a
    ),ylab="Losses; daily and 30 minutes 5% VaR")
lines(var5_seb_np[531:551],t="l",col="red")
lines(var5,t="l",col="green")
lines(losses,t="h",col="black")
legend(0.74*length(losses), 1.03*a, c("daily VaR", "30 min
    VaR"), lwd=2,col=c("red","green"),cex=0.9)
dev.off()