An evaluation of subjective logic for trust modelling in information fusion

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I hereby certify that all material in this dissertation which is not my own work has been identified and that no work is included for which a degree has already been conferred on me.

Signed: ________________________________
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Abstract

Information fusion is to combine information from a variety of sources, or sensors. When the sources are uncertain or contradicting conflict can arise. To deal with such uncertainty and conflict a trust model can be used. The most common ones in information fusion currently is bayesian theory and Dempster-Shafer theory. Bayesian theory does not explicitly handle ignorance, and thus predetermined values has to be hard coded into the system. This is solved in Dempster-Shafer theory by the introduction of ignorance. Even though Dempster-Shafer theory is widely used in information fusion when there is a need for ignorance to be modelled, there has been serious critique presented towards the theory. Thus this work aims at examining another trust models utility in information fusion namely subjective logic. The examination is executed by studying subjective logic using two scenarios from the literature. The results from the scenarios points to subjective logic being a reasonable approach for modelling trust in information fusion.

Keywords: Information fusion, trust model, subjective logic.
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1 Introduction

Appriou et al. (2001, p. 1109) present an informal definition of information fusion as:

“[Information] fusion consists in conjoining or merging information that stems from several sources and exploiting that conjoined or merged information in various tasks such as answering questions, making decisions, numerical estimation, etc.”

The definition has two main ingredients, first it focuses on the combination of information, second it emphasises the aims of the fusion. In the situations where information fusion is used, according to Walts and Llinas (1990), the inherent risks, time pressure and large volume of data have led to the need for computerised aids performing automated information fusion.

When combining information from uncertain and possibly contradictory sources, it has shown to be a good idea to present the uncertainty and contradictions to the receiver of the fusioned view, i.e. the decision maker (see Andre & Cutler 1998, and Bisantz et al. 1999). A trust model can be used to model uncertainty in information through out the fusion process to be able to track contradictions and uncertainty from the sensors.

To trust an agent is, according to Gambetta (2000) to believe that it will perform in a way that we will gain from, or at least that will not not harm us or our goal, that is to believe that if it is assigned a task it will perform the task in a way that will help our interests. He also states that the opposite, to distrust, is to believe that it will perform in a way that will be harmful for our goal, or even deceive us.

There are several types of models for modeling trust relationships, called trust models. The most common ones currently used in information fusion are bayesian theory and Dempster-Shafer theory. Bayesian theory is the probability theories normally used by statisticians that evaluates probabilities for binary outcomes, either true or false, based on statistical probability theories. Dempster-Shafer theory is a newer theory that adds some important concepts and allows for modeling situations where the outcomes are not limited to binary outcomes but can be defined as a set of hypotheses. Bayesian theory does not have ignorance, that is the lack of knowledge, explicitly in its model and thus is not a good model if this is required. The difference between ignorance and uncertainty is further described in section 2.2.

Dempster-Shafer theory of evidences (simply called Dempster-Shafer theory hence forth) is the most common used in information fusion when ignorance is required to be explicit. But Dempster-Shafer theory has been criticised for not always producing intuitive results (Zadeh 1984 and Cohen 1986),
and it has even been stated that Dempsters’ consensus rule is fundamentally flawed (Jøsang 1998).

In his PhD thesis Audung Jøsang (1998) presents a fundamentally new trust model which he calls subjective logic. It is as the name suggests a logic system. Jøsang claims to have a solid base for his theories in bayesian theory and thus that subjective logic does not have the flaws of Dempster-Shafer theory. As subjective logic has its base in bayesian theory it too models binary events.

The alleged flaws of Dempster-Shafer theory and the claimed sound mathematical basis of subjective logic, leads us to believe that information fusion could gain from using the subjective logic model, instead of relying on the Dempster-Shafer model.

1.1 Aim

The aim of this work is to evaluate the utility of subjective logic in information fusion. With utility is meant that it is reasonable, i.e. has the properties that are needed from a trust model for information fusion.

To do such an evaluation two research questions are posed.

<table>
<thead>
<tr>
<th>Research question 1</th>
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<tr>
<td>Does subjective logic have the properties which are desired from a trust model by information fusion applications?</td>
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<tr>
<th>Research question 2</th>
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<tbody>
<tr>
<td>Does subjective logic provide any advantages over Dempster-Shafer theory as a trust model in information fusion?</td>
</tr>
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</table>

To be able to formulate an answer to these two research questions two objectives are evaluated, from the results of the objectives a discussion about the answers to the research questions is given.

1.2 Objectives

1.2.1 Desired properties for a trust models use in information fusion applications

In order to get an idea what is desirable from a trust model to be useful in information fusion, a list of desired properties for the trust models use in information fusion is presented. The desired properties are based on a literature study on the use of trust models in information fusion, and a
discussion based on that study. The desired properties can be used as a metric in order to evaluate a trust models utility in information fusion.

1.2.2 Information fusion scenarios for evaluation of trust models

In order to evaluate whether subjective logic has the desired properties and how well their fulfillment in subjective logic compares to that of Dempster-Shafer theory, two scenarios where trust models are used within information fusion are elaborated on and executed as experiments.
2 Background

Information fusion is the activity of combining information from different sources to get one view of a situation from data originating from a number of sources. When sources are uncertain or contradict one another, the problem of combining the information becomes more complex. To handle the uncertainties and conflicts, a trust model is used. Here follows a description of information fusion, followed by a definition of trust and opinions together with some other important concepts for trust. Then, a brief description of some trust models is given.

2.1 Information fusion

Two terms commonly used in the literature are data fusion and information fusion, which are more or less used interchangeably and there is no accepted definition of the difference between them. For the rest of this report the term information fusion is used.

In military context, information fusion has been identified as a means to perform assessments of identities, situations, and threat potential based on information derived from multiple electronic and intelligence sources (Bisantz et al. 1999).

"Current attempts to bring more information to commanders are doomed to failure due to cognitive overload" (Yu et al. 2004)

Yu et al. (2004) make this statement from a military view, but it is equally true in a civilian context, pointing out the need for information fusion and especially automated information fusion. They also claim that it is impossible to analyse all data manually, which holds true in many information systems today. Appriou et al. (2001) mention querying multiple databases, robot localisation, and stock index analysis as examples. Those examples also strengthen the claim by Yu et al. (2004), that mechanisms to allow modelling and assessing dynamic situations based on data from multiple sources is required. Yu et al. (2004) present information fusion as one possible solution to this problem, Appriou et al. (2001) extend this by further classifying different types of information fusion, based on different aims of the fusion and different types of information.

The Joint Directors of Laboratories (JDL) model presents a view of information fusion presenting four levels, or layers, of information fusion. Two additional layers have later been added. The JDL model is presented with the additional layers in Llinas et al. (2004). This is a military view of information fusion which according to The Data Fusion Server (2005) is not
2.1 Information fusion

universally applicable in the civil domain. The six (0–5) levels, also seen in figure 1 are:

**Level 0** *Preprocessing*, this level is sometimes ignored and viewed as being outside the information fusion process. On this level the data is preprocessed to be used in the fusion, this is often done by the sensor itself.

**Level 1** *object refinement*. On this level objects are identified and associated with properties.

**Level 2** *situation refinement*. On this level the situation is refined, that is to collect objects into clusters, such as for example battalions and platoons.

**Level 3** *threat refinement*. On this level simulation is conducted in order to evaluate what might happen in the future, and what threat the objects may pose.

**Level 4** *process refinement*, can be seen as a meta level where the process of the information fusion itself is refined.

**Level 5** *Cognitive refinement* aims at presenting the result form the information fusion. It is seen as an interface level as it is the output of the information fusion.

Llinas et al. (2004) state that the JDL model should not be used as an architecture for information fusion, more as a pedagogical tool used to divide the research area. They claim that the actual issue is much more complex than the JDL model shows it to be.

The database management system is not seen as a level in the JDL model, it is used by all levels to handle the data needs.

According to Yu et al. (2004) sensor networks are emerging as a new trend in information technology for monitoring information in both military and non-military applications. They describe a number of military applications mainly containing a type of radar technology for target tracking.

When discussing information fusion in this work it will mainly be the type of information fusion used in sensor networks, which is when we fuse sensor information in order to get a view of the current situation, thus it can be said to be mainly on level 2 of the JDL model. This special case of information fusion is called *sensor fusion*, but the term information fusion will be used for the rest of this report. It is important to state that there are many other areas of information fusion and that even the types of trust
2.2 Trust and opinions

As social agents humans deal with trust every day. And according to Gambetta (2000) trust is required for any kind of cooperation, and the rationale for trusting someone is the need to cooperate. More formally he defines trust as:

“trust (or, symmetrically, distrust) is a particular level of the subjective probability with which an agent assesses that another agent or group of agents will perform a particular action, both before he can monitor such action (or independently of his capacity ever to be able to monitor it) and in a context in which it affects his own action” (Gambetta 2000, p.217)

This means that if there is a notion of trust, there is also the notion of distrust as the opposite of trust. To distrust is not the same as having a lack of knowledge whether to trust or not, it is to know not to trust. To have a lack of knowledge whether to trust or distrust is ignorance. Trust or distrust are extremes of a continuous scale. The scale depends on the trust model and the distance of the value from the actual extremes is called uncertainty. Thus the further from blind trust and blind distrust an opinion is the higher
2.3 Trust models

the uncertainty. This can be explained by looking at the information value of the trust. If we are far from knowing what outcome to expect then we have high uncertainty, but that does not necessarily mean we have ignorance. So uncertainty is an aspect of the output probabilities and ignorance is an aspect of the input evidences.

Jøsang (1998) has a more computer scientific approach and defines trust from an IT security point of view as the belief that an IT system will resist malicious attacks, and trust in a human agent as the belief that the agent will cooperate and not behave maliciously. If taken in a wider sense this definition is compatible to the one given by Gambetta, and belief is thus defined as: the subjective probability that a particular action will be preformed, before it can be monitored, or independently of the ability to ever monitor it.

Beliefs are subjective, that is they do not contain truth in a traditional sense, they only contain a subjective or experienced probability, and they focus on what we have evidence to support not what the actual outcome is. A belief is held by an agent. The term opinion is used to denote such subjective beliefs held by an agent. An opinion is not shared by agents, they might have the same amount of belief in the same thing but they each have their own opinion.

2.3 Trust models

A trust algebra is an algebra that allows one to build expressions of how to model trust, that is the rules to build a trust model. There are a number of trust algebras in use and some of the most common in information fusion are bayesian theory and Dempster-Shafer theory. These two models are compared from a practical view by Hoffman and Murphy (1993), they argue that bayesian theory is best suited for applications where there is no need to represent ignorance, and where prior probabilities of the outcomes are available. They also argue that Dempster-Shafer theory is a good model to use in information fusion applications. Haenni and Lehmann (2003) strengthen the argument of why ignorance is needed to be explicit, but also mentions the negative aspects that theories using ignorance introduces problems to a practical implementation, such as a high computational complexity.

There are some critiques presented to Dempster-Shafer theory for sometimes producing counter intuitive results (Zadeh 1984, and Cohen 1986) and even being fundamentally flawed (Jøsang 1998), for a discussion on more trust models and their pros and cons see Jøsang (1998).

Another trust algebra is subjective logic (Josang 1998). It is partly based on Dempster-Shafer theory and it aims at not having the problems with Dempster-Shafer theory. Subjective logic has been applied to e-commerce
and security applications (see for example Jøsang & Tran 2000, and Håkansson 2004). It too has the inherent abilities of Dempster-Shafer theory argued for by Hoffman & Murphy (1993), this is further elaborated on in section 4.1.

The most relevant rule in trust algebras for information fusion applications is the rule of combination, also called consensus.

The focus of this work is on Dempster-Shafer theory and subjective logic. Thus a brief introduction to these theories follows, with a focus on their respective rules for combination.

2.4 Dempster-Shafer theory

Dempster-Shafer theory is presented in Shafer (1976).

Dempster-Shafer theory is not limited to binary outcomes of events but enables to model a set of hypotheses about the outcome. The set of hypotheses is mutually exclusive and exhaustive, i.e. one and only one of the hypotheses is the outcome of a given event. The total set of possible outcomes of an event is called the frame of discernment (FOD) denoted Θ. An observation provides evidence for one of more subsets, called focal elements, a set of such elements is called focal set. Evidence for a focal element takes a value between 0 and 1. The evidence for all prepositions of the FOD from an observation must be 1.0. As the focal elements may be any subset of the FOD, the entire set of propositions, denoted θ may be assigned evidence, thus expressing ignorance. θ is thus defined as every possible subset of the FOD Θ. A focal set where the sum of beliefs is 1.0 is called normalised.

An opinion held by an agent is held as one or more such observations, called belief functions, denoted Bel. The probability for an element or set of elements in the focal set is denoted m(X).

To model an agents opinion about a throw with classic six sided die and its outcomes the FOD would be every outcome of the die that is Θ_{die} = \{1, 2, 3, 4, 5, 6\}. As the FOD holds every possible outcome it is natural to deduce that the combined probability for the outcomes is 1, that is that it is an absolute truth, this is expressed as equation 1.

\[
Bel(Θ) = \sum_{θ \subseteq Θ} m(θ) = 1
\]

Suppose and agent, A1, has an opinion about the outcome of throwing a die once, Bel1, his opinion is that there is an equal probability for every possible outcome that is that every focal element of Bel1, m(X) where X \in \{1, 2, 3, 4, 5, 6\}, has the possibility of \frac{1}{6} and the ignorance m(θ) for Bel1 is 0.

The probabilities can be combined to get the beliefs for either of two outcomes, this is done by adding the probabilities; m(1 or 2) = m(1)+m(2) =
Using Dempster's rule of combination, in equation 2 two belief functions can be combined, that is a consensus between two opinions can be reached.

\[
Bel_3 = Bel_1 \oplus Bel_2 = \frac{\sum_{A_i \cap B_j = C_k; C_k \neq \emptyset} m_1(A_i)m_2(B_j)}{1 - \sum_{A_i \cap B_j = \emptyset} m_1(A_i)m_2(B_j)}
\] (2)

Where the focal elements are \(Bel_1 = A = \{A_1, ..., A_i\}\), \(Bel_2 = B = \{B_1, ..., B_j\}\), and \(Bel_3 = C = \{C_1, ..., C_k\}\) respectively, where \(C\) is all subsets produced by \(A \cap B\).

To give a simple example of the use of Dempsters' rule of combination we use the opinion of agent \(A_1\) about the outcome of a throw of a die, but let's discount it slightly to get some ignorance into it. And combine it to another agent, \(A_2\), opinion about the outcome of a throw of a die the agents opinions about the probability of each outcome can be seen in table 1, with the belief in the whole focal set, ignorance, denoted as \(\theta\).

<table>
<thead>
<tr>
<th>Outcome</th>
<th>Belief</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.12</td>
</tr>
<tr>
<td>2</td>
<td>0.12</td>
</tr>
<tr>
<td>3</td>
<td>0.12</td>
</tr>
<tr>
<td>4</td>
<td>0.12</td>
</tr>
<tr>
<td>5</td>
<td>0.12</td>
</tr>
<tr>
<td>6</td>
<td>0.12</td>
</tr>
<tr>
<td>(\theta)</td>
<td>0.28</td>
</tr>
</tbody>
</table>

By using Dempster's rule of combination from equation 2 to reach a consensus between the two agents we get the opinion in table 2.

<table>
<thead>
<tr>
<th>Outcome</th>
<th>Belief</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.1271</td>
</tr>
<tr>
<td>2</td>
<td>0.1271</td>
</tr>
<tr>
<td>3</td>
<td>0.1271</td>
</tr>
<tr>
<td>4</td>
<td>0.1271</td>
</tr>
<tr>
<td>5</td>
<td>0.1271</td>
</tr>
<tr>
<td>6</td>
<td>0.1271</td>
</tr>
<tr>
<td>(\theta)</td>
<td>0.2373</td>
</tr>
</tbody>
</table>

Note that the reached consensus has less ignorance then the opinions of the agents, thus a consensus helps to reach a less ignorant opinion.
2.5 Subjective Logic

Subjective logic is presented in Jøsang (1998) and elaborated in many later works mainly by Jøsang. The description given here is based on his Ph.D. thesis from 1998. Some interesting later additions are also described below.

Subjective logic is a trust algebra based on bayesian theory and boolean logic. It presents an opinion as three interdependent values. An opinion, denoted as \( \omega \) about preposition \( p \) is expressed as:

\[
\omega_p = \{ b_p, d_p, i_p \} \text{ where } b_p + d_p + i_p = 1.0
\]  

(3)

Where \( b_p \) is the belief in preposition \( p \) and \( d_p \) is the corresponding disbelief and \( i_p \) is the ignorance of the opinion. Equation 3 corresponds to equation 1 in Dempster-Shafer theory for a FOD like \( \Theta = \{ P, \neg P \} \). This means that in contrast to Dempster-Shafer theory, subjective logic only models binary prepositions that is with either true or false outcomes. So to be able to model a die as in the example from the previous section, it must be modeled as positive or negative outcomes, this is shown in an example later.

The set of all possible opinions is denoted as \( \Omega \) and can be seen as the whole opinion triangle in figure 2. Thus, any given opinion \( \omega \) can be presented graphically as a point within the opinion triangle.

![Figure 2: The opinion triangle. (From Jøsang 1998).](image)

Subjective logic allows for building an opinion from a set of evidences about preposition \( p \), as:

\[
\omega_p = \begin{cases} 
    b_p = \frac{r_p}{r_p + s_p + 1} \\
    d_p = \frac{s_p}{r_p + s_p + 1} \\
    i_p = \frac{1}{r_p + s_p + 1}
\end{cases}
\]

(4)

Where \( r_p \) is the number of positive evidences about \( p \) and \( s_p \) is the number
of negative evidences about $p$.

To model a die as in the previous section we need to build an opinion for every possible outcome so we get the set of outcomes $\{\omega_1, \omega_2, \omega_3, \omega_4, \omega_5, \omega_6\}$. If we've never thrown the die before, we have no preconception about the outcome and thus our opinion about it would be total ignorance, to just give an example let's look at the possibility of getting the outcome 3 when throwing the die the opinion would be $\omega_3 = \{0, 0, 1\}$. Then by throwing the die we can build an opinion about it based on the outcomes. Say we throw the die 100 times and get a 3 approximately $\frac{1}{6}$ of the time, that is 17 times. This gives us 17 positive evidence for $\omega_3$ and 83 evidence against $\omega_3$, using equation 4 we can calculate an opinion about the probability of getting a 3 when throwing a die once as:

$$ \omega_3 = \begin{cases} 
 b_3 &= \frac{17}{17 + 83 + 1} \approx 0.168 \\
 d_3 &= \frac{83}{17 + 83 + 1} \approx 0.822 \\
 i_3 &= \frac{1}{17 + 83 + 1} \approx 0.010 
\end{cases} \quad (5) $$

If another observer does another observation of a die and gets an equal opinion about the probability of getting the outcome 3 when throwing the die, then a stronger, that is less ignorant, opinion can be calculated by using the consensus rule in subjective logic shown in equation 6

$$ \omega_p^A \oplus \omega_p^B = \begin{cases} 
 b_{p, A, B} &= \frac{b_{p, B}^{A,B} + b_{p, B}^B}{k} \\
 d_{p, A, B} &= \frac{d_{p, B}^{A,B} + d_{p, B}^B}{k} \\
 i_{p, A, B} &= \frac{i_{p, B}^{A} - i_{p, B}^B}{k} \quad \text{such that } k \neq 0 
\end{cases} \quad (6) $$

where $k = i_{p, B}^A + i_{p, B}^B - i_{p, B}^{A,B}$ such that $k \neq 0$.

That $k$ can not be 0 means that we can’t reach a consensus if both opinions has 0 ignorance this is explained by if a an opinion is totally certain it has no room for influence and therefor consensus would be impossible. This is the consensus rule for independent opinions, that is the opinions are not based on observations of the same event, if we have dependent or partially dependent opinions there are additional consensus rules for those cases (for a description of those rules see Jøsang 1998).

By applying this rule to our die example using two agents, let’s call them $A$ and $B$, who made the above observation about the probability of get the outcome 3 when throwing a die then we get the consensus opinion:

$$ \omega_3^A \oplus \omega_3^B = \begin{cases} 
 b_{3, A, B} &= 0.169 \\
 d_{3, A, B} &= 0.826 \\
 i_{3, A, B} &= 0.005 
\end{cases} \quad (7) $$
This shows that we get a more certain opinion about the probability of getting the outcome 3 when throwing a die by reaching a consensus between opinions about the same proposition.

### 2.5.1 Later additions

There are some later additions to the theory that can be seen as key in the evolution of subjective logic. The main concept added later is *atomicity* (Jøsang 2001) that can be seen as a variable to interpret ignorance in opinions. If an opinion have an atomicity value of 0.5 then an opinion with total uncertainty is interpreted as having an equal chance of producing a true or false output but if the atomicity value is 0.25 then the chance of getting a positive outcome is 25%, so the atomicity value is a way of changing the probability interpretation of ignorance, called expectation.

Atomicity can also be used as a base rate to express mutually exclusive, exhaustive sets of opinions. That means to allow for the power of expression of Dempster-Shafer theory. To give an example let’s go back to the die when expressing a normal die we know from domain knowledge that the outcome is either 1, 2, 3, 4, 5, or 6, this is a set of mutually exclusive and exhaustive set of hypotheses, meaning that one and only one is the true outcome. To express this in subjective logic a base rate of $\frac{1}{6}$ is used, meaning that there are six possible outcomes one and only one of which is true. So by setting the atomicity of all opinions about the hypotheses to $\frac{1}{6}$ we use domain knowledge to express that if we have no evidence we could expect one of six throws to be the outcome which the opinion is about. This value can be used even if we have evidence by weighting it in to our expectancy about the proposition to be true, thus calculating expectation about proposition $A$ as shown in equation 8.

$$E(A) = b^A + i^A a^A \quad (8)$$

Atomicity can also be combined by adding to the consensus rule of independent opinions in equation 6 so it is redefined as in equation 9.

$$\begin{align*}
\omega_p^A \oplus \omega_p^B &= \begin{cases} 
  b_p^A, B &= \frac{i^A p^B + b_p^B i^A p}{i^A p + i^B p} \\
  d_p^A, B &= \frac{d^A p^B + a_p^B i^A p}{i^A p + i^B p} \\
  i_p^A, B &= \frac{i^A p^B}{i^A p + i^B p} \\
\end{cases} \\
\text{where } k &= i^A p + i^B p - i^A p i^B p \text{ such that } k \neq 0.
\end{align*} \quad (9)$$

If $k = 0$ then he opinions still can reach a consensus using the addition
to consensus presented by Jøsang (2002a). The addition states that if \( k \) as defined for the consensus rule is 0 then an alternative consensus rule can be used as defined in equation 10.

\[
\omega_p^A \oplus \omega_p^B = \begin{cases} 
    b_p^{A,B} = \frac{\gamma b_p^A + b_p^B}{\gamma + 1} \\
    d_p^{A,B} = \frac{\gamma d_p^A + d_p^B}{\gamma + 1} \\
    i_p^{A,B} = 0 \\
    a_p^{A,B} = \frac{\gamma a_p^A + a_p^B}{\gamma + 1}
\end{cases}
\]  

(10)

Where \( \gamma = \frac{\gamma}{i_p^B} \), that is the relative ignorance between \( \omega_p^A \) and \( \omega_p^B \). This can be used to calculate consensus in the case where the opinions have \( i = 0 \) as long as a relative ignorance can be defined from domain knowledge.
3 Desired properties

In order to find the desired properties for trust models use in information fusion, a literature study is conducted. The study tries to evaluate as much of the current literature as possible, and uses a systematic approach to the literature search as described in Berndtsson et al. (2002). A discussion on the study is presented here and a list of the desired properties can be seen in appendix A. The desired properties here are divided into structural properties, implementational properties, correctness properties, and domain knowledge properties.

3.1 Structural properties

The process of trust derivation for information fusion to get one view of a scenario from multiple sources can be seen as the type of evidential-reasoning that is presented by Lowrance and Garvey (1986) which they divide into four parts; a) specifying a set of distinct propositional spaces, each of which delimits a set of possible world situations, b) specifying the interrelationships among these propositional spaces, c) representing bodies of evidence as belief distributions over these propositional spaces, and d) establishing paths for the bodies of evidence to move through these propositional spaces by means of evidential operations, eventually converging on spaces where the target question can be answered. For all these specified properties and all for all the belief distributions a trust model can be utilised to keep track of the subjective probability, given by a set of evidences and the reasoning around said evidences, and the relationships of the probabilities.

Zhu and Li (2003) look at different structures of the node networks in information fusion. They identify two main types of networks, from which an arbitrary fusion network can be built, the networks are built from sensors that senses information about the environment and a fusion center that fused the information and outputs an hypothesis or a statement about the state of the observed phenomenon. The first type is a parallel network where nodes work separately and report to a fusion center see figure 3 a, a special case is if the fusion center is a sensor itself this case is called a modified parallel network. The other network structure is called tandem network see figure 3 b, in a tandem network each sensor sends its information together with any information it got from other sensors to the next sensor in the network and the last sensor is the fusion center. By mixing these two types of networks in the same structure an arbitrary network, called tree network, can be built see figure 3 c.

The interesting thing, according to Zhu & Li (2003), about identifying
3.1 Structural properties

Figure 3: The three types of networks presented by Zhu and Li (2003). a) Parallel network, b) Tandem network, and c) Tree network.

these two types of networks is that if we handle these two types with our fusion rules then we can prove them to be able to handle an arbitrary sensor network. This means that a trust model should be able to handle trust building in these two types of networks, if it does it will be able to model trust for an arbitrary sensor network.

**Desired property 1: Arbitrary network structure**
The model should be able to handle arbitrary network structures.

When modeling trust in an information fusion system it is vital that the trust model supports the whole chain of trust, from the source to the decision maker. There are studies done that shows positive effects of presenting uncertainty in both graphical (as fuzzy icons and position circles or ellipses) and verbal (as estimations like good, average, and bad or percentages) and other ways (e.g. Kirschbaum & Arruda 1994, Andre & Cutler 1998, and
3.1 Structural properties 3 DESIRED PROPERTIES

Bisantz et al. (1999). To be able to make a good presentation of uncertainty, ignorance and probabilities of information, the model should support and aid the presentation of trust properties.

**Desired property 2: Aid presentation**
The model should aid presentation of trust properties.

Bisantz et al. (1999) state that decision aids based on information fusion may be applied to support decision making in complex and dynamic environments such as military command and control, non-destructive testing and maintenance, and intelligent transportation. They state that the aids in such applications provide the decision maker with situational estimates which can aid the decision making process, this means that the trust model should be flexible and powerful enough to handle such complex and dynamic scenarios.

**Desired property 3: Complex and dynamic scenarios**
The model should be able to handle complex and dynamic scenarios.

As Appriou et al. (2001) state information fusion is the activity of combing information from several sources, so is of course also the activity of fusing trust to get an opinion about the fused information. When fusing in several steps it is also important to take into account the dependencies of the views so that not one opinion gets weighed in several times if the views are dependent, that is share a common ancestor view (Lowrance & Garvey 1986).

**Desired property 4: Several sources**
The model should be able to handle information and opinions from several sources and to be able to fuse opinions is several steps.

It is stated by Hoffman & Murphy (1993) as a wanted feature that the implementation of the model not always rely on prior knowledge, but allows for the model to dynamically adapt as more information on the scenario is given.

**Desired property 5: Prior knowledge**
The model should not rely on prior knowledge about the environment (in particular when such knowledge is hard to find).

In order for a system to truly be able to handle dynamic situations, and to dynamically adapt, it should be able to learn how to deal with new situations from its past experience. Appriou et al. (2001) mention such learning systems
and although they state that it is not a fusion problem in itself, a good information representation should aid such learning in fusion systems. Thus it is a good idea to also let the trust model be able to build a knowledge base to learn how to build trust and how to derive trust in way that helps the subjective expected utility. This type of learning is incorporated in the JDL model as level 4: process refinement.

**Desired property 6: Learning**
The model should support learning systems applications.

### 3.2 Implementational properties

In this type of learning and adapting the concept of ignorance plays an important role, as shown by Hoffman & Murphy (1993) as well as many others such as Yu et al. (2004), Schubert (2001), Sheridan (1988), Andre and Cutler (1998), Haenni & Lehmann (2003), and Appriou et al. (2001). Ignorance is important in order to know what is unknown and how good base in evidence there is for a statement. Ignorance can also be used as a null data value, that is to show that we know nothing about the proposition and have no reason to assume anything. Ignorance is needed if we are to be able to not give default values to assumptions, which we don’t always have.

**Desired property 7: Ignorance**
The model should handle ignorance (if needed to be modelled).

Both Hoffman & Murphy (1993), and Haenni & Lehmann (2003) discuss one issue with the models supporting ignorance as well as complex and dynamic scenarios, as being computationally expensive. This leads to problems when using them in real-time applications such as real-time decision support systems. To make such implementations possible some shortcuts and cheats has been developed, but it is wanted from the model to aid an as computationally cheap implementation as possible.

**Desired property 8: Computationally cheap**
The model should be as computationally cheap as possible to aid a useful implementation.

An important issue to be handled by the trust model in information fusion is when information sources directly contradict one another or when they both make statements which can not be simultaneously true, both these cases leads to a conflict. This is shown by for example Lowrance & Garvey
Implementational properties

(1986), Schubert (2001), and Hoffman & Murphy (1993). To handle such conflicts Appriou et al. (2001) lists four solutions:

- Take one of the two sources, and discard the other one (unilateral resolution)
- Make a compromise, i.e. merge to a certain degree the information conveyed by the two sources (compromise resolution)
- Create a new world; in this case, it is considered that the two sources do not observe the same world.
- Delay the decision, waiting for more information.

A trust model should help the system figure out which way to go in general but in particular when doing an unilateral resolution to help figure out which one to throw (the least probable, if there is one) and in the compromise resolution to make trust a weight-factor to the degree of compromise.

**Desired property 9: Disagreement**

The model should be able to handle disagreement (conflict) between opinions and help to reach consensus and support other ways to resolve conflict.

Another type of conflict, not directly derived from the evidences is discussed by Cohen (1986). He discusses the use of evidence in trust models and identifies that evidence aren’t always independent, just because they come from different sources. This means that by treating evidences as independent can lead to conflicts later in the reasoning chain, for example evidence might point to, after reasoning about them and deriving opinions from them, both A and B even if both can’t possibly be true at the same time, this type of conflict might be, as discussed by Cohen (1986), due to that evidence are interdependent. This means that one should be able to detect such conflicts and then reason about the evidence leading to the conflict, which might lead to a discrediting of some evidences as less trustworthy. This type of reasoning should be supported by the trust model.

**Desired property 10: Interdependency between evidences**

The model should allow not only to handle opinions but also handle what the evidence (or the subject for the evidence) says about other evidence.

Yu et al. (2004) in their experiments in information fusion shows some good rules to have in the arsenal, the one that sticks out is the conditional probability. That is to be able to say not just that there is a probability of $B$
but also to express given that $A$ is true, or given the probability of $A$, then what is the probability of $B$ ($B$ given $A$).

**Desired property 11: Conditional probabilities**
The model should be able to model conditional probabilities, that is the probability of $B$ given the probability of $A$.

Another need that Yu et al. (2004) point to is the need to be able to give trust not only to one hypothesis but also to state that different hypotheses have a relation to one another. That if one is true so is another, or that a set of hypotheses are exclusive, that only one of them can be true.

**Desired property 12: Union of hypothesis**
The model should not require to give probability only to independent hypotheses, but allow to give probability to unions of hypotheses and express dependencies between hypotheses.

Hoffman & Murphy (1993) present a comparison between bayesian and Dempster-Shafer theory for an information fusion scenario. When doing this they state that both these theories can be used with similar degrees of success if correctly formulated, and that although the results might be equivalent, the implementations may require vastly different amounts of effort and information. By this is meant what type of preknowledge about the scenario is needed by the model and how hard it is to implement the scenario into the model. It is a desired property that the model is powerful enough to model a vast number of different scenarios, but also that it is understandable enough so that the model can be somewhat easily applicable to the scenario.

**Desired property 13: Easy implementation**
The model should require as little amount of effort and information about the environment as possible for the implementation.

### 3.3 Correctness properties

When modeling trust it is important to note that it is not the chance that a hypothesis is true that is reflected by the model, but instead the chance that the evidence means that the hypothesis is true. This moves the focus from what might be true about the world to what do we have evidence to believe about the world. Given this, according to Cohen (1986), the focus is on the interpretation of the evidence. This makes it very important that the trust model reflects the evidence. It is important to note, according to Bogler
(1987), that the interpretation of the evidence must not change the meaning of the evidence itself, so evidence for A and evidence for A gives B must not be interpreted directly as evidence about B. Lowrance & Garvey (1986) also state that evidence are characteristically uncertain and allow for multiple possible explanations, they also state that evidence is often incomplete, that one source of evidence rarely has a full view of the situation and they also state that evidence may be completely or partially incorrect. It is important to know this when building a model so to not give to high praise to evidence and to consider the source.

Desired property 14: Reflect the evidence
The model should not focus on the chance that a proposition is true, but on the chance that the evidence proves that it is true.

As a trust model should be a reflection of the evidence, not necessarily the truth about the world, it is hard to define how to say that a trust model is ‘good’. To be able to tell whether a model is good Sheridan (1988) defines the term subjective expected utility, this is defined as usefulness multiplied by reliability. So to get a good subjective expected utility the model should be useful and reliable, these are not easy to measure, perhaps, but they still can be compared between models.

Desired property 15: Reflect the subjective expected probability
The model should reflect what can be expected for the trusted entity in an intuitive way. That is to be able to calculate what is the subjective probability and that subjective probability relates to the evidence given about the proposition.

Sheridan (1988) states that “It is elementary that we don’t trust (ourselves to use) what we don’t understand”. This is interesting when evaluating trust models as the trust model should not be unnecessarily complex, or hard to understand.

Desired property 16: Understandability
The model should not be unnecessarily complex, but if possible be easy to understand.

3.4 Domain knowledge properties
Appriou et al. (2001) define the terms revision and update. Where revision is to modify prior information about the world at time t by new information
about time \( t \), and update is to modify prior information about the world at time \( t \) by new information about time \( t + 1 \), either by evolution of prior information or by prediction about future information. These two ways of manipulating information should be supported in the trust model as well, so the opinions should be updatable and revisionable. \( ? \) also argues for the need to update and revise as a mean to verify and correct opinions.

**Desired property 17: Revision and update**

The model should be able to both revise and update opinions based on new knowledge.

Blatt in both her articles (2004 and 2005) discusses the issue of operational trust. Operational trust is defined as “the trust that is required from every person and earned from every entity to accomplish an endeavor” (Blatt 2004, p. 5), this means that operational trust is not an opinion but rather a goal state for an opinion that it must reach in order for it to be useful. Blatt (2004 and 2005) shows many situations where operational trust is important, this can be that a trustee must be confident to a certain degree of proposition \( A \) in order for him to act. To express operational trust in the trust model the trust algebra must be able to express comparisons between opinions to compare it to the operational trust level, that is to say greater then, less then, and equal between an opinion and an expression of operational trust, be it another opinion or an other mean of measuring operational trust.

**Desired property 18: Operational trust**

The model should support operational trust, by allowing trust to be compared to metrics for operational trust, be it as goal opinions or other means of measuring operational trust.

Lowrance & Garvey (1986) discuss domain knowledge as a set of loosely interconnected concepts, this can be seen a vague preconceptions about the world. A good trust model should be able to express such preconceptions without mixing it with what the evidence is saying, as the preconceptions is not based on evidence but rather can be seen as ‘hunches’ to be used if no evidence is found or to help a weak set of evidences.

**Desired property 19: Domain knowledge**

The model should be able to use domain knowledge in trust models without confusing it as evidence.
4 Scenarios

From the list of properties and from the literature study two scenarios were selected to evaluate the fulfillment by subjective logic of the desired properties, as well as to compare it to Dempster-Shafer theory.

The scenarios were chosen as they reflect the research questions posed. Scenario 1 is a simple information fusion scenario which help us to answer the first question whether subjective logic has the properties desired form an information fusion application, even though it does not include all properties. Scenario 2 looks at some of the critique towards Dempster-Shafer theory and thus helps answer the second research question whether subjective logic has any advantages over Dempster-Shafer theory.

The scenarios are connected directly to a number of the desired properties from section 3. Subjective logics fulfillment of these, and other, properties is summarised and discussed further in section 5.

4.1 Scenario 1: Fantasy airport

The scenario presented by Hoffman & Murphy (1993) is a fictive example called fantasy airport, which concerns the operation of an airport at a fantasy island. The scenario is described by Hoffman & Murphy (1993) as so:

The airport is not equipped with a radio for communication with arriving airplanes. Instead, it uses a four sided tower built to a considerable height and oriented north-south, east-west. The tower staff consists of four ‘spotters’ whose sole task is to observe the sky for approaching aircraft and make reports of their observations. One spotter is assigned to each side of the tower, north, south, east, and west. Spotters make independent reports of their observations to the airport operations center at the base of the tower. The operations manager then attempts to make sense of this information so appropriate instructions can be given to the airport’s ground crew.

To further clarify and simplify the situation, Hoffman & Murphy (1993), assume that there is no schedule of arrivals and that aircraft can appear from any direction at any time of day without preference. Thus is to say that both the time and direction of arrival for aircraft follows a uniform distribution. Additionally, assume all spotters to be equally proficient in their ability to observe and report and that only one aircraft will ever be in the airport’s vicinity at any time. Now consider the actions of the airport operations manager in judging whether or not the island’s baggage handlers and welcome committee should be dispatched based on spotters’ reports.

Hoffman & Murphy (1993) use this scenario to compare Dempster-Shafer theory to bayesian theory, thus they executes the scenario using Dempster-
4.1 Scenario 1: Fantasy airport

Shafer theory, so here the execution is only done using subjective logic, and compared to their results. So for further details on the execution of the scenario using Dempster-Shafer theory and bayesian theory see Hoffman & Murphy (1993).

This scenario includes the desired property of several sources of opinions (desired property 4). Also it looks at the desired properties that Hoffman & Murphy (1993) discuss, that is easy implementation (desired property 13), ignorance (desired property 7), prior knowledge (desired property 5), and disagreement (desired property 9).

4.1.1 Fantasy airport using subjective logic

Hoffman & Murphy (1993) use the events shown in table 3.

<table>
<thead>
<tr>
<th>Event</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P$</td>
<td>Event that an airplane is in the vicinity of the airport.</td>
</tr>
<tr>
<td>$s_k$</td>
<td>Event that a geometric line of sight (LOS) exists between the $k^{th}$ spotter and an arriving airplane, where $k = {\text{north, south, east, west}}$.</td>
</tr>
<tr>
<td>$e_k^i$</td>
<td>Event that the $k^{th}$ spotter reports $i^{th}$ evidence.</td>
</tr>
<tr>
<td>$\neg P$</td>
<td>Complementary events in the usual way.</td>
</tr>
</tbody>
</table>

When implementing the scenario into Dempster-Shafer theory Hoffman & Murphy (1993) start by defining what they call a naive approach, where every spotter reports his belief that there is an aeroplane in the vicinity. In this approach simple FOD $\Theta = \{P, \neg P\}$ is used. This can be translated to subjective logic as the opinion $\omega_P$. So every spotter has his own opinion about $P$ as $\omega_P^k$. So by using the numbers for these opinions given by Hoffman & Murphy (1993) if a spotter $k$ observes an airplane he has the opinion given in equation 11 and if he does not observe an airplane he has the opinion given in equation 12.

$$\omega_P^1 = \begin{cases} b_P^1 = 0.99 \\ d_P^1 = 0.01 \\ i_P^1 = 0.0 \end{cases} \quad (11)$$

$$\omega_P^0 = \begin{cases} b_P^0 = 0.01 \\ d_P^0 = 0.98 \\ i_P^0 = 0.01 \end{cases} \quad (12)$$

As noted by Hoffman & Murphy (1993) this formulation is sufficient. It
4.1 Scenario 1: Fantasy airport

makes it possible to combine the opinion from each spotter. However when getting positive $(\omega^1_p)$ reports from two spotters and negative $(\omega^0_p)$ reports from the two other spotters the result is counter intuitive, one problem is that subjective logic does not allow for consensus with opinions without ignorance (without using the additions to the consensus rule) so for the sake of argument let $i^1_p = 0.002$. Then the positive observations, say from spotter south and east, can be combined as shown in equation 13 and the negative observations, from the north and west spotters, can be combined as shown in equation 14.

$$\omega^\text{south}_p \oplus \omega^\text{east}_p = \begin{cases} b = 0.991 \\ d = 0.01 \\ i = 0.001 \end{cases}$$ (13)

$$\omega^\text{north}_p \oplus \omega^\text{west}_p = \begin{cases} b = 0.01 \\ d = 0.985 \\ i = 0.005 \end{cases}$$ (14)

These can now be combined as shown in equation 15.

$$\omega^\text{south-east}_p \oplus \omega^\text{north-west}_p = \begin{cases} b = 0.829 \\ d = 0.172 \\ i = 0.0 \end{cases}$$ (15)

This shows the counter intuitive result shown in Dempster-Shafer by Hoffman & Murphy (1993) that the belief $b$ for a plane in the vicinity is lowered by the opinions of spotters that entertain no physical possibility of making the observation. The problem as discussed by Hoffman & Murphy (1993) is that the approach is over simplified and the FOD used is not refined. To solve this they refine the FOD to the FOD shown in equation 16.

$$\Theta = \begin{cases} \{P_{nw}\} & \{\neg P_{nw}\} & \{P_{nw}, \neg P_{nw}\} \\ \{P_{ne}\} & \{\neg P_{ne}\} & \{P_{ne}, \neg P_{ne}\} \\ \{P_{se}\} & \{\neg P_{se}\} & \{P_{se}, \neg P_{se}\} \\ \{P_{sw}\} & \{\neg P_{sw}\} & \{P_{sw}, \neg P_{sw}\} \\ \{P_{nw}, P_{ne}, P_{se}, P_{sw}\} & \{\neg P_{nw}, \neg P_{ne}, \neg P_{se}, \neg P_{sw}\} \end{cases}$$ (16)

This FOD is not as easily translated to subjective logic. But instead in subjective logic we can refine the handling of reports in a similar way by making a refined model of how to build opinions about whether or not there is a plane in the vicinity.
4.1 Scenario 1: Fantasy airport

To start and make the refinement we look at what the reports from the spotters actually says. If getting a report only from the east spotter affirming the the presence of an airplane in the south-east quadrant. The opinion of the operations manager about there being an aeroplane in the south-east quadrant can is shown in equation 17, here he uses information from two spotters to get a view of the situation in one quadrant (desired property 4: several sources).

\[
\omega_{P_{sc}}^{\text{south}} \oplus \omega_{P_{sc}}^{\text{east}} = \begin{cases} 
  b = 0.828 \\
  d = 0.172 \\
  i = 0.002
\end{cases}
\] (17)

As stated by Hoffman & Murphy (1993) this makes the the totality of all belief partitioned equally among the four quadrants meaning that \( \frac{1}{4} \) of all basic belief is assigned to each quadrant, thus it is necessary to multiply the belief by \( \frac{1}{4} \) to compute the basic belief assignments for the refined model, this is one way to use domain knowledge about the environment to reach a result as stated by desired property 19: domain knowledge. So a mean value of all quadrants is used to get the belief table in table 4, the beliefs using Dempster-Shafer theory (from Hoffman & Murphy 1993) is inserted for reference.

A direct comparison of the values from using subjective logic and the values from using Dempster-Shafer theory has little value, but the key thing is to note that both methods reflect an increase in belief as additional evidence accumulates, which is the key point as given by Hoffman & Murphy (1993). The decrease in belief between rows \{6, 7\}, \{9, 10\}, and \{11, 12\} is due to the increase in disagreement which outweighs the increase in evidence, both in subjective logic and in Dempster-Shafer theory, in accordance to desired property 9: disagreement.

The implementation presented here is based on the one using Dempster-Shafer theory presented by Hoffman & Murphy (1993) and uses no additional information about the environment thus subjective logic can handle the scenario with the same amount of prior knowledge that Dempster-Shafer theory requires thus they are comparable on desired property 5: prior knowledge. The implementation of the scenario into the trust model also seems to be very similar in difficulty, in this example, in accordance to desired property 13: easy implementation. Subjective logic also shows to handle ignorance in this example as stated by desired property 7: ignorance.
4.2 Scenario 2: Conflict of evidence

Zadeh (1984) presents some critique towards Dempster-Shafer theory when dealing with conflict of opinions. The critique is further formalised and discussed by Cohen (1986). It interesting to examine how the reasoning by Cohen (1986) applies to subjective logic.

The scenario is that there are three mutually exclusive hypotheses $H_1$, $H_2$, and $H_3$. And there are two experts, $E_1$ and $E_2$, giving their opinion about the truth of these hypothesis their opinion can be seen in table 5.

Table 5: The experts belief in the probability of the hypotheses.

<table>
<thead>
<tr>
<th></th>
<th>$E_1$</th>
<th>$E_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_1$</td>
<td>0.99</td>
<td>0</td>
</tr>
<tr>
<td>$H_2$</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>$H_3$</td>
<td>0</td>
<td>0.99</td>
</tr>
</tbody>
</table>

Combining these opinions using Dempsters rule of combination, from equation 2, the result is that the belief in $H_1$ is 0 and the belief in $H_3$ is...
4.2 Scenario 2: Conflict of evidence

and the belief in $H_2$ is 1. This is a consequence of the fact that $E_1$ rules out $H_3$ and $E_2$ rules out $H_1$ therefor $H_2$ must be true. This is independent of the belief given by the experts to $H_1$ and $H_2$, the argument might seem to make sense, but is it really what the evidence points to?

The problem in this reasoning as given by Cohen (1986) is that the experts are assumed to be perfectly reliable, even though this is rarely the case in real world scenarios, and the experts disagree even though both have no doubt in their opinions so there clearly is a conflict. One way to solve this is to discount the experts opinions, by adding ignorance, but it is still a problem how much to discount the opinions and as Cohen (1986) shows a relatively small and unbalanced (between the experts) discounting gives a big impact on the final result of the combined opinion. And this leads to the argument that the problem is not in the theory but in the initial allocation of support for the experts.

By reasoning about the experts opinions it can be assumed that there is a conflict in the evidence, given that the evidences are independent this could be used to reason about the evidences. This according to Cohen (1986) shows a problem in Dempster-Shafer theory, that knowledge is to be assessed as if we had no knowledge of the evidence provided, it is thus not permitted to use the conflict between the experts as a clue regarding their capabilities and use it as a guide to discount their opinion.

So even though Dempster-Shafer theory provides discounting it does not support the use of conflict to allow for such discounting, for further discussion on this problem see Cohen (1986).

It is easy to see this scenario appearing in a information fusion scenario where the fusion process receives conflicting evidence from highly trusted sources.

This scenario includes the desired properties of disagreement (desired property 9), as the experts disagree on the propositions, which leads to a conflict. It also looks at whether or not the model reflects the evidence (desired property 14), that is does the outcome reflect what the experts are stating and also the subjective expected probability (desired property 15) as whether the outcome is intuitive to our expectation from the scenario. It also looks at using interdependency between evidences (desired property 10) to help resolve conflict as discussed by Cohen (1986).

4.2.1 Conflict of evidence using subjective logic

To examine this scenario in subjective logic it is recreated in subjective logic. The two experts $E_1$ and $E_2$ has given their opinion about $H_1$, $H_2$, and $H_3$ and the experts assessments where expert $E_1$ says there is a 99% chance of
4.2 Scenario 2: Conflict of evidence

\( H_1 \) and a 1% chance of not \( H_1 \). As there are three mutually exclusive and exhaustive hypotheses we use a base rate (atomicity) of \( \frac{1}{3} \), this shows that subjective logic can express union of hypothesis as being codependent as stated by desired property 12: union of hypothesis. So \( E_1 \)'s opinion about hypothesis \( H_1 \) is shown in equation 18.

\[
\omega_{E_1}^{H_1} = \begin{cases} 
  b_{E_1}^{H_1} = 0.99 \\
  a_{E_1}^{H_1} = 0.01 \\
  i_{E_1}^{H_1} = 0.00 \\
  d_{E_1}^{H_1} = \frac{1}{3}
\end{cases}
\]

(18)

This can be extended to both experts statements, but how do we handle the things they don't give a statement about? If they state that either one of the two statements they have an opinion about is true then they actually deny that there is a chance of any other mutually exclusive hypothesis being true, thus their opinion about any other mutually exclusive hypothesis is total disbelief. This gives us a set of opinions as given in table 6.

Table 6: The experts opinion about the hypotheses, given as \{belief, disbelief, ignorance, atomicity\}.

<table>
<thead>
<tr>
<th>Hypothesis</th>
<th>( E_1 )</th>
<th>( E_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( H_1 )</td>
<td>{0.99, 0.01, 0.0, \frac{1}{3}}</td>
<td>{0.0, 1.0, 0.0, \frac{1}{3}}</td>
</tr>
<tr>
<td>( H_2 )</td>
<td>{0.01, 0.99, 0.1, \frac{1}{3}}</td>
<td>{0.01, 0.99, 0.1, \frac{1}{3}}</td>
</tr>
<tr>
<td>( H_3 )</td>
<td>{0.0, 1.0, 0.0, \frac{1}{3}}</td>
<td>{0.99, 0.01, 0.0, \frac{1}{3}}</td>
</tr>
</tbody>
</table>

As these are dogmatic beliefs, as the experts has no doubt in their assessments, the extended consensus operator in equation 10 is used to reach a consensus. To use that consensus operator we must set \( \gamma \) as the relative uncertainty of the opinions, say that we trust both experts equally then we have \( \gamma = \frac{1}{7} \), this is to use domain knowledge as stated in desired property 19: domain knowledge. Then we get the expectations, as defined by equation 8, for the hypotheses shown in table 7.

Table 7: The expectations for the hypothesis after consensus between expert \( E_1 \) and \( E_2 \).

<table>
<thead>
<tr>
<th>Hypothesis</th>
<th>Expectation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( H_1 )</td>
<td>0.495</td>
</tr>
<tr>
<td>( H_2 )</td>
<td>0.01</td>
</tr>
<tr>
<td>( H_3 )</td>
<td>0.495</td>
</tr>
</tbody>
</table>
This shows that first the identification of the opinions as being dogmatic and still contradicting shows that there is a conflict, so subjective logic helps detect a conflict as stated in desired property 9: disagreement. Then we can use the extended consensus operator to reach a consensus by stating a relative uncertainty between the two opinions, from domain knowledge about them, in this case by simply stating that we trust them equally, even though they state a contradiction, this is to use the interdependency between evidences as stated in desired property 10. The extended consensus operator helps us to use the conflict as a mean to identify the need to use the conflict to discount the statements, and then $\gamma$ is used to weight the experts as how to discount their opinions. This is the type of reasoning that is presented by Cohen (1986) as a possible solution to the problem of conflicting evidence.

Given that we trust both experts equally the resulting discounting of their statement by using the conflict the results seems to reflect the evidence, as stated by desired property 14: reflect the evidence, and also seems to reflect what we can expect from the experts given their conflict, as stated in desired property 15: reflect the subjective expected probability.
5 Discussion

The implementation of scenario 1 using subjective logic, in section 4.1, shows that subjective logic is applicable in an information fusion scenario. Even though the scenario is quite simple it is designed to show how a trust model can be applied to an information fusion problem (Hoffman & Murphy 1993). To be able to model the scenario in a similar manner as in Dempster-Shafer theory, using subjective logic shows that the desired properties discussed by Hoffman & Murphy (1993) also hold for subjective logic. Those properties are the use of information from several sources (desired property 4), as the spotters each is a different source of information from which the controller builds one view of the environment in this case whether there is an aeroplane approaching or not. Also the implementation of the scenario in subjective logic uses the same amount of prior knowledge about the environment as the implementation in Dempster-Shafer theory, and thus it is comparable in the desired property of prior knowledge (desired property 5) and also easy implementation (desired property 13). It also handles disagreement in a similar manner as Dempster-Shafer theory (desired property 9) as can be seen in table 4 where the dips in belief where there are more reports from less quadrants due to more disagreement in the reports. Thus the conclusion that Dempster-Shafer theory is a good model when ignorance is needed to be explicit (desired property 7) given by Hoffman & Murphy (1993) also seem to extend to subjective logic. Thus subjective logic seems to be a reasonable approach to model trust in information fusion applications, even a good approach by the reasoning of Hoffman & Murphy (1993).

This shows that subjective logic has a lot of the properties desired by an information fusion application, which partially answers research question 1.

This result is not surprising as subjective logic is partly based on Dempster-Shafer theory and there has been other comparisons between the two models, even though not in information fusion applications, see Jøsang (1998), Jøsang (2001), Jøsang (2002b), and Jøsang (2002a) for example.

Scenario 1 uses a fusion network of type parallel network as seen in figure 3 a, this shows subjective logics ability to use that type of network to model trust (desired property 1).

The implementation of scenario 2, in section 4.2, shows that the later extensions to subjective logic makes the theory very potent as a competitor to Dempster-Shafer theory. It shows that subjective logic has the mechanisms to solve some of the shortcomings of Dempster-Shafer theory, as it allows using conflict of opinions to discount the opinions as the extended consensus operator, from equation 10, can be used to reach consensus between dogmatic opinions, this corresponds to the desired property of interdependency.
between evidences (desired property 10) as this reflects what evidences for one proposition says about the evidence for another proposition given the relationship between the propositions. It also shows how to use atomicity to express mutually exclusive, and exhaustive sets of hypotheses as the FOD in Dempster-Shafer theory thus shows that subjective logic has the desired property of union of hypotheses (desired property 12). The use of atomicity as a base rate also shows that subjective logic can use domain knowledge in trust models without confusing it as evidence as stated as desired property domain knowledge (desired property 19). The results also seem to reflect our expectation of the outcome, given that we trust both experts equal, thus it reflects the evidence which is a desired property (desired property 14), it also seem to reflect the subjective expected probability (desired property 15) as the outcome seems probable given the evidence.

Scenario 2 shows that subjective logic has capability to handle some some scenarios where Dempster-Shafer theory has been criticised for giving counter intuitive results. This shows that subjective logic provides advantages over Dempster-Shafer theory as a trust model in information fusion, this answers research question 2.

It can also be said that the articles comparing subjective logic to Dempster-Shafer theory, referenced above, supports this as they show how to translate many situations from Dempster-Shafer theory to subjective logic, and also the claimed sound mathematical basis of subjective logic. Even though one should be careful not to be too reliant on these articles as they are all presented by the research team at DSTC Security1, lead by Dr. Audun Jøsang the creator of subjective logic.

As discussed above the scenarios shows that subjective logic has the ten desired properties that are examined, that is property 4, 5, 7, 9, 10, 12, 13, 14, 15, and 19, and also one of the network types of desired property 1 the properties and whether they are examined is summarised in appendix A. It would of course be interesting to evaluate on all desired properties but it would be a too extensive task for this work. But to have the desired properties listed is still an interesting result in it self as it can work as a base for future examinations of subjective logic or other trust models utility in information fusion applications.

Given this discussion the research questions can be answered.

Research question 1: Does subjective logic have the properties which are desired from a trust model by information fusion applications?

1DSTC Security research homepage: http://security.dstc.edu.au
It has at least the ten of the nineteen desired properties, which are examined in the scenarios. However nine of the properties are not examined, and it would be interesting to examine them as well.

**Research question 2:** Does subjective logic provide any advantages over Dempster-Shafer theory as a trust model in information fusion?

The execution of scenario 2 shows that subjective logic has mechanisms to handle a scenario where Dempster-Shafer theory has been criticised for not producing intuitive results. This shows that subjective logic provides an advantage over Dempster-Shafer theory, but how big that advantage is depends on the application.
6 Conclusion and future work

As presented in the discussion both research questions got answered. However it should be stated that further experimentation and even a practical implementation is needed to give more fulfilling answers to the questions. The results from the work show that such further investigations and implementation would be interesting and probably lead to further strengthening the arguments for using subjective logic in information fusion applications.

The research area of information fusion is large, and the list of desired properties for a trust models use in information fusion, presented in section 3, is made to be general to the field. It would be interesting to extend it and specialise it to the many different areas of information fusion to be able to use it as a metric of which trust model to use in different scenarios. As Hoffman & Murphy (1993) state the different trust models are not always good or bad in themselves but they are more or less appropriate for different problems.

It would also be interesting to examine more trust models in the light of the desired properties. When facing an information fusion problem in the future the list of desired properties could be matched to the problem. From that match and the examinations of trust models the appropriate model for the problem could be chosen.

A future addition to trust modelling in information fusion might be to cooperate with other trust modelling applications. For example if a trust model is used in the security protocol, as presented by Håkansson (2004), and in the information fusion protocol the two protocols can cooperate and share trust. So if a node is distrusted by the security protocol the information from that node can be discounted also in the information protocol. This can be applied between other protocols as well.

Acknowledgement

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7 References


### Desired properties

Table 8: Desired properties for a trust model to be used in information fusion systems.

<table>
<thead>
<tr>
<th>Desired property</th>
<th>Description</th>
<th>Examined</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Arbitrary network structure</td>
<td>Partially in scenario 1</td>
</tr>
<tr>
<td>2</td>
<td>Aid presentation</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Complex and dynamic scenarios</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Several sources</td>
<td>Scenario 1</td>
</tr>
<tr>
<td>5</td>
<td>Prior knowledge</td>
<td>Scenario 1</td>
</tr>
<tr>
<td>6</td>
<td>Learning</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Ignorance</td>
<td>Scenario 1</td>
</tr>
<tr>
<td>8</td>
<td>Computationally cheap</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>Disagreement</td>
<td>Scenario 1</td>
</tr>
<tr>
<td>10</td>
<td>Interdependency between evidences</td>
<td>Scenario 2</td>
</tr>
<tr>
<td>11</td>
<td>Conditional probabilities</td>
<td></td>
</tr>
</tbody>
</table>

The model should be able to handle arbitrary network structures.

The model should aid presentation of trust properties.

The model should be able to handle complex and dynamic scenarios.

The model should be able to handle information and opinions from several sources and to be able to fuse opinions in several steps.

The model should not rely on prior knowledge about the environment (in particular when such knowledge is hard to find).

The model should support learning systems applications.

The model should handle ignorance (if needed to be modelled).

The model should be as computationally cheap as possible to aid a useful implementation.

The model should be able to handle disagreement (conflict) between opinions and help to reach consensus and support other ways to resolve conflict.

The model should allow not only to handle opinions but also handle what the evidence (or the subject for the evidence) says about other evidence.

The model should be able to model conditional probabilities, that is the probability of $B$ given the probability of $A$. 
Table 9: Desired properties for a trust model to be used in information fusion systems (continued).

<table>
<thead>
<tr>
<th>Desired property</th>
<th>Description</th>
<th>Examined</th>
</tr>
</thead>
<tbody>
<tr>
<td>Union of hypothesis</td>
<td>The model should not require to give probability only to independent hypotheses, but allow to give probability to unions of hypotheses and express dependencies between hypotheses.</td>
<td>Scenario 2</td>
</tr>
<tr>
<td>Easy implementation</td>
<td>The model should require as little amount of effort and information about the environment as possible for the implementation.</td>
<td>Scenario 1</td>
</tr>
<tr>
<td>Reflect the evidence</td>
<td>The model should not focus on the chance that a proposition is true, but on the chance that the evidence proves that it is true.</td>
<td>Scenario 2</td>
</tr>
<tr>
<td>Reflect the subjective expected probability</td>
<td>The model should reflect what can be expected for the trusted entity in an intuitive way. That is to be able to calculate what is the subjective probability and that subjective probability relates to the evidence given about the proposition.</td>
<td>Scenario 2</td>
</tr>
<tr>
<td>Understandability</td>
<td>The model should not be unnecessarily complex, but if possible be easy to understand.</td>
<td></td>
</tr>
<tr>
<td>Revision and update</td>
<td>The model should be able to both revise and update opinions based on new knowledge.</td>
<td></td>
</tr>
<tr>
<td>Operational trust</td>
<td>The model should support operational trust, by allowing trust to be compared to metrics for operational trust, be it as goal opinions or other means of measuring operational trust.</td>
<td></td>
</tr>
<tr>
<td>Domain knowledge</td>
<td>The model should be able to use domain knowledge in trust models without confusing it as evidence.</td>
<td>Scenario 1 &amp; 2</td>
</tr>
</tbody>
</table>