

Seminar Paper No. 515

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WITH RETURNS THAT FOLLOW POISSON
JUMP-DIFFUSION PROCESSES

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THE INTERTEMPORAL CAPITAL ASSET PRICING MODEL WITH RETURNS THAT FOLLOW POISSON JUMP-DIFFUSION PROCESSES

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ABSTRACT

The capital market equilibrium is derived in a model where asset returns follow a mixed Poisson jump – diffusion process, rather than a simple diffusion process as in the traditional ICAPM. In the resulting JCAPM (CAPM with Jumps) expected returns are still linear in beta, but in addition premia have to be paid for jump risk. When jump risk is diversifiable in the market portfolio the JCAPM reduces to the standard ICAPM, as in Jarrow and Rosenfeld (1984).

Jumps are found to be prevalent in the daily returns of the market indices in the 18 countries investigated, during the time period 1985–89. However, when the year of the crash, 1987, is excluded from the sample, the simple diffusion process gives an adequate description of the market returns in seven countries.

1. Introduction

Is it realistic to model asset prices as diffusion processes as in the Intertemporal Capital Asset Pricing Model, introduced by Merton (1973)? Would the ICAPM hold if we allowed for jumps in asset prices? Jarrow and Rosenfeld (1984) address these questions. They give sufficient conditions for the ICAPM to hold for asset prices having discontinuous sample paths. The sufficient condition is that the jump risk be diversifiable in the market portfolio. They do not, however, solve the model completely for the case when this condition is not met. The purpose of this paper is to derive the capital market equilibrium when asset returns follow jump–diffusion processes. The strategy will be to add enough restrictions to the model to be able to get an explicit solution.

Jarrow and Rosenfeld (1984) also test if there is undiversifiable jump risk in the market portfolio. Using U.S. stock market indeces, they find evidence of jumps in *daily* returns, but not in weekly returns. They conclude that jump risk exists and is not diversifiable. In contrast to Jarrow and Rosenfeld, Jorion (1988) finds strong evidence of jumps in the *weekly* returns on the CRSP value–weighted index, even after taking account of ARCH effects. On the other hand, a simple diffusion model provide an adequate description of monthly stock returns. An additional purpose of this paper will be to collect more evidence on the existence of undiversifiable jump risk by investigating if we can detect jumps in the market indices of 18 OECD countries.

Previous studies that have found evidence in favor of discontinuous sample paths for (individual) stock prices include Oldfield, Rogalski, and Jarrow (1977), Ball and Torous (1983,1985), and Ho, Perraudin, and Sorensen (1992). Oldfield, Rogalski, and Jarrow (1977) study the returns on 20 NYSE stocks during the 22 trading days in September 1976. They find that transactions returns (omitting over–night returns) follow an autoregressive jump process. Ball and Torous' (1983) data set consists of 500 daily return observations of 47 NYSE listed stocks. Over 78 percent of the stocks indicated the presence of jumps at the one percent significance level. This result is

confirmed in Ball and Torous (1985) for a different data set and time period. Ho, Perraudin, and Sorensen (1992) test various versions of a continuous-time Arbitrage Pricing Model using daily data on the S&P index and eight individual stocks. They find evidence in favor of a model containing both jumps and ARCH effects.

The plan of the paper is as follows. In section 2 the equilibrium pricing relationships are derived for the intertemporal Capital Asset Pricing Model with Jump risk (JCAPM). Section 3 tests the hypothesis that there is undiversifiable jump risk in the stock market indices of 18 OECD countries. Finally section 4 gives some conclusions.

2. Asset Pricing with Jump Risks

We consider an economy of the type developed in Merton (1973) and modified by Jarrow and Rosenfeld (1984). It is a pure exchange economy with one good, which serves as numeraire. The initial set of assumptions are:

- Assumption 1.* There are N risky assets and one risk-free asset. All assets are marketable and perfectly divisible. There are no taxes, transactions costs, or restrictions on short sales.
- Assumption 2.* Investors take prices as given.
- Assumption 3.* Trading takes place continuously in time at equilibrium prices.
- Assumption 4.* There is a risk-free rate of interest, r , for borrowing and lending.
- Assumption 5.* Investors have homogeneous expectations about asset prices, which satisfy the stochastic differential equations

$$(1) \quad \frac{dP_i}{P_i} = \mu_i dt + \sigma_i dZ_i + \epsilon_i dY - \lambda \epsilon_i dt \quad , i = 1, \dots, N;$$

where P_i is the price of asset i , μ_i represents the instantaneous expected rate of return (including the jump), Z_i is a Wiener process, σ_i is the instantaneous standard deviation of the rate of return, Y is a Poisson process with parameter λ , ϵ_i is the

stochastic jump amplitude with expected value equal to e_i ; and Z_i , Y , and ϵ_i are assumed to be independent. The last two terms in (1) together represent the unexpected rate of return connected with the rare event. The price dynamics could also be set up to separate between diversifiable and nondiversifiable risk. In this case we are considering

$$(1a) \quad \frac{dP_i}{P_i} = \mu_i dt + f_i d\Psi + g_i d\eta_i + \epsilon_i dY - \lambda e_i dt, \quad i = 1, \dots, N;$$

where $\sigma_i^2 = f_i^2 + g_i^2$ and $\sigma_i dZ_i = f_i d\Psi + g_i d\eta_i$.

Assumption 6. Investors maximize their von Neumann–Morgenstern expected utility of lifetime consumption functions,

$$(2) \quad E_t \int_t^\infty U(C(s), s) ds,$$

where E_t is the conditional expectation operator given the information available at time t and $C(s)$ is the rate of consumption. Investors have instantaneous utility functions that exhibit Constant Relative Risk Aversion,

$$(3) \quad U(C(t), t) = \frac{C^{1-\gamma}}{1-\gamma} e^{-\rho t}, \quad \gamma > 0, \gamma \neq 1,$$

where γ is the Arrow–Pratt measure of relative risk aversion and ρ is the utility rate of time preference.

Assumptions 1–4 are standard. In assumption 5, in addition to the diffusion component, we let the returns be affected by a rare event that can cause the prices to jump. The probability of a jump caused by the event in the time interval dt is λdt , where λ is a constant. When the event occurs, there is an instantaneous jump in the return on asset i of size ϵ_i . For a homogeneous Poisson counting process with intensity λ the interarrival times, i.e. the time interval between two successive events, are independently and identically distributed. This may not be totally realistic for some events. For example, we would expect the probability of a devaluation to be smaller just

after a devaluation has occurred.¹ However, we can look at the jump process as a generic rare event, in which case homogeneity will be less of a problem. I.e. one type of rare event can be followed by another rare event, which is independent of the first event having taken place. For example, a devaluation could be followed by a strike in the steel industry. Other examples of the types of rare events we have in mind are stricter environmental legislation, raised energy taxes, inventions, a defaulting bank or some other news that typically will affect more than one company. The specification in assumption 5 is slightly different than the one adopted in Jarrow and Rosenfeld (1984). They let each price process have its own independent jump component. We have chosen to look at a rare event as something that affects more than one stock although each stock may be affected in a different way. Our assumption 6 on preferences is more restrictive than in Jarrow and Rosenfeld's model. They merely assume a twice differentiable, strictly increasing and strictly concave instantaneous utility function. We need to make the assumption of constant relative risk aversion in order to get an explicit solution to the Bellman equation.

Under assumptions 1–6, the investor chooses a portfolio rule, $\{w_i(s)\}_{i=1}^N$, and a consumption rule, $C(s)$, so as to maximize

$$E_t \int_t^\infty U(C(s), s) ds ,$$

subject to

$$(4) \quad dW = \sum_{i=1}^N w_i W \left[\frac{dP_i}{P_i} - r dt \right] + (rW - C) dt ,$$

where W is the investor's wealth.

The investor's problem is solved by the use of dynamic programming. First, we define the maximum value function,

¹See Bertola and Svensson (1990) for an exchange rate model along these lines. Shimko (1989) values cash flows generated by Poisson processes. The jump itself can be diversified, but the expected frequency of jumps cannot.

$$(5) \quad J(W, t) = \max_{\{C, \mathbf{w}\}} \left\{ E_t \int_t^\infty U(C(\tau), \tau) d\tau \right\}.$$

We can now write the Hamilton–Jacobi–Bellman equation for this problem,²

$$(6) \quad \begin{aligned} 0 = \max_{C, \mathbf{w}} & \left\{ \frac{C^{1-\gamma}}{1-\gamma} e^{-\rho t} + J_t(W, t) + J_W(W, t) \left[W\mathbf{w}'(\boldsymbol{\nu} - \mathbf{e}\lambda) + Wr - C \right] \right. \\ & \left. + \frac{1}{2} J_{WW}(W, t) W^2 \mathbf{w}' \Sigma \mathbf{w} + \int_A \lambda \left[J(W + W\mathbf{w}'\epsilon(a), t) - J(W, t) \right] \right\} f(a) da, \end{aligned}$$

where J_x is the partial derivative of the indirect utility function with respect to argument x , $f(a)$ is the density function for the jump amplitudes, and the other notation is the following:

- \mathbf{w} \equiv the $(N \times 1)$ vector of portfolio shares;
- $\boldsymbol{\nu}$ \equiv the $(N \times 1)$ vector of excess rates of return,
i.e. $\nu_i = \mu_i - r$ for asset i ;
- ϵ \equiv the $(N \times 1)$ vector of jump amplitudes;
- \mathbf{e} \equiv the $(N \times 1)$ vector of expected jump amplitudes;
- Σ \equiv the $(N \times N)$ covariance matrix of asset rates of return.

To make the solution simpler we will make a simplifying assumption regarding the jump amplitudes:

Assumption 7. The jump amplitudes are nonstochastic, $\epsilon_i = e_i \ \forall i$.

This assumption is not essential for the solution. It will merely simplify by getting rid of the integration over A in the following expressions. With $J(W, t)$ linear in wealth, we could simply integrate over A and get the expected jump amplitudes. Unfortunately $J(W, t)$ is not linear in W , as we will see shortly.

The investor's problem has a well-defined solution if the following transversality condition holds,

$$\rho > h + \lambda [(1 + \mathbf{w}'\mathbf{e})^{1-\gamma} - 1],$$

where h is defined as

$$h = (1-\gamma)[\mathbf{w}'(\boldsymbol{\nu} - \lambda\mathbf{e}) + r] - \frac{1}{2} \gamma(1-\gamma) \mathbf{w}'\Sigma \mathbf{w}.$$

The solution, in terms of the time-independent indirect utility function $I(W) \equiv e^{\rho t} J(W, t)$, can then be found to be

²See Malliaris and Brock (1982): Ch. 2, Section 12.

$$(7) \quad I(W) = p W^{1-\gamma},$$

where

$$p = \frac{1}{1-\gamma} \left\{ \frac{1}{\gamma} \left[\rho - h - \lambda [(1+\mathbf{w}'\mathbf{e})^{1-\gamma} - 1] \right] \right\}^{-\gamma},$$

which will be constant if the optimal portfolio, \mathbf{w} , is constant over time. So, let us next turn to the choice of an optimal portfolio rule.

We find that the domestic investor's optimal portfolio rule is implicitly given by the following first-order condition:

$$(8) \quad \mathbf{w} - \frac{1}{\gamma} \left[\Sigma^{-1}(\nu - \mathbf{e}\lambda) - \lambda (1+\mathbf{w}'\mathbf{e})^{1-\gamma} \Sigma^{-1} \mathbf{e} \right] = \mathbf{0}.$$

Since there are no time-dependent variables in this equation we conclude that the optimal portfolio will be constant through time. This is consistent with the assumption that p is a constant and thus the conjectured solution in (7) is indeed valid.

The portfolio rule is clearly nonlinear. This means that there is no easy way of aggregating portfolios. It is clear from (8) that investors with different degrees of relative risk aversion will demand different portfolios. In order to get an explicit equilibrium pricing relationship we will have to make an additional assumption at this stage.

Assumption 8. All investors have the same relative risk aversion, γ .

Under assumption 8 all investors will hold the same portfolio according to (8). Hence, all investors will have to hold the market portfolio, \mathbf{w}_m . If we substitute $\mathbf{w} = \mathbf{w}_m$ into (8) we get the equilibrium relationship

$$(9) \quad \nu = \gamma \Sigma \mathbf{w}_m + \lambda [(1+\mathbf{w}_m'\mathbf{e})^{1-\gamma} - 1] \mathbf{e}.$$

From this expression we can derive the security market lines for the intertemporal *Capital Asset Pricing Model with Jump risk* (JCAPM).³ The equilibrium pricing relationship for asset i will be

$$(10) \quad \nu_i - \xi_i = \beta_i (\nu_m - \xi_m),$$

where

³See the Appendix for the derivation.

$$\begin{aligned}
\beta_i &\equiv \sigma_{im} / \sigma_m^2 \text{ is the covariance of asset } i \text{ with the world market portfolio} \\
&\quad \text{divided by the variance on the market portfolio;} \\
\nu_m &\equiv \mathbf{w}_m' \boldsymbol{\nu} \text{ is the excess rate of return on the market portfolio;} \\
\xi_m &\equiv -\lambda e_m [(1+e_m)^{1-\gamma} - 1] \text{ is the risk premium for jumps in} \\
&\quad \text{the return on the market portfolio;} \\
\xi_i &\equiv -\lambda e_i [(1+e_m)^{1-\gamma} - 1] \text{ is the risk premium for jumps in} \\
&\quad \text{the return on asset } i; \\
e_m &\equiv \mathbf{w}_m' \mathbf{e} \text{ is the jump in the return on the market portfolio;}
\end{aligned}$$

Equation (10) states that the JCAPM holds for expected rates of return that have been adjusted by the risk premia for jump risk, ξ_i ($i=1, \dots, N, m$). The risk premia may be positive or negative since the jump amplitudes may be negative or positive. If jump risk is diversifiable in the market portfolio, $e_m = 0$, then $\xi_i = 0$ ($i=1, \dots, N, m$) and the standard ICAPM will hold, which is the result in Jarrow and Rosenfeld (1984). In the case of logarithmic utility ($\gamma=1$) we also get $\xi_i = 0$ ($i=1, \dots, N, m$) and the JCAPM again reduces to the standard ICAPM.

Note that two assets with the same beta can have different expected rates of return because of different expected jumps in their prices. For example, consider an economy with $\gamma > 1$. A rare event that has a negative effect on both asset i and the market portfolio ($e_i < 0$ and $e_m < 0$) will result in a positive risk premium for asset i ($\xi_i > 0$). If the same event has a positive effect on the return to asset j ($e_j > 0$) it will result in a negative risk premium for asset j ($\xi_j < 0$). Thus, the required rate of return will be higher on asset i than on asset j even if they have the same beta value ($\beta_i = \beta_j$).

The risk premia can also be written as

$$(11) \quad \xi_i = - \frac{I_W(W(1+e_m)) - I_W(W)}{I_W(W)} \lambda e_i, \quad i=1, \dots, N, m;$$

where the first factor is the relative jump in the marginal utility of real wealth if a rare event occurs.⁴ An event that results in a negative jump in the return on the market portfolio will cause a positive relative jump in the marginal utility of wealth.

⁴The same type of risk premium is derived in Svensson (1990). In that paper the risk premium is due to exchange rate uncertainty caused by devaluations.

Let us next say something about the practical implications of (10). Our results indicate that if one is to use the CAPM in evaluating the required rate of return for a project one should attempt to take the risk premia for jump risk into account. One should ask questions of the following type. What is the probability of, for example, a devaluation and what effect would it have on the return to the project *and* on the return to the market portfolio? The answers should be stated in terms of expected returns to the project, λe_i , and to the market portfolio, λe_m , respectively. The CAPM can then be modified along the lines of (10) by computing ξ_i and ξ_m and making an assessment of the coefficient of relative risk aversion, γ .

3. Is There Jump Risk in the Market Portfolio?

3.1 Test Procedure

To investigate if there is jump risk in the market portfolio let us look at the market portfolio dynamics. The market portfolio consists of the market weighted values, and equals $M = \sum_{i=1}^N w_{mi} P_i$. Using (1a), the return on the market portfolio can be written as

$$(12) \quad \frac{dM}{M} = \sum_{i=1}^N w_{mi} \left[\mu_i dt + \sigma_i dZ_i + (-\lambda e_i dt + \epsilon_i dY) \right].$$

If jump risk is diversifiable the condition for the ICAPM to hold⁵, using (1a), can be stated as

$$(13) \quad \sum_{i=1}^N w_{mi} \left[g_i d\eta_i + (-\lambda e_i dt + \epsilon_i dY) \right] = 0,$$

where $g_i d\eta_i = \sigma_i dZ_i - f_i d\Psi$, i.e. dZ_i has been divided up into a common factor $d\Psi$ and residuals $d\eta_i$. Condition (13) says that the market portfolio shares $\{w_{mi}\}_{i=1}^N$ must be such that the stochastic components of the returns from the assets are eliminated, except for the common factor $d\Psi$.

⁵Derived by Jarrow and Rosenfeld (1984).

The hypothesis to be tested is (13) and if accepted we have the return on the market portfolio

$$(14) \quad \frac{dM}{M} = \mu dt + f d\Psi,$$

using (12) with $\mu = \sum w_{mi} \mu_i$ and $\sigma = \sum w_{mi} f_i$. If jump risk is not diversifiable we have the alternative

$$(15) \quad \frac{dM}{M} = \mu dt + f d\Psi + dq,$$

where μ represents the drift of the process, σ is the standard deviation (conditional on no jump), $d\Psi$ and dq are independent Wiener and Poisson processes.

The likelihood function corresponding to (14) is

$$(14a) \quad L_c = \prod_{i=1}^N \frac{1}{\sqrt{2\pi\sigma^2 h}} \exp \left[\frac{-(\ln x_i - \mu h)^2}{2\sigma^2 h} \right],$$

and the log-likelihood function is

$$(14b) \quad \ln L_c = -(N/2)\ln(2\pi) + \sum_{i=1}^N \frac{1}{\sqrt{\sigma^2 h}} \exp \left[\frac{-(\ln x_i - \mu h)^2}{2\sigma^2 h} \right].$$

where N is the number of returns, h is the increment of time between observations, and $x_i = (M_i/M_{i-1})$.

Now let jumps arrive according to a Poisson process with mean number of jumps equal to $\lambda > 0$. We assume the jump size, e , be a sequence of independent identical lognormal distributed random variables with parameters (a, b^2) . The jump size and the Poisson process are independent (see Karlin & Taylor (1981)).⁶ We can write the likelihood function corresponding to (15) as⁷

$$(15a) \quad L_u = \prod_{i=1}^N \sum_{j=0}^{\infty} \frac{e^{-\lambda h} (\lambda h)^j}{j!} \frac{1}{\sqrt{2\pi(\sigma^2 h + b^2 j)}} \exp \left[\frac{-(\ln x_i - \mu h - \theta j)^2}{2(\sigma^2 h + b^2 j)} \right],$$

and the log-likelihood function is

⁶Generally the diffusion process can be described as $\ln(M_t/M_{t-1}) = \mu + \sigma dZ + \sum Y_k$. The jump size, Y_k , all identically lognormally distributed, gives the following sentence for the jump size, $\sum Y \sim$

$N(na, nb^2)$, and the final process can be described as $P(\lambda)N(\mu + na, \sigma^2 + nb^2)$ with a mean time between jumps given by $E(T) = \lambda^{-1} \sum N(\mu + na, \sigma^2 + nb^2)$.

⁷Basawa and Rao (1980).

$$(15b) \quad \ln L_u = -N\lambda h - (N/2)\ln(2\pi) \\ + \sum_{i=1}^N \left[\sum_{j=0}^{\infty} \frac{(\lambda h)^j}{j!} \frac{1}{\sqrt{\sigma^2 h + b^2 j}} \exp \left[\frac{-(\ln x_i - \mu h - \theta j)^2}{2(\sigma^2 h + b^2 j)} \right] \right],$$

where $\mu = \alpha - \sigma^2/2$, $\theta = a - b^2/2$.⁸

A likelihood ratio test given by $LR = -2(\ln L_c - \ln L_u)$ can be used to test the hypothesis, jump risk is diversifiable with likelihood as L_c , versus the alternative that jump risk are not diversifiable with likelihood as L_u . LR has a X^2 distribution with degrees of freedom equal the difference in the number of parameters between the two models.

3.2 Description of data

The empirical tests were performed on samples from 18 countries (and a "World" index). The indices used were collected from Morgan Stanley Capital Market Indices. The indices consists of daily observations from a value weighted portfolio on stocks included in the indices. The period covered are from January 1985 to December 1989 and from each indices we have 1303 observations. A technical description of the indices can be found in Morgan Stanley (1986).

A simple investigation of a possible jumps in the indices is given by the empirical distributions in table 1. In table 1 simple summary statistics are given for each indices. Besides means, standard deviations, skewness and kurtosis, the number of observations with respect to sigma limits are reported.

In order to investigate the event in October 1987 the indices has been divided in a part including the whole period from 1985–1989, and another part where 1987 has been excluded.

From table 1 it is seen that all means are positive, and all skewness are negative. This means, that the distributions are all skewed to the left, compared with a normal

⁸As pointed out by Ball and Torous (1985), the infinite sum in (15) has to be truncated so that sufficient accuracy is achieved. The actual truncation depends on the value of λ . The estimation was carried out in double precision and the infinity sum was truncated at $j=10$.

distribution with a skewness of zero. The kurtosis are all positive and has a kurtosis well above a normal distribution where kurtosis is 3.

If we let a possible jump be identified by a return above (+/−) 4 times the standard deviations, the results in table 1 shows several possible jumps in the indices. Not all jumps belongs to a single year, 1987 (in parenthesis), but a lot of jumps does. The World index shows 8 possible jumps and 5 of these jumps belong to a single year, 1987. The only country where no jumps are identified in 1987 is Italy.

It seems resonable to make the empirical investigation in two parts. One which include 1987 (table 2), and a second which exclude 1987 (table 3), and then compare the two investigations.

3.3 Estimation of parameters

The parameters in the model has been estimated using the collected data. Table 2 covers the period from 1985–1989 and the estimated parameters from the two stochastic processes are displayed together with the corresponding standard errors, and the likelihood ratio tests.

In table 2 the first part gives the result from the jump process. As can be seen, the estimates of the mean number of jumps per year, λ , are all significant different from zero. This suggest the existence of infrequent discrete movements. The mean number of jumps from Austria (256.98), Italy (133.66) and Spain (181.30) shows some large numbers in the mean number of jumps in these indices. In the Austrian market there is a jump practically every day. In contrast the mean number of jumps in the U.K. market is small (8.46).

The estimates for the jump size shows an expected negative value for several indices. We would with these negative values expect that after the arrival of a jump the jump size would be negative, that is a negative return in the indices. The positive values are found in indices from Austria, Italy, Japan and Spain. In these contries we would expect a positive return after the arrival of a jump. The estimated standard deviation from the jump size also shows large numbers. At least the values indicate, that the

values for the mean, although negative, they could as well be positive, and vice versa.

The estimates for the diffusion part shows all positive values for the mean, and compared with the standard deviation we would expect positive returns in periods where there are no jumps.

The simple Wiener process shows positive mean in all indices, and for some of the means the standard errors indicate, that the mean is not significantly different from zero. On the other hand the computed standard deviations are all significant different from zero, and none of the standard deviations looks like an abnormal value.

The final likelihood ratio test, all very significant compared with a X^2 with 3 degrees of freedom, indicates jumps in the indices. With the sample from 1985–1989 we reject the hypothesis that jump risk is diversifiable. If we take a look at the period which has excluded 1987 we get the results in table 3.

The results in table 3 show, as indicated in the simple summary statistics, that a single year, 1987, could be the cause to jumps in the indices. The result in table 3 show this is the case for several indices. The likelihood ratio test support our hypothesis, that jump risk is diversifiable for some indices. The hypothesis is accepted for the following indices World, Denmark, Germany, Sweden, UK, Canada and Hong Kong. For the remaining indices we reject the hypothesis and have to accept that jump risk is nondiversifiable.

A closer look at table 3 shows, that if we do not expect a jump in an index, for example the World, then the jump component is zero and the jump size, if a jump should arrive, is very large (6.18). This is the case for all countries where we have rejected the Jump process. In Germany we would indeed expect a very large positive return if a jump should arrive. But the standard deviations are still very large.

6. Conclusion

In this paper we have investigated the Intertemporal Capital Asset Pricing Model as developed by Merton (1973), and further expanded by Jarrow & Rosenfeld (1984) to cover jump risk.

We have used indices from 18 OECD countries covering the period 1985–1989. For the sample period 1985–1989, compared with Jarrow & Rosenfeld (1984), we come to the same results 1) that the market portfolio contains a jump component and 2) that risk is not diversifiable.

If we exclude a single year, 1987, we are able to accept the hypothesis that jump risk is diversifiable for some countries.

Appendix

Derivation of equations (10):

Premultiplying (9) by \mathbf{w}_m' we get

$$(A1) \quad \mathbf{w}_m' \boldsymbol{\nu} = \gamma \mathbf{w}_m' \boldsymbol{\Sigma} \mathbf{w}_m - \sum_{k=1}^K \lambda_k [(1 + \mathbf{w}_m' \mathbf{e}_k)^{-\gamma} - 1] \mathbf{w}_m' \mathbf{e}_k.$$

Simplifying the notation,

$$(A2) \quad \nu_m = \gamma \sigma_m^2 + \xi_m,$$

where the notation can be found after equation (10) in the main text. Combining (9)

and (A2) we get

$$(A3) \quad \boldsymbol{\nu} = \frac{1}{\sigma_m^2} (\nu_m - \xi_m) \boldsymbol{\Sigma} \mathbf{w}_m + \boldsymbol{\xi}.$$

From this equation we get the scalar forms of (10) in the main text.

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TABLE 1

Summary statistics and number of observations
with respect to sigma limits. Period 1985–1989.

INDEX	Mean	Std. Dev.	Skew.	Kurt.	Less than +/-4*Std	More than +/-4*Std
WORLD	0.00085	0.0085	-2.0	37.4	1295 (255)	8 (6)
AUSTRALIA	0.00071	0.0132	-6.4	120.9	1293 (252)	10 (9)
AUSTRIA	0.00104	0.0099	-0.6	11.6	1291 (257)	12 (4)
BELGIUM	0.00080	0.0104	-0.9	24.0	1287 (254)	16 (7)
DENMARK	0.00060	0.0099	-0.1	21.8	1295 (255)	8 (6)
FRANCE	0.00082	0.0121	-1.2	15.2	1291 (254)	12 (7)
GERMANY	0.00057	0.0140	-1.2	15.6	1290 (250)	13 (11)
ITALY	0.00087	0.0137	-0.9	15.5	1293 (260)	10 (1)
JAPAN	0.00088	0.0115	-1.9	44.8	1297 (257)	6 (4)
NETHERLAND	0.00039	0.0124	-0.8	22.7	1287 (250)	16 (11)
NORWAY	0.00065	0.0161	-2.8	45.3	1294 (254)	9 (7)
SPAIN	0.00085	0.0123	-0.3	14.6	1291 (256)	12 (5)
SWEDEN	0.00091	0.0120	-1.2	12.4	1292 (251)	11 (10)
SWITZERLAND	0.00047	0.0115	-2.3	33.0	1291 (252)	12 (9)
UK	0.00054	0.0109	-2.3	28.7	1293 (253)	10 (8)
CANADA	0.00038	0.0088	-1.9	36.9	1292 (251)	11 (10)
HONG KONG	0.00075	0.0191	-8.7	161.7	1296 (259)	7 (2)
SINGAPORE	0.00043	0.0153	-4.7	81.7	1296 (256)	7 (5)
USA	0.00055	0.0134	-4.3	77.9	1291 (257)	12 (4)

TABLE 2

Estimates of the Mixed Poisson Jump – Diffusion Model and the Simple Wiener Process Model of Daily Returns to the Stock Market Indices of 18 Countries, 1985–1989.

Equation (15b) and (14b) are estimated using daily returns, where for the first model the mean rate of return is given by $\mu = \alpha - \sigma^2/2$, while σ is the standard deviation of the return. λ is the mean number of jumps per year, a is the expected jump amplitude and b is the standard deviation of the amplitude. α , σ , μ , a , and b are reported as ten times their estimated value. Standard errors are reported in parenthesis below the point estimates. The final column gives the Likelihood Ratio Test for the hypothesis that jump risk is diversifiable vs. the hypothesis that it is not.

	Wiener – Poisson Jump Process			Wiener Process			L.R. Test			
	$\ln L_u$	α^*10	σ^*10	λ	a^*10	b^*10		$\ln L_c$	μ^*10	σ^*10
WORLD	4619.35	2.99 (0.47)	0.99 (0.03)	9.29 (0.31)	-0.75 (0.05)	29.80 (0.05)	4361.39	2.30 (0.61)	1.37 (0.03)	515.91
AUSTRALIA	4180.27	3.66 (0.65)	1.35 (0.04)	11.83 (0.30)	-1.39 (0.06)	39.26 (0.06)	3786.93	1.84 (0.95)	2.13 (0.04)	786.69
AUSTRIA	4390.24	0.12 (0.45)	0.38 (0.08)	256.98 (4.29)	0.11 (0.00)	9.06 (0.01)	4164.24	2.71 (0.71)	1.60 (0.03)	451.99
BELGIUM	4484.16	2.50 (0.45)	0.69 (0.04)	75.51 (1.30)	-0.04 (0.01)	16.43 (0.02)	4104.26	2.09 (0.75)	1.67 (0.03)	759.78
DENMARK	4349.64	2.07 (0.52)	1.03 (0.03)	27.74 (0.75)	-0.13 (0.03)	27.79 (0.06)	3852.35	1.56 (0.91)	2.03 (0.06)	994.57
FRANCE	4126.46	3.37 (0.68)	1.13 (0.08)	55.70 (1.82)	-0.19 (0.02)	20.25 (0.03)	3908.35	2.13 (0.87)	1.94 (0.04)	436.22
GERMANY	3911.51	4.27 (0.81)	1.35 (0.07)	55.50 (1.25)	-0.46 (0.02)	22.90 (0.02)	3716.88	1.48 (1.01)	2.25 (0.04)	389.26
ITALY	3939.11	2.10 (0.96)	10.00 (0.11)	133.66 (3.25)	0.03 (0.01)	16.01 (0.02)	3745.20	2.26 (0.98)	2.20 (0.04)	387.80
JAPAN	4291.14	2.21 (0.57)	0.94 (0.05)	57.53 (1.21)	0.04 (0.01)	19.13 (0.02)	3972.40	2.29 (0.83)	1.85 (0.04)	637.47
NETHERLAND	4200.80	2.29 (0.61)	1.20 (0.04)	23.14 (0.50)	-0.47 (0.03)	32.30 (0.04)	3875.38	1.01 (0.89)	1.99 (0.04)	650.83
NORWAY	3838.16	3.75 (0.83)	1.39 (0.10)	51.68 (1.65)	-0.34 (0.02)	27.60 (0.04)	3527.69	1.68 (1.16)	2.60 (0.05)	620.93

Table 2 continued.

	Wiener - Poisson Jump Process			Wiener Process			L.R.			
	$\ln L_u$	α^*10	σ^*10	λ	a^*10	b^*10	$\ln L_c$	μ^*10	σ^*10	Test
SPAIN	4144.99	0.11 (.58)	0.61 (.06)	181.30 (2.30)	0.13 (.01)	13.10 (.01)	3877.29	2.21 (.89)	1.99 (.04)	535.40
SWEDEN	4126.08	4.26 (.68)	1.17 (.05)	45.25 (1.02)	-0.38 (.02)	22.00 (.03)	3918.94	2.37 (.86)	1.93 (.04)	414.28
SWITZERLAND	4342.84	3.37 (0.55)	1.02 (0.04)	31.10 (0.69)	-0.65 (0.02)	25.20 (0.03)	3968.59	1.21 (0.83)	1.85 (0.04)	748.50
UK	4278.45	2.61 (0.61)	1.30 (0.03)	8.46 (0.23)	-1.25 (0.07)	38.40 (0.06)	4039.98	1.41 (0.79)	1.76 (0.03)	476.94
CANADA	4658.34	1.88 (0.43)	0.82 (0.04)	30.45 (0.80)	-0.27 (0.02)	19.20 (0.03)	4313.64	0.97 (0.64)	1.42 (0.03)	689.41
HONGKONG	3828.88	4.10 (0.84)	1.71 (0.07)	14.03 (0.52)	-1.27 (0.08)	54.70 (0.13)	3307.70	1.95 (1.38)	3.08 (0.06)	1042.34
SINGAPORE	4004.21	2.44 (0.74)	1.51 (0.06)	13.41 (0.43)	-0.79 (0.07)	47.90 (0.08)	3597.30	1.11 (1.10)	2.47 (0.05)	813.83
USA	4214.40	2.79 (0.60)	1.12 (0.07)	30.18 (0.96)	-0.38 (0.03)	29.90 (0.05)	3773.32	1.43 (0.96)	2.15 (0.04)	882.16

TABLE 3

Estimates of the Mixed Poisson Jump – Diffusion Model and the Simple Wiener Process Model of Daily Returns to the Stock Market Indices of 18 Countries, 1985–1986, 1988–1989.

Equation (15b) and (14b) are estimated using daily returns, where for the first model the mean rate of return is given by $\mu = \alpha - \sigma^2/2$, while σ is the standard deviation of the return. λ is the mean number of jumps per year, a is the expected jump amplitude and b is the standard deviation of the amplitude. α , σ , μ , a , and b are reported as ten times their estimated value. Standard errors are reported in parenthesis below the point estimates. The final column gives the Likelihood Ratio Test for the hypothesis that jump risk is diversifiable vs. the hypothesis that it is not.

	Wiener – Poisson Jump Process				Wiener Process				L.R.	
	$\ln L_u$	α^*10	σ^*10	λ	a^*10	b^*10	$\ln L_c$	μ^*10	σ^*10	Test
WORLD	1906.44	3.27 (.45)	1.00 (.02)	0.00 (.00)	6.18 (2.69)	10.39 (4.16)	1906.44	3.22 (.45)	1.00 (.01)	0.00
AUSTRALIA	1779.99	3.85 (.65)	1.13 (.03)	20.58 (3.36)	0.02 (.02)	0.15 (.01)	1760.95	3.44 (.59)	1.33 (.01)	38.09
AUSTRIA	1725.72	4.99 (0.62)	1.36 (0.03)	2.20 (0.56)	0.40 (0.02)	0.03 (0.03)	1703.64	4.01 (0.66)	1.48 (0.03)	44.16
BELGIUM	1816.18	1.98 (0.47)	0.72 (0.04)	76.48 (10.06)	0.02 (0.01)	0.13 (0.01)	1737.51	3.10 (0.62)	1.39 (0.02)	157.34
DENMARK	1729.74	0.24 (0.63)	1.41 (0.03)	0.00 (0.00)	5.56 (2.31)	17.71 (16.60)	1729.74	0.13 (0.63)	1.41 (0.02)	0.00
FRANCE	1672.53	3.34 (0.67)	1.04 (0.06)	63.26 (13.16)	0.01 (0.01)	0.18 (0.02)	1607.73	3.69 (0.80)	1.78 (0.03)	129.59
GERMANY	1562.88	3.40 (0.87)	1.94 (0.03)	0.00 (0.00)	52.69 (108.34)	34.96 (36.41)	1562.88	3.21 (0.87)	1.94 (0.03)	0.00
ITALY	1488.51	8.20 (0.91)	1.72 (0.04)	28.18 (4.07)	0.07 (0.04)	0.38 (0.04)	1391.82	5.93 (1.22)	2.70 (0.07)	193.38
JAPAN	1765.09	2.28 (0.54)	0.77 (0.04)	81.40 (10.76)	0.01 (0.01)	0.14 (0.01)	1690.55	2.95 (0.68)	1.52 (0.02)	149.08
NETHERLAND	1742.03	1.00 (0.61)	1.26 (0.03)	6.24 (1.17)	0.06 (0.06)	0.40 (0.04)	1660.38	1.28 (0.72)	1.61 (0.02)	163.29
NORWAY	1622.80	1.73 (0.82)	1.08 (0.06)	90.82 (10.43)	0.00 (0.01)	0.16 (0.01)	1585.48	4.31 (0.86)	1.91 (0.02)	74.64

	Wiener – Poisson Jump Process				Wiener Process				L.R. Test	
	$\ln L_u$	α^*10	σ^*10	λ	a^*10	b^*10	$\ln L_c$	μ^*10	σ^*10	
SPAIN	1614.82	0.45 (.59)	0.76 (.11)	127.75 (34.51)	0.03 (.01)	0.16 (.02)	1543.32	4.25 (.90)	2.02 (.04)	142.99
SWEDEN	1628.55	3.35 (0.77)	1.71 (0.03)	0.00 (0.00)	24.38 (7.61)	9.37 (1.17)	1628.55	3.21 (0.77)	1.71 (0.03)	0.00
SWITZERLAND	1813.39	4.08 (0.50)	0.91 (0.02)	16.85 (2.20)	0.03 (0.02)	0.18 (0.01)	1777.39	2.56 (0.58)	1.29 (0.02)	72.01
UK	1767.83	1.80 (0.59)	1.31 (0.03)	0.00 (0.00)	0.85 (16.77)	39.62 (10.72)	1767.83	1.71 (0.59)	1.31 (0.02)	0.00
CANADA	1934.47	1.21 (0.43)	0.95 (0.02)	0.00 (0.00)	1.09 (4.24)	12.51 (4.83)	1934.47	1.16 (0.43)	0.95 (0.01)	0.00
HONG KONG	1584.81	4.14 (0.83)	1.86 (0.02)	0.00 (0.00)	5.06 (12.38)	32.57 (9.99)	1584.81	3.96 (0.83)	1.86 (0.02)	0.00
SINGAPORE	1631.16	1.81 (0.81)	1.18 (0.06)	76.16 (10.91)	0.03 (0.01)	0.15 (0.01)	1602.26	0.33 (0.80)	1.80 (0.02)	57.80
USA	1778.23	2.17 (0.81)	0.96 (0.06)	39.72 (10.91)	0.00 (0.01)	0.17 (0.01)	1700.32	1.88 (0.67)	1.49 (0.02)	155.83