Seminar Paper No. 349

EMPIRICAL EXAMINATIONS OF THE INFORMATION SETS
OF ECONOMIC AGENTS

by

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Seminar Papers are preliminary material circulated to stimulate discussion and critical comment.

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1. Introduction

Most economic models postulate that agents base their behavior on expectations about some variables. Empirical applications of such models must therefore make explicit assumptions about: (1) What information do agents rely on in their expectations formation? (2) Given this information, how is it exploited when expectations are formed? As for the second question, there is now growing consensus that the most appealing assumption is the one of rational expectations—i.e., agents use their available information efficiently. Rational expectations is taken as one point of departure in our paper.

Regarding the first question there is much less consensus. Because theory offers little guidance as to what information agents use, empirical applications of rational expectations models typically rely on quite arbitrary assumptions about agents' information sets. A general presumption in the empirical literature is that the economic agents, whose expectations and behavior are being modeled, have access to no less information than the econometrician. This may either be because these agents see more variables than those specified by the econometrician or because they see the same variables more frequently.
This general presumption is taken as another point of departure in our paper.

In Section 2 we present the following result: Suppose that agents who form a rational expectations forecast about a particular variable $y$, in the form of a linear least square projection, have an information set at least as rich as that specified by an outside econometrician. Then, the econometrician's best guess about the agents' forecast is a weighted average of the linear projection of $y$ on the specified information set and $y$ itself, where the two weights sum to unity. Furthermore, the weight on $y$ is a natural measure of agents' information advantage over the econometrician.

The general implications of this result are discussed in Section 3. We show that the weights can be estimated and identified from joint estimation of the forecasting equation and a behavioral equation incorporating the forecast. This means that one can explicitly test either the hypothesis that agents have better information than the econometrician or the hypothesis that agents have perfect information. While the structure of the information set available to agents might be of direct interest, it may also be of indirect interest in coming to grips with other economic phenomena. As an example, we show that a test of perfect information can be equivalent to a test for the existence of "buffer stock effects" due to significant intervals between agents' portfolio revisions or between price setting and production decisions. Finally, we discuss some econometric issues that arise when an econometrician estimates a behavioral model jointly with a forecasting equation relying on a smaller information set than the actual one. Although this will lead to unbiased estimates of the behavioral coefficients, we argue that the estimates will be more efficient if one
instead uses the alternative guess about agents' forecast we suggest in Section 2.

In Section 4 we report on a particular application of the general methods suggested in the paper. We present estimates of a demand function for reserves by the Swedish bank sector jointly estimated with relevant forecasting equations, and show how these estimates can be used to test for the existence of buffer stock effects.

Section 5 discusses the relation between our approach and some related literature. This final section also contains some qualifications and suggestions for further research.

2. Guessing the forecasts of economic agents

We consider a set-up where some agents are interested in forecasting the random variable $y$. These agents all have an identical information set containing the $(K \times 1)$ vector $x$ of random variables $(x_k)_k$, $k = 1, \ldots, K$, and the $(L \times 1)$ vector $z$ of random variables $(z_{\xi})_{\xi}$, $\xi = 1, \ldots, L$. Agents have rational expectations in the sense that their forecast of $y$ is an optimal linear least-squares projection of $y$ on the variables in the information set.\(^1\) This projection

$$P(y|x,z) = a'x + b'z,$$

where $a$ and $b$ are vectors of projection coefficients, by definition fulfills the orthogonality conditions

$$\begin{align*}
(2.2a) \quad & E \{[y - (a'x + b'z)]x_k\} = 0, \quad k = 1, \ldots, K, \\
(2.2b) \quad & E \{[y - (a'x + b'z)]z_{\xi}\} = 0, \quad \xi = 1, \ldots, L.
\end{align*}$$
The econometrician has a more limited information set, however, in
that he observes only \( x \). The conventional approach, when forming a
guess about agents' expectations, would be to approximate \( P(y|x,z) \) in
(2.1) with the linear projection

\[
(2.3) \quad P(y|x) = a'x,
\]

which fulfills the orthogonality condition

\[
(2.4) \quad E ((y - \hat{a}'x)x_k) = 0, \quad k = 1, \ldots, K.
\]

This is in fact the econometrician's best guess of \( P(y|x,z) \) given
\( x \), since by the law of iterated projections (see, for example, Sargent
(1979, p. 208)), we have \( P[P(y|x,z)|x] = P(y|x) \).

What we argue in this paper is that the conventional approach
disregards information that is normally available to the econometrician,
namely the ex post outcome of \( y \), and that exploiting this information
results in an alternative guess about \( P(y|x,z) \) which can be very useful.
The remainder of this section presents some results regarding this
alternative guess. To simplify the exposition of the results, let us
adopt the shorthand notation \( y^e = P(y|x,z) \) for agents' expectations,
\( \hat{y}^e = P(y|x) \) for the conventional approach and \( \hat{y}^e = P[P(y|x,z)|x] \) for
our approach, and define the error terms \( \varepsilon, \tilde{\varepsilon} \) and \( \hat{\varepsilon} \) by

\[
(2.5a) \quad \varepsilon = y - y^e = y - P(y|x,z)
\]

\[
(2.5b) \quad \tilde{\varepsilon} = y^e - \hat{y}^e = P(y|x,z) - P(y|x)
\]

\[
(2.5c) \quad \hat{\varepsilon} = y^e - \hat{y}^e = P(y|x,z) - P[P(y|x,z)|x,y],
\]
where \( E(y^e \cdot \hat{e}) = E(\hat{y}^e \cdot \hat{e}) = E(\hat{e}^2) = E(\hat{e}) = 0 \) by the properties of optimal linear projections. The properties of \( \hat{y}^e \), that is of our proposed guess about agents' expectations, can then be summarized in the following proposition:

**Proposition:** In the above set-up the optimal linear projection of \( y^e \) on \( x \) and \( y \) satisfies

\[
\hat{y}^e = (1 - m)\tilde{y}^e + my
\]

The weight \( m \), which is given by

\[
m = \frac{\text{Var} \tilde{y}}{\text{Var} \tilde{y} + \text{Var} \tilde{e}}
\]

measures in a natural way the information advantage of agents over the econometrician, rising from zero when \( \text{Var} \tilde{e} = 0 \) - that is, when \( z \) does not improve the forecast of \( y \) given \( x \) - to unity when \( \text{Var} \tilde{y} = 0 \) - that is when agents can forecast \( y \) perfectly given \( x \) and \( z \) (or, alternatively, when they can observe \( y \) directly).

We now prove the first part of the proposition. Projecting \( y^e \) recursively on \( x \) and \( y \) (see Sargent pp. 206-08), we get

\[
(2.6) \quad \hat{y}^e = P(y^e|x) + P[y^e - P(y^e|x)]y - P(y|x)].
\]

Applying the law of iterated projections as above yields \( P(y^e|x) = P(y|x) = \hat{y}^e \). But this allows us to rewrite (2.6) as

\[
(2.7) \quad \hat{y}^e = \hat{y}^e + P(y^e - \hat{y}^e|y - \hat{y}^e)
\]

\[
= \hat{y}^e + m(y - \hat{y}^e)
\]
which is the desired result.

To verify the second part of the proposition, we need to determine the unknown projection coefficient $m$ in (2.7). From (2.5), we have

$$P(y^e - \hat{y}^e | y - \hat{y}^e) = P(\tilde{\varepsilon} | \varepsilon + \tilde{\varepsilon}),$$

so that

$$m = \frac{\text{Cov}(\tilde{\varepsilon}, \varepsilon + \tilde{\varepsilon})}{\text{Var}(\varepsilon + \tilde{\varepsilon})},$$

but since $E(\tilde{\varepsilon}) = 0$ it follows that

$$m = \frac{\text{Var} \tilde{\varepsilon}}{\text{Var} \varepsilon + \text{Var} \tilde{\varepsilon}}.$$

(2.8)

This formula bears out the close analogy between our procedure and the standard signal-extraction problem. We have two noisy signals $\hat{y}^e$ and $y$ from which we are trying to forecast $y^e$ optimally.

3. **Implications**

In this section we discuss some implications of the results in Section 2. Our proposition says:

$$y^e = \hat{y}^e + \hat{\varepsilon} = (1 - m)\hat{y}^e + m y + \hat{\varepsilon},$$

(3.1)

where $\hat{\varepsilon}$ is orthogonal to $\hat{y}^e$ and $y$, by the property of linear least squares projections. But since $y^e$ is unobservable we cannot estimate $m$ in (3.1) directly from the data.\(^2\) Suppose, however, that we have the behavioral relation

$$v = \beta y^e + \eta,$$

(3.2)
where \( \eta \) is a white noise residual and \( \text{Cov}(\varepsilon, \eta) = 0 \). Then, we can estimate the system

(3.3a) \[ v = \beta(1 - m)\tilde{y}^e + \beta my + (\beta\tilde{\varepsilon} + \eta) \]

(3.3b) \[ y = \tilde{y}^e + (\tilde{\varepsilon} + \varepsilon) = \sum_{k=1}^{K} a_k x_k + (\tilde{\varepsilon} + \varepsilon), \]

from which we can clearly identify both \( \beta \) and \( m \). In particular, we can test either the hypothesis that agents have perfect information - that is, \( H_0: m = 1 \) - or the hypothesis that they have no information advantage above the econometrician - that is, \( H_1: m = 0 \).

Although we may sometimes be interested in evidence on agents' information set in its own right, such evidence may also be important for shedding light on other economic phenomena. To illustrate this point we now formulate a simplified set-up, where there are potentially significant lags between agents' decisions, giving rise to buffer stock behavior.

Assume that time can be measured in elementary time periods of length \( \Delta \), where \( \Delta \) measures the interval between the representative agent's decisions as well as the interval between the arrival of new information about the variables in the agents' information set.\(^3\) We assume that there are \( n \) elementary time periods in a unitary time period; that is \( n\Delta = 1 \), where \( n \) is treated as an integer. As before, the random variable \( y \) is to be forecasted.

Consider the following behavioral model:

(3.4a) \[ v_t = v_{t-\Delta} + \eta_t = \beta_{t-\Delta} y_t + \eta_t \]

(3.4b) \[ w_t = y_t - v_t = (y_t - y_{t-\Delta}) + (1 - \beta)_{t-\Delta} y_t - \eta_t, \]
where \( t-\Delta v_t \) are decisions at \( t-\Delta \) about \( v \) at \( t \), and \( t-\Delta y_t \) are expectations at \( t-\Delta \) about \( y \) at \( t \). Thus agents decide at \( t-\Delta \) about \( v \) given their expectations about \( y \), and unanticipated changes in \( y \) are "buffered" by \( w \). One interpretation of (3.4) is that \( y \) is wealth and that \( v \) and \( w \) are the two assets in a portfolio, \( w \) being the more liquid one which acts like a buffer stock. Another interpretation treats \( y \) as sales, \( v \) as production and \( w \) as decumulation of inventories.

The econometrician has information about outcomes only at unitary intervals, however. He thus observes some variables affecting \( y_t \) at \( t - 1, t - 2, \ldots \); these are the variables in \( x \) in our previous notation. Agents in addition observe the same variables at \( t - \Delta, t - 2\Delta, \ldots, t - (n - 1)\Delta, t - (n + 1)\Delta, \ldots \); these (and maybe some other variables) are the variables in \( z \). In analogy with our previous notation we may thus denote the optimal forecast given information at \( t - 1 \), by \( y^e \) and \( t-\Delta y_t \) by \( y^e \). The conventional approach in estimating a model like (3.4) would be to estimate one of (3.4a) or (3.4b) and a forecasting equation, say,

(3.5a) \[ w = y - \beta y^e - (\beta \varepsilon + \eta) \]

(3.5b) \[ y = \tilde{y}^{\varepsilon} + (\tilde{\varepsilon} + \varepsilon) = \tilde{a}'x + (\tilde{\varepsilon} + \varepsilon) \]

(where we have dropped time subscripts to simplify the notation). A possible alternative would be to ignore the decision lags altogether and estimate

(3.6) \[ w = (1 - \beta)y - \eta. \]

Both (3.5) and (3.6) are, of course, totally arbitrary in that (3.5), à priori, sets \( \Delta = 1 \) and (3.6), à priori, sets \( \Delta = 0 \).
Our approach would instead call for using the approximation (3.1)

\[ w = y - \beta \bar{m} - \beta (1 - m) \hat{y}^e - (\beta \hat{\varepsilon} + \eta), \]

together with the forecasting equation (3.5b). With this approach we can explicitly test for the existence of buffer stock effects by testing the hypothesis that \( m = 1 \). By analogy with the discussion in section 2 it follows that the value of \( m \) is monotonically related to \( \Delta \), and rises towards 1 as \( \Delta \) falls towards 0.

We also want to discuss some econometric issues in connection with our results. Both \( \hat{y}^e \) and \( \hat{y}^e \) are obviously imperfect measures of the true value of \( y^e \) and would thus seem to create an errors-in-variables problem when used in a behavioral equation. But given the way \( \hat{y}^e \) and \( \hat{y}^e \) are constructed, it turns out that estimates of behavioral coefficients, such as \( \beta \) in equation (3.2), are (asymptotically) unbiased whichever of the two imperfect measures we use. However, we argue that using \( \hat{y}^e \) rather than \( \hat{y}^e \) results in more efficient estimation by lowering the variance in the estimate of \( \beta \).

Let us show these points formally in the context of the model (3.2). Using \( \hat{y}^e \) instead of \( y^e \) in that equation, the resulting estimate \( \hat{\beta} \) satisfies

\[ \text{plim} \ \hat{\beta} = \frac{\text{Cov}(\hat{y}^e, \hat{y}^e + \beta \hat{\varepsilon} + \eta)}{\text{Var} \ \hat{y}^e} = \beta, \]

since \( \text{Cov} (\hat{\varepsilon}, \eta) = 0 \) by assumption and \( \text{Cov} (\hat{y}^e, \hat{\varepsilon}) = 0 \) by property of optimal linear projections. Similarly, when using \( \hat{y}^e \) to estimate \( \hat{\beta} \), we have
(3.8b) \[ \text{plim } \beta = \frac{\text{cov}(\hat{y}, \hat{y} + \beta \hat{e} + \eta)}{\text{Var } \hat{y}} = \beta. \]

Intuitively, the assumption of rational expectations rules out any correlation between our imperfect measures of expectations \( \hat{y} \) and \( \hat{y} \) and the corresponding error terms in the estimated relation \( \beta \hat{e} + \eta \) and \( \beta \hat{e} + \eta \).

The variance of these two estimators are

(3.9a) \[ \text{Var } \beta = \frac{\text{Var}(\beta \hat{e} + \eta)}{\text{Var } \hat{y}} \]

and

(3.9b) \[ \text{Var } \beta = \frac{\text{Var}(\beta \hat{e} + \eta)}{\text{Var } \hat{y}} \]

From these expressions, and the fact that \( \text{Var } \hat{y} = \text{Var } y + \text{Var } \hat{\epsilon} \) and \( \text{Var } \hat{y} = \text{Var } y + \text{Var } \hat{\epsilon} \), it follows that the condition for \( \text{Var } \beta > \text{var } \beta \) is that \( \text{Var } \hat{\epsilon} > \text{Var } \hat{\epsilon} \). But this condition is always fulfilled (unless \( m = 0 \)). To see this, note that \( \hat{\epsilon} = (1 - m) \hat{\epsilon} - m \hat{\epsilon} \). Using this expression and the formula for \( m \) in (2.8) it is easy to show that

(3.10) \[ \text{Var } \hat{\epsilon} - \text{Var } \hat{\epsilon} = \frac{(\text{Var } \hat{\epsilon})^2}{\text{Var } \hat{\epsilon} + \text{Var } \epsilon} = m \text{ Var } \hat{\epsilon}. \]

Hence, proxying \( y \) in (3.2) with \( \hat{y} \) rather than with \( \hat{y} \) should lead to more efficient estimates of \( \beta \).
4. An application

In this section we report some results from an econometric study of the Swedish financial sector which is an application of the methods discussed in the previous section. The study is based on quarterly data from 1969 to 1982 and is discussed in Gottfries, Palmer and Persson (1984), which study we refer to for details on underlying assumptions and data. Here, we present only the estimates of the demand for reserves by banks.

Like in Section 3, banks are assumed to make decisions and gather new information at discrete intervals $\Delta$. The representative bank's plan at $t - \Delta$ for adjusted reserves (total reserves less net borrowing in the Central Bank) at $t$ is formulated in a conventional way, as

\begin{equation}
M^e_t = [k^e_t + \beta_1 + \beta_2 (r^p_t - r^b_t)] [D^e_t + S^e_t].
\end{equation}

Here, $k$ is the required cash ratio, $r^p$ and $r^b$ are the interest rates on borrowing in the Central Bank and short-term government securities, while $D$ and $S$ are ordinary and special deposits in the bank. Bank holdings of bonds $B$ and supply of loans $L$ at $t$ are also determined at $t - \Delta$. Banks do not control the demand for ordinary and special deposits, however. Ruling out unexpected changes in banks' wealth $A$, the portfolio identity

\begin{equation}
A_t = M_t + L_t + B_t - D_t - S_t,
\end{equation}

implies that actual reserve holdings satisfy

\begin{equation}
M_t = M^e_t + (D_t - D^e_t) + (S_t - S^e_t).
\end{equation}

Thus, unanticipated changes in the uncontrolled parts of the portfolio, from $t - \Delta$ to $t$, are buffered by changes in reserves.
To obtain proxies for the expectations in (4.1), we estimated forecasting equations for k, D and S at t based on an assumed information set at t - 1 (for interest rates we arbitrarily imposed perfect foresight to keep down the number of parameters to be estimated).\(^5\) The implied forecasts \(\tilde{k}^e\), \(\tilde{D}^e\) and \(\tilde{S}^e\) were used together with the actual outcomes to form guesses

\[(4.4a) \quad k_t^e = (1 - m_k)\tilde{k}_t^e + m_k k_t + \hat{\varepsilon}_k\]

\[(4.4b) \quad D_t^e = (1 - m_d)\tilde{D}_t^e + m_d D_t + \hat{\varepsilon}_d\]

\[(4.4c) \quad S_t^e = (1 - m_s)\tilde{S}_t^e + m_s S_t + \hat{\varepsilon}_s\]

along the lines in Section 3.

The behavioral equation to estimate follows from substitution of (4.4) and (4.1) into (4.2), yielding

\[(4.5) \quad M_t = \left[ k_t^e + m_k (k_t - k_t^e) + \beta_1 + \beta_2 (r^p_t - r^b_t) \right] \left[ D_t^e + m_d (D_t - D_t^e) + \tilde{S}_t^e + m_s (S_t - S_t^e) \right]
+ (1 - m_d) (D_t - D_t^e) + (1 - m_s) (S_t - S_t^e) + u_t,\]

where the error term \(u_t\) contains cross-terms involving the \(\hat{\varepsilon}\)'s in (4.4). We ignored any problems caused by the implied non-linearity of \(u_t\), however. Equation (4.5) and the forecasting equations for k, D and S were estimated jointly with non-linear three-stage least squares.\(^6\)

The estimates of the parameters in (4.5) are reported in Table 4.1. Of the \(m\)-coefficients, only \(m_d\) is significantly below 1. The estimates thus suggest that the lags in banks' decisions on reserves are very short (relative to a quarter). By contrast, the estimates of similar \(m\)-coefficients in the private sector's behavioral equations in the same
coefficients in the private sector's behavioral equations in the same study are consistently significantly smaller than unity, suggesting that the lags in the private sector's portfolio decisions are indeed significant enough to give rise to buffer stock effects.

5. Discussion

Above we have described a relatively simple method whereby an econometrician studying the behavior of some economic agents can estimate a coefficient, measuring how well informed these agents are relative to the information set he has specified. The econometrician can also carry out explicit tests regarding the the agents' information set. Since strong conclusions tend to go hand in hand with strong assumptions, it may be instructive to spell out the precise assumptions underlying our approach once more and compare them to conventional assumptions in some related work.

As explained in Section 3 our tests require that we estimate a model where agents' behavior depend on their expectations about some variable y. In line with virtually all empirical work we assume:

$H_0$: The postulated behavioral model is true, but we also require:

$H_1$: Given their information, agents form their expectations about y rationally in the sense of projecting y optimally on the variables in the information set.

$H_0$ and $H_1$ are treated as maintained hypotheses throughout our analysis of agents' information sets. To clarify the latter, let us introduce some new notation. Let $\Omega^e$ denote the information set of the econometrician, let $\Omega^a$ denote the information used by economic agents, and let $\Omega^y$ be the information set necessary to make a perfect (zero-
Table 4.1

Three stage least squares estimates of the parameters in Equation (4.5).

<table>
<thead>
<tr>
<th>$\beta_1$</th>
<th>$\beta_2$</th>
<th>$m_k$</th>
<th>$m_d$</th>
<th>$m_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.014</td>
<td>0.005</td>
<td>1.055</td>
<td>0.701</td>
<td>0.942</td>
</tr>
<tr>
<td>(0.005)</td>
<td>(0.001)</td>
<td>(0.270)</td>
<td>(0.131)</td>
<td>(0.427)</td>
</tr>
</tbody>
</table>

Standard errors in brackets.
known information but may be arbitrarily restricted by the 
econometrician to include a small number of variables.

In Figure 5.1 we illustrate a case without any particular 
restrictions on the relations between $\Omega^e$, $\Omega^a$ and $\Omega^y$. In this case a 
guess about agents' true expectations $P(y|\Omega^a)$, along the lines of 
Section 2, would yield

$$P(P(y|\Omega^a)|\Omega^e), \ y) = m_1 P(y|\Omega^e) + m_2 y,$$

but we would have no restrictions on the sum of $m_1$ and $m_2$. For the 
Proposition in Section 2 to hold we have to assume

$$H_2: \text{Agents have no worse information than the econometrician,}$$

that is $\Omega^e \subseteq \Omega^a$,

an assumption which is illustrated in Figure 5.2. When looking at 
related literature, we should look at empirical work that relies on $H_0$ 
and $H_1$. Then two strands of the literature come to mind: the work on 
efficient capital markets and the work on neutrality of aggregate demand 
policy. As far as we can see all work in these areas rely on a 
hypothesis like $H_2$. See Mishkin (1983) for a survey of both these 
strands of literature and for the references. Note that $H_2$ can be 
tested also but that this may require additional assumptions.\textsuperscript{7} In the 
context of the model (3.2), a test of $H_2$, under the maintained 
hypotheses $H_0$ and $H_1$, would amount to a test of $m_1 + m_2 = 1$ in the 
regression

$$v = \beta m_1 P(y|\Omega^e) + \beta m_2 y + (\delta \hat{e} + \eta).$$

But we see that $m_1$ and $m_2$ are not identified in this equation unless we 
are prepared to impose an à priori restriction on $\beta$ (which might be
Figure 5.1
Illustration of arbitrary information structure

Figure 5.2
Illustration of $H_2$

Figure 5.3
Illustration of $H_3$

Figure 5.4
Illustration of $H_4$
reasonable if (5.2) was, say, an arbitrage relation in an unregulated capital market).

With the maintained hypotheses \( H_0, H_1 \) and \( H_2 \) we may, however, go ahead and test the more interesting hypotheses:

\( H_3 \): Agents have no better information than the econometrician that is (given \( H_2 \)) \( \Omega^a = \Omega^e \),

illustrated in Figure 5.3, or

\( H_4 \): Agents have perfect information, that is \( \Omega^a = \Omega^y \),

illustrated in Figure 5.4.

There are also some other qualifications to our approach. One regards identification. To carry out the tests we have proposed we must obviously be able to identify the \( m \)-coefficient from an estimated equation. Although we have given examples when this may easily be done, the situation may not always be so favorable. Suppose that instead of (3.2), we have the relation

\[
(5.3) \quad u = \gamma(y - y^e) + \mu
\]

This "surprise" formulation is in fact quite common both in the capital market efficiency literature and in the neutrality literature. Substituting (3.1) into (5.3), we get

\[
(5.4) \quad u = \gamma(1 - m)(y - \tilde{y}^e) + (\mu - \gamma \hat{s})
\]

Here, \( m \) cannot be identified unless we are willing to make an à priori assumption about \( \gamma \). As an aside, we observe that the converse is also true. Therefore, the estimates of the behavioral coefficients in conventionally estimated surprise models will be downward biased unless \( H_3 \) is true.
Another qualification concerns the interpretation of the m-coefficient as a measure of information availability. Our exact interpretation holds only as long as we can treat the variable to be forecasted as strictly exogenous to the behavioral relation where m is estimated. When this cannot be assumed, say that v affects y in (3.2) for example, we have to resort to instrumental variable methods for estimation. Even with such simultaneity, we believe that similar m-coefficients can be estimated and given a meaningful interpretation as measuring agents' access to information, only that this access of information will be relative to the set of instruments. This is an issue that we will address in detail in future work.

In Section 4 we gave an example of a practical application of our proposed methodology. In that context the major information advantage of agents was that they observed some variables more frequently than the econometrician. We showed how a test of $H_4$ was equivalent to a test for the existence of significant decision lags in banks' portfolio revisions. One could easily think of other similar applications; for instance, tests for significant lags between price setting decisions of firms.

We believe there are also applications where one would be interested in testing whether agents have access to information which is not publicly known, apart from any advantage arising from more frequent sampling of information. In a suitably formulated model, a test of $H_3$ could constitute a test of "inside information" in the stock market.

Finally, we would like to repeat that our method yields an informed guess about agents true expectations ($\hat{y}^e$ above) where the resulting forecast error has a lower variance than that generated by the conventional informed guess ($\tilde{y}^e$ above). We saw in Section 3 that
this made it desirable to use our proposed guess to achieve higher efficiency in the estimation of behavioral coefficients. If one is simply interested in generating one accurate guess about agents' true expectation our method by definition yields an error with lower variance than the conventional approach. In this respect our approach is related to the work, such as that by Hamilton (1985), that tries to uncover a measure of agents' expectations from market behavior by help of Kalman filtering methods.
Footnotes

* We are grateful for comments by the participants in a seminar at the IIES.

1) With a quadratic objective function this is an optimal way to form expectations.

2) Unless we have an independent measure of expectations, of course.

3) A more elaborate model would distinguish decision and information lags and allow for staggered decision making across agents.

4) See, for instance, Modigliani, Rasche and Cooper (1979).

5) The information set was taken to be a set of bank and macro variables. The optimal forecasts of x was formulated as a regression of x on its own four lagged values and any four lagged values of the other variables in the information set that were jointly significant. Seasonal dummies and a constant were also added.

6) Cross-equation restrictions were imposed on the parameters in the forecasting equations. With instrumental variables estimation the interpretation of the m-coefficients has to be slightly modified. See further Section 5.

7) The tests of rational expectations in the literature on capital market efficiency, say, can also be viewed as tests of H2 under the maintained hypothesis of rational expectations.

8) The set of instruments may, of course, contain ex post realizations of variables which were not available to agents when they formed their expectations.

9) There is also a formal analogy in that our approach relies on recursive projection (cf. Section 2), which underlies Kalman filtering techniques.
References


Modigliani, F., R. Rasche, and P. Cooper, 1970, "Central Bank policy, the money supply and the short-term rate of interest", Journal of Money, Credit and Banking 2, 166-278.