Seminar Paper No. 126

DEMAND BEHAVIOR AND THE THEORY
OF INTERNATIONAL TRADE

by
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Demand Behavior and the Theory of International Trade

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The theory of international trade has long been recognized as a rich field of application for general equilibrium models that are sufficiently simple to reveal the crucial role of demand and supply in affecting prices and outputs in a single market or in a few interrelated markets. Although the importance of both blades of the Marshallian scissors has frequently been noted, I think it is fair to say that the role of supply has been singled out for prime consideration, especially in the classical developments of comparative advantage theory associated with Ricardo and Graham, and in the voluminous literature on Heckscher-Ohlin models with its emphasis on relative factor abundance and factor intensities. There are, however, some traditional questions in trade theory in which the specification of demand behavior assumes central importance. The possibility that growth may be immiserizing is one of these. The effect of transfer payments on the terms of trade is another. Perhaps even more basic is the concern with stability in equilibrium and the possibility of multiple free trade equilibria in commodity markets and markets for foreign exchange. It was this issue that Egon Sohmen addressed in his first publication, "Demand Elasticities and the Foreign-Exchange Market." [8]

In this paper I intend to discuss the importance of demand for some propositions in trade theory by explicitly considering two "extreme" forms of demand behavior and one "intermediate" form. My focus is on the degree of substitutability between commodities that consumers express in their
utility functions. In one extreme case indifference curves are right-angled (and homothetic), displaying no leeway for substituting a commodity that has become cheaper for a commodity that has not. At the opposite extreme is the case of parallel, linear indifference curves. The intermediate case is the assumption of Cobb-Douglas utility functions whereby smooth and continuous substitution in demand is always possible and indifference curves do not hit the axes. Figure 1 illustrates these three cases. Each is amenable to simple expressions and numerical illustration, thus making it feasible in what follows to suggest exercises whereby the trade theory propositions I wish to discuss can easily be illustrated.

Section I of the paper discusses the issue of stability and uniqueness of equilibrium which was of concern to Sohmen. Section II concentrates on the Cobb-Douglas form of the utility function to illustrate the crucial distinction in trade theory between demand elasticities and import demand elasticities. In Section III I illustrate how in a Ricardo-Graham model of world trade with many countries and commodities these simple forms of the demand functions delimit possible gains from free trade. Finally, in Section IV, I consider the question of technical progress in a Ricardoian model in order to illustrate how the pattern of potential country gains or losses depends crucially on the nature of demand, as illustrated by these three cases.

I. Stability and Uniqueness of an Exchange Equilibrium

The Edgeworth box-diagram is a useful device to illustrate the equilibrium positions of a two-country world in which each participant is endowed with fixed amounts of two commodities, X and Y. Total world endowments in the box diagrams shown in Figures 2-4 are the same: 10 units
of X and 8 units of Y. The underlying taste patterns differ: Figure 2 illustrates the fixed coefficients case of Figure 1(a), with country 1's utility function: \( U_1 = \text{minimum of } \{2X_1, Y_1\} \) and country 2's utility function: \( U_2 = \text{minimum of } \{X_2, 2Y_2\} \). Figure 3 shows the case of linear indifference curves with \( U_1 = \frac{1}{2}X_1 + Y_1 \) and \( U_2 = 2X_2 + Y_2 \). Finally, Figure 4's smooth contract curve shows man 1 with a taste bias towards good Y, but with underlying Cobb-Douglas utility functions.

The case of fixed coefficients in Figure 2 admits of a wide variety of possible types of competitive equilibrium, depending upon the endowment allocation of the fixed world totals of X and Y. For example, suppose country 1 is endowed with 6 units of X and 2 units of Y, with country 2 possessing the remainder, 4 units of X and 6 units of Y. This is point E. Point A emerges as a unique, stable, equilibrium with price ratio \( p_Y/p_X \) equal to 2. But point A represents a unique stable equilibrium only if the endowment point lies somewhere in the area southeast of A. By contrast, suppose country 1 only possessed Y and country 2 only owned X, as at endowment point E'. The price ratio shown by ray E'A is indeed an equilibrium \( (p_Y/p_X = 1/2) \), but it is unstable. For example, any slightly higher price for good Y shows world excess demand for Y (with ray \( O_1AB \) showing demand points for country 1 and \( O_2AC \) for country 2). As Sohmen argued, such an unstable point would be flanked by two stable equilibria. But in this case they are the extremes of a zero price for Y (with equilibrium consumption indeterminate on the chord \( O_1C \)) and a zero price for X (with consumption along \( BO_2 \)). This kind of solution, with multiple equilibria and the unstable equilibrium represented by point A, is associated with any endowment point lying northwest of point A. Other possible endowment points in the box admit of a single, stable equilibrium price (although
indeterminate consumption), within a range which is the extreme of free $X$ (for endowment points northeast of $A$ such as $E''$) or free $Y$ (for endowment points southwest of $A$).

The contract curve is, in the case of fixed coefficients, the shaded "band" in Figure 2. With substitution effects wiped out by assumption, only income effects are left to determine the issue of stability. As is commonly known, income effects work in a stabilizing manner if the marginal propensity to consume commodity $Y$ on the part of the net buyer exceeds that on the part of the net seller of $Y$. This is a comparison that depends not only upon tastes (in Figure 2 country 1 has a higher marginal propensity to consume commodity $Y$ than does country 2), but also upon the location of the endowment point. As I argued in [4], the presumption in favor of income effects working in a stabilizing fashion is reflected in the fact that the area southeast of $A$ is larger than that northwest of $A$, and that country 1 will import commodity $Y$ (for which it has a taste bias) if the endowment allocation is southeast of $A$.

Whereas Figure 2 depicts taste patterns allowing for zero substitutability in consumption, the opposite extreme is shown in Figure 3. Country 1's linear indifference curves are based on the utility function, $U_1 = \frac{1}{2}X_1 + Y_1$, and country 2's utility function is $U_2 = 2X_2 + Y_2$. All Pareto-optimal points lie on the edges of the box: $O_1 A_1 O_2$. Therefore in any competitive equilibrium at least one country specializes completely in consumption. Corner point $A$ has each country completely specialized, and will represent the competitive equilibrium for a wide range of possible endowment allocations. For example, point $E$, representing the same endowment allocation as in Figure 2 (country 1 possesses 6 units of $X$ and 2 units of $Y$), supports $A$ as a unique, stable, equilibrium with $p_y/p_x$ equal to unity. Country 1's
offer curve, ECA - extended, intersects country 2's offer curve, EBA - extended, at point A. If a cone (not shown) were drawn southeast from point A, with boundaries determined by the two indifference lines through A, any endowment point lying within that cone would support A as the unique, stable competitive equilibrium. But endowment allocations outside this cone exist that will support any Pareto-optimal point as a competitive equilibrium. For example, point B represents a unique, stable equilibrium if the endowments are allocated anywhere along country 1's indifference line through B (e.g. E'). Such an equilibrium would have all the gains from trade accruing to country 2, whereas any equilibrium interior to the 01A - section of the contract "curve" has the first country reaping all the gains from trade. Less extreme divisions of welfare are only possible at point A.

A comparison of these two extreme taste patterns reveals that with zero substitutability positions of unstable equilibria may emerge, whereas with the extreme degree of substitutability represented by linear indifference curves points of unstable equilibrium are ruled out. As the literature on stability makes clear, sufficiently high potential for commodity substitution can guarantee stability. But note that at an equilibrium such as point A in Figure 3 (assume the endowments are allocated at point E), each country specializes in consumption and remains specialized when prices are disturbed from their equilibrium values. That is, substitution between commodities does not take place in either country. That stability is nonetheless guaranteed is assured in an indirect manner: The high potential for commodity substitution represented by linear indifference curves has insured that the net buyer of commodity X (country 2) is the country with the higher marginal propensity to consume commodity X (unity, as opposed to a zero value for country 1 for
$\frac{p_y}{p_x}$ values between 1/2 and 2). This is the criterion for aggregate net income effects to be stabilizing. In Figure 2, for the case of zero substitutability, income effects are stabilizing for equilibrium point $A$ if the endowment point lies southeast of $A$. In Figure 3, the high degree of substitutability has guaranteed that points southeast of $A$ fill the entire box.

The case of Cobb-Douglas utility functions provides an intermediate example. It supports the smooth contract curve shown in Figure 4 if, as I assume, country 1 has a taste bias in favor of $Y$. For endowment points southeast of the contract curve, e.g. point $E$, in Region I, country 1 is an importer of $Y$, and therefore income effects act in a stabilizing manner. By contrast, endowment allocations such as $E'$ in region II lead to a trade posture in which the net seller of $Y$ (country 1) has a higher marginal propensity to consume $Y$ than has the net buyer.\(^1\) Therefore in region II aggregate income effects are destabilizing. However, as I discuss in the next section, the existence of the degree of commodity substitution represented by the Cobb-Douglas case suffices to guarantee elastic offer curves at equilibrium points such as $A$ or $A'$. That is, in region II destabilizing income effects must be outweighed by stabilizing substitution effects.

These three examples can be used to illustrate two propositions in the transfer literature. The first has to do with the question of presumption as to the movement in the terms of trade when, say, country 2

\(^1\)With Cobb-Douglas utility functions (or any homothetic functions) marginal and average propensities to consume are identical.
makes a transfer (of endowments) to country 1. Elsewhere I have argued for the anti-orthodox presumption that in a stable market the terms of trade are more likely to move in favor of the transferor ([4] and [5]). In Figure 4 a transfer towards country 1 moves the equilibrium point northeastwards along the contract curve. This must raise the equilibrium relative price of \( Y \) (this is what causes the \( Y/X \) consumption ratio in both countries to fall). If the endowment point lies in the (larger) region \( I \), country 1 is the importer of \( Y \), and thus the terms of trade have turned against the recipient of the transfer. A similar anti-orthodox result is shown for the fixed-coefficients case in Figure 2 by an endowment re-allocation from point \( E \) to point \( E''' \) or in Figure 3, for the case of linear indifference curves, by a move from \( E \) to \( E''' \). At point \( E' \) in region II of Figure 4 country 1 exports \( Y \). If country 1 receives an extra endowment bundle from country 2, country 1's terms of trade improve (the relative price of \( Y \) rises) - the orthodox result. However, returning to the fixed-coefficients case in Figure 2, a movement of the endowment point from \( E' \) to \( E'' \) represents a transfer to country 1, the exporter of \( Y \) if \( A \) is the equilibrium. If \( A \) remains the equilibrium, the relative price of \( Y \) would fall rather than rise, again leading to the anti-orthodox result. But \( A \) is a position of unstable equilibrium and the presumption argument presupposes a transfer from an initial stable point.

The second proposition in the transfer literature states that in an exchange model if the terms of trade move in favor of the transferor (the anti-orthodox case), their secondary "blessing" cannot be so large as to make the transferor better off. For example, an endowment transfer from \( E \) to \( E''' \) in Figure 4 would improve country 2's terms of trade, but
nonetheless worsen 2's welfare. But both extreme cases shown in Figures 2 and 3 reveal that if actual substitution possibilities are ruled out, the transferor's real income is unaffected by the transfers—the borderline case of the proposition. In Figure 2 the terms of trade change just balances the endowment transfer from \( E \) to \( E' \) to leave both countries at \( A \). In Figure 3 a transfer from \( E \) to \( E'' \) likewise causes the transferor's (country 2's) terms of trade to improve just sufficiently to restore the initial level of real income. Only when commodity substitution actually takes place will the transferor be worse off.

II. Cobb-Douglas Utility Functions and Demand Elasticities

The discussion of stability in the case of Cobb-Douglas utility functions made use of Figure 4's box diagram. Depicted there was the property of offer curves at free trade equilibrium such as \( A \), in which the endowment allocation was in the (larger) region I, or at a point such as \( A' \), with endowments in region II. In either case, Cobb-Douglas utility functions support elastic offer curves and, therefore, unique and stable equilibria.

Cobb-Douglas utility functions underlie what in the international trade literature are called Graham demand functions: The fraction of income spent on each commodity is a constant. Letting commodity \( X \) serve as numeraire, the simple case is one in which income is derived solely from a fixed endowment of commodity \( X \), \( X_0 \). Thus in the Graham case, 
\[
\frac{p_y}{p_x} \cdot X = \alpha X_0,
\]
where \( \alpha \) is the fraction of income spent on \( Y \). Figure 5 illustrates the vertical offer curve for this case. The quantity demanded of \( Y \) varies inversely with its price, with a constant outlay,
\(aX_0\), spent on \(Y^2\). Demand elasticity is unity.

Of more general interest for trade theory is the case of a mixed endowment bundle. For this case a simple exercise usefully serves to draw out the properties of the Graham demand function: Suppose we are given only one piece of information: With reference to Figure 5 suppose that if the endowment were to consist only of \(X_0\), as shown, consumption point \(A\) would be chosen at the appropriate price. Armed with this clue, draw the offer curve appropriate for the same utility function but with the new mixed endowment bundle shown by point \(E\) in Figure 5. The construction is spelled out in Figure 6.

Given the new mixed endowment point, \(E\), in Figure 6, it is useful, first, to construct the linear offer curves which would be appropriate if instead of endowment point \(E\) the endowment consisted (i) only of \(X_1\), the quantity of \(X\) in \(E\), or (ii) only of \(Y_1\), the quantity of \(Y\) in \(E\). In case (i) the vertical offer curve can be found by drawing a budget line through \(X_1\) parallel to line \(AX_0\). Point \(A\) was chosen when \(X_0\) was the endowment bundle at these prices, and, since Cobb-Douglas utility functions support homothetic indifference curves, \(A'\) would be chosen at the same relative price if the endowment bundle were \(X_1\), where \(A'\) and \(A\) lie on the same ray from the origin. Thus the vertical line through \(A'\) is the offer curve supported by endowment bundle \(X_1\). Now suppose prices were such that \(X_1\) and \(Y_1\) had the same value (this would require a higher relative price

\(^2\)The offer curve is well behaved for all price ratios except a zero relative price for \(X\). When \(X\) is free the utility level drops to zero since no \(Y\) can be obtained by exchange and none is available in the endowment bundle. For further discussion see Chipman, J., [1].
of $Y$, shown by the slope of a line connecting $\overline{Y}_1$ and $\overline{X}_1$). Then point $B$ would be chosen if the endowment bundle consisted only of $\overline{X}_1$, and $B$ would also be chosen if the endowment bundle were $\overline{Y}_1$ instead. That is, the offer curve for case (ii) would be the horizontal line through $B$.

With mixed endowment bundle $E$ (equal to $(\overline{X}_1, \overline{Y}_1)$) the offer curve is neither of these lines, but they serve as asymptotes for the new offer curve. To see this first find the slope of the offer curve at $E$. This is the same as the slope of the indifference curve at $E$. A ray from the origin through $E$ cuts the vertical asymptote at $C$. The slope of the indifference curve at $C$ is the slope of a line (not drawn) connecting $C$ with $\overline{X}_1$. By homotheticity the slope of the indifference curve at $E$ (and therefore the offer curve) is the same.

To find any other point on the offer curve draw a ray from the origin, e.g. ray $\lambda$. This intersects the vertical asymptote at $D$, and the slope of the indifference curve at $D$ would be given by the line (not drawn) connecting $D$ with $\overline{X}_1$. Now draw a budget line through endowment point $E$ with this same slope. It intersects the $\lambda$-ray at $F$, and $F$ is a point on the offer curve. In such a fashion can the offer curve through $E$ be drawn.

The offer curve in Figure 6 is everywhere elastic, as contrasted with Figure 7's portrayal of an offer curve which becomes inelastic for points above $B$ or to the right of $A$. That is, if relative prices are shown by the slope of a line through $E$ in Figure 7, steeper than $BE$, a reduction

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\(^3\)This follows from careful construction or from the explicit algebraic proof given below.
in \( Y \)'s relative price would increase net demand for \( Y \) but reduce net
exports of \( X \) required to obtain imports of \( Y \). This inelastic behavior
for the offer curve is never present in the Graham case.

Turn, now, to a simple algebraic calculation of demand elasticity
for the Cobb-Douglas utility function. Letting \( p \) represent \( Y \)'s relative
price, the demand function for \( Y \) is:

\[
Y = \frac{a(X + pY)}{p}
\]

The elasticity of demand for \( Y \), defined so as to be positive, is

\[
\frac{-p}{Y} \frac{dY}{dp} = \frac{X}{X + pY}
\]

This elasticity is everywhere less than unity. The key to reconciling
this result with Figure 6's illustration that the offer curve is everywhere
elastic is the distinction which must be made between the elasticity of
demand for a commodity and the elasticity of excess demand (which is what
the offer curve reveals). This distinction, in the exchange model, has
nothing to do with supply (or endowment) responding to price. Instead,
the original weights are different. The elasticity of demand for salt
may be quite low, but if imports are small (because demand does not
exceed endowment by very much), the elasticity of excess demand can be
quite high. In the Graham case, assuming \( Y \) is in positive excess demand, the
elasticity of excess demand is given by:

\[
\frac{-p}{(Y-Y)} \frac{dY}{dp} = \frac{X}{X - \left(\frac{1-a}{a}\right) pY}
\]
This elasticity always exceeds unity. \(^4\)

In the theory of international trade it is often the elasticity of excess demand that is crucial. Certainly this is the case in Sohmen's analysis of the foreign exchange market. \(^5\) The Cobb-Douglas utility function possesses the interesting property that if the endowment bundle is mixed, the elasticity of excess demand for imports always exceeds unity, while the elasticity of demand for the imported commodity always falls short of unity. If the endowment of the imported commodity were to shrink to zero, both elasticities would converge to unity. Suppose \(Y\) is imported. Regardless of the composition of endowments, a fall in \(Y\)'s price would always encourage greater demand. But the income effect, also encouraging greater demand as \(p\) falls, depends on the composition of the endowment bundle. If this consists just of \(X\), demand for \(Y\) is unit elastic. However, if the endowment bundle is a mixture of \(Y\) and \(X\), real income will not be as favorably affected by the fall in \(p\), and the increase in quantity demanded would be less than proportional to the price fall. But, as shown, the increase in excess quantity demanded would be more than proportional to the fall in price.

III. World Gains from Trade in a Ricardian Model

Gains from trade can be obtained even if no resources are allowed to shift into activities in which a country possesses a comparative

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\(^4\)The denominator is positive if \(Y\) exceeds \(Y\). Note that the aggregate elasticity of demand in the Cobb-Douglas case depends only upon endowments and not at all upon the single parameter of tastes, \(a\). Let \(\beta\) represent the proportion of income earned from ownership of \(X\), just as \(a\) represents the proportion of income spent on \(X\). Then the elasticity of demand for \(X\) in the Cobb-Douglas case equals \((1 - \beta)\), which depends on \(p\) but not on \(a\). Tastes do, however, influence the elasticity of excess demand.

\(^5\)The Marshall-Lerner stability condition, that the sum of import demand elasticities exceeds unity, is couched in terms of excess demand.
advantage. The model of commodity exchange is designed to bring out the nature of these consumption gains. The Ricardian model, by contrast, focusses on the gains in world output made possible by bringing countries together to face a single set of free trade prices. The concept of a world efficiency frontier, as discussed by Whitin, [9], and McKenzie, [7], displays maximal world output levels achievable under free trade. How much of an improvement does such a frontier represent compared with autarky world production levels? As I discussed in [3], such a comparison depends upon assumptions as to demand behavior in the various countries.

The case of two countries and two commodities can be used to illustrate properties that generalize to Ricardian-Graham models with any number of countries and commodities. In each country technology is summarized by a pair of \(a_{ij}\) labor coefficients, showing, for country \(i\), the quantity of man-hours required to produce a unit of commodity \(j\). These numbers are invariant with respect to output levels and different from country to country. No other inputs are required. Given the labor force in each country, the linear transformation schedules can be aggregated to yield the world efficiency frontier CBA in Figure 8. The method of aggregation involves summing outputs from each country as they face a common set of prices. Thus if country (1) has a comparative advantage in producing commodity \(X\), as illustrated, with free trade country (2) would produce no \(X\) until country (1) was already completely specialized in \(X\).

Although the frontier CBA displays output levels that would be observed with free trade, world outputs when countries do not engage in trade may also lie on this locus. For example, suppose country (2)'s tastes are so biased towards commodity \(Y\) that in a state of autarky it consumes
nothing but $Y$, whereas in country (1) tastes are less extreme, and
cconsumption (and production) of both commodities takes place. Then world
output levels under autarky would be shown by a point along the CB
section of CBA in Figure 8. A move to free trade might shift output
along the frontier, but would not show world output moving up to the
frontier from some inefficient point.\footnote{For example, with free trade more $X$ might be produced if at the relatively
low price for $X$ represented by the slope of CB country (2) is encouraged
to move away from specialization in consumption of $Y$.}

By contrast, taste patterns between
countries might differ in the opposite direction. For example, point D,
a point on the worst locus, CDA could represent autarky if tastes in (1)
were sufficiently biased towards $Y$ and in (2) biased enough in the opposite
direction towards $X$.\footnote{Points below the CDA locus could only be obtained if there exists some
unemployment.} Thus the potential for world output gains represented
by the opening of trade depends sensitively on assumptions about demand.

Suppose we impose the condition that countries have identical
taste patterns, and consider possible world autarky points for the
two extreme and one (Cobb-Douglas) intermediate type of utility functions
considered earlier. Start with the Graham demand functions supported by
identical Cobb-Douglas utility functions. World demand and production
of commodity $X$ under autarky would be shown by;

$$\sum \alpha \frac{w_i L_i}{p_x}$$

where $w_i$ and $p_x$ represent the wage rate and autarky price of $X$ in country
i and $L_i$ is i's fixed labor force. $\alpha$ is the fraction of total income spent
on $X$, constant for the Graham case and identical between countries by
assumption. But labor's marginal product in $X$, \((w_i/p_x^i)\), is the inverse
of the labor input coefficient, \( a_i^X \). Furthermore, dividing \( a_i^X \) into i's total labor force, \( \bar{L}_i \), shows the maximal output of \( X \) in country i, (assuming all labor is devoted to \( X \) production). Call this \( \bar{X}_i^* \). Therefore world output of \( X \) under autarky is \( a \) times \( \sum \bar{X}_i^* \), where \( \sum \bar{X}_i^* \) is distance \( OA \) in Figure 8. Similarly, \( Y \) production would be \( (1 - a) \sum \bar{X}_i^* \).

Depending on the value of the common taste parameter, \( a \), world output under autarky in the Graham case must lie somewhere on the linear chord \( CA \).

In a real, visual, sense, the Cobb-Douglas case is "intermediate" in the sense of yielding autarky outputs mid-way between the worst and the best set of world outputs.

Consider, now, the extreme case of linear indifference curves, but of the same slope in each country. Depending on the common value of this slope, obviously some point on the world efficiency frontier \( CBA \) is obtained. If taste patterns are similar, high degrees of commodity substitutability in demand already will force countries to concentrate labor forces fairly heavily into those commodities in which they possess a comparative advantage. Moving to free trade adds relatively little to world outputs.

By contrast, the case of rigid coefficients, but identical between countries, biases autarky production towards the locus of minimal world outputs, \( CDA \). This locus is never reached (except at the end-points), as tastes would have to be biased in a different fashion between countries, as illustrated earlier. Instead, world output would lie somewhere on the strictly bowed-in locus in Figure 8. The proof of the curvature of this locus is straightforward and not reproduced here.

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8 For each country require \( Y_i = \beta X_i^* \) for some common factor of proportionality, \( \beta \). As well, the transformation line has \( a_i^X X^* + a_i^Y Y^* = \bar{L}_i \). As \( \beta \) varies, \( Y = \Sigma Y_i \) can be derived as a function of \( X = \Sigma X_i^* \), since each is a function of \( \beta \). The second derivative of this locus must be positive if the slopes of the transformation lines differ. That is, radial aggregation of downward sloping straight lines of different slopes must always lead to an aggregate curve bowed in towards the origin.
But Figure 9 shows directly why this locus lies below the linear chord of the Cobb-Douglas case. The line $AB$ is country 2's transformation schedule and $AC$ is country 1's. $\tilde{X}$ and $\tilde{Y}$ represent, respectively, the maximal world outputs of $X$ and $Y$ and chord $\overline{XY}$ corresponds to the locus of autarky world outputs if each country shares identical Cobb-Douglas utility functions. For one such common function the vector $OE$ would represent autarky consumption and production for country 1 and $OD$ for country 2. World output would be given by the sum vector $OF$. Now suppose taste patterns were of the fixed coefficient variety and suppose in each country consumption and production adjust to the proportions shown by ray $OF$. Thus in country 1 output moves from $E$ to $H$, while in country 2 output adjusts from $D$ to $G$. These vectors of change are also shown by $FI$ and $IJ$, so that world output shrinks from $OF$ to $OJ$. In each country the identical fixed coefficient taste pattern has deflected labor resources towards the commodity in which that country has a comparative disadvantage.  

IV. Technical Progress and the International Distribution of Income in a Ricardian Model

The role of low demand elasticities in allowing growth in one country so adversely to affect its terms of trade that real income in the growing country is actually reduced has been thoroughly discussed in the trade literature under the heading of "immiserizing growth". Typically a two-commodity model is used in analyzing this issue. A three commodity model

\[9\text{ Requiring symmetry between countries at the input level instead of the output level leads to a better autarky locus. If all countries devote the fraction } \alpha \text{ of their labor force to good } X \text{ and } (1 - \alpha) \text{ to good } Y, \text{ world output levels lie on the } \tilde{X} \tilde{Y} \text{ chord in Figure 9. This is equivalent to the Graham demand function. For details see Jones, [3].} \]
can usefully reveal how large values of elasticities may not only guarantee that the growing country gains but that other countries may lose even if they rely completely on imports for the commodity in which growth has occurred.

More precisely, consider these issues in the context of a Ricardian model in which the home country produces two commodities (1 and 2) and the foreign country is specialized to a different commodity (3), and in which growth at home takes the explicit form of technical progress (a reduction of the labor coefficient) in producing the first commodity. Letting commodity 2's price be fixed, it is clear that the price of commodity 1 must be reduced by the extent of the technical progress. If the world market for commodity 3 were not disturbed (i.e. its price did not change), both countries would benefit by the technical progress at home, in amounts proportional to each nation's consumption of commodity 1. But the market for the single commodity (3) produced abroad may be affected, and changes in p₃ involve further real income adjustments. The nature of these required changes depends in large part on the nature of demand in both countries. I now analyze this issue with the use of the three types of demand behavior considered in this paper.

First, suppose utility functions in each country are of the Cobb-Douglas variety, supporting Graham demand functions, not necessarily the same in each country. Abroad let α₃* indicate the constant fraction of income spent on commodity 3:

\[ p₃ D₃ = α₃^* Y \]

10 For details of this problem see Jones, [6], Chapter 17.
D₃* refers to foreign demand for commodity 3 and Y* indicates the value of foreign income, which is derived solely from its production of commodity 3. With a given technology and labor force abroad, Y* would be unchanged at given p₃ when p₁ is reduced (because of home technical progress in the first industry). That is, the reduction in the price of imported commodity 1 abroad would not serve to shift either the foreign supply curve (vertical) for good 3 or the foreign demand curve.

At home let a₃ denote the constant fraction of income spent on commodity 3:

\[ p₃D₃ = a₃Y \]

By the same reasoning, the home demand curve for commodity 3 will shift to the right as p₁ is lowered if Y increases, to the left if Y decreases, and remain undisturbed if Y is constant. Y is given by \( p₁x₁ + x₂ \), where commodity 2 has been chosen as numeraire. Technical progress in the first commodity swings the home country's transformation curve (line) outwards around the vertical x₂-axis. That is, Y, the value of national income produced in units of the second commodity, remains unchanged, as does D₃ at initial p₃. Thus the Graham case leads to a situation in which technical progress in one commodity does not disturb the markets (and prices) for others.¹¹

Consider, now, the case of right-angled indifference curves. As Section I reveals, stability rests entirely on income effects, and in order to insure stability it is necessary to assume that the importer's marginal propensity to consume commodity 3, m₃, exceeds that of the foreign exporters, m₃*. If so, the world's demand curve for commodity 3, \( D₃ + D₃* \) in Figure 10, is

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¹¹This characteristic of Graham demand function was skillfully employed by Dornbusch, Fisher, and Samuelson, [2], to allow a simple graphical device in the case of a continuum of commodities in a Ricardian world.
negatively sloped, while the foreign supply curve, $x^*_3$, is vertical. This world demand curve must shift to the right if all substitution effects vanish. At initial $p_3$, the reduction in $p_1$ raises real incomes throughout the world to all consumers of the first commodity. Barring the case of inferiority, both $D_3^*$ and $D_3^*$ shift rightwards and $p_3$ must rise. Clearly if the shift in world demand towards foreign commodity 3 raises $p_3$ by a large amount, the home country, for whom this represents a terms-of-trade deterioration, may end up with a net loss of real income. With right-angled indifference curves, this case of immiserization must take place. To see this, suppose $m_3^*$ is at the boundary value of zero so that the only source of shift in the $(D_3 + D_3^*)$ curve comes from the home country. In such a case $p_3$ must still rise (say from $A$ to $B$ in Figure 10) by exactly enough to choke demand back to its initial level. But the only source of the increased demand at initial $p_3$ ($A$) was the increase in home real income. Therefore at $B$ the price of commodity 3 has risen just enough to reduce home real income to its initial level. If $m_3^*$ were positive, as I now assume, $p_3$ must rise to a level above $B$ in Figure 10, and home real income must be reduced. Thus the possibility that the home country can escape immiserization depends on sufficiently high substitution response in the two countries.

As technical progress lowers the price of the first commodity, income effects in both countries (at initial $p_3$) spill over to exert upward pressure on the foreign price, $p_3$, but substitution effects serve to syphon demand away from good 3 towards lower-priced good 1. In the Cobb-Douglas case these two effects just balanced each other. With stronger substitution effects, foreigners would find the price of their exports reduced, thus also finding some of the gains in real income foreigners achieve as technical progress lowers their import price, $p_1$, eroded. Can $p_3$ be reduced sufficiently
that foreigners actually lose? As I prove in [6], Chapter 17, large values for substitution effects can lead to such a paradoxical lowering of real incomes abroad as a consequence of the initial foreign improvement in its terms of trade (as $p_1$ falls).

The extreme case in which indifference "bowls" are linear reveals the issues involved. If both countries initially consume all three commodities, prices must correspond to the slopes of common indifference planes. In such a case a lowering of costs of producing commodity 1 at home drives out all good 2 production. Foreign producers of commodity 3 can stay in business only if foreign wages fall sufficiently that $p_3$ is lowered by the same relative extent as $p_1$. Foreign real incomes are completely unaffected by such a change. Therefore extremely large values for substitution terms biases the international division of real income gains associated with the technical progress in one commodity at home towards most (or all) of the gain accruing to the country experiencing the technical progress.

An asymmetry between the degree of substitutability between the foreign commodity (3) and the commodity in which the home country experiences technical progress (1) on the one hand and goods (3) and (2) on the other allows an even greater asymmetry in the real income spillovers. Suppose indifference surfaces are such that, once again, commodities 3 and 1 are perfect substitutes in both countries but commodity 2 is less than a perfect substitute for either commodity. In this case a small technical improvement in producing commodity 1 does not wipe out the second industry at home, although it may drastically curtail demand. But foreign price, $p_3$, must fall by the same relative extent as does $p_1$. The change in foreign real income is obtained by balancing off two opposing terms-of-trade effects. On the one hand foreign real income rises by an amount proportional to foreign
imports of commodity 1. On the other, foreign real incomes fall by an
amount proportional to foreign exports of commodity 3. Since foreign
exports of commodity 3 (its only export) must exceed its imports of
commodity 1 if it also keeps importing some of commodity 2, and since
p₁ and p₃ fall by the same proportional amounts, foreign real incomes
must fall.

The character of the substitution possibilities in demand is thus
crucial in explaining how the world gains associated with technical progress
are distributed between the home country experiencing the progress and
the foreign country, which imports this commodity. Low substitution
possibilities suffice to cause the home country's growth actually to be
immiserizing. "Intermediate" values for substitution, represented by the
Graham case, call for a division of real income gains between countries
according to their consumption of the commodity which has become cheaper.
Extremely high values for substitution possibilities in demand serve to
cause the price of foreign exports to fall and tend to wipe out the
possibility of foreign gains. Asymmetry in substitution possibilities
of the appropriate kind can cause foreigners on balance actually to lose.

V. Concluding Remarks

In this paper I have discussed some of the issues in the pure theory
of international trade in which the role of demand is crucial by illustrating
how results differ in the two extreme cases of zero and infinite substitutability
between commodities in consumers' taste patterns and in the intermediate
case of the Cobb-Douglas utility function. The latter case, leading to
the Graham-type demand function, was shown to be "intermediate" in several
senses: (1) If countries have identical Cobb-Douglas tastes, autarky world
production lies on a chord exactly intermediate between the best and the
worst set of possible world outputs, (ii) If countries have mixed endowment bundles, offer curves are always elastic, whereas ordinary demand elasticities are always less than unity. The degree of substitutability embodied in the Cobb-Douglas function is more than sufficient to guarantee unique, stable, equilibria; (iii) Even without similar tastes between countries, Cobb-Douglas utility functions always support the neutral proposition that (in a Ricardian trading world) an improvement in technology anywhere in the world benefits people everywhere. Indeed, in the Ricardian case discussed in Section IV, Cobb-Douglas utility functions guarantee that the only price change is that of the good in which technology has been improved relative to all other goods. If all other goods were produced at constant labor costs in one country, this lack of disturbance to other prices would be guaranteed by the technology. But in the Cobb-Douglas case this independence spreads also to markets in countries producing dissimilar commodities.

The possibility of multiple equilibria, with some equilibria locally unstable, is introduced if substitution response in demand is particularly weak. Furthermore, even if the market is stable, growth may turn out to be immiserizing as low demand response requires large price changes in order that markets clear. Perhaps less obviously, section IV demonstrated that high response of demanders to price changes brought about by technical progress in one country could cause other countries actually to suffer losses even if no import-competing industries are directly threatened.

Finally, the Graham demand assumptions illustrate how a transfer can cause the terms of trade to move in either direction, but nonetheless with a presumption that they turn in favor of the transferor - an "anti-orthodox" result. In the extreme cases either of no substitutability of demand or of linear indifference curves this presumption becomes a certainty:
If the equilibrium is locally stable, the transferor cannot suffer a "secondary burden" in the form of deteriorated terms of trade. Indeed, a transfer leads to a "secondary blessing" to such an extent that real incomes in each country are unaffected.
(a) Fixed Coefficients
(b) Linear
(c) Cobb-Douglas

Figure 1
Figure 5
Figure 6
Figure 10
REFERENCES


