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THE MEASUREMENT OF WELFARE LOSS
FROM TARIFFS: SOME FURTHER EXPLORATIONS

by

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THE MEASUREMENT OF WELFARE LOSS FROM TARIFFS: SOME FURTHER EXPLORATIONS

by Ephraim Kleiman and Michael Michaely

Wide currency has been gained in recent years by the so-called "triangles method" for estimating the welfare impact of the imposition of a tariff or its removal. While this approach faces a few well-known conceptual problems, it seems that no better substitute has yet been devised for this purpose. In the present paper we shall adopt this method, overlooking its difficulties, and attempt to widen its scope by applying it to issues which seem to have hitherto been subject to only little exposition. Three topics will be dealt with. The first, that of the welfare impact of a customs union, relates the measurement of the resultant gains or losses to the theoretical concepts of trade creation and trade diversion. The second, that of the welfare impact of tariffs imposed on the import of a capital good, incorporates into the analysis the treatment of the dynamic process of adjustment: it will be shown that though protection of a capital good does not affect the eventual size of the capital stock, it causes a welfare loss through its effect on the adjustment path. Finally, we analyze the welfare loss in an industry where protection results in a negative value added, so that the estimated effective rate of protection is negative. It will be shown that while the effective rate itself does not appear in the loss measure, all the components of which it consists participate in determining the size of the loss.

First, however, to provide a suitable framework, we shall briefly reconstruct the basic skeleton of the method under consideration.

1. The application of this approach to the theory of tariffs was started mainly in the contributions of W.M. Corden (1957) and H.G. Johnson (1960).

2. For a fuller exposition, E.E. Leamer and R.M. Stern (1970, Chapter 8) is probably the best reference.
1. Measurement of Welfare Losses: The "Triangles Method"

We shall assume throughout a situation in which free trade is unequivocally the best policy, and trade restriction a policy which reduces welfare. We deal, thus, with an economy in which all Pareto-optimum conditions are fulfilled—no domestic distortions are found; and which is small, having no effect on its terms of trade with the outside world. The welfare loss from trade restriction (or the gain from its removal) has to be given a quantitative expression, in terms of the amount of goods made available to the economy. In the following, a two-good world is assumed. The welfare effect is presented by means of Figure 1, where exports are measured along the horizontal, and imports along the vertical, axis.³

PQ is, then, the transformation curve. With free trade conducted at the world prices represented by exchange line ww, production will be at point B, and consumption somewhere to its left on the budget-restraint line ww. This is precisely equivalent to a situation in which a quantity OR of X is produced by the economy (or in any other way made available to it), and the economy is free to move along exchange line ww—that is, every individual is free to trade at the given world price. For this full equivalence to be maintained, however, it is required that the hypothetical production of OR of X will involve the same income distribution as at production point B. For simplicity, we shall assume for the moment that the community consists of a single consumer, so that this problem is circumvented—at least at the present stage.

With a tariff on import good M, tt represents the domestic exchange line, and A will be the production locus. The consumption locus is at some point C, to the left of A on the cum-restriction budget-restraint line w'w'. The economy (the single consumer) could be

³ Of the various possible measures of the welfare effect, we look here for the Hicksian "compensating variation". See the discussion in J. Bhagwati and H.G. Johnson (1960).
at this consumption locus had it produced (or had it otherwise received) the quantity OS of X, and traded part of it freely, at the existing world prices. The difference of SR is thus a measure of loss to the economy: while employing all its resources, as under free trade, it acquires a bundle of goods worth SR units of X less than the bundle of goods acquired under free trade. This is defined as the production cost (or loss) of trade restriction.

But suppose the economy had indeed produced OS of X, and traded it freely at world prices represented by exchange line w'w'. The price of the import good facing home consumers would then be lower, and of the export good higher, than at the cum-tariff price ratio tt. Hence, consumers (our individual consumer, under the present assumption) would not consume the bundle C, but some combination to the left of it – such as C' – representing more of M and less of X. Since C would still be an open consumption possibility, this would mean that C' is superior to C; that is, a welfare gain will be realized from the removal of distortion faced by consumers. To find a position which is not superior to C but equivalent to it, we look for a quantity like OT. Had the economy produced OT of X, and traded it freely at the world price, consumption locus would be at D – which is thus the best possible location on w"w"; and this, we assume (so was point T selected) provides the consumer with just the welfare level afforded by consumption locus C. Thus, TS is an added loss from trade restriction – defined as the consumption cost (or loss). The welfare of the community is equal, under trade restriction, to what it would have been with the production (the availability) of OT of X, and the free trading at world prices. The total loss from trade restriction is thus TR of X, divided between a production loss of SR and a consumption loss of TS.

With information about the transformation curve (which is derived, in turn, from knowledge about production functions and the economy's factor endowment) and the consumer's preferences, as well as data about international prices and the tariff rate, we are now able to measure the cost of protection – in the sense just illustrated. It will greatly facilitate the measurement, however, if this information is presented by way of supply and demand schedules, which
may be derived from the transformation curve and the consumer's preference function. This is done in Figure 2, where M, the import good, is represented on the quantity axis, and the price of M in units of X on the price axis. The quantity of X, which we have adopted as the measuring yardstick for losses or gains, is thus given by the relevant area in each case (the area being M times X/M, thus yielding units of X).

The curve DD, derived from the consumer's preference function, represents a "compensated" demand curve for M, the import good. We know that with a demand curve so constructed, the area bound by the curve above any designated price represents (in terms of units of X) the amount of "consumer's surplus" at this price.

The price OP of M is the world price of M (derived from exchange line WW in Figure 1). At this price, the consumer would have bought OЕ (=PЈ) of M. The tariff raises the home price to OP' (derived from exchange line tt in Figure 1). At this higher price, consumption of M will fall to ОF (=P'L). It may now be shown that the number of units of X yielded by the (shaded) area JKL is identical with the amount TS in Figure 1; that is, this area represents the consumption loss involved in the imposition of the tariff.4

SS, in Figure 2, is the supply curve of M, derived from the transformation curve of Figure 1: the quantity produced is "read off" the transformation curve, as the price of M in terms of X varies. At the free-trade price, OP, home production of M will be ОH (= PN). The tariff, which increases the home price to OP', raises production to ОG (= P'V). It may again be shown, in a similar way, that the (shaded) area NUV is equal to SR units of X (in Figure 1); that is, this area measures the production loss. Thus, the total loss from protection - TR units of X, in Figure 1 - is yielded by the combination of the shaded areas JKL and NUV.

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FIGURE 2
An alternative way of arriving at this conclusion — used in the original development of the measurement by Corden (1957) and Johnson (1960) — is as follows: The imposition of the tariff, which raises the home price from $OP$ to $OP'$, reduces consumer's surplus by the area $PP'LJ$. At the same time, however, producer's surplus rises, with this increase in price, by the area $PP'VN$. In addition, a "surplus" is accrued to the government, which collects now revenues from the import duty: this revenue (again, in terms of units of $X$) is equal to the size of imports ($UK = GP$) times the tariff per unit ($PP'$), that is, to the rectangle $UKLV$. When these additions — to producers and to the government — are offset against the consumer's loss, the net loss to the community is found to be the combination of the two shaded areas, $JKL$ and $NUV$. For some purposes it would be more convenient to use this exposition rather than the former. Accordingly, we shall alternate between the two in our further explorations.

Assume, for simplicity, that $DD$ and $SS$, the demand and supply curves, are straight lines at the relevant range (which they will approximately be if the range is relatively short); the two areas under consideration are thus approximated by two triangles, the size of which is to be estimated. Defining world price ($OP$) to be unity and designating the free-trade demand ($OE$) by $Co$, the free-trade production of $M(OH)$ by $Qo$, the change in price by $t$ — the tariff rate (since originally $P = 1$), the elasticity of demand (defined to be positive) by $\eta$, and the elasticity of supply by $\epsilon$, we may easily see that:

\[(1) \quad \text{area } JKL = \frac{1}{2} t^2 \eta Co,\]

and

\[(2) \quad \text{area } NUV = \frac{1}{2} t^2 \epsilon Qo,\]

The total loss from the imposition of the tariff — the combination of the two areas — becomes therefore:

\[(3) \quad JKL + NUV = \frac{1}{2} t^2 (\eta Co + \epsilon Qo).\]
We may prefer to put this loss in relative terms - relative, that is, to either production or consumption. If we chose, for instance, the former, the relative loss will become $t^2/2 \left( \eta C_0/Q_0 + \epsilon \right)$. It is a function of the tariff rate, of the two elasticities, and of the ratio of free-trade consumption to home production of the import good.\(^5\)

We have assumed, for simplicity, that the community consists of a single consumer. But the argument, and the conclusions, will not be affected by the admission of many consumers - so long as for each of them the same demand experiment is carried out, namely: each individual's demand curve is "compensated", leaving him on the same welfare level regardless of price changes, thus yielding the change in quantity demanded by him due to the substitution effect. Such individual demand curves could be aggregated, to an income-compensated demand curve for the community, information about which will form - as it did in the case of a single consumer - the basis for the calculation of the consumption loss from protection.

2. The Welfare Impact of a Customs Union

In this section we shall extend the measurement of welfare losses and gains to the case when a country's tariff is not removed universally but in a geographically discriminating fashion - the "customs union" case. All the former assumptions are retained. In addition, the outside world is assumed to consist of two countries: country B, with which the home country (A) enters into a union by which the tariff on A's imports from B is abolished; and country C - the rest of the world - the tariff on imports from which

\[^5\text{It may easily be seen that}\]

$$n_m = \frac{n C_0 + \epsilon Q_0}{M_0}$$

where $n_m$ is the elasticity of demand for imports and $M_0$ is the free-trade size of imports. Hence, (3) may be re-written in the form

$$(3') \ JKL + NUV = \frac{1}{2} t^2 n_m M_0.$$  

The size of the loss appears here as the function of the tariff rate; the elasticity of demand for imports; and the size of imports.
is unaffected by the union. The home country is "small" vs. both foreign countries, its trade flows having no effect on the prices in either. As before, the measurement pertains to the welfare of the home country alone. It is assumed, to make the analysis of any relevance, that in the absence of the union trade is conducted with country C; whereas the union leads to the shift of the source of imports to country B.

In Figure 3, the pre-union home price (with a tariff t) is represented by the slope of tt; home production is at E; and trade with country C takes place to its left along c'c'. Following the union, home production shifts to G and trade with country B takes place to its left, with home consumption somewhere on bb. Since we have already shown the measurement of the consequences of a shift from restricted trade to free trade, or vice versa, it will be convenient now to view the act of the union as if it consisted of two separate steps. First, the tariff t is abolished altogether, leading to free trade; and second, the tariff is re-imposed on imports from country C, leading to the shift of trade from C to B. The first step having been analyzed before, we only have now to measure the consequence of the second.

Under free trade, home production is at F, and consumption somewhere to its left on budget-restraint line cc. With the shift from free trade to the union (the imposition of a tariff on imports from C), home production moves to G. With free-trade international prices, this is equivalent to a loss of RS units of X: had this amount been added to the country's product, the economy would be again on its free-trade budget-restraint line cc. Distance RS thus measures the production loss, in units of X, of moving from free trade to the union.
Figure 3
With the union, however, the economy's budget restraint is not \( c_1c_1 \), but rather \( bb \), since trade now takes place with \( B \). Suppose home consumption is now at \( H \), with \( GH \) being the trade vector. Had the country been still trading with \( C \), at the latter's prices, a quantity of \( OU \) of \( X \) would have been just sufficient to allow the country — with a budget restraint line \( cc_2 \) — to secure the consumption basket \( H \). The distance \( SU \) is thus an added loss — a trade-shift loss — due to the move from free trade to the union. It is easily seen that this loss is equal to the amount of imports represented by the trade vector \( GH \) times the excess of country \( B \)'s price of imports \( M \), in units of \( X \), over the price in country \( C \).

But, finally, had the economy been indeed on budget-restraint line \( cc_2 \), with free trade, the consumption-basket selected would have been not \( H \) but another basket (to its left). Since \( H \) would still be a possibility, it must be inferior (or, at the limit, just equal to) the alternative basket. To make consumers just as well off as at \( H \), with free-trade prices, the amount \( OV \) of \( X \) would be sufficient: the best location on \( cc_3 \) would just yield the same welfare as consumption-basket \( H \). The distance \( UV \) is hence this added loss — the consumption loss from the shift from free trade to the union. The combined loss of this shift — the production, trade shift, and consumption losses added together — thus amounts to \( RV \). When this is set against the gain of the first step — the movement from trade restricted by universal tariff to free trade — the net gain or loss of the union will be found. 6 The combination of all these elements will now be done, as before, by the use of Marshallian supply and demand curves, the supply curve being derived from the transformation curve. This is done by means of Figure 4.

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6. When \( cc_3 \) is to the right of \( cc' \), as shown here, the net result of the union is certainly a gain. When the former is to the left of the latter, there could still be a gain. Home prices before the union are those represented by \( tt \), rather than by \( cc' \); whereas with the union, the home price is equal to the international price (\( bb \)). Hence, a budget-restraint line left of \( cc' \) within a certain range would be sufficient to secure the welfare level reached under the restricted trade (this reflects, of course, the consumption loss of the tariff).
The supply and (compensated) demand curves are, respectively, SS and DD. $P_c$ is the price of $M$, in units of $X$, in trade with country $C$, and $P_b$ the price in trade with $B$. The size of the tariff per unit of $M$ is $t$, and the pre-union home price, $P'_c$, is equal to $P_c + t$. Home consumption of $M$ before the union is $P'_c L$, home production is $P'_c V$, and the difference $VL$ is provided by imports (from C).

We now consider, as before, the act of the union as if it consisted of two separate steps. First, the tariff is abolished altogether, and free trade is introduced. Home price will become $P_c$, home production $P_c E$, home consumption $P_c Z$, and imports (from country $C$) $EZ$. The gain from this step, measured in units of $X$, is the combination of the two triangles $EGV$ and $ZRL$. The second step involves a re-imposition of a tariff on imports from $C$ (whether at the size $t$ or any other size which would make the union effective). This will shift the source of imports to country $B$. The new home price will become $P_b$, home production will be $P_b N$, home consumption $P_b J$, and imports (now from country $B$) $NJ$. This will involve losses of two kinds. First, like the imposition of a tariff at the rate equal to the excess of $P_b$ over $P_c$, production and consumption losses equal to the triangles $EFN$ and $ZTW$, respectively, will be incurred; these are equal, respectively, to the production loss $RS$ and the consumption loss $UV$ in Figure 3. Second, the fact that the amount of imports $NJ$ is now bought at the price $P_b$ rather than $P_c$ (as it would have a uniform tariff equal to $P_b - P_c$ been indeed imposed), involves an additional loss equal to the amount of imports times the price differential, that is, to the rectangle $FJNW$; this area is the equivalent of the trade-shift loss $SU$ in Figure 3. The combined loss from the shift from free trade to a union thus amounts to the area $EZJN$. When this loss is set against the gain of the first step - the move to free trade - the net result appears as follows. Triangles $NUV$ and $JKL$ represent gains from the union: these are, respectively, in terms of the conventional classification in customs unions theory, the (positive) trade-creation and consumption effects of the union. The rectangle $GRKU$, on the other hand, represents a loss: this is the (negative) trade-diversion effect, equal to the amount of pre-union imports ($VL$, or $UG$) times the excess price ($P_b - P_c$, or $UG$) borne now by
each unit of these imports. In terms of the quantities and elasticities involved, using the former designations and adding $M_o$ as the pre-union import of $M$, these sums may be written as:

\[(4) \quad \text{NUV} = \frac{1}{2} (P'_c - P_b)^2 \varepsilon Q_o ; \]

\[(5) \quad \text{JKL} = \frac{1}{2} (P'_c - P_b)^2 nC_o ; \]

\[(6) \quad \text{GRKU} = (P_b - P_c) M_o , \]

The combined area of these three sums, measuring the net gain and designated by $W$, is thus:

\[(7) \quad W = \frac{1}{2} (P'_c - P_b)^2 (\varepsilon Q_o + nC_o) - (P_b - P_c) M_o ; \]

or, since $P'_c = P_c + t$

\[(8) \quad W = \frac{1}{2} \left( t - (P_b - P_c) \right)^2 (\varepsilon Q_o + nC_o) - (P_b - P_c) M_o . \]

This net change could obviously be either positive or negative. We may easily verify, by use of (8), some propositions regarding the likelihood of the union resulting in a gain or a loss. Thus, we may first see immediately that the higher is $t$, the pre-union tariff, the higher is the gain component and the more likely the result of a net gain. Second, the lower is $M_o$ in relation to $Q_o$ or $C_o$ - the lower, that is, the pre-union relative size of imports - the lower is the loss component and the more likely a net gain. Third, the closer $P_b$ is to $P_c$ - the less the excess price of imports in the union partner over the price in the rest of the world - the higher will be the positive component, the lower the negative component, and the more likely a net gain. Fourth, the higher the elasticities of home supply and home demand of the import good, $\varepsilon$ and $\eta$ (in a two-good world, this also implies high elasticities for the other good), the higher the positive element and the more likely a net gain from the union. Finally, the fuller the tariff elimination in the union is, the more likely is a net gain: if the tariff were not removed completely but only partially, $t$ in (8) would have to be replaced by $t[1 - (1-r)P_b]$ where $r$ is the proportion of the tariff removed; the positive
component of (8) would then be lowered, whereas the negative element would be unaffected - provided, of course, that this partial reduction of the tariff still makes the union effective and the analysis relevant.\(^7\)

3. The Cost of Protecting Capital Goods

Hitherto a world of two final goods has been assumed. We shall now expand this framework, and introduce capital goods. In some important senses, capital goods are similar to other intermediate inputs. Had space permitted, a proper procedure would have probably been to introduce first an intermediate good, describing the method of estimating the loss from its protection, and then proceed to the special features of a capital good. We skip, however, the first stage of such a procedure and merely draw attention to the fact that the analysis of the cost of protecting intermediate goods could easily be gathered from the elements of the discussion of protecting capital goods.

\(^7\) This last conclusion is strictly applicable only to the two-good world assumed in this analysis. The accepted convention in customs-unions theory, due to the contributions of J.E. Meade (1955) and R.O. Lipsey (1960), asserts that a union is more likely to result in a welfare gain with a partial preferential reduction of the tariff rather than with a full (preferential) tariff elimination. This proposition has been derived explicitly or implicitly, within a multi-good world. It may be shown, however, that even in that framework it cannot be demonstrated as a general rule.

It may be mentioned that an application of the "triangles method" to the estimate of the welfare impact of a customs union has essentially been made by H.G. Johnson (1962, pp. 63-74). Johnson's analysis there is, however, rather involved, mainly due to the assumption of variable international prices; as a result, the relationship of the estimating method to the basic concepts of the trade-creation effect, the consumption effect, and the trade-diversion effect, does not come out as clearly as in the present analysis.

In a recent book, R.E. Caves and R.W. Jones (1973, pp. 294-298) have also presented a measurement of the welfare impact of a customs union; but their measurement is incorrect, since they erroneously regard unions as if they were exclusively either trade creating or trade diverting - rather than treat trade creation and trade diversion as forces both of which are working in any given union.
Capital goods differ from intermediate inputs in two inter-related ways. First, it is not the good itself but the service which it provides which is used in the production process; and second, the amount of services provided in a given period is not a function of the current production of a capital good during the period, but of the existing stock of the good, of which the current production supplies only a part. We shall make a few simplifying assumptions about the relationship of the service stream to the stock of a capital good and to the process of production of the final good. First, we shall assume an eternal life of the good (i.e., no depreciation). Under competitive equilibrium conditions, the value of the annual service provided by the stock of the capital good is then

\[ S = \frac{iK}{k} \]

where \( i \) is the annual rate of interest, \( K \) is the number of units in the existing stock of capital goods and \( P_k \) is their price per unit. We assume that this service is used in the production of just one final good, and that it is required there in a fixed proportion (to be designated by \( g \)) of the quantity produced. Our analysis will be handled throughout by means of the flow of the service, once we see how it is related to the stock of the capital good. As we assume the rate of interest to be constant, the analysis from now on will be that of a partial, rather than of a general equilibrium.

A problem peculiar to the study of capital goods is that the existence of any demand for the good implies - inevitably, if depreciation is assumed away - a departure from an equilibrium position: the demand for the good must be associated with an increase in the demand for the service (due to, say, an increase in the demand for the final good). Consequently, an evaluation of the welfare impact of the tariff must be concerned not with a comparison of the pre- and post-tariff positions, but of the ultimate ("steady state") equilibrium attained, respectively, in the absence and in the presence of the tariff.

To separate out clearly the issue under consideration - as well as to avoid a very awkward exposition - we shall assume that a tariff
is imposed on the capital good alone, the final good using its service not being traded internationally.  

Two alternative situations will be considered. First, we shall examine a situation in which no domestic production of the capital good takes place at the relevant price range of the good. From there we shall move to the more complex situation in which domestic production is called forth by the tariff.

I. Figure 5 describes the market for the final good after the original shift in demand. Demand for the good is now \( d_d \). In the absence of a tariff on the capital good, supply is \( L_0 \), \( P_0 \) is the price and \( Q^* \) the equilibrium quantity produced.

The imposition of a tariff on the capital good will shift domestic supply of the final good from \( L_0 \) to \( L_1 \), due to the added charge on the capital service used in the production process. We shall define units of this service by their starting-position value.

Before the tariff the total value of the service (used annually) is \( S_s = i K_s P_k \), where \( K_s \) is the initial stock of units of capital goods; and this is also the number of units of the service, its price being unity. The tariff on the capital good amounts to \( T_k = t P_k \) per unit of the good where \( t \) is the tariff rate; and to

\[
\frac{ip_k}{iK_k} = t
\]

per unit of the capital service. The cost of production of a unit

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8. With the final good imported freely, changes in its domestic demand will be satisfied by imports, and will not raise the demand for the capital service. The latter change could still be caused by changes in world demand for the final good or in the supply of the other factors used in its production. While the results in these cases are identical with those presented below, we refrain from presenting them because they would considerably clutter up the diagrams. Similarly for the simultaneous imposition of tariffs on both the capital and the final goods.
of the final good will hence rise by the size \( T_f = gT_s = gt \) (\( g \) being, we recall, the coefficient of the capital service in production of the final good). This will thus be the vertical distance between \( L_1 \) and \( L_1' \). The price of the final good will rise to \( p_f' \), and the quantity produced decrease to \( Q_1' \).

We may show the same movement in the market for the capital service, which is described in Figure 6. The domestic demand curve DD for the service is derived from the demand for the final good in Figure 5. Each quantity \( S \) of the service is associated with a quantity \( Q \) of production of the final good by the coefficient \( g \) (so that \( S = gQ \)). The demand price for the service is derived by deducting marginal costs of all factors other than capital, represented in Figure 5 by ZZ, from the demand price of the final good (becoming zero when ZZ intersects the demand curve dd).

We assume, we recall, a price range in which no local production of the capital good exists. In the absence of a tariff on the capital good, the supply price of the capital service provided by new additions to capital - necessarily through imports - is \( P_s (= 1) \) in Figure 6. With the tariff \( T_k \) on the capital good, this supply price of the service becomes \( P_s' = P_s + T_s = 1 + t \).

At either price, supply is infinitely elastic, by the assumption of the home country being small. In addition, however, capital services are supplied (this supply being of zero elasticity) by the existing capital at the beginning of each period. Suppose that this amounts to \( S \) in Figure 6. In the absence of tariff, the capital service provided by new additions from imports to the capital stock will be \( S^{*} \); and at the beginning of the next period no more imports of the capital good will be required, services out of the then existing stock amounting to \( S^{*} \). With the tariff, the provision of additional capital services will be lowered, by the lowering of imports of the capital good, to \( S \) \( S^{*} \); and the new steady state flow of the service will be \( S' \). Consistency in drawing the price scales in Figures 5 and 6 will assure consistency of equilibrium positions: \( S^{*} \) is the amount of capital services required for the production of \( Q^{*} \) and \( S' \) of the service is required for the production of \( Q_1' \) of the final good.
Having established the association between the two markets, we may now turn to the estimate of the welfare loss from the tariff. As a result of the tariff, the steady-state equilibrium quantities of the home production of the final good and of the use of the capital service change, respectively, from $Q^*$ and $S^*$ to $Q_1$ and $S_1$. The shaded areas $P'_{j} \text{NVP}'_j$ and $TMNP'_j$ in Figure 5 represent the loss of, respectively, consumers' and producers' surpluses — consumers and producers, that is, of the final good — which from now onward will be sustained in each production period. But not all of it is a net loss for the economy. To examine this, we move to Figure 6, and start by observing that the sum of the aforementioned welfare losses to producers and consumers of the final good is equal to the area $P\text{FGP}'_s$ in Figure 6. This latter area is reminiscent of the measure of loss of consumer's surplus. But, of course, this loss is shared by both producers and consumers of the final good in their respective capacities as users of the capital service and consumers of the final good. We shall designate it, hence, as a loss of users' surplus. (The distribution of the loss between consumers and producers depending on the extent to which the elasticity of demand enables producers of the final good to shift the rise in the prices of their inputs to the ultimate consumers.) The equality of the values of the areas $TMNP'_s$ in Figure 5 and $P\text{FGP}'_s$ in Figure 6 is thus derived from the relationship between the two figures. Under the linear approximation assumption, area $TMNP'_s$ of Figure 5 is:

\begin{equation}
(10) \quad TMNP'_s = T_s Q_1 + \frac{1}{2} T_s (Q^* - Q_1)
\end{equation}

since $T_s = \rho T_s$, $Q^* = \frac{S^*}{g}$, and $Q_1 = \frac{S_1}{g}$, (10) may be written as:

\begin{equation}
(11) \quad TMNP'_s = T_s S_1 + \frac{1}{2} T_s (S^* - S_1),
\end{equation}

where the right-hand side is, of course, the user's loss — area $P\text{FGP}'_s$ — in Figure 6. From here on, having established this identity, we shall conduct the analysis strictly in terms of the market for the capital service.

Keeping in mind that, in the present case, the user's loss we have just discussed is a "permanent" loss, i.e., one which will repeat
itself in all future production periods, we may now consider the offsetting gains. These consist of:

(i) The rent accruing to the owners of the original stock of the capital good due to the rise in the price of the service. This amounts (per annum) to $T_s S_s$, represented by the area $P_s ABP'_s$, and will be repeated in all future periods.

(ii) The interest accruing to government on the tariff revenue on the capital goods imported. This amounts to $T_s(S_1 - S_s)$, represented by the area $AECB$, per annum and, again, will be repeated in all future periods. 9

We are thus left with the "triangle" EFG as representing the pure, unoffset welfare loss per annum, equal to

$$\frac{1}{2} T_s (S^* - S_1) \tag{12}$$

or, in terms of Figure 1, to the shaded triangle area MNV, equal to $\frac{1}{2} T_f (Q^* - Q_1)$.

This loss will repeat itself year after year. It can be capitalized to the present value of

$$T_s (S^* - S_1) / 2i \tag{13}$$

or, in terms of the tariff on the capital good itself,

$$T_k (K^* - K_1) / 2 \tag{14}$$

where $K^*$ and $K_1$ are the stocks of the capital good yielding annual service flows of $S^*$ and $S_1$, respectively, and $K^* - K_1$ is the tariff-caused decrease in the expansion of the capital stock. This case appears to be remarkably similar to that of an imposition of a tariff on a final good not produced at home, with the difference

---

9 The size of the once-and-for-all tariff revenue is $T_k (K_1 - K_s)$, and the annual interest flow derived from it is hence $i T_k (K_1 - K_s)$, where $K_1$ and $K_s$ are the capital stocks yielding respectively annual services $S_1$ and $S_s$. It may be easily seen that this is equivalent to $T_s (S_1 - S_s)$.
that instead of a consumption cost - the loss of consumer's surplus
- only, we have here a loss to both consumers and producers of the
final good. To distinguish from production costs in the production
of the capital good itself we may regard these aforementioned losses
as a cost in use, which has been termed here the loss of "user's
surplus". The size of the loss will be determined, first, by the
height of the tariff; and, second, by the elasticity of demand for
the capital service, which is in turn a function of the elasticity
of demand for the final good and the elasticity of supply of pro-
ductive services other than the capital service.

II. In the case just examined, the welfare loss due to the im-
position of a tariff on imports of the capital good was caused by
the new steady-state positions in the markets both for the ser-
vice-of the capital good and for the final good differing from
what they would have been in the absence of tariffs. We will now
turn to the opposite case, where the tariff has no effect on the
new steady-state position, but does affect the adjustment process
through which this position is ultimately attained. For conveni-
ence in handling, we shall assume that at the pre-tariff price of
the capital good, its domestic production was just on the verge of
becoming profitable. This is illustrated in Figure 7, where the
supply curve of the service contains a positively sloped segment
BB', representing the supply of the service from new domestically
produced capital goods, starting at the price P, the pre-tariff
supply price of service from newly imported assets. In the ab-
sence of a tariff, equilibrium in the service market would have
been obtained at S*, the excess demand for the service over the
amount supplied by the initially existing stock (S* - S) being
satisfied by imports of the capital good. As the equilibrium quan-
tity of any one period is also the one supplied by the existing
stock in the subsequent period, equilibrium in the second period
will, therefore, be repeated at S* without any further importation
taking place. Thus, in the absence of a tariff, S* would have
been the new steady-state position. In the market for the final
good, this would correspond to production of Q* in Figure 5.
A tariff imposed on the capital good at the beginning of the first period raises the home price of the service to \( P'_s \), with an equilibrium quantity \( S'_1 \). In addition, domestic production of the capital good now becomes profitable, only \( S_1 - S_{h1} \) of the service now comes from newly imported assets, whereas \( S_{h1} - S_s \) comes from newly domestically produced assets. At the start of the subsequent production period, the service supplied by the stock existing then is \( S_{1} \). Were there no domestic production, this would have been the ultimate position, with the price of the service corresponding to the import-cum-tariff price of the capital good. But domestic production is still profitable at even lower prices: \( EE' \) now represents the additional amount of the service to be obtained during the period through new domestically produced assets. Thus, at the end of this period the equilibrium position will be \( S_{2} \) at a price of \( P''_s \) - of which \( S_{1} \) will come from the stock existing at the beginning of this period. (In the market for the final good this will result in a downward shift, not shown in Figure 5, of the supply curve, some way back in the direction of its original position, with a corresponding increase in domestic production.) Thus, at the beginning of the third period, the then existing stock will yield services to the amount of \( S_{2} \). With a domestic production of the capital good still taking place, the equilibrium position at the end of this period will be \( S_{3} \) - which will also be the amount supplied out of the existing stock at the beginning of the subsequent period, and so on. This process will continue until the stock has been expanded to yield the amount of services \( S^* \) per annum. The price then will fall back to \( P'_s \), and there will be no incentive for capital assets to be either imported or produced locally.

Thus, in the case illustrated in Figure 7, the imposition of a tariff will not affect the ultimate new steady-state position, which will be \( S^* \). It will, however, affect the path by which \( S^* \) is reached. In the absence of a tariff the adjustment will be instantaneous, involving imports of capital goods yielding \( S^* - S_s \) of the service. In the presence of a tariff imports will be smaller, corresponding to \( S_1 - S_{h1} \), the rest of the original excess demand of \( S^* - S_s \) being satisfied by domestic production of the
capital goods. Furthermore, the adjustment will be gradual, so that until it has been completed the quantity of the service (and, of course, of the final good produced with it) will fall short of what it would have been in the absence of the tariff.

To evaluate the welfare effects of the tariff it will suffice if we compare the situation in each of the first three steps of the adjustment process described above with that which would have obtained in its absence - i.e., with the steady-state positions of $S^*$ in the capital-service market and $Q^*$ in the final good market. As has been demonstrated in the previous section, the welfare losses due to the contraction in the domestic production of the final good can also be represented as a loss of user's surplus in the market for the service of the capital good. Thus, the welfare loss incurred by the consumers and the producers of the final good in the first step of the adjustment process is represented by the area $P_{FGP^*_s}$ of Figure 7. Offsetting this loss there will be:

(i) the rent to owners of the original stock, represented by the area $P_{BIP^*_s}$;

(ii) the annual interest on the gain in producers' surplus on the capital goods produced domestically in this period, represented by the area $BHI$.

(iii) the annual interest on tariff revenue from capital goods imported in this period, represented by the area $CECH$.

The last two, it should be pointed out, are "permanent" flows, which will repeat themselves in all subsequent periods.

The unoffset welfare loss in this period thus appears to be the two "triangle" areas $BCH$ and $EFG$. Of this $EFG$ may be termed the loss in use (parallel to the consumption loss in the analysis of tariffs on final goods) due to the quantity of the service produced in this period being $S_1$ instead of $S^*$, and its price exceeding $P_{S^*_1}$ i.e., to the decrease in the quantity of the final good produced domestically from $Q^*$ to $Q_{1h}$ (in Figure 5). And $BCH$ is the per annum production loss due to part of the increase in the stock of the capital good (corresponding to $S_1 - S_{1h}$) being produced locally, rather than imported.
For the estimate of the loss in the second period we construct, for convenience, a separate diagram – Figure B. The service supplied by the now existing stock is $S_1$, and local production of the capital good expands it to $S_2$, with a corresponding decline in its price. The domestic supply curve of the final good is shifted some of the way (but not all of it) back to its ultimate steady-state position, with a corresponding decline in the loss of consumers' and producers' surpluses. In the market for the service the gross loss in this period is represented in Figure B by the area $P F R P''$. We count the following offsetting gains or added losses, starting with four elements of gain:

(i) The rent to the owners of the original stock, which will now be lower than in the preceding stage, but will still be positive, represented by the area $P B L P''$. 

(ii) The interest on the producer's surplus on the capital assets produced domestically in the preceding period, represented – as before – by the triangle area $B H I$. 

(iii) The interest on the producer's surplus on capital assets produced in the current period, represented by the area $E E' R$. 

(iv) The interest on the original tariff revenue, which just as in the earlier period amounts to $C E G H$. 

Against these, a loss to be added is 

(v) The capital loss borne by owners of capital assets who have acquired these assets in the preceding period, whether from imports or from domestic production, at a price exceeding the current one by an amount corresponding to the difference $P''$ in the price of the service. This loss amounts to the area $L N C I$. 

Adding up all these elements of loss and gain, a net loss will be found, amounting to the areas of the triangles $B C H$ and $E F F$. The
latter triangle should best be viewed as a combination of two - EE'R and E'FR. We are thus left with a net loss consisting of three elements:

a. The current interest on the loss due to inefficient domestic production of the earlier period, BCH;

b. The current interest on the loss due to inefficient production in the present period, EE'R; and

c. The current loss in use, E'FR, due to the quantity of the service (and of the final good) falling short of the steady-state position.

It will be noted that the loss in (b) is added to the production loss in (a), which has been found to exist in the earlier period; whereas the user's loss in (c) is smaller than (and replaces, rather than is added to) the user's loss in the earlier period (EFG).

In a similar way, it may be shown that in the subsequent, third period, when the price of the service falls to $P''_s$, the welfare loss will consist of the following components:

(a) The current loss due to inefficient production in the first period, BCH;

(b) The current loss due to inefficient production in the second period, EE'R;

(c) The current loss due to inefficient production in the present, third period, E'E''Z; and

(d) The current loss in use, E''EZ, due to the quantity of the service in this last period falling short of the steady-state one.

And so on in the following periods. Comparing the net losses incurred in each of the stages, we notice that the losses caused in the various periods by the inefficiency of the domestic production of the capital good - the shaded triangles in Figure 3 - keep repeating themselves in the subsequent periods as well. It can be easily
verified that they will continue to be incurred also after the service market has reached its new steady-state position. The adjustment process thus generates a series of "permanent" losses, due to the substitution of inefficient domestic production for imports. On the other hand, the loss in use due to the quantity of the service during any one of the adjustment stages falling short of the steady-state one does not repeat itself in the subsequent periods but, rather, is reduced from one stage to another, vanishing completely once the new steady-state position has been reached. The welfare losses caused by the imposition of a tariff can thus be divided in this case into two types: the permanent losses incurred in the production of the capital good, which are due to the partial substitution of imports by (relatively inefficient) domestic production; and transitional losses in the use of the capital good (or the capital service), due to the stock of the capital good adjusting itself only gradually to the increased demand for the service caused by the original shift in the demand for the final good.

Thus, when a new steady-state equilibrium position is reached (and only "permanent" losses are realized) the outcome in this case appears to be diametrically opposite to that established in the former case. There, only a user's loss was found to be created; whereas now, only a production loss in the market for capital goods is found, and no (permanent) user's loss exists at all. The contrast between the two cases is due, of course, to the contrast in assumptions. In the former case domestic production of capital goods was assumed away, so that no costs could be incurred in such production. Without domestic production, the new steady-state price of the capital service must have risen by the size of the tariff, so that a loss to the user of this service must be involved. In the latter case, on the other hand, the tariff gives rise to domestic production of the capital good, leading to a loss from this production; whereas the tariff has, once the new steady-state equilibrium is reached, no effect on the price of the capital service, so that no loss to the user of this service is to be expected.

These were obviously two boundary situations. In all situations in between, the outcome will be a mixture of the two cases. If, in Figures 7 and 8, domestic supply starts not just at the price $P_s$ but at a price higher than it and lower than $P'_s$, the final outcome will involve permanent welfare losses in both production of the
capital good and the use of the capital service. The diagrammatic exposition of welfare losses in such situations will be basically similar to the one which has been presented here, although more complex (the simple "triangles" will be superseded by ones mounted on rectangles).

Restricting ourselves again to the situation analyzed here, in which the steady-state equilibrium price of the capital good is unaffected by the tariff, what could be said about the determinants of the magnitude of the welfare loss from the tariff? Obviously, this loss will be higher the higher the rate of the tariff. It is also clear that the permanent production loss will be greater the greater the domestic production relative to imports in the expansion of the capital good's stock. As can be seen from Figures 7 or 8, given the new "steady state" position, the ratio of domestic production to imports will be higher, and hence the loss bigger, the higher the elasticity of demand for the capital service (which is in turn, we recall, a function of the elasticity of supply of factor services other than that of capital which participate in production of the final good). The effect of the elasticity of supply of capital goods (from domestic production) is, however, less obvious.

As would be seen from a diagrammatic representation easily constructed from Figure 8, the effect of a higher elasticity of supply on the level of the (permanent) loss in production will be different between the loss in production in the first and in subsequent periods. The loss generated in the first period is due to some import substitution taking place at a domestic price which exceeds the free trade one by the amount of the tariff. The elasticity of the domestic supply curve affects the extent of

---

10 If the domestic supply curve starts at a price higher than $P'$, we are obviously at the first case: no domestic production of the capital good exists either before or after the tariff. On the other hand, $S$, being the initial equilibrium quantity of the service, before the shift in demand implies that $P'$ is the lowest price at which domestic production of the capital good may become profitable.
import substitution which takes place at this stage, but (as long
as the tariff is not a prohibitive one) not the excess of the
domestic price over the free trade one. The higher the domestic
supply elasticity, the larger the import substitution in this stage
and the greater the permanent welfare loss generated in it. The
production losses in all subsequent periods are, on the other hand,
due to a given amount of import substitution - satisfying the ex-
cess of the steady-state quantity of the capital good over the stock
existing by the end of the first stage - taking place at prices
above the free trade one. The extent of import substitution is
determined here by the demand conditions. But the elasticity of
supply affects the average of the marginal costs at which the
import substitutes are produced. The higher the domestic supply
elasticity the lower, on the average, the price at which
the import substitutes are produced and the smaller the resultant
welfare loss. Thus, from this reasoning, it would seem that the
effect of the degree of elasticity on the loss generated in all
periods combined cannot be ascertained.

Yet a more rigorous analysis shows that this effect can indeed be
determined; and that the higher the elasticity of domestic supply
of capital goods, the higher is the expected (permanent) loss from
production of these goods. As developed in the appendix, the annual
value of the permanent production loss is shown to be equal to

\[ W(t) = \frac{1}{2} t^2 \left( \frac{\varepsilon S^{\ast} + n^3}{s \varepsilon s + 2nS^*} \right)^2 \]

where \( t \) is the rate of the tariff on the capital good, \( n \) is the
elasticity of demand for the service at the steady-state position,
\( S^* \), and \( \varepsilon_S \) is the elasticity of its supply at the initially existing
stock, \( S^* \). Differentiating (15) with respect to \( n \) and to \( \varepsilon_S \), we
find first that

\[ \frac{dW}{dn} > 0 \]

which confirms our intuitive conclusion that the loss will be
bigger the higher the demand elasticity. And secondly that

\[
\frac{dW}{d\epsilon} > 0
\]

which shows the effect of changes in the supply elasticity on the extent of import substitution and, therefore, on the permanent loss incurred in the first adjustment period to outweigh those on losses incurred in all the later stages. So that the higher this elasticity, the greater the loss incurred in the adjustment process as a whole.

The transitory losses in use are shown in the appendix to sum up to

\[
W' = \frac{1}{2} t^2 \frac{\eta^*}{\epsilon S_s} \left( \frac{\epsilon S_s + \eta^*}{\epsilon S_s + 2\eta^*} \right)^2
\]

Again, we find that

\[
\frac{dW'}{d\eta} > 0
\]

for the higher the demand elasticity, the greater the deficiency in the service, caused by the tariff. However, in contrast to the case of the permanent production losses

\[
\frac{dW'}{d\epsilon} < 0
\]

The losses in use are due, we recall, to the output of the capital service during the adjustment period both falling short of its steady-state level and being sold at a higher than the steady-state price. In the first adjustment period, both the quantity and the price of the service will be unaffected by the elasticity of domestic supply - provided, of course, that the tariff is not prohibitive. In all subsequent periods, on the other hand - just as with the loss in production - the (transitory) losses in use will be smaller the higher is the elasticity of domestic supply, since the faster is then the adjustment; and at each period, the larger is the quantity of the service and the lower is its price. Hence this is true also for all periods (including the first) combined.
4. The Cost of Industries with Negative Value Added

In this final section we shall discuss the measurement of the welfare loss from a tariff which leads to production in an industry with negative value added. Since intermediate goods are necessarily involved here, we shall be dealing again with a world of many goods in a partial-equilibrium model.

As would be easy to realize, and has been shown in the literature, the introduction of intermediate goods leads to the following adjustment in the loss measurement in the market for the final good: while the consumption loss is determined, as in the two-ffinal-goods world, by the nominal tariff on the final good, the production cost is determined by the effective protective rate, which motivates the changes in the amount of value added in the domestic production of the final good. A well-recognized phenomenon of protection is that it may (and quite often does) lead to the establishment of an industry in which domestic value added - when both output and imported inputs are valued at international prices - is negative. This raises no particular problem as far as the consumption loss is concerned. But the production loss can no longer be estimated here by the use of the effective protective rate: when defined in the conventional way this rate will be found to be negative in such industry, and is patently useless for the purpose on hand. The production cost of this industry thus requires a separate method of estimation.

The production loss from protection consists here of two parts. First, the negative value added (in international prices) is itself a cost: it is the excess of what the economy pays abroad, for the intermediate inputs, over what it would have paid for the final good. Second, all the payments to domestic productive factors involved in the activity are a sheer waste, thus an added element of the loss. These will be referred to as payments for "the local value added" - the value added in local prices in distinction from the value added in international prices which is here negative.

It will be assumed, in the present measurement, that no substitution between imported inputs and the services of domestic factors of production takes place. This is an important assumption, in the present context: it rules out the possibility of the negative value added being found only under protection - due to substitution - and not under the free-trade method of production.

In Figure 9 $P_j$ is the international price of a unit of final good $j$, with the unit normalized to make $P_j$ unity; and $P_i$ is the price of a unit of imported input $i$ which is required for the production of a unit of $j$. $P_i > P_j$, so that the value added in international prices is negative. $S_i$ is the supply curve of domestic value added. The addition of this supply schedule to $S_i$, the supply of the intermediate input (infinitely elastic at price $P_i$) yields $S$, the domestic supply curve of final good $j$. At the free-trade price $P_j$, obviously, no domestic production of $j$ will take place. Assume now, however, that a tariff $t_j$ is imposed on the import of final good $j$; for simplicity (this assumption will be removed later without difficulty), we shall assume that no tariff is imposed on the import of input $i$. The tariff $t_j$ of the size (and rate, since $P_j$ is assumed to be unity) $P_j P_j'$, raises the domestic price of $j$ to $P_j'$. At this new price, home production of $j$ will be $Q_p (= P_j' M)$.

The production cost of protection is equal to the shaded area $P_j K M N$, which is the combination of the two aforementioned elements: $P_j K L F_i$ is the excess of payments for the imported input over what the final good would have cost to import (that is, the negative value added at international prices); and $P_i L M N (= O Q U V)$ is the local value added - the cost of domestic productive services involved in the activity of producing the amount $Q_p$.\[12 Call these two elements, respectively, foreign-exchange loss (FEL) and domestic-resource loss (DRL). The first element is quite simple:

\[
(17) \quad \text{FEL} = Q_p (P_i - P_j) = Q_p (a_{ij} - 1),
\]

12 Another way would be to take as this cost $P_i L M P_j'$ - the total payment to domestic productive factors; but deduct from it the area $M P_j' M$, which is a producers' surplus.
where \( a_{i,j} = \frac{P_i}{P_j} \).

Examine the second element (DRL) first within a simplified framework, in which the elasticity of supply of value added (that is, the elasticity of the curve \( S_v \)) is assumed to be unity. \( S_v \) will then be a straight line from the origin \( O \); likewise, \( S \) will be a straight line originating at \( P_i \). The area \( P_iLMN \) will then reduce to a triangle \( P_iLM \), which will be:

\[
(18) \quad \text{DRL} = \frac{Q_p(P_i - P_j)}{2} = \frac{Q_p[t_j - (a_{i,j} - 1)]}{2}
\]

The combination of these two elements, (17) and (18), will then yield:

\[
(19) \quad \text{Total loss} = \frac{Q_p}{2}(t_j + a_{i,j} - 1)
\]

Take, now, the general case, where the elasticity of supply of value added is any positive number (but still assuming that \( S_v \) - hence also \( S \) - is a straight line in the relevant range). Denote the average (arc) elasticity of supply of \( S_v \), in this range, by \( \varepsilon \). We now want to find the area of the triangle \( MNP \), in order to deduct it from area \( P_iLMMP \), thus finding the area (DRL) \( P_iLMN \). \( P_iN \) (= \( OV \)) is the price of value added at which no local production will take place. \( P_v \) is the price of value added when \( Q_p \) is produced, and \( \Delta P_v \) is the change (reduction) of this price which will reduce this production to zero. By definition,

\[
\varepsilon = \frac{\Delta Q}{Q_p} \cdot \frac{\Delta P_v}{P_v}
\]

and we look for the \( \Delta P_v \) which will make \( \Delta Q = Q_p \); that is, where

\[
\varepsilon = \frac{\Delta P_v}{P_v}.
\]

Hence, we know that this particular price change \( \Delta P_v \) (= \( P_j'N \)) is:

\[
\Delta P_v = \frac{P_v}{\varepsilon}; \quad \text{and since} \quad P_v (= P_i'P_j') = t_j - (a_{i,j} - 1),
\]

\[
\Delta P_v (= P_j'N) = \frac{t_j - (a_{i,j} - 1)}{\varepsilon}.
\]

---

13 This necessarily implies an elasticity of supply above unity, since the straight line \( S_v \) starts above the origin.
We are now in the position to find DRL:

\[
DRL = P_{i LMN} = Q_p \left( P_{ij} P_{ij}^T \right) - \frac{Q_n}{2} \left( P_{ij} P_{ij}^T \right)
\]

\[
= Q_p \left[ t_{ij} \left( a_{ij} - 1 \right) \right] - \frac{Q_n}{2} \frac{t_{ij} \left( a_{ij} - 1 \right)}{\varepsilon} ; \text{ or}
\]

\[
(20) \quad DRL = Q_p \left[ t_{ij} \left( a_{ij} - 1 \right) \right] \left( 1 - \frac{1}{2\varepsilon} \right)
\]

Combining this now with FEL, and rearranging, we get:

\[
(21) \quad \text{Total loss} = \frac{Q_n}{2} t_{ij} (2\varepsilon - 1) + (a_{ij} - 1)
\]

From (21), expression (19) above could be derived for the special case in which \( \varepsilon = 1 \).

Finally, let us admit not one imported input but many; and, at the same time, assume that tariffs \( t_{ij} \) may be imposed on these inputs as well as on the final output. Without redrawing the diagram, it is clear that the effect of such input tariffs would be to change the price received by local productive factors - reducing it by the amount of tariff paid on the imported input. Hence, (19) will take the form:

\[
(22) \quad \text{Total loss} = \frac{Q_n}{2} \left( t_{ij} - \sum a_{ij} t_{ij} \right) (2\varepsilon - 1) + \frac{\sum a_{ij}}{\varepsilon} (a_{ij} - 1)
\]

The production loss in an industry with a negative value added is thus seen to be a rising function of the nominal tariff on the import of the final good, of the amount of (protected) local production, of the elasticity of supply of domestic value added, and of the excess of the (international) cost of the input over the cost of the output - the size, that is, of the negative value added; and a declining function of the tariffs imposed on the imported inputs. Although all the ingredients which make the effective protective rate - output duties, input duties, and input coefficients - participate in determining the size of the loss, the effective rate itself does not appear, as such, in this function.

\[14\] It is clear that \( t_{ij} > \sum a_{ij} t_{ij} \), or else the tariff system will not lead to any local production.
**APPENDIX**

*Estimation of Welfare Loss from Protection of Capital Goods*

Consider first the "permanent" losses, i.e., those due to the substitution of inefficient domestic production for imports of the capital good. As shown in Figures 7 and 8, in a linear approximation this loss is equal to half the product of the amount of the service derived from the domestically produced capital goods substituted for imported ones and the excess of its price over the free trade one.

Denoting quantities of the service by the symbols used in Figures 7 and 8 we see that, unless the tariff is prohibitive, import substitution in the first adjustment period will amount to

\[ S_{n1} - S_{1} = T_{s}/b \]

where \( T_{s} \) is the (absolute) tariff per unit of the service and \( b \) is the slope of the supply curve with respect to the horizontal axis (\( b = \Delta P/\Delta S \)). And the resultant welfare loss amounts to

\[ \tilde{W}_1 = \frac{1}{2} \frac{T_{s}^2}{b} \]

In the later adjustment periods, further import substitution will take place, to the amount of \( S^* - S_{1} \). However, the degree of inefficiency in this substitution (i.e., the excess of the price over \( P_{s} \), that corresponding to capital assets being imported at world prices) will not be uniform. Assuming the demand curve for the service to be linear, and denoting the absolute value of its slope by \( a \), the second stage's equilibrium requires that the demand price for the service

\[ P''_{s} = P_{s} + T_{s} - a(S_{2} - S_{1}) \]

be equal to its supply price

\[ P''_{s} = P_{s} + b(S_{2} - S_{1}) \]

yielding
(5) \[ S_2 - S_1 = \frac{T_s}{a+b} \]

Substituting (5) in (3), we have

(6) \[ P''_s = P_s + T_s \left( 1 - \frac{a}{a+b} \right) = P_s + T_s \frac{b}{a+b} \]

And the loss caused in this stage is equal to

(7) \[ \frac{1}{2}(S_2 - S_1)(P''_s - P_s) = \frac{1}{2} T_s^2 \frac{b}{(a+b)^2} \]

Similarly, for the next stage, we have

(3a) \[ P'''_s = P_s + T_s \frac{b}{a+b} - a(S_3 - S_2) \]

(4a) \[ P'''_s = P_s + b(S_3 - S_2) \]

yielding

(5a) \[ S_3 - S_2 = T_s \frac{b}{(a+b)^2} \]

and

(6a) \[ P'''_s = P_s + T_s \frac{b^2}{(a+b)^2} \]

and the corresponding loss is

(7a) \[ \frac{1}{2}(S_3 - S_2)(P'''_s - P_s) = \frac{1}{2} T_s^2 \frac{b^3}{(a+b)^4} \]

and similarly for later stages. The welfare loss generated in the second and later stages, i.e., in the adjustment from \( S_1 \) to \( S^* \) amounts, therefore, to

(8) \[ W_2 = \frac{1}{2} T_s^2 \left[ \frac{b}{(a+b)^2} + \frac{b^3}{(a+b)^4} + \frac{b^5}{(a+b)^6} + \cdots \right] \]

\[ = \frac{1}{2} T_s \frac{b}{(a+b)^2} \left\{ 1 + \left( \frac{b}{a+b} \right)^2 + \left[ \left( \frac{b}{a+b} \right)^2 \right]^2 + \cdots \right\} \]

\[ = \frac{1}{2} T_s \frac{b}{(a+b)^2} \left[ 1 - \left( \frac{b}{a+b} \right)^2 \right] \]
which by rearranging and simplifying reduces to

\[(9) \quad W_2 = \frac{1}{2} T_s \frac{b}{a^2 + 2ab}\]

Summing (2) and (9), the annual value of the total permanent loss is equal to

\[(10) \quad W = \frac{1}{2} T_s \left[ \frac{1}{b} + \frac{b}{a^2 + 2ab} \right] = \frac{1}{2} T_s \frac{(a + b)^2}{b(a^2 + 2ab)}\]

We may prefer to express this loss in terms of elasticities and of the initial and ultimate steady-state quantities of the service, _s_ and _s^*_. As these quantities are unaffected by the tariff, we can thus estimate the loss which would have been incurred had it been imposed (or the gain due to it being removed). Denoting the elasticity of demand for the service at the post-adjustment situation by \(-\eta\), and the elasticity of its supply at the initial situation by \(\varepsilon\), we have

\[a = \frac{P_s}{\eta s^*} \quad \text{and} \quad b = \frac{P_s}{\varepsilon s_s}\]

Units of the service being defined in value terms, so that \(P_s = 1\), it has been shown in the text that \(T_s = T_r/P_r = t\), where \(t\) is the rate of the tariff on the capital good.

Substituting in (10), we obtain

\[(11) \quad W = \frac{1}{2} t^2 \frac{(\varepsilon s_s + \eta s^*)^2}{s_s + 2\eta s^*}\]

Differentiating (11) with respect to \(\eta\) and to \(\varepsilon\), yields

\[\frac{dW}{d\eta} = t^2(s^*)^2 \eta \frac{\varepsilon s_s + \eta s^*}{s_s + 2\eta s^*} > 0\]

and

\[\frac{dW}{d\varepsilon} = \frac{1}{2} t^2 s_s \left[ 1 - \left( \frac{\eta s^*}{s_s + 2\eta s^*} \right)^2 \right] > 0\]

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1 Note that \(\varepsilon\) is the elasticity of the total supply (from all sources) of the service at the initial situation, and not the elasticity of the supply of the service from new domestically produced capital goods which, at this point, is infinite (the quantity supplied from this source being zero).
The "transitional" losses are those due to the amounts of the service produced throughout the adjustment process falling short of the steady-state one of $S^*$. In the first stage this deficiency is

\[(14) \quad S^* - S_1 = T_s / a\]

and the welfare loss

\[(15) \quad \frac{1}{2} (S^* - S_1)(P'_s - P_s) = \frac{1}{2} T_s \frac{2}{a} \frac{1}{a}\]

Deducting from the initial deficiency the increase in domestic supply at subsequent stages, as shown in equations (5) and (5a) we obtain the corresponding deficiencies for these stages:

\[(15a) \quad S^* - S_2 = T_s \left( \frac{1}{a} - \frac{1}{a+b} \right)\]

\[(15b) \quad S^* - S_3 = T_s \left[ \frac{1}{a} - \frac{1}{a+b} - \frac{b}{(a+b)^2} \right]\]

etc. Similarly, from equations (6) and (6a) we derive the excess of the price over $P_s$.

\[(16a) \quad P''_s - P_s = T_s \frac{b}{a+b}\]

\[(16b) \quad P'''_s - P_s = T_s \left( \frac{b}{a+b} \right)^2\]

etc. The sum of the "transitional" losses in all the adjustment stages is then

\[(17) \quad W' = \frac{1}{2} T_s \frac{2}{a} \left\{ \frac{1}{a} + \left[ \frac{1}{a} \frac{b}{a+b} - \frac{b}{(a+b)^2} \right]\right.\]

\[\left. + \left[ \frac{1}{a} \frac{b^2}{(a+b)^2} - \frac{b^2}{(a+b)^3} - \frac{b^3}{(a+b)^4} \right] + \left[ \frac{1}{a} \frac{b^3}{(a+b)^3} - \cdots \right] \right.\]

\[\left. + \cdots \cdots \cdots \cdots \right\}\]
which rearranged and simplified becomes

\[ W' = \frac{1}{2} T_s^2 \frac{(s+b)^2}{a(a^2+2ab)} \]

Expressed in terms of elasticities and of the tariff rate, this is equal to

\[ W' = \frac{1}{2} t^2 \frac{\eta \eta^*}{s \left( \frac{\eta_s + \eta_s^*}{s} \right)^2} \]

Differentiating (19) with respect to \( \eta \) and to \( \varepsilon \), we obtain

\[ \frac{dW'}{d\eta} = \frac{1}{2} \frac{t^2 \eta \eta^*}{s \left( \frac{\eta_s + \eta_s^*}{s} \right)^2} \left[ 1 - \frac{s \eta \eta^*}{s \left( \frac{\eta_s + \eta_s^*}{s} \right)^2} \right] > 0 \]

and

\[ \frac{dW'}{d\varepsilon} = -t^2 \frac{\eta \eta^*}{s \left( \frac{\eta_s + \eta_s^*}{s} \right)^2} \frac{s \eta \eta^*}{s \left( \frac{\eta_s + \eta_s^*}{s} \right)^2} < 0. \]
References


