Seminar Paper No. 326

TAX REFORM AND HOUSING DEMAND:
THE DISTRIBUTION OF WELFARE GAINS AND LOSSES

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Seminar Papers are preliminary material circulated to stimulate discussion and critical comment.

April, 1985

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1. Introduction

A tax reform bill was presented to the Swedish parliament in Spring 1982 with two major elements: an overall reduction in marginal tax rates, and a limitation of the deductibility of interest payments against taxable income. The reform was intended to reduce labor supply disincentives and the asymmetric tax treatment of owner-occupied houses. While these goals were generally accepted, the critical debate surrounding the bill focused on (a) the effects on the demand for (and the prices of) owner-occupied houses, and (b) the effects on the income distribution. One group of critics claimed that the reduction in marginal tax rates unequivocally favoured the high-income groups and dismissed the reform as anti-egalitarian. Another group of critics claimed that the reduction in marginal tax rates, together with the limitation of the interest deductibility, would cause a sharp decrease in the prices of owner-occupied houses, and would thereby harm the house owners. The protagonists of the reform claimed that since these two groups (i.e. high-income earners and homeowners) tend to overlap, the redistributional effects of the tax reform can not a priori be said to have an anti-egalitarian bias.
The direct effects on the income distribution implicitly assuming that household behavior is unaffected - i.e. the cash gain in King's (1983) terminology - have been investigated by the Swedish National Central Bureau of Statistics (1982). That report concludes that the tax reform is regressive in the sense that the cash gain tends to increase with household disposable income. Since these calculations are limited to reporting cash gains they ignore any behavioral responses on part of the households or tax-induced changes in asset values. In Brownstone et al. (1984) we report evidence that these responses may be important: the aggregate demand for owner-occupied housing is predicted to decrease by 15 per cent.

This decrease is the net result of two effects: a slight increase in the predicted number of home-owners and a marked decrease in the average size of house demanded by the owner-occupiers. In calculating welfare effects it is important to account properly for both the dichotomous choice aspect (to own or to rent) and the continuous demand for housing. Small and Rosen (1981) show how the standard Harberger-type calculations can be adapted to handle such cases and we rely on a modification of their analysis for our calculations.

The paper is organized as follows, In section 2 we will make a brief presentation of the 1983–85 Swedish tax reform, and discuss how it affects the consumer's optimization problem. In section 3 we present the microsimulation model used for the empirical study of housing demand, and in sections 4 and 5 the welfare formulas used for the analysis of the redistributional
effects of the tax reform will be derived. In section 6, finally, we report the numerical results of this analysis; it is concluded that the tax reform tends to favor high-income groups. This effect, although significant, is minor in numerical magnitude.

2. The Consumer's Optimization Problem and the 1983-85 Tax Reform

Consider a household which consumes housing and a composite commodity (denoted by z) representing all other goods. Housing must be either owner-occupied (denoted by \( h_o \)) or rented (denoted by \( h_R \)). The household is assumed to maximize its utility function, \( u(z, h_R, h_o) \), subject to a budget constraint and the constraint that it must either rent or own (i.e. \( h_R \cdot h_o = 0 \)). If the household chooses to rent, then the budget constraint can be written:

\[
z + Rh_R = y \tag{1}
\]

where the price of z is normalized to unity, R denotes the rent per unit of housing, and y is the household's disposable income, which depends on exogenous gross income and the tax system.

If the household chooses to own instead of rent it does not face an ordinary linear budget constraint like (1). Since a large part of housing costs consists of interest payments - which to some extent are tax deductible - disposable income becomes a function of \( h_o \). We can formalize this by writing the owner-occupant's budget constraint as
\[ z = B(x + rW, h_0) \]  \hspace{1cm} (2)

where \( x \) is the household's exogenous labor income and \( W \) is exogenous wealth. Denoting the interest rate by \( r \), the sum \( x + rW \) is thus gross income. The function \( B(x + rW, h_0) \) represents the tax system's treatment of owner-occupied housing and tells us how much there will be left after tax for consumption of \( z \), given gross income \( x + rW \) and housing consumption \( h_0 \). The \( B \) function is general enough to encompass various rules for interest deductions and the calculation of taxable imputed income; the specification of it will not be discussed here.\(^1\) It will be decreasing in \( h \) – i.e. the marginal price of housing will be positive – and, due to the progressivity of the tax system (higher marginal tax rates for higher taxable incomes), it will also be concave in \( h \). With this notation, the renter's budget constraint (1) can be written equivalently as:

\[ z + Rh_R = B(x + rW, 0). \] \hspace{1cm} (1')

The consumer's problem of maximizing \( u(z, h) \) subject to (1') or (2) is illustrated in Figure 1, where the concave curve AA represents the owner's budget set (2) while the straight line BB represents the renter's budget set (1').

Without going into details of the 1983-85 Swedish tax reform, which can be found in Brownstone et al. (1984), it implies an increase in the intercept of most households budget sets, together with an increase in the slope for owner-occupants at all levels of \( h_0 \) above a certain low value. This means that
Figure 1: Budget constraints

Figure 2: Pre-reform (solid curves) and post-reform (dashed curves) budget constraints
the curves change from the solid curves in Figure 2 to the dashed ones. The changes in the budget sets vary with income and asset holdings in a complicated fashion. They can be computed, for each household, by the microsimulation model to be described in the following section. It is evident that, as a general feature of the reform, renters gain from the reduction of the marginal tax rates and are not harmed by the reduction in the interest deductibility. Renters thus gain unequivocally from the reform, which is shown by the fact that the straight dashed line in Figure 2 everywhere lies outside the solid line.

For owner-occupants the question is somewhat more complicated. We see that for owner-occupants which choose a low level of \( h_0 \), the relevant part of the post-reform budget set is located outside the pre-reform budget set; thus such people will gain from the tax reform. For owners with preferences for a large quantity of housing it is the other way around. The relevant part of the new budget set lies inside the old one, and these households will thus be harmed by the reform. Finally we have the people who, as a result of the reform, will switch from owner-occupancy into renting or vice versa. These could both gain and lose from the reform. Using our microsimulation model we will compute the sizes of the gains and losses for each household in our sample.

It should be emphasized that our analysis is based on the original reform proposal to parliament (government bill 1981/82:197). Bracket creep associated with only partially indexed tax schedules will make the actual 1985 marginal tax rates higher than those stated in the bill. Also it is important
to bear in mind that the proposed reform will imply reduced tax receipts. The original bill did not clearly specify additional tax changes needed to restore budget balance. Since our main aim is to study effects on income distribution, we have chosen to base our analysis on the tax schedule found in the bill. In the end of section 6 we will briefly consider the effects of a particular scheme of offsetting changes in lump sum taxes that will restore budget balance.

3. The Model

When estimating the dichotomous choice of a household facing a nonlinear budget constraint like AA in Figure 1 above, one should ideally use as independent variables a set of parameters that completely describe the budget constraint. Since this is not feasible for a budget constraint with many kinks, we have to limit ourselves to a small number of parameters. We have chosen to represent each household's tax situation by the ordinate (i.e. the z value) and the slope at two points on the AA curve of that particular household. The first point is at the intercept (cf Figure 3) where we denote by RDI ("Renter's Disposable Income") the variable that tells us how much would be left for the consumption of rental housing and of other goods if the consumption of owner-occupied housing were equal to zero. This variable could be regarded as a reasonable measure of (exogenous) income, and in section 6 below, where we calculate the households' gains and losses, we rank them according to their respective RDI's. At the intercept we also calculate the slope
Figure 3
of the budget constraint (denoted by RMP for "Renter's Marginal Price") which tells us the price for the first unit of owner-occupied housing. The variables RDI and RMP are the ones used by e.g. Feldstein and Clotfelter (1976) in their empirical study of tax-deductible charity contributions with non-linear budget constraints. In order to achieve a better representation of the choice situation we have included one more exogenous point on the budget curve, namely that where the consumer would spend half his RDI on interest payments for the house. This approximation is equivalent to approximating each household's nonlinear budget set by a cubic polynomial. Denoting the rate of interest by $r$, we thus say that the exogenously chosen $h_o$ should be such that $rh_o = RDI/2$. With a rate of interest $r = 0.12$ this implies $h_o = RDI/0.24$ which is the abscissa of the second point we have chosen to represent the household's tax situation (c.f. Figure 3). We denote the $z$ coordinate of that point by ODI ("Owner's Disposable Income") and the slope by OMP ("Owner's Marginal Price").

Based on cross-sectional data from 1978-79 on 2,950 households, we have computed RDI, RMP, ODI, OMP and a set of geographic (Regional Price Index, Percentage of co-ops and condos in the region) and demographic (Proxy for both spouses working, Age of head of household, Number of children, Number of adults, Educational dummies, Dummy for self-employed) variables. These have all been used as independent variables in the estimation. The dependent variables are a choice index indicating whether the household is an owner or a renter, and the market value of the house for owner-occupiers, calculated from the assessed taxable
value. This means that we use the market value as an index of the "quantity of housing". With these data we have estimated a dichotomous choice model of the demand for owner-occupied housing which tells us

(a) the probability that a particular household will choose to own its home instead of renting it, as a function of household characteristics, and

(b) the household's demand for owner-occupied housing (defined as the value of the house demanded), conditional on its decision to be an owner.

For future reference, we denote the probability of owning, as described in (a), by $\pi$. Similarly we denote the conditional demand for owner-occupied housing, as described in (b), by $\tilde{h} (= h_0$ in earlier sections).

The estimated system can be written as:

$$\log \tilde{h}_i = X_i \beta + \sigma u_i$$  \hspace{1cm} (3)

$$h_i = \tilde{h}_i \hspace{1cm} \text{if } D_i = 1$$  \hspace{1cm} (4)

$$D_i = 1 \hspace{1cm} \text{if and only if } X_i \gamma \geq \varepsilon_i$$  \hspace{1cm} (4b)

$$\pi_i \equiv \text{Prob} (D_i = 1 \mid X_i \gamma) = [1 + \exp(X_i \gamma)]^{-1}$$  \hspace{1cm} (5)

where the dichotomous variable $D_i$ is unity if household $i$ owns and zero otherwise. The stochastic variable $\varepsilon_i$ follows a Weibull distribution and $u_i$ is $N(0,1)$. The estimation procedure allows for a non-zero correlation between $\varepsilon$ and $u$. The vector $X_i$ consists of the exogenous variables of household $i$, and $\beta$ and $\gamma$ are parameter vectors to be estimated.
The quantity of housing \( h \) is defined as the market value of the house, which can be calculated from the assessed taxable value in our data base. Using the market value as a measure of quantity, we have to assume that the supply of housing be infinitely elastic to be able to identify the demand side of the model. The model yields simultaneous estimates of \( \pi \) and \( \tilde{h} \) as functions of the tax system variables (RDI, RMP, ODI and OMP) and of the geographic and demographic variables. These estimates can then be used to analyze changes in the behavior of the individual household as a response to changes in the tax system.

In the present study of welfare changes we need numerical estimates of some income and price effects, i.e. we need to know the values of the derivatives

\[
\frac{\partial \pi}{\partial y}, \frac{\partial \tilde{h}}{\partial y}, \frac{\partial \pi}{\partial p} \text{ and } \frac{\partial \tilde{h}}{\partial p},
\]

where \( y \) and \( p \) are some income and price measures to be defined below. Since the budget sets are non-linear, the meaning of these derivatives is not entirely clear, and we will end this section with a brief digression about them.

In Figures 1-3 the owner-occupant's budget set is depicted as a smooth, strictly convex set. In reality, however, the tax system is piecewise linear, and the budget set consists of 10-20 linear segments forming a convex but not strictly convex set. To illustrate the principle, it can thus be depicted as the one in Figure 4. The \( \pi \) and \( \tilde{h} \) functions are estimated as functions of (among other variables) RDI and ODI. Increasing the owner's disposable income by one dollar means shifting the whole budget
set vertically upward by one dollar, i.e. increasing RDI and ODI by one dollar. Provided the point of tangency (Q in Figure 4) is not at a kink and that it is still located on the same linear segment after the dollar transfer as it was before, this experiment is conceptually the same as shifting an ordinary, linear budget set upward by one dollar. Thus, for an infinitesimal change, the derivatives \( \frac{\delta \tilde{h}}{\delta y} \) and \( \frac{\delta \pi}{\delta y} \) can be written as

\[
\frac{\delta \tilde{h}}{\delta y} = \frac{\delta \tilde{h}}{\delta \text{RDI}} + \frac{\delta \tilde{h}}{\delta \text{ODI}}
\]

(6)

and

\[
\frac{\delta \pi}{\delta y} = \frac{\delta \pi}{\delta \text{RDI}} + \frac{\delta \pi}{\delta \text{ODI}}
\]

(7)

where \( \frac{\delta \pi}{\delta y} \) and \( \frac{\delta \tilde{h}}{\delta y} \) are thought of as the changes that would occur if an ordinary, linear budget set were shifted.

For the price derivatives, things are a bit more complicated. We would like to know how \( \tilde{h} \) and \( \pi \) change if the slope of an ordinary, linear budget constraint changes. But since prices in our model, as well as the reality that the model describes, are highly non-linear, this cannot be immediately calculated. We approach this problem by linearizing the budget constraint around the equilibrium point Q and denote by \( \hat{p} \) the (linearized) price and by \( \hat{y} \) the (linearized) income that result from this procedure (cf. Figure 4).

Assume now that the whole non-linear budget set changes in such a way that the marginal price of housing, \( -\frac{\delta \pi}{\delta \tilde{h}} \), is increased by \( \delta \) units at each level of \( \tilde{h} \). This would yield a new
equilibrium at $Q_1$, a new linearized price $\hat{p}_1$ and a new linearized income $\hat{y}_1$. We will assume that $Q_1$ is at the same linear segment as $Q$, which will be the case for small $\delta$, again ignoring households at the kink points. It then follows\textsuperscript{7} that $\hat{p}_1 = \hat{p}_0 + \delta$ and $\hat{y}_0 = \hat{y}_1$.

The effect on demand from a unit change in the slope of the actual nonlinear budget constraint is identical to the effect from an equal change in the appropriate linear price. Hence, we can use our estimated model, which captures the non-linearities, to predict the effects from changing an ordinary (linear) price. The predicted effects are given by

$$\frac{\hat{d}h}{\hat{d}p} = \frac{\hat{d}h}{\hat{d}RMP} + \frac{\hat{d}h}{\hat{d}OMP} - \frac{\hat{d}h}{\hat{d}ODI} \frac{RDI}{0.24} \quad (8)$$

and

$$\frac{\hat{d}\pi}{\hat{d}p} = \frac{\hat{d}\pi}{\hat{d}RMP} + \frac{\hat{d}\pi}{\hat{d}OMP} - \frac{\hat{d}\pi}{\hat{d}ODI} \frac{RDI}{0.24} \quad (9)$$

The first two terms in these expressions reflect that the slope is changed equally everywhere. The third terms reflect the fact that the level of the curve at the hypothetical housing demand, $h = RDI/0.24$, is decreased by $h \cdot dp$.

The derivatives (6) - (9) based on our estimated model and computed for each household will be used in the welfare calculations to be analyzed in the following sections.
4. Welfare Theory with Discrete Choice

The application of welfare theory to models with discrete choice has been analyzed by Small and Rosen (1981). In our application to the Swedish 1983-85 tax reform, we will follow their approach. Due to the non-linearity of the budget set, however, we will have to modify their formulas somewhat. When linearizing the budget set in the way outlined at the end of section 3 above, we will have to take into account not only the change in the linearized price \( \hat{p} \), but also the change in the linearized income \( \hat{y} \).

Let us start with the following linearized formulation of the household's choice problem.

\[
\begin{align*}
\text{Max } & u(z, h_o, h_R) & \quad (10a) \\
\text{s.t. } & z + D\hat{p}h_o + (1-D)R\hat{h}_R = D\hat{y}_o + (1-D)\hat{y}_R & \quad (10b) \\
& h_o \cdot h_R = 0 & \quad (10c) \\
& D = 1 \text{ if } h_o > 0 & \quad (10d) \\
& D = 0 \text{ if } h_R > 0 & \quad (10e)
\end{align*}
\]

where \( h_o \) is the quantity of owner-occupied housing, and \( h_R \) is the quantity of rental housing consumed. Here we have written the budget constraint as an ordinary linear constraint with prices \( \hat{p} \) and \( \hat{R} \), respectively.
The only difference between our problem and that of Small and Rosen (1981) is that we allow for the income to differ depending on which tenure mode the household chooses. If it chooses owner-occupancy, its income will be \( \hat{y}_o \) while if it chooses renting, the income will be \( y_R \) (corresponding to the variable RDI).

The optimization problem (10) can be broken down into two conditional optimization problems:

Max \( u(z, h_o, 0) \) \hspace{1cm} (11a)

s.t. \( z + p \cdot h_o = \hat{y}_o \) \hspace{1cm} (11b)

and

Max \( u(z, 0, h_R) \) \hspace{1cm} (12a)

s.t. \( z + R \cdot h_R = y_R \) \hspace{1cm} (12b)

The solution to the unconditional problem (10) is identical to the solution to that of the two problems (11) and (12) which has the highest indirect utility. Denoting the unconditional indirect utility of (10) by \( v(\hat{p}, R, \hat{y}_o, y_R) \) we thus have

\[
v(\hat{p}, R, \hat{y}_o, y_R) = \text{Max} \left[ \tilde{v}_o(\hat{p}, \hat{y}_o), \tilde{v}_R(R, y_R) \right]
\]

where \( \tilde{v}_o(p, \hat{y}_o) \) is the conditional indirect utility of the owner-occupant's problem, while \( \tilde{v}_R(R, y_R) \) is the conditional indirect utility of the renter's problem. Assume now that \( v(\hat{p}, R, \hat{y}_o, y_R) = u \). Following Small and Rosen (1981) we can define the
unconditional expenditure function \( e(\hat{p}, R, u) \) as well as the conditional expenditure functions \( \tilde{e}_o(\hat{p}, u) \) and \( \tilde{e}_R(R, u) \). Obviously

\[
e(\hat{p}, R, u) = \min[\tilde{e}_o(\hat{p}, u), \tilde{e}_R(R, u)].
\]

Since we also want to allow income to vary between the two choice alternatives it is convenient to define the two conditional surplus functions

\[
\tilde{s}_o(\hat{p}, \hat{y}_o, u) \equiv \tilde{e}_o(\hat{p}, u) - \hat{y}_o \tag{13}
\]

and

\[
\tilde{s}_R(R, y_R, u) \equiv \tilde{e}_R(R, u) - y_R \tag{14}
\]

The \( \tilde{s}_o \) function answers the following question: How much money do we have to give to the owner-occupant, in excess of his income \( \hat{y}_o \), to make it possible for him to attain the utility level \( u \)? The corresponding interpretation applies to the \( \tilde{s}_R \) function. We can also define the unconditional surplus function

\[
s(\hat{p}, R, \hat{y}_o, y_R, u) \equiv \min[\tilde{s}_o(\hat{p}, \hat{y}_o, u), \tilde{s}_R(R, y_R, u)].
\]

It must hold that

\[
s(\hat{p}, R, \hat{y}_o, y_R, u) = 0.
\]

If the household chooses owner-occupancy, then

\[
\tilde{s}_o(\hat{p}, \hat{y}_o, u) = 0
\]

\[
\tilde{s}_R(R, y_R, u) > 0
\]
while $\tilde{s}_o \geq 0$ and $\tilde{s}_R = 0$ if rental housing is the preferred alternative.

Let us now assume that a tax reform takes place. This affects the price of owner-occupied housing $\hat{p}_o$, the owner-occupant's income $\hat{y}_o$, and the renter's income $\hat{y}_R$. Disregarding general equilibrium effects, we assume that the price of rental housing $R$ is unchanged. In analogy with standard use [see e.g. King (1983)], we define the compensating gain (CG) of the tax reform as

$$
CG = s(\hat{p}_o, R, \hat{y}_o, \hat{y}_R, u^0) - s(\hat{p}_1, R, \hat{y}_o, \hat{y}_R, u^0)
$$

(15)

where the triple $(\hat{p}_o, \hat{y}_o, \hat{y}_R)$ describes the pre-reform tax system, the triple $(\hat{p}_1, \hat{y}_o, \hat{y}_R)$ describes the post-reform tax system, and $u^0$ is the pre-reform utility level. Equation (15) is the basic relation to be used in the welfare comparisons. As our estimations do not give us utility function parameters we must make a Taylor approximation of this formula.

5. The Welfare Formulas

In the previous section we derived the surplus function for one household, where it was presumed that the researcher has exact knowledge about his utility function and hence can say with certainty which alternative he will choose. However, econometric studies of discrete choice problems only give estimated probabilities for a particular choice. The usual interpretation of this, see e.g. McFadden (1981) and Hanemann (1984), is that there
are unobservable factors reflecting taste differences among observationally equivalent individuals. With this interpretation each consumer in the econometrician's sample is assumed to represent a large number of consumers. Assuming that unobservable taste differences can be represented by an index \( \alpha \) with an associated distribution function \( F(\alpha) \), the aggregate unconditional surplus function\(^8\) can be written as:

\[
\tilde{s}(p, R, y_o, y_R, u) = \int_{-\infty}^{\alpha^*} s_o(p, \hat{y}_o, \hat{y}_R, u, \alpha) dF(\alpha) + \int_{\alpha^*}^{\infty} s_R(R, y_R, u, \alpha) dF(\alpha) \tag{16}
\]

where \( s_o(p, \hat{y}_o, u, \alpha) \) is the surplus function of an owner-occupant with an unobservable taste prameter \( \alpha \). Here \( \alpha^* \) is the switchpoint at which a household is indifferent between consuming \( h_o \) and consuming \( h_R \). Needless to say, \( \alpha^* \) is not exogenous, but is a function \( \alpha^*(p, R, \hat{y}_o, y_R, u) \). Changing the tax system will also change the switchpoint, i.e. will induce some people who earlier were renters into owner-occupancy, or vice versa.

In many situations there is no direct knowledge about the utility function parameters needed to compute CG exactly according to (15), so some approximations based on compensated price derivatives are needed. From (16), (14) and (13) we have, using Shephard's lemma and assuming that \( \alpha \) does not enter into the compensated demand functions, that\(^9\)
\[
\frac{\partial \tilde{s}}{\partial p} = \pi^c(\hat{p}, R, \hat{y}_o, y_R, u) \ast \tilde{h}_o^c
\]  
\text{(17a)}

\[
\frac{\partial \tilde{s}}{\partial R} = [1 - \pi^c(\hat{p}, R, \hat{y}_o, y_R, u)] \ast \tilde{h}_R^c
\]  
\text{(17b)}

\[
\frac{\partial \tilde{s}}{\partial y_o} = -\pi^c(\hat{p}, R, \hat{y}_o, y_R, u)
\]  
\text{(17c)}

\[
\frac{\partial \tilde{s}}{\partial y_R} = [-1 - \pi^c(\hat{p}, R, \hat{y}_o, y_R, y)]
\]  
\text{(17d)}

where

\[
\pi^c(\hat{p}, R, \hat{y}_o, y_R, u) \equiv \int_{-\infty}^{\alpha^*} dF(\alpha).
\]

The number \(\pi^c\) is the compensated choice probability and \(\tilde{h}^c\) is the compensated conditional demand. Given that all individuals are compensated so that they, under the tax system \((\hat{p}, \hat{y}_o, y_R)\), attain the utility level \(u\), a fraction \(\pi^c(\hat{p}, R, \hat{y}_o, y_R, u)\) of the observationally equivalent individuals will choose owner-occupancy \((h_o > 0)\), while a fraction \([1 - \pi^c(\hat{p}, R, \hat{y}_o, y_R, u)]\) will choose rental housing \((h_R > 0)\).

To approximate the CG of equation (15) one also needs the second-order derivatives

\[
\frac{\partial^2 \tilde{s}}{\partial p^2} = \pi^c \frac{\partial^2 \tilde{h}_o^c}{\partial p^2} + \frac{\partial \tilde{h}_o^c}{\partial p} \frac{\partial \pi^c}{\partial p},
\]  
\text{(18a)}
We are now ready to write down the formula for the compensating gain (CG) applied to the 1983-85 Swedish tax reform. In the terms used above we want to study a move from the tax system \((^82 \ o \ y^o, y^R)\) to \((^85 \ o \ y^o, y^R)\). For this purpose we calculate, for each individual household, the pre-reform \(y^R = RDI^82\) and linearized price and income \(p^82 \ o y^o\) based on predicted conditional housing demand \(h_o^82\), which is defined by equation (3) above. We then use our prediction model to generate a new, post-reform consumption vector for each household \((z^85, h^85)\) and compute the linearized price \(p^85\) and income \(y^o^85\) at that new equilibrium. We also calculate the pre-reform choice probabilities \(\pi^82\), defined by eq. (5) above, and use (19) to obtain the compensated derivatives. Finally we compute the post-reform income \(y^R = RDI^85\). With these numbers, and the numerical values of the derivatives (18) and (19), we can calculate an approximation of the average CG for each observationally equivalent household, defined by

\[
CG = s(p, R, y^o, y^R, u) - s(p, R, y^o, y^R, u^82).
\]

Looking at the derivatives in (18) it may appear that we do not have information on \(\frac{\delta^2s}{\delta y_o\delta y^R} = -\frac{\delta\pi^c}{\delta y^R} = \delta y^o\). However, by Young's theorem (applied to \(\frac{\delta^2s}{\delta p\delta y^R}\)), we have

\[
\frac{\delta\pi^c}{\delta y^R} = \frac{1}{h_o^82} \frac{\delta\pi^c}{\delta p}.
\]

Expanding the expression for \(CG\) around \((p^82, y^o^82, y^R^82)\) and collecting terms yields
\[
\frac{\delta^2 s}{\delta y_o^2} = - \frac{\delta \pi^c}{\delta y_o},
\]

(18b)

\[
\frac{\delta^2 s}{\delta y_R^2} = \frac{\delta \pi^c}{\delta y_R},
\]

(18c)

\[
\frac{\delta^2 s}{\delta p \delta y_o} = \frac{\delta \pi^c}{\delta y_o} \frac{\delta y_o}{\delta y_R} = - \frac{\delta \pi^c}{\delta p},
\]

(18d)

\[
\frac{\delta^2 s}{\delta p \delta y_R} = \frac{\delta \pi^c}{\delta y_R} \frac{\delta y_R}{\delta y_o} = \frac{\delta \pi^c}{\delta p},
\]

(18e)

\[
\frac{\delta^2 s}{\delta y_o \delta y_R} = \frac{\delta \pi^c}{\delta y_R} \frac{\delta y_R}{\delta y_o} = \frac{\delta \pi^c}{\delta y_o}.
\]

(18f)

The derivatives of \(\pi^c\) are given from the definition

\[
\pi^c(\hat{p}, R, \hat{y}_o, \hat{y}_R, u) \equiv \pi[\hat{p}, R, \hat{y}_o + \tilde{s}(\hat{p}, R, \hat{y}_o, y_R, u), y_R + \tilde{s}(\hat{p}, R, \hat{y}_o, y_R, u)].
\]

Thus,

\[
\frac{\delta \pi^c}{\delta p} = \frac{\delta \pi}{\delta p} + \pi^c \frac{\delta \pi}{\delta y_o} + \pi^c \frac{\delta \pi}{\delta y_R},
\]

(19a)

\[
\frac{\delta \pi_c}{\delta y_o} = \frac{\delta \pi}{\delta y_o} - \pi^c \left[ \frac{\delta \pi}{\delta y_o} + \frac{\delta \pi}{\delta y_R} \right],
\]

(19b)

\[
\frac{\delta \pi_c}{\delta y_R} = \frac{\delta \pi}{\delta y_R} - (1 - \pi^c) \left[ \frac{\delta \pi}{\delta y_o} + \frac{\delta \pi}{\delta y_R} \right].
\]

(19c)

The exact way in which these derivatives will be useful in deriving an approximation of CG depends on the particular application. As we will see below it is e.g. not necessary to have information on \(\delta \pi/\delta y_o\) and \(\delta \pi/\delta y_R\) separately, but it is sufficient to know their sum.
\[ \bar{CG} = -\pi \overset{\sim}{\gamma}_o (p^5 - \overset{\sim}{p}^2) + \pi \cdot (y^5_o - \overset{\sim}{y}_o^2) + 
\]
\[ + (1 - \pi) \cdot (y^5_R - \overset{\sim}{y}_R^2) - \frac{1}{2} \left[ \pi \frac{\delta \overset{\sim}{\gamma}_o}{\delta p} + \overset{\sim}{h}_o \frac{\delta \pi^c}{\delta p} \right] \cdot 
\]
\[ \cdot (p^8 - \overset{\sim}{p}^2)^2 + \frac{1}{\overset{\sim}{h}_o} \frac{\delta \pi^c}{\delta p} \left[ (y^5_o - \overset{\sim}{y}_o^2) - (y^5_R - y^8_R) \right]^2 + 
\]
\[ + 2 \frac{\delta \pi^c}{\delta p} (p^8 - \overset{\sim}{p}^2) \left[ (y^5_R - y^5_R) - (y^5_o - y^8_o) \right] \] (20)

6. The Results

We can now use our model to compute \( \bar{CG} \) according to (20) for each one of the 2950 households in our sample. Before interpreting the results, a caveat should be mentioned. Since we are comparing long-run equilibria, transient capital gains and losses are neglected. In reality, the fear of large capital losses has been an important factor in the debate surrounding the reform. Our own previous analysis, in Brownstone et al. (1984), indicates that such fears may be well founded, since we predicted that aggregate demand would fall by 15 per cent, and the demand for large houses would tend to fall more sharply. Hence, the short-run effects of the reform are more adverse for high-income earners than the present calculations of long-run welfare effects indicate.

Our calculations of compensating gains show that some households gain from the reform and some households lose; but a majority are on the winning side. This is mainly due to the fact that the reform does not maintain budget balance, which implies positive income effects for most households. We will come back
to the issue of budget balance later in this section; results in Figures 5-10 refer to the unbalanced reform. Figure 5 displays the distribution of $\bar{G}$ over households. The distribution is rather skewed; hardly any households have lost more than 500 Swedish crowns\(^{11}\) while there is a long tail on the positive side.\(^{12}\)

A diagram like that in Figure 5 tells us nothing about who the winners and losers are. In Figure 6 we have therefore arranged the households in order of disposable income, measured by the RDI variable. The households have been divided into deciles according to their respective RDI, and for each decile we have computed the average $\bar{G}$ over the households in that decile.\(^{13}\) For each decile, this average $\bar{G}$ is indicated by a little "x". To give a visual impression of how $\bar{G}$ varies over income groups, the averages have been connected by straight lines. Since the households within the same income class can be rather different, however, we have also computed a 90% "confidence interval". This has been accomplished by deleting the 5% of the households with the highest $\bar{G}$ and the 5% with the lowest $\bar{G}$ within each decile. The resulting spread is illustrated by the vertical lines in Figure 6. There is a clear tendency that $\bar{G}$ increases with income; the curve connecting the ten averages increases almost monotonically. Also the variation within classes is larger than the variation between classes.\(^{14}\)

One would like to see whether this pattern prevails if we limit ourselves to a more homogeneous group of households. In Figure 7 we have therefore computed the same statistics as in Figure 6, but only for the subgroup in the sample that consists
Figure 6: Mean and 90 per cent spread of $\bar{GG}$ for each decile. 1982 Swedish kronor. Entire sample.
of households with two adults and two children - a kind of "standard" household. We see that the general image is slightly different from that of the entire sample: the variation within deciles is somewhat smaller and the tendency for the average \( \bar{CG} \) to increase is less evident.\(^{15}\) Still, the variation within deciles is much larger than the variation between decile averages.

Does this mean that the tax reform will tend to increase the inequalities in Sweden? Without going into the difficult question of how to measure the degree of inequality, we have seen that the high-income earners make larger gains in absolute terms than the low-income earners. The picture is the same if we study the gains in relative terms. Let us for each decile \( J \) define the average relative \( \bar{CG} \) by

\[
\text{RCG}_J = \frac{\sum_{i \in J} \bar{CG}_i}{\sum_{i \in J} \text{RDI}_i}.
\]

The resulting numbers for the entire sample are depicted in Figure 8. We see that now the picture is less clear. Fitting a straight line would yield a positive slope, but there are obviously two distinct subgroups in the sample: the low-income earners in the first three deciles (RDI below 47,814 Swedish kronor in 1982 prices, which implies either less than full-time work or misreporting of income), and the rest of the sample. For the top seven deciles one can in fact see a slight tendency for RCG to fall with income. Computing the average relative \( \bar{CG} \) for the "standard" households consisting of two adults and two
Figure 7: Mean and 90 per cent spread of \( \bar{CG} \) for each decile. 1982 Swedish kronor. Households with two adults and two children.
Figure 8: Average relative $\bar{G}$ for each decile. Entire sample.
children (Figure 9) we see that there is no clear tendency for the relative gain to increase.

The reform has been criticized for being unfavourable to home owners. We have for that reason split the sample by mode of tenure, where we have defined as owners those who were observed as owners in our data from 1979. It is seen (see figures in the Appendix) that owners in general gain more in both absolute and relative terms than do renters. This is to some extent related to the fact that renters in general are poorer. But it also holds, somewhat surprisingly, controlling for income and family size; for a two children - two adults owner family with an RDI above 80,000 kronor the compensating gain is on average 1.3 per cent of disposable income, while the corresponding percentage for renters is 0.8. An explanation for this is that not only home-owners but households in other categories as well make interest deductions for various purposes.

In the paragraphs above we tried to isolate the income distribution aspect by looking at a subsample of "standard" households (Figures 7 and 9). Of course, we can control for more factors that could account for the seemingly anti-egalitarian profile of the reform. We have therefore run an OLS regression on \( \Delta C \) with the same demographic and geographic variables that were used in the original estimation of our model (see section 3 above), plus income RDI, as explanatory variables. The result of this regression is given in the left columns of Table 1. The first variable is RDI, and we see that its estimated coefficient is positive in sign and highly significant. Thus households which were high-income earners in 1982 have made larger gains
Figure 9: Average relative CG for each decile. Households with two adults and two children.
than low-income earners in general, even when we control for a number of other factors. The effect is however minor. A difference in income between two households of, say, 100,000 Swedish kronor would imply a ceteris paribus difference in \( CG \) of only 82 kronor.

We have also run an OLS regression using the relative compensating gain, \( CG/RDI \), as a dependent variable (right columns of Table 1). We see that the general picture remains; the high-income groups have gained more also in relative terms, when we keep other factors constant. The coefficient of RDI is still significantly positive, although almost negligible in magnitude. It indicates that 100,000 kronor in income difference will imply a .00026 per cent difference in relative compensating gain.

Many studies of the redistributational effects of tax reforms take only the first-order effects into account; household behavior is assumed to be unaffected. Such an approach studies the cash gain in King's (1983) terminology. An investigation of this tax reform published by the National Central Bureau of Statistics (1982) based its conclusions on this concept. However, in the long run people will adjust to the new tax system, and the cash gain concept tends to overestimate the actual welfare loss (or underestimate the actual welfare gain). Returning to our basic formula (20), we see that the first three terms correspond to the cash gain (i.e. \( \bar{h}_0 \) and \( \pi \) are assumed to be constant and the welfare changes are due to changes in \( p \) and \( y \) only). The third term (within curled brackets) describes the allocation effect, i.e. the fact that households can adjust to
the new conditions. It is instructive to calculate the value of this third term separately to see whether it is of any numerical importance.

We have calculated the second-order term in eq. (20) for each household, computed averages and 90 per cent spreads for each decile, and displayed the result in Figure 10. It is evident that the allocational effects are not of major importance. On the average, the size of the allocation term is around 4 per cent of the (absolute value of the) total compensating gain \( \Delta \bar{c} \). An exception is the highest decile, where the allocation term is on average 14 per cent of the total \( \Delta \bar{c} \). Thus the use of the cash gain concept will underestimate the gain more for the high-income groups than for the low-income groups.

The calculations reported above rest on the assumption that most households pay less taxes as a result of the reform. Estimated tax payments, underlying the calculations, fall from 24,543 kronor per household to 24,105 kronor. The study of an unfinanced tax reform makes little sense, however. In reality the economy's resource constraint has to be adhered to, and any tax schedule change will always be "financed" by compensating tax changes or inflation.

A simple scheme for restoring budget balance is to let each household pay an extra lump-sum tax that is proportional to RDI. Denoting this proportion by \( \alpha \), it means that the following equality must hold:

\[
\sum_i T^{82}(h_i, \pi_i, X_i) = \sum_i T^{85}(h_i, \pi_i, X_i) + \alpha \sum_i \text{RDI}_{i}^{85} \quad (21)
\]
Figure 10: Mean and 90 per cent spread of the second-order term in eq. (20) for each decile. 1982 Swedish kronor. Entire sample.
In this expression $T^8_2$ and $T^8_5$ are the tax schedules analyzed above. $X$ is a vector of exogenous incomes plus demographic and other variables affecting the tax treatment of the household and $\tilde{h}_i$ and $\pi_i$ are predicted conditional demand and choice propabilities. $\tilde{h}^8_5$ and $\pi^8_5$ are calculated accounting for the fact that the tax payment, $\alpha \cdot RDI$, will affect each household's demand.

Solving (21) by iteration gives $\alpha = 0.0047$. Presuming that the households after this tax payment are at the same marginal tax bracket as in the unfinanced reform, the relevant values of RCG can be read off Figures 8 and 9 by deducting 0.47 percentage points. It is seen that all decile averages for two adults - two children families are still positive.

7. Concluding Comments

In this paper we have employed a microsimulation model to study the welfare effects of a particular reform of the tax treatment of owner-occupied housing. This has several advantages compared with the more common practice of evaluating reforms and reform proposals only for hypothetical "representative" households. First, the mean forecasts presented differ substantively from the forecast for a household with mean values of the independent variables. As discussed in more detail in Brownstone et al. (1983), this is largely due to the non-linearity of the discrete choice model. Second, we have been able to get a picture of the distributional aspects. Just looking at the decile means one gets the impression of a fairly moderate
vertical redistribution. But as we have seen there is a large variation of the effects, horizontal redistribution, within any single decile.

Analyses of actual tax reforms are complicated by two factors: the strong non-linearity of most actual tax schedules and the importance of discrete choice aspects (to work or not, to own or to rent etc.). For econometric reasons the researcher has to choose between estimating a model based on very restrictive assumptions but enabling the identification of utility function parameters or estimating a more flexibly formulated model. In the former case the calculation of welfare effects is straightforward. In this paper we have demonstrated the tractability of Harberger-type approximations in a more flexible model.

The numerical results of this study show that there are no easily identifiable winners or losers from the proposed tax reform. For groups segregated according to income and/or family structure, the variation of the welfare gains within the groups is larger than the variation between groups. It is difficult to know whether this was an intentional feature of the reform or whether it was simply due to the highly complex structure of the underlying Swedish tax system. From a politician's standpoint, however, this lack of easily identifiable winners or losers makes it very difficult to organize effective opposition to the reform. This may be one reason why this particular reform faced remarkably little political opposition.
Footnotes

*. This research was done as a part of a research program on taxation at the Stockholm School of Economics financed by the Bank of Sweden Tercentenary Foundation. We wish to thank the Foundation for its generosity and Bo Nordin for research assistance.


2. Unless, of course, the renter has borrowed money for some other purpose than housing (for example art or consumer durables) and makes large interest deductions for that reason.

3. Alternatively, one might try to adapt the maximum-likelihood estimation technique developed by Burtless and Hausman (1978) and employed, among others, by Blomquist (1983). This has the advantage that utility function parameters may be identified, but it would be very complicated to compute given the other aspects of our model.

4. This is the standard assumption in studies of the housing market. Cf. the survey by Rosen (1983).

5. See Brownstone et al. (1983) for details.

6. The exact number varies over households.

7. That $\hat{y}_0 = \hat{y}_1$ can be seen as follows. The equation for the original linearized constraint is

$$\text{\hat{y}_o} = B_o (x + r\text{W}, h_o) + \text{\hat{p}_o h}_o$$

By assumption the new equilibrium will be at the same linear segment of $B$, so the linearization can still be performed around $h_o$, even if that is no longer at equilibrium. Since the slope of $B$ is everywhere changed by $\delta$ it is clear that
\[ B_1(x + rW, h_o) = B_0(x + rW, h_o) - \delta h_o \]

Hence, since \( \hat{p}_1 = \hat{p}_0 + \delta \)
\[ \hat{y}_1 = B_0(x + rW, h_o) - \delta h_0 + (\hat{p}_0 + \delta)h_o = \hat{y}_0. \]

8. In (16) \( u \) is a scalar, i.e. it is presumed that \( u \) is the same for all households. In this case \( \alpha \) reflects a choice propensity unrelated to the utility derived by the consumers. Alternatively, \( \alpha \) is assumed to be a factor that is additive to the conditional indirect utility functions. In this case \( u \) would be a function of \( \alpha \). The choice between these interpretations only affects notation.

9. When taking the derivatives of (16) to obtain (17) we note that there will be terms of the type
\[ [\hat{s}_o(\hat{p}, \hat{y}_o, u, \alpha^*) - \hat{s}_R(R, y_R, u, \alpha^*)] \cdot \frac{\partial \alpha^*}{\partial p} \]
and
\[ [\hat{s}_o(\hat{p}, \hat{y}_o, u, \alpha^*) - \hat{s}_R(R, y_R, u, \alpha^*)] \cdot \frac{\partial \alpha^*}{\partial R} \]
etc. These terms will cancel because \( \hat{s}_o(\hat{p}, \hat{y}_o, u, \alpha^*) = \hat{s}_R(R, y_R, u, \alpha^*) \), since households at the switchpoint (i.e. with \( \alpha = \alpha^* \)) are by definition indifferent between owning and renting.

10. To economize on space we have deleted all derivatives with respect to \( R \), since the rent level is assumed to be unaffected by the tax reform. The generalization to a reform which also changes \( R \) is straightforward.
11. All numbers are in 1982 prices. In fact, the minimum value of \( \bar{G} \) is \(-7,108\) Swedish crowns, and some people have lost between 7,000 and 500 crowns. These people are so few that they are not discernible in the diagram.

12. The maximum gain is \(60,549\) crowns, but people with gains in excess of \(16,000\) crowns are less than \(0.1\) per cent of the population.

13. The decile limits are as follows:

1st decile: \(\text{RDI} \leq 32,203\) Skr

2nd decile: \(32,203 - 39,929\)

3rd decile: \(39,929 - 47,814\)

4th decile: \(47,814 - 56,397\)

5th decile: \(56,397 - 67,834\)

6th decile: \(67,834 - 82,374\)

7th decile: \(82,374 - 100,610\)

8th decile: \(100,610 - 114,394\)

9th decile: \(114,394 - 133,439\)

10th decile: \(\text{RDI} \geq 133,439\) Skr

14. The lowest deciles are an exception to this statement. For these groups - mainly pensioners, students etc. - the tax reform means an increase in the marginal tax rate by one per cent. They are in too low income brackets to be affected by the limitation of the deductibility of interest payments.

15. In this subgroup, the households are on the average richer than in the entire sample. The decile limits are:

1st decile: \(\text{RDI} \leq 80,832\) Skr

2nd decile: \(80,832 - 93,381\)

3rd decile: \(93,381 - 100,799\)

4th decile: \(100,799 - 107,655\)

5th decile: \(107,655 - 113,409\)

6th decile: \(113,409 - 119,524\)

7th decile: \(119,525 - 126,210\)

8th decile: \(126,210 - 138,086\)

9th decile: \(138,086 - 157,082\)

10th decile: \(\text{RDI} \geq 157,082\) Skr
References


Compensated gains for owners and renters.
Figure A1: Mean and 90 per cent spread of CG for each decile. 1982 Swedish kronor. Households observed as owners in 1979.
Figure A2: Mean and 90 per cent spread of $\bar{CG}$ for each decile. 1982 Swedish kronor. Households observed as renters in 1979.
Figure A3: Average relative $\bar{G}$ by decile for households observed as owners in 1979.
Figure A4: Average relative CE by decile for households observed as renters in 1979