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VOTING OVER THE SIZE OF GOVERNMENT

by

Mats Persson

INSTITUTE FOR INTERNATIONAL ECONOMIC STUDIES

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Institute for International Economic Studies
S-106 91 Stockholm
Sweden
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Mats Persson
Institute for International Economic Studies
University of Stockholm
S-106 91 Stockholm
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Abstract

This paper studies the provision of a public good from three points of view: (i) The maximum level of production of the public good consistent with a Pareto optimum, (ii) The level of production of the good maximizing a social welfare function, and (iii) The level of production of the public good that will be the result of a democratic system with majority voting. The paper analyzes how these three levels of production differ from each other.
VOTING OVER THE SIZE OF GOVERNMENT*

I. Introduction

Many studies of the growth of government rely on the assumption of some agents in the economy being able to exercise monopoly power in one way or another, i.e. being able to manipulate the political process in their favor. Logrolling, the cost for taxpayers of collecting information about the true consequences of various political decisions, the growth of organized pressure groups and the competition among these groups for political influence can be regarded as expressions for such imperfections in the political market.\(^1\) Another approach relies on the assumption of perfectly rational, well-informed and atomistic agents with no opportunity to manipulate the economy.\(^2\) In the present paper we will follow the latter approach, and we will use the Median Voter Theorem to show that even in such a world the size of the public sector can deviate from what would be optimal.

The paper is organized as follows. In section II we set up the formal model, which analyzes the optimal provision of a public good. The well-known Samuelson (1954) condition for a Pareto optimum is analyzed, and it is shown that of all efficient allocations there is one allocation where the volume of the public good attains a maximum \(y_{\text{max}}\). This is the "egalitarian" allocation where all agents enjoy the same level of utility. We thus interpret \(y_{\text{max}}\) as the largest size of the public sector that is compatible with a Pareto optimum. In Section III we discuss what we will call a "social optimum", i.e. the allocation which maximizes a social welfare function. It is demonstrated that for
a given shape of the tax function, such an allocation exists and is unique. This allocation is associated with a particular size \( y^* \) of the public sector. We then in section IV turn our attention to the public-choice aspects of the model; it is shown that under a system of majority voting, a political equilibrium exists. This equilibrium is in general different from the Pareto-optimal allocations analyzed in section II and from the social optimum derived in section III. Denoting the size of production of the public good that will be the outcome of the political process by \( y(x_m) \), we show that for a symmetric wealth distribution it holds that \( y_{\text{max}} = y(x_m) \). This means that a system with majority voting will produce a size of the public sector which is equal to the maximal size compatible with a Pareto-efficient allocation; for all Pareto-efficient allocations except the "egalitarian" one, the public sector will be smaller than the one produced by the political system. We also show that within our model, \( y^* < y(x_m) \), i.e. the size of the public sector which maximizes the social welfare function is smaller than the size of the public sector preferred by the median voter. In section V we discuss the consequences of introducing other tax functions than the lump-sum tax studied in the previous sections. It is concluded that the basic problem, i.e. the fact that the political process will generate a size of the production of the public good which is different from the optimal one, is not resolved by allowing more general tax functions.

II. The Model: Properties of Pareto Optimum

We consider an economy with only two goods, one public good and one private. The public good is produced from the private
good by a production function $g(\cdot)$. We assume that each agent $i$ is endowed with a certain quantity $x_i$ of the private good and that he pays a tax $t_i$ to finance the production of the public good. Denoting the total quantity of the public good by $y$, we thus have that each agent attains a level of utility $u^i(y, x_i - t_i)$, where $y$ is the same for all agents. Note that $x_i$ is the agent's endowment; to make our main argument, we do not have to assume that there is a market for private goods but can take each $x_i$ as exogenous. Of course we could make the model more complicated by considering several private goods, the various agents' supply and demand functions, and a market equilibrium together with the political equilibrium that determines the production of the public good, but such complications would not yield any additional insights to the model.

Throughout this paper we will assume that the utility function is additively separable of the form

$$u^i(y, x_i) = v^i(y) + w^i(x_i - t_i).$$

This somewhat limiting assumption is in no way necessary for the analysis - the results to be derived below still hold for a much larger set of utility functions$^3$ - but it facilitates the exposition considerably since it allows us to disregard the cross derivatives. We will therefore confine the analysis to the case of additive separability only. However, to remind the reader that the results are applicable also to non-separable utility functions, we will not write the function as $v^i(y) + w^i(x_i - t_i)$ in the following, but will retain the somewhat more general notation $u^i(y, x_i - t_i)$.

The resource constraint of the economy is written as

$$y = g(T) \quad (1)$$
where $T = \int t_i dF(i)$ is the total amount of taxes collected.

Samuelson (1954) showed that the necessary condition for a Pareto optimum in such an economy can be written as

$$\int \frac{u_i^y[g(T), x_i - t_i]}{u_i^x[g(T), x_i - t_i]} dF(i) = \frac{1}{g'(T)}$$

(2)

where $u_i^y$ and $u_i^x$ are the partial derivatives of $u_i$ with respect to $y$ and $x$. A condition like (2) does not give a unique solution for the economy, but defines a whole surface, the utility-possibility frontier, which gives us the various Pareto-efficient utility configurations over individuals which can be attained.

This frontier, which defines a convex set if the usual assumptions on the utility functions are applied, is depicted for the two-person case in the upper half of Figure 1.

Pareto optimum, as defined by (2), is hardly of any practical importance. This is so mainly because preferences vary over individuals, and it is considered impossible in general for the authorities to make the individuals reveal their true preferences. Therefore a tax system satisfying (2), where the individual taxes $t_i$ depend both on wealth levels and on preferences, can hardly be imposed in practice. To simplify the exposition, we will from now on assume that all agents have identical preferences, which means that we can write the tax $t_i$ as a function of wealth, $t_i = t(x_i)$. This simplification does not mean, however, that we will consider Pareto-optimal allocations as feasible in practice; we will still only regard them as benchmark cases, and although we have assumed preferences to be identical for the sake of simplicity, we should bear in mind that they do vary over individuals in reality.
Allocations satisfying (2) will define a surface like that in the upper half of Figure 1, and with identical preferences this surface will be symmetric around the multi-dimensional equivalent of the 45° line shown in the figure. If we move over this surface, the total production y of the public good will not remain constant. We will in particular study the level of y which satisfies (2) at the point where all individuals attain the same utility, i.e. at the 45° line. This is so because, for a large set of utility functions, the Pareto-efficient level of production of the public good attains its maximum at that point.

This fact is easily proved for the two-consumer case and can be generalized to any number of agents.

With two individuals, (2) can be written as:

\[
\frac{u_y(y, x_1 - t_1)}{u_x(y, x_1 - t_1)} + \frac{u_y(y, x_2 - t_2)}{u_x(y, x_2 - t_2)} = \frac{1}{g'(t_1 + t_2)}. \tag{3}
\]

For each given utility level of consumer 2, say \(u_2\), equation (3) defines a unique solution \((y, t_1, t_2)\) which in turn defines a unique utility level \(u_1\) for consumer 1. We can therefore write (3) as

\[
m(y, u_1^-) + m(y, u_2^-) = c(y) \tag{4}
\]

where \(m(y, u_1^-)\) and \(c(y)\) are defined by the system

\[
m(y, u_1^-) \equiv \frac{u_y(y, x_1 - t_1)}{u_x(y, x_1 - t_1)} \tag{5}
\]

\[
c(y) \equiv \frac{1}{g'(t_1 + t_2)}
\]

\[
u(y, x - t) = u_1^- \tag{6}
\]
for \( i = 1, 2 \). That is, the \( m(y, u_i) \) are the marginal rates of substitution between public and private goods along the utility-possibility frontier, and \( c(y) \) is the marginal cost of producing \( y \) along that frontier.

Differentiating (5) and the resource constraint \( y = g(t_1 + t_2) \), and applying (4), yields after some manipulations

\[
\frac{-1}{\frac{du}{du} - 2} = -\frac{u_x(y, x_1 - t_1)}{u_x(y, x_2 - t_2)}. \tag{6}
\]

Let us now differentiate (4):

\[
\frac{m_i}{y} \frac{dy}{du} + \frac{1}{u} \frac{m_i}{y} - 1 + \frac{2}{u} \frac{m_i}{y} - 2 = c'(y)dy
\]

where \( m_i \) is shorthand for \( m(y, u_i) \), \( i = 1, 2 \). Applying (6) and rearranging terms yields

\[
\frac{dy}{du} = \frac{\frac{1}{u}}{m_1} \frac{u_x(y, x_1 - t_1)}{u_x(y, x_2 - t_2)} - \frac{\frac{2}{u}}{m_1} \frac{\frac{m_i}{y}}{m_i}. \tag{7}
\]

We see that \( \frac{m_1}{u} > 0, i = 1, 2 \) and that \( \frac{m_1}{y} + \frac{2}{m_1} \frac{m_i}{y} - c'(y) < 0 \). Thus

\[
\text{sgn} \frac{dy}{du} = \text{sgn} \left( \frac{\frac{2}{u}}{m_1} - \frac{\frac{m_i}{y}}{m_i} \right) \frac{u_x(y, x_1 - t_1)}{u_x(y, x_2 - t_2)} \tag{8}
\]

Assume that both individuals have the same utility: \( u_1 = u_2 \) so that \( x_1 - t_1 = x_2 - t_2 \). Then \( m_u = m_u \) and therefore

\[
\frac{dy}{du} = 0.
\]

\[
|_{u_1}^{u_1} - |^{2}_u = 0
\]
Thus the level of $y$ attains an extremum, which could be either a minimum or a maximum, at the $45^0$ line in Figure 1. We will now proceed to derive a sufficient condition for it to be a maximum.

Assume that individual 1 is assigned a lower utility level than individual 2. Thus $u_1 < u_2$ so that $x_1 - t_1 < x_2 - t_2$ and $u_x(y, x_1 - t_1) > u_x(y, x_2 - t_2)$. We immediately see from (8) that in such a case, a sufficient condition for $dy/du_2$ to be negative is that

$$\frac{m_1}{m_u} > \frac{m_2}{m_u}.$$  \hfill (9)

If $dy/du_2$ is negative for $u_1 < u_2$, then by symmetry the value of $y$ must reach a maximum at $u_1 = u_2$.

Let us now study what kind of utility functions $u(y, x)$ would satisfy a condition like (9), i.e. what kind of utility functions would be associated with a function $m_u$ which is decreasing in $\bar{u}$. For a given utility level $\bar{u}$ we have

$$u(y, x) = \bar{u}$$

which implicitly gives us the level of $x$ required to attain $\bar{u}$:

$$x = \phi(y, \bar{u}).$$

By definition,

$$u(y, \phi(y, \bar{u})) \equiv \bar{u}.$$  \hfill (10)

Differentiating this identity for $du = 0$ yields

$$\phi_y \equiv - \frac{u_y}{u_x} \equiv - m(y, \bar{u})$$
by our definition of $m(y, \tilde{u})$ in (5) and (6) above. Thus $m_u \equiv -\frac{\partial}{\partial y} \phi_u$ and, for (9) to hold, we want the function $\phi_u$ to be increasing in $\tilde{u}$. We have

$$\phi_{yu} = \frac{y \cdot u_{xx}}{(u_x)^2} \cdot \phi_u = m(y, \tilde{u}) \cdot \phi_u(y, \tilde{u}) \cdot u_{xx} \frac{u_x}{u_x}$$

(11)

where we, by the assumption of additive separability, have dropped the cross derivative $u_{yx}$. Now, of the three terms in (11) we know that $m(y, \tilde{u})$ is increasing in $\tilde{u}$. By taking the derivative of (10) with respect to $\tilde{u}$ it is easily shown that the term $\phi_u$ is also increasing in $\tilde{u}$. Thus a sufficient condition for $\phi_{yu}$ to be increasing in $\tilde{u}$ is that $u_{xx}/u_x$ is increasing in $\tilde{u}$, i.e. that the Arrow-Pratt measure of absolute risk aversion $R_A \equiv -u_{xx}/u_x \equiv -w''(x)/w'(x)$ is decreasing in wealth. Although this is not a property which characterizes all conceivable utility functions, it is a property which is confirmed by most empirical studies of risk aversion.\(^7\) In fact, the whole HARA family of utility functions used in the finance literature (which includes the widely employed power function $w(x) = x^{1-r}/(1 - r)$ was constructed as a response to the need for realistic and yet manageable functions, where "realistic" meant (among other things) "displaying decreasing absolute risk aversion".

Thus, for an additively separable utility function $u(y, x) = v(y) + w(x)$, where the $w(x)$ part displays decreasing absolute risk aversion, the Pareto-efficient provision of the public good attains a maximum at the 45\(^0\) line, as is depicted in the lower panel of Figure 1. This maximum will serve as a benchmark in the following analysis.\(^8\)
Before ending this section, however, we will characterize $y_{\text{max}}$ a little closer.

If all individuals enjoy the same utility, this means that $(x_i - t_i)$ are the same for all individuals. Thus the basic condition for a Pareto optimum (2) can be written

$$s \cdot \frac{u_y[g(T), x_i - t_i]}{u_x[g(T), x_i - t_i]} = \frac{1}{g'(T)}$$

(12)

where $i$ now refers to any individual and where $s$ is the size of the population, i.e. $s = \int dF(i)$. With no loss of generality we can set $s = 1$. Since (12) holds for any individual, we can pick the one who pays the average tax $\bar{t}$. With a total tax bill $T$ and an average tax of $\bar{t}$, we must have that $s \cdot \bar{t} = T$ and that, with $s = 1$, $\bar{t} = T$. Since we will consider only the "egalitarian" point on the utility-possibility surface where each individual enjoys the same utility, disposable wealth must be the same for everybody: $x_i - t_i = \alpha$ for all $i$. Thus the marginal tax rate must be 100 per cent at this particular Pareto allocation: $t(x_i) = x_i - \alpha$. This means that the tax function is linear in income, and therefore the person who pays the average tax $\bar{t}$ will also be the person who has the average (gross) wealth $\bar{x}$. Thus we can write (12)

$$\frac{u_y[g(T), \bar{x} - T]}{u_x[g(T), \bar{x} - T]} = \frac{1}{g'(T)}$$

(13)

Equation (13), which gives us the maximum level $y_{\text{max}}$ of the Pareto-optimal provision of the public good, will be more discussed in the following sections. Although it is conceptually self-evident that a unique solution $T_{\text{max}}$ to (13) exists — there
must exist a Pareto-efficient allocation at the $45^0$ line in

**Figure 1** — a formal proof should of course be given. We
will however postpone that proof somewhat; in the next section the
existence of a solution to an equation which is formally very
similar to (13) will be proved, and the reader will then find it
easy to apply the same kind of proof to (13).

III. **Social Optimum**

The Pareto optimum of the previous section is hardly feasi-
ble in practice; even if we have assumed identical preferences
in our model, peoples’ actual tastes will differ and be unobserv-
able in reality. Thus the actual tax schedule will have to be a
(preferably simple) function of some observable magnitude, for
example wealth. We will therefore treat the $y_{\text{max}}$ derived in the
previous section as a point of reference only, and we will
proceed to analyze how to maximize a social welfare function in
the presence of a simple tax rule.

The most common tax function used in the literature is the
linear function with an intercept: $t(x_i) = T + t \cdot x_i$. To make
the formal structure as simple as possible, we will disregard the
proportional part $t \cdot x_i$ and assume that all agents pay a lump-
sum tax $T$, which is the same for all individuals. To avoid
corner solutions, we assume that the parameters of our problem
are such that $x_i - T > 0$ for all agents $i$.

To economize on notation, we will from now on drop the
subscript $i$ denoting the individual agents. The utility function
is thus written $u(y, x - T)$, where the endowment $x$ varies over
agents, but where $y$ and $T$ are the same for all individuals.
The optimal size of the public sector (i.e. the production of y) is given by the programme

$$\begin{align*}
\text{Max} & \int_{T} u(y, x - T)dF(x) \\
\text{s.t.} & \quad y = g[\int_{T} dF(x)] = g(sT) = g(T).
\end{align*}$$

(14)

(15)

The objective function (14) could be derived either from a benevolent social planner with Utilitarian preferences or from a social contract argument, where the agents have to decide on T behind a veil of ignorance, before they know their endowments x. With the second interpretation, the solution to (14) can be regarded as an \textit{ex ante} Pareto optimum. In fact, an objective function which is linear in the individual utilities, like (14), is the only one that can be reconciled with any of these two main interpretations of what is really behind the concept of "social preferences", and it therefore seems to be preferred to other forms of social welfare functions.\textsuperscript{10}

Maximizing (14) subject to (15) gives rise to the first-order condition

$$\frac{\int_{T} u[y(g(T), x - T)dF(x)]}{\int_{x} u[g(T), x - T]dF(x)} = \frac{1}{g'(T)}. \quad (16)$$

With the standard concavity assumptions on u(.), one can easily see that the left-hand side of (16) is decreasing\textsuperscript{11} in T. Similarly, a concave production function g(.) implies that the right-hand side of (16) is increasing in T. If we further add the Inada conditions that

$$\lim_{y \to 0} u_y = +\infty \quad \text{and} \quad \lim_{x \to \infty} u_x = +\infty \quad (17)$$

together with
\lim_{T \to 0} g'(T) = +\infty \quad \text{and} \quad \lim_{t \to \infty} g'(T) = 0

we see that (16) will be guaranteed to have a unique solution \( T^* \).
This solution corresponds to a socially optimal size \( y^* \) of the public sector in the economy.\(^{12}\)

We thus have two measures of the optimal size of the public sector. First we have the various \( y \) values corresponding to the Pareto-efficient allocations; for a wide class of utility functions these have a maximum \( y^{\max} \) which was analyzed in the previous section. Second we have the unique value \( y^* \) which is derived from the maximization of a Utilitarian social welfare function. In the next section we will study how \( y^{\max} \) and \( y^* \) are related to the size of the public sector which will be the result of a political process with majority voting.

IV. Political Equilibrium

Let us now regard an individual agent with wealth \( x \). He is neither concerned about the Pareto-efficient allocations analyzed in section II nor about the social optimum \( T^* \) derived in section III. Instead, he prefers the value of \( T \) which maximizes his own utility, i.e. solves the problem

\[
\begin{align*}
\text{Max } u(y, x - T) \\
\text{s.t. } (15).
\end{align*}
\]

Note that the agent is perfectly rational in the sense that he realizes that the economy is subject to the resource constraint (15). Solving this maximization problem yields the first-order condition

\[
\frac{u_y[g(T), x - T]}{u_x[g(T), x - T]} = \frac{1}{g'(T)}.
\]
By an argument similar to that of the previous section, we see that (19) has a unique solution which we will denote by $T(x)$, where the $x$ serves to remind us that this is the $T$ value preferred by a particular agent, namely the one with the endowment $x$.

It is easily shown that our assumptions about the utility function imply that $T(x)$ is an increasing function of $x$, i.e., that $y$ is a normal good. Thus rich people prefer a larger public sector than poor people do. This may seem surprising against the casual observation that rich people complain more about high taxes than poor people do. In making this observation, however, one should not confuse the progressive, redistributive tax systems of the real world with the simple lump-sum tax system analyzed here. In order to concentrate on the main argument of this paper, i.e. the size of the public sector, we have disregarded the redistributive functions of tax system. If actual tax systems were of the type analyzed here, most rich people would probably be in favor of higher taxes. In fact, our result that $T'(x) > 0$ has an intuitive economic explanation. With lump-sum taxes and a decreasing marginal utility of consumption, most people who have a large endowment of the private good would prefer to reduce the consumption of $x$ somewhat to be able to increase the consumption of $y$ instead. Since $y$ could be the building of art museums and opera houses, it is hardly surprising that the rich would like to increase the supply of such goods while the poor (with high marginal utility of consumption of $x$) will rather prefer more consumption of private goods.
The fact that $T(x)$ is increasing in $x$ means that for a given $T = \tilde{T}$, people with wealth $x < \tilde{x}$, where $\tilde{x} = T^{-1}(\tilde{T})$, will always prefer a lower $T$ while people with $x > \tilde{x}$ will prefer a higher $T$. Thus preferences are single-peaked and a democratic political system with majority voting will result in the size of the public sector that is preferred by the median voter. Let us denote the wealth of the median voter by $x_m$. The political equilibrium will then be determined by equation (19) with $x$ substituted by $x_m$:

$$\frac{u_y[g(T), x_m - T]}{u_x[g(T), x_m - T]} = \frac{1}{g'(T)}.$$  \hspace{1cm} (20)

This is the basic relation to be compared to the formally very similar equations (13) and (16). The question is how the resulting tax system $T(x_m)$ is related to the optimal tax systems $T_{max}$ and $T^*$. That is, whether the political system will produce a too small or a too large public sector.

Let us start by comparing $T(x_m)$ to $T_{max}$, which defines the maximum size of the public sector which is in accordance with a Pareto optimum. To begin, we simplify matters by assuming that the distribution of wealth is symmetric, i.e. that $x_m = \bar{x}$. Comparing (13) and (20), with $x_m = \bar{x}$, we see that they are exactly the same equation, and thus they must have the same solution. We can immediately establish our first result:

$$T_{max} = T(\bar{x})$$

Since no allocation along the utility surface will have a larger public sector than at the "egalitarian" point at the $45^0$ line, the political system with majority voting will produce a public
sector which is larger than any Pareto-optimal allocation — except for the egalitarian allocation, where the public sector is of exactly the same size.\textsuperscript{15} Thus, for an additively separable utility function with decreasing absolute risk aversion, the political system will in general produce too large a public sector if $x_m = \bar{x}$.

For a skewed wealth distribution such that $x_m > \bar{x}$ the conclusion is strengthened; then $T_{\max} < T(x_m)$ and no Pareto-optimal allocation will have a public sector as large as the one which results from the political process. If the distribution is skewed the other way around ($x_m < \bar{x}$), which is the kind of skewness we actually observe in real data, however, it could happen that $T_{\max} > T(x_m)$. For such a case, the political process will produce too small a public sector.\textsuperscript{16}

This much for the Pareto-optimal allocation. From the point of view of practical economic policy the allocations that maximize the social welfare function (14) are more interesting, and we therefore proceed to compare the solutions of (16) and (20). To answer the question of whether $T(x_m)$ is greater than or less than $T^\ast$, we begin by assuming as before that the distribution of wealth is symmetric, i.e. that the median $x_m$ is equal to the mean $\bar{x}$. Then the political equilibrium is given by (20) with $x_m = \bar{x}$. By Jensen's Inequality, and by our standard assumptions about the utility function from section II above, we have that\textsuperscript{17}

$$\int u_x[g(T), x - T]dF(x) > u_x[g(T), \bar{x} - T]$$ \hspace{1cm} (21)

for all values of $T$. We also have that\textsuperscript{18}

$$\int u_y[g(T), x - T]dF(x) \leq u_y[g(T), \bar{x} - T]$$ \hspace{1cm} (22)
for all values of $T$. The inequalities (21) and (22) together imply that

$$\int u_y[g(T), x - T]dF(x) < \int u_x[g(T), \bar{x} - T]dF(x)$$

i.e. that

LHS of eq. (16) $<$ LHS of eq. (20)

for all values of $T$. This is depicted in Figure 2, and we can thus state our second result:

$$T(\bar{x}) > T^*.$$

This means that a political system with majority voting will produce a public sector which is larger than the one which is optimal in the sense of maximizing the social welfare function.\(^{19}\)

Let us end this section by relaxing the assumption of a symmetric distribution of wealth. Assume instead that the distribution of $x$ is skew in such a way that the median $x_m$ is greater than the mean $\bar{x}$. We know that the left hand side of (16) is increasing in $x$, and we denote this by LHS'$(x) > 0$. Thus LHS$(\bar{x}) < \text{LHS}(x_m)$ for $x_m > \bar{x}$, and a glance at Figure 2 would then tell us that $T(x_m) > T(\bar{x}) > T^*$ for such a distribution. Thus our conclusion, that the political process will result in a too large production of the public good, is actually strengthened.

Unfortunately, the wealth distributions we observe in the real world are skewed in the opposite manner: $\bar{x} > x_m$. It is however not clear whether "endowments", however defined, are distributed in such a way. We know that physical and intellectual characteristics are in general symmetrically Gaussian, while the skew distributions of wealth observed in the actual world may well be endogenous to some economic process that we have
Figure 2
In any case, we see that for distributions skewed in such a way that \( \bar{x} > x_m \) we may still get the result that \( T(x_m) > T^* \) if the degree of skewness is "small". For some degree of skewness we will have that \( T(x_m) = T^* \) while for more skewed distributions \( T(x_m) < T^* \), that is, majority voting will produce too little of the public good. Which one of these cases that obtains in reality is of course an empirical matter.

IV. The Role of the Tax Function

In the above analysis we have assumed a lump-sum tax. This is of course somewhat arbitrary, but although the tax function was chosen to emphasize the main point of this paper rather than to give a realistic picture of actual tax systems, it is not altogether unrealistic. The lump-sum function simply says that the total bill should be split \textit{equally between the agents}. Such an arrangement is of course not conformable to the Lindahl solution in the theory of public finance, but nevertheless we frequently encounter its practical application in various contexts. Further, the lump-sum tax is partly chosen to allow us to make a simple comparison with the Pareto-optimal allocation \( y_{\text{max}} \), which would otherwise be more difficult.

To get some perspective on the importance of the assumption, we first note that the economic explanation behind our result on the non-optimality of the voting process relies on concavity. With concave utility functions, poor people have high marginal utilities with respect to the private good, while rich people have low marginal utilities. This discrepancy is taken account of by the social welfare function (14) which adds the
utilities of all individual agents. The median voter, however, does not care about the fact that the marginal disutility of an extra tax dollar is higher for the poor than for the rich, and therefore there is a tendency for the political process to result in too high taxes. Only if the wealth distribution is "very" skew, the median voter will be sufficiently poor himself to decide on a tax that does not harm the poor.

With other (more progressive) tax functions this problem may not be as severe as for the lump-sum case, but the general problem still remains. Assume for example that the tax is proportional to wealth. The first-order condition for a social optimum is then

\[
\frac{\int u_y[g(tx), x(1 - t)] \cdot x \, dF(x)}{\int u_x[g(tx), x(1 - t)] \cdot x \, dF(x)} = \frac{1}{x \cdot g'(tx)}
\]  

(23)

while the political equilibrium (assuming a symmetric distribution over x) is

\[
\frac{u_y[g(tx), x(1 - t)] \cdot x}{u_x[g(tx), x(1 - t)] \cdot x} = \frac{1}{x \cdot g'(tx)}
\]

(24)

With an additively separable utility function, the numerator of the LHS of (24) is equal to the numerator of the LHS of (23). Thus a necessary and sufficient condition for \( \bar{T}(x) > T^* \) is that

\[
u_x[g(tx), x(1 - t)] \cdot x < \int u_x[g(tx), x(1 - t)] \cdot x \, dF(x).
\]

Let us define \( f(x) \equiv u_x[g(tx), x(1 - t)] \) and note that \( f'(x) < 0, f''(x) > 0 \). We want to know whether

\[
f(\bar{x}) \cdot \frac{\bar{x}}{x} < E[(f(x) \cdot x)]
\]

(25)
where $E[\cdot]$ is the expectations operator. From elementary statistics we know that

$$E[f(x) \cdot x] = \text{Cov}[f(x), x] + \bar{x} E[f(x)]$$

where $E[f(x)] > f(\bar{x})$. Thus a sufficient condition for (25) to hold is that $\text{Cov}[f(x), x] > 0$. This is however not the case; the covariance must be negative since we have assumed decreasing marginal utility, and we can therefore not say anything in general about whether (25) holds or not.

It therefore seems as if a proportional tax is less likely than a lump-sum tax to produce too large a public sector in the economy. But still the main problem remains. The political process will produce a public sector which is too large, or it will produce a public sector which is too small, but only by sheer luck it will produce a public sector which is of the optimal size. This brings the attention to the unsatisfactory practice in public finance models of choosing the functional form of the tax system exogenously. One could say that this paper leads to the conclusion that society should choose the functional form of the tax system (or, in more realistic models, the functional form and the tax base) in such a way that the median voter will choose the scale of production of $y$ which corresponds to a social optimum.

This is a persuasive argument, but there are two problems connected with it. First, when talking about choosing a func-
tional form \( t(x) \) so as to maximize social welfare, we are not any longer discussing only the allocative branch of government (i.e. how much resources should be devoted to the production of \( y \)), but also the redistributive branch. The choice of the degree of progressivity in the tax system has an interest of its own, quite apart from the problem of choosing the optimal production of public goods. I have elsewhere (Persson [1983]) studied the redistributive branch of government, and I think it is instructive to try to separate it from the discussion of the allocative branch.

Second, even if we want to study allocative and redistributive questions simultaneously, we must remember that it is not some abstract but omnipotent "society" which decides about the functional form of the tax system. The voters do. And there is no reason to believe that the median voter\(^{22}\) will prefer the particular functional form which leads to a social optimum. In fact, if we restrict our analysis to linear tax functions with an intercept, i.e. \( t(x) = T + tx \), it is evident from the median voter's first order conditions that he will never set \( T = 0 \), that is he will never vote for a proportional tax.

Let us finally say a few words about constitution versus legislation. For that purpose we return to the concept of people deciding about the tax system behind a veil of ignorance. With such a perception of the world, we can resolve the dilemma of the median voter imposing the "wrong" functional form of the tax system on the economy.
A constitution is a decision made by all individuals behind the veil of ignorance, before each one knows his future wealth. This decision concerns the functional form of the tax system (or, more generally, the various tax bases and the functional form for each tax base). Since all individuals are identical ex ante, such a decision will be unanimous, and it is taken conditional on everyone's knowledge that once the veil of ignorance has been removed, the person who turns out to be the median voter will decide upon the size of \( y \) in his own favor. Ex post, the veil of ignorance is then removed, and the agents will differ in realized wealth. Legislation is then the process where the median voter, within the functional form of the tax system stated in the constitution, decides about the quantity of the public good to be produced.

If each agent, behind the veil of ignorance, wants to maximize ex ante expected utility, and if the probability distribution of future wealth is identical to the ex post distribution of realized wealth (i.e., the agents have rational expectations), the constitutional constraint on legislation will imply a functional form of the tax system such that the median voter ex post will choose \( y(x_m) = y^* \). And thus, provided that the constitution will not be changed after the distribution of realized wealth has been revealed, the dilemma has been resolved.

One should however bear in mind that this view of constitutions builds on a parable, viz. that of unanimous citizens behind a veil of ignorance. In reality both constitutions and legislation are decided upon by roughly the same citizens, with ample information about their realized wealth. And it is hard to see,
at least in the Western world, what executive powers can prevent
the removal of a constitution which is considered undesirable by
a majority of the electorate.
Footnotes

* Helpful comments by Peter Englund, Karl-Göran Måler, Lars E.O. Svensson, Lars Werin and Hans Wijkander are gratefully acknowledged.


2. See e.g. Bowen (1943) and Foley (1967).

3. It is easily verified that most of the results also hold for e.g. CES utility functions.

4. With an additively separable utility function, we thus have

\[ u \equiv v'(y) \quad \text{and} \quad u \equiv w'(x - t_i) \]

5. Various devices have been constructed to encourage the agents to reveal their true preferences; cf. Bohm (1984) for a discussion of this issue.

6. Note that we have dropped the superscript i of u, assuming identical preferences.

7. See e.g. Friend and Blume (1975).

8. Note that the assumption of additive separability is only sufficient, and not necessary. In fact, y attains a maximum \( y_{\max} \) for many widely used non-separable utility functions, including e.g. CES functions with an elasticity of substitution \( \sigma \) such that \(-\infty < \sigma < -1\). For the special case of \( \sigma = -1 \) (the Cobb-Douglas function) the Pareto-efficient level of \( y \) is constant along the whole utility-possibility surface. For CES functions with a low degree of substitutability, i.e. for \( \sigma > -1 \), y attains a minimum at the egalitarian point.
9. The assumption of a lump-sum tax \( T \) is in no way essential to the main point of this paper. The argument would go through equally well a linear tax \( T + tx_1 \), but the mathematics would be somewhat more tedious, and the interpretation will be somewhat less clear-cut. Cf. the discussion in section V below.

10. Needless to say, the Utilitarian welfare function (14) is not necessary for our argument.

11. This is in no way restricted to additively separable utility functions. A sufficient, although still not necessary condition for the LHS to be decreasing in \( T \) is that the two goods are complements in the technical sense of the cross derivative \( u_{yx} \) being non-negative.

12. The reader can immediately verify that the same method of proof can be used to demonstrate the existence of a unique solution \( T_{\text{max}} \) to equation (13) in the previous section.

13. For applications of public-choice theory to income redistribution, see e.g. Meltzer and Richard (1981) and Persson (1983).


15. Note that although \( y_{\text{max}} = y_{m} \), the egalitarian Pareto allocation will not be the same as the political equilibrium. Even if the sizes of the public sectors are the same, the distribution of income over individuals will be different; in the former case there will be a 100% marginal tax rate with \( t(x_1) = x_1 - \alpha \), while in the latter case there will be a zero per cent marginal tax rate with \( t(x_1) = T \).
16. As pointed out in footnote 8 above, CES utility with \( \sigma \geq -1 \) implies that the Pareto-efficient level of \( y \) attains a minimum at the egalitarian point. For such a case, a skew wealth distribution with \( x_m < \bar{x} \) will produce a political equilibrium \( y(x_m) \) which is smaller than any Pareto-efficient level of \( y \), and thus the public sector will always be too small.

17) For (21) to hold, we do not have to assume decreasing absolute risk aversion; it is sufficient to make the much weaker assumption that \( u_{xxx} > 0 \).

18) For an additively separable utility function, (22) holds as an equality. For a CES utility function, (22) holds as a strict inequality.

19. This result seems to be in striking contrast with that of Bowen (1943), who showed that majority voting would result in an optimal size of the public sector. Note however that Bowen assumed the marginal rates of substitution between the public and the private goods to be symmetrically distributed, while we have assumed that endowments are symmetrically distributed. With a symmetric distribution of endowments, concave utility functions will produce a skew distribution of marginal rates of substitution.

20. See e.g. Atkinson (1975, chapter 8) and Rosen (1981).

21. This illustration relies on the assumption of additive separability. With non-separable functions, the numerator of the LHS of (24) might well be larger than the numerator of the LHS of (23), which makes it more likely that \( T(x_m) > T^* \). It is interesting to note, however, that for the particular case of a Cobb-Douglas utility function, we get the borderline result that
a proportional tax will always produce a political equilibrium

\[ T(x_m) = T^*, \text{ regardless of whether the distribution } F(x) \text{ is skewed or symmetric.} \]

22. Assuming single-peakedness, which is admittedly a much less convincing assumption now, when we maximize over functions.
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