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DEFICITS AND INTERGENERATIONAL WELFARE

IN OPEN ECONOMIES

by

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Abstract

This paper deals with public debt in open economies, extending Diamond's overlapping generations model to a small open economy as well as an international equilibrium of two large economies. It focuses on the intergenerational welfare redistributions caused by an increase in public debt triggered by a temporary government budget deficit, and shows how these redistributions differ in open and closed economies. The interplay between the deficits in the government budget and the current account is also analyzed. It is shown how a single period with a government deficit can be followed by a sequence of periods with a deficit in the current account.

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1. **Introduction**

This paper deals with public debt in open economies. The question of how public debt issue affects the well-being of future generations, sometimes referred to as the "burden of the public debt", has a long tradition in economics going back as far as Ricardo. Not surprisingly, the discussion seems to have been particularly active after periods with substantial government budget deficits. Indeed, the sizeable and growing budget deficits in many countries during recent years is a strong empirical motive for looking further at this issue today. The present close international integration of goods and capital markets, as well as the observation that deficits in the government budget tend to go hand in hand with deficits in the current account, makes it interesting to analyze the international aspects of the problem.

One intensive round in the debate, initiated by Buchanan (1958), took place in the late fifties and early sixties; for a summary of the heated, and sometimes confused, debate, see Ferguson (1964). Much of the confusion was resolved with the pathbreaking work of Diamond (1965). Diamond's analysis, which extended an ingenious model developed by Samuelson (1958), was particularly useful in two respects: First, by treating a sequence of overlapping generations, it allowed for a precise discussion of intergenerational welfare redistributions in a relatively simple way. Second, by relying on an explicit general equilibrium framework, it made it possible to pose the central question about the burden of national debt in terms of lifetime utilities of welfare-maximizing consumers.

Diamond indeed included external debt in his analysis. However, he chose a formulation where only the government but not the private sector can borrow
abroad. While sufficient to bring out certain differences between internal and external debt, this is not very plausible today from an empirical point of view. Moreover, it makes the interesting interplay between the deficits in the government budget and the current account very trivial.

In this paper Diamond's analysis is extended to open economies where all agents, private and public, have access to perfectly integrated world markets. We look at the short and long-run effects of a period with government deficits, which manifest themselves in a permanent increase in the public debt. The analysis is carried out first in the context of a small open economy, then in a full world equilibrium of two large economies.

There already exist some open-economy versions of the Diamond-Samuelson overlapping generations model in the literature. In particular, there are two recent studies by Buitre (1981) and Dornbusch (1984). Buitre's model does only include private borrowing and lending and thus cannot be used to address the questions pursued here. The work by Dornbusch does include public debt, but the process whereby this debt is generated -- namely government deficits -- is not explicitly modelled and only the long run effects of debt issue are analyzed. In addition, production is left exogenous in the analysis, and there is neither capital nor investment.

The paper proceeds as follows. Section 2 presents the Diamond model of an economy in autarky, and we look at the short and long run consequences for factor rewards and welfare of public debt. This section covers material most of which now can be found in textbooks; see for example Atkinson and Stiglitz (1980). It is included for the sake of completeness and as a background to the
subsequent analysis, however, and can be skimmed through by readers already familiar with the standard overlapping generations model.

Section 3 extends the analysis to a small open economy facing perfect world markets. We look at the consequences of public debt issue for current and future generations' welfare and indicate the differences to the closed economy results. The effects on the economy's external debt in the short and the long run, and the interrelation between the deficits in the budget and the current account, are also discussed.

A full equilibrium in a world with two major economies is studied in Section 4, where we analyze the domestic as well as the foreign adjustment of factor prices, external debts and welfare levels in response to a deficit period in one of the economies.

An important qualification to the analysis is the absence of private intergenerational transfers. It is well-known since the study by Barro (1974), that such transfers may lead to public debt neutrality. That is discussed further in Section 5, which also summarizes the main points in the paper and includes some suggestions for further research.

2. The Diamond Model and Public Debt in Autarky

The economy's population grows at the exogenous rate \( n \). All people live for two periods, so at each point in time there is a young and an old generation living side by side. Young people in period \( t \) supply a fixed amount of labor at wage \( \tilde{w}_t \) and pay a lump-sum tax \( t_t \) to the government. They consume part of the resulting income and save the remainder for their old age. Old people, who are retired, earn principal and interest on their savings. They pay no taxes.
Each young person's consumption in period \( t \) is denoted by \( c_t \), while \( d_t \) denotes consumption by each old. The decision problem for young people born at \( t \) is to maximize the well-behaved utility with function \( U(c_t, d_{t+1}) \) subject to
\[
c_t + \frac{1}{1 + r_{t+1}} d_{t+1} = \bar{w}_t - \tau_t,
\]
where \( r_{t+1} \) is the return on assets held over to period \( t+1 \). The solution gives consumption demand as a function of the after-tax wage \( w_t = \bar{w}_t - \tau_t \) and the interest rate; \( c_t = C(w_t, r_{t+1}) \). Savings are thus \( w_t - c_t \). It is useful to introduce a variable \( a_{t+1} \), which expresses private wealth per worker in the beginning of period \( t \). Because there are no bequests and the growth rate is \( n \), \( a_{t+1} \) will be savings of the young at \( t \) discounted by the factor \( 1+n \), that is
\[
(1+n)a_{t+1} = w_t - c_t. \tag{1}
\]
Since the savings decision is already bygone, old people have a trivial decision problem when retired. Each old person simply consumes all his income, or
\[
d_t = (1+r_t)(w_{t-1} - c_{t-1}).
\]
For future reference we note that the welfare of a member of the \( t^{th} \) generation may be expressed either by the direct utility function above, or by the indirect utility function
\[
v_t = V(w_t, r_{t+1}), \tag{2}
\]
which is increasing in both its arguments.

There are two factors of production, (non-depreciating) capital and labor; because the economy has only one sector, capital is simply non-consumed output. Production in the \( t^{th} \) period, \( X_t \), is carried out according to a well-behaved, linearly homogenous, neo-classical production function \( X_t = F(L_t, K_t) \), where \( L_t \) is the labor force and \( K_t \) is capital carried over from period \( t-1 \). Production per worker can thus be expressed by
where \( k_t \) is capital per worker. With profit maximizing behavior investment in period \( t \), or more precisely the gross amount of capital held over to period \( t+1 \), fulfills

\[
f_k(k_{t+1}) = r_{t+1}.
\]

(4)

Given constant returns to scale, the factor returns are related by

\[
\tilde{w}_t = f(k_t) - k_t f'_k(k_t),
\]

(5)

which together with (4) constitutes the standard factor price frontier.

The government may enter the economy in several ways but to keep the analysis as simple as possible, we shall assume that its consumption and production are both zero, and that its only expenditure is interest payments on the public debt. In each period the government collects lump-sum taxes \( \tau_t \) from each worker. Any deficit in the government budget has to be covered by borrowing which increases its debt. The government's debt instruments are one period bonds that pay the current interest rate and principal in the next period. Denoting the amount of debt outstanding in the beginning of period \( t \) by \( G_t \), the government's deficit is therefore \( G_{t+1} - G_t = r_t G_t - \tau_t L_t \). It will be convenient to express this per worker in what follows. Dividing through by \( L_t \), we have

\[
(1+n)g_{t+1} - g_t = r_t g_t - \tau_t
\]

(6)

We see that if \( \tau_t \) is set to \((r_t-n)g_t\), net debt per worker is kept constant. We shall assume that \( r \) is strictly higher than \( n \) throughout, which makes taxes
positive in this case. The assumption that the interest rate is higher than the rate of growth also rules out equilibria that are "dynamically inefficient," that is equilibria with over-accumulation in the sense that \( k \) is above its golden rule value given by \( f_k(k) = n \).

In each period a temporary, or perhaps better a momentary, equilibrium is established. That equilibrium is recursive in the following sense. Last period's investment and the labor endowment in period \( t \) has already fixed the economy's capital-labor ratio and its present return on capital. Then the before-tax real wage is also determined via the factor price frontier.

Given \( k_t, r_t \) and \( \tilde{w}_t \), the individual consumption, savings and investment plans of young and old consumers, firms, and the government must be made consistent in the goods and the capital markets. One of the clearing conditions is redundant, so it suffices to state the requirement for equilibrium in the capital market. This is that savings is equal to investment

\[
[(1+n)a_{t+1} - a_t] - [(1+n)\tilde{g}_{t+1} - \tilde{g}_t] = (1+n)k_{t+1} - k_t,
\]

which is here expressed in per capita form. However, \( a_t, \tilde{g}_t \) and \( k_t \) are all given by previous decisions and since \( a_t \) must always equal \( \tilde{g}_t + k_t \) in autarky (7) can be restated as

\[
a_{t+1} = k_{t+1} + \tilde{g}_{t+1}.
\]

Equation (8) states that savings by the young must be sufficient to absorb the total amount of capital and government debt carried over to the next period. Both \( a_{t+1} \) and \( k_{t+1} \) are functions of the real interest rate; so this condition uniquely determines \( r_{t+1} \). In the following we shall assume what might be termed (static) Walrasian stability, namely
\[
\frac{1}{(1+n)k^1 + C_r} < 0.
\]  
(9)

When (9) is fulfilled, excess demand in the capital market leads to an increase in the interest rate.\(^1\)

The capital stock carried over to the next period gives rise to a new momentary equilibrium and so the process continues. If it is assumed that the government sets taxes so as to keep net debt per worker constant in the long run,\(^2\) the following condition ensures that the economy's development over time ends up in a steady state:\(^3\)

\[
-1 < - (k+g) \frac{(1-C_w)}{(1+n)^{f_k} + C_r} < 1.
\]  
(10)

Given that (9) holds (and goods are normal so \(C_w < 1\)), the sign of the expression between inequalities is always positive which guarantees asymptotic (as opposed to oscillatory) convergence.

In a steady state all variables grow at \(n\) -- the rate of population growth.

Such an equilibrium is described by

\[
\tilde{w} = f(k) - kf_k(k)
\]  
(11)

\[
r = f_k(k)
\]  
(12)

\[
w = \tilde{w} - r
\]  
(13)

\[
r = (r-n)g
\]  
(14)

\[(1+n)a = w - C(w,r)
\]  
(15)

\[a = k + g
\]  
(16)

\[v = V(w,r)
\]  
(17)

Let us turn to the effects of increased public debt. We assume that the government chooses to finance a transfer to the young generation born at date \(t\)
by running a deficit rather than by increasing taxes. (The analysis of debt rather than tax financing of either a transfer to the old generation or increased government consumption or investment in the t^{th} period would be slightly more complicated but yield qualitatively similar results.) With tax finance net taxes paid by the young are unchanged and there are no effects either in the long or in the short run. Therefore, the difference between the two policies can be found by looking at debt finance only.

What we consider is thus a one-shot decrease in taxes in period t, \( \Delta t_t < 0 \), and a corresponding increase in debt, which from the government budget constraint (6), satisfies \((1+n)d_g_{t+1} = -\Delta t_t\). From period t+1 and onwards taxes are set so as to keep the public debt per capita constant. Hence, the increase in debt remains for all future periods and we may use the simplified notation \(d_g_{t+1} = d_g_{t+2} = \ldots = d_g\).

Let us first look at the effects on factor prices over time. The impact effects are easily determined. Since \(k_t\) is already given by history, so are \(r_t\) and \(\bar{w}_t\), and the only effect on factor rewards is the rise in net wages due to the fall in taxes. Increased government borrowing also creates an excess demand in the capital market. Differentiating the capital market equilibrium condition (8) for \((1+n)dg\) and \(\Delta t_t = -(1+n)dg\), we get

\[
ds_{t+1} = - \frac{C_{w}(1+n)}{(1+n)f_{kk}^{-1} + C_r} dg. \tag{18}
\]

The numerator measures the excess demand at the initial interest rate. An increase in the interest rate reduces investment, since \(f_{kk} < 0\), and may stimulate or reduce savings, since \(C_r > 0\). Our stability assumption (9) guarantees that the net effect on private excess demand (given by the denominator in (18)) is negative, however, which means that the interest rate rises.
In the next period, we already know that the capital-labor ratio is lower due to the rise in $r_{t+1}$. This decreases gross wages along the factor price frontier. Taxes, now adjusted to keep government debt constant, increase both because the interest rate is higher and because government debt is higher. The total effect on net wages $dw_{t+1} = -(k_{t+1} + g_{t+1})dr_{t+1} - (r_{t+1} - n)dg$ is thus negative. Differentiating (8) once again, one obtains

$$dr_{t+2} = -\frac{1 + n - (1-C_w)(dw_{t+1})/dg}{(1+n)f_{kk}^{-1} + C_r} \ dg,$$

which is clearly positive and greater than $dr_{t+1}$. The reason is that the young generation at $t+1$ has a lower income than the young at $t$ but nevertheless has to absorb the same amount of government debt.

From the earlier stability discussion we know that the increase in interest rates and the associated decrease in wages will continue in subsequent periods as the economy approaches its steady state. The long-run effects on factor prices are easily found from (10) to (14) as

$$dw = -(k+g)dr - (r-n)dg < 0 \quad (19)$$

and

$$dr = -\frac{1 + n + (1-C_w)(r-n)}{(1+n)f_{kk}^{-1} + (1-C_w)(k+g) + C_r} \ dg > 0. \quad (20)$$

We now turn our attention to the welfare effects. The generation born at date $t$ experiences an increase in its net wage as well as in its interest rate. It follows from (2) that each member's welfare goes up by $dv_t = V_w dw_t + V_r dr_{t+1}$. 
As for the welfare of the next generation, it may either rise or fall because
\[ dv_{t+1} = V_w dv_{t+1} + V_r dr_{t+2} \]
has an ambiguous sign due to the opposite influences of the falling net wage and the rising interest rate. In other words, it is not clear whether in fact there is a burden on that generation.

The change in steady-state welfare, from (17) and (19), fulfills
\[ dv = [-V_w (r-n) - V_w (k+g) \frac{dr}{dg} + V_r \frac{dr}{dg}] dg. \tag{21} \]
Each of the terms within brackets has a clear interpretation. The first is the negative welfare effect of increased taxes to service the higher government debt. The second, which is also negative, is the combined effect on net wages of the changes in gross wages and taxes, because of the rise in the interest rate brought about by the rise in g. The third effect, finally, is positive and measures the gain of a rise in the real interest rate (all young people are net lenders).

One might think that the net effect is ambiguous. That is not the case, however. By Roy's identity we know that \[ V_r \frac{d}{dV_w} = \frac{d}{1+r}. \] Using that together with \[ d = (w-c)(1+r), \] as well as (15) and (16), we may reformulate (21) as
\[ dv = -(r-n)V_w [1 + \frac{a}{1+r} \frac{dr}{dg}] dg. \tag{22} \]
Given our assumption that \( r > n \), the sum of the last two terms in (21) is thus negative and the net effect is an unambiguous fall in steady state welfare. To understand this result we note that the economy under-accumulates capital relative to its golden rule capital-labor ratio given by \( f_k(k) = n \), already in its initial position. The increase in the interest rate reduces \( k \) further, accentuating this under-accumulation. The resulting welfare loss must be added to the negative effect of higher taxes.
Summarizing, we have thus shown that with an increase in the government debt to finance a transfer payment to the currently young generation, this generation gains, its nearest descendants may either gain or lose, while all generations in the new steady state must necessarily lose. Which generation that starts to experience a fall in welfare is thus an open question.

3. **Public Debt in a Small Open Economy**

Let us take a look at the same economy when it is open for trade. All agents act in the same way as before, but they now have access to a perfect world capital market with a given rate of interest, denoted by $r^*$, as well as a perfect world market for its single good.

In autarky private wealth per worker in each period $a_t$ must be identically equal to the capital stock $k_t$ plus government debt $g_t$. With international capital mobility, this happens only if countries were identical in all respects, however, and we generally expect a non-zero foreign debt (per worker) $e_t$, given by

$$e_t = (k_t + g_t) - a_t.$$  \hspace{1cm} (23)

We may then readily define the current account deficit for period $t$, $q_t$ say, as the increase in foreign debt during that period

$$q_t = (1+n)e_{t+1} - e_t.$$  \hspace{1cm} (24)

Using (23), the current account deficit may be expressed either as the sum of the private sector's and the government's accumulation of financial debt or, equivalently, as
\[ q_t = [(1+n)k_{t+1} - k_t] - [((1+n)a_{t+1} - a_t) - ((1+n)\sigma_{t+1} - \sigma_t)]; \]

The total investment minus the sum of the two sectors' savings. The economy's trade balance deficit \( b_t \) is also easily found. Since the difference between \( q_t \) and \( b_t \) is interest payments abroad, we have
\[ b_t = q_t - r_t e_t = (1+n)e_{t+1} - (1 + r_t)e_t. \]  \( (25) \)

The equations describing a momentary equilibrium in the small open economy are restated here for convenience
\[ \tilde{w}_t = f(k_t) - k_t f'_k(k_t) \]  \( (5) \)
\[ r^*_{t+1} = f'(k_{t+1}) \]  \( (6) \)
\[ r_t = r^*_t \]  \( (26) \)
\[ \tau_t = \tilde{w}_t - r_t \]  \( (27) \)
\[ (1+n)a_{t+1} = \tilde{w}_t - C(\tilde{w}_t, r^*_{t+1}) \]  \( (1) \)
\[ e_t = (k_t + \sigma_t) - a_t \]  \( (23) \)
\[ q_t = (1+n)e_{t+1} - e_t \]  \( (24) \)
\[ v_t = V(\tilde{w}_t, r^*_{t+1}) \]  \( (2) \)

In addition to the issues we have just discussed, there is yet a new feature in this setting. With a given interest rate (and constant returns to scale), the economy's capital-labor ratio is in fact determined independently of domestic conditions and so also is the gross wage.

The dynamics will therefore be on the economy's external debt, rather than on its capital stock cum factor rewards and if the foreign interest rate is constant, the economy will converge to a steady state. The conditions for a steady-state equilibrium are (11) through (15), (17),...
\[ r = r^* \] (28)
\[ e = (k+g) - a, \] (29)
and
\[ q = ne \] (30)

Thus the current account is in deficit (surplus) in steady state to maintain the economy's external debt (asset) constant per worker. However, from (25) we see that under our assumption \( r > n \) the economy runs a surplus (deficit) on its trade account to service its external debt (spend the interest income from its external assets) in the steady state.

We are now prepared to look at an expansion of the public debt. As before, we consider a government deficit in period \( t; (1+n)dg_{t+1} = -d\tau_t > 0 \), and set \( dg_{t+1} = dg_{t+2} = \ldots = dg \) in later periods. To simplify matters, we assume a horizontal path for the foreign interest rate; \( r^*_t = r^* \), for all \( t \).

First consider the adjustment of the external debt. The impact effect is an increased current account deficit. From (23), (24) and (27):
\[ dq_t = (1+n)de_t_{t+1} = c_w(1+n)dg > 0. \] (31)

The young generation who faces an increase in their net wage will absorb some but not all of the new government debt by increasing its savings. This results in an aggregate excess demand for credit of \( c_w(1+n)dg \), which, instead of driving up the interest rate as in the closed economy (cf. equation (18)), here is satisfied via increased foreign borrowing at the given world interest rate (whether it is the government or the private sector that actually borrows abroad is, of course unimportant).
In the period after the government deficit, workers face increased taxes to service the higher government debt, but no change in their gross wage. We find the effect on the current account as

$$dq_{t+1} = (1+n)de_{t+2} - de_{t+1} = [(1+n) + (1-C_w)(r^*-n) - C_w]dg,$$

which is clearly positive. Young people in this period save less because their income is lower, and old people dissave more because they saved more when young in period $t$. Both these reasons make total private savings in $t+1$ lower than in the period before. This again results in an increased current account deficit despite that government savings is slightly higher.

As for the long-run effects, the increase in the foreign debt, from (13), (14), (15) and (29), satisfies

$$de = dg - da = \frac{1}{1+n}[1 + n + (1-C_w)(r^*-n)]dg$$

The derivative $de/dg$ thus exceeds unity. This is explained by the private sector not only being unwilling to hold the higher public debt in the long run, but also saving less since the taxes have been increased to cover a higher debt service in the new steady state. (The expression in brackets on the RHS of (33) is the analog to the numerator in (20).)

If we use (31) and (32) to solve for $de_{t+2}$, we find that $de_{t+2} = de$. In other words, the whole adjustment to the higher long run foreign debt is accomplished by the current account deficits we have already investigated, and the economy has reached its new steady state only two periods after the initial deficit episode. We may note in passing that in this new steady state the economy has a larger current account deficit and a larger trade surplus due to the larger interest payments abroad (cf. above).
Since there are no effects on gross factor returns, the only sources for welfare changes are the changes in taxes over time. It is thus clear that the first generation gains and all future generations lose as a result of having to pay higher taxes. The steady state welfare effect is given by

$$dv = -\frac{V_r r^* dg}{\psi} < 0;$$

equivalent to the first negative term of the bracketed expression in (22) giving the welfare loss in autarky. The second negative term in (22) does not appear here, of course, since the capital-labor ratio stays constant.

Comparing the intergenerational redistribution of welfare with that in autarky, we thus find two differences. First, there is no ambiguity about gainers and losers in the small open economy; the young generation in the deficit period being the only one to gain and all future generations having to bear the burden of the higher public debt. Second, because there are no magnifying effects on gross factor rewards, both the welfare gain of the first generation and the long run welfare losses are smaller than in the closed economy.\(^5\) The opportunity of intertemporal trade at a given interest rate thus eliminates the downward adjustment of the economy's capital-labor ratio that was necessary in autarky and was seen to constitute part of the burden on future generations of an increased public debt.

4. World Equilibrium

We now turn to a full international equilibrium which marries together two large countries. Differences in size and growth rates are not of prime interest
for the problems addressed here, so we simplify matters by assuming that the two countries are identical in the size of their labor endowments, which both grow at the common rate \( n \). With respect to technology, tastes, and initial government debt, we allow differences, however. The two countries are referred to as the home and foreign country and the same notation as before is adopted, but with a *-superscript on foreign variables.

In a momentary equilibrium the home country still obeys equations (1), (2), (4) through (6), (23), (24), and (27), and there are analogous expressions for the foreign country; (1*), ..., etc. With perfect financial capital mobility there must be one single interest rate in the two countries, \( r_t = r^*_t \), and we denote this common rate by \( r_t \). Finally, market clearing requires world savings equal to world investment, \( q_t + q^*_t = 0 \); current accounts must sum to zero. Since \( e_t \) and \(-e^*_t\) are equal by definition, and already predetermined in period \( t \), we can alternatively express the equilibrium condition for the world capital market as

\[
e_{t+1} + e^*_{t+1} = 0. \tag{34}
\]

We assume that the previously discussed conditions (9) and (10) and analogous conditions for the foreign country \((9*)\) and \((10*)\) continue to hold, and that both the home and foreign government set taxes so as to maintain its debt per worker constant in the long run. Then the world economy will asymptotically converge to a steady state.\(^6\) A long-run equilibrium is defined by equations (11) through (14), (16), (17), and (28) through (30) (with and without *), and by the condition
\[ e + e^* = 0. \]  

When discussing the comparative statics, it is useful to recall that the world economy is a large closed economy. Therefore we expect the adjustment to be an intermediate case between that in the single, closed economy and that in the small open economy. Indeed, below we find effects on factor prices smaller than in the closed economy case, and effects on external debts and current accounts smaller than in the small economy case.

Consider then as before an issue of public debt cum tax cut in the home country; \((1+n)dg = - \frac{dt}{t} \). As before, we start by discussing the macroeconomic adjustment process. The gross wages in period \( t \) both at home and abroad is given by previous decisions. As in autarky, the public borrowing creates an excess demand in the capital market, and from (34) we find the effect on the world interest rate, namely

\[
dr_{t+1} = - \frac{C_w (1+n)}{(1+n)(f_{\text{kk}}^{-1} + f_{\text{rr}}^{-1}) + (C_r + C^*_r)} dg, \]

which is positive (by (9) and (9*)) and less than under autarky; cf. (18).

The home country's current account deficit increases by

\[
d_q = (1+n)de_{t+1} = [(1+n)C_w + (1+n)f_{\text{kk}}^{-1} + C_r \frac{dr_{t+1}}{dg}] dg. \]

Comparing this to (31), the effect is clearly less than in the small country case, in the sense that the derivative \( dq/dg \) is smaller. The explanation is that the rise in the interest rate affects home savings and investment. In the foreign country there is a corresponding improvement in the current account, of course.
In period $t+1$ gross as well as net wages fall in both countries because the higher $r_{t+1}$ leads to a higher return on capital as well as on government securities. It is easy to show that the interest rate continues to rise and that the home country suffers a deterioration in its current account also in this period. Instead of developing the rather messy expressions involved, we go to the steady state effects, however.

In the new steady state wages are lower both at home and abroad, viz.

$$dw = -(k+g)dr - (r-n)dg$$  \hspace{1cm} (36) \\

and

$$dw^* = -(k^*+g^*)dr,$$  \hspace{1cm} (36*)

and the interest rate is higher

$$dr = -\frac{(1+n) + (1-C_w)(r-n)}{(1+n)(r_{kk}^{-1}+r_{rr}^{-1}) + (C_r+C^*) + (1-C_w)(k+g) + (1-C^*)(k^*+g^*)} \frac{dg}{dg}.$$  \hspace{1cm} (37)

We may also derive the steady state effects on the two countries net external debts as

$$de = \frac{1}{1+n}[(1+n) + (1-C_w)(r-n) + ((1+n)f_{kk}^{-1}-C_r + (1-C_w)(k+g))\frac{dr}{dg}]dg > 0$$  \hspace{1cm} (38) \\

and

$$de^* = \frac{1}{1+n}[(1+n)f_{kk}^{-1} + C^* + (1-C^*)(k^*+g^*)\frac{dr}{dg}]dg < 0$$  \hspace{1cm} (38*)

Comparing (38) with (33), we find (recalling (10)) that the effect is smaller than in the small economy case. In the same way as for the impact effects, it is the rise in the interest rate that works as a cushion.

When it comes to the dynamics of foreign assets, there is a further interesting difference with the small open economy. Like the factor rewards, e
converges only asymptotically to its new equilibrium. The upshot is thus that a
single period of government deficit may lead to an extended adjustment process
with deficits in the current account.

The initial welfare effects are favorable both at home and abroad. In both
countries the young in period t gain from the higher interest rate, and the
young in the home country also from the lower taxes. The nearest descendants
experience falling wages and rising interest rates. One can show that the
welfare of the young born at t+1, both at home and abroad, can either rise or
fall in the same way as in autarky.

From (17) and (36) we get the welfare change for future home generations in
the steady-state

\[ dv = \left[-V_w(r-n) - V_w(k+g)\frac{dr}{dg} + V_r \frac{dr}{dg}\right]dg. \]  

(21)

This is the same expression as in the autarky case and the three terms have the
same interpretation, capturing the increase in taxes due to the higher debt (at
given r), the decrease in net wages due to lower gross wages and higher taxes,
and the increase in the interest rate, respectively. In autarky we could
verify that the net effect was always negative implying a burden on future
generations. Here, such an assertion can not be made, however. To see this,
note that by (29) and the properties of \( V(\cdot) \), (21) can be reformulated as

\[ dv = -(r-n)V_w[1 + \frac{a}{1+r} \frac{dr}{dg}]dg - V_r \frac{dr}{dg}dg, \]  

(39)

where the first term, which corresponds to (22), is negative, but the second
term is negative only if e is positive.

The economic significance of this is clear. A change in the interest rate
redistributes income from workers/taxpayers to wealth holders. If e is
positive, some of these are foreigners and hence the consumption possibilities for the economy as a whole are reduced. In this case the increase in debt unambiguously lays a burden on future generations at home. If the home country is a net creditor, on the other hand, this intertemporal terms-of-trade effect instead redistributes income in its favor which alleviates the burden of increased taxes and a lower capital-labor ratio on future generations. When -e becomes sufficiently high, the net result is even a welfare gain.

The probability of the home country being a long-run creditor is higher, the lower the rate of time preference of consumers — cf. Buijer (1981) — the lower the initial government debt, and the worse the investment opportunities in the home country; all relative to the foreign country. Since the countries may very well differ on all these accounts, there is no reason to look upon a positive steady-state welfare effect in the home country as a degenerate special case.

Similarly, we derive the long-run welfare effect in the foreign country as

\[ dv^* = \frac{v^*_w}{w} \left[ -a^*_y \frac{r - n}{1 + r} \frac{dr}{dg} - e^*_y \frac{dr}{dg} \right] dg, \]  \hspace{1cm} (39*)

which is equivalent to that in the home country, except that the (direct) tax burden of higher debt is absent. We see that the outcome depends on the negative effect of the lower capital-labor ratio plus the ambiguous intertemporal terms of trade effect. Thus, the result is even more uncertain than for future generations in the home country.7

Let us conclude this section by comparing our findings regarding the welfare redistributions among generations in the two-country case to our
previous results. The currently young generations in both countries gain from a deficit in the home country. As in autarky, but unlike the small open economy, their immediate descendants in the home country may either gain or lose. The ambiguity extends to the foreign country. Both in autarky and the small open economy there was a definite burden on future generations in the steady state. Here, however, this is no longer a necessary result. If the home country is a creditor, the rise in the interest rate may actually redistribute income in its favor to such an extent that future generations gain. Future generations in the foreign country may either gain or lose. However, it is clear from (39) and (39\*) that although welfare may well decline in the long run in both countries, a welfare improvement in both countries is not possible.\(^8\)

5. Final Remarks

This paper studied the effects of public debt issue in open economies with overlapping generations.

With regard to the intergenerational welfare distribution, we showed the following: In a small open economy with access to a perfect world capital market, the welfare effects are smaller than in autarky both in the short and the long run. But the direction of the redistribution; from future to the present generation is essentially the same. In an economy that is large enough to affect world market prices these results need no longer hold, however. Future generations in an economy that increases its public debt may then actually gain because of an intertemporal terms-of-trade effect that redistributes resources from the rest of the world.
We also discussed some interesting dynamics in the current account. The adjustment towards the higher external debt implied by a higher public debt was shown to involve an extended period of current account deficits following an initial government budget deficit. This adjustment period was longer in the large economy.

It should be pointed out that both these sets of results hinge crucially on the absence of operative private gifts between generations. The discussion in Barro (1974) showing how private, non-market, intergenerational transfers can compensate for government, non-market, intergenerational transfers and thereby leave the welfare of future generations unaffected applies equally well to open economies, of course. As is well known, such "dynastic" savings behavior turns the decision problem for each generation into that of infinitely-lived consumers, meaning that a substitution of debt for taxes would leave consumption unaffected. In an open economy context, this means that the current account would be unaffected by public debt-issue (for given government expenditures) -- cf. Sachs (1982).

However, it is well known that a number of quite restrictive assumptions have to be fulfilled for such private compensating transfers to occur. Given this, and the unclear empirical support for the dynastic savings hypothesis, an exploration of overlapping generations models with life-cycle savings behavior is still of interest.

An application of such models to problems in international trade and macro theory is of particular interest because they include maximizing agents with finite planning horizons that overlap with each other. This means that there
are agents with marginal propensities to spend ranging from zero (the unborn) to one (the presently old) and with some in between (the presently young). As a result the adjustment of such economies to various shocks will be quite different from the adjustment of economies with agents that have infinite planning horizons. The effects of terms-of-trade changes on the current account is one example where the results in an overlapping generations framework -- see Persson and Svensson (1985) -- differ a great deal from those in an infinite horizon framework -- see Obstfeld (1982), and Svensson and Razin (1983).

The assumption in the present paper that future deficit policies in the home country and current and future deficit policies in the foreign country do not respond to a current deficit in the home country is obviously unrealistic. In particular, we have shown how public debt issue can give rise to intergenerational as well as international redistributions. There is thus scope for interesting extensions of the present model that allow for various forms of strategic behavior in the design of fiscal policies over time.
Footnotes

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1. Diamond (1965) also discusses the case when (9) does not hold.

2. With other rules for government behavior the model would have very different properties. See Burbidge (1983) for an elaboration.

3. Substituting the factor price frontier and the government budget constraint into (8), and imposing the assumed constancy of government debt, one gets a first-order difference equation in k, and (10) expresses the stability requirement $|d k_{t+1}/d k_t| < 1$.

4. This result might also be understood by viewing the rise in the interest rate and the associated fall in the wage as a movement along the steady-state after-tax factor price frontier, obtained by substituting (11), (12), and (14) into (13). The slope of this in $(w,r)$ space is $-(k+g) = -a$. However, since the indirect utility function is flatter than that -- its slope being $-V_r/V_w = -(w-c)/(1+r) = -a(1+n)/(1+r)$ -- it follows that a rise in the interest rate must lower welfare.

5. Strictly, this requires that the two economies are compared at the same initial interest rate, since $V_w$ might be sufficiently higher at a different
interest rate in the small open economy to change the conclusion.

6. The condition for monotonic convergence is

\[ 0 < \frac{(1 - C_w)(k + g) + (1 - C^*_w)(k^* + g^*)}{(1+n)(f_{kk} + f^{*}_{kk})^{-1} + (C_r + C^*_r)} < 1 \]

which is satisfied if (9), (9*), (10) and (10*) hold.

7. In the two-country model studied by Dornbusch (1984) an increase in home country debt decreases home welfare and increases foreign welfare without ambiguity. This crucially depends on the assumption that all debt is in the form of consols, meaning that debt service is coupon payments independent of the interest rate. Therefore a change in the interest rate cannot redistribute consumption possibilities across countries as it does here. This assumption and the fact that Dornbusch leaves production and capital exogenous also makes his results quantitatively different, since a change in the interest rate does not change the economies' capital-labor ratios.

8. An appropriate measure of the change in world welfare is the sum of the wealth equivalents of the two welfare changes. Hence, if we substitute from (39) and (39*) into

\[ \frac{dv}{V_w(w,r)} + \frac{dv^*/V^*_w(w^*,r)}, \]

we find that world welfare unambiguously declines.

References


Dornbusch, R., 1984, "Intergenerational and International Trade," unpublished manuscript, MIT.


Lindbeck, A. and J. Weibull, 1984, "Intergenerational Aspects of Public Transfers, Borrowing and Debt," unpublished manuscript, IIES.


Obstfeld, M., 1982, "Aggregate Spending and the Terms-of-Trade: Is there a
Laursen-Metzler effect?" Quarterly Journal of Economics 97, 251-270.