Seminar Paper No. 273

TIME-CONSISTENT FISCAL POLICY AND GOVERNMENT

CASH-FLOW: A NOTE

by

Lars E.O. Svensson and Torsten Persson

INSTITUTE FOR INTERNATIONAL ECONOMIC STUDIES

University of Stockholm
Seminar Paper No. 273

TIME-CONSISTENT FISCAL POLICY AND GOVERNMENT

CASH-FLOW: A NOTE

by

Lars E.O. Svensson and Torsten Persson

Seminar Papers are preliminary material circulated to stimulate discussion and critical comment.

February, 1984

Institute for International Economic Studies
S-106 91 Stockholm
Sweden
1. Introduction

Lucas and Stokey (1983) present a pathbreaking analysis of the time-consistency of optimal fiscal and monetary policy in an economy without capital. Their main result is that for a barter economy, an optimal policy is time-consistent, if government debt of sufficiently rich maturity structure is issued. For a monetary economy, an optimal policy is not time-consistent.

Although several illuminating examples are presented, there is no simple yet general intuitive interpretation for their main result, except that it is the possibility of "'devaluing' the debt that provides an incentive for the (benevolent) government at t=1 to deviate from the optimal (at t=0) tax policy" (p. 66).

In this short paper we provide one such simple interpretation of their results, as well as of their somewhat complicated optimality conditions. This is done by focussing on what we call government "cash-flow", which is in each period the excess of tax revenues over government expenditure and the sum of interest and amortization payment on government debt that the government has inherited from previous governments.

In addition to interpreting Lucas and Stokey's optimality conditions and their time-consistency result, we provide some extensions. We show that a
positive government cash-flow in the first period is associated with less tax
distortion for governments in later periods, and that later governments then
inherit lower levels of debt of each maturity (and contingency, if there is
uncertainty).

The paper deals with a barter economy only. The discussion attempts
to be brief but reasonably self-contained; we refer to Lucas and Stokey (1983)
for a thorough presentation of the problem, the model and the various qualifica-
tions to the approach and the results. Section 2 derives and interpretes opti-
mal fiscal policy with commitments. Section 3 discusses time-consistent optimal
policy when later governments are committed to honour previously issued debt but
not previous tax decisions. Section 4 notes the modifications when uncertainty
is introduced. Section 5 concludes the paper.

2. Optimal fiscal policy with commitment

Lucas and Stokey (1983) present a model with stochastic government
expenditure. They note, though, that most of the main points can be developed
in the context of perfect foresight. Here we choose to first look at a deter-
minstic variant of their model. Modifications due to uncertainty are discussed
in Section 4.

There is one produced good. Government consumption of this good in
period $t = 0, 1, 2, \ldots$ is given by $g_t$. There is a representative consumer with an
endowment of one unit of labor in each period. The technology is such that one
unit of labor results in one unit of output. Let $c_t$ and $x_t$ be private consump-

tion of the good and leisure, respectively. Then all feasible allocations satisfy

\[(2.1) \quad c_t + x_t + g_t \leq 1, \quad t = 0,1,2,\ldots\]

The consumer has a standard additively separable utility function given by

\[(2.2) \quad \Sigma_o^{\infty} U(c_t, x_t), \quad 0 < \beta < 1.\]

Lump-sum taxation is not feasible. Government expenditure is financed by proportional taxation on labor, with tax rate \(\tau_t\) in period \(t\). At time 0 government debt to the consumer is predetermined and given by \(o_o b = (o_o b_0, o_o b_1, \ldots)\), where \(o_o b_t\) denotes the consumer's claim to goods in period \(t\). Then the consumer's budget constraint is

\[(2.3) \quad \Sigma_o^{\infty} p_t c_t \leq \Sigma_o^{\infty} p_t (1 - \tau_t)(1 - x_t) + \Sigma_o^{\infty} p_t o_o b_t,\]

where \(p_t\) is the present value at time 0 of a good at \(t\). Maximization of (2.2) subject to (2.3) gives the first order conditions.

\[(2.4a) \quad U_x(c_t, x_t) = (1 - \tau_t) U_c(c_t, x_t) \quad \text{and}\]

\[(2.4b) \quad \beta^U c_t(c_t, x_t) = p_t, \quad \text{for} \quad t = 0,1,2,\ldots\]

where we choose units of utility as the numeraire.

The government at time 0 faces the intertemporal budget constraint

\[(2.5a) \quad \Sigma_o^{\infty} p_t y_{ot} > 0,\]

where \(y_{ot}\) in period \(t\) is given by

\[(2.5b) \quad y_{ot} = \tau_t (1 - x_t) - g_t - o_o b_t \quad t = 0,1,2,\ldots,\]

that is, the excess of tax income over the sum of government expenditure and the liability to the consumer (the sum of interest payments and repayment of
maturing debt). In general, \( y_{ot} \) is thus not equal to the budget surplus as it is conventionally defined.\(^1\) Lacking a better terminology, we shall refer to \( y_{ot} \) as government "cash-flow". This concept will be crucial in interpreting and extending Lucas and Stokey's result.

The government at time 0 has inherited the predetermined debt \( o^b = (o^b_0, o^b_1, \ldots) \). It determines the new debt structure \( b = (b_1, b_2, \ldots) \) that the government at time 1 will inherit subject to the following constraint:

\[
(2.6) \quad y_{oo} = \sum_{t=1}^\infty p_t (o^b_t - b_t)/p_0.
\]

That is, the period 0 cash-flow equals (the negative of) the real value of the net issue of new debt which, we note, is not identical to net government saving in the present context.\(^2\)

Combining (2.1), (2.4) and (2.5) gives the constraints

\[
(2.7a) \quad \sum_0^\infty \beta^t U_c(c_t, x_t) y_{ot} \geq 0 \quad \text{and}
\]

\[
(2.7b) \quad U_c(c_t, x_t)(c_t - o^b_t - y_{ot}) + U_x(c_t, x_t)(x_t - 1) = 0.
\]

The government maximizes (2.2) subject to (2.1) and (2.7). The first-order conditions for an optimum give, after some manipulations,

\[
(2.8) \quad U_{ct} - U_{xt} + \lambda_o [U_{ct} - U_{xt} + (U_{cct} - U_{cxt})(c_t - o^b_t - y_{ot}) +
\]

\[
+ (U_{xct} - U_{xxt})(x_t - 1)] + \lambda_o (U_{cct} - U_{cxt})y_{ot} = 0,
\]

where \( U_{ct} \) denotes \( U(c_t, x_t) \), etc. and where \( \lambda^o \) is the Lagrange multiplier of (2.7a). This condition is identical to the condition resulting from Lucas and Stokey's equations (2.9a) and (2.9b). The only difference is that the
term \( \lambda_0(U_{ct} - U_{xt})y_{ot} \) has been added and subtracted (note that the terms before \( y_{ot} \) cancel in (2.8)).

The advantage with writing the condition as (2.8) becomes apparent when we multiply (2.8) by \( dc_t \), using that by (2.1) \( dx_t = -dc_t \). Then we get the expression

\[
(2.9) \quad \beta^t dU_t + \lambda_0 p_t d[\tau_t(1 - x_t)] + \lambda_0 y_{ot} dp_t = 0.
\]

Here, \( dU_t \) is equal to \( (U_{ct} - U_{xt})dc_t \). It is not difficult to show that the second term in (2.8) multiplied by \( dc_t \) equals the second term in (2.9). The third term follows from (2.4b).

The expression (2.9) has a neat interpretation: In an optimum, suppose the government considers changing private consumption in period \( t \) by \( dc_t \). This implies that leisure, by (2.1), changes by \( dx_t = -dc_t \). The direct effect on utility given the existing distortion is \( \beta^t(U_{ct} - U_{xt})dc_t = \beta^t dU_t \), which is the first term in (2.9). The change in consumption and leisure also changes tax revenues in period \( t \) by \( d[\tau_t(1 - x_t)] \). If taxes increase in period \( t \), they can be lowered in other periods, and still balance the present value budget constraint (2.5a). If taxes decrease they must be raised in other periods.

The effect on utility of the tax change is given by \( \lambda_0 p_t d[\tau_t(1 - x_t)] \), the second term in (2.9). Indeed, \( \lambda_0 \) measures the distortionary effect of proportional taxation, more precisely the effect on utility of switching from proportional to lump sum taxation at constant government expenditure. We shall refer to it as the (level of) "tax distortion". Finally, the change in consumption and leisure in period \( t \) by (2.4b) implies a change in the present value (period
0 value) price of period $t$, $dp_t$. This in turn implies that government "net wealth" $\sum_{t=0}^{\infty} y_{ot} dp_t$, the product of government cash-flow and the change in the present-value price.\(^5\) If government net wealth increases (decreases), taxes can be lowered (must be raised). The effect on utility is hence given by $\lambda y_{ot} dp_t$, the third term in (2.9). In an optimum, the "direct" effect on utility, the "tax" effect due to changes in tax revenues (at constant intertemporal prices), and the "net-wealth revaluation" effect due to changes in intertemporal prices (at constant tax rates), must sum to zero.

If future governments are committed to set the future tax rates in accordance with the optimal policy calculated in period 0, any debt structure $\ldet = (b_1, b_2, \ldots)$ fulfilling (2.6) will do. If future governments are not committed to set the same tax rates but can choose them at their discretion, there is a unique debt structure $\ldet$ that, if honoured, makes successive governments still choose these tax rates, and hence makes fiscal policy time-consistent. We now turn to this issue.

3. Time-consistent policy

We first rewrite (2.8) by dividing with $(U_{cct} - U_{cxt})$, as

\[(3.1a)\] $\lambda y_{ot} + \lambda^0 A_t = B_t, \quad t = 0, 1, \ldots$, where

\[(3.1b)\] $A_t = [U_{cct} - U_{xxt} + (U_{cct} - U_{cxt})(1 - \tau_t)(1 - x_t) \]

$\quad + (U_{cxt} - U_{xxt})(x_t - 1)]/(U_{cct} - U_{cxt})$ and

\[(3.1c)\] $B_t = -(U_{cct} - U_{cxt})/(U_{cct} - U_{cxt})$.

Here we have used that the term $c_t - \ldet b_t - y_{ot}$ equals $(1 - \tau_t)(1 - x_t)$.

We thus see that $A_t$ and $B_t$ depend on $c_t$ and $x_t$ only.
Consider now a government at time 1, which tries to maximize
\[ \sum_{t=1}^{\infty} b_t^{-1} U(c_t, x_t) \] subject to (2.1) for \( t = 1, 2, \ldots \) and the analogue of (2.7),

\begin{align*}
(3.2a) & \quad \sum_{t=1}^{\infty} b_t^{-1} U(c_t, x_t) y_{1t} \geq 0 \\
(3.2b) & \quad U(c_t, x_t) + U(x_t, x_t, x_t, x_t, x_t) = 0, \quad t = 1, 2, \ldots ,
\end{align*}

where the cash-flow \( y_{1t} \) for the government at time 1 is given by

\[ y_{1t} = r_t (1 - x_t) - e_t - b_t, \quad t = 1, 2, \ldots , \]

with \( b_t \) predetermined. By similar reasoning as above, with \( \lambda_1 \) being the

Lagrange multiplier of (3.2a), the condition

\[ \lambda_1 y_{1t} + \lambda_1 A_t = B_t, \quad t = 1, 2, \ldots , \]

holds. Time consistency requires that the government at time 1 chooses the

same \( c_t \) and \( x_t \) for each \( t = 1, 2, \ldots \) as the government at time 0, hence that the

\( A_t \) and \( B_t \) in (3.1) and (3.3) are the same. Let us see what this involves.

The conditions (3.1) and (3.3) give

\[ \lambda_1 y_{1t} = \lambda_1 y_{0t} - (\lambda_1 - \lambda_0) A_t, \quad t = 1, 2, \ldots \]

Multiplying (3.4) with \( p_t \), adding for \( t = 1, 2, \ldots \), adding \( \lambda_0 p_0 y_{00} \) to both sides

of the resulting equation, and using \( \sum_{t=1}^{\infty} p_t y_{0t} = \sum_{t=1}^{\infty} p_t y_{1t} = 0 \), we finally get an

explicit solution to the tax distortion at \( t = 1 \),

\[ \lambda_1 = [1 - p_0 y_{00}/\sum_{t=1}^{\infty} p_t A_t] \lambda_0. \]

Multiplying (3.3) with \( p_t \), adding and using \( \sum_{t=1}^{\infty} p_t y_{1t} = 0 \), we get

\[ \sum_{t=1}^{\infty} p_t A_t = (\sum_{t=1}^{\infty} p_t B_t)/\lambda_1 > 0 , \]

where we assume \( B_t > 0 \), which holds if consumption and leisure are normal goods.\(^7\)

It follows from (3.5) that

\[ \lambda_1 \leq \lambda_0 \quad \text{if and only if} \quad y_{00} \leq 0 . \]

That is, if there is a positive cash-flow at time zero, the tax distortion at

time 1 is less than the tax distortion at time 0, which seems intuitive. By
combining (3.1) and (3.3) as

\[ (3.7) \quad \lambda_0 (y_{ot} + A_t) = \lambda_1 (y_{1t} + A_t) = B_t > 0, \quad t = 1, 2, \ldots \]

we also find that

\[ (3.8) \quad y_{1t} \geq y_{ot}, \quad t = 1, 2, \ldots, \quad \text{if and only if } \lambda_1 \geq \lambda_0. \]

Hence, by (3.6) and (3.8) a positive cash-flow in period 0 is distributed over all future periods, in the sense that the cash-flow \( y_{1t} \) is larger than the cash-flow \( y_{ot} \) for all \( t \).

The debt structures are related by

\[ (3.9) \quad b_{lt} = b_{0t} - (y_{1t} - y_{ot}) = b_{0t} + (1 - \lambda_0 / \lambda_1) (y_{ot} + A_t), \quad t = 1, 2, \ldots, \]

from (2.5b), (3.2c) and (3.4). In other words, the government at time 0 must choose a new debt structure \( b_{0t} \) that fulfills (3.9) in order to induce the government at time 1 to continue the optimal tax policy. An increase in cash-flow in period \( t (y_{1t} > y_{ot}) \) with both government spending and tax revenue given, of course corresponds to a decrease in interest payments and debt maturing in period \( t (b_{lt} < b_{0t}) \). Thus, a positive cash-flow in period 0 means that debt is restructured so that the obligations to consumers are lowered in all future periods.\(^8\)

When the structure of cash flows is changed in this particular way the different tax distortion of the time 1 government is exactly matched by a different incentive to engage in debt devaluations. By a restructuring of the debt according to the principles described by (3.4) (or (3.9)), the government at time 0 can thus indeed bind the hands of its successor so that it finds it optimal to continue the optimal policy.

Now, exactly the same reasoning can be applied to any pair of governments, and a complete description of the sequence of time consistent policies is
obtained just by substituting the subscripts $t$ and $t+1$ for 0 and 1, respectively, in equations (3.1) - (3.9).

4. Uncertainty

Let us also see how the above is modified when government expenditure is stochastic. Let the history $g^t = (g_0^t, \ldots, g_t^t)$ denote the sequence of government expenditure from time 0 to time $t$, and let $F^t(g^t)$ denote its distribution function. Consumption, leisure and taxes at time $t$ will be functions of the history $c_t(g^t), x_t(g^t)$ and $\tau_t(g^t)$, as will be the time 0 value $p_t(g^t)$ of a good whose delivery at time $t$ is conditional upon the history $g^t$ (under the assumption of a complete set of markets). Preferences at time zero are

$$\sum_0^\infty g_t^t \int U(c_t(g^t), x_t(g^t))dr_t(g^t).$$

Let $g^t_s = (g_s^t, g_{s+1}^t, \ldots, g_t^t)$ denote the sequence of government expenditure from time $s$ to time $t$. Claims issued at $t$ will be denoted $b_{t+1}^s(g^t, g_{t+1}^s), s = t+1, t+2, \ldots$, where $b_{t+1}^s(g^t, g_{t+1}^s)$ denotes claims to goods at time $s$, conditional upon $g_{t+1}^s$, given $g^t$. The cash-flows of the government at $0, y_{0t}(g^t), t = 0, 1, \ldots$, and the tax distortion $\lambda_0(g_0^t)$, will be functions of $g^t$ and $g_0$, respectively, while the cash-flows and the tax distortion of the government at time 1 will be functions $y_{1t}(g^t), t = 1, 2, \ldots$, and $\lambda_1(g^1)$, respectively.

With these modifications, (3.1) will hold for all $g^t, t = 0, 1, \ldots$, and (3.3) will hold for all $g^t, t = 1, 2, \ldots$. Equation (3.5a) holds for all $g^1$, given $g^0$, and the sum in the expression is $p_1(g^1)A_1(g^1)$.
\[ \sum_{t=2}^{\infty} p_t(g_t, g_{t-1}) \lambda_t(g_t, g_{t-1}) \alpha_t(g_t, g_{t-1}) dt(g_t|g_{t-1}) \] which is positive under the assumption of normal goods. Although \( \lambda_1 \) depends on \( g_1 \), (3.6) is true for all \( g_1 \), given \( g_0 \). Similarly (3.8) - (3.10) apply for all \( g_t \), given \( g_1 \).

Hence, it is still true that a positive cash-flow at time zero implies a lower tax distortion at time 1, at each given \( g_1 \). Similarly, a positive cash-flow at time zero results in higher cash-flows and lower levels of debt obligations for future governments (for each \( t \) and each \( g_t \)).

5. Summary and Conclusion

We have rewritten Lucas and Stokey’s equilibrium conditions by emphasizing government cash-flow and government net wealth, the latter being the present value of current and future cash-flows (and equal to zero in equilibrium). This way the optimality condition for an optimal policy with commitments can be conveniently interpreted as consisting of the sum of a direct effect on utility, an indirect tax effect due to changes in tax revenues, at constant intertemporal prices, and an indirect net-wealth revaluation effect due to changes in intertemporal prices, at constant tax rates. The latter two effects are weighted with the level of tax distortion – the marginal utility of (a hypothetical) shift from proportional to lump-sum taxation.

Governments at different periods face this optimality condition, but with different levels of tax distortion; less tax distortion if previous governments have had cash-flow surpluses. To induce a succeeding government to continue pursuing the time-consistent tax policy, the preceding government has to choose a debt structure such that any change in the tax distortion is matched by
changes in future cash flows that change the wealth devaluation incentives in an appropriate way. More specifically, a cash-flow surplus in one period, giving rise to a lower tax distortion for later governments, requires a cash-flow increase in all future periods (and states of the world), hence to a decrease in the level of debt obligations of each maturity (and contingency).

Lucas and Stokey, although not expressing their results in this relatively straight-forward way, provide several illuminating examples and a very thorough and thoughtful discussion of the qualifications to the results, for instance why they do not hold if there is capital, why it is assumed that previous debt but not tax decisions are honoured, why the results break down if there is money, etc. There is no point in repeating that discussion here. Let us only note that for an open economy, the existence of a national debt to the rest of the world, and the possibility for a government of a large economy to deliberately devalue its debt to the rest of the world, do present additional problems for time consistent policies. Our current research is directed towards those problems.
Footnotes

1. Only in the special case when all debt is in the form of consols so that \( o^{b_t} \) consists only of coupon payments, does \( y_{ot} \) correspond to the budget surplus.

2. To see that government (net) saving is different from cash-flow, let us do the following accounting exercise. Let \( W_t \equiv \sum_{s=t}^{\infty} \tau_s(1-x_s) - t b_s / p_t \) be government (gross) wealth in period \( t \). Then the government budget constraint at time \( t \) is \( \sum_{s=t}^{\infty} p_s g_s / p_t \leq W_t \). Define the (real) rate of interest \( r_t \equiv p_t / p_{t+1} - 1 \) between period \( t \) and \( t+1 \). Let \( Y_t \equiv r_t W_t / (1 + r_t) \) be government "income" (including capital gains) in period \( t \) (defined as the level of government expenditure that holds wealth constant, follows from rewriting the budget constraint as \( g_t + W_{t+1} / (1 + r_t) = W_t \)). Define government saving as \( S_t \equiv Y_t - g_t \). (Then saving fulfills \( S_t = (W_{t+1} - W_t) / (1 + r_t) \)). From this follows that government saving can be written

\[
S_t = [r_t / (1 + r_t)] \sum_{s=t}^{\infty} \tau_s (1 - x_s) - t b_s / p_t - g_s,
\]

which is different from the cash-flow (of government \( t \) in period \( t \))

\[
y_{tt} = \tau_t (1 - x_t) - g_t - t b_t = \sum_{s=t+1}^{\infty} \tau_s b_s - t+1 b_s / p_t.
\]

3. We have \( p_t d[\tau_t (1 - x_t)] = \beta^t U_{ct} d[(1 - \tau_t)(x_t - 1) + (1 - x_t)] = \beta^t U_{ct} d[U_{xt} (x_t - 1)/U_{ct} + (1 - x_t)] = \beta^t U_{xt} - U_{xt} + (U_{cxt} - U_{cxt})(1 - \tau_t)(1 - x_t) + (U_{xct} - U_{xxt})(x_t - 1),
\]
where we can use \((1 - \tau_t)(1 - x_t) = c_t - o b_t - y_{ot}\).

4. To see this, consider the static problem of maximizing \(U(c, x)\) subject to 
\[U_x/U_c = (1 - \tau), \quad \tau(1 - x) - g + T = 0\] and \(1 - x - c - g = 0\), where \(T\) is lump-sum taxation. The first two constraints can be written \(U_c(1 - x - g + T) + U_x(x - 1)\) and \(U_c(x) = 0\). The Lagrangian is 
\[L = U(c, x) + \lambda[U_c(1 - x - g + T) + U_x(x - 1)] + \mu(1 - c - x - g)\. It follows that the marginal utility of increasing exogeneous lump-sum taxation (and decreasing endogenous proportional taxation) at constant government expenditure is \(\partial L/\partial T = \lambda \cdot U_c\). (The marginal utility of increasing government expenditure at constant lump-sum taxes is \(\partial L/\partial g = -\lambda U_c - \mu\), consisting of the pure resource effect, \(-\mu\), and the pure effect of increased proportional taxation, \(-\lambda U_c\).)

5. This is an "intertemporal terms of trade effect", an intertemporal analogue to the "terms of trade effect" in international trade theory, and the "real income effect" in standard consumer theory.

6. Equation (3.4) is the analogue to Lucas and Stokey's (2.10), the latter expressed in terms of the debt structures \(o b_t\) and \(1 b_t\). We note that our \(a_t\) is closely related but not identical to their \(a_t\), the difference being that their \(a_t\) results if \(c_t\) is substituted for our \((1 - \tau_t)(1 - x_t) = c_t - o b_t - y_{ot}\).

7. We have \(U_{ct} - U_{xt} = \tau_t U_{ct} > 0\) if \(0 < \tau_t < 1\). If goods are normal, 
\[(\partial^2 L)(U_c/U_x) = (U_x U_{cc} - U_x U_{xc})(U_x)^2 < 0\] for all \((c, x)\), which implies 
\(U_{cc} - U_{xc} < 0\).

8. We can also write (3.4) as \(y_{1t} = (\lambda_o/\lambda_1)y_{ot} + (1 - \lambda_o/\lambda_1)(-A_t)\). Here
we see that the structures of net wealth of the two governments are such that government of time 1 inherits a "portfolio" consisting of a linear combination of the portfolio inherited by the government of time 0 and the portfolio $-A_t$, $t = 1, 2, \ldots$. The weights are $\lambda_0/\lambda_1$ and $1 - \lambda_0/\lambda_1$ and sum to zero, but are not always nonnegative. If $\lambda_0 > \lambda_1$, the government of time 1 inherits a short position in $-A_t$. 
Reference