Investigation of antennas for RFID tags on paperbased products

Peter Jonsson och Johan Sidén, Mitthögskolan
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Mid Sweden University
Fibre Science and Communication Network
SE-851 70 Sundsvall, Sweden

Internet: http://www.mh.se/fscn
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Investigation of antennas for RFID tags on paperbased products

Peter Jonsson and Johan Sidén *
* Mid-Sweden University, S-851 70 Sundsvall, Sweden

Abstract

The trend in the development of information systems is towards more and more detailed information about objects, persons, animals, etc. which can be registered electronically at an individual level. In recent years RFID is a technique that has been increasingly used for these purposes. Mid Sweden University requested a study to be made regarding the possibilities of using the RFID-technique for identification of paperbased products. This motivated this Master thesis. Examples of applications are control of authentication of valuable documents and searching for paper documents in archives. One of the goals is to print antennas and construct circuits directly on the paper. The aim of this report is to investigate and describe suitable antennas for transmitting/receiving radiosignals in the objects that are to be identified. A great part of the work has consisted of studies in the areas of RFID-technique and antenna theory, as this was a new area for us.
Sammanfattning

Trenden inom utvecklingen av informationssystem går mot att allt mer detaljerad information om föremål, personer, djur etc. kan registreras elektroniskt på individnivå. RFID är en teknik som på senare år börjat användas allt mer för dessa ändamål.

Mitthögskolan hade vid detta examensarbets början önskemål om studier av möjligheterna till användning av RFID tekniken för identifiering av pappersbaserade produkter. Exempel på användningsområden är kontroll av äkthet hos värdepapper och sökning av pappersdokument. En av målsättningarna är att kunna trycka antenner och konstruera kretsar direkt på papperet.

Syftet med den här rapporten är att undersöka och beskriva lämpliga antenner för sändning/mottagning av radiosignaler i de föremål som ska identifieras.

En stor del av arbetet har bestått av studier i RFID tekniken och antennteori då dessa områden vid examensarbets början var helt nya för oss.

Preface

This 20 credit thesis is the compulsory completion of the Systemteknik master education program 180 credits, at Mid Sweden University. A successful completion of the program leads to a Master of Science in Electrical Engineering.

The work has been carried out at the Department of Information Technology (ITE) at Mid Sweden University, Sundsvall.

The purpose of the Master thesis work is to achieve more in-depth knowledge in the area of the task and to solve problems without predestinate results.

We would like to thank our supervisors Bengt Oelmann and Youshi Xu at Mid Sweden University for their guidance and support.

Sundsvall 2000-10-12

Peter Jonsson  Johan Sidén
Introduction

What is RFID?

RFID stands for Radio Frequency Identification and is a widely varied collection of technologies used for contactless transfer of power and data. A contactless transfer of data is far more flexible than the most common form of automatic identification procedure used today, namely the smart card based upon a contact field.

RFID systems are closely related to the smart cards based upon a contact field. Data is stored on an electronic data-carrying device, commonly called a transponder or tag. However, in RFID systems, data transfer and sometimes the transfer of power required to operate the transponder, are achieved using magnetic or electromagnetic fields instead of the use of galvanic contacts.

The use of magnetic or electromagnetic fields in RFID systems enables energy to penetrate certain goods. Therefore in order to identify a tag, it doesn’t have to be visible.

The components of a RFID system are the transponders, the interrogator or reader and a computer system to process the information. The reader typically contains a transmitter and a receiver.

![Components of an RFID system.

Figure 1-1 Components of an RFID system.

Generally there are two different kinds of RFID systems:

- Magnetic (inductive) coupled.
- Electric field coupled.

The coupling elements in magnetic coupled systems are usually a coil in the reader and a coil in the transponders. When a transponder with a self-resonant frequency corresponding to the
transmission frequency is placed in an alternating magnetic field generated by the reader’s antenna coil, energy from the magnetic field will be drawn. This will result in a measurable voltage drop in the reader. By switching on and off a load resistance at the transponder’s antenna, the voltage in the reader’s antenna will change correspondingly and thus the voltage is amplitude modulated. This method is called load modulation and is a typical method used to receive data back from the transponder in magnetic coupled systems.

Electric field coupled transponders operate in the far field instead of the near field as in the case of magnetic coupled transponders. This means that the electromagnetic waves no longer have a direct effect on the antenna from which they were generated. For this reason electric field coupled systems are often called propagation coupled systems. The antenna structures for electric coupling start to become practical for operating-frequencies above 100 MHz. Since the near field region increases with an increase of the wavelength, magnetic coupling is used for frequencies below 100 MHz, which also allows the use of antenna systems that comprise small coils.

RFID tags are also categorized as either active or passive. Active RFID tags are powered by an internal battery and have typically read/write memory to store the data. Passive RFID tags operate without a separate external power source and all power required for operation must be drawn from the electrical/magnetic field of the reader. Passive tags are therefore much lighter than active tags, less expensive and offer a virtually unlimited operational lifetime. However, the absence of an external power source results in shorter read ranges than the active tags have and thus a higher-powered reader is needed.

Magnetic coupled transponders are almost always operated passively. The reader’s antenna coil generates an electromagnetic field. A small part of the emitted field then penetrates the coil of the tag and hence by induction a voltage is generated in the tag’s antenna coil. After this voltage is rectified it can be stored in a capacitor to operate the transponder.

Passive electric field transponders also collect their power from the reader’s radiated field via its antenna. However, this power is usually too small to operate the transponder as an ordinary transmitter. So in order to communicate the data from the tag to the reader a principle called backscatter modulation, developed by Lawrence Livermore Laboratories in the USA in the 1960’s, is usually used. The energy from the reader is collected, rectified and powers the transponder. The transponder then generates a data stream comprising a clock signal and the data to be transmitted. In synchronism with the data stream a transistor across the antenna of the transponder is switched on and off to vary the antenna load. By varying the antenna load the antenna matching changes and hence the radar cross section (reflection characteristics) of the antenna changes, causing varying amounts of power to be reflected back to the reader. A receiver with a mixer can, by using the transmitter signal as local oscillator, extract the data from the received energy. By using backscatter modulation, the communication can be implemented relatively simply at low cost. Tags in this kind of systems usually have read only memory since writing consumes a lot of power.

In a new project, Mid Sweden University intends to study the possibilities of electronic marking of paper products combined with contactless reading. The underlying technology for
the system should build on RFID technology. One objective is to design the transponders as antennas printed on paper and circuits constructed directly on paper.

With electronic ID-marking, the following applications are interesting for this project:

- Effective localization and searching of documents in archives.
- Authentication of documents e.g. signed contracts, valuable documents and bills.
- Tracking of a document’s origin.
- Identification of a package during transport.
- Identification of a package and automatic sorting during recycling.

**Task**

The task of this Master thesis work is to:

- Study a draft for RFID.
- Investigate antennas for RFID tags, suitable for the project. The reader should preferably have one antenna module and therefore the radiation pattern from a tag should be as isotropic as possible.

**Object**

The object of the documentation is to describe the methods used to carry out the task, the results and the experiences and conclusions we made during our work. Thus it can be used as guidance in Mid Sweden University’s continuing work with RFID.

**Test of RFID Equipment from SCS Inc.**

To be able to compare our theoretical results and maybe get some ideas how to carry out our work, we bought RFID equipment from an American company named SCS Inc.

The system makes use of passive RFID tags operating in 2.45 GHz that are very small and thin and we think that this is the product that is closest to our project at present. The tag is about $50 \times 10 \times 0.5$ mm and seems to consists of one half-wave folded dipole antenna and one magnetic coupled small loop antenna with some parasite elements in it to range gain. The use of the small loop antenna instead of using two orthogonal placed folded dipoles is to keep the tag in one dimension since the small loop acts as a dipole with direction through the loop.

The antenna in the reader, Fig. 2-1 and 2-2, is one variant on a Gregorian antenna, the Gregorian antenna is studied in [1], which means that it radiates using two reflectors. In this case it is made up of a double layered circuit board where the current is first led in on the reverse of the board. Then it radiates through the board with a non-center-fed antenna and through isolated orthogonal slits on the front to get two linear polarizations. The waves are firstly reflected in an isolated elliptical shaped metal plate and then reflected again at the front of the board, which is also the ground plane, to finally radiate out into the air.
The conclusions of the measurements are that the reading capabilities are even worse than the specifications (see appendix E). As you can see by our detection patterns in Fig. 2-3, the range for reading is very small. In the best case about 24 cm if the tag is placed with the dipole in a horizontal polarization and about 2/3 of that when placed vertically. To compensate for this SCS uses 6 antennas placed uniformly around a reading area which could perhaps be sufficient to scan a basket of merchandise when shopping, but not much more. The advantage of the system is, of course, the small size of the tags with the antennas printed on thin plastic film so that they could for example replace barcodes on different products.
Choice of Operating Frequency

There are many points to consider when selecting an RFID system. One is the operating frequency. Available frequency bands are 100 – 135 kHz, 3 – 30 MHz, 2.45 and 5.8 GHz [14]. Generally, RFID systems in the first two frequency bands operate using inductive coupling while microwave systems (2.45 or 5.8 GHz) operate using electromagnetic fields for coupling. In the US, an allocation from 902 MHz to 928 MHz has also been allocated for RFID. In Europe, the GSM cell phones are allocated to the 900 MHz band but a frequency allocation at 869 MHz has recently appeared. This frequency is to be available throughout the EU.

High frequencies mean short wavelengths, which in turn allow the construction of antennas with smaller dimensions and greater efficiency. Also, the energy demand decreases with increasing frequency. Thus by increasing the carrier frequency the read range is improved. By considering these facts and the fact that it is more likely that inductive systems have to deal with electromagnetic interference fields than microwave systems, microwave systems have a distinct advantage. However, lower frequency systems offer better penetration through objects and water. In the GHz-band, buildings and other obstacles behave as good reflectors and damp an electromagnetic wave very strongly at transmission. Higher frequencies also mean more directionality problems and a maximum transmission power of only 0.5 W ERP. The last is due to regulations in Europe that permit a maximum transmission power of 0.5 W ERP for RFID applications operating in the UHF frequency range. This 0.5 W barrier makes it very challenging to find a way to provide the transponder with enough energy to operate a microchip.

Since the goal of the project is to integrate small passive transponders into paper products and the read range should be quite large, our suggestion is to use a microwave system with an operating frequency of 2.45 GHz.

Transponders in microwave systems use backscatter modulation exclusively and the antennas used are primarily dipoles and antenna formats derived from these.

Choice of Antenna

The major task of this project is to choose an antenna that is appropriate for the tag according to different demands. The tag is being oriented in all directions in the room, which means that ideally it should have a spherical radiation pattern. Since the tag should also be kept small and preferably in one dimension, we had to compare the advantages of large antennas with the disadvantages of small antennas. In the following, some antennas and antenna systems are investigated. We have chosen to investigate resonant antennas and combinations of these because of their simple structure and good input impedance over a narrow band of frequencies. The easiest way to get a spherical pattern is simply to put up two orthogonal dipoles (or folded dipoles for higher antenna impedance). However you are now in two dimensions and there is much unused space between the dipoles. Thus we also investigated the consequences of adding two additional pairs of dipoles in the same area. However, first we give an introduction to the straight wire dipoles.
At the end of our thesis work, we had at our disposal an antenna modeling program called NEC-2. To confirm some of our results, we allowed the NEC program to process some of the antennas. The Numerical Electromagnetics Code (NEC-2) has been developed at the Lawrence Livermore Laboratory, Livermore, California, under the sponsorship of the Naval Ocean Systems Center. It is a computer code for analyzing the electromagnetic response of an arbitrary structure consisting of wires and surfaces in free space or over a ground plane. The analysis is accomplished by the numerical solution of integral equations for induced currents. The wire diameters have been chosen to $0.001\lambda$ in all the following computations made by NEC-2. The wires have also been assumed to be perfect conductors.

The Ordinary Dipole Antenna

![Figure 5-1](image-url)  
A center-fed straight dipole antenna.

One of the most common antennas used is the dipole antenna, also called Hertz antenna, Fig. 5-1. The reason that this antenna is so common is because of the ease of construction and its radiation pattern, which we will see, is omnidirectional. This antenna is most often fed into the center and its length is measured according to the used wavelength. It belongs to the category of resonant antennas since the length is matched to some multiplicity of an even part of the wavelength. The current distribution on a dipole is easily achieved by taking the current to be zero at the ends and then to vary it sinusoidal towards the center and it is then symmetric from both ends, see equation (5-2). Some current distributions for different lengths of the dipole are shown in Fig. 5-2.
If we take a cartesian coordinate system and put the antenna along the z-axes, the standard formula for the magnetic vector potential (B-1) will be simplified to

$$A = \hat{z} \frac{\mu_0}{4\pi} \int I(z')e^{j k z'} dz'$$

(5-1)

where

$$I(z) = I_0 \sin \left( \beta \left( \frac{L}{2} - |z| \right) \right) ; \quad |z| \leq \frac{L}{2}$$

(5-2)

$$\hat{r} = \hat{x} \sin \theta \cos \phi + \hat{y} \sin \theta \sin \phi + \hat{z} \cos \theta$$

(5-3a)

$$r' = z'\hat{z}$$

(5-3b)

$$\hat{r} \cdot r' = z' \cos \theta$$

(5-3c)

The current distribution in (5-2) is for an infinitely thin dipole. $I_0$ is the maximum current attained, $L$ the length of the antenna and $\beta = \frac{2\pi}{\lambda}$ the phase constant.

Figure 5-2  Current distribution for center-fed straight dipoles of length L.
Putting (5-2) and (5-3) into (5-1) we get

\[ A_z = \frac{\mu e^{-j\beta}}{4\pi r} \left( \int_{L/2}^{L} I_0 \sin \left[ \beta \left( \frac{L}{2} + z' \right) \right] e^{j\beta \cos \theta} dz' + \int_{0}^{L/2} I_0 \sin \left[ \beta \left( \frac{L}{2} - z' \right) \right] e^{j\beta \cos \theta} dz' \right) \]

\[ = \frac{\mu e^{-j\beta}}{4\pi r} I_0 \left[ \frac{e^{j\beta \cos \theta}}{\beta^2 - \beta^2 \cos^2 \theta} \left( j\beta \cos \theta \sin \left( \beta \left( \frac{L}{2} + z' \right) \right) - \beta \cos \left( \beta \left( \frac{L}{2} + z' \right) \right) \right) \right]_{L/2}^{L} + \left[ \frac{e^{j\beta \cos \theta}}{\beta^2 - \beta^2 \cos^2 \theta} \left( j\beta \cos \theta \sin \left( \beta \left( \frac{L}{2} - z' \right) \right) + \beta \cos \left( \beta \left( \frac{L}{2} - z' \right) \right) \right) \right]_{0}^{L/2} \]

\[ = \frac{\mu I_0 e^{-j\beta}}{4\pi r \beta} \left[ \frac{1}{1 - \cos^2 \theta} \left( j \cos \theta \sin \left( \beta \frac{L}{2} \right) - \cos \left( \beta \frac{L}{2} \right) \right) \right] - \left[ \frac{e^{-j\beta \frac{L}{2} \cos \theta}}{1 - \cos^2 \theta} \left( j \cos \theta \sin \left( \beta \frac{L}{2} \right) + \cos \left( \beta \frac{L}{2} \right) \right) \right] \]

\[ = \frac{\mu I_0 e^{-j\beta}}{2\pi r \beta} \frac{\cos \left( \beta \frac{L}{2} \cos \theta \right) - \cos \left( \beta \frac{L}{2} \right)}{\sin^2 \theta} \]

(5-4)

Since the source is z-directed \( E_\phi \) is zero and \( E_\theta \) is obtained from (B-4) as

\[ E_\theta = j \omega \sin \theta A_z = j \omega \sin \theta \frac{\mu I_0 e^{-j\beta}}{2\pi r \beta} \frac{\cos \left( \beta \frac{L}{2} \cos \theta \right) - \cos \left( \beta \frac{L}{2} \right)}{\sin^2 \theta} = j \frac{\eta I_0 e^{-j\beta}}{2\pi r} \frac{\cos \left( \beta \frac{L}{2} \cos \theta \right) - \cos \left( \beta \frac{L}{2} \right)}{\sin \theta} \]

(5-5)

The \( \theta \)-variation in this formula determines the radiation pattern and by putting in different lengths in (5-5) we will get functions \( F(\theta) \) for the normalized radiation patterns. For example for the half and the one-wavelength antennas we will get

\[ F(\theta) = \frac{\cos \left( \frac{\pi}{2} \cos \theta \right)}{\sin \theta} \quad L = \frac{\lambda}{2} \]

(5-6)
\[ F(\theta) = \frac{\cos(\pi \cos \theta) + 1}{2 \sin \theta} \quad L = \lambda \] (5-7)

The radiation patterns from (5-6), (5-7) and for some other antenna lengths are plotted in Fig. 5-3.

\[
L = \frac{\lambda}{2}
\]

\[
L = \frac{3}{4} \lambda
\]

\[
L = \lambda
\]

\[
L = \frac{5}{4} \lambda
\]

\[
L = \frac{3}{2} \lambda
\]

**Figure 5-3** Normalized radiation patterns for center-fed straight dipoles of length L.
As you can see in Fig. 5-3 the radiation pattern for the half-wave dipole has the widest main beam and is therefore the preferred antenna for this application if we are going to use an ordinary dipole. All the figures in Fig. 5-3 are plotted in the zy-plane and since the pattern is omnidirectional (independent of $\phi$) they can all merely be rotated around the z-axis to show the three dimensional pattern. For the half wave dipole the radiation pattern will become a torus, or a doughnut as can be seen in Fig. 5-4.

![Figure 5-4 Radiation pattern for the half-wave dipole.](image)

Now when we know $E_\theta$ we can calculate the radiated power from (B-9).

$$P = \frac{1}{2\eta} \left| \int_{\theta=0}^{2\pi} |E_\theta|^2 r^2 \sin \theta d\theta d\phi \right| = \frac{1}{2\eta} \left| \int_{\theta=0}^{2\pi} \eta I_0 e^{-j\phi} \frac{\cos \left( \frac{\beta L}{2} \cos \theta \right) - \cos \left( \frac{\beta L}{2} \right) }{2\pi \sin \theta} r^2 \sin \theta d\theta d\phi \right|^2$$

$$= \frac{2\eta I_0^2}{8\pi^2} \left[ \frac{\cos \left( \frac{\beta L}{2} \cos \theta \right) - \cos \left( \frac{\beta L}{2} \right) }{\sin \theta} \right]^2 d\theta d\phi$$

Substituting $\cos \theta = \tau$, $d\tau = -\sin \theta d\tau$, gives us

$$P = \frac{\eta I_0^2}{4\pi^2} \int_{\tau=-1}^{1} \frac{\cos \left( \frac{\beta L}{2} \tau \right) - \cos \left( \frac{\beta L}{2} \right) }{\sin^2 \theta} d\tau$$

$$= \frac{\eta I_0^2}{2\pi} \int_{\tau=-1}^{1} \frac{\cos \left( \frac{\beta L}{2} \tau \right) - \cos \left( \frac{\beta L}{2} \right) }{1 - \tau^2} d\tau$$

$$= \frac{\eta I_0^2}{2\pi} \int_{\tau=-1}^{1} \left[ \cos \left( \frac{\beta L}{2} \tau \right) - \cos \left( \frac{\beta L}{2} \right) \right]^2 d\tau$$  \hspace{1cm} (5-9)
By using the partial fraction identity
\[
\frac{1}{1-u^2} = \frac{1}{2} \left( \frac{1}{1+u} + \frac{1}{1-u} \right)
\] (5-10)

on (5-9) the result is
\[
P = \frac{\eta I_0^2}{4\pi} \int_0^1 \left[ \cos\left(\frac{\beta L}{2}\tau\right) - \cos\left(\frac{L}{2}\right) \right] \frac{1}{1+\tau} d\tau + \left[ \cos\left(\frac{\beta L}{2}\tau\right) - \cos\left(\frac{L}{2}\right) \right] \frac{1}{1-\tau} d\tau
\] (5-11)

This integral can be evaluated in terms of sine and cosine integral functions but we prefer to simply show the evaluation for the case when \( L = \frac{\lambda}{2} \) which gives \( \beta L/2 = \pi/2 \), and calculate the other antennas by numerical methods using a computer.

So, putting \( L = \frac{\lambda}{2} \) gives us
\[
P = \frac{\eta I_0^2}{4\pi} \left[ \cos^2\left(\frac{\pi}{2}\tau\right) + \cos^2\left(\frac{\pi}{2}\tau\right) \right] d\tau
\] (5-12)

Further, substituting \( v = 1 - \tau, \ dv = -d\tau, \ w = 1 + \tau, \ dw = d\tau \) gives us
\[
P = \frac{\eta I_0^2}{4\pi} \left[ -\sin^2\left(\frac{\pi}{2}\right) \right] dv + \left[ \sin^2\left(\frac{\pi}{2}\right) \right] dw = \eta I_0^2 \int_0^1 \frac{1 - \cos\pi v}{2v} dv
\] (5-13)

To be able to use the standard integral (C-3) we then substitute \( t = \pi v, \ dt = dv \), which leads to
\[
P = \frac{\eta I_0^2}{8\pi} \int_0^{2\pi} \frac{1 - \cos t}{t} dt = \frac{\eta I_0^2}{8\pi} \text{Cin}(2\pi) = 2.44 \cdot \frac{\eta I_0^2}{8\pi}
\] (5-14)

where we have calculated \( \text{Cin}(2\pi) \) by using (C-4). Assuming \( \eta = 120\pi \) for air we finally end up with
\[
P = 2.44 \cdot \frac{120\pi I_0^2}{8\pi} = 36.6 I_0^2
\] (5-15)

From (B-11) we then get the radiation resistance
The $\frac{\lambda}{2}$-dipole antenna also has a reactive impedance component, which is inductive, and the complete input impedance is

$$Z_A = 73 + j42.5 \, \Omega$$  \hspace{1cm} (5-17)$$

If the length of the half-wave dipole is reduced to about $L = 0.48 \lambda$, the impedance is about $Z_A = 73 + j0 \, \Omega$ and resonance is achieved.

The Half-Wave Folded Dipole

A folded dipole of length $L = \frac{\lambda}{2}$ is basically two parallel half-wave dipoles, connected at the ends and separated by a distance $d$, as shown in Fig. 6-1.

The input current is easy to obtain. Since the current, instead of being reflected at the end, as in the case of a dipole, is travelling back in the parallel transmission line, the same total current is obtained in the folded dipole case as in the ordinary dipole case. The difference is that the folded dipole has two separated currents equal in size. So

$$I_f = \frac{I_d}{2}$$  \hspace{1cm} (6-1)$$

where $I_f$ and $I_d$ is the amplitude of the input current in the folded dipole respectively the ordinary dipole case.

Now the input power for the folded dipole is

$$P_f = \frac{Z_fI_f^2}{2}$$  \hspace{1cm} (6-2)$$
and for the ordinary dipole

\[ P_d = \frac{Z_d I_d^2}{2} \]  \hspace{1cm} (6-3)

where \( Z_f \) and \( Z_d \) are the antennas input impedance.

As mentioned above the total input currents are the same and therefore the radiated powers are also the same, which gives

\[ P_f = P_d \quad \Leftrightarrow \quad Z_f I_f^2 = Z_d I_d^2 \]  \hspace{1cm} (6-4)

Using (6-1) and (6-4) the final result is

\[ Z_f = 4Z_d \]  \hspace{1cm} (6-5)

Since the half-wave dipole at resonance has about 73\,\Omega of input impedance the half-wave folded dipole has about 292\,\Omega which is very close to the common 300\,\Omega of a twin-lead transmission line. For a parallel-wire line with round conductors with radii \( a \) and separated by a distance \( d \), the characteristic impedance is given by

\[ Z_o = \frac{120}{\sqrt{\varepsilon}} \ln\left(\frac{d}{a}\right) \]  \hspace{1cm} (6-6)

where \( \varepsilon \) is the relative permittivity on the insulating material between and around the conductors. For air \( \varepsilon \approx 1 \). Setting \( Z_o = 292\,\Omega \) gives

\[ \frac{d}{a} = \exp(292/120) \approx 11.4 \]  \hspace{1cm} (6-7)

while setting \( Z_o = 73\,\Omega \) gives

\[ \frac{d}{a} = \exp(73/120) \approx 1.8 \]  \hspace{1cm} (6-8)

The conclusion is that by using a half-wave folded dipole antenna, impedance matching is much easier compared to that of an ordinary half-wave dipole. In fact, the half-wave folded dipole has a wider bandwidth than an ordinary half-wave dipole.
The Two Dipoles Antenna

One simple dipole will not radiate in the direction of the antenna axis. To achieve a more isotropic radiation pattern an antenna containing two orthogonal placed half-wave dipoles as in Fig. 7-1 can be used. The gap in the corner between the dipoles is assumed to be very small and negligible. The dipoles are center-fed with the current distribution shown as solid gray curves.

![Figure 7-1](Image)

**Figure 7-1** Two orthogonal placed half-wave dipoles.

The current and the position vectors to each dipole are, according to Fig. 7-1 and (5-2), given by

\[
I_1 = \hat{y}I_0 \cos(\beta y') \quad |y'| < \frac{\lambda}{4} \\
I_2 = -\hat{x}I_0 \cos(\beta x') \quad |x'| < \frac{\lambda}{4}
\]  

(7-1a) (7-1b)

\[
r_1' = \frac{\lambda}{4} \hat{x} + y' \hat{y} \\
r_2' = x' \hat{x} + \frac{\lambda}{4} \hat{y}
\]  

(7-2a) (7-2b)

Using the vector potential from (B-1) and the expansion of the unit vector for \( \mathbf{r} \) from (A-1) along with (7-1) and (7-2) gives

\[
A = \frac{\mu_0 e^{-j\beta r}}{4\pi} \int I e^{j\mathbf{k} \cdot \mathbf{r'}} d\mathbf{r'} = \frac{\mu_0 e^{-j\beta r}}{4\pi} \int_0^{\lambda/4} \hat{y} \int_{-\lambda/4}^{\lambda/4} \cos(\beta y') e^{j\mathbf{k} \cdot \mathbf{r'}} dy' - \hat{x} \int_{-\lambda/4}^{\lambda/4} \cos(\beta x') e^{j\mathbf{k} \cdot \mathbf{r'}} dx'
\]  

(7-3)

\[
\hat{r} \cdot \mathbf{r}_1' = \frac{\lambda}{4} \sin \theta \cos \phi + y' \sin \theta \sin \phi \\
\hat{r} \cdot \mathbf{r}_2' = x' \sin \theta \cos \phi + \frac{\lambda}{4} \sin \theta \sin \phi
\]  

(7-4)
\[
A = \frac{\mu e^{-j\beta r}}{4\pi r} I_0 \left[ j\pi \sin \theta \cos \phi + j\beta' \sin \theta \sin \phi \right] \int_{-\lambda/4}^{\lambda/4} \cos(\beta y') e^{j\beta' \sin \theta \cos \phi} \sin \theta \sin \phi \, dy' - \hat{x} \int_{-\lambda/4}^{\lambda/4} \cos(\beta x') e^{j\beta' \sin \theta \cos \phi} \cos \theta \cos \phi \, dx'
\]

\[
= \frac{\mu e^{-j\beta r}}{4\pi r} I_0 \left[ j\pi \sin \theta \cos \phi + j\beta' \sin \theta \sin \phi \right] \int_{-\lambda/4}^{\lambda/4} \cos(\beta y') e^{j\beta' \sin \theta \cos \phi} \sin \theta \sin \phi \, dy' - \hat{x} \int_{-\lambda/4}^{\lambda/4} \cos(\beta x') e^{j\beta' \sin \theta \cos \phi} \cos \theta \cos \phi \, dx'
\]

Putting \( \cos \gamma = \sin \theta \cos \phi \) and \( \cos \Omega = \sin \theta \sin \phi \) in (7-5), yields

\[
A = \frac{\mu e^{-j\beta r}}{4\pi r} I_0 \left[ j\pi \cos \gamma \left( \frac{e^{j\beta' \cos \gamma}}{(j \beta \cos \Omega)^2 + \beta^2} \left(j \beta \cos \Omega \cos(\beta y') + \beta \sin(\beta y') \right) \right) \right]_{-\lambda/4}^{\lambda/4}
\]

\[
- \hat{x} e^{j\pi \cos \gamma} \left( \frac{e^{j\beta' \cos \gamma}}{(j \beta \cos \Omega)^2 + \beta^2} \left(j \beta \cos \gamma \cos(\beta x') + \beta \sin(\beta x') \right) \right) \left[ \int_{-\lambda/4}^{\lambda/4} \right]
\]

\[
= \frac{\mu e^{-j\beta r}}{4\pi r} I_0 \left[ j\pi \cos \gamma \left( \frac{e^{j\pi \cos \gamma}}{1 - \cos^2 \Omega} \left(j \cos \Omega \cos \left(\frac{\pi}{2}\right) + \sin \left(\frac{\pi}{2}\right) \right) \right) - \hat{x} e^{j\pi \cos \gamma} \left( \frac{e^{j\pi \cos \gamma}}{1 - \cos^2 \gamma} \left(j \cos \gamma \cos \left(\frac{\pi}{2}\right) + \sin \left(\frac{\pi}{2}\right) \right) \right) \right] =
\]
The E-field is obtained from (B-4).

\[
E_\theta = - j \omega (A_x \cos \theta \cos \phi + A_y \cos \theta \sin \phi)
\]

\[
= - j \eta e^{-j/\beta} I_0 \cos \theta \left[ - e^{j \pi / 2} \cos \frac{\pi}{2} - \cos \theta \cos \phi + e^{j \pi / 2} \cos \frac{\pi}{2} \sin \phi \right]
\]

\[
E_\phi = - j \omega (-A_x \sin \phi + A_y \cos \phi)
\]

\[
= - j \eta e^{+j/\beta} I_0 \cos \theta \left[ e^{j \pi / 2} \cos \frac{\pi}{2} \sin \phi + e^{j \pi / 2} \cos \frac{\pi}{2} \cos \phi \right]
\]

Field patterns in the principal planes can be seen in Fig. 7-2. As shown, the patterns are quite circular in these planes. One of the disadvantages is the antenna size required to achieve them.
Figure 7-2  Plane patterns for the two half-wave dipoles organized as in Fig. 7-1.

The radiated power is obtained from (B-9).

\[
P = \frac{1}{2\eta} \int_0^{2\pi} \int_0^{\pi} \left( |E_\theta|^2 + |E_\phi|^2 \right) r^2 d\Omega
\]

\[
P = \frac{\eta I_0^2}{8\pi^2} \int_0^{2\pi} \int_0^{\pi} \left[ \frac{\cos \left( \frac{\pi}{2} \cos \Omega \right) + \sin \left( \frac{\pi}{2} \cos \gamma \right)}{1 - \cos^2 \gamma} \cos \theta \cos \phi + \frac{\cos \left( \frac{\pi}{2} \cos \phi \right) + \sin \left( \frac{\pi}{2} \cos \phi \right)}{1 - \cos^2 \Omega} \cos \theta \sin \phi \right]^2 d\theta d\phi
\]

By using numerical techniques a value of

\[
P = 84.3193 I_0^2
\]
for free space has been calculated.
The radiation resistance is then found by (B-11), which yields

$$ R_{ri} = \frac{2P}{I_A^2} = \frac{2P}{(2I_0)^2} = \frac{2 \cdot 84.3193I_0^2}{4I_0^2} = 42.1097 \, \Omega $$

(7-11)

In (7-11) above we have chosen to define the current $I_A$ at the antenna input terminal point as the sum of the currents at all the feed points. By doing so we can make a fair comparison between different antenna systems later on.

The preceding calculations have been carried out for infinitely thin dipoles. It has also been assumed that the two dipoles don’t interact with each other and alter the currents compared to that if the dipoles were isolated. That is, mutual coupling has been neglected. This is because the dipoles are orthogonal and placed in such way that the interaction effects will be very small. Hence, the calculations can be simplified.

To verify our calculations for this antenna, we show some results obtained from the NEC antenna modeling program. Radiation patterns in the principal planes are shown in Fig. 7-3. The patterns are very identical to the ones in Fig. 7-2. Thus, the effects of mutual coupling and the fact that the dipoles no longer are infinitely thin have very little effect on the normalized field patterns.

![Figure 7-3](image)

Figure 7-3 Plane patterns for the two half-wave dipoles antenna obtained by the NEC program. Wire radii is $0.0005 \lambda$. 


The radiation resistance obtained from the NEC program is

\[
R_n = 56.3745 \, \Omega
\]  \hspace{1cm} (7-12)

The value in (7-12) differs significantly from the value in (7-11) and can be explained by the use of a finite thin wire in the NEC simulations enabling a more exact current distribution to be obtained. The dipoles also need to be separated by a fraction of a wavelength to avoid being treated as connected by the program.

The input impedance of the two dipoles obtained from NEC-2 is \(112.75 + 111.68 \, j \, \Omega\).

**The Four Dipoles Antenna**

The two dipoles antenna have as shown a fairly spherical radiation pattern. However, as also mentioned earlier one disadvantage is the antenna size. There will be much unused space. Therefore we also have investigated what happens if two additional half-wave dipoles are placed as in Fig. 8-1 to the two dipoles antenna configuration. The gaps in the corners are assumed to be very small and negligible. The dipoles are center-fed with the current distribution shown as solid gray curves. This means that the currents in dipole 1 and 2 and also in 3 and 4 are opposite in phase. The distributions have been chosen like this to achieve maximum possible radiation in the z-direction.

**Figure 8-1** Four half-wave dipoles.
Since the current distribution along a center-fed half-wave dipole of length L is assumed to be sinusoidal and can according to (5-2) be written as

\[ I(d) = I_0 \sin \left( \beta \left( \frac{L}{2} - |d| \right) \right) \quad |d| < \frac{L}{2} \]  

(8-1)

where d is the distance from the center of the dipole, the currents from Fig. 8-1 can, neglecting mutual coupling effects, be expressed as

\[ I_1 = I_2 = -\hat{x}I_0 \sin \left( \beta \left( \frac{\lambda}{4} - |x| \right) \right) = -\hat{x}I_0 \sin \left( \frac{\pi}{2} - \beta |x| \right) \quad ; \quad |x| < \frac{\lambda}{4} \]  

(8-2a)

\[ I_3 = I_4 = \hat{y}I_0 \sin \left( \beta \left( \frac{\lambda}{4} - |y| \right) \right) = \hat{y}I_0 \sin \left( \frac{\pi}{2} - \beta |y| \right) \quad ; \quad |y| < \frac{\lambda}{4} \]  

(8-2b)

The vectors from origin to positions on each side are

\[ r_1' = x\hat{x} - \frac{\lambda}{4} \hat{y} = x\hat{x} - \frac{\pi}{2\beta} \hat{y} \]  

(8-3a)

\[ r_2' = x\hat{x} + \frac{\lambda}{4} \hat{y} = x\hat{x} + \frac{\pi}{2\beta} \hat{y} \]  

(8-3b)

\[ r_3' = -\frac{\lambda}{4} \hat{x} + y\hat{y} = -\frac{\pi}{2\beta} \hat{x} + y\hat{y} \]  

(8-3c)

\[ r_4' = \frac{\lambda}{4} \hat{x} + y\hat{y} = \frac{\pi}{2\beta} \hat{x} + y\hat{y} \]  

(8-3d)

As before the next step is to calculate the vector potential

\[ A = \mu \frac{e^{-j\beta r}}{4\pi r} \int e^{j\beta \cdot \mathbf{r}} dl = \mu \frac{e^{-j\beta r}}{4\pi r} \left[ -\hat{x} \int_{-\lambda/4}^{\lambda/4} \sin \left( \frac{\pi}{2} - \beta |x| \right) e^{j\beta x \cos \theta \sin \phi} \left( e^{-j(\pi/2)\sin \theta \sin \phi} + e^{j(\pi/2)\sin \theta \cos \phi} \right) dx + \hat{y} \int_{-\lambda/4}^{\lambda/4} \sin \left( \frac{\pi}{2} - \beta |y| \right) e^{j\beta y \sin \theta \sin \phi} \left( e^{-j(\pi/2)\sin \theta \cos \phi} + e^{j(\pi/2)\sin \theta \cos \phi} \right) dy \right] \]  

(8-4)
By using the relation $e^{\pm j\alpha} = \cos \alpha \pm j \sin \alpha$, the substitutions $\beta x = x'$ and $\beta y = y'$ and the integral

$$\int \sin(a + bx)e^{cx} \, dx = \frac{e^{cx}}{b^2 + c^2} \left[ c \sin(a + bx) - b \cos(a + bx) \right]$$  \hspace{1cm} (8-5)$$

the evaluation of (8-4) will lead to

$$A = \mu \frac{e^{-j\beta}}{\pi \sigma} \frac{I_0}{\beta} \left[ -\cos \left( \frac{\pi}{2} \sin \theta \sin \phi \right) \cos \left( \frac{\pi}{2} \sin \theta \cos \phi \right) \cos \left( \frac{\pi}{2} \sin \theta \cos \phi \right) \cos \left( \frac{\pi}{2} \sin \theta \sin \phi \right) \right] + \frac{1}{1 - \sin^2 \theta \cos^2 \phi} \hat{x} + \frac{\cos \left( \frac{\pi}{2} \sin \theta \cos \phi \right) \cos \left( \frac{\pi}{2} \sin \theta \sin \phi \right)}{1 - \sin^2 \theta \sin^2 \phi} \hat{y}$$  \hspace{1cm} (8 - 6)$$

$$E_\theta = -j \omega \left( A_x \cos \theta \cos \phi + A_y \cos \theta \sin \phi \right) = j \omega \mu \frac{e^{-j\beta}}{\pi \sigma} \frac{I_0}{\beta} \left[ \cos \left( \frac{\pi}{2} \sin \theta \sin \phi \right) \cos \left( \frac{\pi}{2} \sin \theta \cos \phi \right) \cos \left( \frac{\pi}{2} \sin \theta \sin \phi \right) \cos \left( \frac{\pi}{2} \sin \theta \cos \phi \right) \right] \cdot \cos \theta \cos \phi - \frac{1}{1 - \sin^2 \theta \sin^2 \phi} \cdot \cos \theta \sin \phi = j \eta \frac{e^{-j\beta}}{\pi \sigma} I_0$$

$$= \frac{\cos \left( \frac{\pi}{2} \sin \theta \sin \phi \right) \cos \left( \frac{\pi}{2} \sin \theta \cos \phi \right) \cos \left( \frac{\pi}{2} \sin \theta \sin \phi \right) \cos \left( \frac{\pi}{2} \sin \theta \cos \phi \right)}{1 - \sin^2 \theta \cos^2 \phi} \cdot \cos \theta \cos \phi - \frac{1}{1 - \sin^2 \theta \sin^2 \phi} \cdot \cos \theta \sin \phi$$  \hspace{1cm} (8 - 7a)$$

We then calculate the far-zone electric field components.
\[ E_\phi = -j \omega (-A_x \sin \phi + A_y \cos \phi) = -j \omega \mu \frac{e^{-j\theta}}{\pi r} I_0 \begin{bmatrix} \cos \left( \frac{\pi}{2} \sin \theta \sin \phi \right) & \cos \left( \frac{\pi}{2} \sin \theta \cos \phi \right) \\ 1 - \sin^2 \theta \cos^2 \phi & 1 - \sin^2 \theta \sin^2 \phi \end{bmatrix} \cdot \sin \phi \]

\[ = -j \eta \frac{e^{-j\theta}}{\pi r} I_0 \begin{bmatrix} \cos \left( \frac{\pi}{2} \sin \theta \sin \phi \right) & \cos \left( \frac{\pi}{2} \sin \theta \cos \phi \right) \\ 1 - \sin^2 \theta \cos^2 \phi & 1 - \sin^2 \theta \sin^2 \phi \end{bmatrix} \cdot \sin \phi + \begin{bmatrix} \cos \left( \frac{\pi}{2} \sin \theta \cos \phi \right) \cos \left( \frac{\pi}{2} \sin \theta \sin \phi \right) \\ 1 - \sin^2 \theta \sin^2 \phi \end{bmatrix} \cdot \cos \phi \]

\[ (8 - 7b) \]

Radiation patterns for the principal planes have been plotted in normalized form in Fig. 8-2 by using the components in (8-7). In the xy-plane, that is the plane of the dipoles, the main beams will be directed diagonally in one direction and almost zero at the opposite diagonal. This is due to E-field cooperation and counteracts that arising from the chosen phase-relationship between the currents in the antennas.

![Figure 8-2](image-url)  
(a) Normalized field pattern magnitude in the xy-plane.  
(b) Normalized field pattern magnitude in the yz-plane.  
(c) Normalized field pattern magnitude in the yx-plane.

**Figure 8-2** Plane patterns for four half-wave dipoles organized as in Fig. 8-1.
How well will this antenna combination radiate? Again the radiated power is calculated using a computer from (B-9), yielding for free space

\[ P = \frac{41_0 I_0^2}{32\pi^2} \cdot 101.5322 \approx 121.1951 I_0^2 \] (8-8)

Since the four antennas are fed separately the sum of the current amplitudes at the feed points is \( 4I_0 \) and hence we end up with the radiation resistance

\[ R_n = \frac{2P}{I_0^2} = \frac{2P}{(4I_0)^2} \approx 15.1494 \, \Omega \] (8-9)

A comparison between this value and the value in (7-11) and between the radiation patterns for the two-and-four dipoles antennas, shows that the adding of two additional dipoles is detrimental to the performance. However, the effects of mutual coupling can be quite noticeable for this antenna configuration since there are two pairs of parallel dipoles. The value in (8-9) should therefore not be taken too seriously.

This antenna has also been processed by the NEC antenna modeling program. The radiation patterns obtained in the principal planes are shown in Fig. 8-3.

![Figure 8-3](image)

**Figure 8-3** Plane patterns for the four half-wave dipoles antenna obtained by the NEC program. Wire radius is 0.0005 \( \lambda \).
The radiation resistance acquired from the NEC program is

\[ R_{ri} = 16.4773 \, \Omega \]  

and confirm along with the patterns in Fig. 8-3 that the two dipoles antenna outperforms the four dipoles antenna.

**The Large Loop Antenna**

In the previous section we investigated what happens if four half-wave dipoles are organized as a square. We saw that the radiation resistance was poor. But if the dipoles are connected and form a loop with only one fed point, will the system do better?

Obviously if each side is \( \lambda/2 \) the current on two parallel sides will be in phase and therefore they will fully counteract on the x-and y-axis where there will be no radiation. On the z-axis there will however be full cooperation. We have chosen not to investigate this case, but instead a square loop with sides of length \( \lambda/4 \) since the size of the antenna is of some concern in this project. The one-wavelength square antenna is shown in Fig. 9-1. To simplify the calculations of the radiation properties the current distribution is assumed to be sinusoidal and is shown as gray solid curves in Fig.9-1.

![Figure 9-1](image)  
**Figure 9-1** A square loop antenna with length \( \lambda \).

The current can according to Fig. 9-1 be expressed as

\[
\begin{align*}
I_1 &= I_2 = -\hat{x}I_0 \cos(\beta x), \quad |x| \leq \lambda/8 \\
I_4 &= -I_3 = \hat{y}I_0 \sin(\beta y), \quad |y| \leq \lambda/8
\end{align*}
\]  

(9-1a)  

(9-1b)
The vectors from origin to positions on each side are

\[ \mathbf{r}_2' = x\mathbf{\hat{x}} + \frac{\lambda}{8}\mathbf{\hat{y}} \]  
(9-2a)

\[ \mathbf{r}_1' = x\mathbf{\hat{x}} - \frac{\lambda}{8}\mathbf{\hat{y}} \]  
(9-2b)

\[ \mathbf{r}_3' = -\frac{\lambda}{8}\mathbf{\hat{x}} + y\mathbf{\hat{y}} \]  
(9-2c)

\[ \mathbf{r}_4' = \frac{\lambda}{8}\mathbf{\hat{x}} + y\mathbf{\hat{y}} \]  
(9-2d)

The method used to find the radiation properties is the same as before, so first we calculate the vector potential.

\[ \mathbf{A} = \mu \frac{e^{-j\beta r}}{4\pi} \int I e^{j\beta r} \mathbf{r} dl = \mu \frac{e^{-j\beta r}}{4\pi} I_0 \left[ -\mathbf{\hat{x}} \int^\lambda_{-\lambda} \cos(\beta x) e^{j\beta x \sin \theta \cos \phi} \left( e^{-j(\pi/4)\sin \theta \sin \phi} + e^{j(\pi/4)\sin \theta \sin \phi} \right) dx 
+ \mathbf{\hat{y}} \int_{-\lambda/4}^{\lambda/4} \sin(\beta y) e^{j\beta y \sin \theta \cos \phi} \left( -e^{-j(\pi/4)\sin \theta \cos \phi} + e^{j(\pi/4)\sin \theta \cos \phi} \right) dy \right] \]  
(9-3)

By using the relation \( e^{\pm j\alpha} = \cos \alpha \pm j \sin \alpha \), the substitutions \( \beta x = x' \) and \( \beta y = y' \), the integral from (8-5) and the integral

\[ \int \cos(ax)e^{bx} = \frac{e^{bx}}{a^2 + b^2} (b \cos(ax) + a \sin(ax)) \]  
(9-4)

the evaluation of (9-3) will lead to

\[ \mathbf{A} = \mu \frac{e^{-j\beta r}}{\sqrt{2}\pi} \frac{I_0}{\beta} \left[ \mathbf{\hat{x}} \frac{\sin \left( \frac{\pi}{4} \sin \theta \sin \phi \right)}{1 - \sin^2 \theta \cos^2 \phi} \left( \sin \theta \cos \phi \sin \left( \frac{\pi}{4} \sin \theta \cos \phi \right) - \cos \left( \frac{\pi}{4} \sin \theta \cos \phi \right) \right) 
- \mathbf{\hat{y}} \frac{\sin \left( \frac{\pi}{4} \sin \theta \cos \phi \right)}{1 - \sin^2 \theta \sin^2 \phi} \left( \sin \theta \sin \phi \cos \left( \frac{\pi}{4} \sin \theta \sin \phi \right) - \sin \left( \frac{\pi}{4} \sin \theta \sin \phi \right) \right) \right] \]  
(9-5)
The computing of the far-zone electric field components yields

\[
E_\theta = -j\omega (A_x \cos \theta \cos \phi + A_y \cos \theta \sin \phi) = j\omega \mu \frac{e^{-j\phi} I_0}{\sqrt{2\pi r}} \cos \theta \left[ \frac{\sin \theta \sin \phi (\frac{\pi}{4} \sin \theta \cos \phi)}{1 - \sin^2 \theta \sin^2 \phi} \right]
\]

\[
\cdot \left[ \sin \theta \sin \phi \cos \left( \frac{\pi}{4} \sin \theta \sin \phi \right) - \sin \left( \frac{\pi}{4} \sin \theta \sin \phi \right) \right] - \frac{\cos \phi \cos \left( \frac{\pi}{4} \sin \theta \sin \phi \right)}{1 - \sin^2 \theta \cos^2 \phi} \left[ \sin \theta \cos \phi \sin \left( \frac{\pi}{4} \sin \theta \cos \phi \right) - \cos \left( \frac{\pi}{4} \sin \theta \cos \phi \right) \right] =
\]

\[
= j\eta e^{-j\phi} \frac{I_0}{\sqrt{2\pi r}} \cos \theta \left[ \frac{\sin \theta \sin \phi \cos \left( \frac{\pi}{4} \sin \theta \sin \phi \right)}{1 - \sin^2 \theta \sin^2 \phi} \right] - \frac{\cos \phi \cos \left( \frac{\pi}{4} \sin \theta \sin \phi \right)}{1 - \sin^2 \theta \cos^2 \phi} \left[ \sin \theta \cos \phi \sin \left( \frac{\pi}{4} \sin \theta \cos \phi \right) - \cos \left( \frac{\pi}{4} \sin \theta \cos \phi \right) \right]
\]

\[(9 - 6a)\]
\[
E_\phi = -j \omega \left( A_x \sin \phi + A_y \cos \phi \right) e^{-j \beta} \frac{I_0}{\sqrt{2\pi}} \left[ \sin \theta \cos \phi \sin \left( \frac{\pi}{4} \sin \theta \cos \phi \right) - \cos \left( \frac{\pi}{4} \sin \theta \cos \phi \right) \right]
\]

\[
+ \frac{\cos \phi \sin \left( \frac{\pi}{4} \sin \theta \cos \phi \right)}{1 - \sin^2 \phi \cos^2 \phi} \left[ \sin \theta \sin \phi \cos \left( \frac{\pi}{4} \sin \theta \sin \phi \right) - \sin \left( \frac{\pi}{4} \sin \theta \sin \phi \right) \right]
\]

\[
= j \eta e^{-j \beta} \frac{I_0}{\sqrt{2\pi}} \left[ \sin \phi \cos \left( \frac{\pi}{4} \sin \theta \sin \phi \right) \right]
\]

\[
+ \frac{\cos \phi \sin \left( \frac{\pi}{4} \sin \theta \cos \phi \right)}{1 - \sin^2 \phi \cos^2 \phi} \left[ \sin \theta \sin \phi \cos \left( \frac{\pi}{4} \sin \theta \sin \phi \right) - \sin \left( \frac{\pi}{4} \sin \theta \sin \phi \right) \right]
\]

\[\text{(9 - 6b)}\]

Radiation patterns for different planes are plotted in normalized form in Fig. 9-2 by using above components. From this we see that radiation is maximum along the z-axis and where it is polarized parallel to the x-axis.

Next we calculate the radiated power from (B-9) using computer assistance, yielding for free space

\[P = \eta_0 I_0^2 \approx 57.7379 I_0^2\]

\[\text{(9-7)}\]

The radiation resistance is then found by using (9-7) in (B-11), which gives

\[R_{ri} = \frac{2P}{I_A^2} = \frac{2P}{I_0^2} \approx 115.4758 \Omega\]

\[\text{(9-8)}\]
Since the current distribution was assumed to be sinusoidal the radiation patterns in Fig. 9-2 are not accurate but can be shown to agree very well to more exact solutions obtained with the NEC antenna modeling program. Radiation patterns in the principal planes produced from data by NEC-2 are shown in Fig. 9-3. In the xy-plane the polarization is linear, while it is elliptic in the xz-plane and yz-plane. A comparison between Fig. 9-2 and Fig. 9-3 shows that the plots look almost identical in the xz-plane and differ slightly in the two other planes. The radiation resistance obtained from NEC-2 is

$$R_{r/l} = 100.8211 \, \Omega$$  \hspace{1cm} (9-9)

The conclusion is that the large square loop antenna with side length $\lambda/4$ does not produce as attractive radiation pattern for the project as does the two dipoles antenna. The relatively small size of this loop antenna is however a very attractive property and the large loop antenna has a higher radiation resistance than the antennas investigated so far.
Figure 9-3  Plane patterns for the one-wavelength square loop antenna obtained by the NEC program. Wire radius is $0.0005\lambda$.

The Small Loop Antenna

For completeness, we have also investigated the small loop antenna. A small loop antenna is small in the sense that it is small compared to the wavelength. Its radiation field is independent of the shape of the loop and depends only on the area. Now consider a small loop antenna as in Fig. 10-1 with each side of length $l$. Since the small loop is small compared to the wavelength it is reasonable to assume that the current has constant amplitude $I$ and zero phase all around the antenna.

Figure 10-1 Small loop antenna.
With current as in Fig. 10-1 it can be expressed as

\[ I_1 = -I_2 = -j \hat{x} \]  
\[ I_4 = -I_3 = -j \hat{y} \]  \hspace{1cm} (10-1a)

\[ I_3 = -I_1 = j \hat{x} \]  \hspace{1cm} (10-1b)

The vectors from origin to positions on each side are

\[ \mathbf{r}_1' = x \hat{x} - \frac{l}{2} \hat{y} \]  \hspace{1cm} (10-2a)

\[ \mathbf{r}_2' = x \hat{x} + \frac{l}{2} \hat{y} \]  \hspace{1cm} (10-2b)

\[ \mathbf{r}_3' = -\frac{l}{2} \hat{x} + y \hat{y} \]  \hspace{1cm} (10-2c)

\[ \mathbf{r}_4' = \frac{l}{2} \hat{x} + y \hat{y} \]  \hspace{1cm} (10-2d)

Using the vector potential from (B-1) and the expansion of the unit vector for \( \mathbf{r} \) from (A-1) along with (10-1) and (10-2) gives

\[
\mathbf{A} = \mu \frac{e^{-j\beta'}}{4\pi r} \left[ \int_{-\frac{l}{2}}^{\frac{l}{2}} e^{j\beta' r} \, dl' = \mu \frac{e^{-j\beta'}}{4\pi r} \int_{-\frac{l}{2}}^{\frac{l}{2}} \left( e^{-j\frac{l}{2} \sin \theta \cos \phi} - e^{-j\frac{l}{2} \sin \theta \cos \phi} \right) \frac{dx}{j\beta' \sin \theta \cos \phi} \right]
\]

Now since \( l \) is small compared to the wavelength and \( \beta = \frac{2\pi}{\lambda} \)

\[
\sin \left( \frac{\beta l}{2} \right) \approx \frac{\beta l}{2} \]  \hspace{1cm} (10-4)

So (10-3) can be written as

\[
\mathbf{A} = \mu \frac{e^{-j\beta'}}{4\pi r} \left[ (\mathbf{y}j \beta S \sin \theta \sin \phi - \mathbf{y}j \beta S \sin \theta \cos \phi) \right] \]  \hspace{1cm} (10-5)

where \( S = l^2 \) is the loop area.
From (B-4) we can find the E-field vector.

\[
E = E_\theta \hat{\theta} + E_\phi \hat{\phi} = -j \omega A_\theta \hat{\theta} - j \omega A_\phi \hat{\phi} = -j \omega (A \cdot \hat{\theta}) \hat{\theta} - j \omega (A \cdot \hat{\phi}) \hat{\phi} \\
= -j \omega (A_x \hat{x} + A_y \hat{y} + A_z \hat{z}) - j \omega (A_x \hat{x} + A_y \hat{y} + A_z \hat{z})
\]

which gives by using the vector representations in appendix A and (10-5) the electric field components

\[
E_\theta = -j \omega (A_x \cos \theta \cos \phi + A_y \cos \theta \sin \phi) = \omega \mu \frac{e^{-j\beta r}}{4\pi r} I B S (\sin \theta \cos \phi - \sin \theta \cos \phi) 
\]

\[
E_\phi = -j \omega (-A_x \sin \phi + A_y \cos \phi) = -\omega \mu \frac{e^{-j\beta r}}{4\pi r} I B S \sin \theta 
\]

\[
= -\frac{\beta}{\sqrt{c \mu}} \frac{e^{-j\beta r}}{4\pi r} I B S \sin \theta = \sqrt{\mu / \varepsilon} \frac{e^{-j\beta r}}{4\pi r} I B^2 \sin \theta
\]

The radiation pattern is plotted in Fig. 10-2. Since the E-field vector is independent of \( \phi \), the complete radiation pattern can be visualized by rotating the pattern around the z-axis. As shown the antenna will not radiate in the direction of the axis normal to the loop plane and it has maximum radiation in the xy-plane. This is quite different from the large loop antenna, which directs a great amount of its radiated power in the direction of the normal to the loop plane.

![Figure 10-2 Radiation pattern for a small loop antenna.](image)

To calculate the radiated power we use (B-9), leading to

\[
P = \frac{1}{2\eta} \int \int |E|^2 r^2 \sin \theta d\theta d\phi = \frac{\eta S^2 \beta^4 I^2}{32\pi^2} \int_0^\pi \int_0^\pi \sin \theta d\theta d\phi = \frac{\eta S^2 \beta^4 I^2}{32\pi^2} \int_0^{2\pi} \int_0^4 \sin \theta d\theta d\phi = \frac{\eta S^2 \beta^4 I^2}{12\pi}
\]
For free space the impedance is approximately $120\pi$ and thus the radiated power is

$$P = 10S^2 \beta^4 I^2 = 160\pi^4 \frac{S^2 I^2}{\lambda^4}$$  \hspace{1cm} (10-9)

By using the radiated power the radiation resistance can be found, which with (B-3) yields

$$R_n = \frac{2P}{I^2} = 320\pi^4 \frac{S^2}{\lambda^4} \Omega$$ \hspace{1cm} (10-10)

For example, if we consider a square small loop antenna with each side of length $\lambda / 20$ the radiation resistance found from (10-10) is approximately $0.2 \Omega$. This is of course a very small value. By using multiple turns the radiation resistance can be increased significantly, but this will of course lead to a thicker antenna. Small loop antennas also have considerable ohmic resistance, which gives lower radiation efficiency (the ratio between the radiation resistance and the antenna resistance) compared to a dipole of equal total length.

**The Two Dipoles T-Shaped Antenna**

To satisfy our curiosity, we also investigated how two orthogonal center-fed half-wave dipoles, placed in a T-shape according to Fig. 11-1, will radiate.

![Figure 11-1](image-url)  \hspace{1cm} \text{Figure 11-1 Two orthogonal half-wave dipoles placed in a T-shape.}
The current distribution and position vectors for this antenna, according to Fig 11-1, are given by

\[ I_1 = y' \hat{y} I_0 \cos(\beta y'), \quad |y'| \leq \frac{\lambda}{4} \quad (11-1a) \]

\[ I_2 = -\hat{x} I_0 \cos(\beta x'), \quad |x'| \leq \frac{\lambda}{4} \quad (11-1a) \]

\[ r_1' = y' \hat{y} \quad (11-2a) \]

\[ r_2' = x' \hat{x} + \frac{\lambda}{4} \hat{y} \quad (11-2b) \]

Using the vector potential from (B-1) along with (11-1) and the expansion of the unit vector for \( r \) from (A-1) along with (11-2) gives

\[ A = \frac{\mu e^{-j \beta r}}{4\pi r} \int e^{j k r} dl' = \frac{\mu e^{-j \beta r}}{4\pi r} I_0 \left[ \hat{y} \int_{-\lambda/4}^{\lambda/4} \cos(\beta y') e^{j k r} dy' - \hat{x} \int_{-\lambda/4}^{\lambda/4} \cos(\beta x') e^{j k r} dx' \right] \quad (11-3) \]

\[ \hat{r} \cdot r_1' = y' \sin \theta \sin \phi \quad \hat{r} \cdot r_2' = x' \sin \theta \cos \phi + \frac{\lambda}{4} \sin \theta \sin \phi \quad (11-4) \]

That is

\[ A = \frac{\mu e^{-j \beta r}}{4\pi r} I_0 \left[ \hat{y} \int_{-\lambda/4}^{\lambda/4} \cos(\beta y') e^{j k r} \sin \theta \sin \phi \ dy' - \hat{x} \int_{-\lambda/4}^{\lambda/4} \cos(\beta x') e^{j k r} \sin \theta \cos \phi + j \frac{\pi}{2} \sin \theta \sin \phi \ dx' \right] \]

\[ = \frac{\mu e^{-j \beta r}}{4\pi r} I_0 \left[ \hat{y} \int_{-\lambda/4}^{\lambda/4} \cos(\beta y') e^{j k r} \sin \theta \sin \phi \ dy' - \hat{x} e^{j \frac{\pi}{2} \sin \theta \sin \phi} \int_{-\lambda/4}^{\lambda/4} \cos(\beta x') e^{j k r} \sin \theta \cos \phi \ dx' \right] \]

\[ = \frac{\mu e^{-j \beta r}}{4\pi r} I_0 \left[ \hat{y} \int_{-\lambda/4}^{\lambda/4} \frac{e^{j k r} \sin \theta \sin \phi}{(j \beta \sin \theta \sin \phi)^2 + \beta^2} \left( j \beta \sin \theta \sin \phi \ cos(\beta y') + \beta \sin(\beta y') \right) \ dy' - \hat{x} e^{j \frac{\pi}{2} \sin \theta \sin \phi} \int_{-\lambda/4}^{\lambda/4} \frac{e^{j k r} \sin \theta \cos \phi}{(j \beta \sin \theta \cos \phi)^2 + \beta^2} \left( j \beta \sin \theta \cos \phi \ cos(\beta x') + \beta \sin(\beta x') \right) \ dx' \right] \quad (11-5) \]

Putting \( \cos \gamma = \sin \theta \cos \phi \) and \( \cos \Omega = \sin \theta \sin \phi \) in (11-5), yields
\[ A = \frac{\mu e^{-j\beta r}}{4\pi r} - I_0 \left[ \hat{y} \left( \frac{e^{j\beta r} \cos \Omega}{(j\beta \cos \Omega)^2 + \beta^2} \left( j\beta \cos \Omega \cos (\beta r) + \beta \sin (\beta r) \right) \right)^{1/4} \right. \]
\[ - \hat{x} e^{-j\frac{\pi}{2} \cos \Omega} \left( \frac{e^{j\beta r} \cos \gamma}{(j\beta \cos \gamma)^2 + \beta^2} \left( j\beta \cos \gamma \cos (\beta r) + \beta \sin (\beta r) \right) \right)^{1/4} \]
\[ = \frac{\mu e^{-j\beta r}}{4\pi \beta} I_0 \left[ \frac{j\pi}{2 \cos \Omega} \left( j\cos \Omega \cos \left( \frac{\pi}{2} \right) + \sin \left( \frac{\pi}{2} \right) \right) \right. \]
\[ - \hat{x} e^{-j\frac{\pi}{2} \cos \gamma} \left( j\cos \gamma \cos \left( \frac{\pi}{2} \right) + \sin \left( \frac{\pi}{2} \right) \right) \]
\[ = \frac{\mu e^{-j\beta r}}{2\pi \beta} I_0 \left[ \frac{1}{1 - \cos^2 \Omega} \left( \cos \left( \frac{\pi}{2} \cos \Omega \right) - \hat{x} e^{-j\frac{\pi}{2} \cos \gamma} \cos \left( \frac{\pi}{2} \cos \gamma \right) \right) \right. \]
\[ (11-6) \]

The E-field is obtained from (B-4) and (11-6).

\[ E_\theta = -j \omega (A_x \cos \theta \cos \phi + A_y \cos \theta \sin \phi) \]
\[ = -j \omega e^{-j\beta r} \frac{1}{2\pi \beta} \left[ - I_0 \cos \theta \left( \frac{e^{j\beta r} \cos \gamma}{1 - \cos^2 \gamma} \cos \left( \frac{\pi}{2} \cos \gamma \right) \cos \phi + \frac{1}{1 - \cos^2 \Omega} \cos \left( \frac{\pi}{2} \cos \Omega \right) \sin \phi \right) \right. \]
\[ - \hat{x} e^{-j\frac{\pi}{2} \cos \gamma} \left( \frac{e^{j\beta r} \cos \gamma}{1 - \cos^2 \gamma} \cos \left( \frac{\pi}{2} \cos \gamma \right) \cos \phi + \frac{1}{1 - \cos^2 \Omega} \cos \left( \frac{\pi}{2} \cos \Omega \right) \sin \phi \right) \]
\[ (11-7a) \]
Field patterns acquired from (11-7) can be seen in Fig. 11-2.

\[
E_\phi = -j\omega(-A_\gamma \sin \phi + A_\gamma \cos \phi) \\
= -j \eta e^{-j\phi} \frac{e^{-j \theta}}{2\pi r} I_0 \left[ \frac{e^{j \theta \cos \Omega}}{1 - \cos^2 \gamma \cos \left(\frac{\pi}{2} \cos \gamma\right) \sin \phi + \frac{1}{1 - \cos^2 \gamma} \cos \left(\frac{\pi}{2} \cos \Omega\right) \cos \phi} \right]
\]

\[(11-7b)\]

Figure 11-2 Plane patterns for the two half-wave dipoles T-shaped antenna from Fig. 11-1.
The radiated power is obtained from (B-9) and (11-7).

\[
P = \frac{1}{2\eta} \int \int \left( |E_\phi|^2 + |E_\psi|^2 \right) r^2 d\Omega
\]

\[
= \frac{\eta I_0^2}{8\pi^2} \int_0^{2\pi} \int_0^{\pi} \left[ \frac{e^{jz \cos \Omega}}{1 - \cos^2 \gamma} \cos \left( \frac{\pi}{2} \cos \gamma \right) \cos \phi + \frac{1}{1 - \cos^2 \Omega} \cos \left( \frac{\pi}{2} \cos \Omega \right) \sin \phi \right]^2 \sin \theta \, d\theta \, d\phi
\]

\[
= \left[ \frac{e^{jz \cos \Omega}}{1 - \cos^2 \gamma} \cos \left( \frac{\pi}{2} \cos \gamma \right) \sin \phi + \frac{1}{1 - \cos^2 \Omega} \cos \left( \frac{\pi}{2} \cos \Omega \right) \cos \phi \right]^2 \sin \theta \, d\theta \, d\phi
\]

(11 - 8)

By using numerical techniques to solve (11-8) for free space, we find

\[
P = 73.1309 I_0^2
\]

(11-9)

The radiation resistance is then found by (B-11), which with (11-9) yields

\[
R_a = \frac{2P}{I_0^2} = \frac{2P}{(2I_0)^2} = \frac{2 \cdot 73.1309 I_0^2}{4I_0^2} = 36.5655 \Omega
\]

(11-10)

Since the two dipoles are placed in a T-shape, they will obviously affect each other. More accurate radiation patterns with respect to this are shown in the principal planes in Fig. 11-3. These patterns have been obtained from NEC-2 calculations. The radiation resistance obtained from NEC-2 is

\[
R_a = 43.7891 \Omega
\]

(11-11)

From Fig. 11-2 and Fig. 11-3 it’s evident that this particular antenna will perform worse than the two dipoles antenna investigated in section 7.
The Yagi-Uda Antenna

Figure 11-3 Plane patterns for the two dipoles T-shaped antenna obtained by the NEC program. Wire radius is 0.0005 $\lambda$.

Another important and commonly used antenna is the Yagi-Uda antenna (Fig. 12-1). This antenna consists of one dipole called a driver element and several non-connected parallel elements called parasitic elements. The parasitic elements are divided into two classes...
depending on their length compared to the driver element. If the parasitic element is longer than the driver element it is called a reflector element and it acts mainly as a reflector for incident radiowaves and there is therefore usually just one reflector element.

If it on the other hand is shorter than the driver element it is called a director element and will direct the radiation from the driver. The spacing of the elements depends on the directivity required for the antenna and maximum directivity is obtained at about 0.15-0.25 $\lambda$. The typical length of the driver element is at its resonant frequency just below $\lambda/2$. The director elements are typically 10-20% shorter than the driver element and the reflector element usually have the length of $\lambda/2$. Increased directivity, and with that increased gain, is also achieved for every director element added but there is a smaller increase in directivity for each element added so no more than 5-6 director elements are usually needed to obtain maximum directivity. The radiation pattern of a simple three-element Yagi-antenna is shown in Fig 12-2.

![Figure 12-2 Radiation pattern for the three-element Yagi-Uda Antenna.](image)

If we are to use a Yagi-Uda antenna in our project we would like to have quite a wide radiation pattern so the element spacing would probably be of the order of 0.04$\lambda$ and because of the directivity we would probably want to use several antennas placed uniformly around the circuit.

A proposal for how this arrangement is shown in Fig 12-3 where we have placed four antenna arrays uniformly in the xz-plane.
Figure 12-3 An array of four Yagi-Uda antennas.

The radiation pattern in the xz-plane will be quite circular around the y-axis. But the main disadvantage of this construction for our purpose is of course that if the antennas are placed in the xz-plane the radiation pattern along the y-axis will be approximately zero and therefore we conclude that ordinary Yagi-Uda antennas will not be appropriate for this project.

Radar Cross Section of the Two Dipoles Antenna

As shown previously, the two dipoles antenna give an almost isotropic power pattern. But in order to have a passive tag, it is suggested that backscatter modulation is used. Thus the antenna’s radar cross section, RCS, is of significant importance. RCS can be seen as a measure as to how well a target re-radiates incident power. To achieve a great read range the RCS should be as large as possible. There must also be a way to change the RCS of a tag in order to modulate an answer. In this section we investigate the monostatic RCS of the two dipoles antenna and if it can be improved by adding a thin conducting plate to the configuration. The calculations have been done with the NEC antenna modeling program.

The antenna system is shown in Fig. 13-1. The plate is shown as a gray square.

The placement of the half-wave dipoles, with wire radius 0.0005\(\lambda\), are specified by (x, y, z) coordinates for wire ends:

Dipole 1, end 1: (-0.25\(\lambda\), 0.253\(\lambda\), 0)
Dipole 1, end 2: (0.25\(\lambda\), 0.253\(\lambda\), 0)
Dipole 2, end 1: (0.253\(\lambda\), .0.25\(\lambda\), 0)
Dipole 2, end 2: (0.253\(\lambda\), 0.25\(\lambda\), 0)

The coordinates for the corners specify the placement of plate:
Corner 1: \((-0.25\lambda, -0.25\lambda, 0)\)
Corner 2: \((0.247\lambda, -0.25\lambda, 0)\)
Corner 3: \((0.247\lambda, 0.247\lambda, 0)\)

Figure 13-1  Two half-wave dipoles and a conducting plate.

The simulations are done by allowing the incident plane wave be right-hand circular polarized. This is to avoid a complete polarization mismatch which arises for certain incidence angles when a linear polarized incident plane wave is used. A left-hand circular polarized incident plane wave would of course give similar results. Further, the dipoles have a short circuit at the feed (straight wire). The results are shown in Fig. 13-2 where the monostatic RCS:s for the cases with and without a plate are plotted versus the angle of
(a) xy-plane

(b) xz-plane
As one might expect the RCS increases in the principal planes when a conductive plate is added. Due to symmetry the plots in the xz-plane and the yz-plane look the same and it is in these two planes the plate will be of most use. Calculations show a 32% increase of the overall RCS compared to the configuration with no plate. In the xy-plane the benefits of adding a plate will not be so drastic. Here the overall RCS will increase by approximately 11% compared to the configuration with no plate.

One disadvantage of the two-dipole configuration is that the scattered E-fields will be, depending on the incidence angle of the incident plane wave, right-hand elliptic polarized or left-hand elliptic polarized. So to avoid a total polarization mismatch, a rather complex receiver is needed.

In this section it has been shown that adding a conductive plate can increase the overall monostatic RCS in the principal planes of the two-dipole configuration. But in order to use backscatter modulation with this particular antenna system, one must first investigate whether changes to the dipoles input impedance (or some other way to change the RCS) will give a consistent increase/decrease of the antenna system’s RCS throughout all incidence angles of the incident plane wave.

**Figure 13-2** Monostatic RCS versus incidence angle for the two dipole configuration.
Conclusions and experiences

RFID has become very popular over the last few years. The technique is not yet ready to conquer the consumer market on grand scale. To really get a market for paperbased RFID tags, progress in the area of polymer technique is required. Today reasonable resistances, inductances and capacitances can be printed directly on paper but until semiconductors can be printed, paperbased RFID will be very rare. Solutions based on silicon can of course be used but can no way compete with polymer techniques which are non-polluting, cheaper and not sensitive to bending.

The equipment from SCS Inc. which is in the market now, is probably the product, that is closest to our intended applications. However, the technique is still very primitive, like short reading distance, simple microprocessor and is also expensive.

Of the investigated antennas, the two dipoles antenna has caught our attention. It has a simple construction, a decent radiation pattern and high radiation resistance. If higher antenna input impedance is preferred, folded dipoles can be used instead. However, our way of defining the current at the antenna input doesn’t necessarily result in a fair comparison between the radiation resistance between the different antennas we investigated. Interesting investigations would be to look at the RCS when backscatter modulation techniques are applied and then do an analysis of the resulting spectrum.

We have also seen that by adding a conducting plate to the two dipoles antenna the overall RCS can be slightly increased in the major planes. This leaves many unanswered questions such as how to achieve modulation.

The verifying of the two dipoles antenna with the NEC antenna modeling program shows that mutual coupling effects have very little influence on the normalized radiation patterns. Mutual coupling effects do however affect the radiation resistance.

The task has been very challenging. RFID was a completely new area for us at the beginning of this thesis work. Our knowledge in antenna theory was also very limited. This has resulted in laborious but interesting and stimulating work and has encouraged us to continue our research in this field.
References

Book references

Paper references

Internet reference sites
Appendix A

Unit vectors

Figure A-1 Definitions.

\[ \mathbf{r} = \hat{x} \sin \theta \cos \phi + \hat{y} \sin \theta \sin \phi + \hat{z} \cos \theta \quad (A-1) \]
\[ \mathbf{\hat{r}} = \hat{x} \cos \theta \cos \phi + \hat{y} \cos \theta \sin \phi - \hat{z} \sin \theta \quad (A-2) \]
\[ \mathbf{\hat{\phi}} = -\hat{x} \sin \phi + \hat{y} \cos \phi \quad (A-3) \]

\[ \hat{x} = \hat{r} \sin \theta \cos \phi + \hat{\theta} \cos \theta \cos \phi - \hat{\phi} \sin \phi \quad (A-4) \]
\[ \hat{y} = \hat{r} \cos \theta \cos \phi + \hat{\theta} \cos \theta \sin \phi - \hat{\phi} \cos \phi \quad (A-5) \]
\[ \hat{z} = \hat{r} \cos \theta - \hat{\theta} \sin \theta \quad (A-6) \]
Appendix B

Relationships and Equations used

If assuming parallel ray approximation the vector potential can be found from

\[ A = \mu e^{-j\beta r} \frac{1}{4\pi} \iint_{\Omega} J e^{j\beta r'} dr' \]  \hspace{1cm} (B-1)

where \( \mu \) is the permeability of the medium, \( r \) is the distance from origin of a selected coordinate system to a field point, \( \beta \) is the phase constant, \( J \) is the current density vector and \( r' \) is the vector from origin to a source of volume \( \Omega \).

The permeability is related to the intrinsic impedance of the medium \( \eta \) by

\[ \eta = \sqrt{\frac{\mu}{\varepsilon}} \]  \hspace{1cm} (B-2)

where \( \varepsilon \) is the medium’s permittivity.

The phase constant is related to the permeability and permittivity by

\[ \beta = \omega \sqrt{\mu \varepsilon} = \frac{2\pi}{\lambda} \]  \hspace{1cm} (B-3)

where \( \omega \) is the radian frequency.

The E-field vector can be found from the vector potential by

\[ E = -j\omega (A_\theta \hat{\theta} + A_\phi \hat{\phi}) \]  \hspace{1cm} (B-4)

\[ H = \frac{1}{\eta} \hat{r} \times E \]  \hspace{1cm} (B-5)

The power radiated by an antenna can be calculated from the surface integral

\[ P = \frac{1}{2} \Re \iint (E \times H^*) \cdot ds \]  \hspace{1cm} (B-6)

where \( ds = d\Omega \hat{n} \) and \( \hat{n} \) is the unit normal to the surface and the power density or Poynting vector is

\[ S = \frac{1}{2} E \times H^* \]  \hspace{1cm} (B-4)
Appendix B

From (B-5)

\[ H_\phi = \frac{E_\phi}{\eta} \]
\[ H_\theta = -\frac{E_\theta}{\eta} \]  

(B-8)

which with (B-6) gives

\[ P = \frac{1}{2\eta} \int \int_0^{2\pi} (|E_\phi|^2 + |E_\theta|^2) r^2 \sin \theta d\theta d\phi \]  

(B-9)

where \( \sin \theta d\theta d\phi \) is the element of solid angle, \( \Omega \), with unit square radian or steradian, sr.

From the radiated power, the radiation resistance \( R_r \) is usually defined as

\[ R_r = \frac{2P}{I_m^2} \]  

(B-10)

where \( I_m \) is the current distribution maximum, which doesn’t have to occur on the antenna.

\( R_r \) is related to the radiation resistance at the antenna input terminal point, \( R_{ri} \), by

\[ P = \frac{I_m^2 R_r}{2} = \frac{I_{ri}^2 R_{ri}}{2} \]  

(B-11)

where \( I_{ri} \) is the current distribution at the antenna input terminal point. If ohmic losses are neglected, \( R_{ri} \) equals the total antenna input resistance \( R_A \).
Appendix C

Integrals Used

\[ \int \sin(a + bx)e^{cx} \, dx = \frac{e^{cx}}{b^2 + c^2} \left[ c \sin(a + bx) - b \cos(a + bx) \right] \quad (C-1) \]

\[ \int \cos(a + bx)e^{cx} \, dx = \frac{e^{cx}}{b^2 + c^2} \left[ c \cos(a + bx) + b \sin(a + bx) \right] \quad (C-2) \]

\[ Cin(x) = \int_0^x \frac{1 - \cos \tau}{\tau} \, d\tau \quad (C-3) \]

\[ Cin(x) = 0.5772 + \ln(x) - Ci(x) \quad (C-4) \]
## Symbols and Constants

### The different symbols

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<td>Wavelength</td>
<td>Meter</td>
<td>m</td>
</tr>
<tr>
<td>γ</td>
<td>Propagation constant</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>α</td>
<td>Attenuation constant</td>
<td>Neper/meter</td>
<td>Np/m</td>
</tr>
<tr>
<td>β</td>
<td>Phase constant</td>
<td>Radians/meter</td>
<td>rad/m</td>
</tr>
<tr>
<td>u₀</td>
<td>Propagation velocity</td>
<td>Meter/second</td>
<td>m/s</td>
</tr>
<tr>
<td>p</td>
<td>Electric dipole moment</td>
<td>Coulombmeter</td>
<td>Cs</td>
</tr>
<tr>
<td>m</td>
<td>Magnetic dipole moment</td>
<td>Amperesquaremeter</td>
<td>Am^2</td>
</tr>
<tr>
<td>Q</td>
<td>Qualityfactor</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>L</td>
<td>Length of the antenna</td>
<td>Meter</td>
<td>m</td>
</tr>
<tr>
<td>D, G₀</td>
<td>Directive gain</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>G₀</td>
<td>Power gain</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>η₀</td>
<td>Radiation efficiency</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>φ</td>
<td>Angle in horizontal plane relative to the x-axis</td>
<td>Radians</td>
<td>Rad</td>
</tr>
<tr>
<td>θ</td>
<td>Angle in horizontal plane relative to the z-axis</td>
<td>Radians</td>
<td>Rad</td>
</tr>
</tbody>
</table>
## Appendix D

### Constants

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Value</th>
<th>Unit</th>
<th>Unit shortening</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c$</td>
<td>Speed of light</td>
<td>$3 \cdot 10^8$</td>
<td>Meter/second</td>
<td>m/s</td>
</tr>
<tr>
<td>$\mu_0$</td>
<td>Permeability constant</td>
<td>$4\pi \cdot 10^{-7}$</td>
<td>Voltsecond/amperemeter</td>
<td>Vs/Am</td>
</tr>
<tr>
<td>$\varepsilon_0$</td>
<td>Permittivity constant</td>
<td>$8.85 \cdot 10^{-12}$</td>
<td>Amperesecond/voltmeter</td>
<td>As/Vm</td>
</tr>
</tbody>
</table>
# Specifications for RFID Equipment from SCS Inc.

## Scanner Specification Version S416

<table>
<thead>
<tr>
<th>Specification</th>
<th>Details</th>
</tr>
</thead>
</table>
| **Nominal Physical Dimensions:**                  | Length: 13 in (33 cm)  
Width: 7 in (18 cm)  
Height: 4.5 in (11 cm)  
Weight: 4.5 lbs. (2 kg) |
| **Operating Temperature:**                         | 32°F - 122°F (0°C - 50°C) (non-condensing)                              |
| **Power Requirements:**                            | Internal Power Supply, External Cable provided  
85-264 VAC, 47-63 Hz, less than 0.5 Amps                                     |
| **External Environment:**                          | Protection from relative moisture and humidity                          |
| **Certification:**                                 | FCC Part 15                                                             |
| **Serial Interface:**                              | 9-pin Female RS-232 connector                                           |
| **Antenna Ports:**                                 | 6                                                                      |
| **Application Program Interface:**                 | Allows interface with host applications. Standard C interface; currently supports DOS or Windows applications. Facilitates optimized label read and write operations and provides macros for frequently used command scripts |

## System Specifications

<table>
<thead>
<tr>
<th>Specification</th>
<th>Details</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>RF Characteristics:</strong></td>
<td>ISM band (no license required) 2.400 - 2.483 GHz @ +28 dBm (0.7 W), using frequency-hopping spread spectrum (FHSS)</td>
</tr>
<tr>
<td><strong>Read Distance:</strong></td>
<td>Nominal 14 inches (35 cm)*</td>
</tr>
<tr>
<td><strong>Write Distance:</strong></td>
<td>Nominal 1 inch (2.5 cm)*</td>
</tr>
<tr>
<td><strong>Host Interface:</strong></td>
<td>RS-232 at up to 19,200 baud</td>
</tr>
</tbody>
</table>

*Obstructing metals, other conducting material and label orientation may significantly affect read and write distance.

## Protocol Specifications

<table>
<thead>
<tr>
<th>Specification</th>
<th>Read</th>
<th>Write</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Word (16 bits)</td>
<td>10ms</td>
<td>200ms</td>
</tr>
<tr>
<td>Unique Serial Code (32 bits)</td>
<td>20ms</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(preprogrammed)</td>
</tr>
</tbody>
</table>

Multiple label identification (per label, 2 words): 84 ms
Appendix E

**DL-1000 Label Specifications**

The DL-1000 Dura-label consists of a special laminate material designed to withstand even the most demanding conditions. Each label comes with a unique serial code and 928 user programmable bits. Utilizing the proprietary (Interactive Identification) protocol and an SCS InstaScan scanner, multiple labels can be identified simultaneously and within seconds.

<table>
<thead>
<tr>
<th>RF Characteristics:</th>
<th>ISM band (no license required) 2.45 GHz Passive; RF Backscatter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Read Distance:</td>
<td>Nominal 14 inches (35 cm)*</td>
</tr>
<tr>
<td>Average Read Times:</td>
<td>1 word (16 bits) in less than 10 ms</td>
</tr>
<tr>
<td>Write Distance:</td>
<td>Nominal 1 inch (2.5 cm)*</td>
</tr>
<tr>
<td>Average Write Times:</td>
<td>1 word (16 bits) in less than 200ms</td>
</tr>
<tr>
<td>Memory:</td>
<td>Reserved – 1 Word (16 bits) Unique Serial Code - 5 Words (16 bits each) User Defined – 58 Word (16 bits each) (User Programmable)</td>
</tr>
<tr>
<td>Physical Characteristics:</td>
<td>Flexible substrate. Length: 2.4 in (6cm) Width: 0.4 in (1 cm) Thickness: .03 in (.8 mm)</td>
</tr>
<tr>
<td>Operating Temperature:</td>
<td>-5° to 149° F (-20° to 65° C)</td>
</tr>
<tr>
<td>Storage Temperature:</td>
<td>-22° to 167° F (-30° to 75° C)</td>
</tr>
</tbody>
</table>

*Obstructing metals, other conducting materials, and label orientation may significantly affect read distance. Read/Write distance measured using a FCC Part 15 certified InstaScan scanner.