Power Control with Time Delay Compensation

Fredrik Gunnarsson, Fredrik Gustafsson

Division of Communications Systems
Department of Electrical Engineering
Linköpings universitet, SE-581 83 Linköping, Sweden
WWW: http://www.comsys.isy.liu.se
Email: fred@isy.liu.se, fredrik@isy.liu.se

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AUTOMATIC CONTROL

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Abstract

Closed-loop power control is considered as an important component in the management of radio resources in cellular radio systems. The algorithms are typically based on feedback information, which for practical reasons is outdated. These time delays in the system, hamper the performance and might even result in unstable systems. Several power control strategies have been proposed in order to improve the capacity of cellular radio systems, but time delays are usually neglected. Here, time delay compensation is introduced as a means to improve the dynamical behavior of power controlled cellular systems, despite time delays. The improvements are validated both in theory with respect to global convergence and stability and in some illuminating simulations.

Keywords: Cellular radio systems; Power control; Time delays; Time delay compensation; Convergence analysis; Stability

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Fredrik Gunnarsson, Fredrik Gustafsson
Div. of Communications Systems
Dept. of EE, Linköping University
SE-581 83 Linköping, SWEDEN
http://www.comsys.isy.liu.se
fred, fredrik@isy.liu.se

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Closed-loop power control is considered as an important component in the management of radio resources in cellular radio systems. The algorithms are typically based on feedback information, which for practical reasons is outdated. These time delays in the system, hamper the performance and might even result in unstable systems. Several power control strategies have been proposed in order to improve the capacity of cellular radio systems, but time delays are usually neglected. Here, time delay compensation is introduced as a means to improve the dynamical behavior of power controlled cellular systems, despite time delays. The improvements are validated both in theory with respect to global convergence and stability and in some illuminating simulations.

1 Introduction

While the demand for access to services in wireless communications systems is exponentially growing, an increased interest in utilizing the available resources efficiently can be observed. A consequence of the limited availability of radio resources is that the users have to share these resources. Power control is seen as an important means to reduce mutual interference between the users, while compensating for time-varying propagation conditions.

The powers are controlled using feedback, and feedback results in a dynamical behavior that critically affect the performance. Among the most troublesome are effects from using outdated feedback information, resulting in time delays in the control loops. The objective with this paper is to develop methods for eliminating or at least reducing these effects, and to study stability and convergence properties.

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Power control has been an area subject to extensive research in recent years. Some surveys of previous work include [14, 12, 7].

If full information of the propagation conditions between mobiles and base station are known, the transmitter powers of every transmitter could be computed in a centralized fashion. Aein [1] focused on satellite communications systems, and introduced the term *SIR balancing*, for a power control strategy aiming at the same *signal-to-interference ratio (SIR)* at every receiver. Zander [19] interpreted these algorithms as "optimal solutions" in the sense that there exist no other transmission powers yielding a higher SIR for *all* receivers (i.e., no higher balanced SIR). If such transmission powers exist providing the required SIR:s at the receivers, the power control problem is said to be *feasible*. In the former paper, ideal receivers are assumed. When the receivers are non-ideal, a fraction of the transmitted power cannot be utilized by the connected receiver. This fraction is instead experienced as interference – *auto-interference* [5], resulting in a smaller optimal balanced SIR.

To actually implement a centralized power control solution is not plausible in practice due to the signaling overhead. Instead, such schemes serve as performance bounds, to implementationally appealing distributed solutions. These include the *Distributed Power Control (DPC)* algorithm [4], which converge to the centralized solution if the power control problem is feasible. Other important decentralized proposals include [6, 15, 18, 2, 17] aiming at different perspectives of power control, such as constrained power levels, fixed-step power updates, measurement related issues and problems when the power control problem is infeasible.

These algorithms perform well, when relying on the latest measurements. However, when subject to outdated measurements, stability problems and oscillatory behaviors can be observed [8]. In that analysis, the distributed power control algorithms are seen as interacting local control loops, providing a framework in which the time delays are naturally expressed. It is argued that stability of these local loops is a necessary condition for stability of the overall, global system. A natural design approach is therefore to optimize local performance with respect to stability, see e.g., [11], where linear design methods are used.

Global properties of the distributed power control algorithms, such as stability and convergence, are therefore of upmost importance when subject to time delays. Yates [18] provides a framework for analyzing convergence, but it is restricted to situations when the latest information is available (i.e., no time delays). Therefore, it is necessary to extend the convergence analysis to also consider time delays. Convergence properties for the specific case of fixed-step power updates subject to time delays is in focus in [13].

Primarily, time delays hamper the performance in two different ways

1. Delayed reactions to changes in external disturbances.

2. Internal dynamics of the power control loops.

In this paper, the focus is on the dynamical behavior and stability issues (second item above). The first item (delayed reactions) is addressed in [16, 7].

Based on the system model and notation in Section 2, *Time Delay Compensation (TDC)* is introduced in Section 3 as a means to reduce effects of time delays. Theoretical stability results considering time delays are presented in Section 4 were the benefits of TDC are founded in theory. The dynamical behavior of power controlled cellular systems with and without TDC are illuminated by simulations in Section 5, followed by some concluding remarks in Section 6.

2 System Model

To emphasize that the discussion applies to both the up- and downlink, we consider a system of m transmitters and m receivers. In an uplink situation, the transmitters and the active mobile stations are equivalent, while the base stations are seen as equipped with a number of receivers – one per connected mobile station, and vice versa in the downlink. Thereby, there is a on-to-one correspondence between transmitters and the connected receivers. The base station assignments are assumed fixed over the time frame of the analysis, which is natural, since updates are much more infrequent than power level updates.

2.1 Notation

Most quantities in this paper can be expressed using either logarithmic (e.g. *dB* or *dBm*) or linear scale. To avoid confusion we will employ the convention of indicating linearly scaled values with a bar. Thus $\bar{g}_{ij}$ is a value in linear scale and g_{ij} the corresponding value in logarithmic scale.

Assume that the m transmitters are transmitting using the powers $\bar{p}_i(t)$, $i = 1, \dots, m$. The signal between transmitter i and receiver j is attenuated by the power gain $\bar{g}_{ij}(t)$ (< 1). Thus the corresponding connected receiver will experience a desired signal power $\bar{C}_i(t) = \bar{p}_i(t)\bar{g}_{ii}(t)$ and an interference from other connections plus noise $\bar{I}_i(t)$. The *signal-to-interference ratio (SIR)* at receiver i can be defined by

$$\bar{\gamma}_i(t) = \frac{\bar{C}_i(t)}{\bar{I}_i(t)} = \frac{\bar{g}_{ii}(t)\bar{p}_i(t)}{\sum_{j\neq i}\bar{\theta}_{ij}\bar{g}_{ij}(t)\bar{p}_j(t) + \bar{\nu}_i(t)}, \qquad (1)$$

where $\bar{\theta}_{ij}$ is the normalized cross-correlation between the waveforms of user i and j, and $\bar{\nu}_i(t)$ is thermal noise. In a system with orthogonal signals, e.g., a FDMA or a TDMA system, $\bar{\theta}_{ij} \in \{0, 1\}$, while in a system with non-orthogonal signals, e.g., uplink DS-CDMA, $\bar{\theta}_{ij} \in [0, 1]$. To keep the notation clear, we define $\bar{\theta}_{ii} = 1$, and redefine the power gain to incorporate the cross-correlations:

$$\bar{g}_{ij} := \bar{\theta}_{ij}\bar{g}_{ij}.$$

Depending on the receiver design, propagation conditions and the distance to the transmitter, the receiver is differently successful in utilizing the available desired signal power $\bar{p}_i\bar{g}_{ii}$. Assume that receiver i can utilize the fraction $\bar{\delta}_i(t)$ of the desired signal power. Then the remainder $\left(1 - \bar{\delta}_i(t)\right)\bar{p}_i\bar{g}_{ii}$ acts as interference, denoted *auto-interference* [5]. We will assume that the receiver efficiency changes slowly, and therefore can be considered constant. Hence, the SIR expression in Equation (1) transforms to

$$\bar{\gamma}_i(t) = \frac{\bar{\delta}_i\bar{g}_{ii}(t)\bar{p}_i(t)}{\sum_{j\neq i}\bar{g}_{ij}(t)\bar{p}_j(t) + \left(1 - \bar{\delta}_i\right)\bar{p}_i(t)\bar{g}_{ii}(t) + \bar{\nu}_i(t)}. \qquad (2)$$

From now on, this quantity will be referred to as SIR. For efficient receivers, $\bar{\delta}_i = 1$, and the expressions (1) and (2) are equal. In logarithmic scale, the SIR expression becomes

$$\gamma_i(t) = p_i(t) + \delta_i + g_{ii}(t) - I_i(t). \qquad (3)$$

We will only discuss the *Quality of Service* (QoS) in terms of SIR. The individual quality objectives at each receiver i are assumed expressed as target SIR:s $\bar{\gamma}_i^t(t)$, possibly reconsidered regularly by outer control loops [7]. The outer loop update rate is typically orders of magnitude slower, and the target SIR:s will therefore be considered constant.

2.2 Power Control Algorithms

The distributed power control algorithms are based on local feedback information. Different protocols may be used to feed back these measurements. We will consider two important feedback situations:

- **Information feedback.** The mobile feeds back the exact SIR measurements or the error

$$e_i(t) = \gamma_i^t - \gamma_i(t), \ \left(\bar{e}_i(t) = \frac{\bar{\gamma}_i^t)}{\bar{\gamma}_i(t)}\right). \qquad (4)$$

The important characterization is essentially that real numbers are fed back.

- **Decision feedback.** In a power control setting, we typically associate this with feedback of the sign of the error in (4)

$$s_i(t) = \text{sign}(\gamma_i^t - \gamma_i(t)) = \text{sign}(e_i(t)) = \text{sign}(\bar{e}_i(t)) \quad (5)$$

  Thus, only one bit is needed for command signaling, which makes the scheme bandwidth efficient.

Initially, we will consider a simple integrator as control algorithm. Depending on feedback assumptions, the powers will thus be controlled as

$$\text{Information feedback:} \quad p_i(t+1) = p_i(t) + \beta e_i(t) \quad \text{(6a)}$$

$$\text{Decision feedback:} \quad p_i(t+1) = p_i(t) + \beta s_i(t) \quad \text{(6b)}$$

where $\beta > 0$. Yet simple, these two algorithms include most of the proposed algorithms to date. For example information feedback and $\beta = 1$ yield the DPC algorithm [4] (in linear scale)

$$\bar{p}_i(t+1) = \bar{p}_i(t)\frac{\bar{\gamma}_i^t}{\bar{\gamma}_i(t)}. \quad (7)$$

Furthermore, decision feedback in 6b yield the *Fixed-Step Power Control (FSPC)* algorithm discussed in [15] with step-size β dB

$$\begin{aligned} s_i(t) &= \text{sign}(\gamma_i^t(t) - \gamma_i(t)) \\ p_i(t+1) &= p_i(t) + \beta s_i(t). \end{aligned} \quad (8)$$

The actual time between consecutive power updates, the *sample interval* T_s, varies from systems to system. For example $T_s = 0.48$ s in GSM and $T_s = 1/1500$ s in WCDMA. To avoid confusion, we define the sample instants t as the power level updates in the transmitters. Seemingly, this notation is equal to the assumption of synchronous updates, but the only needed assumption is that all transmitters update their power levels within the time frame of one sample interval.

As seen above, the controller itself contains a delay of one sample interval. However, both measuring and control signaling take time, resulting in additional time delays in the system. A relevant model is that the measured SIR available to the algorithm at time instant t, depends on the power level at $t-1$. This is typically the situation in GSM and WCDMA. The SIR at receiver i is thus given by

$$\gamma_i(t) = p_i(t-1) + \delta_i + g_{ii}(t) - I_i(t). \quad (9)$$

In general, time delays are primarily of two kinds. Firstly, it takes some time to measure and report the measurements to the algorithm, resulting in a time delay of n_m samples. Secondly, we have a time delay of n_p samples due to the time it takes before the computed power level is actually used in the transmitter. These time delays should be considered as additional time delays, since the discrete-time control algorithm is assumed to always provide a delay of one sample. For example, when subject to time delays, DPC in (7) becomes

$$\bar{p}_i(t+1) = \bar{p}_i(t)\frac{\bar{\gamma}_i^t}{\bar{\gamma}_i(t-n_m)}. \quad (10)$$

As stated before, the typical example is $n_m = 0$ and $n_p = 1$. Note that n_p and n_m are integers since they refer to update instants at the transmitter.

The output powers are considered to be unconstrained. Appropriate modifications when considering constrained output powers are discussed in [6, 10].

2.3 Power Control from a Control Theory Perspective

Since the time delays affect the dynamical behavior, they have to be naturally represented by the framework. Introduce the *time-shift operator* q as

$$q^{-n}p(t) = p(t-n),\ q^n p(t) = p(t+n) \quad (11)$$

For a more rigid discussion on a q-operator algebra, the reader is referred to [3]. The intuitive relations to the complex variable z of the z-transform are also addressed.

The integrator utilizing information feedback in (6a) can be rewritten using the time-shift operator

$$p_i(t) = \frac{\beta}{q-1}e_i(t) = R(q)e_i(t) \quad (12)$$

A general log-linear power control algorithm utilizing information feedback can thus be defined as a rational function $R(q)$ in the time-shift operator q. For design methods of $R(q)$, see [11, 7]. When subject to time delays, the local power control loop (or *local loop*) can be depicted as in Figure 1. That figure also include models of quantization via the additive quantization noise $d_i(t)$ and additive measurement errors $w_i(t)$. The local loop dynamics can thus be described by transfer functions between input and output signals. The relation between target SIR and resulting SIR will be referred to as the *closed local loop* $\gamma_i(t) = G_{ll}(q)\gamma_i^t(t)$. Straightforward computations yield

$$G_{ll}(q) = \frac{B_G(q)}{A_G(q)} = \frac{R(q)}{q^{n_p+n_m} + R(q)}. \quad (13)$$

The following theorem addresses stability of the local loop

Theorem 1 (Asymptotic Stability)
The local loop in Figure 1 is said to be asymptotically stable *if the roots q_i to the characteristic equation*

$$0 = A_G(q) = q^{n_p+n_m} + R(q) \quad (14)$$

lie within the unit disc $|q_i| < 1$ [8, 7]. The roots q_i are referred to as the local loop poles.

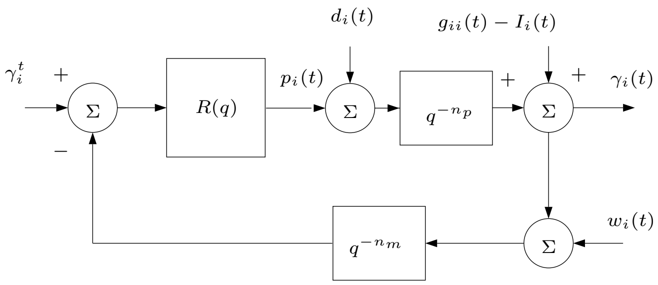

Figure 1. The local loop dynamics when employing the general linear control algorithm $R(q)$. The measurements are subject to additive errors $w_i(t)$, and the output powers by quantization noise $d_i(t)$.

For example consider the integrator utilizing information feedback in Equation (12), subject to the typical delay situation $n_p = 1$ and $n_m = 0$. The corresponding closed local loop is thus given by

$$\gamma_i(t) = \frac{\beta}{q^2 - q + \beta}\gamma_i^t(t), \qquad (15)$$

with closed loop poles within the unit circle if and only if $\beta < 1$, since the product of the two complex conjugated roots is equal to β. Hence, DPC ($\beta = 1$) is not locally stable when subject to time delays, and consequently not globally stable either.

Local dynamics in case of decision feedback is further explored in [8, 9]. One central conclusion is that the up-down device of FSPC (the sign function) together with the dynamics result in an oscillatory behavior.

3 Time Delay Compensation (TDC)

The core problem with time delays is that the measurements do not reflect the most recent power level updates. However, these are known to the algorithm, and can be to adjust the measurements. In this section we discuss such a compensation strategy with respect to specific as well as general power control algorithms. In essence, TDC monitors the powers to be used by the transmitter (by considering the issued power control commands that not yet have been effective), to adjust the measurements in the receiver by internal feedback. TDC is further discussed in [7].

3.1 FSPC algorithm

The main idea is best illustrated by an example. Consider the typical case $n_m = 0$ and $n_p = 1$ and the FSPC algorithm in (8). In a delayless situation, the SIR is given by (2)

$$\gamma_i(t) = p_i(t) + \delta_i + g_{ii}(t) - I_i(t).$$

When subject to time delays as in this case, we instead get the expression in (9)

$$\gamma_i(t) = p_i(t-1) + \delta_i + g_{ii}(t) - I_i(t).$$

Since the output powers are known, the following compensation is straightforward to eliminate the effect of the outdated power level

$$\breve{\gamma}_i(t) = \gamma_i(t) + p_i(t) - p_i(t-1) = \gamma_i(t) + \beta s_i(t-1).$$

In the general case with arbitrary time delays, $\gamma_i(t - n_m)$ is the latest available measurement for power control. Therefore, the following generalization is straightforward (now also considering a limited dynamic range)

Algorithm 1 (FSPC with TDC I)

i) Adjust measurements:
$\breve{\gamma}_i(t) = \gamma_i(t - n_m) + \breve{p}_i(t) - \breve{p}_i(t - n_p - n_m)$.

ii) Issue power control command:
$s_i(t) = \text{sign}\,(\gamma_i^t - \breve{\gamma}_i(t))$.

iii) Monitor output powers to be used:
$\breve{p}_i(t{+}1) = \max\,(p_{\min},\, \min(p_{\max},\, \breve{p}_i(t) + \beta s_i(t)))$.

By introducing the transfer function

$$H(q) = 1 - q^{-n_m - n_p},$$

TDC can be seen as internal feedback in the receiver, see the block diagram of the local control loop in Figure 2. To

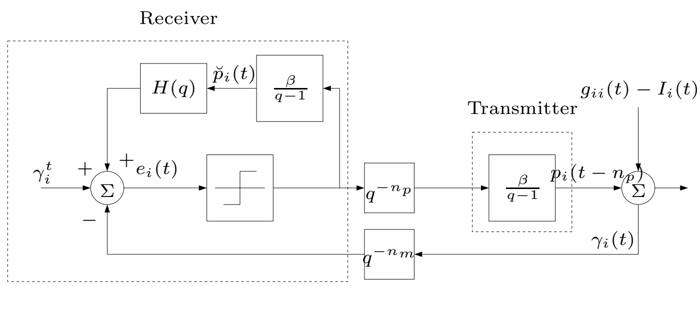

Figure 2. Time delay compensation (TDC) can be implemented as an internal feedback by monitoring the powers to be used by the transmitter, $\breve{p}_i(t)$. The feedback transfer function $H(q) = 1 - q^{-(n_p+n_m)}$ is obtained from Algorithm 1i.

further illuminate the effect of TDC, some block diagram

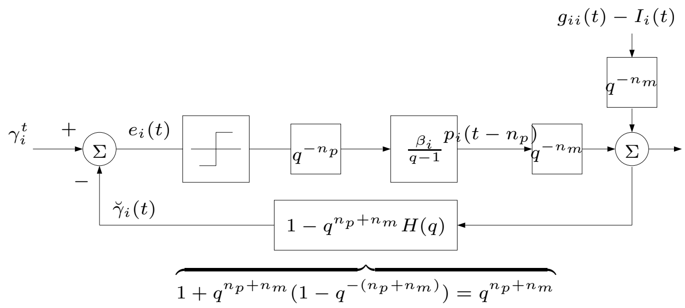

Figure 3. By rewriting the diagram in Figure 2, it is evident how TDC cancels the round-trip delays in the control loop. External signals and disturbances are still delayed before they are reflected in $\gamma(t)$.

algebra exercise is instructive. Most of the blocks in Figure 2 commute. Hence, the block diagram is easily rewritten as in Figure 3. The merits of TDC are evident, since the internal round-trip delays are cancelled in the loop. However, external signals and disturbances are still delayed, and for example it takes some time before changes in $\gamma_i^t(t)$ are reflected in the measurements $\gamma_i(t)$.

Using geometric series, it is easy to verify that

$$\breve{p}_i(t) - \breve{p}_i(t - n_p - n_m) = \beta \sum_{j=1}^{n_p+n_m} s_i(t-j).$$

Algorithm 1 can thus be rewritten as

Algorithm 2 (FSPC with TDC II)

i) Adjust measurements:
$\breve{\gamma}_i(t) = \gamma_i(t - n_m) + \beta \sum_{j=1}^{n_p+n_m} s_i(t-j)$.

ii) Issue power control command:
$s_i(t) = \text{sign}\,(\gamma_i^t - \breve{\gamma}_i(t))$.

Further details of the dynamical behavior of FSPC with and without TDC can be found in [9].

3.2 DPC algorithm

TDC for the DPC algorithm can essentially be implemented as in Algorithm 1.

Algorithm 3 (DPC with TDC)

i) Adjust measurements:
$\breve{\gamma}_i(t) = \gamma_i(t - n_m) + \breve{p}_i(t) - \breve{p}_i(t - n_p - n_m)$.

ii) Update the transmission power:
$p_i(t+1) = p_i(t) + \beta\,(\gamma_i^t - \breve{\gamma}_i(t))$

iii) Monitor output powers to be used:
$\breve{p}_i(t{+}1) = \max\,(p_{\min},\, \min(p_{\max},\, \breve{p}_i(t) + \beta\,(\gamma_i^t - \breve{\gamma}_i(t)))$.

It is instructive to transform the unconstrained case of TDC to linear scale, and combine with DPC in (10):

$$\bar{p}_i(t+1) = \bar{p}_i(t)\frac{\bar{\gamma}_i^t}{\breve{\bar{\gamma}}_i(t)} = \bar{p}_i(t - n_m - n_p)\frac{\bar{\gamma}_i^t}{\bar{\gamma}_i(t - n_m)} \tag{16}$$

3.3 General Case

In a general case, power control commands $s_i(t)$ are computed in the receiver by the control algorithm R_i using SIR measurements and target SIR:s, i.e., $s_i(t) = R_i\{\gamma_i^t, \gamma_i(t - n_m)\}$. The control commands are interpreted at the transmitter side by the device D_i to produce an updated power level, $p_i(t - n_p) = D_i\{s_i(t - n_p)\}$. Time delay compensation is then implemented as

Algorithm 4 (Time Delay Compensation (TDC))

i) Adjust measurements:
$\breve{\gamma}_i(t) = \gamma_i(t - n_m) + \breve{p}_i(t) - \breve{p}_i(t - n_p - n_m)$.

ii) Issue power control command:
$s_i(t) = \mathrm{R}_i\{\gamma_i^t, \breve{\gamma}_i(t)\}$.

iii) Monitor output powers to be used:
$\breve{p}_i(t+1) = \mathrm{D}_i\{s_i(t)\}$.

In the linear case, the powers are controlled using a linear controller $p_i(t) = R(q)e_i(t)$ (see Section 2.3). With TDC implemented as in Algorithm, the closed local loop becomes

$$G_{ll}(q) = \frac{R(q)}{q^{n_p+n_m}\,(1 + R(q))}. \tag{17}$$

According to Theorem 1, TDC result in $n_p + n_m$ local loop poles at the origin (i.e., stable), and the remaining poles identical to the poles of a system without delays (cf. the characteristic equation in (14)). Hence, it is possible to design the controller $R(q)$ to meet the requirements of a system without delays, and with TDC the system subject to delays will be locally stable.

As an example, consider DPC subject to the typical delay situation as in Section 2.3, but now with TDC. The closed loop system is obtained from (17) as

$$\gamma_i(t) = \frac{\beta}{q(q - 1 + \beta)}\gamma_i^t(t), \tag{18}$$

which is locally stable for $\beta < 2$ according to Theorem 1. TDC thus stabilizes DPC locally (and globally, which is proven in the following section).

4 Global Stability

4.1 Definitions

Introduce the matrix $\bar{\boldsymbol{Z}}$ and the vectors $\bar{\boldsymbol{p}}$ and $\bar{\boldsymbol{\eta}}$ as below

$$\bar{\boldsymbol{Z}} = [\bar{z}_{ij}] \triangleq \left[\frac{\bar{g}_{ij}}{\bar{g}_{ii}}\right] , \ \bar{\boldsymbol{p}} \triangleq [\bar{p}_i] , \ \bar{\boldsymbol{\eta}} = [\bar{\eta}_i] \triangleq \left[\frac{\bar{\nu}_i}{\bar{g}_{ii}}\right] .$$

To include auto-interference and individual target SIR:s (possibly provided by individual outer control loops) in the framework, we define the following diagonal matrices of receiver efficiencies $\bar{\boldsymbol{\Delta}}$ and target SIR:s $\bar{\boldsymbol{\Gamma}}_t$:

$$\bar{\boldsymbol{\Delta}} \triangleq \mathrm{diag}\left(\bar{\delta}_1, \dots, \bar{\delta}_m\right)$$
$$\bar{\boldsymbol{\Gamma}}_t \triangleq \mathrm{diag}(\bar{\gamma}_1^t, \dots, \bar{\gamma}_m^t)$$

When individual target SIR:s are considered, feasibility has to be defined more careful than in the balanced situation. The *feasibility margin* as defined below, can be used to discuss this matter.

Definition 2
Given a power control problem ($\bar{\boldsymbol{Z}}, \bar{\boldsymbol{\eta}}, \bar{\boldsymbol{\Delta}}, \bar{\boldsymbol{\Gamma}}_t$), the <u>*feasibility margin*</u> $\bar{\Gamma}_m \in \mathbb{R}^+$ *is defined by*

$$\bar{\Gamma}_m = \sup\left\{\bar{x} \in \mathbb{R} \ : \ \bar{x}\bar{\boldsymbol{\Gamma}}_t \ \textit{ is feasible}\right\}$$

The concept has been adopted from Herdtner and Chong [13], where similar proofs of similar and additional theorems covering related situations also are provided. The authors used the term *feasibility index* R_I and omitted auto-interference. The motivation for introducing the name *feasibility margin* is to stress the similarity to stability margins, as will be seen below. In the following theorem we capture the essentials regarding feasibility margins.

Theorem 3
Given a power control problem ($\bar{\boldsymbol{Z}}, \bar{\boldsymbol{\eta}}, \bar{\boldsymbol{\Delta}}, \bar{\boldsymbol{\Gamma}}_t$), the feasibility margin (of both up- and downlink) is obtained as

$$\bar{\Gamma}_m = 1/\bar{\mu}^*$$

where $\bar{\mu}^*$ *is*

$$\bar{\mu}^* = \max \mathrm{eig}\left\{\bar{\boldsymbol{\Gamma}}_t(\bar{\boldsymbol{\Delta}}^{-1}\bar{\boldsymbol{Z}} - \boldsymbol{E})\right\}.$$

The power control problem is feasible if and only if $\bar{\Gamma}_m > 1$. *If the power control problem is feasible, there exists an optimal downlink power assignment, given by*

$$\bar{\boldsymbol{p}} = \left(\boldsymbol{E} - \bar{\boldsymbol{\Gamma}}_t(\bar{\boldsymbol{\Delta}}^{-1}\bar{\boldsymbol{Z}} - \boldsymbol{E})\right)^{-1}\bar{\boldsymbol{\Gamma}}_t\bar{\boldsymbol{\Delta}}^{-1}\bar{\boldsymbol{\eta}}.$$

The corresponding uplink power assignment is obtained by replacing $\bar{\boldsymbol{Z}}$ *by* $\bar{\boldsymbol{Z}}^T$.

Proof See [7]. □

The feasibility margin can also be related to the load of the system. When the feasibility margin is one, the system clearly is fully loaded (only possible when unlimited transmission powers are available). Conversely, when the feasibility margin is large, the load is low compared to a fully loaded system. Thus the following load definition is logical.

Definition 4 (Relative Load)
The <u>*relative load*</u> $\bar{L}_r$ *of a system is defined by*

$$\bar{L}_r = \frac{1}{\bar{\Gamma}_m} \ (= \bar{\mu}^* \textit{ in Theorem 3}).$$

Feasibility of the power control problem is thus equivalent to a relative load less than unity.

4.2 FSPC algorithm

The FSPC algorithm result in an oscillative local loop, as discussed in Section 2.3. Therefore, it will never converge to an equilibrium. The local loop oscillations are reduced when employing TDC [9]. Moreover, as will be shown in this section, FSPC with TDC converges to a smaller region than without. A comparative comment is provided in the following remark.

Theorem 5 (Convergence of FSPC)
Consider the FSPC algorithm subject to a total time delay of $n = n_p + n_m$ *samples. Then there exists a* $\tau > 0$*, such that*

FSPC, no TDC:	$\lvert\gamma_i^t - \gamma_i(t)\rvert \leq \beta(2n+2), \ \forall t \geq \tau$
FSPC, with TDC:	$\lvert\gamma_i^t - \gamma_i(t)\rvert \leq \beta(n+2), \ \forall t \geq \tau$

Hence, the longer the time delay, the more emphasized improvements using TDC. Note, however, that the fact that the bounds are tighter does not imply that the error variance is smaller.

Proof The convergence proof of FSPC without TDC can be found in [13], while the result for FSPC with TDC is provided in [7]. □

4.3 DPC algorithm with TDC

The convergence of the DPC algorithm in (7) in a delayless case is proven formally in [18, 7]. As disclosed in Section 2.3, the DPC gets unstable when subject to time delays. TDC is seen in Section 3.2 to locally stabilize DPC, but local stability is only a necessary, and not a sufficient condition for global stability. Instead, global stability of DPC with TDC is established for an arbitrary time delay in the following theorem.

Theorem 6 (Convergence of DPC with TDC)
The DPC algorithm employing TDC (16) converges if the power control problem is feasible to a unique equilibrium $\bar{\boldsymbol{p}}_\infty$ that meets the target SIR requirements with equality for any initial power vector $\bar{\boldsymbol{p}}_0$. The convergence rate is $n+1$ times slower compared to the DPC algorithm applied to the corresponding delayless power control problem.

Proof The updating procedure of the DPC algorithm with TDC in (16) is

$$\bar{p}_i(t+1) = \bar{p}_i(t - n_m - n_p)\left(\frac{\bar{\gamma}_i^t}{\bar{\gamma}_i(t-n_m)}\right) = \frac{\bar{\gamma}_i^t \bar{I}_i(\bar{\boldsymbol{p}}(t-n_m-n_p))}{g_{ii}}.$$

Introduce $n = n_p + n_m$. Clearly, the update at time instant $t+1$ only depend on the powers at time instant $t-n$. This is essentially the same as having $n+1$ algorithms operating independently in parallel. Thus if the sample interval is T_s, each of these algorithms update every $n+1$ sample interval. In essence, the situation is the same as in the delayless case, but the sample interval is longer, resulting in a factor $n+1$ slower convergence. □

4.4 Linear Power Control Algorithms

The techniques in the previous section apply primarily to structurally simple algorithms. When considering delays, filters and general log-linear controllers, it is desirable to relate to the local loop analysis and design in Section 2.3 and in [7]. The challenge is the nonlinear interconnections between the log-linear control loops. In this section we apply a robust stability approach, based on a linearized interference. Essentially, we aim at rewriting the global system, to associate it with a setting where the *Small Gain Theorem* applies. Further details are provided in [7]. Omitting the details, the following theorem can be proven

Theorem 7
Assume that the powers are controlled by the general linear control algorithm $R(q)$ as in Figure 1. Then the global system is stable if and only if the following holds

$$\sup_w |G_{ll}(e^{iw})| \leq 1$$

Proof See [7]. □

The important conclusion is that it is possible to guarantee global stability by proper local loop design. The criterion in Theorem 7 tends to be rather conservative. Therefore, the following relaxation is provided in [7]

$$|G_{ll}(e^{iw})| < \frac{1}{\bar{L}_r} = \bar{\Gamma}_m. \qquad (19)$$

Note that this only should be considered as a rule of thumb. However, it further motivates the name *feasibility margin*. This means that systems operating close to maximum capacity ($\bar{L}_r$ close to unity) are much more sensitive to the local loop behavior, while it is possible to employ more aggressive control actions in sparse systems.

5 Simulations

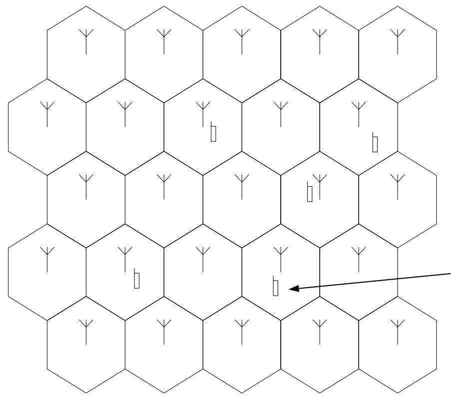

Figure 4. Configuration in the simulation example, where four mobile stations initially are allocated to the same channel. At time instant $t = 0$ a fifth mobile station (indicated by the arrow) is allocated to the channel.

Consider the uplink situation in Figure 4, where four mobile stations are connected using the same channel in a system with orthogonal channels. The simulation is initiated at time instant $t = -10$, and the four mobile stations are initially using the optimal transmission powers corresponding to a target SIR of 12 dB. The signal power gains $\bar{g}_{ij}$ are constant throughout the simulation. At time instant $t = 0$, the same channel is allocated to a fifth mobile station, initially using a transmission power of 1 dBW. The target SIR is still feasible in the five mobile station case. Power control is employed as either DPC described by (7) or FSPC (step size β=1 dB) given by (8). It is illustrative to see how these algorithms recover in the changing environment when subject to delayed power control commands. In this case, the power control command (FSPC) or the transmission power itself (DPC) are delayed by one sample interval.

The performance using the two algorithms with and without TDC is depicted in Figure 5. We note in a) that DPC without TDC is unstable in the sense that the oscillations in SIR are aggravated over time. Up to $t = 0$, the powers are not updated since the target SIR and the measured SIR are identical. The system is thus at rest in an equilibrium point. Then, the admitted mobile increases the interference at each receiver. This disturbance reveal the instability of the equilibrium point, and the SIR fluctuations get worse over time. The difference when employing TDC

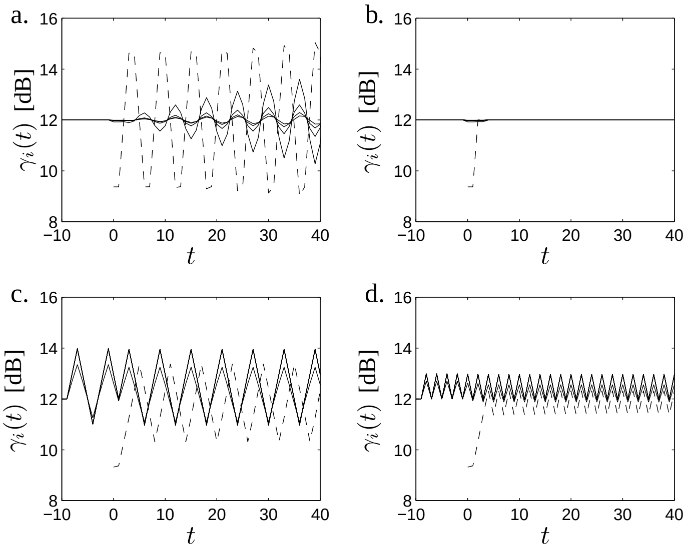

Figure 5. The recovering ability of a. DPC, b. DPC with TDC, c. FSPC, and d. FSPC with TDC, when subject to the situation in Figure 4 and a time delay of one sample.

is significant, as seen in Figure 5b). TDC stabilizes DPC and the original four mobiles are essentially not affected by the entering mobile.

Despite being perfectly initialized, FSPC results in oscillations in the case of four mobiles. That is due to the decision feedback, where the powers are either increased or decreased each time instant. The oscillations are not that much affected by the entering mobile station. However, one might expect that this up-down device would result in small oscillations (approximately 1 dB peak-to-peak) around the target value. As seen in the plot in Figure 5c), the amplitude is larger (3 dB peak-to-peak) than expected. When employing TDC, the oscillations are significantly reduced in Figure 5d), enabling the use of less fading margin.

6 Conclusions

Time delays hamper the performance of any system using feedback. In this paper, the effects of time delays on stability and convergence of distributed power control algorithms is studied. It is seen that the effects of time delays can be eliminated or at least reduced by utilizing the introduced time delay compensation (TDC). It is a general technique, applicable to arbitrary power control algorithms within a general class of algorithms. The benefits of using TDC are exemplified for popular power control algorithms, and the performance improvements are significant. Moreover, results on global stability and convergence are provided, to establish the merits in theory. The operation of TDC is further illuminated by simulations.

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