

# Dynamical Effects of Time Delays and Time Delay Compensation in Power Controlled WCDMA

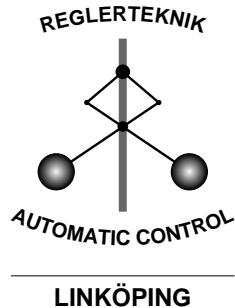
Fredrik Gunnarsson, Fredrik Gustafsson and Jonas Blom

Department of Electrical Engineering  
Linköpings universitet, SE-581 83 Linköping, Sweden

WWW: <http://www.control.isy.liu.se>

Email: [fred@isy.liu.se](mailto:fred@isy.liu.se), [fredrik@isy.liu.se](mailto:fredrik@isy.liu.se), [jb@isy.liu.se](mailto:jb@isy.liu.se)

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## Abstract

Transmission power control is essential in WCDMA in order to optimize the bandwidth utilization which is critical when variable data rates are used. One remaining problem is oscillations in the output powers due to round-trip delays in the power control loops together with the power up-down command device. The oscillations are naturally quantified using discrete-time describing functions, which are introduced and applied. More importantly, Time Delay Compensation (TDC) is proposed to mitigate the oscillations. When employing TDC, the power control algorithm operates more stable, which is important from a network perspective. Simulations illustrate the oscillations and the benefits of TDC. Moreover, the fading tracking capability is improved and thus less fading margin is needed. The latest news from standardization is to try to operate without additional delay when close to a base station. Nevertheless, there will still be more distant mobile stations for which TDC will be beneficial.

**Keywords:** Power control, Time delay compensation, Delay analysis, Dynamics, WCDMA, CDMA, IS-95, Oscillations, Describing functions



# 1 Introduction

In order to utilize the available resources in cellular radio systems efficiently, different radio resource management schemes are needed. One such technique is to control the output powers of the transmitters. In systems based on CDMA, this is particularly important since all terminals are communicating using the same spectrum. Most power control algorithms proposed to date strive to balance the *carrier-to-interference ratio* (C/I) or the *signal-to-interference ratio* (SIR) [1].

Fast fading has to be mitigated when possible, and therefore it is desirable to choose a high updating interval of the power control algorithm. The signaling bandwidth is kept low by utilizing only one bit for signaling, where the power is stepwise increased or decreased. Viterbi [23] proposed a scheme where the transmitter power is increased or decreased based on the comparison of received SIR and a threshold. The scheme was further investigated by Ariyavisitakul [5].

Signaling and measuring takes time resulting in time delays in the power control loop, which in turn affects the dynamics of the closed-loop. This has primarily been considered as imperfect power control and Sim et. al. [20] concluded that power control is more sensitive to the delay than to the SIR estimator performance. Chockalingam et. al. [10] indicated, using simulations and analytically approximated second-order statistics, that the performance is degraded when subject to delays in the power control loop.

Leibnitz et. al. [18] proposed a Markov chain model to describe the power control dynamics. The horizon of transitions was, however, chosen too short to reveal the presence of oscillations.

The intuitive behavior of a power control algorithm in operation is that the received SIR oscillates up and down around the threshold  $\gamma_{tgt}$  as in Figure 1a. When subject to delays, however, the amplitude of the oscillations is larger as seen in Figure 1b. Primarily, time delays results in oscillations in two different ways

1. Delayed reactions to changes in external disturbances.
2. Internal dynamics of the power control loop.

In this paper, *time delay compensation* (TDC) [16] is proposed to mitigate oscillations due to internal dynamics (second item above). As seen in Figure 1a, which represents the same situation as in Figure 1b, but with TDC in operation, the oscillations are significantly reduced. This means that the capacity can be better utilized, which is critical when using variable data rate.

TDC is described using the loglinear model introduced in [8], which is similar to the model used in [10]. The dynamics are quantified using discrete-time describing functions to predict the different modes of oscillations.

The rest of the paper is organized as follows. Section 2 introduces the system models, which are used to describe the power control algorithm in operation. The closed-loop system is intuitively depicted in a block diagram capturing the dynamics. Time delay compensation is presented in Section 3, and the dynamics is quantified using discrete-time describing functions in Sections 4 and 5. The performance improvements using TDC are further illuminated in simulations in Section 6. Section 7 provide the conclusions.

A reader who only wants to grasp the most important results can peruse Sections 2 and 3, Proposition 4 in Section 4 and Section 5.

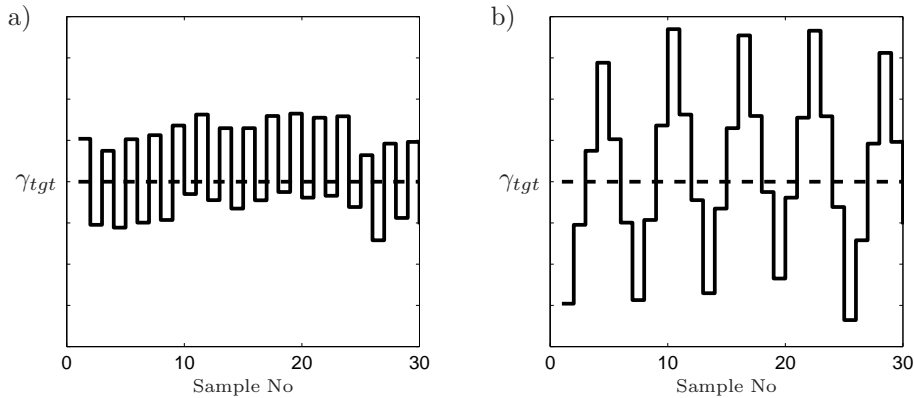


Figure 1: Received SIR in a typical WCDMA situation, where the power control commands are delayed by one slot. a) TDC employed, b) no TDC.

## 2 System Model

Initially, it will be assumed that each mobile station is connected to only one base station, i.e. any soft handover scheme is not considered. This is chosen to keep the notation simple and to emphasize the core ideas. The case of soft handover will be considered in more detail in the end of this section and in Section 3.2. Moreover, the nomenclature is settled to focus on the uplink (reverse link). However, the downlink (forward link) is treated analogously, except when stated.

### 2.1 Notation

All values will be represented by values in logarithmic scale (e.g., dB or dBW). Assume that  $m$  active mobile stations transmit using the powers  $p_i(t)$ ,  $i = 1, \dots, m$ . The signal between mobile station  $i$  and base station  $j$  is attenuated by the signal gain  $g_{ij}(t)$  ( $< 0$ ). Moreover, mobile station  $i$  is connected to base station  $j_i$ . Thus the corresponding connected base station will experience a desired carrier signal

$$C_i(t) = p_i(t) + g_{ij_i}(t)$$

and an interference plus noise  $I_i(t)$ . The *carrier-to-interference* ratio at base station  $j_i$  is defined by

$$\gamma_i(t) = p_i(t) + g_{ij_i}(t) - I_i(t).$$

Some authors prefer to use the *signal-to-interference* ratio (SIR), which in a CDMA setting is defined by

$$[\text{SIR}]_i = [C/I]_i + \text{PG},$$

where PG is the processing gain. As will be seen below, the two quality measures result in the same dynamic behavior.

**Remark.** Note that the notation identifies each uplink connection by the number of the mobile station  $i$ .

## 2.2 Closed-loop Power Control in WCDMA

In order to avoid extensive signaling in the networks, distributed power control algorithms are desirable. Furthermore, fast power updates are of interest in order to mitigate the fast fading when possible. In the WCDMA proposal, the signaling bandwidth is kept low despite the high update rate by utilizing single-bit signaling [3, 4]. The C/I is estimated at the receiver and compared to a threshold  $\gamma_{tgt,i}(t)$ . Then the power control command  $s_i(t) = -1$  is sent to the transmitter when above the threshold and  $s_i(t) = +1$  when below. The updating procedure can thus be described as

$$\begin{aligned} \text{Receiver : } \quad e_i(t) &= \gamma_{tgt,i}(t) - \gamma_i(t) \\ s_i(t) &= \text{sign}(e_i(t)) \end{aligned} \tag{1a}$$

$$\text{Transmitter : } \quad p_i(t+1) = p_i(t) + \Delta_i s_i(t) \tag{1b}$$

The step size  $\Delta_i$  might be adapted as well. Here, only updates at a much slower rate than the power updates are considered, which means that it can be considered constant in the analysis. The effects of updates at faster rates, will be discussed in Section 5.2.

In the actual proposals, SIR is utilized instead of C/I. However, since only the difference between the estimate and a target value is considered in (1a), this will result in the same sequence of power control commands  $s_i(t)$ . Therefore, C/I will be used in the sequel, but the reader should remember that the analysis is identical when utilizing SIR.

The target value,  $\gamma_{tgt,i}(t)$ , is provided by an outer control loop operating at a lower rate [3, 8, 23]. Therefore, this value may be regarded as constant on a short term. Related to the choice of target values is the following definition [24].

### Definition 1

The vector of target values,  $[\gamma_{tgt,i}]$ , is feasible if there exist a power vector,  $[p_i]$ , that result in these target values.

In a real system, signaling and estimating takes some time, resulting in time delays in the closed loop. These time delays can be described using the *delay operator* in the time domain, defined by<sup>1</sup>

$$\begin{aligned} s_i(t-1) &= q^{-1} s_i(t) \\ s_i(t+1) &= q s_i(t) \\ s_i(t-n) &= q^{-n} s_i(t). \end{aligned}$$

Applying this scheme to the power update in the transmitter 1b yields

$$p_i(t) = \frac{\Delta_i}{q-1} s_i(t) \tag{2}$$

Assume that the power control commands are delayed by  $n_p$  samples before they are actually considered in the transmitter. Moreover, the estimated C/I is delayed by  $n_m$  samples before it is compared to the threshold. Since the command signaling is standardized, these delays are known exactly in number of samples (or slots). Typical situations in WCDMA are depicted in Figure 2. As seen in Figure 2b, the transmitter power can be controlled without excessive loop delay by shifting the slot configurations. This is only possible for mobile stations relatively close to the base station. IS-95 have an excess delay of 2-3 samples [21].

<sup>1</sup>Note that  $t$  represents *time index* and not the actual time. An alternative is to include the power update interval (or sample interval)  $T_s$ , and consequently define  $s_i(t - nT_s) = q_T^{-n} s_i(t)$ .

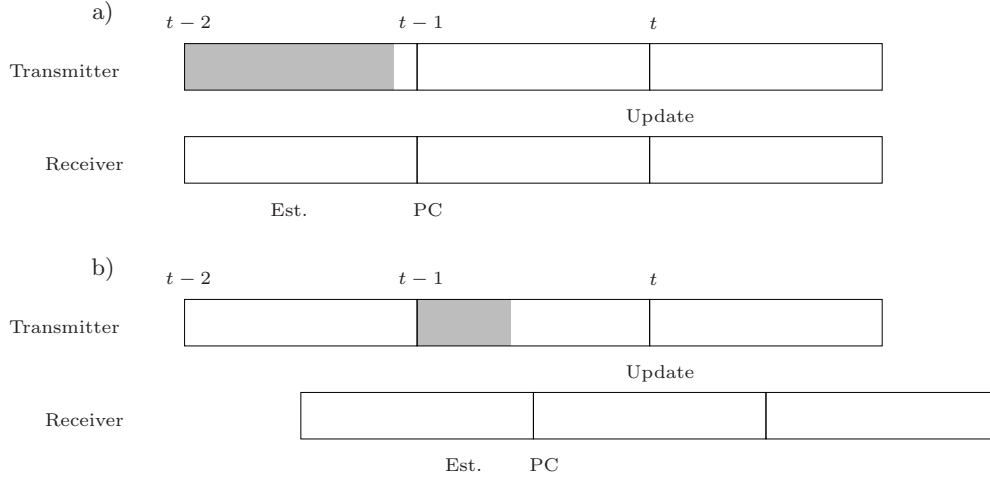


Figure 2: Typical situations in WCDMA describing time of estimation, power command signaling (PC), and power update. a)  $n_p = 1$ ,  $n_m = 0$ , b) By shifting the slot configurations,  $n_p$  can be brought down to zero when the mobile station is close to the base station.

Using 2, the closed-loop behavior of the power control algorithm can be depicted as in Figure 3. This *local loop* captures the dynamics if the interference  $I_i(t)$  can be treated as independent of the transmitter power  $p_i(t)$ , which is a reasonable approximation in most cases. However, when the target values  $\gamma_{tgt,i}(t)$  are infeasible (see Definition 1), then the transmitter powers will ramp up until one or several transmitters are using maximum powers. This effect is referred to as the *party effect* and is further discussed in [2]. In this article, it is assumed that the target values are feasible.

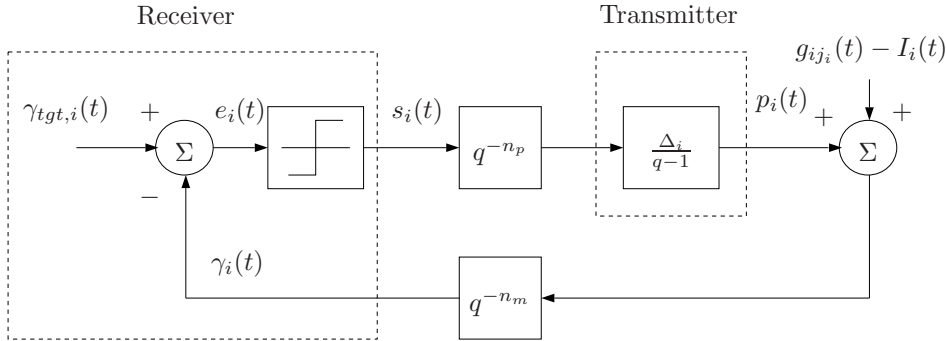


Figure 3: WCDMA inner loop power control, where the power control commands are delayed by  $n_p$  samples and the measurements by  $n_m$  samples.

In a real systems, the power control commands may be corrupted by disturbances resulting in command errors. This is modeled by the stochastic multiplicative disturbance  $x_i(t)$  with probability function

$$p_X(x) = (1 - p_{CE})\delta(x - 1) + p_{CE}\delta(x + 1),$$

i.e., the command error probability is  $p_{CE}$ . Moreover, the output powers are limited to the set  $[p_{\min}, p_{\max}]$ . A more detailed block diagram incorporating these components is given in Figure 4.



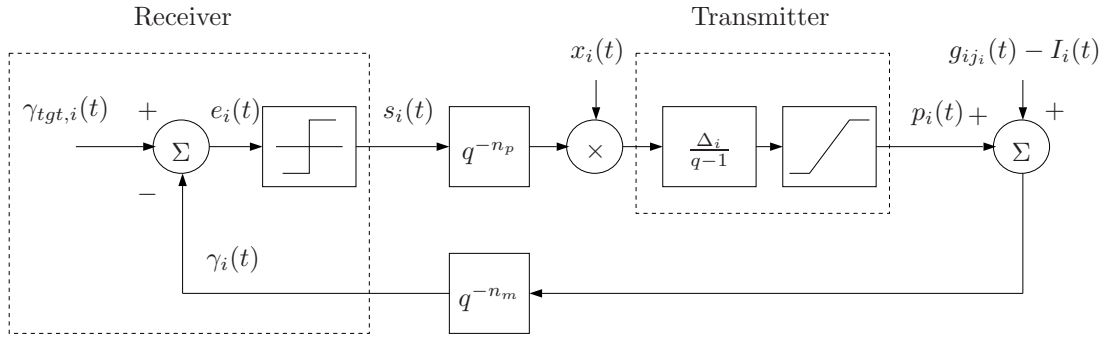


Figure 4: WCDMA inner loop power control as in Figure 3, but with power command errors  $x_i(t)$  and limited dynamic range.

### 2.3 Comparison to Carrier-based Power Control

In some systems, e.g., IS-95, a similar scheme is employed, but the power control commands are determined by comparing the received carrier power to a threshold [22]. As in the previous section, the closed power control loop can be described by the block diagram in Figure 5. Since the dynamics is equivalent to the dynamics in Figure 4, analysis of the C/I based power control will hold in this case as well. We can thus without loss of generality focus on the C/I-based control.

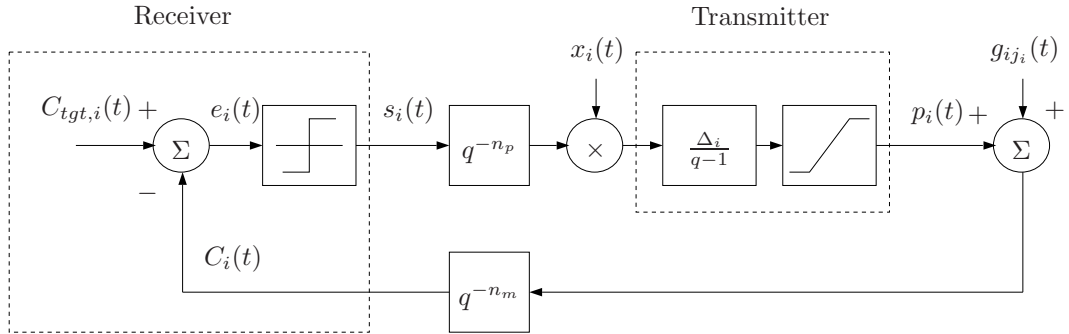


Figure 5: Carrier-based inner loop power control, which is dynamically equivalent to Figure 4.

### 2.4 Soft and Softer Handover

When the quality of service is degraded, it may be beneficial to connect to several base stations. A typical situation is when the mobile station is moving from one cell to another. During a transition phase, the mobile station will be connected to both base stations to preserve an acceptable connection. The number of connected base stations is referred to as the *active set* (AS). This active set includes the base station with the strongest received signal and the base stations within a window of size *hm* (*handover margin*). However, the active set contains at most *ASmax* number of base stations. For uplink power control, the mobile station should adjust the power with the largest step in the “down” direction ordered by the power control commands received from each base station in the active set [3].

A related strategy is to connect to several (normally two) sectors at the same base station.

This is referred to as *softer handover*.

### 3 Time Delay Compensation

As indicated by Figure 1, round-trip delays in the power control loop result in oscillations. Basically, the main reason is that the algorithm overreacts since the receiver has not seen the effect of the previous issued command when the next one is to be issued. A solution is to compensate for the delay when determining which power control command to issue. The core idea is to adjust the measured C/I according to the power control commands that have been sent but whose effect have not yet been experienced by the receiver [16].

#### 3.1 Algorithms and Implementations

As discussed above, the commands not yet experienced have to be compensated for. This is accomplished by adjusting the measured C/I as

$$\tilde{\gamma}_i(t) = \gamma_i(t) + \Delta_i \sum_{j=1}^{n_p+n_m} q^{-j} s_i(t)$$

Time delay compensation can thus be implemented as

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#### Algorithm 1 (Time Delay Compensation I)

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1. Adjust the measured C/I

$$\tilde{\gamma}_i(t) = \gamma_i(t) + \Delta_i \sum_{j=1}^{n_p+n_m} q^{-j} s_i(t).$$

2. Compare to the threshold

$$e_i(t) = \gamma_{tgt,i}(t) - \tilde{\gamma}_i(t).$$

3. Compute the power control command

$$s_i(t) = \text{sign}(e_i(t)).$$


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**Remark.** In the typical WCDMA case  $n_p = 1$ ,  $n_m = 0$ , the measurement is adjusted simply by  $\tilde{\gamma}_i(t) = \gamma_i(t) + \Delta_i s_i(t - 1)$ .

Rewrite the compensation term in order to see the effects of TDC more clearly.

$$\Delta_i \sum_{j=1}^{n_p+n_m} q^{-j} s_i(t) = \Delta_i q^{-1} \frac{1 - q^{-n_p-n_m}}{1 - q^{-1}} s_i(t) = (1 - q^{-(n_p+n_m)}) \cdot \frac{\Delta_i}{q-1} s_i(t) \quad (3)$$

Introduce  $\tilde{p}_i(t)$  to monitor the powers to be used in the transmitter (cf. (1b))

$$\tilde{p}_i(t+1) = \tilde{p}_i(t) + \Delta_i s_i(t) \iff \tilde{p}_i(t) = \frac{\Delta_i}{q-1} s_i(t)$$

Together with (3), this yields

$$\Delta_i \sum_{j=1}^{n_p+n_m} q^{-j} s_i(t) = (1 - q^{-(n_p+n_m)}) \tilde{p}_i(t) = \tilde{p}_i(t) - \tilde{p}_i(t - n_p - n_m)$$

The compensation can thus be written as

$$\tilde{\gamma}_i(t) = \gamma_i(t) + \tilde{p}_i(t) - \tilde{p}_i(t - n_p - n_m).$$

Basically the compensation can be seen as operating in two phases. Monitoring the powers to be used in the transmitter,  $\tilde{p}_i(t)$ , and subtracting the old power and add the new power to the measured C/I. This can be formulated as the algorithm below. Note that when consider this as an implementation, the limited dynamic range must be considered.

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**Algorithm 2 (Time Delay Compensation II)**

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1. Adjust the measured C/I

$$\tilde{\gamma}_i(t) = \gamma_i(t) + \tilde{p}_i(t) - \tilde{p}_i(t - n_p - n_m).$$

2. Compare to the threshold

$$e_i(t) = \gamma_{tgt,i}(t) - \tilde{\gamma}_i(t).$$

3. Compute the power control command

$$s_i(t) = \text{sign}(e_i(t)).$$

4. Monitor in the receiver the powers to be used by the transmitter

$$\tilde{p}_i(t+1) = \max(p_{\min}, \min(p_{\max}, \tilde{p}_i(t) + \Delta_i s_t(t))).$$


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Introduce

$$H(q) = -(1 - q^{-(n_p+n_m)}).$$

Based on Figure 3, the operation of this algorithm is naturally represented by the block diagram in Figure 6. Note that limited dynamics range is not included for simplicity. Readers with a background in control recognize the relations to the Smith-predictor, discussed e.g., in [6]. The benefits of TDC are illuminated by rewriting this block diagram. After some “block diagram algebra” exercise, the diagram can be rewritten as in Figure 7. The merits of TDC are then evident, since it cancels the internal round-trip delays in the loop. However, external signals and disturbances are still delayed, and it takes some time before changes in  $\gamma_{tgt,i}(t)$ ,  $g_{ij_i}(t)$  and  $I_i(t)$  are reflected in the measurement  $\gamma_i(t)$ .

### 3.2 Applicability

TDC is applicable to both uplink and downlink power control in WCDMA and IS-95, as well as systems where the transmitter powers are controlled analogously. One problem arises for the uplink when employing soft handover. While in operation, a base station is unaware of whether an issued control command is applied or not. As stated in Section 2.4, the mobile station decreases the power if at least one base station has issued a down command. Therefore, TDC should be disabled in the uplink while in soft handover. The applicability to the downlink when in soft handover as well as in softer handover is depending on the combining strategy in the receiver.

A similar situation is prevalent when the power command error probability is high. The benefits of TDC will then be degraded. This is further discussed in Section 6.

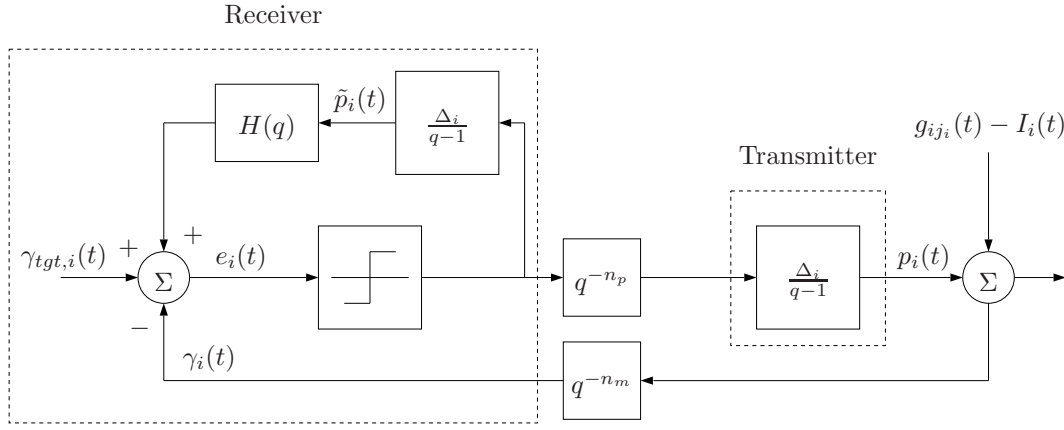


Figure 6: Time delay compensation (TDC) can be implemented as an internal feedback by monitoring the powers to be used by the transmitter,  $\tilde{p}_i(t)$ .

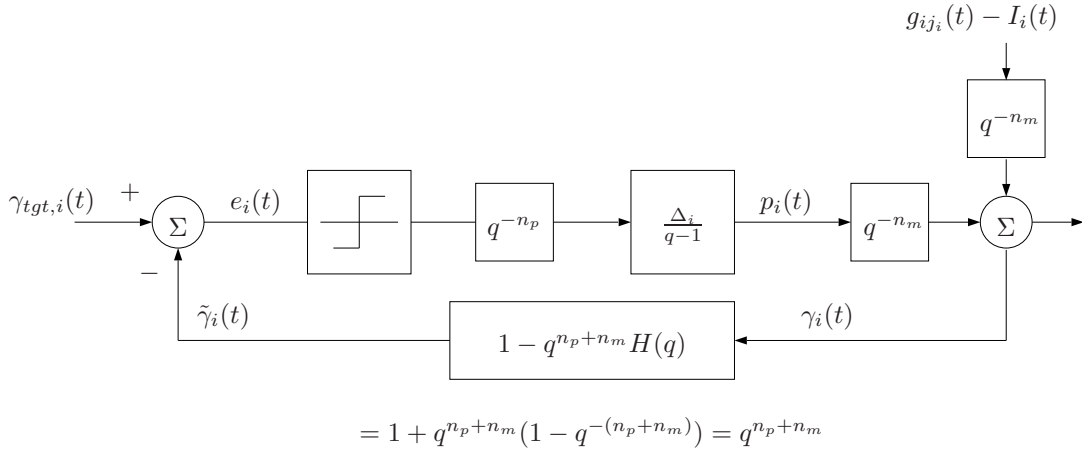


Figure 7: By rewriting the diagram in Figure 6, it is evident how TDC cancels the round-trip delays in the control loop. External signals and disturbances are still delayed before they are reflected in  $\gamma(t)$ .

## 4 Describing Functions

In this section we develop the underlying theory, first in a simple case to stress the main ideas, and then with focus to the power control local loops. The theory of describing functions or *limit cycles* are further discussed by Glad [12], Atherton [7] and Phillips and Nagle [19]. The applicability to power control in cellular radio systems have been addressed by Gunnarsson et. al. in [8, 9, 14, 15]

### 4.1 A Local Loop with one Nonlinearity

Basically, we are focusing on loops that consist of a linear part with transfer function  $G(q)$  and a static nonlinearity described by the function  $f(\cdot)$  resulting in a loop as in Figure 8. Note that we have assumed a zero input to the loop, and nonzero inputs are further studied in Section 4.2.

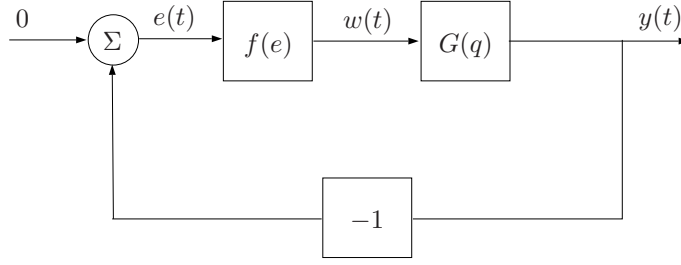


Figure 8: Block diagram of a nonlinear system, separated in one linear and one nonlinear component.

Nonlinearities in the loop normally result in an oscillatory behavior. This will be studied by assuming that there is an oscillation in the error signal  $e(t)$ , and then try to verify this assumption.

We proceed by making the  $N$ -periodic hypothesis

$$e(t) = E \sin(\Omega_e t) = E \sin\left(\frac{2\pi}{N}t\right),$$

where  $E$  is the amplitude of the oscillation and  $\Omega_e$  is the normalized angular frequency. In order to simplify the calculations we assume for a moment that  $f(\cdot)$  is odd. The computations using a general static nonlinearity are analogous, but a little bit more complicated, and we will return to a general  $f(\cdot)$  further on. Since the nonlinearity is static,  $w(t)$  is  $N$ -periodic as well. Using discrete time Fourier series expansion, the signal  $w(t)$  is decomposed into its Fourier components as

$$w(t) = f(E \sin(\Omega_e t)) = A_1(E, N) \sin(\Omega_e t + \phi_1(E, N)) + A_2(E, N) \sin(2\Omega_e t + \phi_2(E, N)) + \dots$$

Recall that a sinusoid  $\hat{X} \sin(\Omega_X t)$  fed through a linear system  $H(q)$  results in the output

$$\hat{X} |H(e^{i\Omega_X})| \sin(\Omega_X t + \arg(H(e^{i\Omega_X}))),$$

after the transients have decayed. Now let us make the assumption that the linear system  $G(q)$  will attenuate the harmonics much more than the fundamental frequency. This is the only approximation we will make, and it yields

$$y(t) \approx A_1(E, N) |G(e^{i\Omega_e})| \sin(\Omega_e t + \phi_1(E, N) + \arg(G(e^{i\Omega_e}))). \quad (4)$$

Figure 8 yields

$$y(t) = -e(t) = -E \sin(\Omega_e t) = E \sin(\Omega_e t + \pi). \quad (5)$$

Recall that  $\Omega_e = \frac{2\pi}{N}$ . By combining Equations (4) and (5) we state that we will get an oscillation if there exists a solution to the following equations

$$A_1(E, N) |G(e^{i\frac{2\pi}{N}})| = E \quad (6a)$$

$$\phi_1(E, N) + \arg(G(e^{i\frac{2\pi}{N}})) = \pi + 2\pi\nu. \quad (6b)$$

By utilizing the complex Fourier coefficient  $C_1(E, N)$ , this can be written more compact by defining the complex number

$$Y_f(E, N) = \frac{A_1(E, N)e^{i\phi_1(E, N)}}{E} = \frac{2i}{E}C_1(E, N), \quad (7)$$

which will be referred to as the *describing function*. Then (6) can be rewritten as

$$Y_f(E, N)G(e^{i\frac{2\pi}{N}}) = -1, \quad (8)$$

## 4.2 Nonzero Input

When considering nonzero inputs in general, the analysis becomes more complex. In some cases, however, such as the power control case, some simplifications are readily available. Consider the power control update in (1) and include the delays as in Figure 3. This yields

$$p_i(t+1) = p_i(t) + \Delta_i \text{sign}(\gamma_{tgt,i} - p_i(t - n_p - n_m) - g_{ij_i}(t - n_m) + I_i(t - n_m)).$$

Introduce  $\tilde{p}_i(t) = p_i(t) - \gamma_{tgt,i} + g_{ij_i}(t + n_p) - I_i(t + n_p)$ , and assume that the power gain and the interference are constant over the delay horizon ( $n_p + n_m$  samples). Hence

$$\tilde{p}_i(t+1) = \tilde{p}_i(t) + \Delta_i \text{sign}(-\tilde{p}_i(t - n_p - n_m)),$$

which is the zero input case. The imperfect assumption of constant gain and interference over the delay horizon can be considered by incorporating an unknown phase shift in the definition of the describing function. Note that this phase shift  $\delta_e \in [0, 2\pi/N[$  will not affect the issued power control commands, since

$$s_i(t) = \text{sign}(E \sin(\Omega_e t)) = \text{sign}(E \sin(\Omega_e t + \delta_e)).$$

Therefore, the following definition of the describing function is plausible when the input can be assumed to be nonzero, and the imperfection of that assumption is captured by an unknown phase.

### **Definition 2 (Discrete-Time Describing Functions)**

The describing function of the static nonlinearity  $f(\cdot)$  is defined (cf. (7)) by

$$Y_f(E, N) = \frac{2i}{E} C_1(E, N),$$

where the complex Fourier coefficient  $C_1(E, N)$  is given by

$$C_1(E, N) = \frac{1}{N} \sum_{t=0}^{N-1} f(E \sin(\Omega_e t + \delta_e)) e^{-i(\Omega_e t + \delta_e)}.$$

The definition can be written more compact

$$Y_f(E, N) = \frac{2i}{NE} \sum_{t=0}^{N-1} f(E \sin(\Omega_e t + \delta_e)) e^{-i(\Omega_e t + \delta_e)}.$$

When employing the definition, several solutions might satisfy the equations due to the unknown phase. For relays, the following is intuitive

### **Proposition 3 (Even-period oscillations)**

For a stable oscillation in the relay case, the period  $N$  has to be even. While in operation, single cycles of odd periods might be present, but they will be regarded as transitions between even-period cycles.

The discrete-time describing functions analysis is summarized in the following proposition

### **Proposition 4 (Discrete-Time Describing Functions Analysis)**

Consider the situation in Figure 8, where the loop is separated in a linear ( $G(q)$ ) and a nonlinear ( $f(\cdot)$ ) part. Then the oscillation in the error signal  $e(t)$  is approximated by the procedure

1. Determine the discrete-time describing function of the nonlinearity as

$$Y_f(E, N) = \frac{2i}{NE} \sum_{t=0}^{N-1} f(E \sin(\Omega_e t + \delta_e)) e^{-i(\Omega_e t + \delta_e)},$$

where  $N$  is even,  $\Omega_e = \frac{2\pi}{N}$  and  $\delta_e \in [0, 2\pi/N[$ .

2. Compute  $G(q)|_{q=e^{2\pi i/N}}$ .

3. Solve the following equation for  $E$  and  $N$ .

$$Y_f(E, N)G\left(e^{i\frac{2\pi}{N}}\right) = -1 \quad (9)$$

If a solution exist, then the oscillation is approximated by

$$e(t) = E \sin\left(\frac{2\pi}{N}t + \delta_e\right).$$

If several solutions exist, then several modes of oscillation is possible.

### 4.3 Describing Function of the Relay

The describing function is given by Definition 2. In the relay case, the complex Fourier coefficient  $C_1(E, N)$  in Definition 2 can be computed as

$$\begin{aligned} C_1(E, N) &= \frac{1}{N} \sum_{t=0}^{N-1} f(E \sin(\Omega_e t + \delta_e)) e^{-i(\Omega_e t + \delta_e)} = \\ &= \frac{1}{N} \sum_{t=0}^{N/2-1} e^{-i(\Omega_e t + \delta_e)} - \frac{1}{N} \sum_{t=N/2}^{N-1} e^{-i(\Omega_e t + \delta_e)} = \\ &= \frac{1}{N} e^{-i\delta_e} [1 - e^{-i\pi}] \sum_{t=0}^{N/2-1} e^{-i\Omega_e t} = \\ &= \frac{2}{N \sin\left(\frac{\pi}{N}\right)} e^{i\left(\frac{\pi}{N} - \frac{\pi}{2} - \delta_e\right)} \end{aligned}$$

The describing function is then given by Definition 2 as

$$Y_f(E, N) = \frac{4}{NE \sin\left(\frac{\pi}{N}\right)} e^{i\left(\frac{\pi}{N} - \delta_e\right)} \quad (10)$$

## 5 Describing Functions Analysis

The dynamical behavior of the local loop with a relay can be analyzed using *describing functions* based on assumptions of oscillations in the loop.

### 5.1 Quantitative Analysis

Consider the process outlined in Proposition 4:

1. The describing function of the relay is provided in (10).

2. The linear part of Figure 3 is given by

$$G(q) = \frac{\Delta_i}{q^{n_p+n_m}(q-1)}$$

Hence

$$G(q)|_{q=e^{2\pi i/N}} = \frac{\Delta_i}{2 \sin(\pi/N)} e^{-i(\frac{\pi}{2} + \frac{\pi}{N} + \frac{2\pi}{N}(n_p+n_m))} \quad (11)$$

3. Equations (9), (10) yield and (11)

$$\frac{2\Delta_i}{NE \sin^2 \frac{\pi}{N}} e^{-i(\frac{\pi}{2} + \delta_e + \frac{2\pi}{N}(n_p+n_m))} = -1 = e^{-i(\pi+2\pi\nu)} \quad (12)$$

Solve for  $N$  by comparing the arguments of each side. The procedure is illustrated for the typical WCDMA case  $n_p = 1$ ,  $n_m = 0$  in Figure 9. Note that the left hand

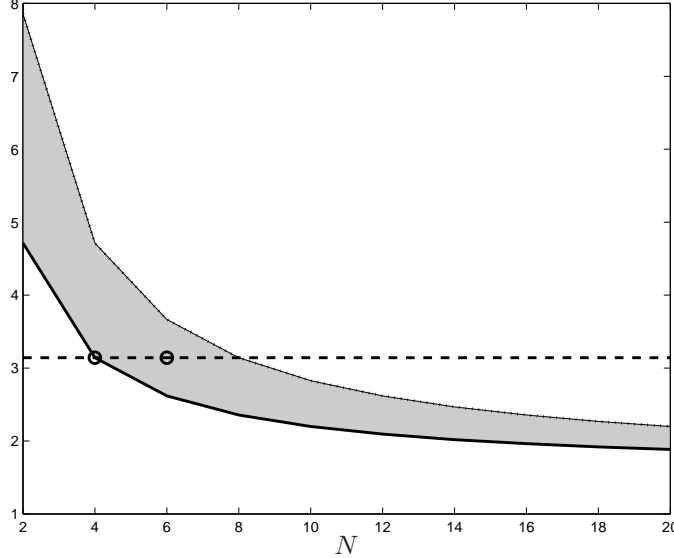


Figure 9: The argument of the left hand side of Equation (12)

is represented by a shaded area due to the unknown phase  $\delta_e$ . The right hand side argument is described by the dashed line. As seen in the graph, there are two possible solutions,  $N_0 = 4$  and  $N_0 = 6$ .

side is depending on the unknown phase  $\delta_e$ , resulting in the shaded area in the figure. The right hand side is described by the dashed line. From the figure, we conclude that there are two modes of oscillations in the closed-loop:  $N_0 = 4$  and  $N_0 = 6$ . The corresponding amplitudes can be computed by solving (12) for  $E$ , which yields

$$E_0 = \frac{2\Delta_i}{N_0 \sin^2 \frac{\pi}{N_0}}. \quad (13)$$

Table 1 summarizes some predicted oscillations for various delays.

The approximation of the amplitude  $E$  turns out to be not as accurate as the approximation of the period  $N$ . As an alternative to (13), one may consider to use

$$E_0 = \frac{N_0}{4} \Delta_i. \quad (14)$$

The approximation is intuitive, since this is exactly the amplitude of a discrete-time triangular wave with period  $N_0$  and step size  $\Delta_i$ .



$n = n_p + n_m$	Oscillation modes
0	$N_0 = 2, E_0 = \Delta_i$
1	$N_0 = 4, E_0 = \Delta_i$ $N_0 = 6, E_0 = 1.33\Delta_i$
2	$N_0 = 8, E_0 = 1.7\Delta_i$ $N_0 = 10, E_0 = 2.1\Delta_i$
3	$N_0 = 12, E_0 = 2.5\Delta_i$ $N_0 = 14, E_0 = 2.9\Delta_i$

Table 1: Predicted oscillation modes for various delays.

## 5.2 Discussion

The benefits of employing TDC is further illuminated by Table 1. Since TDC cancels the internal round-trip delays in the loop, as concluded in Section 3.1, the period of oscillations is reduced to a minimum.

Assume that the power gain and the interference are relatively slowly varying compared to the updating rate of the power control algorithm. Then the dominating oscillation originates from the internal dynamics together with the relay. This oscillative behavior is typical when nonlinear elements are present [7]. Therefore, relays should be avoided when possible. In the power control algorithm implementation, however, the benefits of a low command signaling bandwidth justifies the use of a relay.

One possible way of utilizing two bits is to use one for power up/down commands and one for step size up/down commands. This will incorporate yet another relay in the control loop resulting in an even more complex oscillative behavior for which compensation is complicated. From a dynamics point of view, it is better to increase/decrease the step size more seldom.

## 6 Simulations

In this section, system simulations will back-up and illuminate the obtained analytical results and approximations from the previous sections. The simulation model will primarily be used to generate a realistic interference environment, and the focus will be on a specific user. Power gain is modeled as proposed by Hata [17] (path loss), Gudmundson [13] (shadow fading) and Clarke [11] (fast fading). Some of the important parameters<sup>2</sup> are summarized in Table 2

Antennas	Sectorized
Cell radius	2000 m
Control sample interval	$T_s = 0.625$ ms
Slots per frame	16
Power command delay	1 slot
Shadow fading std.dev.	10 dB
Shadow fading corr. dist.	29 m
Mean mobile station speed	10 m/s

Table 2: System simulation parameters for a typical WCDMA case.

<sup>2</sup>The simulator has not yet been adapted to the new standard with 15 slots per frame.

First, consider the system in operation without TDC. As seen in Figure 10a, the oscillation is fairly stable and the correspondence with the predicted oscillation in Table 1 is good. When employing TDC, the oscillations are more or less mitigated, and the algorithm tracks the target value much better. Less fading margin is needed to reside permanently above some critical level, which in turn increases the capacity. Furthermore, a more stable operation result in a more stable overall system.

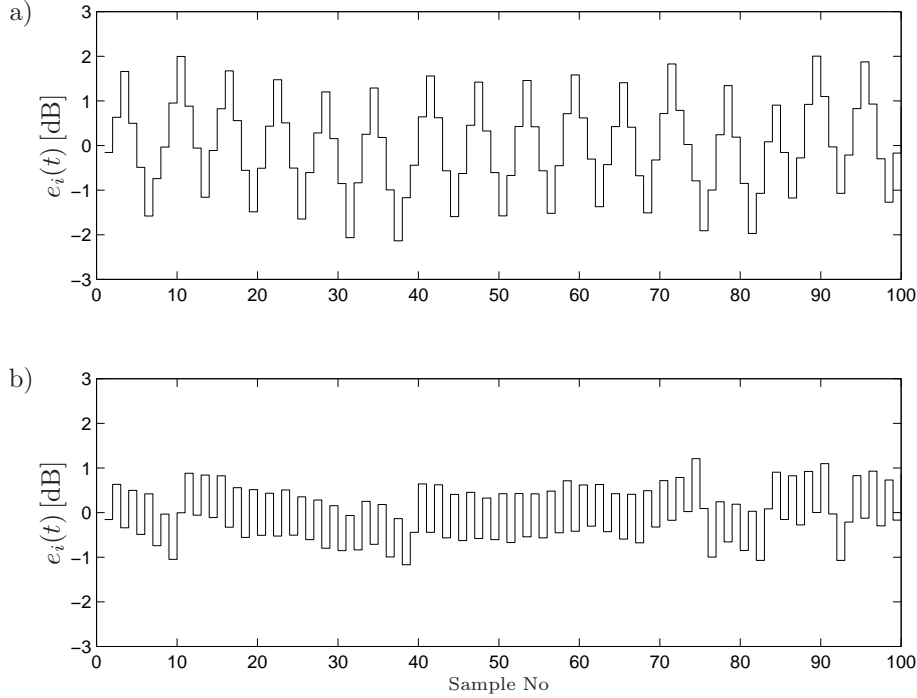


Figure 10: Control error  $e_i(t) = \gamma_{tgt,i}(t) - \gamma_i(t)$  for the typical case of one slot delay. The correspondence to the predicted oscillations in Table 1 is good. a) No TDC, b) TDC.

As stated in Section 3.2, power command errors may reduce the performance of TDC. Consider the case described in Figure 4 with command error probability  $p_{CE} = 0.05$ . The error signals together with the multiplicative disturbance  $x_i(t)$  are depicted in Figure 11. As expected, command error will result in bursty errors in C/I, but the algorithm recovers fast.

Increased power command error probability degrades the performance gradually. By using the error standard deviation, the variations in the error signals can be quantified. Figure 12 captures the degradation in the performance of TDC with increased command error probability, but it is still beneficial compared to without TDC.

From the simulations we conclude that TDC is beneficial to employ despite power command errors. The benefits are more emphasized when the command error probability is low. Furthermore it was illuminated that describing functions provide relevant predictions of oscillations in the local loops.

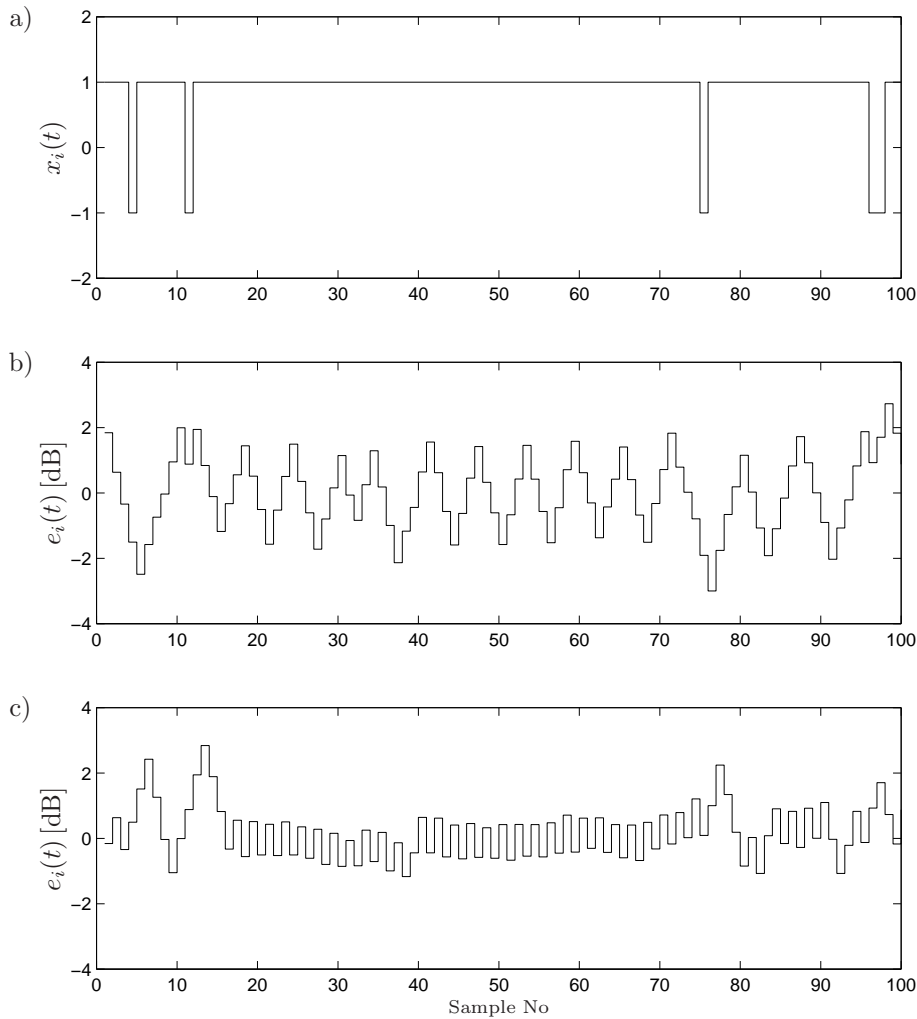


Figure 11: Same case as in Figure 10, but with command errors ( $p_{CE} = 0.05$ ). a) Multiplicative disturbance  $x_i(t)$ , b) error without TDC c) error with TDC.

## 7 Conclusions

A common implementation of power control in many DS-CDMA cellular systems is to utilize a power up-down command device. This device together with round-trip delays in the power control loops result in oscillations in output powers as well as in received SIR. By utilizing a loglinear model and the introduced discrete-time describing functions we quantify the different modes of oscillations. Both period and amplitude of the oscillation can be determined with good accuracy.

More importantly, Time Delay Compensation (TDC) is proposed to mitigate the oscillations. When employing TDC, the power control algorithm operates more stable, which is important from a network perspective. The scheme is simple to implement, and the main idea is to adjust the measured SIR according to the power control commands that have been sent but whose effect have not yet been experienced by the receiver. Moreover, the fading tracking capability is improved and thus less fading margin is needed.

Simulations illustrate the oscillations and the accuracy of predicted oscillations obtained

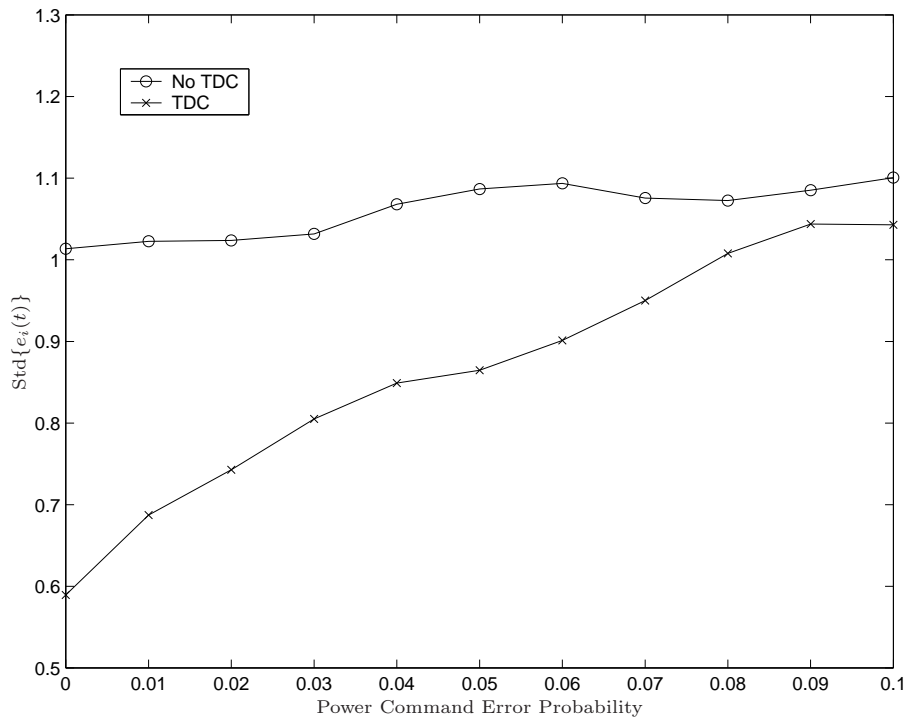


Figure 12: Standard deviation of the received SIR error in a typical WCDMA situation, where the power control commands are delayed by one slot. a) TDC employed, b) no TDC.

from describing function analysis. TDC is beneficial to employ despite power command errors. Furthermore it was illuminated that describing functions provide relevant predictions of oscillations in the local loops.

The latest news from standardization is to try to operate without additional delay when close to a base station. Nevertheless, there will still be more distant mobile stations for which TDC will be beneficial.

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