Relationships Between Epistemological Beliefs and Properties of Discourse: Some Empirical Explorations

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In this paper, I investigate what types of epistemologies are conveyed through properties of mathematical discourse in two lectures. A main purpose is to develop and explore methods for a type of analysis for this investigation. The analysis focuses on the types of statements and types of argumentations used in explicit argumentations in the lectures. This type of analysis proves to be useful when characterizing epistemological aspects of lectures. However, some limitations are also noted, in particular that it was common to use more implicit types of argumentations in the lectures, which was not included as data in the present analysis.

Introduction

There seems to be an agreement in educational research about the importance of beliefs for understanding the processes of teaching and learning. For example, there exist plenty of quantitative empirical studies showing a connection between variation in students' epistemological beliefs and variation in different kinds of ability or comprehension (Schommer, 1990). Results about connections between beliefs and teaching seem to be more tentative, which have highlighted some problematic theoretical and methodological issues (Pajares, 1992; Skott, 2005; Speer, 2005). A general problem with this kind of research is the focus on such a "large" construct as teaching practice since this practice can be influenced by many factors, in particular that it is not only beliefs that influence the decisions a teacher makes during lessons (Skott, 2005).

Thus, there is a need to study the relationship between beliefs and teaching at a more detailed level. My choice in this endeavor is to focus on epistemological beliefs and on some aspects of communication in teaching situations. Epistemological belief here refers to belief *about knowledge* (what knowledge is) and *about knowing* (how knowledge is acquired), which is sometimes referred to using different notions such as "personal epistemology, epistemological beliefs or theories, ways of knowing, or epistemic cognition" (Hofer, 2002, p. 3).

A study of relations between epistemological beliefs and communication can include how beliefs can affect, or be affected by, communication, including how one expresses oneself or how one interprets something expressed by someone else (e.g. in writing or orally). Another perspective on the relations between beliefs and communication is not to see them as two separate “objects” that can affect each other, but as more integrated aspects of cognition and/or behavior. These different perspectives on relationships between beliefs and communication are discussed more in the section Theoretical perspectives.

Purpose

In this paper, I examine the types of epistemological beliefs that are highlighted, more or less implicitly, through how one talk about mathematics. Thus, an objective is to examine what types of epistemologies are conveyed through properties of mathematical discourse. In particular, mathematical lectures are analyzed in this paper.

This study is a first attempt at empirically analyzing connections between epistemological beliefs and communication. Therefore, a main purpose is to develop and explore methods for this type of analysis. What is meant by “highlighted” is therefore, at least for now, defined through the types of analyses I use in the present study.

Theoretical perspectives

The overall interest in this paper is to study the relationship between epistemological beliefs and communication. In the present study I have an empirical focus, but in my previous studies the focus was on theoretical aspects. For example, theories describing epistemological beliefs and communication have been compared (Österholm, 2009). This comparison revealed some differences between theories regarding assumptions about cognition and discourse, but it also revealed some possibilities to develop existing theories in order to create a coherent framework for forthcoming studies about relationships between epistemological beliefs and communication. However, in the present paper I will not use a specific theory, but in the following I argue that the type of analysis performed here is relevant for different types of theories, including theories with a cognitive perspective and theories with a discourse perspective.

In general, the students taking part in a lecture are (or could be) influenced in some way by what is presented. This influence can include aspects of epistemology, through what types of epistemologies are highlighted in the presentation, implicitly or explicitly.

From a cognitive perspective, a lecture could be described using the notions of sender and receiver. The lecturer’s beliefs can be seen as a cause for how s/he presents the mathematics, for example that epistemological beliefs are a basis for how it is argued that one knows something. How the lecturer presents the mathematics is then influencing how the students think about mathematics, including epistemological aspects. In this perspective, focus is on cause and effect, where the study of properties of discourse in the lecture can be relevant both as a sign of the lecturer's beliefs and also as a potential cause for students' beliefs. The focus on cause and effect seem common in educational research about beliefs, for example when studying teachers' *attributed* beliefs (Speer, 2005) or when *explaining* students' differences in performance through differences in beliefs (Schommer, 1990).

From a discourse perspective, or a social perspective, a lecture could be described using the notions of participation and enculturation. The lecturer's statements are then not seen as a reflection of some cognitive structure, but as being constitutive themselves, as a part of the social situation (Skott, 2009). In this perspective, beliefs could be defined in terms of discourse practices (Edwards & Potter, 2005). The study of properties of discourse in the lecture can then be relevant both in the process of defining belief in this manner and also by seeing it as a part of students' enculturation (including becoming familiar with the discourse on epistemological aspects of mathematics).

Method

Two mathematics lectures at university level are analyzed in this study. At this point, the main purpose is not to compare the lectures or lecturers, but to develop and test some analytical tools for the analysis of epistemological properties of mathematical discourse (i.e. the analysis of what types of epistemologies are conveyed). For this purpose, the chosen lectures have different lecturers and different types of mathematical content, in order to have more differentiated discourses for analyses. One lecture is part of a course in calculus and this particular lecture is about improper integrals, while the other lecture is part of a course in statistics for natural scientists and this particular lecture is about some examples of discrete probability distributions. Both lectures are approximately two times 45 minutes long, but in this paper only the first part of each lecture is analyzed, since the plan is to use the other half for some other type of analysis for comparison. Only the lecturers' activity is analyzed, in order to focus on one type of discourse; the one used in lecturing, and not in for example dialogue. The lectures were recorded with audio and video, but students' statements are not audible in the recordings and the camera is always focusing on the lecturer's activity at the whiteboard.

The analysis in this paper focuses on the lecturers' auditory communication, and the lectures were transcribed from the audio recording, but using the video recording in case of doubt in the process of transcription and in case of unclear references in the lecturers' statements (e.g. referring to "this" or "that" when pointing to something on the whiteboard).

Basis for data analysis

The type of analysis used in this paper is somewhat inspired by the framework of epistemological resources (Hammer & Elby, 2002), which utilizes a bottom-up type of analysis when observing children's behavior in situations when they decide how they know something. Other frameworks seem to have a more top-down perspective, when describing categories of epistemological beliefs that can be discerned theoretically or philosophically (e.g. Schommer, 1990). Such types of categories seem difficult to apply to the type of data used in the present paper. In addition, my previous analyses show that the bottom-up perspective seems to be the best starting point in the study of relationships between epistemological beliefs and communication (Österholm, 2009). Therefore, I create my own structure for how to analyze epistemological aspects of discourse in mathematics lectures, but relate to other relevant frameworks in the creation of this structure.

Two central aspects of epistemology are the nature of knowledge (what knowledge is) and the nature of knowing (how knowledge is acquired). The *types of statements* used in a lecture could highlight the first epistemological aspect, regarding the nature of knowledge, and the *types of argumentations* used could highlight the other aspect, regarding the nature of knowing. Instead of analyzing all statements in a lecture, I choose to focus on those statements that are part of an explicit argumentation, that is, statements that are used when (at least) one statement is explicitly given as an argument for another statement. For example, when stating that "function f looks the same to the left as it does to the right *since* it is an even function", the second statement is given as an explicit argument for the first statement through the use of 'since'. The main reason for the choice to limit the analysis to these explicit argumentations is to have a clear focus in the type of data I use and also that both epistemological aspects can be included in the analysis. However, it should be noted that this choice excludes some aspects of verbal communication as well as other forms of communication that could be relevant from an epistemological perspective, but for now this choice is seen as suitable in order to have a clear focus regarding type of data.

When focusing on the *types* of statements and the *types* of argumentations, the analysis does not focus on the mathematical content of the statements or the argumentations, and the purpose of the analysis is not didactical, in the sense that the focus is not on aspects of teaching and learning the mathematical content nor on the teaching and learning of argumentation or proving. Instead, the analysis of types of statements and types of argumentations in the lectures is used in order to draw conclusions about what is conveyed *about* mathematics, in particular regarding epistemological aspects.

In order to create an a priori categorization of *types of statements* relevant from an epistemological perspective, I relate to a central distinction in mathematics education regarding different aspects of knowledge; conceptual and procedural knowledge (Hiebert, 1986). Thus, in the analysis of statements used in a lecture I separate two main types; statements about the use of mathematics objects, labeled *use-statements* (related to procedural knowledge) and statements about properties of mathematical objects, labeled *object-statements* (related to conceptual knowledge). Here I choose to use the general notion 'mathematical object', which can refer to concepts as well as procedures. The difference between the two types of statements is therefore that they describe either properties of objects or the use of such objects, which is seen as a central aspect regarding the difference between conceptual and procedural knowledge. For example, the statement "The derivative of ln x is one over x" is an object-statement while the statement "When you take the derivative of ln x you get one over x" is a use-statement.

Regarding the *types of argumentations*, you could use some elaborate framework for the analysis, such as that of Toulmin (1958). However, for the more exploratory type of purpose in this paper, I choose to use a more simplified structure for my analysis, consisting of a conclusion that is drawn (or a *claim*, using Toulmin's vocabulary) together with statement(s) used as argument for this conclusion. In order to locate the argument, the words or wordings used to make explicit the argumentative relationship between statements are of great importance (e.g. words such as 'therefore' and 'since'). As an abbreviation, these words or wordings are labeled *connect-words*, and the analysis in this paper will focus on these types of words.

From these main aspects of my intended analysis, some more specific areas of interest can be outlined as a guide for the exploration of the results from the analysis:

- Regarding the types of statements, focusing on the use- and object-statements
    - If there is a tendency to use different types of statements in the different lectures, which could highlight properties of different areas of mathematics or of purposes of different types of courses.
    - If statements of one kind easily could be re-formulated to become a statement of the other kind, which highlights the possibility to choose how to express yourself.
- Regarding the types of argumentations, focusing on the connect-words
    - What types of connect-words that are used and how they are used, e.g. if they are used in a consistent and clear way.
    - How chains of arguments are created, i.e. argumentations consisting not only of the relationship between two statements.

Procedure of data analysis

Statements from the lectures are analyzed in several steps, in order to create a clear structure in the analysis and also to make certain that only relevant statements are analyzed. However, it is not as certain that *all* relevant statements are analyzed, but the purpose of this paper is not to create a complete picture of each lecture or lecturer. Instead, the focus of this paper is on the creation and exploration of the method of analysis.

The first step in the analysis is to mark use- and object-statements, and also connect-words in the transcription. Each coherent section of the transcription is then extracted from the transcription, for further analysis. A coherent section refers to a set of statements that are connected through the use of connect-words. Such a section can for example be only one conclusion together with an argument, as in the following example from the lecture in calculus, where the connect-words are in italics: “The derivative of ln x is one over x, which is larger than zero, *which means that* it grows all the time”. Note that there is actually a linguistic ambiguity about exactly what the word ‘which’ in ‘which means’ refers to, but logically all information given before the conclusion is needed, and this full statement is therefore regarded as the argument. A section can also include several argumentations, as in the following example from the lecture in statistics:

> And you can show that the expected value is one over p. This can be seen as. Yes if we imagine that for example p is zero point two. Then this means that we will succeed on average each fifth time. And this also means that the expected value then becomes one over zero point two, which is five. So it is exactly that we will have to do on average five tries in this case.

A next step in the analysis, which is mostly relevant for sections that do not consist of a single argumentation, is to extract the relevant statements from the excerpt and arrange them in a structured manner, which for the latest example can be done in the following way, where the connect-words are in italics:

1. You can show that the expected value is one over p.
2. P is zero point two.
3. *Means that*: We will succeed on average each fifth time.
4. *Means that*: The expected value is one over zero point two, which is five.
5. *So*: We have to do on average five tries.

From this structure it is easier to analyze how the statements are related according to the connect-words, although the analysis has to include some considerations to what is reasonable, as was done in the previous example about the derivative, since it is not always clear exactly what is referred to as being the argument for the conclusion. In such situations, the logically necessary statements previously stated are listed as included in the argument. In this example, we see that line 3 is only based on line 2 as an argument, while line 4 cannot only be based on the previous line, although the exact same type of connect-words are used. The result of this type of analysis is then summarized in a three column table with a conclusion, the argument(s) for this conclusion, and the connect-words used in the argumentation. From the example above, one line in the table thus becomes:

We will succeed on average each fifth time	P is zero point two	Means that

An exploratory analysis can then be performed on the content of these tables, one for each lecture, in relation to the areas of interest outlined at the end of the previous section.

Results from data analysis

Before discussing the areas of interest in the exploration of the results from the data analysis, it can be noted that many statements from the lectures are not part of this exploration since they do not have an explicit connection to another statement in an argumentative way. For example, there are many statements in the process of formal calculations for which no explicit connect-words are used, but where one can assume that everybody knows that one step in the process is seen as the argument for the next step. However, there are also statements that are not part of such a process but still are not explicitly used in an argumentation. This is the case in situations when statements are listed one after the other, where perhaps it is meant that the second statement is a conclusion based on the first, like the following example from the calculus lecture: “f is an even function, it looks the same to the left as it does to the right”. In this example there is no explicit connection between these two statements, and a reason for this might be that they are seen as synonymous, but logically the argumentation could go in any direction between these statements.

Through the described procedure of analysis, the produced tables for each lecture consist of 39 lines for the calculus lecture and 43 lines for the statistics lecture. Each line in the table corresponds to one argumentation, which consists of a conclusion, the statement(s) used as argument for the conclusion, and the connect-words.

Use- and object-statements

When studying how common the different types of statements are in the two lectures, a clear difference between these lectures appears: In the calculus lecture, use-statements appear as a conclusion on four lines in the table (10 %) and as an argument on two lines (5 %), while in the statistics lecture, use-statements appear as a conclusion on 22 lines (51 %) and as an argument on 18 lines (42 %). Thus, in the calculus lecture object-statements are most common, while in the statistics lecture the two types of statements are about equally common.

There are several examples of statements of one type that can easily be reformulated in order to turn it into a statement of the other type. For example, in the statistics lecture there is the use-statement “if you add a constant to all values of the function, this will not change the variation”, which can be reformulated into a statement saying that a property (the variation) of two functions is the same (i.e. an object-statement). An example of the opposite type of reformulation is taken from the calculus lecture, where there is the object-statement "(the graph of) one over x has a similar appearance (as the graph of one over x squared)", which can be reformulated into "if we sketch the graph of one over x, the result is similar as when we sketch the graph of one over x squared" (i.e. a use-statement). At the moment no more in-depth analysis has been made regarding this aspect of the relationship between these types of statements, mainly because there is vagueness in the "easiness" of reformulation. The easiness has so far been seen as a sort of reasonable type of reformulation in the sense that you could imagine someone using this new formulation, so that it is not merely something that is grammatically correct but could be seen as part of mathematical discourse.

Connect-words

A common connect-word is 'so'; it is used in about half of all argumentations. In the performed analysis this word has been interpreted as a word that can signal an argumentative relation. However, there are also several examples where this word is primarily used as a sort of temporal transition in the monolog. This observation creates some doubts whether there are any real differences between to use this kind of connect-word and to only give statements without any connect-word, where it could be meant or assumed that one statement follows from a previous one.

There are several occasions where it is unclear which statement(s) is/are referred to as argument for a specific conclusion, of the same kind presented through an example in the section Procedure of data analysis. This type of uncertainty occurs in chains of arguments, where one statement is part of several single argumentations within a section of a lecture. I have found only one example where the lecturer talks more explicitly about the relationships between statements in a chain of arguments, while all other arguments are signaled by more simple connect-words, if signaled explicitly at all. This one example is from the statistics lecture, where the lecturer refers to what they did recently (they told what type of random variable x was) and to what assumptions have been made (that a certain probability equals 0.7) as arguments for what parameters the random variable has.

Conclusions

The study of use- and object-statements shows a potential difference between the two lectures regarding some epistemological aspects. However, since many statements seem easy to reformulate into the other type of statement, there is some arbitrariness, but not necessarily randomness, regarding what type of statement is used. These observations highlight the questions if or how these properties of discourse can be seen as tied to the individual, to the mathematical content, to the type of course, or to other aspects of the situation. Such questions seem possible to examine in more detail using the type of method for data analysis presented in this paper. However, it is also necessary to relate to a theoretical perspective since the interpretation of the results from the data analysis depends on a chosen theory. For example, differences in discourse can be seen as mainly caused by "properties" of the lecturer or as constitutive in the situation (see the section about theoretical perspectives).

The analysis in this paper has focused on explicit argumentative connections between statements in the discourse of mathematics lectures. The fact that many statements in the lectures are not part of this analysis together with the unclear uses of connect-words that have been observed show that more implicit types of argumentation seem common, at least in the lectures studied here. Other types of analyses are therefore needed in order to characterize these implicit types of argumentation. More generally, as Duval (1999) points out; "argumentation cannot actually be reduced to the use of a single argument". Thus, in order to capture also more implicit aspects of argumentation, the analysis cannot focus only on linguistic aspects but needs to take into account for example contextual aspects, including "the position of the person being spoken to relative to the arguer […], the motivation of the argumentation […] and its objective" (Duval, 1999).

If students are mostly exposed to the more implicit types of argumentation it would be interesting to examine how students interpret these and how they handle situations where more explicit argumentations are demanded from them, which might be the case in exams. In addition, it would be interesting to compare epistemological characterizations of different settings (not only lectures) for the same person and of communication of different persons (in particular to include also students) within one setting.

In conclusion, on the one hand I have shown the usefulness of the type of analysis presented in this paper, but on the other hand I have also noted a need for other types of analyses in order to better characterize epistemological aspects of mathematical discourse.

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