An experimental study of fiber suspensions between counter-rotating discs

by

Charlotte Ahlberg

November 2009
Technical Reports from
Royal Institute of Technology
KTH Mechanics
SE-100 44 Stockholm, Sweden

©Charlotte Ahlberg 2009
Universitetsservice US–AB, Stockholm 2009
An experimental study of fiber suspensions between counter-rotating discs
Charlotte Ahlberg 2009,
KTH Mechanics
SE-100 44 Stockholm, Sweden

Abstract

The behavior of fibers suspended in a flow between two counter-rotating discs has been studied experimentally. This is inspired by the refining process in the papermaking process where cellulose fibers are ground between discs in order to change performance in the papermaking process and/or qualities of the final paper product.

To study the fiber behavior in a counter-rotating flow, an experimental setup with two glass discs was built. A CCD-camera was used to capture images of the fibers in the flow. Image analysis based on the concept of steerable filters extracted the position and orientation of the fibers in the plane of the discs. Experiments were performed for gaps of $0.1 - 0.9$ fiber lengths, and for equal absolute values of the angular velocities for the upper and lower disc. The aspect ratios of the fibers were 7, 14 and 28.

Depending on the angular velocity of the discs and the gap between them, the fibers were found to organize themselves in fiber trains. A fiber train is a set of fibers positioned one after another in the tangential direction with a close to constant fiber-to-fiber distance. In the fiber trains, each individual fiber is aligned in the radial direction (i.e. normal to the main direction of the train).

The experiments show that the number of fibers in a train increases as the gap between the discs decreases. Also, the distance between the fibers in a train decreases as the length of the train increases, and the results for short trains are in accordance with previous numerical results in two dimensions. Furthermore, the results of different aspect ratios imply that there are three-dimensional fiber end-effects that are important for the forming of fiber trains.

Descriptors: Fiber suspension flow, rotating discs, shear flow, self-organization of fibers.
Enjoy!
Preface

Parts of this work has been presented at

- SIAMUF (Swedish Industrial Association for Multiphase Fluids) conference May 13–14, 2009 in Älvkarleby, Sweden.
- ASME (American Society of Mechanical Engineering) Fluid Engineering Division Summer Meeting August 3–6, 20091 in Vail, Colorado, USA.

The supervisors have been L. D. Söderberg2 and F. Lundell3.

November 2009, Stockholm

Charlotte Ahlberg

---

1Written contribution to the conference proceedings: "Self-organization of fibers in flow between two counter-rotating discs", C. Ahlberg, F. Lundell and L. D. Söderberg.
2at Innventia AB and KTH Mechanics.
3at KTH Mechanics.
## Contents

Abstract iii  
Preface v  

**Chapter 1. Introduction**  
1.1. Short description of the papermaking process 1  
1.2. Aim and scope of this thesis 5  

**Chapter 2. Background and previous work**  
2.1. Non-dimensional numbers 9  
2.2. Similarity solution for flow between rotating discs 10  
2.3. Characteristics of flow between rotating discs 14  
2.4. Settling velocity 15  
2.5. Fiber suspensions 16  
2.6. Orientation of fibers in shear flow 17  
2.7. Cylinders in shear flow 20  
2.8. Pattern formations of particles in shear flow 22  

**Chapter 3. Experimental set-up, methods and data evaluation** 25  
3.1. Experimental set-up 25  
3.2. Evaluation techniques 29  
3.3. Experimental procedures and parameters 31  
3.4. Characteristics of the flow 34  
3.5. A note on statistical reliability of the data 35  

**Chapter 4. Results and discussion** 39  
4.1. Flow conditions 39  
4.2. Orientation and position of fibers 40  
4.3. Behavior and occurrence of fiber trains 45
4.4. Distance between fibers in trains 51
4.5. Angle deviation of fibers in trains 55
4.6. Effect of fiber length 56

Chapter 5. Conclusions and outlook 59

Acknowledgements 61

References 63

Appendix A. Measurement data 67

Appendix B. Position-orientations pictures 71
This licentiate thesis deals with the orientation and behavior of fibers suspended in the shear flow between two rotating, closely spaced and parallel flat discs. Experiments have been performed and show a complicated interaction between the flow, the solid walls and the solid fibers, where the fibers are seen to form patterns. The results have been analyzed and compared to other published studies.

This research has been inspired by the papermaking industry, where the refining stage in the papermaking process includes fiber suspension flow in narrow conduits. The work can also be related to other areas where pattern-forming of rod-like particles are in focus. For example, lubrication of bearings is one area where pattern formation of particles can be of importance. There, minimization of energy loss due to friction is the goal and present research concerns liquid crystals forming patterns that improve lubrication and minimize the friction, see Abrahamson (2008).

1.1. Short description of the papermaking process

In order to be able to view this thesis as part of a greater whole, a brief description of the pulp and papermaking process follows. A simplified and schematic picture of the papermaking process is shown in Fig. 1.1.

First, the pulp is produced and the process starts with removing the outer insulating and protective layer on the tree trunk, i.e. the bark, which is done in a debarker. The wood is then chopped into chips and treated in order to liberate the individual cellulose fibers. For mechanical pulp this is done by grinding and heat treatment, and for chemical pulp, chemicals are used instead of grinding. There can also be a combination of these treatments. At this point in the manufacturing process, the pulp, i.e. the suspension, consists of cellulose fibers and water that can be delivered to the papermaking plant.

In the papermaking process, Fig. 1.1, the delivered pulp is worked up to a pulp suspension. Chemicals are added in order to modify the fibers to improve desired qualities, e.g. the dewatering ability within the paper process or tensile index and light-scattering abilities of the product, see e.g. Norman & Fellers (1998). The suspension is then ground in a refiner to nap the surfaces of the
fibers. There are two types of refiners, high consistency (HC) and low consistency (LC) refiners, where the pulp concentration in a HC refiner is typically $25 - 35\%$ and $2 - 6\%$ in a LC refiner. When the refining and fractionation steps have produced a satisfactory suspension, it is jetted out, through a headbox and out onto a wire, as a thin suspension sheet. The dewatering on the wire makes the fibers form a network on it, giving a fiber mat that starts to look like paper. Mechanical dewatering squeezes out the remaining water through pressing and after that, water still trapped in the fibers is evaporated in the drying section. After the dewatering stages, the surface of the paper can be improved in mainly two ways, either by calendering, where the surface is improved mechanically by being pressed between several cylinders, or by coating, where a coat is added onto the paper surface. The final stage is the winding of the finished paper onto a reel at the reel-up.
1.1. SHORT DESCRIPTION OF THE PAPERMAKING PROCESS

Figure 1.3. Schematic picture of (a) a typical rotor-stator refiner, with (b) a close-up of the bars and grooves.

Figure 1.4. Example of pattern of grooves and bars for a refiner segment (US Patent no. 5362003).

Focusing on the refining stage, the fiber suspension is ground between discs in order to nap the surface of the fibers and/or to cut the fibers. This changes processes and also final properties of the paper product, e.g. napping makes the dewatering process more difficult but improves the strength and formation of the paper. Images of chemically treated fibers before and after the refining are shown in Fig. 1.2. Comparing (a) and (b), it is clearly seen that the refiner effects the fibers and that smaller pieces, e.g. fines, are created.

Depending on the type of refiner it can consist of a rotor and a stator, or two stators and one rotor in between or of other combinations. A schematic picture of a refiner with one rotor and one stator is shown in Fig. 1.3(a). The inflow of the suspension is located in the center of one disc and the outflow is at the periphery of the discs, where the suspension continues to the next step in the papermaking process. On the surfaces of the discs, which are divided into segments, there are grooves and bars, that can be found in many different patterns and sizes depending on the purpose and the manufacturer,
1. INTRODUCTION

Figure 1.5. The general flow pattern in grooves according to Fox et al. (1979). The secondary flow is indicated with dashed curves and the tertiary flow with dotted curves.

see Figure 1.4 for one example. Fibers in the suspension can get entangled and form flocs and Fig. 1.3(b) shows a close-up of how fiber flocs might be caught between the bars of the rotor and stator. Besides the size and layout of the bars and grooves, there are other parameters to consider, e.g. refiner diameter, rotating speed and gap distance. A real industrial LC-refiner has a diameter in the order of 1 m and a speed of rotation in the order of 1000 rpm. When two bars, one on the stator and one on the rotor, are crossing each other the gap in between is very small, only in the order of a couple of tenths of millimeters, and the fibers in the suspension are exposed to a high pressure pulse and considerable shear forces. The flow in the grooves becomes a fast swirling flow, driven by each bar crossing, the centrifugal forces and the overall flow through the refiner.

The energy consumption in the refining stage is substantial, typically in the order of 200 kWh/t for copy paper and 10 times higher for bank notes (made of cotton), which is much greater than the energy that is actually needed to treat the fibers. A large amount of energy is dissipated to heat, since both the water and the refiner get hot during the process.

The general flow behavior of a fiber suspension inside a refiner was investigated by Fox et al. (1979). They used a laboratory refiner, with a diameter of 12 in (≈ 30.5 cm), made of clear plastic and filmed the flow of the cellulose fiber suspension. The speed of the rotor was up to 1750 rpm. They observed an inward flow in the grooves of the stator and an outward flow in the rotor, creating a recirculating flow, except near the outlet, see Fig. 1.3(a), where both flows were outwardly directed. These flows together with the small circumferential flow due to the rotor were called primary flows. The secondary flow, see Fig. 1.5, was the swirling flow in the grooves of both discs, consisting of a large recirculation cell and smaller recirculation zones in the corners. The final flow
or tertiary flow, see Fig. 1.5, was directed from the corner of a groove (where the smaller recirculation zones are) in the stator towards and past the corner of the nearest bar. In addition to this qualitative description of the flow of the suspension, the behavior of the fibers was also qualitatively described by Fox et al. (1979). They observed that at a specific radius, the fibers were stapled to the bars of either the rotor or the stator, and concluded that the tertiary flow held the fibers to the bars and it was mainly these stapled fibers that were refined.

Later, Koskenhely et al. (2005) studied the effects of different ways of refining different fractions of bleached softwood kraft pulp; a 56/44% mixture of pine/spruce fibers. Fiber length, fiber cross-sections, fines content, amount of water in the cell wall of the cellulose fiber and cell wall thickness were measured for the different pulp fractions. Several connections between paper properties and selective refining could be seen, e.g. that the fiber shortening was most severe for higher intensities. However, it was not clear whether the cutting was due to high tension or to the bar edges, or if it was a fatigue process or a single-event phenomenon.

Though there are many more studies with cellulose fibers and refiners, they deal mainly with the material (pulp) input contra the material output of the refiner and not the treatment inside it. At this point in the review it can be concluded that it is not known what physical events that treat the fibers nor the details of how the fiber suspension behaves inside a refiner. It is a complex environment with many parameters. Due to this, the refining process in industry can, at the present time, be considered to be an art, based on knowledge and experience. The refiner at a plant is adjusted based on the information of the input pulp and by knowing the behavior of the refiner at that particular plant. Then, the output of the refiner becomes the desired, perhaps after some iterative adjustments.

1.2. Aim and scope of this thesis

The aim of this thesis is to investigate what the flow looks like and how the fibers behave in a basic geometry, similar to what can be found inside a refiner, but at lower velocities. The vision is of course to completely understand the flow inside the real refiner and more important the treatment of the fibers. Then, it would be possible not only to predict the result of the refining without trial and error, but also to improve the process and to save energy. One can also hypothesize that improved understanding of the refining process will reveal the physical reasons behind the effects. With such knowledge, the desired physical fiber treatments might be possible to achieve more efficiently in some completely different device.
1. INTRODUCTION

As described in the previous section, a real refiner with all its parameters is a very complicated environment. In order to increase the knowledge and understanding, it is necessary to start with a basic case with few parameters that can be controlled. Only then is it possible to introduce more parameters one after another and gradually build up a more complicated environment and to understand it. However, there seem to be a lack of fundamental research of fibers in shear and rotating flows in the literature.

In order to approach our vision we proceed according to the following strategy regarding the experimental set-up and parameter range:

1. Basic geometry, low Reynolds numbers
2. More complex geometry, low Reynolds numbers
3. More complex geometry, increasing Reynolds numbers

This thesis is the first step of the strategy, where the basic geometry of the experimental set-up consists of two coaxial flat discs, exactly counter-rotating and separated by a small gap distance. The gap between the two flat discs are often simply called ‘the gap’. Single phase flow in this geometry has been studied extensively. Thus, the flow in the present study is well established and will be described in the review in section 2.2. To understand the behavior of fibers in a suspension, parts dealing with a falling fiber in quiescent fluid, characteristics of fiber suspensions, cylinders in shear flow and pattern formations are included in this work.

Furthermore and concerning the fibers in the suspension, real cellulose fibers would have shown too large deviations in length, thickness and stiffness, which in turn would have introduced too many unknown parameters. Therefore, artificial fibers with a well-defined length and thickness have been used in this work.

During the initial observations of the fiber behavior in this thin and rotating environment, we discovered that the fibers tend to form patterns, and the latter part of this work came to focus on the occurrence and structure of these fiber patterns.

This thesis continues with a review of relevant fluid mechanical studies of flow between rotating discs and particles in shear flow in chapter 2. The experimental apparatus and methods, including data evaluation, are described in chapter 3. In chapter 4 the results are presented and discussed before the conclusions are stated in chapter 5 together with an outlook. Two appendices

---

1 Using Reynolds number is one way to characterize the flow and is the ratio between the inertial and viscous terms in the Navier-Stokes equations for an incompressible fluid.

2 with exactly counter-rotating is meant that the absolute values of the angular velocities of the upper and lower disc are the same.
are included: the first is a complete overview of all parameter combinations studied and the second is the complete data on fiber position and orientation.
CHAPTER 2

Background and previous work

This review chapter starts with single-phase flow between counter-rotating discs and continues with fiber suspensions and orientations of slender bodies in confined shear flow. Finally, work concerning pattern formation of particles in shear flow are reviewed.

2.1. Non-dimensional numbers

In order to study the problem of a fiber suspension in between counter-rotating discs, schematically illustrated in Fig. 2.1, we define the following set of non-dimensional numbers:

- \( R_{H} = \frac{2hR\Delta\Omega}{\nu} \) as a gap Reynolds number, where \( 2h \) is the vertical gap distance between the discs, \( R \) is the radial coordinate, \( \Delta\Omega = |\Omega_{up}| + |\Omega_{low}| \) is the difference between the angular velocity of the upper disc, \( \Omega_{up}(>0) \), and the angular velocity of the lower disc, \( \Omega_{low}(<0) \). \( \nu \) is the kinematic viscosity of the fluid.
- \( R_{\Omega} = \frac{\Omega}{\Delta\Omega/2} \) as a second gap Reynolds number, where \( \Omega = \Delta\Omega/2 \).
- \( R_{e\phi} = \frac{\Delta\Omega R^2}{\nu} \) as a rotational Reynolds number.
- \( R_{ep} = \dot{\gamma} \frac{a^2}{\nu} \) as a particle Reynolds number, where \( a \) is the radius of a fiber and \( \dot{\gamma} \) is the shear rate, \( \dot{\gamma} = R\frac{\Omega}{h} \).
- \( G = \frac{2h}{R_2} \) as a gap ratio, where \( R_2 \) is the radius of the upper disc.
- \( \lambda = \frac{2h}{L} \) as the ratio between the gap and the length of a fiber, \( L \).
- \( \alpha = \frac{L}{d} \) as the aspect ratio for a fiber, where \( d = 2a \).
- \( \xi = \frac{\rho_p}{\rho_f} \) as the ratio between the density of the particles, \( \rho_p \) and the density of the fluid, \( \rho_f \).

Having a two disc problem with a gap in between, we also define

\[
    s = \frac{\Omega_{up}}{\Omega_{low}}
\]  

(2.1)

Hence, \( s = -1 \) in the case of exactly counter-rotating discs, i.e. when \( |\Omega_{up}| = |\Omega_{low}| \), and \( s = 1 \) is the case of equally rotating discs, i.e. the case where \( \Omega_{up} = \Omega_{low} \) and we would obtain a solid body rotation as the steady state solution.
2. BACKGROUND AND PREVIOUS WORK

2.2. Similarity solution for flow between rotating discs

The flow between two counter-rotating discs is assumed steady and rotationally symmetric and the dimensional cylindrical coordinates are \((R, \phi, Z)\). The centers of the discs are at \(Z = h\) and \(Z = -h\), respectively. If we denote the dimensional velocity vector \(\mathbf{u} = u \mathbf{e}_r + v \mathbf{e}_\phi + w \mathbf{e}_z\), the Navier-Stokes equations are:

\[
\frac{u}{R} \frac{\partial}{\partial R} \left( R \frac{\partial u}{\partial R} \right) + \frac{w}{\partial Z} \left( \frac{\partial u}{\partial Z} \right) = - \frac{1}{\rho} \frac{\partial p}{\partial R} + \nu \left( \nabla^2 u - \frac{u}{R^2} \right) \quad (2.2)
\]

\[
\frac{u}{R} \frac{\partial}{\partial R} \left( R \frac{\partial v}{\partial R} \right) + \frac{w}{\partial Z} \left( \frac{\partial v}{\partial Z} \right) = \nu \left( \nabla^2 v - \frac{v}{R^2} \right) \quad (2.3)
\]

\[
\frac{u}{R} \frac{\partial}{\partial R} \left( R \frac{\partial w}{\partial R} \right) + \frac{w}{\partial Z} \left( \frac{\partial w}{\partial Z} \right) = - \frac{1}{\rho} \frac{\partial p}{\partial Z} + \nu \nabla^2 w \quad (2.4)
\]

The continuity equation is:

\[
\frac{\partial}{\partial R} (Ru) + \frac{\partial}{\partial Z} (Rw) = 0 \quad (2.5)
\]

and the Laplace operator is:

\[
\nabla^2 = \frac{1}{R} \frac{\partial}{\partial R} \left( R \frac{\partial}{\partial R} \right) + \frac{\partial^2}{\partial Z^2} \quad (2.6)
\]

The pressure can be eliminated by cross-differentiation of eqs. (2.2) and (2.4), giving:

\[
\frac{\partial}{\partial Z} \left\{ u \frac{\partial u}{\partial R} + w \frac{\partial u}{\partial Z} - \frac{v^2}{R} - \nu \left( \nabla^2 u - \frac{u}{R^2} \right) \right\} =
\]

\[
= \frac{\partial}{\partial R} \left( u \frac{\partial w}{\partial R} + w \frac{\partial w}{\partial Z} - \nu \nabla^2 w \right) \quad (2.7)
\]
2.2. SIMILARITY SOLUTION FOR FLOW BETWEEN ROTATING DISCS

The continuity equation will be satisfied if we take:

\[
\begin{align*}
  u &= \frac{1}{R} \frac{\partial \Psi}{\partial Z} \\
  w &= -\frac{1}{R} \frac{\partial \Psi}{\partial R}
\end{align*}
\]  

Putting eqs. (2.8) and (2.9) into eqs. (2.7) and (2.3), gives us two equations with two unknowns; \( v \) and \( \Psi \).

Assuming that both discs are of radius \( R_2 \) and that the gap ratio \( G \ll 1 \), we can assume that the edge effects can be neglected, i.e. assuming that the discs are infinitely large in the sense that \( G \approx 0 \). The radial and vertical coordinates are non-dimensionalized with the length \( h \), i.e. \( r = R/h \) and \( z = Z/h \) where \( r \) and \( z \) are non-dimensional coordinates. The velocities are scaled with \( h\Omega \), i.e. \( \hat{u} = u/h\Omega, \hat{v} = v/h\Omega \) and \( \hat{w} = w/h\Omega \). One can now assume that the non-dimensional velocity field \( \hat{u} = \hat{u}_r e_r + \hat{u}_\phi e_\phi + \hat{u}_z e_z \) is of the following form, using the similarity ansatz of von Kármán (1921), see e.g. Greenspan (1990) or Hoffman (1974):

\[
\begin{align*}
  \hat{u} &= -\frac{1}{2} \Re r \frac{d\mathcal{H}}{dz}(z) \\
  \hat{v} &= r \mathcal{G}(z) \\
  \hat{w} &= \Re \mathcal{H}(z)
\end{align*}
\]

where \( \mathcal{G}(z) \) and \( \mathcal{H}(z) \) are non-dimensional functions and eqs. (2.10)–(2.12) automatically satisfies the non-dimensional continuity equation, which is eq. (2.5), that after substitution to the non-dimensional variables becomes the same but with the non-dimensional coordinates and velocities.

Substituting eqs. (2.10)–(2.12) into the Navier-Stokes equations gives the following expressions for \( \mathcal{G} \) and \( \mathcal{H} \):

\[
\begin{align*}
  \frac{d^4\mathcal{H}}{dz^4} &= 4\mathcal{G} \frac{d\mathcal{G}}{dz} + \Re^2 \mathcal{H} \frac{d^3\mathcal{H}}{dz^3} \\
  \frac{d^2\mathcal{G}}{dz^2} &= \Re^2 \left( \mathcal{H} \frac{d\mathcal{G}}{dz} - \mathcal{G} \frac{d\mathcal{H}}{dz} \right)
\end{align*}
\]

The no-slip condition at the discs \( (z = \pm 1) \) in terms of \( \mathcal{G} \) and \( \mathcal{H} \) becomes, for exactly counter-rotating discs:

\[
\begin{align*}
  \mathcal{G}(\pm 1) &= \pm 1, \quad \mathcal{H}(\pm 1) = 0, \quad \frac{d\mathcal{H}}{dz}(\pm 1) = 0
\end{align*}
\]

The algorithm for solving this system of equations was outlined by Hoffman (1974), but has here been solved using spectral collocation method and
2. BACKGROUND AND PREVIOUS WORK

Figure 2.2. Non-dimensional velocity profiles in the (a) radial, (b) tangential and (c) vertical direction, for the single-phase flow between two exactly counter-rotating discs. Profiles are for $10^{-1} \leq \mathcal{R} \leq 10^{0.5}$, where the solid curves represent the highest Reynolds number.

Chebychev polynomials, see Weideman & Reddy (2000) for more details of this numerical method.

The non-dimensional velocity profiles $(u/R\Omega, v/R\Omega, w/h\Omega)$ are plotted in Figs. 2.2 and 2.3 for $10^{-1} \leq \mathcal{R} \leq 10^{0.5}$ and $10^{0.5} \leq \mathcal{R} \leq 10^2$, respectively.

For low $\mathcal{R}$, the velocity profile in the tangential direction, see Fig. 2.2(b), is practically a simple shear flow profile with the same velocity as the lower disc at $z = -1$ and as the upper disc at $z = 1$. In the middle, at $z = 0$, the tangential velocity is zero. At higher $\mathcal{R}$, see Fig. 2.3(b), the profile becomes less linear and a core region with $v/h\Omega \approx 0$ develops, and two separate boundary layers are seen near the discs, where the velocity gradient is large.

The radial velocity, $u/R\Omega$ in Fig. 2.2(a), is zero at the rotating discs because of the no-slip condition, and is outward directed (positive) close to each disc due to the centrifugal force. To balance the outward flow, the flow is directed inwards (negative values) in the middle. It can be seen in the same figure that the zero velocity points between the inward and the two outward directed flow regions stay at the same vertical positions $|z| \approx 0.5$ for increasing but still relatively low $\mathcal{R}$. It is the value of the maximal velocity in each of the three regions that increase with increasing $\mathcal{R}$. When $\mathcal{R}$ increases further, see Fig. 2.3(a), the zero velocity points come closer to the discs and the maximal velocity in the middle region (in this scaling) reaches a maximum and then
2.2. SIMILARITY SOLUTION FOR FLOW BETWEEN ROTATING DISCS

starts to decrease, creating a core growing in height with $\mathcal{R}$ and with equal radial velocity, and two separate boundary layers near the discs.

Finally, the vertical velocity for low $\mathcal{R}$, $w/h\Omega$ in Fig. 2.2(c), is zero in the middle of the gap at $z = 0$, and directed towards the lower disc for $z < 0$ and towards the upper disc for $z > 0$, creating two circulation cells in the $rz$-plane. At the disc surfaces the vertical velocity is zero since the discs are not permeable. For increasing $\mathcal{R}$, see Fig. 2.3(c), the profile of the vertical velocity is similar to the vertical velocity profile for lower $\mathcal{R}$ (see Fig. 2.2(c)), but with the maximal absolute value of the velocity occurring at a position closer to the disc for each circulation cell.

Furthermore, Fig. 2.2 shows that for small $\mathcal{R}$ the dominant velocity is the tangential velocity component, $v/R\Omega$. The radial component $u/R\Omega$ is at least one tenth smaller in magnitude. Moreover, $w/h\Omega$ has been scaled with $h$ instead of $R$. Multiplying $w/h\Omega$ with $h/R$ and knowing that $R \gg h$ for narrow gaps between large discs (as in our case) excluding the hub region where $h \sim R$, implies that $w/h\Omega \geq w/R\Omega$. This in turn means that $w/R\Omega$ is much smaller than $v/R\Omega$, and also smaller than $u/R\Omega$.

**Figure 2.3.** Non-dimensional velocity profiles in the (a) radial, (b) tangential and (c) vertical direction, for the single-phase flow between two exactly counter-rotating discs. Profiles are for $\mathcal{R} = 10^{0.5}$ (dotted curves), $\mathcal{R} = 10^1$ (dash-dotted curves), $\mathcal{R} = 10^{1.5}$ (dashed curves) and $\mathcal{R} = 10^2$ (solid curves).
2.3. Characteristics of flow between rotating discs

A lot of research has been done concerning two rotating discs with a large gap and at high Reynolds numbers. At these high Reynolds numbers fully developed boundary layers can be assumed on both discs, but for small distances the two boundary layers are joined as was shown in Fig. 2.2.

Table 2.1. Flow classification according to Daily & Nece (1960). Regimes I and III have merged boundary layers, whereas they are separate in Regimes II and IV. Low Reynolds numbers indicate laminar flow.

<table>
<thead>
<tr>
<th>small clearance</th>
<th>large clearance</th>
</tr>
</thead>
<tbody>
<tr>
<td>low $Re_\phi$</td>
<td>Regime I</td>
</tr>
<tr>
<td>high $Re_\phi$</td>
<td>Regime II</td>
</tr>
<tr>
<td></td>
<td>Regime III</td>
</tr>
<tr>
<td></td>
<td>Regime IV</td>
</tr>
</tbody>
</table>

A useful classification of these types of flow was done by Daily & Nece (1960), who carried out experiments in a liquid-filled rotor-stator configuration. The flow was classified into four regions according to the size of the gap distance and $Re_\phi$, see table 2.1. Regime I is a laminar flow with merged boundary layers existing for all $Re_\phi$ if the gap is sufficiently small. Regime II is also laminar but with separate boundary layers and may never exist for very small gaps, whereas Regime III and IV are turbulent, with merged and separated boundary layers, respectively. Regime III may never exist for very large gaps and Regime IV exists for all gaps if $Re_\phi$ is large enough. The transition to Regime IV occurs at lower $Re_\phi$ as the gap size increases.

Szeri et al. (1983) studied the velocity profiles of a rotor-stator system for a gap ratio of $G = 0.050$, where they used water as fluid and carried out measurements by Laser Doppler Velocimetry. The results were compared with infinite-disc solutions and they found the results to be in good agreement for $Re_\phi \leq 2500$, with a gap size of $2h = 1.26$ cm (0.5 in). They also studied the end-effects for exactly counter-rotating discs with and without boundary modifications in the form of flow-straightener devices. For $Re_\phi \approx 10^5$ the tangential velocity profiles for the two cases coincided for $R/R_2 \leq 0.7$, but it was less apparent to draw a conclusion regarding the effects of boundary modifications on the radial velocities.

In order to see the influence of the boundary effects, Brady & Durlofsky (1987) studied the flow between two large but finite ($G \ll 1$) rotating discs by a combined asymptotic-numerical analysis. They compared their results with the similarity solutions associated with infinite discs and concluded that, to a first approximation, for $\mathcal{R} < 29.95$, it does not matter whether the enclosure
at the periphery rotates or not, since the end-effects are important only to a region of $O(G)$, corresponding to a very small portion of the disc area if the gap is small. Their results obtained with an open boundary show that for $\mathcal{R} = 2.5$ the end-effects are important only for $R/R_2 \geq 0.92$ and therefore, the similarity solutions in Fig. 2.2 are valid throughout the bulk of the flow domain. By valid it is meant that the solution agree for all axial positions at a given radial position with an error of at most 2%. Results obtained with a closed boundary showed that end-effects were important for $R/R_2 \geq 0.8$ at $\mathcal{R} = 10$. Thus, when $\mathcal{R}$ increases the end-effects increase and the area where the similarity solutions are valid is reduced.

Dijkstra & van Heijst (1983) did experiments for $G = 0.036$, where $2h = 35$ mm, and measured the velocity fields by tracing polystyrene spheres of a diameter of 0.3 mm. They covered $50 \leq \mathcal{R} \leq 250$ and several values of $s$, but never $s = -1$ (counter-rotating discs). They did however see a tendency towards symmetric flow solutions for $s = -0.825$.

Gauthier et al. (2002) studied instability patterns in a rotating two-disc-system and for different values of the gap distance, down to $G \approx 0.048$, which in their case implied $h = 3.4$ mm. Anisotropic flakes were added to the water-glycerin mixture to visualize the instabilities, appearing as spiral patterns. They concluded that the threshold for these instabilities depended on the difference in absolute angular velocities between the discs, and therefore would be non-existent for the exactly counter-rotating case at low Reynolds numbers.

Some years later, Moisy et al. (2004) investigated experimentally the onset of instability for counter-rotating discs with a gap ratio of $0.048 \leq G \leq 0.5$. The flow field was visualized with anisotropic flakes in the fluid, and they also carried out measurements using Particle Image Velocimetry and spherical tracer particles. It was found that in the limit of very small $G$, the cell radius $R_2$ has vanishingly small influence on the flow, leading to the conclusion that the relevant Reynolds number was the gap Reynolds number, $\mathcal{R}$.

**2.4. Settling velocity**

It was shown in Fig. 2.2 that the vertical velocity is small, but not zero, and it is therefore necessary to compare the vertical velocity to the settling velocity of a particle in order to be able to interpret its importance. The settling velocity, $W_s$, is the velocity with which a particle would fall in a quiescent liquid with a viscosity $\mu = \rho_f \nu$ as the particle has stopped accelerating, i.e. the drag force, $F_D$, induced by the particle in the liquid equals the gravity force, $F_g$, of the particle in the liquid:

$$F_g = (\rho_p - \rho_f)V_p g$$  (2.16)
Here $V_p$ is the particle volume and $g$ is the gravity constant.

If $A$ is a representative area (perpendicular to the flow velocity vector) of the particle, the definition of the drag coefficient is, see e.g. Acheson (1990):

$$C_D = \frac{F_D}{\frac{1}{2} \rho f W_s^2 A}$$  \hspace{1cm} (2.17)

Using the fact that the drag force and mass force balance each other and that $A = 2aL$ and $V_p = \pi a^2 L$, for a cylindric particle that is aligned perpendicularly to the flow direction, we get an expression for the settling velocity, $W_s$, by combining eqs. (2.16) and (2.17):

$$W_s = \sqrt{\frac{\pi ag}{C_D} (\xi - 1)}$$  \hspace{1cm} (2.18)

As expected, a particle that is neutrally buoyant, i.e. when $\xi = 1$, have a settling velocity equal to zero.

Several empirical formulas describing $C_D$ for a cylinder can be used in order to approximate the drag a fiber particle is subjected to, is found in White (1991). Two accurate formulas for $10^{-4} < Re_D < 2 \cdot 10^5$ are by White (1991) and by Sucker & Brauer (1975), respectively:

$$C_D \approx 1 + \frac{10.0}{Re_D^{2/3}}$$  \hspace{1cm} (2.19)

$$C_D \approx 1.18 + \frac{6.8}{Re_D^{0.89}} + \frac{1.96}{Re_D^{0.5}} - \frac{0.0004Re_D}{1 + 3.6364 \cdot 10^{-7}Re_D^2}$$  \hspace{1cm} (2.20)

These formulas will be used when evaluating the effects of gravitational sedimentation in the present experiments.

2.5. Fiber suspensions

A suspension of fibers behaves differently compared to a suspension consisting of spheres, since the fibers can withhold mechanical forces at lower packing. Also, a suspension of fibers behaves differently depending on the fiber concentration. Kerekes & Schmell (1992) introduced the concept of a crowding factor for fibre suspensions where the crowding factor, $Cr$, is the number of fibers contained in a sphere of diameter $L$. The sphere then represents the volume in which a completely contained fiber can reach all positions by rotating around its center position. The crowding factor is defined as follows:

$$Cr = \frac{2}{3} c_v a^2$$  \hspace{1cm} (2.21)
Table 2.2. Characterization of a fiber suspension flow regimes by the crowding factor, $Cr$, according to Kerekes & Schnell (1992).

where $c_v$ is the volume concentration of fibers and $\alpha$ is the aspect ratio of the fibers. Sometimes it is more convenient to start from the mass concentration of the suspension, $c_m = m_f/m_l$ where $m_f$ is the mass of dry fibers and $m_l$ is the mass of liquid. Equation (2.21) can then be rewritten, with $\sigma$ being the coarseness [mass/length] of the fibers:

$$Cr = \frac{\pi}{6} c_m \frac{L^2}{\sigma} \quad (2.22)$$

The level of collisions of fibers in a suspension, and thus the tendency to form flocs, can therefore be characterized by the crowding factor, as specified in table 2.2.

Moreover, the presence of fibers has rheological implications, which are beyond the scope of the present thesis. The interested reader can find a review on this topic in Petrie (1999).

2.6. Orientation of fibers in shear flow

Consider a slender body or fiber, as sketched in Fig. 2.4, in a shear flow in the $XY$-plane. In this figure, $\Phi$ is the angle between the projection of the fiber in the $XY$-plane and the $X$-axis, and $\beta$ is the angle between the fiber and the $Z$-axis. The origin of the coordinate system is always at the center of the fiber, and hence, the coordinate system moves with the translation velocity of the fiber.

The equations of motion of a solitary ellipsoid suspended in an unbounded laminar simple shear flow, as in Fig. 2.4 with the shear flow in the $XY$-plane, were first solved theoretically by Jeffery (1922), and therefore the orbits, in which the slender body will move are called Jeffery orbits. In the solution by Jeffery (1922), the orbits are characterized by a constant $C$, where $C = 0$ represents a rolling fiber with its long axis aligned with the $Z$-axis in Fig. 2.4. A large value of $C$ corresponds to the angle $\beta$ being close to $90^\circ$, and hence the fiber is almost in the $XY$-plane. In Fig. 2.5(a) the trace of one end of the fiber is plotted for $C = 0.05$, 0.25, 1 and 25, where $C = 0.05$ is the smallest orbit.
The orbits, except the rolling one, performs a flipping motion, since the angle $\Phi$ changes rapidly with time as soon as it deviates from the $X$-direction. The evolution of $\Phi$ as a function of time is plotted in Fig. 2.5. For intermediate $C$-values, the orbit motion is referred to as a kayaking motion, since the fiber motion is similar to the motion of a kayak paddle. For the highest $C$-value, the motion is pure flipping when no walls are present. If a wall is present and the end of the fiber touches it, the motion is modified and referred to as pole-vaulting.

Stover & Cohen (1990) observed neutrally buoyant rodlike particles with aspect ratio of $\alpha = 12$ and suspended in a pressure-driven flow in a Hele-Shaw cell, i.e. between two flat plates, with $\lambda = 3, 6 \text{ and } 20$, and at low Reynolds numbers. Both the orientation and position of the particles were recorded in three dimensions by combining pictures from two different cameras. It was observed that particles oriented close to their vorticity axis and placed less than $L/2$ from the wall, remained close to the wall and had a somewhat longer period of rotation than predicted by theory with no wall present. Furthermore, the flipping particles also had a longer period of rotation and were forced by the interaction of the wall to a position of $L/2$ from the wall after a pole-vaulting motion.

Later, Carlsson et al. (2007) carried out experimental studies of orientation of fibers suspended in shear flow near a wall. In this study, the fibers situated close to a smooth wall mostly oriented themselves perpendicularly to the flow, and fibers positioned more than $L/2$ from the wall were oriented in the direction of the flow.
2.6. ORIENTATION OF FIBERS IN SHEAR FLOW

Figure 2.5. In (a): Jeffery orbits for $C = 0.05$, $0.25$, $1$ and $25$. In (b): the angle $\Phi$ showing the flipping behavior of a fiber in shear flow.

Holm & Söderberg (2007) experimentally studied a laminar fiber suspension flow down an inclined plane, and the fiber orientation was extracted. Fibers were found to be mostly oriented perpendicularly to the flow direction near the wall. Moreover, it was found that the number of such fibers decreased as the
2. BACKGROUND AND PREVIOUS WORK

Figure 2.6. Sketch of rotating cylinders in linear shear flow.

fiber aspect ratio increased. Fibers far from the wall were mostly aligned in the flow direction.

Slender bodies are here interpreted as fibers, which are the interest for the papermaking industry, but slender bodies are also of interest to other research areas, e.g. geology. Ventura et al. (1996) investigated the orientation of plagioclase crystals in high-viscosity lava flows. The lava flow is similar to a pipe flow and plagioclase are crystals with a blade shape and rectangular with an approximate length-to-width ratio of $3.5 - 4$. The investigation was carried out by taking photos of thin sections of the solidified lava and analyzing these pictures. It was found that the angle between the crystal and the ground in the flow direction (corresponding to $\Phi$ in Fig. 2.4) was close to zero in the proximity of the lower boundary (the ground), and increased quickly with the distance. This relationship was seen to an angle of approximately $40^\circ$ before a random distribution was seen for crystals in the middle between the ground and the upper boundary (crust). Near the crust, the angle between the crystal and the crust decreased as the distance between the crystal and the crust decreased. Ventura et al. (1996) concluded that the crystal alignments could be used to identify different deformation zones in the lava flow and that the textural features of the lava could be explained by the flow profile. However, it should be noted that a lava flow is complicated, e.g. the final deformation event modifies the previous crystal alignment and the pressure increases in the middle as the lava solidifies.

2.7. Cylinders in shear flow

In a flow between two closely spaced and counter-rotating discs, the flow in the tangential direction at radius $R$ is a linear shear flow between two walls, see
2.7. CYLINDERS IN SHEAR FLOW

Fig. 2.6, where the upper and lower walls have the velocities $R\Omega$ and $-R\Omega$, respectively. Radially aligned fibers can then be viewed as rotating cylinders, as illustrated in the figure. Studies can be found regarding one rotating cylinder in a linear shear flow, but the torque in the literature is not always zero, i.e. the cylinder is not necessarily rotating freely. Other properties to take into consideration is whether the flow is bounded by walls or not and how far away these walls are positioned. Also, if the flow is co- or counter-flowing and if there are one or several cylinders and finally if the cylinders are neutrally buoyant (if not fixed) or not.

Zhao et al. (1998) studied a freely rotating and translating cylinder with a diameter $d$ in a pipe flow in a 2D channel with a height of $2h = 10d$. A volume-based finite-element method was used and the results (considering only $Z \leq 0$) showed that, for $Re_z = 2hU/\nu = 1$ ($U$ is the mean velocity in the channel), the lift was positive for $Z/h < -0.7$ but negative for other values ($-0.7 < Z/h < 0$), except in the middle of the channel $Z/h = 0$ where it also was zero. For increasing Reynolds numbers, up to 50, the zero lift point moved somewhat closer to the wall.

For particles heavier than the fluid, the gravity force have to be balanced by a lift force in order for the particles to find stable positions where their vertical velocity is zero. This was studied, with cylindrical particles of diameter $d$ in a 2D co-flowing linear shear flow by Feng & Michaelides (2003) They used the lattice Boltzmann method and did calculations for particle Reynolds numbers $0.5 \leq Re_p \leq 5$ and for density ratios $1.001 \leq \xi \leq 1.1$. The channel used had a height of $10d$ and a width of $20d$, and the shear rate was $\dot{\gamma} = U_W/(2h)$. The upper wall moved with velocity $U_W$ and the lower wall was stationary. They calculated the equilibrium positions when both the lift created by the shear and by the proximity of the wall were balanced by the gravity force of the particle. If the gravity force was too large, the cylinders were not able to migrate away from the wall but settled instead. A settled particle is defined as having a clearance of $< 0.2d$ to the lower wall. Feng & Michaelides (2003) proposed the following equation:

$$Re_{p,cr} = \frac{58}{4} (\xi - 1)^{0.59}$$

(2.23)

in order to describe the curve separating the two regions of settling and migrating particles. It is plotted in Fig. 2.7 for a larger range of $\xi$ than investigated in the study by Feng & Michaelides (2003). In this figure, it can be seen that lighter particles, but still heavier than the fluid, need a smaller $Re_p$ in order to overcome its weight and start migrating. In their study it was also observed that lighter particles migrates to a higher equilibrium position, compared to heavier particles.
If a neutrally buoyant particle in counter-flowing shear flow is exactly in the middle, the lift force is zero, i.e. the centerline is a stable position. This can be deduced from symmetry and was confirmed by Yan et al. (2007), who made 2D-calculations for the flow case as in Fig. 2.6 based on the lattice Boltzmann method for particulate flow. In their case, the two cylinders were neutrally buoyant and freely rotating in the shear flow where the upper and lower walls were bounding the counter-flowing flow, $h/a = 2$ and $4$. In addition to confirming that the centerline between the two walls is a stable position to which the rotating cylinders migrate, they also looked at the lateral plane and found that there is a critical separation distance of $6.5d$ between the centers of the two cylinders for $Re_H < 384$ and $Re_p \leq 3.0$. Beyond that critical distance the center line is always a stable position, but if the cylinders were placed closer than that distance they would repel each other, both vertically and horizontally, and then migrate towards the centerline.

2.8. Pattern formations of particles in shear flow

An experimental observation of self-organization of particles was reported by Matas et al. (2004). They studied the behavior of neutrally buoyant spherical particles (diameter $d$) in a pipe flow, for Reynolds numbers, based on the mean axial velocity and the diameter of the pipe ($D$), $Re < 2500$ and for a particle
Reynolds number, $Re_p = Re(d/D)^2$, of approximately up to 10. By taking pictures of the flow and by identifying and counting the particles by eye, they observed long-lived trains of spheres aligned with the flow. The mean surface separation distance between the particles decreased with increasing $Re_p$. The trains could be very long and were observed to form at a position 0.6 times the radius of the pipe, known as the Segré-Silberberg equilibrium annulus, for details see Segré & Silberberg (1962).

Similar effects as those of Matas et al. (2004) has also been observed in a numerical simulation by Chun & Ladd (2006). They used a square duct flow in which they simulated neutrally buoyant spherical particles and their migration, by using the lattice-Boltzmann method. The particle diameter was about 1/10 of the channel dimension and trains were observed to form at equilibrium positions if the suspension was dilute and if the Reynolds number, based on the width of the channel and the mean velocity, was in the range 100 to 1000. At higher Reynolds numbers the trains broke up and the particles migrated towards an equilibrium position in the middle ($Re \sim 1000$) and formed clusters.

Di Carlo et al. (2007) investigated positioning of particles with different geometries by doing experiments in laminar flow in microchannels, and unexpectedly observed pattern formations both longitudinally (along the streamlines) in regular chains and rotationally for discoidal red blood cells. By changing the symmetry of the channel, they then succeeded in ordering spherical particles of diameter 9 µm with good accuracy in three spatial dimensions. They concluded that the ordering was independent of particle density for their cases, 0.85 < ξ < 1.35, and that the migration was due to lift forces on the particles. The experiments of blood cells were done within a 50 µm square channel (width $w$ and height $H$). For a Reynolds number of 60, based on the hydraulic diameter ($D_h = 2wH/(w+H)$) and maximum velocity of the channel, the cells formed trains at the bottom and top of the channel with their disc axis, around which they rotated, parallel to the wall. This motion would look something like a rolling tyre on the ground.

Self-organization of particles is also found when a thermally convecting fluid interacts with particles, as Liu & Zhang (2008) have investigated by experiments in a Rayleigh-Bénard convection cell. They observed a cyclic behavior of particles reorganizing from one side of the convection cell to the other.
CHAPTER 3

Experimental set-up, methods and data evaluation

3.1. Experimental set-up

An experimental apparatus has been set up in order to study the flow of a fiber suspension between two perfectly counter-rotating discs and a schematic overview of the complete experimental set-up in the water laboratory at KTH Mechanics is shown in Fig. 3.1. The laptop computer controls the camera (CAM) as well as the power generators (V), that give voltage to the motors \( M_1 \) and \( M_2 \). Two sensors (S) read the speed of the discs and their signals are monitored by the oscilloscope (OSC) and sent to the computer via an A/D-converter. The light source is also indicated, with its three-headed gooseneck lamp.

3.1.1. The discs

A sketch of the main parts of the experimental apparatus is shown in Fig. 3.2(a) with a close-up in Fig. 3.2(b). The apparatus consists of two flat and horizontal discs made of glass, mounted inside a cylindrical acrylic container with an inner diameter of 45 cm. The upper disc is hanging under a flat 5 mm-thick aluminum ring, which fits into the acrylic container and has an inner diameter of 26 cm. The aluminum ring is in turn hanging on the brim of the acrylic container by long screws and hooks at three points and thus, there is no central axis going through the gap between the discs.

The upper glass disc has a diameter of 30 cm and can be moved vertically by adjusting the long screws, in order to obtain the desired gap distance between the discs. The lower disc is fixed vertically and at the edge of the lower disc, i.e. at radius 20 cm, there is a small wall to prevent the suspension (light gray area in Fig. 3.2(a)) from escaping. Both discs have a thickness of 1 cm. The surface of the upper disc was measured with high-accuracy-shims (accuracy of 0.001 mm) and was found to not vary more than 0.01 mm during one rotation. This variation was mostly due to a small play of the track, in which the disc slides by the aid of a belt connected to the motor. The lower disc is fixed to another disc, denoted A-disc in Fig. 3.2(b) that is mounted on the central axis. The position of the surface of the lower disc was measured with a laser.
3. EXPERIMENTAL SET-UP, METHODS AND DATA EVALUATION

3.1.2. Gap distance

The gap distance \( (2h) \) is obtained by measuring the distance \( A \), see Fig. 3.2(b), by manually sticking a slide calliper through the hole \( C \) and measuring \( A \). The height \( B \) is known and hence the gap distance \( 2h = A - B \). \( A \) and \( B \) were measured at the same three points (indexed 1, 2 and 3) as the positions of the adjustments screws. The screws were adjusted until \( 2h_1 = 2h_2 = 2h_3 \) within the accuracy of the calliper, which is 0.01 mm. The total accuracy of the gap distance is estimated to be \( \pm 0.045 \) mm.

However, due to the fact that the adjustments of the gap were carried out at three points, these points might be in unfortunate positions because of the unevenness of the discs, thus giving a bad starting level of the gap distance. To minimize this error, \( 2h_1 - 2h_3 \) were measured several times, rotating the lower disc between measurements. The distances were also measured before as well as after each measurement series. If the gap distance adjustment turned out to be poor, which also could be seen by a wobbling behavior of the fiber groups, the data was discarded and the measurement series was redone.
3.1. EXPERIMENTAL SET-UP

3.1.3. Motors

Each disc was driven by a separate DC-motor that could make the disc rotate with the desired angular velocity. The lower disc was mounted directly on the shaft of the lower motor, $M_1$ in Fig. 3.1, while the upper motor, $M_2$, was connected to the upper disc via a driving belt. $M_1$ was a prototype Maxon® motor M98300A-15 without specifications, while $M_2$ was a commercial Maxon® motor (366500 A-max26 GB 11W) with graphite brushes and an assigned power rating of 11 W. $M_2$ was put together with a 14:1 reduction planetary gear (Maxon GP32A 2.25NM) with a maximal continuous torque of 2.25 Nm.
3.1.4. **Angular velocity control**

The rotational velocity of each disc was measured with an optical system. The side walls of the discs were striped black and white and two reflective object sensors (Photologic® OPB715), S in Fig. 3.1, were used to obtain square wave signals, generated by the stripes when the discs were rotating, from which the angular velocities were calculated.

The upper disc rotated counter clockwise and the lower one clockwise when viewed from above, and the absolute values of the angular velocities were the same for each measured case, with an accuracy of ±0.015 rad/s. This gave an accuracy of 2% at the lowest speed, 0.75 rad/s. The angular velocities were set by the voltage generated by the power generators, which in turn were controlled by the laptop. Once the preset velocities were obtained, the camera was used to obtain images.

3.1.5. **Camera and light source**

The CCD-camera (Basler PiA1900, 32 fps, 1080×1920, 8 bit/pixel) was mounted above the discs and captured images of the fibers in the suspension through the upper glass disc. The captured area included the radius from approximately 3 to 12 cm. The exposure time was set to 1 ms and for illumination a cold light source (Schott KL2500LCD) was used at 3000 – 3200 K equivalent temperature. The light source had a three-headed gooseneck that made it possible to illuminate from three directions at the same time. The captured images were transferred to a laptop computer.

3.1.6. **Fiber suspension**

The fibers in the suspension between the discs were white cellulose acetate fibers with a nominal length of $L = 1$ mm with small deviations from this value. They were stiff, straight and had a density, $\rho_p \approx 1.3$. The fiber length was measured using the FiberMaster at Innventia AB (former STFI-Packforsk) where the mean length was measured to be 1.061 mm and where 91.6% of the fibers were in the length interval $0.9 \leq L \leq 1.2$ mm. Also, the mean diameter was measured to be $d = 69.7\mu$m for 88.2% of the fibers in the length interval $0.5 < L < 1.5$ mm.

Experiments with fibers of nominal length $L = 0.5$ mm and $L = 2$ mm were also performed. These fibers have not been measured with FiberMaster, but were considered to have the same diameter and density as the 1 mm fibers, because they came from the same thread which was cut in different lengths. They were also supposed to have the same accuracy regarding their respective lengths. The aspect ratios for the fibers $L = 0.5$ mm, $L = 1$ mm and $L = 2$ mm were $\alpha = 7$, 14 and 28, respectively.
3.2. EVALUATION TECHNIQUES

For all experiments, regardless of fiber length, 0.1 g of cellulose acetate fibers was suspended in 0.7 l of water at room temperature, resulting in a mean mass concentration of approximately $c_m = 0.014\%$. This is equivalent to a mean volume concentration $c_v \approx 0.011\%$ or a crowding factor $C_r \approx 0.01$ for $L = 1$ mm, which is a very dilute suspension according to the classification by Kerekes & Schnell (1992) in table 2.2. It should be noted that different aspect ratios of the fibers give different $C_r$ for the same concentration, see eq. (2.21).

It should also be noted that the artificial fibers differ from real chemically treated cellulose fibers used when making paper, especially regarding the stiffness, where the artificial fibers are much less supple.

3.2. Evaluation techniques

3.2.1. Image processing, filtering and fiber detection

The images captured by the CCD-camera were transformed in Matlab from .bin-files to .tif-files. All the images in one measurement series were used to obtain an average image representing the background. This background image was then subtracted from all the other images in order to remove some of the background noise (reflections e.t.c.) and unevenness of the illumination. The next steps in the treatment were to normalize the value of the pixels to be between 1 and 0, and to invert the image.

A portion of a typical captured and treated image is shown in Fig. 3.3, where arcs have been added to indicate the approximate radial positions. The fibers are clearly seen as short black lines and many fibers can also be seen forming trains. A train is a set of radially aligned fibers with a narrow center-to-center distance between the fibers, and with the fibers neatly lined up at a constant radial position, so that the direction of the whole train becomes tangential. One fiber train has been encircled at $R \approx 9.2$ cm in Fig. 3.3. The processed images were analyzed to obtain the orientation and positions of the fibers, and from this information the train formations were quantified.

To locate the fibers in an image, such as Fig. 3.3, the steerable filter used by Carlsson et al. (2007), which is based on the designs by Jacob & Unser (2004) and Freeman & Adelson (1991), was used also in the present investigation. The accuracy of the filter was analyzed by Carlsson (2009) and was shown to be very good for images with low noise level, such as Fig. 3.3. The filtration of the images for each measuring series provided information for every single fiber, such as its position, length and orientation in the plane of the discs. This data was saved for further processing.
3. EXPERIMENTAL SET-UP, METHODS AND DATA EVALUATION

**Figure 3.3.** Part of inverted picture of fibers for $2h = 0.3$ mm and $\Omega = 1.0$ rad/s, where several fiber trains can be seen and where one is encircled at $R \approx 9.2$ cm. The arcs indicate constant radii. The angle $\Theta$ is also defined in the upper part of the $R = 8$ cm arc.

### 3.2.2. Train detection

To classify fiber train formations, we will denote those consisting of only two aligned fibers as *pairs*, while fiber train formations containing three or more fibers will be denoted *trains*. We also introduce an orientation angle, $\Theta$ (illustrated in Fig. 3.3 at the upper part of the 8 cm radius), which is the angle between a fiber and the radial direction. Hence, $\Theta = 0^\circ$ for radially aligned fibers and $\Theta = 90^\circ$ for those tangentially aligned.

The information about every fiber that was detected by the filter was processed by an algorithm in order to find the train formations. The conditions in the algorithm for detection of a train were the following:

1. The orientation angle of the fibers have to be within $-45^\circ < \Theta < 45^\circ$.
2. The radially alignment condition is complemented by the condition that the distance between two equivalent ends of two adjoining fibers has to be less than one fiber length. This implies that the center points of two adjoining fibers are closer to each other than one fiber length, which is also expressed as a supplementary condition.
3. The last condition is that one fiber can only be part of one train, if any.
3.3. Experimental procedures and parameters

The camera was mounted and adjusted to capture an appropriate area of the disc, that covered the radius from approximately 2–11 cm. The aim was to have one radial line along the long centerline of the image, see Fig. 3.4. In order to do this calibration, a ruler was placed on the upper disc along a line with its zero position at the center of the disc. A picture was taken with the camera and this image was imported to Matlab, where the positions (in pixels) of all visible centimeter markings of the ruler were recorded by hand, the number of pixels per centimeter was calculated as well as the hub position. This procedure was carried out every time the camera’s position was adjusted.

Long series of triplets of images were captured, where the time between starts of triplets was 10 s and the time between pictures in one triplet was
10 ms. For this study, the first image of each triplet is the one that has been analyzed. The reason for taking triplets was to get information about velocities of fibers and trains within a short range of time, but this has not yet been performed and is subject to future research.

The numbers of triplets taken for each case involving 0.5 mm fibers typically were 100 or 150 triplets. For 1 mm fibers, which is the most thoroughly investigated fiber length, typically 300 triplets were taken for each case. Finally, typical numbers for \( L = 2 \) mm are 67 or 150 triplets. The maximal variation of parameters are shown in table 3.1. The exact parameter variations and numbers of triplets taken for each series are shown in tables in appendix A. The measurements were performed during a period of approximately 7 months.

Between the measurement series the fiber suspension was mixed in order to achieve an even initial distribution of the fibers in the gap. The mixing
3.3. EXPERIMENTAL PROCEDURES AND PARAMETERS

Procedure was performed by manually adjusting the rotating speeds of the upper and lower disc until the fiber distribution was uniform, as observed by the eye. For smaller gap distances this took longer time than for larger gaps, where the fibers could move more freely.

Figure 3.5. General migration behavior of fibers.

The cases were not run in any specific order, but as one gap distance had been set, all the cases for that gap were run. The order of the cases for that gap was then alternated between high and low values of the angular velocity. The reason for this was the fact that fibers tended to gather at different radii and the mixing process could somewhat be shortened by this procedure. The general migration behavior of the fibers depending on size of gap and magnitude of angular velocity is sketched in Fig. 3.5. This behavior of the fibers can be related to the radial velocity profile in Fig. 2.2 where the velocity is directed inward for vertical positions near the middle of the gap and directed outward for positions near the lower disc. It can be seen in Fig. 3.5 that for small gaps and/or higher angular velocities the fibers are positioned near the middle of the gap, since they are migrating inwards towards the hub of the discs, i.e. the same direction as the radial velocity. For large gaps and/or lower velocities it can be deduced that the fibers are closer to the lower disc, since they are heavier than the fluid and because of the outward migration direction.

A captured series of 300 images took about 50 min to run if no problems arose. However, it can be seen in the tables in appendix A, that some of the series are shorter than others and the reasons for this are mainly three. One being that problems with the camera-computer transfer occurred, which led to a stop of the capturing process. This was quite common for the first sets of measurements series. Another reason is that the fibers in some cases, mostly those of larger gap distances, were quickly transported inwards or outwards causing either flocculation in the center, or fibers moving out of sight. At these points it was not sensible to take long series. The third reason is that for some cases, especially at high angular velocities and large gaps, no trains were
formed. As might be expected, analysis of the images series has been corrected according to the number of images taken, if pertinent.

3.4. Characteristics of the flow

With our maximal value of the gap ratio, \( G = 0.006 \), being half of the smallest gap ratio that Daily & Nece (1960) used, in their experiments to classify the flow, and with our maximum rotational Reynolds number \( Re_\phi \approx 1.1 \cdot 10^5 \), the flow will always belong to Regime I and thus be laminar according to table 2.1. The study by Gauthier et al. (2002), concerning instability patterns, also implies that the flow in our parameter domain should be laminar.

The qualitative behavior of the single-phase flow in the apparatus was also investigated with Iriodin\(^1\) added to water. For the investigated combinations of angular velocities and gap distances, there were no instability patterns visible. The gap distance was increased beyond the investigated range, in order to see if instabilities occurred. The gap distance had to be increased to approximately 3 mm for this to happen at high angular velocities. For small gaps turbulent flow was not achievable with the existing motors.

According to Brady & Durlofsky (1987), the edge-effects would only influence an area of \( \mathcal{O}(2h/R_2) \), which would be very small in our case. It would also be outside the camera view. The image reaches at most to \( R \approx 0.8 R_2 \). Furthermore, the study by Szeri et al. (1983) shows that the infinite-disc solutions are valid for \( R/R_2 < 0.7 \) for a relatively large gap ratio, \( G = 0.05 \). Also, according to Moisy et al. (2004), the influence of the disc radius, \( R_2 \), is reduced when \( G \) decreases and is vanishingly small when \( G \ll 1 \).

It can thus be established that the flow field, in absence of fibers, between the two rotating discs is laminar, without edge-effects in the camera view and that the single-phase flow follows the similarity solutions in section 2.2.

The vertical velocity \( w/R\Omega \) in Fig. 2.2 is small, but must be compared to the settling velocity in order to understand its importance. We choose a fiber positioned perpendicularly to the settling direction, thus having a representative area of \( 2aL \), since such a fiber would have the lowest settling velocity. By iterating eq. (2.18) and eq. (2.20), we get a dimensionless settling velocity \( U_s = U/(h\Omega) \) of order \( \mathcal{O}(10^{1}) - \mathcal{O}(10^{2}) \), which is much larger than the vertical flow velocity, \( w/(h\Omega) \) that is approximately of order \( \mathcal{O}(10^{-2}) \), see Fig. 2.2. Therefore, it can be concluded that for \( z > 0 \), the vertical velocity in the flow field is not large enough (by several orders of magnitude) to overcome the settling velocity giving lift to fibers. For \( z < 0 \), \( w/(h\Omega) \) and \( U_s \) are both acting in the same direction, i.e. down towards the lower disc.

---

\(^1\)Iriodin is here 120 Lustre Satin from Merck, consisting of small flakes that orient themselves according to the flow and that can reflect light. They give an image of flow field structures.
3.5. A note on statistical reliability of the data

As mentioned earlier, long series of images were captured even though the distribution of the number of fibers in the gap became stabilized very quickly. By stabilization we do not imply that the number of fibers found remains constant, but that the number of fibers and their general behavior are fairly constant and only changes slowly with time.

Graphs showing the number of detected fibers in the first image of each triplet, as a function of clock time, can be seen in Fig. 3.6, where all the graphs are for the case where \(2h = 0.2\) mm and the fiber length is \(L = 1\) mm. Ignoring the high frequency oscillations for the time being, it can be seen that if there are significant changes in the number of detected fibers, e.g. for the curve where \(\Omega = 1.25\) rad/s, they take place in the beginning, and that the number of detected fibers then becomes approximately constant or only slowly changing with time. If the concentration of fibers would be uniform, only about 20 fibers would be in the field of view of the camera. If instead, the
fibers were evenly distributed right on the lower disc, such an idealized surface mean concentration \( [\text{fibers/m}^2] \) of fibers would imply about 630 fibers in view of the camera. 630 fibers in the smallest gap implies a \( Cr \approx 0.33 \) for the whole volume, which still is a very dilute suspension according to table 2.2. It is relevant to look at the surface mean concentration since the fibers are heavier than the fluid and will settle, especially for low angular velocities. This most probable vertical position of the fibers near the lower disc was mentioned in section 3.3 when discussing the migration behavior. Therefore, it is not surprising to see that the number of fibers in Fig. 3.6 for the lowest angular velocity, \( \Omega = 0.75 \text{ rad/s} \), stays at a high value around 700 identified fibers per image, which can be linked to the surface mean concentration.

Now considering the rapid variations in Fig. 3.6, they are due to the fact that the surfaces of the discs are not exactly horizontal and thus makes the fibers favor a specific tangential position. This can be deduced by the fact that the peaks appear periodically and when a new image has been taken after a number of full rotations. For example, there are exactly 10 full rotations between the peaks for \( 0.75 \text{ rad/s} \) and about 30 full rotations between peaks for \( 1.75 \text{ rad/s} \), which are the two angular velocities with the largest rapid variations in number of fibers found.

After analyzing the position and orientation of each fiber found, they could be sorted and summed up into a matrix, \( Re_H \times \Theta \). The values in the matrix were then divided by the largest number in order to normalize the values to the range \( 0–1 \). The resulting position-orientation matrix for the case where \( \Omega = 1.25 \text{ rad/s} \) and \( \lambda = 0.2 \) is shown in Fig. 3.7(e) where black areas indicate where most fibers are and white ones where there are no fibers detected. In this figure, there are thus a lot of radially aligned fibers \( (\Theta = 90^\circ) \), in the range \( 20 < Re_H < 55 \), whereas there are no tangentially aligned fibers.

Since this case \( (\Omega = 1.25 \text{ rad/s} \) and \( \lambda = 0.2) \) showed the largest decline in number of detected fibers in Fig. 3.6, position-orientation matrices for the first 30 images (Period 1) and the last 30 images (Period 3), respectively, were extracted. They are showed in Fig. 3.7(a) and (c) and it can be seen that the distributions do not vary significantly in appearance, even though the numbers of fibers found are not the same. They are also similar to (e), where all 300 images have been summed up to create the position-orientation matrix.

Figures (b), (d) and (f) show the same kind of distributions, but for the case where \( \lambda = 0.4 \) and \( \Omega = 1.25 \text{ rad/s} \). In these figures, both radially and almost tangentially \( (\Theta \approx 0^\circ) \) aligned fibers are present. It can also be seen that the distributions for the different periods are, as in the previous case, similar. It can therefore be assumed that the results concerning fiber positions and orientations are not notably influenced by the initial effects when summarizing over a whole series of images. Thus, the data is consistent even though the
3.5. A NOTE ON STATISTICAL RELIABILITY OF THE DATA

FIGURE 3.7. The distribution of fibers, by $\Theta$ and $Re_H$ for $\Omega = 1.25$ rad/s. In (a), (c) and (e) for $\lambda = 0.2$, and in (b), (d) and (f) for $\lambda = 0.4$. Included periods as indicated in the figures.

number of fibers found in each image varies. Furthermore, this implies that the shorter series of images contribute with accurate data.

An additional argument for data consistency is that fiber trains and their structures are quite stable, and thus independent of the rapid variations of the fiber distributions, which are mainly due to movements of single fibers.
CHAPTER 4

Results and discussion

This chapter begins with a discussion about the flow as deduced from the parameter range, §4.1. Then the results concerning the orientation of individual fibers are presented in §4.2, followed by the behavior and occurrence of fiber trains in §4.3. In the latter section, qualitative observations concerning the forming, destruction and movement of trains are described. Furthermore, the length of trains and their radial position are presented. Finally, the results from closer investigations of the structure of the trains are presented, §4.4-4.5. It should be noted that all results shown to this point are based on data from fibers of with $\alpha = 14$. The last section, §4.6, presents results obtained from investigations with $\alpha = 7, 14$ and 28.

4.1. Flow conditions

As concluded in section 3.4 the flow field between the two discs is laminar, if no fibers are present. In order to interpret and understand the results to come, it is worthwhile to discuss the interaction between fibers and such a flow field.

In section 3.4 it was also concluded that for $z > 0$, the vertical velocity, see Fig. 2.2(c), in the flow field is not large enough to overcome the settling velocity and to lift the fibers, and for $z < 0$, $w/(h\Omega)$ and the settling velocity ($U_s$) are both acting in the same direction, i.e. down towards the lower disc. Thus, all vertical positions of the fibers, other than on the lower disc, are not due to the vertical flow profile between the discs.

The velocity direction with the largest magnitude is the tangential one, see $v/R\Omega$ in Fig. 2.2(b), where the flow shows a linear shear profile for the relevant range of the Reynolds number, i.e. $10^{-1} < \mathcal{R} < 10^{0.5}$.

The radial velocity is also important and the fibers are observed to migrate radially according to Fig. 3.5, which might be possible to explain by the vertical positions of the fibers. The vertical positions are not possible to record directly in the present set-up. Nevertheless, a comparison between the migration direction and the radial velocity profile indicates that for small gaps, when the fibers are almost stationary in the radial direction, the fibers are believed to be located just below the middle $z = 0$, at the zero velocity point of $u/R\Omega$, see Fig. 2.2(a). Their rotational speed, the wall effect and the fiber end-effects are
very likely large enough to create a lift force that makes the particles migrate to that point. Migration of particles was also investigated by Feng & Michaelides (2003), but in their study, the density ratio between particle and fluid was less than in our case, and using their formula for separation between migrating and settling particles, eq. (2.23), we would in fact have a settling particle.

In addition to this, the tangential velocity of a fiber can be an indication of its vertical position. It was seen in Fig. 2.2 that the tangential velocity profile is linear and a fiber following the fluid at a certain \( z \)-position would travel with the corresponding velocity. Since we do not have quantitative data on the vertical position it is not possible to conclude this discussion at this point.

Still, for increasing gaps the lift force is not strong enough to transport the particle to or above the lower zero velocity point, \( u = 0 \) in Fig. 2.2(a), and the fibers remain in the region below where the flow is directed outwards. Hence, outward migration of fibers is dominant for large gaps, which is seen in Fig. 3.5.

### 4.2. Orientation and position of fibers

The positions and orientations of the fibers depend on the angular velocities of the discs and the gap distance. Each detected fiber has a radial position, which can be translated to the Reynolds number, \( Re_H \), which includes the angular velocity difference, \( \Delta \Omega \), between the discs. The fiber also has an orientation and according to these two properties the fiber is placed in a matrix. All fibers found, for one case, are added to the matrix at their appropriate position. The matrix is then divided by the highest number in order to have values between 0 and 1. This is now the distribution of fibers in the \( Re_H \)-\( \Theta \) plane. The accuracy of the angle was \( \pm 0.5^\circ \) and the radial position of the fiber was determined with an accuracy of approximately \( \pm 0.4 \text{ mm} \).

Such distributions are seen in Figs. 4.1 and 4.2 for two different gaps and varying \( \Omega \). In Fig. 4.3 results for \( \Omega = 1.75 \text{ rad/s} \) and varying gap distances are shown. In these figures, black areas indicate where most fibers are. It can be seen that the radially aligned fibers seemed to be dominating for approximately \( Re_H < 70 \) in the case where \( \lambda = 0.4 \), see Fig. 4.1. As the angular velocity is increased the Reynolds number range increases, but high amount of radially aligned fibers (\( \Theta = 0^\circ \)) around \( Re_H = 60 \) remains. Furthermore, tangentially aligned fibers (\( \Theta = 90^\circ \)) appear at higher \( Re_H \).

An interesting feature in Fig. 4.1 is that the tangentially aligned fibers tend to have an orientation angle, \( \Theta \), of either a little higher or a little lower than \( 90^\circ \) for the intermediate \( Re_H \), i.e. before \( Re_H \) becomes high enough for all such fibers to be almost exactly tangentially aligned, \( \Theta = 90^\circ \). This phenomenon produces the rocket-like shape in the images for \( \Omega = 1.75 \text{ rad/s} \) and \( \Omega = 2.00 \text{ rad/s} \). For a larger gap, \( \lambda = 0.6 \) in Fig. 4.2, the rocket-like shape of the
4.2. ORIENTATION AND POSITION OF FIBERS

Figure 4.1. Position, $Re_H$, and orientation, $\Theta$, of fibers where black areas represent areas where most fibers are. $\lambda = 0.4$ and $\Omega = (a) 0.75$, (b) 1.0, (c) 1.25, (d) 1.5, (e) 1.75, (f) 2.0, (g) 2.25 and (h) 2.5 rad/s.

Distribution of the tangential fibers ($\Theta = 90^\circ$) is less distinct, but is still thicker towards low $Re_H$.

In Fig. 4.3, the effect of increasing gap distance is seen for $\Omega = 1.75$ rad/s. Most fibers are radially aligned for the smallest gap, see (a), and as the gap increases, more fibers are tangentially oriented.
Figure 4.2. Position, $Re_H$, and orientation, $\Theta$, of fibers where black areas represent areas where most fibers are. $\lambda = 0.6$ and $\Omega =$ (a) 0.75, (b) 1.0, (c) 1.25, (d) 1.5, (e) 1.75, (f) 2.0 and (g) 2.25 rad/s.

The portion of radially and tangentially aligned fibers divided with the total number of fibers found, versus the dimensionless gap $\lambda$ is plotted in Figs. 4.4(a) and 4.4(b), respectively, for fibers with an aspect ratio $\alpha = 14$. Radially aligned fibers are fibers with an orientation of $\Theta = 0^\circ \pm 5^\circ$ and tangentially aligned fibers have $\Theta = 90^\circ \pm 5^\circ$. Figure 4.4(a) shows that for small gaps, $\lambda < 0.5$, a
large portion of the fibers are oriented radially. Together with the results for the tangential fibers in Fig. 4.4(b), it can also be seen that approximately half of the total amount of detected fibers is either tangentially or radially aligned for small gaps.
4. RESULTS AND DISCUSSION

Figure 4.4. Portion of (a) radially aligned fibers and (b) tangentially aligned fibers, divided with the total number of fibers found, as function of the dimensionless gap distance. All cases are included, i.e. $0.75 < \Omega < 2.50$ rad/s.

When the gap increases, less fibers tend to be oriented in the radial direction, Fig. 4.4(a), which also was concluded from Fig. 4.3 ($\Omega = 1.75$ rad/s). For $\lambda > 0.5$ the tangentially aligned fibers, Fig. 4.4(b), are in general more frequent than the radially aligned fibers. This is in agreement with the results of Holm & Söderberg (2007) and Carlsson et al. (2007), who found that fibers
were aligned with the flow direction for large distances from the wall ($> L/2$), and perpendicularly aligned with the flow direction close to the wall ($< L/8$). Since each disc is a wall, one would thus expect the majority of the fibers to be perpendicular to the flow direction (fibers oriented radially in the present experiments) if

$$2h < \frac{L}{8} + \frac{L}{8} = \frac{L}{4} \Leftrightarrow \lambda < \frac{2h}{L} < 0.25$$

(4.1)

This can be seen in Fig. 4.4, since it is at $\lambda < 0.25$ that the portion of radially aligned fibers is very large. The tangentially aligned fibers becomes dominating when $\lambda > 0.5$.

Since the gap distance is less than one fiber length in all experimental cases, the fibers cannot flip around vertically, i.e. the pure flipping motion described in section 2.6. Instead, they are mainly oriented in the horizontal plane, where they can do a resemblance of a kayaking motion of a solid ellipsoid in simple shear flow or be rolling around their own axis. A selection of these motions (i.e. Jeffery orbits) were shown in Fig. 2.5(a).

The majority of the fibers appear to prefer either to be tangentially aligned or to be radially aligned in the horizontal plane. The fact that the radially aligned fibers rotate around their own axis can be seen by the eye for fibers that are slightly bent. For tangentially aligned fibers, it has not been investigated whether the fibers rotate or not around their own axis, due to the difficulty of visualizing this. However, the shear of the radial velocity is non-zero, except at $z = 0$ and at the maxima of the velocity near each discs (see Fig. 2.2), and that shear is probably strong enough to start rotating a tangentially aligned fiber around its own axis.

### 4.3. Behavior and occurrence of fiber trains

#### 4.3.1. Qualitative observations

4.3.1a. **Train forming and destruction.** As already shown in Fig. 3.3, the fibers form regular patterns, here called fiber trains, in a similar manner as reported from the experiments by Matas et al. (2004) for spheres in pipe flow and by Di Carlo et al. (2007) for spheres and oblate spheroids in pipe flow, and from the computations by Chun & Ladd (2006) for spheres in square pipe flow. The phenomenon of pattern forming is similar, even though there are differences regarding the geometries of the particles, density ratios, ambient flow field and the influence by walls. Fibers creating a fiber train are radially aligned and hence most fiber trains form when the portion of radially aligned fibers is large. A qualitative observation of the fiber trains is that both the number of trains and their lengths decrease as the gap increases. Also, there seem to be a preferred radial position where most of the trains form. When the angular
4. RESULTS AND DISCUSSION

Figure 4.5. Sequence of pictures showing the evolution of a train crash and its reforming for $\Omega = 1.5$ rad/s and $\lambda = 0.2$. The time between pictures is 0.5 s, and arrows are pointing at the small fiber cluster causing the crash.

velocities are increased this radial position migrates a little outwards and then bifurcates into two preferred radial positions for high velocities and large gaps.

For very small gaps, the trains are preferable formations for the fibers to be in and they are very stable. As a consequence, there are few fast-moving fibers (tangentially aligned or doing a kayaking motion) that can collide with a train, causing it to be destroyed. If a train does get destroyed (crashes) in a narrow gap it quickly reforms into one or several (shorter) trains.

A sequence of pictures showing the evolution of a train crash is shown in Fig. 4.5. In this figure, trains are moving slightly with the lower disc, i.e. upwards. A small fiber cluster, at which the arrows point, causes the train to crash and moves in the opposite direction, except between (c) and (d), where the cluster has moved in the same direction as the lower disc, hence having a lower vertical position. In (a) the cluster touches a long train, and disturbs its side alignment. In (b), this disturbance has caused the train to completely
crash and the fibers are oriented in all directions. But already a half a second later, in (c), the fibers have started to reorganize themselves in smaller train-like formations and the cluster has moved on. Finally, in (d), some of the smaller train-like formations have joined each other, forming longer trains. In addition to this, it can be seen that the last 4 + 2 fibers of the destroyed train in (c) seem to be still affected by the disturbance, since they are not exactly aligned at a radial position, whereas they are in (d).

4.3.1b. Motion of trains. The radial motion of trains (and fibers) are almost non-existent, and the tangential motion of a train, as a whole, is in the same direction as the lower disc and very slow. The tangential velocities of whole trains vary slightly and it is possible for one train to join another. In that case, the faster train approaches the slower one from behind and docks the first one, creating an even longer train. Occasionally, a train is divided into two shorter ones, but this event has not been observed often and a decrease in train length is mostly due to the formation of shorter trains after a crash. The length of the train can also be increased or decreased by single fibers by the same manner of approach or separation. The departure of a single fiber at the end of a train is a quite common observation.

The trains are in general neatly lined up with the center of one fiber directly behind the center of the next fiber, as could be seen in Fig. 3.3. When the gap increases this meticulous behavior vanishes and the alignment becomes less perfect, and hence the train crashes more easily. For intermediate gap distances, the trains have more difficulties to reform, since the fibers are free to move in other orbits. They also move faster, as a whole, in the direction of the lower disc. This would indicate that they are positioned closer to that disc than the upper one, which seem natural because the fibers are heavier than the fluid.

For the largest gaps in this study, radially aligned fibers seemed to be rolling on the lower disc, but still being able to form trains. The fibers and fiber trains were observed by eye to have a tangential velocity almost the same as the lower disc at their specific radial positions, which is a higher speed compared to the observed velocities of the trains for smaller and intermediate gap distances. These higher tangential velocities implies a vertical position near the lower disc when looking at the velocity profiles in Fig. 2.2.

4.3.2. Length of trains
The length of a train, $N$, is defined as the number of fibers it contains. A case is a measurement series for one $\Omega$ and one gap distance. The amount of detected trains of length $N$ is denoted $M(N)$, the number of detected fibers is denoted $F_{tot}$ and the number of fibers constituting trains is denoted $F_{tr}$. 
4. RESULTS AND DISCUSSION

Figure 4.6. In (a): average number of trains and pairs found versus $N$ for $\Omega = 1.5$ rad/s and the dimensionless gap distance as indicated in the legend. $M(N)$ has been normalized with $F_{\text{tot}}/N$, i.e. the maximal number of trains with length $N$ that could have been formed with the fibers found, $F_{\text{tot}}$. In (b): average number of trains found versus $\lambda$, averaged over all $\Omega$. Train lengths as indicated in the legend.
A plot of the number of pairs and trains, $M(N \geq 2)$, found versus the train length, $N$, for $\Omega = 1.5$ rad/s and with a separate curve for each gap distance, is seen in Fig. 4.6(a). The horizontal axis has been limited to $N \leq 30$ for better visualization and the number of pairs and trains on the vertical axis has been normalized by the sum of fibers found for each gap and $\Omega = 1.5$, divided by $N$, i.e. normalization by the maximal number of $N$-trains that could have been formed with $F_{tot}$ fibers. The curve over the number of pairs and trains steepens as the gap distance increases which means that the maximum train length decreases when $\lambda$ increases and that long trains only occur for smaller gap distances.

A closer view at the number of trains, containing exactly $N = 3, 4, 5$ and 8 fibers as function of the dimensionless gap can be seen in Fig. 4.6(b). The number of trains has been normalized by the sum of fibers found divided by $N$, and averaged over all cases with equal $\lambda$. The number of $N$-fiber-trains decreases as the gap increases and shorter trains are more common for larger gaps, which is in line with the qualitative observations in section 4.3.1. The decrease can be divided into a primary decrease at $0.2 < \lambda < 0.3$ and a secondary decrease at $0.5 < \lambda < 0.6$. For $M(N = 3)$ a primary decrease cannot be seen and the reason for this may be that 3-fiber-trains at small gaps are primarily waiting for another train, with which it may form a longer train. The chance of joining another train and creating an even longer one decreases as $N$ increases. For larger gaps, 3-fiber-trains are primarily waiting to get destroyed, as are the longer trains. Hence, a secondary decrease can be seen for all $N$ when $\lambda > 0.5$, which is when the tangentially aligned fibers are dominating, see section 4.2.

To quantify the maximum train lengths for our cases, a typical longest train length, $L_{90}$, can be defined as the lowest $L_{90}$ for which

$$\sum_{N=3}^{L_{90}} p_N > 0.90$$

(4.2)

where $p_N$ is the probability that an identified train has the length $N$ and where the series $p_N$ is normalized so that

$$\sum_{N=3}^{\infty} p_N = 1$$

(4.3)

A plot of $L_{90}$ versus $\lambda$ is shown in Fig. 4.7 and it can be seen that for $\lambda = 0.2$ there are lots of longer trains, which was also seen in Fig. 4.6(a). When the gap increases, $L_{90}$ directly decreases to $4 - 6$ fibers, and a final decrease has occurred at $\lambda = 0.7$ to $3 - 4$ fibers, which is the shortest possible train length.
4.3.3. Radial position

The ratio between the number of fibers constituting trains, $F_{tr}$, and the total number of fibers found, $F_{tot}$, is plotted as function of the Reynolds number $Re_H$ in Fig. 4.8 for $\Omega = 1.5$ rad/s and $0.2 \leq \lambda \leq 0.8$. For each plotted case, the fibers constituting trains tend to mainly appear at a certain radius, see Fig. 4.8. The maximum of the ratio $F_{tr}/F_{tot}$ for each case is denoted $(F_{tr}/F_{tot})_{max}$ and occurs at $Re_{H,\text{max}}$. The values of the maxima are lower and less distinct for larger gaps, since fewer trains are formed, and therefore only cases where

$$\left(\frac{F_{tr}}{F_{tot}}\right)_{\text{max}} \geq 4\% \quad (4.4)$$

are considered in the following analysis.

Values of $Re_{H,\text{max}}$ versus $\lambda$, are plotted in Fig. 4.9. For small gaps, $0.2 \leq \lambda < 0.5$, $Re_{H,\text{max}}$ is around 50 for all rotational speeds. At $\lambda = 0.5$, a shift in $Re_{H,\text{max}}$ is seen. For $\lambda > 0.5$ maxima occur at both $Re_{H,\text{max}} \approx 50$ and $Re_{H,\text{max}} \approx 200$. This confirms the qualitative observations in section 4.3.1, where a radial position bifurcation was mentioned.
4.4. DISTANCE BETWEEN FIBERS IN TRAINS

Figure 4.8. $F_{tr}/F_{tot}$ versus $Re_H$ for $\Omega = 1.5 \text{ rad/s}$ and (a) $\lambda = 0.2$, (b) $\lambda = 0.4$, (c) $\lambda = 0.6$ and (d) $\lambda = 0.8$.

However, since the discs in the present study are finite, large $Re_H$ cannot be obtained for small gaps and there is a possibility that there might be a second maximum of $F_{tr}/F_{tot}$ at $Re_H \approx 200$ for small $\lambda$. The approximate limit of $Re_H$ is plotted as a dashed line in Fig. 4.9.

4.4. Distance between fibers in trains

The fibers in a train are lying next to each other and the distance from center-to-center point between fiber $i$ and fiber $i+1$ is denoted $\delta_{cc,i}$. We will present the mean distance between the surfaces of two adjacent fibers, denoted surface separation distance, $\delta_{ss,i} = \delta_{cc,i} - d$. Furthermore, $\delta_{ss}$ is the mean value of the $(N-1)$ values of $\delta_{ss,i}$ that exist for each train of length $N$:

$$\delta_{ss} = \frac{\sum_{i=1}^{N-1} \delta_{ss,i}}{N-1} \quad (4.5)$$
Figure 4.9. The $Re_{H,max}$ where each maximum of the portion of fibers constituting trains occurs, as function of the gap. A shift in $Re_{H,max}$ is clearly seen at $\lambda = 0.5$. The dashed line indicates the approximate limit for $Re_H$ that can be obtained in the present experimental set-up, as function of $\lambda$.

Figure 4.10(a) shows the distribution of $\delta_{ss}$ for trains of different $N$-ranges, for the case where $\lambda = 0.2$ and $\Omega = 1.5$ rad/s. As before, short trains are more numerous than long trains and it can be seen that the distribution of $\delta_{ss}$ narrows as the train length increases, and also that the maximum of each curve appears at lower values of $\delta_{ss}$ for increasing $N$. Hence, the fibers lie closer to each other in a long train than they do in a shorter train. This can in fact be perceived by the eye in Fig. 3.3.

A plot of the same information as in Fig. 4.10(a), but with $N$ on the horizontal axis and $\delta_{ss}$ on the vertical axis, is shown in Fig. 4.10(b), where $\delta_{ss}$ for each and every train is represented by a dot. This plot shows that the lower limit of $\delta_{ss}$ is very distinct for all train lengths ($N \geq 3$). The pairs ($N = 2$) are also included in the graph and their uncertainty in $\delta_{ss}$ is clearly seen as a very large spread. It should be noted that the upper limit for $\delta_{ss}$ originates from the second condition in the train-detection algorithm, see section 3.2.2.

The results shown in Figure 4.10 are for the case where $\lambda = 0.2$ and $\Omega = 1.5$ rad/s, but the same tendencies are observed for the other cases and the lower limit of $\delta_{ss}$ is approximately four fiber diameters for all cases.
Having defined $\delta_{ss}$ according to equation 4.5, we can calculate an average $\delta_{ss}$ of a certain train length, $N$, over all cases. Let $\delta_{ss,N,k,j}$ be the value of $\delta_{ss}$ for a train $j$ of length $N$ and for case $k$. There are $M_k(N)$ number of $N$-trains.
in case $k$. Then, $\delta_{ss,N}$ is the average of $\delta_{ss}$ for trains of length $N$ and of all cases.

$$\delta_{ss,N} = \frac{\sum_{k=1}^{C} \sum_{j=1}^{M_k(N)} \delta_{ss,N,k,j}}{\sum_{k=1}^{C} M_k(N)} \quad (4.6)$$

where $C$ is the number of cases. Hence, different cases are weighted according to the number of trains found in each particular case.

If $\delta_{ss,N}$ is calculated for each $N$, using the information from all cases and measurements, and plotted versus $N$, the solid line in Fig. 4.11 is obtained. For comparison, the results of one of the first measurement series, taken in October 2008 and one of the last series, taken in January 2009, are extracted and plotted separately in the same figure. The results are similar and demonstrate the reproducibility of the train formation, also in terms of quantitative data. In Fig. 4.11, there is a decrease in $\delta_{ss,N}$ as $N$ increases for $3 \leq N < 20$ fibers. Thereafter $\delta_{ss,N}$ levels out and stays around $3.5 - 4.0$ fiber diameters ($d$). $\delta_{ss,N=2}$, i.e. for pairs, is seen to be approximately $6 \ d$. As mentioned before, the pairs contain significant uncertainties, and one could expect the value to be around $6.7 \ d$, if extrapolating from the other values.

The expected value of $\delta_{ss,N=2}$, i.e. $\delta_{ss,N=2} \approx 6.7 \ d$, can be compared with the distances found by Yan et al. (2007) between two freely rotating and neutrally buoyant cylinders in a simple shear flow. We can note that the our distance is larger than the critical separation distance of $\delta_{ss,N=2,crit} = 5.5 \ d$ found by Yan et al. (2007) for $Re_H < 384$. Their critical surface separation distance $\delta_{ss,N=2,crit}$ is defined as the distance beyond which the cylinders remain in place, rotating on the centerline between the upper and lower wall. Thus, since $\delta_{ss,N=2,crit} < \delta_{ss,N=2}$ it indicates that the fibers are positioned at a distance from each other that admits stable positions on the centerline, according to Yan et al. (2007). In our study, we have shown that most trains form for $\lambda \leq 0.5$ and at a $Re_H \approx 50$, a parameter range that is within the one investigated by Yan et al. (2007). However, the lower limit of $\delta_{ss,N}$ in our study is approximately $4-5 \ d$, see Fig. 4.10. This distance is, according to Yan et al. (2007), not a stable distance on the centerline. The reason for this discrepancy is not yet understood.

It is clear from Fig. 4.11 that the length of the train has an impact on the average $\delta_{ss}$. Matas et al. (2004) found a dependence between $\delta_{ss}$ and $Re_p$ in their experiments with spheres in a pipe flow. In our study, we have seen a dependence on $N$ and not on $Re_p$. 
4.5. Angle deviation of fibers in trains

As explained before in section 3.2.2, the condition for the angle deviation from the radial direction is set very generously in the algorithm for finding trains. The reason for this is to be sure to find fibers that might be part of a train. The candidate fibers of course have to be more numerous than the actual fibers belonging to trains. The algorithm described in section 3.2.2 searches for fiber train structures among the candidate fibers and the rest of the fibers are discarded.

Figure 4.12 shows the distribution of orientation for the fibers found in trains for different ranges of $N$, where the sum of the distributions has been normalized to 1. It is seen that fibers, identified as being passengers in a train, only deviate a few degrees from the radial direction. A fiber with a large deviation is in most cases a fiber at one end of the train, thus in the process of leaving or joining the train. This explains the large spread for shorter trains, where the end fibers constitute a large portion of the whole train.

An interesting feature in Fig. 4.12 is that the peaks of the PDF’s are not exactly at $\Theta = 0^\circ$ but at higher values.
4. RESULTS AND DISCUSSION

Figure 4.12. Probability density function (PDF) of the angle deviation from the radial direction for $\lambda = 0.2$ and $\Omega = 1.5$ rad/s. Different ranges of train lengths are indicated in the legend. The PDF is normalized to 1 for the four graphs together.

4.6. Effect of fiber length

As mentioned earlier, two other fiber lengths (0.5 mm and 2 mm) with aspect ratios $\alpha = 7$ and $\alpha = 28$, respectively, have been investigated. Here, the results of those will be compared to the results for $\alpha = 14$ ($L = 1$ mm). This section will highlight some interesting observations and results, related to fiber length variation.

4.6.1. General observations

The general migration behavior, described in section 4.1, was modified according to Fig. 4.13 when the fiber aspect ratio changed. Fibers with lower aspect ratio tend to migrate inwards to a larger extent, thus moving the border line between the two areas (i.e. outward and inward migration) down and to the left. This implies that the fibers of lower aspect ratio, to a larger extent, are positioned at $z$-values where the radial velocity is directed inward, see Fig. 2.2(a). Thus, the fibers experience a stronger lift force, possibly due to fiber end-effects.

The opposite migration behavior was seen for fibers with a higher aspect ratio, which explains Fig. 4.13 for all three fiber lengths.
4.6. EFFECT OF FIBER LENGTH

4.6.2. Ability to form trains

The shorter fibers required a smaller gap distance to form long trains, which is consistent with the results of e.g. Carlsson et al. (2007) where the fibers mainly orient themselves perpendicularly to the flow if their center is located less than \( L/8 \) from the wall. It was seen that the shorter fibers formed trains when \( \lambda = 0.5 \) or less. The second condition in the algorithm for finding trains, described in section 3.2.2, were modified for the 0.5 mm fibers, where the distances between two neighboring fibers and their ends were extended to 1.5 \( L \). The reason for this is that \( \delta_{cc} \) became too close to the value of \( L \) and fibers should not be discarded as train passenger because of their length.

For the longer 2 mm fibers a new difficulty arose because of the fact that they were not completely straight, but often a little bent. Bent fibers easily caused its own train or the train they were joining to crash since the fibers are rotating, and therefore there were few long trains for long fibers.

4.6.3. Surface separation distance

Calculations for the average surface separation distance, \( \delta_{ss,N} \), were done for the shorter and longer fiber lengths and the results can be seen in Fig. 4.14 together with the results for \( \alpha = 14 \). The results for pairs have been omitted because of the large errors of those results. The trend of decreasing \( \delta_{ss,N} \) for increasing \( N \) can be seen for all fiber lengths, and it is interesting to see that the graphs for \( \alpha = 14 \) and \( \alpha = 28 \) almost coincide, but that the graph for the shorter fibers is moved downward to smaller \( \delta_{ss,N} \), still having the same slope. This implies that for \( \alpha < 14 \), end-effects and 3D effects become important and lead to shorter \( \delta_{ss,N} \). A typical case where 3D effects cannot be neglected is train forming of spheres. For such a case, Matas et al. (2004) found that the surface separation distance was much shorter \( (0.5–3.5 \, d) \). For \( \alpha = 7 \) the \( \delta_{ss,N=2} \) is extrapolated to be approximately 5.5 \( d \) (i.e. \( \delta_{cc} = 6.5 \, d \)), which is the value
Figure 4.14. The average surface separation distance for all cases versus $N \geq 3$. Three different fiber aspect ratios are represented, as indicated in the legend.

found by Yan et al. (2007) to be the critical distance for cylinders. However, $\delta_{ss,N=2}$ was found to be approximately $6.5 \, d$ for longer fibers ($\alpha = 28$), which one would expect to be more similar to cylinders.
The behavior of fibers suspended in a liquid flow between two flat and counter-rotating discs has been studied experimentally. The fibers had aspect ratios $\alpha = 7, 14$ and $28$. The investigated range of the dimensionless gap distance, $\lambda$, between the discs was $0.2 \leq \lambda \leq 0.9$ and the gap Reynolds number range was $9 < Re_H < 540$. The position and orientation of fibers in the plane of the discs were measured. Train formations, i.e. self-organization of a set of radially oriented fibers at a constant radial position, with short fiber-to-fiber distance in between, were observed to be a common feature in this set-up. The behavior, occurrence and structure of these train formations have been studied and discussed.

Train formations require fibers to be aligned in the radial direction and it has been observed that a larger portion of the fibers are radially aligned for gaps less than half of a fiber length. Most trains were also formed at small gaps, with exceptionally long ones (up to 70 fibers) for a dimensionless gap of $\lambda = 0.2$.

The surface separation distance, $\delta_{ss,N}$, between two adjacent fibers in a train decreases as the length of the train, $N$, increases, i.e. contains more fibers. The surface separation distance for a pair of $\alpha = 14$ fibers is extrapolated to approximately 6.7 fiber diameters. The distance decreases with increasing train length to around 4 fiber diameters for trains of approximately 20 fibers. The results for fibers with $\alpha = 28$ were approximately the same, though the amount of data was less, and $\delta_{ss,N=2}$ was extrapolated to approximately 6.5 $d$.

The surface separation distance for fibers with $\alpha = 7$ were found to be smaller for all values of $N$, compared to $\delta_{ss,N}$ for longer fibers. Pairs of the shorter fibers have $\delta_{ss,N=2} \approx 5.5 \ d$. However, the surface separation distance decreases with the same slope as the other fiber lengths with increasing $N$.

The results for a pair of fibers are in agreement with Yan et al. (2007), who found that if $\delta_{ss} \geq 5.5 \ d$ between cylinders in a 2D flow, then the cylinders will remain in their positions.
As mentioned in the beginning this study is the first undertaking in an effort aimed at understanding what happens inside a refiner, used in the papermaking process. We have seen several interesting features, which deserve more attention and that will be investigated further, e.g. the velocities of whole trains in an effort of determining their vertical positions. The work will then continue according to the strategy in section 1.2 with a more complex geometry of the experimental set-up, where it will be possible to have a throughflow of fiber suspension and/or bars and grooves on the surfaces of the discs.
Acknowledgements

This project has been financially supported by the Swedish Energy Agency (Energimyndigheten) and is greatly acknowledged. Bengt Ingeströms stipendiefond is also acknowledged for funding participation at international conferences.

I would like to thank my supervisors Daniel Söderberg and Fredrik Lundell for support and guidance.

I would also like to thank my colleagues at the Department of Mechanics, who have supported me and who create a good working atmosphere. Among these, I would like to mention Fredrik, Fritz, Göran, Ingunn, Kim, Lisa, Monika, Olle, Ramis and Shiho. Thanks also to my office mate Gabriele Bellani for patience and good laughs.

Moreover, I would like to thank the people at Innventia AB, who kindly have supported me.

Huge thanks to my brother Mikael Ahlberg for enlightening me on many computer mysteries and for always being supportive. Finally, I very much thank my friends and family for love and encouragement.
References


REFERENCES

GREENSPAN, H. P. 1990 The Theory of Rotating Fluids. Breukelen Press, MA, USA.


APPENDIX A

Measurement data

Tables over the number of triplets for each measurement series, with their identification numbers indicated in the captions. The measurements were taken in the water laboratory at KTH, Stockholm, over a period of approximately 7 months, from October 2008 to April 2009.
### A. MEASUREMENT DATA

<table>
<thead>
<tr>
<th>gap [mm]</th>
<th>Ω [rad/s]</th>
<th>0.9</th>
<th>0.8</th>
<th>0.7</th>
<th>0.6</th>
<th>0.5</th>
<th>0.4</th>
<th>0.3</th>
<th>0.2</th>
<th>0.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.50</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.25</td>
<td></td>
<td>100</td>
<td>50</td>
<td>50</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.00</td>
<td></td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.75</td>
<td></td>
<td>100</td>
<td>150</td>
<td>150</td>
<td>150</td>
<td>80</td>
<td>80</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.50</td>
<td></td>
<td>150</td>
<td>150</td>
<td>150</td>
<td>150</td>
<td>80</td>
<td>80</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.25</td>
<td></td>
<td>100</td>
<td>150</td>
<td>50</td>
<td>150</td>
<td>150</td>
<td>150</td>
<td>80</td>
<td>80</td>
<td></td>
</tr>
<tr>
<td>1.00</td>
<td></td>
<td>150</td>
<td>150</td>
<td>150</td>
<td>150</td>
<td>80</td>
<td>80</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.75</td>
<td></td>
<td>100</td>
<td>150</td>
<td>150</td>
<td>150</td>
<td>80</td>
<td>80</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Table A.1.** Number of triplets taken for 0.5 mm fibers (series nos. 05mm0902 and 05mm0903). 10 s between triplets and 1 ms between pictures in one triplet.

<table>
<thead>
<tr>
<th>gap [mm]</th>
<th>Ω [rad/s]</th>
<th>0.9</th>
<th>0.8</th>
<th>0.7</th>
<th>0.6</th>
<th>0.5</th>
<th>0.4</th>
<th>0.3</th>
<th>0.2</th>
<th>0.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.50</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.25</td>
<td></td>
<td>300</td>
<td>200</td>
<td>37</td>
<td>200</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.00</td>
<td></td>
<td>300</td>
<td>300</td>
<td>263</td>
<td>200</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.75</td>
<td></td>
<td>300</td>
<td>300</td>
<td>300</td>
<td>200</td>
<td>34</td>
<td>24</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.50</td>
<td></td>
<td>300</td>
<td>300</td>
<td>300</td>
<td>300</td>
<td>56</td>
<td>100</td>
<td>48</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.25</td>
<td></td>
<td>300</td>
<td>300</td>
<td>300</td>
<td>200</td>
<td>100</td>
<td>100</td>
<td>48</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.00</td>
<td></td>
<td>300</td>
<td>200</td>
<td>160</td>
<td>300</td>
<td>100</td>
<td>100</td>
<td>82</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.75</td>
<td></td>
<td>300</td>
<td>300</td>
<td>300</td>
<td>200</td>
<td>49</td>
<td>100</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Table A.2.** Number of triplets taken for 1 mm fibers (series no. 1mm1008). 10 s between triplets and 1 ms between pictures in one triplet.
### A. Measurement Data

<table>
<thead>
<tr>
<th>gap [mm]</th>
<th>Ω [rad/s]</th>
<th>0.9</th>
<th>0.8</th>
<th>0.7</th>
<th>0.6</th>
<th>0.5</th>
<th>0.4</th>
<th>0.3</th>
<th>0.2</th>
<th>0.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.50</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.25</td>
<td></td>
<td>200</td>
<td>300</td>
<td>300</td>
<td>300</td>
<td>300</td>
<td>300</td>
<td>300</td>
<td>237</td>
<td></td>
</tr>
<tr>
<td>2.00</td>
<td></td>
<td>250</td>
<td>300</td>
<td>300</td>
<td>300</td>
<td>300</td>
<td>300</td>
<td>300</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.75</td>
<td></td>
<td>300</td>
<td>300</td>
<td>300</td>
<td>300</td>
<td>300</td>
<td>300</td>
<td>300</td>
<td>279</td>
<td></td>
</tr>
<tr>
<td>1.50</td>
<td></td>
<td>300</td>
<td>300</td>
<td>300</td>
<td>300</td>
<td>300</td>
<td>300</td>
<td>300</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.25</td>
<td></td>
<td>300</td>
<td>300</td>
<td>300</td>
<td>300</td>
<td>300</td>
<td>300</td>
<td>300</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.00</td>
<td></td>
<td>300</td>
<td>300</td>
<td>300</td>
<td>300</td>
<td>300</td>
<td>300</td>
<td>300</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.75</td>
<td></td>
<td>300</td>
<td>300</td>
<td>300</td>
<td>300</td>
<td>300</td>
<td>300</td>
<td>300</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Table A.3.** Number of triplets taken for 1 mm fibers (series no. 1mm0901). 10 s between triplets and 1 ms between pictures in one triplet.

<table>
<thead>
<tr>
<th>gap [mm]</th>
<th>Ω [rad/s]</th>
<th>0.9</th>
<th>0.8</th>
<th>0.7</th>
<th>0.6</th>
<th>0.5</th>
<th>0.4</th>
<th>0.3</th>
<th>0.2</th>
<th>0.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.50</td>
<td></td>
<td>67</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.25</td>
<td></td>
<td>67</td>
<td>67</td>
<td>67</td>
<td>67</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.00</td>
<td></td>
<td></td>
<td>150</td>
<td>150</td>
<td>150</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.75</td>
<td></td>
<td>67</td>
<td>150</td>
<td>67</td>
<td>150</td>
<td>67</td>
<td>150</td>
<td>67</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.50</td>
<td></td>
<td>67</td>
<td>150</td>
<td>67</td>
<td>150</td>
<td>67</td>
<td>150</td>
<td>67</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.25</td>
<td></td>
<td>67</td>
<td>150</td>
<td>67</td>
<td>150</td>
<td>67</td>
<td>150</td>
<td>67</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.00</td>
<td></td>
<td>67</td>
<td>150</td>
<td>67</td>
<td>150</td>
<td>67</td>
<td>150</td>
<td>67</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.75</td>
<td></td>
<td>67</td>
<td>150</td>
<td>67</td>
<td>150</td>
<td>67</td>
<td>150</td>
<td>67</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Table A.4.** Number of triplets taken for 2 mm fibers (series nos. 2mm0902 and 2mm0904). 10 s between triplets and 1 ms between pictures in one triplet.
APPENDIX B

Position-orientations pictures

Pictures over the positions and orientations of fibers (α = 14) for gaps $0.2 \leq \lambda \leq 0.9$ and angular velocities $0.75 \leq \Omega \leq 2.50$. The pictures for $\lambda = 0.4$ and $\lambda = 0.6$ were shown in section 4.2, but are also included here in order to have the complete set of figures in the appendix.
Figure B.1. Positions and orientation distribution of fibers for $\lambda = 0.2$ and $\Omega = (a) 0.75$, (b) 1.0, (c) 1.25, (d) 1.5, (e) 1.75, (f) 2.0 and (g) 2.25 rad/s.
Figure B.2. Positions and orientation distribution of fibers for $\lambda = 0.3$ and $\Omega = (a) 0.75$, (b) 1.0, (c) 1.25, (d) 1.5, (e) 1.75, (f) 2.0 and (g) 2.25 rad/s.
Figure B.3. Positions and orientation distribution of fibers for $\lambda = 0.4$ and $\Omega = (a) 0.75$, (b) 1.0, (c) 1.25, (d) 1.5, (e) 1.75, (f) 2.0, (g) 2.25 and (h) 2.5 rad/s.
Figure B.4. Positions and orientation distributions of fibers for $\lambda = 0.5$ and $\Omega = (a) 0.75$, (b) 1.0, (c) 1.25, (d) 1.5, (e) 1.75, (f) 2.0 and (g) 2.25 rad/s.
Figure B.5. Positions and orientation distributions of fibers for $\lambda = 0.6$ and $\Omega = (a) 0.75$, (b) 1.0, (c) 1.25, (d) 1.5, (e) 1.75, (f) 2.0 and (g) 2.25 rad/s.
Figure B.6. Positions and orientation distributions of fibers for $\lambda = 0.7$ and $\Omega =$ (a) 0.75, (b) 1.0, (c) 1.25, (d) 1.5, (e) 1.75, (f) 2.0 and (g) 2.25 rad/s.
Figure B.7. Positions and orientation distributions of fibers for $\lambda = 0.8$ and $\Omega = (a) 0.75$, (b) 1.0, (c) 1.25, (d) 1.5, (e) 1.75 and (f) 2.0 rad/s.
Figure B.8. Positions and orientation distribution of fibers for $\lambda = 0.9$ and $\Omega = (a) 0.75$, (b) 1.0, (c) 1.25, (d) 1.5, (e) 1.75 and (f) 2.0 rad/s.