A High Frequency Transformer Winding Model for FRA Applications

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To my mother
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Abstract

Frequency response analysis (FRA) is a method which is used to detect mechanical faults in transformers. The FRA response of a transformer is determined by its geometry and material properties, and it can be considered as the transformer’s fingerprint. If there are any mechanical changes in the transformer, for example if the windings are moved or distorted, its fingerprint will also be changed so, theoretically, mechanical changes in the transformer can be detected with FRA.

The purpose of this thesis is to partly create a simple model for the ferromagnetic material in the transformer core, and partly to investigate the high frequency part of the FRA response of the transformer winding. To be able to realize these goals, two different models are developed separately from each other. The first one is a time- and frequency domain complex permeability model for the ferromagnetic core material, and the second one is a time- and frequency domain winding model based on lumped circuits, in which the discretization is made finer and finer in three steps. Capacitances and inductances in the circuit are calculated with use of analytical expressions derived from approximated geometrical parameters.

The developed core material model and winding model are then implemented in MATLAB separately, using state space analysis for the winding model, to simulate the time- and frequency response.

The simulations are then compared to measurements to verify the correctness of the models. Measurements were performed on a magnetic material and on a winding, and were compared with obtained results from the models. It was found that the model developed for the core material predicts the behavior of the magnetic field for frequencies higher than 100 Hz, and that the model for the winding predicts the FRA response of the winding for frequencies up to 20 MHz.

Index terms: complex permeability, frequency response analysis, time domain reflectometry, high frequency modeling, transformer diagnosis.
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List of publications

Journal publications


Conference publications


Contents

Abstract

Acknowledgements

List of publications

1 Introduction ............................................................................................................... 13
  1.1 Background ..................................................................................................... 13
  1.2 Aim ................................................................................................................. 13
  1.3 Outline of the thesis ........................................................................................ 13

2 Power transformers and Frequency Response Analysis (FRA) ........................... 15
  2.1 Transfer function ............................................................................................. 15
  2.2 Frequency response measurements ................................................................. 15
     2.2.1 Impulse response method .................................................................... 16
     2.2.2 Frequency sweep method .................................................................... 16
  2.3 Mechanical faults in a transformer .................................................................. 16
  2.4 Diagnosing mechanical faults in a transformer with the help of FRA ............ 17

3 Core model and measurements ................................................................................ 19
  3.1 Background ..................................................................................................... 19
  3.2 Complex-permeability model ......................................................................... 20
  3.3 Detailed hysteresis model ............................................................................... 21
     3.3.1 Static hysteresis ................................................................................... 21
     3.3.2 Excess losses ....................................................................................... 23
  3.4 Measurement setup ......................................................................................... 23
  3.5 Comparison between model and measurements ............................................. 26

4 High frequency winding model ................................................................................ 29
  4.1 Transformer winding model for high frequency applications ......................... 29
  4.2 Three different resolutions of the model ......................................................... 29
  4.3 Calculation of the capacitances ....................................................................... 31
  4.4 Calculation of the inductances and resistances ............................................... 31
  4.5 State space model for a single winding ........................................................... 33
  4.6 Comparison between the three models ........................................................... 37

5 Time Domain Reflectometry (TDR) ........................................................................ 39

6 Frequency response measurements ......................................................................... 43
  6.1 Impedance measurement device and dimensioning of the experimental setup 43
  6.2 Impedance measurement results ..................................................................... 45
7 Model verification ..................................................................................................... 47
  7.1 Comparison of model with measurements ...................................................... 47

8 Interpretation ............................................................................................................ 51
  8.1 Explanation of the different oscillation modes .............................................. 51
     8.1.1 Radial resonance modes ................................................................. 51
     8.1.2 Azimuthal resonance modes ............................................................. 53
  8.2 Azimuthal resonances independent of number of discs in the winding ....... 54
  8.3 Using reluctance network for proximity effect ............................................. 55

9 Summary, conclusions and future work ................................................................. 61

References .............................................................................................................................. 63
1 Introduction

In this Chapter, a short background information about power transformers is given, and the aim, outline and structure of the thesis is presented.

1.1 Background

A power transformer is an electric device used for transmission and distribution of electric power, and it is mainly used when there is a need for a voltage transformation. In a transformer, electric energy is transferred between different electrical circuits by the use of electromagnetic induction. Power transformers are very large and expensive so replacement of old ones with new ones, in order to increase the reliability, is often not economically justified; therefore, they are supposed to be used for maximum number of years.

In most of the power transformers, copper or aluminum is used for the windings and each conductor in the winding is individually insulated with some kind of insulation paper. The low voltage winding is usually placed close to the grounded core, since insulation requirement is less. High voltage and low voltage windings are insulated from each other with insulation oil and key spacers. The core is made of thin silicon steel laminations, of which the magnetic properties are best along the rolling direction. Each lamination is insulated to minimize eddy current losses in the core. Between the windings and the core, oil is used as insulation since oil has better dielectric strength than air and at the same time serves as cooling medium for removing the heat generated by the losses in the transformer.

1.2 Aim

The aim of this project is to develop models for the ferromagnetic core, and the winding. The magnetic core material model shall be computationally time effective and include magnetic static hysteresis and dynamic hysteresis effects like eddy currents and excess eddy currents, and the winding model shall be able to include the higher part of the frequency spectrum. The models have to be worked out so that they can be used in both time- and frequency domains and the goal is to create models which are realistic when compared to measurements.

1.3 Outline of the thesis

The thesis is structured as follows.

Chapter 2 presents a general overview of Frequency Response Analysis (FRA) and measurement techniques, mechanical faults in transformers and the way these faults can be detected by FRA.

In Chapter 3, which deals with the core model, the simple complex permeability model, the detailed hysteresis model and the measurements are presented and compared to each other.

Chapter 4 presents the developed lumped element winding model, the state space equation which is used for calculating the currents and voltages in the model, and the formulas for the
lumped element parameters. This Chapter is concluded by a comparison between the impedance of the three models with different levels of discretization in the frequency spectrum.

Chapter 5 presents the time domain method Time Domain Reflectometry (TDR) and the way the winding model is adapted for use in TDR for diagnosis of transformer winding faults. Chapter 6 presents the measurement setup and the performed measurements.

In Chapter 7, the winding models are verified by comparison with measurements. Chapter 8 deals with the explanation and interpretation of the different oscillation modes of a single disc, and is concluded by the development of the reluctance network model for compensation of the proximity effect.

Finally, Chapter 9 gives a summary, conclusions and some suggestions for future work.
2 Power transformers and Frequency Response Analysis (FRA)

In this Chapter, the method of FRA and the measurement techniques are explained. Further, the different mechanical faults in transformers and the way they can be detected by FRA is described.

2.1 Transfer function

A transfer function is generally defined as a mathematical representation of the relation between the input and output of a linear time-invariant system, and for this system it is independent of the applied input signal. The input and output signals give the transfer function its physical interpretation. If, for example, the output is a voltage and the input is a current, then the transfer function will be an impedance with the unit $\Omega$. For the transfer function there will be a magnitude and phase which both vary with frequency and which can be measured experimentally.

2.2 Frequency response measurements

Frequency Response Analysis (FRA) is a powerful method for characterizing a system by analyzing its frequency response, which is uniquely defined by the system parameters; this means that FRA can be used to either design a system or to analyze an existing one. It is the phase and magnitude response of a system when subjected to sinusoidal inputs, and it has become a popular method for evaluating the mechanical condition of the windings and the clamping structure of power transformers [1–3].

The first technical work describing the possibility of using frequency response analysis technique for diagnosing mechanical faults inside power transformers was published by Dick and Erven [5] in 1978. Since then, FRA has been gaining popularity among researchers and utilities as a potential method to detect mechanical integrity of power transformers. The frequency range for FRA is generally from 10 Hz to some 10 MHz and the evaluation is based on the fact that the frequency response of a transformer is defined by its capacitance and inductance distributions, which are determined by the geometrical construction of the transformer and characteristics of materials used. Therefore, mechanical deformations change the capacitive and inductive parameters, yielding deviations in the FRA spectrum. This means that FRA is basically a comparative method, in which a fingerprint measurement taken at an earlier stage is compared with a measurement taken at a later stage, perhaps after relocation or during a maintenance operation. Then the changes in characteristics of the response are analyzed to detect mechanical changes inside the transformer.

Frequency response can either be measured directly by sweeping the frequency (sweep frequency method) or be estimated from impulse response measurements. Both methods have advantages and disadvantages. For example, the impulse response method needs less
measuring time, but it is very noise sensitive. On the other hand, the frequency sweep method takes a little longer time for the measurements, but it is not so noise sensitive.

2.2.1 Impulse response method

In the impulse response method, an impulse voltage that has adequate frequency content is applied to the test object and the corresponding response signal and the applied signal are simultaneously measured. This is based on the definition of the transfer function which says that the transfer function is independent on the applied signal when the system is linear and time invariant. Then both of the measured signals are numerically transformed into the frequency domain using Fast Fourier Transform (FFT). The ratio between the FFT of the response signal and the applied signal becomes the frequency response of the corresponding transfer function.

This method has been used by many researchers for diagnosing mechanical faults in power transformers [3]. Limitation of the excitation source that can produce enough energy in the whole frequency band of interest, reduced energy level of injected impulse at higher frequencies limiting the upper limit of the calculated frequency response, and the need for good noise prevention techniques are some of the disadvantages of the impulse response method.

2.2.2 Frequency sweep method

In this method, a constant amplitude sinusoidal signal is applied and the magnitude and phase shift measurements are taken at predefined frequency points. This means that this is a direct method of determining the frequency response, since the result is ready and available after sweeping the predefined frequency range.

2.3 Mechanical faults in a transformer

A transformer can be damaged due to transportation [4], installation, and the forces inside it due to the interaction of the current and the (leakage) magnetic flux density according to:

$$F \propto I \times B$$  \hspace{1cm} (1)

where \(F\) is the force, \(I\) is the current in the winding and \(B\) is the magnetic flux density. According to Eq. (1) there will be heavy mechanical stresses in the transformer in case of a sudden short circuit fault, as the current flowing through the winding at that time is enormous. Eq. (1) also means that there will be two kinds of force vectors generated by the axial component of the leakage flux density (radial force) and by the radial component of the leakage flux density (axial force). The radial forces tend to squeeze the inner winding and expand the outer winding, resulting in circlot deformation in the outer winding and circlot buckling in the inner winding due to imbalance of the radial pressure. In contrast the axial forces tend to displace the windings axially in relation to each other or to abrade the key spacer separating two turns in the same winding from each other. It may bend conductors between rigid axial spacers, and during winding movements the insulation between the turns could be abraded, which can lead to short circuiting and damaging of the windings in the same layer, the same disk, different layers or different windings. Short circuit faults can cause great harms, because if the clamping pressure is not capable to counteract the involved forces,
significant winding deformation or even break down of windings can happen almost immediately, often convoyed with shorted turns.

2.4 Diagnosing mechanical faults in a transformer with the help of FRA

It is generally said that FRA has the capability of identifying faults of the types: core movement, winding deformation, winding movement, broken or loose winding or clamping structure, partial collapse of the winding and short-circuited turns or open circuit windings. Interpretation of FRA results, used for detecting mechanical changes inside a transformer, has neither been standardized nor fully agreed among researchers yet. Therefore, one may find several different types of transfer functions (in-impedance, transfer-impedance, voltage transfer ratio, etc.), different measurement techniques and different ways of interpreting FRA results. But as mentioned before, FRA is essentially a comparative method and therefore, a fingerprint response measurement of the same transformer which is going to be diagnosed or a sister unit should be available for comparison with the present measurement. When the measurement is compared with the reference set, then the changes in the frequency response which could be identified as mechanical faults are as follows:

- Abnormal shifts in the existing resonances.
- Emergence of new resonances or evaporation of the existing resonances.
- Considerable changes in the overall shape of the frequency response.

By the comparison of the new and the old measurements, an expert can identify possible faults.
3 Core model and measurements

In this Chapter, the core model developed separately from the winding model is presented. For efficient magnetic field calculations in electrical machines and transformers, the hysteresis and eddy current losses in laminated electrical steel must be modelled in a simple and reliable way. Therefore, in this thesis, a frequency dependent complex permeability model and a more detailed model (describing hysteresis, classical eddy current effects, and excess losses separately) are compared with single sheet measurements. It is discussed under which circumstances the simple complex-µ model is an adequate substitute for the more detailed model.

3.1 Background

Recent research has resulted in detailed models of the magnetic hysteresis and loss mechanisms in a wide frequency range [6−8]. Although these models provide a good description of magnetic material properties or of simple reluctance circuits based on them, they are too demanding numerically to be incorporated into a full-scale magnetic field simulation of a realistic geometry, as with a FEM or FDM calculation tool. In other words, while such a detailed simulation of the $H$-$B$ relation of a single or a few interacting cells is still perfectly feasible, simulating thousands or ten thousands of them simultaneously may be inconvenient or impossible.

Moreover, in many practical situations a detailed description is not required either. Often the goal is to obtain a good estimate of some local or global quantity containing much less information than the detailed local $H$-$B$ relation, such as the local losses causing dangerous hot spots, or simply the total losses in a machine relevant for cooling or economic reasons. For such applications it is desirable to use a simple model of magnetic hysteresis and losses, which can easily be incorporated in field calculation tools but which at the same time is sufficiently close to reality, within the frequency range of interest for the specific application. Such a model is the description of laminated magnetic materials by a suitable frequency dependent complex permeability, which is the most general linear description of a local and isotropic $H$-$B$ relation. If desired, it can easily be extended to a nonlocal and/or anisotropic $H$-$B$ relation by turning $\mu$ from a scalar function of position into a distance dependent integral kernel and/or tensor, respectively [9].

In this thesis, it is discussed to which extent the measurement results obtained with a single-sheet tester on strips of electrical steel can reliably be described by a simple complex permeability function of frequency. Both the resulting $H$-$B$ curves and the effective complex permeability are compared to the measured data at different frequencies. For comparison, simulation results obtained with a much more detailed model of the magnetic hysteresis, eddy current and excess losses are also reported.
3.2 Complex-permeability model

Reduced to its simplest terms, hysteresis introduces a time phase difference between $B$ and $H$. $B$ is assumed to lag $H$ by a constant angle $\theta_h$, called the hysteresis angle. In such a description, harmonics introduced by saturation are ignored, and the hysteresis loop becomes an ellipse whose major axis is inclined by an angle $\theta_h$ relative to the $H$-axis. Using complex field components $\hat{B}$ and $\hat{H}$, a low-frequency complex permeability including hysteresis can be defined as

$$\hat{\mu}_h = \frac{\hat{B}}{\hat{H}} = \mu_0 \mu_i e^{-j\theta_h}. \quad (2)$$

In addition to this, eddy currents in the lamination sheets introduce frequency dependence. The well-known procedure [10] for deriving the effective frequency dependent complex permeability is briefly sketched below. Faraday’s law

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad (3)$$

and Ampere’s law

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}, \quad (4)$$

in combination with the constitutive relations

$$\mathbf{J} = \sigma \mathbf{E}, \quad \mathbf{D} = \varepsilon \mathbf{E}, \quad \mathbf{B} = \hat{\mu}_h \mathbf{H}, \quad (5)$$

and time-harmonic assumption lead to

$$\nabla^2 \mathbf{H} = \hat{\alpha}^2 \mathbf{H} = j \omega \hat{\mu}_h (\sigma + j \omega \varepsilon) \mathbf{H}. \quad (6)$$

For lower frequencies when wave propagation can be ignored (i.e., $\sigma \gg \omega \varepsilon$), one has $\hat{\alpha}^2 \approx j \omega \hat{\mu}_h \sigma$.

---

Fig. 1. Laminate infinite in $z$ direction, with a width in $y$ direction much larger than its thickness $2b$, exposed to $a H$ field in $z$ direction.

For analysis of the magnetic field in a laminate, the simple geometry illustrated in Fig. 1 is appropriate. The magnetic field is applied in the $z$ direction, hence the only component of the magnetic field strength is $H_z$ which varies only in the $x$ direction, $H_z = H_z(x)$. In one dimension, Eq. (6) reduces to
\[
\frac{\partial^2 H_z}{\partial x^2} = \hat{\alpha}^2 H_z,
\]
which has the general solution
\[
H_z(x) = A_1 e^{\hat{\alpha}x} + A_2 e^{-\hat{\alpha}x}.
\]

The field strength on the both sides of the laminate is assumed to be \( H_0 \). For the reason of symmetry the following condition is obtained
\[
H_z(b) = H_z(-b) = H_0.
\]

The final expression for the magnetic field strength then becomes
\[
H_z(x) = H_0 \frac{\cosh(\hat{\alpha}x)}{\cosh(\hat{\alpha}b)}.
\]

The effective, complex permeability of a lamination is given as the average magnetic flux density \( \bar{B} \) in the laminate normalized to the surface magnetic field strength \( H_0 \),
\[
\hat{\mu}_{\text{eff}} = \mu'_{\text{eff}} - j\mu''_{\text{eff}} = \frac{\bar{B}}{H_0} = \frac{1}{H_0 2b} \int_{-b}^{b} H_0 H_z(x) dx = \hat{\mu}_0 \frac{\tanh(\hat{\alpha}b)}{\hat{\alpha}b}.
\]

This expression accounts for the effect of hysteresis without saturation, and the effect of eddy currents. It is assume here that additional (or “excess”) losses are either negligible or have a similar frequency dependence so that they can be incorporated in the expression (11) for \( \hat{\mu}_{\text{eff}} \).

### 3.3 Detailed hysteresis model

Later in this Chapter some results obtained with a more detailed model of the magnetic hysteresis, eddy current and excess losses will be reported, and this is therefore described here in some detail.

The total hysteresis is a combination of three different phenomena, namely, static hysteresis, eddy current effects and excess eddy currents. For the detailed hysteresis model, the following approach has been used. The static hysteresis is modelled using Bergqvist’s lag model [11, 12], the classical eddy currents are modelled using Cauer circuits [13, 8], and the excess losses are modelled using an approach by Bertotti [6].

#### 3.3.1 Static hysteresis

The Bergqvist’s lag model of static hysteresis starts from the idea that the magnetic material consists of a finite number of pseudo particles \( n_p \), i.e., volume fractions with different magnetization. The total magnetization is then a weighted sum of the individual magnetization of all pseudo particles.
The hysteresis curve for one particle is introduced by applying a “play operator” with a play equal to the “pinning strength” $k$ (which will determine the width of the hysteresis curve) on the an-hysteretic curve, see Fig. 2, where $m$ is the magnetization of the actual pseudo particle, and $\eta$ is the back field i.e. the field that will give the magnetization $m$ if no hysteresis is present.

Using a population of pseudo particles with different pinning strengths allows constructing minor loops. An individual pinning strength $\lambda_i k$ is assigned to every pseudo particle, where $k$ is the mean pinning strength, and $\lambda_i$ is a dimensionless number for particle $i$. The total magnetization is then given by a weighted superposition of the contributions from all pseudo particles (Fig. 3).

The expression

$$M_{an}(H) = m \arctan \left( \frac{\pi H \chi}{2M_s} \right)$$

is used for the an-hysteretic magnetization, where $M_s$ is the magnetization saturation and $\chi$ is the susceptibility at $H = 0$. For infinite number of pseudo particles, the total magnetization of the material is then given by

$$M = c M_{an}(H) + \int_0^\infty M_{an}(P_{\lambda k}(H)) \xi(\lambda) d\lambda,$$

where $c$ is a constant that governs the degree of reversibility, and the integral describes the hysteretic behaviour (irreversible part). $P_{\lambda k}$ is a play-operator with the pinning strength $\lambda k$, and $\xi(\lambda)$ is a density function describing the distribution of the pseudo particles. Finally, the magnetic flux density is obtained from $B = \mu_0 (H + M)$. 

Fig. 2. Anhysteretic curve (left), play operator (middle), and resulting hysteresis curve (right); figure taken from [7].

Fig. 3. Weighted superposition of the contributions from pseudo particles describes a minor loop; figure taken from [7].
3.3.2 Excess losses

Excess losses are caused by microscopic eddy currents induced by local changes in flux density due to domain wall movements. For the detailed model an approach described by Bertotti [6] is used. In this approach, a number of active correlation regions are assumed randomly distributed in the material. The correlation regions are connected to the microstructure of the material like grain size, crystallographic textures and residual stresses. In Bertotti’s model, the resulting contribution to the magnetic field strength is given by

$$H_{\text{excess}} = \frac{n_0 V_0}{2} \left( \sqrt{1 + \frac{4 \sigma G 2 b w}{n_0 V_0} \left| \frac{dB}{dt} \right|} - 1 \right) \text{sign} \left( \frac{dB}{dt} \right),$$

(14)

where $w$ is the width of the laminate and $2b$, as before, its thickness. $G$ is a parameter depending on the structure of the magnetic domains. $n_0$ is a phenomenological parameter related to the number of active correlation regions when the frequency approaches zero, whereas $V_0$ determines to which extent micro-structural features affect the number of active correlation regions.

The parameters $n_0$ and $V_0$ are by definition frequency independent, but they are expected in reality to depend on the amplitude of the $B$ field [14]. Since the precise form of this dependence is unknown, their values are usually adjusted empirically for a given amplitude.

The parameters $n_0$ and $V_0$ are used, although the amplitude of the $B$ field varies slightly in the measurements.

3.4 Measurement setup

The magnetic measurements were carried out using a Single Sheet Tester. It consists of two equal U-shaped yokes placed face-to-face to each other (Fig. 4). The magnetic sheet to be tested is placed between the yokes and most of the flux is forced through it due to its high permeability. For the measurement of the flux in the test material a coil is surrounding the strip which is connected to a flux meter. The magnetic field strength is measured with a Hall probe placed close to the surface of the sample and connected to a Tesla meter. A sinusoidal $H$ field was applied to the sample; the $H$ and $B$ field values were measured for 100 periods and numerically filtered. Thereafter, the mean values at different phase angles of the $B$ and $H$ fields were calculated. These values were then used in the thesis.
The measured $H$-$B$ curve is approximated with a complex-$\mu$ ellipse characterized by the permeability $\mu_{\text{meas}}$ by matching both its peak values $H_p$, $B_p$ and its area $A$ to the measured results. This is of course appropriate as long as the shape of the measured $H$-$B$ curve is close to an ellipse, i.e., if saturation effects are not too pronounced. The area $A$, which measures the power loss per cycle, is given by the integral

$$A = \oint B_{\text{meas}} \, dH_{\text{meas}} = \int_0^T B_{\text{meas}} \frac{dH_{\text{meas}}}{dt} \, dt,$$

(15)

where $B_{\text{meas}}$ and $H_{\text{meas}}$ are the time dependent measured $B$ and $H$ fields, respectively, and $T$ is the duration of a period. If the measured magnetic field strength is assumed to vary sinusoidally,

$$H_{\text{meas}}(t) = \text{Re}(H_p e^{j\omega t}) = H_p \cos(\omega t),$$

(16)

then its derivative becomes

$$\frac{dH_{\text{meas}}(t)}{dt} = -\omega H_p \sin(\omega t),$$

(17)

and the measured magnetic flux density

$$B_{\text{meas}}(t) = \text{Re}(\mu_{\text{meas}} H_p e^{j\omega t}) = \text{Re}((\mu'_{\text{meas}} - j\mu''_{\text{meas}}) H_p e^{j\omega t}) = H_p (\mu'_{\text{meas}} \cos(\omega t) + \mu''_{\text{meas}} \sin(\omega t)).$$

(18)

By inserting Eq. (18) and (17) into Eq. (15) one gets

$$\mu''_{\text{meas}} = -\frac{A}{\pi H_p^2}.$$  

(19)
Furthermore, from the relation \( \hat{\mu}_{\text{meas}} | H = B \), one obtains

\[
|\hat{\mu}_{\text{meas}}|^2 = (\mu'_{\text{meas}})^2 + (\mu''_{\text{meas}})^2 = \left( \frac{B_p}{H_p} \right)^2, \tag{20}
\]

which implies

\[
\mu'_{\text{meas}} = \sqrt{\left( \frac{B_p}{H_p} \right)^2 - (\mu''_{\text{meas}})^2}. \tag{21}
\]

Both \( \mu'_{\text{meas}} \) and \( \mu''_{\text{meas}} \) are functions of frequency.

Fig. 5 compares the measured \( H-B \)-curves with complex-\( \mu \) ellipses, generated with the adapted \( \hat{\mu}_{\text{meas}} \) at frequencies \( f = 50 \) Hz and 400 Hz.

\( \hat{\mu}_{\text{meas}} \) as defined in Eq. (11) is a function of frequency and of a vector \( x = (\mu, \theta, \sigma b^2) \) containing the model parameters. It is adjusted to measured data by numerically minimizing the expression

\[
\sum_{i=1}^{N} |\hat{\mu}_{\text{eff}}(x, \omega_i) - \hat{\mu}_{\text{meas}}(\omega_i)|^2 \tag{22}
\]

with respect to \( x \). \( \hat{\mu}_{\text{meas}}(\omega_i) \) are the measured complex permeability values, defined by Eq. (19) and (21), at \( N \) different frequencies \( \omega_i = 2\pi f, \ i = 1, \ldots, N \). Measurements at \( N = 9 \) different frequencies ranging from 50 Hz to 2 kHz (see Figs. 6 and 7 below) were performed on a 100 mm \( \times \) 3.2 mm strip of the non-oriented magnetic material M600 with a thickness of \( 2b = 0.5 \) mm.
3.5 Comparison between model and measurements

Since the measurement setup was quite sensitive to noise, the measurements had to be numerically filtered. Adjusting $\hat{\mu}_{\text{eff}}$ to the filtered data using Eq. (22), the following model parameter values are obtained: $\mu_r = 3366$, $\theta_h = 0.477$ rad, and $\sigma^2 = 0.243$ Sm, i.e., $\sigma = 3.89 \times 10^6$ S/m which is somewhat larger than the true dc conductivity $\sigma_{\text{dc}} = 3.33 \times 10^6$ S/m since excess losses were included in the classical phenomenological form (11). In Fig. 6, the real and imaginary parts of the measured complex permeability are compared at different frequencies with the adjusted $\hat{\mu}_{\text{eff}}$.

![Graph showing the comparison between model and measurements](image)

Fig. 6. Real and imaginary parts of the measured complex permeability (symbols) and of the fitted permeability function (curves).

The agreement is quite satisfactory considering the simplicity of the model, especially at higher frequencies. The deviation between $\mu'_{\text{meas}}$ and $\mu'_{\text{eff}}$ at the lowest frequencies is probably due to saturation effects which are not properly taken into account by the expression (11) for $\hat{\mu}_{\text{eff}}$, see for instance the measurement at 50 Hz (Fig. 5(a)). The amplitude had to be chosen large enough for the signal not to be covered by noise. Below the $H-B$ hysteresis curves are shown for all measured frequencies. Measurement, simple model, and detailed model are represented by solid green lines, dashed blue lines and dotted red lines, respectively.
The above way of defining a “best fit” of ellipses to the more complicated $H$-$B$ hysteresis relations approximately preserves both $H$ and $B$ amplitudes and magnetic losses in the whole frequency range. This is illustrated in the Fig. 7, where the measured $H$-$B$ curves are compared with the corresponding complex-$\mu$ ellipses and the detailed model at different frequencies. As can be seen the simple model agrees very well with the measurements as long as saturation is not too strong, which means for low amplitude fields and/or for frequencies higher than about 200 Hz.
4 High frequency winding model

In this Chapter, a winding model based on a lumped element approach with three different levels of discretization has been developed. The developed models are analyzed using state space analysis in the frequency domain and the impedances for the three different models are compared to each other to detect the new phenomena emerging for higher frequencies as the model discretization is made finer and finer.

4.1 Transformer winding model for high frequency applications

In the field of transformer winding modeling, various approaches and tools are available. Among the most common tools are lumped element circuits \([15–16]\). Usually all the turns of one or two discs are lumped together into one inductive element (“segment”) of the model, which leads to a decreased computation time but also to a reduction of the model’s upper frequency limit, typically to values around some 100 kHz.

In this thesis, lumped element models with much higher resolution (up to 4 segments per turn) of a single winding with \(n\) turns in every disc are studied, in order to increase their upper frequency limit.

Each lumped element represent a part, a section of the physical geometry with similar quantities like magnetic flux, electric potential, resistance and etc, and these lumped elements are connected together to represent the whole geometry. For power transformers, the windings are divided into finite sections represented by lumped resistance, inductance, capacitance and conductance where each section should be small enough so that it can be assumed that the current through it is constant and not influenced by the displacement current which will be noticeable at higher frequencies.

Up to a few hundreds of kilo hertz, the displacement current will not be so remarkable and can be approximated to zero, so that a winding can merely be modeled by means of its self and mutual inductances and resistance. But at higher frequencies, the aforementioned approximation is no longer valid, and the displacement currents from a section to other sections or to conductive bodies have to be accounted for, for the model to be realistic, and this is done by means of capacitors. The total capacitance for a particular section is then split into two halves and located at both ends of the section. As mentioned above, the complete winding model is made by connecting all the sections together.

In this thesis, the transformer winding is a single phase continuous disc winding (Fig. 9), composed of quadratic discs as in Fig. 8. Also, since the low voltage winding is on a much lower voltage than the high voltage winding, it (the LV-winding) is replaced by ground in the models developed here.

4.2 Three different resolutions of the model

Three different model resolutions are studied: in the models labeled 1, 2, and 3, each turn in the discs is modeled by one, two, or four segments, respectively. This implies that there will be one, two, or four capacitances between any two neighboring turns in the winding,
respectively (see Fig. 8 from right to left). Every segment consists of one resistance in series with one self inductance, as it is depicted in Fig. 10, and there are mutual inductances between any two parallel segments of the winding. For simplicity, self inductances and resistances are shown in Fig. 8 on one segment only, and some of the mutual inductances are indicated by arrows. In case of model 1, also the connection of the voltage source for impedance measurement for one single disc is shown. All the inductance and capacitance parameters (i.e., all model parameters except for the damping resistances) are estimated from the physical winding geometry, and not fitted to measurements, and they are discussed in the next section. The models are analyzed by solving their state space equations [15–17] in the frequency domain, which is discussed in section 5 of this Chapter.

Fig. 8. The three different levels of discretization with the nodes numbered in an increasing sequence from one winding end to the other. The resolution increases from model 1 to model 3.

Fig. 9. The cross section of the continuous disc winding.

Fig. 10. One segment and its electrical circuit equivalence.
4.3 Calculation of the capacitances

The capacitance, which reflects the electric field energy stored in a system, is defined by both its geometry and the relative permittivity of the dielectric material used. The relation for the capacitance between two planar surfaces, capacitance = permittivity × area / distance, is used. The total capacitance between two turns $C_{tt}$ in the quadratic disc is then approximately given by

$$C_{tt} = 4\varepsilon_0 \varepsilon_1 (d + d') \frac{h + 2\tau_i}{\tau_i}$$  \hspace{1cm} (23)

where $h$ is the height of the conductor, $\tau_i$ is twice the insulation thickness, $\varepsilon_1$ is the relative permittivity of the conductor insulation, $(d + d')$ is the mean side lengths of the disc (see Fig. 9), and the addition of $2\tau_i$ to $h$ accounts for the fringing effect. The total capacitance between two discs, if they are close enough to each other and if air is used as insulation between them, is approximately given by

$$C_{dd} = 4\varepsilon_{air} \varepsilon_0 \frac{(d')^2 - d^2}{\tau_i + \tau_{ks}}$$  \hspace{1cm} (24)

where $\varepsilon_{air}$ is the relative permittivity of air (=1) and $\tau_{ks}$ is the distance between two discs. The total capacitance between one disc and the outer ground wall is given by

$$C_{og} = 8\varepsilon_{air} \varepsilon_0 \frac{d'h'}{d_0 K}$$  \hspace{1cm} (25)

and the total capacitance between one disc and the inner ground wall is given by

$$C_{ig} = 8\varepsilon_{air} \varepsilon_0 \frac{dh'}{d_i K}$$  \hspace{1cm} (26)

where $K$ is the total number of discs in the winding, $h'$ is the total height of the winding, and $d_0$ and $d_i$ are the outer and inner distances between the winding and the ground respectively.

4.4 Calculation of the inductances and resistances

For the calculation of the self and mutual inductances, formulas from Ref. [18] are used. The self inductance $L_{self}$ of each straight segment with the length $l$, height $h$ and width $w$ in the disc is

$$L_{self} = \frac{\mu_0}{2\pi} \left[ \ln \left( \frac{2l}{0.2235(w + h)} \right) - 1 \right] .$$  \hspace{1cm} (27)

The mutual inductance $M$ between two segments which are perpendicular to each other is zero. That between two parallel segments of length $l$ in Fig. 11, separated by a distance $x$, is given by
When the segments are parallel but have different lengths, as in Fig. 12, the mutual inductance is given by

$$2M = \left(M_{m+p} + M_{m+q}\right) - \left(M_p + M_q\right),$$

(29)

where for example $M_{m+p}$ is the mutual inductance between two straight wires both having the length $m+p$ and being placed relative each other as in Fig. 11, and which for the symmetric case $p = q$ reduces to

$$M = M_{m+p} - M_p.$$  

(30)

Of course, in the formulas for the mutual inductances it is assumed that the conductors have very small cross section areas, which is just an approximation. The resistance $R$ of each segment is assumed to be of the form

$$R = \alpha l \left(\frac{1}{\sigma w h} + \frac{1}{2(w+h)} \sqrt{\frac{\mu_0 \pi f}{\sigma}}\right),$$

(31)

where $\sigma$ is the conductivity of the conductor and $f$ is the frequency. The first term is the DC resistance and the second term accounts for the skin effect at higher frequencies. Since proximity losses are not included in the model, a numerical factor $\alpha > 1$ has been introduced and adjusted so that a realistic level of resonance damping is obtained.
4.5 State space model for a single winding

The circuit model for the three different winding models in Fig. 8 and 9 of the single winding consists of $K \cdot n_i$ winding sections resulting in $K(n_i + 1)$ nodes and $K \cdot n_i$ inductive branches and associated capacitances and resistances, where $n_i = 2^{i-1} n$, for $i = 1, 2, 3$ for model 1, 2 and 3 respectively, and where $n$ is the number of turns in one disc. $K$ is the number of discs used in the winding and hence one will arrive at the following two matrix equations by considering the voltage difference between the nodes of inductive branches and the current conservation at the nodes:

$$\Gamma I = C \frac{d}{dt} V \tag{32}$$

$$-\Gamma'V = L \frac{d}{dt} I + RI \tag{33}$$

Here $V$ and $I$ are the vectors containing the voltages at the nodes and the currents in the inductive branches

$$V = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_{Kn_i} \\ v_{K(n_i+1)} \end{bmatrix}$$

$$I = \begin{bmatrix} i_1 \\ i_2 \\ \vdots \\ i_{Kn_i} \\ i_{K(n_i+1)} \end{bmatrix}$$

(34)

The matrix $\Gamma$ connects the currents and voltages and consists of 1, $-1$ and 0, and $\Gamma^t$ is the transpose of $\Gamma$.

$$\Gamma = \begin{bmatrix} S & 0 & \cdots & \cdots & 0 \\ 0 & S & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ \vdots & \vdots & \cdots & S & 0 \\ 0 & \cdots & \cdots & 0 & S \end{bmatrix}$$

and

$$S = \begin{bmatrix} -1 & 0 & \cdots & \cdots & 0 \\ 1 & -1 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ \vdots & \vdots & \cdots & 0 & 1 \\ 0 & \cdots & \cdots & 0 & 1 \end{bmatrix} \tag{35}$$

The resistance matrix $R$ for the whole winding is a diagonal matrix

$$R = \begin{bmatrix} R_{\text{disc}} & 0 & \cdots & \cdots & 0 \\ 0 & R_{\text{disc}} & 0 & \vdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & R_{\text{disc}} & 0 \\ 0 & \cdots & \cdots & 0 & R_{\text{disc}} \end{bmatrix} \tag{36}$$

composed of the resistances of the discs where
\[
R_{\text{disc}} = \begin{bmatrix}
R_{\text{seg,1}} & 0 & \cdots & \cdots & 0 \\
0 & R_{\text{seg,2}} & 0 & \vdots & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
0 & \cdots & 0 & R_{\text{seg,n-1}} & 0 \\
0 & \cdots & \cdots & 0 & R_{\text{seg,n}}
\end{bmatrix}_{n \times n}
\]  

(37)

is composed of the resistances of each segment in one disc. The inductance matrix \(L\) for the whole winding is composed of smaller matrices

\[
L = \begin{bmatrix}
L_{\text{disc}} & L_{12} & L_{13} & \cdots & L_{1K} \\
L_{21} & L_{\text{disc}} & L_{23} & \cdots & L_{2K} \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
L_{K1} & L_{K2} & \cdots & \cdots & L_{\text{disc}}_{K_{n-1}}
\end{bmatrix}_{K_{n} \times K_{n}}
\]  

(38)

where the off-diagonal matrices \(L_{ij}\) are \(n \times n\) matrices for the mutual inductance between disc \(i\) and \(j\), and the matrix in the diagonal \(L_{\text{disc}}\) is the inductance matrix for a single disc, and it is composed of the mutual inductances \(M\) and self inductances \(L_{\text{self}}\) of the segments in one disc.

\[
L_{\text{disc}} = \begin{bmatrix}
L_{\text{self,1}} & M_{12} & M_{13} & \cdots & M_{1K} \\
M_{21} & L_{\text{self,2}} & M_{23} & \cdots & M_{2K} \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
M_{K1} & M_{K2} & \cdots & \cdots & L_{\text{self,n-1}}_{K_{n-1}}
\end{bmatrix}_{n_{i} \times n_{i}}
\]  

(39)

The total capacitance matrix \(C\) in Eq. (32) is

\[
C = \begin{bmatrix}
C^{(i)} + C_{DD}^{(i)} & -C_{DD}^{(i)} & \cdots & \cdots & \cdots & 0 \\
-C_{DD}^{(i)} & C^{(i)} + 2C_{DD}^{(i)} & -C_{DD}^{(i)} & \ddots & \ddots & \vdots \\
0 & \vdots & \ddots & \ddots & \ddots & \vdots \\
\vdots & \ddots & \ddots & \ddots & \ddots & 0 \\
0 & \cdots & 0 & -C_{DD}^{(i)} & C^{(i)} + 2C_{DD}^{(i)} & -C_{DD}^{(i)}
\end{bmatrix}_{K_{(n-1)} \times K_{(n+1)}}
\]  

(40)

where \(i = 1, 2, 3\) for model 1, 2 and 3 respectively as mentioned before. \(C^{(i)}\) is the “specific” capacitance matrix for model \(i\), and for model 1, the roughest model, it is
\[
C^{(1)} = \begin{bmatrix}
\frac{1}{2}C_u^{(1)} + C_{ig}^{(1)} & -\frac{1}{2}C_u^{(1)} & 0 & \cdots & \cdots & \cdots & 0 \\
-\frac{1}{2}C_u^{(1)} & \frac{3}{2}C_u^{(1)} & -C_u^{(1)} & 0 & \cdots & \cdots & 0 \\
0 & -C_u^{(1)} & 2C_u^{(1)} & -C_u^{(1)} & 0 & \cdots & 0 \\
\vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & \cdots & \cdots & 0 & -C_u^{(1)} & 2C_u^{(1)} & -C_u^{(1)} \\
0 & \cdots & \cdots & \cdots & \cdots & \cdots & 0 \\
\end{bmatrix}
\]

(41)

where \( C_u^{(1)} = C_u, C_{ig}^{(1)} = C_{ig}, C_{og}^{(1)} = C_{og} \). For model 2, the next finer model, it is

\[
C^{(2)} = \begin{bmatrix}
\frac{1}{2}C_u^{(2)} + C_{ig}^{(2)} & 0 & \frac{1}{2}C_u^{(2)} & 0 & \cdots & \cdots & \cdots & 0 \\
0 & C_u^{(2)} + C_{ig}^{(2)} & 0 & -C_u^{(2)} & 0 & \cdots & \cdots & 0 \\
-\frac{1}{2}C_u^{(2)} & 0 & \frac{3}{2}C_u^{(2)} & 0 & -C_u^{(2)} & 0 & \cdots & \vdots \\
0 & -C_u^{(2)} & 0 & 2C_u^{(2)} & 0 & -C_u^{(2)} & \ddots & \cdots \\
\vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & \cdots & \cdots & 0 & -C_u^{(2)} & 0 & \frac{3}{2}C_u^{(2)} & 0 \\
0 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & 0 \\
0 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & 0 \\
\end{bmatrix}
\]

(42)

where \( C_u^{(2)} = \frac{1}{2}C_u, C_{ig}^{(2)} = \frac{1}{2}C_{ig}, C_{og}^{(2)} = \frac{1}{2}C_{og} \). For model 3, the finest model, it is

\[
C^{(3)} = \begin{bmatrix}
\frac{1}{2}C_u^{(3)} + C_{ig}^{(3)} & 0 & 0 & \frac{1}{2}C_u^{(3)} & 0 & \cdots & \cdots & \cdots & 0 \\
0 & C_u^{(3)} + C_{ig}^{(3)} & 0 & 0 & -C_u^{(3)} & 0 & \cdots & \cdots & \cdots & \vdots \\
0 & 0 & C_u^{(3)} + C_{og}^{(3)} & 0 & 0 & -C_u^{(3)} & 0 & \cdots & \cdots & \ddots \\
0 & 0 & 0 & C_u^{(3)} + C_{og}^{(3)} & 0 & 0 & -C_u^{(3)} & 0 & \cdots & \ddots \\
-\frac{1}{2}C_u^{(3)} & 0 & 0 & 0 & \frac{3}{2}C_u^{(3)} & 0 & 0 & -C_u^{(3)} & 0 & \cdots \\
0 & -C_u^{(3)} & 0 & 0 & 0 & 2C_u^{(3)} & 0 & 0 & 0 & -C_u^{(3)} \\
\vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & \cdots & \cdots & 0 & -C_u^{(3)} & 0 & 0 & 2C_u^{(3)} & 0 & 0 \\
0 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
0 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
\end{bmatrix}
\]

(43)

where \( C_u^{(3)} = \frac{1}{4}C_u, C_{ig}^{(3)} = \frac{1}{4}C_{ig}, C_{og}^{(3)} = \frac{1}{4}C_{og} \).

The matrix \( C_D^{(i)} \) in Eq. (40) accounts for the capacitive coupling between to neighbouring discs and it has the form
\[
C_{dd}^{(i)} = \begin{bmatrix}
\frac{1}{2} C_{dd}^{(i)} & 0 & \ldots & \ldots & 0 \\
0 & C_{dd}^{(i)} & 0 & \ldots & \vdots \\
\vdots & \ddots & \ddots & \ddots & \ddots \\
\vdots & \vdots & 0 & C_{dd}^{(i)} & 0 \\
0 & \ldots & \ldots & 0 & \frac{1}{2} C_{dd}^{(i)} \\
\end{bmatrix}_{(n_i+1) \times (n_i+1)}
\]  
(44)

where \(C_{dd}^{(i)} = \frac{1}{n_i} C_{dd}\) for all three models i.e. \(i = 1, 2, 3\).

The pre-factors \(\frac{1}{2}\) in the capacitance matrices are due to the fact that the capacitance connected to the first and the last nodes in a disc account only for a half segment, and the pre-factors \(1, 2\) and \(\frac{3}{2}\) in the diagonals are due to the fact that, if there is no capacitance to ground, the sum of the elements in a row/column must be equal to zero.

When an external voltage source is connected to a node \((k)\), its node voltage is no longer unknown. The voltage at that node \(V_k\) and its time derivative \(\frac{d}{dt} V_k\) should therefore be separately inserted in (32) and (33) as an input vector accompanied by the corresponding columns of matrices \(C\) and \(\Gamma\). Eq. (32) and (33) are then transformed to

\[
\Gamma I = C \frac{d}{dt} V + Q \frac{d}{dt} V_k
\]

\[
-PV_k - \Gamma' V = L \frac{d}{dt} I + RI.
\]  
(46)

Here, \(Q\) consists of one column taken out from \(C\) matrix corresponding to index \(k\), and \(P\) consists of the \(k\):th column taken out from \(\Gamma'\) (transpose of \(\Gamma\)). By rearranging terms in these equations and putting them in one matrix equation, Multi Input Multi Output (MIMO) state space model of the lumped parameter circuit can be formulated as:

\[
\frac{d}{dt} X = AX + BV_k
\]  
(47)

where

\[
X = \begin{bmatrix} V \\ I \end{bmatrix}, \quad A = \begin{bmatrix} O & C^{-1} \Gamma \\ -L^{-1} \Gamma' & -L^{-1} R \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} -C^{-1}Q \frac{d}{dt} \\ -L^{-1} P \end{bmatrix}.
\]  
(48)

The state vector \(X\) consists of all the nodal voltages (except the applied ones) and inductor currents of the lumped circuit. By taking the Laplace transformation of the equation system and selecting all state variables as outputs, one will arrive at
\[ H(s) = \frac{X(s)}{V_k(s)} = (sI - A)^{-1} B', \]  
\[ \text{where } s \text{ is the Laplace variable, } I \text{ is the identity matrix with the same size as } A, \text{ and} \]
\[ B' = \begin{bmatrix} -C^{-1}Qs \\ -L^{-1}P \end{bmatrix}. \]  

\[ H(s) \] contains all the transfer functions of the nodal voltages and inductor currents with respect to the applied voltage \( V_k \).

### 4.6 Comparison between the three models

The first comparison between the three models is made for a single disc winding and the new phenomena that emerge with increasing model resolution are studied. The disc in the models consist of \( n = 10 \) turns of varnished copper wire with the conductivity \( \sigma = 5.8 \times 10^7 \text{ S/m} \), the conductor height \( h = 7 \text{ mm} \) and width \( w = 3 \text{ mm} \). The inner sides of the square disc have a length of \( 1.2 \text{ m} \) (\( 2d = 1.2 \text{ m} \)), and the gap between any two neighboring conductors (turns) i.e. twice the insulation thickness is \( \tau_i = 0.4 \text{ mm} \), and there is no ground wall which means that \( d_0 \) and \( d_i \) are set to infinity in the calculations (see Fig. 9 for a geometrical illustration of the parameters). The same dimensions are used for the experimental setup which will be explained in Chapter 6, and the reason for the dimensions chosen will be explained in the same Chapter.

The magnitudes of the calculated impedances \( Z(f) = \frac{U(f)}{I(f)} \) (see Fig. 8) are compared to each other in Fig. 13.

![Impedance magnitude for the different models.](image)

It can be seen that several resonances occur: the first, pronounced impedance maximum is the fundamental resonance of the winding, due to the total inductance and series capacitance of the whole disc. As it will be argued in Chapter 8 section 1, the three following resonances can
be interpreted as “radial” resonance modes, and the two after that, which form a pronounced impedance minimum above 10 MHz and do not appear in the lowest-resolution model 1, as “azimuthal” resonance modes.

As can be seen in Fig. 13, for model 1 the impedance becomes purely capacitive i.e. it becomes of the form $Z = (j\omega C)^{-1}$ after the radial modes i.e. for frequencies higher than about 6 MHz. This is not a physical reality and it means that model 1 is for sure not valid for that part of the frequency spectrum. For model 2 and 3, the impedance becomes purely capacitive in the end of the frequency spectrum after the azimuthal modes, which as expected would mean that with finer discretization the model becomes valid for higher frequencies.
5 Time Domain Reflectometry (TDR)

Time Domain Reflectometry or TDR is a measurement technique used to determine the characteristics of electrical lines by observing reflected waveforms. It is a powerful tool for the analysis of electrical or optical transmission media such as coaxial cables [20] and optical fibers [21], or for the measurement of soil characteristics in geology and soil science [22] or etc.

The TDR analysis begins with the propagation of a step or impulse of energy into a system and the subsequent observation of the energy reflected back by the system. When the launched wave reaches the end of the cable/transmission line or any impedance change along it, part or all of the pulse energy is reflected back.

By analyzing the magnitude, duration and shape of the reflected wave, the nature of the impedance variation in the transmission system can be determined. The impedance change of the discontinuity can be determined from the amplitude of the reflected signal. The distance to the reflecting impedance change can also be determined from the time that a pulse takes to return.

The possibilities for the TDR method for use in transformer faults diagnostics has, as far as it is known to the author, not been developed nor studied, so in this thesis, the possibilities this method can have to offer for detection of transformer winding faults will be explored.

The idea is, as mentioned above, that a pulse or step signal wave is sent into the transformer winding and winding faults are detected through the reflected wave or waves. TDR would then detect reflections coming from mechanical changes and deteriorations, but it would also record reflections related to normal geometrical irregularities along the winding length. So the solution is that one takes the difference between TDR measurements before and after changes have occurred. Thus the signals from the mechanical changes and faults can be distinguished from other reflections that are constant and normal. In other words, the difference between the TDR measurements before and after mechanical faults would contain information only about the mechanical faults and changes; hence the method is called differential TDR or DTDR (see Fig. 14).
The same models developed above (model 1, 2 and 3) are then solved in time domain with the only difference that the frequency dependent part of the resistance which counts for the skin effect is removed, and this is because the models can’t be used in the time domain if this part is present. Removing the frequency dependent part of the resistance affects only the damping of the high frequency components of the pulse sent into the winding, and it doesn’t have any greater impact on the overall response of the models.

Solving the models in the time domain means that Eq. (47), with $V_k$ being a pulse or step voltage, is solved in Matlab with the ode functions which solve ordinary differential equations. With the in-voltage $V_k$ being fixed and known, the current in the in-port of the winding is checked for the interesting reflections.

A realistic winding geometry with 60 discs (connected together as in Fig. 9) and 10 turns per disc, with the parameters listed in the table below, is used as an illustration. Notice that for this case, oil is used as insulation instead of air, so $\varepsilon_{\text{oil}}$ is used instead of $\varepsilon_{\text{air}}$ in the previous equations.

<table>
<thead>
<tr>
<th>$d$ [mm]</th>
<th>$w$ [mm]</th>
<th>$h$ [mm]</th>
<th>$\tau_1$ [mm]</th>
<th>$\tau_{\text{ks}}$ [mm]</th>
<th>$\sigma$ [S·m⁻¹]</th>
<th>$\varepsilon_{\text{oil}}$</th>
<th>$\varepsilon_{\text{i}}$</th>
<th>$d_v$ [mm]</th>
<th>$d_i$ [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>150</td>
<td>2.5</td>
<td>9</td>
<td>0.4</td>
<td>4.5</td>
<td>$5.8 \cdot 10^7$</td>
<td>2.2</td>
<td>2.95</td>
<td>70</td>
<td>28</td>
</tr>
</tbody>
</table>

The damaged winding is simulated as the tenth disc being displaced, which is simulated by changing the disc-to-disc capacitance between that disc and the two neighboring discs with 10%.

The in-voltage is $V_k = 0.001 \left( \frac{1}{e^{\frac{\tau_1}{\tau_2}}} - \frac{1}{e^{\frac{\tau_1}{\tau_2}}} \right)$, with $\tau_1 = 0.5 \mu$s and $\tau_2 = 1000 \tau_1$. The simulations are done with model 1, and the result for the DTDR for the current in the in-port is plotted in Fig. 15.
The first (negative) pulse in Fig. 15 is due to the disc displacement and the second (positive) pulse with larger magnitude is due to the reflection at the grounded end of the winding. The first one occurs for the time $t_1 \approx 2 \mu s$ and the second one occurs for the time $t_2 \approx 11.5 \mu s$. The distance to the displaced disc and to the end of the winding is given by $s_1 = v \cdot t_1$ and $s_2 = v \cdot t_2$ respectively, where $v$ is the velocity of the wave traveling along the winding. This would give

$$\frac{s_1}{s_2} = \frac{t_1}{t_2} = 0.174$$

But as it is known, the 10:th of the 60 discs is the displaced one, so the distance ratio is $10/60 \approx 0.167$, which is close enough to the value calculated in Eq. (51). So in this way, the location of the fault can be determined. The effect of the type of fault on the shape of the reflected wave, its bandwidth and sign and etc. will be studied further in the future in the doctoral thesis.

On more thing that can be noticed in Fig. 15 is that even though the pulse sent into the winding is very steep, the reflected pulses have a softer shape, a wider bandwidth, and this is due to the fact that the capacitances between the turns and between the discs, and the mutual inductances introduce the effect of dispersion of the signal when it travels forth and back in the winding. This has been observed in the simulations, and it has been noticed that if the capacitances and inductances mentioned above are removed, then there will be no dispersion of the signal. The dispersion effect will also be studied and examined in the doctoral thesis.

![Graph of the DTDR current in the in-port](image)
6 Frequency response measurements

In this Chapter, the measurement device and the experimental setup are introduced and the choice of the dimensions of the setup is explained. Also, the different measurements performed will be presented.

6.1 Impedance measurement device and dimensioning of the experimental setup

A network analyzer Bode 100 from Omicron Electronics [19] (frequency range 1 Hz – 40 MHz) was used for the impedance measurements (see Fig. 16).

Fig. 16. Impedance measurement device Bode 100 from Omicron Electronics.

As mentioned before, the shape of the discs is chosen quadratic so that all the self and mutual inductances can be calculated by simple analytic formulas from [18]. The location of the resonances of the winding in the frequency spectrum depends on the physical geometry and material properties of the winding, and generally, larger dimensions of the winding leads to larger inductance and capacitance values, which in turn leads to the resonances occurring for lower frequencies.

The measurement device can measure up to the frequency of 40 MHz, but since the measurements will be more sensitive to the effects of environmental noise and measurement
cables for the higher part of that frequency range, the measurements will be more disturbed and unreliable for that part (the higher part) of the frequency range, and due to this fact, the geometrical size of the discs had to be chosen so that all the interesting phenomena and resonances occur for frequencies below approximately 20 MHz. The most effective way to satisfy this requirement is to design the square discs with large side lengths. So the constructed discs consist of $n = 10$ turns of varnished copper wire with rectangular cross section (7 mm $\times$ 3 mm). The inner sides of the square discs have a length of 1.2 m, and the gap between any two neighboring conductors (turns) is varying between 0.4 mm (= twice the insulation thickness) and about 1 mm because of manufacturing irregularities. The cross section dimensions of the varnished copper wires (7 mm $\times$ 3 mm) are regular wire dimensions used in power transformers, and the number of turns can not be chosen too high (the discs will be too heavy and impractical to handle), or too low (there will be too few resonances), so $n = 10$ turns seemed reasonable and was chosen.

Ten units of these discs were manufactured by the transformer manufacturing company Nordtrafo AB (see Fig. 17).

Fig. 17. Ten separate disc units manufactured by Nordtrafo AB
6.2 Impedance measurement results

The first measurements were performed on each of the ten different single disc units separately (Fig. 18).

By comparison between the impedances of the different discs, it could be affirmed that due to the manufacturing irregularities mentioned above, the impedances of the discs differ from each other more and more as the frequency is increased and this seems consistent, since for higher frequencies, the capacitances between the turns in the discs play a larger part in shaping the impedance, and since a capacitance is per definition sensitive for small distance changes between two conductors, and since (as mentioned earlier) the gap between any two neighboring conductors (turns) is varying between 0.4 mm (= twice the insulation thickness \( r_i \)) and about 1 mm (because of manufacturing irregularities), then the discs have different capacitive features for higher frequencies, leading to different high frequency impedances. In Fig. 19, the measured impedance for four of the discs is plotted to illustrate the differences.
It can from Fig. 19 be seen that for frequencies up to approximately 300 kHz, the discs have the same impedance value, and this is the so called inductive regime where the impedance behaves as $Z = j\omega L$, and since the inductance is not so sensitive for small irregularities in the physical geometry, hence the discs have all the same impedance. But as the frequency is approached to the fundamental coil resonance frequency, the capacitance starts to play in, and it can be seen that from now on the impedances do not coincide with each other. The “radial” and “azimuthal” modes are still there for all of the discs, but they do not have exactly the same location, shape and amplitude.

Measurements were then performed on units with two, three and four discs respectively. The distance between two neighbouring discs $\tau_{ks}$ was changed between approximately 1 mm to 5 cm, and different types of connections between the disc were tried, and all these for comparison between models and measurements. It was found that as the distance between the discs is decreased, the models deviate more and more from the measurements, and this will be discussed and explained in Chapter 7 and 8.

Next, the model simulations will be compared to measurements for verification.
7 Model verification

In this Chapter, model 3 is verified by comparison with measurements. The model is compared to measurements for one, two, three and four discs respectively, and it is shown that the resonances predicted by the model also occur in the measurements.

7.1 Comparison of model with measurements

The impedance magnitude of model 3 for one single disc is compared to the measured one in Fig. 20.

Fig. 20. Comparison between measurement and model 3 for a single disc.

Comparison between measurement and model (Fig. 20) shows that the radial modes and the high-frequency azimuthal modes, which model 1 is unable to produce and which model 2 produces partially, are no model artifacts but real physical phenomena. In the next Chapter, the physical meaning of the radial and azimuthal resonances will be explained and discussed. In the measurements, the radial modes are shifted toward somewhat higher frequencies compared to the simulations, but they are fully recognizable. Such a shift is expected to occur due to the proximity effect which has not been taken into account in the model calculations reported here, and which is going to be discussed in Chapter 8.

Next, the impedance magnitude of model 3 for a two, three and four disc winding, with the distance between two discs \( r_{ks} \) being approximately 1 mm, is compared to the measured ones in Figs. 21 to 23 respectively.
Fig. 21. Comparison between measurement and model 3 for a two disc winding.

Fig. 22. Comparison between measurement and model 3 for a three disc winding.
As it is apparent from all three figures above, the first resonance after the fundamental coil resonance in the model (which also is very pronounced in amplitude) is displaced to lower frequencies very articulately when compared to the same resonance for the measurements. The reason for this is, as for the resonance shifts for one single disc, the proximity effect which is neglected in the models and which is going to be discussed in the next Chapter.

Fig. 23. Comparison between measurement and model 3 for a four disc winding.
8 Interpretation

In this Chapter, the physical meaning of the different resonances will be explained, and it will be shown that the azimuthal resonances are independent of the number of discs in the winding. Further, a model based on a reluctance network is proposed for simulation of the proximity effect.

8.1 Explanation of the different oscillation modes

As it was mentioned in Chapter 4 section 6, the three resonances somewhere between 1 MHz to 8 MHz in Fig. 13 for a single disc are called radial resonances and the ones somewhere between 10 MHz and 20 MHz are called azimuthal resonances, and it could in Fig. 20 be seen that these resonances also occur in the measurement, which means that these are physical realities and not model artifacts and now the physical meaning of these resonances will be investigated and explained.

8.1.1 Radial resonance modes

Those resonances whose node voltages vary rapidly in the radial direction, but slowly in the azimuthal direction are called “radial”. By radial and azimuthal directions, the $\rho$ and $\varphi$ directions in polar coordinates are meant respectively (see Fig. 24 for model 3).

![Fig. 24. Geometry of the disc, showing the definitions of coordinates $\varphi$ and $\rho$.](image-url)
In Figs. 25–27, instantaneous node voltages for a sinusoidal excitation voltage $U$ are depicted for different resonance frequencies, each at two different instants of time during an oscillation period, obtained from simulations of model 3.

A linear voltage profile along the whole winding (which is the low-frequency limiting behavior) is subtracted, so that the values at both end nodes of the winding are equal to zero. The green line depicts the geometry of the winding disc and the location of the nodes, and the thin horizontal red line shows the zero level of the voltage as a reference. Black lines connect voltage levels in radial direction, and vertical blue lines indicate the correspondence between voltage levels and nodes.

The voltage distribution in the disc for the first, second and third radial resonance of Fig. 13 is depicted in Figs. 25, 26, and 27, respectively. $f_k (k = 1, 2, 3)$ denotes the frequency for that particular resonance, and $1/f_k$ is the corresponding period time. The radial resonances appear as standing voltage waves which can approximately be described by the formula

$$
V_k(\rho, \varphi, t) \approx \cos(2\pi f_k t) \left[ A_k \sin \left( k2\pi \frac{\rho - \rho_0}{\rho_1 - \rho_0} \right) + B_k \left( \rho - \frac{\rho_0 + \rho_1}{2} \right) \right]
$$

(52)

for $\rho_0 < \rho < \rho_1$, where $\rho_0$ and $\rho_1$ are the inner and outer “radii” of the disc, respectively (see Fig. 24). The resonance voltage amplitudes $A_k$ and $B_k$ are damping dependent. Note that the approximate expression (52) is independent of $\varphi$. It can be seen in Figs. 25–27 that the approximation (52) is best for low resonance order $k$. The amplitude $B_k$ is close to zero for $k = 1$ and increases with increasing resonance order $k$. 

Fig. 25. Voltage profile of the first radial resonance $(k = 1)$, at times $t = 0.7/f_1$ (left) and $t = 1.4/f_1$ (right).

Fig. 26. Voltage profile of the second radial resonance $(k = 2)$, at times $t = 0.4/f_2$ (left) and $t = 0.8/f_2$ (right).
8.1.2 Azimuthal resonance modes

For “azimuthal” resonances, just like the “radial” ones, the node voltages vary rapidly in the $\rho$ direction, but the difference is that there are also significant node-voltage variations in the $\phi$ direction. This pattern can be seen in Figs. 28 and 29 which depict the instantaneous node voltages for the two dominant azimuthal resonances, appearing in Figs. 13 and 20 as pronounced minima close to $f_{az}$, at two different instants of time. Again, model 3 has been employed and a linear voltage profile has been subtracted.

In contrast to the radial resonances which are spread out in frequency, the azimuthal resonances are “clustered” (at least when viewed on a logarithmic frequency scale) around a characteristic frequency $f_{az}$ slightly above 10 MHz. They cannot be described by a simple formula like that for the radial resonances (52), but their common characteristics is approximated by the expression

\[
V_{az}(\rho, \phi, t) \approx A_{az} \cos(2\pi f_{az} t) \left(1 - \cos(\phi)\right) \left(\rho - \frac{\rho_0 + \rho_1}{2}\right).
\]

This fundamental behavior is indicated in Figs. 28 and 29 by dotted lines. Individual azimuthal resonance modes differ from it by additionally superposed short-wavelength modulations. The mode in Fig. 28 resembles more closely to the fundamental expression (53) than the one in Fig. 29.
8.2 Azimuthal resonances independent of number of discs in the winding

As it was reported in Chapter 7 and illustrated in Figs. 21, 22 and 23, measurements were also performed on a winding with more than one disc. In Figs. 30 and 31, measurements and simulations are shown for a winding consisting of two, three and four discs (with the same dimensions as the single disc measured in Fig. 20) respectively, connected together in a continuous way as in Fig. 9.

It can be seen that the azimuthal resonances occur around the same frequency $f_{az}$ somewhere between 10 and 20 MHz, no matter how many discs are connected together in the winding. This supports the picture that azimuthal resonances are internal oscillations in every individual disc, and are roughly independent of the other discs. Furthermore, measurements and simulations show that they are very sensitive to small changes in the winding geometry (e.g. mechanical winding deformations). For instance, in the measurements on several winding discs of identical design but with small manufacturing differences (see Fig. 19), the precise location, relative strength and shape of the two dominating azimuthal resonances varied noticeably, each disk thus having its individual “finger print”.

![Fig. 30. Measurements on windings with two, three, and four discs.](image)
8.3 Using reluctance network for proximity effect

As it was mentioned earlier in Chapter 4 section 4, the formulas for the inductances are for thin filaments and also frequency independent. This is a good approximation as long as the distance between two conductors is much larger than the largest cross sectional dimension of the conductors, and as long as the frequency is low enough. But when the frequency is high and the two conductors are close to each other, the so called skin effect and proximity effect will be present respectively.

Skin effect – when a time-varying current flows in a conductor it creates a time-varying magnetic field which in turn induces eddy currents i.e. induced currents that counteract the original current. The consequence is that the total current tends to be confined to the surface of the conductor. This effect becomes stronger as the frequency is increased and the effective current carrying area of the conductor becomes restricted to a thin layer below the surface, which is called skin depth and is defined by

\[
\delta = \frac{1}{\sqrt{\pi \mu_0 \mu_r \sigma f}}
\]

In Fig. 32 (a), a simulation by Dr. Nilanga Abeywickrama is depicted which shows the influence of skin effect on the distribution of the current density and the magnetic flux density. It can be noticed that due to the skin effect, the current density becomes non-uniform in the radial direction \( \rho \), but it is still uniform in the azimuthal direction \( \phi \) when the conductor has cylindrical symmetry.

Proximity effect – unlike the skin effect, proximity effect is eddy currents which are induced in a conductor due to time-varying magnetic flux density produced by the current in other
conductors in the vicinity. Due to proximity effect, the current density and magnetic flux distributions become unsymmetrical in both \( \rho \) and \( \varphi \) directions (see Fig. 32 (b)).

![Fig. 32](image)

**Fig. 32.** Distribution of magnetic flux density (arrow plot) and current density (surface plot) for circular conductors; in (a) there is only the skin effect present while in (b) both the skin effect and the proximity effect are present (for the currents in the conductors flowing in the same direction). This figure is borrowed from [23].

The impact of skin effect on the self inductance is that the *internal* inductance of the conductor decreases, but since the internal inductance is a much smaller part of the total self inductance of a single conductor, the influence of skin effect on the self inductance can be neglected.

But the proximity effect (in combination with the skin effect) is more serious, and this is due to the fact that when the frequency is very high and the conductors are really close to each other, the current is not only confined to the surface of the wires but is distributed around the axis in conformity with the law of distribution of the charges in the corresponding electrostatic problem [18]. This means that if current flows in opposite directions in two parallel conductors, the current density in each conductor is a maximum at the nearest points of the cross sections of the conductors. This has the effect of a reduction of the effective spacing of the conductors, and it means that the mean distance between the effective current carrying areas will be considerably smaller than the distance between the centers of the two conductors which is the distance used in the thin filament approach (see Fig. 11 and 12). This in turn has the effect that the actual mutual inductances for turns close to each other will deviate considerably from the ones calculated in expressions (28) – (30).

So the solution proposed here is the so called reluctance network model, which is a high frequency approximation and requires short computation time. Consider the cross section of one side of a single quadratic disc depicted in Fig. 33 below.

![Fig. 33](image)
The dimension of the air gaps (due to the manufacturing uncertainty and irregularities) between the conductors is exaggerated in the figure above to make it possible to illustrate the problem picture. It is assumed that the frequency is infinitely high, which has the consequence that the current in the conductors flows only on the surface, and this is a good approximation for the cases in this thesis, since the lowest frequency dealt with here is 10 kHz, and already at that frequency the skin depth \( \delta \) is very small compared to the smallest cross sectional dimension of the conductors (\( w = 3 \text{ mm} \)).

With the current being confined to the surface of the conductors, there will be no magnetic flux density inside the conductors, and it is assumed that it (the magnetic flux density) only flows through the network reluctances which are defined as

\[
\text{reluctance} = \frac{\text{length}}{(\text{permeability} \times \text{area})}. \tag{55}
\]

Expression (55) is accurate for the reluctance between the turns (\( \mathcal{R}_2 \)) since the length and area used in (55) can be defined accurately enough for \( \mathcal{R}_2 \), but this is not the case for the reluctances “outside” the turns (\( \mathcal{R}_0 \) and \( \mathcal{R}_1 \)) since there is no easy way to define an area which the magnetic flux density passes through and a length which it travels along. So the solution is to adjust these reluctances, and a good approximation is to set

\[
\mathcal{R}_0 \approx \mathcal{R}_1 \approx 0.01 \mathcal{R}_2, \tag{56}
\]

since the magnetic flux density outside the turns has a much wider area to flow through, which in turn leads to a much smaller reluctance. The reluctance between the turns is given by

\[
\mathcal{R}_2 = \frac{h}{\mu_0 \tau_i} \tag{57}
\]

where \( h \) is the height of the conductors as before, \( \tau_i \) is the distance separating two turns and \( l \) is the length of the straight conductors which is the same as the length of the sides of the quadratic disc, and it is approximately equal to \( 2d \) (\( = 1.2 \text{ meters} \)). The value for \( \tau_i \) is not exactly known (due to the manufacturing uncertainty and irregularities) and it has to be adjusted.

It is assumed that it flows a current \( I_i \) in each of the conductors for \( i = 1, 2, 3, \ldots, n \). The relation then between the voltages and currents in the conductors becomes

\[
U = L_{\text{rel}} \frac{d}{dt} I, \quad \text{or} \quad \begin{bmatrix} U_1 \\ U_2 \\ \vdots \\ U_n \end{bmatrix} = L_{\text{rel}} \frac{d}{dt} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_n \end{bmatrix}, \tag{58}
\]

where \( L_{\text{rel}} \) is the inductance matrix calculated using the reluctance network model and which it is looked for. Now each of these voltages are connected to the magnetic flux \( \Phi_{si} \) generated by the individual conductors by the relation
The equation for the “full” magnetic flux vector is

$$\Phi = \Phi_s,$$  \hspace{1cm} (60)

where $\Phi'$ is the transpose of the flux connection matrix $\Phi$ with the expression

$$\Phi' = \begin{bmatrix} -S_1 \\ S_2 \\ S_1 \end{bmatrix}, \text{ where } S_1 = \begin{bmatrix} 1 & 0 & \ldots & \ldots & 0 \\ 0 & 1 & 0 & \ldots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \ldots & 0 & 1 & 0 \\ 0 & \ldots & \ldots & 0 & 1 \end{bmatrix} \text{ and } S_2 = \begin{bmatrix} -1 & 0 & \ldots & \ldots & 0 \\ 1 & -1 & 0 & \ldots & 0 \\ 0 & \vdots & \ddots & \ddots & \vdots \\ \vdots & \ldots & 0 & 1 & -1 \\ 0 & \ldots & \ldots & 0 & 1 \end{bmatrix}_{n \times n} \hspace{1cm} (61)$$

$\Phi_k$ (which is element number $k$ of $\Phi$) contains the amount of magnetic flux that flows through the reluctance having the same number $k$ in red colour in Fig. 33, for $k = 1, 2, \ldots, 3n+1$.

Using Ampere’s law in matrix form, the current matrix can be written as

$$I = \Phi' \Phi_s,$$ \hspace{1cm} (62)

where $\Re$ is the reluctance matrix expressed as

$$\Re = \begin{bmatrix} \Re_{00} & 0 & 0 \\ 0 & \Re_{1,2} & 0 \\ 0 & 0 & \Re_{00} \end{bmatrix}_{(3n+1) \times (3n+1)} \text{ with } \Re_{00} = \begin{bmatrix} \Re_0 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \Re_0 \end{bmatrix}_{n \times n} \text{ and } \hspace{1cm} (63)$$

$$\Re_{1,2} = \begin{bmatrix} \Re_1 & 0 & 0 & 0 & 0 \\ 0 & \Re_2 & 0 & 0 & 0 \\ 0 & 0 & \ddots & 0 & 0 \\ 0 & 0 & 0 & \Re_2 & 0 \\ 0 & 0 & 0 & 0 & \Re_1 \end{bmatrix}_{(n+1) \times (n+1)}$$

Taking the time derivative of (62), one arrives at

$$\frac{d}{dt} I = \partial \Re \partial \Phi_s,$$ \hspace{1cm} (64)

which in combination with (58) and (59) leads to
\[ L_{\text{rel}} = \left( \partial R \partial' \right)^{-1}. \] (65)

\( L_{\text{rel}} \) is a symmetric matrix as it is expected for an inductance matrix. Now in the new modified inductance matrix \( L_{\text{mod}} \) which is going to be used, the inductances calculated in Chapter 4 using the thin filament approach \( (L_{\text{fil}}) \) and the reluctance network model \( (L_{\text{rel}}) \) will be combined.

For the self inductances, the values obtained by the thin filament approach will be used

\[ L_{\text{mod,ii}} = L_{\text{fil,ii}} \] (66)

This is because the thin filament expression (27) for the calculation of the self inductances is accurate enough. The mutual inductances between conductors located on the same side of the disc have to be modified in a way in which the total inductance for two turns \( (L_{\text{tot}} = L_{ii} + L_{jj} - 2M_{ij}) \) in the new modified model has the same value as in the reluctance network model. This means that

\[ L_{\text{mod,ii}} + L_{\text{mod,ij}} - 2M_{\text{mod,ij}} = L_{\text{rel,ii}} + L_{\text{rel,ij}} - 2M_{\text{rel,ij}}. \] (67)

By rearranging the terms in (67) and using (66) one will arrive at

\[ 2M_{\text{mod,ij}} = L_{\text{fil,ii}} + L_{\text{fil,ij}} - L_{\text{rel,ii}} - L_{\text{rel,ij}} + 2M_{\text{rel,ij}}. \] (68)

For the mutual inductance between conductors on different sides of the disc the ones calculated with thin filament approach will still be used, and this is due to the fact that the distance between them is far enough for the proximity effect to be neglected.

In Fig. 34, measurement on a single disc (black dotted line), results from model 3 using the inductance matrix obtained with the thin filament approach only (red line), and obtained with the thin filament approach in combination with the reluctance network model (blue line) are depicted.

Fig. 34. Measurement on a single disc (black dotted) compared to results from model 3 with the thin filament approach only (red), and thin filament approach combined with the reluctance network model (blue).
It can be noticed in the figure above that for the new, combined model, the first two radial resonances now are placed correctly in the frequency domain when compared to measurement. The third radial resonance is relatively unchanged and it can be seen that a new azimuthal resonance (which was absent in the model with the thin filament approach) appears in the frequency range between the third radial resonance and the azimuthal resonances which form the pronounced impedance minima. This new azimuthal resonance is absent in the measurement on a single disc in Fig. 34, but it has to be recalled that the single discs don’t have exactly the same impedance for the frequency range around $f_{az}$, and this is illustrated in Fig. 35, where the measurement on another single disc is depicted, and it can be seen that the new azimuthal resonance predicted by the model using the modified inductance matrix is present in some form here.

![Graph showing impedance vs. frequency](image)

**Fig. 35.** Measurement on a single disc.

The reluctance network model is going to be investigated more in the future work and the model will be developed further to include several discs in the winding.
9 Summary, conclusions and future work

In this thesis, a simple, frequency dependent complex-$\mu$ model of magnetic core material has been developed and adjusted to measurements. Its real and imaginary parts were compared to measurements in a wide frequency range. The agreement was found satisfactory, especially for higher frequencies, which makes the complex-$\mu$ model a very convenient starting point for the estimation of flux distribution and losses in complicated core geometries. Furthermore, $H-B$ curves from the measurements, the simple complex-$\mu$ model and the detailed hysteresis model were compared for different frequencies. Again the results from the complex-$\mu$ model were found to agree well with measurements at higher frequencies. At low frequencies and high field amplitudes the complex-$\mu$ model deviates from measurements and detailed hysteresis model, since it does not take saturation effects properly into account. This is, however, not expected to affect its usefulness for loss estimation.

Also, a quadratic winding model based on the lumped element approach has been developed. The model has three steps of discretization, and these three models can be simulated both in time- and frequency domain. When simulated in time domain, the models are intended to be used for Time Domain Reflectometry (TDR), which is a method in which damages in the winding are detected by reflection of pulses sent into the winding. This method is going to be investigated more in the doctoral thesis, where the models are going to be further developed, and the possibilities for their use in transformer winding diagnostic will be explored.

When simulated in the frequency domain, the models are used in Frequency Response Analysis (FRA) in which the impedance of the winding is calculated in a wide frequency range. The frequency domain simulations are compared to measurements and the model has been verified. Two classes of internal resonance modes of a single transformer winding disc have been defined, the “radial” modes at lower frequencies and the “azimuthal” modes at higher frequencies (above around 10 MHz in the case in this thesis), and have been studied by measurements and model simulations.

The radial modes are characterized by a rather constant voltage amplitude within every turn, whereas the azimuthal modes describe electrical oscillations of different parts of the same turn against each other and therefore can only be seen in models with more than one segment per turn.

When several discs are connected together to a multi disc winding, simulations and measurements show that the number and distribution of the radial modes varies strongly with the number of discs, which shows that they are “global” modes of the whole winding consisting of interacting discs. In contrast, the azimuthal modes always cluster around the same frequency $f_{az}$, leading to a very pronounced impedance minimum, which suggests that they are “local” modes, depending only on the geometry of the individual discs. As such, they are expected to be useful for probing the integrity of individual discs in frequency response (FRA) measurements on transformers. Moreover, measurements and simulations show that they are highly sensitive to small changes in the winding geometry. The application of these
mode signatures to winding fault detection should therefore be further explored in the future work.
Also, a reluctance network model which has the purpose to account for the proximity effect in the calculation of the mutual inductances has been partially developed, and in the doctoral thesis, this model will be further developed for simulation of winding models with an arbitrary number of discs.
References

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