Risk management based on GARCH and Non-parametric stochastic volatility models and some cases of Generalized Hyperbolic distribution

Master’s Thesis in Financial Mathematics

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Abstract

The paper is devoted to the modern methods of Value-at-Risk calculation using different cases of Generalized Hyperbolic distribution and models for predicting volatility. In our research we use GARCH-M and Non-parametric volatility models and compare Value-at-Risk calculation depending on the distribution that is used. In the case of Non-parametric model corresponding windows are proved by the Cross Validation method. Furthermore in our work we consider adaption of the method to intraday data using ACD and UHF-GARCH models. The project involves also application of the developed methods to real financial data and comparable analysis of the obtained results.
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Chapter 1

Introduction

Nowadays risk is an essential part of the financial world. Estimation of the market risk is one of the main problems that different types of financial institutions face every day. This kind of risk arises because of fluctuations of interest rates, commodity prices, stock prices, exchange rates etc. As there is direct relation between these movements and investor’s gains or losses, we get the necessity to estimate the market risk as precise as possible. There exist a lot of different risk measures. But most of them do not provide exact information about magnitude of losses. For example if we consider one of the most well-known risk-measures standard deviation (volatility), we obtain information just about the magnitude of changes in price, but not about the amount of possible losses. Therefore today such instrument as Value-at-risk became one of the most preferable methods to quantify risk on financial markets. In the present financial world VaR, a quantile measure, has become the most widespread approach to estimate the market risk and for the last several years it has been a really popular and useful tool in risk management. But VaR measure had been known for quite a long time before it came into use in financial lexicon. In 1952 Harry Markowitz used VaR in his paper ”Portfolio selection” to evolve methods of portfolio optimization. Later since middle 1980s till 1990s a lot of financial institutions adopted VaR measure to deal with capital allocation and market risk. In 1994 J.P. Morgan, one of the biggest investment companies of the United States released it’s free risk management service Risk Metrics that represented VaR as a basic approach for market risk estimation. Even now values that are obtained using this service are considered to be a kind of a standard of VaR calculation. Finally in 1995 the Basel Committee on Banking Supervision set requirements concerning market risk capital reserves based on VaR measure. After all these stages of development this approach has become one of the main methods for estimating market risk.
There exist three main approaches for VaR calculation: historical, variance-covariance and Monte-Carlo Simulations. The main aim of this research is to investigate variance-covariance method and to introduce some improvements by using non-normal density functions.

In general VaR is a probabilistic measure that gives us opportunity to measure the risk of a portfolio or a single asset with just one number that accumulates all the information about the risk that is taken. To compute Value-at-Risk we need to have some model for describing price behavior. In this work we use a model that is based on two main components: Stochastic Volatility and Levy processes. Such model was investigated in the paper of Eberlein, Kallsen and Kristen (2003)[10]. The first component, Volatility, is a very important characteristic of the market. To estimate the market risk sufficiently precise one should take it into account. In contrary of the Black-Scholes approach the idea of the stochastic volatility price model is based on the empirical fact that volatility is not constant and in the real world it changes randomly. Stochastic volatility is very well-known and widely used in financial world. There are a lot of works where it was described and investigated, such as Hull and White (1998)[21], Barone-Adesi et al. (1998, 1999)[2], Guermat and Harris (2002)[18], Venter and de Jongh (2002)[26], McNeil and Frey (2000)[23].

The second component is Levy processes. More precisely speaking to achieve our goals we will take into consideration some cases of Generalized Hyperbolic distributions that were introduced in papers of Brandorff-Nielsen (1998)[1], Eberlein (2001)[8], Eberlein and Prause (2002)[9]. These kinds of distributions are much more flexible then normal distribution. That is why they can provide better fit to empirical data.

There are three main goals that should be achieved at the end of the work. The first one is to compare two markets, the comparably small Stockholm and the rather big Frankfurt exchange markets. We will do this by analyzing the DAX and the OMXS30 indexes correspondingly. We will investigate the behavior of the volatility on both markets and try to find if they are correlated or not. The second goal is to calculate VaR using the same indexes from two exchange markets that were mentioned. To do so we will compare different types of generalized hyperbolic distributions to decide which one will provide the best fit to the empirical data. Also we will compare VaR results obtained by using different generalized hyperbolic distributions with VaR calculated using Normal Gaussian distribution. And finally the third aim of this paper is to apply a model for estimation of volatility using tick data and conditional duration for VaR calculation.

The paper is composed in the next consecution: we will start with presentation of Value-at-Risk calculation methods overview. Later in Chapter 1
two basic components, stochastic volatility and generalized hyperbolic dis-
tributions, with their partial cases and basic notions are introduced. Further
in Chapter 2 we describe methods of volatility prediction and estimation of
distribution parameters for logarithmic returns. At the end of the chapter we
will suggest methods to access the accuracy of calculations. The results that
were obtained in this research and description of the used data are presented
in Chapter 3. Finally the meaning of these results and ideas for further
investigation are discussed in Conclusion section.
Chapter 2
Models description

2.1 General model

Consider the price process of such type $S_t = S_0 e^{X_t}$. We suggest to model log-return process $\Delta X_t = X_t - X_{t-1}$ as it was done in the paper Eberlein, Kallsen and Kristen (2003)\textsuperscript{[10]}

$$\Delta X_t = \sigma_t \Delta L_t$$ (2.1)

where $\sigma_t$ is a stochastic volatility series and $\Delta L_t$ - white noise.

**Definition (Wide sense white noise)**: The sequence $\epsilon_t$ is called white noise in the wide sense if $E\epsilon_t = 0$, $E\epsilon_t^2 < \infty$ and

$$E\epsilon_k \epsilon_m = 0, \quad \forall k \neq m$$

In the above mentioned paper Eberlein, Kallsen and Kristen (2003)\textsuperscript{[10]}, authors argue that such model is rather natural. They considered Dow Jones index returns data of very large time period and have found that real residuals

$$\log \Delta X_t^2 - \log \sigma_t^2$$

look very similar with simulated standard normal white noise. But in fact, during the given comparison very common effects of price were recognized

1. Large positive value of residuals appear much more rarely in the Gaussian sample than for real data. This illustrates the well-known fact that return data has heavier tails than normal distribution.

2. We can observe additionally some clusters of extremely high values. Which tell us that there may be a very short-lived volatility component in the data.
3. The time series

\[ \log \Delta X_t^2 = \log \sigma_t^2 + \log \Delta L_t^2 \]

can be considered as a signal \( \log \sigma_t^2 \) perturbed by an white noise, which has a mean \( E \log \Delta L_t^2 \).

Due to this observations the suggested general model seems to be very robust. In the paper the authors describe residuals white noise using Hyperbolic distribution instead of Normal. In our research we consider a wide class - General Hyperbolic distributions, which are tailor-made and flexible enough for fitting real data. Let’s take a closer look to the basic components of the model.

### 2.2 Stochastic volatility

In modern financial mathematics volatility is considered to be nearly the most investigated phenomenal occurrence because of two reasons. First of all for the last several years the usage of financial derivatives has increased dramatically and volatility is involved in their calculation. The second reason is directly related with the subject of this paper and consists in that the volatility of the stock returns plays the central role in risk management and particularly in such method as VaR. From the beginning Black-Scholes (1973)[3] the volatility was assumed to be constant but the existence of such phenomena as the volatility smile showed that it appears to vary indeed.

Stochastic volatility models were introduced to obtain the possibility to describe such empirical tendency that exists on financial markets and is not taken into account by the constant volatility models. The main idea of the stochastic volatility is to present the volatility itself as a stochastic process.

**Definition (Stochastic volatility)**: *A process in which the return fluctuation includes an unobservable component which cannot be predicted using current available information.*

In other words it is a model in which volatility is presented as a randomly changing process described by a discrete random process or a stochastic differential equation.

In the paper Eberlein, Kallsen and Kristen (2003)[10] introduced an exploratory analysis of stochastic volatility based on the Dow Jones Industrial Average index. In that research the authors substantiated the necessity of using stochastic volatility models by performing a qualitative analysis of volatility fluctuations and showed that besides the slowly varying stochastic volatility there exist really short-period volatility constituents. Also he practically showed that a widely known fact that the stock returns data have
heavier tails rather than if it was normally distributed really appears to be. We have already mentioned the model for our returns:

$$\Delta X_t = \sigma_t \Delta L_t$$

Where the volatility can be determined as shown below:

- **ARCH**

  $$\sigma_t^2 = a_0 + \sum_{i=1}^{p} a_i \Delta X_{t-i}^2 \quad a_0 > 0, \quad a_i \geq 0$$

  From the equation we can see that the volatility is a fully deterministic function of previous returns. Also it is necessary to mention that previous big returns result in big value of the volatility and small returns give us small volatility. This is called Autoregressive Conditional Heteroscedastic model or ARCH(p). This model was very widely used because it explained such phenomena as heavy tails of return data, clustering etc. that linear stochastic models could not explain (AR, MA, ARMA etc.) cf. Shiryaev (1998)[25].

- **GARCH**

  As a further development there appeared a generalized form of this model called GARCH(p, q):

  $$\sigma_t^2 = a_0 + \sum_{i=1}^{p} a_i \Delta X_{t-i}^2 + \sum_{j=1}^{q} b_j \sigma_{t-j}^2$$

  Here $$a_0 > 0, \quad a_i \geq 0, \quad b_i \geq 0$$. In this case the volatility depends not only on the previous values of returns, but also on the previous values of the volatility itself. Now let’s consider a particular case of this model that we will use for our research called GARCH(1.1)-M. (cf. Shephard (1996)[24], Shiryaev (1998) [25] section II. 3a(10)):

  $$\sigma_t^2 = a_0 + a_1 \sigma_{t-1}^2 (\Delta L_{t-1}^2 - m)^2 + b_1 \sigma_{t-1}^2$$

  Where $$a_0 > 0, \quad a_1, b_1 \geq 0, \quad m = E\Delta L_1$$. This is a really widely used stochastic volatility model that shows that big values of the volatility appear because of the large absolute values of previous returns.

- **Non–parametric**

  Volatility modeling is possible not just with parametric models. We can suppose that the volatility is changing according to some arbitrary stochastic process but slowly. We assume that the endurance of volatility
changes is longer than the interval of the historical observations. So we can say that tomorrow volatility is more likely to keep its tendency and in such case we can use a moving average to predict volatility one day ahead. Our goal is to revile volatility signal masked with noise from the historical data. To do this we use a non-parametric smoothing (cf. Härdle (1991)) [19].

- **UHF – GARCH**

So far we considered price as a process which varies in fixed time intervals and described its randomness by log-return sequence $\Delta X_t$. But in fact real stock price stays of the same value for a some period of time and then jumps at a random moment $\tau$. Such price movements are called ticks and moments $\tau$ - stopping times.

**Definition (Stopping Time)**: For a stochastic basis $B = (\Omega, F, (F_n)_{n \geq 0}, P)$ non-negative random variable $\tau = \tau(\omega)$ is called stopping time, which does not depend on future if $\forall t \geq 0$

$$\{\omega : \tau(\omega) \leq t\} \in F_t$$

Thus trajectories of $X_t$ can be presented as piece-wise constants with jumps at times $\tau_k$ as it is shown in the Figure 2.1.

In other words price is a process which randomness can be described by two random sequences $\Delta X_{\tau_k}$, log-returns and $\Delta_k = \tau_k - \tau_{k-1}$, the duration of period between $k - 1$ and $k^{th}$ ticks.

![Figure 2.1: Price ticks](image-url)
In the above section we discussed how to model log-returns. But the question is how to model this random sequence $\Delta_k$ and it is interesting to receive some past dependence for them, because in reality duration between ticks are correlated.

For our research we use the model which was investigated in Engle, Russel (1998)\cite{Engle98}. In this model durations assumes to be correlated and all past dependence is included in function $\psi$, where $\frac{\Delta}{\psi} \sim i.i.d.$.. Also one can realize that process in the Figure 2.1 looks similar with well-known Poisson process, where the periods of time between ticks are exponentially distributed. It could be transformed for modeling desirable durations distribution and seems to be a good starting point.

Thus first simple case of the model has exponential distribution with parameter $\lambda = \frac{1}{\psi}$. It is called Exponential ACD or just EACD

$$p(\Delta_k|\psi_k) = \frac{1}{\psi_k} e^{-\frac{\Delta_k}{\psi_k}}$$

where

$$\psi_k = \omega + \sum_{i=1}^{p} \alpha_i \Delta_{k-i} + \sum_{j=1}^{q} \beta_j \psi_{k-j}$$

and

$$\psi_0 = \frac{\omega}{1 - \beta}$$

This formula for $\psi$ is very similar with GARCH(p,q) and it is called Autoregressive Conditional Duration model or ACD(p,q). As in the GARCH it is more convenient to work with ACD(1,1)

$$\psi_k = \omega + \alpha \Delta_{k-1} + \beta \psi_{k-1}$$

The second more general case of the model assumes that conditional distribution of durations is Weibull, where $(\frac{\Delta_k}{\phi_k})^\gamma$ is exponential. It is called Weibull ACD or just WACD model. The density function then looks like that

$$g(\Delta_k) = \frac{\gamma}{\Delta_k} \left( \frac{\Delta_k}{\phi_k} \right)^\gamma e^{-\left(\frac{\Delta_k}{\phi_k}\right)^\gamma} \forall \Delta_k > 0$$

where

$$\phi_k = \tilde{\omega} + \tilde{\alpha} \Delta_{k-1} + \beta \phi_{k-1}$$

$$\phi_k = \frac{\psi_k}{\Gamma\left(1 - \frac{1}{\gamma}\right)}$$
and $\Gamma$ is a Gamma function. Another reasonable question arises - how can we use these durations for our risk management purposes?

It was noticed in Engle (2000)\cite{Engle2000} that there exists some correlation between durations of periods when price is a constant and volatility. Indeed, when durations between price ticks are very long, it corresponds to the situation when trading transactions occur very rarely. According to that, the volatility of the price will be very low and vice versa. This fact tells us about negative correlation between these two processes. Define the conditional variance of log-returns per unit of time. In order to do that we divide log-returns by the square root of the current duration

$$\text{Var} \left( \frac{\Delta X_t}{\Delta t} \bigg| \Delta t \right) = \sigma^2_t$$

Define new sequence

$$\frac{\Delta X_t}{\Delta t} = \epsilon_t$$

which is consist of log-returns per unit of time. Then GARCH(1,1) volatility model for such process looks like that

$$\sigma^2_t = \omega + \alpha \epsilon^2_{t-1} + \beta \sigma^2_{t-1}$$

This model can be applied if we think that current duration does not provide any information about volatility. Another more interesting case of the volatility model due to negative correlation effect seems to be more realistic

$$\sigma^2_t = a + b \epsilon^2_{t-1} + c \sigma^2_{t-1} + d \Delta_t^{-1}$$

Usually durations between price movements are very small time intervals measured in seconds that is why equation \cite{2.3} is called Ultra-High Frequency or just UHF-GARCH(1,1). Estimation parameters procedure and one-step prediction for duration models will be presented in the subsequent chapter.

As it was shown in Eberlein, Özkan (2002)\cite{Eberlein2002} in the case when the time scale is very small, estimations of the white noise $\Delta L_t$ with Generalized Hyperbolic distribution is still consistent. Hence we can apply above defined UHF-GARCH volatility approach to our general model.

### 2.3 Levy processes

Today investigation and application of Levy processes has become very widespread in financial analysis. It was named in honor of famous French mathematician Paul Levy.
Definition (Levy process): Levy process is any stochastic process $X$, which satisfy to the following conditions:

1) $X_0 = 0$
2) $X$ has independent increments. It means that for every $n$ and $m$ ($n \neq m$)
   \[ \Delta X_n \perp \Delta X_m, \text{ where } \Delta X_i = X_i - X_{i-1} \]
3) $X$ has stationary increments. It means that distribution of the $X_n - X_m$ for any $n \neq m$ depends only the difference $n - m$. Stationarity of increments seems to be a very natural and convenient property for our purposes because it admits to behavior of price.
4) $X$ is stochastically continuous
   \[ \lim_{n \to m} P(|X_n - X_m| > \epsilon) = 0, \quad \forall m \geq 0, \quad \forall \epsilon \geq 0 \]

Sometimes in the definition of Levy process it is also mentioned about regularity of trajectories. The most well-known Levy representatives are the Wiener processes and the Poisson processes. It is very evident fact that the distribution of log-returns differs essentially from the normal one. That is why we will consider Generalized Hyperbolic distributions an its subclasses.

Generalized Hyperbolic distribution is defined by five parameters, which gives an opportunity to perform very good fit to empirical distribution of data. Density function of the Generalized Hyperbolic distribution $\forall x \in \mathbb{R}$ looks like that

\[ f_{GH}(x; a, b, \mu, \beta, \nu) = c_3(a, b, \beta, \nu) \frac{K_{\nu-\frac{1}{2}}\left(\alpha \sqrt{b + (x - \mu)^2}\right)}{(\sqrt{b + (x - \mu)^2})^{\frac{\nu}{2}} - e^{\beta(x - \mu)}} \]  

(2.4)

where

\[ \alpha = \sqrt{a + \beta^2} \]
\[ c_3(a, b, \beta, \nu) = \frac{\left(\frac{\xi}{2}\right)^{\frac{\nu}{2}} \alpha^{\frac{1}{2} - \nu}}{\sqrt{2\pi}K_{\nu}(\sqrt{ab})} \]

And $K_{\nu}(y)$ is a modified Bessel function of the third kind of order $\nu$

\[ K_{\nu}(y) = \frac{1}{2} \int_0^\infty t^{\nu-1}e^{-\frac{y}{2t}} dt \]

Parameters description:
\mu - location parameter
b - scale parameter
Chapter 2. Models description

$a$ and $b$ - responsible for the shape of the density function. If we consider process of price in such form

$$S_t = S_0 e^{X_t}$$

then Generalized Hyperbolic distribution describes empirical distribution of log-returns, which are presented as follows

$$\Delta X_t = X_{t+1} - X_t = m + d\sigma_t^2 + \sigma_t \epsilon_t$$

Here $\epsilon \sim N(0, 1)$ and $\sigma^2 \sim GIG$ - Generalized Inverse Gaussian random variable ($\sigma_t^2 \perp \epsilon_t$). So we can say that GH is a Normal variance-mean mixture

$$GH = E_{\sigma^2}N(m + d\sigma^2, \sigma^2)$$

Denote

$$GH(a, b, \mu, \beta, \nu) = N \circ GIG(a, b, \nu)$$

In this sense we can distinguish following cases of Generalized Hyperbolic distribution:

- **Hyperbolic**
  For the Generalized Inverse Gaussian distribution with parameters $a \geq 0$, $b > 0$, $\nu = 1$ we obtain Positive Hyperbolic distribution

$$GIG(a, b, 1) = H^+(a, b)$$

And hence for the Generalized Hyperbolic distribution

$$GH(a, b, \mu, \beta, 1) = N \circ GIG(a, b, 1) = N \circ H^+(a, b)$$

which is called Hyperbolic distribution. It has the density function of such type

$$f_{Hyp}(x; a, b, \mu, \beta) = \frac{a}{2b\alpha K_1(\sqrt{ab})}e^{-a\sqrt{b+(x-\mu)^2} + \beta(x-\mu)}, \ \forall x \in \mathbb{R}$$

The logarithm of this density function is a hyperbola, which gives the name for this distribution.

- **Normal Inverse Gaussian (NIG)**
  When $a > 0$, $b > 0$, $\nu = -\frac{1}{2}$, the Generalized Inverse Gaussian is just Inverse Gaussian

$$GIG(a, b, -\frac{1}{2}) = IG(a, b)$$
And it follows that
\[ GH(a, b, \mu, \beta, -\frac{1}{2}) = N \circ GIG(a, b, -\frac{1}{2}) = N \circ IG(a, b) \]
which is called Normal Inverse Gaussian distribution. Its density has such form
\[ f_{NIG}(x; a, b, \mu, \beta) = \frac{ab}{\pi} e^{\sqrt{ab}K_1(\alpha\sqrt{b + (x - \mu)^2})} e^{\beta(x - \mu)}, \forall x \in R \]

It is worth mentioning that NIG distribution has such property:
if \( \xi_1 \sim NIG(a, b_1, \beta, \mu) \) and \( \xi_2 \sim (a, b_2, \beta, \mu) \)
then \( \xi_1 + \xi_2 \sim (a, (\sqrt{b_1} + \sqrt{b_2})^2, \beta, \mu) \)

**Variance – Gamma (VG)**
Generalized Inverse Gaussian distribution with parameters \( a > 0, b = 0, \nu > 0 \) is called Gamma distribution
\[ GIG(a, 0, \nu) = Gamma\left( \frac{a}{2}, \nu \right) \]

\( GH(a, 0, \mu, \beta, \nu) = N \circ GIG(a, 0, \nu) = N \circ Gamma \)
- Variance-Gamma distribution. Its density has a form
\[ f_{VG}(x; a, 0, \mu, \beta) = \frac{a^\nu}{\sqrt{\pi} \Gamma(\nu)(2\alpha)^{\nu - 1}} |x - \mu|^{\nu - \frac{1}{2}} K_{\nu - \frac{1}{2}}(\alpha |x - \mu|) e^{\beta(x - \mu)}, \forall x \in R \]

Variance-Gamma distribution has the same property as NIG for the sum of random variables.

For further details see Shiryaev (1998) [25].
Also as a benchmark we will consider skewed Student-t distribution, which is very well-known limiting case of Generalized Hyperbolic distribution. The density of skewed Student-t distribution has more pointed shape than normal one and with heavier tails. This density has following form
\[ f(x) = \frac{\Gamma\left( \frac{\nu + 1}{2} \right)}{\sqrt{\pi b \Gamma\left( \frac{\nu}{2} \right)}} \left( 1 + \frac{(x - \mu)^2}{b^2} \right)^{-\frac{\nu + 1}{2}} \]
where \( \nu \) is a number of degrees of freedom and \( \Gamma(z) \) - gamma function
\[ \Gamma(z) = \int_0^\infty t^{z-1}e^{-t}dt \]
2.4 Risk management

Today risk management is presented by many methods, but one of the most prevailing is considered to be Value-at-Risk. To give an exact definition we can say, that VaR is an estimate of the level of possible loss on a portfolio or a single asset, which may appear over some certain period of time and may be exceeded with a relatively small probability that is also known.

Definition (Value-at-Risk) Given some confidence level $\alpha \in (0,1)$, then

$$VaR_{\alpha} = \inf (l \in R : P(L > l) \leq 1 - \alpha)$$

So VaR tells us that we are $\alpha\%$ sure, that our loss will not exceed $l$ amount of money during N days. In this statement $l$ value is VaR itself. It is amount of loss or loss percentage that is a function of two parameters: $N$ - time horizon (VaR horizon) and $\alpha$ - the confidence level. In other words VaR value gives us the worst scenario that can occur with a relatively high confidence, usually 95% or 99%, over a period of time that we choose. It may be a day, a month or a year.

There are three basic methods of calculating VaR. They are the historical method, the variance-covariance method, and Monte-Carlo Simulation.

1. Historical method
   In this method we assume that the future returns will repeat the tendency of the historical data. We simply build a histogram of real historical returns and get a number of bars with different hight that show us the frequency of particular returns. Then if we consider for example 95% confidence level, we take 5% of the worst returns and assume that with 95% confidence such losses will not occur.

2. Monte-Carlo Simulation
   Using this method we develop a model that predicts future returns of an asset and runs multiple possible outcomes. Actually a Monte-Carlo Simulation is any method that acts like a generator of random results. We run the process 100 times, then take the worst 5 outcomes and obtain the result.

3. The variance-covariance method
   About this method we will speak more precisely as it is one of the main topics of this paper and we will use it at the final stage of our research. This method assumes that our returns are distributed according to some density function. Usually it is a normal distribution but we will use generalized hyperbolic distributions as they fit the real data much
better. As in the first method we also use historical returns but just for the estimation of the parameters of the chosen distribution. After fitting it to our historical data we plot a distribution curve. Next we find the $\alpha\%$ quantile and obtain $VaR_\alpha$.

Let’s assume that log-returns of stock price $x_t$ are distributed with some distribution function $F = F(x)$. And $f(x)$ its density function. Then for the Value-at-Risk of log-returns with some confidence level $\alpha$ we have a formula

$$VaR_\alpha(x) = -q_\alpha(f(x))$$

where

$$q_\alpha(f(x)) = F^{-1}(x)$$

quantile of the level $\alpha$ for $x$ distribution function. And

$$\alpha = \int_{-\infty}^{q_\alpha} f(x)\,dx$$

As it shown in Figure 2.2 the quantile with confidence level $\alpha\%$ can be considered as a value which separates distribution function curve into two parts. And area of the part which lies in the left side of this separator shold be equal to $\alpha\%$ from the total area of the figure between the density function curve an horizontal axis. In our research we use R program package and calculate quantiles through numeric integration procedure.
Chapter 3

Methods

3.1 Volatility estimation

For our risk management purposes we will use some models as a volatility estimator. They were introduced in the chapter above. And here we take a closer look to the models, show how to estimate parameters and make one-step forecast using them.

- Non – parametric

  Nonparametric regression is a fast growing direction in statistical analysis. There are many theoretical developments made in this field. A very interesting thing about the nonparametric models is that it is possible to use them to uncover the volatility of a process that is masked by some noise.

  The information that we do know is the historical log-returns $\Delta X_t$ that consist of the signal $\sigma_t$ and a noise $\Delta L_t$:

  $$\Delta X_t = \sigma_t \Delta L_t$$

  To make further estimations easier let’s put the equation to the power of two and pass to the logarithm:

  $$\log (\Delta X_t)^2 = \log \sigma_t^2 + \log (\Delta L_t)^2$$

  Our goal is to estimate the hidden volatility from the observations. The strength of fluctuations of log-returns mainly depends on the volatility and the noise gives just some extra perturbation. So in such situation it is convenient to use a non-parametric smoothing (cf. Härdle (1991)) which will consider the noise as some error. In our research we use a
moving average method:

\[
\log \hat{\sigma}_t^2 = \frac{1}{k} \sum_{i=0}^{k-1} \log (\Delta X_{t-i})^2
\]

Where \( k \) is a correctly chosen smoothing parameter. There are several possibilities to choose the \( k \). It is possible to set it "by eye" by trying several values and choosing the best result. But such method is too subjective and depends on a researcher very much. In our work we use cross-validation method which idea is in minimizing the function:

\[
CV(k) = \frac{1}{T} \sum_{t=1}^{T} (\log (\Delta X_t)^2 - (\log \hat{\sigma}_t^2)^{-t})^2
\]

Here \( T \) is a total number of available data points (in our case it is 499) and:

\[
(\log \hat{\sigma}_t^2)^{-t} = \frac{1}{k} \sum_{i=1}^{k} \log (\Delta X_{t-i})^2
\]

is the same moving average but with the window moved one observation back as if the information at time \( t \) is missing. So according to this method the optimal window is such "\( k \)" that minimizes the \( CV \) function and \( CV \) is actually a sum of errors.

We applied this method to our 499 data set taken from the DAX and OMX indexes. In the paper of Eberlein, Kallsen and Kristen (2003)[10] authors were using the same data set and tried windows varying from 5 to 80 with step 5. As a result they obtained the optimal window that turned out to be 40. We created an application using the R package that calculates the \( CV \) function for the same set of windows from 5 to 80 but with step 1. As a result we obtained a more precise result. It is necessary to mention that in fact to be able to examine this set of windows we had to extend the data set from 499 to 579. It was necessary to do so because the moving average method needs 80 observations before the point that we want to start with to use the maximum window. The code of the application is presented in the appendix. But lets get back to our results. For the date set taken from the DAX index the optimal window is 35 with \( CV \) equal to 9.68452. While the window 40 showed \( CV \) equal to 9.705831. After applying this method to the date set taken from the OMX index the optimal window turned out to be 36 with \( CV \) equal to 7.409330 and the window 40 performed \( CV \) equal to 7.445528. To say in advance while calculating VaR we used both 35
and 40 for DAX and 36 and 40 for OMX to be able to compare the results and to find out whether such slight difference in windows will affect the final amount of overlaps. But first we used these windows to estimate the volatility and obtained next graphs in Figure 4.3 and Figure 4.4.

We should not forget that $\sigma_t^2$ is affected by the expectation of $(\log \Delta L_t)^2$ and for different Levy processes it is different. But since the Levy process can be multiplied by any constant we do not have to compensate for this affection.

Finally we have estimated the volatility of our log-returns, but we have not introduced any model for $\sigma$. So how are we going to create a one-period prediction for $\sigma_{t+1}$. In chapter 1 we have assumed that the endurance of volatility changes is longer than the interval of historical observations. So it is logical to say that tomorrow volatility is more likely to be the same as a day before $\sigma_t$.

- **GARCH(1, 1) – M**

  Remind the basic formula for the model

  $$\sigma_t^2 = a_0 + a_1 \sigma_{t-1}^2 (\Delta L_{t-1}^2 - m)^2 + b_1 \sigma_{t-1}^2$$  \hspace{1cm} (3.1)

  Thus we have to find just four unknown parameters $\omega, \alpha, \beta, m$. Usually estimation procedure is performed by using Maximum Likelihood method. Let’s assume that $\Delta L_t$ are distributed with some distribution, which has density function $f(\cdot, \theta)$, where $\theta$ is a set of parameters of this distribution.

  Suppose we have an initial vector of log-returns $x_1, \ldots, x_T$ Then the likelihood function for the given data will have a form

  $$Lhood_T = \prod_{t=1}^{T} l_t = \sum_{t=1}^{T} l_t$$

  where

  $$l_t = \log l_t = l_t(x_t, (x_s)_{s=1, \ldots, t-1}, \omega, \alpha, \beta, m, \theta) = \log f\left(\frac{\Delta x_t}{\sigma_t}, \theta\right)$$

  We should maximize function $Lhood_t$ by the vector of model parameters $\omega, \alpha, \beta, m$ and such optimal vector gives an estimator for desirable parameters. In our research we use the R program which perform optimization and parameter estimation using SQP algorithm. It is based on the normal distribution, i.e. supposes $\Delta L_t$ to be normally distributed. If we consider non-normal increments, then as it was shown Gouriéroux
Chapter 3. Methods

(1984)[17] in this case nevertheless Quasi-Maximum likelihood estimator (QMLE) appears to be consistent.

For our Value-at-Risk calculation procedure we need one step of prediction for the volatility time series. Value $\sigma_{T+1}$ can be obtained directly from the equations 2.1 and 3.1

- **UHF – GARCH(1, 1)**
  
  In order to estimate parameters of this model and receive one step forecast for the volatility we should find estimator of duration predicted value first. It means we have to investigate ACD model. Remind the main formula for EACD(1,1)

  $$p(\Delta_t | \psi_t) = \frac{1}{\psi_t} e^{-\frac{\Delta_t}{\psi_t}}$$

  where

  $$\psi_t = \omega + \alpha \Delta_{t-1} + \beta \psi_{t-1}$$

  (3.2)

  We apply the MLE method and use similar procedure as for the GARCH(1,1) model. EACD(1,1) log-likelihood function for the given vector of durations $\Delta_1, ..., \Delta_T$ looks like that

  $$Lhood_T = \sum_{t=1}^{T} \left[ \log \psi_t + \frac{\Delta_t}{\psi_t} \right]$$

  The estimation procedure will be similar in the case of Weibull ACD

  $$g(\Delta_k) = \frac{\gamma}{\Delta_k} \left( \Delta_k / \phi_k \right)^{\gamma - 1} e^{-\left( \Delta_k / \phi_k \right)^\gamma} \forall \Delta_k > 0$$

  where

  $$\phi_k = \tilde{\omega} + \tilde{\alpha} \Delta_{k-1} + \tilde{\beta} \phi_{k-1}$$

  (3.3)

  $$\phi_k = \frac{\psi_k}{\Gamma \left( 1 - \frac{1}{\gamma} \right)}$$

  Then log-likelihood function for WACD

  $$Lhood_T = \sum_{t=1}^{T} \left[ \log \frac{\gamma \Delta_t}{\Delta_t} + \gamma \log \frac{\Delta_t}{\phi_t} - \left( \frac{\Delta_t}{\phi_t} \right)^\gamma \right]$$

  Maximizing $Lhood_t$ we can receive vector of parameters $\omega, \alpha, \beta$. Here our risk management purposes limits us with the estimation time. Because it will be useless at all if the program search for an estimator
of duration longer than this duration period of time itself. We need quicker algorithms, that is why we use another parameter estimation method in R, call Nlminb, which is fast enough.

Conditional mean of \( \Delta_t \) is \( \psi_t \). Hence we take \( \psi_t \) as a predictor the next duration. This value can be obtained through the formulas 3.2 and 3.3 for EACD and WACD correspondingly. Now we are ready to estimate parameters of the UHF-GARCH(1,1).

\[
\sigma_t^2 = a + b\epsilon_{t-1}^2 + c\sigma_{t-1}^2 + d\Delta_t^{-1}
\]  

(3.4)

where \( \epsilon \) is as in 2.2. Log-likelihood function in this case looks like that

\[
Lhood_T = -\frac{1}{2} \sum_{t=1}^{T} \left[ \log 2\pi\sigma_t^2 + \frac{\epsilon_t^2}{\sigma_t^2} \right]
\]

We apply QMLE method and as before receive vector of desirable parameters \( a, b, c, d \). One step forecast for the volatility \( \sigma_{T+1} \) can be obtained directly from the equation 3.4

### 3.2 The driving Generalized Hyperbolic process estimation

In order to obtain consistent prediction Value-at-Risk we should know distribution of the increment \( \Delta L_t \) from the equation 2.1. Further we will see how it can be applied for the VaR calculation.

Advantages of Generalized Hyperbolic class of distributions and reasons to take it as a distribution of the driving process were described in the previous chapter. With such assumption for the process \( \Delta L_t \) we have to estimate five parameters \( a, b, \mu, \beta, \nu \) of the density function \( f_{GH} \) from the equation 2.4.

Standard procedure to do it is Maximum Likelihood estimation method. Thus for the given vector of log-returns \( x_1, \ldots, x_T \) we obtain log-likelihood function of the following form

\[
Lhood_T = \sum_{t=1}^{T} f_{GH}(x; a, b, \mu, \beta, \gamma)
\]

Common MLE procedure supposes search for the optimal set of parameters through the maximization of log-likelihood function over desirable parameters. However it is rather complicated to use this one for our purposes because we have to access values of five parameters and perform maximization procedure also over covariance matrices.
In the R program this estimation of parameters is performed by using the iteration MCECM scheme and BFGS optimization algorithm. For further details about these procedures see Breymann, Luthi (2008) [5]. In our risk management approach we assume that the next day log-return has the same distribution, which was estimated using previous T days data. Since estimated values of parameters vary essentially depending on T, we take the same value as in Eberlein, Kallsen and Kristen (2003) [10] T=500.

3.3 Value-at-Risk calculation

The last step in our risk management method is to calculate VaR. We want to quantify risk concerned with changes of log-returns $\Delta X_t$. Since Value-at-Risk can be found from the expression

$$VaR_\alpha(\Delta X_{T+1}) = -q_\alpha(\Delta X_{T+1})$$

we need to know tomorrow distribution of log-returns. In order to obtain it, following the general model [2.1] it is enough to define the distribution of $\sigma_{T+1}\Delta L_{T+1}$. Therefore we use a well-known property of Generalized Hyperbolic distributions class, that it is closed under linear transformation. If $\xi \sim GH(a, b, \mu, \beta, \nu)$ then $\forall A, B \in R, A \neq 0$

$$A\xi + B \sim GH(a, b, A\mu + B, A\beta, \nu)$$

Assume $\Delta L_{T+1}$ has $GH_{T+1}(a, b, \mu, \beta, \nu)$ distribution, and $\sigma_{T+1}$ is the one-step forecast of volatility then due to above mentioned fact distribution of $\Delta X_{T+1}$ will be $GH_{T+1}(a, b, \sigma_{T+1}\mu, \sigma_{T+1}\beta, \nu)$ Now we are ready to calculate $\alpha$-quantile as it was shown in the section Risk management.
Chapter 4

Backtestings and conclusions

In the practical part of this paper we are going to apply the discussed models to the historical data which was taken from two financial indexes, the DAX Index and the OMX Stockholm Index (OMXS30).

The DAX Index is a Blue Chip stock market index which is the most used indicator for measuring the returns of stocks that are traded on the Frankfurt Stock Exchange. It was started in 1984 and consist of the 30 largest and most traded stocks on the exchange. The DAX belongs to performance-based indexes, which means that dividends and other add-ins are included into the index’s final calculation. The DAX index is calculated using prices from the ”Xetra” electronic trading system from 9:00 AM to 5:30 PM Central European Time. Daily trading volume of the DAX market is approximately 200000 contracts, and daily price range is approximately 160 ticks. The DAX Index includes stocks of such big and famous companies as Adidas, Bayer, BMW, Commerzbank, DaimlerChrysler, Deutsche Bank, Deutsche Boerse, Lufthansa, MAN, METRO, Siemens, Volkswagen etc.

The OMX Stockholm 30 (OMXS30) is the most known and widely used index in the Nordic region. It is a capitalization-weighted index and consists of the 30 most liquid stocks that are traded on Stockholm Stock Exchange which is relatively small comparing to the Frankfurt Stock Exchange. The OMXS30 displays the general movements in the stock market. The share of each stock of the index is specified by the market capitalization of each company. The OMXS30 index is calculated accordingly to the situation on September 30, 1986, when the index was developed with a base level of 125. This Index is represented by stocks of such world known companies as Volvo Group, TeliaSonera, Svenska Handelsbanken, SEB, SCANIA, Nordea, Ericsson, Nokia, Hennes & Mauritz etc.

Using data from these two indexes we are going to test the introduced
volatility models and calculate the VaR. In order to do it and compare the results we take DAX and OMX indexes daily closing data for the same period of time as in [10] which is 2nd January 1992 - 29 June 1999. Totally - 1875 observations. First 500 observations we use for the estimation routine of Volatility and Hyperbolic processes. The remaining 1375 are used for backtesting. Thus we will be able to access preciseness of the methods. Also it is interesting to mention such detail that we were able to find dependence in price changes between these two financial markets. This effect easy to see in the following graphics of losses. Note that near to 1200 observation in the DAX index very large losses occurred and little bit later the same situation happened in the Nordic market.

Figure 4.1: Losses of DAX(top) and OMX(bottom) indexes during observed period
Next the estimated volatility time series for backtesting period is presented on the Figure 4.2, Figure 4.3 and Figure 4.4. After that we show Generalized Hyperbolic fit statistics for the first 500 observations data on the Figure 4.5 and Figure 4.6. Finally on the Figures 4.7 and 4.8 Value-at-Risk illustrations are considered.

We preferred not to show graphics of all the fit and Value-at-Risk for all cases of distributions. We illustrated only the best result of calculated VaR for OMX and DAX data. All the rest of the results are presented in tables and a corresponding analysis is given.

![Figures 4.2 and 4.3 showing volatility estimated using GARCH model for DAX and OMX indexes](image-url)

**Figure 4.2:** Volatility estimated using GARCH model for the DAX (top) and OMX (bottom) indexes
Figure 4.3: DAX Volatility estimated using Nonparametric approach with the windows: 35(top), 40 (bottom)
Figure 4.4: OMX Volatility estimated using Nonparametric approach with the windows: 36(top), 40(bottom)
Figure 4.5: Fit of Generalized Hyperbolic distribution to DAX index data (500 obs.)

Figure 4.6: Fit of Generalized Hyperbolic distribution to OMX index data (500 obs.)
Figure 4.7: $VaR_{99\%}$ and $VaR_{95\%}$ calculated for DAX index using GARCH (top) and Nonparametric (bottom) models (Normal Inverse Gaussian case)
Figure 4.8: $VaR_{99\%}$ and $VaR_{95\%}$ calculated for OMX index using GARCH (top) and Nonparametric (bottom) models (Normal Inverse Gaussian case)
Before discussing the preciseness of VaR calculation depending on the type of the distribution we would like to go back to the windows that were used in the Non-parametric model. As we mentioned before in the "Volatility estimation" section the optimal windows according to our research for DAX and OMX are 35 and 36 correspondingly. As provided by the Cross Validation method the window 40 that was obtained in the paper of Eberlein, Kallsen and Kristen (2003)[10] showed a bit worse result. To check it practically we calculated VaR for DAX and OMX using volatility calculated both with windows 35, 36 and 40. The results are performed in the Table 4.9.

According to this table windows 35 and 36 showed better result in 9 cases and window 40 in 7 cases. So we can say that a slight change in the length of the window in Non-parametric model while calculating the volatility affects the preciseness of VaR calculation.

In our research we have found that VaR preciseness very substantially depends on the distribution which is used for $\Delta L_t$. An example which is shown below illustrates the difference in DAX $VaR_{99\%}$ estimation for the Generalized Hyperbolic and Gaussian cases. As a result for the same backtesting period investigated models showed following behavior:
• GARCH volatility model and \( L \sim \) Generalized Hyperbolic distributed - 16 overlaps

• GARCH volatility model and \( L \sim \) Normally distributed - 22 overlaps

• Non parametric volatility model and \( L \sim \) Generalized Hyperbolic distributed - 17 overlaps

• Non parametric volatility model and \( L \sim \) Normally distributed - 25 overlaps

Here "overlap" means underestimation of returns. This example demonstrates us the fact that the accuracy of the VaR estimation depends on the type of chosen distribution. It is obvious in the case of Normal Gaussian distribution because even the example shows that density function differs dramatically. But in fact such differences we can observe inside the Generalized Hyperbolic distributions class.

<table>
<thead>
<tr>
<th>Frequency of excessive losses</th>
<th>N</th>
<th>NIG</th>
<th>GH</th>
<th>Hyp</th>
<th>VG</th>
<th>Stud-T</th>
</tr>
</thead>
<tbody>
<tr>
<td>OMX 99%</td>
<td>14</td>
<td>9</td>
<td>10</td>
<td>10</td>
<td>11</td>
<td>10</td>
</tr>
<tr>
<td>Non-parametric</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GARCH-M</td>
<td>15</td>
<td>12</td>
<td>12</td>
<td>13</td>
<td>13</td>
<td>12</td>
</tr>
<tr>
<td>DAX 95%</td>
<td>25</td>
<td>17</td>
<td>17</td>
<td>17</td>
<td>17</td>
<td>17</td>
</tr>
<tr>
<td>Non-parametric</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GARCH-M</td>
<td>22</td>
<td>16</td>
<td>16</td>
<td>16</td>
<td>16</td>
<td>16</td>
</tr>
<tr>
<td>OMX 95%</td>
<td>51</td>
<td>50</td>
<td>50</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Non-parametric</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GARCH-M</td>
<td>64</td>
<td>62</td>
<td>62</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DAX 99%</td>
<td>67</td>
<td>57</td>
<td>58</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Non-parametric</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GARCH-M</td>
<td>65</td>
<td>59</td>
<td>59</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

N - Normal
NIG - Normal Inverse Gaussian
GH - Generalized Hyperbolic
Hyp - Hyperbolic
VG - Variance-Gamma
Stud-T - Skewed Student-T

- best results among distributions

Figure 4.10: Frequency of Losses Table
Finally due to our research and results presented on the tables above we can conclude that the Normal Inverse Gaussian distribution satisfies our risk management proposes and shows the highest level of accuracy while calculating VaR.

In order to check performance of the model introduced for tick data we apply it to the one day Dow Jones index data. More precisely we consider movements of the index’s price during the 1st September 2004, total 10135 tick observations. From the previous tests of the general model on DAX and OMXS30 indexes we have found that Normal Inverse Gaussian (NIG) distribution gives the best results. That is why we use it again as an assumed distribution of returns for the tick data. The fit of the distribution to empirical data looks as follows.
The Figure 4.12 confirms the assumption that the quality of model’s fit is still sufficiently high in the case of tick returns. Next the 99% and 95% VaR curves obtained for Dow Jones index are presented.

Figure 4.12: Fit of Normal Inverse Gaussian distribution to Dow Jones index tick data
Figure 4.13: $VaR_{95\%}$ (upper picture) and $VaR_{99\%}$ (bottom picture) curves calculated for Dow Jones index tick data using UHF-GARCH model.

The Figure 4.13 reflects calculation result of Value-at-Risk for 500 tick points which corresponds to approximately 30 minutes of trading. In the case of 95% VaR there were 40 overlaps and in the case of 99% VaR, 12
overlaps occurred. We can see that presented method is rather robust and gives consistent results. A higher level of preciseness can be reached by using more complex types of ACD model. The developed method could be useful for those investors, who wants to make profit on very short movements of the price.

In conclusion of our research we would like to enumerate those results that we obtained:

- OMX and DAX indexes are highly correlated. Furthermore losses of Nordic market repeat the tendency of losses of German market with a small lag. This effect can be easily seen when one compares two pictures on the Figure 4.1. If very high losses occur in DAX then after some small period of time such losses occur in OMX (pictures reflect losses of both indexes for the same period of time).

- Estimated volatility which was obtained both using GARCH and Non-parametric models describes the shape of losses very well. It can be observed on the Figures 4.1, 4.2, 4.3 and 4.4. This property of calculated volatility is especially important for getting precise Value-at-Risk estimation.

- In Non-parametric model the windows 35 and 36 were used for DAX and OMX indexes correspondingly. They were obtained by the Cross Validation method and during backtesting performed more precise comparing to the window 40. Respective information is shown in the Figure 4.9.

- During backtesting Normal distribution performed worse than others and Normal Inverse Gaussian showed the best outcome. It was one of our main goals to compare Value-at-Risk calculation depending on the distribution that is used. It is evident that Normal distribution can not describe returns of an index as precise as Generalized Hyperbolic distributions does. But it was interesting to obtain as a result that Normal Inverse Gaussian distribution due to its special properties performs better than others.

- Value-at-Risk sensitivity to the choice of distribution becomes higher as the level of confidence increases.

- We have developed a model which allows us to calculate Value-at-Risk for an intraday data. Also we have applied this model to Dow Jones index and have received consistent results. Actually it is possible to
model tick data using different cases of ACD and UHF-GARCH models. For example using Weibull ACD model or UHF-GARCH model with long run volatility variable included. These improvements of the suggested model can be considered as a subject for further research.
Appendix

Program code in R

Value-at-Risk for DAX using GARCH-M model

## Take 500 work data points
z <- read.table("H:\dax.csv",sep=""," skip=1511)  
y <- z[,5]
b=c(length(y),NA)
for (i in 1:length(y))
b[i]=y[length(y)-i+1]
x<-diff(log(b))
print(x)

## Backtesting 459 data points
2009
z1<- read.table("H:\dax.csv",sep=""," skip=133)
y1<- z1[,5]
b1=c(length(y1),NA)
for (i in 1:length(y1))
b1[i]=y1[length(y1)-i+1]
print(b1)
x1<-diff(log(b1))
print(x1)
plot(x1, type="p")

## Excessive loses sequence for the Backtesting data
loses=c(1378, NA)
for (i in 500:1877)
{if (x1[i]<0)
loses[i-499]=-x1[i]
else loses[i-499]=0
}
plot(loses, type="h")

##Prices loses
z2<- read.table("H:\dax.csv",sep=""," , skip=133)
y2<- z2[,5]
b2=c(length(y2),NA)
for (i in 1:length(y2))
b2[i]=y2[length(y2)-i+1]
print(b2)
x3<-diff(b2)
print(x3)
plot(x3, type="h")

## Excessive loses sequence for the Backtesting data
Pdloses=c(1378, NA)
for (i in 500:1874)
{if (x3[i]<0)
Pdloses[i-499]=-x3[i]
else Pdloses[i-499]=0
}
plot(Pdloses, type="h")
lines(loses, type="h", col="blue")

##----------------------------
## Volatility GARCH sequence
fit = garchFit(~garch(1, 1), x)
print(fit)
Interactive Plot:
plot(fit1, which=2)
plots, results
Batch Plot:
plot(fit, which = 2)
summary(fit)

##devolatilized returns
L=x/fit@sigma.t
print(L)

##Normal Inverse Gaussian fit
NIGfit<-fit.NIGuv(L)
hist(NIGfit, ghyp.col="blue")
qqghyp(NIGfit, gaussian=FALSE)

##---------------------
## Quantity of overlaps
Calc=function(varf)
{
num=c(2008,NA)
k=0
d=0
for (i in 500:1877)
if (varf[i]<=loses[i-499])
{k=d+1
d=k
num[k]=i}
print(k)
print(num)
Calc=k
}

##-----------------
## myGarch 1 step
mygarch=function(sigma1,L1,m,c,a,b)
{
mygarch=sqrt(c+a*sigma1*sigma1*(L1-m)*(L1-m)+ b*sigma1*sigma1)
}

##----------------
## VaR 1:499
var<-c(2008,NA)
for (i in 1:499)
{
fitgh<-transform('NIGfit',0,fit@sigma.t[i])
lambda=gghyp(0.01,fitgh)
for (i in 499:2008)
{
  j=i-498
  window<-x1[j:i]
  g=garchFit(~garch(1, 1), window)
  L=window/g$sigma.t
  NIGfit<-fit.NIGuv(L)
  sd=mygarch(g$sigma.t[499],L[499],g$fit$coef[1],
         g$fit$coef[2],g$fit$coef[3],g$fit$coef[4])
  fitgh<-transform('NIGfit',0,sd)
  lambda=qghyp(0.01,fitgh)
  var[i+1]=-lambda;
}
print(Calc(var))
plot(losses, type="h")
lines(var, type="l",col="red")

jpeg(filename="H:\QQplot for NIG.jpg")
qqghyp(NIGfit,gaussian=FALSE)
dev.off()
## Take 500 work data points
z <- read.table("H:\omx1.txt", sep="", skip=4)
y <- z[,5]
b=c(length(y),NA)
for (i in 1:length(y))
b[i]=y[length(y)-i+1]
x<-diff(log(b))
print(x)

## Backtesting 459 data points
z <- read.table("H:\omx1.txt", sep="", skip=4, nrow=500)
y <- z[,2]
x<-diff(log(y))
print(x)

z1 <- read.table("H:\omx1.txt", sep="", skip=4)
y1 <- z1[,2]
x1<-diff(log(y1))
x2<-diff(y1)

## Excessive loses sequence for the Backtesting data
loses=c(1378, NA)
for (i in 500:1877)
{if (x1[i]<0)
 loses[i-499]=-x1[i]
 else loses[i-499]=0
}
plot(loses, type="h")

## Excessive PRICE loses sequence for the Backtesting data
Ploses=c(1378, NA)
for (i in 500:1877)
{if (x2[i]<0)
 Ploses[i-499]=-x2[i]
 else Ploses[i-499]=0
}
plot(Ploses, type="h")

## Volatility GARCH sequence
fitOmx = garchFit(~garch(1, 1), x)
Interactive Plot:
plot(fitOmx, which=2)

## devolatilized returns
OmxL=x/fit@sigma.t
print(OmxL)

## GH fit
fitted<-fit.ghypuv(OmxL, silent=TRUE)
hist(OmxL, ghyp.col="blue")
qqghyp(Omxfit, gaussian=FALSE)

## Quantity of overlaps
Calc=function(varf)
{
 num<-c(2008,NA)
k=0
d=0
for (i in 500:1877)
if (varf[i]<=loses[i-499])
{k=d+1
 d=k
 num[k]=i
}
print(k)
print(num)

Calc=k

myGarch=function(sigma1,L1,n,c,a,b)
{
 myGarch=sqrt(c+a*sigma1*sigma1*(L1-m)*(L1-m)+ b*sigma1*sigma1)
}

var<-c(1878,NA)
for (i in 1:499)
{
 fitgh<-transform('fitted',0,fitOmx@sigma.t[i])
 lambda=qghyp(0.01,fitgh)
 var[i]=lambda;
}

var<-c(1878,NA)
Curve<-c(1378,NA)
for (i in 499:1877)
{
 j=i-498
 window=x1[j:i]
 g=garchFit(~garch(1, 1), window)
 L=window/g@sigma.t
 Ggfit<-fit.gphypuv(L)
 sd=mygarch(g@sigma.t[499],L[499],g$fit$coef[1],
 g$fit$coef[2],g$fit$coef[3],g$fit$coef[4])
 Curve[j]=sd
 fitgh<-transform('Ggfit',0,sd)
 lambda=qghyp(0.01,fitgh)
 var[i+1]=lambda;
}

plot(Curve)
Calc(var)
Rvar<-var[500:1877]
Losses<-c(1879,NA)
Losses<-loses[1:1378]
Losses[1379]=0.15
plot(Losses, type="h", col="white")
lines(loses,type="b")

jpeg(filename="H:\QQplot for GH.jpg")
qqghyp(fitted,gaussian=FALSE)
dev.off()
Value-at-Risk for Dow Jones tick data

```r
library(fGarch)
library(ghyp)

##Data
hh<-read.table("D:\dj_tick.csv",sep="", skip=1, nrow=10000)
pr<-hh[,4]
plot(pr,type="l")
ff<-(log(pr))
plot(ff,type="l")
r<-diff(ff)
plot(r,type="l")

Losses<-c(500)
for(i in 500:9999)
{
  if (r[i]<0)
    Losses[i-499]=-r[i]
  else
    Losses[i-499]=0
}

tm<-hh[,3]
ttt<-timeDate(paste(tm), format = "%H:%M:%S")
dt<-diff(ttt)
d<-as.numeric(dt)

rLosses<-c(500)
eps=r/sqrt(d)
for(i in 500:9999)
{
  if (r[i]<0)
    rLosses[i-499]=-r[i]
  else
    rLosses[i-499]=0
}
plot(rLosses[1:9499],type="h")

##---------------------------------
## ACD parameters estimation

dur<-d[1:499]
lhood<-function(teta,dur)
{
  psi<-c(499,NA)
  if (teta[1]/(1-teta[3])>0)
  {
    psi[1]=teta[1]/(1-teta[3])
    for(i in 2:499)
    lhood=sum(log(psi)+dur/psi)
  }
  else
    lhood=Inf
}
predict<-function(dur,a,b,c)
{
  psi<-c(500,NA)
  psi[1]=a/(1-c)
  for(i in 2:500)
    psi[i]=a+b*psi[i-1]
  lhood=sum(log(psi)+dur/psi)
}
```

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psi[i]=a+b*dur[i-1]+c*psi[i-1]

predict=psi[500]

###-----------------------------
## UHF-GARCH parameters estimation

ret<-r[1:499]
eps<-ret/sqrt(dur)

Vlhood<-function(theta,eps,dur)
{
  sig<-c(499,NA)
  if( theta[1]*(1-theta[2]-theta[3])> 0)
  {
    sig[1]=sqrt(theta[1]/(1-theta[2]-theta[3]))
    for(i in 2:499)
    {
      if ((theta[1]+theta[2]*eps[i-1]^2+theta[3]*sig[i-1]^2+theta[4]/dur[i])>0)
      else sig[i]=Inf
    }
    Vlhood=sum(log(2*pi*sig^2)+(eps/sig)^2)/2
  }
  else
    Vlhood=Inf
}

predictVol<-function(vect,eps,d1)
{
  sig<-c(500,NA)
  for(i in 2:500)
  predictVol=sig[500]
}

Vol<-function(vect,eps,d1)
{
  sig<-c(500,NA)
  for(i in 2:500)
  Vol=sig
}

###----Shape of the distribution---------

shape<-function(f)
{
  x<-c(200000,NA)
  y<-c(200000,NA)
  x[1]=-0.0003
  y[1]=dg hyp(0.0001,fitg h)
  for ( i in 2:40)
  { 
    k=-0.0001*i*0.00001
    x[i]=k
    y[i]=dg hyp(k,fitg h)
  }
  plot(x,y,type="l")
}
shape(fitgh)

## My Quantile

\[
f(x) = \text{dghyp}(x; fitgh)
\]

\[
\text{quant1}(q) = \begin{cases} 
  k = 0 \\
  p = 1 \\
  a = -0.1 \\
  b = 0 \\
  x = (a+b)/2 \\
  \text{ind} = 0 \\
  \text{while}((\text{abs}(q-p)>1e-6) && (k<1000)) \\
  \text{if} \ (\text{ind}==0) \\
  \{ 
  k1 = k \\
  k = k1 + 1 \\
  x = (a+b)/2 \\
  \} \\
  \text{if} (\text{integrate}(f, -0.1, x, \text{stop.on.error} = \text{FALSE})$\text{message}$=="OK") \\
  \{ 
  \text{ind} = 0 \\
  I = \text{integrate}(f, -0.1, x, \text{stop.on.error} = \text{FALSE})$\text{value}$ \\
  \text{if} \ (q<p) \\
  \{ 
  b = x \\
  \text{side} = -1 \\
  \} \\
  \text{else} \\
  \{ 
  a = x \\
  \text{side} = 1 \\
  \} \\
  \} \\
  \text{else} \\
  \{ 
  x1 = x \\
  \text{if} \ (\text{side}==1) x = x1 + 0.0001 \\
  \text{else} x = x1 - 0.0001 \\
  \text{ind} = 1 \\
  \} \\
\]

\text{print(k)}
\text{print(p)}
\text{quant1} = \begin{cases} 
  \text{q} < \text{quant1}(0.01) \\
  \end{cases}

\text{q}

\text{square(x)} = \begin{cases} 
  \text{integrate}(f, -0.1, 0.1)$\text{value}$ \\
  \end{cases}

\text{Calc} = \text{function(var, n1, n2)
num<-c(501,NA)
k=0
d=0
for (i in n1:n2)
if (var[i]<rLosses[i])
{k=d+1
d=k
num[k]=i
}
print(k)
print(num)
Calc=k
##--------------------------------------------
##------VaR-----
tvar<-c(10000,NA)
for (i in 499:9999)
{
j=i-498
w<-d[j:i]
r499<-r[j:i]
eps499<-r499/sqrt(w)
sg0=sum(eps499^2)/499
teta0<-c(sg0,1/4,1/4)
#teta0<-c(1/2,1/4,1/4)
opt<-nlminb(start = teta0, lhood, d = w)
vect1<-c(opt$par[1],opt$par[2],opt$par[3])
d500<-w
d500[500]=predict(w,vect1[1],vect1[2],vect1[3])
theta0<-c(sg0,3/16,3/16,2/16)
#theta0<-c(1/2,1/6,1/6)
opt1<-nlminb(start = theta0, lhood, eps=eps499, d = w, lower=1e-10, upper=5)
vect2<-c(opt1$par[1],opt1$par[2],opt1$par[3],opt1$par[4])
sig=Vol(vect2,eps499,d500)
sigma=sig*sqrt(d500)
L=eps499/sigma[1:499]
NIGfit<-fit.ghypuv(L)
fitgh<-transform('NIGfit',0,sigma[500])
shape(fitgh)
lambda<-quant1(0.01)
tvar[j]=lambda;
}
tvar[1:9501]
plot(rLosses[1:9501],type="h")
lines(tvar[1:9501], type="l", col="blue")
Calc(tvar,1,9500)
-------------
Finding the optimal window for Non-parametric model using CV method

## Working with data from DAX ------------------------------------------------------

### Loading 500 data points from the file --------------------------------------------

```r
z <- read.table("D:\dax.csv",sep="", skip=1431)
y <- z[,5]
b <- c(length(y),NA)
for (i in 1:length(y))
b[i]=y[length(y)-i+1]
```

### Calculating log-returns ----------------------------------------------------------

```r
x<-diff(log(b))
print(x)
```

### After calculating log-returns in some points we obtain zeros that we can not
use, so we replace them with a very small value 1e-10 -------------------------------

```r
zeros<-c(579,NA)
id=1
for(i in 1:579)
if (x[i]==0)
{
x[i]=1e-10
zeros[id]=i
id=id+1
}
zeros
```

### This function is ((log(b^2)^-t) and is used to calculate CV function ------------

```r
sigma=function(k,x,l)
{
d=0
vec<x[1:l]
vec1<-log(vec^2)
for(i in 1:k)
{
a=l-i
d=d+vec1[a]
}
sigma=d/k
}
mas<-log(x^2)
```

### This is CV function -------------------------------------------------------------

```r
CV=function(k,mas,x,T)
{
s=0
for(j in 1:T)
{
l=j+80
s=s+(mas[l]-sigma(k,x,l))^2
}
CV=s/T
}
```
This shows all the values of CV function calculated for different \textit{k} and then finds the optimal window by choosing the smallest one -----------------------------

\texttt{T=499}
\texttt{ff<-c(200,NA)}
\texttt{min=CV(5,mas,x,T)}
\texttt{window=5}
\texttt{min}
\texttt{k=5}
\texttt{while(k<80)}
\texttt{\{}
\texttt{ff[k]=CV(k,mas,x,T)}
\texttt{if (ff[k]<min)}
\texttt{\{}
\texttt{min=ff[k]}
\texttt{window=k}
\texttt{\}}
\texttt{k=k+1}
\texttt{\}}
\texttt{ff}
\texttt{window}

Working with data from OMX ----------------------------------------------------

Algorithm and the body of the program are the same, we change only the part that loads data points from the file -----------------------------

\texttt{z <- read.table("D:\omx1.txt",sep="", skip=4, nrow=580)}
\texttt{y <- z[,2]}
\texttt{print(y)}
\texttt{x<-diff(log(y))}
\texttt{print(x)}
Value-at-Risk for DAX using Non-parametric model

```r
## Loading data points from the file------------------------------------------------
z1<- read.table("D:\dax.csv",sep="", skip=2)
y1<- z1[,5]
b1=c(length(y1),NA)
for (i in 1:length(y1))
b1[i]=y1[length(y1)-i+1]
print(b1)

## Calculating log-returns ----------------------------------------------------------
x1<-diff(log(b1))
print(x1)
plot(x1, type="h")

## Finding excessive loses sequence for Backtesting and obtaining it's graphic ------
loses=c(length(x1), NA)
for (i in 1:length(x1))
{if (x1[i]<0)
loses[i]=-x1[i]
else loses[i]=0
}
plot(loses, type="h")
lines(loses, type="h", col="blue")

## A function that predicts volatility one step forward ----------------------------
sigma=function(k,x,t)
{
d=0
vec<-x[1:t]
vec1<-log(vec^2)
for(i in 1:k)
if(is.finite(vec1[t-i]))
d=d+vec1[t-i]
sigma=d/k
}

## A function that calculates volatility for distribution's parameters estimation --
sigmad=function(k,x,t)
{
d=0
vec<-x[1:t]
vec1<-log(vec^2)
for(i in 0:k-1)
if(is.finite(vec1[t-i]))
d=d+vec1[t-i]
sigma=d/k
}

## Calculating volatility using window 35 to find VaR ------------------------------
volatility35<-c(1973,NA)
k=35
for (i in 36:2008)
volatility35[i-35]=sqrt(exp(sigma(k,x1,i)))
}

## Calculating volatility using window 35 to estimate distribution's parameters ----
volatilityd35<-c(1973,NA)
k=35
```

50
for (i in 35:2008)
  volatilityd35[i-34]=sqrt(exp(sigmad(k,x1,i)))

## Calculating VaR 1% using GENERALIZED HYPERBOLIC DISTRIBUTION

var35GHYP<-c(2008,NA)
for (i in 533:2008)
{
  j=i-498
  a=j-34
  b=i-34
  c=i-33
  window<-x1[j:i]
  v1<volatilityd35[a:b]
  l=window/v1
  GHYPfit<-fit.ghypuv(L)
  fitgh<-transform('GHYPfit',0,volatility35[c])
  lambda=qghyp(0.01,fitgh)
  var35GHYP[i+1]=mean(fitgh)-lambda;
}

## Calculating the number of overlaps and finding points where they occurred

Calc=function(varf)
{
  num<-c(2008,NA)
  k=0
  d=0
  for (i in 1:1474)
    if (varf[i]<=Loses[i])
      { k=d+1 d=k num[k]=i }
  print(k)
  print(num)
  Calc=k
}

## Drawing graphics of losses and VaR 1%

var35GHYPplot<-c(1475,NA)
var35GHYPplot<-var35GHYP[534:2007]
Losses<-c(1475,NA)
Losses<- losses[534:2007]
plot(Loses, type="h", col="white")
lines(Loses, type="h")
lines(var35GHYPplot, type="l",col="red")
Calc(var35GHYPplot)
var35GHYPplot

## To calculate VaR 5% using GENERALIZED HYPERBOLIC DISTRIBUTION we follow the same
## steps that were shown above for calculating VaR 1% but the line
## "lambda=qghyp(0.01,fitgh)" should be changed to "lambda=qghyp(0.05,fitgh)"

## To calculate VaR using other distributions we take the same procedure as shown
## above but with following changes

## VaR 1% using HYPERBOLIC DISTRIBUTION
for (i in 533:2008)
{
  j=i-498
  a=j-34
  b=i-34
  c=i-33
  window<-x1[j:i]
  v1<-volatilityd35[a:b]
  L=window/v1
  HYPfit<-fit.hypuv(L)
  fith<-transform('HYPfit',0,volatility35[c])
  lambda=qghyp(0.01,fith)
  var35HYP[i+1]=mean(fith)-lambda;
}

## VaR 5% using HYPERBOLIC DISTRIBUTION --------------------------------------------

lambda=qghyp(0.01,fith) -----> lambda=qghyp(0.05,fith)

## VaR 1% using NORMAL INVERSE GAUSSIAN --------------------------------------------

... var35NIG<-c(2008,NA)
for (i in 533:2008)
{
  j=i-498
  a=j-34
  b=i-34
  c=i-33
  window<-x1[j:i]
  v1<-volatilityd35[a:b]
  L=window/v1
  NIGfit<-fit.NIGuv(L)
  fitnig<-transform('NIGfit',0,volatility35[c])
  lambda=qghyp(0.01,fitnig)
  var35NIG[i+1]=mean(fitnig)-lambda;
}

## VaR 5% using NORMAL INVERSE GAUSSIAN --------------------------------------------

lambda=qghyp(0.01,finig) -----> lambda=qghyp(0.05,fitnig)

## VaR 1% using NORMAL GAUSSIAN DISTRIBUTION ---------------------------------------

... var35GAUSS<-c(2008,NA)
for (i in 533:2008)
{
  j=i-498
  a=j-34
  b=i-34
  c=i-33
  window<-x1[j:i]
  v1<-volatilityd35[a:b]
  L=window/v1
  par<-coef(GAUSSfit)
  fitgauss<-gauss(par$mu, volatility35[c]*par$sigma)
  lambda=qghyp(0.01,fitgauss)
  var35GAUSS[i+1]=mean(fitgauss)-lambda;
}

...
## VaR 5% using NORMAL GAUSSIAN DISTRIBUTION ---------------------------------------

\[
\lambda = \text{qghyp}(0.01, \text{fitgauss}) \quad \rightarrow \quad \lambda = \text{qghyp}(0.05, \text{fitgauss})
\]

## VaR 1% using Variance Gamma -----------------------------------------------------

\[
\text{var35VG} \leftarrow \text{c}(2008, \text{NA})
\]

\[
\text{for} \ (i \ \text{in} \ 533:2008) \\
\{
\quad j = i - 498 \\
\quad a = j - 34 \\
\quad b = i - 34 \\
\quad c = i - 33 \\
\quad \text{window} \leftarrow x1[j:i] \\
\quad v1 \leftarrow \text{volatility35}[a:b] \\
\quad L \leftarrow \text{window}/v1 \\
\quad \text{VGfit} \leftarrow \text{fit.VGuv}(L) \\
\quad \text{fitvg} \leftarrow \text{transform('VGfit', 0, volatility35[c])} \\
\quad \lambda = \text{qghyp}(0.01, \text{fitvg}) \\
\quad \text{var35VG}[i+1] = \text{mean(fitvg)} - \lambda;
\}
\]

## VaR 5% using Variance Gamma -----------------------------------------------------

\[
\lambda = \text{qghyp}(0.01, \text{fitvg}) \quad \rightarrow \quad \lambda = \text{qghyp}(0.05, \text{fitvg})
\]

## VaR 1% using STUDENT -------------------------------------------------------------

\[
\text{var35St} \leftarrow \text{c}(2008, \text{NA})
\]

\[
\text{for} \ (i \ \text{in} \ 533:2008) \\
\{
\quad j = i - 498 \\
\quad a = j - 34 \\
\quad b = i - 34 \\
\quad c = i - 33 \\
\quad \text{window} \leftarrow x1[j:i] \\
\quad v1 \leftarrow \text{volatility35}[a:b] \\
\quad L \leftarrow \text{window}/v1 \\
\quad \text{Stfit} \leftarrow \text{fit.tuv}(L) \\
\quad \text{fitst} \leftarrow \text{transform('Stfit', 0, volatility35[c])} \\
\quad \lambda = \text{qghyp}(0.01, \text{fitst}) \\
\quad \text{var35St}[i+1] = \text{mean(fitst)} - \lambda;
\}
\]

## VaR 5% using STUDENT -------------------------------------------------------------

\[
\lambda = \text{qghyp}(0.01, \text{fitst}) \quad \rightarrow \quad \lambda = \text{qghyp}(0.05, \text{fitst})
\]
Value-at-Risk for OMX using Non-parametric

```r
## Loading data points from the file---------------------------------------------
z1<- read.table("D:\omx1.txt",sep="",skip=4)
b1<- z1[,2]
print(b1)

## Calculating log-returns ---------------------------------------------------
x1<-diff(log(b1))
print(x1)
plot(x1, type="h")

## Finding excessive loses sequence for Backtesting and obtaining it's graphic ----
loses=c(length(x1), NA)
for (i in 1:length(x1))
  if (x1[i]<0)
    loses[i]=-x1[i]
  else loses[i]=0
plot(loses, type="l")

## A function that predicts volatility one step forward ----------------------
sigma=function(k,x,t)
{
  d=0
  vec<x[1:t]
  vec1<-log(vec^2)
  for(i in 1:k)
    if(is.finite(vec1[t-i]))
      d=d+vec1[t-i]
  sigma=d/k
}

## A function that calculates volatility for distribution's parameters estimation --
sigmad=function(k,x,t)
{
  d=0
  vec<x[1:t]
  vec1<-log(vec^2)
  for(i in 0:k-1)
    if(is.finite(vec1[t-i]))
      d=d+vec1[t-i]
  sigma=d/k
}

## Calculating volatility using window 36 to find VaR ------------------------
volatility36<-c(1877,NA)
k=36
for (i in 37:1877)
  volatility36[i-36]=sqrt(exp(sigma(k,x1,i)))
volatility36

## Calculating volatility using window 36 to estimate distribution's parameters ----
volatilityd36<-c(1877,NA)
k=36
for (i in 36:1877)
  volatilityd36[i-35]=sqrt(exp(sigmad(k,x1,i)))
volatilityd36
```

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## Calculating VaR 1% using GENERALIZED HYPERBOLIC DISTRIBUTION --------------------

```
# Calculating VaR 1% using GENERALIZED HYPERBOLIC DISTRIBUTION
var36GHYP<-c(1877,NA)
for (i in 534:1877)
{
  j=i-498
  a=j-35
  b=i-35
  c=i-34
  window<-x1[j:i]
  v1<-volatilityd36[a:b]
  L=window/v1
  GHYPfit<-fit.ghypuv(L)
  fitgh<-transform('GHYPfit',0,volatility36[c])
  lambda=qghyp(0.01,fitgh)
  var36GHYP[i+1]=mean(fitgh)-lambda;
}

for (i in 535:1876)
{
  if(is.na(var36GHYP[i])==TRUE)
    var36GHYP[i]=var36GHYP[i-1]
}

## Calculating the number of overlaps and finding points where they occurred -------
Calc=function(varf)
{
  num<-c(1342,NA)
  k=0
  d=0
  for (i in 1:1342)
  {
    if (varf[i]<=Loses[i])
    {
      k=d+1
      d=k
      num[k]=i
    }
  }
  print(k)
  print(num)
  Calc=k
}

## Drawing graphics of losses and VaR 1% -------------------------------------------
var36GHYPplot<-c(1342,NA)
var36GHYPplot<-var36GHYP[535:1876]
Loses<-c(1342,NA)
Loses<-loses[534:1876]
Loses[1342]=0.1
Loses[1342]=loses[534:1876]
plot(Loses, type="h", col="white")
lines(Loses, type="h")
lines(var36GHYPplot, type="l",col="red")
Calc(var36GHYPplot)

## To calculate VaR 5% using GENERALIZED HYPERBOLIC DISTRIBUTION we follow the same
steps that were shown above for calculating VaR 1% but the line
"lambda=qghyp(0.01,fitgh)" should be changed to "lambda=qghyp(0.05,fitgh)"

## To calculate VaR using other distributions we take the same procedure as shown
above but with some changes -----------------------------------------------------
```

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## VaR 1% using HYPERBOLIC DISTRIBUTION

```r
... var36HYP<-c(1877,NA) for (i in 534:1877) {
  j=i-498
  a=j-35
  b=i-35
  c=i-34
  window<x1[j:i]
  v1<-volatilityd36[a:b]
  L=window/v1
  HYPfit<-fit.hypuv(L)
  fith<-transform('HYPfit',0,volatility36[c])
  lambda=qghyp(0.01,fith)
  var36HYP[i+1]=mean(fith)-lambda;
}
...
```

## VaR 5% using HYPERBOLIC DISTRIBUTION

lambda=qghyp(0.01,fith) -----> lambda=qghyp(0.05,fith)

## VaR 1% using NORMAL INVERSE GAUSSIAN

```r
... var36NIG<-c(1877,NA) for (i in 534:1877) {
  j=i-498
  a=j-35
  b=i-35
  c=i-34
  window<x1[j:i]
  v1<-volatilityd36[a:b]
  L=window/v1
  NIGfit<-fit.NIGuv(L)
  fitnig<-transform('NIGfit',0,volatility36[c])
  lambda=qghyp(0.01,fitnig)
  var36NIG[i+1]=mean(fitnig)-lambda;
}
...
```

## VaR 5% using NORMAL INVERSE GAUSSIAN

lambda=qghyp(0.01,fitnig) -----> lambda=qghyp(0.05,fitnig)

## VaR 1% using NORMAL GAUSSIAN DISTRIBUTION

```r
... var36GAUSS<-c(1877,NA) for (i in 534:1877) {
  j=i-498
  a=j-35
  b=i-35
  c=i-34
  window<x1[j:i]
  v1<-volatilityd36[a:b]
  L=window/v1
  GAUSSfit<-fit.gaussuv(L)
  par<coef(GAUSSfit)
  fitgauss<-gauss(par$mu, volatility36[c]*par$sigma)
  ...
```
\(\lambda = \text{qghyp}(0.01, \text{fitgauss})\)
\(\text{var36GAUSS}[i+1] = \text{mean}(\text{fitgauss}) - \lambda;\)

\[\text{...}\]

### VaR 5% using NORMAL GAUSSIAN DISTRIBUTION ---------------------------------------
\(\lambda = \text{qghyp}(0.01, \text{fitgauss})\) -----> \(\lambda = \text{qghyp}(0.05, \text{fitgauss})\)

### VaR 1% using Variance Gamma -----------------------------------------------------
\[\text{...}\]
\(\text{ar36VG} \leftarrow c(1877, NA)\)
\(\text{for} \ (i \ in \ [534:1877]) \{\)
  \(\text{j} = i - 498\)
  \(\text{a} = j - 35\)
  \(\text{b} = i - 35\)
  \(\text{c} = i - 34\)
  \(\text{window} \leftarrow x1[\text{j}:\text{i}]\)
  \(\text{v1} \leftarrow \text{volatility36[}\text{a:b}]\)
  \(\text{L} = \text{window}/\text{v1}\)
  \(\text{VGfit} \leftarrow \text{fit.VGuv}(\text{L})\)
  \(\text{fitvg} \leftarrow \text{transform}(\text{VGfit}, 0, \text{volatility36[c]})\)
  \(\lambda = \text{qghyp}(0.01, \text{fitvg})\)
  \(\text{var36VG}[i+1] = \text{mean}(\text{fitvg}) - \lambda;\)
\(\text{...}\)

### VaR 5% using Variance Gamma -----------------------------------------------------
\(\lambda = \text{qghyp}(0.01, \text{fitvg})\) -----> \(\lambda = \text{qghyp}(0.05, \text{fitvg})\)

### VaR 1% using STUDENT -----------------------------------------------------------
\[\text{...}\]
\(\text{var36St} \leftarrow c(1877, NA)\)
\(\text{for} \ (i \ in \ [534:1877]) \{\)
  \(\text{j} = i - 498\)
  \(\text{a} = j - 35\)
  \(\text{b} = i - 35\)
  \(\text{c} = i - 34\)
  \(\text{window} \leftarrow x1[\text{j}:\text{i}]\)
  \(\text{v1} \leftarrow \text{volatility36[}\text{a:b}]\)
  \(\text{L} = \text{window}/\text{v1}\)
  \(\text{Stfit} \leftarrow \text{fit.tuv}(\text{L})\)
  \(\text{fitst} \leftarrow \text{transform}(\text{Stfit}, 0, \text{volatility36[c]})\)
  \(\lambda = \text{qghyp}(0.01, \text{fitst})\)
  \(\text{var36St}[i+1] = \text{mean}(\text{fitst}) - \lambda;\)
\(\text{...}\)

### VaR 5% using STUDENT -----------------------------------------------------------
\(\lambda = \text{qghyp}(0.01, \text{fitst})\) -----> \(\lambda = \text{qghyp}(0.05, \text{fitst})\)
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