How reliable is implied volatility 
A comparison between implied and actual volatility on an index at the Nordic Market

Master's Thesis in Financial Mathematics

Maria Kozyreva

School of Information Science, Computer and Electrical Engineering
Halmstad University
How reliable is implied volatility
A comparison between implied and actual volatility on an index at the Nordic Market

Maria Kozyreva

Halmstad University
Project Report IDE0740

Master's thesis in Financial Mathematics, 15 ECTS credits
Supervisor: Ph.D. Jan-Olof Johansson
Examiner: Prof. Ljudmila A. Bordag
External referees: Prof. Krzysztof Szajowski

October 22, 2007

Department of Mathematics, Physics and Electrical engineering
School of Information Science, Computer and Electrical Engineering
Halmstad University
Preface

I would like to thank all people who were helping me during my work. Especially, a lot of thanks to my supervisor Ph.D. Jan-Olof Johansson for this advices and support. I also thank the SIX Company for given opportunity to get useful data.
Abstract
Volatility forecast plays a central role in the financial decision making process. An intrinsic purpose of any investor is profit earning. For that purpose investors need to estimate the risk. One of the most efficient methods to this end is the volatility estimation.

In this theses I compare the CBOE Volatility Index, (VIX) with the actual volatility on an index at the Nordic Market. The actual volatility is defined as the one-day-ahead prediction as calculated by using the GARCH(1,1) model. By using the VIX model I performed consecutive predictions 30 days ahead between February the 2nd, 2007 to March the 6th, 2007. These predictions were compared with the GARCH(1,1) one-day-ahead predictions for the same period. To my knowledge, such comparisons have not been performed earlier on the Nordic Market.

The conclusion of the study was that the VIX predictions tends to higher values then the GARCH(1,1) predictions except for large prices upward jumps, which indicates that the VIX is not able to predict future shocks. Except from these jumps, the VIX more often shows larger value than the GARCH(1,1). This is interpreted as an uncertainly of the prediction. However, the VIX predictions follows the actual volatility reasonable well. I conclude that the VIX estimation can be used as a reliable estimator of market volatility.
Contents

1 Introduction 1

2 Methods 3
   2.1 Implied Volatility 5
   2.2 Actual Volatility 12

3 Results 17
   3.1 Empirical Results 17

4 Conclusions 23

Notation 27

Appendix 31
Chapter 1

Introduction

Volatility forecast plays a central role in the financial decision making process. An intrinsic purpose of the investor is profit earning. For that purpose investors need to estimate the risk and then make a decision with respect to this estimation. This means that risk estimation constitutes an important task for investors. The concept of risk is closely connected to the concept of volatility and one of the most used methods to estimate the risk is to estimate the volatility, cf. [1]. High volatility is normally connected with high risk, which means that investors are able to either get high profit or high loss. Low volatility, on the other hand, implies that the risk is not so large and investors possible profit or loss is moderate.

By its implication implied volatility is not an observable artifact calculated on basis of this or that formula. Since option price depends on the oscillation degree of the underlying asset, that is the risk based on the market data about last deal, prices can be calculated ”what does market think about volatility” or which risk does traders mean.

The implied volatility appears in several option pricing models i.e. the Black-Sholes model or the binomial model and is derived from the quoted price for an option, cf. [14]. A quite different method to compute the implied volatility is realized by the CBOE Volatility Index (VIX), see [5]. This method is free from any model and uses only the prices of the S&P 500 index calls and puts options for its calculations. The VIX reflects the market expectation about volatility 30 day ahead and estimates expected volatility from the prices of stock index options in a wide range of strike prices. I apply the VIX calculations to the OMX Stockholm 30 instead of the S&P 500 index. To my knowledge, this method of volatility calculation is never used before in the Nordic Market.

The actual volatility can also be calculated by different methods. A common method uses the closing price returns, \( R_t = \log(P_t/P_{t-1}) \), where \( P_t \)
is the closing price day $t$. Based on the returns, $R_t$, the standard deviation is calculated for a specified number of days and the volatility is then expressed as the annualized standard deviation, see [10].

In this study I concentrate on the Generalized Autoregressive Conditional Heteroscedastic model (GARCH). The one-day ahead prediction of the GARCH(1,1) model, $\hat{\sigma}_t^2$, coincides with the current volatility at time $t$, which is a constant plus an exponentially weighted moving average of the past squared returns, $P_t^2$, see [1].

The aim of this thesis is to compare the VIX volatility with actual or the current volatility on an index at the Nordic Market in order to estimate its reliability as an risk predictor.

In this section the background is given. In Section Methods the data, methods and models are presented. All results are presented in Section Results and discussed and interpreted in section Conclusion. Finally complete calculation and simulation program code may be found in the Appendix.
Chapter 2

Methods

The data for my empirical research consists of OMX Stockholm 30 Index at the Nordic Exchange Market closing prices during the period 4 May 2000 to 3 May 2007. From the original dataset, which includes prices recorded for every trading day, I extract data during the period 27 September 2005 to 3 May 2007. Thus I get 400 datapoint, that is quite enough for my calculation.

In finance it is well known that the prices, the magnitude of financial indexes fluctuate stochastically. In Figure 2.1 I graph the fluctuation of the OMX Stockholm 30 Index over the period 27 September 2005 to 3 May 2007.

Volatility is one of the basic tools of the risk measure of a financial active. To the present time in a trader arsenal, financial analysts and risks-managers the set of volatility models is saved, cf. [11]

A record of stock price movements can be used to estimate volatility.

I take the closing price and calculate the daily return series $R_t$ as the natural logarithm of ratio of successive prices.

$$R_t = \log \frac{P_t}{P_{t-1}}, \quad (2.1)$$

where $P_t$ denotes the closing price on trading day $t$, with $t = 1, \ldots, T$.

Also, I calculate the squared return series $R_t^2$. In the Figure 2.2 I graph the daily return series $R_t$ together with the squared return series $R_t^2$ over the full sample period and I report the summary statistics of both series in the table 2.1, see [7].

From the graph, see Figure 2.2, can be discerned more volatile periods which occurred toward the middle of 2006 and during the first quarter of 2007. As is easy to see, there are bunching of high and low volatility episodes. It
Figure 2.1: Fluctuation of the OMX Stockholm Index over the period 27 September 2005 to 3 May 2007

is called clustering effect, cf. [6].

An estimate, $s$, of the standard deviation of the $R_t$’s is given by

$$s = \sqrt{\frac{1}{T-1} \sum_{t=1}^{T} R_t^2 - \frac{1}{T-1} \left( \sum_{t=1}^{T} R_t \right)^2}, \quad (2.2)$$

where $T$ is the number of observations and $R_t$ is defined as in equation 2.1.

The standard deviation of the $R_t$’s is $\sigma \sqrt{\tau}$, where $\sigma$ is the volatility of the index price and $\tau$ is the length of time interval in years. If follows that $\sigma$ itself can be estimated as $\hat{\sigma}$, where

$$\hat{\sigma} = \frac{s}{\sqrt{\tau}} \quad (2.3)$$

The standard error of this estimate can be shown to be approximately equal to $\frac{\hat{\sigma}}{\sqrt{2T}}$, [8].

Choosing an appropriate value for $T$ is not easy. More data generally lead to more accuracy in parameter estimation, but $\sigma$ does change over time.
and data that are too old may not be relevant for predicting the future. A compromise which seems to work reasonably well is to use closing prices from daily data over the most recent days. There is an important issue concerned with whether time should be measured in calendar days or trading days when volatility parameters are being estimated and used.

2.1 Implied Volatility

By its implication implied volatility is not an observable artifact calculated on basis of this or that formula. Since option price depends on the oscillation degree of index price, that is risk based on the market data about last deal,
prices can be calculated “what does market think about volatility” or which risk does traders mean.

Implied volatility can be calculated by using different methods. One method uses the Black-Scholes formula for option pricing. For example the price of a call option is given by,

\[
C(P,t) = P N(d_1) - Ke^{-r(T-t)} N(d_2),
\]

where \( N(\cdot) \) denotes the standard normal distribution and

\[
d_1 = \frac{\log(S/E) + (r + \frac{1}{2}\sigma^2)(T-t)}{\sigma \sqrt{T-t}}
\]

and

\[
d_2 = \frac{\log(S/E) + (r - \frac{1}{2}\sigma^2)(T-t)}{\sigma \sqrt{T-t}}.
\]

The parameters of the formula are the strike price \( K \), the interest rate \( r \), the expire \( T \), the volatility \( \sigma \) and \( P \), the value of the underlying asset.

If for example, all observable parameter values, including the market price, \( C(P,t) \), at time \( t \), of a call option are inserted into the Black-Scholes formula the the remaining unobserved value of the volatility can be computed, see [6]. The computation is possible due to the one-to-one correspondence between option price and volatility, which is manifested by the monotonically increase of price with the volatility, see also [14].

Another method, which is not dependent on any pricing model has been developed by the Chicago Board of Options Exchange, (CBOE) and is called Volatility Index, VIX.
How reliable is implied volatility

This index was first introduced in 1993 and at that time calculated based on a small number of stocks on the S&P 100 index. In 2003 it was reorganized by using all stocks in a broader index, the S&P 500 and using new formulas for calculating its value.

Other examples of volatility indices are VXN, which is CBOE Nasdaq Volatility index and VXD which is based on the Dow Jones Industrial Average. These indices serve as benchmarks for stock market volatility.

In this thesis I have chosen the VIX implied volatility computation method because this index expresses the market’s expectation of 30 day volatility. The 30 days future prediction will be compared with 30 days historical based actual volatility. For the actual volatility, I have used a formula based on the GARCH(1,1) model with 30 historical data and parameters estimated on 400 historical data. The choice of 30 historical return values assures sufficient precision of the values, cf. [5].

The VIX calculations are based on the prices of stock index option. It uses several strike prices of the option in a near term and in a nest near term. The two resulting volatilities are weighted and interpolated to get the final estimation of volatility, cf. [13], cited in the following:

"The formula for calculating the volatility, \( \sigma \), for one future term is given by,

\[
\sigma^2 = \frac{2}{T} \sum_i \frac{\Delta K_i}{K_i^2} e^{rT} Q(K_i) - \frac{1}{T} \left( \frac{F}{K_0} - 1 \right)^2,
\]

where

- \( T \) is the time to expiration,
- \( F \) is forward index level derived from index option prices,
- \( K_i \) is the strike price of \( i^{th} \) out-of-the-money option; a call option if \( K_i > F \) and a put option if \( K_i < F \),
- \( \Delta K_i \) is the interval between strike prices - half the distance between the strike on either side of \( K_i \):

\[
\Delta K_i = \frac{K_{i+1} - K_{i-1}}{2},
\]

\( K_0 \) is the first below the forward index level, \( F \),
- \( r \) is the risk-free interest rate to expiration,
- \( Q(K_i) \) is the midpoint of the bid-ask spread for each option with strike \( K_i \) and \( \text{VIX} = \sigma * 100 \), cf. [13].

The calculation of the volatility uses only out-of-the-money options with exception for \( K_0 \), where \( Q(K_0) \) is the average of the call and put option prices at \( K_0 \). But \( K_0 \leq F \) which means the at this strike the in-the-money call is
used. The last term in (2.7) is then necessary to convert this in-the-money call into an out-of-the-money put by using the put-call parity.

The calculation uses all call options at strikes greater than $F$ and put options at strike lower than $F$. However, the bid prices of these options must be strictly positive. At the extreme strikes the definition for the interval $\Delta K$ is modified in the following way. For the lowest strike, the $\Delta K$ is defined as the difference between the lowest strike and the next lowest strike and for the highest strike, the $\Delta K$ is the difference between the highest strike and the next highest strike, see [5].

The new VIX usually uses options in the two nearest-term expiration months to bracket the coming 30-day calendar period. But if only 8 days are left to expiration, the new VIX rolls over to the second and third contract months. This action is performed to minimize pricing anomalies which might occur close to expiration.

The time in the VIX calculation is measured in minutes, in order to replicate the precision most often used by professional option and volatility traders. The time to expiration is given by the following expression:

$$T = \frac{M_{\text{Current Day}} + M_{\text{Settlement Day}} + M_{\text{Other Days}}}{\text{Minutes in a Year}},$$

where

$M_{\text{Current Day}} =$ number of minutes remaining until midnight of the current day,

$M_{\text{Settlement Day}} =$ number of minutes from midnight until time of the VIX calculation on settlement day,

$M_{\text{Other Days}} =$ total number of minute in the days between current day and settlement day.

The new VIX calculation can be divided on three steps. The first step is to select the options to be used in the calculation. For each contract month the forward index level, $F$, is determined based on at-the-money option prices. The at-the-money strike is the strike price at which the difference between the call and put prices is smallest. Therefore, the CBOE chooses a pair of put and call options with prices that are closest to each other, cf. [5]. Then, the forward price is derived via the put-call parity relation:

$$F = \text{Strike Price} + e^{rT} \times (\text{Call Price} - \text{Put Price}).$$

Next determine $K_0$ - the strike price immediately below the forward index level, $F$.

Then sort all of the option in ascending order by strike price. Select call options that have strike prices greater than $K_0$ and a non-zero bid price.
After encountering two consecutive calls with a bid price of zero, do not select any other calls. Next, select put options that have strike prices less than \( K_0 \) and non-zero bid price. After encountering two consecutive puts with a bid price of zero, do not select any other puts. The CBOE select both the put and call with strike price \( K_0 \). Then average the quoted bid-ask prices for each option.

Notice that two options are selected at \( K_0 \), while a single option, either a put or call, is used for every other strike price. This is done to center the strip of options around \( K_0 \). In order to avoid double counting the put and call prices at \( K_0 \) are averaged to arrive at a single value, see [13].

The second step of the new VIX calculation is to calculate the volatility for both near term and next term option.

Applying Formula 2.7 for calculating the new VIX to the near term and the next term options with time to expiration of \( T_1 \) and \( T_2 \) respectively, yields:

\[
\sigma_1^2 = \frac{2}{T_1} \sum_i \frac{\Delta K_i}{K_i^2} e^{r T_1} Q(K_i) - \frac{1}{T_1} \left( \frac{F_1}{K_0} - 1 \right)^2, \tag{2.11}
\]
\[
\sigma_2^2 = \frac{2}{T_2} \sum_i \frac{\Delta K_i}{K_i^2} e^{r T_2} Q(K_i) - \frac{1}{T_2} \left( \frac{F_2}{K_0} - 1 \right)^2. \tag{2.12}
\]

The new VIX is a combination of the information reflected in the prices of all of the options used. The contribution of a single option to the new VIX value is proportional to the price of that option and inversely proportional to the option’s strike price.

And the last, third step of the new VIX calculation is to interpolate \( \sigma_1^2 \) and \( \sigma_2^2 \) to arrive at a single value with a constant maturity of 30 days to expiration. Then take the square root of that value and multiply by 100 to get VIX.

\[
\sigma = \sqrt{\left( T_1 \sigma_1^2 \left[ \frac{N_{T_2} - N_{30}}{N_{T_2} - N_{T_1}} \right] + T_2 \sigma_2^2 \left[ \frac{N_{30} - N_{T_1}}{N_{T_2} - N_{T_1}} \right] \right) \times \frac{N_{365}}{N_{30}}}, \tag{2.13}
\]

where

\( N_{T_1} = \) number of minutes to expiration of the near term options,

\( N_{T_2} = \) number of minutes to expiration of the next term options,

\( N_{30} = \) number of minutes in 30 days (30 \( \times \) 1, 440 = 43,200),

\( N_{365} = \) number of minutes in a 365-day year (365 \( \times \) 1, 440 = 525,600).

And the new VIX is equal:
\[ VIX = 100 \times \sigma. \] 

These computation are applied to real data from the OMX Stockholm Index. I take 1 day, 2 January 2007, and make prediction 30 days ahead, that is 2 February 2007. In computation is used such two nearest-term expiration as 26 January 2007 and 23 February 2007. Therefore, in the beginning of calculation numbers of days remaining until first and second maturity date are equal 24 and 52 days, respectively. The time of the new VIX calculation is assumed to be 14:00. The risk-free interest rate for the beginning of 2007 year is equal to 3.00%.

As shown in the Figure 2.3, the difference between the call and put prices is smallest at the 1160 strike in the near term and at the 1180 strike in the next term.

Table 2.2 lists the values for number of minutes in a 365-day year \( (N_{365}) \), the number of minute in 30 days \( (N_{30}) \), the number of minutes remaining until midnight of the current day \( (M_{Current\ Day}) \), the number of minutes from midnight until 14:00 on settlement day \( (M_{Settlement\ Day}) \), the total number of minutes in the days between current day and settlement day \( (M_{Other\ Days}) \).
How reliable is implied volatility

the time to expiration \((T)\), the forward index level \((F)\) and the strike price immediately below the forward index level \((K_0)\).

<table>
<thead>
<tr>
<th>Expiration term</th>
<th>Near term</th>
<th>Next term</th>
</tr>
</thead>
<tbody>
<tr>
<td>(N_{365})</td>
<td>525600</td>
<td>525600</td>
</tr>
<tr>
<td>(N_{30})</td>
<td>43200</td>
<td>43200</td>
</tr>
<tr>
<td>MCurrent Day</td>
<td>600</td>
<td>600</td>
</tr>
<tr>
<td>MSettlement Day</td>
<td>840</td>
<td>840</td>
</tr>
<tr>
<td>MOther Days</td>
<td>31680</td>
<td>72000</td>
</tr>
<tr>
<td>Interest rate</td>
<td>3.00%</td>
<td>3.00%</td>
</tr>
<tr>
<td>(T)</td>
<td>0.063014</td>
<td>0.139726</td>
</tr>
<tr>
<td>(F)</td>
<td>1167.01</td>
<td>1192.05</td>
</tr>
<tr>
<td>(K_0)</td>
<td>1160</td>
<td>1180</td>
</tr>
</tbody>
</table>

Table 2.2: Numbers of minutes for different time periods and calculated values of \(T\), \(F\) and \(K_0\)

Then, select call option that have strike prices greater than \(K_0\) and a non-zero bid price. And select put option that have strike prices less than \(K_0\) and a non-zero bid price. In the Figure 2.4 shown results of this selection and computation of average the quoted bid-ask prices for each option. Also, in this figure represented the contribution of the near term and the next term of each option. Contribution is given by

\[
\frac{\Delta K_i}{K_i^2} e^{rT} Q(K_i)
\]

Figure 2.4: Options used to calculate the new VIX on 2nd January 2007

Insert all known values in the Equation 2.7 I get values of \(\sigma_1^2 = 0.0237\) for near expire term and \(\sigma_2^2 = 0.0196\) for next expire term. And from the
Equation 2.13 I compute $\sigma = 0.1483$ value and $VIX = 100 \times \sigma = 14.83$.

Analogously compute the new VIX index for every day during period 2 January 2007 to 31 January 2007. But for period 18 January 2007 to 31 January 2007 instead the near term (26 January 2007) and the next term (23 February 2007) use the next term (23 February 2007) and term, follow after the next term (30 March 2007), respectively.

### 2.2 Actual Volatility

Actual or current volatility can be calculated by using some different methods. When intraday data are available the actual volatility can be modeled as the integrated volatility and approximately calculated by using the realized volatility, cf. [2].

In this work I concentrate on the Generalized Autoregressive Conditional Heteroscedastic model (GARCH). this model involves in calculations the volatility value, computed on the previous steps. Since market has a memory that should be taken into consideration. It is reasonable to assume that current volatility depends on previous volatility, see [12].

The GARCH class of models is the daily time-varying volatility model, where volatility is explicitly modeled as the second moment of daily returns.

To describe the evolution of the variables $R_t = \log \frac{P_t}{P_{t-1}}$, consider the conditionally Gaussian model with $R_t = \sigma_t \varepsilon_t$. As regards the volatility $\sigma_t$ assume that

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i R_{t-1}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2,$$

where $\omega > 0$, $\alpha_i \geq 0$, and $\beta_j \geq 0$, see [3].

In the following I will assume that the mean is zero which is common in risk management, at least when short horizons are considered, cf. [4].

The simple symmetric GARCH(1,1) model is written as

$$\sigma_t^2 = \omega + \alpha R_t^2 + \beta \sigma_{t-1}^2.$$

In this work I concentrate on the GARCH(1,1) case. Repeated substitution in the Equation 2.17 readily yields,

$$\sigma_t^2 = \frac{\omega}{1 - \beta} + \alpha \sum_j \beta^{j-1} R_{t-j}^2,$$
so that the GARCH(1,1) process implies that current volatility is an exponentially weighted moving average of past squared return, see [1].

GARCH(1,1) model estimation will be based on the 400 daily returns from 27 September, 2005 to 3 May, 2007. The Statistical Data Analysis Packages "R" provides procedures for estimating the model parameters. The results are shown in the Figure 2.5.

The choice of GARCH(1,1) model can be commented by the following. If on the same data apply another models, for example GARCH(3,3), GARCH(2,2), GARCH(1,2), GARCH(2,1), GARCH(0,2) and GARCH(2,0) and with help of the "R" Packages produce estimation, then can be obtain result, which has bad parameter estimation. And according to this results, number of parameters should be decrease to (1,1).

Figure 2.5: Estimation results for the GARCH(1,1) model for the OMX Stockholm 30 index over the period 27 September, 2005 to 3 May, 2007.

Using the received parameters of estimation volatility can be counted for any period which is included in given period from 27 September, 2005 to 3 May, 2007. In case of my research it is the period 2 February, 2007 to 6 March, 2007.
The necessary calculations are made in the R program. According to program code, such values of estimation parameters as $\omega$, $\alpha$ and $\beta$ are written down in variable $\hat{\omega}$, $\hat{\alpha}$ and $\hat{\beta}$, respectively. The lines

\[ n < - 338 + j \]

and

\[ k < - 317 + j \]

are written for selection of the necessary period from the period 27 September, 2005 to 3 May, 2007. For calculation the value of volatility use the Equation 2.18 and for representation it in percentage use the following formula

\[ \sigma_t = \sqrt{\sigma_t^2 \ast 252 \ast 100}. \]

(2.19)

The complete code of this program is in the Appendix B. And the Volatility value for the OMX Stockholm Index over the period 2 February, 2007 to 6 March, 2007 is shown in Table 2.3
<table>
<thead>
<tr>
<th>Date</th>
<th>Volatility value</th>
</tr>
</thead>
<tbody>
<tr>
<td>02.02.2007</td>
<td>12.274</td>
</tr>
<tr>
<td>05.02.2007</td>
<td>12.154</td>
</tr>
<tr>
<td>06.02.2007</td>
<td>12.558</td>
</tr>
<tr>
<td>07.02.2007</td>
<td>12.285</td>
</tr>
<tr>
<td>08.02.2007</td>
<td>13.623</td>
</tr>
<tr>
<td>09.02.2007</td>
<td>13.466</td>
</tr>
<tr>
<td>12.02.2007</td>
<td>14.883</td>
</tr>
<tr>
<td>13.02.2007</td>
<td>13.899</td>
</tr>
<tr>
<td>14.02.2007</td>
<td>17.879</td>
</tr>
<tr>
<td>15.02.2007</td>
<td>16.809</td>
</tr>
<tr>
<td>16.02.2007</td>
<td>15.617</td>
</tr>
<tr>
<td>19.02.2007</td>
<td>14.804</td>
</tr>
<tr>
<td>20.02.2007</td>
<td>14.778</td>
</tr>
<tr>
<td>21.02.2007</td>
<td>13.909</td>
</tr>
<tr>
<td>22.02.2007</td>
<td>13.029</td>
</tr>
<tr>
<td>23.02.2007</td>
<td>13.636</td>
</tr>
<tr>
<td>26.02.2007</td>
<td>14.214</td>
</tr>
<tr>
<td>27.02.2007</td>
<td>28.905</td>
</tr>
<tr>
<td>01.02.2007</td>
<td>31.201</td>
</tr>
<tr>
<td>02.02.2007</td>
<td>27.717</td>
</tr>
<tr>
<td>05.02.2007</td>
<td>25.269</td>
</tr>
<tr>
<td>06.02.2007</td>
<td>22.318</td>
</tr>
</tbody>
</table>

Table 2.3: Volatility value for the OMX Stockholm Index over the period 2 February, 2007 to 6 March, 2007, calculated by GARCH(1,1) model
Chapter 3

Results

In this chapter the results obtained with two volatility models described in previous chapter are presented. A prediction calculated by the CBOE Volatility Index (VIX) model was performed for 30 day ahead over the period 2 February, 2007 to 6 March, 2007. For the same time period the GARCH(1,1) model was used to calculate the actual volatility value for each day. The two different methods of calculated the volatility are compared by using statistical methods.

3.1 Empirical Results

In Table 3.1 the values of the volatility for the OMX Stockholm Index over the period 2 February, 2007 to 6 March, 2007, calculated by the CBOE Volatility Index (VIX) and by GARCH(1,1) model are shown.

The graphical representation of the volatility values are given in Figure 3.1. The leverage effect is observed in that figure. When the price is falling, the volatility increases, cf. for example [9] or [4]. Also, the GARCH make reacts on the shocks which the VIX model does not, see [6]. Table 3.2 represents the statistical results for the two given models over the settled period. The mean, the standard deviation and the skewness of the GARCH model is large than the mean, the standard deviation and the skewness of the VIX model. This is a consequence of the sensitivity of the GARCH model with respect to the shocks.

As can be seen in Figure 3.1, the difference between the VIX predicted values and the GARCH actual volatility are reasonable small except for periods when shocks occur. At these occasions the actual volatility reacts immedi-
## Chapter 3. Results

<table>
<thead>
<tr>
<th>Date</th>
<th>VIX</th>
<th>GARCH</th>
</tr>
</thead>
<tbody>
<tr>
<td>02.02.2007</td>
<td>14.834</td>
<td>12.274</td>
</tr>
<tr>
<td>05.02.2007</td>
<td>15.725</td>
<td>12.154</td>
</tr>
<tr>
<td>06.02.2007</td>
<td>16.570</td>
<td>12.558</td>
</tr>
<tr>
<td>07.02.2007</td>
<td>15.750</td>
<td>12.285</td>
</tr>
<tr>
<td>08.02.2007</td>
<td>16.620</td>
<td>13.623</td>
</tr>
<tr>
<td>09.02.2007</td>
<td>14.958</td>
<td>13.466</td>
</tr>
<tr>
<td>12.02.2007</td>
<td>13.327</td>
<td>14.883</td>
</tr>
<tr>
<td>13.02.2007</td>
<td>14.778</td>
<td>13.899</td>
</tr>
<tr>
<td>14.02.2007</td>
<td>13.071</td>
<td>17.879</td>
</tr>
<tr>
<td>15.02.2007</td>
<td>13.836</td>
<td>16.809</td>
</tr>
<tr>
<td>16.02.2007</td>
<td>13.515</td>
<td>15.617</td>
</tr>
<tr>
<td>19.02.2007</td>
<td>14.713</td>
<td>14.804</td>
</tr>
<tr>
<td>20.02.2007</td>
<td>15.133</td>
<td>14.778</td>
</tr>
<tr>
<td>21.02.2007</td>
<td>12.968</td>
<td>13.909</td>
</tr>
<tr>
<td>22.02.2007</td>
<td>15.711</td>
<td>13.029</td>
</tr>
<tr>
<td>23.02.2007</td>
<td>15.541</td>
<td>13.636</td>
</tr>
<tr>
<td>27.02.2007</td>
<td>13.874</td>
<td>28.905</td>
</tr>
<tr>
<td>01.02.2007</td>
<td>14.351</td>
<td>31.201</td>
</tr>
<tr>
<td>02.02.2007</td>
<td>12.875</td>
<td>27.717</td>
</tr>
<tr>
<td>05.02.2007</td>
<td>14.661</td>
<td>25.269</td>
</tr>
<tr>
<td>06.02.2007</td>
<td>14.672</td>
<td>22.318</td>
</tr>
</tbody>
</table>

Table 3.1: Volatility value for the OMX Stockholm Index over the period 2 February, 2007 to 6 March, 2007, calculated by the CBOE Volatility Index (VIX) and by GARCH(1,1) model
ately but the VIX knows nothing about the shock and does not react on this.

![Graph of price change and volatility forecasts](image)

Figure 3.1: Price change and volatility forecasts. In the upper panel represent price change over the period 2 February, 2007 to 6 March, 2007. In the bottom panel shown the both of the volatility forecasts. The VIX index represent by line with triangular markers, and the GARCH model represent by line with circle markers.

In Figure 3.2 plots of the autocorrelation functions for both of models are shown. The autocorrelation function of the VIX time series is submitted on the left and the autocorrelation function of the GARCH time series is submitted on the right. Due to the construction of the GARCH model, see Equation 2.18, the nearest lag will be highly correlated.

In Figure 3.3 represented histograms for both of models. For VIX the histogram is submitted on the left, for GARCH the histogram is submitted on the right.
Chapter 3. Results

Moments | VIX | GARCH |
---------|-----|-------|
Mean     | 14.652 | 17.056 |
Stdev    | 1.102  | 5.926  |
Skewness | 0.044  | 1.237  |
Kurtosis | -0.956 | -0.012 |

Table 3.2: Summary statistics of the CBOE Volatility Index (VIX) and GARCH(1,1) model over the period 2 February, 2007 to 6 March, 2007

The histogram of the VIX data seems to be symmetric while the GARCH histogram is not. The number of observations is however too small to make too extensive conclusions.

In Figure 3.4 the boxplots for both of models are presented. For the VIX data, the boxplot is submitted on the left and for the GARCH data, the boxplot is submitted on the right. Here, it is evident that the GARCH data has larger variation than the VIX data and react immediately on the price variation.

In the results from the estimation of the parameters of the GARCH model, see Table 2.5, it can be noted that the Box-Jung test results with a p-value equal to 0.8872. This value justifies the choice of the GARCH model, see [4].
Figure 3.3: Histogram for VIX and GARCH models

Figure 3.4: Boxplot for VIX and GARCH models
Chapter 4

Conclusions

In this thesis I have compared the VIX volatility against actual volatility on an index at the Nordic Market. For computation of implied volatility I have used the CBOE Volatility Index (VIX) and for computation of actual volatility I have used the Generalized Autoregressive Conditional Heteroscedasticity (GARCH) model. I empirically investigate the forecasting performance of the methods for the OMX Stockholm 30 index over the period 2 February to 6 March 2007.

The conclusion is that the VIX tends to higher values than the GARCH except when the prices make large jumps. The GARCH model takes care about jump, the VIX does not.

Except from the jumps, the VIX is normally larger than GARCH, which can be interpreted as reflected uncertainty of the VIX estimation. However, the VIX estimation performs well except for the jumps, see [5].

I have used the GARCH model one-day ahead prediction as actual volatility. Another choice could be standard volatility estimation by calculating the standard deviation of logarithmic returns.

In this thesis I have used the GARCH one-day ahead prediction volatility because it takes long time volatility behavior and short time price into account.

It is possible to make computer program for calculation VIX volatility. For that, all data should be written in text file. From that file make selection useful data, such as all calls and puts with the same near expiry day and the next expiry day. For each expiry day calls and puts write down in the same
text file. And also match calls and puts with the same strike price to each other. Then that data should be well regulated by increasing of strike prices. On the next step again make selection calls and puts with the same strike prices and non-zero bid prices. Save in arrays useful strike prices, last prices for call and last prices for put. The interest rate, number of minutes in year and in 30 days are constant, time the VIX calculation known and always the same. The nearest expiry date, the next expiry date and current date are set manually.

Then it is not so difficult to compute $\sigma^2$ value and the VIX value by using given formulas.
Notation

$P_t$ The closing price on trading day $t$, $t = 1, \ldots, T$.
$T$ the time to expiration.
$T_1$ the time to near-term expiration.
$T_2$ the time to next-term expiration.
$R_t$ the daily return series.
$R_t^2$ the squared return series.
$s$ an estimate of the standard deviation of the $R_t$’s.
$\sigma$ the volatility of the index price.
$\sigma_1$ the volatility of the index price for near-term expiration.
$\sigma_2$ the volatility of the index price for next-term expiration.
$F$ the forward index level derived from index option prices.
$F_1$ the forward index level derived from index option prices for near-term expiration.
$F_2$ the forward index level derived from index option prices for next-term expiration.
$K_0$ the first below the forward index level $F$.
$K_i$ the strike price of $i^{th}$ out-of-the-money option.
$\Delta K_i$ the interval between strike prices - half the distance between the strike on either side of $K_i$.
$R$ the risk-free interest rate to expiration.
$Q(K_i)$ the midpoint of the bid-ask spread for each option with strike $K_i$.
$M_{\text{Current Day}}$ number of minutes remaining until midnight of the current day.
$M_{\text{Settlement Day}}$ number of minutes from midnight until time of the VIX calculation on settlement day.
$M_{\text{Other Days}}$ total number of minute in the days between current day and settlement day.
$N_{T_1}$ the number of minutes to expiration of the near term options.
$N_{T_2}$ the number of minutes to expiration of the next term options.
$N_{30}$ the number of minutes in 30 days ($30 \times 1,440 = 43,200$).
$N_{365}$ the number of minutes in a 365-day year ($365 \times 1,440 = 525,600$).
$N_{90}$ the number of minutes in 30 days ($30 \times 1,440 = 43,200$).
$N_{365}$ the number of minutes in a 365-day year ($365 \times 1,440 = 525,600$).
Bibliography


[12] V. Tvardovsky. Volatility as tool for define a market minimum. ZAO,
Investment company ”IT Invest”, 2005. In Russia.


Appendix

Here represented programming code of R packages for calculation the GARCH(1,1) model and create estimation parameters for that models.

```r
a <- read.table("data400.txt", sep="\t")
s <- a[1:399,3]
s2 <- a[2:400,3]
r <- log(s/s2)
plot(r, type="l")
library(tseries)
garch.r <- garch(r, order=c(1,1))
summary(garch.r)
plot(garch.r)
rr <- r * r
plot(rr, type="l")

\hat{\omega} = 1.072e-05
\hat{\alpha} = 1.755e-01
\hat{\beta} = 7.376e-01

sigma2 <- rep(0, times=22)
for (j in 1:22) {
    bt <- rep(0, times=22)
    for (i in 1:22) { bt[1] = \hat{\beta}^{(i-1)} } 
    n <- 338 + j
    k <- 317 + j
    r2 <- rr[n:k]
    sum <- bt %*% r2
    sgm2 <- \hat{\omega} / (1 - \hat{\beta}) + \hat{\alpha} * sum
    sigma2[j] = sqrt(sgm2 * 252) * 100 }
grch <- - sigma2[1:22]
```

31