METACOGNITION AND READING – CRITERIA FOR COMPREHENSION OF MATHEMATICS TEXTS

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This study uses categories of comprehension criteria to examine students' reasons for stating that they do, or do not, understand a given mathematics text. Nine student teachers were individually interviewed, where they read a text and commented on their comprehension, in particular, why they felt they did, or did not, understand the text. The students had some difficulties commenting on their comprehension in this manner, something that can be due to that much of comprehension monitoring, when criteria for comprehension are used, might be operating at an unconscious cognitive level. Some specific aspects of mathematics texts are examined, such as the symbolic language and conceptual and procedural understanding.

INTRODUCTION

Problem solving is of course a major aspect of mathematics and mathematics education research. Also when discussing reading, this seems often to be done in relation to problem solving (Hubbard, 1990), for example, by examining word problems (Hershkovitz & Nesher, 2001) or when studying symbolic expressions (Ferrari & Giraudi, 2001). However, in this paper, reading comprehension is studied in the context of reading a text for learning, using texts that describe and try to explain something to the reader, where no specific task to solve is given.

Some of my previous research studies (Österholm, 2004, in press) have focused on the creation of a mental representation when reading mathematics texts, that is, on how the reader understands a text. An open question in these studies is to what extent the reader believes that the text is understood – a metacognitive aspect. My previous studies have used a specific "measure" of comprehension, which may rely on criteria for comprehension that do not need to agree with what the reader views as important when understanding (mathematics) texts. Thus, different criteria might be used to decide if a text has been understood. This is a methodological problem when trying to investigate comprehension monitoring ability (Glenberg & Epstein, 1985).

This paper reports on an exploratory empirical study about what kind of criteria for comprehension university students use when reading mathematics texts.

METACOGNITION AND READING COMPREHENSION

There are different parts of metacognition, for example, knowledge about cognition and self-regulation (Brown, 1985; Schoenfeld, 1987). Comprehension monitoring is included in self-regulation and consists of two parts, evaluating comprehension using some kind of criterion and "repairing" lack of comprehension using some type of strategy (Baker, 1985).

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Comprehension monitoring

There exist several different methods for examining comprehension monitoring, some of which do not separate the use of criteria and strategies (see Ling, 2000). But there are some results that show a general weakness in evaluating one's own comprehension, for example, that students "seem not to gain information concerning the actual memorial consequences of their study behavior until they are tested on the material" (Pressley & Ghatala, 1990, p. 23), which is sometimes called *the test effect*. While there is some debate over the methods used in this type of research (Ling, 2000), some results can be explained by the domain familiarity hypothesis, according to which the evaluation of comprehension can be "based on these general beliefs [about the level of one's knowledge in a specific domain], rather than on experience with the particular texts" (Glenberg & Epstein, 1987, p. 90).

It has also been noted that much of comprehension monitoring and self-regulation seems to occur at an unconscious level (Brown, 1985; Fitzsimons & Bargh, 2004), which could explain some results showing poor monitoring, since some research methods rely on students' awareness of their own comprehension.

Criteria for comprehension

Baker (1985) gives a comprehensive description of possible criteria for reading comprehension, here presented in abbreviated form, and somewhat reformulated, with a label for each criterion together with a description of what this criterion focuses on:

Lexical	Individual words
Syntactic	Grammar
Semantic criteria:	
Propositional	Integration of ideas in text (e.g., when one part of the text refers to another part)
Structural	Thematic compatibility of ideas in text (e.g., if a part of the text fits with the main theme of the text)
External	Consistency with prior knowledge
Internal	Consistency of ideas in text (e.g., that two parts of the text are not contradicting each other)
Clarity	Necessary information to achieve a specific goal

A person's epistemological beliefs seem to be a natural source for comprehension criteria, and for metacognitive processes in general (Hofer, 2004). However, in this paper, criteria are taken for granted as existing, how they are created and how they evolve will not be discussed.

PURPOSE

The purpose of this study can be divided into three main parts. However, since this is my first study that has a metacognitive approach to reading comprehension for

mathematics texts, all three parts are of an exploratory type, where a purpose is to generate questions and hypotheses about the studied phenomena, which are planned to be studied in more detail in future studies.

Firstly, due to what has previously been discussed about to what extent processes of metacognition can be unconscious, one purpose of this study is to see how much students are able to describe parts of their comprehension processes, that is, to describe *why* they regard themselves as understanding a text or not.

Secondly, the types of criteria given by Baker (1985) will be used and tested as a tool for characterizing students' criteria for comprehension. In particular, since Baker's criteria are general in nature, it is of interest to see whether there is a need to describe more specific criteria for mathematics texts, for example, about symbolic expressions and algorithmic/procedural aspects.

Finally, one purpose is to investigate similarities and differences between criteria used in different situations: When focusing on macro- or microstructures in the text (i.e., larger or smaller parts of the text), when reading different types of texts, and when focusing on symbolic or natural language.

METHOD

Nine student teachers voluntarily participated in this study, where they individually read one or two texts and orally commented on their comprehension. The students were studying to become mathematics teachers for the Swedish upper secondary level, and had studied some mathematics courses at the university level (in algebra, geometry, and analysis). The texts, which are more thoroughly described later, describe something that was new to the students. This procedure was part of a larger data collecting session with other activities (reading other types of texts and answering questions), therefore, some students read only one text while others read two different texts. But the activities when reading the texts where the same: The student read the whole text and then commented on their comprehension, then the text was divided into sections that were shown in order one by one to the student, where their comprehension was commented on after each section (comments about macrostructure). Finally, a few single statements from the text were given one by one, and the students' comprehension was commented on once again, after each statement (comments about microstructure). When commenting on their comprehension, the students got to decide to what extent they had understood the text in question, and were then asked to explain and give reasons for why they felt that they had or had not understood (some part of) the text.

The conversations with the students were audio recorded and transcribed. The transcripts were analysed by noting where comments were made about reasons for (lack of) comprehension, and these comments were then categorized using Baker's (1985) types of criteria. At this moment, no testing of reliability of the coding process has been performed. Also, it should be noted that this methodology does not directly examine the criteria that actually have been used when reading the texts, but implicitly gives criteria from the way students talk about their comprehension.

The texts

Two different texts were used in this study, one describing basic concepts of group theory (*mathematical system* and *group*) and the other describing Newton-Raphson's method for numerically solving equations. Neither text takes up more than one page.

The text about group theory can be said to focus on conceptual understanding, while the text about Newton-Raphson presents a sort of algorithm, and can be said to focus more on procedural understanding (at least when compared to the other text). For the text about group theory, a total of 13 occasions occurred when the reader was prompted to comment on their comprehension (for the whole text, six sections, and six statements). Twelve occasions occurred for the text about Newton-Raphson (the whole text, seven sections, and four statements).

Three students read both texts (starting with the text about group theory), three read only the text about group theory, and three read only the text about Newton-Raphson.

RESULTS

When asked to give motives for their judgments of their comprehension, the students sometimes simply pointed to a smaller part of the text, stating that this part was (not) understood, but did not give any motive for this statement. Also, sometimes the students seemed somewhat uncomfortable with the situation, when asked for motives for their judgments of their comprehension. Therefore, this question was not repeated as often as planned, instead the students could sometimes more freely comment about their comprehension of the text.

To locate statements that refer to motives for (lack of) comprehension among students' comments were not experienced as problematic, but to categorize a specific statement was sometimes difficult. One reason for this is that when giving comments about why they did (not) understand, these were sometimes of a much general nature, for example, that the symbols in the text made it more difficult to understand or that the text was easy to understand because they had studied mathematics courses at the university and were familiar with the type of language. Such comments could fit many different types of criteria, since they do not refer to any specific content (i.e., meaning) of the text, which makes these types of comments seem compatible with the domain familiarity hypothesis.

Examples of students' comments

The following is an excerpt from the text about Newton-Raphson (originally in Swedish, but translated for this paper):

> If $f'(x_1) \neq 0$, then the tangent intersects the *x*-axis in a specific point. As the next approximation x_2 we choose the *x*-coordinate of the intersection point. See picture: [picture omitted due to space limitations]
>
> We can determine x_2 by letting $x = x_2$ and $y = 0$ in the tangent's equation. This gives the formula
>
> $$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}.$$

The first part of this excerpt, up to and including the picture, is section 4 of the text, and the remaining is section 5. The first sentence of the excerpt is statement 1, used when presenting single statements from the text to the students. Table 1 shows some examples of students' motives for (not) understanding some part of the given excerpt.

General aspects

Although it was not a purpose of this study, the connection between students' beliefs and criteria for comprehension sometimes became evident. Some students continuously claimed to in principle understand everything read, but clearly had some difficulties to grasp the contents of the texts. These students said that they regarded learning by reading as virtually impossible in mathematics, and that one needs to do some calculations in order to understand. Their beliefs thus made them use somewhat superficial criteria for reading comprehension, and they felt that they had understood *the text*, but in some sense not the *content of the text* (i.e., the mathematics described).

Other students did not reject the possibility of learning mathematics by reading, but regarded it as quite difficult, often using the criterion for comprehension that one should be able to *use the text* (to do some calculations on what the text is about). Therefore, they often commented on the need for concrete examples of "how to do", something that corresponds to the criterion of clarity (see example in Table 1).

Student	Text	Criterion	Student's comment
A	Section 4	Clarity	Difficult to see how to do the calculations
A, B	Section 5	Clarity	Now I understand what to do
C	Section 5	Propositional	It was the one shown before [about the tangent's equation]
C	Section 5	External	Or is this something I should know
A, C, D	Statement 1	Propositional	[Argumentation that it is true]

Table 1: Examples of categorization of students' comments.

Comparisons

Table 2 shows that there are no clear differences between students' comments about macro- and microstructures in the texts, but that some differences exist between comments about the two different texts, where the external criterion is more frequently used for the text about group theory and the clarity criterion is more frequently used for the text about Newton-Raphson. Since different persons have read different texts, this could be due to that different students mainly use different criteria. Only the external criterion shows the same pattern when looking at the three students who read both texts. However, the small number of comments makes it generally difficult to analyse one particular criterion for individual persons.

When comparing comments about symbolic expressions and sentences expressed in natural language, some qualitative differences and similarities emerge. The following

Criterion	All comments	Text		Part of text	
		Group th.	Newton-R.	Macro	Micro
Lexical	33 %	37 %	28 %	28 %	41 %
Syntactic	3 %	4 %	1 %	1 %	6 %
Propositional	13 %	8 %	20 %	14 %	11 %
Structural	10 %	9 %	13 %	12 %	8 %
External	20 %	32 %	6 %	24 %	14 %
Internal	5 %	5 %	4 %	3 %	8 %
Clarity	15 %	5 %	28 %	17 %	13 %
Total (100 %)	*163*	*92*	*71*	*99*	*64*

Table 2: Distribution (in each column) of number of occasions among all students that a criterion has been used.

is an excerpt from the text about group theory, and was one of the single statements given to students to comment on (originally in Swedish, but translated for this paper):

> The set of all whole numbers together with addition is a group

Several students understood this statement by "accepting it", without commenting on the including concepts and the relations between them. I would argue that this corresponds to a type of syntactic criterion, since only the statement's grammatical structure is taken into account, that the statement "makes sense". This can be compared with a purely symbolic statement, which was part of the text about Newton-Raphson and also given as a single statement to comment on:

$$y = f(x_1) + f'(x_1)(x - x_1)$$

Most of the students wanted to know what the symbols "stand for" for comprehension (a type of lexical criterion), often that a specific function and specific points (x and x_1) needed to be known in order to understand this expression. Nobody ever "accepted" this relationship; they always requested a context in order to understand, in which it was possible to do the calculations given in the formula. However, some students were satisfied with that the calculations *could* be done if one knew the function and the points, but all focused on the knowledge of how to do the calculations as a criterion for comprehension, which is somewhat similar to the criterion for the single statement in natural language, since it focuses on the grammar of the expression (i.e., that it "makes sense" and can be calculated).

CONCLUSIONS

In general, the students seem to have difficulties in articulating their motives for feeling that they have understood a text, or not. Perhaps this difficulty makes them often comment on the meaning of individual words (which is done about one third of the time), since this could be seen as a common cause for difficulty when reading. The cause for their difficulties could be that most of the monitoring activity takes

place at an unconscious level, that the use of some comprehension criteria has been cognitively automatized.

The major problem in this study seemed to be the collection of data, to get the students to comment on their comprehension. However, Baker's (1985) criteria for comprehension, which were originally not created as a tool for categorizing empirical data, could be necessary to refine in order to make them more easily usable in empirical analyses, especially when using them for mathematics texts. For example, Baker (1985, p. 165) refers to the clarity criterion as a "residual, encompassing dimensions that cannot be subsumed under any of the other headings", but this criterion seems quite useful, and commonly used, for mathematics texts, perhaps especially symbolic parts of texts and texts focusing on algorithmic and procedural understanding. Also, these criteria have not been created based on mathematics, and therefore these should be related to some specific theories about comprehension in mathematics, which in itself is a complex concept (see Sierpinska, 1994).

No clear differences were found between criteria used for macro- and microstructures in the texts, but some differences were found between the two texts. Is this showing an adaptive behaviour among readers, when using different types of criteria for different types of texts, and is this done consciously or at a more unconscious level? This should be examined in more detail. The clearest difference between the texts was that the external criterion was much used for the "conceptual text" (about group theory) but hardly ever for the "procedural text" (about Newton-Raphson). However, it is unclear whether this is due to the conceptual and procedural aspects of the texts.

When comparing criteria used for statements in natural and symbolic language, there exist both similarities (using a sort of syntactical criterion) and differences (accepting statements or not). However, since the syntactical criterion is used quite differently, in a conceptual manner for natural language and in a procedural manner for the symbolic statement, it seems necessary to refine or elucidate the categories of criteria, especially when using them for mathematics texts.

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