

A Reading Comprehension Perspective on Problem Solving

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Abstract: The purpose of this paper is to discuss the bi-directional relationship between reading comprehension and problem solving, i.e. how reading comprehension can affect and become an integral part of problem solving, and how it can be affected by the mathematical text content or by the mathematical situation when the text is read. Based on theories of reading comprehension and a literature review it is found that the relationship under study is complex and that the reading process can affect as well as act as an integral part of the problem solving process but also that not much research has focused on this relationship.

Introduction

All over the world, textbooks seem to play an important role in mathematics education at all levels (Foxman, 1999), and thereby students' reading activity and reading comprehension also play important roles. However, the reading activity can be of different kinds, for example, reading an expository text that tries to explain something to the reader or reading a problem text in order to attempt to solve the given problem. This paper focuses on the second type of reading situation; reading comprehension when trying to solve a given problem. It seems like most research in mathematics education about reading comprehension has been done in a manner that reduces reading to a potential obstacle for learning (Borasi & Siegel, 1990), for example, by focusing on how limitations in reading ability affect learning in mathematics or on readers' misunderstandings of a written task and how this can influence the solving of the task. This paper is an attempt to start from a more nuanced view of reading comprehension, and analyze problem solving from this perspective.

Purpose

The purpose of this paper is not merely to add a detailed view of reading comprehension as an important component of a problem-solving situation, but to discuss the bi-directional relationship between reading comprehension and problem solving. Therefore, the purpose of this paper consists of two parts:

- A. To theorize about how reading comprehension can *affect* problem solving and how reading comprehension can be added as an *integral part of* problem solving.

- B. To analyze how reading comprehension can be affected by the mathematical text content or by the mathematical situation when the text is read. In this paper, the problem-solving situation is the particular type of mathematical situation that will be studied.

Method and structure of the paper

Theories of reading comprehension will be the starting point in this paper (in section 2), that is, these theories will be used as theoretical tools when discussing the relationships between reading comprehension and problem solving. However, the two parts of the purpose will be handled differently, where the theorizing in part A will be done in a more unrestricted and exploratory way (in section 3), while the analysis in part B will be done with the help of a literature survey (in section 4). The main purpose of this survey is not to find and describe all possibly relevant references, but to find some references (of course as many as possible) that are highly relevant for part B of the purpose of this paper, in order to discuss these references from a reading comprehension perspective. A description of the literature survey follows.

The search for literature was made in two different ways. First, databases were used to search for references containing (part of) words such as ‘problem solving’, ‘reading’, ‘semantics’, and ‘linguistic’ in relevant combinations. This search was made in order to find literature that specifically deals with a relationship between problem solving and reading comprehension. Second, a less specific search was performed, for studies dealing with word problems. This second type of search for literature was made in order to find such references that potentially could be relevant for the purpose of this paper, without explicitly dealing with a relationship between reading comprehension and problem solving. There are two reasons for focusing on word problems. Firstly, a more general search for problem solving, and not specifically word problems, was presumed to yield too many irrelevant references. Secondly, a literature survey by Österholm (2004, section 3.1) found that word problems were often discussed in literature that focuses on texts and reading in mathematics.

A general search in the MathDi database¹ (using ‘basic index’) for references dealing with word problems gave 1424 results². This number had to be reduced in order to be able to complete this task in a reasonable time. Therefore, it was chosen to limit the search to the titles of references (to get references that more directly focus on word problems) and to only include those published in journals (to ensure generally high quality). This resulted in 199 references, and all these

¹ <http://www.emis.de/MATH/DI/>

² All mentioned searches in MathDi were performed on 6 September 2005 and were limited to references in English. The search words used were ‘word probl*’, which resulted in references including ‘word problem’ or ‘word problems’ that were published in the years 1976-2005.

were included when the abstracts were analyzed. More details of this analysis and the results from it can be found in section 4 of this paper.

Theories of reading comprehension

Mental representations

When reading a text, a mental representation of the text is created by the reader, which describes how the reader understands the text. Many studies about reading comprehension show, or support the conclusion, that “multiple levels of representation are involved in making meaning” (van Oostendorp & Goldman, 1998, p. viii). In particular, the work of Walter Kintsch (e.g., see Kintsch, 1992, 1998) seems to have had a great influence on research on reading comprehension (Weaver, Mannes, & Fletcher, 1995). Kintsch (1998) distinguishes between three different levels, or components, of the mental representation created when reading a text: the surface component, the textbase, and the situation model.

When the words and phrases themselves are encoded in the mental representation (possibly together with linguistic relations between them), and not the *meaning* of the words and phrases, one can talk about a surface component of the mental representation.

The textbase represents the meaning of the text, that is, the semantic structure of the text, and it “consists of those elements and relations that are directly derived from the text itself [...] without adding anything that is not explicitly specified in the text” (Kintsch, 1998, p. 103). Since the textbase consists of the meaning of the text and the same meaning can be expressed in different ways, a textbase can be created without any memory of the exact words or phrases from the text.

A pure textbase can be “impoverished and often even incoherent” (Kintsch, 1998, p. 103), and to make more sense of the text, the reader uses prior knowledge to create a more complete and coherent mental representation. A construction that integrates the textbase and relevant aspects of the reader’s knowledge is called the situation model. Of course, some prior knowledge is also needed to create a textbase, but this knowledge is of a more general kind that is needed to “decode” texts in general, while the prior knowledge referred to in the creation of a situation model is more specific with respect to the content of the text.

Content literacy

As defined by McKenna and Robinson (1990), content literacy refers to the ability to read, understand and learn from texts from a specific subject area. They also distinguish between three components of content literacy: general literacy skills, content-specific literacy skills, and prior knowledge of content. Both the general and the content-specific literacy skills can be assumed to refer to some more general type of knowledge that is not dependent on the detailed content of a

specific text. This type of knowledge is primarily used to create a textbase in the mental representation. The third component, prior knowledge of content, refers to knowledge that is connected to the content of a specific text, and is thus primarily used to create a situation model in the mental representation.

It is not clear to what extent mathematics in itself creates a need for content-specific literacy skills and how much of reading comprehension in mathematics depends on more general literacy skills and prior knowledge. However, the symbolic language used in mathematics seems to be a potential cause for the need of content-specific literacy skills. Also, in a study by Österholm (2006), comprehension of one mathematical text not using mathematical symbols (i.e., written in a natural language) mainly depended on the use of more general literacy skills.

Cognitive processes

Thinking about one's own reading process it seems clear that a skilled reader usually does not need to actively think very much to create a mental representation when reading. The use of syntactic and semantic rules together with the activation of more specific prior knowledge thus happens quite automatically, on a more unconscious level. In general, different cognitive processes can be more or less conscious. Perception can refer to highly automatic and unconscious processes, for example when you see a dog and directly recognize it as a dog; you are aware of the result of the process (that you see *a dog*) but no active and conscious thought processes were needed for this recognition. Problem solving on the other hand can be said to deal with active thinking, a more resource demanding process, for example when trying to remember the name of a person you meet and recognize. Thus, when reading a text without experiencing any difficulties in understanding what you read, the process has more in common with perception than with problem solving, in that the process of understanding is mainly unconscious. This is a situation representative for Kintsch's (1998) concept of *comprehension*, which "is located somewhere along that continuum between perception and problem solving" (Kintsch, 1992, p. 144).

Problem solving and reading comprehension

Problems that need to be solved can arise in different ways, but here focus is on given problems with a specific question, in particular mathematics problems given in writing. Specific theories about the problem-solving process sometimes include the reading of the problem text as an important part (e.g., see Pólya, 1990), which seems natural since one surely needs to start by reading the given problem text in order to try to *understand the problem*. Thereby, a mental representation of the text is created, that is, a mental representation of *the problem* is created. But in order not to limit the description of the result of this reading process to that the reader either has understood the text or not, and what kind of (negative) effects this might have on the solution process, a more integrated view

is suggested of (1) reading the problem, (2) understanding the problem, and (3) solving the problem.

It seems quite obvious that deficiencies in literacy skills, general or content specific, can affect the attempt to solve a given problem, since direct reading errors (i.e., problems in creating a textbase) increase the risk that the mental representation contradicts the text. However, the mental representation created in the reading process does not only serve as *background* to solving the problem, but the solving process has already started, since also prior knowledge is activated in the reading process, including more specific types of prior knowledge that can be suitable for solving the problem, that is, the comprehension of the problem need not only consist of a pure textbase in the mental representation but also a situation model can be created. It could even be the case that the problem in principle has been solved through the reading process (or at least the problem *is believed* to be solved). In such a case, the problem is solved using mainly unconscious cognitive processes, that is, the problem is solved through pure *comprehension* (Kintsch, 1998) of the problem/situation. Davis (1984, p. 207) gives an empirical example of this type of solution by comprehension, where an existing mental representation of a similar problem was activated, and the person “had done this unconsciously, but had been able to reconstruct some of the process by determined introspection afterwards.” Thus, this is not only a theoretical possibility, and it has also been shown that these types of unconscious comprehension processes can be used to explain behavior in such situations as action planning (Mannes & Kintsch, 1991) and decision making (Kitajima & Polson, 1995). Perhaps some observed student behavior when solving problems also can be explained by assuming that the student is relying mostly on these types of comprehension processes when trying to solve the problem, for example, when Lithner (2000, p. 165) reports that “focusing on what is familiar and remembered at a superficial level is dominant over reasoning based on mathematical properties of the components involved.”

To generate the answer to the posed question in a given problem can be seen as a natural *goal* of the situation, and in order to reach that goal one needs to regulate one's behavior, that is, self-regulating processes are active. The given question can thus play a very important role also in the creation of the mental representation in the reading process since it can influence what kind of prior knowledge is activated, that is, the self-regulation seems to start already in the reading process. It has also been shown that self-regulating processes (which usually are considered as metacognitive processes) can operate at an unconscious level (Fitzsimons & Bargh, 2004). Therefore, it could be of particular interest to examine how variations of *questions* in problem texts can influence the comprehension and solution of a problem.

The literature survey

First, the titles and abstracts from the 199 references about word problems are analyzed, where the references are categorized with respect to type of research (empirical or theoretical, and what is being studied/discussed). Thereafter, in the second section, reading comprehension and problem solving will be discussed, using results from relevant references (from both types of literature searches, see above).

Word problems

Table 1 presents the number of some different types of research studies found about word problems. To only study titles and abstracts does have its limitations, and for some references it has also been difficult to decide exactly what type of research is being discussed in the full article. Some duplicates do also exist in the database, and it cannot be guaranteed that all have been found. The conclusion from these remarks is to not to take the numbers too exactly, but to see the overall distribution. Also, and more importantly, the purpose of the different categories is to find relevant literature (the named categories) for the purpose of this paper, and not to make a complete categorization and description of all references.

Overall, not many studies exist that in a direct manner examines the relation between reading and problem solving among the 199 references about word problems. However, studies that vary the wording of a text and examine the effect on the solution or solution process can also be of interest in order to see how the comprehension of the text is related to the solving.

Table 1. Hierarchy of categories of references among 199 articles about word problems, with the number of references in each category given. Named categories include references that have been studied in more detail in section 4.2. All subcategories are not necessarily disjoint.

Empirical studies	115
Variation of variables	75
<i>Effect on performance (i.e., right or wrong solution)</i>	52
Effect of text formulation [Category EP1]	19
Effect of reading ability [Category EP2]	4
<i>Effect on the solving process</i>	15
Effect of text formulation [Category ES]	4
Case (no structured variation of variables)	40
Discussions or theoretical studies	77
Types/properties of problems (including how solving can be affected) [Category D1]	12
Types of factors affecting solving problems [Category D2]	7

Reading comprehension and problem solving

This discussion will focus on the *results* from different studies, where different types of students and problems have been used, which of course can affect the results in different ways. However, this discussion will settle with the conclusion that there *exist* problems/students for which these types of results emerge, which then will be interpreted from a reading comprehension perspective. Therefore, no full meta-analysis of purposes, methods and results of studies will be performed. Several studies show that the performance in solving problems can be negatively affected by a higher complexity of the language used in the problem text (Category EP1 in table 1, e.g., Abedi & Lord, 2001) as well as by a relatively lower reading ability among students (Category EP2, e.g., Jordan & Hanich, 2000). Although these results can be seen as quite obvious, theoretically they can be interpreted as showing the need for more general literacy skills also when reading and solving problems in mathematics.

Both task context and situation context (Wedegé, 1999) have been studied in the analyzed references. For word problems, the task context has been varied in different ways in empirical studies, for example, by trying to make the text more personal or interesting for the reader (Bates & Wiest, 2004). These types of studies focus on the effect the context may have on the performance among students (Category EP1), and different types of effects have been found, but it is not clear how these results should be interpreted. Does an increase in interest cause an increase in the effort that the student puts into trying to solve the problem, or does it activate more relevant prior knowledge that can be helpful when solving the problem? These types of questions have not been answered in the reviewed literature. Also, as the study by Renninger, Ewen and Lasher (2002) shows, there are many different types of interests that come into play at the same time in a rather complex way, such as interest for reading, for mathematics, and for the context described in the problem text. More generally, these results seem to depend on a rather complex, and seemingly not thoroughly investigated, interplay between properties of the text, the reader, and the situation.

Several studies show that students often seem to ignore realistic considerations when solving mathematical problems (e.g., Yoshida et al., 1997). However, other studies have altered the physical and social situation when solving a problem (i.e., the situation context), which resulted in more realistic answers among the students (Roth, 1996; Wyndhamn & Säljö, 1997). Thereby, how the student experiences the situation will affect the problem-solving process, that is, the comprehension of the situation is a relevant factor when solving (word) problems. Others (e.g., De Corte, Verschaffel, & De Win., 1985) describe a word problem as a quite peculiar type of text that can include ambiguous statements, which in the given situation need to be interpreted in a particular way (e.g., a statement that a person has \$5 could in general be interpreted as either that the

person has *exactly* \$5 or that the person has *at least* \$5). Thus, one needs specific types of prior knowledge about how statements in this situation should be interpreted, that is, one needs a type of content-specific literacy skill in this type of situation. However, as a side effect, as these type of skills evolve they seem to cause students to produce unrealistic answers in certain situations.

Another content-specific skill that seems to evolve among some students is to focus on numbers and keywords in the problem text (Hegarty, Mayer, & Monk, 1995). This surely seems to be a reading strategy specific to mathematics, since Bilsky, Blachman, Chi, Mui and Winter (1986) show that students' reading strategies can be influenced by making them read a text either as a mathematics problem or as a telling of a story. When read as a problem, the text was read with a focus on quantitative aspects and as a story it was read with a focus on more qualitative and temporal aspects.

Studies that in a more direct manner examine both the mental representation (often by letting students retell the text) and the solving of the problem consistently show a strong connection between these two aspects (Category ES, e.g., Cummins, Kintsch, Reusser, & Weimer, 1988), that is, the students solve the problem as they have understood it. Another possibility would be that one creates an elaborate mental representation of the text but bases the solution on something else (e.g., parts of the text itself and not the mental representation of the text). More detailed studies of the relationship between the mental representation and the solution show that better problem solvers mostly remember the semantic structure of the text while worse problem solvers mostly remember details in the text (Hegarty et al., 1995), and that the retelling of a problem text sometimes is made in another order than what was presented in the given text, an order that more closely resembles the calculation that is used when solving the problem (Hershkovitz & Nesher, 2001). This last result appeared both when the retelling was performed before the solving of the task (i.e., directly after reading the text) and when it was performed after the problem had been solved. Thus, the solving of the problem seems to have already begun in the reading process, a possibility discussed earlier in this paper. The existence of a specific question in the text as an important aspect was also discussed earlier. Therefore, in order to more clearly see a possible more direct effect of the *mathematical situation* (and not the existence of a question) it would be interesting to examine the mental representation before a question is given. However, no such studies have been found in this literature survey.

Conclusions

From the discussions in this paper it becomes evident that the relationship between reading comprehension and problem solving is complex. First, the reading process can affect the problem solving process, but can also act as an integral

part of the solving process. However, not much research seems to have been done involving the latter relationship. Second, the literature survey has given examples of how the problem-solving situation seems to affect the reading process. However, not much research seems to have been done that directly focuses on this relationship, but the results discussed in the literature survey seem to be able to explain by assuming that the situation affects the reading process in certain ways, for example, that the reader uses specific strategies (or literacy skills) in this type of situation.

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