Cylinder-by-Cylinder Torque Model of an SI-Engine for Real-Time Applications

Master’s thesis
performed in Vehicular Systems
by
Mohit Hashemzadeh Nayeri

Reg nr: LiTH-ISY-EX- -05/3830- -SE

19th December 2005
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Linköping, 19th December 2005
In recent years Hardware-in-the-Loop HiL, has gained more and more popularity within the vehicle industry. This is a more cost effective research alternative, as opposed to the tests done the traditional way, since in HiL testing the idea is to test the hardware of interest, such as an electronic control unit, in a simulated (or partially simulated) environment which closely resembles the real-world environment.

This thesis is ordered by Daimler Chrysler AG and the objective of this thesis is the developing of a cylinder-by-cylinder model for the torque of an SI—engine in real time. The model will be used for the purpose of emulation of misfire in a four-stroke SI—engine. This purpose does not demand a precise modelling of the cylinder pressure but rather an adequate modelling of position and amplitude of the torque produced by each cylinder. The model should be preferably computationally tractable so it can be run on-line. Therefore, simplifications are made such as assuming the rule of a homogenous mixture, pressure and temperature inside the cylinder at all steps, so the pressure model can be analytical and able to cope with the real-time demand of the HiL. The model is implemented in Simulink and simulated with different sample rates and an improvement is to be seen as the sample rate is decreased.
Abstract

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Keywords: CCEM, HiL, Simulink, Wiebe Function, Cylinder Pressure, Torque Balance
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Chapter 1

Introduction

As a result of electronic progresses in recent decades, the impact of electronic and software programming in vehicles, in order to achieve better performance, is continuously growing. The better performance can be achieved in various fields, such as the drive ability, better safety, reduced emissions thanks to a better fuel economy etc. One of the key issues for achieving some of the goals mentioned above is a better understanding of engine performance in order to control the engine as much as possible. Mostly for control purposes a Mean Value Engine Model, $MVEM$, is used where the torque produced by all cylinders combined are regarded as one. But for some aspects such as misfire detection or backlash caused by play between gears a Cylinder-by-Cylinder Engine Model, $CCEM$, is required. Generally a theoretical model is build and later implemented in a suitable interface. In this thesis an on-line applicable model, meaning a computationally tractable model is implemented in Simulink. In order to achieve this, simplifications has been made capturing the essence of a cylinder pressure. So the central question here is to see how well a simple model can capture this process. The Simulink model itself is later implemented in Hardware in the Loop HiL, which in recent years has become more and more popular within the vehicle industry. Hardware-in-the-loop testing merges software simulation with actual hardware testing, allowing real-world testing to take place without the overhead of in-vehicle testing. To assure that the behavior of the real hardware components reflect the behavior in an actual vehicle, HiL assures that all software components must execute in real-time. It offers a cost effective way to test and optimize a mechanical or electrical device of a system through computer-simulation in some interface. This method has the obvious advantage of saving time and not having to develop costly prototypes. It also gives the benefit of being able to test any single part of a device on its own before putting it all together. The model is later on, if corrected and approved, implemented in an Engine Control Unit, ECU.
1.1 Objectives

The objective of this thesis is to develop a model for the torque of the individual cylinders in a four-stroke, four-cylinder SI-engine (Spark Ignited) that will be run in real time. The purpose of this model is to simulate misfire. It should be emphasized that it is not in the task of this thesis to present a protocol for discovering a misfire but rather to have a CCEM reflecting the torque of each cylinder during the course of every cycle i.e. two crankshaft revolutions.

1.2 Approach

First a model describing piston motion, speed and acceleration based on crankshaft angle is build. Thereafter based on this model a torque equation is presented and developed in dependence of the pressure inside the cylinder. Simplifications are made in modelling of the cylinder pressure since calculating the chemical and the thermodynamical interactions in between cylinder, inlet manifold and the exhaust valve are complex and it would add a huge deal to required amount of calculations. This is important in order to make the model computationally tractable and practical for on-line use. Thereafter the torque model is implemented in Simulink for the purpose of later on being implemented in HiL, which leads to the execution time limits since the model should be able to run in real-time.

1.3 Outline

The background theories are constantly presented in relation to their field of use. In chapter two the modelling of an SI—engine is discussed. The torque sum is divided into subtorques. These subtorques are calculated based on the piston motion equations. Further, the combustion torque requires simulation of the cylinder pressure. In the third chapter the four stroke engine divided into high and low pressure cycle is described using an analytical model based on the behaviour of an ideal Otto-cycle. In the fourth chapter a summary of the equations, resulting in the final torque are given. In the fifth chapter simulations of the model presented in the three previous chapters are showed and discussed. In the last chapter the final conclusions are presented and some suggestions for improvements are made.
Chapter 2

The Operation of SI-Engines

There are two commonly used combustion engines in production:

1. The SI-engines: (Spark Ignited) used for vehicles using gasoline.
2. The CI-engines: (Compression Ignited) used for diesel engines.

What is common for both of these engines is, that they both work in two cycles and thereby within four strokes. In this work the focus is solely on the SI-engines. In the following sections we will have a look at what happens during these strokes in a SI-engine out of a physical point of view, meaning a look at things that are considered important and included in the computer model.

2.1 Four Strokes

A thorough discussion of the ideal cycle can be found in [4] and [1]. The pressure produced by the SI-engine keeps repeating itself within two cycles or four strokes. When the piston is at its top position, Top Dead Center \( TDC \), the crank angle is considered as zero and after two cycles, i.e. \( 720^\circ \), the pressure cycle starts all over. The four strokes can be seen in Figure 2.1 and are described briefly as follows:

1. Intake phase: Intake valve opens and the fresh air plus the fuel mixture gets sucked in as the piston is moving downwards towards Bottom Dead Center \( BDC \). Therefore the cylinder pressure in this phase is at the same level as the pressure inside the inlet manifold \( p_{im} \).

2. Compression phase: Both valves are closed, the piston moves up towards \( TDC \), thereby compressing the gas. Combustion starts (\( SOC \), Start Of Combustion) due to an ignition usually around \( 15^\circ - 35^\circ \) before the \( TDC \).
3. Expansion phase: Both valves are still closed and the piston is moving from $TDC$ towards $BDC$. The combustion finishes ($EOC$, End Of Combustion) usually about $40^\circ$ after the $TDC$.

4. Exhaust phase: Exhaust valve is opened and the burned gases are pushed out of the cylinder as the piston is once again moving up from $BDC$ towards $TDC$.

Strokes number two and three constitute the high pressure cycle and the strokes number four and one constitute the low pressure cycle. The work done during these two crankshaft revolutions are usually described with a $pV$-diagram as illustrated in Figure 2.2.

The effective work is produced during the high pressure cycle, i.e. when both valves are closed and the combustion takes place. The mechanical work can be obtained with an integration of the $pV$-diagram in Figure 2.2 and thereafter normalized through a division by the displacement volume $V_d$, see [1]:

$$w_i = \frac{1}{V_d} \int \sum_{j=1}^{CYL} (p_j V_j - p_{atm}) dV_j$$  \hspace{1cm} (2.1)

where $CYL$ and $w_i$ are the number of cylinders and the normalized indicated specific work respectively. The total displacement volume of all the cylinders combined, $V_d$ is:

$$V_d = CYL \cdot (V_BDC - V_TDC)$$
2.2 The Torque Production

In this section a classification of the torques will be done due to their sources and origins. The net torque produced is very much related to the amount of fuel injected. This fuel is mixed in the inlet manifold and gets sucked in as a homogeneous air-fuel mixture. The torque output should be a sum of the torque produced due to the combustion $T_{comb}$, the torque due to the reciprocating masses $T_{mass}$, the torque due to the friction $T_{fric}$ and the load torque $T_{load}$, which acts on the output side of the clutch as the result of the load the engine is exposed to. An intuitive equation for derivation of the torque balance can be found in [1], [3]:

$$T_{net} = T_{comb} - T_{load} - T_{mass} - T_{fric} \quad (2.2)$$

$$T_{net} = J \cdot \ddot{\alpha} \quad (2.3)$$

where $T_{net}$ is the net torque. Finally $J$ and $\ddot{\alpha}$ are the moment of inertia of the whole crankshaft and the crankshaft angle acceleration. If $\ddot{\alpha} = 0$ the
equation (2.2) is said to be in balance. The equation for torque balance is then given by:

\[ T_{comb} - T_{load} - T_{mass} - T_{fric} = 0 \]  \hspace{1cm} (2.4)

In order to describe the torques mentioned above, details regarded the piston motion will be looked at in the following section. Thereafter based on the derived equations, the torque equations above will be constructed.

### 2.3 Piston Motion

As it has been mentioned in introduction the aim of the thesis is to looke for a model describing the torque through a cycle as opposed to knowing the mean value of the torque of a cycle. Therefor, basing the torque on the crankshaft angles is the idea that is anticipated.

![Figure 2.3: Geometric properties of a cylinder.](image)

Figure 2.3 describes the geometric properties of a piston. The piston stroke \( s \), can be seen as a function depending on \( \alpha \) and \( \beta \). According to Figure 2.3:

\[ s(\alpha, \beta) = l(1 - \cos \beta) + r(1 - \cos \alpha) \]  \hspace{1cm} (2.5)

Further from Figure 2.3 this relationship can be hold:

\[ l \sin \beta = r \sin \alpha \iff \cos \beta = \sqrt{1 - \frac{r^2}{l^2} \sin^2 \alpha} \]  \hspace{1cm} (2.6)
By use of (2.6) and (2.5) the piston stroke can be written as a function of $\alpha$:

$$s(\alpha) = r \left( 1 - \cos \alpha + \frac{l}{r} \left( 1 - \sqrt{1 - \frac{r^2}{l^2} \sin^2 \alpha} \right) \right)$$  \hspace{1cm} (2.7)

We can thereafter derive the piston stroke velocity $\frac{ds}{d\alpha}$, and piston stroke acceleration $\frac{d^2 s}{d\alpha^2}$ as the derivatives of the piston stroke:

$$\frac{ds}{d\alpha}(\alpha) = r \left( \sin \alpha + \frac{r}{l} \cdot \frac{\sin \alpha \cos \alpha}{\sqrt{1 - \frac{r^2}{l^2} \sin^2 \alpha}} \right)$$  \hspace{1cm} (2.8)

and

$$\frac{d^2 s}{d\alpha^2}(\alpha) = r \left( \cos \alpha + \frac{r}{l} \cdot \frac{(1 - 2 \sin^2 \alpha) + \frac{r^2}{l^2} \sin^4 \alpha}{\left(\sqrt{1 - \frac{r^2}{l^2} \sin^2 \alpha}\right)^3} \right)$$  \hspace{1cm} (2.9)

The time derivatives of $\frac{ds}{d\alpha}$ and $\frac{d^2 s}{d\alpha^2}$ are then obtained through chain derivation:

$$\dot{s} = \frac{ds}{dt} = \cdots = \frac{ds}{d\alpha} \cdot \dot{\alpha}$$  \hspace{1cm} (2.10)

$$\ddot{s} = \frac{d}{dt} \frac{ds}{dt} = \cdots = \frac{d^2 s}{d\alpha^2} \cdot \dot{\alpha}^2 + \frac{ds}{d\alpha} \cdot \ddot{\alpha}$$  \hspace{1cm} (2.11)

## 2.4 Torque Equations Based on Crankshaft-Angle

The indicated specific work (2.1) can according to [1] be rewritten as:

$$w_i = \frac{1}{V_d} \int \sum_{j=1}^{CYL} (p_j(\alpha) - p_{atm}) A_p \frac{ds_j(\alpha)}{d\alpha} d\alpha = \frac{1}{V_d} \int T_{comb}(\alpha) d\alpha$$

where $A_p$ is the area of the piston. The combustion torque is then defined as:

$$T_{comb} = \sum_{j=1}^{CYL} (p_j(\alpha) - p_{atm}) A_p \frac{ds_j(\alpha)}{d\alpha}$$  \hspace{1cm} (2.12)

Generally, according to [7], the following relation between kinetic energy of the reciprocating masses $E_{mass}$, and their moment of inertia $J$, holds:

$$E_{mass} = \frac{1}{2} J \dot{\alpha}^2$$
and the torque mass can according to [1] be derived as:

\[ E_{\text{mass}} = \int_0^{2\pi} T_{\text{mass}} d\alpha \] (2.13)

The torque mass is then a derivative of the kinetic energy of the reciprocating masses with regard of \( \alpha \):

\[
T_{\text{mass}} = \frac{dE_{\text{mass}}}{d\alpha} = \frac{1}{2} \left( \frac{dJ}{d\alpha} \ddot{\alpha}^2 + J \frac{d}{dt} (\dot{\alpha}^2) \cdot \frac{1}{d\alpha/dt} \right) \\
= J\ddot{\alpha} + \frac{1}{2} \frac{dJ}{d\alpha} \dot{\alpha}^2
\] (2.14)

where the first term represents the rotational masses and the second term the oscillating ones. This leads to the need of separating the total mass into an oscillating and a rotational portion according to Figure 2.4.

Figure 2.4: a) Two-mass model for oscillating and rotating masses. b) Rotational model at the crankshaft.

- an oscillating portion

\[ m_{\text{rod, osc}} = m_{\text{rod}} \cdot \frac{l_{\text{osc}}}{l} \] (2.15)

- and a rotational portion

\[ m_{\text{rod, rot}} = m_{\text{rod}} \cdot \frac{l_{\text{rot}}}{l} \] (2.16)
2.4. Torque Equations Based on Crankshaft-Angle

The two lengths $l_{osc}$ and $l_{rot}$:

$$l = l_{osc} + l_{rot}$$

are defined based on the center of gravity CoG of the connecting rod. The oscillating and rotating mass of each cylinder is then:

$$m_{osc} = m_{piston} + m_{rod,osc} \quad (2.17)$$

$$m_{rot} = \frac{m_{crank}}{CYL} + m_{rod,rot} \quad (2.18)$$

With help from Figure 2.4 the equations for the rotational motion can be listed:

$$x_{rot,j} = r \sin \alpha$$

$$\dot{x}_{rot,j} = r \dot{\alpha} \cos \alpha$$

$$\ddot{x}_{rot,j} = r (\ddot{\alpha} \cos \alpha - \dot{\alpha}^2 \sin \alpha)$$

$$y_{rot,j} = r (1 - \cos \alpha)$$

$$\dot{y}_{rot,j} = r \dot{\alpha} \sin \alpha$$

$$\ddot{y}_{rot,j} = r (\ddot{\alpha} \sin \alpha + \dot{\alpha}^2 \cos \alpha) \quad (2.19)$$

The equation for kinetic energy of the reciprocating masses $E_{mass}$, can now also be written as:

$$E_{mass} = \frac{m_{rot}}{2} \sum_{j=1}^{CYL} v_{rot,j}^2 + \frac{m_{osc}}{2} \sum_{j=1}^{CYL} v_{osc,j}^2 \quad (2.20)$$

where $v_{rot,j}$ is the rotational speed defined as:

$$v_{rot,j} = [\dot{x}_{rot,j}, \dot{y}_{rot,j}]$$

$$|v_{rot,j}|^2 = \dot{x}_{rot,j}^2 + \dot{y}_{rot,j}^2 \quad (2.21)$$

and the oscillation speed is the time derivative of the respective piston stroke defined as in equation (2.10).

$$v_{osc,j} = \dot{s}_j \quad (2.22)$$

From (2.14) we have:

$$\frac{dE_{mass}}{dt} = \frac{dE_{mass}}{d\alpha} \cdot \frac{d\alpha}{dt} = T_{mass} \cdot \dot{\alpha} \quad (2.23)$$

On the other hand the derivation of (2.20) using (2.11), (2.19), (2.21) and (2.22) gives:
\[
\frac{dE_{\text{mass}}}{dt} = m_{\text{rot}} \sum_{j=1}^{\text{CYL}} (\dot{x}_{\text{rot},j} \cdot \ddot{x}_{\text{rot},j} + \dot{y}_{\text{rot},j} \cdot \ddot{y}_{\text{rot},j}) + m_{\text{osc}} \sum_{j=1}^{\text{CYL}} \dot{s}_j \dot{s}_j
\]

\[
= \text{CYL} \cdot m_{\text{rot}} \cdot r^2 \cdot \dot{\alpha} \cdot \ddot{\alpha} + m_{\text{osc}} \sum_{j=1}^{\text{CYL}} \frac{ds_j}{d\alpha} \cdot \dot{\alpha} \left( \frac{d^2 s_j}{d\alpha^2} \cdot \ddot{\alpha}^2 + \frac{ds_j}{d\alpha} \dddot{\alpha} \right)
\]

(2.24)

The first term after the last equal-sign, is a result of the trig identity and the fact that \(\sin \alpha \cdot \cos \alpha\) terms, have the opposite signs. Further, rewriting (2.24) with (2.23) and (2.14) in mind leads to the following conclusions:

\[
\frac{dE_{\text{mass}}}{dt} = \begin{bmatrix}
J \\
\text{T}_{\text{mass}}
\end{bmatrix}
\]

\[
= \begin{bmatrix}
J \\
\text{T}_{\text{mass}}
\end{bmatrix} = \begin{bmatrix}
\text{CYL} \cdot m_{\text{rot}} \cdot r^2 + m_{\text{osc}} \sum_{j=1}^{\text{CYL}} \left( \frac{ds_j}{d\alpha} \right)^2 \ddot{\alpha} + \frac{1}{2} \left( 2m_{\text{osc}} \sum_{j=1}^{\text{CYL}} \frac{ds_j}{d\alpha} \cdot \frac{d^2 s_j}{d\alpha^2} \right) \ddot{\alpha}^2
\end{bmatrix}
\]

(2.25)

We have now \(J\) and \(\frac{dJ}{d\alpha}\), which in combination, constitute the mass torque. In the next subsection the friction and the load torque will be discussed.

### 2.4.1 Friction- and the Load Torque

As the piston moves up- and downwards the friction torque \(T_{\text{fric}}\), is generated due to the contact between piston and the cylinders inner walls. This torque is then given by the Coulombs law\(^1\) and can be found in [1] and [7]:

\[
T_{\text{fric}} = \sum_{j=1}^{\text{CYL}} c_f \dot{s}_j \frac{ds_j}{d\alpha}
\]

\[
= c_f \sum_{j=1}^{\text{CYL}} \left( \frac{ds_j}{d\alpha} \right)^2 \dot{\alpha}
\]

(2.26)

\(^{1}\text{f} = \mu \cdot N:\) where \(f\) and \(\mu\) are the friction force respective the friction coefficient. The normal force \(N\), is proportional to \(\dot{s}_j\).
The torque balance (2.4) can now for cylinder $j$ be written as:

$$
(p_j(\alpha) - p_{atm}) \cdot A_p \cdot \frac{ds_j(\alpha)}{d\alpha} - \left( m_{rot} \cdot r^2 + m_{osc} \cdot \left( \frac{ds_j(\alpha)}{d\alpha} \right)^2 \right) \ddot{\alpha} - \frac{1}{2} \left( 2m_{osc} \cdot \frac{ds_j(\alpha)}{d\alpha} \cdot \frac{d^2s_j(\alpha)}{d\alpha^2} \right) \cdot \dot{\alpha}^2 - c_f \cdot \left( \frac{ds_j(\alpha)}{d\alpha} \right)^2 \cdot \dot{\alpha} - T_{load,j}(\alpha) = 0
$$

(2.27)

The load torque varies usually around 30 Nm for engine-speeds at low and medium range (meaning an upper limit of 3000 to 4000 rpm). In this thesis the load torque is set to be an inparameter.

### 2.5 Some Implementation Aspects

The model in this thesis, as mentioned earlier, is implemented for a four-cylinder engine. Since the model should be able to be run in real-time, it is important to avoid executing the same calculations at the same time in different parts of the model. Therefore it is necessary to realize that the geometrical cylinders 1 and 4 respective 2 and 3, see Figure 2.5, have the same volume $V$, moment of inertia $J$ and the derivative of the moment of inertia $\frac{dJ}{d\alpha}$, see Figure 2.5. In

![Figure 2.5: Ignition order in a four cylinder SI-engine.](image)

This can also be realized looking at the equations (2.7), (2.8) and (2.9), since they are equal for cylinders 1 and 4 respective cylinders 2 and 3.
Chapter 3

Cylinder Pressure

In the previous section the four strokes are divided into high- and low pressure cycles. The pressure keeps repeating the same pattern during these four strokes, assuming that combustion is taking place. However, even if a misfire occurs during the high pressure cycle, the pressure rises and falls as the piston moves up and down, see Figure 3.1. The cylinder pressure during the low pressure cycle is directly dependent on the pressure in the intake and exhaust manifolds.

The approach used for simulating the cylinder pressure is based on the model in [2] and will be described in following sections. The analytical model presented is relatively fast in order to meet the demands of being able to be run on-line. This model, as opposed to a two- or multizone model, assumes a homogeneous pressure and temperature inside the cylinder which suits the real time purpose, since a single zone model is less computationally demanding compared to a double or multiple-zone model, see [10].

3.1 High Pressure Cycle

The intake phase ends when the inlet valve closes. Then, the fuel-air mixture is compressed and ignited by a spark plug just before the piston reaches TDC. Under regular driving conditions, the mixture is ignited 15° – 35° before TDC. The maximum pressure then occurs around 20° after TDC as shown in Figure 3.1.

The simulation of the high pressure cycle here consists of three basic parts, which will be explained in greater details later:

Compression part: This phase can be described as a polytropic process. The key idea here is based on the observation of the ideal Otto cycle, see Figure 2.2 and its log-log diagram in Figure 3.2. What can be
Figure 3.1: This is the pressure at one cylinder both when there is a missfire (dashed) and a combustion (line). The expansion asymptote (dashed-dot) is also shown here. The engine speed is at $2000 \text{rpm}$ and it is a four cylinder engine with a total cylinder volume of $1.8 \text{l}$.

observed is that the slopes at the compression and expansion part of the diagram in Figure 3.2 provide information about the behaviour through these phases.

**Interpolation part:** Since the pressure ratio between a firing cycle\(^1\) and a motored cycle\(^2\), is quite similar to the burn profile function, this function is used for interpolating between the polytropic processes of compression and expansion.

**Expansion part:** This phase, can also in the similar way as in the compression part be described with a polytropic process. The pressure and temperature references are approximated through phasing.

---

\(^1\)Cycle where no combustion takes place.

\(^2\)Cycle where combustion takes place.
3.1.1 Compression

Once the inlet valve closes, the process inside is considered reversible\(^3\) and isentropic\(^4\). However, there are no truly reversible isentropic processes in practice, so this is off course a simplification. In a true reversible system no mechanical friction is allowed, there is no leakage and the temperature differences between the working fluids and its surroundings should be infinitely small. A reversible, isentropical process is also an adiabatic process, defined as a process in which no heat is supplied to or rejected from the working fluid. An isentropic process is described by the following law: \(P \nu^\kappa = \text{Constant}\), where \(\kappa\) is the polytropic exponent. Therefore it is considered, that the pressure and temperature in the compression phase can be modeled by polytropic processes with a good accuracy giving:

\[
\begin{align*}
    p_c(\alpha)V(\alpha)^{\kappa_c} &= K1 \quad (3.1) \\
    T_c(\alpha)V(\alpha)^{\kappa_c-1} &= K2 \quad (3.2)
\end{align*}
\]

These constants are provided using the pressure and temperature in inlet

---

\(^3\)When a state of working fluid and its surroundings can be restored to the original ones.

\(^4\)The entropy stays the same through the whole process.
3.1. High Pressure Cycle

Valve closing, $p_{ivc}$ respective $T_{ivc}$ and the volume of cylinder when $IVC$:

$$p_{ivc}V_{ivc}^{\kappa_c} = K1 \quad (3.3)$$

$$T_{ivc}V_{ivc}^{\kappa_c-1} = K2 \quad (3.4)$$

Combining now (3.1), (3.2), (3.3) and (3.4), the following equations are obtained:

$$p_c(\alpha) = p_{ivc} \left( \frac{V_{ivc}}{V(\alpha)} \right)^{\kappa_c} \quad (3.5)$$

$$T_c(\alpha) = T_{ivc} \left( \frac{V_{ivc}}{V(\alpha)} \right)^{\kappa_c-1} \quad (3.6)$$

These traces describe the cylinder pressure up to $SOC$. Holding a track over the temperature is also important, since it has a direct impact over the reference pressure in the expansion part simulation.

**The initial pressure** for compression pressure can initially intuitively be set to the pressure in inlet manifold just before the valve gets closed:

$$p_{ivc} = p_{im}(IVC)$$

But since the crank angle at the closing of the intake valve $IVC$, is not exactly known due to production tolerances, some tuning parameters are used to compensate for the pressure drops over valves etc. An additional correction because of engine speed contributes also to an improvement of the accuracy of the compression pressure model:

$$p_{ivc} = p_{im}(IVC) + c_1 + c_2 * n_{eng}$$

But to maintain simplicity the evaluation is concentrated on the first model.

**The initial temperature** is more difficult to estimate. Here the fresh air inside the inlet manifold is heated from $T_{im}$ to $T_a$, where $T_a$ is the temperature of the air, after it has passed by the hot intake valve and the locally high heat transfer coefficients in the cylinder before that inlet valve closes. Fuel is also added in the ports with the fuel temperature $T_f$ and undergoes an evaporation which also influences the temperature. To this mixture also heat is added, giving the following equation for the initial air/fuel mixture temperature:

$$T_{af} = \frac{m_a \cdot c_{p,a} \cdot T_a + m_f \cdot c_{p,f} \cdot T_f - m_f \cdot h_{v,f} + Q}{m_a \cdot c_{p,a} + m_f \cdot c_{p,f}}$$

where $h_{v,f}$ is the vaporization enthalpy for the fuel and $Q$ is the heat added to the fresh mixture. The mixture temperature in the cylinder, after that the
air/fuel mixture is mixed with the residual gases:\(^5\) is:

\[ T_{ivc} = \frac{m_{af} \cdot c_{p,af} \cdot T_{af} + m_r \cdot c_{p,r} \cdot T_r}{m_{af} \cdot c_{p,af} + m_r \cdot c_{p,r}} \]

Since this model is complex and has several unknown variables, which are difficult to determine, the central question is if there could be a more simple model who could capture the process, so obviously there is a need of simplifications. First it is assumed that the same specific heat \( c_p \), yields for the residual gas and the air/fuel mixture:

\[ T_{ivc} = T_{af}(1 - x_r) + x_r T_r \quad (3.7) \]

where \( x_r \), the residual gas fraction, is defined as:

\[ x_r = \frac{m_r}{m_a + m_f + m_r} \quad (3.8) \]

Further the terms representing heat transfer to the fresh fluid and the fuel evaporation are neglected and the temperature of the fresh fluid is set equal to the temperature in the intake manifold:

\[ T_{af} = T_{im} \quad (3.9) \]

Finally the heat transfer from the residual gas is neglected and the residual gas temperature \( T_r \) is set equal to the temperature at the end of the cycle, \( T( EVO) \). This approach is mainly justified because of its simplicity, but there are some effects that cancel out. Such as the heat lost due to the fuel evaporation and the heat lost from the residual gases to the chamber walls cancel out the heat transfer from the intake valve to the fresh mixture. According to [4] page 102, the \( x_r \) is around 7% at high load and 20% at low load. Both the values of \( x_r \) and \( c_p \) used in this thesis will be discussed later in 3.3.

### 3.1.2 Expansion

In analogy with the compression phase, the expansion phase is also modeled as a polytropic process with polytropic exponent \( \kappa_e \):

\[ p_e(\alpha) = p_3 \left( \frac{V_3}{V(\alpha)} \right)^{\kappa_e} \quad (3.10) \]

\[ T_e(\alpha) = T_3 \left( \frac{V_3}{V(\alpha)} \right)^{\kappa_e^{-1}} \quad (3.11) \]

The determination of \( V_3, p_3 \) and \( T_3 \) that refer to state three in the ideal Otto cycle will be discussed later in this section. In this approach the air-to-fuel ratio and the ignition timing both have an impact on the results and they

---

\(^5\)Here, the gas that is left in the cylinder after that the exhaust phase has taken place.
3.1. High Pressure Cycle

are covered by this approach. From state 2 to state 3 in the \( pV \)-diagram, see figure 2.2, the temperature increase is determined by:

\[
\Delta T_{\text{comb}} = \frac{m_f \cdot q_{HV} \cdot \eta_f(\lambda)}{c_v \cdot m_{\text{tot}}} = \frac{(1 - x_r) \cdot q_{HV} \eta_f(\lambda)}{\lambda \cdot (A/F)_s + 1} \cdot c_v
\]  

where the appreciation of fuel conversion \( \eta_f \) comes from [2] with this formula:

\[
\eta_f(\lambda) = 0.95 \cdot \min(1 : 1.2\lambda - 0.2)
\]

As it can be seen the second equality in the equation 3.12 rewrites the impact of the fuel mass through using the residual gas fraction \( x_r \) and the mass of air to mass of fuel ratio by using the following formula:

\[
\frac{m_a}{m_f} = \lambda \left( \frac{m_a}{m_f} \right)_s
\]

Thereby the last equality implies that it is actually the proportionality of the fuel mass \( m_f \) and the total mass \( m_{\text{tot}} = m_a + m_f + m_r \) that determines the temperature rise due to combustion. An Exhaust Gas Recirculation EGR\(^7\), enters the model the same way as the residual gas and it would also influence \( T_{ivc} \) and the dilution \( x_r \). In reality the thermodynamic properties of the fluids, such as \( c_v, \kappa_c \) and \( \kappa_e \) of the burned and unburned gases are dependent of \( \lambda \) but in order to simplify things, those facts are not considered here. The temperature after the combustion is:

\[
T_3 = T_2 + \Delta T_{\text{comb}}
\]

As it can be seen in figure 3.2, the volume of the states two and three are the same. The fact that the pressure and the temperature are the only variables changing (assuming a combustion taking place) and that the ideal gas law \( PV = nRT \) holds, provides the following formula for calculation of \( p_3 \):

\[
p_3 = p_2 \frac{T_3}{T_2}
\]

where \( p_2 \) and \( T_2 \) are determined from equations 3.5 and 3.6, meaning:

\[
p_2(\alpha) = p_{ivc} \left( \frac{V_{ivc}}{V_2} \right)^{\kappa_c}
\]

\[
T_2(\alpha) = T_{ivc} \left( \frac{V_{ivc}}{V_2} \right)^{\kappa_c - 1}
\]

where the phasing of the volumes at the states 2 and 3 are explained in the next subsection after an introduction, describing the burning angles and the burn profile function.

\(^6\)The abbreviation \( s \) implies stoichiometric relation ruling.
\(^7\)An emission control method that involves recirculating exhaust gases from an engine back into the intake and combustion chambers. This lowers combustion temperatures and reduces oxides of nitrogen.
Chapter 3. Cylinder Pressure

Flame Characteristics And Combustion Phasing

Combustion starts with an ignition and ends as either the fuel or the oxygen in the cylinder chamber are finished. This course of event is called flame development and its characteristics and the mass fraction burn rate can be seen in Figure 3.3. The terminology in the figure is usually defined as follows:

Figure 3.3: Definition of burning angles and mass fraction burned versus crank angle curve.

\( x_b \) : Burn profile or mass fraction burn describes how many percent of the fuel has been burned. A functional formula often used to represent the mass fraction burned versus crank angle curve is the Wiebe function:

\[
x_b(\alpha) = 1 - \exp \left[ -a \left( \frac{\alpha - SOC}{\Delta\theta} \right)^{m+1} \right]
\]  

(3.17)

where \( a \) and \( m \) are calculated as:

\[
a = -\ln(1 - 0.1) \left( \frac{\Delta\theta}{\Delta\theta_d} \right)^{m+1}
\]  

(3.18)

\[
m = \frac{\ln \left( \frac{\ln(1-0.1)}{\ln(1-0.85)} \right)}{\ln(\Delta\theta_d) - \ln(\Delta\theta_d + \theta_b)} - 1
\]  

(3.19)

\( \Delta\theta_d \) : Flame Development Angle is the crank angle interval during which flame kernel develops after spark ignition (is usually set to 10% of the mass-fraction burned).
3.1. High Pressure Cycle

\( \Delta \theta_b \): Rapid Burning Angle is the crank angle required to burn most of the mixture. Defined here as the interval in between the end of the flame development and the end of the flame propagation (here set to mass fraction burned around 85%).

\( \Delta \theta \): Combustion Duration is the crank angle interval in between \( SOC \) and \( EOC \). Here \( \Delta \theta \) is approximated as \( \Delta \theta = 2 \cdot \Delta \theta_d + \Delta \theta_b \).

\( EOC \): End Of Combustion is defined here as: \( EOC = SOC + \Delta \theta \).

The burn angles are dependent of variables such as engine speed, EGR, kinematic viscosity, laminar flame speed, the bore, fuel-to-air ratio and density [3] but these variables are set as inputs.

**Method to account for combustion phasing.** The ignition timing and combustion phasing influence the final pressure. Here the combustion phase is adjusted to the mass fraction burned profile. The position for combustion \( \theta_c \), is chosen to be at \( TDC \) when 50\% of the fuel-mass burned \( mf_{50} \), matches its optimal value \( MFB_{50,OPT} \). The variables \( \theta_c, mf_{50} \) and \( MFB_{50,OPT} \) are defined as:

\[
\theta_c = mf_{50} - MFB_{50,OPT} \tag{3.20}
\]

\[
 mf_{50} = \Delta \theta_d + \Delta \theta_b / 2 \tag{3.21}
\]

\[
 MFB_{50,OPT} = 8^\circ ATDC \tag{3.22}
\]

The model above is motivated by the following observations given in [2]:

- The cycle with the best combustion phasing has best efficiency and lowest exhaust temperature.

- The best phased real cycles have their 50\% mass fraction burned position around \( 8^\circ ATDC \).

- The Otto cycle has the best efficiency and lowest exhaust temperature if the combustion is at \( TDC \).

These statements couple the mass fraction burned trace to \( \theta_c \) in the ideal Otto cycle that defines the volumes at states 2 and 3 to \( V_2 = V_3 = V(\theta_c) \).

3.1.3 Combustion

The pressure ratio is defined as the ratio between the pressure from a firing cycle \( p(\alpha) \), and the pressure from a motored cycle (a cycle without combustion) \( p_c(\alpha) \) :

\[
 PR(\alpha) = \frac{p(\alpha) - p_c(\alpha)}{p_c(\alpha)}
\]
Traces produced by the pressure ratio are similar to the mass fraction burned profiles, for example the position for \( \frac{PR(\theta)}{\max(\theta)} = 0.5 \) differs only around 1\(^{-2}\)° from the position for 50% mass fraction burned [6]. This implies that in order to simulate pressure, similar functions as the mass fraction burned can be used. So for the interpolation between \( p_c \) and \( p_e \) the well known Wiebe function is used:

\[
PR(\alpha) = x_b(\alpha)
\]

which gives the following expression for the pressure:

\[
p(\alpha) = (1 - PR(\alpha)) \cdot p_c(\alpha) + PR(\alpha) \cdot p_e(\alpha)
\]

After EOC till EVO the pressure will be equal to \( p_e \), see figure 3.1. The high pressure cycle can now shortly be summerized like this:

\[
p(\alpha) = \begin{cases} 
p_c(\alpha) = p_{ivc} \left( \frac{V_{ivc}}{V(\alpha)} \right)^{\kappa_c} & IVC < \alpha < SOC \\
(1 - PR(\alpha))p_c + PR(\alpha)p_e(\alpha) & SOC < \alpha < EOC \\
p_e(\alpha) = p_3 \left( \frac{V_3}{V(\alpha)} \right)^{\kappa_e} & EOC < \alpha < EVO \end{cases}
\]

### 3.2 Low Pressure Cycle

How the pressure and the temperature changes during the low pressure phase is not important for simulation of combustion and misfire. However out of a combustion and misfire simulation perspective, it is important that the variables \( T_{im}(IVC) \) and \( p_{im}(IVC) \) are captured within a good range of precision. In reality with a discrete step. Therefore measures are taken to smooth the transitions at EVO and IVO. In the following sections a closer description will follow.

#### 3.2.1 The Exhaust Phase

In Figure 2.1 (d, e and f) three different possible course of the exhaust phase are shown, namely:

1. If \( IVO = EVC \), then there will be a direct jump from exhaust phase into inlet phase. In Figure 2.1 this can be seen as a direct jump from d to a.

2. If \( IVO < EVC \), then there will be a pressure exchange between the inlet- and the exhaust manifold. In Figure 2.1 this can be seen as a path through d, e and finally a.

3. If \( IVO > EVC \), then for a few degrees both valves will be closed. This is modeled with a polytropic process since \( pV = Constant \) is applicable. In Figure 2.1 this can be seen as a path through d, f and finally a.
More details on these pathes will now follow.

**Simply The Exhaust Pressure**

After $EVO$, the pressure inside the cylinder will go towards $p_{exh}$. Here the
difference between the pressure at the end of the cycle $p_e(EVO)$ and
$p_{exh}$ will be multiplied with an expression going from one to zero. In [2]
an interpolation between the two phases through a cosine function is
mentioned. Here an interpolation function with the first two terms of the Taylor
series, recalling $\cos(x) = 1 - x^2 + O(x^4)$, is used. As the engine speed in-
creases the transition duration $TD$ increases too. Therefore the $TD$ is set to
$TD = TD_{const} + n_{eng} \cdot TD_{coeff}$, where $TD_{const}$ and $TD_{coeff}$ are tuning
parameters. If $IVO = EVC$ the pressure is then modeled:

$$
p(\alpha) = p_{exh} + \frac{(p_e(EVO) - p_{exh})(1 - (\frac{EVO + TD - \alpha}{TD})^2)}{p(\alpha) = p_{exh}} \quad EVO < \alpha < EVO + TD
$$

$$
p(\alpha) = p_{e}(EVO) \quad EVO + TD < \alpha < IVO
$$

(3.25)

**The Exhaust-Inlet Manifold Pressure**

Here analog to 3.26 the pressure tunes into $p_{exh}$ and stays that way till $IVO$.
If $IVO < EVC$, then:

$$
p(\alpha) = p_{exh} + \frac{(p_e(EVO) - p_{exh})(1 - (\frac{EVO + TD - \alpha}{TD})^2)}{p(\alpha) = p_{exh}} \quad EVO < \alpha < EVO + TD
$$

$$
p(\alpha) = p_{exh} \quad EVO + TD < \alpha < IVO
$$

(3.26)

After $IVO$, since both valves are open, the pressure is set to the medium
pressure of the exhaust and the inlet manifold pressure:

$$
p(\alpha) = \frac{p_{exh}V_{exh} + p_{im}V_{im}}{V_{exh} + V_{im}} \quad IVO < \alpha < EVC
$$

(3.27)

**The Polytropic Pressure**

Here is $EVC < IVO$ and the polytropic pressure occurs as a result of both
valves being closed:

$$
p(\alpha) = p_{exh} + \frac{(p_e(EVO) - p_{exh})(1 - (\frac{EVO + TD - \alpha}{TD})^2)}{p(\alpha) = p_{exh}} \quad EVO < \alpha < EVO + TD
$$

$$
p(\alpha) = p(EVC) \left(\frac{V(EVC)}{V(\alpha)}\right)^{\kappa} \quad EVC < \alpha < IVO
$$

(3.28)
The $\kappa$ here is chosen to a value inbetween $\kappa_c$ and $\kappa_e$. The relevance of the accuracy of these variables are very limited as it has been mentioned earlier, since they do not determine whether if we do have a misfire or not.

### 3.2.2 The Intake Phase

Similar to the exhaust phase here the pressure is tuned into the pressure ruling in the inlet-manifold $p_{im}$, with a similar interpolation function:

$$p(\alpha) = p_{im} + (p_{exh}(\max(IVO, EVC)) - p_{im})(1 - \left(\frac{IVO + TD - \alpha}{TD}\right)^2)$$

$$\max(IVO, EVC) < \alpha < IVO + TD$$

Then all the way till the low pressure cycle is finished and the high pressure cycle can start all over again.

### 3.3 Some Implementation Aspects

There are several of the factors in equation 3.12, who could be quite complicated variables to calculate when they have to be very precise. In this section the calculation of specific heat and residual gases are more specifically discussed.

**Specific Heat**

In order to have correct $c_v$, information about the gas components, and the temperature of the gas is needed. Knowing the gas components would require knowing the fraction of the burned and unburned gases. In this thesis full combustion and stoichiometric relation are assumed. For gas mixtures, once the composition is known, mixture properties are determined either on a mass or molar basis:

$$c_v = \sum x_i c_{v,i}$$
$$c_p = \sum x_i c_{p,i}$$

$$\tilde{c}_v = \sum \tilde{x}_i \tilde{c}_{v,i}$$
$$\tilde{c}_p = \sum \tilde{x}_i \tilde{c}_{p,i}$$

(3.29)

where $x_i$ and $\tilde{x}_i$ are the mass and the mole fraction. Further relations of relevance are:

$$\tilde{c} = cM$$
$$\tilde{R} = RM$$

$$\frac{\tilde{c}}{R} = \frac{c}{R}$$
$$\tilde{c}_p - \tilde{c}_v = \tilde{R}$$

(3.30)

where $M$ is the molecule weight and $\tilde{R} = 8314.3[J/(kmol \cdot K)]$ is the universal gas constant. Here $c$ can be replaced with $c_v$ or $c_p$. Now gasoline is a complicated mixture of hydrocarbons boiling between 50 and 200 degrees celsius, with chemical formulas between $C_6H_{14}$ and $C_{12}H_{26}$, but a good "average" compound is $C_8H_{18}$. These react in an ideal situation to produce...
carbon dioxide and water, but in an actual automobile engine they also produce some amount of undesirable compounds including carbon monoxide, oxides of nitrogen, and sulfur-containing compounds. But here, as already mentioned, an ideal situation with full combustion and stoichiometric relation is assumed:

$$2C_8H_{18} + 25O_2 + 100N_2 \rightarrow 16CO_2 + 18H_2O + 100N_2$$

In [4] Table 4.7 the burned gas mole fraction and the molecular weight of the burned mixture is given as:

$$CO_2 : 0.125 \quad H_2O : 0.14 \quad N_2 : 0.735 \quad M_{bm} = 27.3224 \left[ \frac{kg}{kmol} \right]$$

(3.31)

where the subscript $bm$, marks burned mixture. Further in [4] Table 4.9, for each species $i$ in its standard state at temperature $T(K)$, the specific heat $\tilde{c}_{p,i}$ is approximated by:

$$\frac{\tilde{c}_{p,i}}{R} = a_{i1} + a_{i2}T + a_{i3}T^2 + a_{i4}T^3 + a_{i5}T^4$$

(3.32)

The coefficients $a_{ij}$ for species $CO_2$, $H_2O$ and $N_2$ among some other species, are given for two different temperature ranges: 1) $300 – 1000K$ for unburned mixtures, 2) $1000 – 5000K$ for burned mixtures. Since full combustion is assumed the second alternative is chosen. This can be seen in table 3.33.

<table>
<thead>
<tr>
<th>Species</th>
<th>$CO_2$</th>
<th>$H_2O$</th>
<th>$N_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_{i1}$</td>
<td>0.446(+1)</td>
<td>0.272(+1)</td>
<td>0.290(+1)</td>
</tr>
<tr>
<td>$a_{i2}$</td>
<td>0.310(-2)</td>
<td>0.295(-2)</td>
<td>0.152(-2)</td>
</tr>
<tr>
<td>$a_{i3}$</td>
<td>-0.124(-5)</td>
<td>-0.802(-6)</td>
<td>-0.572(-6)</td>
</tr>
<tr>
<td>$a_{i4}$</td>
<td>0.227(-9)</td>
<td>0.102(-9)</td>
<td>0.998(-10)</td>
</tr>
<tr>
<td>$a_{i5}$</td>
<td>-0.155(-13)</td>
<td>-0.485(-14)</td>
<td>-0.652(-14)</td>
</tr>
</tbody>
</table>

(3.33)

During combustion, temperatures around $2000K$ are quite common and using the formula 3.32 with the constants provided in 3.33, the following $\frac{\tilde{c}_{p,i}}{R}$ are achieved:

$$\frac{\tilde{c}_{p,CO_2}}{R} (2000) = 4.46 + \cdots = 7.27$$

$$\frac{\tilde{c}_{p,H_2O}}{R} (2000) = 2.72 + \cdots = 6.18$$

$$\frac{\tilde{c}_{p,N_2}}{R} (2000) = 2.90 + \cdots = 4.33$$

Now using 3.29 and 3.30 the specific heat for the burned gases at $2000K$ can
with the help of tables 3.31 be calculated as:

\[
\tilde{c}_p = \tilde{R} \cdot \sum x_i \tilde{c}_{p,i} \\
= 8.31 \cdot (0.125 \cdot 7.27 + 0.14 \cdot 6.18 + 0.735 \cdot 4.33) \\
= 41.2 \left[ \frac{kJ}{kmol \cdot K} \right] \quad (3.34)
\]

Now considering 3.30 and table 3.31 \(c_v\) can be calculated:

\[
\tilde{c}_v = \tilde{c}_p - \tilde{R} = 41.2 - 8.31 = 32.1 \left[ \frac{kJ}{kmol \cdot K} \right] \quad (3.35)
\]

\[
c_v = \frac{\tilde{c}_v}{M_{bm}} = \frac{32.9}{27.3} = 1.21 \left[ \frac{kJ}{kg \cdot K} \right] \quad (3.36)
\]

In analogous way the specific heat for some other temperatures are calculated and listed here:

\[
c_v = 1037 \quad T = 1000^\circ \\
c_v = 1265 \quad T = 3000^\circ
\]

Since the difference is not of any greater proportions for the discovery of a misfire the \(c_v\) for \(T = 2000K\) is chosen as a constant input.

**Residual Gases**

Recall from 3.1.2 that the influence of the fuel mass \(m_f\), is acknowledged as a mass quote between the consumed air and fuel, which gets rewritten as the residual gas fraction, \(x_r\). The residual gas mass fraction \(x_r\) (or burned gas fraction if EGR is used) is usually determined by measuring the \(CO_2\) concentration in a sample of gas extracted from the cylinder during the compression stroke. Then

\[
x_r = \frac{(\tilde{x}_{CO_2})_C}{(\tilde{x}_{CO_2})_e}
\]

where the subscripts C and e denote compression and exhaust, and \(\tilde{x}_{CO_2}\) are mole fractions in the wet gas. This is far to complicated to be considered in this thesis and that is why the residual gases are considered as an input. In [4], it is however mentioned that \(x_r\) is about 0.2 at low load and 0.07 at high load and that is why the \(x_r\), in my simulations, are set to 0.1.
Chapter 4

Summary Of The Equations

In this chapter the summary of the equations calculating the torque and the pressure will be presented. The net torque is then given by:

\[
T_{\text{net}} = (p_j(\alpha) - p_{\text{atm}}) \cdot A_p \cdot \frac{ds_j(\alpha)}{d\alpha} \\
- \left( m_{\text{rot}} \cdot r^2 + m_{\text{osc}} \cdot \left( \frac{ds_j(\alpha)}{d\alpha} \right)^2 \right) \ddot{\alpha} \\
- \frac{1}{2} \left( 2m_{\text{osc}} \cdot \frac{ds_j(\alpha)}{d\alpha} \cdot \frac{d^2s_j(\alpha)}{d\alpha^2} \right) \cdot \dot{\alpha}^2 \\
- c_f \cdot \left( \frac{ds_j(\alpha)}{d\alpha} \right)^2 \cdot \dot{\alpha} - T_{\text{load,j}}
\]

where the pressure \( p_j(\alpha) \) during every cycle is given by the following sum up of the high pressure:

\[
p(\alpha) = \begin{cases} 
  p_c(\alpha) = p_{\text{ivc}} \left( \frac{V_{\text{ivc}}}{V(\alpha)} \right)^{\kappa_c} & \text{IVC} < \alpha < \text{SOC} \\
  (1 - PR(\alpha))p_c + PR(\alpha)p_e(\alpha) & \text{SOC} < \alpha < \text{EOC} \\
  p_e(\alpha) = p_3 \left( \frac{V_3}{V(\alpha)} \right)^{\kappa_e} & \text{EOC} < \alpha < \text{EVO} 
\end{cases}
\]

and the low pressure cycle consisting of the exhaust and the intake phase:

If \( IVO = EVC \), then the exhaust pressure is modeled with:

\[
p(\alpha) = p_{\text{exh}} + \\
( p_e(\text{EVO}) - p_{\text{exh}})(1 - \left( \frac{EVO + TD - \alpha}{TD} \right)^2) \quad EVO < \alpha < EVO + TD \\
p(\alpha) = p_{\text{exh}} \quad EVO + TD < \alpha < IVO
\]
If $IVO < EVC$ then the exhaust pressure is modeled with:

$$p(\alpha) = \begin{cases} p_{exh} + (p_c(EVO) - p_{exh})(1 - \left(\frac{EVO + TD - \alpha}{TD}\right)^2) & \text{EVO} < \alpha < EVO + TD \\ p_{exh} & \text{EVO} + TD < \alpha < IVO \end{cases}$$

If $IVO > EVC$ then the exhaust pressure is modeled with:

$$p(\alpha) = \begin{cases} p_{exh} + (p_c(EVO) - p_{exh})(1 - \left(\frac{EVO + TD - \alpha}{TD}\right)^2) & \text{EVO} < \alpha < EVO + TD \\ p_{exh} & \text{EVO} + TD < \alpha < EVC \\ p(EVC) \left(\frac{V(EVC)}{V(\alpha)}\right)^\kappa & \text{EVC} < \alpha < IVO \end{cases}$$

The intake phase is modeled with:

$$p(\alpha) = p_{im} + (p_{exh}(\max(IVO, EVC)) - p_{im})(1 - \left(\frac{EVO + TD - \alpha}{TD}\right)^2)$$

\[ \max(IVO, EVC) < \alpha < IVO + TD \]
Chapter 5

Simulations and Discussions

Before taking a look into the results and the summation of the two previous chapters, it is important to have an insight in the speed of calculation required, in order to keep up with the running engine. The most important course of action, in respect to whether if there is a missfire or not, is the course of an eventual combustion. The duration of combustion is not long and the burn angles can be converted into times (seconds) with for example the following formula:

\[ t_{90\%} = \frac{\Delta \theta_{90\%}}{n_{\text{eng}}} \text{[s]} \]

The table below gives an idea of some of the engine speeds and hence the calculation speeds required:

<table>
<thead>
<tr>
<th>Engine Speed</th>
<th>formula</th>
<th>Deg/Calculation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard car at idle</td>
<td>500</td>
<td>16.7</td>
</tr>
<tr>
<td>Standard car at max power</td>
<td>4000</td>
<td>2.1</td>
</tr>
<tr>
<td>Formula car at max power</td>
<td>19000</td>
<td>0.4</td>
</tr>
</tbody>
</table>

The model was due to its implementing purpose in HiL, in beginning, demanded to give an accurate enough picture of the course of event, originally through a calculation every millisecond, \([ms]\). But this demand later on got loosen up and changed to 0.6 \([ms]\), meaning that now the model should only give a good reflection of the reality through an interpolation between calculations every 0.6 \([ms]\) in stead of every \([ms]\). Now for example, the common engine speed of 2000 \([rpm]\) with the sampling time of 0.6 \([ms]\) per calculation entails:

\[ 2000[rpm] = 2000 \cdot 360 \cdot \frac{1}{60}[\text{degrees/second}] = 12000[\text{deg/s}] \]

\[ 12000 \cdot 0.0006[\text{degrees/calculation}] = 7.2[\text{deg/cal}] \]

\[ \frac{1}{2000[rpm]} = \frac{60}{2000 \cdot \frac{1}{2}}[\text{seconds/cycle}] = 0.06[\text{s/cyc}] \]
where \( [\text{deg}] \), \( [\text{cal}] \) and \( [\text{cyc}] \) represent degrees, calculations and cycles. With similar calculations for the sampling time \( 0.6 \ [ms] \) the following table can be obtained:

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1000</td>
<td>6000</td>
<td>3.6</td>
<td>0.12</td>
</tr>
<tr>
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<td>14.4</td>
<td>0.03</td>
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This table is good to have in mind as a rule of thumb in the coming simulations. When there is a stepsize of \( 0.06 \ [ms] \) the figures in the calculation duration column can be devided by ten and in the same way when the stepsize is \( 0.006 \ [ms] \) the numbers in that column can be devide by 100.

### 5.1 Simulations

Unfortunately not all the needed data were provided. That is why for the following simulations, some common and probable cylinder datas are used:

- The total displacement volume, \( V_d : 1.8 \ [l] \). (given)
- Cylinder inner diameter, \( d_{cyl} : 0.082 \ [m] \). (given)
- Crank shaft radie, \( r : 0.0425 \ [m] \). (given)
- Clearance volume per cylinder, \( V_c : 0.05281 \ [l] \). (given)
- Compression ratio, \( \epsilon : 9.5 \). (given)
- Connecting rod length, \( l : 0.12 \ [m] \). (guessed)
- Piston mass, \( m_{piston} : 0.5 \ [kg] \). (guessed)
- Rod mass, \( m_{rod} : 0.35 \ [kg] \). (guessed)
- Crank mass, \( m_{crank} : 14 \ [kg] \). (guessed)
- Friction constant, \( c_f : 0.05^1 \). (guessed)

The coming simulation results will present the achievements made in the two previous chapters.

---

^1In Table T-1.4 in [9], the friction between lubricated steel is given in a range of 0.01 and 0.1. The constant 0.05 is chosen while no other datas were provided.
5.1.1 Cylinder Pressure During Combustion

Simulations with the common engine speed of 2000 [\text{rpm}] and the cylinder specifications given in 5.1 are made here in order to demonstrate the pressure model. The following simulations are done with a fixed step size. The reasons given for the fix step size can be summarized like this:

- In the industry no chances are taken for missing detection of a possible calculation overflow i.e. that there would be more calculations needed during an intervall than what is permitted.

- One might think this could be fixed by a choice of minimum stepsize. The answer: Don’t fix what’s not broken.

Naturally with a stepsize 10 times faster than the company recommended 0.6 [\text{ms}] i.e. 0.06 [\text{ms}], the interpolation function in \textit{MATLAB}, i.e. the linear function connecting every two discrete values, provides a more consistent picture and a better continuity is maintained as if an infinitely small step size would have been possible. To prove this, simulations with stepsizes 0.6, 0.06 and 0.006 [\text{ms}] have been simulated. Better continuity as a result of higher resolution is understandable. But as it can be seen in Figures 5.1 and 5.2 some minor dents can be noticed in the pressure curves with the longest stepsize. These dents occur also for curves simulated with smaller stepsizes, but cannot be noticed with the naked eye. The dents occur while leaving one state and entering the next state. The reason is that the same value is set to output as one state is exited and the next one is entered. There can be more work done to avoid this either by not implementing in Stateflow at all or evade this phenomenon by a different implementation in Stateflow, given that this is not a software bug.

The set of the flame characteristic angles $\Delta \theta_d$ and $\Delta \theta_b$, (recall from 3.1.2), are given in this thesis as inputs. These angles in combination with \textit{SOC} are crucial to the positioning and the shape of the pressure curve and therefore on the outcome of the engine torque. Further as it can be seen both Figures 5.1 and 5.2 confirm a good performance from the model, since it does not miss any information of importance out of a misfire-simulation point of view.

5.1.2 Torque Simulations

The engine torque is the sum of the torque of all the individual cylinders:

$$T_{\text{engine}} = \sum_{j=1}^{\text{CYL}} T_{\text{net},j}$$

These individual cylinder torques $T_{\text{net},j}$ are then the sum of the torques mentioned in (2.2). In the coming simulations the behaviour of the cylinder torques are mostly tested in respect to different engine speeds with the
Chapter 5. Simulations and Discussions

Figure 5.1: Pressure curves simulated with different burning angles and steprates.

Figure 5.2: Pressure curves simulated with different burning angles and steprates.

to:

sampling rates of 0.6 \([ms]\) (straight lines), 0.06 \([ms]\) (dashed-dot lines) and 0.006 \([ms]\) (dashed lines) respectively. Similar to the previous simulations, a certain distinction will be noticed, especially in the torque peaks, due to the chosen stepsize. The explanation lies in what has already been mentioned in 5.1.1. In Figure 5.3 a single cylinder torque, acting over one cycle 720°, is shown with the engine speeds of one and two thousands \([rpm]\). The company recommended stepsize does not differ anything particular from the faster steprates. But since the model is static, there should not be any difference in the same point at all. The known reason for the distinction within different steprates is as a result of the reasons given regarding the Stateflow in 5.1.1. The influence of the oscillating masses grow as the engine speed is increased. This is especially noticed when the crankshaft angle is between 300° to 500°. In Figure 5.4 the influence of the oscillating masses are big. At the crankshaft
angle around zero degrees the torque curve with a slow steprate differs from the torque curves with faster steprates, which is due to the fast oscillations during the combustion. Thereafter in all the figures, all the four cylinders acting simultaneously are shown in a) and their sum i.e. the engine torque, is shown in b). The impact of $T_{\text{mass}}$ grows rapidly as a result of higher engine speeds, see equation (2.24). Since the engine speed $n_{\text{eng}}$ is fix in the simulations done here, this gives $\ddot{\alpha} = 0$. This means that the rotational part of the $T_{\text{mass}}$ is also equal to zero, see equation (2.25). Besides what has been explained for Figure 5.3, there is not much difference between the Figure 5.5 and the Figure 5.6. In Figure 5.7 the influence of $T_{\text{mass}}$ increases which is a result of an increasing $n_{\text{eng}}$. This for Figure 5.8 b) makes it difficult to separate misfire from combustion by looking at the engine torque.

![Figure 5.3](image1.png)  
**Figure 5.3:** Torque output from one cylinder with different engine speeds.

![Figure 5.4](image2.png)  
**Figure 5.4:** Torque output from one cylinder with different engine speeds.
Figure 5.5: a) Four cylinders acting simultaneously b) The engine torque. The first low-top indicates a misfire.

Figure 5.6: a) Four cylinders acting simultaneously b) The engine torque. The first low-top indicates a misfire.
Figure 5.7: a) Four cylinders acting simultaneously during one cycle b) The engine torque is mapped for two cycles. The low-tops indicate misfire.

Figure 5.8: a) Four cylinders acting simultaneously b) The engine torque. The first low-top indicates a misfire.
Chapter 6

Conclusions

A cylinder-by-cylinder torque model of a four stroke SI—engine has been presented. The model has not been validated due to absence of validation data, but it behaves as expected by common consent of the experts. The model seems to be satisfying out of a misfire simulation point of view. The torque-model for the purpose of misfire-simulation does not demand a severely precise modelling of the cylinder pressure. That is why some simplifications are made in modelling of the cylinder pressure since calculating the chemical and the thermodynamical interactions in between cylinder, inlet manifold and the exhaust valve are quite complicated. In this thesis there is this assumption, as opposed to a two- or multizone model, that the pressure and the temperature inside the cylinder is homogeneous. This suits the real time purpose, since a single zone model is less computationally demanding compared to a double or multiple-zone model. The pressure model used is static, including no differential equations. However, a certain difference can be seen within curves with different steprates. Further work with an implementation, perhaps excluding Stateflow, might prevent this distinction. This model is analytical which is beneficial in order to make it computationally tractable and practical for an on-line use. For high engine speeds, it shows that the model is suffering by its growing oscillation part of $T_{\text{mass}}$ as expected. Further, the accuracy of the torque model is naturally dependent on the accuracy of the given inputs. For this thesis, simulations have partly been based on probable datas and inputs. These, plus tuning parameters can be changed for adjustments to different verification datas. Also in this thesis $T_{\text{load}}$ has been given a fix value. This torque is not fix in reality but the approximation done for the purpose of this thesis is acceptable.
References


[5] Lars Eriksson *Spark Advance Modeling and Control*


[8] L.Nielsen, L.Eriksson *Course material, Vehicular Systems*


### Notation

#### Abbreviations, Constants and Variables

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Units</th>
<th>Description</th>
<th>Set Values</th>
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<td>$A_p$</td>
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<td>Piston area</td>
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<td>Bottom Dead Center</td>
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<td>$c_{p,i}$</td>
<td>$[J/kg \cdot K]$</td>
<td>Specific heat at constant pressure for substance $i$</td>
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Appendix A

Simulink Model

The essential features of the simulink model:

Figure A.1: a) An overview of the whole model. b) An overview of the pressures.

Figure A.2: Look inside the pressure block of the first cylinder.
Figure A.3: Look inside the Puppet Master of cylinder one.
Figure A.4: a) Look inside the left chart of the previous figure. b) Look inside the right chart of the previous figure.

Figure A.5: Look inside the High-Pressure of cylinder one.
Figure A.6: Look inside the Low-Pressure of cylinder one.

Figure A.7: The four cylinders and the summation of all the torques.
Figure A.8: Look inside cylinder one.
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