Simultaneous Measurement of Transmitter and Receiver Amplitude and Phase Ripple
Simultaneous Measurement of Transmitter and Receiver Amplitude and Phase Ripple

David Wisell\textsuperscript{1,2,3} and Peter Händel\textsuperscript{2,3}

\textsuperscript{1}Ericsson AB, SE-16480 Stockholm, Sweden, email: david.wisell@ericsson.com, Phone: +46 730 477942
\textsuperscript{2}Royal Institute of Technology, Signal Processing Lab, SE-100 44 Stockholm, Sweden, email: peter.handel@s3.kth.se, Phone: +46 8 790 7595, Fax: +46 8 790 7260
\textsuperscript{3}University of Gävle, Dept. of Electronics, SE-801 76 Gävle, Sweden

Abstract

This paper describes a method to simultaneously measure the amplitude and phase ripple of two band pass linear systems in cascade by changing the center frequency for one of the systems. Extensive measurements are presented to support the theory. The method has a wide applicability for measurements on RF power amplifiers, transmitters, receivers, etc. The accuracy of the method is in the order of a few hundredths of a dB and one degree for most realistic measurement set-ups.

\textit{Key words}: Amplitude measurement, calibration, measurement system, phase measurement.

1. Introduction

Measurement techniques based on digitalization of the measured data are becoming more and more common and are making inroads in domains that have traditionally been considered analog. For example, characterization and testing of radio frequency (RF) power amplifiers (PAs), which was usually done in the analog domain, is now shifting towards digital. This shift is part of a general trend of a shift from analog to digital within the field of telecommunications, where digital modulation formats have replaced analog modulation. Large parts of both transmitters and receivers, which were once implemented in the analog domain, are now implemented using digital techniques. An illustrative example of this is the PA, which in a modern design, gets its performance only in combination with a suitable digital predistortion (DPD) algorithm. The use of advanced DPD algorithms has resulted in an interest in modeling of PAs, and therefore in measurement systems capable of extracting data that can be used for such modeling. A system for sampled input-output measurements for PAs has been described in [1], [2] and [3], with similar systems being described in [4], [5] and [6].

The basic principle of these techniques is shown in Fig. 1. The measurement system consists of a vector signal generator (VSG) and a vector signal analyzer (VSA). The output signal of the amplifier is measured and compared to a reference (input)
signal, which is chosen somewhat differently in the different measurement set-up designs of the studies mentioned above. In [1] and [4] only the output of the PA was measured and the internal digital signal in the VSG was used as reference. In [6], the input and output signals were both measured using a VSA with two channels, while in [2], they were measured sequentially using a single VSA and a repetitive test signal.

Other similar set-up designs have a receiver or a transmitter whose characteristics shall be measured as the device under test (DUT). When the DUT is a receiver, the measurement set-up will be a VSG, and when the DUT is a transmitter, the measurement set-up will be a VSA. In both cases the measured signal is compared to a theoretical reference signal, i.e. the internal digital signal in either the VSG (receiver testing), or the transmitter (transmitter testing).

![Fig. 1. A measurement set-up consisting of a vector signal generator (VSG) and a vector signal analyzer (VSA). Here the device under test is a power amplifier. The VSG and VSA are frequency locked using a reference clock signal.](image-url)

One of the most common problems in sampled input-output measurement systems is that the amplitude and phase responses of neither the VSG nor the VSA are ideal. This limits the usable bandwidth of the devices considerably and introduces distortion in the measurements. Often an amplitude deviation of several tenths of a dB, and sometimes more than one, and a phase deviation of several degrees can be seen over bandwidths as narrow as 10 MHz. This problem is even more pronounced if a measurement system is built of individual blocks, e.g. a combination of a board with analog-to-digital converters (ADCs), a board with digital-to-analog converters (DACs), an RF front-end, etc. Errors of this magnitude can often not be tolerated.

While the list of applications is certainly not limited to the following examples, some examples encountered by the authors are given below.

- Generation of wideband signals for testing of receivers and amplifiers. In certain specific applications for third generation (3G) cellular systems, considerably more than 60 MHz of bandwidth is needed typically.
- Accurate modeling of PAs requires accurate instrumentation and measurements. It is important to assure that the amplitude and phase ripple of the measurement instruments do not become a part of the amplifier models.
Design, verification and production testing of wideband transmitters and receivers benefit considerably from the ability to obtain accurate complex envelope data.

When using the signal generator together with DPD on a power amplifier, if the predistorted data is transmitted through a non-ideal signal generator, the performance of the predistorter will be degraded in addition to wrong conclusions being drawn about the characteristics of the amplifier.

Thus, the problems caused by amplitude and phase ripple in measurement set-ups addressed in this paper constitute a problem in numerous different measurement set-ups and applications. The solution to overcome these problems is to perform some type of calibration of the measurement system. The amplitude response can be calibrated by using a continuous wave (CW), i.e. a sine-wave, which is stepped in frequency over the frequency band of interest. The major problem is to obtain an accurate phase calibration. To calibrate or measure the amplitude response using a CW signal is also tedious and time consuming, and thus faster methods based on multiple tone signals are of interest, especially for production testing applications where measurement time is always an important factor.

Calibration methods based on using digital sampling oscilloscopes (DSOs) and external comb filters as references have been proposed for somewhat similar applications [7]. An obvious drawback with these techniques is the extra equipment and instrumentation that are needed. The method presented here has the advantage of being able to carry out the calibration in-line, i.e. no cables have to be disconnected etc, which is often the case when external equipment is used. This can also be of interest in a production environment. Calibrating both the signal generator and the signal analyzer for amplitude and phase ripple by using the other is not straightforward. The problem is illustrated in Fig. 2, where \( H_1(\omega) \) and \( H_2(\omega) \) are the unknown linear transfer functions of the signal generator or transmitter \( H_1(\omega) \), and the signal analyzer or receiver \( H_2(\omega) \), which have to be determined. In Fig. 2, \( H_1(\omega) \) and \( H_2(\omega) \) are time discrete complex envelope filters and \( u(n) \) and \( y(n) \) are input and output signals respectively, which are sampled on a complex envelope form. For an RF signal \( s(t) = r(t)\cos(2\pi F_c t + \phi(t)) \), where \( r(t) \), \( \phi(t) \) are the envelope and phase of the signal respectively, and \( F_c \) is the carrier frequency in Hz, the complex envelope \( z(t) \) is given by [8]

\[
z(t) = (s(t) + j\tilde{s}(t))e^{-j2\pi F_c t} = r(t)e^{j\phi(t)}, \quad (1)
\]

where \( \tilde{s}(t) \) is the Hilbert transform of \( s(t) \).

A detailed investigation of the origins of linear distortion in the VSG and VSA are outside the scope of this paper. Briefly, the main contributor in the VSA is normally the resolution bandwidth filter (RBW), while the VSG can have several contributors.
like the DACs, the IQ-modulator, reconstruction filters, etc. depending on the hardware implementation and the settings of the instrument.

An attempt to solve this problem would be to insert a nonlinear component between the VSG and the VSA and then consider the system as a Wiener-Hammerstein system, see Fig. 3, where the inserted nonlinear memoryless element is denoted by $N(\cdot)$ [9]. Methods to identify the different parts of the Wiener-Hammerstein system are treated in [9], [10] and [11]. These methods are built on the use of carefully designed multiple tone signals and can be used to find both $H_1(\omega)$ and $H_2(\omega)$ simultaneously.

In this paper an alternative method is derived for the problem at hand and supported by measurements using a set-up similar to that shown in Fig. 1. The presented method also makes use of multiple tone signals, but in a quite different way. Namely by introducing different frequency shifts between the two systems, equations to find the ripple of both are obtained. The method was first outlined in [12], but is in this paper derived and supported by extensive measurements.

The paper is organized as follows. In Section 2 some basic theory that is needed for the derivation of the method proposed in this paper is introduced. The proposed method is then derived in Section 3. The experimental set-up is treated in Section 4. Measurement results that support the method are presented and discussed in Section 5. Finally, some conclusions are drawn in Section 6.

Fig. 2. Illustration of the problem at hand. Two unknown linear bandpass systems in series modeling the VSG and the VSA respectively.

Fig. 3. The Wiener-Hammerstein system.

2. Prerequisites

This Section introduces some definitions and tools that will be used in Section 3 to derive the main algorithm proposed in this paper. In Section 2.1 the term ripple as used here is first defined. The design of the input signal used throughout this paper is discussed in Section 2.2. In Sections 2.3 through 2.5, a method for finding the amplitude and phase ripple of a linear single-
block system $H(\omega)$ using multitones is discussed for reference. This method is then used in Section 3 to derive a method to find the amplitude and phase ripple of two linear systems in series. Cascaded linear systems are finally introduced in Section 2.6.

2.1. Definition of Ripple

The terms amplitude and phase ripple used in this paper are defined as a deviation from constant amplitude and linear phase respectively. The amplitude ripple $R_A(\omega)$ of a filter $H(\omega)$ over a frequency interval $[\omega_{\text{Start}}, \omega_{\text{Stop}}]$ is given by

$$R_A(\omega) = \frac{|H(\omega)|}{G} , \quad \omega_{\text{Start}} \leq \omega \leq \omega_{\text{Stop}},$$

where $\max(\cdot)$ and $\min(\cdot)$ are taken over the interval $[\omega_{\text{Start}}, \omega_{\text{Stop}}]$. The phase ripple $R_p(\omega)$ in the same interval is defined as the deviation of the phase of $H(\omega)$ from the ideal linear phase, that is

$$R_p(\omega) = \angle H(\omega) - a_0 - a_1 \omega ,$$

where $a_0$ and $a_1$ are constants that are to be determined. There are several approaches for determining $a_0$, $a_1$, such as by a least-squares fit of the unwrapped phase of $H(\omega)$. In this work, a two point fit of the parameters is used as will be outlined in Section 2.4.

The amplitude and phase ripple as defined in (2) and (4) are illustrated in Fig. 4 and Fig. 5 respectively. In Fig. 4 the gain towards which the ripple is measured is defined as $G$ according to (3). In Fig. 5 the two frequency points that are used to find $a_0$, $a_1$ are labeled $\omega^0, \omega^1$. The phase ripple at the frequency point $\omega^2$ is also illustrated.

The amplitude ripple is most often converted to power ripple and expressed in dB while the phase ripple is expressed in degrees. Often, the maximum absolute values of the amplitude and phase ripple are used to state the amplitude and phase ripple of $H(\omega)$ respectively.
2.2. The Input Signal

The input signal that will be used throughout this paper is an $M$ tone complex-valued signal, where $M$ is an even integer. The input signal has equal amplitudes and known phases, and is described on complex envelope form by

$$u(n) = \frac{1}{M} \sum_{k=0}^{M-1} e^{j(\omega_k n + \phi_k)}, \quad n = 0,1,\ldots \tag{5}$$

where the $\{\phi_k\}$ are the initial phases of the tones, and the normalized sub-carriers $\{\omega_k\}$ are referenced to the RF carrier which has frequency zero in the complex envelope notation. The tones are symmetrically distributed around the RF carrier and are equally spaced with normalized tone-spacing $2\omega_d$, i.e. $\omega_k = (2k - (M - 1))\omega_d$, where $\omega_k$ and $\omega_d$ are normalized angular frequencies. The constant $\omega_d$ is given by $\omega_d = 2\pi F_d / F_s$, where $F_s$ is the clock frequency of the DACs in the VSG and $F_d$ is half the tone separation in Hz. Thus,
The amplitudes do not necessarily need to be equal, but in the derivation presented here equal amplitudes are used for notational convenience. The phases on the other hand should be chosen with some care. Since all practical systems have a limited dynamic range, a signal with a large peak-to-average power ratio (PAPR) gives rise to nonlinear distortion which will disturb the measurement. The worst case is obviously if all tones at some point add in-phase. Then, \( \text{PAPR} = 10 \log_{10}(M) \) dB. In many practical cases the PAPR can be made sufficiently low by using random phases. If a low PAPR is needed, Schroeder multisines that use parabolic phases are simple to generate and conventional to use \([13],[14]\). An even lower PAPR is obtained using the method in \([15]\) which takes the Schroeder multisine as a starting point and manipulates it further using a nonlinear Chebyshev approximation method. Methods to find multisines that minimize the distortion in nonlinear systems rather than the PAPR are treated in \([16]\). This is often more important than the PAPR since it is the nonlinear distortion, not the PAPR as such, that distorts the measurement results. Although the method in \([16]\) gives a lower PAPR than the Schroeder multisines, it is not as low as the method presented in \([15]\). However, if the number of tones is small in relation to the dynamic range of the system, the choice of the phases is not significant, especially if the worst cases when all tones add in phase can be avoided. For example, with \( M = 100 \), the PAPR of \( u(n) \) in (6) is typically about 7.5 dB using random phases, which may not be significant in a measurement system such as the one in Fig. 1 where the dynamic range often is in excess of 50 dB. This is also the case for the measurements presented in this paper.

### 2.3. Finding the Response of a Single Linear Bandpass System Using Least-Squares Modeling

Here, a method to find the amplitude and phase ripple of a linear system is treated. The method will be used in the forthcoming section as a tool to find the amplitude and phase ripple of two linear systems in series.

First, let’s define a time discrete filter on complex envelope form with frequency function \( H(\omega) \). Let the input signal to this filter be given by (6). The output signal of the filter is then passed through an unknown delay \( \tau \), which corresponds to delays in the measurement set-up, and then sampled using a clock that is frequency locked but not phase locked to the clock that generates the input signal. This gives a model of the measurement set-up similar to that in Fig. 1, which can be used for the calculations. The sampled and delayed output signal is then given by

\[
u(n) = \frac{1}{M} \sum_{k=0}^{M-1} e^{j(2k-(M-1))\omega_0 n + \phi_k}.
\]
where $\alpha$ is the phase difference between the two clocks, and the delay $\tau$ introduces a phase shift of $\omega_k \tau$. Modeling errors and thermal noise are gathered in the stochastic term $w(n)$. The delay $\tau$ is unknown as previously mentioned, but it is not necessary to determine it entirely in order to find the amplitude and phase distortion in $H(\omega)$. Let $H(\omega) = |H(\omega)|e^{j\angle H(\omega)}$, and introduce $\theta_k$ as

$$\theta_k = |H(\omega_k)|e^{j(\phi_k + \omega_k \tau + \alpha + \angle H(\omega_k))},$$

then (7) can be rewritten as

$$y(n) = \frac{1}{M} \sum_{k=0}^{M-1} \theta_k e^{j\omega_k n} + w(n).$$

With $N$ samples of the output, a linear system of equations for $n = 0, 1, ..., N-1$ can be formed as

$$y = H\theta + w,$$

where $y$ is a column vector with $N$ samples containing the measured and sampled output signal $y(n)$ on complex envelope format, that is

$$y = \begin{bmatrix} y(0) & y(1) & \cdots & y(N-1) \end{bmatrix}^T.$$

In (11), the superscript $T$ denotes transpose. Furthermore $\Theta$ is an $M$ terms complex-valued column vector given by

$$\Theta = \begin{bmatrix} \theta_0 & \theta_1 & \cdots & \theta_{M-1} \end{bmatrix}^T.$$

The matrix $H$ has a Vandermonde structure and is given by

$$H = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ e^{-j\omega_0} & e^{-j\omega_1} & \cdots & e^{-j\omega_{M-1}} \\ \vdots & \vdots & \ddots & \vdots \\ e^{-j\omega_0(N-1)} & e^{-j\omega_1(N-1)} & \cdots & e^{-j\omega_{M-1}(N-1)} \end{bmatrix}.$$
and the vector $w$ contains the noise samples.

Provided that $N \geq M$, the least-squares estimate (LSE) of $\theta$ in (10) is given by [17]

$$\hat{\theta} \triangleq \left[ \hat{\theta}_0 \quad \hat{\theta}_1 \quad \ldots \quad \hat{\theta}_{M-1} \right]^T = (H^H H)^{-1} H^H y,$$

(14)

where $H^H$ is the conjugate transpose of $H$ and $(\cdot)^{-1}$ denotes the inverse. Due to the user chosen $\omega_d$, it is guaranteed that the inverse of $H^H H$ exists and that a numerically stable solution can be obtained. However, it is obvious that $H^H H$ looses rank in degenerated cases like $\omega_d = 0$, for which the multiple tone waveform collapses to a single tone.

In practice, there are many factors that may influence the accuracy of the proposed method, such as the characterization of the VSG, the measurement time, precision and synchronization of sampling clocks, external or thermal noise, et cetera. A full robustness analysis is beyond the scope of this paper. However, the analysis techniques introduced in another context in [18] may be applicable here. A first attempt to analyze the behavior is to describe all the imperfections as additive complex-valued white Gaussian noise with variance $\lambda^2$, hence $w \sim CN(0, \lambda^2 I)$.

Under the given assumptions the covariance matrix of $\hat{\theta}$ is given by [17]

$$Cov(\hat{\theta}) = \lambda^2 (H^H H)^{-1}.$$

The diagonal elements of $(H^H H)^{-1}$ are $1/N$, while the off-diagonal elements can be made zero by choosing $N$ as an integer number of whole periods of all the sinusoids that are measured, sometimes called coherent sampling [19]. In general, the off-diagonal elements decrease as $1/N^2$. Therefore, for large values of $N$, $Cov(\hat{\theta})$ is well approximated by a diagonal matrix with $\lambda^2 / N$ on the diagonal, regardless of whether an integer number of periods of the sinusoids are measured or not. Thus,

$$Var(\hat{\theta}_k) \approx \lambda^2 / N \quad \text{and} \quad Cov(\hat{\theta}_k, \hat{\theta}_l) \approx 0, \quad k \neq l.$$

2.4. Determination of Pseudo Delay and Phase Difference in Clocks

It should now be noted in (14) that the $\hat{\theta}_k$'s in $\hat{\theta}$, in addition to the sought $H(\omega_k)$, contains the original phase $\phi_k$ of the tone, the phase shift caused by $\omega_k \tau$ and the phase $\alpha$. Since $\phi_k$ is known, it can easily be compensated for. The compensation for the
unknown \( \omega_k \tau \) and \( \alpha \) is somewhat more complicated and is described below.

Since (6) is a repetitive signal with periodicity \( \tau_u = 2\pi / \omega_d \), it is not necessary to compensate for the entire \( \tau \), but only for the part of it that is not an entire period of \( u(n) \). Hence, \( \tau = L \tau_u + \tau_p \), where \( L \) is the unknown and uninteresting number of whole periods of \( u(n) \) in \( \tau \), and \( \tau_p \) is the remainder. In the sequel, \( \tau_p \) is labeled as the pseudo delay.

Consider \( \theta_k \) according to (8) with \( \tau = L \tau_u + \tau_p \). Due to the periodicity of the test signal \( u(n) \), \( \exp(j \omega_k \tau_u) \equiv 1 \), and thus

\[
\angle \theta_k = \phi_k + \omega_k \tau_p + \angle H(\omega_k) + \alpha
\]

where \( \phi_k \) and \( \omega_k \) are the known initial phase and input frequency respectively. The quantities \( \tau_p \) and \( \alpha \) are the unknown pseudo delay due to the measurement set-up and the phase difference between the VSG/VSA clocks. Accordingly, the phase ripple \( R_p(\omega) \) at the input frequencies \{ \omega_k \} according to (4) is given by

\[
R_p(\omega_k) = \angle \theta_k - \phi_k - \omega_k \tau_p - \alpha - a_0 - a_1 \omega_k.
\]

Rewriting (17) as

\[
R_p(\omega_k) = \angle \theta_k - \phi_k - \frac{(\alpha + a_0) - (\tau_p + a_1) \omega_k}{b}
\]

gives the phase ripple on the form in (4). The two constants \( a \), \( b \) are determined by letting \( R_p(\omega) = 0 \) for the two frequency bins \( \omega = \pm \omega_d \), that is for \( \omega = \omega_k \), determined by \( k = M/2-1 \) and \( k = M/2 \) respectively. Then, for \( k = M/2-1 \) and \( k = M/2 \),

\[
0 = \angle \theta_k - \phi_k - a - b \omega_k.
\]

Now, replacing \( \theta_k \) with its estimate \( \hat{\theta}_k \) from (14) for \( k = M/2-1 \) and \( k = M/2 \) it is straightforward to solve for the unknown \( a \) and \( b \), that is solving

\[
\begin{bmatrix}
1 & \omega_d \\
1 & -\omega_d
\end{bmatrix}
\begin{bmatrix}
a \\
b
\end{bmatrix} = \begin{bmatrix}
\angle \hat{\theta}_{M/2} - \phi_{M/2} \\
\angle \hat{\theta}_{M/2-1} - \phi_{M/2-1}
\end{bmatrix}.
\]

By solving the above set of equations, estimates are obtained for the pseudo delay and phase difference between the VSG/VSA clocks relative to the two point fit linear phase. The estimates are denoted by \( \hat{a} \) and \( \hat{b} \) in the sequel.
2.5. Extraction of the Amplitude and Phase Ripple

The phase ripple can now be found as follows. From (18) an estimate of $R_p(\omega)$ can be formed as

$$\hat{R}_p(\omega_k) = \angle \hat{\theta}_k - \phi_k - \hat{a} - \hat{b} \omega_k$$

(21)

where the hat over $R_p(\omega)$ denotes that the phase ripple is estimated from the least-squares estimate $\hat{\theta}_k$ and the sequential estimates $\hat{a}, \hat{b}$. By construction $\hat{R}_p(\omega_{M/2-1}) = \hat{R}_p(\omega_{M/2}) = 0$, whereas for the remaining $M-2$ values of $k$ it will provide a value of the phase ripple.

An estimate of the amplitude ripple follows directly from the definition in (2) and (3) by noting that $|\hat{H}(\omega_k)| = |\theta_k|$, i.e.

$$\hat{R}_A(\omega_k) = \frac{2|\hat{\theta}_k|}{\max_k \left( |\hat{\theta}_k| \right) + \min_k \left( |\hat{\theta}_k| \right)}.$$  

(22)

Now, a vector $\hat{r}$ with the estimated amplitude and phase ripple at the frequency bins $\{\omega_k\}$ can be defined as

$$\hat{r} = \begin{bmatrix} \hat{r}_0 \\ \hat{r}_1 \\ \vdots \\ \hat{r}_{M-1} \end{bmatrix} = \begin{bmatrix} \hat{R}_A(\omega_0)e^{j\hat{R}_p(\omega_0)} \\ \hat{R}_A(\omega_1)e^{j\hat{R}_p(\omega_1)} \\ \vdots \\ \hat{R}_A(\omega_{M-1})e^{j\hat{R}_p(\omega_{M-1})} \end{bmatrix}.$$  

(23)

The vector $\hat{r}$ in (23) will be an essential tool for simultaneous determination of the ripple of the cascaded systems $H_1(\omega)$ and $H_2(\omega)$ in Section 3.

This completes the algorithm to find the amplitude and phase ripple of a linear system using a multiple tone signal in a measurement set-up similar to that in Fig. 1. The algorithm is summarized in Table I.
<table>
<thead>
<tr>
<th>Step</th>
<th>Procedure</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Generate a multiple tone signal as in (6) and pass it through the unknown system $H(\omega)$.</td>
</tr>
<tr>
<td>2.</td>
<td>Measure the sampled output $y(n)$, $n = 0, 1, \ldots$, $N-1$, (7).</td>
</tr>
<tr>
<td>3.</td>
<td>Calculate the response vector $\hat{\theta}$ using (14).</td>
</tr>
<tr>
<td>4.</td>
<td>Find the linear phase parameters $\hat{a}$ and $\hat{b}$ using (20).</td>
</tr>
<tr>
<td>5.</td>
<td>Calculate all $\hat{R}_p(\omega_k)$ using (21).</td>
</tr>
<tr>
<td>6.</td>
<td>Calculate all $\hat{R}_q(\omega_k)$ using (22).</td>
</tr>
<tr>
<td>7.</td>
<td>Construct the augmented ripple vector $\hat{r}$ using (23).</td>
</tr>
</tbody>
</table>

2.6. Cascaded Systems

Now, consider for a moment the case when $H(\omega)$ can be decomposed as the product $H(\omega) = H_1(\omega)H_2(\omega)$, see Fig. 2. Even though $H(\omega)$ has been found, the individual contributions of the two subsystems $H_1(\omega)$ and $H_2(\omega)$ are still unknown. If the ripple of one of $H_1(\omega)$ and $H_2(\omega)$, say $H_1(\omega)$, is sufficiently small compared to the other, $H_2(\omega)$, then the amplitude and phase ripple of $H_2(\omega)$ has been found and can be applied to the measurements. However, in many cases the ripple of $H_1(\omega)$ and $H_2(\omega)$ are of the same order, and hence both have to be found. This case will be examined in the following Section.

3. A Technique for Measuring Ripple of Cascaded Systems

This section describes a method to find the amplitude and phase ripple of two cascaded linear bandpass systems under some constraints. These constraints are met in practice by most commercially available VSGs and VSAs, as well as by most other transmitters and receivers. The limiting constraints can be put on either system, that is on $H_1(\omega)$ or $H_2(\omega)$.

3.1. Principle

The proposed method to find the individual amplitude and phase ripple of $H_1(\omega)$ and $H_2(\omega)$ makes use of three or more measurements of the amplitudes and phases as described in Section 2, taken at different center frequencies. The method is derived here for the case of three measurements, but it is emphasized that the extension to include the case of additional measurements is straightforward. However, in the application at hand, three measurements are a reasonable trade-off between measurement time and accuracy.
The first measurement, consisting of the procedure in Table I, is taken at a first RF center frequency $F_c$ [Hz], i.e. where the center frequencies of the VSG and VSA are equal. The second measurement is taken at a second center frequency $F_c + 2F_d$ [Hz], i.e. the center frequency of the VSG has been increased by $2F_d$ [Hz]. The third measurement is taken at a third center frequency $F_c - 2F_d$ [Hz], i.e. the RF carrier of the VSA is moved exactly one tone spacing between the measurements. In the derivation presented here, the center frequency of the VSG is changed, but the extension to the case when the center frequency of the VSA is changed is straightforward. From here on all measurements will be related to the center frequency of the first measurement done at $F_c$ [Hz], which as previously mentioned corresponds to a frequency of zero in the complex envelope notation.

Furthermore, normalized angular frequencies will be used. In the normalized frequency notation of the first measurement, the second and the third measurements are taken at center frequencies of $-2\omega_d$ and $2\omega_d$ respectively. The three measurements are illustrated in Fig. 6 for the case of a test signal (6) comprising four tones, i.e. $M = 4$. In Fig. 6, the frequency of the transmitter (or VSG) $H_1(\omega)$ is raised (Fig. 6b) and lowered (Fig. 6c) while the frequency of the receiver (or VSA) $H_2(\omega)$ remains constant.

### 3.2. Derivation

Again consider the system in Fig. 2. Let the input signal be an $M$-tone signal as in (6). Since $H(\omega) = H_1(\omega)H_2(\omega)$, a measure of the ripple of $H(\omega)$ can be given by

$$R^0(\omega) = R_1(\omega)R_2(\omega),$$  \hspace{1cm} (24)

where $R(\omega) = R_A(\omega)e^{jR_A(\omega)}$ is obtained at the grid frequencies $\{\omega_k\}$ by applying the procedure in Table I. The superscript ‘0’ denotes that there is a zero difference between the center frequencies of the VSG and the VSA.

Now a frequency shift of $2\omega_d$ between $H_1(\omega)$ and $H_2(\omega)$ is introduced according to Fig. 7. In practice, the frequency shift is introduced by changing the center frequency, i.e. the RF carrier, of either the VSG or the VSA.
Fig. 6. Measurements are taken at three different center frequencies spaced $2\omega_d$ apart. Here the oscillator frequency of the receiver $H_r(\omega)$ remains constant while the frequency of the transmitter $H_t(\omega)$ is raised (b) and lowered (c) in relation to the first measurement taken in (a).

![Diagram](Image)

Fig. 7. A frequency shift of $\pm 2\omega_d$ between the transmitter and receiver is introduced.

![Diagram](Image)

This yields

$$R^+(\omega_k) = R_1(\omega_k - 2\omega_d)R_2(\omega_k)$$

for a frequency shift of $+2\omega_d$ of the center frequency of the VSG and

$$R^-(\omega_k) = R_1(\omega_k + 2\omega_d)R_2(\omega_k)$$
for a frequency shift of -2ω_d, respectively. The superscripts ‘+’ and ‘-’ on R(ω) are used to denote the positive and negative frequency shifts respectively.

In the above, two tones in the multi-tone input have been allocated to find (a, b) in (18) for each of the three measurements as was described in Section 2.4. Here, the linear phase parameters are denoted by (a, b)^-, (a, b)^0 and (a, b)^+ respectively.

From the constraints R_p(±ω_d) = 0 in (19), without loss of generality, the phase ripple for the two systems in cascade is constrained according to:

\[
\begin{align*}
\angle R_1(-\omega_d) &= 0, \\
\angle R_1(\omega_d) &= 0, \\
\angle R_2(-\omega_d) &= 0, \\
\angle R_2(\omega_d) &= 0.
\end{align*}
\] (27)

These constrains do not impose any limitation as long as the only parameter of interest is the phase distortion and not the absolute delays of H_1(ω) and H_2(ω). It should be noted that the three measurements now in general will have three different sets of estimated linear phase parameters \(\hat{a}, \hat{b}\). This can be handled in one of two ways. First, if a sufficient amount of memory is available in the VSA, the signal can be sampled continuously during the frequency shifts. The definition of sufficient is determined by the sampling rate used and the settling time of the oscillator. In this case, all three measurements can be related to the \(\hat{a}, \hat{b}\) of the first measurement. If this procedure is not possible, a second approach can be used, which introduces additional mild conditions on one of the systems, i.e. H_1(ω) or H_2(ω). If at least one of the systems can be assumed to have phase that is linear or close to linear over the centered frequency range \([-3\omega_d, 3\omega_d]\) i.e. for four tones, then all three pairs \(\hat{a}, \hat{b}\)^-, \(\hat{a}, \hat{b}\)^0 and \(\hat{a}, \hat{b}\)^+ for the three different measurements will essentially be equal. This becomes a necessary condition for the proposed method to work properly.

Now for a moment consider the complex valued \(z = z_1 z_2\), which expressed in Nepers yields \(z^{\text{Np}} = |z|^{\text{Np}} + j\angle z = |z_1|^{\text{Np}} + j\angle z_1 + |z_2|^{\text{Np}} + j\angle z_2\). Further, recall that \(\hat{r}\) in (23) is an estimate of \(R_1(\omega)\) at the frequency grid determined by (6). According to (23), the notations \(\hat{r}^0, \hat{r}^+\) and \(\hat{r}^-\) are now introduced for the vectors corresponding to a zero, a positive and a negative carrier frequency shift respectively and expressed in Nepers. Starting with the case of a zero frequency shift, the following (under-determined) set of linear equations is obtained for (24),
\[ [ \mathbf{I}_M \mathbf{I}_M ] \mathbf{b} = [ \hat{\mathbf{x}}^0 ]^Np, \]  

(28)

where \( \mathbf{I}_M \) denotes the unit matrix of order \( M \), the vector \( \mathbf{b} \) is of length \( 2M \) and contains the unknown and sought amplitude and phase ripple of the two filters. It is given by

\[
\mathbf{b} = \begin{bmatrix}
R_1(-M-1)\omega_d)^{Np} \\
R_1(-M-3)\omega_d)^{Np} \\
\vdots \\
R_2(M-1)\omega_d)^{Np} \\
R_2(-M-1)\omega_d)^{Np} \\
\vdots \\
R_2(M-3)\omega_d)^{Np} \\
R_2(M-1)\omega_d)^{Np}
\end{bmatrix} ,
\]  

(29)

where \( R(\cdot)^{Np} = |R(\cdot)|^{Np} + j\angle R(\cdot) \).

Similar sets of equations hold true for the two remaining cases, i.e. (25) and (26). That is, let

\[
\hat{\mathbf{f}}^{Np} = \begin{bmatrix}
[ \hat{\mathbf{f}}^0 ]^{Np} \\
[ \hat{\mathbf{f}}^- ]^{Np} \\
[ \hat{\mathbf{f}}^+ ]^{Np}
\end{bmatrix} ,
\]  

(30)

then

\[ \mathbf{A} \mathbf{b} = \hat{\mathbf{f}}^{Np}, \]  

(31)

where \( \mathbf{A} \) is a \((3M-2) \times 2M\) matrix given by

\[
\mathbf{A} = \begin{bmatrix}
\mathbf{I}_M & \mathbf{I}_M \\
\mathbf{I}_{M-1} & \mathbf{0}_{M-1} & \mathbf{0}_{M-1} & \mathbf{I}_{M-1} \\
\mathbf{0}_{M-1} & \mathbf{I}_{M-1} & \mathbf{I}_{M-1} & \mathbf{0}_{M-1}
\end{bmatrix} ,
\]  

(32)

where \( \mathbf{0}_{M-1} \) is a zero column vector of length \( M-1 \). It is noted that when setting up (32) one equation is lost for each of the two frequency shifts, see Fig. 6.

Since only the amplitude ripples are of interest here rather than the absolute amplitude values, the amplitude of one of the filters is set to 1 at one point in frequency. Here,

\[ |R_1(\omega_d)| = 1 \Rightarrow |R_1(\omega_d)|^{Np} = 0 , \]  

(33)
is used before solving the system. If the absolute values are of interest, they can be found by doing additional measurements.

When solving (31) with respect to $\mathbf{b}$ it shall be observed that the definitions of delay and amplitude ripple described in (27) and (33) put some constrains on the solution.

These constraints are introduced below noting that (31) can be rewritten as

$$
\begin{bmatrix}
\mathbf{A} & 0 \\
0 & \mathbf{A}
\end{bmatrix}
\begin{bmatrix}
\operatorname{Re} \{ \mathbf{b} \} \\
\operatorname{Im} \{ \mathbf{b} \}
\end{bmatrix}
= 
\begin{bmatrix}
\operatorname{Re} \{ \tilde{\mathbf{F}}^{Np} \} \\
\operatorname{Im} \{ \tilde{\mathbf{F}}^{Np} \}
\end{bmatrix},
$$

(34)

where $\operatorname{Re}\{\cdot\}$ and $\operatorname{Im}\{\cdot\}$ denote the real and imaginary parts of the quantity within parentheses respectively.

Now, the constraints can be introduced in a straightforward manner by deleting the terms that correspond to the variables that have already been defined as zero, which are the amplitude ripple for one tone (33) and the phase ripple for at least the two central tones at $+/−\omega_i$ for both $R_1(\omega)$ and $R_2(\omega)$ according to (27). This gives the system

$$
\begin{bmatrix}
\mathbf{A}_A & 0 \\
0 & \mathbf{A}_\phi
\end{bmatrix}
\begin{bmatrix}
\mathbf{b}_A \\
\mathbf{b}_\phi
\end{bmatrix}
= 
\begin{bmatrix}
\operatorname{Re} \{ \tilde{\mathbf{F}}^{Np} \} \\
\operatorname{Im} \{ \tilde{\mathbf{F}}^{Np} \}
\end{bmatrix},
$$

(35)

which can be viewed as two systems, one for the amplitude and one for the phase. The matrices $\mathbf{A}_A$ and $\mathbf{A}_\phi$ are similar to $\mathbf{A}$ in (32), but with the columns corresponding to the predefined variables removed. The vectors $\mathbf{b}_A$ and $\mathbf{b}_\phi$ are as $\operatorname{Re}\{\mathbf{b}\}$ and $\operatorname{Im}\{\mathbf{b}\}$, but with the predefined values removed. The number of unknowns in (35) is then $4M-5$ in the case of continuous sampling, see Section 3.2. This is found by first noting that since (31) has $2M$ complex unknowns, (34) has $4M$ real unknowns.

From this, we can subtract the four unknowns that are set in (27) and the one that is set in (33). In the case of non-continuous sampling, two additional unknowns are set as discussed in Section 3.2, giving a total of $4M-7$ unknowns.

The unknown ripple of the filters in the vectors $\mathbf{b}_A$ and $\mathbf{b}_\phi$ are now given by the least-square solution

$$
\begin{align*}
\hat{\mathbf{b}}_A &= (\mathbf{A}_A^H \mathbf{A}_A)^{-1} \mathbf{A}_A^H \tilde{\mathbf{F}}^{Np} \\
\hat{\mathbf{b}}_\phi &= (\mathbf{A}_\phi^H \mathbf{A}_\phi)^{-1} \mathbf{A}_\phi^H \tilde{\mathbf{F}}^{Np}
\end{align*}
$$

(36)

where $\tilde{\mathbf{F}}^{Np} = \operatorname{Re}\{\tilde{\mathbf{F}}^{Np}\}$ and $\tilde{\mathbf{F}}^{Np} = \operatorname{Im}\{\tilde{\mathbf{F}}^{Np}\}$. This completes the algorithm for finding the amplitude and phase ripple of both $H_1(\omega)$ and $H_2(\omega)$, and is summarized in Table II.
TABLE II
SUMMARY OF THE ALGORITHM PROPOSED TO FIND THE AMPLITUDE AND PHASE RIPPLE OF TWO LINEAR SYSTEMS IN SERIES

<table>
<thead>
<tr>
<th>Step</th>
<th>Procedure</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Take three measurements as shown in Table I, at three different center frequencies as shown in Fig. 6.</td>
</tr>
<tr>
<td>2.</td>
<td>Form the set of equations in (31) using (24) to (26).</td>
</tr>
<tr>
<td>3.</td>
<td>Remove constrained values from (31) using (27) and (33) to form the set of equations in (35). Solve (35) as shown in (36) to obtain $\hat{b}<em>A$ and $\hat{b}</em>\phi$ which contain the estimated amplitude and phase ripple of $H_1(\omega)$ and $H_2(\omega)$ (see (29)), at the frequency bins ${ \omega_k }$.</td>
</tr>
</tbody>
</table>

3.3. Increasing the Number of Measurements

As the number of tones $M$ is increased, (36) becomes more and more ill-conditioned, which may lead to a “leakage” between the filters. One countermeasure is to increase the number of RF center frequencies $P$ at which the measurements are taken.

Equations (30) - (32) have to be modified accordingly. Fig. 8 shows the condition number $\eta = \lambda_{\text{max}}/\lambda_{\text{min}}$, where $\lambda_{\text{max}}$ and $\lambda_{\text{min}}$ are the largest and smallest eigenvalues of $\mathbf{A}_A^\dagger \mathbf{A}_A$ respectively, versus the number of measurements and the number of tones. As shown, the condition number decreases substantially for a larger number of tones if more than three measurements are taken. For example, $M = 24$ and $P = 3$ gives $\eta \approx 400$, while $M = 64$ and $P = 11$ gives $\eta \approx 365$. Hence, if it is desirable to use a large number of tones in the test signal, the number of center frequencies should be chosen accordingly. It is not possible to give a precise recommendation on what condition number can be allowed in different situations, but in general, a condition number of the order of $10 - 20$ works well whereas a condition number of 100 should be treated with some caution [20]. In the practical measurements taken in this paper, condition numbers as high as several hundreds were used with results showing high accuracy.

A drawback with taking more measurements is that if the memory of the VSA does not allow all measurements to be taken during continuous sampling, then the requirement for linear phase increases on one of either $H_1(\omega)$ or $H_2(\omega)$. Then in general, the phase of one of the systems must be linear or close to linear over the interval $[-P \omega_d, P \omega_d]$. In addition, more measurements will also increase the measurement time, which is often undesirable. Different numbers of measurements can be used for the amplitude and phase calculations.
Fig. 8. Plot of the condition number $\eta$ versus the number of tones $M$ and the number of measurements $P$. It is clear that the number of tones can be increased from 24 to 64 if the number of measurements is increased from three to eleven, without increasing the condition number.

4. Experimental Set-Up

The measurement system used was similar to that featured in Fig. 9, consisting of only one VSG and one VSA. The VSG was an SMU 200A and the VSA was an FSQ, both from Rohde & Schwarz GmbH. Both the SMU 200A and the FSQ should be considered state-of-the art in their respective fields. Despite this, additional calibration similar to that proposed here is required for some measurements. For simpler VSGs and VSAs the need for calibration is even more pronounced.

First it is important to discuss the origin of the amplitude and phase ripple in the VSA and VSG respectively. All distortion generated after the mixer at which the frequency shift is introduced will erroneously be attributed to the other instrument. This situation is illustrated in Fig. 9, where $H_{\text{VSG,RF}}(\omega)$ and $H_{\text{VSA,RF}}(\omega)$ are the distortion at RF for the VSG and VSA respectively. Hence, at least one of $H_{\text{VSG,RF}}(\omega)$ and $H_{\text{VSA,RF}}(\omega)$ must have insignificant amplitude and phase ripples over the frequency range of interest. This is often the case over reasonable bandwidths especially for VSAs, and can be checked with traditional analog measurement techniques if necessary. The transfer function $H_{\text{VSG,RF}}(\omega)$ is often a function of both power level and center frequency, which makes it necessary to calibrate the system for the intended power level and the intended center frequency. $H_{\text{VSG}}(\omega)$ and $H_{\text{VSA}}(\omega)$ on the other hand are normally more stable versus power and frequency. This is quite natural since they are physically at the same frequency, baseband or an intermediate frequency (IF), regardless of the RF.
Fig. 9. Illustration of the different contributions of the measurement setup to the total amplitude and phase ripple. For each of the two instruments, i.e. the VSG and the VSA, there is an RF contribution to the linear distortion, labeled $H_{VSG,RF}(\omega)$ and $H_{VSA,RF}(\omega)$ respectively. This linear distortion at RF for one of the instruments will enter the calculations as an error term, while the linear distortion at the baseband or at an IF will be accounted for and determined using the proposed algorithm.

5. Results and Discussion

The proposed method was evaluated and validated using actual measurements on the system in Fig. 9.

5.1. Test Signals

Some different measurement signals were first designed:

i.) An ideal signal $Testsignal_0$ on complex envelope form, given by

$$u(t) = \frac{1}{M} \sum_{k=0}^{M-1} \alpha_k e^{j(2\pi F_t t + \phi_k)}$$

with $M = 20$. The frequency $F$ belongs to the set \{-4.75, -4.25, ..., 4.75\} [MHz]. Further, all $\alpha_k$ equals $\alpha_k = 1$ and all $\phi_k$ equals $\phi_k = 0$. The total signal bandwidth at RF thus equals 9.5 MHz.

ii.) $Testsignal_1$. Same as $Testsignal_0$, but with the difference that $\alpha_k$ now belongs to the set
\[ \alpha = \{ 1 \, 1 \, 1 \, 1 \, 1 \, 1 \, 1 \, 1 \, 1 \, 1 \, 1 \, 1 .98 \, .95 \, .85 \, .8 \, .7 \} \], while \( \phi_k \) belongs to the set

\[ \phi = \{ .1 \, .2 \, .3 \, .4 \, .5 \, .6 \, .7 \, .8 \, 0 \, 0 \, 0 \, 0 \, 0 \, 9 \, 1 \, 2 \, 3 \, 4 \, 5 \, 6 \, 7 \} \, [\text{deg}] \].

iii.) **Testsignal\_2.** Same as **Testsignal\_0**, but with the difference that \( \alpha_k \) now belongs to the set

\[ \alpha = \{ .5 \, .6 \, .7 \, .8 \, .9 \, 1 \, 1 \, 1 \, 1 \, 1 \, 1 \, 1 \, 1 \, 1 \, 1 \, 1 \, 1 \, 1 \, 1 \} \],

while \( \phi_k \) belongs to the set

\[ \phi = \{ \quad -1 \, -7 \, -3 \, -4 \, -.5 \, -.6 \, -.1 \, -3 \, 0 \, 0 \, 0 \, 0 \, 3 \, 1 \, .6 \, .5 \, 4 \, 3 \, 7 \, 1 \} \, [\text{deg}] \].

### 5.2. Initial Measurements

Two evaluation measurements using **Testsignal\_1** and **Testsignal\_2** were done at a center frequency of 2140 MHz using a sampling rate of 81.6 MHz, which is the maximum sampling rate of the FSQ. In both measurements an RBW of 10 MHz was used in order to force \( H_{\text{VSA}}(\omega) \) to have some linear distortion, see Fig. 9. The transfer function \( H_{\text{VSG}}(\omega) \) will be close to the values of the test signals given above as the ripple of the baseband part of the SMU 200A can be neglected for this bandwidth compared to the amplitudes introduced in the two test signals along with the distortion in the set-up. This is also true for the ripple of \( H_{\text{VSA,RF}}(\omega) \), i.e. it can be neglected for this bandwidth (9.5 MHz). The quantity \( H_{\text{VSG,RF}}(\omega) \) will be non-zero and will be measured. The results are shown in Sections 5.3 (amplitude ripple) and 5.4 (phase ripple). The bandwidth and the amplitude ripple are increased in Section 5.5 and a result for a filter with more than 25 dB attenuation is given in Section 5.6.

For the measurements presented here, as all the measurements at the different center frequencies could not be taken during continuous sampling, the VSA was assumed to have linear phase over the interval \([ -P\omega_d, \, P\omega_d \]). Here \( P = 3 \) (the number of center frequencies at which measurements were taken) in all cases except in Section 5.7, where the case of \( P > 3 \) is treated.

### 5.3. Amplitude Ripple

The results for the amplitude ripple of the VSG, i.e. the amplitudes assigned to the different tones in Section 5.1 are shown using **Testsignal\_1** in Fig. 10 and Fig. 11 and using **Testsignal\_2** in Fig. 12. The result for the amplitude ripple of the VSA is shown in Fig. 13.
Fig. 10. Measured amplitude ripple of the VSG using Testsignal_1. The result obtained using the proposed algorithm is compared to the theoretical amplitude of Testsignal_1, labeled “Unmodified Reference”, and to the amplitude at the output port of the VSG as measured by a spectrum analyzer, labeled “Modified Reference”. The frequency is related to the RF carrier center frequency, which is 2140 MHz.

Fig. 11. Amplitude ripple difference for the VSG using Testsignal_1. Difference between the modified reference and the proposed algorithm in Fig. 10. The frequency is related to the RF carrier center frequency, which is 2140 MHz.
Fig. 12. Measured amplitude ripple of the VSG using Testsignal_2. The result obtained using the proposed algorithm is compared to the theoretical amplitude ripple of Testsignal_2, labeled “Unmodified Reference”, and to the amplitude ripple at the output port of the VSG as measured by a spectrum analyzer, labeled “Modified Reference”. The frequency is related to the RF carrier center frequency, which is 2140 MHz.

In Fig. 10 the result is compared both to the theoretical amplitude given by Testsignal_1, labeled ‘Unmodified Reference’, and to the amplitudes of the different tones at the output of the VSG as measured by a spectrum analyzer, labeled ‘Modified.
Reference’. Clearly, the result of the algorithm deviates considerably from the theoretical, while it is in close agreement (+/- 0.02 dB) with the amplitude measured by the spectrum analyzer, as shown in Fig. 11. The difference is attributed to $H_{VSG,RF}(\omega)$. A relative amplitude measurement using a state-of-the-art spectrum analyzer such as the FSQ has accuracy in the order of at most a few hundredths of a dB.

The result for the amplitude of the VSG in Fig. 12 using Testsignal_2 shows similar performance.

Fig. 13 shows the amplitude ripple of the VSA for both Testsignal_1 and Testsignal_2, obtained in the same measurements as the amplitude ripple of the VSG in Fig. 10 and Fig. 12. A measurement using a multiple tone signal with equal amplitudes at the input port of the VSA, i.e. to within +/-0.01 dB as measured with the FSQ in swept mode, is also shown as reference. The deviation between the reference and Testsignal_1 is at most 0.02 dB. For Testsignal_2 on the other hand, an amplitude deviation of about 0.1 dB is seen at some frequencies. This error can be overcome by increasing $P$ as discussed in Section 3.3 and supported by measurements in Section 5.7.

5.4. Phase Ripple

The phase ripple of the VSA for the two measurements shown in Fig. 14 differs by less than 0.5 degrees. For a reference measurement performed under the assumption that the phase response of the VSG is ideal over this bandwidth, the difference is about 1.5 degrees. (This assumption was confirmed to be reasonably valid to within at least 1 degree through measurements using Testsignal_0.). While the origin of this difference remains unknown, it is probably due to a combination of the assumption made about the phase of the VSG and an error introduced by the algorithm.

The phase ripple of the VSG for the two test signals in Fig. 15 and Fig. 16 differs from the theoretical phase ripple of the test signals by less than 1 degree. How much of this difference is due to the algorithm and how much is due to the VSG could not be determined. A first guess is that there is a difference in the estimation of $\hat{a}, \hat{b}$ in the three measurements since the slope of the phase is different for the measured data and the reference. However, the error of less than 1 degree is small.
Fig. 14. Measured phase ripple of the VSA obtained using Testsignal_1 and Testsignal_2. A reference measurement obtained using a multiple tone signal with known phases at the input port of the VSA is also shown. The frequency is related to the RF carrier center frequency, which is 2140 MHz.

Fig. 15. Measured phase ripple of the VSG using Testsignal_1. The theoretical phase for Testsignal_1 is also shown for reference. The frequency is related to the RF carrier center frequency, which is 2140 MHz.
5.5. A Measurement Bandwidth of 28.5 MHz

A test over a larger bandwidth and at another center frequency of 1500 MHz was performed with Testsignal_0, but with $F$ now belonging to the set \{-14.25, -12.75, ..., 14.25\} [MHz], giving a total signal bandwidth of 28.5 MHz. To accommodate the signal, the RBW of the VSA was set to 50 MHz. The result for the amplitude is shown in Fig. 17. The total ripple was obtained using (14) on the measurement taken at 1500 MHz, which as discussed previously, gives an estimate with high accuracy. A reference measurement for the amplitude ripple of the VSG was taken using the VSA in swept mode. As mentioned earlier, the relative amplitude error of the VSA in swept mode is of the order of a few hundredths of a dB and can thus be neglected. The difference between the reference and the measured result for the VSG is about +/- 0.02 dB, as is the difference between the measured result for the VSA and the total deviation minus the reference of the VSG (not shown explicitly in the figure). Despite the substantial deviation from an ideal flat amplitude response, the phase of both the VSA and the VSG is still measured to be within +/- 1 degree (not shown).


A final test is done using Testsignal_2 but with $F$ now belonging to the set \{-19, -17, ..., 19\} [MHz], giving a total bandwidth of 38 MHz. An RBW of 20 MHz and a sampling clock of 81.6 MSamples/s were used in the VSA. This means that a substantial attenuation can be expected at the band edges. That is, $H_{VSA}(\omega)$ will have a very significant ripple. The result is shown in Fig. 18 for the amplitude ripple of the VSA and in Fig. 19 for the amplitude ripple of the VSG.
Fig. 17. Measured amplitude ripple of the VSA and VSG. The amplitude ripple of the VSG measured with a spectrum analyzer is also shown for reference. The frequency is related to the RF carrier center frequency, which is 1500 MHz.

Fig. 18. Measured amplitude ripple of the VSA using Testsignal_2, essentially the 20 MHz RBW filter. The frequency is related to the RF carrier center frequency, which is 1500 MHz.
Fig. 19. Measured amplitude ripple of the VSG using Testsignal_2 as obtained with the proposed algorithm compared to the theoretical amplitude for Testsignal_2 and to the amplitude ripple measured using a spectrum analyzer, labeled “Reference”. The frequency is related to the RF carrier center frequency, which is 1500 MHz.

It is noted in Fig. 18 that the attenuation at the edges is close to 30 dB. The amplitude of the VSG was also compared with a spectrum analyzer measurement, labeled “Reference” in Fig. 19. The phases of both the VSG and the VSA are shown in Fig. 20 together with the initial phases of Testsignal_2. It is clear from Fig. 20 that despite the substantial attenuation at the band edges and a phase shift of more than 40 degrees in the VSA, the phase measurement of the VSG is still only off from the theoretical by less than 1.5 degrees, a slight increase from the 1 degree difference for the measurement in Fig. 16. Hence, the proposed method is concluded to perform well even when one of the filters is far from ideal.
Fig. 20. Measured phase ripple of the VSA and the VSG as obtained using Testsignal_2. The phase ripple of the VSG is compared to the theoretical phase ripple for Testsignal_2 labeled “reference phase of the VSG”. The difference is less than 1.5 degrees. The frequency is related to the RF carrier center frequency, which is 1500 MHz.

5.7. Measurements with $P > 3$

In order to stabilize the solution in (36) and get a well conditioned system, measurements were taken at 11 different center frequencies. This corresponds to lowering the condition number from 284 for three measurements to 77 for 11 measurements, see Fig. 8. The result for the VSG is shown in Fig. 21. The solution changes considerably when the number of measurements is increased. A swept measurement of the output of the VSG is also shown. The agreement is well within the limits of the spectrum analyzer and considering the smooth, nice looking result of the algorithm, the difference is probably mostly due to measurement errors in the reference measurement.
6. Conclusion

In this paper a method has been proposed to simultaneously measure the amplitude and phase ripple of an RF transmitter and a receiver. By shifting the center frequency of either the transmitter or the receiver, enough equations to find the amplitude and phase ripple of both systems can be obtained. The method is summarized in Table I and II. The method has been supported by extensive measurements on a VSG and a VSA and its effectiveness has been demonstrated. The method is useful for numerous measurement situations including sampled measurements on RF power amplifiers and amplitude and phase ripple measurements on RF receivers and transmitters. The amplitude accuracy of the method rivals that of stepped tone measurements, but can be carried out at a fraction of the time needed for the former. In addition the phase ripple is also measured.

It is important to notice that the frequencies of the sinusoidal stimuli signal have to be known, which means that the transmitter and the receiver have to be frequency locked. If they are not, the method is still applicable, but the frequencies must be incorporated in the estimation procedure.

The precise absolute accuracy of the method cannot be determined through measurements and a possible future work could include looking into derivation of theoretical error limits for the method under different circumstances.

**Acknowledgment**

This work has been supported by the University of Gävle, the Knowledge Foundation (KKS), Ericsson AB, the Graduate
School of Telecommunications (GST), the Royal Institute of Technology (KTH), Note AB, Syntronic AB, Racomna AB, and Rohde & Schwarz GmbH. The authors especially wish to thank Rohde & Schwarz GmbH of Germany for providing the measurement instruments.

The authors further wish to thank Mr. Peder Malmlöf, Mr. Patrik Stenvard, Mr. Olav Andersen, Mr. Tobias Johansson, Dr. Leonard Rexberg, Dr. Niclas Keskitalo and Mr. Richard Berg, all with Ericsson AB, for valuable discussions on the subject of the paper.

References


