



JÖNKÖPING UNIVERSITY

School of Engineering

**A numerical analysis of different scenarios that
affect slope stability for part of project Lappen
19 in Munksjöstaden**

**En numerisk analys av olika scenarier som påverkar
släntstabiliteten för en del av projekt Lappen 19 i
Munksjöstaden**

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This thesis project has been carried out at the Institute of technology in Jönköping within the main area of construction technology. The authors themselves are responsible for expressed opinions, conclusions, and results.

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Abstract

The purpose of this study is to perform a numerical analysis using the Plaxis LE 2D software on a section of a slope being constructed as part of Project Lappen 19 in Munksjöstad. The study will be conducted using the results of various geotechnical tests that yield different parameters and measurements. These parameters and measurements are input into the software, where multiple scenarios are executed to achieve a satisfactory factor of safety. Sensitivity analysis is conducted to examine the impact of each parameter on the factor of safety.

The study is conducted in order to examine the slope along the shoreline in Munksjöstad. Document study is the data collection technique used to obtain data from previous site investigations of the study area.

This study has presented several results. The most significant finding indicated a significant variation in the factor of safety of the slope when the friction angle is altered. The higher the friction angle, the higher the factor of safety. Another result also demonstrated that the addition of crushed material in the form of an embankment increases the factor of safety based on its placement.

The results were compiled in various diagrams, allowing for easy interpretation of the changes in the factor of safety. The analysis of the results involved comparing the new improved section with the old one to assess the impact of crushed material in the form of an embankment on stability. Subsequently, different scenarios were created, modifying parameters to determine which parameter had the greatest influence on the factor of safety. The load on the slope was also studied, simulating the situation using the Plaxis LE 2D tool to gain insight into the relationship between load and factor of safety.

Keywords:

Slope stability, factor of safety, embankment, Plaxis LE 2D, numerical analysis.

Sammanfattning

Syftet med denna studie är att utföra en numerisk analys med hjälp av programvaran Plaxis LE 2D på en sektion av en slänt som byggs som en del av projekt lappen 19 i munksjöstaden. Studien kommer att utföras med hjälp av olika geotekniska tester som ger ut olika parametrar och mått. Dessa parametrar och mått sätts in i programvaran där flera olika scenarios utförs för att uppnå en tillfredställande säkerhetsfaktor. Känslighetsanalys är utförd för att undersöka vilken inverkan varje parameter har på säkerhetsfaktorn.

Studien utförs för att studera en slänt längs strandlinjen vid munksjöstaden. Dokumentstudie är den datainsamlingstekniken som används för att erhålla data från tidigare markundersökningar av det studerade området.

Denna studie har presenterat flera resultat. Det viktigaste resultatet indikerade på en stor förändring av säkerhetsfaktorn på slänten när friktionsvinkeln ändras. Ju högre friktionsvinkel, desto högre säkerhetsfaktor. Ett annat resultat visade även att när krossmaterial är tillagt i form av tryckbank så ökar säkerhetsfaktorn baserat på placeringen av materialet.

Resultatet sammanställdes i olika diagram där man kan enkelt läsa av förändringen i säkerhetsfaktorn. Resultatet analyserades genom att jämföra den nya förbättrade sektionen med den gamla för att se hur krossmaterial i form av tryckbank har påverkat stabiliteten. Därefter, skapades olika scenarier där parametrar modifierades för att se vilken parameter som har störst påverkan på säkerhetsfaktorn. Lasten på slänten har även studerats där situationen simulerades i Plaxis LE 2D verktyget för att skapa en uppfattning av förhållandet mellan last och säkerhetsfaktor.

Nyckelord:

Släntstabilitet, säkerhetsfaktor, tryckbank, Plaxis LE 2D, numerisk analys.

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1 Introduction

In order to create an example of a safe and stable ground improvement, it is necessary to carefully consider different factors and study different geotechnical tests. Safety is one of the most important aspects the world has been focusing on in construction engineering (Han, 2015). To achieve this safety for the study that will be presented in the next part of this report, different factors, variables and equations will be used to reach satisfactory results. The result of this study will be in the form of a numerical model presented using geotechnical analysis in a computer software. Another result that is expected to be achieved in this study is an improved factor of safety.

1.1 Background and problem

The human ambition in building a more sustainable, safe living and working environment has increased during the past years. Therefore, it is necessary to perform studies and analysis of the ground to have a sustainable environment (Zavadskas et al., 2018). According to Baskar (2023), geotechnics is the foundation of all building projects which means that geotechnical analysis should be performed with good competence and sufficient skills. Analysis and investigations in this technology field creates a frame for how the characteristics of soil and rock can be used which is a foundation for choice of method and construction solutions (Baskar, 2023).

For this thesis project, the slope in which the new cycling path will be built on is going to be studied. The new path is planned to be built along the shoreline between Munksjöstad and the central part of Jönköping city, (see figure 1.0). According to Jönköping municipality, the surface level is approximately +90,3 closest to the shoreline. As the road to be built will be connected to central Jönköping, the slope is expected to be sufficiently stable and durable to handle the walking and cycling traffic load, (Jönköpings kommun, 2022).



Figure 1.0 walking and cycling paths.

When constructing a road or a building on slope especially with high water table level and adjacent to a lake, additional analysis and stabilization may be required. In such cases, stabilization is needed to prevent slope instability and to support the edge of the outer part of the construction. This prevents the structure from collapsing due to

extreme load or unsuitable soil parameters and increases safety. A case study will be carried out in the area Munksjösten, Jönköping, Sweden.

1.2 Objectives and issues

The purpose of this study is to perform a numerical analysis using the Plaxis LE 2D software. An analysis of the current slope stability will be executed to identify different factors of safety achieved in different sections of the slope.

- How does the construction load affect the slope in part of project Lappen 19?
- How does the factor of safety change if crushed material is added to the slope as an embankment?
- What is the effect of different soil parameters on slope stability and factor of safety in part of project Lappen 19?

1.3 Delimitation

The main delimitation for this study is that the focus is on a specific slope located in, Jönköping, Sweden. Also, the analysis will be performed based only on the following parameters:

- Cohesion
- Friction angle
- Normally consolidated ratio (k)
- Minimum shear strength
- Load

In addition, an example for an improvement using crushed material will be presented as a new slope model. The model will be analyzed by the limit equilibrium method (Morgen Stern-price) using Plaxis LE software.

2 Theoretical framework

According to Zhang (2015), at the beginning of the 1960s, it became clear that slope stability studies needed more than just geological analysis. Zhang (2015) noted that the limit equilibrium method was widely used to examine slope stability during this time. Additionally, Zhang (2015) highlights that mathematical modeling of soil/rock behavior has improved slope predictions compared to traditional methods. The adoption of numerical analysis techniques for slope stability analysis has been made possible in large part thanks to advances in computing technology (Zhang, 2015)

The negative effects from load, water and potential contaminated soil, the study needs to be based on scientific trials and methods to measure different factors. Results that contribute to strengthening the slope create a positive impact such as increasing shear strength which contributes to improve the value of the factor of safety. According to Skredkommissionen (1995) the factor of safety for the slope is expected to be at 1,5 – 1,7.

The Commission on Slope Stability, also known as "Skredkommissionen," is a committee within the Royal Swedish Academy of Engineering Science (IVA) that was founded in 1988. Its primary objective is to oversee research, development, and dissemination of information related to landslides. The commission is responsible for initiating and coordinating research on slope stability, landslides, and preventive measures. It aims to provide information and promote knowledge in the field of slope stability to facilitate effective preventive measures (Skredkommissionen, 1995).

According to Principles and practice of ground improvement, there are three different types of slope failures (Han, 2015).

- **Local failure** happens at local areas of the slope, usually because of poorly compacted geomaterial, triggering factor or because of water intruding.
- **Surficial failure** is the most common for slopes. It occurs more in areas subjected to rainfall. The cause for this kind of failure is weak geomaterial, loss of cohesion and seepage force.
- **Toe slope failure** happens through the lowest part of the slope on the outside. This failure occurs mostly because of the large slope angle and low soil strength.

These different forms of failures would be helpful if understood in order to reach optimal results. Consequently, the understanding and study of different forms of slope failure may be helpful to determining what kind of soil improvement methods should be used. Optimal results for this study can be defined as the solution that's leads to a higher factor of safety.

According to Soleymani et al. (2019), the factor of safety is a quantitative measurement of the ratio of the resisting forces to the driving forces applied on a slope. The resisting forces are presented as the soil and the driving forces are presented as the load applied on the slope structure. Factor of safety is also used to determine possibilities for landslides and other forms of failures in soils. The information provided by the factor of safety is used to create more accurate decisions during the planning phase of a project.

2.1 Crucial parameters for stability analysis

The stability of the slope is determined by the following parameters:

- **Friction angle:** is the parameter of internal friction in the slope and can be determined by performing CPT-test, triaxial test, direct shear test and SPT test, (Axelsson, K., & Mattsson, 2021). According to Das (2014) the friction angle is a parameter that directly affects the shear strength based on the type of soil in the existing slope. Shear strength is determined by two important parameters, friction angle and cohesion. The friction angle is considered to be a crucial parameter for assessing the strength of a geotechnical structure (Das, 2014).
- **Unit weight:** is the weight per unit volume of the material. Can be understood as a physical quantity similar to density (Knappett J., R.F. Craig, 2020).
- **Cohesion:** The term cohesion, represented by "c", is typically measured through laboratory testing using the Direct Shear Test which is one of a few methods to determine this parameter. Cohesion can be affected by different factors, such as soil type, humidity, density, pressure and deformation (Ahmadi-Naghadeh & Toker, 2018). The higher the cohesion, the higher the soil's strength and resistance to deformation (Knappett J., R.F. Craig, 2020).
- **Water table:** The position where the pore water is under atmospheric pressure (Knappett J., R.F. Craig, 2020).
- **Pore pressure:** is the pressure created by fluids like water, oil, or gas that are stuck in the small spaces between rocks or soil below the ground. This pressure affects the stability of things that are underground, like buildings or dams and the movement of subsurface fluids (Blomen, 2017).
- **Embankment:** According to (Skogen, nd), an embankment is a form of filling that intends to provide stability with its counterweight and prevent landslides. By building an artificial support or berm along the base of the slope, embankments can provide stability to a variety of slope sizes and inclinations (Meehan et al., 2013). Embankments can often be constructed using local materials, reducing costs and environmental impact. Another is that they can improve drainage and reduce erosion along the slope, which can increase stability and decrease the risk of slope failure (Meehan et al., 2013).
- **Soil type:** describes how soil is categorized according to its physical, chemical and biological characteristics (Brady & Weil, 2016). Different types of soils have different properties, such as texture, structure, composition and water holding ability. Soil types are divided into groups such as sand, silt and clay, but also more specific subgroups such as sandy clay, sandy silt, silty sand etc. Since it helps in understanding soil behaviour, erosion potential and other factors that's impact building and land use, soil classification is significant in engineering (Knappett J., R.F. Craig, 2020).
 - **Sand:** Soil that contains 85% or more sand and has particle size between 0.063 mm and 2 mm (Knappett J., R.F. Craig, 2020)

- **Filling material:** The filling mainly consists of loosely deposited sand and gravel material, often with a mixture of organic soil, as well as remnants of bricks, tiles, slag, etc.
- **Crushed material:** material that has been broken down or crushed from larger pieces into smaller ones. Examples of crushed materials include rocks, stones, gravel, and sand. It can be used to create stable foundations for buildings and infrastructure. Material used have particle size 0-200 mm.
- **Organic sediment:** is a type of sediment that consists primarily of organic material, such as dead plant or animal matter. It may also contain organic compounds, such as oils, fats, and other chemicals of organic origin (Knappett J., R.F. Craig, 2020). Within the water area, there are organic sediments with thicknesses up to approximately 17 meters in the current area. The upper part of the sediment consists of material that is similar to mud and silt with very low undrained shear strength.

2.2 Shear strength

Determining the shear strength of soils plays a crucial role in the geotechnical projects (Ahmadi-Naghadeh and Toker, 2019; Ahmadi-Naghadeh, 2016). The ability of a material to resist forces that cause its internal structure to slide against itself is measured by its shear strength. more simply, it measures a material's resistance to sliding or shearing forces that are applied perpendicular to the force's direction (Hibbeler, 2010).

A shear test, in which a sample of the material is subjected to a shear force until it fails, is typically used to determine a material's shear strength. The material's shear strength is the greatest force it can withstand before breaking (Hibbeler, 2010).

2.2.1 Drained shear strength

This term describes the shear resistance of a soil under fully drained conditions where the excess pore water pressure is dissipated. Drained shear strength is extremely important for slope stability which depends on balancing shear strength of soil and the shear stress on the slope model. Drained shear strength can be obtained through performing lab tests such as the direct shear test, the triaxial shear test, or the unconfined compression test (Das, 2014).

2.2.2 Undrained shear strength

The difference between drained and undrained shear strength is that undrained shear strength is when the excess pore water pressure is not allowed to dissipate. In many cases, when the load is too high the water in the soil is forced to dissipate which leads to the maximum shear strength of a soil before failure occurs (Das, 2014).

Undrained shear strength is an important parameter specifically for soft clays and saturated soils. Undrained loading conditions lead to soil liquefaction which significantly reduces soil strength and increases risk of failure. This parameter can be determined through the unconsolidated undrained triaxial test which is executed by applying load on a soil without allowing pore water pressure to dissipate (Das, 2014).

To investigate the stability of a slope, different tests must be conducted to determine the existing situation of the current slope. the results of the geotechnical tests will provide details such as water level, water content, shear strength etc.

2.3 Geotechnical tests performed in situ

The following are geotechnical tests performed in situ:

- **CPT-test:** Performed to determine three different parameters: tip resistance, sleeve friction and dynamic pore water pressure (Axelsson, K., & Mattsson, 2021).
- **Screw sample test:** Used to determine soil type (Axelsson, K., & Mattsson, 2021).
- **Vane shear test:** Used to determine the shear strength of soil. The test can be carried out either on site or in a laboratory and is suitable for soils such as silt, clay and sand, with maximum particle size of up to 20mm (Axelsson, K., & Mattsson, 2021).
- **CRS-test:** Used to explore the deformation properties and permeability of the soil (Axelsson, K., & Mattsson, 2021).

2.4 Short-term analysis

It analyses the stability in case its affected by the temporary immediate load applied on the soil, an earthquake for example. Short term analysis helps with predicting the conditions that might cause slope failure due to drawdown of water levels or other causes that affect stability. Using the information about strength and stability, short term analysis helps in obtaining the factor of safety. In general, this method is important for evaluating a specific situation for a soil or slope (Bell & Simons, 2013).

2.5 Long-term analysis

This analysis predicts potential deformation or failure on the soil. Slopes are subjected to long-term loading conditions where failures are more likely to occur after a long period of time. The long-term deformation and behaviour of a slope is crucial and has great impact on the factor of safety which makes the long-term parameter a very important one (Bell & Simons, 2013).

2.6 Numerical analysis

Numerical analysis for slope stability is a method used to access the stability information of a slope. The method involves mathematical modelling and simulation of scenarios in order to evaluate properties such as external loads and parameters that affect the slope. Calculation methods such as the limit equilibrium method are used to discretize the slope into smaller elements and models in order to calculate the factor of safety (Huang & Leung, 2018).

Geotechnical engineering software like PLAXIS and GEO-SLOPE GeoStudio rely heavily on numerical analysis. For complex geotechnical problems like slope stability analysis, these software packages are widely used in the industry. In slope stability analysis, PLAXIS can be used to create 2D and 3D numerical models of slopes and assess the potential for slope failure under different loading conditions. The software can also be used to evaluate different stabilization measures such as soil nails and geotextiles (Zhang, 2015).

2.6.1 Mohr-Coulomb

Mohr-Coulomb is a theory used to define the maximum shear stress that a homogeneous, isotropic, and porous material can withstand before it starts to deform. The theory assumes that shear stress depends on two factors: The cohesion between particles in the material and the internal friction of the particles. The Mohr-Coulomb theory is used to calculate the stability of soil masses and to evaluate the risk of landslides or slope failures (Knappett J., R.F. Craig, 2020).

2.6.2 SHANSEP

According to Plaxis (2016), the undrained strength of a soil is often directly related to the effective overburden stress. This relation can be represented by a ratio of shear strength to vertical effective stress. Ladd and Foot developed the Stress History and Normalized Soil Engineering Property (SHANSEP) method in 1974 to account for the stress history of the soil, specifically the over-consolidation ratio (OCR). This method allows for a more precise estimation of the soil's undrained strength and can be implemented in software such as PLAXIS LE.

The following are parameters that the SHANSEP analysis is based on:

- **OCR**, which is a measure of the degree of consolidation that a soil has undergone in the past.
Minimum shear strength, parameter that is used to design different geotechnical models. This parameter refers to the lowest shear resistance of soil girding and is obtained by the direct shear test or triaxial test.
- **Power (M)**, a parameter that is used to measure the change in stiffness or strength in a soil as the stress increases. A high m value indicates that the soil stiffness changes rapidly as stress increases and conversely, lower m value indicates more constant rate of change in stiffness.
- **Normally consolidated ratio (K)**, which is a parameter that describes the soils bearing capacity to its own weight over time. The parameter provides information about how much the soil has settled. A high k value indicates that the soil is more compressible.

2.7 Limit equilibrium method

The limit equilibrium method is widely used in geotechnics and other earth structures. The basic principle of this method is that if all forces applied on a structure are balanced, the structure remains stable. This analysis method involves dividing the structure or soil into different slices and analysing them individually. The stability of the structure is evaluated by comparing the forces acting on the structure with the resisting forces along the surface of the structure (Huang & Kim, 2008).

2.7.1 Morgenstern-Price

The Morgenstern-Price method is a popular limit equilibrium method for analyzing slope stability in geotechnical engineering. By using a slice-by-slice approach, it compares driving and resisting forces to determine a slope's stability. The ratio of driving forces to resisting forces is used to calculate the factor of safety, which shows how stable the slope is. The method has some limitations, like not considering how soils change over time and assuming uniform soil strength (Zhu et al., 2005).

2.8 Connection between research question and theoretical field

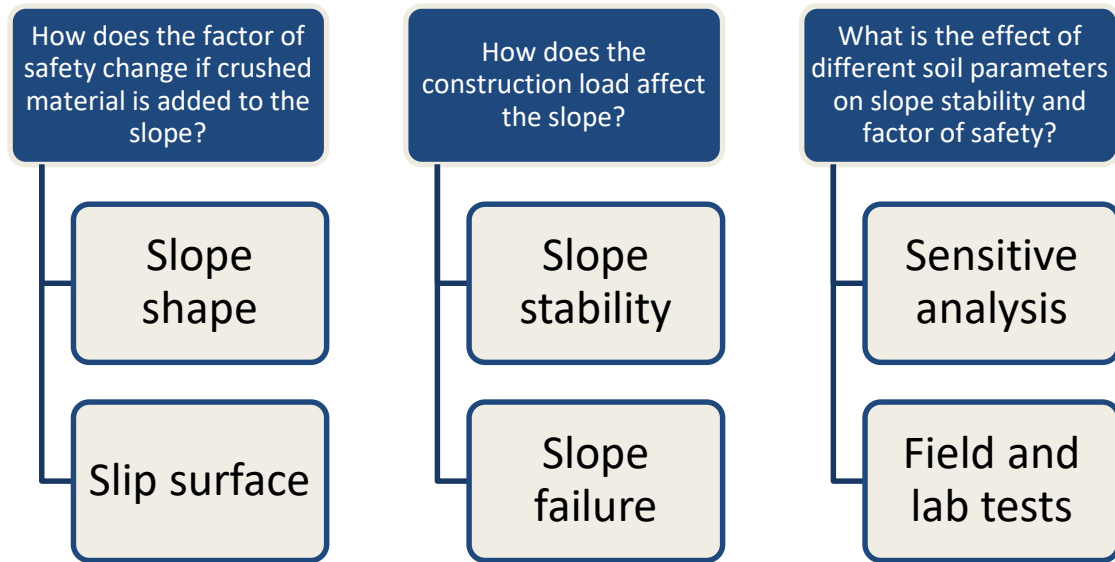


Figure 2.0 relation between issues and theory.

3 Method and implementation

The case study provided by SWECO is reviewed and related geotechnical parameters derived from related performed tests. Subsequently, slope stability has been evaluated using the geotechnical analysis methods in Plaxis LE 2D software. Analysis of the current situation of the slope have been executed to determine the changes and improvements on the slope model. The study was quantitative due to the tests that have been used. These tests include data about the situation of the soil in the area, such as water content, pore water pressure, soil type, loads, friction angle and shear strength.

The different geotechnical documents, containing lab and field tests have been analysed and used for creating a numerical model with an accepted factor of safety according to the Skredkommissionen (1995). However, the model was modified in terms of shape and soil parameters which helped finding a better factor of safety. Different parameters have been slightly modified using different combinations to help identify the parameters that created the biggest impact on the factor of safety.

PLAXIS LE is a software used to simulate and evaluate projects related to geotechnical engineering. This tool is specifically designed for slope stability analysis and analysis of groundwater flow using the limit equilibrium method. They offer efficient and thorough analysis for a wide range of conditions.

3.1 The connection between research questions and choice of methodology

The research questions have been answered through gathering data about the different parameters and their impact on the soil. Parameters have been modified and calculations have been made through the software Plaxis LE. In order to present correct calculations, the software had to be calibrated by inserting earlier parameters presented by the lab and field tests. The factor of safety has also been improved by adding crushed material to the slope body under the water table. That has been done in order to determine the amount of crushed material that was needed for achieving a factor of safety over 1,5 which is considered safe according to Skredkommissionen (1995)

In order to answer the research questions, a simulation of the slope situation and how water, load and soil parameters affect it have been carried out. A document analysis have been carried out in order to obtain measurements from previous ground surveys of the studied area. The data was fed into the PLAXIS software to create a numerical model that simulates the slope's existing and future (after measures have been taken) situation. Various hypotheses have been tested to finally find a solution that meets the factor of safety requirements for each of them.

3.2 The conditions of the study

The case study area is situated in Munksjöstaden. For several years, the municipality has been studying and planning to develop the area which is a cultural historical area near the lake Munksjö lake. As a result, Project Lappen 19 was planned by Jönköping municipality to transform the industrial area of Munksjöstaden into residency areas, workplaces, and service. As a part of project Lappen 19, walking and cycling paths are planned to be constructed on the slope of the lake, (see figure 1.0).

A section of the slope has been chosen due to the relatively weak strength values according to the document analysis. The soil type is similar in every section along the shoreline. Section D has been chosen as the section to be studied and analysed because it had a factor of safety of 1,118 which is considered dangerously low according to Skredkommissionen (1995). Therefore, the section has been analysed and studied using the Plaxis LE software. Section D currently has no support from other materials in order to stabilize the slope.

3.3 Data collection

A document analysis of the geotechnical report of the area of the case study was carried out to collect the data. The reports have been created by the consulting company studying the area and obtained from the municipality website. The quantitative data obtained from the reports have been used to create a simulation to study the slope stability. The data gathered will provide information about the soil and the geotechnical situation in the project. These reports contain parameters provide by tests such as CPT-tests, Screw sample test, Vane shear test and CRS-test.

3.4 Work process



Figure 3.0 Work process for this study.

- **Review of previous research**

In earlier stages of the current study, previous research and tests have been examined. The purpose of this examination was to gain insight into the geotechnical data that are essential for determining an appropriate factor of safety for the slope. The analysis provided by the documents helped to better understand the context and background of the project, which also helped to identify the specific requirements and parameters for the construction of the cycling path. These initial investigations were crucial for laying the foundation for the current study and ensuring that all relevant factors were taken into consideration during the subsequent phases of the study.

- **Execution**

In this study, a case study method is used with document analysis as the data collection technique. The aim of the document analysis is to collect data and information from various documents, including soil investigation reports, to create an identical section (Section D) that appears in the documents. The selected section is then modeled using the same parameters and analysis methods to achieve a similar factor of safety.

To reinforce the slope, embankment measures were taken using a specific material type (crushed material). After a satisfactory factor of safety was achieved according to Skredkommissionen (1995), changes were made to different parameters in the various soil layers to determine which parameter has the greatest impact on the factor of safety in that specific slope. By using case study and document analysis as the method and data collection technique, this research can provide valuable insight into how different parameters affect the factor of safety in slopes.

- **Analysis**

The analysis is based on examining the different sections generated using the Plaxis LE 2D software. After each parameter change, a new analysis of the slope was conducted to obtain a factor of safety that is based on the modifications made.

Through this approach, the study was able to determine the effect of different parameters on the factor of safety of the slope. The use of Plaxis LE 2D software for analysis allowed for a comprehensive evaluation of the slope's behavior under different conditions. By changing parameters such as Cohesion, friction angle, and reinforcement measures etc. the study was able to assess the influence of each parameter on the slope's stability.

- **Results**

Throughout the process of completing our thesis project, one of the most important aspects was documenting all results. The results from the use of the crushing material have been presented as a model of a slope with the approved factor of safety, considering the conditions of the water situation and the soil properties in this project.

3.5 Method of analysis

The first step of the method analysis was to perform a case study with data collection techniques such as document analysis in order to achieve deeper understanding of the project generally and the slope model specifically. Thereafter, based on the data from lab and field tests gained from the geotechnical report, soil parameters were chosen in order to insert them into the Plaxis LE software and create a similar model of the slope section. After the analysis was performed, it was evident that the result of the analysis is calibrated and comparable to the results from the consulting company. After various measures have been taken, different analysis with different scenarios were executed and through that, primary data is obtained and is sufficient to answer the research questions of the study.

4 Results and analysis of empirical data

The following images and graphics represent the existing condition in the selected slope created based on the data obtained through the document analysis. Each figure shows a description of a specific analysis situation using different parameters.

The following section (section D) is representing a slope starting at a height of +90,3 meters and ending at a height of +50 meters. The slope below contains different layers of soil such as sand, filling material, organic sediment, and crushed material. The following figure shows the existing state of the slope with a factor of safety of 1.118. Figure 4.0.

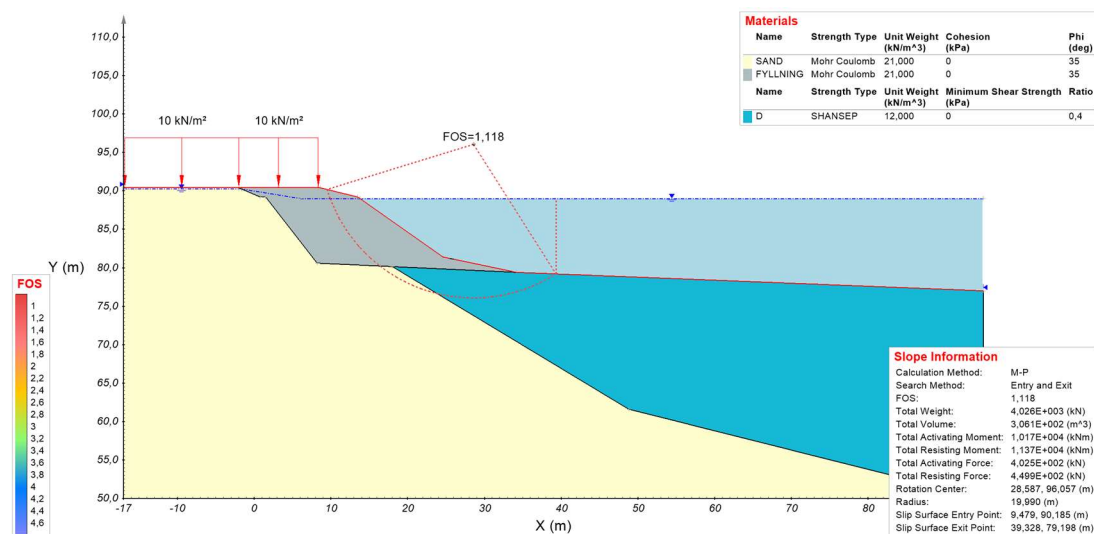


Figure 4.0 the slope section in its existing state with a factor of safety 1,118.

4.1 How does the factor of safety change if crushed material is added to the slope as an embankment?

A section has been created in Plaxis LE to model crushed material on the slope in the structure of an embankment in order to observe the behavior of the factor of safety and improve it. The slope model without any modification had a factor of safety of 1,118 which is substantially low compared to the standards of Skredkommissionen (1995), figure 4.0 Also, the slip surface appeared to be wider which indicated that if a failure would occur, a bigger part of the slope model will collapse.

A few attempts to increase the factor of safety have been performed where the organic material have been affected through removing part of it in order to support the slope from the lower part. That showed a factor of safety of 1,658 and the slip surface decreased significantly. For the other attempt, the crushed material was added to the slope model without affecting the organic material which showed a factor of safety of 1,651 which is lower but comparing the amount of material added is more effective for the stability, figure 4.1.

The cohesion (0), unit weight (21kNm^2) and friction angle (42°) for the crushed material were assumed to certain values according to the site investigation report provided by the consulting company. The crushed material has a rock cleavage of 0-200 mm.

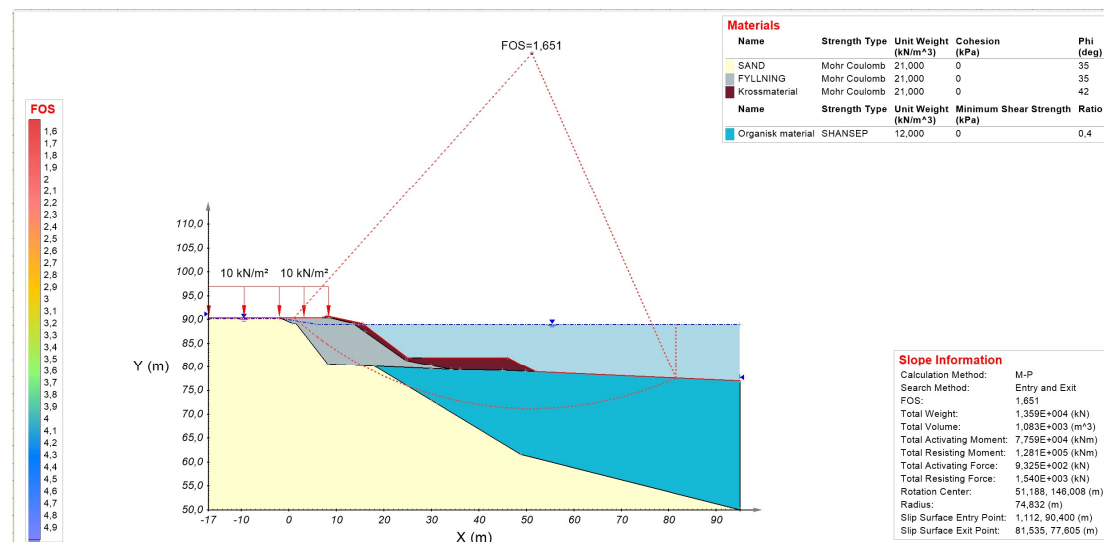


Figure 4.1 the slope after reinforcement measures have been taken with a factor of safety of 1,651.

4.2 What is the effect of different soil parameters on slope stability and factor of safety?

For this question, different soil types with different parameters have been tested and modified in order to study the behaviour of the factor of safety and to increase the stability of the slope.

Soil type: SAND (C=0 kpa)

Sand has no cohesion and in order to observe the differences of the factor of safety, the friction angle has been altered. As shown in figure 4.2, the factor of safety increases in relation to increases internal friction in the soil. The average increase of factor of safety between the friction angles of 25° and 35° is approximately 0,216 per 5° of internal friction. When the friction angle is 35° or more, the increase in the factor of safety gradually loses the constant rate of impact on the factor of safety. As seen in figure 4.2, the increase of factor of safety between angle 35° and 38° is 0,007 which is considered to be very minor.

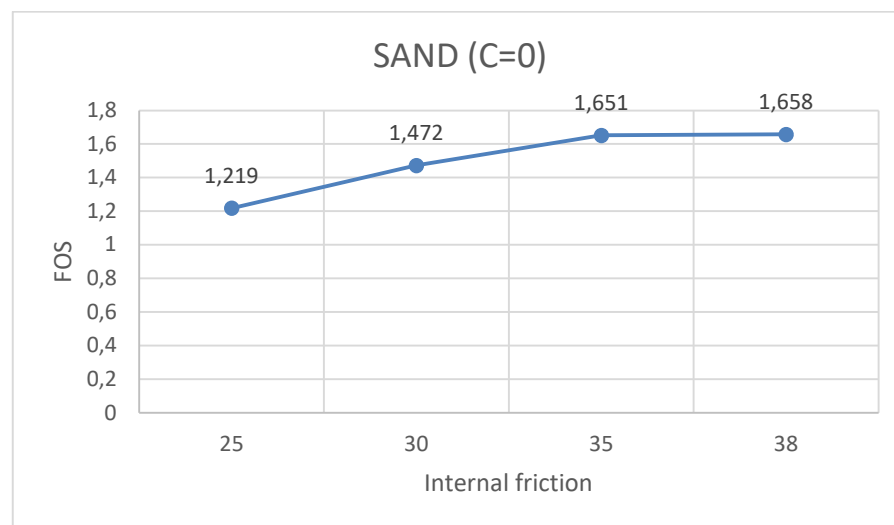


Figure 4.2 the relationship between the factor of safety and the friction angle of sand material.

Soil type: FILLING

Two different friction angles have been used in order to study the impact of the filling material on the slope stability. The parameter that has been tested in this case is the cohesion of the soil.

For the first test, figure 4.3, the existing friction angle of (35°) has been used and the cohesion has been changed in an interval between 0 – 15 kpa. The starting value of the factor of safety is 1,651. When the cohesion was increased from 0 to 10 kpa, a constant change in the factor of safety has been observed. The average increasing value is calculated to 0,0535 where the factor of safety at cohesion of 10 kpa was 1,758. At cohesion of 15 kpa, a very minor improvement (0,003) of the factor of safety has been observed.

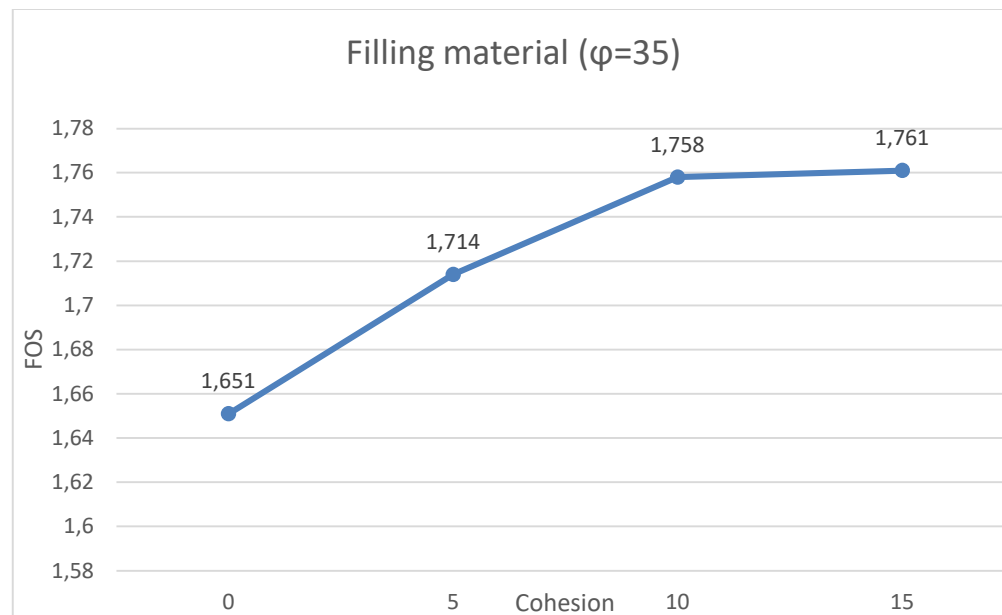


Figure 4.3 the relationship between the factor of safety and cohesion appears in the fill material at a constant friction angle of 35 degrees.

For the second test, figure 4.4, the new proposed friction angle of (20°) has been used. The cohesion value interval is identical to the earlier test. The starting value of the factor of safety in this case is 0,929 which is a significant difference compared to the starting value of the first test. The average increase of factor of safety between 0 and 10 kpa cohesion is calculated to 0,276. At 15 kpa cohesion, the increase value is 0,097.

For both tests, the same pattern is noticed where the increase value of factor of safety becomes minor when the cohesion reaches 15 kpa. Also, the friction angle affected the starting value of the factor of safety majorly as seen in figure 4.3 and figure 4.4. The increase of factor of safety when the friction angle is 35° is calculated to 0,11 (1,651 to 1,761). However, the increase when the friction angle is 20° is significantly higher and is calculated to 0,649 (0,929 to 1,578).

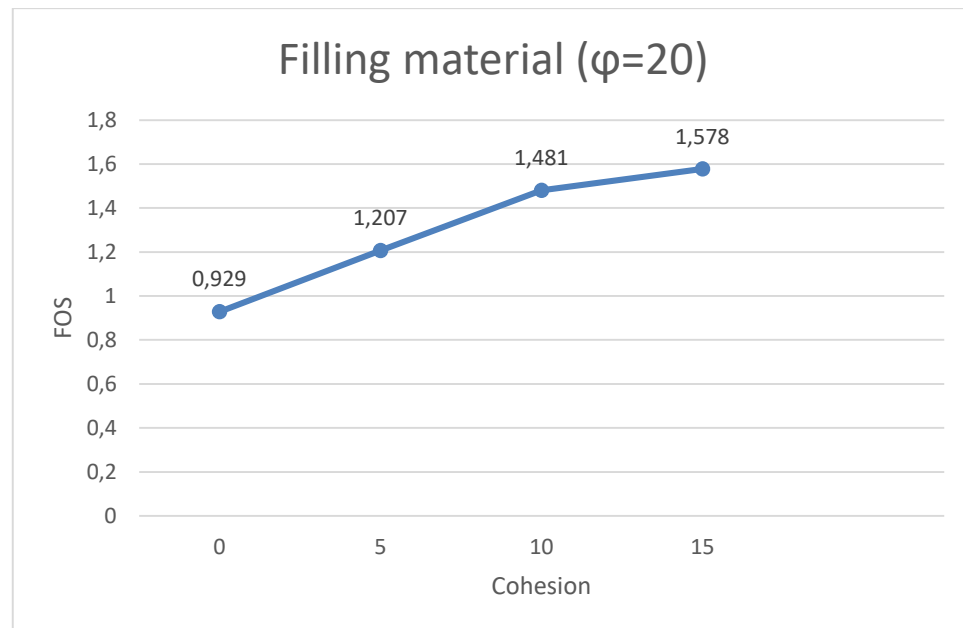


Figure 4.4 the relationship between the factor of safety and cohesion appears in the fill material at a constant friction angle of 20 degrees.

Soil type: CLAY

Two different friction angles have also been used for this analysis of clay material. The parameter that has been studied and analysed is the cohesion of the soil.

For the first test of clay material where the friction angle is 25° , figure 4.5, the change of cohesion showed a slight positive impact on the factor of safety. the initial value of the factor of safety was decreased by 0,44. The cohesion has been changed in an interval between 0 – 15 kpa. The final value of the factor of safety is 1,770.

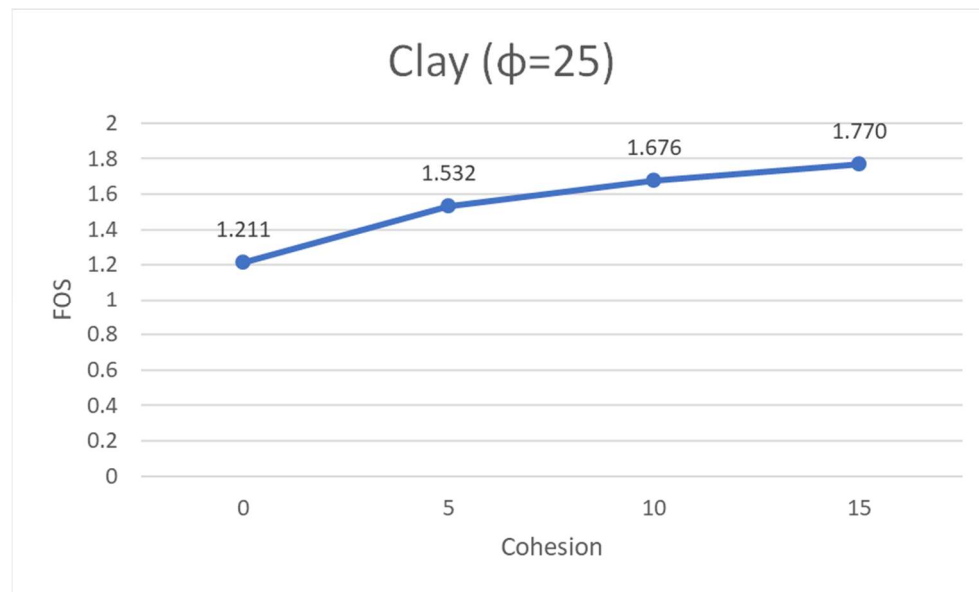


Figure 4.5 the relationship between the factor of safety and cohesion appears in the clay material at a constant friction angle of 25 degrees.

The friction angle was proposed to be 32° for the second test, figure 4.6, the interval values of the soil cohesion is identical to the earlier test (0 – 15 kpa). A change of the initial value has been noticed in the factor of safety as it decreased by 0,098 after a new proposed friction angle. The change of factor of safety has been slightly constant as it increases for every increase of the cohesion. The final value of the factor of safety is 1,842.

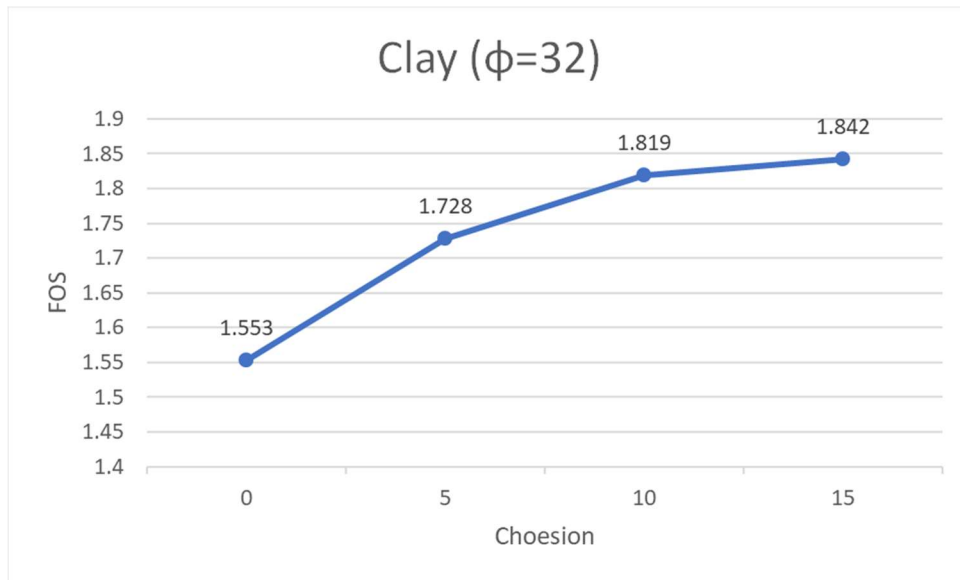


Figure 4.6 the relationship between the factor of safety and cohesion appears in the clay material at a constant friction angle of 32 degrees.

Soil type: ORGANIC SEDIMENT

For this test, three different parameters have been changed in order to find out which one has the biggest impact of the stability and factor of safety.

Normally consolidated ratio, K figure 4.7

The starting of the factor of safety in this scenario is 1,073 with a k value of 0,2. The average change in factor of safety between 0,2 and 0,4 k is calculated to 0,289. It is seen that the greatest impact is between 0,2 and 0,3 k and the increase between 0,2 and 0,4 is approximately 50% lower. After the k value of 0,4, no significant change is seen. The increasing value is 0,007 and after that the factor of safety is a constant 1,658.

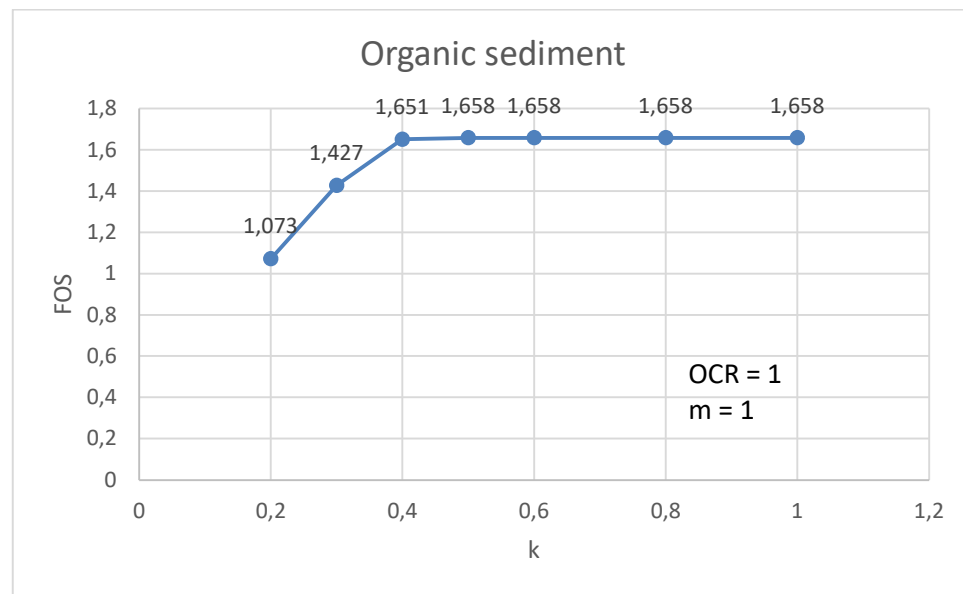


Figure 4.7 the relationship between the parameter k (normally consolidated ratio) and the factor of safety appears in organic sediment with a constant OCR (overconsolidation ratio) and m (power).

Minimum shear strength (kPa), figure 4.8

The starting value of the factor of safety in this case is 1,651 with a minimum shear strength of 0 kPa. When the minimum shear strength has a value of 1 kPa, the factor of safety increases with 0,042 and stays constant for up to 10 kPa. The final value of the factor of safety is calculated to 1,693.

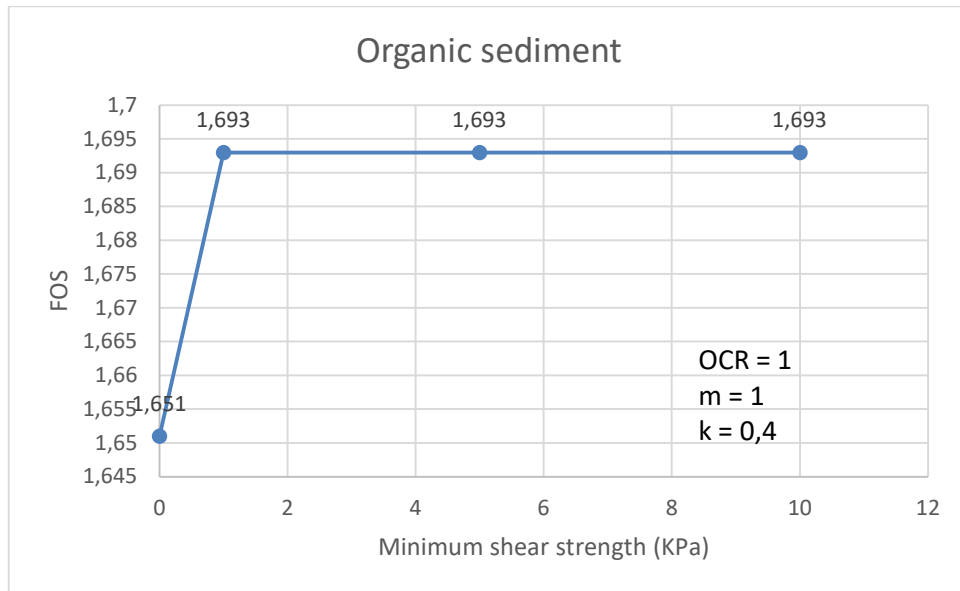


Figure 4.8 the relationship between the minimum shear strength parameter and the factor of safety appears in organic sediment with constant OCR (overconsolidation ratio), k (NCR), and m (power).

SHANSEP parameter M (Power), figure 4.9

The value of the factor of safety in this case is 1,651 and no change has been noticed on the factor of safety nor the stability of the model.

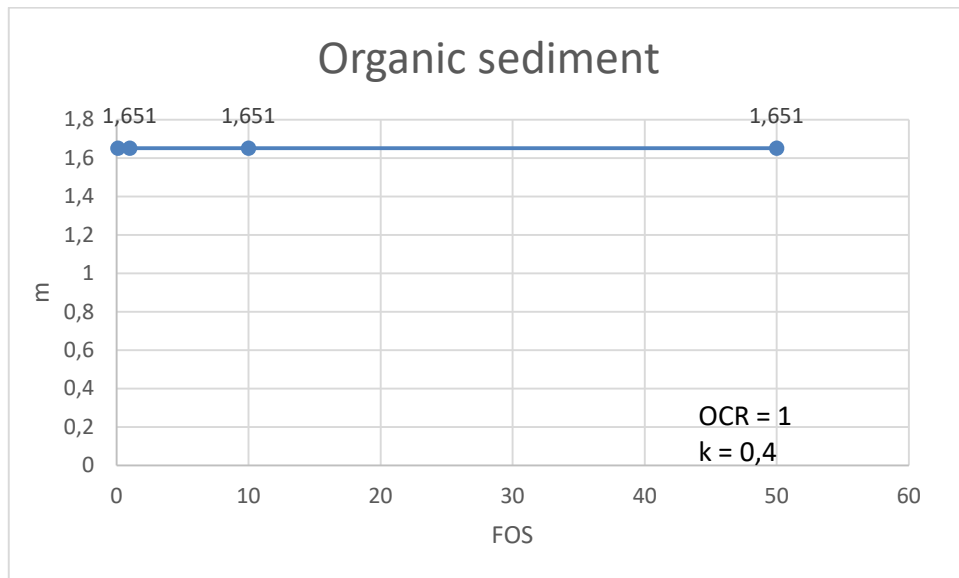


Figure 4.9 the relationship between the parameter m (power) and the factor of safety appears in organic sediment with constant OCR (overconsolidation ratio) and k (NCR).

4.3 How does the construction load affect the slope?

To answer this question, different loads with a range between 0 – 25 kPa have been applied on the slope model after the application of reinforcement measures. The results presented in figure 4.10 show that there is no difference in the factor of safety when the applied load is less than 5 kPa. However, the factor of safety starts decreasing when the applied load exceeds 5 kPa. A minimal deviation is seen to occur with the factor of safety as the load increases.

The average value that the factor of safety is affected by is calculated to be 0,0145 for every 5 kPa load increase. This difference could be seen as insignificant due to the minor change in factor of safety. Furthermore, the Skredkommissionen (1995) requirements are fulfilled even with 25 kPa of maximum load on the slope model. The factor of safety at 5 kPa was 1,665 and at 25 kPa was 1,607. This marginal difference gives the possibility to apply more load without significantly disturbing stability.

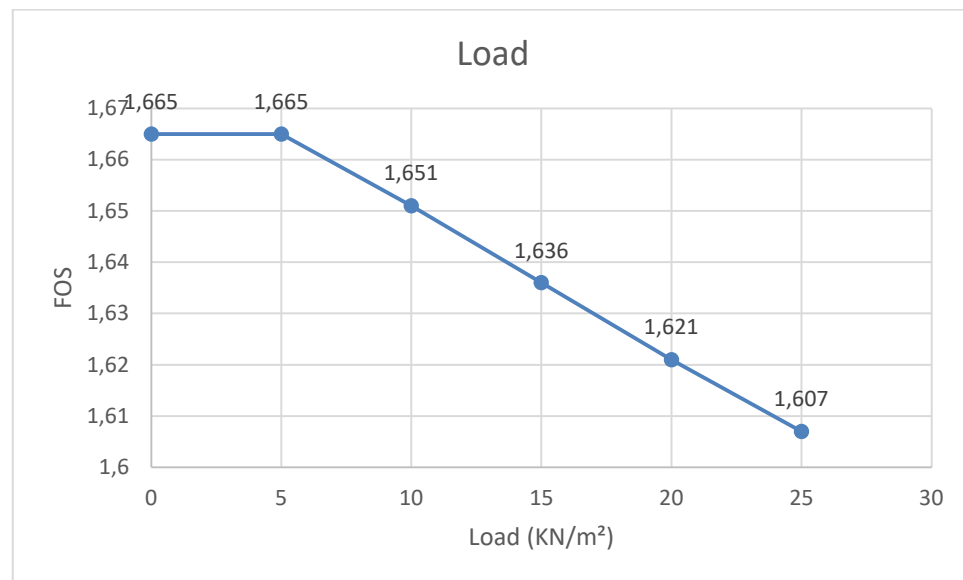


Figure 4.10 the relationship between the load and the factor of safety appears.

4.1 Analysis of empirical data

In the following chapter, the results of case study findings will be analysed with regard to the questions of issue for this study.

- **How does the construction load affect the slope?**

To answer this question, different loads with a range between 0 – 25 kPa have been applied on the slope model after the application of reinforcement measures. The results presented in figure 4.10 show that there is no difference into the factor of safety when the applied load is less than 5 kpa. However, the factor of safety starts decreasing when the applied load exceeds 5 kpa. A minimal deviation is seen to occur with the factor of safety as the load increases.

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- **How does the factor of safety change if crushed material is added to the slope as an embankment?**

A section has been created in Plaxis LE to model crushed material on the slope in the structure of an embankment in order to observe the behavior of the factor of safety and improve it. The slope model without any modification had a factor of safety of 1,118 which is substantially low compared to the standards of Skredkommissionen (1995), figure 4.0 Also, the slip surface appeared to be wider which indicated that if a failure would occur, a bigger part of the slope model will collapse.

A few attempts to increase the factor of safety have been performed where the organic material have been affected through removing part of it in order to support the slope from the lower part. That showed a factor of safety of 1,658 and the slip surface decreased significantly. For the other attempt, the crushed material was added to the slope model without affecting the organic material which showed a factor of safety of 1,651 which is lower but comparing the amount of material added is more effective for the stability, figure 4.1.

The cohesion (0), unit weight (21kNm²) and friction angle (42°) for the crushed material were assumed to certain values according to the site investigation report provided by the consulting company. The crushed material has a rock cleavage of 0-200 mm.

- **What is the effect of different soil parameters on slope stability and factor of safety?**

For this question, different soil types with different parameters have been tested and modified in order to study the behaviour of the factor of safety and to increase the stability of the slope.

Soil type: SAND (C=0 kpa)

Sand has no cohesion and in order to observe the differences of the factor of safety, the friction angle has been altered. As shown in figure 4.2, the factor of safety increases in relation to increases internal friction in the soil. The average increase of factor of safety between the friction angles of 25° and 35° is approximately 0,216 per 5° of internal friction. When the friction angle is 35° or more, the increase in the factor of safety gradually loses the constant rate of impact on the factor of safety. As seen in figure 4.2, the increase of factor of safety between angle 35° and 38° is 0,007 which is considered to be very minor.

Soil type: FILLING

Two different friction angels have been used in order to study the impact of the filling material on the slope stability. The parameter that has been tested in this case is the cohesion of the soil.

For the first test, figure 4.3, the existing friction angle of (35°) has been used and the cohesion has been changed in an interval between 0 – 15 kpa. The starting value of the factor of safety is 1,651. When the cohesion was increased from 0 to 10 kpa, a constant change in the factor of safety has been observed. The average increasing value is calculated to 0,0535 where the factor of safety at cohesion of 10 kpa was 1,758. At cohesion of 15 kpa, a very minor improvement (0,003) of the factor of safety has been observed.

For the second test, figure 4.4, the new proposed friction angle of (20°) has been used. The cohesion value interval is identical to the earlier test. The starting value of the factor of safety in this case is 0,929 which is a significant difference compared to the starting value of the first test. The average increase of factor of safety between 0 and 10 kpa cohesion is calculated to 0,276. At 15 kpa cohesion, the increase value is 0,097.

For both tests, the same pattern is noticed where the increase value of factor of safety becomes minor when the cohesion reaches 15 kpa. Also, the friction angle affected the starting value of the factor of safety majorly as seen in figure 4.3 and figure 4.4. The increase of factor of safety when the friction angle is 35° is calculated to 0,11 (1,651 to 1,761). However, the increase when the friction angle is 20° is significantly higher and is calculated to 0,649 (0,929 to 1,578).

Soil type: CLAY

Two different friction angles have also been used for this analysis of clay material. The parameter that has been studied and analysed is the cohesion of the soil.

For the first test of clay material where the friction angle is 25° , figure 4.5, the change of cohesion showed a slight positive impact on the factor of safety. the initial value of the factor of safety was decreased by 0,44. The cohesion has been changed in an interval between 0 – 15 kpa. The final value of the factor of safety is 1,770.

The friction angle was proposed to be 32° for the second test, figure 4.6, the interval values of the soil cohesion is identical to the earlier test (0 – 15 kpa). A change of the initial value has been noticed in the factor of safety as it decreased by 0,098 after a new proposed friction angle. The change of factor of safety has been slightly constant as it increases for every increase of the cohesion. The final value of the factor of safety is 1,842.

Soil type: ORGANIC SEDIMENT

For this test, three different parameters have been changed in order to find out which one has the biggest impact of the stability and factor of safety.

- **Normally consolidated ratio, K figure 4.7**

The starting of the factor of safety in this scenario is 1,073 with a k value of 0,2. The average change in factor of safety between 0,2 and 0,4 k is calculated to 0,289. It is seen that the greatest impact is between 0,2 and 0,3 k and the increase between 0,2 and 0,4 is approximately 50% lower. After the k value of 0,4, no significant change is seen. The increasing value is 0,007 and after that the factor of safety is a constant 1,658.

- **Minimum shear strength (kPa), figure 4.8**

The starting value of the factor of safety in this case is 1,651 with a minimum shear strength of 0 kpa. When the minimum shear strength has a value of 1 kpa, the factor of safety increases with 0,042 and stays constant for up to 10 kpa. The final value of the factor of safety in calculated to 1,693.

- **SHANSEP parameter M (Power), figure 4.9**

The value of the factor of safety in this case is 1,651 and no change has been noticed on the factor of safety nor the stability of the model.

5 Discussion

In this chapter, results, methods, and delimitations will be analysed and discussed.

Different factors that might have affected the result of this study will be discussed and reasoned. The result that has been presented will lead the study to a conclusion of which parameter creates the biggest difference in the safety of a slope model. The result thus includes a comparison between different soil parameters. One of the factors that have been limiting the accuracy of the result is that the different sections of the slope have been created in GeoStudio which is calibrated differently compared to Plaxis LE.

This analysis approach can provide valuable insights into the design and evaluation of slopes in geotechnical engineering. By evaluating the impact of different parameters on the factor of safety of the slope, engineers can make informed decisions about the most effective measures to implement to reinforce and stabilize slopes.

5.1 Result discussion

The result of this study helps understand the behaviour of the factor of safety which is very important in order to determine the parameter with the biggest influence on the stability. The results of this quantitative study immediately showed clear results about which of the soil parameters has had the biggest effect on the factor of safety.

As shown in the empirical data the friction angle showed a major effect on the stability of the slope structure and the factor of safety. It is clear that there is a big impact on the factor of safety when the friction angle of the filling material is changed from 35° to 20° with an increasing value of 0,649. That result shows that the friction angle in this particular section of the slope in the filling material is mostly dominant regarding the change of factor of safety compared to other parameters that have been analysed. Numerical analysis has been the most important method in order to determine the crucial parameter for slope stability.

Regarding how the result of this study relates to Das (2014), the result is mostly predictable and that is due to the impact and importance of internal friction in a soil. Das (2014) agree that the factor of safety is impacted widely by the shear strength which is calculated using the internal friction value of the slope.

It has been shown that the embankment has shown a positive impact on the slope stability. It also showed that it is not the amount of material that is mostly critical for increasing the factor of safety, it is rather the placement of the material. The embankment works as a support for the slope model which prevents it from failing, as Meehan et al. (2013) describes. This has been proven by seeing the difference in factor of safety when the embankment is added to the model. More simply, the embankment works as a counterweight for the load coming from the slope.

5.2 Method discussion

The methodology during the work with this study has been generally simple to walk through. One of the challenges that impacted the study majorly is the calibration of Plaxis LE software and the measuring of different sections using Bluebeam. This is because of the lack of exact measurement in the sections provided by the consulting company. Therefore, the solution for that was to insert the slope section model into Bluebeam and then calibrate it by measuring according to the scaling of the section.

However, the measurements made in Bluebeam seem to be accurate for the majority of instances and that is due to the scale calibration made before measuring. Another challenge that affected gaining an identical section to the one made by the consulting company is that two different analysis softwares have been used. That caused the starting value of the factor of safety to be slightly different. On the other hand, the existing differences did not compromise the final result and the particular reason for this circumstance is that only the change of factor of safety is being measured and not the value itself.

By using different charts, we were able to clearly show the relationship between the factor of safety and the changed parameters. It was important for us to be transparent with our methods to ensure that our results were reliable. Finally, the use of charts was an effective method to present our data in an easily understandable way. This research approach can contribute to the field of geotechnical engineering, particularly in the design and evaluation of slopes. The case study approach allows a thorough investigation of the specific slope under consideration, while the document analysis technique enables the researcher to gather relevant information from various sources.

When performing a study that includes making measurements and conducting analysis, the result is expected to be affected by other error sources such as human error, analysis method and assumption of values of different parameters. In geotechnics, many methods of analysis are used to determine the factor of safety or stability of a certain geotechnical structure. Nonetheless, if this analysis would be performed using another analysis method, the result may vary depending on the method and on the software used. This has been a limitation for this study but also provides higher reliability for the limit equilibrium method as the focus area in this study.

6 Conclusions and suggestions for further research

The performed analysis on the slope model proves that the load on the slope surface, in this case the load of the cycling path, does not affect the stability of the slope majorly. The slope model in this case shows higher bearing capacity for up to at least 25 kpa while keeping a factor of safety that is accepted according to Skredkommissionen (1995) standards. This result shows that if a stable slope model is created, the load bearing capacity increases.

In terms if the effect of adding crushed material to the slope model, it shows a positive impact depending mainly on how the crushed material is placed. It has been shown in the analysis section that it is the positioning of the crushed material is the most important and not the amount added. There has been no need to interfere with the organic material under the slope. Finally, the factor of safety showed an improvement and change in behavior when a smaller amount of crushed material was added to the slope. That change is a result of the placement of the crushed material which concludes that the biggest impact is created by the placement and not the amount of material.

The numerical analysis performed showed many different results on how the factor of safety is impacted by changing different parameters. The initial and final factor of safety for each analysis have been compared which in the end concluded that the internal friction (friction angle) of the filling material showed the biggest impact on the slope stability and the factor of safety. In conclusion, the friction angle is proved to be one of the most important and impactful parameters that affect the factor of safety. this could also be connected to the fact that the bearing capacity of a soil is calculated using the friction angle. Furthermore, this study shows coherence with earlier research regarding the impact and the importance of the friction angle.

6.1 Suggestion for further research

Geotechnics is a very wide and complicated field that could be studied in a very high level. In this study, only the limit equilibrium method has been used to perform the analysis. The same analysis could be performed using other softwares or other methods such as the finite element method etc. however, it must have the same conditions. The results of this study apply only to this specific slope with these specific conditions. The biggest difference would probably be noticed if other methods such as Bishop's method, Janbu's method and Spencer's method. These are different limit equilibrium methods mentioned. These different methods would increase the reliability of this kind of study.

What could have been done better for the work process and the methodology of this study is to create more complicated analysis where more complicated combinations are used to determine how different parameters work together in order to either improve or compromise the factor of safety.

Apart from that, this analysis proves once again that the friction angle is a crucial parameter. It also proves that the factor of safety is dependent mostly on the bearing capacity as well as the positioning of added crushed material. Finally, there is no critical

need to perform further analysis on how the factor of safety behaves based on different friction angles because as mentioned earlier in this chapter, there have been many studies performed that proved that the internal friction of a soil is crucial for stability.

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