



JÖNKÖPING UNIVERSITY

*School of Engineering*

# **Street-lights LED Lens Design Optimization using Machine Learning**

**PAPER WITHIN** *Product development*

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## Abstract

Street lighting continues to face a range of challenges on roads with twisted and curved shapes. To address this issue, this study proposes that machine learning algorithms, specifically deep neural networks and multi-output random forests, can be utilized to find the optimal LED lens design. The use of machine learning can significantly speed up the design process, reducing time and cost for lens manufacturing companies. Currently, it is difficult to achieve the desired light distribution with a lens design due to the complexity of the design and the long optimization process, which typically requires the use of three expensive and complicated software programs and intensive human supervision over a period of several weeks. By streamlining this process through the use of machine learning, factories can save time and money while also improving comfort, reducing glare, minimizing visual discrimination, and maximizing illumination performance for drivers.

This study employs a mixed-method approach in order to achieve a machine learning model structure with accurate performance. That succeeds at giving a solution for the addressed lighting problem for streetlight LED lens design optimization. The ultimate goal is to replace the existing process and lens manufacturing models, which are inadequate in addressing these issues, with a more effective solution. Specifically, the study aims to find the geometric parameters of the lens shape that produce the desired size and shape of the illumination distribution. Optimizing the lens design is crucial for minimizing light pollution and energy waste.

keywords: lens optimization, machine learning, deep learning, neural network, multi-output, regression.

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# 1 Introduction

Ensuring proper lighting on roads is a critical concern worldwide, as it impacts the comfort and safety of both drivers and pedestrians. In order to provide the best possible lighting experience, it is essential to take into consideration all aspects of road lighting. As demonstrated in Figure 1.1, the use of a lens can significantly impact the distribution of light on the road. It is important to design an optimal LED lens in order to achieve the desired lighting outcomes.

To design an effective LED lens, it is necessary to understand the setting specifications and principles of lens design. This includes understanding the purpose of creating specific shapes and the concept of total internal reflection (Yi Luo, Z.F. and Han, Y. 2017). As it is critical for achieving the desired level of output uniformity, efficiency, and illumination.

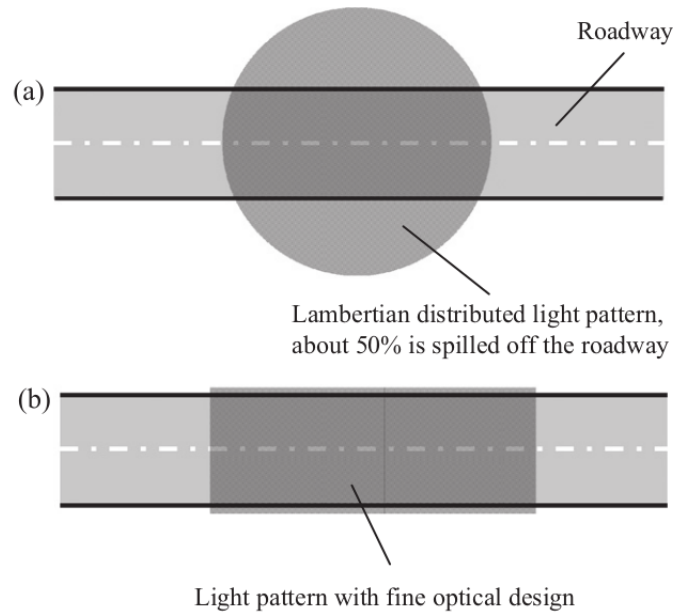


Figure 1.1: Light pattern with and without using a lens

The accurate design of LED lenses is essential for achieving the desired level of visual acuity. Machine learning algorithms can significantly improve the accuracy of making a lens design (Abbasi, M.A. et al. 2022). This can be achieved by utilizing sophisticated modeling geometry and large volumes of training data. These algorithms have been successfully applied in a variety of domains to address complex multivariate and nonlinear problems. The performance of LED lenses has greatly improved the distribution of light. This helps to address issues such as light pollution and visual discrimination. By using machine learning, it is possible to achieve precise beam distribution and light patterns more easily, which can enhance visibility, comfort, and the overall performance of the LED lens in lighting.

LEDs are a smart choice for a variety of lighting applications, including street lights,

spotlights, interior lighting, and car lights. However, the use of LEDs alone may not be sufficient to achieve the desired intensity and focus of light. By adding an optical component, such as a lens, it is possible to create an integrated lighting system that can adaptively emit light. Figure 1.2 demonstrates the impact of using different lenses with the same LED source and area. This shows that different lens designs can produce different light spot focuses.

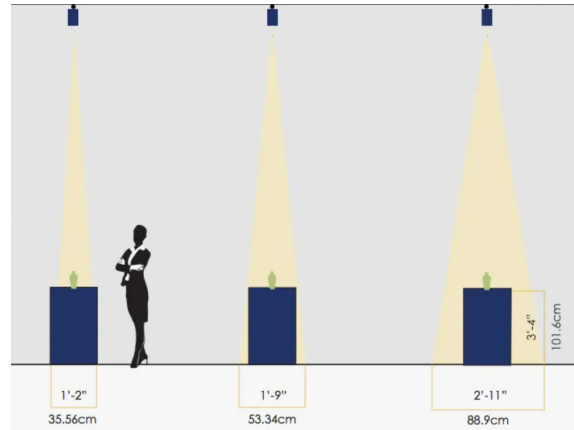


Figure 1.2: The effect of using different lenses

Manufacturing of LED lenses involves several stages and various factors can affect the distribution of light. Some of these factors are:

- Water bubbles inside the lens
- High temperatures that may cause cracking or discoloration
- Broken bond wire, die-attach delamination
- Cracking

The performance of LED lenses and systems depends on multiple factors such as thermal and mechanical loading. The sophisticated nature of LED lens light distribution goes beyond simple fluctuations in luminous output. This study aims to examine the relationship between geometric parameters and the distribution of light by LED lens products.

## 1.1 Problem and research questions

### 1.1.1 Problem

LED lenses are an important aspect of modern lighting technology, as they can help reduce energy consumption and contribute to a more sustainable environment. In order to

maximize the benefits of LED lenses, it is necessary to carefully consider the distribution of light and ensure that it is evenly distributed on the appropriate surfaces. The process of configuring LED lens light distribution can be complex and requires consideration of various elements, including the dimensions of the street, the intensity of the light, and other factors. If any of these elements are overlooked, it can compromise the overall quality of the LED lens light distribution. In recognition of the importance of LED lenses, many governments around the world have invested heavily in energy-efficient LED technology and related products, as it offers an opportunity to reduce environmental and financial burdens.

The design process can be complex, requiring specialized software, materials catalogs, and an in-depth understanding of surface properties. Once all necessary properties have been identified, the optimization process begins in order to create a lens that is suitable for a specific road scenario (Chen, W.-C. et al. 2012). However, this process can be time-consuming and require intensive human supervision. This makes the use of machine learning algorithms an attractive option to improve the efficiency and accuracy of the design process.

The current method used in the manufacturing industry involves a multi-step process that involves careful consideration and expert knowledge. The first step involves manually selecting the most appropriate base-lens model by human experts who possess the necessary expertise. The second step involves undergoing an optimization process in order to find the best design. This optimization process can be quite time-consuming, taking several weeks to complete. It is also costly due to the use of three specialized software programs. The first software program is used for simulating designs and analyzing their performance and stress levels. The second software is used to optimize manufacturing processes. The third is used to design and optimize the performance of LED lighting systems, while taking into consideration factors such as luminous intensity, luminance, and illuminance.

### **1.1.2 Aim and objective**

This research aims to investigate the potential of integrating machine learning and artificial intelligence techniques into the optimization process of LED street light lens designs. The ultimate goal is to develop an algorithm that can effectively replace current optimization methods and produce a lens design that is adaptable to various road scenarios. In order to achieve this, the study will examine the impact of various geometric parameters on lens designs.



### 1.1.3 Research questions

**Can AI/ML be used to improve the area of lighting and optical designs?**

- *What is the most suitable machine learning algorithm to use for lens optimization to achieve efficient light distribution?*
- *Find the possibility of using the multi-output ML model in a real world problem like our optical design optimization problem and replace the current manual lens optimization process?*

## 1.2 Limitations

While this research can potentially be applied to various environments, such as office spaces, factories, and homes, this is not the primary focus of the study. Instead, the research specifically examines the optimization of street-light lens designs. This research will not cover the full scope of the solution due to a lack of data. Our data-set does not have sufficient samples to accurately represent the final optimization of the product. We did not reach the final stage of the optimization process during the data gathering period, so the final product is not present in the current data-set. As a result, this research only covers one road scenario

## 1.3 Outline

This thesis is structured as follows. The first chapter introduces the fundamental concepts of LED lenses and the real-world challenges faced by this technology in the field of street lighting. The second chapter provides a theoretical overview of artificial intelligence and the various types of LED lenses. We will delve into the details of specific models that are also used in this thesis.

In the third chapter, we describe the architecture of our models in detail and explain how they operate. The fourth chapter presents the results of the model's performance and includes an analysis of the obtained values. Finally, in chapter five we conclude with a discussion of the findings, including the conclusion and proposed future plans.

## 2 Theoretical background

### 2.1 What is AI?

AI, or artificial intelligence, is the capability of an algorithm to make decisions based on a large set of data. The accuracy of these decisions is determined by the type of data the algorithm was trained on. The widespread use of AI in various industries can be attributed to its ability to process and analyze large data sets and make accurate decisions. AI can also be utilized to automate tasks that may be too complex or dangerous for humans to perform, resulting in both time and life-saving benefits (Du-Harpur, X. et al. 2020). The ability of AI to automate specific tasks enables individuals to concentrate on other vital activities, leading to improved efficiency, advancement, and the ability to explore creative possibilities.

AI is a versatile technology that is applied or researched in various fields such as computer vision, natural language processing, robotics, machine learning, and deep learning. Within machine learning, there are two main approaches: supervised learning and unsupervised learning. Supervised learning involves training a model on a labeled data-set, where the correct output or label is provided for each input. Common algorithms used in supervised learning include decision trees, random forest, and neural networks. On the other hand, unsupervised learning involves training a model on an unlabeled data-set, with the goal of discovering patterns or relationships within the provided data-set. Algorithms commonly used in unsupervised learning include k-means, and hierarchical clustering (Berry, M.W. et al. 2020).

Artificial intelligence (AI) holds a great deal of promise for solving a diverse range of challenges, such as data analysis, automating complex tasks, Optimizing processes, image improving, and video analysis.

### 2.2 Deep neural network

Deep neural networks are a subset of machine learning, which in turn is a subset of artificial intelligence. They are inspired by the structure of the human brain, and the architecture that is used in deep learning is known as an artificial neural network. In this type of machine learning, the neural network extracts features without human intervention. To achieve good results with deep learning, it is necessary to have a large data-set to train the algorithm. A diagram of a deep neural network can be seen in Figure 2.1. It comprises of input and output layers, n hidden layers, and varying numbers of neurons in each layer (Liu, W. et al. 2017).

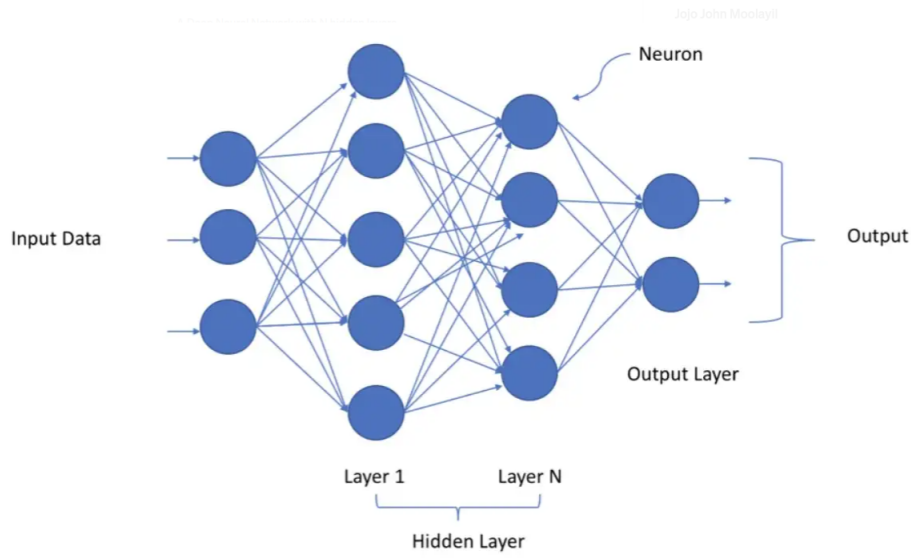


Figure 2.1: The structure of DNN

## 2.3 Multi-output Random forest regression

Random forest is an algorithm used for both classification and regression tasks in machine learning. It is a type of ensemble method, meaning that it combines multiple decision trees to make predictions, rather than relying on a single one. A Multi-Random Forest (MRF) is a variation of the Random Forest that is specifically developed to handle multi-label classification tasks, where an instance can belong to multiple classes simultaneously. In an MRF, each decision tree in the forest can predict multiple labels for a given instance, and the final prediction is based on the majority vote of all the trees in the forest (Borchani, H. et al. 2015). MRFs are particularly useful for tasks like image classification, where an image can have multiple objects that need to be identified, and text classification, where a document can belong to multiple categories.

## 2.4 The challenge of handling Multi-Output Parameters in Machine Learning

Deep learning models are often used to predict a single feature at the output, and these models have demonstrated a high level of accuracy in many such situations. Multi-output parameters, the scenario where a machine learning model needs to predict multiple outputs, can present a significant challenge in various applications. Handling multi-output parameters requires the model to learn multiple, potentially complex, and diverse relationships between inputs and outputs. This can be a difficult task, especially when the

relationship between inputs and outputs is non-linear or when the number of outputs is high (W. Liu. et al. 2019). Additionally, when handling multi-output parameters, the model's performance is often evaluated by looking at the performance of each output separately. However, it could be possible to achieve good performance for these problems by using Multi-output Decision trees, Multi-output Random Forests, or deep neural networks.

## 2.5 LED Lens design challenges

Designing and manufacturing optical lenses that effectively and accurately emit and distribute light is a significant challenge in the field of lighting design. There are various technologies that are currently being utilized in the creation of LED lenses for street lighting, with the goal of achieving optimal light emission and distribution. However, controlling the distribution of light is not always easy, and LED sources can be characterized by a range of defects, including light pollution, an upward reflection of light, non-uniform light distribution and patterns, energy waste, and glare. These issues can be particularly problematic in certain environmental settings. Additionally, LED lighting technologies have been known to cause visual discomfort and eye strain for pedestrians and drivers (Lee, X.-H. et al. 2013).

To address these challenges, the concept of adaptive light distribution has been introduced, whereby the amount of light cast onto the road by the LED lens can be manipulated. Ensuring uniform illumination is a critical factor in this process, as is studying how the shape of the LED lens can be adapted to the shape of the road. By carefully considering these factors, it may be possible to overcome the difficulties associated with using LED technology and create more effective and efficient lighting systems.

This study also delves into the quality of lighting and beam control. This includes investigating how optics can effectively and evenly distribute light on the road surface. A number of factors have been taken into consideration in this research, such as the road width, the distance between light poles, the mounting height, and the overhang, which are all properties of the roadway. The parameters of the geometric lens shape, including both the outer and inner sections of the lens, have also been studied. All of these elements have a direct influence on the distribution of light in street lighting, which can be seen in the uniformity of the illumination on both dry and wet roads, the efficiency of the light, and the reduction of glare. Figure 2.2 shows the appearance of the lens, including the outer and inner parts.

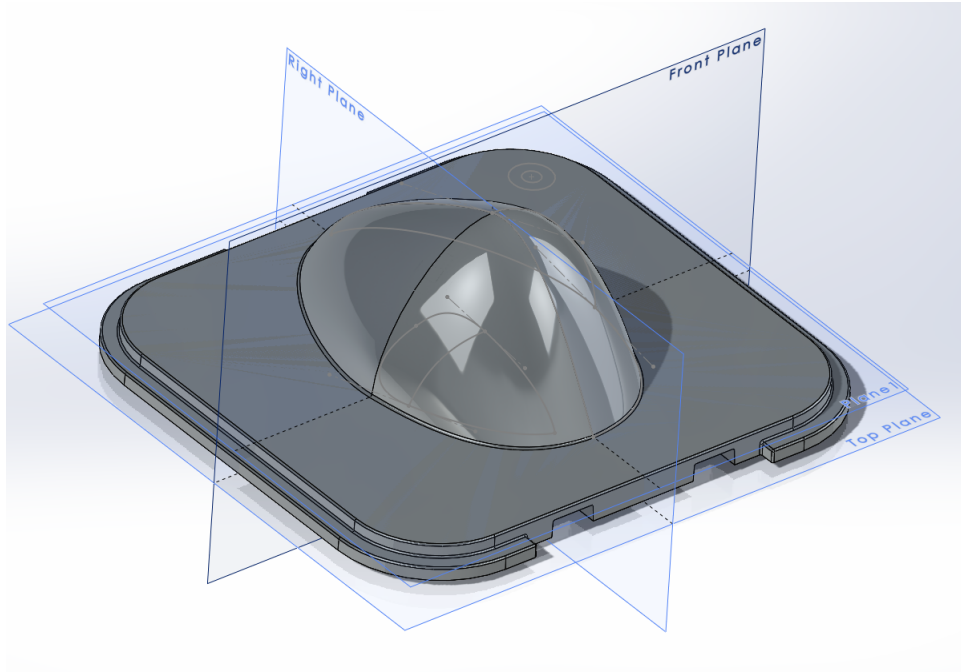


Figure 2.2: The lens geometric shape

In this research, we are exploring the use of machine learning algorithms to predict multiple output parameters.

Before delving further into the use of machine learning algorithms, it is important to first introduce some fundamental concepts from optics, specifically Snell's law (Kovalenko, S. et al. 2001), which is considered the most important principle for light reflection and refraction. Understanding this law will make it easier to grasp the concept of how a lens manipulates the light traveling through it and adapts it for use on the road.

Snell's law is a key formula that describes how a light beam can be altered as it passes through the medium of a lens. It explains the behavior of light traveling through the lens and how the light beam can be manipulated, such as focusing on a small area or spreading out over a wider area, depending on the intended purpose or use of the lens. Figure 2.3 illustrates how light refraction occurs based on the shape and type of lens. The overall concept of Snell's law involves determining the degree of refraction when the light hits the surface of the lens and travels through it, and the second refraction occurs when the light exits the lens medium. (Kovalenko, S. 2001).

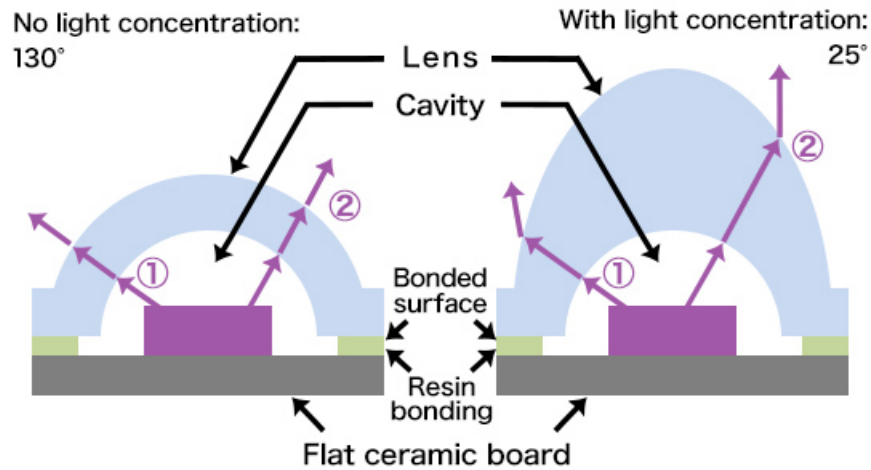


Figure 2.3: Light refraction in different lenses

## 2.6 Adaptive LED lens

Adaptive LED lenses as in Figure 2.4, are designed to be flexible and adaptable to a variety of lighting situations. They can be fabricated in a range of shapes and designs to suit the needs of the user. The internal structure of an adaptive LED lens typically includes an optics design that is simple and effective. The LED light is directed through a total internal reflection (TIR) lens, which helps to collimate the light and provide a high-quality distribution (C. -C. Sun et al. 2017).

One of the key features of an adaptive LED lens is the microlens, which is narrowed to improve the quality of the light. However, this design is not perfect and may require a housing box to protect the LED lens. The housing box can help to improve the reflective capacity of the lens and increase its optical efficiency.

In addition to these features, adaptive LED lenses may also include special microlenses that promote the shaping and homogenization of the light. This can be especially useful in applications where a more uniform light distribution is desired. Overall, adaptive LED lenses offer a range of benefits and can be an effective solution for a variety of lighting needs.

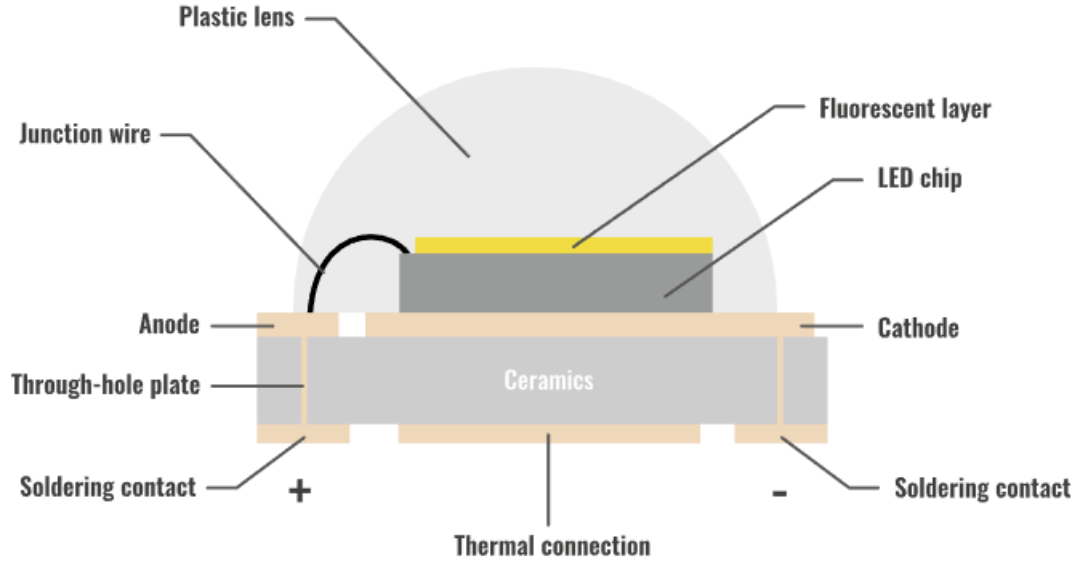


Figure 2.4: LED Lens model/System Structure

## 2.7 Special microlens

The special microlens is an important aspect of the LED lens because it helps to shape the light distribution. This microlens has the ability to distribute light in a specific pattern, thanks to its adaptive optical element. This element helps to tailor the light distribution of the lens to the specific characteristics of the road. For example, special microlens may be used to produce a beam of light that covers a particular section of the road, ensuring that the light is not wasted (Ottevaere, H. et al. 2006).

When the road is straight, the microlens is typically parabolic in shape with a rectangular aperture. By bending the aperture, it is possible to create an illumination pattern with a curved light distribution. This type of light distribution is known for its high efficiency, high uniformity, and low light waste.

Through the use of artificial intelligence, it is possible to optimize the shape of the microlens for maximum efficiency. The width, length, and height of the lens can all be fine-tuned using machine learning algorithms to increase the adaptability and efficiency of the light distribution based on the shape of the road (Mahmoud et al. 2022). In this way, the special microlens helps to improve the performance of the adaptive LED lens.



## 2.8 Lens optimization

Finding the significant parameters is an important key to optimizing the lens design and structure to enhance the performance of the LED lens. Various reflecting cavities have traditionally been used to define light distribution by LED lenses. The following illustration shows a top view, three-dimensional, and light distribution in a rectangular aperture, as (Kamoji et al. 2020) shows in Figure 2.5.

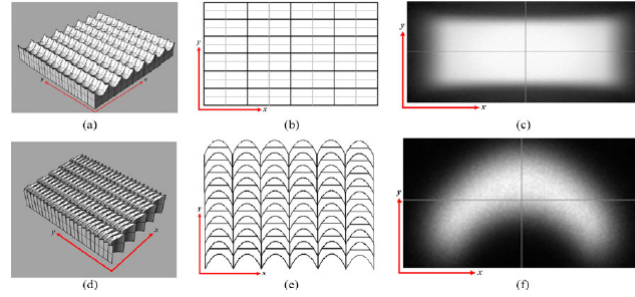


Figure 2.5: Three-dimensional view of lens light distribution

The process of bending the aperture described above is used to create different illuminance profiles for the road. Each aperture has a different microlens that is used to distribute the light in a unique way. For example, Figure 2.5 shows how a curved light distribution can be achieved. The parameters of the lens, such as its depth, width, and size, all influence the shape and size of the light distribution. Additionally, the optical efficiency of the lens may vary depending on the specific aperture used.

(Sun et al. 2017) have demonstrated that LED lenses can be modified to produce precise beams of light, highlighting the many benefits of using optimized lenses in street lighting. Traditional street road lighting, as shown in the illustration provided by (Sun et al. 2017), is just one example of how LED lenses can be used to improve the efficiency and effectiveness of lighting systems. However, it is important to note that different lenses may have varying levels of illumination efficiency and illumination distribution. (Wang et al. 2020) also, highlight the role of microlens in influencing road illumination distribution.

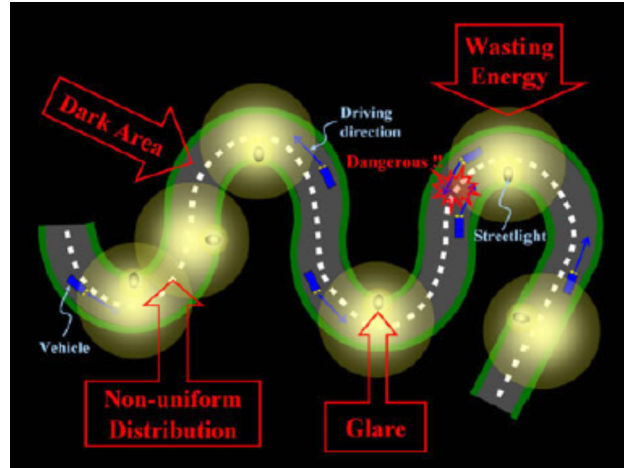


Figure 2.6: Limitations of traditional lighting

As shown in Figure 2.6, traditional lighting systems are often unable to provide precise, targeted lighting. In contrast, the use of machine learning algorithms can significantly improve the performance of adaptive road and street lighting systems. A comparison between adaptive lighting and traditional lighting illustrates that the latter results in significant light loss. This is particularly problematic when lighting roads with irregular shapes, as it can cause discomfort for drivers and pedestrians.

On the other hand, machine learning algorithms have the ability to direct light in a way that is tailored to the shape of the road or street. This can help to enhance eye comfort, reduce glare, and minimize visual discrimination. By using machine learning to design and control luminaires, it is possible to meet the specific needs of a road or street with a free-form shape. This is essential for optimizing the performance of the lighting system and ensuring that it provides the desired level of illumination and visual comfort.

(Lévai & Bánhelyi 2015) identified the issue of unnecessary lighting as a problem that needs to be addressed, particularly in order to conserve energy. They suggest that LED technologies may offer a solution to this problem, but acknowledge that it can be challenging to design LED configurations that are effective and efficient. They propose that machine learning may be the best way to address this challenge, as it can help to influence the pattern of LED light in a more precise and controlled way.

Other researchers have also recognized the potential of machine and deep learning to improve the quality of street lighting. (Kamoji et al. 2020) suggest that deep learning can be used to enhance the performance of street lights, while (Nascimento et al. 2018), (Mahmoud et al. 2021), and (Alvarez et al. 2022) all highlight the role of machine learning in developing adaptive street and road lighting systems. Overall, it seems that machine learning may hold the key to achieving more effective and efficient lighting solutions, particularly in the context of LED technologies.

### 3 Method and implementation

This study aims to identify which machine learning algorithms are effective at predicting multiple output parameters. We treat this problem as a supervised machine learning regression problem due to the nature of the data-set, which contains pure numerical continuous data and multiple outputs. To address this issue, we will compare the performance of two machine learning algorithms: a multi-output random forest and a deep neural network to determine which one is the most suitable for our research problem. The goal is to determine if these algorithms can be used to automatically design LED lenses that meet standard light requirements for different road scenarios (Lai, W. et al. 2011).

Furthermore, the study will explore the possibility of using artificial intelligence algorithms to replace the existing optimization process. Therefore, the research centers on achieving the best possible light distribution using multiple lens geometric parameters. The process of optimizing lenses can differ among companies and designs and tend to be time-consuming, require significant resources, and involve a lot of human involvement. Commonly used software in this process include SolidWorks ("3D CAD Design Software. Solidworks". (n.d.). Retrieved from <https://www.solidworks.com/>), ISight ("CAD/CAE Integrated Framework based on isight optimization platform". (n.d.). Retrieved from <https://www.researchgate.net/>) and Light Tools ("LightTools: Synopsys Optical Solutions Group". SOLIDWORKS. (n.d.). Retrieved from <https://www.solidworks.com/partner-product/lighttools/>). Additionally, different algorithms using Python are implemented to achieve lens optimization. Therefore, it is important to establish the best fit implementation process that helps achieve lens optimization through machine learning. G classification and M-Classification will determine the best light distribution (Putrada et al., 2022). The study will establish how to use machine learning for street-light led lens optimization.

Additionally, the study will try to investigate the relationship between light parameters and the impact of small changes in geometric lens shape on light distribution on the road. The goal is to understand the properties of all parameters and prioritize them in order to identify the key ones for the proposed solution. The model will be instrumental in creating a light pattern and distributed computation. The main challenge is the complexity of global optimization.

The study will proceed as follows: it will present the data-set used in the research, analyze the data to uncover relationships between features, and present the proposed solution through the conduct of three experiments.

### 3.1 Data set and analysis

A data-set containing around 50,000 samples was collected for this study, consisting only of numeric features. The data was obtained from Fagerhult, a company that specializes in the production of modern lighting materials, including lenses. Data cleaning was performed by removing NaN values and duplicates from the data-set. In order to evaluate the accuracy of the models, the performance of each model was tested using the data. The models were tested using different proportions of the data, with the first experiment using approximately 20% of the data, the second experiment using approximately 60% of the data, and the final experiment using the entire data-set.

It was observed that the performance and results of both machine learning algorithms improved with each increment. (Côté et al. 2021) noted that it can be difficult to obtain reliable results from a data-set of 500,000 samples. The data-set used in this study consists of 66 features, including 13 light features, 48 lens geometric shape features, and 5 road features. These road features were not included in the experiments because the data is only available for one road scenario. These road features, including road width, number of lines, the distance between light poles, mounting height, and overhang, can be considered a road setup or scenario and each time one of these features changes it is considered a new road scenario.

The data was collected over a period of more than three months. As previously mentioned, the geometric lens shape consists of an outer and inner lens (cavity), as shown in Figure 2.2, and each is determined by three sketches. The most important light features are glare, illumination efficiency, and uniformity. The study also acknowledges the differences between the expected manufacturing outcomes and the assumed distributions. The aim of this study is to identify an algorithm that can help achieve the best lens with the best distribution of light on the road and performs a linear regression analysis to demonstrate how a data-set relating to street light features can be used to optimize the shape and lighting features of the lens within street lights. The shape of the lens is the dependent variable in the presence of various variables, including light and shape parameters.

### 3.2 Method

At the start of the project, a linear model was used to generate two model scenarios in order to determine which scenario is the most efficient and provides a suitable solution for our research problem. The first scenario focuses on lens geometric output, while the other focuses on light distribution as the output. The results of these two models will help identify the optimal shape and lighting parameters for the street light lens. After analyzing the output of both scenarios and consulting with experts in the field, it was de-

terminated that the scenario that produces lens geometric shape as output is more efficient because it eliminates the need for human intervention throughout the entire process. In contrast, the second scenario requires some human oversight after obtaining light measurements as output to filter and compare to light standards to find the best lens match. The chosen model, which has a geometric lens shape as output, was also trained and tested using the data-set to predict lens distribution patterns. The data was split, with 80% used for training and 20% used for testing, and it was randomly selected to minimize bias.

Given the many parameters that need to be taken into account in lens-making, as well as the need to generate multiple outputs from the light features parameters, it was important to use an algorithm that could handle the wide range of variables to produce the desired outputs. To this end, the study chose two algorithms: a multi-output random forest algorithm and a deep neural network. To determine which algorithm was best suited for the task, a survey was conducted to evaluate which machine learning algorithm could predict the output parameters most effectively. Additionally, since the problem at hand is a regression problem, it was important that the algorithm was able to predict continuous outcomes, which aligned with the data-set we have. Due to these limitations, there were few options to choose from, as most ML algorithms are not designed to handle multi-output predictions.

The two algorithms selected were well-suited to create a ML model that incorporated a wide range of elements. The multi-output random forest algorithm is effective at combining different data classifications and generating multiple regression responses. Similarly, deep neural networks function similarly to the human brain by processing information through layers between inputs and outputs. These networks can be trained to identify and relate specific features to a given object and can also be configured to generate multi-output predictions. This makes them well-suited to determining the optimal lens properties for given features. Both algorithms' outputs were evaluated to determine which provided the most accurate and suitable solution for the research problem. When the model is created successfully, the features of the roads and the desired light can be fed as input parameters into the model, and a virtual lens will be generated based on the model's output which gives the lens geometric shape parameters. This will aid in generating the most optimal prediction for lens-making based on key aspects, and serve as the foundation for improving the street lights' lenses.

### 3.3 Experiments

Three experiments were conducted using various settings, configurations, and scenarios to study different output characteristics. The first experiment focused on the attributes of light distribution, analyzing how they affect the overall distribution. The second experiment includes the geometric shape parameters to study and analyze the performance of the model. The third experiment utilized a combination of multiple models to provide a multi-level prediction.

The aim of these experiments is to gain a deeper understanding of the dynamics and capabilities of our models in order to improve their performance and functionality in different applications. The results of these experiments will guide future research and development efforts, leading to more effective and efficient solutions. We will delve into the details of the three experiments in the coming paragraphs.

#### 3.3.1 Experiment one: Predicting light features

This experiment aims to forecast the characteristics of light through a model that utilizes lens geometric shape parameters as input. The input data for this model consists of 48 parameters, and the output data contains 13 parameters. It is important to note that road features were not taken into consideration during this experiment, as the data set used in this study only includes data for one specific road scenario, thus making the road features identical across all samples. The model is thus trained based on the geometric parameters of the lens and predicts the light features.

#### Setup and implementation

This experiment was conducted three times using three different sized data sets. During the first run, the experiment included around 10,000 samples, the second run included around 30,000 samples, and the final run included the entire data set, which consisted of approximately 50,000 samples. The purpose of this experiment was to compare the performance of two machine learning algorithms, deep neural network (DNN) and multi-output random forest (MRF), by implementing each of them twice. This approach was done in order to find the best model structure for both algorithms that suit the type of data we have. To achieve that, a process called hyper-parameters tuning was applied, where different parameters of the model structure were adjusted and tuned to optimize the model performance. By comparing the performance of the two algorithms under different data set sizes and with different model structures, this experiment provides valuable insights into the most suitable algorithms and model structures for the given data set and scenario. The DNN model architecture that was obtained after applying hyperparameter tuning is

composed of the following components, as shown in Figure 3.1.

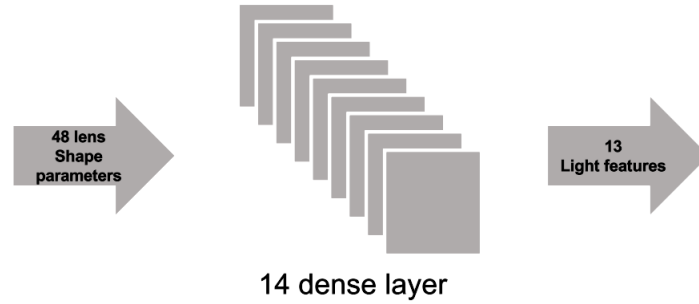


Figure 3.1: The first experiment DNN architecture

The DNN model architecture, obtained after applying hyperparameter tuning, is composed of 14 dense layers with a succession of neurons in each layer, specifically (96, 320, 288, 256, 384, 320, 384, 320, 96, 256, 288, 64, 128, 192). The number of epochs and batch size used were 350 and 32 respectively. The activation function used is linear, the mean absolute error is used as the loss matrix, and the optimizer used is Adam with a learning rate of 0.001.

In addition, the architecture of the Multi-output Random Forest (MORF) model after applying hyperparameter tuning was found to be best suited for the data-set features. The parameters used in this model include 400 estimators, which represent the number of trees, and 30 max depth, representing the number of branches per tree. This configuration can be seen in Figure 3.2.

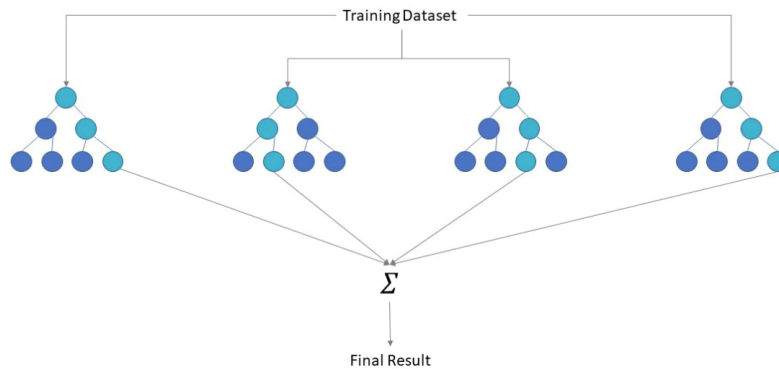


Figure 3.2: Diagram of Multi-output Random Forest

The data-set was divided into two parts, where 80% was used for training and 20% for testing.

### 3.3.2 The second experiment: Predicting lens geometric shape parameters

The second experiment was designed to test a different hypothesis than the first experiment. Specifically, the hypothesis was formulated as follows: The input for this experiment was composed of 13 light distribution parameters, and the output was determined by 48 parameters for the lens geometric shape shown in Figure 3.3. Given the substantial discrepancy between the number of input and output features, an additional study was conducted on the light data in order to identify any potential relationships that could be incorporated into the original input to aid in the model's prediction.

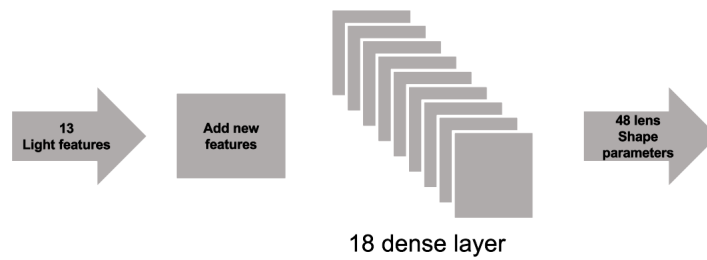


Figure 3.3: Second experiment DNN architecture

In this scenario, the ML models were presented with a significant challenge in trying to predict a large number of features, nearly three times more than the number of features used as input. After the study and the analysis of the relationships between the light features, two new features were added to the input, the G-Class and M-Class. These standards were defined by the traffic authority in Sweden as requirements that street lights must meet (Traupmann, P. 2017). The G Classification, for example, comprises of six levels, each level indicating the allowed glare on the road for light beam angles of 80 and 90 degrees. The G standard levels are G1, G2, G3, G4, G5, and G6, where G6 represents the most optimal level of glare that can be on the road.

Similarly, the M Standard classification is defined by 6 levels for light distribution on the road; M0, M1, M2, M3, M4, and M5, and each one of these levels fit different types of roads, it could give a better match for the road needs. It is worth noting that the inclusion of these additional features helped the models to better predict the output parameters for the lens geometric shape.

This exemplifies how models should be tweaked and fine-tuned to achieve the best results from the data-set and the problem at hand.



## Setup and implementation

To ensure the most accurate results, the data-set was divided into 80% for training and 20% for testing. Through the process of hyperparameter tuning, an optimal architecture was determined for the deep neural network (DNN) model, which consists of 18 dense layers, with a varying number of neurons in each layer (96, 320, 288, 128, 64, 128, 256, 384, 320, 384, 320, 96, 256, 128, 288, 64, 128, 192). Additionally, the model was trained for 220 epochs with a batch size of 32, used linear activation function, mean absolute error as the loss matrix, and the Adam optimizer (0.001) as the optimization method.

Similarly, the architecture of the multi-output random forest algorithm was also determined through hyperparameter tuning, and includes 200 estimators (number of trees) and a maximum depth of 25.

The architecture of DNN and multi-output random forest was selected based on the best performance during the training phase, and it was found that this configuration of layers, neurons, epochs and batch size gives the best results in terms of accuracy, and could be optimal in predicting the output parameters for the lens geometric shape.

### 3.3.3 The third experiment: Multi-level prediction

The approach used in this experiment was similar to the second experiment. The input for this experiment was the 13 light distribution parameters and the output was the 48 parameters for the lens geometric shape. To improve the performance of the model, two additional features were added to the input, exactly like in the previous experiment. This approach is aimed to achieve better results by using a multi-level prediction process, which allows for greater accuracy and more insight from the same amount of data.

## Setup and implementation

In this experiment, only one of the previous models was utilized. The Multi-output Random Forest (MRF) was chosen as it was found to provide better results than the Deep Neural Network (DNN) model in previous experiments. This approach employed a multi-level prediction process using different MRF structures for each level. However, the size of the data-set limited the prediction to only 3 levels.

It was observed from the second experiment that some output features were significantly far from their actual values when predicted. To address this, the output features were divided into three categories: easy, normal, and difficult features, based on the prediction error interval. An error interval allowance of 5% was chosen with the guidance of

an expert in the field of lens optimization. Figure 3.4 illustrates the architecture of the multi-level prediction model that was constructed. It depicts how the different layers of the MRF were arranged and configured to form the multi-level prediction model.

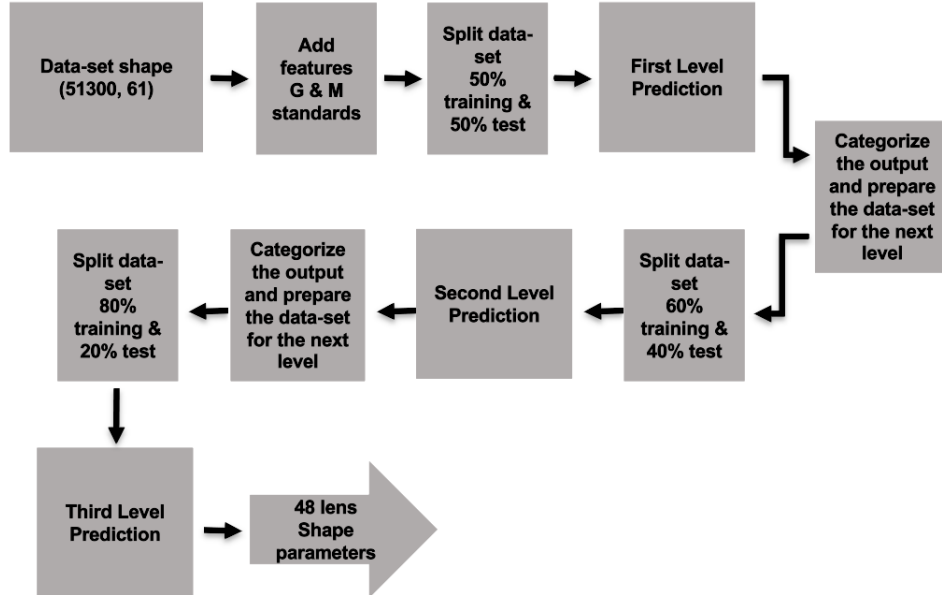


Figure 3.4: Third experiment model architecture

Initially, the same MRF model architecture from the second experiment was used as the first level prediction. However, the data-set was split differently, with 50% of the data being used for training (DTrain1) and the remaining 50% for testing (DTest1). The output was then categorized into three types (easy, normal, and difficult predicted features) based on the 5% error interval allowance.

For the second level prediction, the test data from DTest1 was used as input and the easy predicted features were added to it, creating new input data. Hyperparameter tuning was conducted on this level two model, as the new shape of the input data required adjustments to the model's architecture. The resulting level two model architecture consisted of 180 estimators, and a maximum depth of 25. The final step was repeated one more time to prepare the input for the third level prediction. Figure 3.4 illustrates the multiple steps for this model. After applying hyperparameter tuning, the final model architecture contains 200 estimators and a maximum depth of 15.

## 4 Results

In this chapter, we will provide an extensive examination of the outcomes from the three experiments conducted in the preceding chapter. The experiments were executed to collect information and understanding of a particular research question, and their results will be presented in a detailed but succinct format with the use of tables that will aid in the comprehension of the findings.

### 4.1 Predicting light features

The goal of this experiment is to predict the characteristics of light, based on the geometric shape parameters of the lens. The study utilizes a model with 48 input parameters to predict 13 light characteristics, using only lens geometric shape parameters, with no consideration of road features as the data-set only includes a single road scenario.

#### Results and interpretation

The performance of the deep neural network model is assessed using the R-square score as the evaluation matrix, while the multi-output random forest model is evaluated using a combination of the mean absolute error and R-square as the evaluation matrix.

Table 4.1: The results for both ML algorithms

Number of samples	DNN R2	MORF R2
10000	-13649%	10%
30000	-128.36%	67%
50000	4.7%	93%

By analyzing the results of both algorithms which are presented in Table 4.1. The table displays the outcomes of training the model three times using varying amounts of data, ranging from approximately 10,000 to 50,000. The second column represents the R-square values of the deep neural network (DDN) model, while the third column shows the R-square values of the multi-output random forest model. We observe that each time we increase the size of the data-set the results which presented as R Square get better. Actually, the experiment succeeds to achieve a pretty good results. Also the MORF model gives more accurate results than DNN and that is not only according to the evaluations matrix we used, because additionally we did a visual analysis for the prediction of both models and after comparing the output between both of the models, we notice that the results for MORF it is much closer to the real values of the tested sample.

## 4.2 Predicting lens geometric shape parameters

The second experiment was designed to investigate a different hypothesis than the one explored in the first experiment. The input for this experiment comprised of 13 light distribution parameters and the output was determined by 48 parameters for the lens geometric shape. The large discrepancy between input and output features prompted us to conduct an additional study on the light data to identify any potential relationships that could be incorporated into the input to improve the model's prediction capabilities. To resolve the issue of limited input parameters, two new features were added to the input, G-Class and M-Class. The inclusion of these features helped the models to improve their predictions of the output parameters for the lens geometric shape. Overall, this experiment highlights the importance of fine-tuning and customizing models to achieve optimal results for the specific data-set and problem.

### Results and interpretation

The evaluation of results for this experiment involved both visual analysis of the predicted values in comparison to the actual values from the test data, and an analysis of the R-square values, determined by utilizing the multi-output function "uniform average" as the evaluation matrix.

Table 4.2: The results for both ML algorithms

samples	DNN R2	MRF R2
10000	-498.36%	-24.28%
30000	-22.36%	12%
50000	3.38%	62%

The results of the second experiment did not match the success of the first experiment, as can be seen in Table 4.2. Both the deep neural network (DNN) and the multi-output random forest (MRF) model performed less well, with lower R2-scores. Furthermore, when visually comparing the predicted target features to the actual values, it was clear that the predictions were not as close. By taking into account the 5% error interval between the actual values and the predicted values, as determined by experts in the field of lighting systems. However, it was observed that similar to the first experiment, as the size of the data-set increased, the results, as measured by R-squared, improved. Additionally, it was also noted that the MRF model performed better than the DNN model when using the largest data-set, which included around 50,000 samples. The results of this experiment demonstrate the importance of increasing data-set size in order to improve model performance, as well as the fact that different types of models may work better with certain

types of data.

## 4.3 Multi-level prediction

In this experiment, we employed a multi-level prediction approach to enhance the outcomes achieved in previous experiments. This method of prediction involved breaking down the problem into smaller sub-problems, and making predictions at each level before consolidating the predictions at the final level. It allowed us to work with the data more efficiently, and increase the accuracy of the predictions. By using this approach, we were able to extract more information from the data and improve the overall performance of the model. Overall, this experiment demonstrated the importance of fine-tuning the models and adapting them to the specific problem and data-set.

### Results and interpretation

In this experiment, the model was run using the entire data-set as input. After the initial level of prediction, the outcomes were categorized, resulting in 24 easily predicted features. The 24 easily predicted features were added as an additional component to the input data for the next level of prediction. In the second level of prediction, the model was provided with a data-set of 37 input features, which included the original input of 13 features along with the easy predicted features from the previous level. With this data, the model was able to predict an additional 7 features with an error prediction interval of less than 5%. These features were categorized as easy predictions and were taken into consideration to be added to the input for the next stage of the prediction process.

The third level of prediction resulted in the identification of 10 more easy features, bringing the total number of good predicted features with an error interval of less than 5% to 41 out of 48 features. This can be seen in Table 4.3.

Table 4.3: The number of the easy features predicted in each prediction level

Prediction level	Number of the easy predicted features
1	24
2	31
3	41

This method of multi-level prediction allows the model to break down the problem into smaller sub-problems, and make predictions at each level before consolidating the predictions at the final level. This approach enables the model to work more efficiently with the data and increase the accuracy of the predictions. In this specific case, it allowed the

model to extract more information from the data and improve the overall performance of the model. Additionally, by classifying the outcomes and focusing on easy predicted features the model was able to achieve better results in the second level of predictions. This experiment demonstrates the importance of fine-tuning models, adapt them to the specific problem and data-set and to approach the prediction in multiple levels.

## 4.4 Research questions

The research effectively addresses the key question and sub-questions by providing thorough and detailed responses. Through in-depth analysis and experimentation, the study has effectively responded to the questions at hand, delivering insights and findings that are relevant and useful to the field. Overall, the research has demonstrated that it has adequately and effectively responded to the questions and sub-questions being studied, and has provided a solid foundation for further inquiry and exploration in this area.

- **Can AI and ML be used to improve the area of lighting and optical designs?**

- As claimed, the results obtained from our experiments, show that AI and machine learning can serve as powerful tools for enhancing lens design optimization in our research case. This can be seen as an important step forward for the application of AI and ML in other areas of optics. The results suggest that AI and ML are promising for optimization in lens design and for advancement in this field.

- **What is the most suitable machine learning algorithm to use for lens optimization to achieve efficient light distribution?**

- Based on the results obtained from the three experiments, when evaluated in light of the type and size of the data-set used and the performance of the two algorithms studied, indicate that the Multi-output Random Forest algorithm outperformed the deep neural network.

- **Find the possibility of using the multi-output ML model in a real-world problem like our optical design optimization problem and replace the current manual lens optimization process?**

- We explored the potential of using a multi-output machine learning model to improve the current manually-driven lens optimization process in our optical design problem. Although we were not able to fully optimize the lens design using machine learning, we made progress by identifying starting points for optimization. This serves as a promising foundation for future work in this field.

## 5 Discussion and conclusions

### 5.1 Discussion

#### 5.1.1 Predicting light features

##### **Result discussion**

The results of the experiment are presented in a Table 4.1 that shows the R-square values of both models while increasing the size of the data-set from 10,000 to 50,000. The results show that as the size of the data-set increases, the R-square values for both models improve. Additionally, the MORF model is observed to give more accurate results than DNN as observed from both the evaluation matrices and visual analysis of the predictions of both models, MORF is much closer to the real values of the tested sample.

The aim of this research is to identify a straightforward and practical solution that can be applied in real-world scenarios. However, some experts in the field have raised concerns that the proposed solution may not be as effective as hoped. The designed model is used to anticipate the light distribution parameters and then filter the results by software to identify the best sample that meets the desired requirements for road lighting. Then trace back the sample index to find the input sample which includes the geometric lens shape parameters. In order to implement this solution, it is essential to incorporate both software and human involvement. The software is used for model generation and result filtering, while human input is needed to validate the accuracy of the predictions and make adjustments for optimal results. This is why we persisted in searching for alternative solutions, despite obtaining favorable results from one of our models.

#### 5.1.2 Predicting lens geometric shape parameters

##### **Result discussion in relation to the other experiments**

The second experiment attempted to explore a different hypothesis than the first experiment and utilized 13 light distribution parameters as input and 48 lens geometric shape parameters as output. However, the experiment was challenging for the machine learning models due to the large number of input features. The R-squared scores for both DNN and MRF models were much lower than the first experiment and the visual comparison of predicted target features to actual values also showed that predictions were not as close. With increasing data-set size, the results improved and the MRF model performed better than the DNN model using a data-set of 50,000 samples. Table 4.2 shows the low R

square values for both models which means that to analyze the results we need an additional way. That is why we did the visual comparison for the result of the testing samples between the actual and predicted values.

The second experiment produced less precise predictions of the output compared to the first experiment. Although the results were not entirely satisfactory, they were still useful as a starting point for lens optimization. This experiment demonstrates that it is possible to create a lens that can provide the desired distribution of light on roads with an abstract shape, even if it may not be the optimal solution. This experiment has the potential to reduce the time required for optimization and minimize human involvement, especially if the model is further improved by increasing the size of the training data. particularly if the model continues to demonstrate improved performance by being trained with a larger data-set. Actually, the model shows us its potential for future work and development.

### 5.1.3 Multi-level prediction

#### Result discussion and relation to the other experiments

This paragraph describes an experiment in which a machine learning model is used to predict certain features of a data-set. The model starts by using the entire data-set as input, and in the first level of prediction, it identifies 24 easily predicted features it shows in Table 4.3. These features are then added as additional inputs for the second level of prediction, where the model is able to predict an additional 7 features with a low error prediction interval of less than 5%. This process is repeated in a third level of prediction, resulting in the identification of 10 more easy features, bringing the total number of good predicted features to 41 out of 48. This approach of breaking down the problem into smaller sub-problems and making predictions at each level, before consolidating the predictions at the final level, increases the accuracy of the predictions and allows the model to work more efficiently with the data and improve overall performance.

In order to fully comprehend the outcome of the experiment, it is important to first explain the three base settings that were utilized.

- The reason for only utilizing three-level predictions was due to limitations in the size of the data-set.
- A specific split ratio was employed between the prediction levels to ensure that there would be sufficient data for the final prediction level. It should be noted that as the number of prediction levels increases, the input data for each subsequent level becomes the testing data for the previous stage.
- Additionally, it's worth mentioning that the 5% error allowance is a ratio determined



by a select group of experts, and may be viewed differently by other experts in the field or by professionals working within different organizations.

The multi-level model, which employed multi-output random forest algorithms, was able to predict 41 of the total 48 features for the geometric lens with an error interval of less than 5%. These findings indicate that utilizing a larger data-set improves the overall quality of the results, as the prediction capacity is significantly greater in comparison to the second experiment. In the second experiment, it was used as the first level, and subsequently, better results were obtained for each prediction level that was implemented. In this specific case, it allowed the model to extract more information from the data and improve the overall performance of the model. Additionally, by classifying the outcomes and focusing on easily predicted features the model was able to achieve better results in the second level of predictions. This experiment demonstrates the importance of fine-tuning models, adapting them to the specific problem and data set, and approaching the prediction at multiple levels.

### **5.1.4 General discussion and reflections**

The experiments carried out in this study indicate the potential of machine learning and artificial intelligence to enhance the optimization of LED street light lenses and in the general lighting domain. By using the data-set we have, the first experiment demonstrated that machine learning can yield accurate results in predicting the light distribution produced by LEDs and optics. Despite the limitations of using few machine learning algorithms that support multi-output predictions, the second and third experiments showed promising results. Though these models may not be able to solve all lens optimization problems single-handedly by this small size of data-set, but at least they can provide a starting point for the optimization processes, helping to save time, resources, and costs compared to traditional lens optimization techniques.

The focus of this paper is to explore the potential of using machine learning and artificial intelligence to optimize the design of LED street light lenses. The aim is to evaluate how these technologies can be applied to achieve optimal light distribution and lens design. The research aims to determine the geometric lens parameters that result in the optimal light distribution on roads and to find an alternative to current lens optimization methods.

The study's results indicate that machine learning can be used to enhance the accuracy of LED street light lenses, which lead to improved driver comfort, reduced glare, better visual discrimination, and optimal illumination performance. Overall, the study highlights that using machine learning in lens optimization holds great promise for enhancing the performance of LED street light lenses.

The study aimed to investigate the integration of machine learning and artificial intelli-

gence in the optimization of LED street light lens designs, with the goal of examining how these technologies can be used to achieve efficient light distribution and optimize lens design. By utilizing the power of machine learning, lens manufacturers can improve the performance of their products and potentially expand in the field of adaptive road lighting. The results of the study can be useful for the development of new adaptive road lighting prototypes, which can be tested to evaluate the proposed concepts. The use of machine learning has the potential to enhance the capabilities of adaptive road lighting systems, leading to a range of benefits for road users.

## 5.2 Conclusions

This research proposed and confirmed that machine learning can be effectively utilized in optimizing the design of LED street-light lenses. This optimization can contribute to reducing pollution and energy waste, as well as enhancing driver comfort, reducing glare, and minimizing visual discrimination. The proposed adaptive LED lens design includes an optical lens that distributes light based on the shape of the road. Hence, the study confirmed that machine learning can help attain efficient light distribution. Among the two algorithms studied, Multi-output Random Forest algorithm proved to be more efficient in achieving this goal than the neural network algorithm, under the given conditions and limitations.

The secondary objective of the research is to determine if the data at hand can aid lens manufacturing companies in creating a lens with optimal light distribution for use on roads. The log-likelihood function will be utilized to evaluate the optimal value of the estimates. The outcome of the model is of interest to lens manufacturing companies to see if it can achieve the desired lens design.

This proposed solution is limited by the data-set used in this study. If another company were to use a different approach for collecting data and different measurement parameters for the lens, it is uncertain how the model would perform and if it would achieve similar results or potentially worse or better results. As a conclusion, the study did not result in an optimal lens design. However, it serves as a starting point for future research and can be useful in taking into account the limitations encountered and finding ways to overcome them in future work.

Finally, future studies can enhance the current solution by collecting more data that includes various road scenarios, which can be interesting to evaluate the performance of the model over time. Reinforcement learning can be an appropriate approach for this problem, as it is based on the interaction between AI/ML systems and their environment. Through this interaction, the model receives feedback in the form of rewards or penalties for its predictions (Du et al., 2022) which can aid in optimizing the lens design over time.

Furthermore, it allows real-time optimization and can help to find the best lens design for different scenarios.

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