On Some Properties and Applications of Patterned Ferromagnetic Thin Films

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Abstract

A microwave reflection method has been used to measure the spin excitations corresponding to the translational mode of magnetic vortices in samples containing either one or two vortices. Experimental findings are complemented by micromagnetic simulations.

One-vortex systems are investigated in micron-sized circular and elliptical cylinders. For ellipses, the resonance frequency can effectively be tuned by applying static magnetic fields and the field dependence of the frequency is significant for fields applied along the short axes but negligible when applied along the long axes of the ellipses. This is contrary to the circular case, where virtually no field dependence was found. This can be understood by considering the shape of the vortex potential well. Further, it is found that the resonance frequency is independent on the direction of the excitation field for the one-vortex systems.

Ellipses containing two interacting vortices are also investigated. It is shown that the relative vortex core polarizations dominate the vortex translational mode and cause, in the case of opposite polarizations, a dependence on the excitation field direction. For parallel core polarizations, no dependence on the excitation field direction is found. The dependence of the resonance frequencies on applied static fields along the long and short axes are also experimentally mapped out and compared with micromagnetic simulations, where the possible eigenmodes are determined.

Another section of the thesis introduces the dawn of a device based on patterned magnetic elliptical elements for the manipulation and movement of magnetic particles on a surface. The controlled movement and separation of individual particles are successfully demonstrated.

Contributions to micromagnetic standard problems and simulations on magnetization switching in nanoscale particles have also been performed. The standard problems highlight some important aspects of choosing the discretization cell sizes and the finite temperature simulations show that thermal fluctuations can alter the magnetization reversal paths.

Keywords: micromagnetics, vortex, spin dynamics, domains, MFM, microwave, patterned, permalloy

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To my parents
List of Papers included in the thesis


**List of Papers not included in the thesis**


**Contributions to micromagnetic standard problems**


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List of Important Symbols

$A$: Exchange stiffness constant
$E_d$: Demagnetization energy
$E_{ex}$: Exchange energy
$E_k^c$: Cubic magnetocrystalline anisotropy energy
$E_k^u$: Uniaxial magnetocrystalline anisotropy energy
$E_Z$: Zeeman energy
$\vec{F}$: Force
$\vec{H}$: Magnetic field (often used as the applied field in the text)
$\vec{h}$: Reduced magnetic field
$\vec{H}_{App}$: Applied magnetic field
$\vec{H}_e$: Effective field
$\vec{h}_c$: Reduced effective field
$\vec{H}_{ex}$: Exchange field
$\vec{H}_d$: Demagnetizing field
$\vec{H}_k$: Anisotropy field
$\vec{H}_z$: Zeeman field
$k_B$: Boltzmann constant
$K_{c1}$: First cubic magnetocrystalline anisotropy constant
$K_{c2}$: Second cubic magnetocrystalline anisotropy constant
$K_m$: Magnetostatic energy density constant
$K_{u1}$: First uniaxial magnetocrystalline anisotropy constant
$K_{u2}$: Second uniaxial magnetocrystalline anisotropy constant
\( L \): Dot thickness
\( \vec{M} \): Magnetization
\( M_s \): Saturation magnetization
\( \vec{m} \): Reduced magnetization
\( \vec{N} \): Demagnetizing tensor
\( p_{1,2} \): Core polarization (cores 1 and 2)
\( R \): Dot radius
\( \vec{r} \): Position
\( t \): Time
\( V \): Volume
\( \vec{X}_0 \): Vortex equilibrium position
\( \alpha \): Damping parameter
\( \Delta t \): Discretization time step
\( \gamma_0 \): Gyromagnetic ratio
\( \kappa_{x,y} \): Spring constants
\( \mu_0 \): Permeability in vacuum
\( \phi_d \): Magnetic scalar potential
\( \xi \): Dynamic vortex displacement
1. Introduction

Ever since the first experiments by Barkhausen in 1919 [1], Sixtus and Tonks in 1931 [2], and later the direct imaging by Bitter [3, 4] (the originator of the Bitter pattern) and the works by Hamos and Thiessen [5], the existence of magnetic domains is an experimental fact, thus proving the domain concept that was already conjectured by Weiss as far back as in 1907 [6]. The interpretation of the domain images was, however, not well understood at the early stages mainly due to damaged sample surfaces and the use of comparably rough powders for imaging. Originally, domains were merely postulated in order to give a mechanism for demagnetization. A more detailed attempt for a theoretical description came in 1935 by Landau and Lifshitz [7], who used the concept of domain walls that had previously been suggested by Langmuir [2, 8] and put into a quantum mechanical form by Bloch [9]. In their paper they presented a classical calculation of a domain wall in which the spontaneous magnetization is of constant magnitude but where the direction of the magnetization varies continuously with position (which is really the seed to the field of micromagnetics). Landau and Lifshitz were also able to show that minimization of the total energy gives rise to flux-closure domains and that the stray field energy (a term that previously had been much neglected) plays a decisive role in forming the equilibrium domain structure. The further development of this continuum approach was mainly performed by William Fuller Brown, who was greatly inspired by the work of Landau and Lifshitz and who coined the name micromagnetics [10, 11, 12, 13]. Experimentally, the work really showing the correspondence between the existing domain theories and experimental observations was presented in 1949 by Williams, Bozorth and Shockley [14] using well oriented and carefully prepared crystals in combination with well dispersed magnetic colloids. The same year, Kittel wrote a review of domain theory and the up-to-date experiments at that time with his own further additions to the theory [15]. It seems that all this really opened up the floodgates to the research in this field, resulting in a vast body of previously unconceived magnetic microstructures (see the book by Hubert and Schäfer [16]). Since these times, a large set of experimental tools have been developed for the imaging of magnetic microstructures, such as for instance magneto-optical methods, magnetic force microscopy, transmission electron microscopy methods and many more (see [16] for an overview). Further, the recent advances in lithographical techniques have opened up the possibility to produce samples on the nanometer scale, thus making it possible to study
magnetism in more confined geometries and to produce a vast variety of magnetic devices (see e.g. [17, 18, 19]). Moreover, with the emergence of fast computers, numerical micromagnetics is today a well established and commonly used approach in the study of magnetic microstructure, yielding important contributions to the basic understanding of magnetic microstructure as well as to the development of novel magnetic devices. It is in this ever-growing flood that hopefully this thesis can contribute with a drop of new knowledge. The studies presented in this work are as in most cases quite specific and the thesis can more or less be said to consist of four parts. The first part is a description of some theoretical concepts, numerical and experimental techniques, basically the necessary ingredients for the research that forms this thesis. The second part deals with some numerical experiments using micromagnetics with a code developed here, focusing on two of the so called µMAG standard problems [20] and with some possible scenarios of thermal fluctuations in small particles. These illustrative numerical experiments have been performed since they yield valuable insight into discretization issues and possible scenarios for switching of the magnetization in small particles under the influence of thermal fluctuations. The third part concerns magnetic vortex states [21], which appear as flux-closure states or as part of flux-closure states in a vast body of magnetic systems. Mainly, the dynamical behaviour of either one or two such vortices under the influence of perturbing fields have been studied in experiments and numerical calculations. A motivation for these studies, apart from the interest in the fundamental aspects of magnetic vortex dynamics, is that vortices are such prominent magnetic entities in samples of sizes typically used in magnetic devices and it is thus important to understand their dynamical behaviour. Finally, the fourth part demonstrates an application of patterned magnetic elements for the manipulation and motion of magnetic particles on a surface, indicating potential industrial applications such as separation of specific bio-molecules from mixed solutions.
2. Micromagnetics

The general approach of micromagnetics is to replace magnetic spins with a continuous magnetization vector. The length scales involved are in between those of lattice constants and domains, i.e. intermediate lengths, small enough to resolve domain walls but large enough to use a continuous magnetization vector. The basis of the continuum description is that the forces involved exert only a small perturbation on the parallel alignment of neighboring magnetic spins. In effect, the spin directions can only change by small angles from lattice point to lattice point. In the words of Brown: *It therefore seems legitimate to approximate the direction angles of the spins or magnetic moments with continuous functions of position* [10]. A rigorous treatment of the general micromagnetic theory was put forth by William Fuller Brown in his famous book *Micromagnetics* in 1963 [10]. The basic idea is that within the magnet, the magnetization $\mathbf{M} = M_s \mathbf{m}(\mathbf{r})$ varies continuously along the coordinate axes and has magnitude $M_s$ (saturation magnetization), as determined by the temperature $T$. The forces acting on $\mathbf{M}$ are due to fields $\mathbf{H}(\mathbf{r})$ arising from all the magnetic interactions (to be described in subsequent sections). These fields exert torques on $\mathbf{M}$ tending to align it along the local field directions. Equilibrium states may be computed through energy minimization of the total energy. However, the most common approach today is to solve the Landau-Lifshitz (LL) equation of motion (see section on magnetization dynamics), which is a torque equation relating the time evolution of $\mathbf{M}$ to the torques exerted by $\mathbf{H}$. In this way the magnetization dynamics is also captured and an equilibrium state is reached when the magnitude of the torques vanish in every lattice point. The numerical micromagnetics used in this work is based on the solution of the LL-equations. For a summary of the basic principles of micromagnetics, see also Fidler [22]. What will follow now is a brief description of the energy terms involved in a ferromagnet, then moving on to the associated effective fields and computer implementation issues.
2.1 Energy terms

2.1.1 Exchange energy

The exchange energy is the interaction responsible for parallel or anti-parallel alignment of magnetic spins and is usually written as [23]

\[ E_{\text{ex}} = -\sum_{ij} J_{ij} \vec{S}_i \cdot \vec{S}_j, \quad (2.1) \]

where \( \vec{S}_i \) and \( \vec{S}_j \) are two magnetic spins located on neighbouring sites \( i \) and \( j \) and \( J_{ij} \) is the exchange coupling term. One sees here that if \( J_{ij} > 0 \) a parallel alignment (ferromagnetic coupling) is preferred, whereas if \( J_{ij} < 0 \), anti-parallel alignment (anti-ferromagnetic coupling) is favoured. Assume that \( J_{ij} \) is constant throughout the material, that \( \vec{S}_i \) and \( \vec{S}_j \) are equal in magnitude and that the magnetization varies slowly spatially. The interaction energy of one spin with its nearest neighbours can then be written as:

\[ E_{\text{ex}}^i = -J \left| \vec{S} \right|^2 \sum_{nn} \cos \phi_{ij} \approx E_{\text{ex}}^0 + \frac{1}{2} J S^2 \sum_{nn} \phi_{ij}^2 + \cdots, \quad (2.2) \]

where the first term \( E_{\text{ex}}^0 = -zJS^2 \) if \( z \) is the number of nearest neighbours (for instance in a simple cubic (sc) lattice \( z = 6 \)), \( \phi_{ij} \) is the angle between two spins located at sites \( i \) and \( j \) and \( J \) is the nearest neighbour interaction and the summation is carried out over nearest neighbours (nn). It is convenient to set \( E_{\text{ex}}^0 \) to zero since what is important is the associated increase or decrease in exchange energy when \( \phi_{ij} \) is varied. For small angles \( \left| \vec{m}_i - \vec{m}_j \right|^2 \approx \left| \phi_{ij} \right|^2 \)

where \( \vec{m} \) is a unit vector along the local spin direction. Taylor expansion up to first order yields [23]

\[ \left| \vec{m}_i - \vec{m}_j \right| = \left| \left( \vec{a}_i \cdot \nabla \right) \vec{m} \right|, \quad (2.3) \]

where \( \vec{a}_i \) is the position vector from lattice point \( i \) to \( j \). Equation (2.3) into (2.2) and summing over the whole material for the total energy give

\[ E_{\text{ex}} = \frac{1}{2} JS^2 \sum_i \sum_{\vec{a}_i} \left[ \left( \vec{a}_i \cdot \nabla \right) \vec{m} \right]^2, \quad (2.4) \]

where the second summation is over nearest neighbours. By writing out the sum over nearest neighbours and taking the continuous limit by letting the summation over all sites \( i \) be replaced by an integral over the material volume, one arrives at [10, 23]

\[ E_{\text{ex}} = A \int_V \left[ \left( \nabla m_x \right)^2 + \left( \nabla m_y \right)^2 + \left( \nabla m_z \right)^2 \right] dV, \quad (2.5) \]

where \( A \), known as the exchange stiffness constant, is given by \( A = JS^2/a \) and \( a \) is the lattice constant. A neat thing is that although Eq. 2.5 was derived for a sc lattice it can be applied to any cubic lattice; body centered cubic (bcc) and face centered cubic (fcc) as well as to hexagonal close packed (hcp) structures. This is done by multiplying \( A \) by a characteristic structure factor \( c \), such that the new \( A \) can be obtained as \( A = \frac{JS^2}{a} c \) [10, 23, 24].
2.1.2 Magnetocrystalline anisotropy energy

The magnetocrystalline anisotropy (MA) energy results from deviations of the magnetization from preferred crystallographic directions (easy directions) in the material. These easy directions stem from spin-orbit interactions and are intrinsic properties of the undisturbed crystal (i.e. structural defects not included). The typical MA considered are the cubic and the uniaxial ones. For the cubic case the MA energy is

\[ E^c_k = \int_V \left[ K_{c1} (m_1^2 m_2^2 + m_1^2 m_3^2 + m_2^2 m_3^2) + K_{c2} (m_1^2 m_2^2 m_3^2) \right] dV, \]

where the \( m_i \) are the magnetization components along the x,y,z-axes, and \( K_{c1} \) and \( K_{c2} \) are the anisotropy constants. Usually, \( K_{c1} \) takes on values in the range \( \pm 10^4 \text{J/m}^3 \) for different materials and \( K_{c2} \) and higher order terms can in many cases be neglected [16]. The uniaxial anisotropy energy is usually written as

\[ E^u_k = \int_V \left[ K_{u1} \sin^2(\phi) + K_{u2} \sin^4(\phi) \right] dV, \]

where \( K_{u1} \) and \( K_{u2} \) are uniaxial anisotropy constants and \( \phi \) is the angle between \( \vec{M} \) and the preferred (easy) direction. In many cases \( K_{u2} \) is negligible in comparison to \( K_{u1} \) and in the coming numerical implementation of uniaxial MA only the \( K_{u1} \) term is considered (see section on Illustrative numerical experiments). Uniaxial anisotropies can be much stronger than cubic anisotropies with values reaching \( 10^7 \text{J/m}^3 \) for some materials [16]. A schematic of possible magnetization distributions for different MA situations is shown in Fig. 2.1.

2.1.3 Magnetostatic energy and domains

For any magnetization distribution there will be an associated magnetostatic or demagnetizing field, \( \vec{H}_d \), directed opposite to the magnetization, \( \vec{M} \). For a uniformly magnetized ellipsoid this internal field can be expressed exactly as
\[ \vec{H}_d = -\vec{N} \vec{M}, \]  

(2.8)

where \( N \) is the demagnetizing tensor whose elements are functions of the ratios of the lengths of the ellipsoid principal axes. The demagnetizing energy of such a body is [25]

\[ E_d = -\frac{\mu_0}{2} \int_V \vec{M} \cdot \vec{H}_d dV. \]  

(2.9)

In the event that \( M \) lies along any of the principle axes, the demagnetizing tensor \( N \) reduces to numbers, known as demagnetizing factors [16]. Notice that \( E_d \) is always greater or equal to zero because the demagnetizing field is always directed opposite to the magnetization. It can be seen that a way to minimize Eq. (2.9) is to minimize the total \( M \), i.e. that the vectorial sum of the magnetization within the entire body should be as small as possible (see concept of magnetic domains below). The internal field of a uniformly magnetized body other than an ellipsoid is not uniform [23]. But as shown by Brown [11, 12], one can by considering the appropriate average (like the volume average) of the internal field still define demagnetizing factors that correspond to an ellipsoidal body with appropriate dimensions. In effect, Eq. (2.8) can as well be used for non-ellipsoidal bodies. The magnetostatic field \( \vec{H}_d \) can be calculated from the magnetic scalar potential \( \phi_d \) [16, 22, 23] from the relation \( \vec{H}_d = -\nabla \phi_d \) (a result of \( \vec{H}_d \) being irrotational). The scalar potential itself is given by

\[ \phi_d(\vec{r}) = \frac{1}{4\pi} \left[ -\int_V \frac{\nabla \cdot \vec{M}(\vec{r}')}{|\vec{r} - \vec{r}'|} dV' + \int_S \frac{\vec{M}(\vec{r}') \cdot \hat{n}}{|\vec{r} - \vec{r}'|} dS' \right], \]  

(2.10)

where \( \hat{n} \) is the body outward normal. The integrals in Eq. (2.10) can be interpreted as fields arising from volume (-\( \nabla \cdot \vec{M} \)) and surface (\( \vec{M} \cdot \hat{n} \)) charge densities, respectively. It is therefore common to use the notion of magnetic sources and sinks that act like positive and negative magnetic charges, just like in electrostatics but with the difference that magnetic charges never occur isolated but always come in balancing pairs. They just act as magnetic charges. The corresponding demagnetizing field obtained from Eq. (2.10) is [22]

\[ \vec{H}_d(\vec{r}) = \frac{1}{4\pi} \left[ -\int_V \frac{(\vec{r} - \vec{r}') \nabla \cdot \vec{M}(\vec{r}')}{|\vec{r} - \vec{r}'|^3} dV' + \int_S \frac{(\vec{r} - \vec{r}') \vec{M}(\vec{r}') \cdot \hat{n}}{|\vec{r} - \vec{r}'|^3} dS' \right]. \]  

(2.11)

The demagnetizing tensor \( N \) of a uniformly magnetized non-ellipsoidal body may be obtained from the evaluation of the demagnetizing field arising from surface charges, i.e. the surface part of Eq. (2.11), then taking the appropriate average of that field (see Newell et al. [26]).

A magnetically uniform state, in the absence of any external field, is unfavourable from a demagnetizing energy point of view. This is easily understood by looking at Eq. (2.9). Clearly by arranging the magnetization within
the sample in such a way that $\vec{M}$ vectorially is zero should effectively minimize the demagnetizing energy. A material may therefore break up into regions called magnetic domains, where within each such domain the magnitude of the magnetization is $M_s$. The domains are then oriented in such a way that the vectorial sum of the $M_s$ cancel out as much as possible, thus minimizing Eq. (2.9) (see Fig. 2.2). The first theoretical treatment of such a flux closure domain state was pioneered by Landau and Lifshitz in 1935 [7]. The transition region between two domains requires a gradual rotation of the magnetization. This will give rise to energy penalties in both exchange and MA energies. The thicker the wall, the more effectively the exchange energy is minimized [24], since then the relative angles between neighboring magnetic spins are very small. However, the thicker the wall, the higher is the MA energy penalty since a large section of magnetic spins have directions deviating from the easy axes. In effect, the equilibrium domain configuration will be the result of a minimization of magnetostatic, exchange and MA energies.

The formation of domains cannot occur for very small particle sizes. At a certain size limit, the so called single domain limit (SDL), it becomes energetically unfavourable to create domain walls because the lowering of the magnetostatic energy is not sufficient to motivate the energy stored in a possible domain wall. Thus, there exists a critical size (material dependent of course) above which domain structures can form. Needless to say, if a particle has a non-uniform magnetization distribution, complications arise as to how the calculation of the demagnetizing tensor is to be performed. The system will then have to be divided into sets of subvolumes where each subvolume is small enough to be a single domain entity and mutual demagnetizing tensors can then be calculated to obtain the magnetostatic fields (see section Effective fields and Numerical technique). The number of different domain configurations that can be found are vast and are functions of both intrinsic (e.g. exchange stiffness, saturation magnetization, MA, etc.) and extrinsic materials properties (shape, lateral extension and thickness). It is of course not the object of this summary to even begin to go through the different types encountered or predicted, but for a thorough overview on that topic the reader is referred to the monumental book by Hubert and Schäfer [16] (see also for instance [27, 28]). However, one of the greatest contributions to the study of magnetic...
microstructure provided from micromagnetics is the calculation of the domain walls, i.e. the structure of the transition regions separating domains. It therefore seems appropriate to say something about the most characteristic domain walls cited. Again, the different types of domain walls are numerous but there are two pure characteristic cases that constitute the basis of how the transition regions between domains are constructed. These are the so called Bloch and Neel walls. For an overview of the different varieties and/or hybrides of such walls the reader is again referred to Hubert and Schäfer [16]. Consider two domains, where the magnetizations within each of these are oriented anti-parallel with respect to each other as in Fig. 2.3. Then, in a Bloch wall, the change of spin direction between domains occurs gradually over many atomic planes. The wall is named after Felix Bloch who was the first to study this type of transition layer [9]. However, Neel [29] realized that the theory of Bloch walls breaks down for thin films, if by thin one means thicknesses that are comparable to the wall width. The argument goes as follows: If one approximates the wall by an elliptical cylinder having width $W$ and height $D$ (cf. Fig. 2.4), then the demagnetizing factor for a Bloch wall $N_{\text{Bloch}} = W/(W + D)$ [16]. If $W > D$, the demagnetizing energy of the wall is large and the wall magnetization would rather flip into being in-plane (cf. Fig. 2.4), a so called Neel wall. The demagnetizing factor for such a wall is $N_{\text{Neel}} = D/(W + D)$ and $N_{\text{Neel}} < N_{\text{Bloch}}$ for $W > D$.

### 2.1.4 Zeeman energy

The Zeeman energy is due to the interaction of the magnetization with an external magnetic field $\vec{H}_{\text{App}}$. By aligning the magnetization along the applied field direction, the system can lower its energy. The Zeeman energy is given by [25]

$$E_z = -\mu_0 \int_V \vec{M} \cdot \vec{H}_{\text{App}} dV.$$  \hspace{1cm} (2.12)
2.1.5 Thermal energy

One difficult fact is that the saturation magnetization $M_s$ is temperature dependent. This causes problems in the evaluation of the energy terms since if there are spin fluctuations present caused by thermal energy, there is no real justification in assuming that the temperature is exactly the same at every point in a material. Setting $M_s$ constant is only true when considered as an average quantity over some volume. As particles shrink in size, the single domain state becomes favourable. When the superparamagnetic limit is reached, thermal fluctuations exceed the energy barriers that block reversal and including thermal effects in calculations becomes a necessity. In micromagnetic simulations, thermal fluctuations are usually taken into account by adding a random fluctuation field term to the effective field (see next section for information about the effective field). This stochastic random field accounts for the interactions of the magnetization with for instance phonons, conduction electrons, nuclear spins, etc. The statistical characteristics of the stochastic field is assumed to be that of a Gaussian random process. For a thorough treatment of thermal fluctuations, see the work by Palacios and Lazaro [30]. How to include thermal fluctuations in micromagnetic simulations will briefly be mentioned in the numerical technique section. For now, focus is put on the deterministic parts.

2.2 The effective field

Adding up all the energy terms described above, the total energy is

$$E_{tot} = \int_V \left\{ A \left[ (\nabla m_x)^2 + (\nabla m_y)^2 + (\nabla m_z)^2 \right] + \epsilon_k - \mu_0 \vec{M} \left( \frac{1}{2} \vec{H}_d + \vec{H}_{App} \right) \right\} dV,$$

(2.13)
where $e_k$ is the MA energy density ($E_k/V$). The total field acting on $\vec{M}$ is the sum of the fields originating from all the different interaction contributions. This field or what is called the effective field can be calculated from the energy density [31, 32]

$$\vec{H}_e = -\frac{1}{\mu_0 V} \frac{\partial E_{tot}}{\partial \vec{M}}.$$  \hspace{1cm} (2.14)

In this way one gets expressions for the exchange, anisotropy, magnetostatic and applied fields. The exchange field is [31, 32]

$$\vec{H}_{ex} = \frac{2A}{\mu_0 M_s} \nabla^2 \vec{m},$$  \hspace{1cm} (2.15)

and the (uniaxial) anisotropy field becomes [25, 32]

$$\vec{H}_k = \frac{2K_u}{\mu_0 M_s} (\vec{m} \cdot \hat{u}) \hat{u},$$  \hspace{1cm} (2.16)

where $\hat{u}$ is a unit vector along the easy direction. The applied field is simply $\vec{H}_{App}$. The magnetostatic or demagnetizing field can generally be evaluated from Eq. (2.11). It is the most cumbersome field term and several techniques exist in numerical micromagnetics to evaluate it. One approach, which is used in this work, is to rely on the fact that the demagnetizing field for a uniformly magnetized body can be written as in Eq. (2.8). The main problem is then that even in single domain particles the magnetization is not really uniform [26]. However, in the case of a material exhibiting a spatially varying magnetization distribution one can by dividing the model into subvolumes small enough to assume uniform magnetization within each subvolume, find expressions for the demagnetizing tensor corresponding to the magnetostatic interactions between these subvolumes. This tensor will be a function of the difference coordinates between subvolumes and the geometry of the considered subvolumes. A generalization of such a tensor has been obtained by Newell, Williams and Dunlop [26] and constitute the expressions used in the evaluation of the demagnetizing fields in the numerical micromagnetics part of this thesis. The generalized tensor handles both the self and mutual magnetostatic interactions. For a complete derivation, see [26]. In summary, the effective field acting on $\vec{M}$ is then

$$\vec{H}_e = \vec{H}_{ex} + \vec{H}_k + \vec{H}_{d} + \vec{H}_{App}$$  \hspace{1cm} (2.17)

### 2.3 Micromagnetic length scales

Micromagnetics suffers from a wide range in length scales, from the short-range exchange interaction to the long-range dipolar interaction. The range of
relevant length scales is also a function of the magnetic state in the material. For example, if one considers the vortex structure (see section on magnetic vortices), then one of the smallest features is the region where the magnetization orients itself perpendicular to the surface. Another typical example is the characteristic features of Neel walls in thin films. This characteristic length is $\sqrt{A/K_m}$ [16], where $K_m$ is the stray field energy constant defined as $K_m = \frac{1}{2} \mu_0 M_s^2$. Another length is the Bloch-wall width parameter $\sqrt{A/K}$ [16], where $K$ is the anisotropy constant. Both these lengths are typically referred to as the exchange length. In general (neglecting some other contributions such as magnetostriction) it is advisable to keep below or equal to the smallest of these characteristic lengths in micromagnetic calculations (for Permalloy it is about 5 nm), in order to properly resolve structural features. An example of the dependence of the magnetic state in a material on the relevant length scale is indicated by the micromagnetic standard problem no. 4 (see section Illustrative numerical experiments). Depending on what one wants to compute it is a good strategy during a situation of uncertainty to also try using discretization cell sizes below the exchange length and see if there is mesh independence or not.

2.4 Magnetization dynamics

The dynamical behaviour of the magnetization vector in the presence of an effective field can be described by the Gilbert-equation of motion [31]:

$$\frac{d\vec{M}(t)}{dt} = -\gamma_0 \left[ \vec{M}(t) \times \vec{H}_e(t) \right] + \frac{\alpha}{M_s} \left[ \vec{M}(t) \times \frac{d\vec{M}(t)}{dt} \right],$$

(2.18)

where $t$ is time, $\alpha$ is a phenomenological damping parameter and $\gamma_0$ is the gyromagnetic ratio. Or, there is the equivalent Landau-Lifshitz form [31]

$$(1 + \alpha^2) \frac{d\vec{M}(t)}{dt} = -\gamma_0 \left[ \vec{M}(t) \times \vec{H}_e(t) \right] - \frac{\alpha \gamma_0}{M_s} \left\{ \vec{M}(t) \times \left[ \vec{M}(t) \times \vec{H}_e(t) \right] \right\},$$

(2.19)

after the earlier work of Landau and Lifshitz in 1935 [7]. The first term on the right-hand side describes the precession of $\vec{M}$ around $\vec{H}_e$, whereas the second term describes the dissipation, causing $\vec{M}$ to spiral in towards the direction of $\vec{H}_e$ to finally align in parallel. Finally, for computational purposes, it is convenient to work in dimensionless units. If one lets $\tau = \gamma_0 M_s t$, $\vec{m} = \vec{M}/M_s$ and $\vec{h}_e = \vec{H}_e/M_s$, Eq. (2.19) can be cast into the dimensionless form [31]

$$(1 + \alpha^2) \frac{d\vec{m}(t)}{d\tau} = -\left[ \vec{m}(t) \times \vec{h}_e(t) \right] - \alpha \left\{ \vec{m}(t) \times \left[ \vec{m}(t) \times \vec{h}_e(t) \right] \right\}.$$  

(2.20)

A schematic showing the directions of the vector products in Eq. (2.20) is displayed in Fig. 2.5, while Fig. 2.6 shows a plot of the numerical solution of Eq. (2.20) for a single magnetization vector in a magnetic field.
Figure 2.5: Schematic of the vectors in Eq. (2.20).

Figure 2.6: Numerical solution of Eq. (2.20) for a single magnetization vector $\vec{m}$ in a field with $\alpha=0.1$ and $\vec{h}=[001]$. The initial magnetization was set to $\vec{m}=[010]$. 
3. Numerical technique

3.1 Discretization

It is now time to use Eq. (2.20) in the modelling of a (in this case) ferromagnetic system. Since one wants to model it numerically one will need to decide how to discretize the geometry. Two ways of discretizing are commonly used; one is a finite difference (FD) division where the space is discretized into identical rectangular boxes (see Fig. 3.1) and the other one is a finite element (FE) scheme where the discretization cells can take practically any shape. For an overview of both of these, see for instance the review by Fidler [32]. There are both advantages and disadvantages with both methods. The FD method allows for advantages in computational speed due to the possibility of using Fast Fourier Transform (FFT) methods in evaluating the demagnetizing field [33, 34, 35, 36, 37]. On the other hand, if one wishes to model a perfect material with curved boundaries, the FD method will approximate the boundary stepwise whereas the FE method allows for cell shapes that can more accurately follow the shape of the boundaries. Which method to use should be decided by the problem that one wishes to study. Associated errors in the computed exchange and stray fields due to stepwise approximations of curved boundaries have been explored by Cervera [38]. However, in direct comparisons with experimental results it is not so evident that a more perfect boundary is always to be preferred. For instance, lithographically defined dots (which are being studied here) cannot be made perfect and will have jagged boundaries. In fact, in order to make simulations more directly comparable to experiments it could possibly be an advantage to create imperfect boundaries, by removing the magnetization within cells in a random fashion along the boundary. Another possibility is to use a high-resolution image of the sample as the template for the geometry (see for instance [39]). However, even if samples with very smooth boundaries cannot be manufactured as of yet, it is still important to compute the magnetization dynamics for perfect model systems in order to get detailed insight into the more pure nature of the problem. If one is uncertain about how imperfections in the discretization scheme are influencing the results of the modelling, one can perform several simulations, gradually reducing the cell dimensions while keeping track of the partial energy densities. As an example; if one is discretizing a circular cylinder by a FD scheme one typically ends up with +/- poles all along the sample boundary and the associated increase in demagnetization energy is then just an artifact due to discretization. Lastly, one other advantage of the FE method
that should be mentioned is the possibility of using mesh refinements in those locations where the solutions to the LL-equations are most interesting by allowing the mesh to move with the trajectory of a magnetic feature such as for instance a domain wall. For examples of this see Ref [40]. However, in general the FE method is in comparison to the FD technique much more complicated to implement, and since in this work satisfactory results have been produced by the FD scheme, there has been no need for an FE implementation.

3.2 The effective field and its discretization

When computing the effective fields, the continuum expressions have to be discretized which in the case here is handled by using a FD scheme. The exchange field is given by Eq. (2.15) and on discretized form, the Laplacian can be approximated by [41, 42, 33, 43, 44]

$$\nabla^2 \vec{m} \approx \frac{\vec{m}_{i+1,j,k} - 2\vec{m}_{i,j,k} + \vec{m}_{i-1,j,k}}{(\Delta x)^2} + \frac{\vec{m}_{i,j+1,k} - 2\vec{m}_{i,j,k} + \vec{m}_{i,j-1,k}}{(\Delta y)^2} + \frac{\vec{m}_{i,j,k+1} - 2\vec{m}_{i,j,k} + \vec{m}_{i,j,k-1}}{(\Delta z)^2}.$$  \hspace{1cm} (3.1)

For the uniaxial anisotropy field, Eq. (2.16) is directly applicable and the external field is as mentioned before directly implemented as $\vec{H}_{App}$. The most cumbersome of the field terms is the demagnetization field $\vec{H}_d$, which essentially is given by the interaction between dipoles. The cumbersome aspect is that it is long-range and it is advisable to take into account the interaction of all dipoles in the system and since it is a pairwise interaction it is an expensive
evaluation with respect to computational effort. Luckily, it turns out that the mathematical form of $\vec{H}_d$ enables a procedure that substantially speeds up this field evaluation. If the magnetization is discretely represented by the distribution $\{\vec{M}_{i,j,k}\}$ at points $\{\vec{r}_{i,j,k}\}$, then the magnetostatic field at point $\vec{r}_{i,j,k}$ due to all points $\vec{r}_{i',j',k'}$ is given by [26]

$$\vec{H}_d^{i,j,k} = - \sum_{i',j',k'} \vec{N} \left( \vec{r}_{i,j,k} - \vec{r}_{i',j',k'}, \Delta x, \Delta y, \Delta z \right) \cdot \vec{M}_{i',j',k'}.$$  \hspace{1cm} (3.2)

Here $\vec{N}$ is an interaction tensor called the demagnetizing tensor and is a function of the difference coordinates $\vec{r}_{i,j,k} - \vec{r}_{i',j',k'}$ and the discretization cell dimensions $\Delta x, \Delta y$ and $\Delta z$. The demagnetization tensor is quite complicated to evaluate and there exists several works that have derived analytical expressions for the tensor elements between interacting rectangular or cubic slabs [45, 26, 35]. In the implementation herein expressions derived by Newell et al. [26] have been used. Since the dipole-dipole interaction is pairwise, the computational time cost to evaluate Eq. (3.2) is of the order of $X^2$, if the computational space is subdivided into $X$ cells. However, one can see that Eq. (3.2) is in the form of a convolution and that it corresponds to a discrete convolution in each dimension. This opens up the possibility to reduce the computational effort by using the convolution theorem [42, 43, 46] to transform the expression into a product in frequency space. Standard FFT’s assume that the data is periodic. However, this is not the case when one has a confined geometry, which is the case in this work. For this reason, the magnetization array has to be padded with zeros in such a way that the buffer area between a physical cell and the first image cell in an adjacent period is larger than the range of magnetostatic interactions. This requires a void area of at least two times the dimension in each direction of the physical area [35, 36, 37]. By doing this, stray fields from artificial structures will not affect the computational region of interest. Another necessity is that for the FFT to work, the number of discretization cells after zero padding has to be a power of two [42] (greater or equal to twice the number of physical cells in each dimension). Of course, if one wishes to perform computations on a continuous film, the computation region of interest is usually much smaller than the extension of the film and so periodic boundary conditions can be assumed and the zero paddings would not be necessary. But in this work, confined geometries such as lithographically defined structures are always considered, so zero-padding is always implemented. In effect, the space where to evaluate the stray field grows to at least twice in each dimension but nevertheless gives a huge computational speedup as the size of the system grows. The demagnetizing tensor has to be Fourier transformed only once at the beginning of the simulation since it only depends on the geometry of the system. At every iteration, the field is then evaluated by Fourier transforming the magnetization vector, performing the product implied by Eq. (3.2) and then taking the inverse Fourier
transform of that product. It is not the object of this thesis to reiterate standard numerical techniques, so for computational methodologies in general involving the convolution and deconvolution of matrices and zero-padding the reader is referred to the nearly all-covering reference in computational methods, Numerical Recipes [42]. And for ideas about implementations to compute the demagnetizing field in magnetic systems, see for instance the works of Bagnères, Hayashi and Champion [35, 36, 37].

3.3 Codes used in this thesis

The numerical micromagnetic work performed in this thesis is the result of using several codes. In total, three codes have been utilized:

1. The code developed here.
2. The Object Oriented Micromagnetic Framework (OOMMF) [47].
3. The LLG Micromagnetics Simulator package [48].

Characteristics of the code developed herein and work performed with it

The code developed here has basically been described in the preceeding sections. To summarize, it is a standard 3D finite difference code. It takes into account the basic energy contributions; exchange, magnetostatic, MA and Zeeman energies, with the possibility of adding thermal fluctuations (see paper I). The exchange interactions are modeled by a nearest neighbour interaction, using the standard finite difference expansion of the Laplacian [41, 42, 33]. Magnetostatics are computed with FFT methods [42] (see also [35, 36, 37]) using expressions for the demagnetizing tensor elements by Newell et al. [26]. The LL-equations are solved by a fourth order Runge-Kutta scheme [42, 49] (in the case of the work performed in paper I, a Heun scheme [50] was implemented). The equilibrium criterion employed in deterministic simulations is a standard minimum-torque criterion (a typical value used is $10^{-6}$). Finally, routines for computing MFM images are included.

Several computations have been made against the OOMMF Oxsii solver [47] with excellent agreement on the results. Computations performed by using the code developed herein are:

- Standard $\mu$MAG Problem no. 3 (Illustrative numerical experiments section).
- Standard $\mu$MAG Problem no. 4 (Illustrative numerical experiments section) and Ref. [20].
- All work on the effect of thermal fluctuations (paper I).
- Computed MFM-images in Fig. 6.2.
• Computed magnetization distributions and cross-sections in Figs. 6.5, 6.6 and 6.19.

For information concerning the OOMMF and the LLG Micromagnetics Simulator packages, the reader is referred to their respective manuals or websites [47, 48]. What should be mentioned though is that they both are FD codes and that comparative runs with them have yielded the same results, which is why it is legitimate to mix the usage of them. Wherever OOMMF or LLG Micromagnetics Simulator packages have been used, it will be mentioned and cited in the forthcoming sections.
4. Illustrative numerical experiments

In this section some illustrative numerical work is presented and discussed. The problems considered here highlight some of the troubles that should always be in the back of the mind when performing micromagnetic simulations. The first two problems are two of the so called \( \mu \mathrm{MAG} \) standard problems. At the National Institute of Standards and Technology (NIST)/Center for Theoretical and Computational Materials Science (CTCMS), several micromagnetic standard problems have been issued with the objective to let researchers compare and test their computational methods and codes, and to identify problems and bugs in their programs [20]. The herein developed code has been used to leave contributions to problems no. 3 and 4. Apart from being an excellent opportunity to compare results with contributions from other groups, it gives valuable insights into the possible problems arising in these types of computations. The reason for choosing problem no. 3 is to do a full 3D computation of static magnetic configurations and to test the effect of using different mesh sizes. To also test spin dynamics and predictions for the magnetization reversal path, problem no. 4 was chosen. The study of the latter problem has been completed and published, while the results from the study of problem no. 3 has not yet been submitted. The reason for this is simply to get a few more data points in the necessary fitting and extrapolation procedures. But already at this stage, parameters obtained from the extrapolations are in excellent agreement with the contributions of Rave et al. and Martins et al. [20]. Finally, some possible scenarios for small particle switching at finite temperatures are included as the last example in this section.

4.1 Standard \( \mu \mathrm{MAG} \) problem no. 3

The objective of this problem is to calculate the single-domain (SDL) limit for a cube with edge \( L \) consisting of a ferromagnetic material exhibiting uniaxial MA. The transition structure considered is the vortex state. The two states considered in the transition are schematically shown in Fig. 4.1. This problem uses reduced units and thus no specific material parameters need to be defined. Instead \( L \) is expressed in terms of the intrinsic length scale \( l_{\text{ex}} = \sqrt{A/K_m} \), where \( K_m \) is the magnetostatic energy density and is given by \( K_m = (1/2)\mu_0 M_s^2 \). The MA is expressed in terms of an anisotropy parameter \( Q = K_u/K_m \) and the energy densities are expressed in units of \( K_m \). The desired outputs are; the SDL limit, i.e. the value of \( L \) for which the flower and vortex
states are degenerate, the partial energy densities at the SDL and the spatially averaged magnetizations along the cube axes. The anisotropy parameter used in this simulation is $Q = 0.1$, but calculations on cubic particles using other values of $Q$ have been reported by Rave [51]. The discretization setup is displayed in Fig. 4.2 and vector plots in different planes of the vortex state are shown in Fig. 4.3. Notice the twisting of the vortex close to the cube faces. Interestingly enough, the SDL is found to be inversely proportional to the square of the number of cells in each dimension, i.e. SDL $\propto 1/N^2$ as can be seen in Fig. 4.4. The same dependence was found for most quantities extracted from this problem study. The critical edge size as well as the values displayed in Table 4.1 were obtained by fitting a $1/N^2$-dependence to the computed results and extrapolating to $N \rightarrow \infty$. Also to be mentioned is that, for increasing
Figure 4.3: Computed magnetization distribution for the vortex state in various planes of the cube.

Table 4.1: Results for μMAG problem no. 3; total and partial reduced energy densities (magnetostatic, exchange and MA energies), and reduced magnetization components along the x and z-axes. The computed critical edge length at infinitely fine meshing is $L = 8.47(96)$. 

<table>
<thead>
<tr>
<th>$e$ and $\langle m \rangle$</th>
<th>Flower</th>
<th>Vortex</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e_{demag}$</td>
<td>0.27939</td>
<td>0.07827</td>
</tr>
<tr>
<td>$e_{exchange}$</td>
<td>0.01771</td>
<td>0.17231</td>
</tr>
<tr>
<td>$e_k$</td>
<td>0.00561</td>
<td>0.05213</td>
</tr>
<tr>
<td>$e_{total}$</td>
<td>0.30271</td>
<td>0.30271</td>
</tr>
<tr>
<td>$\langle m_z \rangle$</td>
<td>0.97095</td>
<td>-</td>
</tr>
<tr>
<td>$\langle m_x \rangle$</td>
<td>-</td>
<td>0.35162</td>
</tr>
</tbody>
</table>
edge length, an intermediate state, the twisted flower state, was found by Hertel and Kronmüller [52]. In this state, the magnetization twists along the easy axis.

4.2 Standard $\mu$MAG problem no. 4

The objective of this problem is three-fold. Firstly, from a seed state consisting of an S-state [20] and for two different field cases to calculate the time evolution of the spatially averaged magnetization of the sample. The two fields, $\mu_0\vec{H}_1$ and $\mu_0\vec{H}_2$ will henceforth be referred to as the field 1 and field 2 cases, respectively. Secondly, to identify the magnetization reversal path for the two cases and lastly, to record the influence of using different discretization cell sizes (here called $\Delta_1$ and $\Delta_2$). The sample geometry is shown in Fig. 4.5 and all input parameters used in the simulations are listed in Table 4.2. The initial S-state was created by slowly reducing a saturating field to zero along the [111]-direction. In this problem, 3D spins on a 2D mesh was used, since the sample thickness is well below the exchange length. The results are shown in Fig. 4.6.

In Fig. 4.7(a) one can see that mesh independence has been achieved, and thus using $5\text{nm}\times5\text{nm}\times3\text{nm}$ cells is enough to properly resolve all structural features. The magnetization distribution in Fig. 4.6(a) shows that for the field

Figure 4.4: The SDL plotted as a function of $1/N^2$. The dashed line is a linear fit, yielding the SDL $L=8.47\cdot l_{ex}$ at infinitely fine meshing.
Figure 4.5: Geometry of sample in standard problem no. 4.

Simulation parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>0.02</td>
</tr>
<tr>
<td>$M_s$</td>
<td>800 kA/m</td>
</tr>
<tr>
<td>$K$</td>
<td>0.0</td>
</tr>
<tr>
<td>$\gamma_0$</td>
<td>$2.211 \cdot 10^5$ m/As</td>
</tr>
<tr>
<td>$L$</td>
<td>500 nm</td>
</tr>
<tr>
<td>$d$</td>
<td>125 nm</td>
</tr>
<tr>
<td>$t$</td>
<td>3 nm</td>
</tr>
<tr>
<td>$\mu_0\vec{H}_1$</td>
<td>-24.6x+4.3ŷ+0.0ẑ mT</td>
</tr>
<tr>
<td>$\mu_0\vec{H}_2$</td>
<td>-35.5x-6.3ŷ+0.0ẑ mT</td>
</tr>
<tr>
<td>$\Delta_1$</td>
<td>5nm×5nm×3nm</td>
</tr>
<tr>
<td>$\Delta_2$</td>
<td>2.5nm×2.5nm×3nm</td>
</tr>
</tbody>
</table>

Table 4.2: Material and simulation parameters in $\mu$MAG no. 4.

1 case reversal starts through counter-clockwise rotation at the end domains of the S-state and that the central portion of the particle lags behind. Figure 4.7(b) clearly shows a difference in the oscillations for the two meshes. Notice the narrow domain walls in Fig. 4.6(b). A plausible explanation for this is that the 5nm cells are not small enough to properly resolve the details of these narrow walls and that in this case the relevant length scale is determined by the domain wall features. The same finding of a significant mesh dependence for the field 2 case has been found by all the other contributing groups [20]. The magnetization reversal displayed in Fig. 4.6(b) shows two narrow domain walls formed by counter-clockwise rotation of the magnetization at the ends but clockwise rotation in the center of the slab. This is also a clear demonstration of that the switching of the particle magnetization in a delicate way
Figure 4.6: (a) Time evolution of spatially averaged magnetization components for field 1, cell size $\Delta_2$. (b) Time evolution of spatially averaged magnetization components for field 2, cell size $\Delta_2$. The inserts show the magnetization distributions corresponding to $\langle m_x \rangle = 0$. 
Figure 4.7: (a) Effect of cell size for field 1. (b) Effect of cell size for field 2.
depends on the applied field direction, which is an important aspect in applications where speed in reversing the magnetization of a device is of outmost importance. Figure 4.8 displays the time evolution of \( \langle m_y \rangle \), including results of all groups having contributed to \( \mu \)MAG problem no. 4. One sees a close correspondence between all results for the field 1 case, but after the second oscillation for field 2, clear deviations in results are observed, despite the fact that the reversal path snapshots (inserts in Figs. 4.6 (a) and (b)) correspond well among the various contributions. The reason for the observed discrepancies for the field 2 case is not clear. One possibility to consider is that the different contributions have not always used the same discretization cell sizes (although everyone has kept them smaller than the exchange length), and since simulation results indicate a significant mesh dependence for the field 2 case it could show up as clear differences in the time evolution of the magnetization components. Another possibility is that not all have used the same computational methods, especially considering the evaluation of the demagnetizing field. Then there is also the seed state; it is possible that different contributions have created slightly different S-states due to setting different stability criteria. The result may be a slight variation in the magnetization distribution, which perhaps becomes visible in the results for the field 2 case. The goal is of course that, provided that the approximations used from a physical point of view are valid and that there are no programming errors, the different methods used should produce the same results, which is why this is something that needs to be investigated in detail. This is, however, not within the scope of this thesis work.
4.3 Thermal fluctuations

Effects of temperature are (as briefly stated in a previous section) introduced in the LL-equations by adding a stochastic field, \( \vec{H}_t \), to the effective field in Eq. (2.2). This random field is assumed to be a delta correlated Gaussian distributed random series of field pulses with variance \([39, 53]\)

\[
\sigma^2 = \frac{2\alpha k_B T}{\gamma_0 M_s V \left(1 + \alpha^2\right) \Delta t},
\]

(4.1)

where \( V \) is the mesh element volume and \( \Delta t \) is the timestep used in the simulation. One way to generate the random numbers is through the Box-Müller algorithm [42], which is the method used in paper I. Some work on small particle switching under finite temperature conditions were made to make the point that thermal fluctuations can cause alterations in the switching path, yielding different switching fields and time scales. Presented in this section is a summary of results for a 100 nm \( \times \) 50 nm \( \times \) 5 nm particle (yet another particle size was in addition considered in paper I). In short, it is found that thermal fluctuations can cause alterations in the switching path thus influencing the switching field and time scales. Similar results have later been found by Liu and Cervera [54]. One example is displayed in Fig. 4.9. Notice the relatively large jump in coercivity between 50 K and 100 K. This is found to be the result of a thermally induced alteration in the switching path (see paper I for details). At 50 K (and below) the particle chooses to go through a C-state and at 100 K an S-state switching is enabled. Such states are shown in Fig. 4.10. For these particle sizes and geometries it can be seen that going through an S-state significantly reduces the switching field. An important aspect of stochastic simulations is the time spent at each field point, \( t_{obs} \), when performing a hysteresis...
Figure 4.10: Left: Remanent states at different temperatures. Middle: Magnetic states at a field of -30 kA/m. Right: Schematics of flower, C- and S-states.

curve calculation. In the deterministic case one can always use an equilibrium condition such as a minimum torque criterion. When thermal fluctuations are involved it is not evident how to include a stability criterion since the magnetizations will always be subject to random field pulses. In general it is always best to try to mimic the experimental conditions as much as possible. Here, for instance, hysteresis curves are computed for various $t_{obs}$; Fig. 4.11 displays the coercivity as a function of $t_{obs}$ for one of the particles in paper I. $t_{obs}$ is accordingly choosen to be in the interval where the coercivity within statistical errors remains constant. The interpretation of the change in switching path can be that the S-state is slightly higher in energy than the C-state and when sufficient energy is given to the system (at some elevated temperature), the S-state can be formed and the switching then occurs through this state. In any case, the objective of this numerical experiment is not to determine the characteristics of a certain material. Parameters for Permalloy were chosen just to have a system to work with. The objective is simply to show that thermal fluctuations can play a significant role as far as switching characteristics are concerned and that it is possible to simulate such behavior, which is why this example is put in the section presenting illustrative numerical experiments. A possible scenario is as shown in Fig. 4.12, where below a certain temperature the particle predominantly switches via a C-state (to the left of the dotted vertical line), while above this temperature the particle favors going through an S-state (to the right of the dotted vertical line).
Figure 4.11: Coercivity as a function of time spent at each field point, T=100 K.

Figure 4.12: Coercivity as a function of temperature.
5. Experimental techniques

5.1 Sample preparation
All samples studied in this work have been made with electron-beam (e-beam) lithography. A brief description of the e-beam lithography technique is as follows: A substrate (in this case Si) is covered with a thin layer of photo-resist. This is done by applying a drop of the photo-resist onto the substrate, followed by spinning of the substrate using a resist spinner. The thin layer of resist thus obtained is then baked by placing the substrate on a hot-plate. If a bilayer resist method is used then the process includes a second layer of another photo-resist. In this work ZEP 520A [55] resist is always used as the imaging layer (except in section 'Implementation of possible application of patterned magnetic thin films' and in papers II, IV and VIII, where PMMA [56] was used). The bilayer resist method in addition uses PMGI [56] resist as underlayer. Desired patterns are drawn in a program available on the computer controlling the path of the e-beam. The e-beam is then scanned according to the patterns over the sample surface. Wherever the beam hits the photo-resist the bondings of the resist will be weakened. For the bilayer method, one resist is more sensitive to the e-beam than the other (see Fig. 5.1). The next stage is to develop the sample whereby the exposed parts of the photo-resists are dissolved and rinsed away. In the bilayer case, two different developers are used, one for each type of photo-resist. For information about developer fluids, see the websites [55, 56]. After the developing procedure, metal (in this case Permalloy) is deposited by either sputtering or thermal evaporation. At this stage one has metal filling up the empty areas in the photo-resist and on top of the remaining resist. The latter along with the resist remains are removed during the last process step (lift-off step) by placing the sample in a strip solution [55, 56]. The final result consists of elevated metallic structures with shapes corresponding to the pattern formed by the e-beam writing. A simple schematic illustrating the bilayer situation is depicted in Fig. 5.1.

5.2 Magnetic force microscopy (MFM)
For MFM images taken in this work a Digital Instruments Nanoscope Dimension 3100 has been used [57]. In this section the main principles of imaging will briefly be described.
5.2.1 Principles of MFM imaging

In MFM, a cantilever with a tip coated with a magnetic material is first scanned over the surface in a tapping mode fashion. The tip is then raised to some height over the surface and scanned over the surface. The tapping procedure yields topographic data while in the lift scan the influence of magnetic forces are measured by the principle of force gradient detection. With no magnetic forces present each cantilever has a natural resonant frequency. When the tip experiences magnetic forces this frequency shifts by some amount, proportional to the vertical magnetic force gradient felt by the tip [58]. There will also be associated shifts in the oscillation amplitude and in the phase difference between the probe oscillation and the driving signal [59]. Thus, there are several ways to obtain magnetic contrast (frequency modulation, amplitude detection and phase detection [58]). For the imaging done for this thesis work, the phase detection mode has been used whereby the phase difference between the cantilever's oscillation relative to the piezo drive is measured. In order to apply magnetic fields and do in-situ measurements an XY-magnet was constructed (Fig. 5.2). The currents in the coils are controlled by a LabView [60] program, thus enabling an efficient way of applying in-plane fields at arbitrary angles.
5.2.2 Magnetic interactions between tip and sample

The magnetic tip will experience repulsive or attractive forces when scanned over the sample. This is due to interaction with the sample stray field \( \vec{H}_s \). The force \( d\vec{F}_t \) acting on a volume element \( dV' \) of the tip at the internal coordinate position \( \vec{r}' \) (see Fig. 5.3) is given by [61]

\[
d\vec{F}_t = \mu_0 \nabla \left[ \vec{M}_t(\vec{r}') \cdot \vec{H}_s(\vec{r} + \vec{r}') \right] dV',
\]

(5.1)

where \( \vec{M}_t(\vec{r}') \) is the magnetization of the tip volume element \( dV' \) and \( \vec{H}_s(\vec{r} + \vec{r}') \) is the stray field from the sample. If \( \hat{n} \) is a unit vector normal to the cantilever then the force along that direction is \( \hat{n} \cdot d\vec{F}_t \). In effect, the force gradient along \( \hat{n}, dF'_{t,\hat{n}} \), is then given as [61]

\[
dF'_{t,\hat{n}} = \mu_0 \hat{n} \cdot \nabla \left\{ \hat{n} \cdot \nabla \left[ \vec{M}_t(\vec{r}') \cdot \vec{H}_s(\vec{r} + \vec{r}') \right] \right\} dV'.
\]

(5.2)

For the case when the cantilever is parallel to the sample surface (\( \hat{n} = \hat{z} \)) and the tip can be approximated by a point dipole, it can be shown that the expression of the force gradient reduces to [61]

\[
F'_{t,\hat{z}} = \mu_0 \hat{n} \cdot \nabla \left\{ \hat{n} \cdot \nabla \left[ \vec{M}_t(\vec{r}') \cdot \vec{H}_s(\vec{r} + \vec{r}') \right] \right\} dV'.
\]

(5.3)

where \( m_{t,x}, m_{t,y} \) and \( m_{t,z} \) are the \( x, y \) and \( z \)-components of the tip magnetic moment, respectively. For large tip-sample separations, such as those typically used in magnetic force microscopy, one can then think of the tip as a magnetic...
Figure 5.3: Schematic of tip-sample geometry, adapted from Rugar [61].

A dipole interacting with the stray fields from all the dipoles in the sample [61]. Denoting the position of the tip magnetic moment with respect to a sample dipole as \( \vec{r}' \), the field seen by the tip can be written as [25]

\[
\vec{H}_s(\vec{r}') = \frac{1}{4\pi} \left[ \frac{3(\vec{r}' \cdot \vec{m}_s)\vec{r}'}{r'^5} - \frac{\vec{m}_s}{r'^3} \right],
\]

(5.4)

where \( \vec{m}_s \) is the magnetic moment of a dipole in the sample. The total field from a collection of sample dipoles will then be a sum over all dipole contributions. If one makes the assumption that the tip only has a vertical magnetization, then the force gradient can be evaluated from Eq. (5.3) (with \( m_x, m_y = 0 \)). Performing the appropriate derivatives of Eq. (5.4) yields [62]

\[
\frac{\partial^2 H_{sz}}{\partial z^2} = \frac{3}{4\pi} \sum_i \left( \frac{3m_{ix}A^2 - 15ABz - 15m_{ix}Az^2 + 35Bz^3}{A^{3/2}} \right),
\]

(5.5)

where \( A = x^2 + y^2 + z^2 \) and \( B = xm_{ix} + ym_{iy} + zm_{iz} \), and the summation is carried out over all sample dipoles. Examples of the usage of Eq. (5.5) in computing expected MFM images are shown in section Magnetic vortices.

5.3 Microwave reflection method

Dynamical studies (resonant properties) of lithographically defined magnetic dots have been carried out using a microwave reflection technique. The major components of the experimental setup are schematically shown in Fig. 5.4. A radio frequency (rf) current is used to produce a small amplitude excitation field in a coplanar waveguide (CPW) where magnetic dots have been patterned directly ontop of the central line (see Fig. 5.5). The CPW’s were fabricated by
Figure 5.4: Schematic of experimental setup for microwave reflection experiments.

Figure 5.5: Portion of a CPW showing permalloy elliptical dots on the central line.

Optical lithography and lift-off and are 5 mm long, 5-10 µm wide and consist of 270 nm Au with a 2 nm Cr underlayer. Patterning of magnetic dots were carried out by e-beam lithography, sputtering and lift-off. The rf field is swept in frequency, and the resulting reflected signal is measured by an Anritsu [63] vector network analyzer (VNA) operating in the reflection mode. Further, the CPW is placed in the pole gap of an electromagnet in order to be able to perform measurements under the influence of static applied fields. A Hall probe is also positioned in the pole gap to monitor the field strength of
the static field. The power generator to the electromagnet is controlled by a LabView [60] program. Also within the pole gap of the electromagnet, coils for the production of a low frequency, small amplitude modulation field (see for instance [64]) are positioned. The current to these coils is controlled by the LabView program. A directional coupler between the CPW and the VNA assures that the reflected signal is measured. The VNA essentially measures the complex reflection coefficient that is a function of the circuit impedance [65, 66], i.e. information of both the effective resistance and the reactance are obtained. When the frequency of the rf excitation field corresponds to that of a resonance frequency of the magnetic dots, they (which are an integral part of the circuit) will absorb energy from the rf field. This causes a change in the permeability that in turn causes a change in the circuit impedance [65, 66].

The reason for using the modulation field is to remove magnetic field independent (rf frequency dependent) non-magnetic contributions in the reflected signal. The modulation field is simply superimposed on the rf field resulting in that the physical system parameters will display the periodicity of the modulating signal. Phase sensitive digital lock-in detection [67] is then employed where the modulating waveform is the reference signal and the reflected signal from the CPW is the in-signal to the lock-in routine. Now, if the modulation amplitude is small then a Taylor series expansion gives the derivative of the signal \( \frac{dS}{dH} \) (where \( S \) is the signal and \( H \) is the modulation field). So, when the frequency of the rf field is close to that of a resonance, one will be sensitive to the field dependence of the resonance (due to the modulation field). In this way one gets rid of rf frequency dependence of the circuit and can be more sure that the recorded resonances stem from magnetic contributions from the sample. Finally, an increase of signal to noise ratio is obtained by averaging over many rf frequency scans. An example of an experimental output of the absorption derivative is shown in Fig. 5.6.

![Figure 5.6: Representative experimental output for the absorption derivative.](image-url)
6. Magnetic vortices; statics and dynamics

It has been shown experimentally that the lowest energy state of certain small (micron-sized) soft ferromagnetic systems consists of a curling spin configuration, called the magnetic vortex state [21]. The vortex state is characterized by an in-plane curling spin structure with a core region where the spins tilt out-of-plane, as has been observed in spin-polarized scanning tunneling microscopy experiments by Wachowiak and co-workers [68]. The characteristics of the vortex state can be described in terms of a core polarization \( p = \pm 1 \) that defines whether the out-of-plane component of the magnetization points up \((p = +1)\) or down \((p = -1)\) and a chirality that defines the in-plane curling direction (clockwise or anti-clockwise) of the magnetic spins. An MFM-image of the vortex state in circular cylinders is shown in Fig. 6.4. Even though the chirality and the core polarization are uncorrelated it has been found by Choe and co-workers [69] that the polarization plays a decisive role in controlling the vortex motion during the low frequency vortex translational mode.

In this thesis, some properties of magnetic vortices have been studied in both circular and elliptical cylindric geometries. Depending on the geometry of the sample, a broad variety of states containing vortices can be found (see for instance [16]). In particular, emphasis in this work has been put on systems containing either one or two vortices. One vortex and two vortex states will henceforth be referred to as SV (single vortex) and 2V (two vortex) states, respectively. The study of the SV state has here been performed on both circular cylinders and elliptical cylinders of aspect ratio 2:1, while 2V states have been studied exclusively in 2:1 elliptical cylindric geometries. The energy difference between the two mentioned vortex states is relatively small for these elliptical geometries, which is why in an as-deposited-state one can often find both states represented. This is shown in Fig. 6.1. Predicted MFM contrast maps from micromagnetic computations are shown in Fig. 6.2. The characteristics of these states will be described in subsequent sections. To be mentioned already at this stage is that, in the case of the studied elliptical cylinders, it turns out that one can more or less single out either the SV or 2V states according to one’s desire by a simple field cycling procedure. If the ellipses are saturated along their long axes, a 2V state will predominantly be nucleated upon removal of the field, while saturation along the short axes produces SV states when the field is removed. This can be evidenced by MFM...
Figure 6.1: MFM images of two prominent vortex states (SV and 2V) in 3 µm × 1.5 µm × 0.04 µm ellipses.

studies (see Figs. 6.3 and 6.20 for SV and 2V nucleation, respectively) and is in agreement with previous reports on similar sized elliptical dots [70].

Figure 6.2: Examples of simulated MFM images for 2 µm × 1 µm × 0.04 µm ellipses for two different heights above the surface of the sample. The images are calculated according to Eqs. (5.3) and (5.5). (a): SV, height=90 nm; (b): 2V, height=90 nm; (c): SV, height=50 nm; (d): 2V, height=50 nm. Notice the good qualitative agreement with Fig. 6.1. The corresponding magnetization distributions for the SV and 2V states are shown in Figs. 6.6 and 6.19, respectively.

6.1 The SV state

6.1.1 SV statics in circular and elliptical cylinders

Consider a circular cylinder of a ferromagnetic material with negligible magnetocrystalline anisotropy. Assume the size of the sample to be above the SDL
and having a SV configuration. The in-plane magnetization distribution is then as described above in the text. An MFM image will display the circular cylinder with a blob in the center, indicating the out-of-plane component of the magnetization. This is shown in Fig. 6.4. The diameter of a circular cylinder will henceforth be denoted as $2R$ and the thickness of both circular and elliptical cylinders is denoted by $L$. The in-plane magnetization distribution with the corresponding out-of-plane component as obtained from micromagnetic

**Figure 6.3:** In-field MFM images displaying the nucleation of a SV state in 3 $\mu$m $\times$ 1.5 $\mu$m $\times$ 0.04 $\mu$m Permalloy ellipses. The black arrow in (a) indicates the direction of the applied field. (a) 40 mT, (b) 25 mT, (c) 10 mT and (d) 0 mT.

**Figure 6.4:** MFM image displaying the SV state for circular cylinders, $2R=1$ $\mu$m $L=0.05$ $\mu$m.
calculations are shown in Fig. 6.5. The chirality of the vortex is uncorrelated to the polarity of the core (see for instance paper II and [71]) and both polarity states are degenerate. The magnetization process of such a structure is through movement and annihilation of the vortex [72, 73]. The magnetization distribution of the elliptical cylinder is similar to that of the circular. Due to the aspect ratio, the elliptical case will, however, display MFM-contrast such as that shown in Fig. 6.3(d). The in-plane magnetization distribution of such a state with the corresponding plot of its out-of-plane component are displayed in Fig. 6.6.

6.1.2 SV dynamics in circular cylinders

Dynamical studies of the SV states in circular cylinders have been carried out both experimentally and numerically in this thesis (see paper III). The vortex configuration results in large differences in the nature of spin excitations as compared to those observed for the magnetically saturated state. For the vortex state, a spiral like precession of the vortex around an equilibrium point, known as the vortex translational mode, was previously theoretically predicted [74, 75] and experimentally verified by time-resolved Kerr effect measurements [76, 77, 78]. The handedness of the orbital motion has been demonstrated to be dependent on the polarization of the vortex core by X-ray magnetic circular dichroism [69] measurements. Previous experimental work relating to the determination of the frequencies for this mode have been qualitative or semi-quantitative. In this thesis work, measurements on this mode using a vector network analyzer (see section Experimental techniques) have been performed. The analyzer was operated in the reflection mode, measuring the field derivative of the real and imaginary components of the impedance.
Figure 6.6: Vector plot of calculated in-plane projected magnetization distribution ($m_{x,y}$ - components) of a SV state in a 2 $\mu$m $\times$ 1 $\mu$m $\times$ 0.04 $\mu$m elliptical cylinder and the corresponding out-of-plane magnetization profile through a cut XX’.

The work was carried out for both circular and elliptical cylinders (see next section for the elliptical case) in the SV state. For the circular cylinders, three sizes of diameter $2R$ and thickness $L$ were considered; [2$R$=1.1 $\mu$m, $L$=40 nm], [2$R$=2.0 $\mu$m, $L$=20 nm] and [2$R$=2.2 $\mu$m, $L$=40 nm]. MFM studies confirmed the presence of the SV state and in addition, magneto-optical Kerr hysteresis measurements showed the characteristic features of vortex movement, annihilation and nucleation. Figure 6.7 shows dispersion derivative spectra for all three discs considered. The static field dependence of the vortex translational mode was found to be virtually none, at least for displacements of the vortex far away from the rim of the disc. Notice that, for a given thickness, there is a significant increase in the resonance frequency with decreasing diameter.

To investigate this spin excitation mode in more detail, micromagnetic simulations were performed. The software used for this work was the OOMMF package [47]. Material parameters used were those typical for Permalloy, with the exchange stiffness constant $A$=13 pJ/m, damping $\alpha$=0.01 and the MA constant $K$=0. The saturation magnetization $M_s$ was set to 750 kA/m (see paper III for details). Two types of simulations were performed. One correspond-
Figure 6.7: (a) $2R = 2.0 \ \mu m$, $L = 20 \ \text{nm}$. (b) $2R = 2.2 \ \mu m$, $L = 40 \ \text{nm}$. (c) $2R = 1.1 \ \mu m$, $L = 40 \ \text{nm}$. The number of frequency sweeps averaged are in (a) 320, (b) 160 and (c) 640.

Figure 6.8: Simulated spectrum of the vortex translational mode in a $2R = 1.1 \ \mu m$, $L = 40 \ \text{nm}$ disc. The inset shows the damped oscillations of the magnetization during the vortex core motion around the dot center. This simulation was made using the OOMMF package [47].

Simulating to free oscillation in zero-applied field and one to forced oscillation under the influence of an in-plane time dependent excitation field of the form $H(t) = H_0 \cos(2\pi ft)$. The latter study was performed to mimic the experimental conditions where an exciting rf field drives the dynamics. Figure 6.8 shows a typical spectrum obtained from the micromagnetic simulations for the $2R = 1.1 \ \mu m$, $L = 40 \ \text{nm}$ disc. This spectrum was obtained by initially shift-
Figure 6.9: Comparison of dynamic and static vortex core profiles as obtained from micromagnetic simulations. Cross sections XX’ and YY’ are taken through a static vortex in zero-field equilibrium. Cross sections AA’ and BB’ are through a vortex in steady-state motion, driven at its translational mode resonance frequency by an in-plane rf field. Simulations performed using OOMMF [47].

Simulating the vortex by a static in-plane field of 50 Oe in the +x-direction. Using the shifted state as a seed state the system was then relaxed in zero field and the time evolution of the x-component of the reduced spatially averaged magnetization $m_x$ was recorded (see inset in Fig. 6.8). This time series was then Fourier transformed, yielding a frequency of 275 MHz, in good agreement with the experimental results. The ac simulations (forced oscillation) produced the same results as the free oscillating method but at a slightly larger computational cost. An interesting feature captured by the micromagnetic simulations is the change in vortex core profile during motion. This is shown in Fig. 6.9, where cross-sections through the dot of the $m_z$ component have been plotted. Cross-sections AA’ and BB’ are taken through the center of the core in steady state motion, driven by an ac-field at the resonance frequency, and sections XX’ and YY’ are through a static vortex in zero-field. Notice the asymmetry in the dynamical case. One can therefore conclude that there is a static and a dynamic vortex magnetization distribution. Figure 6.10 shows a 3D blow-up of the dynamical vortex core. Further, it is to be noted that the harder the core is driven, the larger is the deformation.

The eigenfrequencies of the vortex translational mode have previously been analytically obtained [75]. A rigid vortex model has been shown to be able to describe the equilibrium vortex shift induced by a magnetic field [79]. How-
Figure 6.10: A 3D plot showing the $m_z$-component of the dynamic vortex core in Fig. 6.9. Simulations performed using OOMMF [47].

Figure 6.11: Comparison of experimental data (solid marker=current experiments, open markers=measurements by Park et al. [76]), micromagnetic simulations (solid line) and analytical theory (dotted line).

ever, that model assumes that the vortex is rigid, i.e. does not deform during motion in a magnetic field. Such a model overestimates the total energy, in particular the magnetostatic part, and therefore fails to describe the dynamics in a realistic way (see paper III). The dynamic magnetization distribution is quite different from the static one (see e.g. Fig. 6.9), which should be taken into account in the theoretical modelling. Another analytical approach is the surface charge free spin-distribution model [75]. In this model, the magneti-
zation distribution of an oscillating vortex is modified at each point in time so as to eliminate surface charges on the boundary of the dot. Comparisons of the magnetization distributions with numerical micromagnetic results showed that this analytical approach gives a more realistic description than the rigid vortex model. The translational mode of the vortex motion is calculated by Thiele’s equation of motion \[75, 80\],
\[
\vec{G} \times \frac{d\vec{X}}{dt} - \frac{\partial W(\vec{X})}{\partial \vec{X}} = 0,
\]
where \(\vec{X} = (X, Y)\) is the vortex center position and \(W(\vec{X})\) is the potential energy of the shifted vortex (taking into account exchange, magnetostatic, Zeeman and anisotropy contributions). The first term is the gyroforce, which is proportional to the gyrovector \(\vec{G} = -G\hat{z}\), where the gyro-constant is \(G = 2\pi q p L M_s/\gamma_0\). Here, \(q\) is the vorticity (\(q=1\) is a vortex and \(q=-1\) is an anti-vortex) and the other parameters are as before. The second term in Eq. (6.1) describes the restoring force acting on the vortex shifted from the dot center. For small displacements of the vortex center from its equilibrium position \((\vec{X} = 0)\), the potential energy can be written as \(W(\vec{X}) = W(0) + (1/2)\kappa X^2\), where \(\kappa\) is the stiffness or spring constant. In other words, the restoring forces are of the form of those found in springs, and so this is effectively a Hooke’s law representation. For details of the surface charge free spin-distribution model the reader is referred to the work by Guslienko \[75\]. The dependence of the resonance frequencies on the dot aspect ratio \(\beta = L/R\), previously predicted in \[75\], has by micromagnetic simulations and measurements herein been confirmed. This is displayed in Fig. 6.11. The micromagnetic simulation results in Fig. 6.11 were obtained using discs with \(2R=0.5 \mu m\) to save computational time. The open triangles in the figure were taken from time-resolved Kerr measurements by Park et al. \[76\].

6.1.3 SV dynamics in elliptical cylinders
In contrast to the circular discs, the translational mode frequency for elliptical cylinders showed a field dependence (see paper VII). The SV translational mode consists of a vortex core motion in elliptic orbits around its equilibrium position. For the microwave reflection experiments, three samples were manufactured with lateral dimensions; \(A: 3\times1.5 \mu m^2\), \(B: 2\times1 \mu m^2\) and \(C: 1\times0.5 \mu m^2\), all with a thickness of 40 nm. Measurements were mainly done on samples \(A\) and \(B\) since they yielded a better signal. Further, two sets of each sample were produced, having either the long or the short axes of the ellipses parallel to the extension of the waveguide (sample fabrication details can be found in paper VII). The reason for this was to be able to study the effects of the direction of the rf excitation field. In order to study the SV translational mode, the samples were subjected to a saturating field along their
short axes prior to measurements, thus nucleating predominantly SV states (see e.g. Fig. 6.3). The resonance frequency was then measured for different static in-plane magnetic fields $\vec{H}$. Figure 6.12 shows representative real (Re) and imaginary (Im) impedance derivative spectra for sample $A$ under a static field of $H = 75$ Oe, applied along the short axes of the ellipses. The resonance frequency was extracted by averaging the values obtained from the peak position of the real spectra and the zero crossing of the imaginary spectra. Figure 6.13 shows the field dependence of the resonance frequency for static fields applied both along and perpendicular to the long axes of the ellipses. As can be seen, this dependence is highly anisotropic. These data were obtained for static fields applied perpendicular to the rf excitation field. However, the resonance frequencies turned out to be independent of the rf-field direction. This anisotropic field dependence was then further investigated by numerical micromagnetics using both the OOMMF [47] and the LLG micromagnetic simulator [48] packages. Details of the simulation parameters used are found in paper VII. As before, a seed state (the SV state) was first created for each $\vec{H} + \vec{h}$, where $\vec{h}$ is a small field perturbation. After equilibrium is reached for these seed states, $\vec{h}$ is set to zero and the time evolution of the spatially averaged magnetization is recorded during the relaxation process that follows; representative magnetization oscillations are shown in Fig. 6.14. Notice the high frequency ripples superimposed on the low frequency translational mode oscillations. The significance of these spin-waves will be given at a later stage. For now, focus on the translational mode. The resonance frequencies can be extracted as usual (by taking the Fourier transform of the oscillations in Fig. 6.14). Qualitatively, results from micromagnetic simulations agree well with
Figure 6.13: Experimental (solid lines and filled symbols) and simulated (dotted lines and open symbols) resonance frequencies as a function of in-plane static field $\vec{H}$ for samples A and B. Simulations performed using OOMMF [47].

corresponding experimental results, showing almost no field dependence under static in-plane fields applied along the long axes of the ellipses, but a clear dependence for static fields applied along the short axes. Further, the invariance of the frequencies with respect to the perturbation field direction was reproduced. However, overall the simulations give higher values of the frequencies than experimentally detected. The reason for this is probably due to mismatch in ellipse dimensions between experiments and simulations. Apart from mismatches in the lateral and vertical dimensions, the discrepancies in the magnitude of the resonance frequencies is also expected for larger $\vec{H}$, i.e. when the vortex is pushed closer to the rim of the particle, since the edges along the rim of the fabricated samples tend to be sloped. In effect, there is a reduced thickness in the vicinity of the rim and this should lead to a decrease in resonance frequency since this frequency is mainly governed by magneto-statics making it proportional to ellipse thickness [75]. In order to get a more detailed understanding of the anisotropic field dependence of the resonance frequencies, a combination of analytical treatment and further numerical micromagnetics was used. The translational mode resonance of a SV in a circular cylinder has been calculated by Guslienko in Ref. [75] using the Thiele equation of motion [80] (see Eq. (6.1)). This method can also be applied to the description of a vortex confined in elliptical geometries. Following the work described in [75], considering small displacements of the vortex core center $\vec{X}$ from its equilibrium position, the vortex energy for the ellipse case can be approximated as $W(\vec{X}_0, \vec{\xi}) = W(\vec{X}_0) + \kappa_x(\vec{X}_0)\xi_x^2/2 + \kappa_y(\vec{X}_0)\xi_y^2/2$. Here $\kappa_{x,y}$
Figure 6.14: Time evolution of the spatially averaged magnetization for sample B under the influence of a 10 Oe static field applied along y. The inset shows an orbit of the vortex core. Simulations performed using LLG [48].

$(\kappa_x \neq \kappa_y)$ are stiffness coefficients, $\vec{\xi} = \vec{X} - \vec{X}_0$ the dynamic vortex displacement, and $\vec{X}_0$ is the vortex equilibrium position. The motion of the vortex core corresponds to an oscillation in a dynamic potential well with a frequency of $\omega_0(\vec{X}_0) = G^{-1}(\kappa_x(\vec{X}_0)\kappa_y(\vec{X}_0))^{1/2}$ [75]. Thus, the stiffness constants could yield information about the field dependence of $\omega_0$ (since $\kappa_{x,y} = \kappa_{x,y}(\vec{X}_0)$ and $\vec{X}_0 = \vec{X}_0(H)$). The analytical model can be used to quantify the resonance frequencies for $H = 0$ as in papers VI and VII, but becomes unmanageable for finite $H$ (see further details in paper VII). However, through the use of micromagnetic simulations, the vortex potential well can be constructed for different $\vec{H}$. In order to reduce computation time, smaller ellipses of lateral dimensions $1 \times 0.5 \ \mu m^2$ were modelled. Firstly, consider the potential well for a vortex in $H = 0$. This was done by a set of simulations whereby the vortex is shifted from its equilibrium position by applied fields $\vec{H} = (H_x, H_y)$. For each shifted position, the energy (sum of demagnetization and exchange energies) and core position are recorded and plotted against each other. The resulting energy profile $W(\vec{X}) - W(\vec{X}_0)$ and corresponding cross-sections are shown in Figs. 6.15 (a) and (b), respectively. Values of $\kappa_{x,y}$ can then be extracted from quadratic fits to the cross-sections. Further, the same static simulations were used to calculate the energy profile for finite $\vec{H}$. Thus, the new equilibrium core position is defined by $\vec{H}$, i.e. the zero-field position of the vortex is no longer the equilibrium. Here, the exchange and demagnetizing energies are the same as for the zero-field case. However, the Zeeman energy must be added to the energy function. This is done by adding the term $W_\xi(\vec{X}) = -\mu_0 \vec{M}(\vec{X}_0, \vec{\xi}) \cdot \vec{H}V$, where
\( \vec{M}(\vec{X}_0, \xi) \) is the volume \( (V) \) averaged magnetization. In other words, one is here using the energy (sum of exchange, magnetostatic and Zeeman energies) for a given core position \( \vec{X} \) instanteneously after changing the applied field from \( \vec{H} + \vec{h} \) to \( \vec{H} \). Cases for \( H_z = 100 \) Oe and \( H_y = 400 \) Oe are shown in Figs. 6.15(c)-(f). Notice, how the profile in Fig. 6.15(f) (for \( H \) along \( x \)) shifts but otherwise retains its shape along \( x \) and \( y \) when compared to the \( \vec{H} = 0 \)-case. In Fig. 6.15(d), corresponding to \( H \) along \( y \), the profile becomes narrower and asymmetric along \( x \). This profile fits the assumed form for \( W(\vec{X}) \) if \( X^3 \) terms are added (to take into account the asymmetry). However, for small perturbations, the eigenfrequency is still determined by the \( X^2 \)-terms. Fitting the Hooke’s law representation to the profiles, values of \( \kappa_x, \kappa_y \) were extracted and are summarized in Fig. 6.16. Notice that all \( \kappa_x < \kappa_y \). This reflects the geometry of the system in which the vortex is less confined along the \( x \)-axis than along the \( y \)-axis. For \( \vec{H} \) applied along \( x \) (i.e. the long axis), the potential well is compressed along \( y \) and a bit elongated along \( x \), leading to an increase of \( \kappa_y \) but a decrease in \( \kappa_x \). The result of this is that the resonance frequency does not change much, thus reflecting the observed weak field dependence of the translational mode for \( \vec{H} \) parallel to the long axis. In contrast, for \( \vec{H} \) along the short

![Figure 6.15: Potential energy vs. core position for a 1 \( \mu \text{m} \times 0.5 \mu \text{m} \times 0.04 \mu \text{m} \) ellipse.](image)

(a) \( \vec{H} = 0 \), (c) \( \vec{H} = (0, 400) \) Oe and (e) \( \vec{H} = (100, 0) \) Oe, while (b), (d) and (f) are corresponding cross-sections along \( x \) and \( y \) directions. All gray-scale-plots are plotted on the same colorbar as \( W - W_0 \) in units of \( 10^{-10} \) erg.
Figure 6.16: Stiffness coefficients $\kappa_x$ and $\kappa_y$ as a function of $X_0$ and $Y_0$.

axis, i.e. when the vortex core is pushed along $x$, the confinement in both $x$ and $y$ increases, resulting in an increase in both $\kappa_x$ and $\kappa_y$, and thus also an increase of the resonance frequency. In Fig. 6.17, frequencies obtained from full dynamic simulations and as obtained from $\kappa_{x,y}$ in Fig. 6.16 are shown as solid symbols. In comparing dynamic and static simulations, one can see that the overall trend is reproduced by the static simulations but the magnitudes of the resonance frequencies are much too high. To understand this, the

Figure 6.17: Comparison of resonance frequencies obtained by fitting a sinusoidal dependence to dynamic simulation results (solid circles) and by using spring constants $\kappa_{x,y}$ extracted from static potential wells (solid triangles). The open triangles represent resonance frequencies obtained by using $\kappa_{x,y}$ from dynamical potential wells.
energy and vortex core displacement were extracted during a dynamic sim-
ulation; the results are displayed in Fig. 6.18. In the figure there is a time
lag before $W \propto \xi^2$ is established. The reason for this is that initially energy is
also absorbed to excite spin waves that are created in order to reconfigure the
static vortex shape into a dynamical distribution, similar to the one shown in
Fig. 6.9. Once the dynamic vortex shape has been established, the spin-waves
have been damped out. This is visible in Fig. 6.14, where the superimposed
ripple-like oscillations are caused by spin waves. Therefore, by extracting $\kappa_{x,y}$
from the peaks observed in the dynamic simulation (in Fig. 6.18), using data
for which $W \propto \xi^2$ holds, one obtains in this example (for $\vec{H} = 0$) a frequency
of 284 MHz to be compared with 276 MHz obtained from the same simulation
using the standard method (Fourier transform of the oscillations). This high-
lights the importance of choosing a model that takes into account dynamical
aspects, such as dynamic magnetization distributions.

6.2 Two vortex (2V) state

6.2.1 2V statics in elliptical cylinders

The 2V states are characterized by two vortices (with opposite chiralities) sep-
arated by a central diamond shaped domain which is why this state is often
referred to as the diamond state. Figure 6.19 shows the in-plane magnetiza-
tion distribution with corresponding out-of-plane components of a 2V state
(compare with Figs. 6.1 and 6.2). In total, four different combinations of core
polarizations can be considered; (a) $[p_1 = 1, p_2 = 1]$, (b) $[p_1 = -1, p_2 = -1]$, (c) $[p_1 = -1, p_2 = 1]$ and (d) $[p_1 = 1, p_2 = -1]$. Notice that (a) and (b) are ener-
getically equivalent as are (c) and (d). Therefore, as far as core polarizations
Figure 6.19: Top: Vector plot of calculated in-plane projected magnetization distribution \( m_{x,y} \) -components of a 2V state in a 2 \( \mu \text{m} \times 1 \mu \text{m} \times 0.04 \mu \text{m} \) ellipse. Bottom: Corresponding \( m_z \)-component in a cross-section \( XX' \) along the long axis where two cases are shown; \( p_1 p_2 = +1 \) (solid line) and \( p_1 p_2 = -1 \) (dashed line). The two cases are practically degenerate.

are concerned, one needs only to consider two cases that appear to be different in nature, i.e. only look at the two cases \( p_1 p_2 = \pm 1 \). It could be expected that \( p_1 p_2 = -1 \) is energetically favourable over \( p_1 p_2 = +1 \) from a magneto-static point of view. However, for these relatively large sample sizes, the vortex cores are well separated so the static vortex-vortex interaction energy is low. Results from micromagnetic calculations shown in Fig. 6.19 clearly show this. Firstly, the computed total energies are the same within numerical error. Secondly, from a static point of view, the inter-vortex distances are the same for both cases. It will later be seen, however, that the core polarization configuration is a decisive factor for the dynamics.

6.2.2 2V dynamics in elliptical cylinders
The translational mode in the 2V state is complicated by the fact that now there are two vortices with associated core polarizations. Micromagnetic modelling studies have predicted that magnetostatic interactions between SV magnetic
Figure 6.20: MFM image showing the magnetic states after magnetic saturation and field removal of $2\,\mu m \times 1\,\mu m \times 0.04\,\mu m$ ellipses. The result is an increase of the $2V$ population. Notice that the tip has turned some of the elements into $SV$ states (compare to paper II).

dots in proximity to each other will alter the excitation spectrum [81, 82]. Therefore it is expected that a vortex pair confined in an elliptical dot should also modify the spectrum. For dynamical studies, the same samples as used for the study of the $SV$ dynamics were used, but subjected to a saturating field along the long ellipse axes prior to measurements. This will produce mostly $2V$ states (as described previously), which was also checked by MFM imaging. A representative image is displayed in Fig. 6.20. Firstly, we focus on the $3\,\mu m \times 1.5\,\mu m \times 0.04\,\mu m$ (sample A) ellipses and try to understand the basic dynamics.

The same measurement technique as before (i.e. microwave reflection) was used here. A static in-plane field $\vec{H}$ is applied either parallel or perpendicular to the excitation field, together with a low amplitude, low frequency modulation field applied parallel to $\vec{H}$. Figure 6.21 shows representative experimental spectra of the impedance derivatives as a function of frequency, obtained with the excitation field along the short axes and the static field $H=10$ Oe along the long axes of the ellipses. The resonance frequency is taken as the average of the peak position for the real spectrum and the zero crossing of the imaginary spectrum. Notice that in Fig. 6.21 there are two peaks in the real impedance spectrum for the $2V$ state. The $SV$ resonance is also included as the dotted line. As mentioned before in the section on the static vortex
properties, contributions from both $p_1 p_2 = +1$ and $p_1 p_2 = -1$ are expected to appear in the spectrum. This is exactly what is being detected, a conclusion that is also confirmed by micromagnetic simulations (see further on in the text). To be mentioned also is that the field history controlling the nucleation of either a SV or a 2V state (as confirmed by field dependent MFM images and micromagnetic simulations) was in addition tested by this method. When saturating the ellipses along the short axes prior to measurements, the two peaks in Fig. 6.21 disappeared and was replaced by a single resonance appearing at a lower frequency (the dotted line in Fig. 6.21 corresponding to the SV mode). Figure 6.22 shows the static field dependence of the resonance frequencies. The frequencies with $\vec{H}$ applied along the long and short ellipse axes are shown at negative and positive fields. For $\vec{H}$ parallel to the long axes of the ellipses, it was only possible to observe the resonance over a rather small field range, since the vortex annihilation field is much smaller in this case (shape-wise easy direction). It is also found that the frequencies of the two peaks decrease slightly with increasing field. The modes plotted with the up-triangles in Fig. 6.22, were observed to be independent of the relative direction of the excitation field with respect to $\vec{H}$. However, this mode splits up into two branches with increasing $\vec{H}$ along the short axes (see Fig. 6.22). In
Figure 6.22: Field dependence of the resonance frequencies for 2V states in sample A. The directions of the static and rf fields are indicated in the figure (solid and open symbols indicate that the rf field was applied perpendicular or parallel to $\vec{H}$, respectively).

In contrast to this, the data points marked as down-triangles are excitation direction dependent; perturbations along the y-direction excite a high-frequency mode that changes very little with $\vec{H}$ along y, whereas for perturbations along x, a lower frequency mode is excited for applied fields along y and whose resonance frequency clearly increases with increasing field strength.

In order to understand these different excitations, more detailed micromagnetic simulations were performed. For this, the LLG Micromagnetics Simulator package was used [48]. Simulations of the dynamics were carried out by first creating a zero-field equilibrium 2V-state. For the sake of saving computation time, all simulations of the 2V dynamics were done on $1.5 \mu m \times 0.75 \mu m \times 0.02 \mu m$ ellipses (enabling a qualitative comparison to be made). This seed state is then perturbed by a small in-plane static field until equilibrium in the perturbed state is reached. The perturbation field is then removed and the system is allowed to relax (with small damping). It is found that the vortex cores rotate in elliptical orbits around their equilibrium points and the frequencies can then be extracted from either the time evolution of the spatially averaged magnetization or from the time dependent core positions (for material parameters used in the simulations, see paper V). The micromagnetic
modelling shows that different modes (each with its own frequency) are excited depending on the vortex core polarization (i.e. if $p_1p_2=1$ or -1) and the direction of the excitation field. As was described above, the core equilibrium positions are virtually independent of the relative polarizations and from an energy point of view the two $p_1p_2$ combinations are practically degenerate. But from a dynamical point of view there is a large difference. This is due to the fact that the core polarization determines the direction (or handedness) of the vortex orbital motion [69]. Figure 6.23 shows a schematic summary of the different modes found by micromagnetic simulations for $p_1p_2 = \pm 1$. One can characterize the different modes in terms of the phase between the motions of the two vortex cores along $x$ and $y$-directions. For the $p_1p_2 = +1$ case, both cores rotate in the same direction so that the possible modes are either in-phase ($i$) or out-of-phase ($o$) motion along $(x,y)$. In effect, the labeling is ($i,i$) or ($o,o$). If $p_1p_2 = -1$, the cores have to rotate in opposite directions, so they are labeled ($i,o$) or ($o,i$). Figure 6.24 shows the possible vortex shifts when the ellipses are subjected to a static applied field. The effect of a static field along $x$ is that the vortex cores are pushed along $y$ in opposite directions, yielding states that are equivalent upon field inversion (see the figure). The predicted behaviour of the frequencies as a function of $\vec{H}$ from micromagnetic simulations are shown in Fig. 6.25. However, two different cases can arise when the static field is applied along $y$. If the magnetization direction of the central domain is parallel to the applied field, the vortex cores will be pushed further apart and if the field and central domain magnetization are anti-parallel, the cores will be pushed closer together. These two cases will yield different fre-
quencies. It is of course expected that both cases will be present and indeed an apparent frequency splitting is observed (cf. Figs. 6.22 and 6.25). Now look a little closer at the different core motions. Experimental results of modes that give certain frequencies have been obtained, but what do these modes look like? Further, which of these modes do we expect to be able to excite or detect in the experiments? In the micromagnetic simulations, different perturbations are created to map out the possible modes and the results are then compared to the experimental results. For instance, the $p_1 p_2 = +1(i,i)$-mode could be created in the simulations, but cannot be excited in the experiments simply because it arises when the vortex cores are displaced symmetrically along $x$ in the same direction. This cannot be achieved by a uniform external field and will therefore not be excited in the experiments. In effect, for the $p_1 p_2 = +1$ case only the $(o,o)$-mode is excited in the experiments. It was found that this case is independent of the direction of the perturbing field. The splitting in frequency for this mode is as mentioned before due to the relative direction of the magnetization in the central domain with respect to $\vec{H}$. The crossed-over up-triangle in Fig. 6.25 shows the resonance frequency for the $p_1 p_2 = +1(i,i)$-mode at $H=0$ as obtained in the micromagnetics simulations.

Now, move on to the $p_1 p_2 = -1$-mode. It is found from the simulations that a perturbation along $x$ excites the $(i,o)$-mode, while a perturbing field along $y$ excites the $(o,i)$-mode. Further, it is observed experimentally that the mode excited by a perturbing field along $y$ for $\vec{H}$ along $y$ is detected, but the small branching in frequency for this mode due to the cases of $\vec{H} = \pm H_{\parallel}$ found in the simulations (see Fig. 6.25) is not detected in the experiments. The reason for this is not yet clear, but a probable reason is that the branching
Figure 6.25: Results from micromagnetic simulations showing the resonance frequencies as a function of $\vec{H}$. The simulations were performed on a $2V$ state in a $1.5 \, \mu m \times 0.75 \, \mu m \times 0.02 \, \mu m$ ellipse. The crossed-over triangle at $H=0$ represents the mode not excited in the experiments. Solid and open symbols show eigenmodes excited by perturbations perpendicular or parallel to $\vec{H}$, respectively. Diagrams show the direction of $\vec{H}$ and the resulting core displacements giving rise to the branches.

is too small to be resolved in experiments. Further, for perturbations along $x$ (still with $\vec{H} = \pm H_y$), only the upper branch is experimentally detected due to the low-frequency cut-off of the instrumentation. Similarly, the case for $\vec{H} = H_x$ with perturbations along $x$ is not detected in the experimental setup since the frequencies of this mode are also below the low frequency cut-off of the instrumentation (see Fig. 6.22). All in all, the micromagnetic simulations provide a very good qualitative explanation of the experimental results. Previous theoretical investigations, based on the work by Thiele [80], have applied the anisotropic Heisenberg model to similar problems, finding that the relative vortex core polarizations do indeed influence their motions [83, 84]. However, those studies do not consider the magnetostatic energy and the applied magnetic field and should therefore not be relied much upon for predictions concerning the resonance frequencies and core trajectories in systems such as those studied here. In addition to the micromagnetics, some analytical work has been included by solving a system of Thiele equations [80] including magnetostatic interactions in the energy function. Results can be found in paper V, but in summary the product of the two eigenfrequencies for each core polarization combination should be invariant,
i.e. \( \omega^{p_1p_2=+1}_{(i,i)} \omega^{p_1p_2=+1}_{(o,o)} = \omega^{p_1p_2=-1}_{(i,o)} \omega^{p_1p_2=-1}_{(o,i)} \). This agrees well with the micromagnetic results.

6.3 Aspect ratio dependence of the resonance frequencies

As previously mentioned, three different sample sizes were manufactured. Figure 6.26 shows the SV resonance frequencies as a function of vertical aspect ratio \( \beta = \frac{2L}{a+b} \), where \( a \) and \( b \) are the lateral semi-axes of the long and short lateral extensions of the ellipse, respectively. The resonance frequencies in Fig. 6.26 were obtained in a weak magnetic field (5-10 Oe) along the long axes of the ellipses and should therefore be representative of the \( \hat{H} = 0 \) situation. Analytical results (based on the methodology in paper VI) for the SV modes are added in the graph for comparison. The 2V resonance frequencies for samples A and B are included in the inset of Fig. 6.26. The reason for not displaying 2V results for sample C is that it was found to be difficult to stabilize a sufficiently large population of 2V states for these smaller ellipse dimensions.
7. Implementation of possible application of patterned magnetic thin films

From an application point of view, one of the goals here has been to construct a patterned magnetic surface enabling a controlled manipulation of the motion of small (micron-sized) magnetic particles. A commonly used method in separating different bio-molecules in heterogeneous suspensions, is to use magnetic particles with functionalized surfaces that bind selectively to specific target molecules in the suspension. Once this is achieved, separation from the rest of the liquid is done by applying an external magnetic field [85, 86, 87]. However, in this way all the target molecules are separated as an entire group and further distinction of the molecules is not possible. Now, if it were possible to manipulate the movement of single magnetic particles, the different molecules could be separated and sorted amongst themselves with each type gathering at a different location. The preliminary stages of such a device have been developed as part of this thesis, whereby the movement of micrometer-sized magnetic particles is controlled by an external, rotating magnetic field on patterns of thin film magnetic elements. The magnetic surfaces have been tailored to form what herein is denoted as transport lines. Junctions have then been added that allows for separation of individual magnetic particles.

7.1 Principle of operation

If magnetic particles are placed in a perfectly homogeneous magnetic field, there will be no translational motion. To physically move a particle, a field gradient is required. One way to create inhomogeneous fields is to use current carrying wires [88]. Moving magnetic particles on a surface using such an approach was demonstrated in Ref. [89]. Another method (utilized here) is to use the inhomogeneous stray fields from patterned magnetic elements. In this way no wiring to the surface is needed because the elements can be magnetized by an external field. The positioning of magnetic particles onto patterned templates by this method has been demonstrated by Yellen and Friedman [90]. In this work, a way to also govern the translational motion of particles on magnetic patterns utilizing an in-plane rotating magnetic field is demonstrated (see paper IV). The magnetic elements constituting the pattern where chosen so as to be elliptical cylinders and positioned in an inter-elemental base configu-
ration according to Fig. 7.1. The reasons for choosing these particular dimensions and layout of the pattern will gradually become apparent. One key aspect in obtaining a reliable and controllable movement of the particles is that the elliptical elements should be easily magnetized and pushed back into flux-closure states. A saturated ellipse will produce a stray field gradient that in turn will attract a magnetic particle just like a bar magnet attracts iron filings. An ellipse in a flux-closure state, however, produces only weak stray fields and will in comparison to the saturated state only weakly attract a particle. Another important point is the shape anisotropy of the ellipses. Due to its aspect ratio, it is much easier to saturate it along the long symmetry axis than along the short axis. The strength of the externally applied field is chosen so that it will saturate the ellipses when applied along the long axes, but will not do so when applied along the short axes. In the latter case the ellipses will effectively be in a close-fluxure state. To better illustrate the principle, consider Fig. 7.2. Here a horizontal field is applied, saturating the horizontal ellipse and a particle is stuck to the apex of this ellipse (Fig. 7.2(a)). The vertical ellipse is still in a flux-closure state and exerts a relatively low pull on the particle. As the field is rotated the particle follows the perimeter of the ellipse (Fig. 7.2(b)). At some point, when the direction of the in-plane field approaches the magnetically hard direction of the horizontal ellipse, the vertical ellipse becomes highly magnetized while the horizontal one breaks up into a flux-closure state (Fig. 7.2(c)). It should be noted that the field angle resulting in a break-up into a flux-closure state is a function of the applied field strength.
Figure 7.2: Principle of particle movement. (a): A magnetic field $H$ saturates a horizontal ellipse and a particle is attracted to its apex. (b): Upon rotating $H$, the magnetization in the ellipse rotates as well and the particle follows the rim of the ellipse. (c): When the direction of $H$ approaches the magnetically hard direction, the ellipse breaks up into a flux-closure state but the vertical ellipse becomes saturated, thus attracting the particle more than the horizontal ellipse. The particle then jumps to the apex of the vertical ellipse.

as well as the magnetic field history of the ellipse (see paper VIII). The particle is then pulled to the apex of the vertical ellipse. Examples of magnetic responses of the ellipses under applied in-plane fields in various directions are shown in Fig. 7.3. The mentioned process is then subsequently carried out by rotating the field in-plane with the result that the particle moves along the transport line. The experimental setup for realizing this process is schematically shown in Fig. 7.4. A suspension of magnetic particles, often referred to as magnetic beads, is injected in the fluid cell (A) through an inlet (B). In the center of the fluid cell, the patterns (C) containing the ellipses are placed. A pair of perpendicular magnets (D) are then used to create the in-plane field and the events are recorded through an optical microscope (E). The path of a magnetic bead during rotation of the in-plane field is depicted in Fig. 7.5. Due to the geometry of the base-pattern, a particle can only jump from the long side of one ellipse to the apex of another ellipse, making the transport a one-way street so to speak. A useful property is that the net displacement of a particle is constant with respect to the sense of rotation of the applied field, but the path it takes will depend on the rotational direction of the field. For clock-
Figure 7.3: MFM images of the base-pattern. The black arrows indicate the applied field direction. Field strength is 6.3kA/m.

wise field rotation the particle chooses to follow the perimeter of the ellipses on one side and for anti-clockwise field rotation it chooses to travel along the other side. By adding junctions to the transport lines and forming a loop, this property can effectively be utilized whereby a particle can either pass the loop or be caught in it. This is depicted in Fig. 7.6, whereby the particle is manipulated to be either caught in the loop with the application of a clock-wise field or made to go past the loop using an anti-clockwise rotating field. Having demonstrated this process it is possible to take it a step further and consider the separation of individual magnetic particles. Consider two particles trapped in the loop as in Fig. 7.7. While rotating the field clockwise, they will travel along the inside perimeter of the loop. When both particles have passed the first junction the field is switched to anti-clockwise rotation and both particles follow the outside of the transport line as it did in Fig. 7.6(c). When the first of the two particles has passed the second junction, the field is switched to clockwise again and the remaining particle changes its path to be on the inside of the transport line as in Fig. 7.7 (c). The result is that the first particle is out of the loop and can continue along the transport line and the second particle is caught in the loop, as is shown in Fig. 7.7(d). A benefit of such a method is that no wiring to the substrate is needed and it is from a lithographic point of
Figure 7.4: Schematic of experimental setup used for particle movement. (A) fluidcell, (B) inlet for injection of particles, (C) sample holder for patterned magnetic film, (D) computer controlled $xy$ electromagnets and (E) optical microscope.

Figure 7.5: Optical microscope images showing the path of particles along the base-pattern as the applied field is rotated in-plane. The arrow indicates the current field direction in each of the images. The solid line in (f) shows the total path travelled by the lower of the two particles.

...view just a one-step process. A second fact is that by avoiding to produce the necessary stray fields for particle movement by current carrying wires, heat generation is circumvented. This could be of great importance if the particles are carriers of biological material that is sensitive to elevated temperatures.
Figure 7.6: (a): Schematic displaying the path of a particle under the influence of a clock-wise (solid arrow) and anti-clockwise (dotted arrow) rotational field. (b,c): Optical microscope images showing how the particle is either captured by the loop or passes the loop depending on the sense of rotation of the applied field. The ring in the lower left of the images depict the starting point for the particle. In (b) with clockwise field rotation, the particle travels on the inside of the transport line and is then caught by the loop. In (c) with anti-clockwise field rotation, the particle travels on the outside thus passing the loop.

Figure 7.7: Optical microscope images demonstrating the separation of two magnetic beads caught in a loop structure. (a) Both particles travel along the inside perimeter of the loop with a clockwise field. (b) The field is switched to anti-clockwise enabling the first particle to travel past the second junction along the transport line. (c) The field is switched back to clockwise rotation and the second particle travels on the inside of the transport line to later be caught in the loop. (d) The first particle has passed the loop but the second one is caught in the loop.
8. Conclusions and Outlook

8.1 Conclusions

8.1.1 Numerical experiments

Three numerical problems have been studied. The first two are the µMAG problems no. 3 and 4. In problem no. 3, the SDL (and many of the other properties) of a ferromagnetic cube is found to be inversely proportional to the square of the number of discretization cells in one dimension (i.e. SDL $\propto 1/N^2$). The SDL is found to be $L=8.47 \cdot l_{ex}$.

In problem no. 4, the dynamical response to two different reversal fields in a rectangular slab of Permalloy was computed and the influence of meshing cell sizes was investigated. Mesh independence is found for the first field case (for cell sizes below the exchange length) but the results show mesh dependence for the second reversal field. This problem clearly demonstrates the dependence of the relevant length scales on the magnetic microstructure of the sample.

In the final numerical experiment, switching properties of yet smaller samples under the influence of thermal fluctuations are investigated. The results show that not only do the temperature fluctuations speed up the switching events and reduce the switching fields, but they can also alter the magnetization reversal path.

8.1.2 Magnetic vortex dynamics

A study of the vortex translational mode dynamics for SV and 2V states has been performed by using a microwave reflection technique and micromagnetic simulations. In the case of circular cylindrical geometries having a SV state, the frequency of this mode shows a very weak field dependence, while in the case of SV states in elliptical cylinders the translational mode frequency can be tuned by a magnetic field applied along the short axes of the ellipses. The observed field dependence may be understood in terms of the shape of the vortex potential well.

2V dynamics was investigated in elliptical cylinders. Different modes were detected depending on the relative core polarizations of the vortices and the direction of the rf field. By micromagnetic simulations, four different modes corresponding to different combinations of in-phase and out-of-phase core motion along the two ellipse axes were identified. Three of them can be excited in experiments by using a spatially uniform perturbation field.
8.1.3 Application of magnetic thin film elements

A method to control the transportation and separation of individual magnetic particles has been demonstrated using a patterned magnetic surface. Particle motion is facilitated by the element stray fields when a rotating in-plane magnetic field is used to magnetize and demagnetize the elements constituting the magnetic pattern.

8.2 Outlook

Having explored SV and 2V dynamics it is tempting and interesting from a fundamental point of view to try to study three vortex (3V) dynamics. Different types of 3V systems were stabilized in various sizes of elliptical cylinders. An MFM image of one of those systems is shown in Fig. 8.1. Here three vortices are lined up in a row, where the two outer ones have the same chirality and the central one has opposite chirality with respect to the other two. The number of non-degenerate combinations of core polarizations in this case should be three. Considering the fact that the 2V states showed a quite complicated dynamical behaviour, this 3V system should be even more complex but still quite possible to study within the framework of the measurement techniques and micromagnetic simulation methods described in this thesis. Field dependent MFM studies on arrays of ellipses having dimensions as shown in Fig. 8.1 indicated that these 3V states were relatively easily stabilized by ramping a field applied along their short axes.

Another 3V state was also observed for 3 \( \mu m \times 1.5 \mu m \times 0.06 \mu m \) ellipses, consisting of two vortices separated by an anti-vortex [91]. The anti-vortex, also called a cross-tie structure, seems to be stable at these larger thicknesses (as also found for elliptical cylinders of different aspect ratios in Ref. [92]) and was nucleated here upon ramping a magnetic field along the short axes of the ellipses. Ramping the field along the long axes appeared to favour mostly 2V states. An MFM image of this 3V state is shown in Fig. 8.2. For thinner samples such as those considered in previous sections, the only two stable
states were the SV and 2V states. This nicely demonstrates the sensitivity of the magnetic microstructure on the sample thickness. The same complexity of the dynamical behaviour is expected for this structure since it also has three different non-degenerate combinations of core-polarizations. Some preliminary measurements of the vortex-antivortex-vortex states were performed and a spectrum with $H_y=150$ Oe is shown in Fig. 8.3. However, the possibility that the spectrum contains contributions from 2V states can not be ruled out.

Figure 8.2: Left: MFM image of vortex-anti-vortex-vortex states in $3 \mu m \times 1.5 \mu m \times 0.06 \mu m$ ellipses. Right: Schematic of the magnetization distribution of such a state.

Figure 8.3: Dispersion derivative spectrum with $H_y=150$ Oe, possibly corresponding to the vortex-anti-vortex-vortex state.

A substantial amount of micromagnetic simulations on all the core polarization combinations of the vortex-anti-vortex-vortex state in combination with 2V simulations (to single out any possible contributions from such states) will have to be performed in order to properly interpret experimental results. These studies, however, will have to be postponed for the future.

For the application part, the next goal is to attach biomolecules to the particles and make them move along the transport lines. A further step is to device
a way to separate particles carrying different molecules and collecting these particles at different locations on a substrate surface. This could possibly be achieved in more than one way. One thing to try is to use particles that have somewhat different magnetic properties, such that one type of molecule is attached to one type of particle and another molecule is attached to a particle that differs a little in its magnetic properties. Then by creating different magnetic field sequences a certain particle type will choose to go along one particular path on the surface, while another particle will choose another path. The result will then be that different types of particles with their associated attachments will collect at different locations. In such a way a lab-on-chip separation technique can be developed, enabling the user to control the collection and separation of extremely small quantities of a substance.
9. Sammanfattning på svenska

De tekniska framstegen när det gäller mönstring med litografiska metoder har medfört att det idag är möjligt att tillverka strukturer med längdskalor på nanometernivå. Detta i kombination med analytiska metoder som erbjuder magnetisk kontrast såsom magneto-optiska metoder, magnetisk kraftmikroskopi, transmissionselektronmikroskopi (med många flera) har gjort det möjligt att studera magnetism i mikroskopiska geometrier. Denna utveckling har lett till en uppsjö av olika magnetiska produkter (metoder för lagring och läsning av data inom datorindustrin är ett bra exempel på detta). Vidare, så har det med utvecklingen av snabbare datorer blivit möjligt att bedriva mikromagnetiska beräkningar på realistiska geometrier, något som idag utgör en väl etablerad och använd metod inom både grundläggande forskning av magnetisk mikrostruktur och utvecklingsprojekt som syftar till att skapa bättre och snabbare magnetiska komponenter.

Det är i samband med detta som denna avhandling förhoppningsvis kan tillföra lite ny kunskap. De studier som bedrivits inom ramen för denna avhandling är som i de flesta fall relativt specifika och avhandlingen i sig består av fyra delar.

Den första delen ger en del teoretiska begrepp samt beskriver numeriska och experimentella tekniker, i princip de nödvändiga byggstenarna för den forskning som utgör denna avhandling.

Den andra delen behandlar några numeriska experiment med en mikromagnetisk kod som utvecklats inom ramen för avhandlingsarbetet och fokuserar på två stycken av de så kallade mikromagnetiska standardproblemen och på möjliga scenarier då termiska fluktuationer medtas i beräkningar. Dessa problem har studerats dels för att de ger en djupare insyn i vad som är viktigt när man sätter upp en geometrisk modell som skall användas för mikromagnetiska beräkningar och dels för att man i detalj kan följa vad som händer i materialet när magnetiseringen byter riktning. Studierna visar hur pass känsliga resultaten kan vara beroende på den cellstorlek som används i uppsättandet av beräkningsgeometrin och ger ett tydligt exempel på att bara för att man håller cellstorleken i utbyteslägeområdet för materialet så betyder det inte att man nödvändigtvis lyckas upplösa den magnetiska mikrostrukturen ordentligt. Detta är ett bra exempel på situationer då den magnetiska mikrostrukturen i sig kan diktera den nödvändiga cellstorleken. I exemplet då termiska fluktuationer tas med i beräkningen för små partiklar visar resultaten att en ökning i temperatur kan förutom påskyndandet av det magnetiska omslaget
(d.v.s. då magnetiseringen byter riktning från en riktning till en annan) och en reduktion av omslagsfälten, även drastiskt ändra på de transienta magnetiska konfigurationer som passeras under magnetiskt om slag.

Den tredje delen handlar om magnetiska vortex tillstånd, ett tillstånd som ofta påträffas i mikrometerstora magnetiska strukturer som nollfältstillstånd eller som byggstenar i nollfältstillståndet. Ett vortexstillstånd karakteriseras av att provets magnetiseringsfördelning kröker sig på ett cirkulärt vis; se Fig. 9.1(a) som visar magnetiseringsfördelningen för ett cirkulärt cylindriskt prov. Inom ett litet område då man närmar sig centrum av provet börjar magnetiseringen orientera sig ut ur planet och precis i centrum är magnetiseringen helt vinkelrätt mot planet. Området där magnetiseringen börjar att luta ur planet benämns vortexkärnan (Fig. 9.1(b)). Tonvikten har lagts på de dynamiska

![Diagram](image)

**Figure 9.1:** (a): Magnetiseringsfördelning för ett vortexstillstånd i ett cirkulärt cylindriskt prov. (b): Ut-ur plan komponenten av magnetiseringen i ett tvärsnitt genom vortexstillståndet. (c): Translationsmoden för en vortex som först förskjutits från sitt jämviktsläge m.h.a. ett magnetfält och sedan relaxerar i noll-fält.

aspekterna av system innehållande en eller två vortexar. Två olika geometrier av Permalloy (en nickel-järn legering) har studerats; tunna cirkulära och elliptiska (lång/kortaxel-förhållande 2:1) cylindrar. De moder som har mätts och räknats på är vortexens translationsmoder (se Fig. 9.1(c) för ett exempel på translationsmoden för en vortex i ett cirkulärt cylindriskt prov). Mätningar utfördes med en reflektiv mikrovågsteknik. För de cirkulära proverna studeras exklusivt en-vortexstillståndet och för de elliptiska fallen studeras båda en- och två-vortexstillstånd. Det som specifikt har mätts är modernas karak-
teristiska frekvenser (resonansfrekvenser). Vidare så har frekvensernas fältberoende (statiska magnetfält) kartlagts; ett pålagt fält innebär i praktiken att man förskjuter vortexarnas jämviktspositioner i materialet. De cirkulära proverna uppvisar i princip inget fältberoende av resonansfrekvensen (i alla fall inte för förskjutningar där vortexkärnan befinner sig långt från provets periferi). Vidare så har ett tidigare förutsagt geometriskt skalningsberoende av resonansfrekvensen bekräftats både genom mätningar och mikromagnetiska simuleringar. För en-vortex tillstånd i den elliptiska geometrin syntes inget fältberoende av resonansfrekvensen då det statiska fältet appliceras längs med ellipsens långaxel, men ett starkt fältberoende för fält längs med kortaxeln. Mikromagnetiska simuleringar visar att detta kan förklaras i termen av formen på vortexens energilandskap. En viktig konsekvens av denna assymetri i resonansfrekvensernas fältberoende är att det då är möjligt att genom ytterligare fält justera resonansfrekvensen. För två-vortexsystemen visar det sig att vortex-kärnornas relativa polarisationer (polarisationen definieras av magnetiseringens riktning i kärnan och kan antingen ha uppåt eller nedåt orientering vinkelrätt mot planet) dominerar deras dynamiska beteenden och ger en upphov till, för antiparallell orientering av polarisationerna, att olika translationsmoder exiteras beroende på hur man i experimentet riktar mikrovågfältet. För parallell orientering av vortexkärnornas polarisationer uppmättes inget fältberoende av den exiterade moden på mikrovågfältets riktning. I de mikromagnetiska simuleringarna som gjordes kunde fyra möjliga moder exiteras, varav tre av dem kunde exiteras i experimenten. Ett annat fenomen som också noterades var en uppsplittring i resonansfrekvenserna beroende på hur det yttre fältet är riktat i förhållande till magnetiseringsriktningen i den centrala domänen i två vortex strukturen (notera dock att denna ändring av resonansfrekvenserna inte är relaterad till en förändring av själva translationsmoden). De olika modernas beroende på pålagt fält längs med ellipserna kort och långaxlar utfördes systematiskt även för dessa två-vortex strukturer.

I den sista delen i avhandlingen presenteras en metod för att manipulera och förflytta mikrometerstora magnetiska partiklar med hjälp av ströfälten från likaledes mikrometerstora mönstrade magnetiska elliptiska element av Permalloy. De elliptiska elementen magnetiseras och avmagnetiseras successivt med hjälp av ett yttre roterande magnetfält. Under denna process följer de magnetiska partiklarna randen av de mönstrade strukturerna. Även separation av individuella magnetiska partiklar har med framgång påvisats inom ramen för denna metod. Implikationerna av detta är möjligheten att genom att använda partiklarna som bärare av t. e.x. proteiner, kunna flytta dessa på ett kontrollerat sätt mellan olika punkter på en yta och allt detta med mikrometerprecision (Fig. 9.2). När det gäller den fortsatta metodutvecklingen vill man utnyttja den påvisade separationsmetoden för att på ett effektivt sätt kunna separera partiklar bärande olika typer av biomolekyler, d.v.s. att från en blandning av dem kunna separera och samla de olika typerna på olika ställen på ytan, något som kan vara till nytta inom industrier där man vill kunna separ-
Figure 9.2: Framtidsvisionen av partiklar som bär med sig molekyler på en mönstrad yta med magnetiska element.

...era små mängder av sina ämnen (så små som en miljarddel av en miljarddels liter).
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