

Stochastic Optimization of Energy Prosumption Systems

A case study on energy balancing for self-sustainable societies

Jim Allansson

**Industrial and Management Engineering, master's level
2022**

Luleå University of Technology
Department of Social Sciences, Technology and Arts

Acknowledgment

First and foremost I would like to direct a thanks to my supervisor Björn Samuelsson who entrusted me with this master thesis project, I could not have wished for a better topic to end my master's with. I would also like to thank him for inspiring and encouraging me to pursue a PhD long before this thesis even started. He also played an important role in making sure the thesis holds its current level of quality. He was however not the only one that helped me with this thesis, I would like to give a thanks to everyone involved with helping me improve this thesis with valuable feedback and discussion. Thank you.

This has been the final work for my Master in Industrial Logistics within the program of Industrial and Engineering Management at Luleå University of Technical.

Luleå, June 2022:


.....
Jim Allansson

Abstract

To achieve a sustainable future fossil electricity is being replaced with renewable, leading to higher uncertainties in electricity production. This has resulted in an incentive for consumers to produce, sell, and store their own electricity, hence becoming prosumers. Austerland Skags is a Swedish project that explores the possibility to convert a small society into a prosumption system. The system includes solar and wind power as electricity producers and hydrogen-fueled vehicles for commodity transport. To capitalize the most on their produced electricity they want to store excess electricity. This master thesis uses Austerland Skags as a case study to develop a stochastic linear optimization model to determine the optimal energy storage solution for an energy prosumption system with both electricity and hydrogen demand.

The method used in this thesis was the sample average approximation (SAA) algorithm. The results from the SAA were compared to the expected results from the expected value problem (EEV) to show the difference between a stochastic and deterministic solution. The results from the SAA turned out to consistently outperform the EEV for the samples created.

Since hydrogen demand could only be sourced in-house, the model was forced to use an electrolyzer and hydrogen tank. The final result from the SAA showed that both a battery and fuel cell was used in addition to the electrolyzer and hydrogen tank in the optimal solution. All capacities stayed within reasonable levels showing the possibility of realizing a cost-effective prosumption system.

Key words: Stochastic Optimization, Energy Prosumption, SAA, Linear Programming.

Sammanfattning

För att uppnå en hållbar framtid byts fossila bränslen ut mot förnyelsebara energikällor, vilket leder till högre osäkerheter i elproduktionen. Detta har skapat ett initiativ för konsumenter att börja producera, sälja, och lagra sin egen elektricitet och därav bli prosumenter. Austerland Skags är ett svenskt projekt som undersöker möjligheten att konvertera ett litet samhälle till ett prosumentsystem. Systemet är uppbyggt med sol och vindkraft för produktion av elektricitet och planerar att använda vätgasdrivna fordon för transport av råvror. För att utnyttja så mycket som möjligt av den producerade elektriciteten vill de kunna lagra överskotts elektricitet. Den här masteruppsatsen använder Austerland Skag som en fallstudie för att utveckla en stokastisk linjär optimeringsmodell för att avgöra den optimala energilagringens lösningen för ett energiprosumentsystem med både el och vätgasbehov.

Metodvalet i denna uppsats var sample average approximation (SAA) algoritmen. Resultatet från SAA jämfördes med det förväntade resultatet från förväntade värdeproblemet (EEV) för att visa skillnaden mellan stokastiska och deterministiska lösningar. Resultatet från SAA visade sig ständigt ge bättre resultat än EEV för undersökta stickprov.

Eftersom vätgasbehovet endast kunde förses in-house i modellen var den tvingad att dimensionera upp ett elektrolysör och vätgaslager. Slutresultatet av SAA visade att både batterier och bränsleceller var aktuellt tillsammans med elektrolysör och vätgaslager i den optimala lösningen. Alla kapaciteter förhöll sig inom rimliga nivåer vilket påvisar möjligheten att realisera ett kostnadseffektivt prosumentsystem.

Contents

1	Introduction	1
1.1	Problem Background	1
1.2	Problem Formulation	1
1.3	Purpose & Research Questions	2
1.4	Delimitations	3
2	Method	4
2.1	Research Purpose and Approach	4
2.2	Data Collection	4
2.2.1	Sampling process	5
2.3	Analysis Method	5
2.4	Reliability and Validity	6
3	Literature Review	7
3.1	Electricity Prosumption Systems	7
3.2	Technologies in Prosumption Systems	8
3.2.1	Batteries in Prosumption Systems	9
3.2.2	Hydrogen in Prosumption Systems	11
3.3	Stochastic Demand and Supply	12
3.3.1	Fixed Recourse Two-Stage Programs	12
3.3.2	The Newsvendor Approach	15
3.4	Safety Stock Under Uncertainty	16
4	Situation Analysis	17
4.1	The Austerland Skags Case	17
4.1.1	Production and Consumption Patterns	18
5	Analysis	20
5.1	Literature Analysis	20
5.2	Deterministic optimization model	20
5.2.1	Objective function	23
5.2.2	Constraints	24
5.2.3	Linear Optimization Model	26
5.3	Deterministic Analysis Results	28
5.4	Stochastic Optimization Model	31
5.4.1	Explanation of Stochastic Model	32
5.4.2	Linear Optimization Model	32
5.5	Stochastic Analysis Results	33
6	Discussion	36
7	Conclusion	38
8	Future Studies	39
	References	
	Appendix A Deterministic Python Model	i

Appendix B Stochastic Python Model	ix
Appendix C Simplified Model	xx
Appendix D Austerland Skags Final Deterministic Model	xxii

List of Figures

1	Austerland Skags Energy System	2
2	Illustration of research approach	4
3	One Year of Renewable Energy Production	18
4	Yearly Electricity Consumption Pattern	19
5	Average Daily Electricity Consumption Pattern Every Month	19
6	Hydrogen Consumption Pattern	19
7	Cost of increased self-sufficiency	30
8	Relations between capacities and self-sufficiency	30

List of Tables

1	Description of the parameters and variables of the two-stage recourse model	13
2	Costs of technologies	18
3	Relative heating demand per month	19
4	No waste heat vs waste heat collection	28
5	Maximum self-sufficiency optimization	29
6	Normal vs Simplified	34
7	SAA Algorithm Results	34
8	The expected value solution	35

1 Introduction

This chapter presents the studied problems background followed by the problem formulation. The purpose of the thesis is then presented together with the research statement followed by the thesis delimitations.

1.1 Problem Background

In response to the EU:s sustainability goal to be carbon dioxide neutral by 2050, more and more fossil energy sources are being replaced with renewable energy sources. This has led to higher uncertainties of electricity production since a bigger share of renewable energy results in higher fluctuations in production. Wind power, as an example, can range from 0.5-48.8% of Sweden's total electricity production for a specific hour (Svenska Kraftnät, 2022). The result of high variability in electricity production has led to fluctuations in electricity prices.

This has resulted in an incentive for consumers to produce, sell, and store their own electricity. These systems where the consumer both consume and produce is called prosumption. To use an electricity prosumption system efficiently a solution where both production and consumption are balanced needs to be found. One solution to this could be to export electricity during overcapacity and import during under capacity. The problem is that export often occurs during low electricity prices and import during high prices since the locally produced electricity is dependent on the same factors as the national grid. Another solution to balance the system could be to store the electricity during excess capacity and use the stored electricity when needed.

Austerland Skags is a Swedish project that explores the possibility to convert a small society located on Gotland into a prosumption system. The proposed energy system of Austerland Skags includes renewable energy production in the form of a wind and solar power park which, in combination with the electricity grid, will help power 200 households, a local farm, a treatment plant, and more, see figure 1. The system also plans to include rooftop solar panels on the households. This way both the park and households will sometimes produce excess electricity. Excess electricity will either be stored in batteries or/and hydrogen for later use or sold to the electricity grid. The goals of the project are to produce and consume energy locally, create a system for storage and smart control, enable energy sharing within the system, and build a robust system that helps stabilize the regional electricity grid (Nygarn Utveckling AB, 2022). The case of Austerland Skags will be used in this master thesis as a small-scale example of how renewable energy sources can be implemented in combination with energy storage systems.

1.2 Problem Formulation

A common way to store electric energy is in batteries, there are however ways to convert electric energy and store it in other forms. Some examples of such storage mediums are thermal and mechanical storage, this can be done through pumped hydro storage, compressed air, heated lithium fluoride, and more (Ferreira et al., 2013). A lot of these solutions are however dependent on the geographical location and not always possible to achieve (Møller et al., 2017). Hydrogen is another alternative to store energy, this method has high potential since it has an almost negligible self-discharge rate and high storage capacity which makes it an excellent storage medium for long-term storage (Energy Systems and Energy Storage Lab, 2020). The problem with hydrogen is that the energy efficiency of converting electricity to hydrogen and then from hydrogen back to electricity is relatively low. When producing hydrogen through water electrolysis the energy efficiency can vary between 62-90% depending on the conditions and technology (Carmo et al., 2013; Kumar & Himabindu, 2019). When

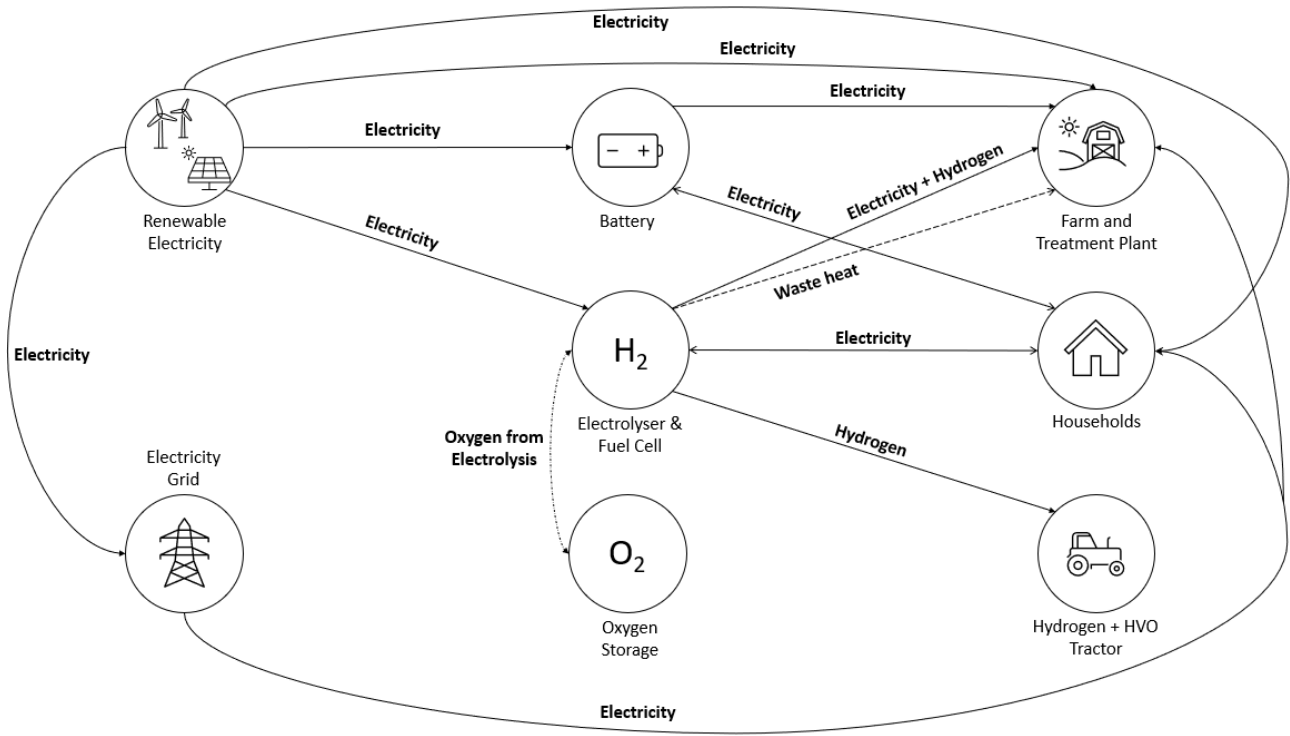


Figure 1: Austerland Skags Energy System

converting back to electricity from hydrogen in a fuel cell the efficiency varies between 38-60% depending on conditions and technology (Parra et al., 2019; Taner, 2018). This gives hydrogen storage a round trip efficiency of 23.6-54% which is relatively low compared to the round trip efficiency of batteries which usually lies between 63-97% depending on the type of battery (X. Luo et al., 2015).

The problem with batteries is that they are less suitable to store energy for longer periods of time due to self-discharge rates, weight, and the size required to store larger capacities (X. Luo et al., 2015). Batteries are however an excellent storage technology during shorter periods and could be suitable to balance daily/weekly energy consumption.

There are several studies that examine energy storage solutions and their applications (Ferreira et al., 2013; X. Luo et al., 2015; Møller et al., 2017), and there exists several studies on prosumption systems (Anthony Jnr et al., 2020; Kubli et al., 2018; Sossan et al., 2016). But there is a lack of studies that focuses on the optimal capacity of multiple energy storage solutions in prosumption systems that consider both stochastic demand and supply. It hence exists an interest to develop a general optimization model that calculates the optimal storage capacities for a prosumption system and what storage mediums to use to achieve an optimal energy balance solution. This master thesis will use the case of Austerland Skags to test and verify the created optimization model.

1.3 Purpose & Research Questions

The purpose of this thesis can be formulated by the following research statement:

Design and implement a linear optimization model which calculates the optimal storage capacity of different storage mediums, with stochastic demand and supply, in an energy prosumption system.

To achieve this the statement has been divided into several research questions:

1. What are relevant storage mediums for energy storage in a prosumption system?

2. What found variables and parameters are relevant to an energy prosumption system?
3. How do different degrees of self-sufficiency affect the model?
4. How will the stochastic demand and supply affect the model?

1.4 Delimitations

The situation analysis will only cover current technology, hence no new solutions of energy storage will be investigated. Renewable energy production capacity will be set, hence no analysis on the optimal capacities of wind and solar power will be considered. The system will be modeled after the case of Austerland Skags hence no connections other than those in figure 1 will be considered. Electricity consumption and production is presented on an hourly resolution. This is not coherent with reality since consumption and production of electricity is continuous, this might result in errors regarding self-sufficiency and self-consumption (Nyholm et al., 2016). Data for intra-hourly demand can however be hard to find and significantly increases computational burden. Transmission losses in the grid will also be neglected since the effect is deemed to small to motivate the computational complexity. The study will not consider the water supply to the electrolyzer, and will assume water to be sufficient to always run the electrolyzer. Excess oxygen produced from the electrolysis of water will also not be treated due to difficulties in approximating the excess oxygens value.

2 Method

This chapter presents the research purpose and approach for the thesis, followed by the data collection process and analysis. The chapter ends with the reliability and validity of the thesis.

2.1 Research Purpose and Approach

According to Saunders et al. (2007) most research purposes follow three different classifications, these classifications are exploratory, descriptive, and explanatory. A research purpose does not necessarily need to follow a specific classification but can according to the author fulfill more than one purpose. The purpose of this thesis was to design a general optimization model to determine the optimal storage capacities of different storage mediums in an electricity prosumption system. This purpose could be classified in several of the classifications. The purpose was primarily explanatory in the way that it showed how different variables in the system correlated to each other and how that affected costs and self-sufficiency. The purpose could be descriptive in the sense that it portrayed an accurate picture of how a real system would operate. One could also argue that the thesis was exploratory since it examined the possibility of including different technologies based on the local conditions of the place examined.

The approach of this thesis has followed an abductive structure. The abductive approach is a mixture of deductive and inductive based analysis, the approach consists of exploring a phenomenon by moving between theory and empirical data to successively build a deeper understanding of the subject (Saunders et al., 2007). The thesis started with searching the literature and analyzing the given case from Austerland Skags to form a first version of the deterministic optimization model. The model was then tested with the data from the case and literature, the results of the model were then analyzed to see if the model acted as intended and gave realistic results. The implementation was updated in cases of unrealistic behaviors, the changes were then noted in the deterministic model. The literature was then revisited to find potential updates for the deterministic model, this process was repeated in an iterative manner until the deterministic model was deemed satisfactory to represent a realistic energy prosumption system. When the deterministic model was finished it was used together with the literature on stochastic optimization to create a stochastic optimization model. The stochastic model was then together with a sample algorithm used to produce the final result. For an illustration of the research approach see figure 2.

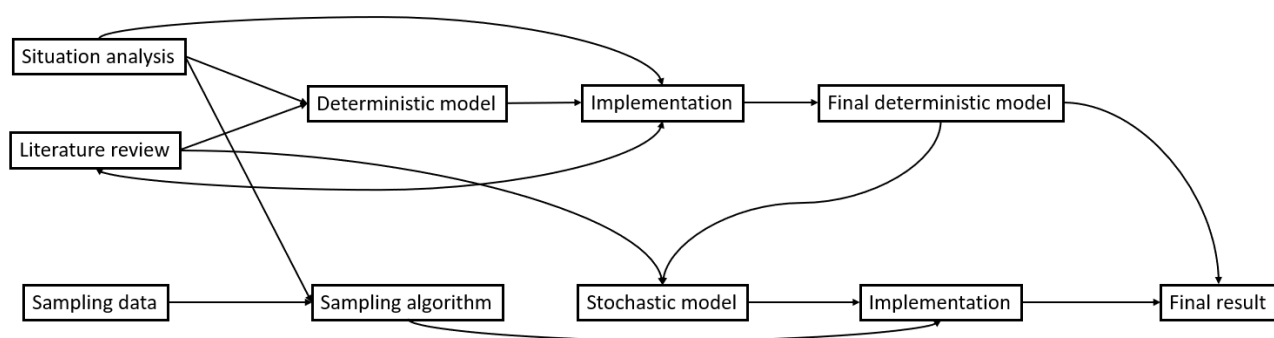


Figure 2: Illustration of research approach

2.2 Data Collection

Collected data for the report consisted of both primary and secondary data. Primary data was provided by the studied case of Austerland Skags and consisted of necessary supply and demand parameters to

run an optimization of the system. It also included estimated prices for the relevant technologies together with technology lifetimes and operating expenditures. Heating demand data was not presented in the original case data, an estimate of annual heating demand was however delivered upon request.

Secondary data was primarily collected through the literature review which mainly consists of scientific articles and books. The studied articles were collected from the databases *Google Scholar* and *Scopus* with keywords such as; "*stochastic optimization*", "*stochastic programming*", "*energy prosumption*", "*pv-battery*", and "*hydrogen energy storage*". The articles on stochastic optimization were primarily sorted by highest number of citations, articles on hydrogen and battery technology were primarily sorted by date since the technological advancements in those fields are moving fast and information on efficiencies and profitability quickly gets outdated. After sorting, articles from renowned journals were prioritized.

2.2.1 Sampling process

For the case of Austerland Skags one year of necessary data was available, corresponding to one sample. Since the stochastic model needed several samples to capture the intermittent nature of renewable energy sources, electricity prices, and electricity demand profiles more samples were needed. To create more samples secondary data including wind speed, global solar irradiation, electricity prices, and demand profiles were collected. The weather data was collected from weather stations on Gotland provided by SMHI (2022). Electricity price data for SE3 (Swedish electricity area 3) was gathered from Nordpool (2022) and consumption profiles for Gotland can be found on Mimer (2022).

The same processes of finding the variance and mean for wind power, electricity prices, and demand profiles were used. The process consisted of sorting the data by year and calculate the average and standard deviation for the same timestamps each year. Sorting of data and calculations of average and standard deviation were done in Microsoft Excel. Missing data was treated as empty cells, in other words it did not affect the calculations other than the sample size. When the average and standard deviation had been found, the relative deviation from the mean was calculated. The data from Austerland Skags case were then used as a mean together with the relative deviations to create distributions for the demand, these distributions were then used to create samples. Since the production of wind energy could drop to zero but not below a lognormal distribution was used to describe the variation. For the sun data, a uniform distribution was applied across years since the data did not follow a continuous model. Either the sun was shining resulting in high radiation or it was blocked by clouds resulting in low. For the electricity prices, a normal distribution was directly applied to the calculated means and standard deviations from the gathered secondary data.

2.3 Analysis Method

To find the optimal storage capacities and answer the research questions, methods within the field of operations research were used. The foundation of the analysis was based on linear programming where real world restrictions are formulated as linear constraints and the objective is formed as a linear cost function that is either minimized or maximized. The formulated linear optimization model, see chapter 5.2, was implemented in Python 3.8 together with the mathematical optimization solver Gurobi version 9.1.0. All optimizations were performed on a Laptop with Intel(R) Core(TM) i5-8250U 1.60GHz processor and 4GB RAM. The python implementation can be found in appendix A.

Since heating demand for the system only existed as annual data it needed to be reworked to fit the same resolution as the rest of the data. To do this the seasonal variation of heating demand was gathered from Energimyndigheten (2022) statistics database. The database included quarterly heating

demand for Swedish households and was used to set the relative seasonal heating demand, which was then assumed to increase/decrease linearly throughout the months. The data was then assumed to be equally distributed over the hours throughout the month.

The analysis continued with a cost optimization of the deterministic model, with and without utilization of waste heat to see how it affected cost and self-sufficiency. The degree of self-sufficiency, with utilization of waste heat, achieved during cost optimization was later used as a minimum degree of self-sufficiency. Another optimization based on self-sufficiency was then performed to find the maximum possible degree of self-sufficiency. The self-sufficiency was then gradually increased 0.5 percentage points at a time by forming a linear constraint on minimum degree of self-sufficiency from the most economically viable to the maximum allowed. Several graphs were then plotted based on the results to show the correlation between self-sufficiency and system variables.

The linear deterministic optimization model was then integrated with the sampling algorithm and sample average approximation (SAA) method, see section 3.3.1, to see how much the stochastic parameters affected the results. Both the sampling algorithm and SAA was written in python script by the author and can be found in appendix B. The N , N' , and M of the SAA was increased until the 4GB RAM was fully occupied and the program stopped due to memory error. The SAA results were then compared to the result from the expected value problem to show the difference between an stochastic and deterministic solution.

2.4 Reliability and Validity

To uphold accurate results and keep good research quality high reliability and validity were of importance. Reliability refers to how consistent the analysis and data collection procedures were. Will the thesis yield the same results on other occasions or by other observers? Was the thesis transparent in how it analyzed the data? Validity refers to if the study actually measured what it claimed to do (Saunders et al., 2007).

Since this thesis was quantitative in its nature and very few subjective decisions were made during the study the reliability should be high. All collected data came from either governmental agencies or recent literature. In other words, it should not matter who performed the study, it should still yield similar results. The study would however yield different results depending on location due to different geographical conditions. This was however not an effect of the methodology but the data the model had to work with. The technology in the field is progressing quickly, meaning that some of the parameters used in the thesis will change over time resulting in a different outcome than the one obtained in this study. The transparency was deemed to be sufficient since established methods in the field of operations research were used to find the results.

3 Literature Review

This chapter presents literature regarding prosumption systems, stochastic optimization, and inventory management under uncertainty. The purpose of the literature review is to provide a better understanding of the thesis topic and lay the foundation of the analysis.

3.1 Electricity Prosumption Systems

With the decreasing costs of solar panels and the possibility to sell electricity to the electricity grid, decentralized electricity production through photovoltaic (PV) panels in households has dramatically increased since the beginning of the 2000's (Bellekom et al., 2016; Ellsworth-Krebs & Reid, 2016). This has created a situation where the households both produce and consumes electricity, hence creating an electricity prosumption system with electricity prosumers. The term prosumption was first introduced by Toffler and Alvin (1980) and is a combination of the words production and consumption. In other words, being a prosumer means that one both produces and consumes a commodity.

A study by Couture et al. (2014) presented different drivers for the emerging prosumption systems, economic drivers, behavioral drivers, and technology drivers. The economic driver, the expected economic performance of the prosumption investment, is usually the primary driver for prosumers to install renewable energy sources such as solar PV panels. This is why Germany for example has a significantly higher installed residential production capacity than France and the US even tho both have higher potential daily production capacity per square meter than Germany. Due to the high electricity taxes in Germany it becomes more profitable to install PV panels compared to France and the US.

Another driver brought up by Couture et al. (2014) is the behavioral driver. This type of driver can motivate consumers to become prosumers when there is little to no financial gain. Factors that influence these behavioral drivers are usually:

- **Environmental values:** The will to reduce electricity produced by fossil fuels to hinder climate change and reduce pollution.
- **Control:** Some prosumers value the possibility of not only controlling how much energy they consume but also how the energy is produced.
- **Self-sufficiency:** The ability to be independent of third-party suppliers.
- **Reliability and safety:** A PV system combined with a battery can act as a backup in case of a power outage.
- **Status and prestige:** Certain people value and take prestige in owning and operating high technology equipment.
- **Interest in technology:** Consumers who like to keep up with newer technology trends might invest in PV cells and become prosumers due to it being a new growing technology.
- **Desire for choice:** Some consumers prefer the possibility to choose where they source their electricity from and might invest in solar PV just because they have the option.

There also exist behavioral drivers that work against becoming a prosumer, such as lack of awareness, lack of trust in the technology, and inconvenience.

The last primary driver that Couture et al. (2014) mentioned was the technology driver. Being an electricity prosumer is usually connected with the system of having an electric vehicle and battery

storage, all being connected with a smart grid. Hence with the technological advancements of other technologies such as batteries and electrical vehicles, it becomes more compelling to start producing your own electricity since the yield from the produced electricity becomes higher the better the infrastructure around it is. Technological advancements in electricity production such as PV panels also play an important role in this driver as well.

Electricity prosumption is however not only important for the prosumer but also important for society as a whole. Having decentralized renewable electricity generation systems both increases redundancy in the grid, lowers carbon emissions, and reduces the need for increased grid infrastructure by producing the electricity needed on sight (Couture et al., 2014; Ellsworth-Krebs & Reid, 2016). Electricity prosumption systems will hence play an important role in climate change, energy security, and electricity affordability (Ellsworth-Krebs & Reid, 2016).

In a typical electricity prosumption system excess energy produced by the renewable energy source is sold to the electricity grid and during deficits electricity is bought from the grid. This creates a scenario where the prosumer is semi self-sufficient and can not utilize the full potential of their production. This is why some prosumption systems include a battery, enabling the possibility to store the energy for later use increasing self-consumption of the renewable energy (Bellekom et al., 2016). To maximize profit from the system there exists a ratio between electricity stored in batteries and the amount sold to the grid. This ratio can be hard to find due to the intermittent nature of renewable energy sources which will be further discussed in section 3.3. The goal is however not always to maximize profit when installing renewable energy sources, some systems are designed to be as self-sufficient as possible hence maximizing self-consumption (Bellekom et al., 2016). Some go as far as disconnecting from the grid completely and produce 100% of their own electricity (Couture et al., 2014). Other systems are designed to even out load levels in the electricity grid, hence reducing network infrastructure costs. In a study by Wang et al. (2013) a battery storage system was implemented to balance the load profiles of the electricity grid while simultaneously balancing the electricity costs of households hence optimizing the storage capacity based on the total costs for both consumers and system operator. The way this was set up was that the consumers controlled a percentage of the battery capacity, say $[x \mid 0 \leq x \leq 1]$, and the system operator controlled the remaining capacity, $1 - x$. By then sending electricity to the battery during low demand in the electricity grid and using stored electricity during high demand the load profile could be balanced resulting in network investment savings for the system operators. The system also worked in a similar way for the consumers where they could charge the battery during low electricity costs and discharge during higher costs evening out the total electricity costs during the day.

3.2 Technologies in Prosumption Systems

In recent years the number of prosumers has seen a rapid increase due to the increase of residential PV panels (Dafalla et al., 2020). It is estimated that by 2050 the residual sector could be able to see up to 89% of their own electricity demand (Gähns et al., 2020). For this to be possible the prosumers have to work together and form collective prosumer networks to maximize self-consumption (Hahnel et al., 2020; Inês et al., 2020). This requires more people to adopt the idea of producing their own electricity. Currently some of the roadblocks for a broader general audience to adopt being a prosumer is, according to Gähns et al. (2020), missing knowledge and acceptance issues. The author thus states that it is important to educate the importance and benefits of being a prosumer. The author continues to point out that policymakers can further haste the adoption of prosumption by reducing legal and administrative barriers, promoting communal self-consumption, and bringing forth financial initiatives.

Most of today's prosumption systems are based on PV solar panels as electricity generators, a smart meter to control the electricity flow within the system, and potentially a battery storage technology to increase self-sufficiency (Couture et al., 2014). Some of the systems are configured in a way where the electricity production is placed after the smart meter hence selling all the electricity produced to the grid, these systems exist because of Feed-in Tariffs (FiTs). FiTs are an extra fee that the grid operator pays the prosumer for producing and selling renewable electricity. These kinds of systems never include a battery since there is no interest in balancing the produced electricity. The more common way however, is to place the production before the meter and utilize the produced electricity for oneself. If the goal for this system is maximizing self-sufficiency then batteries are usually included to balance out the produced electricity throughout the day (Brown et al., 2019). There is speculation on including electric cars in the systems and utilizing the battery of the car as a storage medium for the entire household. This way an external battery is not needed since the electric vehicle can fulfill the role of electricity balancing. This technology is however not available yet (Couture et al., 2014).

There are however speculations on other storage technologies than batteries to store energy over longer periods of time. Batteries are poorly fit to achieve this task due to high costs and capacity restrictions (Møller et al., 2017). These problems do not have a large impact on certain climates where the sun irradiation is somewhat balanced throughout the year, but for countries located in the northern hemisphere the limited capacity and costs of batteries start to show (Puranen et al., 2021a). Batteries also lose some of the electricity stored over time, the phenomenon is called self-discharge and usually lies between 0.1-0.3% per day. There are other storage technologies that do not experience self-discharge and suffer from the same capacity restrictions. The storage technologies mainly discussed in the literature for long time storage are pumped hydro storage, compressed air storage, and chemical storage in the form of hydrogen or methane (Ferreira et al., 2013; X. Luo et al., 2015). The main problem with pumped hydro storage and compressed air storage is that they are heavily dependent on geographical location and require large initial investments (X. Luo et al., 2015). Hydrogen is not restricted by geographical limitations, even though it could benefit from it by storage in caverns, and it has the largest potential for large-scale energy storage (Møller et al., 2017).

3.2.1 Batteries in Prosumption Systems

The inclusion of battery storage in prosumption systems can increase self-sufficiency and help balance the grid load. However depending on the type of battery the system can reach different levels of self-sufficiency. The current most common battery used in prosumption systems is lithium-ion batteries. A part of the reason for the lithium-ion batteries progress in the grid implementations is due to the spillover effect from the electrical vehicle market (Kamiya et al., 2021). The Hornsdale Power Reserve, located in South Australia, was as of 2019 the worlds largest lithium-ion battery reserve with a 100MW discharge capacity and a 129 MWh storage capacity. The power reserve was supplied by Tesla, one of the leading suppliers of electric vehicles, and has due to its capability to help balance the electricity grid provided significant cost savings for the national electricity market (Aurecon, 2020). The battery technology from the electrical vehicle market is not ideal for grid implementation. The focus on the electric vehicle market is usually to create energy-dense batteries with low volume and weight. For stationary batteries, volume and weight are usually secondary considerations. Because of this lithium iron phosphate batteries are becoming more popular in grid-scale implementations due to their lower costs, higher safety, higher durability, and lower material scarcity than batteries more prominent in the electrical vehicle market (Kamiya et al., 2021). Lithium batteries are however not the only battery technology that has been implemented in prosumption systems. Lead-acid batteries are a mature technology that has seen many implementations in prosumption systems in the past. Its main selling point is that it is cheaper than the lithium-ion batteries but it suffers from a general lower efficiency, 80-83% compared to lithium-ions 85-92%, 92-96% charge/discharge efficiency (Gähns et

al., 2020; Han et al., 2022; Puranen et al., 2021b; Zou et al., 2022). Another benefit of lithium batteries over lead acid is the higher depth of discharge (DoD). To prolong the lifetime of batteries they are rarely fully discharged and the minimum state of charge of a battery is decided by the DoD. The DoD of lead-acid batteries is usually 60% (Khiareddine et al., 2018; Parra et al., 2016), while lithium batteries allows a DoD of 80% (Campana et al., 2021; Puranen et al., 2021b; Zou et al., 2022). Another technology that could replace lithium-ion batteries in battery storage projects is flow batteries. They are less sensitive to higher DoD, have fast response times, almost no daily self-discharge, and long life cycles. The problem with flow batteries is that they have a low energy density and low market maturity (Kamiya et al., 2021; X. Luo et al., 2015).

A study by Puranen et al. (2021b) investigated the economic viability of different electric energy storage methods for two prosumer households in Finland. Among these methods was physical battery storage for electricity storage, the study concluded that physical battery storage is not economically viable for either of the two households. Another study by Campana et al. (2021) showed that including a battery for peak shaving and price arbitrage can lead to considerable annual savings. The study examined different geographical locations for their PV prosumer system, Stockholm, Johannesburg, and Rome, and showed that Stockholm had the largest potential for peak shavings but the lowest potential for annual revenues. All geographical locations showed that the net present value of the battery investment was negative, meaning that the annual savings are not high enough to financially motivate the investment of a battery. The study claimed that battery prices need to drop 50%, 250 US\$/kWh (2375 SEK/kWh), for the investment to be profitable. There are however other factors than battery prices that affect the economic value of batteries in PV prosumption systems. Barbour and González (2018) study examined what electricity prices and FiTs are needed for battery storage to be profitable with battery prices ranging from 400-700 US\$/kWh (3800-6650 SEK/kWh). The study concluded that for battery storage solutions in prosumption systems to see economical viability the electricity prices need to exceed 0.40 US\$/kWh (3.8 SEK/kWh) and FiTs below 0.05 US\$/kWh (0.475 SEK/kWh).

Most of the literature examined finds that PV battery systems are not economically viable, there are however some studies that claim that it can be viable. Goop et al. (2021) mentions that the economical viability of a PV battery prosumer system is dependent on local conditions such as annual solar PV electricity generation, electricity prices, PV prices, and battery prices. The author continues to claim that studies on the economical viability of PV battery systems rarely consider the surrounding electricity system. By taking spot price, taxes, and grid fees into consideration when calculating economic viability, the study found that a prosumption system of 2104 Swedish households with a solar PV capacity of 5-20 GW_p (gigawatt peak) could benefit from a 0.5-10 GWh battery system depending on future conditions. The high variability of PV and battery capacity is due to the three different scenarios that the study examined where they varied the investment costs for batteries, 90-300 €/kWh (931.5-3105 SEK/kWh), and solar PV, 900-1200 €/kW_p (9315-12420 SEK/kW_p), for each scenario. Han et al. (2022) examines 26 regions in Switzerland and finds that a battery PV prosumer system can already be more profitable than a PV-only system. The reason for examining multiple regions was because the author found that studies tended to focus on single households/buildings or clusters of households within the same region, which does not give a representative picture of a country's PV battery system potential. The author claims that the battery sizes will continue to expand in the future as the technology matures and becomes cheaper, the prosumers that benefit the most from the system are those with high irradiation and electricity demand.

Even though studies find different results on the economic viability of batteries in PV prosumption systems due to the uncertainties in battery costs, electricity costs, tariffs, and geographical conditions all examined studies are in agreement on the potential of increasing self-sufficiency by implementing a physical battery storage (Campana et al., 2021; Goop et al., 2021; Puranen et al., 2021b). A

study investigating battery storage for single households with PV systems found that using battery storage, 5-15 kWh, could increase self-sufficiency by 20-30 percentage points (Gähns et al., 2020). The study by Puranen et al. (2021b) showed that a 20 kWh battery could increase self-sufficiency by 20-25 percentage points for two different households with a solar PV capacity of 8 kW_p and 21.11 kW_p respectively. Hence increasing the households self-sufficiency from 25% and 40% to 50% and 60%. The study then concluded that the increase in self-sufficiency stagnated with increasing battery capacities above 20 kWh only increasing by a few percentage points from 20 to 100 kWh of battery capacity. Another study found similar results showing that increased self-sufficiency from batteries stagnates at a certain battery capacity. The study found that investments in battery storage, in kWh, larger than four times the annual PV production capacity, in MWh, results in minimal improvements in self-sufficiency (Nyholm et al., 2016).

3.2.2 Hydrogen in Prosumption Systems

Hydrogen as a storage medium for electricity requires three components for the system to work. These are an electrolyzer, hydrogen storage tank, and fuel cell. The electrolyzer uses electricity to split water into hydrogen and oxygen, the hydrogen tank then stores the hydrogen so it can later be converted back into electricity, the fuel cell uses the excess energy that is created when hydrogen reacts with oxygen to create electricity (Zhang et al., 2017). The type of electrolyzer, hydrogen tank, and fuel cell will affect the efficiency and costs of the system. The fuel cell most commonly used in prosumption systems is the proton exchange membrane (PEM) fuel cell due to its quick start-ups and low operating temperatures (X. Luo et al., 2015; US Department of Energy, 2011). The PEM fuel cell can reach electricity efficiencies between 38-60% and if waste heat is utilized the system efficiencies can reach 60-80% (Y. Luo et al., 2021; Parra et al., 2016; Puranen et al., 2021a). PEM electrolyzers are also a popular alternative in prosumption systems, one reason for this is connected with one of the problems with PEM fuel cells. The PEM fuel cell is sensitive to hydrogen impurities and the PEM electrolyzer produces hydrogen with high purity (99.99%). Other advantages of the PEM electrolyzer are quick responses, high efficiency (80-90%), and a compact design (Kumar & Himabindu, 2019).

Lacko et al. (2014) used a hydrogen system to enable an off-grid household to become 100% self-sustainable using only renewable energy sources. This system also utilized the excess heat that is produced during both electrolysis and the fuel cell conversion. The study first analyzed the use of excess heat from the hydrogen system without thermal storage, this resulted in 54% self-sufficiency mainly due to a mismatch in production and consumption of the heat, primarily from the electrolyzer. By adding thermal storage the heat can be stored for later use and the household was able to completely remove their use of fossil fuels, achieving 100% self-sufficiency. Another study that mentioned the potential of utilizing excess heat from the fuel cell was Puranen et al. (2021a). Their study claimed that the excess heat from the fuel cell could cover up to 1 MWh of electricity used for heating, the annual electricity demand for the system was 7.271 MWh. In their case this covered 50 kg, one-fourth of the total demand, of produced hydrogen.

A study by Parra et al. (2016) also studied a stand-alone hydrogen system to increase the self-consumption of solar PV electricity in a community energy system. The study found that the round trip efficiency of the system reached 52% when utilizing waste heat. The system is still less viable than batteries for short-term storage but shows potential for medium to long-term storage. Puranen et al. (2021a) examines the potential of an off-grid household in northern climates utilizing a hybrid energy storage system with both batteries and hydrogen. The hydrogen system is modeled like the previously mentioned with an electrolyzer and fuel cell with a nominal power of 6 kW each and no minimum power output. In practice low power loads, below 20%, might shorten the life span of the technologies and cause problems due to increasing hydrogen crossover to oxygen during electrolysis.

This problem was partially considered since the fuel cell operated to also charge the battery during unmet demand from the renewable energy sources to avoid partial loads below 20% from the fuel cell and minimize the start and stops. The results from the study showed that hydrogen storage needed a capacity of at least 183 kg, 7.21 MWh using the higher heating value of hydrogen, to balance the annual electricity demand. The system did not include a compressor which means that the storage pressure was 50 bar, which is possible to achieve with some PEM electrolyzers, resulting in a storage volume of 45.8 m³. The study also concluded that the results were heavily dependent on geographical location, due to varying solar irradiation, and that the hydrogen storage would most likely decrease for more southern locations.

When looking at larger electricity prosumer systems the balance of consumption and production starts to affect the grid. Usually when evaluating prosumer systems net present value and self-sufficiency ratio are two important factors, but when the system reaches a large enough scale grid power fluctuation also becomes a factor. Hydrogen storage can better help lower the negative grid impacts than battery storage due to the higher capacity of hydrogen storage for the same costs. The hydrogen storage solution also reaches better self-sustainable ratios and net present values when taking grid impact into consideration (Zhang et al., 2017).

3.3 Stochastic Demand and Supply

An energy supply chain with renewable energy has two important technical properties, namely an intermittent energy source, and an intermittent energy demand (Schneider et al., 2016). There exist several approaches to optimize problems with uncertainty, they follow different modeling philosophies such as minimizing deviation from goals, minimization of maximum costs, and optimization over soft constraints (Sahinidis, 2004). One of the approaches to optimizing with uncertainties is stochastic programming. Birge and Louveaux (2011) emphasizes the value of stochastic programming and shows that the problem including stochastic uncertainty, called the recourse problem (RP), will always have at least as good of a solution as the expected results from the expected value problem (EEV). Both the EEV and the RP are then worse than the wait and see solution (WS) which is the best possible value for the given situation since the stochastic values are then no longer stochastic because they already occurred, hence resulting in the relation shown in equation 1:

$$WS \leq RP \leq EEV \quad (1)$$

The WS solution is however not always accessible due to either high costs or due to the information not being available. Which is why the RP can be of value when dealing with uncertainty. How the RP and EEV is calculated will be further discussed in section 3.3.1.

The RP is however not the only way to deal with uncertainties, there exist several different ways to approach stochastic programming in optimization models. Reddy et al. (2017) presents a review of several different stochastic optimization methods and compares the robustness, accuracy, and speed of the different approaches. The author states that recourse models are robust and accurate but can sometimes lead to a high computationally burden resulting in a time-consuming method, especially if the stochastic variables are continuous resulting in non-linear models.

3.3.1 Fixed Recourse Two-Stage Programs

For information on the following variables and parameters see table 1. The fixed recourse two-stage model was first implemented by Beale (1955) and Dantzig (1955) and is the problem of finding:

$$\min z = c^T x + E_{\xi} [\min q(\omega)^T y(\omega)] \quad (2)$$

Subject to

$$Ax = b \quad (3)$$

$$T(\omega)x + Wy(\omega) = h(\omega) \quad (4)$$

$$x \geq 0, y(\omega) \geq 0 \quad (5)$$

Where solving the problem

$$z = \min_x \{c^T x \mid Ax = b, x \geq 0\} \quad (6)$$

is usually referred to as solving the first stage problem. This first stage problem is usually simple to solve, but the second stage can be more difficult. Birge and Louveaux (2011) states that the difficulty derives from the expectation of the second stage objective $q(\omega)^T y(\omega)$ and that it has a linear solution $y(\omega)$ for every ω . To emphasise this the second stage value function for a given realization of ω is sometimes expressed as:

$$Q(x, \xi(\omega)) = \min_y \{q(\omega)^T y \mid Wy = h(\omega) - T(\omega)x, y \geq 0\} \quad (7)$$

Which allows the expected second stage value function to be expressed as:

$$\mathcal{L}(x) = E_\xi Q(x, \xi(\omega)) \quad (8)$$

Implementing equation 8 on the original two-stage recourse model the following deterministic equivalent program is obtained:

$$\min z = c^T x + \mathcal{L}(x) \quad (9)$$

Subject to

$$Ax = b$$

$$x \geq 0$$

Showing that the difference between a deterministic and a stochastic model lies in the recourse function. If the recourse function were to be given the problem then turns into a nonlinear model. For more information on recourse functions and proofs of concepts see Birge and Louveaux (2011).

Table 1: Description of the parameters and variables of the two-stage recourse model

Symbol	Dimension	Description
c	$n_1 \times 1$	The first stage objective $c \in \mathfrak{R}^{n_1}$
A	$m_1 \times n_1$	The first stage matrix
x	$n_1 \times 1$	Deterministic decision variable
b	$m_1 \times 1$	The first stage right hand side $b \in \mathfrak{R}^{m_1}$
E_ξ		The mathematical expectation regarding ξ
ω		A random event $\omega \in \Omega$ where Ω is the set of all random events
ξ	$n_2 + m_2 + (n_2 \times m_2)$	The stochastic components such that $\xi^T(\omega) = (q(\omega)^T, h(\omega)^T, T_1(\omega), \dots, T_{m_2}(\omega))$
$q(\omega)$	$n_2 \times 1$	The second stage objective vector $q(\omega) \in \mathfrak{R}^{n_2}$
$y(\omega)$	$n_2 \times 1$	The second stage decision vector
$T(\omega)$	$m_2 \times n_1$	The technology matrix
$h(\omega)$	$m_2 \times 1$	The right hand side in the second stage $h(\omega) \in \mathfrak{R}^{m_2}$
W	$m_2 \times n_2$	The recourse matrix

Birge and Louveaux (2011) mentions that when considering a nonlinear problem the computational effort tends to increase compared to linear problems. Hence, an interest in finding a linear approximation or deterministic equivalent to $\mathcal{L}(x)$ exists. The problem can be solved with the expected value (EV), $EV = \min_x z(x, \bar{\xi})$, but as previously mentioned the EEV

$$EEV = E_{\xi}(z(\bar{x}(\bar{\xi}), \xi)) \quad (10)$$

is at best equal to the recourse problem. Resulting in interest to find better approximations of the uncertainty than the mean.

A common approximation of the recourse problem, $\mathcal{L}(x)$, is through the sample average approximation (SAA). By taking random samples ξ^1, \dots, ξ^N of the random vector ξ the recourse function for the random sample can be rewritten as:

$$\mathcal{L}(x) = \frac{1}{N} \sum_{k=1}^N Q(x, \xi^k) \quad (11)$$

Hence the non-linear objective function in equation 9 can be rewritten as the deterministic equivalent for a given ξ , see equation 12.

$$z = \min_{x, y_k} \{c^T x + \frac{1}{N} \sum_{k=1}^N q_k^T y_k \mid Ax = b, Wy_k = h_k - T_k x, x \geq 0, y_k \geq 0\} \quad (12)$$

Where N is the sample size and N^{-1} is the probability of a possible realization ξ^k . A study by Santoso et al. (2005) used the SAA to evaluate the expectation in the objective function. They let z_N and \hat{x}_N denote the optimal value and optimal solution vector and then showed that as N increases z_N and \hat{x}_N converges with a probability of one to the stochastic optimal solution exponentially fast. This means that the approximated solution of the recourse function should be fairly accurate with limited sample size, in the study by Santoso et al. (2005) a sample size of $N=20$ were used and the SAA solution came close to the true stochastic optimal solution.

The following SAA algorithm is an adaption of the one presented in Kleywegt et al. (2002):

1. Choose initial amount of samples M and sample sizes N and N' such that $N < N'$.
2. For $j = 1, \dots, M$ repeat steps 2.1 and 2.2.
 - 2.1 Generate a sample $\xi_j^1, \dots, \xi_j^N \in \xi$ and solve the SAA problem in equation 12. Save the optimal objective value $\hat{z}_{N,j}$ and the optimal solution $\hat{x}_{N,j}$.
 - 2.2 Calculate $\hat{f}_{N'}(\hat{x}_{N,j}) := c^T x + \frac{1}{N'} \sum_{k=1}^{N'} Q(x, \xi^k)$ and compare it with $\hat{f}_{N'}(\hat{x}_{N,j'})$ where $\hat{x}_{N,j'}$ is the best solution found thus far and $j' < j$. Then let \hat{x} denote the solution among $\hat{x}_{N,j'}$ and $\hat{x}_{N,j}$ with the best value of $\hat{f}_{N'}(\hat{x})$.
3. Let $\bar{z}_{N,M} = \frac{1}{M} \sum_{j=1}^M \hat{z}_{N,j}$ and $\hat{f}_{N'}(\hat{x})$ be the best solution found in 2.2. The estimated optimality gap can then be calculated with: $gap = |f_{N'}(\hat{x}) - \bar{z}_{N,M}|$. The variance of the gap estimator is then estimated with $\sigma_{gap}^2 = \sigma_{N'}^2(\bar{z}) + \sigma_{\bar{z}_{N,M}}^2$ where $\sigma_{N'}^2(\bar{z}) = \frac{1}{N'(N'-1)} \sum_{n=1}^{N'} (c^T \hat{x} + Q(\hat{x}, \xi(\omega^n)) - f_{N'}(\hat{x}))^2$ and $\sigma_{\bar{z}_{N,M}}^2 = \frac{1}{M(M-1)} \sum_{j=1}^M (\hat{z}_{N,j} - \bar{z}_{N,M})^2$

4. If the optimality gap is large then increase N and/or N' and return to step 2. The number of samples M can also be adjusted if deemed necessary. If the optimality gap is sufficiently small then stop the process, \hat{x} is the best solution.

The SAA algorithm proposed by Kleywegt et al. (2002) can produce good and often optimal solutions with low values of N and M . The gap estimator for the algorithm is however not good enough to verify that the solution is close to optimal when using a small N and M , thus requiring much larger sample sizes. Resulting in a larger computational burden to ensure good estimations.

Another problem with the SAA is that step 2.1 involves solving a two-step stochastic program for every sample. Even though this problem is much smaller than the original it may still require a decent amount of calculations. This is why Birge and Louveaux (2011) proposes to use an approximation of the optimal solution in step 2.1 of the SAA. By doing this less calculation effort is spent on inaccurate samples. One of these approximation algorithm is the L-shaped decomposition algorithm (also known as Benders decomposition algorithm). For further information about the algorithm see Birge and Louveaux (2011).

3.3.2 The Newsvendor Approach

The problem with stochastic programming is that it can be computationally difficult to solve, which is why simpler solutions are frequently used such as solving the deterministic problem instead by assuming that the stochastic parameter is deterministic (Birge & Louveaux, 2011). Several studies investigating stochastic demand and supply use some sort of approach to relax some of the randomnesses in their model (Cristea et al., 2020; Nguyen & Chen, 2019; Schneider et al., 2016). Cristea et al. (2020) and Schneider et al. (2016) in particular chose to assume the demand as deterministic while keeping the supply as stochastic to find a model to optimize electrical energy storage in a system with renewable energy sources. They both modeled their optimization model after an unreliable supplier that can be backed up with a reliable, but more expensive, supplier. Where renewable sources in this case were unreliable and the electricity grid was the more reliable but expensive alternative. Both these studies ended up with an adaptation of the Newsvendor problem, where the optimal electric energy storage capacity C^* was calculated by:

$$C^* = \eta * S^* \quad (13)$$

Where η stands for the conversion losses of the system and S^* is the optimal order up to level decided by:

$$S^* = D - Q_w \quad (14)$$

Q_w is the smallest valid boundary that fulfills the requirements of:

$$F_w(Q_w) \geq \frac{c_o}{c_o + c_u} \quad (15)$$

Where $F_w(\cdot)$ is the cumulative additive distribution function of the yield (supply - demand) and c_o, c_u is the overage and underage costs related to the Newsvendor model. For more information on the Newsvendor problem see Axsäter (2015) and Ghiani et al. (2004).

3.4 Safety Stock Under Uncertainty

Uncertainties in demand, supply, and lead times can create the need for additional inventory to avoid shortages. This additional inventory is called safety stock and is the average amount of inventory on hand that allow for variations in demand and supply (Axsäter, 2015). This safety stock helps satisfy demand even during unfavorable conditions or unexpected peaks in demand. The downside with safety stock is the extra cost of increased storage volume and the increased tied capital (Ghiani et al., 2004).

Safety stock is usually determined based on a specified service level. There exist several different definitions of how the service level is calculated, two of the most popular are:

- S_1 : The probability of no stock out
- S_2 : Fraction of demand that can be fulfilled immediately from stock

Axsäter (2015) argues that S_1 has some disadvantages due to it not considering batch size and can severely overestimate the actual service level if batch quantities are small. As a consequence S_1 is a poor fit for practical applications, it is however very easy to calculate which is why it is still used by some. S_2 on the other hand can give a better picture of the actual service level but will in turn require more calculations. The formulas as presented in Axsäter (2015) for S_1 and S_2 are:

$$P(D(L) \leq R) = S_1 = \Phi\left(\frac{R - \mu'}{\sigma'}\right) = \Phi\left(\frac{SS}{\sigma'}\right) \quad (16)$$

$$S_2 = 1 - F(0) = 1 - \frac{\sigma'}{Q} \left[G\left(\frac{R - \mu'}{\sigma'}\right) - G\left(\frac{R + Q - \mu'}{\sigma'}\right) \right] \quad (17)$$

Where R is the reorder point, μ' is the average during the lead time, σ' is the standard deviation during the lead time, SS represents the safety stock, Q the batch quantity, Φ the distribution function, and $G(x)$ is the loss function of a normal distribution.

4 Situation Analysis

This chapter explains the current situation for the case of Austerland Skags and the state of the surrounding environment. Specific values of energy production and demand has been excluded due to confidentiality.

4.1 The Austerland Skags Case

The Austerland Skags project is a project funded by the Swedish energy authority to examine the possibilities for a smart energy system that can reduce the local carbon dioxide emissions from agriculture, transport, and the local residents (Nygarn Utveckling AB, 2022). This will be achieved by creating a model of a local energy system that considers energy flow, conversion losses, energy storage, aggregator capacities, and more. Deciding appropriate technology for the different components of the system together with a cost calculation and practical feasibility study is also part of the project (Energicentrum Gotland, 2021). The system design for the first simulation is set up in the following way. Wind and solar energy for renewable electricity production. Battery and hydrogen storage for increased self-sufficiency of renewable electricity. Electricity demand from 200 households, an electric carpool, a treatment plant, and a farm. Hydrogen demand from hydrogen vehicles and a potential fuel cell that can help balance annual electricity consumption.

For the first optimization the renewable energy capacity was set to 2.6 MW_p solar and 0.5 MW_p wind. Excess electricity produced by renewable energy sources can be sold to the electricity grid. To do this the local grid needs to expand. The costs of expanding the local grid were set to 1 450 SEK per kW of estimated max load during the optimization. There was also a use fee connected to importing electricity from the electricity grid, the cost was 0.484 SEK/kWh 06:00-22:00 from November to March and 0.16 SEK/kWh otherwise. The difference in cost is due to the higher grid loads during the winter period. There was also another fee connected to the maximum grid load per month named power fee that amounted to 35 SEK per kW of max load during the month. The electricity price, known as SPOT-price, varies every hour and can be found on Nordpool (2022). For the first optimization the estimated electricity SPOT-price for the coming year was used. An electricity tax of 0.36 SEK/kWh was also added to the electricity costs, no additional taxes for self-consumed electricity was used.

With the projected electricity produced by the solar and wind plant, the system of Austerland Skags could reach self-sufficiency of almost 48% without using energy storage. This was under the assumption that:

- Hydrogen is produced with an electrolyzer with 85% efficiency and immediately consumed after production
- Daily hydrogen demand is evenly spread throughout the day
- Renewable electricity is used when available
- Excess electricity is sold to the electricity grid

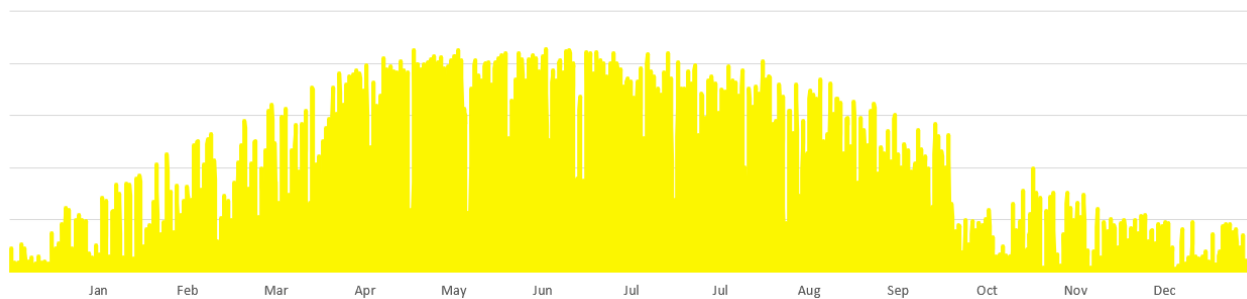
The current state of the project is to evaluate the feasibility of using combined hydrogen and battery storage to increase self-sufficiency from renewable energy sources. The estimated costs and lifetimes of the battery and hydrogen technologies for the case are presented in table 2.

Table 2: Costs of technologies

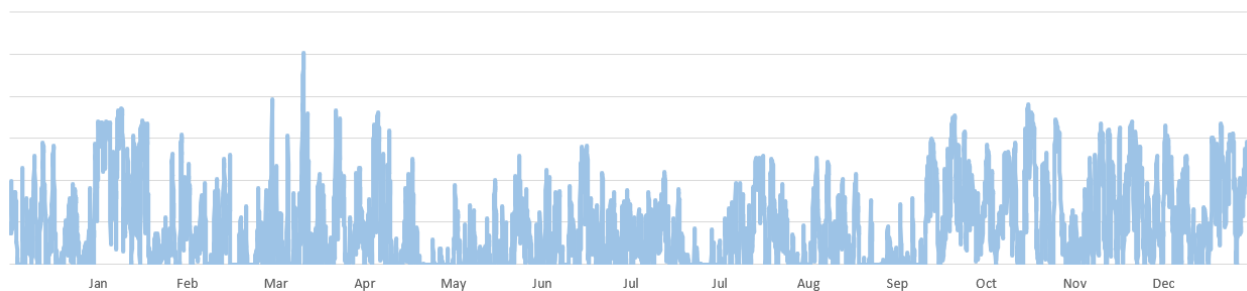
Technology	Investment Cost	Annual Operating Cost (% of total investment)	Lifetime (years)
Batteries	5 500 SEK/kWh	0.1	15
Electrolyzer	5 000 SEK/kW	0.5	15
Hydrogen Tank	240 SEK/kWh	0.5	15
Fuel Cell	5 000 SEK/kW	0.5	15

4.1.1 Production and Consumption Patterns

In the case data from Austerland Skags estimated production data for one year of solar and wind power was presented. The solar data shows great potential from April to August and then significantly drops in October and continues to be relatively low until March, see figure 3a. The wind power was less reliable than the solar power, but it shows a higher potential in the beginning and towards the end of the year, see figure 3b. Making it a good complement to solar power. The production capacity of wind power was however only one-fifth of the solar power production, resulting in a lack of renewable energy in October.



(a) Solar Power Production



(b) Wind Power Production

Figure 3: One Year of Renewable Energy Production

Electricity demand varied depending on the season. During winter the demand was higher and during the summer demand tended to be lower, see figure 4. The electricity consumption also varied significantly depending on the time of the day. Electricity demand tended to peak around 17:00-19:00 and was usually at its lowest around 07:00-09:00, figure 5. There was no real difference in the daily consumption pattern during the year other than the increased consumption during the winter and occasional lows during the summer when rooftop solar panels produced more electricity than consumed.

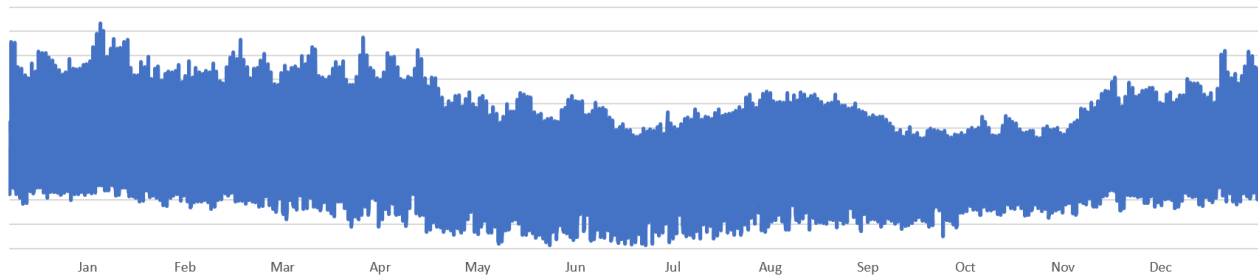


Figure 4: Yearly Electricity Consumption Pattern

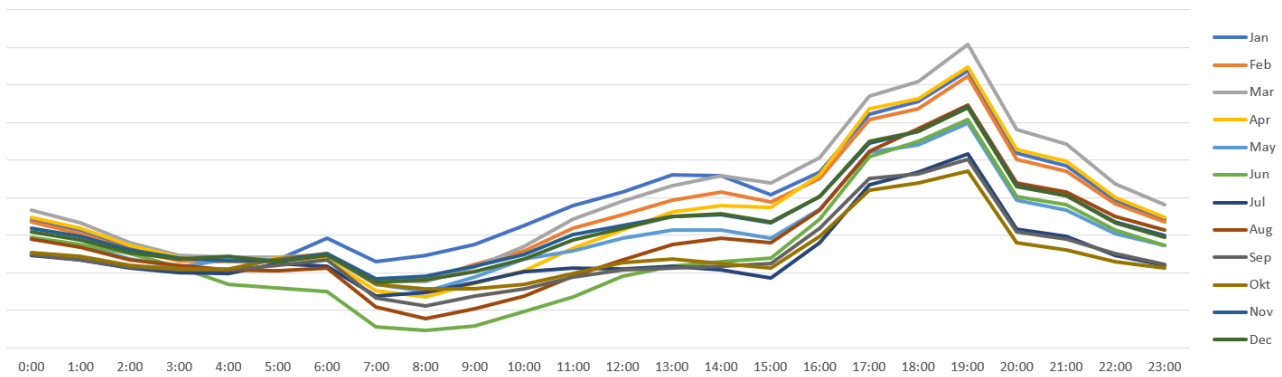


Figure 5: Average Daily Electricity Consumption Pattern Every Month

The hydrogen demand was derived from replacing fossil fuel in the system with hydrogen under the assumption that 100% of fossil fuels could be replaced, resulting in agricultural vehicles being fueled by hydrogen only. The fossil fuel use came primarily from heating and transportation connected to Skags farm and showed significant peaks during seeding and harvest season, see figure 6.



Figure 6: Hydrogen Consumption Pattern

The heating demand that was replaced by hydrogen for Skags farm represented 26% of total hydrogen demand. Table 3 presents the estimated percentage of heating demand per month based of data from Energimyndigheten (2022).

Table 3: Relative heating demand per month

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Okt	Nov	Dec
%	12.4	13.3	10.7	8.2	5.7	5	4.4	3.7	6	8.4	10.7	11.5

5 Analysis

This chapter presents an analysis of the literature followed by the mathematical optimization models for the deterministic and stochastic case and the respective result of applying them to the Austerland Skags case.

5.1 Literature Analysis

From the information gathered in the Situation Analysis, section 4, the primary drivers for Austerland Skags where their environmental values and interest in technology. There was of course an financial aspect involved, hence the interest to find a balance between self-sufficiency and costs. The system of Austerland Skags was modeled to potentially have both an lithium-ion battery and hydrogen storage system with an PEM electrolyzer and fuel cell. For the electrolyzer and fuel cell no consideration regarding minimum load was taken. This might affect the life times of the technologies, as stated by Puranen et al. (2021a), which in turn might affect the annual costs of electrolyzers and fuel cells.

According to the literature batteries needed to cost less than 2375 SEK/kWh to be profitable or electricity prices needed to exceed 3.8 SEK/kWh together with FiTs below 0.475 SEK/kWh. In the case of Austerland Skags there existed no FiTs, the battery costs and electricity prices are however far from reaching the levels stated as profitable by the literature. But as Goop et al. (2021) mentioned the economic viability also depends on local weather conditions and the surrounding electricity system.

A stochastic solution was examined since the WS was not accessible and the EEV is, according to Birge and Louveaux (2011), always equal or worse than the solution considering stochastic uncertainty. When looking at the relative deviation for the supply, solar and wind data, it varies significantly more than the demand. This could motivate the use of the Newsvendor approach, as done in Cristea et al. (2020) and Schneider et al. (2016). However, the demand still varies and the Newsvendor does not take the randomness of electricity prices into consideration. Relaxing both of these conditions was deemed too inexact to motivate the use of the Newsvendor approach. There were also several other factors that the proposed Newsvendor approach did not consider, such as the hydrogen storage and seasonal variations. The SAA was hence chosen as the analysis method since it takes all the stochastic variables into consideration.

5.2 Deterministic optimization model

Indices:

- $t = \{1, \dots, T\}$: Set of time where every increment of t equals one hour
- $t_0 = \{0, \dots, T : t_0 \supset t\}$: Super set of t including period 0
- $t_m = \{0, 744, 1440, 2184, \dots, 8784\}$: Cumulative hours per month (assuming 29 days in February)
- $m = \{0, 1, \dots, M\}$: Set of months where M is the smallest integer that fulfills $t_M \geq T$

Parameters:

Grid:

- $C_{EH}(t)$: Costs of electricity from the grid to the system based on both power fee and electricity spot price in time period t

- $C_{RE}(t)$: Selling price of renewable electricity to the grid in time period t
- C_P : Power fee based on monthly maximum power consumption from the electricity grid
- C_{cf} : Cost of connection fee for scaling up the local electricity grid
- $D_h(t)$: Electricity demand from households in time period t
- $D_s(t)$: Electricity demand from Skags in time period t
- $D_n(t)$: Electricity demand from Nyhagen in time period t
- $P_R(t)$: Produced renewable electricity in time period t
- T_c : Electricity taxes for companies
- T_s : Taxes for self consumed renewable electricity

Batteries:

- C_B : Investment cost per kWh of battery capacity
- C_{OB} : Annual operating cost of the battery storage (in % of total investment)
- D_{is} : Self-discharge rate of battery (in % per hour)
- B_{eff} : Efficiency of the battery
- DoD : Depth of discharge
- C : Maximum charge/discharge rate of the battery
- B_{life} : Lifetime of the battery

Hydrogen:

- C_L : Investment cost per kW of electrolyzer capacity
- C_S : Investment cost per kWh of hydrogen storage capacity
- C_F : Investment cost per kW of fuel cell capacity
- C_{OL} : Annual operating cost of the electrolyzer (in % of total investment)
- C_{OS} : Annual operating cost of the hydrogen storage (in % of total investment)
- C_{OF} : Annual operating cost of the fuel cell (in % of total investment)
- $H_d(t)$: Hydrogen demand for the system in time period t
- $D_{heat}(t)$: Part of hydrogen demand that is derived from heat in time period t
- L_{eff} : Efficiency of the electrolyzer
- F_{eff} : Efficiency of the fuel cell
- L_{heat} : Retrievable waste heat efficiency of the electrolyzer
- F_{heat} : Retrievable waste heat efficiency of the fuel cell

- L_{life} : Lifetime of the electrolyzer
- S_{life} : Lifetime of the hydrogen storage tank
- F_{life} : Lifetime of the fuel cell

Variables:

Grid:

- RE_{max} : Maximum amount of renewable energy sent to the electricity grid
- $E_{max}(m)$: Maximum amount of electricity sent from the electricity grid in month m
- $EH(t)$: Amount of electricity from electricity grid to households in time period t
- $ES(t)$: Amount of electricity from electricity grid to Skags in time period t
- $EN(t)$: Amount of electricity from electricity grid to Nyhagen in time period t
- $RE(t)$: Amount of renewable electricity to the electricity grid in time period t
- $RH(t)$: Amount of renewable electricity directly to households in time period t
- $RS(t)$: Amount of renewable electricity to Skags in time period t
- $RN(t)$: Amount of renewable electricity to Nyhagen in time period t
- $HE(t)$: Amount of electricity from the Houses to the Electricity grid in period t
- $y(t)$: Binary indicator if the house demand is negative for period t

Batteries:

- B_{cap} : Total battery capacity
- $S_B(t_0)$: Amount of electricity stored in batteries in time period t
- $RB(t)$: Amount of renewable electricity to batteries in time period t
- $BH(t)$: Amount of electricity from batteries to households in time period t
- $BS(t)$: Amount of electricity from batteries to Skags in time period t
- $BN(t)$: Amount of electricity from batteries to Nyhagen in time period t
- $HB(t)$: Amount of electricity from the Houses to the Batteries

Hydrogen:

- L_{cap} : Total electrolyzer capacity
- S_{cap} : Total hydrogen storage tank capacity
- F_{cap} : Total fuel cell capacity
- $S_S(t_0)$: Amount of hydrogen stored in time period t
- $RL(t)$: Amount of renewable electricity to the electrolyzer in time period t

- $SF(t)$: Amount of hydrogen from the hydrogen storage to the fuel cell in time period t
- $FH(t)$: Amount of electricity from the fuel cell to households in time period t
- $FS(t)$: Amount of electricity from the fuel cell to Skags in time period t
- $FN(t)$: Amount of electricity from the fuel cell to Nyhagen in time period t
- $HL(t)$: Amount of electricity from households to fuel cell in time period t
- $LW(t)$: Waste heat produced from electrolyzer in time period t
- $FW(t)$: Waste heat produced from fuel cell in time period t

5.2.1 Objective function

When selling excess electricity from the renewable energy sources the electricity has to be transported to the local grid. This means that the local grid needs to be expanded, the cost of expanding the local grid is proportional to the peak of electricity sent from the renewable energy sources to the local electricity grid. This grid expansion was assumed to be a one time cost during the lifetime of the project and was thus divided by the technical lifetime of the other technologies.

$$\frac{RE_{max} * C_{cf}}{B_{life}} \quad (18)$$

The storage technologies used in the system has both an immediate capital expenditure which is proportional to the volume or possible output of the technology and an annual operational expenditure directly proportional to the initial investment of the technology. The capital expenditure for a technology was divided by the technical life time of the technology to estimate an annual cost which was used in the model. The different technologies are represented by indices where B stands for battery, L electrolyzer, S hydrogen storage, and F stands for fuel cell.

$$\sum_{i=\{B,L,S,F\}} C_i * i_{cap} * \left(C_{oi} + \frac{1}{i_{life}} \right) \quad (19)$$

The cost of buying electricity from the electricity grid was based on a power fee that varies between seasons and hours, a electricity spot price, and electricity taxes. The total cost of importing electricity to the system was based of the electricity prices per kWh multiplied with the amount of electricity imported every hour. The total imported electricity costs was thus represent by the sum of costs over all hours.

$$\sum_{t=1}^T (C_{EH}(t) + T_c) * (EH(t) + ES(t) + EN(t)) \quad (20)$$

There also exist taxes for self consumed electricity. This cost amounts to a specified tax tariff multiplied with the renewable energy consumed within the system.

$$\sum_{t=1}^T T_s * (RB(t) + RH(t) + RS(t) + RN(t) + RE(t) + RL(t)) \quad (21)$$

Electricity can also be exported to the electricity grid. The selling price of electricity is equal to the electricity spot price since there are no add on tariffs for selling renewable electricity on the Swedish market. Electricity can both be sold from the renewable energy sources or from households directly

in case of excess energy produced from rooftop solar panels.

$$\sum_{t=1}^T C_{RE}(t) * (RE(t) + HE(t)) \quad (22)$$

The last cost was connected to the load on the local power grid. There exist a power fee connected to the peak electricity grid load every month.

$$\sum_{m=1}^M C_P * E_{max}(m) \quad (23)$$

All components can be finalized in the following objective function:

$$\begin{aligned} \min z = & \frac{RE_{max} * C_{cf}}{B_{life}} + \sum_{i=\{B,L,S,F\}} C_i * i_{cap} * (C_{Oi} + \frac{1}{i_{life}}) + \sum_{t=1}^T (C_{EH}(t) + T_c) * (EH(t) + ES(t) + \\ & EN(t)) + T_s * (RB(t) + RH(t) + RS(t) + RN(t) + RL(t)) - C_{RE}(t) * (RE(t) + HE(t)) \\ & + \sum_{m=1}^M C_P * E_{max}(m) \end{aligned} \quad (24)$$

5.2.2 Constraints

The initial charge of the batteries in period 0 before the optimization started was assumed to be the minimum allowed charge for the battery according to the depth of discharge. This way the model will not overestimate self sufficiency by adding extra electricity to the system.

$$S_B(0) = 1 - DoD \quad (25)$$

The maximum storage capacity of the batteries was determined by the highest amount of electricity stored in a single period.

$$S_B(t) \leq B_{cap} \quad \forall t \quad (26)$$

In every period of time there needs to be a balance condition that ensures that the next periods storage level is coherent with the previous periods. This balance condition is dependant on the amount of electricity charged into the battery, which equals electricity sent to the battery storage times the charge efficiency of the battery. The battery charge level was also dependant on previous periods charge, where daily self-discharge rate per hour was included. Lastly next periods charge level was dependant on how much the batteries where discharged.

$$B_{eff} * (RB(t) + HB(t)) + D_{is} * S_B(t-1) - BH(t) - BS(t) - BN(t) = S_B(t) \quad \forall t \quad (27)$$

The demand and electricity sent to households, Skags farm, and Nyhagen sewage plant required balance conditions. To ensure that demands are met electricity sent from the solar/wind plant, batteries, fuel cell, and electricity grid are matched with demand. The energy sent from the batteries was multiplied with the discharge efficiency to consider the efficiency losses during discharge. Households, equation 28, are slightly different since they include roof top solar panels and can in some periods produce excess energy which results in negative demands. Since all variables can only represent positive values additional reverse flow variables was needed. A balance condition for produced renewable energy and consumed renewable energy was also needed, see equation 31.

$$RH(t) + BH(t) * B_{eff} + EH(t) + FH(t) - HE(t) - HB(t) - HL(t) = D_h(t) \quad \forall t \quad (28)$$

$$RS(t) + BS(t) * B_{eff} + ES(t) + FS(t) = D_s(t) \quad \forall t \quad (29)$$

$$RN(t) + BN(t) * B_{eff} + EN(t) + FN(t) = D_n(t) \quad \forall t \quad (30)$$

$$RB(t) + RH(t) + RS(t) + RN(t) + RE(t) + RL(t) = PR(t) \quad \forall t \quad (31)$$

For the batteries to uphold a long lifespan they should not exceed a depth of discharge above 80%. The following constraint ensures that the battery level was above at least 20% of its maximum capacity at all times.

$$S_B(t) \geq B_{cap} * (1 - DoD) \quad \forall t \quad (32)$$

Since energy will be sent from the renewable energy sources to the electricity grid the grid needs to be scaled up based on the highest achieved load for the power line.

$$RE(t) \leq RE_{max} \quad \forall t \quad (33)$$

Due to the power fee cost per month there was an interest in finding the maximum power consumption from the grid every month. By letting t_m represent the last hour of every month, the span of $[t_m + 1, t_{m+1}]$ represented every hour of a specific month m . By finding the largest sum of electricity grid outputs of any given hour of a month it will represent the largest distributed output from the electricity grid for that month.

$$EH(t) + ES(t) + EN(t) \leq E_{max}(m + 1) \quad \forall m \setminus \{M\}, t \in [t_m + 1, t_{m+1}] \quad (34)$$

Since the future households had their own installed solar panels the electricity demand for the households could be negative. This potential negative demand was checked by equation 35. During negative demand the households needed to either sell the electricity to the grid, send it to the electrolyzer, or store in batteries. This balance condition was upheld by equation 36.

$$0 \leq D_h(t) * (1 - y(t)) \quad \forall t \quad (35)$$

$$HE(t) + HB(t) + HL(t) = -D_h(t) * y(t) \quad \forall t \quad (36)$$

The batteries can in practice not discharge or charge their full capacity in a single hour. The amount that they can charge/discharge is dependant on the specific charge/discharge rate of the battery, C , and the total battery capacity, B_{cap} . Thus all electricity sent to the battery and from the battery is restricted by the total battery capacity multiplied with the specific charge/discharge rate. Equation 37 represented the discharge and equation 38 the charge constraint.

$$BH(t) + BS(t) + BN(t) \leq B_{cap} * C \quad \forall t \quad (37)$$

$$RB(t) + HB(t) \leq B_{cap} * C \quad \forall t \quad (38)$$

To ensure that the system did not produce more hydrogen than it was capable of an upper bound for both the electrolyzer and hydrogen storage was set, equation 39 and 40. The maximum output capacity of the fuel cell was also set to find the required fuel cell capacity, see equation 41. The capacities of the technologies was decided on the same principle as the batteries.

$$RL(t) + HL(t) \leq L_{cap} \quad \forall t \quad (39)$$

$$S_S(t) \leq S_{cap} \quad \forall t \quad (40)$$

$$SF(t) \leq F_{cap} \quad \forall t \quad (41)$$

Much like the battery storage the hydrogen storage level was directly dependant on the previous periods storage level and amount added and extracted from the system. This together with amount of hydrogen used for heating that was replaced by excess heat from the electrolyzer and fuel cell forms the balance condition that determines the next periods hydrogen storage level.

$$L_{eff} * (RL(t) + HL(t)) + S_S(t - 1) + LW(t) + FW(t) - H_d(t) - SF(t) = S_S(t) \quad \forall t \quad (42)$$

There needs to be a balance condition that sets the amount of electric energy produced by the fuel cell equal to the amount sent from the fuel cell to the system.

$$SF(t) * F_{eff} = FH(t) + FS(t) + FN(t) \quad \forall t \quad (43)$$

Since a fraction of the hydrogen demand was derived from the heating demand some of this could be replaced by the waste heat produced from the electrolyzer and fuel cell. The produced waste heat from the electrolyzer was dependant on the amount of renewable energy it received, see equation 44. Fuel cell waste heat was directly dependant on the amount of hydrogen converted to electricity, see equation 45. Total waste heat utilized could not exceed the demand of heat required by the system, see equation 46.

$$LW(t) \leq RL(t) * L_{heat} \quad \forall t \quad (44)$$

$$FW(t) \leq SF(t) * F_{heat} \quad \forall t \quad (45)$$

$$LW(t) + FW(t) \leq D_{heat}(t) \quad \forall t \quad (46)$$

Since there was no quick backup for shortages in the hydrogen storage, like the electricity from the grid for the battery system. The hydrogen storage needs to consider a safety stock to be able to fulfill the needs of the fuel cell vehicles in the system. The variance was based on the average daily hydrogen demand every month since hydrogen consumption patterns was assumed to peak on a daily basis when the hydrogen fueled agricultural vehicles refuel. The safety stock was based on service level S_1 since the agricultural vehicles was assumed to sustain a stock cycle with low fraction of demand fulfilled.

$$SS(t) \geq k * \sigma'_h(m + 1) \quad \forall m \setminus \{M\}, t \quad (47)$$

Much like the batteries there needs to be an initial value for the hydrogen storage for period 0. Hydrogen demand can only be sourced through renewable energy sent to the electrolyzer. In case of low renewable energy production during the first day hydrogen demand would not be met if initial storage volumes would be 0. Thus the initial hydrogen storage was set to fulfill one week of hydrogen demand plus the safety stock.

$$S_S(0) = k * \sigma'_h(1) + \sum_{t=1}^{168} H_d(t) \quad (48)$$

5.2.3 Linear Optimization Model

With the above objective function and constraints the following model was formed:

Objective function:

$$\begin{aligned} \min z = & \frac{RE_{max} * C_{cf}}{B_{life}} + \sum_{i=\{B,L,S,F\}} C_i * i_{cap} * (C_{oi} + \frac{1}{i_{life}}) + \sum_{t=1}^T (C_{EH}(t) + T_c) * (EH(t) + ES(t) + \\ & EN(t)) + T_s * (RB(t) + RH(t) + RS(t) + RN(t) + RL(t)) - C_{RE}(t) * (RE(t) + HE(t)) \\ & + \sum_{m=1}^M C_P * E_{max}(m) \end{aligned}$$

Subject to:

$$\begin{aligned} S_B(0) &= B_{cap} * (1 - DoD) \\ S_B(t) &\leq B_{cap} \quad \forall t \\ B_{eff} * (RB(t) + HB(t)) + D_{is} * S_B(t-1) - BH(t) - BS(t) - BN(t) &= S_B(t) \quad \forall t \\ RH(t) + BH(t) * B_{eff} + EH(t) + FH(t) - HE(t) - HB(t) - HL(t) &= D_h(t) \quad \forall t \\ RS(t) + BS(t) * B_{eff} + ES(t) + FS(t) &= D_s(t) \quad \forall t \\ RN(t) + BN(t) * B_{eff} + EN(t) + FN(t) &= D_n(t) \quad \forall t \\ RB(t) + RH(t) + RS(t) + RN(t) + RE(t) + RL(t) &= PR(t) \quad \forall t \\ S_B(t) &\geq B_{cap} * (1 - DoD) \quad \forall t \\ RE(t) &\leq RE_{max} \quad \forall t \\ EH(t) + ES(t) + EN(t) &\leq E_{max}(m+1) \quad \forall m \setminus \{M\}, t \in [t_m + 1, t_{m+1}] \\ 0 &\leq D_h(t) * (1 - y(t)) \quad \forall t \\ HE(t) + HB(t) + HL(t) &= -D_h(t) * y(t) \quad \forall t \\ BH(t) + BS(t) + BN(t) &\leq B_{cap} * C \quad \forall t \\ RB(t) + HB(t) &\leq B_{cap} * C \quad \forall t \\ S_S(t) &\leq S_{cap} \quad \forall t \\ RL(t) + HL(t) &\leq L_{cap} \quad \forall t \\ L_{eff} * (RL(t) + HL(t)) + S_S(t-1) + LW(t) + FW(t) - H_d(t) - SF(t) &= S_S(t) \quad \forall t \\ SF(t) * F_{eff} &= FH(t) + FS(t) + FN(t) \quad \forall t \\ SF(t) &\leq F_{cap} \quad \forall t \\ LW(t) &\leq RL(t) * L_{heat} \quad \forall t \\ FW(t) &\leq SF(t) * F_{heat} \quad \forall t \\ LW(t) + FW(t) &\leq D_{heat}(t) \quad \forall t \\ S_S(t) &\geq k * \sigma'_h(m+1) \quad \forall m \setminus \{M\}, t \\ S_S(0) &= k * \sigma'_h(1) + \sum_{t=1}^{168} H_d(t) \\ 0 &\leq RE_{max}, B_{cap}, L_{cap}, S_{cap}, F_{cap} \\ 0 &\leq EH(t), ES(t), EN(t), RE(t), RH(t), RS(t), RN(t), HE(t) \quad \forall t \\ 0 &\leq S_B(t_0), RB(t), BH(t), BS(t), BN(t), HB(t) \quad \forall t \end{aligned}$$

$$\begin{aligned}
0 &\leq S_S(t_0), RL(t), SF(t), FH(t), FS(t), FN(t), LW(t), FW(t), HL(t) \quad \forall t \\
0 &\leq E_{max}(m) \quad \forall m \\
y(t) &\in \{0, 1\} \quad \forall t
\end{aligned}$$

5.3 Deterministic Analysis Results

By applying the deterministic model to the data given from Austerland Skags the best possible mix, from a cost perspective, of hydrogen and battery storage was derived. Two scenarios was created with the model, one without recovery of waste heat and one with, see table 4 for results.

Table 4: No waste heat vs waste heat collection

No waste heat		Waste heat recovered	
Total Cost	2 812 487 SEK	Total Cost	2 658 173 SEK
Battery capacity	35 kWh	Battery Capacity	45 kWh
Electrolyzer Capacity	827 kW	Electrolyzer Capacity	777 kW
Hydrogen Storage	37 775 kWh	Hydrogen Storage	32 095 kWh
Fuel Cell Capacity	0 kW	Fuel Cell Capacity	70 kW
Self-sufficiency	55.8%	Self-sufficiency	59.7%
Self-consumption	67.5%	Self-consumption	68.2%

Since producing hydrogen through renewable energy was the only way the system could fulfill the hydrogen demand a relatively large hydrogen tank was formed. This was mainly due to the high hydrogen demand and low hydrogen production potential in October. Without waste heat recovery all produced hydrogen was used to meet the hydrogen demand resulting in no fuel cell being used. With waste heat recovery a fuel cell was built due to the increased efficiency of the fuel cell and already existing infrastructure of electrolyzer and hydrogen storage. Both the electrolyzer and hydrogen storage saw an decrease in capacity due to some of the hydrogen demand for heating being replaced by waste heat. The system also saw an increase in both self-consumption and self-sufficiency when waste heat was recovered.

Since the cost of expanding the grid was relatively low compared to selling excess electricity only a small battery and fuel cell was built in both cases. The reason for this was mainly due to the large hydrogen storage used to fulfill hydrogen demand. The hydrogen storage was used to shave peaks in renewable energy production hence lowering the peak exported electricity. If the hydrogen demand was removed the battery capacity increased to 388 kWh. This result supports the argument from Goop et al. (2021) that a battery storage becomes profitable when including surrounding grid fees. This was because the battery shave peaks in electricity input to the system, since the electricity grid needed to expand based on peak exported electricity the cost became sensitive to peak electricity export. When excluding grid fees, $C_{cf} = 0$, there was no need to avoid export peaks which resulted in no battery storage being the most profitable option. Which goes in line with the literature, battery costs need to drop for battery storage to become economically viable as an energy balancing medium.

By changing the objective function to maximize self-sufficiency the highest possible degree of self-sufficiency was found. The new objective function is presented in equation 49. Results from the self-sufficiency optimization is presented in table 5.

$$\max z = \sum_{t=1}^T RH(t) + RS(t) + RN(t) + BH(t) + BS(t) + BN(t) + FH(t) + FS(t) + FN(t) \quad (49)$$

Table 5: Maximum self-sufficiency optimization

Variable	Value	Unit
Total Cost	18 386 711	SEK
Batteries	19 072	kWh
Electrolyzer	2 275	kW
Hydrogen Tank	545 060	kWh
Fuel Cell	201	kW
Self-sufficiency	86.1	percentage
Self-consumption	100	percentage

The new objective function paid no regard to costs hence the high total cost for the system. A fuel cell was used even though the worse round trip efficiency compared to the batteries. Meaning that the self discharge ratio of the batteries was high enough to motivate seasonal storage in the form of hydrogen. In the case of no heat recovery the efficiency of the fuel cell would not be high enough to motivate a fuel cell resulting in a significantly larger battery size. The self-sufficiency of 86.1% was unreasonable since it needed an hydrogen storage of 545 MWh, ≈ 16 tonnes of hydrogen, which would result in a 3923 m³ hydrogen tank assuming 50 bar of pressure and 20°C.

By gradually increasing self-sufficiency with equation 50, from economic optimum to maximum self-sufficiency, the cost and capacity patterns could be analysed.

$$\begin{aligned}
& \sum_{t=1}^T (RH(t) + RS(t) + RN(t) + BH(t) + BS(t) + BN(t) + FH(t) + FS(t) + FN(t) + LW(t) + FW(t)) \\
& - S_s(0) \geq SelfSufficiency * \sum_{t=1}^T D_h(t) + D_s(t) + D_n(t) + H_d(t) * (SelfSufficiency - 1)
\end{aligned} \tag{50}$$

The relation between self-sufficiency and costs follows a piece wise linear pattern where the costs increases relatively slow until 73% self-sufficiency. Between 73%-85% the cost increase fast and above 85% it increases exponentially, see figure 7. The increase after 73% can primarily be explained by the increase in hydrogen storage volume. Since the hydrogen demand peaks in October when there was low solar electricity production the hydrogen storage gets oversized the rest of the year. As the self-sufficiency rises this excess capacity was utilized during the rest of the year by implementing a larger electrolyzer and fuel cell. This continued until 71% self-sufficiency, after 71% the conversion losses from the fuel cell was too big to achieve higher self-sufficiency so the system replaces fuel cell capacity with bigger batteries. When the system reached 73% self-sufficiency the focus switched from meeting electricity demand to stock hydrogen and make better use of high peaks in electricity production. In this stage both battery, electrolyzer, and fuel cell capacities somewhat stagnate and hydrogen storage capacity rapidly increased. This behaviour then continues until 84% where the fuel cell starts to be completely phased out due the better efficiency of the battery technology. During the last percentage the battery capacity exponentially increases and the electrolyzer and fuel cell sees one last peak on 86% where the losses from self-discharge of the battery exceeded the efficiency losses of the fuel cell. For illustrations of technology behaviours see figure 8.

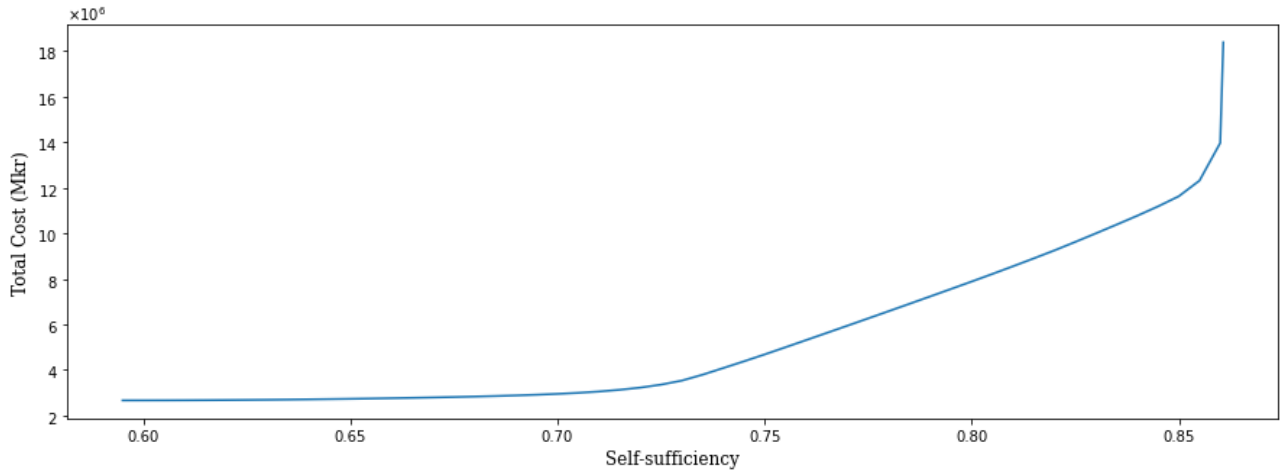
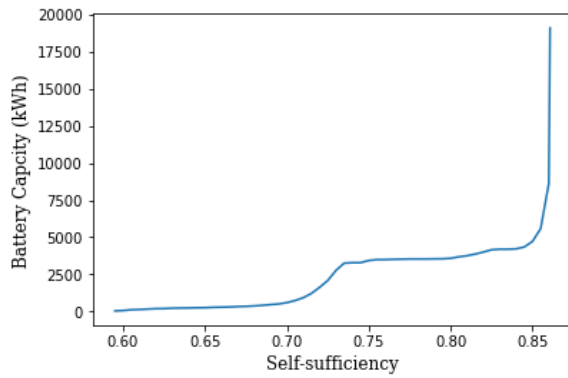
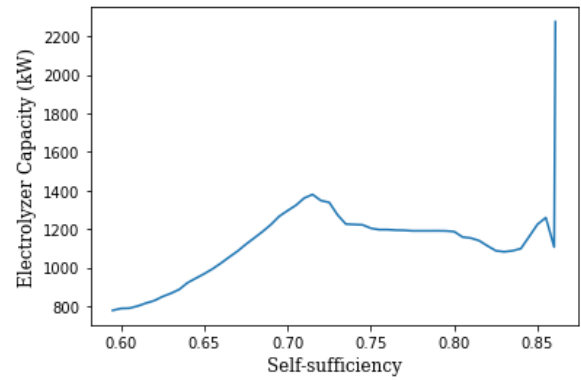


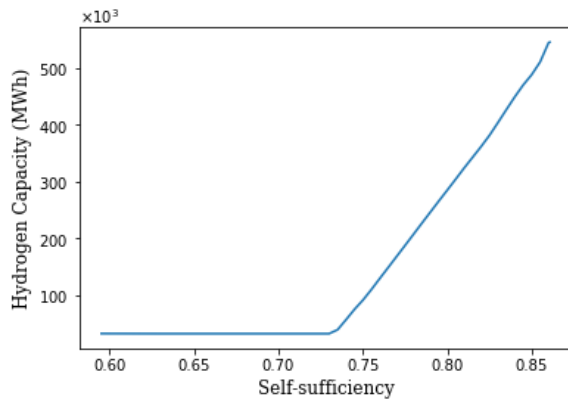
Figure 7: Cost of increased self-sufficiency



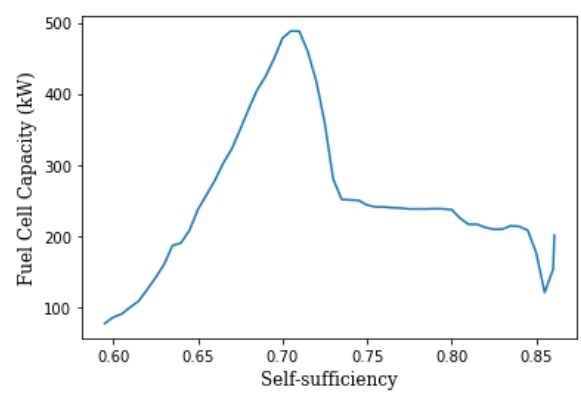
(a) Battery



(b) Electrolyzer



(c) Hydrogen Storage



(d) Fuel Cell

Figure 8: Relations between capacities and self-sufficiency

The results from figure 7 and 8 was partly in agreement with Puranen et al. (2021b) result regarding the stagnating effect of increased storage beyond a certain point. The same goes for Nyholm et al. (2016) connection between annual PV production and battery capacity. The results are however hard to compare since this thesis system included both wind power production and hydrogen supply/demand which neither of the previous mentioned studies considered.

5.4 Stochastic Optimization Model

New Indices:

- $k = \{1, \dots, N\}$: Sample size
- $j = \{1, \dots, J\}$: Amount of samples

Changes in parameters for the stochastic version:

Grid:

- $C_{EH}(t)_k$: Costs of electricity from the grid to households in time period t for sample k
- $C_{RE}(t)_k$: Selling price of renewable electricity to the grid in time period t for sample k
- $D_h(t)_k$: Electricity demand from households in time period t for sample k
- $D_s(t)_k$: Electricity demand from Skags in time period t for sample k
- $D_n(t)_k$: Electricity demand from Nyhagen in time period t for sample k
- $P_R(t)_k$: Produced renewable electricity in time period t for sample k

Changes in variables for the stochastic version:

Grid:

- $EH(t)_k$: Amount of electricity from electricity grid to households in time period t for sample k
- $ES(t)_k$: Amount of electricity from electricity grid to Skags in time period t for sample k
- $EN(t)_k$: Amount of electricity from electricity grid to Nyhagen in time period t for sample k
- $RE(t)_k$: Amount of renewable electricity to the electricity grid in time period t for sample k
- $RH(t)_k$: Amount of renewable electricity directly to households in time period t for sample k
- $RS(t)_k$: Amount of renewable electricity to Skags in time period t for sample k
- $RN(t)_k$: Amount of renewable electricity to Nyhagen in time period t for sample k
- $HE(t)_k$: Amount of electricity from the Houses to the Electricity grid in period t for sample k
- $y(t)_k$: Binary indicator if the house demand is negative for period t for sample k

Batteries:

- $S_B(t_0)_k$: Amount of electricity stored in batteries in time period t for sample k
- $RB(t)_k$: Amount of renewable electricity to batteries in time period t for sample k
- $BH(t)_k$: Amount of electricity from batteries to households in time period t for sample k
- $BS(t)_k$: Amount of electricity from batteries to Skags in time period t for sample k
- $BN(t)_k$: Amount of electricity from batteries to Nyhagen in time period t for sample k
- $HB(t)_k$: Amount of electricity from the Houses to the Batteries for sample k

Hydrogen:

- $RL(t)_k$: Amount of renewable electricity to the electrolyzer in time period t for sample k
- $SF(t)_k$: Amount of hydrogen from the hydrogen storage to the fuel cell in time period t for sample k
- $FH(t)_k$: Amount of electricity from the fuel cell to households in time period t for sample k
- $S_S(t_0)_k$: Amount of hydrogen stored in time period t for sample k
- $FS(t)_k$: Amount of electricity from the fuel cell to Skags in time period t for sample k
- $FN(t)_k$: Amount of electricity from the fuel cell to Nyhagen in time period t for sample k
- $HL(t)_k$: Amount of electricity from households to fuel cell in time period t for sample k

5.4.1 Explanation of Stochastic Model

Aside from changed variables and parameters the stochastic model does not differ from the deterministic other than averaging over the samples in the objective function and the inclusion of $\forall k$ for most constraints. The samples, k , represented possible outcomes of how the conditions could look for a year. In other words k represents the random event ω from table 1, hence with every k comes the second stage objective vector, $q(\omega)$, and the decision vector, $y(\omega)$. These vectors was the presented changed parameters and variables in section 5.4. The deterministic decision variable vector, x , was set as $[RE_{max}, B_{cap}, L_{cap}, S_{cap}, F_{cap}]$.

5.4.2 Linear Optimization Model

Objective function:

$$\begin{aligned} \min z = & \frac{RE_{max} * C_{cf}}{B_{life}} + \sum_{i=\{B,L,S,F\}} C_i * i_{cap} * (C_{oi} + \frac{1}{i_{life}}) + \sum_{m=1}^M C_P * E_{max}(m) + \frac{1}{N} \sum_{k=1}^N \sum_{t=1}^T (C_{EH}(t)_k + \\ & T_c * (ES(t)_k + EN(t)_k + EH(t)_k) + T_s * (RB(t)_k + RH(t)_k + RS(t)_k + RN(t)_k + RL(t)_k) \\ & - C_{RE}(t)_k * (RE(t)_k + HE(t)_k) \end{aligned}$$

Subject to:

$$\begin{aligned} S_B(0) &= B_{cap} * (1 - DoD) \\ S_B(t)_k &\leq B_{cap} \quad \forall t, k \\ B_{eff} * (RB(t)_k + HB(t)_k) + D_{is} * S_B(t-1)_k - BH(t)_k - BS(t)_k - BN(t)_k &= S_B(t)_k \quad \forall t, k \\ RH(t)_k + BH(t)_k * B_{eff} + EH(t)_k + FH(t)_k - HE(t)_k - HB(t)_k - HL(t)_k &= D_h(t)_k \quad \forall t, k \\ RS(t)_k + BS(t)_k * B_{eff} + ES(t)_k + FS(t)_k &= D_s(t)_k \quad \forall t, k \\ RN(t)_k + BN(t)_k * B_{eff} + EN(t)_k + FN(t)_k &= D_n(t)_k \quad \forall t, k \\ RB(t)_k + RH(t)_k + RS(t)_k + RN(t)_k + RE(t)_k + RL(t)_k &= PR(t)_k \quad \forall t, k \\ S_B(t)_k &\geq B_{cap} * (1 - DoD) \quad \forall t, k \\ RE(t)_k &\leq RE_{max} \quad \forall t, k \\ EH(t)_k + ES(t)_k + EN(t)_k &\leq E_{max}(m+1) \quad \forall m \setminus \{M\}, t \in [t_m + 1, t_{m+1}], k \end{aligned}$$

$$\begin{aligned}
0 &\leq D_h(t)_k * (1 - y(t)_k) \quad \forall t, k \\
HE(t)_k + HB(t)_k + HL(t)_k &= -D_h(t)_k * y(t) \quad \forall t, k \\
BH(t)_k + BS(t)_k + BN(t)_k &\leq B_{cap} * C \quad \forall t, k \\
RB(t)_k + HB(t)_k &\leq B_{cap} * C \quad \forall t, k \\
S_S(t)_k &\leq S_{cap} \quad \forall t, k \\
RL(t)_k + HL(t)_k &\leq L_{cap} \quad \forall t, k \\
L_{eff} * (RL(t)_k + HL(t)_k) + S_S(t-1)_k + LW(t)_k + FW(t)_k - H_d(t) - SF(t)_k &= S_S(t)_k \quad \forall t, k \\
SF(t)_k * F_{eff} &= FH(t)_k + FS(t)_k + FN(t)_k \quad \forall t, k \\
SF(t)_k &\leq F_{cap} \quad \forall t, k \\
LW(t)_k &\leq RL(t)_k * L_{heat} \quad \forall t, k \\
FW(t)_k &\leq SF(t)_k * F_{heat} \quad \forall t, k \\
LW(t)_k + FW(t)_k &\leq D_{heat}(t) \quad \forall t, k \\
S_S(t)_k &\geq k * \sigma'_h(m+1) \quad \forall m \setminus \{M\}, t \\
S_S(0) &= k * \sigma'_h(1) + \sum_{t=1}^{168} H_d(t) \\
0 &\leq RE_{max}, B_{cap}, L_{cap}, S_{cap}, F_{cap} \\
0 &\leq EH(t)_k, ES(t)_k, EN(t)_k, RE(t)_k, RH(t)_k, RS(t)_k, RN(t)_k, HE(t)_k \quad \forall t, k \\
0 &\leq S_B(t_0)_k, RB(t)_k, BH(t)_k, BS(t)_k, BN(t)_k, HB(t)_k \quad \forall t, k \\
0 &\leq S_S(t_0)_k, RL(t)_k, SF(t)_k, FH(t)_k, FS(t)_k, FN(t)_k, HL(t)_k \quad \forall t, k \\
0 &\leq E_{max}(m) \quad \forall m \\
y(t)_k &\in \{0, 1\} \quad \forall t, k
\end{aligned}$$

5.5 Stochastic Analysis Results

Due to the system only being able to produce hydrogen from renewable energy sources a problem in the model occurred during stage 2.2 of the SAA. Since the amount of renewable energy varied between samples some suffered from low energy production in October resulting in the predetermined hydrogen storage capacity not being sufficient to supply the hydrogen demand. The reverse problem also occurred since the maximum amount of renewable energy that could be sent to the electricity grid, batteries, and electrolyzer was predetermined. This resulted in cases where the renewable energy production spiked and was too high for the system to handle. This problem was fixed by allowing the model to add electrolyzer and hydrogen tank capacity during stage 2.2, the safety stock restriction was also relaxed in stage 2.2 to open up more storage capacity to help balance production and demand peaks.

Another problem that happened during the stochastic optimization was that the computer ran out of RAM for a relatively small size of N' , $N' = 13$. To reduce the complexity of the model Skags, Nyhagen, and the households were grouped to one unit. This simplified model also created a more general model, for the simplified model see appendix C. To get a comparison of the complexity and results of the normal and simplified model the deterministic case was used, see table 6 for the results of the deterministic runs.

Table 6: Normal vs Simplified

Normal		Simplified	
Solution Method: Primal and dual simplex		Solution Method: Dual simplex and barrier	
Continuous Variables	202 052	Continuous Variables	105 428
Binary Variables	8 784	Binary Variables	0
Simplex Iterations	106 277	Barrier Iterations	87
Run Time	251 Seconds	Run Time	19 Seconds
Results		Results	
Total Cost	2 658 173 SEK	Total Cost	2 658 418 SEK
Battery Capacity	45 kWh	Battery Capacity	45 kWh
Electrolyzer Capacity	777 kW	Electrolyzer Capacity	777 kW
Hydrogen Storage	32 095 kWh	Hydrogen Storage	32 095 kWh
Fuel Cell Capacity	70 kW	Fuel Cell Capacity	70 kW
Max Grid Load	1 476 kW	Max Grid Load	1 476 kW

No differences in capacities were seen between the simplified and normal version, only slight differences in when it sold, stored, and bought electricity. The differences in results were deemed small enough to motivate the reduced complexity of the simplified model, allowing for larger sample sizes in the SAA algorithm. The final settings used for the SAA and the results from running the algorithm are presented in table 7.

Table 7: SAA Algorithm Results

Settings		Results	
M	10	Total Cost	2 449 960 SEK
N	10	Battery capacity	319 kWh
N'	25	Electrolyzer Capacity	917 kW
Performance		Hydrogen Storage	16 606 kWh
Run Time Gurobi	8 435 Seconds	Fuel Cell Capacity	100 kWh
Run Time Sampling	5 725 Seconds	Grid Capacity	1 348 kW
Run Time Total	14 160 Seconds	Gap	2 067
Continuous Variables		Variance N'	11 890 497
N	1 054 120	Variance $Z_{N,M}$	11 564 356
N'	2 635 265	Variance gap	23 454 853

The gap and variance was deemed small enough to give an accurate solution. The solution differed quite a bit from the original simplified solution. The main differences were seen in battery and hydrogen storage capacity. The reason for the decreased hydrogen storage capacity was because the more evenly spread solar power production. From the data gathered by SMHI there was no dip in solar irradiation during October for any of the examined years. This resulted in a smaller hydrogen storage capacity since the model did not need to compensate as much for the lower solar irradiation during October when there was high hydrogen demand. The increased battery capacity was mainly due to the highly fluctuating electricity prices. By letting the EL-spot price follow a normal distribution for every hour there was a higher discrepancy between the daily/hourly peaks and lows in price. The model thus capitalised on this by increasing the battery capacity allowing for more electricity to be sourced in-house during high electricity prices.

Since the samples deviated significantly from the original data provided by Austerland Skags the N' sample batch was used to create an average sample corresponding to $\bar{\xi}$. $\bar{\xi}$ was then used to create the EV solution, the results from the EV can be found in table 8.

Table 8: The expected value solution

Variable	Value	Unit
Total Cost	2 309 746	SEK
Batteries	0	kWh
Electrolyzer	746	kW
Hydrogen Tank	14 570	kWh
Fuel Cell	30	kW
Grid Capacity	1 396	kW

By taking the average of the N' samples the volatility in the data disappears and the model does not need a battery, resulting in 0 kWh of battery capacity. Both the electrolyzer and hydrogen storage capacity was smaller in the SAA solution since the electricity production was more evenly spread. A couple of samples where used to test the EEV compared to the SAA solution and the SAA continuously outperformed the EEV, as expected according to Birge and Louveaux (2011). The EEV also often struggled to find a feasible solution for some samples since the hydrogen storage and electrolyzer was too small to fulfill hydrogen demand.

6 Discussion

This chapter presents a discussion on the method, assumptions, and potential practical problems of the study.

One of the major points that were not considered in this thesis was demand load shifting. If the demand could be evenly spread across the day or even shifted to peak in the middle of the day when solar power production was at its highest the need for energy storage would drastically reduce. Such demand load shifts might be possible in the future when electric vehicles and smart devices are more common so charging vehicle batteries and daily chores can be done during peak solar hours.

During the optimization hydrogen demand was assumed to be evenly distributed across the day and constant throughout the month. A more accurate sample of hydrogen demand would affect hydrogen storage capacity and electrolyzer profiles, it would also give the possibility to calculate a variation. In the optimization, hydrogen demand was assumed to follow a normal distribution with 15% standard deviation from the mean. This assumption could be inaccurate and significantly affect the calculated safety stock. Another factor that can increase the accuracy of the model was the distribution of waste heat. The current distribution of waste heat was assumed to be continuous throughout the day, if more accurate profiles of heating demand were present then different patterns of electrolyzer and fuel cell activity might have occurred and affected the self-sufficiency factor. Another problem was that no real heat infrastructure was modeled, the cost savings from including waste heat might be misleading since no costs for the infrastructure of utilizing waste heat were included.

The annual hydrogen demand of the system amounted to over 38 tons. If this hydrogen were to be produced solely from electrolysis of water then, by using the chemical reaction of electrolysis and molar masses of water and hydrogen, it can be derived that the system would need approximately 345 000 liters of water every year. Worth mentioning is that the electrolyzer is sensitive to impurities meaning that not just any water can be used to produce the hydrogen. With freshwater scarcity being a climate debate, this might constrain hydrogen production. If the hydrogen's only purpose were energy balancing, this would not be a problem since the water produced from the fuel cell would be pure enough for the electrolyzer, meaning that it could be recycled to the electrolyzer in a closed-loop system. Oxygen produced from the electrolyzer could also be part of this closed-loop system. The oxygen could potentially be stored in a separate oxygen tank and used internally by the fuel cell, excess oxygen could also be sold. This alternative profit could have affected electrolyzer capacity which in turn could affect hydrogen tank and fuel cell capacities. Another variable that could have been included was the selling price of hydrogen. By allowing the system to sell excess hydrogen it might have utilized the electrolyzer to produce hydrogen instead of selling the electricity to the grid further increasing self-consumption of the prosumption system.

The results from the stochastic and deterministic versions differed significantly. This was likely a result of the sample creation. When creating the samples the data became more volatile, sometimes one hour could have high solar irradiation and the next hour almost none. Where in reality one hour of low solar irradiation was usually followed by another with low, so instead of creating cloudy days the samples had cloudy hours. This higher volatility in production, demand, and price caused the battery capacity to increase significantly to capitalize on the volatility in price and production. Since we are moving towards a more intermittent electricity market this might be more accurate than the original sample provided by the Austerland Skags case, and since the solar production dictates such a large portion of the result this also gives a more accurate picture of the energy spread throughout the year.

The final run of the applied SAA algorithm took about 4 hours to complete. There exist methods to speed up the process, like the L-shaped decomposition algorithm mentioned by Birge and Louveaux

(2011). By doing an approximation of the optimal solution less time is spent on each sample. But since the optimization was only to be used one time to decide an optimal capacity for future use optimization time was not considered a critical factor, hence no approximations were used in the SAA. There also existed other methods of finding the optimal energy storage capacity in the literature, the most prominent of those where the Newsvendor approach. This method was however, as stated in section 5.1, deemed too inexact to motivate. There are other methods than optimization to find the capacities, one commonly used method in energy systems is HOMER simulations. The purpose of the thesis was however not a simulation but an optimization which was why the HOMER was not considered.

Another thing not considered in the model was partial loads and its effect on efficiencies and safety for the electrolyzer and fuel cell. In reality the efficiency of both the fuel cell and electrolyzer was affected by the current load, higher loads lead to higher efficiencies, for more details see Puranen et al. (2021a). To be able to model such a behavior a nonlinear constraint would need to be formed which in this case was not possible. Another way to consider this was to not allow the system to operate with lower loads, hence creating a lower allowed limit of production. By doing this the problem with hydrogen crossover would also be solved, the problem with this was that always running the systems on max capacity was not realistic due to the uncertainties of renewable energy production. Having the system always running max loads will also affect the utilized waste heat, since every hour has a maximum capacity of utilizable waste heat this would be filled and the rest would be wasted which would produce inaccurate results of how waste heat could be utilized. If heat storage would be present then this would not have been a problem but since it was not then this could cause inaccurate results.

The problems with the SAA where it could not find feasible solutions in stage 2.2 were fixed by allowing the system to "overload" the electrolyzer and hydrogen storage, as stated in section 5.5. For the final run of the SAA the system overloaded the electrolyzer with 13.6 kW and never needed to overload the hydrogen storage. In reality it might be better to spread this overload between the different technologies, batteries, grid, and electrolyzer but by giving the model too much freedom to add slack it might find other cheaper solutions that were not intended hence only allowing for increased electrolyzer and hydrogen tank capacity.

The final model used in this thesis had some wrong assumptions about how the system Austerland Skags was planned. One of these wrong assumptions was regarding waste heat, hydrogen was not used for heating and waste heat can thus not be used to replace a part of the hydrogen demand. In reality wood chips are used to heat the farm and in the final model proposed to Austerland Skags waste heat was used to replace these wood chips instead, resulting in an added cost in the objective function. Another thing that was not in the thesis model was the opportunity to buy hydrogen externally. The changes in waste heat and the opportunity to buy hydrogen can be seen in the model in appendix D. With these changes the SAA showed a 10-25% decrease in electrolyzer, hydrogen tank, and fuel cell capacity. With the most significant decrease being in hydrogen tank capacity. The battery capacity saw a slight increase in capacity and the total cost dropped by almost 5%.

7 Conclusion

This chapter presents the answers to the research questions proposed in section 1.3.

The purpose of this thesis has been to *"Design and implement a linear optimization model which calculates the optimal storage capacity of different storage mediums, with stochastic demand and supply, in an energy prosumption system."* This purpose has been fulfilled by answering the research questions. The first question: *"What are relevant storage mediums for energy storage in a prosumption system?"* was answered by the literature review and analysis where it was derived that battery, hydrogen, and thermal energy storage are relevant storage technologies for energy prosumption systems. The capacities of these different technologies are dependant on the climate of the place investigated. Hydrogen and thermal are more relevant for Northern climates where winters are characterised of low solar irradiation and high heating energy demand, such as the Austerland Skags case, and batteries are more prominent in places with high volatility in electricity prices.

The answer to the second research question: *"What found variables and parameters are relevant to an energy prosumption system?"* can be found in section 5.2 which presented the applied variables and parameters. An interesting finding was that retrieving waste heat from an electrolyzer without thermal storage, which is often not beneficial since the electrolyzer usually produces during renewable energy production peaks, had a significant impact on both cost and self-sufficiency. So for a prosumption system with external hydrogen demand retrieving waste heat from both the electrolyzer and fuel cell can be of importance.

The third research question was: *"How does different degrees of self-sufficiency affect the model?"*. By increasing self-sufficiency the cost increase in a piece-wise linear pattern. The system favoured to increase electrolyzer and fuel cell capacities to begin with until it fully utilized the hydrogen capacity. Then it increased battery capacity because of its better efficiency to later linearly increase hydrogen storage capacity to make better use of electricity peaks. For the last percent of possible self-sufficiency the cost increases exponentially together with battery capacity, the fuel cell falls off until the very end where it comes back a bit due to the self-discharge of the battery storage.

The fourth and last research question was: *"How will the stochastic demand and supply affect the model?"*. The stochastic demand and supply increased the complexity of the model and significantly increased run time depending on how accurate the result sought to be. The stochastic electricity price turned out to be a major factor regarding battery capacity, the more volatile electricity price the higher the battery capacity. A more even spread of hydrogen demand also turned out to affect hydrogen storage and electrolyzer capacity. Using averages and the EV does not capture the intermittent nature of electricity prices and renewable energy production resulting in an on average more expensive solution than a solution that treats every sample individually.

8 Future Studies

This chapter presents potential future advancements of this thesis.

Something that this thesis did not include was the optimal capacities of renewable energy generation. Future studies could expand the model to include optimal capacities of PV solar panels and wind turbines. Another thing to investigate is how to create more accurate samples that can be applied to the SAA. Currently the samples are derived from distributions resulting in about the same average as the original data but more volatile. Hence an interest exist to create more accurate samples following a more natural pattern to see how the results from the SAA would respond.

One of the major findings from the literature was that the inclusion of a heat storage combined with utilizing the excess electrolyzer/fuel cell heat could significantly increase self-sufficiency. By including this in the model a more efficient system might be achievable. The inclusion of a thermal storage will however further increase the complexity of the model, which might require approximations such as the benders algorithm.

References

Articles

- Anthony Jnr, B., Abbas Petersen, S., Ahlers, D., & Krogstie, J. (2020). Api deployment for big data management towards sustainable energy prosumption in smart cities-a layered architecture perspective. *International Journal of Sustainable Energy*, 39(3), 263–289.
- Barbour, E., & González, M. C. (2018). Projecting battery adoption in the prosumer era. *Applied energy*, 215, 356–370.
- Beale, E. M. (1955). On minimizing a convex function subject to linear inequalities. *Journal of the Royal Statistical Society: Series B (Methodological)*, 17(2), 173–184.
- Bellekom, S., Arentsen, M., & van Gorkum, K. (2016). Prosumption and the distribution and supply of electricity. *Energy, sustainability and society*, 6(1), 1–17.
- Brown, D., Hall, S., & Davis, M. E. (2019). Prosumers in the post subsidy era: An exploration of new prosumer business models in the uk. *Energy Policy*, 135, 110984.
- Campana, P. E., Cioccolanti, L., François, B., Jurasz, J., Zhang, Y., Varini, M., Stridh, B., & Yan, J. (2021). Li-ion batteries for peak shaving, price arbitrage, and photovoltaic self-consumption in commercial buildings: A monte carlo analysis. *Energy Conversion and Management*, 234, 113889.
- Carmo, M., Fritz, D. L., Mergel, J., & Stolten, D. (2013). A comprehensive review on pem water electrolysis. *International journal of hydrogen energy*, 38(12), 4901–4934.
- Cristea, M., Cristea, C., Tirnovan, R.-A., & Fagarasan, C. (2020). Optimal sizing of electrical energy storage system for a household with a grid-connected pv system using inventory model. *2020 International Conference and Exposition on Electrical And Power Engineering (EPE)*, 067–071.
- Dafalla, Y., Liu, B., Hahn, D. A., Wu, H., Ahmadi, R., & Bardas, A. G. (2020). Prosumer nanogrids: A cybersecurity assessment. *IEEE Access*, 8, 131150–131164.
- Dantzig, G. B. (1955). Linear programming under uncertainty. *Management science*, 1(3-4), 197–206.
- Ellsworth-Krebs, K., & Reid, L. (2016). Conceptualising energy prosumption: Exploring energy production, consumption and microgeneration in scotland, uk. *Environment and Planning A: Economy and Space*, 48(10), 1988–2005.
- Ferreira, H. L., Garde, R., Fulli, G., Kling, W., & Lopes, J. P. (2013). Characterisation of electrical energy storage technologies. *Energy*, 53, 288–298.
- Goop, J., Nyholm, E., Odenberger, M., & Johnsson, F. (2021). Impact of electricity market feedback on investments in solar photovoltaic and battery systems in swedish single-family dwellings. *Renewable Energy*, 163, 1078–1091.
- Hahnel, U. J., Herberz, M., Pena-Bello, A., Parra, D., & Brosch, T. (2020). Becoming prosumer: Revealing trading preferences and decision-making strategies in peer-to-peer energy communities. *Energy Policy*, 137, 111098.

- Han, X., Garrison, J., & Hug, G. (2022). Techno-economic analysis of pv-battery systems in switzerland. *Renewable and Sustainable Energy Reviews*, 158, 112028.
- Inês, C., Guilherme, P. L., Esther, M.-G., Swantje, G., Stephen, H., & Lars, H. (2020). Regulatory challenges and opportunities for collective renewable energy prosumers in the eu. *Energy Policy*, 138, 111212.
- Khiareddine, A., Salah, C. B., Rekioua, D., & Mimouni, M. F. (2018). Sizing methodology for hybrid photovoltaic/wind/hydrogen/battery integrated to energy management strategy for pumping system. *Energy*, 153, 743–762.
- Kleywegt, A. J., Shapiro, A., & Homem-de-Mello, T. (2002). The sample average approximation method for stochastic discrete optimization. *SIAM Journal on Optimization*, 12(2), 479–502.
- Kubli, M., Looock, M., & Wüstenhagen, R. (2018). The flexible prosumer: Measuring the willingness to co-create distributed flexibility. *Energy policy*, 114, 540–548.
- Kumar, S. S., & Himabindu, V. (2019). Hydrogen production by pem water electrolysis—a review. *Materials Science for Energy Technologies*, 2(3), 442–454.
- Lacko, R., Drobnič, B., Mori, M., Sekavčnik, M., & Vidmar, M. (2014). Stand-alone renewable combined heat and power system with hydrogen technologies for household application. *Energy*, 77, 164–170.
- Luo, X., Wang, J., Dooner, M., & Clarke, J. (2015). Overview of current development in electrical energy storage technologies and the application potential in power system operation. *Applied energy*, 137, 511–536.
- Luo, Y., Shi, Y., & Cai, N. (2021). Bridging a bi-directional connection between electricity and fuels in hybrid multienergy systems. *Hybrid systems and multi-energy networks for the future energy internet*. Elsevier, 41e84.
- Møller, K. T., Jensen, T. R., Akiba, E., & Li, H.-w. (2017). Hydrogen-a sustainable energy carrier. *Progress in Natural Science: Materials International*, 27(1), 34–40.
- Nguyen, D. H., & Chen, H. (2019). Optimization of a perishable inventory system with both stochastic demand and supply: Comparison of two scenario approaches. *Croatian Operational Research Review*, 10(1), 175–185.
- Nyholm, E., Goop, J., Odenberger, M., & Johnsson, F. (2016). Solar photovoltaic-battery systems in swedish households—self-consumption and self-sufficiency. *Applied energy*, 183, 148–159.
- Parra, D., Gillott, M., & Walker, G. S. (2016). Design, testing and evaluation of a community hydrogen storage system for end user applications. *International Journal of Hydrogen Energy*, 41(10), 5215–5229.
- Parra, D., Valverde, L., Pino, F. J., & Patel, M. K. (2019). A review on the role, cost and value of hydrogen energy systems for deep decarbonisation. *Renewable and Sustainable Energy Reviews*, 101, 279–294.
- Puranen, P., Kosonen, A., & Ahola, J. (2021a). Technical feasibility evaluation of a solar pv based off-grid domestic energy system with battery and hydrogen energy storage in northern climates. *Solar Energy*, 213, 246–259.

- Puranen, P., Kosonen, A., & Ahola, J. (2021b). Techno-economic viability of energy storage concepts combined with a residential solar photovoltaic system: A case study from Finland. *Applied Energy*, 298, 117199.
- Reddy, S. S., Sandeep, V., & Jung, C.-M. (2017). Review of stochastic optimization methods for smart grid. *Frontiers in Energy*, 11(2), 197–209.
- Sahinidis, N. V. (2004). Optimization under uncertainty: State-of-the-art and opportunities. *Computers & Chemical Engineering*, 28(6-7), 971–983.
- Santoso, T., Ahmed, S., Goetschalckx, M., & Shapiro, A. (2005). A stochastic programming approach for supply chain network design under uncertainty. *European Journal of Operational Research*, 167(1), 96–115.
- Schneider, M., Biel, K., Pfaller, S., Schaede, H., Rinderknecht, S., & Glock, C. H. (2016). Using inventory models for sizing energy storage systems: An interdisciplinary approach. *Journal of Energy Storage*, 8, 339–348.
- Sossan, F., Namor, E., Cherkaoui, R., & Paolone, M. (2016). Achieving the dispatchability of distribution feeders through prosumers data driven forecasting and model predictive control of electrochemical storage. *IEEE Transactions on Sustainable Energy*, 7(4), 1762–1777.
- Taner, T. (2018). Energy and exergy analyze of PEM fuel cell: A case study of modeling and simulations. *Energy*, 143, 284–294.
- Wang, Z., Gu, C., Li, F., Bale, P., & Sun, H. (2013). Active demand response using shared energy storage for household energy management. *IEEE Transactions on Smart Grid*, 4(4), 1888–1897.
- Zhang, Y., Campana, P. E., Lundblad, A., & Yan, J. (2017). Comparative study of hydrogen storage and battery storage in grid connected photovoltaic system: Storage sizing and rule-based operation. *Applied Energy*, 201, 397–411.
- Zou, B., Peng, J., Li, S., Li, Y., Yan, J., & Yang, H. (2022). Comparative study of the dynamic programming-based and rule-based operation strategies for grid-connected PV-battery systems of office buildings. *Applied Energy*, 305, 117875.

Books

- Axsäter, S. (2015). *Inventory control* (Vol. 225). Springer.
- Birge, J. R., & Louveaux, F. (2011). *Introduction to stochastic programming*. Springer Science & Business Media.
- Ghiani, G., Laporte, G., & Musmanno, R. (2004). *Introduction to logistics systems planning and control*. John Wiley & Sons.
- Saunders, M., Lewis, P., & Thornhill, A. (2007). *Research methods for business students*. Pearson education.
- Toffler, A., & Alvin, T. (1980). *The third wave* (Vol. 484). Bantam books New York.

Other

- Aurecon. (2020). Hornsdale power reserve—year 2 technical and market impact case study. Retrieved February 28, 2022, from <https://hornsdalepowerreserve.com.au/wp-content/uploads/2020/07/Aurecon-Hornsdale-Power-Reserve-Impact-Study-year-2.pdf>
- Couture, T., Barbose, G., Jacobs, D., Parkinson, G., Chessin, E., Belden, A., Wilson, H., Barrett, H., & Rickerson, W. (2014). *Residential prosumers: Drivers and policy options (re-prosumers)* (tech. rep.). Meister Consultants Group; Lawrence Berkeley National Lab.(LBNL), Berkeley.
- Energicentrum Gotland. (2021). Austerland energi – ny energi till östergarnslandet. Retrieved March 15, 2022, from <https://energicentrum.gotland.se/projekt/austerland-energi/>
- Energimyndigheten. (2022). Kvartalsvis energibalans. Retrieved April 12, 2022, from <https://www.energimyndigheten.se/statistik/den-officiella-statistiken/statistikprodukter/kvartalsvis-energibalans>
- Energy Systems and Energy Storage Lab. (2020). Energy storage technologies: Hydrogen. Retrieved January 18, 2022, from www.eseslab.com/ESsensePages/Hydrogen-page
- Gährs, S., Pfeifer, L., Naber, N., Doracic, B., Knoefel, J., Hinsch, A., Assalini, S., van der Veen, R., Ljubas, D., & Lulic, Z. (2020). *Key technical findings and recommendations for prosumer communities (proseu-prosumers for the energy union: Mainstreaming active participation of citizens in the energy transition. iöw, germany* (tech. rep.). Deliverable.
- Kamiya, G., Hassid, S., & Gonzalez, P. (2021). Iea tracking report - energy storage. Retrieved February 28, 2022, from <https://www.iea.org/reports/energy-storage>
- Mimer. (2022). Mimer förbrukningsprofiler. Retrieved March 21, 2022, from <https://mimer.svk.se/ConsumptionProfile/ConsumptionProfileIndex>
- Nordpool. (2022). System price curve data. Retrieved March 15, 2022, from www.nordpoolgroup.com/elspot-price-curves/
- Nygarn Utveckling AB. (2022). Austerland skags – ny teknik och lokal samverkan för smart energiomställning. Retrieved February 22, 2022, from <https://nygarn.se/austerland-skags/>
- SMHI. (2022). Smhi data. Retrieved March 16, 2022, from <https://www.smhi.se/data>
- Svenska Kraftnät. (2022). Kontrollrummet. Retrieved January 17, 2022, from www.svk.se/om-kraftsystemet/kontrollrummet/
- US Department of Energy. (2011). Comparison of fuel cell technologies. Retrieved March 10, 2022, from https://www1.eere.energy.gov/hydrogenandfuelcells/fuelcells/pdfs/fc_comparison_chart.pdf

A Deterministic Python Model

```

1  # -*- coding: utf-8 -*-
2  """
3  Created on Wed Feb  9 08:33:54 2022
4
5  @author: karlj
6  """
7  import math
8  import os
9  import xlrd
10 import xlwt
11 from gurobipy import *
12 from scipy.stats import norm
13
14 book = xlrd.open_workbook(os.path.join("DataFinal.xlsx"))
15 T = 8789 #Time horizon for the model (197 = one week)
16 M = T//720 #Months that the model will run for
17 Start = 5
18 tm = [0,744,1440,2184,2904,3648,4368,5112,5856,6576,7320,8040,8784]
19         #Cumulative hours per month
20 DistH
21     =[0.124,0.133,0.107,0.082,0.057,0.05,0.044,0.037,0.06,0.084,0.107,0.115]
22 sh = book.sheet_by_name("Electricity")
23 sh2 = book.sheet_by_name("Hydrogen")
24 Dates = []
25 DateTime = []
26 Months = []
27 i = 0
28 while i <= M:
29     Months.append(i)
30     i+=1
31 Ds = {} #Electricity demand for skags
32 Dh = {} #Electricity demand households
33 Dn = {} #Electricity demand Nyhagen
34 Dheat = {} #Heat demand for the system
35 Hd = {} #Total hydrogen demand
36 PR = {}
37 CEH = {} #Electricity price + usage fee
38 CRE = {} #Electricity sell price (same as electricity price in data file)
39 HydrogenDemand = [0] #Daily hydrogen demand every month
40 TotHeat = sh2.cell_value(0, 3)*0.261307
41 i = Start
42 j = Start
43 DateTimeExtended = [i-5]
44 while i<=T:
45     try:
46         Dates.append(sh.cell_value(i,1))
47
48         DateTime.append(sh.cell_value(i,0))
49         DateTimeExtended.append(sh.cell_value(i,0))
50
51         c = sh.cell_value(i, 3)
52         Ds[sh.cell_value(i,0)]=c
53
54         c = sh.cell_value(i, 4)
55         Dn[sh.cell_value(i,0)]=c
56
57         c = sh.cell_value(i, 5)

```

```

57         Dh[sh.cell_value(i,0)]=c
58
59         c = sh.cell_value(i, 6)
60         d = sh.cell_value(i, 7)
61         PR[sh.cell_value(i,0)]=c+d
62
63         c = sh.cell_value(i, 8)
64         d = sh.cell_value(i, 9)
65         CRE[sh.cell_value(i,0)]=c/1000
66         CEH[sh.cell_value(i,0)]=d+c/1000
67
68 #The following code is to ensure that the hydrogen demand occurs once a day
69     if i > tm[j-Start+1]+Start: #Swaps months
70         j+=1
71         c = sh2.cell_value(j, 11) #11 is in kg 14 in kWh
72         Hd[sh.cell_value(i,0)]=c*33.6/24
73
74         Dheat[sh.cell_value(i,0)]=TotHeat*DistH[j-Start]/(tm[j-Start+1]-tm[j
-Start])
75         i+=1
76
77     except IndexError:
78         break
79
80 DisDay = 0.002 #Daily self-discharge of batteries
81 Blife = 15 #Life time of batteries
82 CB = 5500 #Investment costs of batteries per kWh
83 Dis = 10**((math.log(1-DisDay,10)/24) #Self discharge rate for lithium ion
    batteries per hour
84 Beff = 0.94 # Charge/discharge efficiencies of lithium ion batteries
85 COB = 0.001 #Annual operation costs for batteries
86 Tc = 0.36 #Electricity taxes for companies
87 Ts = 0 #Electricity taxes for self consumed electricity
88 DoD = 0.8 #Depth of discharge
89 Ccf = 1450/15 #Connection fee for scaling the electricity grid
90 CP = 35 #Power fee
91 C = 0.5 #Charge/discharge rate of the battery
92
93 Llife = 15 #Life time of electrolysis
94 Slife = 15 #Life time of hydrogen storage
95 Flife = 15 #Life time of fuel cell
96 CL = 5000 #Investment costs of electrolyzer per kW
97 CS = 240 #Investment costs of hydrogen storage per kWh
98 CF = 5000 #Investment costs of fuel cell per kW
99 COL = 0.005 #Annual operation costs for electrolyzer
100 COS = 0.005 #Annual operation costs for the hydrogen storage
101 COF = 0.005 #Annual operation costs for the fuel cell
102 Leff = 0.85 #Electrolyser efficiency
103 Feff = 0.5 #Fuel cell efficiency
104 Lheat =0.06 #Waste heat from electrolyzer
105 Fheat = 0.2 #Waste heat from fuel cell
106 ServiceLevel = 0.95
107 k = norm.ppf(ServiceLevel) #Safety factor for safety stock
108 totCost = 0
109 HydrogenVar = 0.15
110
111 selfSuff = 0.65
112
113 # Total demand/supply used to calculate self sufficiency and self
    consumption

```

```

114 TotDemandHSN = sh.cell_value(0,3)+sh.cell_value(0,4)+sh.cell_value(0,5)
115 TotDemandHydrogen = sum(Hd.values())
116 TotProduced = sum(PR.values())
117
118 try:
119     # Create a new model
120     m = Model("StorageOptimization")
121
122     # Create variables
123     # Bcap is total battery capacity
124     Bcap = m.addVar(lb=0, name="Bcap")
125
126     # Maximum amount of renewable energy sent to the electricity grid
127     REmax = m.addVar(lb=0, name="REmax")
128
129     # Emax is the maximum amount of electricity from the grid a specific
month
130     Emax = m.addVars(Months, lb=0, name="Emax")
131
132     # EH is the amount of electricity from the Grid to Houses
133     EH = m.addVars(DateTime, lb=0, name="EH")
134
135     # ES is the amount of electricity from the Grid to Skags
136     ES = m.addVars(DateTime, lb=0, name="ES")
137
138     # EN is the amount of electricity from the Grid to Nyhagen
139     EN = m.addVars(DateTime, lb=0, name="EN")
140
141     # RE is the amount of electricity from the Renewables to Grid
142     RE = m.addVars(DateTime, lb=0, name="RE")
143
144     # RH is the amount of electricity from the Renewables to Households
145     RH = m.addVars(DateTime, lb=0, name="RH")
146
147     # RS is the amount of electricity from the Renewables to Skags
148     RS = m.addVars(DateTime, lb=0, name="RS")
149
150     # RN is the amount of electricity from the Renewables to Nyhagen
151     RN = m.addVars(DateTime, lb=0, name="RN")
152
153     # RB is the amount of electricity from the Renewables to Batteries
154     RB = m.addVars(DateTime, lb=0, name="RB")
155
156     # BH is the amount of electricity from the Batteries to Houses
157     BH = m.addVars(DateTime, lb=0, name="BH")
158
159     # BS is the amount of electricity from the Batteries to Skags
160     BS = m.addVars(DateTime, lb=0, name="BS")
161
162     # BN is the amount of electricity from the Batteries to Nyhagen
163     BN = m.addVars(DateTime, lb=0, name="BN")
164
165     # SB is the amount of electricity stored in the batteries
166     SB = m.addVars(DateTimeExtended, lb=0, name="SB")
167
168     # y is an indicator if the house demand is negativ
169     y = m.addVars(DateTime, vtype=GRB.BINARY, name="y")
170
171     # HE is the amount of electricity from the Houses to the Grid
172     HE = m.addVars(DateTime, lb=0, name="HE")

```

```

173
174 # HB is the amount of electricity from the Houses to the Batteries
175 HB = m.addVars(DateTime ,lb=0, name="HB")
176
177
178 # Lcap is total electrolyzer capacity
179 Lcap = m.addVar(lb=0, name="Lcap")
180
181 # Scap is total hydrogen storage capacity
182 Scap = m.addVar(lb=0, name="Scap")
183
184 # Fcap is total fuel cell capacity
185 Fcap = m.addVar(lb=0, name="Fcap")
186
187 # SS is the amount of hydrogen stored in the hydrogen tank
188 SS = m.addVars(DateTimeExtended, lb=0, name="SS")
189
190 # RL is the amount of renewable electricity to the electrolyzer
191 RL = m.addVars(DateTime, lb=0, name="RL")
192
193 # SF is the amount of hydrogen sent to the fuel cell
194 SF = m.addVars(DateTime, lb=0, name="SF")
195
196 # FH is the amount of electricity sent from the fuel cell to households
197 FH = m.addVars(DateTime, lb=0, name="FH")
198
199 # FS is the amount of electricity sent from the fuel cell to Skags
200 FS = m.addVars(DateTime, lb=0, name="FS")
201
202 # FN is the amount of electricity sent from the fuel cell to Nyhagen
203 FN = m.addVars(DateTime, lb=0, name="FN")
204
205 # HL is the amount of electricity sent from households to the
electrolyzer
206 HL = m.addVars(DateTime, lb=0, name="HL")
207
208
209 # Added from Heat inclusion
210 # LW is the amount of utilized waste heat from the electrolyzer
211 LW = m.addVars(DateTime, lb=0, name="LW")
212 # FW is the amount of utilized waste heat from the fuel cell
213 FW = m.addVars(DateTime, lb=0, name="FW")
214
215
216 # Objective: Minimize costs
217 m.setObjective(REmax*Ccf+CB*Bcap*(COB+(1/Blife))+CL*Lcap*(COL+(1/Llife))
218               +CS*Scap*(COS+(1/Slife))+CF*Fcap*(COF+(1/Flife))+
219               sum((Tc+CEH[t])*(ES[t]+EN[t]+EH[t])+Ts*(RH[t]+RS[t]+RN[t]
220               +RB[t]+RL[t])-CRE[t]*(RE[t]+HE[t]))for t in DateTime)
221               +CP*sum(Emax[m] for m in Months), GRB.MINIMIZE)
222
223 #Constraints
224 # # c-1 Selfsufficiency restraint
225 # m.addConstr(sum(RH[t]+RS[t]+RN[t]+BH[t]+BS[t]+BN[t]+FH[t]+FS[t]+FN[t]
226 #               for t in DateTime) >= selfSuff*TotDemandHSN+
227 #               TotDemandHydrogen*(selfSuff-1))
228
229 # # c0 initial value of battery storage
230 m.addConstr(SB[DateTimeExtended[0]] == Bcap*(1-DoD), "c0")
231 #

```

```

232
233 # c1 ensures that the total battery capacity is sufficient
234 m.addConstrs((SB[t] <= Bcap) for t in DateTime)
235
236 # c2 Sets this periods battery stock level based on previous periods
237 m.addConstrs((Beff*(RB[t]+HB[t])+Dis*SB[t-1]-BH[t]-BS[t]-BN[t] == SB[t])
238               for t in DateTime)
239
240 # c3 Matches demand of households with output
241 m.addConstrs((RH[t]+BH[t]*Beff+EH[t]+FH[t]-HE[t]-HB[t]-HL[t]==Dh[t])
242               for t in DateTime)
243
244 # c4 Matches demand of skags with output
245 m.addConstrs((RS[t]+BS[t]*Beff+ES[t]+FS[t]==Ds[t]) for t in DateTime)
246
247 # c5 Matches demand of Nyhagen with output
248 m.addConstrs((RN[t]+BN[t]*Beff+EN[t]+FN[t]==Dn[t]) for t in DateTime)
249
250 # c6 Matches produced renewable energy with outputs
251 m.addConstrs((RB[t]+RH[t]+RS[t]+RN[t]+RE[t]+RL[t] == PR[t])
252               for t in DateTime)
253
254 # c7 Ensures that depth of discharge is not exceeded
255 m.addConstrs((SB[t]>=Bcap*(1-DoD)) for t in DateTime)
256
257 # c8 checks the maximum electricity grid output
258 m.addConstrs((RE[t] <= REmax) for t in DateTime)
259
260 # c9 finds the maximum electricity output from the grid per month
261 for mnt in Months[:M]:
262     for t in DateTime[tm[mnt]:tm[mnt+1]]:
263         m.addConstr(EH[t]+ES[t]+EN[t] <= Emax[mnt+1])
264
265 # c10 incase the demand is negative
266 m.addConstrs(0<=Dh[t]*(1-y[t]) for t in DateTime)
267 m.addConstrs(HE[t]+HB[t]+HL[t]==-Dh[t]*y[t] for t in DateTime)
268
269 # c11 maximum output and input from the battery system
270 m.addConstrs(BH[t]+BS[t]+BN[t] <= Bcap*C for t in DateTime)
271 m.addConstrs(RB[t]+HB[t] <= Bcap*C for t in DateTime)
272
273 # c12 ensures that the total hydrogen storage capacity is sufficient
274 m.addConstrs((SS[t] <= Scap) for t in DateTime)
275
276 # c13 ensures that the total electrolyzer capacity is sufficient
277 m.addConstrs((RL[t]+HL[t] <= Lcap) for t in DateTime)
278
279 # c14 Sets this periods hydrogen stock level based on previous periods
280 m.addConstrs((Leff*(RL[t]+HL[t])+SS[t-1]+FW[t]+LW[t]-Hd[t]-SF[t]==SS[t])
281               for t in DateTime)
282
283 # c15 Matches fuel cell output with electricity input
284 m.addConstrs((SF[t]*Feff == FH[t]+FS[t]+FN[t]) for t in DateTime)
285
286 # c16 ensures that the total fuel cell capacity is sufficient
287 m.addConstrs((SF[t] <= Fcap) for t in DateTime)
288
289 # c17 sets initial storage level of hydrogen tank Scap*(1-DoD)
290 m.addConstr(SS[DateTimeExtended[0]] == Hd[9]*24*7+k*HydrogenVar*Hd
[9]*24)

```

```

291
292 # c18 sets the safety stock for the hydrogen tank
293 for mnt in Months[:M]:
294     for t in DateTime[tm[mnt]:tm[mnt+1]]:
295         m.addConstr(SS[t] >= k*HydrogenVar*Hd[tm[mnt]+9]*24)
296
297 # c19 sets waste heat
298 m.addConstrs(FW[t] <= SF[t]*Fheat for t in DateTime)
299
300 # c20 Matches heat demand with heat output
301 m.addConstrs(FW[t]+LW[t] <= Dheat[t] for t in DateTime)
302
303 # c21 sets waste heat
304 m.addConstrs(LW[t] <= RL[t]*Lheat for t in DateTime)
305
306 m.optimize()
307
308 #This is where the writing to excel occurs
309 workbook = xlwt.Workbook()
310 sheet = workbook.add_sheet("Summary")
311 sheet1 = workbook.add_sheet("BatteryCapacity")
312 sheet2 = workbook.add_sheet("Hydrogen")
313 # Specifying style
314 style = xlwt.easyxf('font: bold 1')
315 totCost=(REmax.x*Ccf+CB*Bcap.x*(COB+(1/Blife))+CL*Lcap.x*(COL+(1/Llife))
316 +CS*Scap.x*(COS+(1/Slife))+CF*Fcap.x*(COF+(1/Flife))+sum((Tc+CEH[t])*
317 (ES[t].x+EN[t].x+EH[t].x)-CRE[t]*(RE[t].x+HE[t].x) for t in DateTime)+
318 sum(CP*Emax[m].x for m in Months))
319 #Summary sheet
320 sheet.write(0, 0, "TotalCost", style)
321 sheet.write(1, 0, totCost, style)
322 sheet.write(0, 1, "BatteryCapacity", style)
323 sheet.write(1, 1, Bcap.x, style)
324 sheet.write(0, 2, "ElectrolyzerCapacity", style)
325 sheet.write(1, 2, Lcap.x, style)
326 sheet.write(0, 3, "HydrogenTankCapacity", style)
327 sheet.write(1, 3, Scap.x, style)
328 sheet.write(0, 4, "FuelCellCapacity", style)
329 sheet.write(1, 4, Fcap.x, style)
330
331 #Sheet 1
332 sheet1.write(0, 0, 'Period', style)
333 sheet1.write(0, 1, 'BatteryLevel', style)
334 sheet1.write(0, 2, 'HydrogenLevel', style)
335
336 sheet1.write(0, 4, "Renewable_House", style)
337 sheet1.write(0, 5, "Renewable_Skags", style)
338 sheet1.write(0, 6, "Renewable_Nyhagen", style)
339 sheet1.write(0, 7, "Renewable_Batteries", style)
340 sheet1.write(0, 8, "Renewable_Grid", style)
341 sheet1.write(0, 9, "Renewable_Electrolyzer", style)
342 sheet1.write(0, 10, "Renewable_Prod", style)
343 sheet1.write(0, 11, "Grid_House", style)
344 sheet1.write(0, 12, "Battery_House", style)
345 sheet1.write(0, 13, "Battery_Skags", style)
346 sheet1.write(0, 14, "Battery_Nyhagen", style)
347 sheet1.write(0, 15, "Demand_House", style)
348 sheet1.write(0, 16, "Demand_Skags", style)
349 sheet1.write(0, 17, "Demand_Nyhagen", style)
350 sheet1.write(0, 18, "Emax", style)

```

```

351     sheet1.write(0, 19, "Grid_total", style)
352
353     sheet1.write(0, 20, "Heat_demand", style)
354     sheet1.write(0, 21, "GridToHeat", style)
355     sheet1.write(0, 22, "RenewToHeat", style)
356     sheet1.write(0, 23, "BatteryToHeat", style)
357     sheet1.write(0, 24, "FuelCellToHeat", style)
358     sheet1.write(0, 25, "WasteToHeat", style)
359     sheet1.write(0, 26, "HouseToHeat", style)
360
361     #Sheet 2
362     sheet2.write(0, 0, 'Period', style)
363     sheet2.write(0, 1, 'Renewable_Electrolyzer', style)
364     sheet2.write(0, 2, 'FuelCell_Households', style)
365     sheet2.write(0, 3, 'FuelCell_Skags', style)
366     sheet2.write(0, 4, 'FuelCell_Nyhagen', style)
367     sheet2.write(0, 5, 'Storage_FuelCell', style)
368     sheet2.write(0, 6, 'Household_Electrolyzer', style)
369     sheet2.write(0, 7, 'HydrogenDemand', style)
370     sheet2.write(0, 8, 'HydrogenLevel', style)
371
372     RenewHSN = 0
373     BatteryHSN = 0
374     FuelCellHSN = 0
375     ToBatterySum = 0
376     ToElectrolyzerSum = 0
377     ToHeatSum = 0
378     j = 1
379     for v in DateTime:
380         sheet1.write(j, 0, Dates[j-1])
381         sheet1.write(j, 1, SB[v].x)
382         sheet1.write(j, 2, SS[v].x)
383         sheet1.write(j, 4, RH[v].x)
384         sheet1.write(j, 5, RS[v].x)
385         sheet1.write(j, 6, RN[v].x)
386         sheet1.write(j, 7, RB[v].x)
387         sheet1.write(j, 8, RE[v].x)
388         sheet1.write(j, 9, RL[v].x)
389         sheet1.write(j, 10, PR[j])
390         sheet1.write(j, 11, EH[v].x)
391         sheet1.write(j, 12, BH[v].x)
392         sheet1.write(j, 13, BS[v].x)
393         sheet1.write(j, 14, BN[v].x)
394         sheet1.write(j, 15, Dh[j])
395         sheet1.write(j, 16, Ds[j])
396         sheet1.write(j, 17, Dn[j])
397         sheet1.write(j, 19, EH[v].x+ES[v].x+EN[v].x)
398
399         sheet1.write(j, 20, Dheat[j])
400         sheet1.write(j, 25, FW[v].x)
401         sheet1.write(j, 26, LW[v].x)
402
403         sheet2.write(j, 0, Dates[j-1])
404         sheet2.write(j, 1, RL[v].x)
405         sheet2.write(j, 2, FH[v].x)
406         sheet2.write(j, 3, FS[v].x)
407         sheet2.write(j, 4, FN[v].x)
408         sheet2.write(j, 5, SF[v].x)
409         sheet2.write(j, 6, HL[v].x)
410         sheet2.write(j, 7, Hd[j])

```

```

411         sheet2.write(j, 8, SS[v].x)
412
413         RenewHSN += RH[v].x+RS[v].x+RN[v].x
414         BatteryHSN += BH[v].x+BS[v].x+BN[v].x
415         FuelCellHSN += FH[v].x+FS[v].x+FN[v].x+FW[v].x
416         ToBatterySum += RB[v].x
417         ToElectrolyzerSum += RL[v].x
418         ToHeatSum += FW[v].x+LW[v].x
419         j += 1
420     j = 1
421     for v in Months:
422         sheet1.write(j, 18, Emax[v].x)
423         j += 1
424     #Writing to excel occurs here because of previous calculated sums
425     SelfSuff = ((RenewHSN+BatteryHSN+FuelCellHSN+ToHeatSum+TotDemandHydrogen
426                 -(SS[0].x))/(TotDemandHydrogen+TotDemandHSN))
427     SelfCons = (RenewHSN+ToBatterySum+ToElectrolyzerSum)/(TotProduced)
428     sheet.write(0, 5, "SelfSufficiency", style)
429     sheet.write(1, 5, SelfSuff, style)
430     sheet.write(0, 6, "SelfCounsumption", style)
431     sheet.write(1, 6, SelfCons, style)
432     workbook.save("HeatTestElectro.xls")
433
434 except GurobiError as e:
435     print('Error code ' + str(e.errno) + ': ' + str(e))

```

B Stochastic Python Model

```

1  # -*- coding: utf-8 -*-
2  """
3  Created on Sat Mar 19 14:21:26 2022
4
5  @author: karlj
6  """
7  import os
8  import xlrd
9  import xlwt
10 import numpy
11 import math
12 import gc
13 import random
14 from gurobipy import *
15 from scipy.stats import norm
16
17 def main(N, its, prim):
18     book = xlrd.open_workbook(os.path.join("DataFinal.xlsx"))
19     T = 8789 #Time horizon for the model (197 = one week)
20     M = T//720 #Months that the model will run for
21     Start = 5
22     #Cumulative hours per month
23     tm = [0,744,1440,2184,2904,3648,4368,5112,5856,6576,7320,8040,8784]
24     DistH
25     =[0.124,0.133,0.107,0.082,0.057,0.05,0.044,0.037,0.06,0.084,0.107,0.115]
26     sh = book.sheet_by_name("Electricity")
27     sh2 = book.sheet_by_name("Hydrogen")
28     DateTime = []
29     Months = []
30     i = 0
31     while i <= M:
32         Months.append(i)
33         i+=1
34     Ds = {} #Electricity demand for skags
35     Dh = {} #Electricity demand households
36     Dn = {} #Electricity demand Nyhagen
37     Hd = {} #Total hydrogen demand
38     PS = {} #Produced solar pwoer
39     PW = {} #Produced wind power
40     CRE = {} #Electricity sell price
41     CEH = {} #Additional electricity fee
42     Dheat = {} #Heat demand for the system
43     TotHeat = sh2.cell_value(0, 3)*0.261307
44
45     i = Start
46     j = Start
47
48     while i<=T:
49         try:
50             DateTime.append(sh.cell_value(i,0))
51
52             c = sh.cell_value(i, 3)
53             d = sh.cell_value(i, 4)
54             e = sh.cell_value(i, 5)
55             Ds[sh.cell_value(i,0)]=c+d+e
56

```

```

57         Dn[sh.cell_value(i,0)]=c
58
59
60         Dh[sh.cell_value(i,0)]=c
61
62         c = sh.cell_value(i, 6)
63         d = sh.cell_value(i, 7)
64         PS[sh.cell_value(i,0)]=c
65         PW[sh.cell_value(i,0)]=d
66
67         c = sh.cell_value(i, 8)
68         d = sh.cell_value(i, 9)
69         CRE[sh.cell_value(i,0)]=c
70         CEH[sh.cell_value(i,0)]=d
71
72         #Following is because hydrogen demand shifts depending on month
73         if i > tm[j-Start+1]+Start: #Swaps months
74             j+=1 #j points to current motnh
75
76         c = sh2.cell_value(j, 11) #11 is in kg 14 in kWh
77         Hd[sh.cell_value(i,0)]=c*33.6/24
78         Dheat[sh.cell_value(i,0)]=TotHeat*DistH[j-Start]/(tm[j-Start+1]-
tm[j-Start])
79         i+=1
80
81     except IndexError:
82         break
83
84     xlData = [Dh, Ds, Dn, PS, PW, CRE, CEH]
85     Data = [DateTime, Months, Hd, Dheat]
86     SAA(xlData, Data, N, its, prim)
87
88 #SAA is the sample average approximation algoritm
89 def SAA(xlData, Data, N, M, prim):
90     runs = [] #Saves the optimization objects
91     xOpt = 0 #The best found objective function value thus far
92     OptSol = []
93     opts = []
94     primSample = caseData(N+prim, xlData)
95     indexOpt = 0
96     for i in range(M):
97         Samples = caseData(N, xlData)
98         runs.append(OPT(Samples, Data))
99         runs[i].optimize(0)
100        runs[i].updateSamples(primSample)
101        runs[i].optimize(1)
102        opts.append(runs[i].getOptimum())
103        if len(runs) == 1:
104            xOpt = runs[i].getOptimumPrim()
105            OptSol = runs[i].getSolution()
106        elif runs[i].getOptimumPrim()<xOpt:
107            xOpt = runs[i].getOptimumPrim()
108            OptSol = runs[i].getSolution()
109            indexOpt = i
110    write(primSample)
111    Znm = sum(runs[i].getOptimum() for i in range(M))/M
112    gap = runs[indexOpt].getOptimumPrim()-Znm
113    var = runs[indexOpt].getVariance()
114    del Samples, primSample
115    gc.collect()

```

```

116     varN = sum((var[i]**2 for i in range(N+prim)))/((N+prim)*(N+prim-1))
117     varZnm = sum((opts[i]-Znm)**2 for i in range(M))/(M*(M-1))
118     varGap = varN + varZnm
119
120     workbook = xlwt.Workbook()
121     sheet = workbook.add_sheet("Summary")
122     # Specifying style
123     style = xlwt.easyxf('font: bold 1')
124
125     #Summary sheet
126     sheet.write(0, 0, "TotalCost", style)
127     sheet.write(1, 0, xOpt, style)
128     sheet.write(0, 1, "BatteryCapacity", style)
129     sheet.write(1, 1, OptSol[0], style)
130     sheet.write(0, 2, "ElectrolyzerCapacity", style)
131     sheet.write(1, 2, OptSol[2], style)
132     sheet.write(0, 3, "HydrogenTankCapacity", style)
133     sheet.write(1, 3, OptSol[3], style)
134     sheet.write(0, 4, "FuelCellCapacity", style)
135     sheet.write(1, 4, OptSol[4], style)
136     sheet.write(0, 5, "Gap", style)
137     sheet.write(1, 5, gap, style)
138     sheet.write(0, 6, "VariancesN", style)
139     sheet.write(1, 6, varN, style)
140     sheet.write(0, 7, "VariancesZnm", style)
141     sheet.write(1, 7, varZnm, style)
142     sheet.write(0, 8, "Variances", style)
143     sheet.write(1, 8, varGap, style)
144     workbook.save("StochasticTest.xls")
145
146 def caseData(N, xlData):
147     Samples = []
148     j = 0
149     while j < N:
150         j +=1
151         SDs = {} #New demand dictionary for the sample
152         SPS = {} #New solar production dictionary for the sample
153         SPW = {} #New wind production dictionary for the sample
154         SPR = {} #New total renewable production dictionary for the sample
155         SCORE = {} #New el spot dictionary for the sample
156         SCEH = {} #Electricity price + usage fee
157
158         SDs = normalInsert(xlData[1], SDs, 14,0) #Demand sample 0
159         SPS = normalInsert(xlData[3], SPS, 6, 2) #Solar power sample 2
160         SPW = normalInsert(xlData[4], SPW, 2, 1) #Wind power sample 1
161         SCORE = normalInsert(xlData[5], SCORE, 10,0) #Elspot sample 0
162         for k in xlData[0].keys():
163             try:
164                 d = xlData[6][k]
165                 c = SCORE[k]
166                 SCORE[k] = c/1000
167                 SCEH[k]=d+c/1000
168
169                 a = SPW[k]
170                 b = SPS[k]
171                 SPR[k]=a+b
172             except IndexError:
173                 break
174         samp=[SDs, SPR, SCORE, SCEH]
175         Samples.append(samp)

```

```

176     return Samples
177
178 def normalInsert(mean, samp, start, dist):
179     #dist = 0 Normal, 1 = Lognormal, 2 = Uniform, 3 = Normal
180     j = 2 #Excel dokument data starts at 3rd row
181     book = xlrd.open_workbook(os.path.join("Data_AVE_STD.xlsx"))
182     sh = book.sheet_by_name("Sheet1")
183     sh2 = book.sheet_by_name("Sheet2")
184     for i in mean.keys():
185         std = sh.cell_value(j,start)
186         if dist == 0:
187             data = numpy.random.normal(mean[i],abs(mean[i]*std), size=None)
188         elif dist == 1:
189             data = mean[i]*numpy.random.lognormal(0, std/10, size=None)
190         elif dist == 2:
191             if j < 8041:
192                 data = random.choice(sh2.row_values(j-2,0,14))*2.62
193             else:
194                 data = random.choice(sh2.row_values(j-2,0,13))*2.62
195         if data < 0 and mean[i] > 0:
196             data = 0
197         samp[i] = data
198         j +=1
199     return samp
200
201 class OPT:
202     def __init__(self, Samples, Data):
203         self.Samples = Samples
204         self.Data = Data
205         self.Optimum = 0
206         self.OptimumPrim = 0
207         self.variance = 0
208         self.Solution = []
209
210     def optimize(self, mode):
211         N = len(self.Samples)
212         DateTime = self.Data[0]
213         ##### General data for the optimization #####
214         DisDay = 0.002 #Daily self-discharge of li-ion batteries
215         Blife = 15 #Life time of batteries
216         CB = 5500 #Investment costs of batteries per kWh
217         Dis = 10**(math.log(1-DisDay,10)/24) #Self-discharge rate per hour
218         Beff = 0.94 # Charge/discharge efficiencies of lithium ion batteries
219         COB = 0.001 #Annual operation costs for batteries
220         Tc = 0.36 #Electricity taxes for companies
221         Ts = 0 #Electricity taxes for self consumed electricity
222         DoD = 0.8 #Depth of discharge
223         Ccf = 1450/15 #Connection fee for scaling the electricity grid
224         CP = 35 #Power fee
225         C = 0.5 #Charge/discharge rate of the battery
226
227         Llife = 15 #Life time of electrolysis
228         Slife = 15 #Life time of hydrogen storage
229         Flife = 15 #Life time of fuel cell
230         CL = 5000 #Investment costs of electrolyzer per kW
231         CS = 240 #Investment costs of hydrogen storage per kWh
232         CF = 5000 #Investment costs of fuel cell per kW
233         COL = 0.005 #Annual operation costs for electrolyzer
234         COS = 0.005 #Annual operation costs for the hydrogen storage
235         COF = 0.005 #Annual operation costs for the fuel cell

```

```

236     Leff = 0.85 #Electrolyser efficiency
237     Feff = 0.5 #Fuel cell efficiency
238     Lheat = 0.06 #Waste heat from electrolyzer
239     Fheat = 0.2 #Waste heat from fuel cell
240     ServiceLevel = 0.95
241     K = norm.ppf(ServiceLevel) #Safety factor for safety stock
242     HydrogenVar = 0.15 #Assumed standard deviation of hydrogen demand
243     #Accumulated hours for each month
244     tm = [0,744,1440,2184,2904,3648,4368,5112,5856,6576,7320,8040,8784]
245     DateTimeExtended = [0] + self.Data[0]
246     ##### General data for the optimization #####
247
248     try:
249         # Create a new model
250         m = Model("StorageOptimization")
251         # Create variables
252         if mode == 1:
253             # Bcap is total battery capacity
254             Bcap = self.Solution[0]
255
256             # Max amount of renewable energy sent to electricity grid
257             REmax = self.Solution[1]
258
259             # Lcap: total electrolyzer capacity
260             Lcap = self.Solution[2]
261
262             # Scap: total hydrogen storage capacity
263             Scap = self.Solution[3]
264
265             # Fcap: total fuel cell capacity
266             Fcap = self.Solution[4]
267
268         else:
269             # Bcap is total battery capacity
270             Bcap = m.addVar(lb=0, name="Bcap")
271
272             # Lcap: total electrolyzer capacity
273             Lcap = m.addVar(lb=0, name="Lcap")
274
275             # Scap: total hydrogen storage capacity
276             Scap = m.addVar(lb=0, name="Scap")
277
278             # Fcap: total fuel cell capacity
279             Fcap = m.addVar(lb=0, name="Fcap")
280
281             # Max amount of renewable energy sent to electricity grid
282             REmax = m.addVar(lb=0, name="REmax")
283
284             # Emax: max electricity output from the grid a specific month
285             Emax = m.addVars(self.Data[1], lb=0, name="Emax")
286
287             # ES: amount of electricity from the Grid to Skags
288             ES = m.addVars(DateTime, N, lb=0, name="ES")
289
290             # RE: amount of electricity from the Renewables to Grid
291             RE = m.addVars(DateTime, N, lb=0, name="RE")
292
293             # RS: amount of electricity from the Renewables to Skags
294             RS = m.addVars(DateTime, N, lb=0, name="RS")
295

```

```

296     # RB: amount of electricity from the Renewables to Batteries
297     RB = m.addVars(DateTime, N, lb=0, name="RB")
298
299     # BS: amount of electricity from the Batteries to Skags
300     BS = m.addVars(DateTime, N, lb=0, name="BS")
301
302     # SB: amount of electricity stored in the batteries
303     SB = m.addVars(DateTimeExtended, N, lb=0, name="SB")
304
305     # SS: amount of hydrogen stored in the hydrogen tank
306     SS = m.addVars(DateTimeExtended, N, lb=0, name="SS")
307
308     # RL: amount of renewable electricity to the electrolyzer
309     RL = m.addVars(DateTime, N, lb=0, name="RL")
310
311     # SF: amount of hydrogen sent to the fuel cell
312     SF = m.addVars(DateTime, N, lb=0, name="SF")
313
314     # FS: amount of electricity sent from the fuel cell to Skags
315     FS = m.addVars(DateTime, N, lb=0, name="FS")
316
317     # LW is the amount of utilized waste heat from the electrolyzer
318     LW = m.addVars(DateTime, N, lb=0, name="LW")
319     # FW is the amount of utilized waste heat from the fuel cell
320     FW = m.addVars(DateTime, N, lb=0, name="FW")
321
322     # Slack variables for hydrogen storage and electrolyzer
323     slack = m.addVar(lb=0, name="slack")
324     slack2 = m.addVar(lb=0, name="slack2")
325
326     # Objective: Minimize costs
327     m.setObjective(REmax*Ccf+CB*Bcap*(COB+1/Blife)+CL*(Lcap+slack2)
328                  *(COL+1/Llife)+CS*(Scap+slack)*(COS+1/Slife)+
329                  CF*Fcap*(COF+1/Flife)+sum(CP*Emax[m]
330                  for m in self.Data[1])+(1/N)*sum((Tc+
331                  self.Samples[k][3][t])*(ES[t,k])+Ts*(RS[t,k]+
332                  RB[t,k]+RL[t,k])-self.Samples[k][2][t]*
333                  RE[t,k] for t in DateTime
334                  for k in range(N)), GRB.MINIMIZE)
335
336     #Constraints
337     # c0 initial value of battery storage
338     m.addConstrs((SB[DateTimeExtended[0],k] == Bcap*(1-DoD))
339                  for k in range(N))
340
341     # c1 ensures that the total battery capacity is sufficient
342     m.addConstrs((SB[t,k] <= Bcap) for t in DateTime
343                  for k in range(N))
344
345     # c2 Sets this periods battery level based on previous period
346     m.addConstrs((Beff*(RB[t,k])+Dis*SB[t-1,k]-BS[t,k] == SB[t,k])
347                  for t in DateTime for k in range(N))
348
349     # c4 Matches demand of skags with output
350     m.addConstrs((RS[t,k]+BS[t,k]*Beff+ES[t,k]+FS[t,k] ==
351                  self.Samples[k][0][t]) for t in DateTime
352                  for k in range(N))
353
354     # c6 Matches produced renewable energy with outputs
355     m.addConstrs((RB[t,k]+RS[t,k]+RE[t,k]+RL[t,k] ==

```

```

356         self.Samples[k][1][t])for t in DateTime
357             for k in range(N))
358
359 # c7 Ensures that depth of discharge is not exceeded
360 m.addConstrs((SB[t,k]>=Bcap*(1-DoD))for t in DateTime
361             for k in range(N))
362
363 # c8 checks the maximum electricity grid input
364 m.addConstrs((RE[t,k] <= REmax) for t in DateTime
365             for k in range(N))
366
367 # c9 finds maximum electricity output from the grid per month
368 for mnt in self.Data[1][:len(self.Data[1])-1]:
369     for t in DateTime[tm[mnt]:tm[mnt+1]]:
370         m.addConstrs(ES[t,k] <= Emax[mnt+1] for k in range(N))
371
372
373 # c11 maximum output and input from the battery system
374 m.addConstrs(BS[t,k] <= Bcap*C for t in DateTime
375             for k in range(N))
376 m.addConstrs(RB[t,k] <= Bcap*C for t in DateTime
377             for k in range(N))
378
379 if mode == 1: #Allows slack variables for step 2.2
380     # c12 ensures total hydrogen storage capacity is sufficient
381     m.addConstrs((SS[t,k] <= Scap+slack) for t in DateTime
382             for k in range(N))
383
384     # c13 ensures that total electrolyzer capacity is sufficient
385     m.addConstrs((RL[t,k] <= Lcap+slack2) for t in DateTime
386             for k in range(N))
387
388 else: #Normal case
389     # c12 ensures total hydrogen storage capacity is sufficient
390     m.addConstrs((SS[t,k] <= Scap) for t in DateTime
391             for k in range(N))
392
393     # c13 ensures that total electrolyzer capacity is sufficient
394     m.addConstrs((RL[t,k] <= Lcap) for t in DateTime
395             for k in range(N))
396
397 # c14 Sets this periods hydrogen level based on previous periods
398 m.addConstrs(((Leff*(RL[t,k])+SS[t-1,k]+LW[t,k]+FW[t,k]-
399             self.Data[2][t]-SF[t,k] == SS[t,k])
400             for t in DateTime for k in range(N))
401
402 # c15 Matches fuel cell output with electricity input
403 m.addConstrs((SF[t,k]*Feff == FS[t,k]) for t in DateTime
404             for k in range(N))
405
406 # c16 ensures that the total fuel cell capacity is sufficient
407 m.addConstrs((SF[t,k] <= Fcap) for t in DateTime
408             for k in range(N))
409
410 # c17 sets initial storage level of hydrogen tank
411 m.addConstrs((SS[DateTimeExtended[0],k] == self.Data[2][9]*24*7+
412             24*K*HydrogenVar*self.Data[2][9]) for k in range(N))
413
414 if mode == 0: #Relaxes the safety stock constraint for stage 2.2
415     # c18 sets the safety stock for the hydrogen tank

```

```

416         for mnt in self.Data[1][:len(self.Data[1])-1]:
417             for t in DateTime[tm[mnt]:tm[mnt+1]]:
418                 m.addConstrs(SS[t,k] >= K*HydrogenVar*
419                             self.Data[2][tm[mnt]+9]*24
420                             for k in range(N))
421
422     # c19 sets waste heat for electrolyzer
423     m.addConstrs(LW[t,k] <= RL[t,k]*Lheat for t in DateTime
424                 for k in range(N))
425
426     # c20 sets waste heat for fuel cell
427     m.addConstrs(FW[t,k] <= SF[t,k]*Fheat for t in DateTime
428                 for k in range(N))
429
430     # c21 Makes sure that utilized waste heat does not exceed demand
431     m.addConstrs(FW[t,k]+LW[t,k] <= self.Data[3][t]
432                 for t in DateTime for k in range(N))
433
434     m.optimize()
435
436     if mode == 0: #Stage 2.1
437         self.Optimum = m.objVal
438         self.Solution = [Bcap.x, REmax.x, Lcap.x, Scap.x, Fcap.x]
439         del (Bcap,REmax,Lcap, Scap,Fcap,ES,RE,RS,RB,BS,SB,SS,RL,SF,
440             FS,FW,LW)
441         gc.collect()
442     else: #Stage 2.2
443         self.OptimumPrim = m.objVal
444         cTx=((REmax)*Ccf+CB*Bcap*(COB+(1/Blife))+CL*(Lcap+slack2.x)
445             *(COL+(1/Llife))+CS*(Scap+slack.x)*(COS+(1/Slife))+CF*Fcap*
446             (COF+(1/Flife))+sum(CP*Emax[m].x for m in self.Data[1]))
447         var = []
448         for k in range(N):
449             diff = 0
450             for t in DateTime:
451                 diff += ((Tc+self.Samples[k][3][t])*(ES[t,k].x)+
452                         Ts*(RS[t,k].x+RB[t,k].x+RL[t,k].x)-
453                         self.Samples[k][2][t]*(RE[t,k].x))
454             var.append(cTx + diff-m.objVal)
455         self.variance = var
456
457         workbook = xlwt.Workbook()
458         sheet = workbook.add_sheet("Summary")
459         sheets = []
460         for itr in range(N):
461             sheets.append(workbook.add_sheet("Sheet"+str(itr)))
462         # Specifying style
463         style = xlwt.easyxf('font: bold 1')
464
465         #Summary sheet
466         sheet.write(0, 0, "TotalCost", style)
467         sheet.write(1, 0, m.objVal, style)
468         sheet.write(0, 1, "BatteryCapacity", style)
469         sheet.write(1, 1, Bcap, style)
470         sheet.write(0, 2, "ElectrolyzerCapacity", style)
471         sheet.write(1, 2, Lcap, style)
472         sheet.write(0, 3, "HydrogenTankCapacity", style)
473         sheet.write(1, 3, Scap, style)
474         sheet.write(0, 4, "FuelCellCapacity", style)
475         sheet.write(1, 4, Fcap, style)

```

```

476
477     k = 0
478     for sheet in sheets:
479         sheet.write(0, 0, 'Period', style)
480         #Battery section
481         sheet.write(0, 1, 'BatteryLevel', style)
482         sheet.write(0, 2, "Renewable_Batteries", style)
483         sheet.write(0, 3, "Households_Batteries", style)
484         sheet.write(0, 4, "Battery_House", style)
485         sheet.write(0, 5, "Battery_Skags", style)
486         sheet.write(0, 6, "Battery_Nyhagen", style)
487         #Renewable section
488         sheet.write(0, 8, "Renewable_Prod", style)
489         sheet.write(0, 9, "Renewable_Grid", style)
490         sheet.write(0, 10, "Renewable_Electrolyzer", style)
491         sheet.write(0, 11, "Renewable_House", style)
492         sheet.write(0, 12, "Renewable_Skags", style)
493         sheet.write(0, 13, "Renewable_Nyhagen", style)
494         #Grid/demand section
495         sheet.write(0, 15, "Grid_House", style)
496         sheet.write(0, 16, "Grid_Skags", style)
497         sheet.write(0, 17, "Grid_Nyhagen", style)
498         sheet.write(0, 18, "Demand_House", style)
499         sheet.write(0, 19, "Demand_Skags", style)
500         sheet.write(0, 20, "Demand_Nyhagen", style)
501         sheet.write(0, 21, "Grid_total", style)
502         #Hydrogen section
503         sheet.write(0, 23, 'Renewable_Electrolyzer', style)
504         sheet.write(0, 24, 'FuelCell_Households', style)
505         sheet.write(0, 25, 'FuelCell_Skags', style)
506         sheet.write(0, 26, 'FuelCell_Nyhagen', style)
507         sheet.write(0, 27, 'Storage_FuelCell', style)
508         sheet.write(0, 28, 'Household_Electrolyzer', style)
509         sheet.write(0, 29, 'HydrogenDemand', style)
510         sheet.write(0, 30, 'HydrogenLevel', style)
511         sheet.write(0, 31, 'Slack', style)
512         sheet.write(0, 32, 'Slack2', style)
513         sheet.write(1, 31, slack.x)
514         sheet.write(1, 32, slack2.x)
515         sheet.write(0,33, "WasteHeat")
516
517     j = 1
518     for v in DateTime:
519         sheet.write(j, 0, j)
520         #Battery section
521         sheet.write(j, 1, SB[v,k].x)
522         sheet.write(j, 2, RB[v,k].x)
523
524         sheet.write(j, 5, BS[v,k].x)
525
526         #Renewable section
527         sheet.write(j, 8, self.Samples[k][1][j])
528         sheet.write(j, 9, RE[v,k].x)
529         sheet.write(j, 10, RL[v,k].x)
530
531         sheet.write(j, 12, RS[v,k].x)
532
533         #Grid/demand section
534
535         sheet.write(j, 16, ES[v,k].x)

```

```

536         sheet.write(j, 19, self.Samples[k][0][j])
537
538         sheet.write(j, 21, ES[v,k].x)
539         #Hydrogen section
540         sheet.write(j, 23, RL[v,k].x)
541
542         sheet.write(j, 25, FS[v,k].x)
543
544         sheet.write(j, 27, SF[v,k].x)
545
546         sheet.write(j, 29, self.Data[2][j])
547         sheet.write(j, 30, SS[v,k].x)
548
549         sheet.write(j,33,FW[v,k].x+LW[v,k].x)
550         j += 1
551         k += 1
552         workbook.save("logNormTest.xls")
553         del (ES,RE,RS,RB,RL,self.Samples,self.Data,Bcap,REmax,Lcap,
554             Scap,Fcap,SF,SS,FS,BS,SB,FW,LW)
555         gc.collect()
556     except GurobiError as e:
557         print('Error code ' + str(e.errno) + ': ' + str(e))
558
559     def updateSamples(self, Samples2):
560         self.Samples = Samples2
561
562     def getOptimum(self):
563         return self.Optimum
564
565     def getOptimumPrim(self):
566         return self.OptimumPrim
567
568     def getSolution(self):
569         return self.Solution
570
571     def getVariance(self):
572         return self.variance
573
574     def write(Samples):
575         workbook = xlwt.Workbook()
576         sheet = workbook.add_sheet("Samples")
577         # Specifying style
578         style = xlwt.easyxf('font: bold 1')
579         k = 0
580         for v in Samples:
581             sheet.write(0, k, "Demand", style)
582             sheet.write(0, k+1, "Renewable", style)
583             sheet.write(0, k+2, "Elspot", style)
584             sheet.write(0, k+3, "Elspot+fee", style)
585             sheet.write(0, k+4, "", style)
586             for l in v:
587                 j = 1
588                 for m in l.keys():
589                     sheet.write(j, k, l[m])
590                     j += 1
591                 k +=1
592             k +=1
593         workbook.save("logNormTestData.xls")
594
595     #N is the size of each sample

```

```
596 N = 10
597 #M is the number of sample batches
598 M = 10
599 #prim is how much larger the Nprim sample should be than N
600 prim = 15
601
602 main(N, M, prim)
```

C Simplified Model

Simplified variables

- $ET(t)_k$: Total amount of electricity sent form the electricity grid
- $RT(t)_k$: Total amount of renewable energy sent to the system
- $BT(t)_k$: Total amount of electricity sent from the batteries to the system
- $FT(t)_k$: Total amount of electricity sent from the fuel cell to the system

Objective function:

$$\min z = RE_{max} * C_{cf} + \sum_{i=\{B,L,S,F\}} C_i * i_{cap} * (C_{oi} + \frac{1}{i_{life}}) + \sum_{m=1}^M C_P * E_{max}(m) + \frac{1}{N} \sum_{k=1}^N \sum_{t=1}^T (C_{EH}(t)_k + T_c) * ET(t)_k + T_s * (RB(t)_k + RT(t)_k + RL(t)_k) - C_{RE}(t)_k * RE(t)_k$$

Subject to:

$$\begin{aligned} S_B(0) &= B_{cap} * (1 - DoD) \\ S_B(t)_k &\leq B_{cap} \quad \forall t, k \\ B_{eff} * RB(t)_k + D_{is} * S_B(t-1)_k - BT(t)_k &= S_B(t)_k \quad \forall t, k \\ RT(t)_k + BT(t)_k * B_{eff} + ET(t)_k + FT(t)_k &= D_s(t)_k \quad \forall t, k \\ RB(t)_k + RT(t)_k + RE(t)_k + RL(t)_k &= PR(t)_k \quad \forall t, k \\ S_B(t)_k &\geq B_{cap} * (1 - DoD) \quad \forall t, k \\ RE(t)_k &\leq RE_{max} \quad \forall t, k \\ ET(t)_k &\leq E_{max}(m+1) \quad \forall m \setminus \{M\}, t \in [t_m + 1, t_{m+1}], k \\ BT(t)_k &\leq B_{cap} * C \quad \forall t, k \\ RB(t)_k &\leq B_{cap} * C \quad \forall t, k \\ S_S(t)_k &\leq S_{cap} \quad \forall t, k \\ RL(t)_k &\leq L_{cap} \quad \forall t, k \\ L_{eff} * (RL(t)_k + S_S(t-1)_k + LW(t)_k + FW(t)_k - H_d(t)_k - SF(t)_k) &= S_S(t)_k \quad \forall t, k \\ SF(t)_k * F_{eff} &= FT(t)_k \quad \forall t, k \\ SF(t)_k &\leq F_{cap} \quad \forall t, k \\ LW(t)_k &\leq RL(t)_k * L_{heat} \quad \forall t, k \\ FW(t)_k &\leq SF(t)_k * F_{heat} \quad \forall t, k \\ LW(t)_k + FW(t)_k &\leq D_{heat}(t) \quad \forall t, k \\ S_S(t)_k &\geq k * \sigma'_h(m+1) \quad \forall m \setminus \{M\}, t \\ S_S(0) &= k * \sigma'_h(1) + \sum_{t=1}^{168} H_d(t) \\ 0 &\leq RE_{max}, B_{cap}, L_{cap}, S_{cap}, F_{cap} \\ 0 &\leq ET(t_k), RE(t)_k, RT(t)_k \quad \forall t \end{aligned}$$

$$\begin{aligned}
0 &\leq S_B(t_0)_k, RB(t)_k, BT(t)_k \quad \forall t, k \\
0 &\leq S_S(t_0)_k, RL(t)_k, SF(t)_k, FT(t)_k, LW(t)_k, FW(t)_k \quad \forall t, k \\
0 &\leq E_{max}(m) \quad \forall m
\end{aligned}$$

D Austerland Skags Final Deterministic Model

New parameters

- C_W : Price of wood chips used for heating of Skags farm
- C_H : Price of hydrogen that can be bought externally

New variables

- $H_{buy}(t)$: Amount of hydrogen bought into the system in time period t

Objective function:

$$\begin{aligned} \min z = & RE_{max} * C_{cf} + \sum_{i=\{B,L,S,F\}} C_i * i_{cap} * (C_{oi} + \frac{1}{i_{life}}) + \sum_{t=1}^T (C_{EH}(t) + T_c) * (EH(t) + ES(t) + \\ & EN(t)) + T_s * (RB(t) + RH(t) + RS(t) + RN(t) + RL(t)) - C_{RE}(t) * (RE(t) + HE(t)) + \\ & H_{buy}(t) * C_H - C_W * (LW(t) + FW(t)) + \sum_{m=1}^M C_P * E_{max}(m) \end{aligned}$$

Subject to:

$$\begin{aligned} S_B(0) &= B_{cap} * (1 - DoD) \\ S_B(t) &\leq B_{cap} \quad \forall t \\ B_{eff} * (RB(t) + HB(t)) + D_{is} * S_B(t-1) - BH(t) - BS(t) - BN(t) &= S_B(t) \quad \forall t \\ RH(t) + BH(t) * B_{eff} + EH(t) + FH(t) - HE(t) - HB(t) - HL(t) &= D_h(t) \quad \forall t \\ RS(t) + BS(t) * B_{eff} + ES(t) + FS(t) &= D_s(t) \quad \forall t \\ RN(t) + BN(t) * B_{eff} + EN(t) + FN(t) &= D_n(t) \quad \forall t \\ RB(t) + RH(t) + RS(t) + RN(t) + RE(t) + RL(t) &= PR(t) \quad \forall t \\ S_B(t) &\geq B_{cap} * (1 - DoD) \quad \forall t \\ RE(t) &\leq RE_{max} \quad \forall t \\ EH(t) + ES(t) + EN(t) &\leq E_{max}(m+1) \quad \forall m \setminus \{M\}, t \in [t_m + 1, t_{m+1}] \\ 0 &\leq D_h(t) * (1 - y(t)) \quad \forall t \\ HE(t) + HB(t) + HL(t) &= -D_h(t) * y(t) \quad \forall t \\ BH(t) + BS(t) + BN(t) &\leq B_{cap} * C \quad \forall t \\ RB(t) + HB(t) &\leq B_{cap} * C \quad \forall t \\ S_S(t) &\leq S_{cap} \quad \forall t \\ RL(t) + HL(t) &\leq L_{cap} \quad \forall t \\ L_{eff} * (RL(t) + HL(t)) + S_S(t-1) + H_{buy}(t) - H_d(t) - SF(t) &= S_S(t) \quad \forall t \\ SF(t) * F_{eff} &= FH(t) + FS(t) + FN(t) \quad \forall t \\ SF(t) &\leq F_{cap} \quad \forall t \\ LW(t) &\leq RL(t) * L_{heat} \quad \forall t \\ FW(t) &\leq SF(t) * F_{heat} \quad \forall t \end{aligned}$$

$$\begin{aligned}
LW(t) + FW(t) &\leq D_{heat}(t) \quad \forall t \\
S_S(t) &\geq k * \sigma'_h(m+1) \quad \forall m \setminus \{M\}, t \\
S_S(0) &= k * \sigma'_h(1) + \sum_{t=1}^{168} H_d(t) \\
0 &\leq RE_{max}, B_{cap}, L_{cap}, S_{cap}, F_{cap} \\
0 &\leq EH(t), ES(t), EN(t), RE(t), RH(t), RS(t), RN(t), HE(t) \quad \forall t \\
0 &\leq S_B(t_0), RB(t), BH(t), BS(t), BN(t), HB(t) \quad \forall t \\
0 &\leq S_S(t_0), RL(t), SF(t), FH(t), FS(t), FN(t), LW(t), FW(t), HL(t) \quad \forall t \\
0 &\leq E_{max}(m) \quad \forall m \\
y(t) &\in \{0, 1\} \quad \forall t
\end{aligned}$$