Transient Response of Grounding Systems Caused by Lightning: Modelling and Experiments

BY

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**Abstract**


In order to achieve better lightning protection and electromagnetic compatibility (EMC) requirements, the needs for a proper grounding system and the knowledge of its transient behaviour become crucial.

The present work is focused towards developing engineering models for transient analysis of grounding system with sufficient accuracy and simplicity for lightning studies. Firstly, the conventional uniform transmission line approach for a single grounding conductor is modified and extended to grounding grids. Secondly, in order to overcome the drawbacks of all the existing transmission line approaches, for the first time, a non-uniform transmission line approach is developed for modelling the transient behaviour of different types of grounding systems. The important feature of such an approach is in its capability to include the electromagnetic couplings between different parts of the grounding system using space and time dependent per-unit length parameters.

High voltages and currents induced in the grounding systems due to lightning always produce ionization in the soil. This phenomenon should be included during the transient analysis of grounding systems. In the present work, an improved soil ionization model including residual resistivity in ionization region is developed. The fact that there exists residual resistivity in ionization region (7 % of the original soil resistivity) can be proved by the experiments reported in the literature and the experiments carried out at the high voltage lab of Uppsala University. The advantage of including residual resistivity is that the beneficial influence of soil ionization in reducing the potential rise of grounding system will not be overestimated, especially in high resistivity soil.

Finally, the transmission line approaches are adopted for studying the response of grounding systems due to lightning for different applications. These are, influence of soil parameters on the transient behaviour of grounding systems, transient analysis of grounding structures in stratified soils, investigation of the validity of existing definitions for effective length/area of different grounding structures, current distribution in the shields of under ground cables associated with communication tower, and influence of insulator flashover and soil ionization around the pole footing on surge propagation in Swedish railway system.

**Keywords:** Grounding system, Lightning protection, Electromagnetic Compatibility, Transient analysis, Soil ionization, Transmission line approach

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urn:nbn:se:uu:diva-4556 (http://urn.kb.se/resolve?urn=urn:nbn:se:uu:diva-4556)
To my husband Xiangyang Song and my daughter
Erin Jiarui Song and my parents
This thesis is based on the works described in the following journals and conference papers


The works described in the following technical reports are also included in the thesis


The list of papers below are other collaborated works which are not included in the thesis


3. Theethayi N, Thottappillil R, **Y. Liu** and Montano R, “Parameters that influence the crosstalk in multiconductor transmission line”, (paper in review at IEEE Trans. on Power Delivery.)


High Quality Paper Certificate

to

Nelson Theethayi and Ms. Liu

for the presentation of the paper "Modeling Direct Lightning Attachment to Electrified Railway System in Sweden" by N. Theethayi, Y. Liu, R. Montano, R. Thottappillil, M. Zitnik, V. Cooray and V. Scuka

[Signature]
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1 Introduction

1.1 The need for grounding/earthing system in lightning protection

Grounding system refers to metallic wire(s) of different geometrical structures, which are buried in the soil. Fig.1-1 shows the commonly used grounding structures, namely, single horizontal grounding wire, vertical rod, ring conductor and grounding grid with large area, or a suitable combination of above said structures.

![Fig. 1-1 Illustration of common grounding structures](image)

If a system to be protected is totally isolated or shielded from any external coupling mechanisms, there is no need for a grounding system for achieving Electromagnetic Compatibility (EMC). However, many practical systems are not always isolated. Usually, there exist connections with the outside world through power supply, water or gas pipes and communication cables. Consequently, when a lightning event occurs in the vicinity of the system or if the stroke terminates on the system, there will be potential rise and potential difference and transient energy transfer between the system and the external world, which is the primary cause of damage and injury. In order to reduce the above said risks, one has to have an efficient design of grounding system for various objects from the scope of lightning protection and EMC. This is because an efficient grounding system can safely dissipate the stroke currents into the soil as much as possible and reduce the above said risks.
Considering the above said facts, it is certain that grounding system forms an essential part of Lightning Protection System (LPS) for any power system components (like substations, generating stations, transmission lines, etc.), electrified railway systems, communication tower, large buildings, etc. It is also important for protecting the personnel working on site with the above said systems. There are very many damages, which are caused due to inadequate grounding structures by lightning. Here, we present three examples out of many.

1. Consider a transmission line tower where a pole supports a bare wire and an insulated cable as shown in Fig.1-2. If a large lightning current flows into a high impedance grounding system below, there will be a potential rise on the bare wire, which leads to a large potential difference between the bare wire and the cable. The consequence of such a large potential difference could cause a flashover between those two lines.

2. For buried cables in the ground, when lightning strikes in the vicinity of the cable, there will be high potential rise near the buried cables leading to soil breakdown. If there isn’t any proper grounding system beside the cables for their protection, then the buried cables will be damaged as shown in Fig. 1-3.

3. For the structures preserving flammable or explosive materials, the grounding system becomes crucial. Any improper grounding features like if the grounding conductors are not continuous or if the joints are not well bonded, and then there could be sparking due to high potential difference within the structure when a lightning strikes on it. The sparking may cause an explosion.

Fig. 1-2 Illustration of flash over between the bare conductor and the insulated cable due to the high potential rise at the pole grounding foot.
Fig. 1-3 Lightning damage to underground power cables. a) Coaxial cable in an insulating jacket inside a PVC conduit. b) Coaxial cable in an insulating jacket, directly buried. C) Coaxial cable with its neutral directly in contact with earth (adapted from [1])

Having understood the importance of grounding systems, one wonders what are the features of a good grounding system. A good grounding system should satisfy the following criteria.

- To provide a low impedance return path for surge current which is necessary for the timely operation of the over current/voltage protection system.
- To reduce the risks of breakdown hazard to electrical systems or electronic equipments.
- To reduce electric shock hazard to personnel.
- To minimize the cost.

Specifically, the third requirement demands that the touch and step voltages should be within the safe value for the personnel as shown in equation (1-1) [2].

$$V_t = \left(116 + 0.174 \rho_{\text{soil}}\right) / \sqrt{f}$$  \hspace{1cm} (1-1a)

$$V_s = \left(116 + 0.7 \rho_{\text{soil}}\right) / \sqrt{f}$$  \hspace{1cm} (1-1b)
In equation (1-1a and b), $V_t$ is the touch voltage, and $V_s$ is the step voltage. $\rho_{\text{soil}}$ is the soil resistivity and $t$ is the time duration of surge current in seconds. Equation (1-1) was derived based on two postulates. One is the resistance of the human foot to the ground, which is a function of the foot area and the surface soil resistivity, and the other is the permissible current flowing through the human body, which is an inverse function of the time duration.

For the electrical/electronic systems connected to the grounding structure, the voltage/(voltage difference) due to the surge current should not be larger than the breakdown value of the system. It is well known that different systems have different breakdown voltages.

In order to meet the above said criteria, there are several basic rules that should be followed.

- The size of the grounding system should be large enough to reduce the maximum potential rise when the surge current enters it.
- The spacing between the grounding wires should be so arranged that the touch and step voltages will be smaller than the safe value for the personnel.
- The downward conductor should be connected to the grounding system at such points in order to reduce the ground potential rise, for example, at the mid point of the grounding system.
- For different soil structures, the grounding system should be laid in such a way that it can take advantage of the low resistivity part of the soil to reduce the ground potential rise as much as possible.
- The effective length/area should be considered when one tries to minimize the cost.

Consequently, a good grounding system can dissipate the lightning current into the soil as fast as possible, thereby, reduce the potential rise and potential difference within the different parts of the system. And finally, there will be minimum damage to interconnected electronic devices and injury to the personnel working on site. A good design of typical grounding system for a FMV communication tower is shown in Fig.1-4. Here, we have translated the following names from Swedish to English.
1. radiella jordlinor: radial grounding wire
2. ringledare: ring conductor
3. jordlinor: grounding wire
4. stag: stay wire
5. mast: communication tower
6. maskkablage: cable from tower
7. medföljande jordlina som fjärmas från kabeln: follow on grounding wire
8. annat kablage: other cables
9. byggnad: building
10. transientskydd: surge protection devices
For the purpose of designing a good grounding system for lightning protection, one should know certain characteristics and parameters associated with lightning flashes. There are countless articles and some important books, which give a very detailed description of the principle of lightning [1,4-6]. Here, only a summary of it is given.

A typical lightning is an electrical discharge between 1) cloud and the earth (cloud-to-ground flash), 2) within the cloud (intra-cloud flash), 3) different clouds (inter-cloud flash). Out of all the flashes occurring in the nature, only 10% of flashes are cloud-to-ground flashes. A typical cloud-to-ground lightning flash starts with a descending stepped leader after a preliminary discharge in the cloud. As the stepped leader propagates towards the ground, the electric field at the ground or at the tip of grounded objects increases. At a certain instant, the field would be sufficiently high to launch upward connecting leaders from the ground or grounded structures. The upward connecting leaders have a polarity opposite to that of the downward leader. The incepted upward leader propagates in such a way that it seeks the tip of the descending leader. When the gradient between the two leader tips is sufficiently high (500 kV/m), then a final jump between the two occurs. The length of this final jump depends upon the charges in the above said two leaders. Because of the interception between the two leaders, the gap between the cloud and ground is bridged. At this instant, a ground potential wave called the return stroke propagates upward, discharging the leader channel. The return stroke current is the most important parameter for the transient behaviour study of the grounding system. The properties of it have been well summarized in [7]. Usually, the discharge current of return stroke increases from zero to a maximum in few μs (from 0.1 to 10 μs), then declines to half the peak value in about 20 to 1000 μs. The typical value of the
peak current derivative $\frac{di}{dt}$ is about 110 kA/μs. The peak value of the stroke current is about 15-30 kA (median value), and some stroke current could be about 250 kA (probability of occurrence less than 0.1%). The above said lightning return stroke current parameters forms the basis of the impulse current sources which will used as an input for the transient analysis of grounding system in the present study.

Based on the different requirements and applications, there are voluminous literature describing grounding system, methods and practices for making and laying grounding systems and measuring the grounding resistances [8-17]. Those literatures mostly discuss the DC or low frequency behaviour of the grounding system, which is not the topic of interest as far as this thesis is concerned. In this thesis, we will mainly concentrate on the transient behaviour of the grounding system specifically for lightning studies.

1.2 What this thesis contains?

In chapter 2, we will review various existing mathematical models for the transient analysis of grounding systems. The advantages and the disadvantages of each model are summarized. The discussions in this chapter form the basis of the modelling strategy for the present work.

In chapter 3, firstly, the conventional uniform transmission line approach for a single grounding conductor is modified and extended to grounding grids. Secondly, the drawbacks of all the existing transmission line approaches for modelling the transient behaviour of grounding systems are discussed. Based on those discussions, a non-uniform transmission line model is described in detail. The work in this chapter forms the heart of this thesis.

Chapter 4 describes the traditional model and the improved model for soil ionization, which are implemented in conjunction with the transmission line approaches presented in chapter 3. The improved soil ionization model is based on the concept of residual resistivity in the ionization region. Inclusion of soil ionization phenomena in the transient analysis of grounding system makes the model more physical.

Chapter 5 describes various applications of commonly encountered grounding problems.

Chapter 6 discusses the possible scope for the future work/studies.
2 The history of modelling grounding system for lightning studies

Since this thesis is more concentrated on modelling of grounding systems for lightning studies, it is necessary to have a historical overview of various mathematical models for the transient analysis of grounding systems. Those existing models form footsteps and pathways for the present research work. Further, one has to realize that a significant new contribution to the subject basically relies on those pioneering works carried out by previous researchers.

In the following sections of this chapter, we will touch up on different mathematical and engineering models by various research groups starting from 1934.

2.1 Earlier developments in modelling grounding wires — Analytical and empirical methods

Experimental and theoretical investigation of transient behaviour of grounding system under the lightning strikes bloomed for the first time in 1934 from the works of Bewley [18-19]. His work was a part of the research on the lightning protection of power systems, where he derived the impedance of a counterpoise at the injection point for an applied unit-step voltage. This impedance as shown in equation (2-1) was derived with an assumption that the wire is a long lossy transmission line with constant per-unit length parameters.

\[
Z_c(t) = \frac{1}{Gl_c \left\{ 1 - \sum_{k=1}^{\infty} \frac{8e^{-\alpha}}{(2k-1)\pi^2} \left[ \cos \omega_c t + \left( \frac{G}{4\omega_c C} - \frac{\omega_c C}{G} \right) \sin \omega_c t \right] \right\}} \tag{2-1a}
\]

\[
\omega_c = \frac{1}{2} \sqrt{\frac{(2k-1)2\pi^2 - \frac{G^2}{C^2}}{LC_l^2}} \tag{2-1b}
\]

\[
\delta = \frac{G}{2C} \tag{2-1c}
\]

In equation (2-1), \( l_c \) is the length of the grounding wire. \( G, L \) and \( C \) are the per-unit length leakage conductance, inductance and capacitance of the wire, respectively. Equation (2-1) indicates that the transient impedance of
counterpoise wire begins with an initial surge impedance effect \( \sqrt{L/C} \) and ends with the final leakage resistance effect \( \frac{1}{Gt} \), and the time of transition between these two effects depends upon the soil resistivity and the surge voltage. All the above said investigations on the counterpoise wire were later summarized in his book “Travelling waves in transmission systems” [20].

In 1943, Bellaschi and Armitom analytically computed the voltage response of grounding rods at the injection point for the current impulses with different wave shapes [21]. They gave the expressions for the voltage developed at the injection point by a slowly convergent series. For a unit step current impulse, the impulse voltage at injection point is given by (2-2a).

\[
e(t) = \frac{1}{G_i} \left[ 1 + 2 \sum_{n=0}^{\infty} e^{\frac{-n^2 \pi^2 t}{G_i L}} \right]
\]

(2-2a)

For a double exponential current impulse \( I(t) = I_0 \left( e^{-\alpha t} - e^{-\beta t} \right) \), the voltage at injection point is given by (2-2b).

\[
e(t) = I_0 \left[ \frac{L \alpha e^{-\alpha t}}{G_i \tan(\sqrt{G_i L} \alpha)} - \frac{L \beta e^{-\beta t}}{G_i \tan(\sqrt{G_i L} \beta)} + \alpha - \beta \sum_{n=1}^{\infty} \frac{2n^2 \pi^2 e^{\frac{-n^2 \pi^2 t}{G_i L}}}{G_i L} \right]
\]

(2-2b)

For a current impulse with sinusoidal front \( I(t) = A(1 - \cos Bt) \), the voltage at injection point is given by (2-2c).

\[
e(t) = \frac{A}{G_i} - A \left[ \frac{L \beta}{G_i} \cos(0.783 - \tan \left( \frac{\sin\sqrt{2G_i L} B}{\sinh\sqrt{2G_i L} B} \right)) \right]
\]

\[- \frac{2AG_iB^2L^2}{\pi^4} \sum_{n=0}^{\infty} e^{\frac{-n^2 \pi^2 t}{G_i L}} \]

\[- \frac{2AG_iB^2L^2}{\pi^4} \sum_{n=0}^{\infty} \frac{e^{\frac{-n^2 \pi^2 t}{G_i L}}}{\pi^4} \]

(2-2c)

In equation (2-2a, b and c), \( L \) is the total rod inductance in henrys, \( G_i \) is the total ground conductance in mhos, \( I_0 \) is the peak value of injection current. \( \alpha, \beta, A \) and \( B \) are the constants for different injection current wave shapes. It was also mentioned in [21] that for the grounding rod with longer lengths, the equivalent circuit with distributed earth resistance and wire inductance is suitable for the calculation of its impulse impedance. Comparing equation (2-1) and (2-2), it can be seen that Bellaschi et.al. neglected the capacitive effects in their model.

One of the most important and classical textbooks on grounding system was written by Sunde [12], which even today is widely used by many practicing engineers to solve grounding problems. His approach of describing the
grounding system is based on electromagnetic field theory starting from full Maxwell’s equations. He presented not only the calculation of DC resistance of different grounding structures (see chapter 3 in [12]), but also gives an exhaustive theory of the high frequency inductive behaviour of grounding wires (see chapter 4 in [12]). Sunde was perhaps the first to introduce the transmission line concept with frequency dependent per-unit length parameters for modelling the transient behaviour of single horizontal grounding wire on the surface of the soil due to direct lightning strikes using telegrapher’s equations given below.

\[
\frac{dI(x, j\omega)}{dx} = -YV(x, j\omega) \tag{2-3}
\]

\[
\frac{dV(x, j\omega)}{dx} = -ZI(x, j\omega)
\]

In equation (2-3), \(Z\) is per-unit length longitudinal impedance of the wire, and \(Y\) is per-unit length transversal admittance of the wire. Both \(Z\) and \(Y\) for a single horizontal wire are shown in equation (2-4a and b).

\[
Y(\Gamma) = \left[ Y_i^{-1} + \frac{1}{\pi(\sigma_{\text{soil}} + i\omega\varepsilon_{\text{soil}})} \log\left( \frac{1.12}{\Gamma a} \right) \right]^{-1} \tag{2-4a}
\]

\[
Z(\Gamma) = Z_i + \frac{i\omega\mu_0}{2\pi} \log\left( \frac{1.85}{a(\gamma^2 + \Gamma^2)^{1/2}} \right) \tag{2-4b}
\]

In equation (2-4a and b), \(\gamma^2 = i\omega\mu_0(\sigma_{\text{soil}} + i\omega\varepsilon_{\text{soil}})\), \(Z_i\) is the per-unit length internal impedance and \(Y_i\) is the per-unit length admittance of wire insulation, which is zero when the wire is in perfect contact with the soil, and ‘a’ is the radius of the grounding conductor. The propagation constant \(\Gamma\) is given by (2-5).

\[
\Gamma = \left[ Z(\Gamma)Y(\Gamma) \right]^{1/2} \tag{2-5}
\]

It is clear from all the above said models that the modelling of transient behaviour of grounding system started from the principle of transmission line theory, and was derived analytically under certain approximations for quick solutions because of the absence of powerful computer. Hence, those methods/models were limited to simple grounding systems, i.e., counterpoise wire or single grounding rods. For complex grounding systems, such as large grounding grids, only the empirical analysis could be thought of, which was attempted by Gupta et.al. in 1980 [22]. He through experiments found that the response of grounding grids to the unit step injection currents could be represented by equation (2-2a). Since in equation (2-2a), \(L_t\) and \(G_t\) are the parameters of single grounding rod, he gave empirical method for determining the total \(L_t\) and \(G_t\) of the grid based on the experimental results.
2.2 Later developments in modelling grounding systems — Numerical methods

Since the early eighties, the power of computer has increased dramatically, which accelerated almost all the science and engineering research fields in solving complex practical problems based on various powerful numerical methods [23-27]. Consequently, the modelling of complex transient behaviour of grounding system under lightning strikes had a better future specifically because of the following reasons.

- The earlier models described in the previous section has several assumptions in order to arrive at simple equations. But by using numerical methods, most of complex equations can be solved.
- Practical complex grounding systems can be modelled easily because of the large memory and speed of the computer.

The various numerical modelling methods for grounding systems under lightning studies, developed since 1980’s until now, can be classified as follows.

- Circuit approach
- Electromagnetic field approach
  - Method of moment
  - Finite element method
- Hybrid approach
- Transmission line approach

2.2.1 Circuit approach

One of the numerical models often used for modelling the transient behaviour of grounding systems with complex geometries is circuit approach. The main steps involved in this method are as follows.

- Divide the grounding system into many finite segments.
- Create the equivalent lumped circuit for each segment and calculate its parameters, such as self and mutual inductance ($\Delta L$), capacitance ($\Delta C$), conductance ($\Delta G$) and the internal resistance ($\Delta r$).
- Solve the nodal equations of the equivalent circuit that represents the whole grounding system based on Kirchoff’s laws. The nodal equations can be presented in different forms based on the adopted equivalent circuit of the grounding system.

The circuit approach for the transient analysis of grounding system was developed for the first time by Meliopoulos et.al. in 1983 [28]. He used frequency independent parameters for each segment ($\Delta L$, $\Delta C$, $\Delta G$ and $\Delta r$), which are calculated based on the Laplacian equation ($\nabla^2 V = 0$) in the semi-infinite conducting medium of the earth. The interesting part of this work is that each segment of the grounding wire was replaced by a lossless
transmission line and two extra shunt earth leakage conductances as shown in Fig. 2-1a, and it can be transformed to the circuit in Fig. (2-1b).

The nodal equation of the above said equivalent circuit is given by (2-6).

\[
[Y] \cdot [V(t)]= [I_i(t)] + [b(t-t_m,...)]
\]  

(2-6)

In equation (2-6), \([Y]\) is the nodal admittance matrix of the equivalent circuit. \([V(t)]\) is the voltage vector at time \(t\) for the nodes. \([I_i(t)]\) is the external current vector injected at the nodes of the circuit. \([b(t-t_m,...)]\) is the current history vector. The model described by Meliopoulos et al. [28] for the computation of the transient response of grounding system is compatible with the solution methodology of the Electromagnetic Transient Analysis Program (EMTP) [29]. Thus, the above said model for grounding systems can be easily interfaced with EMTP, thereby, adds an advantage to investigate the transient performance of large power systems (substation, surge arresters and transmission lines, etc.).

Later, as an extension of the work of [28], Meliopoulos et al. improved their circuit approach of grounding system for the lightning studies by calculating the response of each segment due to any current excitation based on quasi-static Maxwell’s equations [30], so that the parameters of each segment and the history currents are frequency dependent. A recursive convolution technique was used for the computation of the past history currents.

In 1989, Ramamoorthy et al. developed a simplified circuit approach for the grounding grid [31]. In their approach, after dividing the whole grounding system into \(n\) segments, each segment was only presented by a lumped circuit with self and mutual inductances (\(\Delta L\)) and self earth leakage conduc-
tance ($\Delta G$) as shown in Fig. 2-2. Therefore, the nodal equation of the equivalent circuit of the grounding system is given by (2-7).

$$\frac{d[V]}{dt} = [G] \cdot \begin{bmatrix} \frac{d[I]}{dt} - [L] \cdot [V] \end{bmatrix}$$

(2-7)

In (2-7), $[V]$ is the nodal voltage vector, $[I]$ is the nodal injection current vector, $[G]$ is the nodal conductance matrix, and $[L]$ is the nodal inductance matrix. Even though this model neglected the capacitive coupling, it is still reasonably accurate for the transient analysis of grounding system in low resistivity soils.

In 1999, two modifications of circuit approach based on Meliopoulos works [28, 30] were published by Geri [32] and Otero et al. [33-34], respectively, and both of them even included the soil ionization phenomena in their models (explained in Chapter 4). Instead of lossless transmission line combined with earth leakage conductance, which has been adopted by Meliopoulos et al. [28], Geri [32] used a different equivalent circuit to represent each segment of the grounding wire. As shown in Fig. 2-3, Geri used an equivalent conductance parallel with an ideal voltage-controlled current source to represent every capacitance-conductance and resistance-inductance branches of the circuit. Based on the above said new resistive equivalent circuit, the nodal equation (2-6) of the grounding system can be easily solved.

---

Fig. 2-2 Equivalent circuit of a square mesh of the grid

Fig. 2-3a Equivalent circuits of capacitance-conductance branch of grounding system in Geri’s Model
The equivalent circuit of grounding system by Otero et al. [33-34] is shown in Fig. 2-4. The nodal equation for the equivalent circuit in Fig. 2-4 is in the form of equation (2-8). Equation (2-8) was solved in frequency domain, which was probably the first attempt for the transient analysis of grounding system in frequency domain based on circuit approach.

\[
\mathbf{I}_n = \mathbf{K} \cdot \mathbf{G} \cdot \mathbf{V} + \mathbf{Y} \cdot \mathbf{V} \tag{2-8}
\]

where \(\mathbf{V}_{\text{ave}} = \mathbf{K} \cdot \mathbf{V}\). In equation (2-8), \(\mathbf{I}_n\) is the vector of external current source. \(\mathbf{K}\) is constant matrix which is related with column vector of branch voltages \(\mathbf{V}_{\text{ave}}\) and column vector of node voltages \(\mathbf{V}\). \(\mathbf{G}\) is the matrix that includes conductive and capacitive effects and \(\mathbf{Y}\) is the admittance matrix of the circuit including resistive and inductive effects.
Fig. 2-4 Equivalent circuits of grounding system in Otero’s Model

Circuit approach is easy to understand in the sense that the complex transient behaviour of grounding systems is transformed to a simple transient analysis of equivalent circuits. This transformation makes the problem more observable. Circuit approach can easily incorporate the non-linear soil ionization phenomena. Further, circuit approaches can include all the mutual coupling between the grounding wires. The main drawback of this approach is that it cannot predict the surge propagation delay.

2.2.2 Electromagnetic field approach

Electromagnetic field approach is the most rigorous method for modelling the transient behaviour of grounding system, because it solves full Maxwell’s equations with minimum approximations. This approach can be implemented either by Method of Moment (MoM) or by Finite Element Method (FEM).

The model for the transient behaviour of grounding system based on MoM was first developed by Greev [35-38]. This method starts from electric field Maxwell’s integral equation as given in (2-9) [24].

\[
E^\prime = \frac{1}{4\pi \omega \varepsilon} (\nabla \nabla - \gamma^2) \int J'(r')G_s(r,r') dl \tag{2-9a}
\]

\[
G_s(r,r') = G_i(r,r') + G_j(r,r') + G_k(r,r') \tag{2-9b}
\]

In equation (2-9), \(E^\prime\) is the total scattered electric field along the surface of the conductor. \(J'(r')\) is the current flowing along the conductor.

\[
\dot{\varepsilon} = \varepsilon + \frac{\sigma}{j\omega} \quad \text{is complex medium permittivity.} \quad \gamma = j\omega \mu (\sigma + j\omega \varepsilon) \quad \text{is the wave}
\]
propagation constant in the medium with \( \sigma \), \( \varepsilon \) and \( \mu \) as conductivity, permittivity and permeability, respectively. \( G_i(r,r') \) is the complete Green function. \( G_e(r,r') \) and \( G_h(r,r') \) are Green’s dyadic functions for the electric field at \( r \) due to the current element and its image [39], and \( G_r(r,r') \) is a correction term (expressed in terms of Sommerfeld integrals [40]) due to the air-soil interface, which was necessary for the complete solution of the electric field.

The boundary condition is that the total longitudinal electric field on the surface of the grounding conductor should satisfy equation (2-10).

\[ t \cdot (E_i + E_r) = I Z \Delta l \]  

(2-10)

In equation (2-10), \( E_i \) is the incident electric field, \( Z \) is the per-unit length series internal impedance of the conductor including skin effect.

The numerical treatment of above said equation (2-10) is called Method of Moment (MoM), which is nothing but transforming the associated integral equation to a system of linear algebraic equations with \( N \) unknowns, where the \( N \) unknowns usually represent the coefficients of the current based on some appropriate expansions. If the distribution of the current along the conductor is approximated as piecewise sinusoidal, the corresponding integral equation for the electric field (2-9) is called Reaction Integral Equation (RIE). If the distribution of the current along the conductor is approximated as piecewise constant, the corresponding integral equation for the electric field (2-9) is called as Mixed Potential Integral Equation (MPIE). An example of the linear algebraic expression of equation (2-10) based on MPIE is given by equation (2-11) [24, 41].

\[
\begin{bmatrix}
1 & 0 & \cdots & 0 \\
Z_{21} & Z_{22} + Z \Delta l & \cdots & Z_{2N} \\
\vdots & \vdots & \ddots & \vdots \\
Z_{N1} & Z_{N2} & \cdots & Z_{NN} + Z \Delta l \\
\end{bmatrix}
\begin{bmatrix}
I_1 \\
I_2 \\
\vdots \\
I_N \\
\end{bmatrix}
= 
\begin{bmatrix}
I_g \\
0 \\
\vdots \\
0 \\
\end{bmatrix}
\]  

(2-11)

In equation (2-11), \( Z_{mn} \) refers to mutual impedance, which is equal to the voltage across the \( m \)-th segment due to a unit current through the \( n \)-th segment. \( I_g \) is the injection current in the first segment. \( Z_{mn} \) can be calculated by equation (2-12).

\[ Z_{mn} = \frac{1}{l} \int_{l_n} F^e_n(l_n) \hat{I}_n \cdot E_m dl \]  

(2-12)

In equation (2-12), \( F^e_n(l_n) \) is current expansion function for the segment \( n \), and \( E_m \) is the average electric field on the surface of segment \( n \) due to the current in segment \( m \). The mutual impedance \( Z_{mn} \) is dependent only on the geometry of the system, the frequency and the characteristics of the soil. By solving equation (2-11) in frequency domain, one can get current distribution along the grounding conductor.
If the current sources for every segments of grounding conductor are known, the electric field around the grounding system and the leakage current from the grounding conductor segment to the soil can be easily calculated using fundamental equations for the related source and medium. The potential at different points on the surface of the grounding conductor could be calculated by integration of the normal electric field from the point on the surface of the conductor to the remote earth. The longitudinal component of the electric field is not included, as it is path dependent and its contribution is negligible compared to that of the normal electric field for lightning studies. Finally, to get response in time domain, one could use IFFT.

Since electromagnetic field approach based on Method of Moment solves full Maxwell’s equations in frequency domain, it has minimum assumptions. Consequently, it is believed to be very accurate. The higher is the frequency of input sources, the greater is the accuracy of the electromagnetic field approach. However, this model is too complex to be implemented. Further when the grounding structure is large, the computation time is very large. Another disadvantage of electromagnetic field approach is that, because of its frequency domain solution procedure, it cannot be easily modified to include non-linearity due to soil ionization, and combine other non-linear devices that have time domain models.

Another electromagnetic field approach for the transient analysis of grounding system was developed by Nekhoul et.al. [42-43]. The model starts from electric or magnetic energy equations which involves partial differential Maxwell’s equations with respect to the vector potential ($\vec{A}$) and the scalar potential ($\vec{V}$) in different domains/volumes of the system. It is then implemented using Finite Element Method (FEM) for the solutions based on the physical principle of minimizing the energy in the system. The final $A$-$V$ functions are given in (2-13) (2-13a and b for the field in the soil, 2-13c for the field in air), which involves the weighting function $\vec{W}$ and $\vec{w}$ for the vector potential and scalar potential, respectively.

\[
\int \frac{1}{\mu_0} (\nabla \times \vec{W}) \cdot (\nabla \times \vec{A}) + \frac{1}{\mu_0} (\nabla \cdot \vec{W})(\nabla \cdot \vec{A}) + (\sigma_{\text{rad}} + j\omega\sigma_{\text{rad}})(j\omega\vec{W} \cdot \vec{A} + \vec{W} \cdot \nabla \vec{V}) d\Omega = 0 \tag{2-13a}
\]

\[
\int \sigma_{\text{rad}} + j\omega\sigma_{\text{rad}} \nabla \vec{W} \cdot (j\omega\vec{A} + \nabla \vec{V}) d\Omega = 0 \tag{2-13b}
\]

\[
\int \frac{1}{\epsilon_0} (\nabla \times w) \cdot (\nabla \times \vec{A}) + \frac{1}{\mu_0} (\nabla \cdot \vec{W})(\nabla \cdot \vec{A}) d\Omega = 0 \tag{2-13c}
\]

In order to solve the problems numerically, the above said equations were transformed to linear equations by dividing the whole system into $N$ small volumes or elements. The difficulty in this approach is to transform the open boundaries of both air and earth environment into a closed boundary problem using spatial transformation [44-45], which will reduce the size of the problem. The main advantage of this electromagnetic field approach based on FEM is that the descritization of the domain (geometry or the me-
dium) of the problem can be highly flexible non-uniform patches or elements that can easily describe complex shapes. That is the reason why the soil ionization can be easily included into the above said model [43]. However, this method is even more complicated to understand than the one that is based on Method of Moment, because it is not directly solving the Maxwell’s equations.

### 2.2.3 Hybrid approach

Hybrid approach for the transient analysis of grounding system was first initiated by Dawalibi in 1986 [46-47], and later modified by Andolfato et.al. [48] in 2000. Here, the word “hybrid” means that this approach is a combination of both electromagnetic field approach and circuit approach. The methodology of this model is as follows. The whole grounding system should be divided into \( n \) small segments. The electric field at any point is given by (2-14), which was derived from full Maxwell’s equations.

\[
E = -\nabla V + j\omega \mathbf{A}
\]  
(2-14)

In equation (2-14), \( \mathbf{A} \) is vector potential, and \( V \) is scalar potential. Along each segment \( k \), the above said equation (2-14) can be transformed by equation (2-15).

\[
Z_{ik} I_k + \sum_{i \neq k} (V_{-ave_i} - V_{-ave_i}) + j\omega \sum_{i \neq k} \oint_{i} \mathbf{A}_{i} dl = 0
\]
(2-15)

In (2-15), \( Z_{ik} \) is series internal impedance of the conductor segment, \( k \), which includes the skin effect. \( V_{-ave_i} \) and \( V_{-ave_i} \) are the potential of segment, \( k \) and \( i \). Andolfato et.al. [48] explained in detail that in equation (2-15), \( V_{ik} = V_{ave_k} - V_{ave_i} \) is due to capacitive-conductive coupling, and \( j\omega \oint_{i} \mathbf{A}_{i} dl \) is due to inductive coupling. So that equation (2-15) can be rewritten as (2-16).

\[
Z_{ik} I_k + \sum_{i \neq k} (C - G)_{ik} I_{l \neq k} + j\omega \sum_{i \neq k} L_{i} I_{i} = 0
\]
(2-16)

Equation (2-16) is in the form of a circuit equation, however, the inductive and capacitive-conductive coupling components in (2-16) were evaluated by rigorous electromagnetic field analysis as below.

\[
j\omega L_{ik} = \frac{j\omega \oint_{i} \mathbf{A}_{i} dl}{I_{i}}
\]
(2-17a)

\[
(C - G)_{ik} = \frac{V_{ik}}{I_{l \neq k}} = \frac{1}{4\pi\sigma \text{sol} I_{i}} \int_{l_{i}}^{e_{ik}} \frac{e^{-\tau r}}{r} dl + \zeta \cdot \frac{1}{4\pi\sigma \text{sol} I_{i}} \int_{l_{i}}^{e_{ik}} e^{-\tau r} dl
\]
(2-17b)

In (2-17), \( \mathbf{A}_{ik} \) is the vector potential on segment \( k \) due to the current source on segment \( i \). \( l_{k} \) and \( l_{i} \) are the lengths of segment \( k \) and its image \( k' \). \( I_{i} \) is
the current flowing along the segment $i$. $I_{ik}$ is the dissipation current from the segment $i$ to $k$ through the soil. $r$ and $r^*$ are the distance from the current source and its image to the point where the field is calculated.

\[ \dot{\varphi}_{\text{soil}} = \sigma_{\text{soil}} + j \omega \varepsilon_{\text{soil}} \]

is the complex conductivity of the soil. $\gamma = \sqrt{j \omega \mu_0 (\sigma_{\text{soil}} + j \omega \varepsilon_{\text{soil}})}$ is the propagation constant. $\zeta$ is the capacitive-conductive reflection coefficient.

The merit of hybrid approach is that the frequency influence on series internal impedances, inductive components and capacitive-conductive components are included, which makes the above said approach more accurate than the conventional circuit approach, especially when the injection source frequency is high.

### 2.2.4 Transmission line approach

As we have described in section 2.1 that the transmission line approach was the first approach that was used for simulating transient behaviour of grounding system. However, the development of this approach was not as fast as that of the circuit approach and electromagnetic field approach.

Verma et.al. [49] Mazzetti et.al. [50] and Velazquez et.al. [51] applied lossy transmission line concept on the horizontal grounding wire, which was described by telegrapher’s equations.

\[ \frac{\partial V}{\partial x} + L \frac{\partial I}{\partial t} + rI = 0 \quad (2-18a) \]

\[ \frac{\partial I}{\partial x} + C \frac{\partial V}{\partial t} + GV = 0 \quad (2-18b) \]

The solution of above equation (2-18) was to derive analytically the current and voltage distribution along the grounding wire in s-domain and later converted the s-domain equations to time domain using inverse Laplace transform. Later, Lorentzou et.al. [52] started from the same telegrapher’s equations (2-18a,b), but derived the equation of current and voltage distribution on the wire in time domain directly. The common feature of the above said transmission line approaches is that the per-unit length parameters are uniform along the grounding conductors.

Menter and Grecq’s [53] transmission line approach of transient analysis of grounding system was carried out by implementing Sunde’s frequency dependent lossy transmission line equation (2-3), where the per-unit length longitudinal impedance and transversal admittance $Z(\Gamma)$ and $Y(\Gamma)$ are changing with frequency as shown in equation (2-4a and b). The above said parameters were calculated numerically, which was impossible in the absence of powerful computer. Moreover, Menter et.al. combined this transmission line model for the counterpoise wire with the other parts of a 123 kV substation in EMTP [29].
The reason for transmission line approach being the first approach for modelling the transient behaviour of grounding systems is that it was initially used for simulating the transient behaviour of counterpoise wire. A counterpoise wire has the transient behaviour, which is similar to that of the overhead transmission lines. The only difference is the former is buried in the soil and the latter is in the air. Transmission line approach for modelling transient behaviour of grounding systems can be either in time or in frequency domain, but it is easy to include soil ionization in time domain. Similar to circuit approach, it can also include all the mutual couplings between the different parts of the grounding wires. Moreover, transmission line approach can predict surge propagation delay, which becomes important when the grounding system has large size. Further, the computational time required for transmission line approach is extremely less compared to the electromagnetic field approach.
3 The improved transmission line approaches

After having had an exhaustive literature survey in chapter 2, we found that, for modelling the transient behaviour of grounding systems, different approaches were adopted. Those approaches have their own merits and demerits, which will be summarized in Table 3-1.

Table 3-1 Comparison of different approaches for modelling the transient behaviour of grounding system

<table>
<thead>
<tr>
<th>Approaches</th>
<th>Mathematical expressions</th>
<th>Visualisation</th>
<th>Solution procedure</th>
<th>Requirement of computer power</th>
<th>Accuracy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electromagnetic field approach</td>
<td>Complicated</td>
<td>Difficult to understand</td>
<td>Complicated, difficult to include soil ionization, can predict wave propagation delay</td>
<td>Very powerful computer, large computation time</td>
<td>Believed to be most accurate</td>
</tr>
<tr>
<td>Circuit approach</td>
<td>Simple</td>
<td>Easy to understand</td>
<td>Simple, easy to include soil ionization, can not predict wave propagation delay</td>
<td>Normal PC, small computation time</td>
<td>Reasonably accurate</td>
</tr>
<tr>
<td>Hybrid approach</td>
<td>Complicated</td>
<td>Not very easy to understand</td>
<td>Slightly complicated, can include the soil ionization, capability of predicting wave propagation delay</td>
<td>Normal PC, small computation time</td>
<td>Reasonably accurate</td>
</tr>
<tr>
<td>Transmission line approach</td>
<td>The simplest</td>
<td>Very easy to understand</td>
<td>Very simple, can predict wave propagation delay, easy to include soil ionization</td>
<td>Normal PC, small computation time</td>
<td>Reasonably accurate</td>
</tr>
</tbody>
</table>

From an engineering perspective, a model for the transient analysis of grounding systems for lightning studies should be simple for quick applications, and at the same time, it should predict all the important features of transient behaviour of grounding systems. Based on the above comparisons in table 3-1, it is found that the transmission line approach is a better choice for the present study. Unfortunately, the development of this approach was not as fast as that of circuit and electromagnetic field approaches. As of
now, all the transmission line approaches were limited to model simple
grounding systems, i.e., counterpoise wires or single grounding rods.
Consequently, from scope of this thesis, it becomes necessary to improve
transmission line approach for modelling transient behaviour of grounding
systems for engineering application. Firstly, we will extend the conventional
transmission line approach with constant per-unit length parameters \((L, C, G\) and \(r_e\)) from a single grounding wire/rod to grounding grids. For more
details of this part of work, kindly refer paper I and paper II. Secondly, the
disadvantages of conventional transmission line approaches with uniform
per-unit length parameters will be discussed, and finally a non-uniform
transmission line approach for modelling the transient behaviour of ground-
ing system will be presented. For more details of this part of work, kindly
refer paper III and paper IV.

3.1 Extension of conventional infinite transmission
line approach from single grounding wire to
grounding grids

In order to extend the conventional infinite transmission line approach with
uniform constant per-unit length parameters \((L, C, G\) and \(r_e\)) from a single
grounding wire to grounding grids, we adopted two software packages. One
is Ace [54], a Finite Element Method solver for electromagnetic field analy-
sis. And the other is Alternative Transients Program/ Electro-Magnetic
Transients Program (ATP-EMTP) [55], the software widely used by power
engineer for transient analysis.

3.1.1 Basic Assumptions in the model

The grounding systems are structured with good conductors. Every conduc-
tor is assumed to be a lossy transmission line. It is also assumed that the
radii of the conductors are much smaller than the buried depth and the
length of the wire. The conductor is characterized by its electrical properties
and dimensions. The soil is modelled as a linear and homogeneous half-
space characterized by resistivity, relative permittivity and permeability,
thus making the soil a lossy medium. The current is partly flowing along the
conductors, and partly dissipating from its surface into the soil in a radial
direction. For the moment, the ionization of the soil is not considered, even
though it is included in the model later in chapter 4. The skin effect (internal
loss) of the conductor is neglected, as the losses in the soil are much larger
than the internal loss. An infinite uniform transmission line approach under
quasi-TEM field structure assumption, similar to the one used by Mazzetti
et.al. [50], is used in the simulation. Also, the transmission line per-unit
length parameters are frequency independent. It means the per-unit length
parameters are constant for a given lightning stroke. On the other hand, practical grounding systems have some conducting structure above the ground (e.g., air-termination, and down conductors of the lightning protection system), in addition to the buried conductors. Any electromagnetic coupling between the overground system and underground system are not considered. Besides, the vertical conductor that connects the buried grounding system with the overground system is also not considered. The surge current is assumed to be injected directly into the grounding system.

3.1.2 Calculation of per-unit length parameters of the grounding conductors using FEM

In order to apply the infinite transmission line approach for transient analysis of grounding system, all the per-unit length parameters of the grounding conductors, such as internal resistance, self or mutual inductance, capacitance and conductance to the earth, should be calculated first, which is the primary step of the model. For this purpose, a software package, Ace [54], is adopted to calculate the above said parameters except the internal resistance, according to the configuration of the grounding system. One of the advantages of this software is that it can calculate not only the self per-unit length parameters of grounding wires but also the mutual coupling parameters between them. In Ace, the problem is solved based on Finite Element Method (FEM).

If the medium is inhomogeneous, for example, soil is half space medium, or the soil has stratified structure, or when there is soil ionization around the conductor and the original homogeneous soil becomes inhomogeneous, FEM programs, for instance Ace, will be the good choice for calculating the per-unit length parameters of grounding conductors, because the field distribution due to the inhomogeneous medium can be easily taken into account in FEM program, such as air-soil interface and interface between the different soil layers, and so on.

It should be noticed that all the per-unit length parameters are calculated under static field condition (DC), because the field structure for quasi-TEM mode of propagation is identical to a static field [56]. When the boundary is fixed, the accuracy of the result depends on the condition of the mesh. The finer and more uniform is the mesh around the positions where the field distribution changes dramatically (e.g. in the vicinity of the conductors and interfaces of different mediums), the more accurate is the result. For a simple grounding system, such as a single buried horizontal conductor, there are no mutual per-unit length parameters. For meshed grounding system, such as $1 \times 1$ and $2 \times 2$ grounding grids, mutual coupling is considered between the parallel conductors, but not between the perpendicular conductors as the problem is solved in two-dimension.
3.1.2.I Boundary fixation in FEM
The difficult part in using FEM program, such as Ace, to calculate the per-unit length parameters for the grounding conductors, is to choose a suitable reference boundary. As we mentioned earlier, infinite uniform transmission line approach is used in the simulation, which means that the per-unit length parameters, L, C and G, are length independent. Consequently, in Ace, the boundaries for calculating the per-unit length parameters of horizontal grounding conductors are fixed by thumb rules. For the single horizontal conductor, about 10 times of the buried depth of the grounding conductor is chosen as the reference boundary. For the horizontal grounding grids, the space between two conductors usually is much larger than the buried depth. In this regard, among the 10 times of the buried depth and two to three times of the spacing between two parallel grounding conductors, the larger one is chosen as the reference boundary. It means that only the buried depth of the horizontal grounding conductors and the spacing between them are taken into account for the choice of suitable reference boundary, the longitudinal length of the grounding conductors is not considered. The above said thumb rules are in similar lines to the method of determining the resistance of single vertical rod in IEEE Standard [57].

3.1.2.II Equations for calculating the per-unit length parameters
Once the boundary is fixed in Ace according to the above said principle, the per-unit length parameters could be calculated. The self per-unit length inductances can be calculated by equation (3-1a).

\[ L_s = 2 \frac{\int_0^{y} [H \cdot dB] ds}{I_i^2} \]  (3-1a)

In (3-1a), \( L_s \) is the self-inductance, \( I_i \) is the current flowing along with the \( i-th \) conductor, \( \int_0^{y} [H \cdot dB] ds \) is the magnetic field energy caused by \( I_i \), \( s \) is the cross section area between the \( i-th \) conductor and the reference boundary. Similarly, the mutual per-unit length inductances can be calculated by equation (3-1b).

\[ L_{ij} = \frac{\int_0^{y} [H \cdot dB] ds - \frac{I_i^2 L_{oo} + I_j^2 L_{oo}}{2}}{I_i I_j} \]  (3-1b)

The self and mutual per-unit length capacitances can be calculated by equation (3-2).

\[ C_s = \frac{\int_0^{y} D \cdot ds}{V_r} \]  (3-2)
In (3-2), if \( i = j \), \( C_{ij} \) is self-capacitance, which is sum of all the mutual capacitances and the capacitance to the remote earth. If \( i \neq j \), \( C_{ij} \) is mutual capacitance. \( V_i \) is the potential of the \( i-th \) conductor with respect to the boundary, while the potentials on the other conductors are set to zero. \( \int_{D_i} ds \) is the charge deposited on the \( j-th \) conductor per-unit length due to \( V_i \), \( s_j \) is the integral surface around the conductor. Similarly, the self and mutual per-unit length conductances can be calculated. The equation of self- and mutual per-unit length conductances can be found in paper I.

3.1.3 Transient analysis in ATP-EMTP

After having estimated the per-unit length parameters of grounding conductors, the transient behaviour of grounding systems can be simulated based on conventional transmission line approach. Since a lumped distributed circuit can be used to represent a transmission line if the line is divided into many electrically small sections [56]. The transient behaviour of grounding systems can be simulated in ATP-EMTP [55], based on the above said distributed circuit. In ATP-EMTP, every electrically small section of the transmission line is represented by lumped resistance, inductance, capacitance and other coupling elements. The number of the sections required for an accurate simulation is dependent on the highest frequency of the injected impulse, the higher the frequency, the larger the number of sections required.

An example of application of this model is modelling transient behaviour of 1x1 and 2x2 grounding grids, which have dimensions as shown in Fig.3-1. The diameter of the conductor is 14 mm. All the grids are buried at 0.5 m depth in a homogeneous soil with \( \rho_{soil} = 1000 \, \Omega m \) and \( \varepsilon_{e_{soil}} = 9 \). A double-exponential current impulse, \( I(t) = 1 \times (e^{-27000t} - e^{-560000t}) \) A, is injected at point A, the corner of the grounding grids. All the above parameters of the system are same as that in [58], for comparing the simulation results between the present transmission line approach with constant per-unit length parameters and the more accurate electromagnetic field approach [58].
Fig. 3-1 Dimensions of two different grounding grids

Fig. 3-2 is the transient voltages at the injection point for the above said two grids based on the present conventional infinite transmission line approach. The maximum values are larger than the results in [58] by only 5% ~ 8%. This difference may be due to:

- Injection current may not be exactly the same as that in [58]
- Error due to the calculation of the per-unit length parameters.
- Fringing fields not being considered and its possible influence on the per-unit length parameters.
- The coupling between the perpendicular conductors being neglected
- The mutual conductive coupling between the conductors through the soil being neglected.

Fig. 3-2 Transient voltages of grounding grids $1 \times 1$ and $2 \times 2$ at the injection point

3.2 Non-uniform transmission line approach

If one studies in detail the conventional transmission line approaches, some drawbacks of those approaches can be found. The conventional transmission line approach means that the per-unit length parameters of grounding conductors are constant along the conductor as described in [49-53,59] and in the previous section. In this section, we will first discuss the drawbacks of different existing transmission line approaches, and later in order to over-
come those problems, we developed a non-uniform transmission line approach, a better engineering model for modelling transient behaviour of grounding system. Here, non-uniform means that the per-unit length parameters are varying as function of space and time.

3.2.1 Drawbacks of existing conventional uniform transmission line approach

3.2.1.I Infinite transmission line approach with frequency dependent per-unit length parameters

In [53], Sunde’s equations (see equation (2-4)) of frequency dependent per-unit length longitudinal impedance, \( Z(\Gamma) \), and transversal admittance, \( Y(\Gamma) \), for a single grounding conductor with infinite length on the surface of the soil were used for the buried grounding conductor by changing the radius of the conductor, \( a \), to \( \sqrt{2ad} \) (\( d \) is the buried depth of the conductor). It is not certain that such an extension is valid. Moreover, it is very difficult to derive the mutual \( Z(\Gamma) \) and mutual \( Y(\Gamma) \) corresponding to the mutual coupling between the grounding wires. This means that the model described in [53] is difficult to adopt for simulating the transient behaviour of grounding grids. Further, when the maximum transient voltage of a single counter-poise wire was simulated based on the above said infinite transmission line approach, there were about 13%-40% difference between the simulation results and the experimental results.

In the rest of the thesis, we will assume that the per-unit length parameters of grounding conductors are frequency independent.

3.2.1.II Infinite transmission line approach with constant per-unit length parameters

In [50], the per-unit length parameters (\( L \), \( C \) and \( G \)) were calculated using Sunde’s equation [12] for a one-meter length of wire as shown in equation (3-3).

\[
G = \frac{\pi}{\rho_{soil} \left[ \ln \frac{2}{\sqrt{2ad}} - 1 \right]} \quad (3-3a)
\]

\[
C = \frac{\pi \sigma_{soil}}{\ln \frac{2}{\sqrt{2ad}} - 1} \quad (3-3b)
\]

\[
L = \frac{\mu_0}{\pi} \left[ \ln \frac{2}{\sqrt{2ad}} - 1 \right] \quad (3-3c)
\]

In equation (3-3), \( a \) is the radius of the conductor, and \( d \) is the buried depth of the conductor. Using the above said parameters, the transient analysis of
grounding wires with different length was carried out in time domain. This means it is assumed that the per-unit length parameters are independent of the length. Or, one can say that the infinite transmission line concept is used in this approach [50]. The problem of this method is that it predicts incorrect transient voltages at injection point of the wires. In order to clearly show this incorrect prediction of transient voltage at injection point based on the above said infinite transmission line approach in [50], we give an example.

Fig. 3-3 Illustration of the horizontal grounding conductors with 20 m and 100 m lengths

As shown in Fig. 3-3, the radius of the horizontal grounding conductor is 7.5 mm, and the lengths of the conductors are 20 m and 100 m. The conductors are buried at 0.5 m depth in the soil with $\varepsilon_{\text{soil}}=50$, $\rho_{\text{soil}}=100 \, \Omega \cdot \text{m}$ and $\mu_{r_{\text{soil}}}=1$. Current impulse, $I(t) = 12935 \cdot (e^{-190099t} - e^{-2922879t}) \, \text{A}$, is injected at point A. This injection current has $1/5$ μs wave shape. All those simulation parameters corresponding to geometry, material properties and injection current impulse are same as that in [60], to which we are going to compare.

Fig. 3-4a Transient voltages at different points for the horizontal grounding wires in Fig.3-3 based on infinite transmission line approach in [50]
Fig. 3-4b Transient voltages at different points for the horizontal grounding wires in Fig. 3-3 based on Geri’s circuit model [60].

Fig. 3-4a presents the transient voltages at points A, B and C for both the conductors shown in Fig. 3-3 based on infinite transmission line approach in [50]. Fig. 3-4b shows the transient voltages at same points for the same conductors adapted from Geri’s circuit model [60]. It is found that, based on the transmission line approach in [50], the maximum transient voltage at injection point is about 100 kV. While, based on the Geri’s circuit approach, the maximum transient voltage at injection point is about 160 kV, which is about 50% larger than the previous one. It is believed that the simulation results in [60] are more reliable, because they are comparable with the results based on electromagnetic field approach, except that Geri’s circuit model is incapable of predicting the wave propagation delay.

The infinite transmission line approach presented in section 3.1 does not use equation (3-3) for the per-unit length parameters calculation, as they were calculated based on the Finite Element Method software (Ace) in two-dimension. The boundary condition for the calculation of per-unit length parameters is based on thumb rules. There is no guarantee that the above said thumb rules will be suitable for all kind of grounding arrangements. On the other hand, even though this approach can be used to model the transient behaviour of 1x1 and 2x2 grounding grids, for the grids with 3x3 meshes or larger, it is difficult to create the distributed circuits for those larger grids in ATP-EMTP. Further, it has been realized that the grounding conductors of finite length subjected to lightning impulses do not have a field structure that is really TEM or quasi-TEM, consequently, the per-unit length parameters \((L, C\) and \(G\)) should include the influence of the conductor length.

3.2.1. III Uniform transmission line approach using length dependent per-unit length parameters

Based on the discussion in the previous paragraph, some researchers [49,51,52,59] have computed the total conductance, inductance and capaci-
tance of the finite length wire using Sunde’s equations [12] and distributed the parameters equally on a per unit length basis as shown in equation (3-4).

\[
G = \frac{\pi}{\rho_{\text{soil}}} \left[ \ln \frac{2l}{\sqrt{2ad}} - 1 \right] \tag{3-4a}
\]

\[
C = \frac{\pi \varepsilon_{\text{soil}}}{\ln \frac{2l}{\sqrt{2ad}} - 1} \tag{3-4b}
\]

\[
L = \frac{\mu_{\text{soil}}}{\pi} \left[ \ln \frac{2l}{\sqrt{2ad}} - 1 \right] \tag{3-4c}
\]

In equation (3-4), \( l_c \) is the length of the conductor. All the other parameters are same as that in equation (3-3). Using the above said distributed per-unit length parameters, the transient analysis was carried out by solving telegrapher’s equations. However, this method fails to predict the effective length of the grounding conductors. Here, the effective length is defined as the length of the grounding conductor, beyond which the transient voltage at injection point is length independent, for a given lightning impulse and soil properties. This definition is similar to the definition of the effective area of grounding grid in [58].

As an example, the transient behaviour of horizontal grounding conductors with different lengths, buried in the soil with \( \varepsilon_{r_{\text{soil}}} = 4 \), \( \rho_{\text{soil}} = 1000 \Omega \text{m} \) and \( \mu_{r_{\text{soil}}} = 1 \), are simulated. The current impulses similar with lightning wave shape, \( I(t) = 1 \cdot (e^{-27000t} - e^{-5600000t}) \) A (fast impulse) and \( I(t) = 1 \cdot (e^{-924t} - e^{-401090t}) \) A (slow impulse), are injected at one end of the conductors. The peak amplitudes of above two current impulses are 0.95 and 0.9 A. The first current impulse has 0.36 \( \mu \text{s} \) rise time, and second current impulse has 10 \( \mu \text{s} \) rise time. The lengths of the conductors are 20 m, 40 m, 50 m, 80 m, 100 m and 280 m. The radius of the conductors is 4 mm and the buried depth of the conductors is 0.75 m.

Fig. 3-5(a and b) show the transient voltage at the injection point for all the above said conductors for both fast transient (with 0.36 \( \mu \text{s} \) rise time) and slow transient (with 10 \( \mu \text{s} \) rise time) based on the uniform transmission line approach using length dependent per-unit length parameters. That is the per-unit length parameters, \( L \), \( C \) and \( G \) of the conductor are calculated using equations (3-4a, b, c) and they are the functions of length of the conductor. It is found in Fig. 3-5a and b that the transient voltage at injection point for the longer conductor (e.g. 280 m) is much higher than that of the shorter conductor (e.g. 20 m), especially for the injection current with fast rise time, which is incorrect from grounding system point of view.
After having gone through the drawbacks of the existing uniform transmission line approaches in the literature [49-52,59], and the one that we developed in section 3.1, we found that if one still wants to use transmission line approach for modelling transient behaviour of grounding systems properly, the existing transmission line approaches are not enough. That is why we developed the following non-uniform transmission line approach.

### 3.2.2 Non-uniform transmission line approach

The demonstration of non-uniform transmission line approach of the transient analysis of grounding structures starts from a single horizontal grounding conductor as shown in Fig. 3-6. The conductor is assumed as a thin wire
and divided into \( n \) small segments in order to calculate the per-unit length parameter matrixes

\[
R = \begin{bmatrix} R_{11} & R_{12} & \cdots & R_{1n} \\
R_{21} & R_{22} & \cdots & R_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
R_{n1} & R_{n2} & \cdots & R_{nn} \end{bmatrix}
\quad \text{and} \quad
P = \begin{bmatrix} P_{11} & P_{12} & \cdots & P_{1n} \\
P_{21} & P_{22} & \cdots & P_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
P_{n1} & P_{n2} & \cdots & P_{nn} \end{bmatrix},
\quad \text{and} \quad
L = \begin{bmatrix} L_{11} & L_{12} & \cdots & L_{1n} \\
L_{21} & L_{22} & \cdots & L_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
L_{n1} & L_{n2} & \cdots & L_{nn} \end{bmatrix}.
\]

The segment length should be chosen in such a way that the mutual coupling between the segments is effectively taken into account. This means one can not choose large segment lengths for the simulations as this situation will correspond to the uniform transmission line approach with electrode length dependent per-unit length parameters (See paper III and paper IV). In order to overcome this, in the present model, the segment length should be small enough (<< 1/10 of the wavelength in the soil corresponding to the highest frequency component of the current source) so that each segment during the wave propagation dynamically contributes to the coupling phenomena. Consequently, the number of segments, \( n \), should be large, but at the same time, the length of each segment, \( l_{\text{seg}} \), should satisfy the thin wire assumption, that is, \( l_{\text{seg}} >> a \). In this thesis, a segment length of 0.5 m to 2 m was found to be satisfactory.

\[\text{Fig. 3-6 Illustration of a single conductor, the discretization and the coupling}\]

3.2.2.1 Integral method for calculating the per-unit length parameter matrixes

The per-unit length parameters, such as self and mutual inductance, earth resistance and susceptance, can be derived based on electromagnetic field theory [12]. Those parameters are the function of the soil properties and the geometry of the system. The image principle should be used for the calculation of the earth resistance and susceptance parameters, because the soil is assumed to be semi-infinite medium and it has different conductivity and permittivity compared to the air. For the inductance, the image does not exist, because we assumed that air and soil are non-magnetic materials and they have the same permeability, \( \mu_0 \).

The elements of earth resistance matrix can be calculated by equation (3-5).
\[ R_{ij} = \frac{\nu_{ij}}{I_{ij}} = \frac{\rho_{\text{soil}}}{4\pi i_{ij} l_{ij}} \int_{r_{ij}}^{r_{ij}'} \frac{1}{dldl} + k_{\text{soil}} \frac{\rho_{\text{soil}}}{4\pi i_{ij} l_{ij}} \int_{r_{ij}'}^{r_{ij}''} \frac{1}{dldl} \]  
(3-5)

when \( i = j \), it is self earth resistance of segment, \( i \), when \( i \neq j \), it is the mutual earth resistance between two segments \( i \) and \( j \). Here, \( I_{ij} \) is the uniform dissipation current density flowing from the axis of segment, \( i \), to the soil.

\[ k_{\text{soil}} = \frac{\sigma_{\text{soil}} - \sigma_{\text{air}}}{\sigma_{\text{soil}} + \sigma_{\text{air}}} \]  

is the reflection coefficient due to the different conductivities of the air and the soil. \( \nu_{ij} \) is the average induced voltage per meter on the surface of the segment, \( j \), due to the dissipation current from segment \( i \), and \( \rho_{\text{soil}} \) is the resistivity of the soil. \( l_i, l_j, \) and \( l_i' \) are the length of the segments, \( i, j \) and image, \( i' \). \( r_{ij} \) and \( r_{ij}' \) are the distances between the source point and the field point.

Similarly, the elements of the susceptance and inductance matrices can be calculated (see the equations 3 and 4 in paper III). The only difference is that the reflection coefficient becomes \( k_{\text{epison}} = \frac{\epsilon_{\text{soil}} - \epsilon_{\text{air}}}{\epsilon_{\text{soil}} + \epsilon_{\text{air}}} \) or zero. And \( \rho_{\text{soil}} \) is replaced by \( 1/\sigma_{\text{soil}} \) or \( \mu_0 \). If the segments \( i \) and \( j \) are perpendicular to each other, the mutual inductance is zero.

3.2.2.2 Transient analysis

After knowing the per-unit length parameters of the wire and the injection current source that corresponds lightning impulse, the transient behavior of the grounding conductor can be simulated using modified telegrapher’s equations (3-6a and b).

\[ \frac{\partial V(x,t)}{\partial x} = r_e I(x,t) + l(x,t) \frac{\partial I(x,t)}{\partial t} \]  
(3-6a)

\[ \frac{\partial I(x,t)}{\partial x} = g(x,t)V(x,t) + c(x,t) \frac{\partial V(x,t)}{\partial t} \]  
(3-6b)

In equation (3-6), \( V(x,t), I(x,t) \) are the unknown distributed voltage and current along the grounding wire. \( r_e \) is the per-unit length series resistance. \( l(x,t), g(x,t) \) and \( c(x,t) \) are the effective per-unit length inductance, capacitance and capacitance of the conductor, respectively, at position \( x \) and time \( t \). telegrapher’s equations (3-6a and b) are solved using Finite Difference Time Domain (FDTD) method [25]. The most important stage while solving equation (3-6a and b) is to estimate the effective per-unit length parameters, \( l(x,t), g(x,t) \) and \( c(x,t) \), at each time step for each segment. While solving telegrapher’s equations using FDTD method, \( dx \) is equal to \( l_{\text{seg}} \), the length of each segment. Once the segment length is fixed, the time step, \( dt \), for the simulation should be chosen so that the \( \frac{dx}{dt} \) should be greater than the
maximum velocity of the wave propagation in the soil, \(\frac{v_{air}}{\sqrt{\varepsilon_{r_{soil}}}}\), which ensures the stable and consistent results. Here \(v_{air}\) is the speed of light in air.

Before we describe the methodology on how one calculates the effective per-unit length parameters of segment \(i\), \(l_i(t)\), \(g_i(t)\) and \(c_i(t)\), for each time step, it is necessary to understand how the uniform transmission line approach with electrode length dependent per-unit length parameters works.

Uniform transmission line approach in \([49, 51, 52, 59]\) used equation (3-4a, b, c) to calculate the per-unit length parameters (\(C, L\) and \(G\)), which included the influence of the length of the grounding conductor. The transient analysis was implemented using telegrapher’s equations (2-18 a and b). It has been demonstrated in paper III and paper IV that the above said transient analysis is not physical as explained below.

During the transient period, the current wave propagates from the first segment to the last segment with specific interval of time. Hence, the mutual coupling between any two segments can exist if and only if the two segments are sourced (that is when they carry current or charge) and in addition to that the value of coupling coefficients should be source dependent. So, in the uniform transmission line approach with electrode length dependent parameters, the mutual coupling between different segments has been over estimated during the transient analysis. The consequences of this over estimated mutual coupling between the segments were shown in Section 3.2.1.III, for example, in Fig. 3-5(a and b), the transient voltage at injection point for the longer conductor (e.g. 280 m) is much higher than that of the shorter conductor (e.g. 20 m), which is incorrect from grounding system point of view \([61]\). This means, for the given soil parameters, conductor cross section, and injection current impulse, the conductor with longer length should have transient voltage at the injection point that is either equal to or less than the conductor with shorter length.

The above explanation shows that

a) If the length influence should be included in the per-unit length parameters calculation, only for uniform source distribution, the coupling coefficients between any two segments will be unity.

b) During the transient period, the mutual coupling between different segments should not be always taken into account by unit coupling coefficients, because, in the impulse duration, the source distribution along the conductor will never be uniform.

Consequently, in the transient analysis of grounding systems, the coupling coefficients between any two segments should be varying with time for the effective per-unit length parameter calculations. Thus, in the present model,
the effective per-unit length parameters, \( l(x,t) \), \( g(x,t) \) and \( c(x,t) \), in telegrapher’s equations (3-6a and b) are the function of space and time.

The computation of effective per-unit length parameters is based on three different regions of the transient period. The first region is zero coupling region, which is applicable at \( t=0 \). As the impulse current source is not connected to the grounding conductor, the segments of the conductor are independent, that is, they are not coupled to each other. This situation can be interpreted as if the conductor segments were floating. Under above said condition, for any segment \( i \) (\( i = 1,\ldots,n \)), the effective per-unit length parameters are, \( g_i(t=0) = 1/R_{ii} \) and \( c_i(t=0) = 1/P_{ii} \), and \( l_i(t=0) = L_{ii} \). Here, \( R_{ii}, P_{ii} \) and \( L_{ii} \) are the diagonal values of the parameter matrices \([R]\), \([P]\) and \([L]\).

The second region is corresponding to any time, \( t>0 \), after the impulse source is injected into the grounding conductor, and it is called as transient coupling region. In this region, the mutual coupling between different segments starts to contribute to the calculation of the effective per-unit length parameters, \( g \), \( c \) and \( l \), based on the source distribution along the conductor.

It is important to note that a segment remains floating until it is sourced by the propagating impulse. And there will not be any mutual coupling between a floating segment and a sourced segment. The consequence of taking into account the time varying mutual coupling is to decrease effective \( c \) and \( g \), and at the same time increase effective \( l \) for a given segment. Depending on the type of current impulse, the decrement of \( c \) and \( g \), and increment of \( l \) will be in such a way that each segment can have values of effective per-unit length parameters which are at most, corresponding to the maximum coupling condition, uniform source distribution. Based on this knowledge, it is clear that the coupling coefficients between the segments for the calculation of the effective per-unit length parameters can only vary between zero and one. The third region is the case where the mutual coupling reaches a maximum and the coupling coefficients will be nearly unity. This is ideal situation corresponding to DC, which will never be reached for the current impulse that corresponds to lightning.

According to the above explanation, the straightforward calculation of the effective per-unit length parameters of \( i^{th} \) segment, \( g_i(n\Delta t) \), \( c_i(n\Delta t) \) and \( l_i(n\Delta t) \), at \( t = n\Delta t \) is as follows. Using the effective \( g_i((n-1)\Delta t) \), \( c_i((n-1)\Delta t) \) and \( l_i((n-1)\Delta t) \), the voltage at the \( i^{th} \) node \( V(i,n\Delta t) \) and the current through the \( i^{th} \) segment \( I(i,n\Delta t) \) can be calculated by equation (6a, b). The average voltage of the \( i^{th} \) segment at \( t = n \cdot \Delta t \) will be

\[
V_{ave}(i,n\Delta t) = 0.5 \cdot (V(i,n\Delta t) + V(i+1,n\Delta t)) \quad (i = 1,2\ldots,n)
\]

(3-7)

Then, the longitudinal current along the \( i^{th} \) segment is \( I(i,n\Delta t) \), the dissipation current from this segment and the charge on this segment at \( t = n \cdot \Delta t \) can be calculated as
After knowing the new source distributions, the effective per-unit length parameters of the $i$th segment at $t = n\Delta t$ can be calculated as

$$g_i(n\Delta t) = 1/(\sum_{j=1}^{n-1} R_j \cdot a_{ji})$$ (3-9a)

$$c_i(n\Delta t) = 1/(\sum_{j=1}^{n-1} P_j \cdot b_{ji})$$ (3-9b)

$$l_i(n\Delta t) = \sum_{j=1}^{n-1} L_j \cdot d_{ji}$$ (3-9c)

where,

$$a_{ji} = \left\{ \begin{array}{ll}
1, & (i = j) \\
\left[ I_{\text{dis}}(j,n\Delta t)/I_{\text{dis}}(i,n\Delta t) \right] & (0 < I_{\text{dis}}(j,n\Delta t) < I_{\text{dis}}(i,n\Delta t)) \\
1, & (I_{\text{dis}}(j,n\Delta t) > I_{\text{dis}}(i,n\Delta t) > 0) \\
0, & (\text{else})
\end{array} \right. $$

$$b_{ji} = \left\{ \begin{array}{ll}
1, & (i = j) \\
\left[ Q(j,n\Delta t)/Q(i,n\Delta t) \right] & (0 < Q(j,n\Delta t) < Q(i,n\Delta t)) \\
1, & (Q(j,n\Delta t) > Q(i,n\Delta t) > 0) \\
0, & (\text{else})
\end{array} \right. $$

$$d_{ji} = \left\{ \begin{array}{ll}
1, & (i = j) \\
\left[ I(j,n\Delta t)/I(i,n\Delta t) \right] & (0 < I(j,n\Delta t) < I(i,n\Delta t)) \\
1, & (I(j,n\Delta t) > I(i,n\Delta t) > 0) \\
0, & (\text{else})
\end{array} \right. $$

which means, for the segment $i$, the coupling from other segments, $j$, can be included by using coupling coefficients, $a_{ji}$, $b_{ji}$ and $d_{ji}$. Those coupling coefficients are varying between zero and one based on the source distribution along the conductor. When the ratios of the sources on segments, $j$ and $i$, are larger than one, we assume that the coupling coefficients between them are one in order not to violate the maximum coupling conditions. If one chooses the coupling coefficients equal to the ratios of the sources on the segments, which are larger than one, then the resulting effective per-unit length parameters will be smaller for $c$ and $g$, and larger for $l$ compared to the corresponding values obtained using uniform source distributions.

Once the new effective per-unit length parameters, $g_i(n\Delta t)$, $c_i(n\Delta t)$ and $l_i(n\Delta t)$ are calculated, the voltage at the $i$th node, $V(i,(n+1)\Delta t)$ and the current through the segment $i$, $I(i,(n+1)\Delta t)$, can be estimated by using the same procedure as explained before.
3.2.2.III Verification of the model

In order to verify the present non-uniform transmission line approach, same grounding wires as shown in Fig. 3-3 are used for the transient voltage calculation at point A, B and C. Same current impulse, $I(t) = 12935 \cdot (e^{-190099t} - e^{-2922879t})$ A, is injected at the end of the conductors. The simulation results are compared with circuit theory approach and the electromagnetic field approach [60].

Fig. 3-7 is the transient voltages at points A, B and C of both the conductors. It is clear that the present approach can predict the effective length, which is impossible for the transmission line approach with length dependent per-unit length parameters as shown in section 3.2.1.III.

It is observed that the results in Fig. 3-7 are comparable with that in [60]. Comparing the simulation results in Fig. 3-7 based on the present approach with that in [60] based on circuit and electromagnetic theory approaches (see Fig. 3-4b), it is observed that the difference in the voltages at point A is between 2%-9%. At points B and C, the simulation results of the present approach are similar to that of the circuit theory approach, but for both of the approaches, when compared to electromagnetic field approach, the differences is bigger. The reason could be due to the different domains (time domain and frequency domain) adopted in the models. This phenomenon needs more investigations.

On the other hand, in Fig. 3-7 it is also observed that non-uniform transmission line approach predicts the wave propagation delay, which is same as that of electromagnetic field approach [60], while, the circuit theory approach [60] could not predict the wave propagation delay (see Fig. 3-4b). This is because the non-uniform transmission line approach and electromagnetic field approach use space and time discretization in the simulation, when the telegrapher’s equations or Maxwell’s equations are solved, while the circuit theory approach only involves time discretization in the simulation, when the node equations are solved.
Fig. 3-7 Transient voltage at different points for horizontal grounding conductor 20 m and 100 m for fast impulse, and soil with $\varepsilon_{r_{soil}} = 50$, $\rho_{soil} = 100$ $\Omega m$ and $\mu_{r_{soil}} = 1$ based on the present model.

The present non-uniform transmission line approach can also be easily extended to complex grounding grids with large size under lightning strikes. The examples are shown in paper III, paper IV and paper VIII.
4 The model of soil ionization around the grounding conductors

High impulsive currents associated with lightning return strokes always produce ionization in the soil, and this effect of soil ionization should be accounted while modelling the transient behaviour of grounding systems. The knowledge of the non-linear behaviour of the grounding systems under the lightning strikes is not fully understood, perhaps due to its extremely complicated process, and therefore, some approximations are usually adopted when the effect of soil ionization is to be included. In this chapter, firstly, the traditional soil ionization model will be included into the simulation of transient behaviour of grounding systems. Secondly, an improved soil ionization model that considers residual resistivity in ionization region will be described. For more details of this part of work, kindly refer paper V and VI. The residual resistivity in ionization region is investigated based on the existing literature and the experiments that have been carried out at Uppsala University. For more details of this part of work, kindly refer paper VII.

4.1 Traditional model for soil ionization

Bellaschi in 1942 [62] and Petropoulos in 1948 [63] assumed that the soil ionization is uniform around the conductor and the resistivity of this ionization region decreases to the same value as that of the grounding conductor (almost zero) instantaneously. Since then, this assumption has been utilized by many researchers when the soil ionization phenomenon is to be included into the models for transient analysis of grounding systems [32,34,43,51,53]. Large increase in conductor diameter mathematically increases the per-unit length conductance ($G$) of the conductor, so that the grounding conductor can effectively dissipate more lightning current into the soil, and therefore reduces the potential rise at the injection point and the grounding impedance. This concept also can be adopted to simulate soil ionization phenomenon when the transient behaviour of grounding systems is modelled based on improved transmission line approaches, which have been described in chapter 3. Here, we will give one example to show that
how the traditional soil ionization model is combined with non-uniform transmission line approach.

As described in section 3.2.2.I, the grounding wires should be discretized to \( n \) segments first. Before the soil ionization occurs, the per-unit length parameters can be calculated using the method described in section 3.2.2.I. The radius used for the parameters’ calculation is the original radius of the conductor.

When the lightning current injected at one end of the wire, the transient analysis of grounding system starts with telegrapher’s equations (3-10) as described in section 3.2.2.II. If the dissipation current is high enough for a segment, the electric field intensity on the surface of this segment will exceed the critical electric field intensity value for soil ionization, \( E_0 \), and then, the soil ionization is initiated on that segment. The radius of the ionization region is increased to some certain distance where the electric field intensity finally falls to the critical value, \( E_0 \). This radius is calculated by [51]

\[
a_i = \frac{\rho_{\text{soil}} I_{\text{seg}}}{2\pi \ell_{\text{seg}} E_0}
\]  

In equation (4-1), \( I_{\text{seg}} \) is the dissipation current from the conductor segment to the soil, \( \ell_{\text{seg}} \) is the length of each segment, and \( \rho_{\text{soil}} \) is the pre-ionization soil resistivity. Based on Ballaschi’s soil ionization model [62], the radius of ionization region will be used as fictitious one for the new per-unit length parameters’ calculation. Equation (4-1) is for all the segments of horizontal grounding conductors and vertical rods, except the segment at the tip of the vertical rod. For the segment at the tip of the vertical rod, the radius of ionization region, \( a_i \), can be easily derived by adding a semi-sphere at the tip. It is given by

\[
a_i = \frac{1}{2} \left( -\ell_{\text{seg}} + \sqrt{\ell_{\text{seg}}^2 + \frac{2\rho_{\text{soil}} I_{\text{seg}}}{\pi E_0}} \right)
\]  

After calculating the new per-unit length parameters of grounding wires based on the fictitious radii of all the segments at each time step, the transient voltages and currents for the next time step can be calculated as described in section 3.3.2.II.

An example of transient analysis of grounding wire including soil ionization based on non-uniform transmission line approach is presented below. A horizontal grounding wire with 10 m length is buried at 0.5 m depth in the soil. The radius of the wire is 7.5 mm. The soil resistivity, \( \rho_{\text{soil}} \), and relative permittivity, \( \varepsilon_{r_{\text{soil}}} \), are 50 \( \Omega \)m and 10, respectively. The critical electric field for soil ionization, \( E_0 \), is 350 kV/m based on IEEE standard [64]. The
lightning current, \( I(t) = 10000(e^{-10000t} - e^{-20000t}) \) A, was injected to one of the ends of the wire. The voltages at injection point and at the other end of the wire with and without soil ionization are shown in Fig. 4-1a. It is found that when soil ionization is included into the transient analysis of grounding system, the potential rise due to large transient current is about 86% of that when the soil ionization is not included. This is because when the soil ionization occurs, the grounding wire dissipates more current to the soil. Fig. 4-1b shows the dissipation current of the first one meter of the wire to the soil with and without soil ionization. The simulation results show that if the same grounding wire is in the soil with resistivity of 1000 \( \Omega \)m, for the same injection current, the potential rise at injection point will decrease to about 60% of the one without soil ionization. This means that the influence of soil ionization in reducing the potential rise at injection point is more important in high resistivity soil.

In the present soil ionization model, it is assumed that the inductance and capacitance of each segment are not changing with the soil ionization, because the soil ionization will not change the soil permeability, and only slightly change the soil permittivity. However, the influence of the soil permittivity on the transient behaviour of grounding system is negligible compared with that of the soil resistivity [65].

Fig. 4-1a Transient voltage at injection point and the other end of the 10 m wire with and without soil ionization. \( \rho_{\text{soil}} = 50 \ \Omega \)m.
4.2 Improved model for soil ionization

It is very unlikely that the ionized soil has the same conductivity as that of the metal used for grounding electrode. Therefore, the assumption of equalizing the effect of soil ionization as increase in the diameter of the grounding conductors would be overestimating the beneficial influence of the soil ionization on limiting the ground potential rise, especially when the soil ionization region is very large and in the soil with high resistivity. Our analyses of the experimental results in [62, 66-67] clearly show that the resistivity of the ionized soil always keeps up some certain value, which is larger than that of the grounding conductors. Also, Loboda observed 10% to 30% voltage drop inside the ionization region for different kinds of soil in a coaxial cylindrical experimental setup in 1985 [68]. It means that the resistivity in ionization region could not be the same as that of the metal of the grounding electrode. Moreover, from the theory of soil ionization presented in [69], it can be inferred that soil retains some residual resistivity in the ionization region. The dynamic model for the soil ionization presented by Liew and Darveniza [67] assumed that the soil resistivity in the ionization region is changing with time according to an exponential law. It means the resistivity in ionization region will never go to zero.

4.2.1 Calculation of the residual resistivity in ionization regions

In order to prove the above observation, we first calculated the residual resistivity in ionization region by experimentally determining the transient impedance of the grounding rods or the spherical electrodes in the soil at the peak current when the soil ionization occurs [62, 66-67]. Those works were published by Bellaschi et.al. (1942), Liew and Darveniza (1973) and Oettle
(1988), respectively. Here, the transient impedance is defined as the instantaneous ratio of the voltage at the injection point to the injection current [70]. The choice of the impedance of the grounding electrodes at the peak current is favoured because the highest ground potential rise due to lightning strikes usually happens at or before the peak current.

Bellaschi et al. performed impulse tests for different grounding rods with different radii and lengths, for instance, the rods F, M and K as shown in Table 4-1, in different types of soil. Oettle's experimental arrangement was a hemispherical vessel filled with the soil, a small spherical high voltage electrode at the centre of the vessel with half of the volume buried in the soil (See Fig. 10 in [66]). In Liew and Darveniza's experiments, a single vertical rod buried in the soil with original resistivity of 50.0 Ωm was used for the impulse test in order to verify their dynamic model of grounding rod with impulse current injection. All the parameters that were involved in the above said experiments, such as the dimensions of the electrodes (rod with radius, \( a \) and length \( l_c \), spherical electrode with radius \( a \), hemispherical vessel with radius, \( r_v \)), the resistivity of the soil before ionization, \( \rho_{soil} \), the peak value of injection current, \( I \), and the critical electric field intensity for soil ionization for different soils, \( E_{ion} \), are listed in Table 4-1, 4-2 and 4-3. Based on the above said parameters, the residual resistivities, \( \rho_{res} \), in soil ionization regions were calculated and also listed in the above said Tables.

### Table 4-1 Residual soil resistivity calculation in ionization region for Bellaschi’s experiments.

<table>
<thead>
<tr>
<th>Rod</th>
<th>( a ) (mm)</th>
<th>( l_c ) (m)</th>
<th>( \rho_{soil} ) (Ωm)</th>
<th>( E_0 ) (kV/m)</th>
<th>( I ) (kA)</th>
<th>( \rho_{res} ) (Ωm)</th>
<th>For experimental results</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rod F</td>
<td>2.7</td>
<td>3.0</td>
<td>77.2</td>
<td>1.27</td>
<td>10.84</td>
<td>4.7 (3.3% of ( \rho_{soil} ))</td>
<td>For the experimental results in Fig. 11 [62]</td>
</tr>
<tr>
<td>Rod M</td>
<td>1.94</td>
<td>2.4384</td>
<td>157</td>
<td>2.0</td>
<td>6.8</td>
<td>4.0 (4.6% of ( \rho_{soil} ))</td>
<td>For the experimental results in Fig. 16 [62]</td>
</tr>
<tr>
<td>Rod K</td>
<td>1.94</td>
<td>2.4384</td>
<td>310</td>
<td>0.7</td>
<td>5.6</td>
<td>1.7 (7.5% of ( \rho_{soil} ))</td>
<td>For the experimental results in Fig. 14 [62]</td>
</tr>
</tbody>
</table>

### Table 4-2 Residual soil resistivity calculation in ionization region for Liew and Darveniza’s experiments.

<table>
<thead>
<tr>
<th>Rod dimension</th>
<th>Soil condition</th>
<th>( \rho_{soil} ) (Ωm)</th>
<th>( E_0 ) (kV/m)</th>
<th>( I ) (kA)</th>
<th>( \rho_{res} ) (Ωm)</th>
<th>For the experimental results in Fig. 3 [67]</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a = 6.35 ) mm, ( l = 0.61 ) m</td>
<td>( \rho_{soil} = 5.0 ) Ωm</td>
<td>1.1</td>
<td>3.3</td>
<td>11.6 (23.3% of ( \rho_{soil} ))</td>
<td>For the experimental results in Fig. 3 [67]</td>
<td></td>
</tr>
<tr>
<td>( a = 9.7 ) kA</td>
<td>( \rho_{soil} = 6.4 ) Ωm</td>
<td>(13% of ( \rho_{soil} ))</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( I = 13.0 ) kA</td>
<td>( \rho_{soil} = 4.3 ) Ωm</td>
<td>(23.8% of ( \rho_{soil} ))</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
In order to have additional evidences to support the above said phenomena that there is residual resistivity remaining in ionization region, a series of impulse tests on the sand with different original resistivities before breakdown were carried out in the high voltage laboratory of Uppsala University. The experimental setup is shown in Fig. 4-2. The inner sphere electrodes with diameters of 30 mm and 63 mm were used for the test, and the outer electrode is a vessel, which has a diameter 500 mm. The material of the bigger inner electrode and vessel is brass, and the smaller inner electrode is copper.

The current generator was used as impulse source. The wave shape of the impulse current is depending on the impedance of the testing circuit. For the sample with high impedance, the rise time of the impulse current was about 1 μs, and the decay time was longer than 1 ms. For the sample with low impedance (after the significant ionization has started), the rise time of the current impulse was about 100 – 200 μs, while the wave shape of the voltage impulse only changed the decay time. The parameters of sand samples are listed in Table 4-4. Finally, the residual resistivities in ionization region are calculated based on the above said experimental parameters. The critical electric field intensities for soil ionization in different soil were determined experimentally as described in paper VII.

Table 4-3 Residual soil resistivity calculation in ionization region for Oettle’s experiments.

<table>
<thead>
<tr>
<th>Sample</th>
<th>d (mm)</th>
<th>g (mm)</th>
<th>E0 (kV/cm)</th>
<th>( \rho_\text{fail} ) (Ωm)</th>
<th>( E_1 ) (kV/cm)</th>
<th>( \rho_\text{fail} ) (Ωm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>5</td>
<td>250</td>
<td>8.0</td>
<td>690</td>
<td>76.0</td>
<td>57.4</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>250</td>
<td>8.0</td>
<td>690</td>
<td>157.0</td>
<td>20.0</td>
</tr>
<tr>
<td></td>
<td>1.5</td>
<td>250</td>
<td>8.0</td>
<td>646</td>
<td>30.0</td>
<td>3.5</td>
</tr>
<tr>
<td></td>
<td>1.5</td>
<td>250</td>
<td>8.0</td>
<td>646</td>
<td>60.0</td>
<td>5.5</td>
</tr>
<tr>
<td></td>
<td>2.5</td>
<td>250</td>
<td>8.0</td>
<td>643</td>
<td>34.0</td>
<td>4.1</td>
</tr>
</tbody>
</table>

**Fig. 4-2 Illustration of the experimental setup**
A thorough analysis of the experimental results found in the literature [62,66-67] and the experiments which have been carried out in high voltage laboratory of Uppsala university shows that the soil in ionization region retains some percentage of the original or pre-ionization soil resistivity. Combining all the above analyses, the percentage of the residual resistivity in ionization region at peak current versus soil resistivity for different injection currents and different electrodes is plotted in Fig. 4-3. It shows, for different types of electrode, different soil resistivities ranging from 50 $\Omega$m to 827 $\Omega$m, and different values of injection currents ranging from several amperes to several thousand amperes, the residual resistivity in the ionization region varied in a range from 1.7% to 47% of the original soil resistivity with very large scatter. However, the distribution is approximately lognormal with geometric mean of 6.77%, and the medium value of 6.6%. Consequently, a typical residual resistivity in ionization region of 7% might be chosen to model the effect of soil ionization even in the soil with higher original resistivity than the ones reported here. But, more experiments may be required to find out the full information of the relationship between the original soil resistivity and the residual resistivity in ionization regions, especially for the soil with high resistivity, which could result in more accurate geometric mean value of the residual resistivity in ionization region.

Table 4-4 Parameters of different soil samples

<table>
<thead>
<tr>
<th>Sample's number</th>
<th>Content of the sample</th>
<th>Diameter of the inner electrodes (mm)</th>
<th>Soil resistivity (Gm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sample 1</td>
<td>35 litres dry sand, 3 litres pure water, 18 grams salt</td>
<td>63</td>
<td>174</td>
</tr>
<tr>
<td>Sample 2</td>
<td>35 litres dry sand, 2.5 litres pure water, 6 grams salt</td>
<td>63</td>
<td>355</td>
</tr>
<tr>
<td>Sample 3</td>
<td>35 litres dry sand, 2 litres pure water, 3 grams salt</td>
<td>30</td>
<td>579</td>
</tr>
<tr>
<td>Sample 4</td>
<td>35 litres dry sand, 1.5 litres pure water, 1.5 grams salt</td>
<td>30</td>
<td>827</td>
</tr>
</tbody>
</table>

Table 4-5 Residual soil resistivity in ionization region for the experiments in Uppsala University

<table>
<thead>
<tr>
<th>Sample</th>
<th>Current and residual resistivity in ionization region</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sample 1 ($\rho_{ion}=174$ $\Omega$m)</td>
<td>I=24.7 A (7.3% of $\rho_{soil}$)</td>
</tr>
<tr>
<td>Sample 2 ($\rho_{ion}=256$ $\Omega$m)</td>
<td>I=25.8 A (13.2% of $\rho_{soil}$)</td>
</tr>
<tr>
<td>Sample 3 ($\rho_{ion}=759$ $\Omega$m)</td>
<td>I=5.0 A (15.8% of $\rho_{soil}$)</td>
</tr>
<tr>
<td>Sample 4 ($\rho_{ion}=827$ $\Omega$m)</td>
<td>I=2.6 A (46.7% of $\rho_{soil}$)</td>
</tr>
</tbody>
</table>
4.2.2 Implementation of the improved model for soil ionization

The principles used to include the residual resistivity in ionization region for modelling the soil ionization are as follows. When the current, dissipating from the grounding conductor into the soil, is high enough, the electric field intensity on the surface of the grounding conductor could exceed the critical electric field intensity value, $E_0$, and then, the soil ionization is initiated.

The radius of the ionization region still can be calculated by equations (4-1) and (4-2). Then, we assume that the soil resistivity in this ionization region decreases to 7% of the pre-ionization soil resistivity. It means that, for the same injection current, the lower is the original resistivity of the soil, the lower is the absolute retaining resistivity in the ionization region. The value of 7% was selected based on the discussions in the previous section.

Here, the model of transient analysis of grounding systems is based on infinite transmission line approach, which has been described in section 3.1. The reason for choosing above said approach instead of non-uniform transmission line approach is as follows. As described in section 3.1, the per-unit length parameters of the grounding conductors, such as per-unit length self and mutual inductance, capacitance and the conductance to the earth, are calculated in a field simulation program based on Finite Element Methods (FEM), which is extremely convenient for the field calculation in non-uniform soil. For instance, in the stratified soil and when the soil ionization occurs, the soil in ionization region has lower resistivity than that of the soil without ionization. On Contrary, the analytical equations used for the per-unit length parameters’ calculation in non-uniform transmission line approach, as described in section 3.2.2.1, are only valid for homogeneous soil.
condition, which will become very complicated and mathematically involved if those equations were to be extended to inhomogeneous soil.

According to infinite transmission line approach described in section 3.1, after soil ionization occurs, for different size of ionization region, the per-unit length conductance of the grounding conductor including residual resistivity in ionization region can be calculated using FEM before the transient behaviour of grounding systems is simulated in ATP-EMTP. By using curve fitting, one could get the equation of variable conductance of the grounding conductor as a function of the size of ionization region. As we described at the beginning of this section, the size of ionization region is related to the dissipation current from the grounding conductor to the soil. Consequently, by using Models Language, a feature in ATP-EMTP [71], it is easier to link the dissipation current of each segment to its variable conductance in ATP-EMTP. Based on the same reasons as in section 4-1, we assume that soil ionization will not influence the capacitance and inductance of the grounding conductor. Finally, the transient behaviour of grounding systems including the above said new conductance parameter that is changing at each time step based on soil ionization can be simulated in ATP-EMTP as described in section 3.1.

4.2.3 Importance of residual resistivity in ionization region — comparison between traditional model and the presented model

To verify the importance of modelling the ionization region keeping some residual resistivity, one grounding rod is chosen for simulation. The diameter of the rod is 50 mm, and the length is 1 m. The soil is assumed homogeneous, and the soil resistivity, relative permittivity and permeability are \( \rho_{\text{soil}} = 2000 \, \Omega \cdot \text{m} \), \( \varepsilon_{r, \text{soil}} = 10 \), and \( \mu_{r, \text{soil}} = 1 \), respectively. The critical electric field intensity for soil ionization, \( E_0 \), is 1000 kV/m, which is chosen according to the experimental observation that critical electric field intensity for soil ionization is increasing with the increase of the soil resistivity [72]. The double exponential form current impulse, \( I(t) = 10000(e^{-1.2000t} - e^{-5.46000t}) \) A, is injected from the top of the rod. The peak value of the current impulse is about 10 kA.

Fig. 4-4 shows the injection current and the transient voltages at the injection point of the vertical rod with the resistivity in the ionization region retaining 7% of the original value (case 2). It is compared to the voltages with the resistivity in the ionization region the same as that of the grounding conductor itself (case 1). It is observed that, for the resistivity in ionization region the same as that of the conductor, the amplitude of the transient voltage at the injection point is about 1.71 MV. While, the peak voltage when
the ionization region is assumed to retain 7% of the original soil resistivity is about 2.48 MV, which is 45% higher than the results of case 1. From this example, it is clear that modelling the ionization region as having the same conductivity as that of the electrode could over estimate the beneficial influence of the soil ionization, especially in the soil with high resistivity as shown here. Consequently, the improved model of soil ionization in this chapter is more suitable for modelling the transient behaviours of the grounding system under the lightning stroke when the soil ionization should be included.

![Fig. 4-4 Injection current and transient voltage of the grounding rod at the injection point. Case 1: soil ionization modelled as increasing the size of the conductor. Case 2: soil ionization modelled as decreasing the resistivity in ionization region to 7% of 2000 $\Omega \cdot m$.](image)
5 Applications for different practical problems

The aim of developing mathematical models for grounding systems is not only to analyse the effectiveness of various existing grounding systems in detail, but also to come up with methods of designing effective grounding systems and optimize the design criteria considering safety, space and economics. In this regard, the transmission line approaches are adopted to study the response of grounding systems due to direct lightning strikes for different applications. These applications are

- Estimation of DC resistance of complex grounding structures.
- The influence of soil parameters on the transient behaviour of grounding systems.
- Transient analysis of grounding structures in stratified soils.
- The validity of existing definitions of effective length/area of different grounding structures and their empirical equations.
- Current distribution in the shields of underground cables associated with communication tower.
- Influence of insulator flashover and soil ionization around the pole footing in Swedish railway system.

For more details of the above said works, kindly refer report 1, paper I, paper II, paper V, paper VI, paper VIII, report 2 and paper IX.

5.1 DC resistance of complex grounding structures under communication towers

In general, communication towers are installed on the top of hills or mountains, where, the soil resistivities are usually high. From a safety point of view, the grounding system under the tower should have low DC resistance. A typical grounding system for a communication tower is that the tower legs with foundations connected to a ring conductor forms the main part of the grounding structure. In addition to that, there are four vertical rods connected at the corners of the ring conductor. It is believed that extra vertical rods, which are connected with ring conductor, could be very helpful in reducing the DC resistance of the whole grounding system. In this section, we will investigate
- How much the vertical rods can reduce the DC resistance of the grounding system under the tower? In poorly conducting soil, can the DC grounding resistance of the tower reduce to a value which is below 20Ω as suggested in Swedish Standard [73]?
- What is the influence of tower legs and foundations on the overall DC resistance of grounding system?
- How much the horizontal grounding wires can help in reducing the DC resistance of grounding system?

5.1.1 Method of DC resistance calculation

The estimation of DC resistance of grounding system can be carried out by solving Laplacian equation \( \nabla^2 V = 0 \) [12]. That is, it can be obtained by calculating the electrical coupling between the conductor segments due to the dissipation currents from the grounding conductors into the soil. It is true that the dissipation current from each segment is uniform under DC conditions. It is assumed that the grounding conductors follow the thin wire approximation.

As long as the geometry of grounding system is complex; only numerical methods can be adopted. In order to get more accurate results by including all the mutual coupling between the conductors in the system, the whole grounding system is divided into smaller segments. Then, the per unit length earth resistance matrix of grounding system

\[
R = \begin{bmatrix}
R_{11} & R_{12} & \cdots & R_{1n} \\
R_{21} & R_{22} & \cdots & R_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
R_{n1} & R_{n2} & \cdots & R_{nn}
\end{bmatrix},
\]

is to be calculated using equation (3-5) for each element. Here, \( n \) is the number of segments. Once the per-unit length earth resistance matrix is known, the per-unit length conductance matrix can be calculated by

\[
[G] = [R]^{-1} \tag{5-1}
\]

Having known the per-unit length conductance matrix, the average resistance of the whole grounding system can be obtained by equation (5-2).

\[
R_{DC} = \frac{1}{\sum_{j=1}^{N} (\text{self}G_j \times I_{\text{seg}})} \tag{5-2}
\]

In equation (5-2), \( \text{self}G_j = \sum_{j=1}^{N} G_j, I_{\text{seg}} \) is the length of \( i^{th} \) segment.
5.1.2 Typical example of grounding system under a communication tower

In order to investigate the questions, which we raised at the beginning of this section, a grounding system under the communication tower used by Banverket (see Fig. 5-1) is considered as an example. The grounding system is buried at about 0.5 m depth in the soil. The size of the ring conductor is about 6 m x 10.5 m. The cross section of the ring conductor and the connection between ring conductor and tower legs is 35 mm². The ring conductor forms the main grounding system for the tower and the technical room beside the tower. The length of the tower legs is 1.825 m in the soil, and the foundation holds the tower legs. A bare conductor, buried at the same depth as ring conductor (0.5 m), connects the ring conductor to the s-rail of tracks. The length of this bare conductor is varying from 20 m to 100 m, and the cross section is 50 mm². Assume the soil is uniform with 2500 Ωm resistivity.

![Fig. 5-1a Plan view showing the ring conductor and the horizontal conductor to S-rail of tracks.](image1)

![Fig. 5-1b 3-D view, geometry of grounding system under the tower used in simulations](image2)
After having calculated the DC resistances of above said grounding system for different arrangements, it is found that (see report 1 for details)

- The vertical rods, which are connected to the ring conductor, are not very useful for decreasing the DC resistance of the whole system, while the horizontal conductor, which is connected to s-rail, can decrease the DC resistance of whole system a lot when its length is increasing (see table 5-1 and table 5-2). However, vertical rods can be beneficial if it can reach substantially low resistivity soil underneath the high resistivity layer.

Table 5-1. Influence of four extra vertical rods. In all the cases, there is a horizontal conductor of 18 m, buried at the depth of 0.5 m in the soil. S-rail is not included in the calculations

<table>
<thead>
<tr>
<th>Vertical rod</th>
<th>The length of conductor (m)</th>
<th>The resistance of the whole structure (Ω)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 (no vertical rods)</td>
<td>92.3</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>92.0</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>90.6</td>
<td></td>
</tr>
<tr>
<td>4.5</td>
<td>87.7</td>
<td></td>
</tr>
</tbody>
</table>

Table 5-2. Influence of horizontal conductor (connected to the ring conductor and s-rail). S-rail is not included in the calculations.

<table>
<thead>
<tr>
<th>Horizontal conductor connected to s-rail</th>
<th>The length of conductor (m)</th>
<th>The resistance of the whole structure (Ω)</th>
</tr>
</thead>
<tbody>
<tr>
<td>18 (no vertical rod)</td>
<td>92.3</td>
<td></td>
</tr>
<tr>
<td>30 (no vertical rod)</td>
<td>74.2</td>
<td></td>
</tr>
<tr>
<td>99 (no vertical rod)</td>
<td>44.2</td>
<td></td>
</tr>
</tbody>
</table>

- The foundation and tower legs can decrease the DC resistance of the tower grounding by about 15% (see Table 5-3).

Table 5-3. Influence of the tower legs and the foundation. S-rail not included in the calculations.

<table>
<thead>
<tr>
<th>Horizontal conductor connected to s-rail (m)</th>
<th>Vertical rod (m)</th>
<th>The resistance of the whole structure (Ω)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exclude Foundation</td>
<td>26</td>
<td>No vertical rod</td>
</tr>
<tr>
<td>Include Foundation</td>
<td>18</td>
<td>4.5</td>
</tr>
<tr>
<td>Include Foundation</td>
<td>36</td>
<td>No vertical rod</td>
</tr>
<tr>
<td>Include Foundation</td>
<td>18</td>
<td>4.5</td>
</tr>
</tbody>
</table>

- The vertical rod could be replaced by horizontal conductor with cross section of 35 mm², and the DC resistance of the tower grounding can be decreased by increasing the length of the horizontal conductor (Table 5-4). The foundation is included in the calculation.

Table 5-4. Influence of the horizontal conductors, which replace the extra vertical rods. S-rail is not included in the calculations
5.2 Sensitivity analyses of soil parameters and the dimension of the grounding wire

If the transmission line approaches are used for modelling the transient behaviour of grounding system, all the parameters of the grounding system, such as the soil relative permittivity $\varepsilon_{\text{soil}}$, soil resistivity, $\rho_{\text{soil}}$, and the diameter of the conductor, are related to the electric circuit elements through their per-unit length parameters. Therefore, the influence of these parameters can be explained based on circuit theory.

- **Influence of soil resistivity $\rho_{\text{soil}}$**
  For the same grounding structure that has large size and the same injection current impulse, if the soil has high resistivity, the grounding system will have larger potential difference at different points, especially at the beginning of the current impulse. This can be explained as follows. In the soil with high resistivity, the earth conductance of the grounding wires is much smaller than that in the soil with low resistivity, so, for the same dissipation current, the potential rise of same grounding wire is much higher in high resistivity soil. Thus, at the beginning of current impulse, the part of grounding system from where the current is dissipating into the soil with high resistivity will have higher potential than that in low resistivity soil. On the other hand, the other parts of grounding system where the current impulse has not reached, the potential at those parts will be almost zero. Consequently, there will be big potential difference between different parts of grounding system at the beginning of current impulse. The problem of uneven voltage distribution is very important in multipoint grounding system. If a complex electrical system is connected to the grounding grid with multipoint grounding method, different points will have different potentials under lightning strikes. Then unexpected current from the grounding system might flow into the electric system and could cause serious EMC problems.

- **Influence of the Soil Permittivity $\varepsilon_{\text{soil}}$**
  The influence of the soil relative permittivity $\varepsilon_{\text{soil}}$ is more related to the capacitive coupling between the grounding conductors because the soil is a

<table>
<thead>
<tr>
<th>Horizontal conductor connected to ring conductor</th>
<th>Length of conductor (m)</th>
<th>Horizontal conductor connected to a coil (m)</th>
<th>The resistance of the whole structure (Ω)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>5</td>
<td>18</td>
<td>86.7</td>
</tr>
<tr>
<td></td>
<td>15</td>
<td>18</td>
<td>86.8</td>
</tr>
<tr>
<td>Vertical rod only</td>
<td>4.5</td>
<td>18</td>
<td>87.7</td>
</tr>
</tbody>
</table>
dielectric medium. So, it is obvious that the influence of the soil permittivity on the transient behaviour of the grounding system is effective through the influence of the self and mutual capacitance of the grounding conductors. In the soil with lower resistivity, the conduction current is dominant, so, the influence of the soil permittivity on the transient behaviour of the grounding system compared with that of soil resistivity is very small and could be neglected. In high resistivity soil, the capacitive coupling is more effective because the displacement current is comparable with conduction current, especially, during the rise time of injection current where high frequencies are dominant. Consequently, in the soil with very high resistivity, the influence of the soil permittivity on the transient behaviour of the grounding system may be considered for better accuracy.

- **Influence of the conductor’s conductivity and skin effect**

The conductivity and the skin effect of the conductor mainly influence the per-unit length internal resistance of the conductor. When the radius of the conductor is constant, the per-unit length resistance of the conductor is proportional to the inverse of the conductivity, \( r_e \propto 1/\sigma \). If the injection current impulse has high frequency and the skin depth of the conductor is smaller than the radius of the conductor, the skin effect should be considered. Then, the per-unit length resistance of the conductor will be proportional to the root of the frequency, \( r_{HF} \propto \sqrt{f} r_e \). For example, if one of the frequency component of the lightning current impulse is about 100 kHz, for a grounding wire with 14 mm radius, the per-unit length resistance of the conductor will be 300 times the DC resistance at this frequency. It is found from the simulations that the influence of increasing the resistance of the conductor on the transient behaviour of the grounding system is very small compared to that of the loss in the soil, so that the skin effect can be neglected.

- **Influence of the grounding conductors’ diameter**

The influence of the grounding conductors’ diameter on the transient behaviour of the grounding system is more complex than the influence of the soil resistivity and permittivity, because when the diameter of the conductor changes, the per-unit length resistance, inductance, conductance and capacitance will change. For example, when the diameter of grounding conductor is increasing, the self-inductance decreases, and the resistance of the conductor decreases, while the capacitance increases and the earth conductance increases.

It is found from simulations that increasing the diameter of the conductor can decrease the transient voltage at the injection point. Here, the influence of the decrease in resistance could be the smallest, because the influence of series resistance of the conductor is much smaller than that of the self-
inductance when the current changes very fast, and also much smaller than that of the earth resistance of the conductor. Consequently, increasing the diameter of the conductor is advantageous to decrease the peak value of the transient voltage of the grounding system at injection point mainly because of the decrease of the self-inductance and the increase of the earth conductance of the conductor. This result also indirectly shows that the soil ionization can help the grounding system to dissipate the current and decrease the transient voltage faster, because in general the soil ionization is modelled as homogeneous increase in the diameters of the grounding conductors as described in chapter 4.

5.3 Grounding systems in stratified soil

When the grounding systems have to be designed, soil condition should be considered first. For different soil conditions, one should use different grounding structures. For example, in high resistivity soil, large size of horizontal grounding systems are better than the vertical rods, because in high resistivity soil, it is difficult to install vertical rods with longer lengths. Further, as described in section 5.1, the vertical rod is even not very efficient for decreasing the DC grounding resistance. In practice, the soil is usually having stratified structure rather than being homogeneous. Under those situations, the grounding systems should be arranged in such a way that it can reduce the transient impedance. Here, the infinite transmission line approach is used for modelling the transient behaviour of grounding systems including soil ionization with residual resistivity in ionization region. Based on the simulation results, we will show that, in different stratified soils, what kind of grounding conductor arrangement is most efficient.

One of the typical two-layer soils is that the upper layer soil has high resistivity and the bottom layer soil has very low resistivity. The above said soil structure can represent the soil condition in winter months, where the frozen upper layer has high resistivity compared to that of the unfrozen bottom layer, and the difference of the resistivity in those two layers could be several thousand $\Omega \cdot m$ [57]. Further, this type of two-layer soil could also represent a soil with sand in upper layer and clay in bottom layer. Under the above said soil condition, we found that a vertical rod, which only has small part of the length penetrating into the low resistivity soil (see Fig.5-3a), is very efficient to reduce the transient voltage at injection point.
Another typical two-layer soil is that the upper layer soil has low resistivity and the bottom layer soil has high resistivity. This soil condition can represent the soil situation in summer, when lightning is associated with heavy rain. In that case, the wet top layer could have much lower resistivity than the dry bottom layer [57]. Under this soil condition, a horizontal grounding structure buried in low resistivity soil layer is very efficient.

From the examples in paper VI and paper VII, it is found that the transient voltage at the injection point for the grounding rod in homogeneous high resistivity soil ($\rho_{\text{soil}}=2000 \ \Omega\text{m}$) is about 12 times of that in stratified soil with the 20% length of the same rod penetrating into the low resistivity soil layer (see Fig. 5-3a). It is also observed that the transient voltage of the star-shaped grounding structure (see Fig. 5-3b) at the injection point in stratified soil decreases by 8 times compared to that in homogeneous soil with high resistivity ($\rho_{\text{soil}}=2000 \ \Omega\text{m}$). Consequently, in stratified soil, to make the grounding conductor penetrate the lower resistivity soil layer could help to decrease the ground potential rise at the injection point very much, because most of the current will be dissipated into the lower resistivity soil and cause more soil ionization in this soil layer.

5.4 Investigating the validity of existing definitions for effective length/area of grounding structures and their empirical equations

The effective length/area of grounding wire/grid under fast transients like lightning is one of the most important parameters that must be considered when the grounding system is being designed. This parameter is important in the sense that it not only gives a design that protects personnel and electric facilities, but also brings about considerable saving in the material and laying costs. The main purpose of this section is to investigate the validity of existing definitions and empirical equations for effective length/area of grounding structures using the non-uniform transmission line approach described in chapter 3. Here, the soil breakdown phenomenon, which occurs...
only if the strength of lightning current is sufficiently high, is neglected because this phenomenon only helps to reduce the potential rise of the grounding systems, hence, it does not form the worst situation.

5.4.1 Effective length of horizontal grounding wire

There are several definitions for effective length of horizontal grounding wire available in the literature [52,74,75]. In order to compare them, all the existing definitions in the literature, are listed below.

Definition 1. The effective length of single horizontal grounding wire is from current injection point to a distance at which the voltage reaches 3% of its value at current entrance point [74].

Definition 2. The effective length of single horizontal grounding wire is the length, above which no further reduction of the transient impedance is observed [52].

Definition 3. The effective length is defined as the length of the horizontal grounding conductor, beyond which the maximum transient voltage at injection point is length independent [75].

As shown in paper VIII, the drawback of definition 1 for effective length is that the transient voltage at the far end of the conductor (the point that is far away from the injection point) is usually larger than 3% of the corresponding value at current injection point and this value falls below 3% only when the lengths of conductor are large. But, it is observed that the maximum transient voltage at injection point has already reached some asymptotic value much earlier when the conductor length is shorter, and any additional length of the grounding conductor does not contribute to reducing the potential rise at current injection point. Such phenomenal difference becomes more serious in the high resistivity soil.

Definitions 2 and 3 are somewhat similar. In definition 2, the effective length can be found by comparing the transient impedances at the injection point of grounding wires for increasing lengths, while, in definition 3, transient voltages at injection point are compared. Hence, the determination of the asymptotic behaviour of either the maximum transient impedance or voltage at injection point is the basis of above two definitions (definition 2, 3). In principle, the above said asymptotic values occur when the length of the grounding wire is infinite. Thus, definition 2 and 3 pose a difficulty for determining the effective length of grounding wire in practice.

Based on the investigations in paper VIII, it was found that a practical method to calculate the effective length of grounding wire could be as follows.

Method a). Effective length of single horizontal grounding wire is the length of the wire, at which the maximum transient impedance at injec-
tion point is 3% larger than its final asymptotic value determined assuming that the difference in the maximum transient impedances at injection point between two successive lengths ($l_c$ m, ($l_c$ +1) m) is only 0.001 $\Omega$.

From the simulations, it is found that the effective length based on the definition 2 or 3 is comparable with that of the method a) when the injection current has fast rise time (0.1 $\mu$s). When the rise time of injection current is slow (10 $\mu$s) or when the grounding wire is in high resistivity soil (2000 $\Omega$m), the effective lengths based on the definition 2 or 3 is about 20% larger than that of the method a), under such a circumstance, the transient impedance at the injection point based on method a) is only 3% larger than the corresponding value obtained using definition 2 or 3.

5.4.2 Effective area of grounding grid

For the effective area of grounding grid, there are two definitions available in the literature [22,58].

Definition 4. The effective area of grounding grid is the area beyond which the maximum voltage at the injection point remains constant [58].

Definition 5. The effective area of the grounding grid is the area beyond which the decrement of transient impedance at the injection point has decreased to a value within 3% of the final maximum transient impedance of the grid [22].

For the grids with smaller inner mesh size (6m x 6m or less), definition 5 can be implemented similar to method a). When the inner mesh size is above 6m x 6m, here, again, we have a difficulty to implement definition 5, because it is computationally involved in order to determine the final maximum transient impedance first. Consequently, we have modified the method a) for the grids with larger inner mesh size as follows.

Method b). The effective area of grounding grid is the area, beyond which the decrement of maximum transient impedance at the injection point between the grid with N x N inner meshes and the grid with (N+1) x (N+1) inner meshes has decreased to a value within 1%.

If one estimates the effective area of grid with inner mesh size of 10 m x 10m buried in the soil with 1000 $\Omega$m resistivity. When a current with 10 $\mu$s rise time is injected in the corner of the grid, the effective area of above said grounding grid using method b) would be 22500 m$^2$ (the grid with 15x15 inner meshes), which is about 44% less than the area (40000 m$^2$) corresponding to the asymptotic value of the transient impedance. But, the transient
impedance at the injection point for the above said grid (15x15 inner meshes) is comparable with the corresponding value obtained using definition 5.

5.4.3 The validity of existing empirical equations for the effective length/length of grid edge

For engineering applications, the best way is to come up with some empirical equations to calculate the effective length/area of grounding structures, which has been attempted in [22,74]. Having performed calculations based on the non-uniform transmission line approach in section 3.2.2, it has been found that there are some points worth mentioning here.

Firstly, the empirical equation for the effective length of a single horizontal grounding wire in [74] is

\[ l_e = k_0 \cdot \sqrt{\rho_{\text{soil}} \cdot t_{\text{rise}}} \]  

where, \( l_e \) is the effective length of the wire, \( \rho_{\text{soil}} \) is the soil resistivity, and \( t_{\text{rise}} \) is the rise time of the injection current. According to [74], \( k_0 = 1.4 \), when the current is injected at one end of the wire, and \( k_0 = 3.1 \), when the current is injected in the middle of the wire.

For a single horizontal grounding wire, the simulation results based on the method a), described in section 5.4.1, show that \( l_e \propto \sqrt{t_{\text{rise}}} \) is satisfied both in low resistivity and in high resistivity soils. However, \( l_e \propto \sqrt{\rho_{\text{soil}}} \) was only satisfied when the rise time of the injection current is slow. It is found that for a single horizontal grounding wire with fast injection current \( (t_{\text{rise}} = 0.1 \mu s) \), the empirical equation (5-3) is not valid.

It was also found that equation (5-3) is applicable for the simulation results corresponding to vertical rods in most cases. The only difference we found was in the coefficient, \( k_0 \). Here, \( k_0 \) is equal to 1.3 instead of 1.4. Similarly, for the vertical rod with fast injection current \( (t_{\text{rise}} = 0.1 \mu s) \), equation (5-3) is also not valid.

Secondly, for the grounding grid, the empirical equation of the effective radius of an equivalent circular plate, which has the same area as that of the grid with effective area, is given by [22].

\[ r_{\text{ep}} = k_0 \cdot \sqrt{\rho_{\text{soil}} \cdot t_{\text{rise}}} \]

\[ l_e = \sqrt{\pi r_{\text{ep}}} \]  

(5-4)
In equation (5-4), \( r_{ep} \) is the radius of the equivalent circular plate. 
\[ k_n = 0.6 - 0.025 \cdot s_1 \] (\( s_1 \) is the spacing between the conductors of the grid). And \( l_e \) is effective length of the edge of the grid. Based on the simulations in paper VIII, it is also found that equation (5-4) is not valid for the grounding grid with larger inner mesh size of 10 m x 10 m.

5.5. Current distribution in the shields of underground cables associated with a communication tower under direct lightning strike

Grounding structures are always associated with some interconnected complex systems. It is well known that each part of such a system is not isolated, and they will influence each other under transient conditions. In order to estimate the surge distributions in the whole system to a reasonable accuracy, it is important to simulate the whole system as a single unit/entity so that the mutual influences are not neglected. In this thesis, we have made an attempt to model the transient behaviour of two complex systems under lightning strikes. Firstly, the surge current distribution in the shields of underground cables and grounding wires associated with a communication tower due to direct lightning strikes will be estimated in this section.

5.5.1 Geometry of FMV communication tower complex

The whole communication tower complex is simplified as shown in Fig. 5-4 for the sake of modelling. In Fig. 5-4, the whole system is divided into three main parts.

- Four communication tower legs and 11 associated downward shielded cables
- The continuation of the above said 11 cables in the ground and the horizontal grounding wire, which connects the ring conductors under the tower and the technical room.
- Two ring conductors under the tower and technical room

More details about the geometry of the system can be found in report 2.
5.5.2 Methodology of modelling

In order to simulate the transient behaviour of the whole system, and estimate the lightning current distributions in the shields of the underground cables and the grounding wires for quick solutions, transmission line approach is adopted. In the simulation, the whole system is separated into four groups of MTLs, namely, the connection on the top of the tower, tower legs and parallel cables, ring conductors under the tower and the technical room, and the buried part of the cables with horizontal grounding conductor. The transient behaviour of each coupled MTLs group was simulated by solving telegrapher's equations using FDTD method.

The tower legs and the shields of parallel cables along the tower, as a group of MTLs, are discretized to smaller segments, and their per-unit length parameters are calculated by electromagnetic field integral equations that have been described in chapter 3. It is found that the tower legs and the parallel vertical cables is a non-uniform MTL, because their per-unit length parameters are the function of height of the segment. Here, it is assumed that there is no coupling between the structures above the ground and under the ground.

The ring grounding conductors and the group of the horizontal grounding conductor with the underground 11 cables were simulated as uniform transmission lines. According to [75], when the grounding conductor is much shorter than effective length, uniform transmission line approach is suitable for the transient analysis of the grounding system. One important difference between the grounding wire and the cable is that there is no conductance
parameter for the cable due to the insulation around it. The soil ionization around the grounding wires is included based on IEEE standard model, the same as that in the next section.

### 5.5.3 Simulation results

In the simulation, according to the standard IEC 61312-1 [76], a subsequent return stroke represented by an ideal current source (equation (5-5)) injects the top of the tower.

\[
I(t) = \frac{I_0}{0.993} \cdot \frac{(t / \tau_1)^{10}}{1 + (t / \tau_2)^{10}} \cdot \exp(-t / \tau_2)
\]

In equation (5-5), \( \tau_1 = 0.454 \) μs and \( \tau_2 = 143 \) μs. The simulation results, such as the potentials at different points of the system, the current entering into the grounding structure under the tower, the currents entering into the shields of the cables in the ground and so on are described in detail in report 2. In order to estimate the influence of soil ionization, all the simulations are carried out with and without soil ionization. It was found that each of the shields of the 11 cables in the ground carries 3-7% of the lightning current injected at the tower top. About 40-50% of the total lightning current arrives at the ring conductor under the technical room, which is 20 m away from the tower. The current division between the tower and cable shields is not strongly influenced by the non-linear soil ionization. However, the ground potential rise (GPR) at the technical room is influenced significantly by the same. Soil ionization reduces the GPR by about 40-50%.

### 5.6 Influence of insulator flashover and soil ionization at the pole footing in the Swedish electrified railway system for the case of direct lightning strike

Another complex system that has been analyzed in this thesis is the Swedish electrified railway system for the case of direct lightning strike. In this section, the influence of insulator flashover and soil ionization at the pole footing on the surge propagation in the above said system will be described.

The schematic of a single-track railway system found in Sweden is shown in Fig. 5-5, where the conductors represent a system of multi-conductor transmission line system (MTLs). The different mediums, involved in the system, are also shown in Fig. 5-5. The details of the associated material properties and the conductor nomenclature are described in paper IX. The sleepers between the rail and ballast are neglected and it is assumed that the rails rest
on bed of crushed stones. In Fig. 5-5, there are two conductors marked as R3. It is because the contact and messenger wires are interconnected or shorted to each other at every 7 – 10 m. We have considered them to be a single conductor using the concept of bundled conductors. Thus in the simulations we have 9 conductors instead of 10.

Fig. 5-5. Geometry of the railway system

Fig. 5-6 Schematic for calculation of the equivalent circuit under flashover and ionization at pole footing conditions

In the simulations, the following complexities are included.
• The ground loss, which is a major parameter that determines the surge propagation
• The interconnection between the return conductor (R5 and R6) and S-rail (R1)
• Insulator flashover and soil ionization at the pole footing

The transient voltages/currents propagation in the MTLs for the Swedish single-track railway system above a lossy ground is simulated by solving modified telegrapher’s equations using FDTD method [77]. The effect of finitely conducting ground (ground loss) is included using a more accurate time domain expression for the transient ground impedance, which properly considers the behaviour of the earth to the incident electromagnetic fields in all the three regions, namely, Semlyen’s region (very high frequency), transition region (high frequency) and Carson’s region (low frequency).

Item 2 and 3 forms the major terminations on the multi-conductor lines. Here, we will discuss about the influence of the insulator flashover and soil ionization at the pole footing in the surge distributions on the lines.

The equivalent termination resistance at a given pole location that includes the above said two phenomena is bit complex. It depends on the soil ionization characteristics at the pole footing and insulator flashover behaviour. The S-rail (R1) is connected to the pole footing directly and all the other conductors excepting I-rail (R2) are connected to the pole through the insulators as shown in Fig. 5-6. Note that when the flashover takes place the resistance change from high value 100.0 MΩ to some low value depending on the adopted arc model. We used here a simple empirical arc model proposed by Toepler [78] where the arc resistance is function of the current as shown in equation (5-6). The constant ‘\( k_{arc} \)’ was taken as 4.5e-2 [78] and ‘\( d_{arc} \)’ is the arcing distance, which is approximately taken as the insulator length.

\[
R_{arc} = \frac{k_{arc} d_{arc}}{\int_0^t f(t) dt} \tag{5-6}
\]

Since the S-rail R1 is connected to the pole directly, the pole is always held at the potential of R1. The resistance (\( R_g \)) due to ionization at the pole footing is calculated using equation (5-7) based on the method proposed by the IEEE standard [64]. Where \( R_g \) is the footing resistance measured with low current, \( I_g \) is the lighting current through the footing resistance, \( I_{g}^\prime \) is the current required to produce the electric field gradient, \( E_0 \), at which soil breakdown occurs. The value of \( E_0 \) is about 400 kV/m.

\[
R_g = \frac{R_g^\prime}{\sqrt{1 + \frac{I_{g}^\prime}{I_g}}} \tag{5-7a}
\]
The schematic in Fig 5-6 shows as to how many insulators come in the circuit at a pole. The calculation of equivalent termination matrix is described in detail in paper IX. The flashover computation is done as follows. Once the potential between the conductor holding the insulator (V_i) and that of the R1 (V_1) exceeds or equals the insulator flash over voltage, we say that the flashover has occurred across the insulator surface. At this instant the R_i’s change from the initial operating value (high) to some value as given by Toepler’s law (5-6). These resistances keep on changing until the current through them changes sign (+ to – or – to +), to ensure the arc extinction. At this instant the initial operating values of the insulation resistance is assumed to be regained. Then the process is repeated for checking the flashover re-strike conditions. The insulator flashover and soil ionization levels at pole footing conditions are checked out at every pole in the total line length under analysis.

It was observed, for the direct lightning stroke with 31 kA peak current on the conductor R7, the above said insulator flashover and soil ionization phenomena caused the surge voltages at the very first pole from the point of strike to decay much faster within a fraction of microsecond. This faster decay of surge voltages was not observed when the above said phenomena were not included into the simulations. Thus, it is important to include insulator flashover and soil ionization at the pole footing while analysing the lightning surge propagation in the overhead transmission line system, which represents the electrified Swedish railway system.

\[ I_e = \frac{1}{2\pi} \frac{E_o}{\sigma_{soil} R_o^2} \]  

(5-7b)
6 Discussion and scope for the future work/studies

After having had an overview of the present work, it is certain that there will be doubts, questions and criticism, as these are the sources of inspiration for the future work. Out of many, in the author’s opinion, the following questions are worth mentioning.

- Does the non-uniform transmission line approach proposed in this thesis have any drawbacks?
- Does the uniform soil ionization model in some way represent the reality?

In order to answer the first question, we can refer back to the comparisons among the non-uniform transmission line, Geri’s circuit theory and electromagnetic field approaches [61] in chapter 3. It is very clear that those approaches predict different voltage distribution along the grounding conductors. The difference is minimum at the injection point, and it increases as we move away from the injection point. There could be various parameters/ reasons that are probably associated with this complex phenomenon. One of the interesting parameters could be the influence of frequency dependent per-unit length quantities. It could be interesting to incorporate this influence with the transient analysis of grounding systems based on uniform and non-uniform transmission line approaches. This certainly forms an important study for the future.

The second question is related with soil ionization phenomenon. In this thesis it is assumed that soil ionization around the grounding conductor is uniform. This might be reasonable if the ground conductor is in homogeneous low resistivity soil. Practical soils have organic matter and stones, and therefore, some degree of non-uniformity of the ionization region has to be expected. In addition to that, it has been observed both in triggered lightning experiments and laboratory experiments that discrete discharge channels are formed from the ground conductor both inside the soil and on the air-soil interface, especially in high resistivity soils. Surface arcs up to 100-200 meters long are observed in natural lightning [79-81]. Optical observations during triggered lightning has shown that more than 90% of lightning return
strokes with peak currents above 15 kA produced surface arcing in heavy red clay with a soil resistivity of about 500 Ωm [79]. These discharge channels carry a portion of the lightning current. Presence of discrete discharge channels on the surface or in the soil is not included in the simulations presented here. The experiments and theoretical analysis should address the issues, such as under what conditions discrete channels are initiated, how much percentage of the current is carried by those channels, what is the direction and the speed of progression of those channels, and finally what are the positions on the conductor from where these channels are initiated. Each of those questions forms an exhaustive study for the future.

Any model, whether it is simple or complex or accurate etc., is valid only if it is in agreement with the experimental measurements. It is true that a proper experimental measurement does represent the reality. Bearing this in mind, the future work should be focused not only to develop/improve models but also validate those models through measurements.
7 Sammanfattning

För att erhålla bättre åskskydd och elektromagnetisk kompatibilitet (EMC) är ordentliga jordningssystem och kunskaper om deras transienta beteende av stor vikt.

Detta arbete är inriktat mot utveckling av ingenjörsmodeller för transientanalysering av jordningssystem med tillräcklig noggrannhet och enkelhet för studier av blixtar. För det första har den konventionella transmissionsledningsansatsen för en ensam jordledning modifierats och utvidgats till jordningsnät. För det andra, har för första gången en icke likformig transmissionslinjeansats utvecklats för att modellera de transienta beteendena hos olika typer av jordningssystem. Den viktiga egenskapen hos denna ansats är dess förmåga att inkludera de elektromagnetiska kopplingarna mellan olika delar av jordningssystemet, via normaliserade rums- och tidsberoende längdparametrar.

Höga spänningar och strömmar som har inducerats i jordningssystem av blixtar joniserar alltid jorden. Detta fenomen bör inkluderas i transientanalys av jordningssystem. I detta arbete har en förbättrad jordjoniseringsmodell utvecklats, där hänsyn tas till kvarvarande resistivitet i joniseringsområdet. Det faktum att det existerar en kvarvarande resistivitet i joniseringsområdet (7 % av den ursprungliga resistiviteten i jorden) bevisas av experiment i litteraturen och av experiment utförda i högspänningslaboratoriet vid Uppsaluniversitet. Fördelen med den här modellen är att den fördelaktiga påverkan från markjoniserering vid reducering av potentialökningen hos ett jordningssystem inte överskattas, speciellt för mark med hög resistivitet.

Slutligen, har transmissionslinjeansatserna för att studera jordningssystems respons på blixtar anpassats för olika tillämpningar avseende det svenska järnvägsnätet. Dessa tillämpningar är: uppskattnings av DC-resistans hos komplexa jordningsstrukturer under telekommunikationstorn, inverkan av markparametrar på de transenta beteendena hos jordningssystem, transientanalys av jordningsstrukturer i skiktad jord, undersökning av giltigheten hos existerande definitioner för effektiv längd/area hos olika jordningsstrukturer, strömfördelning i skärmarna hos underjordiska kablar som är knutna till telekommunikationstorn samt inverkan av isolationsöverslag och markjoniserings runt stolpfundament vid överströmsutbredning.
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