Active and Passive Unequally Spaced Reflect-Arrays and Elements of RF Integration Techniques

BY

DHANESH G. KURUP
Abstract

Using an array synthesis tool based on a modified differential evolution algorithm, it is shown that the position-phase synthesis exhibits improved pattern characteristics compared to both the phase only and position only synthesis of uniform amplitude antenna arrays. The design of an unequally spaced planar reflect-array and an active power combining reflect-array are presented. The unit cell of the active reflect-array consists of an amplifying active reflect-antenna designed using a novel dual polarized microstrip-T coupled patch antenna. Two modelling approaches are proposed for the active reflect-antenna and the modelling methods are compared with the experiments.

A computationally efficient analysis of an H-slot in the ground plane of a microstripline is carried out using a transmission line model. To improve the accuracy in the resonant region of the H-slot and retaining the computational efficiency, an artificial neural network is combined with an efficient spectral domain method. An efficient analysis tool for a silicon micromachined H-slot coupled antenna is developed by combining the transmission line models of the H-slot and an aperture coupled antenna. The experimental results are compared with the theory showing good agreement.

The analysis and design of a microwave amplifier based on non-resonant slot matching is carried out. It is seen that the designed slot matched amplifier has decreased layout size, improved gain and noise figure characteristics compared to a stub matched amplifier. An efficient method for the analysis of non-resonant slots is compared with other approaches showing good agreement. This points to the fact that non-resonant slot matched circuits can be designed with the same speed and efficiency as we design the traditional stub based matching circuits.

To address the problem of bandwidth and performance of reflect-arrays we propose a dielectric resonator antenna with slotline stubs. As a preliminary step we design a dielectric resonator antenna with slotline feed and the experimental results are compared with those of a commercial CAD tool. Design and analysis of 3D interconnects based on non-radiative dielectric waveguides is carried out. At millimeterwave, these interconnects are useful for hybrid and multilayer integration techniques.

Keywords: Active-Antenna, Reflect-Arrays, NRD Waveguides, RF Integration.

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ISSN 1104-232X
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To my Parents
Preface

The RF technology has emerged as an area which is too shallow to swim and too deep to dive. This is a small and useful conclusion I have derived after all these years. To clearly understand and visualize how the divergence of the curl of that field component is influenced by the metal-dielectric interface of this structure needs an extra effort and time. On the other hand with some peripheral theoretical concepts and a fair understanding in one of the myriad of computer simulators, one can clearly derive useful conclusions. In the industrial world, as I saw, the best RF Engineers are those who can take the risk of making an extra mouse click in the design phase and attempting an extra screw in the experimental phase. However one should appreciate that diving deep with out the simulators and getting pearls brings a special unparalleled joy. Here is the collision between the fast changing and demanding industrial world and the traditional academic world. The thesis tries to identify some places worth swimming and diving and attempts few swimming and occasional under water diving, definitely not in deep sea.

Dhanesh G. Kurup
Acknowledgements

Many years back during my previous employment, I developed a desire to learn more on the topics which I was involved with. All my thanks and gratitude for fulfilling this desire goes to my principal supervisor Professor Anders Rydberg, who by accepting me as a Ph.D student, also granted me the opportunity to experience this pristine and beautiful scandinavian part of our little globe. Without his supervision, sharing of ideas, constructive criticism and patience in correcting the manuscripts, I could not have reached the various goals set. I would also like to thank Professor Mohamed Himdi of University of Rennes I, France for the research collaboration and hospitality during my visits to Rennes. His creativity and ideas in doing the experiments was a source of inspiration to me. I express my thankfulness to Dr. Rainer Storn of Infineon Technologies for sharing his Differential Evolution optimization software. I am particularly pleased with some improvements in the original code after a series of communications with him. Thanks are also due to my past co-supervisor Dr. Tommy Öberg and Professor Mikael Sternad for the discussions. I also thank Dr. Mats Gustafsson for teaching me Neural Networks which was useful in the research.

Professor Anders Ahlén, the head of our department deserve special thanks as he is instrumental in creating a nice and creative environment in our group. Thanks also to our secretary Ylva Johansson for keeping the infrastructure in good order and computer systems manager Ove Ewerlid for his ready availability. I also thank staff members Caroline Olofsson, Nora Masszi, Dr. Ping Wu, study director Lars Ericsson, Professor Tadeusz Stepinski and others in our group for their help one time or the other. I take this opportunity also to thank all the Ph.D students for the fun and sharing their experiences both academic and non-academic around the coffee table. Special thanks to Nilo C Ericsson for the collaboration during the signal processing courses and Erik Öjefors for the research collaboration in the micro-machined antennas. Also I express my thankfulness to past colleagues Dr. Erik Lindskog, Dr. Mattias Wennström and Dr. Staffan Bruce for their friendship and help in various forms.

And finally I thank my wife Sindhulakshmi for all her love and support and my little son Hemanth for the pure joy.
List of publications

A. Attached


B. Related


Chapter 1

Introduction

The growth of radio frequency (RF) technology has been phenomenal since the prediction of the existence of electromagnetic waves by the genius James Clark Maxwell in the year 1867 and the subsequent experimental proof by the brilliant Heinrich Hertz in 1887 [1]. Thanks to the famous experiments and commercialization efforts by the great visionary Guglielmo Marconi starting from the year 1896, we have today a wide variety of RF applications ranging from cooking to communications. Also the scope and range of RF applications continue to grow with every passing year [2].

Among the three major RF application areas such as the consumer, military and space technology, the consumer applications is the most challenging for an RF Engineer. This is because, for the commercial success of applications such as the wireless personal communications, not only should the terminals be integrated but should also include multiple functionalities and applications [3, 4]. For increasing the data-rates and number of users in the system, multiple antennas and radio channels, such as the adaptive antenna systems and multi-input multi-output systems (MIMO) [5] are required. Also multiple standards and complex modulation schemes dictate large bandwidth for future radio systems which requires the radio part in the terminals to be broadband. Another important criteria, which is perhaps not so important to RF Engineers before is the requirement for power efficient radio front-ends [6]. For instance it is pointed out in [7] that for a typical mobile terminal, the RF circuits alone consumes about 60% of the total battery power and occupy about 50% of the total volume. There are numerous examples from the industry, where the commercial deployment has delayed or never happened
because of the problems in the RF design. Therefore it is very important to continue looking into new design methodologies so that there are new tools and techniques at the disposal of an RF Engineer. The goal of the thesis is based on this motto and the thesis examines diverse aspects in RF design.

Maximizing the radiated power using an antenna array in a given direction with few available resources is a well known concern in applications such as satellite communications and wireless personal communications. In the case of satellite communications, savings in the maximization of the array radiated power helps in relaxing the DC power specifications of the preceding power amplifier stage. This helps in prolonging the on-board battery life. In the case of cellular communication systems, the linearity of the power amplifier is a matter of major concern because of the usage of complex modulation schemes. If there is savings in terms of optimized radiation power pattern in a base station, then the power amplifier can be operated with high linearity. Most of the classical pattern synthesis techniques including the Dolph-Chebychev and Taylor methods achieve the array specifications such as low sidelobe by varying the current amplitude in the equally spaced elements [8]. Amplitude tapering of an array is not optimum as far as the radiated power from the array is concerned since each array element is not operated at the maximum available power from the preceding power amplifier stage. On the other hand, the phase only synthesis of an equally spaced array enables us to operate the elements at maximum uniform amplitudes, thereby maximizing the radiated power. Considering these advantages, the equally spaced antenna arrays designed using phase only synthesis is more attractive than the nonuniform amplitude arrays in applications where available power is a matter of major concern [9, 10]. The main draw back of the phase only synthesis using equal element spacing is the requirement of a large number of elements compared to the amplitude tapered arrays used for achieving lowered sidelobe levels. In chapter 2 of this thesis and [11] [attached paper I] we consider the issue of maximizing the radiated power for uniform amplitude arrays. It is noted that optimizing the position as well as phase of the elements (position-phase synthesis) yields lowered sidelobe levels compared to optimizing only position (position only synthesis) or only phase (phase only synthesis) for a given number of array elements with uniform amplitudes. It is also shown in chapter 2 and [11] that for the position-phase synthesis there is a tradeoff between the range of phases and the size of the array for the same sidelobe level and number of elements. The array optimization tool for the position and/or phases of the unequally
spaced array has been developed using a modified version of the Differential Evolution Algorithm (MDE) which is a variant of the Genetic Algorithm [12]. In chapter 2 we demonstrate that the MDE is more robust than the original Differential Evolution algorithm (DE) described in [13, 14]. In applications such as reflect-arrays [15], the closer the elements the more will be the mutual coupling. On the other hand, the position-phase or position only synthesis of reflect-arrays with high illumination efficiency enables the distances between the elements to vary, thereby helping us to achieve reduced mutual coupling between the elements. The experimental results of a 324 element planar reflect-array designed using the MDE based array optimization tool is presented in chapter 2 and [16] [attached paper II].

The main reason for the interest of researchers and standardization bodies in millimeterwave bands is the spectrum scarcity in microwave bands. Millimeterwave frequency band such as the 60GHz band enables us to have excellent frequency reuse because of the peak rain and oxygen absorption and thereby increasing the user capacity in a communication system [17]. Millimeterwave bands also enables us to produce RF components in small size packages thereby reducing the system size and weight. In addition the antenna beamwidth can be easily narrowed resulting in a higher resolution for obstacle detection applications such as car radars [18]. One of the major problems for millimeterwave applications is the very small available power from solid state devices. On the other hand device level combining results is difficult due to thermal and matching problems. Waveguide power combiners have large insertion loss and are difficult to be fabricated at millimeterwave bands. Antenna based spatial or quasi-optical power combining arrays has been recently proposed to mitigate some of these problems [19]. In a spatial power combining system the power from individual solid state devices combines in free space and basically such systems can be divided into two, spatial power combining oscillators and spatial power combining amplifiers [19, 20, 21, 22]. One of the common way of realizing a spatial power combining amplifier is by locating the transmitting and receiving points on the opposite sides of the array, for instance the reported grid amplifier in [22] and the hard horn feed method reported in [23]. An interesting deviation from this method is described in [24] where an active reflect-array is used to accomplish spatial power combining. The prime advantage of this method is that the real estate consumption is reduced. For the realization of the active reflect-array in [24], dual polarized aperture coupled patch antennas has been used. Due to the double layer structure of aperture coupled patches,
Chapter 1. Introduction

the active reflect-array is difficult to manufacture and assemble. Also the
two orthogonally polarized apertures of the aperture coupled patch located
away from the center of the patch tend to decrease the RF isolation be-
tween the ports. In chapter 3 we describe the design and development of
an unequally spaced single layer power combining active reflect-array. The
active reflect-antenna element in the power combining array is based on a
microstrip-T coupled patch antenna [25] [attached paper III] which is stud-
ied in detail in chapter 4 [26] [attached paper IV]. Apart from its small size,
the dual polarized MTCP element has excellent RF isolation and inherent
DC isolation between the ports. It is shown that the small size of the active
reflect-antenna element helps in achieving decreased inter-element spacing
and the inherent DC isolation helps in avoiding coupling capacitors in the
RF path of the power combining reflect-array.

Model based design methodology is the hallmark of common commercial
simulators for RF integrated circuit design [27][28]. For instance a particu-
lar fabrication process such as the Si or GaAs MMIC technology [29] often
provide the computationally efficient models of various active and passive
components such that the designer can follow the pick and paste approach
for bringing the various components into a design window. The design win-
don in turn provides the user with the option of linear or non-linear simu-
lation. Widely followed numeric quasi-static models of passive components
are normally acceptable as the dimensions of the RF IC’s are quite small and
hence only weakly-dispersive. As the size, band-width and functionalities
of RF IC’s grow, new fabrication process such as multi-layer RF IC design
approaches are needed to meet the stringent system specifications. In such
scenarios, full wave electromagnetic simulation of some blocks in the active
circuit is required mainly because of the inaccuracy or non-availability of the
numeric quasi-static models. For the task of bringing the full-wave electro-
magnetically derived models in the design window, most of the commercial
programs provide pipelining between the circuit and electromagnetic simula-
tion suits. However, fullwave electromagnetic simulations and the subsequent
modelling is currently very inefficient. Therefore it is desirable to have more
and more computationally efficient models of passive components in the data
base of the active circuit simulators. The thesis derive computationally ef-
ficient physics based models of some components which are not available in
the passive component data base of commercial simulators. Efficient trans-
mission line modelling of a slotline combinations such as the I and H-slots in
the ground plane of a microstripline is presented in chapter 4 [30] [attached
paper V]. Once implemented in an active circuit simulator, it is possible to quickly optimize the implemented models along with active circuits for the overall performance improvement. As an active circuit application a non-resonant slot matched amplifier is designed in chapter 5 [31] [attached paper VI]. For non-resonant slot applications such as the slot-matched amplifier the simple transmission line model is sufficient. For increased accuracy for the resonant I and H-slot antennas, chapter 4 presents an improvement in the transmission line model by incorporating an artificial neural network together the spectral domain method. It is found that the proposed method not only has excellent computational efficiency but also agrees very well with that of commercial fullwave simulators. The transmission line model of the H-slot is further used for modelling a millimeterwave silicon micromachined slot coupled patch antenna in chapter 4. It is found that the results agree well with experiments [32] [attached paper VII]. Chapter 4 also presents a study on the microstrip-T coupled patch antenna [25] used for the dual polarized active reflect-antenna element presented in chapter 2 and a dielectric resonator antenna with slot-line feed for broad band reflect-arrays. For emerging millimeterwave applications, the nonradiative dielectric (NRD) waveguides have proven to be attractive due to its low cost fabrication and low loss characteristics [33]. In chapter 5, the design, modelling and experiment of NRD interconnects are presented [34][attached paper VIII].
Chapter 1. Introduction
Chapter 2

Position-phase and position only synthesis of antenna arrays

A concept called competing mating strategies is applied to modify the conventional Differential Evolution Algorithm for the synthesis of uniform amplitude unequally spaced antenna arrays [11]. It is demonstrated that the modified Algorithm retains all the advantages of the Differential Evolution Algorithm such as its efficient global search capability over a multidimensional space as well as further increases the computational efficiency. We demonstrate the robustness of the modified Differential Evolution Algorithm (MDE) by comparing it with the conventional Differential Evolution Algorithm (DE) for a set of test functions. MDE is then applied for the synthesis of uniform amplitude unequally spaced antenna arrays with unequal phases (position-phase synthesis) and equal phases (position-only synthesis). Using position-phase synthesis it is demonstrated that a tradeoff exists between the size of the unequally spaced arrays and the range of phases for the same radiation characteristics. It is also demonstrated that [11] the proposed position-phase synthesis technique decreases the size of the array for the same sidelobe level compared to both the phase only synthesis and position only synthesis but also retains their advantages compared to equally spaced arrays [8, 35]. Design and fabrication of an unequally spaced reflect-array is carried out using inset-fed microstrip patches [16]. It is seen that the usage of unequally spaced inset fed patches yielded larger inter element spacing and improved pattern characteristics compared to that of the equally spaced case.
2.1 The Modified Differential Evolution Algorithm

The modified Differential Evolution Algorithm (MDE) is based on the conventional Differential Evolution Algorithm (DE) [13, 14] which belong to the class of Genetic Algorithms. The method followed for the evolution in DE/MDE is different from the method followed in the conventional Genetic Algorithm (GA) in the sense that, the evolution mechanism in DE/MDE consists of forming the vector differences of floating point parameters instead of the uniform or point crossovers on binary strings followed in the conventional GA. This is one of the most significant advantages of the MDE compared to the conventional GA as it helps us in optimising large antenna arrays and complex microwave optimization problems with higher computational efficiency compared to the conventional GA. It is to be noted that all the essential features of GA such as adaptivity and global search over continues spaces are retained in DE/MDE. The modified Differential Evolution Algorithm (MDE) which is based on the concept of competing mating strategies is well described in the block diagram shown in Figure 2.1. First, a parent population, $\bar{p}_i$, $i \in [1, N_p]$, uniformly distributed in the parameter space is initialized. When the best parent $\bar{p}_b$ in terms of the objective function meets the optimization criteria or the number of iterations $itn$ reaches the maximum specified number of iterations $max$, the optimization is terminated. As shown in Figure 2.1, for each iteration $itn$ and a given member $\bar{p}_i$, either $K$ trial members $\bar{t}_k$, $k \in [1, N_p]$, using $K$ different strategies are generated or the parent $\bar{p}_i$ becomes the child $\bar{c}_i$. Since $urand$ generates a uniformly distributed random number in $[0,1]$, the probability of generating the trial members is $p_e$ and the probability of the parent $\bar{p}_i$ directly becoming the child $\bar{c}_i$ is $(1 - p_e)$, see Figure 2.1. If $K \geq 2$, the trial members generated compete amongst each other and the winner competes with the parent $\bar{p}_i$. This is the concept of competing mating strategies. Each MDE strategy is essentially a linear combination of the vector differences of a subset of the parent population together with the parent $\bar{p}_i$ and(or) $\bar{p}_b$ [13, 14]. The number of members in the subset is much smaller than $N_p$, the total number of members in the parent population. Thus the number of participating parents in the evolution mechanism can be more than two unlike the Genetic Algorithm which uses two parents for the point and uniform crossovers [12]. The vector transformations or the evolution mechanism for the generation of
2.1. The Modified Differential Evolution Algorithm

The two trial members \( \bar{t}_{1,2} \) implemented in the MDE are as follows.

\[
\begin{align*}
\bar{t}_{1a} &= \bar{p}_R + F (\bar{p}_i - \bar{p}_S) \\
\bar{t}_{1b} &= \bar{p}_S + F (\bar{p}_R - \bar{p}_T) \\
\bar{t}_1 &= \bar{t}_{1a} \quad \text{or} \quad \bar{t}_1 = \bar{t}_{1b} \\
\bar{t}_2 &= \bar{p}_b + F (\bar{p}_i - \bar{p}_R)
\end{align*}
\]

where, \( F \) is a real and constant factor which controls the differential variations \( (\bar{p}_i - \bar{p}_S), (\bar{p}_R - \bar{p}_T) \) and \( (\bar{p}_i - \bar{p}_R) \). In the implemented version there is a choice of fixing the first trial member as is described in (2.3). The members \( \bar{p}_R, \bar{p}_S \) and \( \bar{p}_T \) constitutes the subset of parent population satisfying the condition that the indexes \( i, R, S \) and \( T \) are different, see Figure 2.1. More on the different MDE strategies and the index generation technique for \( R, S \) and \( T \) can be found in [13]. It is to be noted that, not all the
components of the vectors $\vec{t}_1$ and $\vec{t}_2$ undergo the transformations given by (2.3) and (2.4). The components which do not undergo the transformations are assigned prior the components of the vector $\vec{p}_i$. As shown in Figure 2.1, once all the children are assigned as described above, they are transferred to the parent population. The difference between the Differential Evolution Algorithm (DE) [13, 14] and MDE is that, the MDE uses $K = 2$ instead of $K = 1$ in [13], see Figure 2.1. The optimization performance of the MDE and its comparison with DE is demonstrated in the next section for a set of test functions with multiple minima and maxima.

**Performance of the MDE Algorithm**

A common yardstick to evaluate the performance of any optimization Algorithm is the number of function calls before the optimum value of parameters are attained. First we compare the performance of the MDE with the conventional Genetic Algorithm (GA) described in chapter 1 of [12]. The function described to demonstrate the GA performance in [12] is the product of a pair of $Sinc$ functions described as,

$$f(x, y) = \left| \frac{\sin(\pi(x - 3))}{\pi(x - 3)} \right| \cdot \left| \frac{\sin(\pi(y - 3))}{\pi(y - 3)} \right|$$

(2.5)

The objective is to find the maximum value of (2.5). It is to be noted that the function has several local maxima. The optimization performance of the MDE for the product of the $Sinc$ functions in a random run is shown in Figure 2.2. The number of individuals in the MDE run is 10 and the specified parameter range is $[0, 8]$. The best fitness of 0.99999 is achieved in just 127 function calls. The average number of function calls needed for the MDE for 20 independent runs was 138 compared to 160 cited in [12] for the GA to achieve a fitness $\geq 0.998757$. The evolution strategies used for the MDE are (2.2) and (2.4). It is to be noted that in Figure 2.2, instead of adding 20 function calls in each generation for the MDE, one generation added has 17 due to the fact that, during the operation of strategies, three individuals was found to fall beyond the parameter range. The performance of the individual strategies in the MDE optimization of Figure 2.2 is shown in Figure 2.3 with respect to the number of times each of them win and the percentage of parents replaced, see Figure 2.1. It can be seen from Figure 2.3 that the strategy 2 is the dominant strategy in terms of number of wins except for one generation. It can also be seen from Figure 2.3 that as the
2.1. The Modified Differential Evolution Algorithm

![Graph showing fitness and generation relationship](image)

**Figure 2.2:** Mean and best fitness in the MDE optimization of product of Sinc function with respect to the number of generations, quantities inside the bracket indicate the number of function calls.

The number of generations increases the percentage of parents replaced become smaller and smaller showing the convergence of the MDE. The modified Differential Evolution Algorithm (MDE) is further evaluated by testing it for three other test functions. These test functions which are popular among the optimization community for their difficulty in attaining the optimum value are,

- **Rosenbrock’s saddle function:**
  \[
  f_r(\vec{x}) = 100.(x_0^2 - x_1)^2 + (1 - x_0)^2, \quad x_1, x_2 \in [-2.048, 2.048].
  \] (2.6)

- **Third De Jong function:**
  \[
  f_d(\vec{x}) = 30 + \sum_{j=0}^{4} \lfloor x_j \rfloor, \quad x_j \in [-5.12, 5.12].
  \] (2.7)

- **Griewangk’s function:**
  \[
  f_g(\vec{x}) = \sum_{j=0}^{9} \frac{x_j^2}{4000} - \prod_{j=0}^{9} \cos \left( \frac{x_j}{\sqrt{j+1}} \right) + 1, \quad x_j \in [-400, 400].
  \] (2.8)
Although the function $f_r(\bar{x})$ has just two parameters, it has the reputation of being a difficult minimization problem due to its multiple local minima. The global minimum is $f_r(\bar{0}) = 0$. For $f_d(\bar{x})$ it is necessary to incorporate the constraints imposed on $x_j$ into the objective function. This function exhibits many plateaus which pose a considerable difficulty for many minimization algorithms. The global minimum for this function is $f_d(\bar{x}_j) = 0$, $\bar{x}_j = -5 - \epsilon$ and $\epsilon = [0.0, 0.12]$. The function $f_g(\bar{x})$ is extremely difficult for many optimization methods. The true global minimum for $f_g$ is $f_g(\bar{0}) = 0$. We optimize each of these functions using both DE and MDE. For the DE we use two separate evolution mechanisms or strategies and for the MDE we apply competition of the DE strategies, see Figure 2.1. We notate the DE evolution mechanism as strategy-1 and strategy-2 and for the MDE we notate the same as strategy-12. In order to appreciate MDE fully, it is necessary to subject the optimization to a large number of independent runs such that the conclusions on the performance are statistically correct. We perform 1000 independent runs of DE and MDE for each function described in (2.6)-(2.8). The number of generations are fixed at 800. If the differ-
2.1. The Modified Differential Evolution Algorithm

Figure 2.4: Histogram of \( N_E \), the required number of function evaluations for the optimization of Rosenbrock’s saddle function using (a) DE strategy-1 (b) DE strategy-2 (c) MDE strategy-12.

ence between the optimized function value and the actual global minimum is \( \leq 10^{-6} \) then we characterize the optimization run as a success. If 500 out of first 1000 independent runs do not end in the global minimum, then we term the optimization method as a failure. On the other hand if there are \( n_f \) failures with \( n_f \leq 500 \) in the first 1000 independent runs, then the number of independent runs will be increased such that there are 1000 successful runs. The evolution mechanisms for strategy-1 and strategy-2 followed are (2.2) and (2.4) respectively. The starting population for each of the strategies are same in a given run so that the performance comparison is correct. Figure 2.4 - 2.6 shows the histogram showing the number of function evaluations, \( N_E \) needed for attaining the global minimum for Rosenbrock’s saddle, third De Jong and Griewangk’s function respectively for each of these strategies. The number of histogram bins is fixed at 50 between the minimum and maximum number of function evaluations. The statistical conclusions of each
of the optimization are further summarized in Table 2.1. It can be seen
2.1. The Modified Differential Evolution Algorithm

Table 2.1: DE/MDE optimization showing the number of individuals \( N_p \) in the population, the number of failures \( n_f \) in the first 1000 independent runs, the mean \( M_{N_F} \) and the variance \( \sigma_{N_F} \) of the number of function evaluations in the 1000 successful runs.

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<th>( N_p )</th>
<th>Strategy</th>
<th>( n_f %))</th>
<th>( M_{N_F} )</th>
<th>( \sigma_{N_F} )</th>
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<td>15</td>
<td>DE: strategy-1</td>
<td>33.1</td>
<td>856.1</td>
<td>297.4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>DE: strategy-2</td>
<td>0.0</td>
<td>581.3</td>
<td>155.3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>MDE: strategy-12</td>
<td>0.1</td>
<td>663.6</td>
<td>177.7</td>
</tr>
<tr>
<td>Third De Jong</td>
<td>35</td>
<td>DE: strategy-1</td>
<td>0.0</td>
<td>1636.4</td>
<td>204.8</td>
</tr>
<tr>
<td></td>
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<td>DE: strategy-2</td>
<td>16.6</td>
<td>2044.6</td>
<td>2148.7</td>
</tr>
<tr>
<td></td>
<td></td>
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<td>1467.1</td>
<td>535.5</td>
</tr>
<tr>
<td>Griewangk’s</td>
<td>50</td>
<td>DE: strategy-1</td>
<td>0.1</td>
<td>29349.2</td>
<td>3282.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>DE: strategy-2</td>
<td>&gt;50.0</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td></td>
<td>MDE: strategy-12</td>
<td>0.2</td>
<td>29651</td>
<td>3139.9</td>
</tr>
</tbody>
</table>

From Figure 2.4 and Table 2.1 that for the Rosenbrock’s saddle function, DE strategy-2 performed best in terms of both the number of failures in 1000 independent runs and the mean number of function evaluations. The performance of MDE strategy-12 is close to DE strategy-2 and far better than DE strategy-1 which has almost 33% failures in the first 1000 runs. For the third De Jong function MDE strategy-12 proved to be the best among all the three strategies. Contrary to the case of Rosenbrock’s saddle function DE strategy-2 performed worst with 16.6% failures in 1000 first independent runs. For the Griewangk’s function DE strategy-1 performed best in terms of both the number of function evaluations before success and number of failures in the first 1000 independent runs. However DE strategy-2 resulted in > 50% failures in the first 1000 independent runs. The performance of MDE strategy-12 was almost close to DE strategy-1. Evaluating the performance of MDE, we see that MDE performance is either close to or better than the best DE strategy for all the treated functions. Also we see that the probability of MDE arriving at the true global minimum is larger than the individual DE-strategies. Therefore the usage of single DE strategy for all the functions do not guarantee the best performance for all them as each DE-strategy has a bias. On the other hand the competitive DE strategies used in the MDE algorithm removes this bias resulting in improved overall
2.2 Position-phase synthesis of arrays using the MDE Algorithm

Consider a linear array of \( N_e \) elements with arbitrary positions. Its array factor, \( A_F(\theta) \) can be written as,

\[
A_F(\theta) = \sum_{i=1}^{N_e} w_i e^{\frac{j2\pi}{\lambda} x_i \sin \theta} \tag{2.9}
\]

where, \( w_i \) denotes the current (in general complex) of the element located at \( x_i \) and \( \lambda \) is the wavelength. We consider elements with constant amplitude, arbitrary phase and arbitrary positions. Therefore (2.9) becomes,

\[
A_F(\theta) = \sum_{i=1}^{N_e} e^{\frac{j2\pi}{\lambda} x_i \sin \theta + \phi_i} \tag{2.10}
\]

where, \( \phi_i \) represents the excitation phase of the element located at \( x_i \). Thus the number of parameters to be synthesized is \( N_e \) for both the position only and phase only cases and \( 2N_e \) for the position-phase case. If the task of the optimization is the minimization of the sidelobe level, then the objective function, \( f^* (\bar{\rho}) \) to be minimized can be written as,

\[
f^* (\bar{\rho}) = \text{Max}_{\theta \in S} \left| \frac{A_F^\rho(\theta)}{A_F^\rho(\theta_0)} \right| \tag{2.11}
\]

where, \( S \) is the space spanned by the angle \( \theta \) excluding the mainlobe and \( \bar{\rho} \) represent the unknown parameter set consisting of the element positions and/or phases which minimizes all the sidelobe levels and maximizes the power in the mainlobe located at \( \theta = \theta_0 \). Therefore \( \bar{\rho} \) can be written as follows,

- **position-phase synthesis,**
  \[
  \bar{\rho} = \{ x_i, \phi_i \}, \quad 1 \leq i \leq N_e. \tag{2.12}
  \]

- **position only synthesis,**
  \[
  \bar{\rho} = \{ x_i \}, \quad 1 \leq i \leq N_e. \tag{2.13}
  \]
2.2. Position-phase synthesis of arrays using the MDE Algorithm

2.2.1 Low sidelobe arrays - comparison of position-phase and position only synthesis

In this section we study the MDE based position-phase and position only synthesis for low sidelobe with upper limit in the distances between the elements as a variable.

Effect of the upper limit in the distances between the elements on the position only and position-phase synthesis

The position-phase and position only synthesis of a symmetric unequally spaced linear array was carried out based on MDE for different $d_{\text{max}}$, the upper limit in the distances between the elements. The number of array elements considered for the MDE based synthesis is 32, hence the number of parameters to be optimized is 16 for the position only synthesis and 32 for the position-phase synthesis. The prior constraint in the synthesis of the element positions for both the cases is $d_{\text{min}} = 0.5\lambda$, where $d_{\text{min}}$ is the minimum distance between two adjacent elements. The upper limit in the distance between the elements, $d_{\text{max}}$ is varied from $0.5\lambda$ to $1\lambda$. Figure 2.7 shows

![Figure 2.7: The maximum sidelobe level of an equal amplitude, unequally spaced 32 element array with the upper limit in the spacing $d_{\text{max}}$ in the MDE synthesis lower limit, $d_{\text{min}} = 0.5\lambda$, ○: position only synthesis. □: position-phase synthesis. ●: uniform phases. ■: phase only synthesis.](image-url)
Table 2.2: The MDE based position only and position-phase synthesis of a 32 element array.

<table>
<thead>
<tr>
<th>i</th>
<th>$x_i/\lambda$</th>
<th>$x_i/\lambda$</th>
<th>$\phi_i/\lambda$</th>
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<th>$\phi_i/\lambda$</th>
</tr>
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<td>0.25</td>
<td>47.6</td>
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<td>0.26</td>
<td>26.1</td>
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<td>2</td>
<td>0.75</td>
<td>0.75</td>
<td>53.1</td>
<td>0.75</td>
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</tr>
<tr>
<td>3</td>
<td>1.25</td>
<td>1.25</td>
<td>57.3</td>
<td>1.25</td>
<td>1.26</td>
<td>28.2</td>
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<td>1.75</td>
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<td>6.52</td>
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<td>7.77</td>
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<td>7.99</td>
<td>40.9</td>
<td>9.24</td>
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<td>61.3</td>
</tr>
<tr>
<td>16</td>
<td>8.66</td>
<td>8.59</td>
<td>55.2</td>
<td>10.14</td>
<td>10.50</td>
<td>28.8</td>
</tr>
</tbody>
</table>

$\{x_i, \phi_i\} = \{-x_{i-16}, \phi_{i-16}\}, i \in [17, 32]$.

The maximum sidelobe level for different $d_{\text{max}}$ for the MDE based position only and position-phase synthesis. As can be seen from Figure 2.7, when $d_{\text{max}}$ is smaller, the maximum sidelobe level for the position-phase synthesis is considerably lower compared to that of the position only synthesis. For example, when $d_{\text{max}}=0.6\lambda$ the maximum sidelobe level for the position-phase synthesis is lower by about 3.5dB compared to that of the position only synthesis. It is to be noted that, when $d_{\text{min}} = d_{\text{max}} = 0.5\lambda$, the position-phase synthesis is equivalent to the phase only synthesis and the position only synthesis is equivalent to a 0.5$\lambda$ uniformly spaced array. From Figure 2.7 we can see that, for $d_{\text{max}} = 0.5\lambda$, position only synthesis gave a maximum sidelobe level of $\approx 13.3$dB which is the maximum sidelobe level of a uniformly excited array with a spacing of 0.5$\lambda$. Also we can see that the maximum sidelobe level for the 0.5$\lambda$ spaced array derived using the phase only synthesis
is lower by about 5dB compared to the case when the array is uniformly exited. From Figure 2.7 we can also see that, when the upper limit in the maximum distance between the adjacent elements, $d_{\text{max}}$ is increased, the maximum sidelobe level decreases for both the cases. When $d_{\text{max}}$ approaches $1\lambda$, there is no significant reduction in the maximum sidelobe level for the position-phase synthesis compared to the position only synthesis. The MDE synthesis results of positions and phases for the cases when $d_{\text{max}}=0.6\lambda$ and $d_{\text{max}}=1\lambda$ are given in the Table 2.2. The array patterns for $d_{\text{max}} = 0.6\lambda$

![Figure 2.8: Array patterns for the MDE based position only synthesis (dashed) and the position-phase synthesis (solid) of the 32 element array for $d_{\text{max}}=0.6\lambda$ in Figure 2.7.](image)

and $d_{\text{max}} = 1\lambda$ are shown in Figure 2.8 and 2.9 respectively. From Figure 2.8 we can see that the maximum sidelobe level for the position-phase synthesis is -19.2dB which is about 3.5dB lower than that for the position only synthesis. The directivity of the patterns shown in Figure 2.8 for the position only and position-phase synthesis are 18.5dB and 17.8dB respectively. When $d_{\text{max}}=1\lambda$, the maximum sidelobe level of the position-phase synthesis and position only synthesis are -23.34dB and -22.53dB respectively, see Figure 2.9. The corresponding directivity are 18.8dB and 18.85dB respectively. Therefore from Figure 2.7, 2.8 and 2.9 we can conclude that for smaller $d_{\text{max}}$, the element phases have a larger effect in lowering the sidelobe level of an unequally spaced array with no significant difference in the directivity. The
relative array efficiency $\eta$ (ratio of the actual peak power density of the main beam to the peak power density of the main beam when all the elements are uniformly exited with maximum power) is 100% for the position only synthesis. But $\eta$ for $d_{\text{max}} = 0.6\lambda$ and $d_{\text{max}} = 1\lambda$ for the position-phase synthesis are 85.5% and 96.2% respectively. Thus an improvement of over 10% in $\eta$ is achieved for $d_{\text{max}} = 1\lambda$ over $d_{\text{max}} = 0.6\lambda$.

### 2.2.2 Non broadside arrays - comparison of position-phase and phase only synthesis

A well known concern for non broad side arrays using phase only synthesis is its low directivity and high sidelobe levels. In order to compare position-phase synthesis with phase only synthesis in this case, we consider a 12 element array. The lower and upper limit in the distances between the elements are $0.5\lambda$ and $0.9\lambda$ respectively for position-phase case where as for the phase only synthesis, the uniform distances between the elements is fixed at $0.5\lambda$. The aim of the synthesis is to steer the beam 20° with broadside as well as keeping the sidelobe level minimum. The positions of the elements are assumed to be symmetric and the phases anti-symmetric with respect to the
2.2. Position-phase synthesis of arrays using the MDE Algorithm

Figure 2.10: Array patterns for the MDE based position-phase synthesis (solid) and phase only synthesis (dotted) of a non broadside array.

Table 2.3: The element positions and phases of the 12 element 20° steered non broadside array using the MDE algorithm.

<table>
<thead>
<tr>
<th></th>
<th>phase-only</th>
<th>position-phase</th>
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<td>i</td>
<td>φ_i</td>
<td>x_i/λ</td>
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<td>0.25</td>
</tr>
<tr>
<td>2</td>
<td>-88.9</td>
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</tr>
<tr>
<td>3</td>
<td>-149.6</td>
<td>1.37</td>
</tr>
<tr>
<td>4</td>
<td>124.7</td>
<td>2.11</td>
</tr>
<tr>
<td>5</td>
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<td>2.74</td>
</tr>
<tr>
<td>6</td>
<td>27.6</td>
<td>3.44</td>
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</table>

\{x_i, φ_i\} = \{-x_{i-6}, -φ_{i-6}\}, i ∈ [7, 12].

array center. The synthesized element parameters for both these cases are shown in Table 2.3. The corresponding patterns are shown in Figure 2.10. The achieved directivity for position-phase synthesis is 14.4dB at a sidelobe level of -15.4dB where as phase only synthesis yielded 13.4dB and -12.5dB for directivity and sidelobe level respectively. Thus we can see that the position-phase synthesis yielded both lower sidelobe level as well as higher directivity.
2.3 Synthesis and experiment of a planar unequally spaced reflect-array

With its low profile nature, low cost and easy fabrication, microstrip reflect-arrays can be a substitute for high gain antennas such as parabolic reflectors [15]. The microstrip reflect-arrays can be broadly divided into two classes, uniformly spaced arrays of patches of same size and variable tuning stubs [36, 37] and uniformly spaced arrays of patches of different sizes [38, 39]. Reflect-arrays with patches of the same size can be tuned for higher radiation efficiency compared to the reflect-arrays of patches with variable sizes. On the other hand, the later can be optimized for larger bandwidth. To the authors knowledge, all the published reflect-arrays reported so far are equally spaced. In this section we present the design and experimental results of an unequally spaced reflect-array of microstrip patches with inset fed stubs [16] using MDE.

2.3.1 Configuration of the reflect-array

We followed the MDE based position only synthesis to design an 18x18 element planar reflect-array. Since the positions of the elements was assumed to be symmetric with respect to the array center, there are 18 elements along each plane or 9 parameters in the optimization. With the constraint that the lower and the upper limit in the distance between the elements are 0.55λ and 1.0λ respectively, the synthesized parameters and the element positions of the planar array in the x-y plane is depicted in Figure 2.11. Assuming uniform illumination and no mutual coupling between the elements, the H-plane radiation pattern of the synthesized unequally spaced array is first compared with that of an equally spaced array. The distance between the elements of the equally spaced array is the lower limit in the distance between the elements in the unequally spaced array such that uniformly spaced array has the minimum sidelobe level. The theoretical radiation pattern for both these cases are shown in Figure 2.12. It can be seen from Figure 2.12 that the maximum sidelobe level of the unequally spaced array is lower by about 7dB compared to that of the equally spaced array. The directivities for the unequally spaced and equally spaced cases are 20.1dB and 19.5dB respectively.
2.3. Synthesis and experiment of a planar unequally spaced reflect-array

whereas the sizes of unequally spaced and equally spaced cases are $11.84\lambda$ and $10.2\lambda$ respectively. Thus with an increase in physical size of about 5cm
at 10GHz, unequally spaced array showed considerable improvement in the beam pattern. Since the distances between most of the elements in the unequally spaced array are longer than the distance between the elements in the equally spaced case, the possible degradation in pattern due to mutual coupling can be expected to be less for the unequally spaced array.

![Array Photograph](image)

*Figure 2.13: Photograph of the array part of the 38X38cm. reflect-array.*

### 2.3.2 Results and discussion

For the array element we used an inset fed microstrip patch resonant at 10GHz [40]. The length of the tuning stubs attached to each element is calculated for a given primary feed position [36]. The feed used is a standard X-band horn which is offset 20° to the array perpendicular along the H-plane at a distance of 60cm. The photograph of the fabricated array part of the unequally spaced reflect-array is shown in Figure 2.13. The measured H-plane and E-plane patterns of the reflect-array are shown in Figure 2.14 and 2.15.
2.3. Synthesis and experiment of a planar unequally spaced reflect-array

Figure 2.14: Measured H-plane pattern of the unequally spaced reflect-array, solid: co-polarization, dotted: cross-polarization.

Figure 2.15: Measured E-plane pattern of the unequally spaced reflect-array, solid: co-polarization, dotted: cross-polarization.

respectively. It can be seen from the patterns that the cross-polarization is below -25dB for the H-plane pattern and below -30dB for the E-plane case. From Figure 2.12 and Figure 2.14 we see that there is an increase in sidelobe level in the measured patterns of the unequally spaced reflect-array compared to the theoretical one. However, it is to be noted that the measured sidelobe level of the unequally spaced array is still below the theoretical sidelobe level of the equally spaced case showing the advantage of position only synthesis over the conventional equally spaced design. The increase
in sidelobe levels of the measured pattern may be attributed to the mutual coupling and alignment errors.
Chapter 3

An unequally spaced power combining active reflect-array

In this chapter, we design and characterize an amplifying active reflect-antenna element based on the microstrip T-coupled patch antenna. Two different modelling methods, one based on a single antenna approach and the other based on a passive reflect antenna approach are compared with experiment. It is noted that the experiment, which is based on a time domain network analysis method, closely follow the passive reflect-antenna modelling approach. The active reflect-antenna element is then utilized in the design of a linear power combining active reflect-array. The element positions of the power combining reflect-array are derived using the position-phase synthesis approach described in chapter 2. Since each element in the power combining reflect-array consists of the microstrip-T coupled patch antenna and an amplifier, it is important to make sure every active element in the array are working with out any fault. For the fault diagnosis, the added power due to the array as well as total bias current for different bias configurations are noted. In order to compare the performance of the power combining reflect-array, a passive reflect-array using the same element configuration as the power combining reflect-array and without the amplifiers is designed. It is noted that the effective isotropic radiated power (EIRP) of the active reflect-array exhibited a 6dB improvement over the passive reflect-array.
3.1 Amplifying active reflect-antenna - the unit cell

The unit cell developed for the power combining active reflect-array is shown in Figure 3.1 [26]. As can be seen from Figure 3.1, the passive antenna part of the active reflect-antenna is the dual polarized microstrip-T coupled patch antenna (MTCP) described in section-4.3 [25]. The normal patch radiation pattern is affected very little by the microstrip-T, since the currents in microstrip-T close to the radiating edge of the patch are orthogonal to the TM mode currents in the patch. The parameters in attaining matching at the ports are the dimensions of the microstrip-T and its spacing from the patch. The microstrip-T junction therefore acts as an impedance transformer, transforming the high radiation resistance of the patch to the desired low impedance which in our case is 50Ω. The gain of the active reflect-antenna is provided by the amplifier sandwiched between the orthogonally
3.1. Amplifying active reflect-antenna - the unit cell

polarized ports. Since there is an inherent DC isolation between the dual polarized ports of the active reflect-antenna, see Figure 3.1, additional coupling capacitors which may increase the cross-polarization and design complexity can be avoided in the RF path of the amplifier. The substrate used for the design has a thickness of 0.5mm, $\varepsilon_r = 3.0$ and $\tan\delta = 0.003$. The dimensions of the microstrip-T coupled patch antenna with identical port configurations, fabricated on a substrate of size approximately 6 X 6cms. are also shown in Figure 3.1. The size of the layout may be further reduced for array applications by bringing the bends closer and with accurate characterization of the element. For the amplifier part of the active reflect-antenna, we used a packaged PHEMPT transistor (Agilent ATF-36077) in an unmatched commonsource configuration. The reason for using the unmatched configuration was to decrease the scattering cross-section from the matching circuits such as open stubs. However it is best to use internally matched amplifier chips for optimum gain and decreased layout size. The stability over the frequency band of interest was a matter of significant concern due to the usage of the dual polarized ports of the single antenna at the input and output of the amplifier. Although the isolation between the dual polarized ports of MTCP is very high as was seen in section 4.3, the stability of the active reflect-antenna is verified experimentally by noting the linear relationship between the reflected and incident power at a given frequency. The experimental verification of stability is highly recommendable because of factors such as the proximity of the active components with the antenna and the strong coupling between the components. Usually these subtle factors are difficult to model using commercial simulators although the FDTD method such as the one described in [41] to study the interaction between the active component and antenna is promising in this context. For large arrays involving the active antenna the modelling of the overall stability should incorporate the mutual coupling between the elements. It is to be noted that the design frequency of 10GHz and the substrate used in the design of the unit cell are same as in the configuration described in section 4.3. Hence the dimensions of the MTCP in section 4.3 is same as that of the unit cell of Figure 3.1. The measured return loss and isolation between the ports are 27dB and 32dB respectively. It is to be noted that the 10dB return loss bandwidth of 2.1% for the MTCP is higher than the bandwidth attainable using common matching techniques such as the quarter wave transformer for an edge coupled patch antenna on a similar substrate. The measured radiation patterns of the isolated microstrip-T coupled patch antenna is shown in section-4.3 as well as
3.1.1 Two modelling approaches for the amplifying active reflect-antenna

![Diagram showing two methods for modelling the amplifying reflect-antenna: (a) single antenna method. (b) passive reflect-antenna method.]

We followed two different approaches to model the effective isotropic power gain (EIPG, [42]) of the amplifying active reflect-antenna depicted in Figure 3.1. In the first method, namely the single antenna method, see Figure 3.2(a), the amplifying reflect-antenna is modelled as an interconnection of the horizontally polarized antenna, amplifier and the vertically polarized antenna. Assuming that the horizontally polarized and the vertically polarized antennas have the same gain $G_d$, the EIPG for the amplifying reflect-antenna $EIPG_a$ using this model can be written as,

$$EIPG_a = G_d^2 G_a$$  \hspace{1cm} (3.1)
where, $G_a$ is the amplifier gain. In the second modelling approach, see Figure 3.2(b), namely the passive reflect-antenna method, a passive reflect-antenna is fabricated similar to Figure 3.1 and of the same size by bridging the ports of the microstrip-T coupled patch antenna with a microstripline and removing the amplifier and its associated bias circuitry. The EIPG of the amplifying reflect-antenna can be written as the product of the EIPG of the passive reflect-antenna $EIPG_p$ and the amplifier gain $G_a$ as,

$$EIPG_a = EIPG_p G_a$$ (3.2)

### 3.1.2 Experimental characterization of the amplifying active reflect-antenna

![Diagram](image-url)

Figure 3.3: (a) Setup for the measurement of the bistatic RCS of the amplifying reflect-antenna (ARA) using a vertically polarized antenna (VPA) and horizontally polarized antenna (HPA). (b) Setup for the measurement of the monostatic RCS using a single dual polarized antenna (DPA).

In order to experimentally determine the EIPG (effective isotropic power gain) of the amplifying or passive reflect-antenna, a monostatic RCS measurement is carried out. The classical measurement setup for the monostatic RCS needs two separate orthogonally polarized probe antennas similar to
the bistatic RCS measurement method, see Figure 3.3(a). In the method followed for the monostatic RCS measurement, we used a single dual polarized antenna (DPA) as shown in Figure 3.3(b). A similar setup using a dual polarized antenna with high degree of isolation between the ports has also been used for the characterization the amplifying reflect-antenna in [24]. The DPA used in our measurement is the microstrip-T coupled patch part of the amplifying reflect-antenna [25]. The forward transmission between the orthogonally polarized ports of the DPA $|S_{21}|$ in the presence of the amplifying reflect-antenna with bias ON can be written using the two way Friss transmission formula as,

$$|S_{21}|^2 = G_r G_a G_t G_d^2 \left( \frac{\lambda}{4\pi R} \right)^4$$  \hspace{1cm} (3.3)

In the above equation, $G_r$ and $G_t$ are the receiving and transmitting antenna gains of the active reflect-antenna respectively and $G_a$ is the amplifier gain. $G_d$ in (3.3) is the broad side gain of the DPA, $\lambda$ is the measurement wavelength and $R$ is the distance between the reflect-antenna and the DPA, see Figure 3.3(b). From the definition of monostatic RCS, $\sigma_m$ [43], we can write,

$$\sigma_m = G_r G_a G_t \frac{\lambda^2}{4\pi}$$  \hspace{1cm} (3.4)

The equations (3.3) and (3.4) yields,

$$\sigma_m = |S_{21}|^2 \frac{(4\pi)^3 R^4}{G_d^2 \lambda^2}$$  \hspace{1cm} (3.5)

The product $G_r G_a G_t$ which is the effective isotropic power gain can be written in terms of $\sigma_m$ as,

$$EIPG_a = \frac{4\pi \sigma_m}{\lambda^2}$$  \hspace{1cm} (3.6)

The same analysis, (3.3)-(3.6), holds for the passive reflect-antenna as well with $G_a = 1$ and $EIPG_p$ replaced with $EIPG_p$ in (3.6). From (3.5) and (3.6) we can see that by measuring the forward transmission between the DPA ports using a vector network analyzer, see Figure 3.3 we can find the monostatic RCS as well as the effective isotropic power gain (EIPG) of the active and passive reflect-antenna element. In the single DPA measurement method the coupling from the transmitting port to the receiving port can result in the phase cancellation between the desired signal and the coupled
3.1. Amplifying active reflect-antenna - the unit cell

3.1.3 Results and discussion

The effective isotropic power gain (EIPG) of the amplifying reflect-antenna calculated using the modelling approaches discussed in section-3.1.1 is compared with the measured results in Figure 3.5. It can be seen from Figure 3.5...
that the single antenna approach, see Figure 3.2(a) yielded more optimistic gain results than the actual measured EIPG. As can also be seen from Figure 3.5 that the EIPG calculated using the passive reflect-antenna method, see Figure 3.2(b), follow the measured EIPG with approximately 1-2dB accuracy over the frequency sweep. The large error in the single antenna method is due to the non-consideration of the drop in actual antenna gain because of the influence of feed line radiation and the bends. On the other hand, measurement of the passive reflect-antenna eliminates these source of errors in the modelling as its monostatic RCS is used first to find the effective isotropic power gain $EIPG_p$. Figure 3.6 shows the comparison of the monostatic RCS of the amplifying reflect-antenna and the passive reflect-antenna normalized to the maximum gain of the passive reflect-antenna. As can be seen from Fig. 3.6, at the tuning frequency of the microstrip-T coupled patch, the measured isolated amplifier gain is almost equal to the normalized gain of the amplifying reflect-antenna. On either side of the maximum active reflect-antenna gain, the amplifier gain calculated from the measurement of passive and active reflect-antenna showed a ripple with respect to the isolated amplifier gain. This is due to the dependance of the amplifier gain on its source and
load impedances. The 50Ω source and load impedances used in the amplifier measurement is attained only at the resonant frequency of the antenna. At frequencies other than the resonant frequency of the antenna, the source and load impedances of the amplifier is equal to the complex impedances of the antenna presented through the sections of transmission lines.

Monostatic RCS pattern of the active reflect-antenna element is an important parameter of interest for designing retrodirective arrays [44]. This is because, the reflected beam from the array is usually steered to the same location as the source of the interrogating signal. On the other hand for the reported amplifying reflect-arrays such as in [24], the bistatic RCS pattern of the active reflect-antenna element is more interesting. We study both the monostatic and bistatic RCS pattern of the designed amplifying reflect-antenna. For the monostatic RCS pattern of the amplifying reflect-antenna, the scattering parameters between the vertical and horizontally polarized ports of the DPA (|S_{21}|), see Figure 3.3(b), is measured at a constant far field distance for various angle θ. The measurement was carried out
Figure 3.7: Normalized monostatic RCS pattern: Measured (circles) and calculated using the measured patterns of single antenna (solid-line).

Figure 3.8: Normalized bistatic RCS pattern: calculated using the measured monostatic RCS pattern and the measured E-plane pattern of the single antenna (circles), the measured H-plane pattern of the single antenna (solid-line).
3.2. The power combining active reflect-array

at 10.034GHz, which is the resonant frequency of the isolated microstrip-T coupled patch antenna. In the single antenna-amplifier approach, the normalized monostatic RCS pattern can be calculated as the normalized sum of measured E-plane and H-plane pattern of the single antenna [25]. This is because, the amplifying active reflect-antenna receive and transmit signal in the E-plane and the H-plane of the microstrip-T coupled patch respectively. The calculated and measured monostatic RCS patterns of the amplifying reflect-antenna are shown in Figure 3.7. It can be seen from Figure 3.7 that the calculated monostatic RCS pattern almost follows the measured monostatic RCS pattern in the broadside. The difference between the measured and calculated monostatic RCS for large angles from the broadside may be attributed to the feed line bends and the interference from the amplifier. Due to the small aperture size of the amplifying reflect-antenna and non-availability of high gain probe antennas of small size, the bistatic RCS of the amplifying reflect-antenna was not separately measured. The bistatic RCS pattern however can be calculated as the difference between the monostatic RCS and the E-plane gain of the passive reflect-antenna. On the other hand, the bistatic RCS pattern using the single antenna approach should be equal to its normalised H-plane pattern. The measured H-plane pattern of the single antenna [25] and the calculated bistatic RCS pattern are shown in Figure 3.8.

3.2 The power combining active reflect-array

The prime motivation to seek unconventional power combining schemes in microwave and millimeter-wave band stems from the drawback of solid state devices in delivering large power. Although for microwave frequency bands the solid state transistors are becoming increasingly mature and are meeting the required power criteria, however this is not true for millimeter-wave and submillimeter-wave bands where the solid state device technology is still immature to meet the required power deliverance. One conventional way to overcome this problem is to combine the power from a large number of solid state devices at circuit level. Basically this scheme requires a large number of power dividers and combiners which results in a large insertion loss and low combining efficiency. Recently several antenna based spatial or quasi-optical arrays has been proposed to overcome the myriad problems faced by circuit level power combining schemes [19]. In a spatial power combining system the
power from individual solid state devices combines in free space and basically such systems can be divided into two, spatial power combining oscillators and spatial power combining amplifiers [19]. Although an early demonstration of the potential for antenna based power combining scheme was demonstrated way back in 1968 by Staiman et al. [45], the renewed interest in millimeter wave bands is the main reason behind the recent research activity in antenna based power combining schemes [20, 21, 22].

Most of the reported spatial power combining amplifiers are double sided in the sense that the transmitting and receiving points of the spatial power combiner are located on the opposite sides of the array, for instance the reported grid amplifier in [22] and the hard horn feed method reported in [23]. A marked deviation from this method is the method described in [24] where an active reflect-array is used to accomplish spatial power combining. Apart from the decreased real estate consumption, this scheme also has the advantage of increased isolation between the transmitting and receiving points. With large number of elements, the use of dual polarized aperture coupled patches described in [24] may be difficult to be manufactured and assembled due to its double layer structure. Also the two orthogonally polarized apertures in the aperture coupled patch located away from the center of the patch tend to decrease the RF isolation between the ports. In the proposed active reflect-array we use the microstrip-T coupled patch (MTCP) as the antenna element [25]. As was seen in section 4.3, the dual polarized MTCP element has excellent RF isolation and inherent DC isolation between the ports which is particularly useful in the proposed spatial combiner. In the proposed power combining array we used the unit cell based on MTCP similar to the configuration shown in Figure 3.1. The following sections describe the design methodology and the experimental results including the fault diagnosis method adapted for the power combining active reflect-array.

### 3.2.1 Synthesis of element positions and phases

The synthesis of element positions as well as the phases of the power combining reflect-array is carried out using the position-phase synthesis based on the MDE (Modified Differential Evolution, see chapter 2). As was described in the chapter 2, the position-phase synthesis of arrays yields least sidelobe level for a given number of elements compared to both the position only as well as phase only synthesis with out any significant sacrifice of directivity. The uniform amplitude assumption in the last chapter for position-phase
3.2. The power combining active reflect-array

3.2.1 The power combining active reflect-array

The power combining active reflect-array will hold for the active reflect-array as well if the f/D ratio of the feed point is selected for high illumination efficiency. The element positions of the unequally spaced reflect-array derived using the Modified Differential Evolution algorithm are given in Table 3.1. The calculated sidelobe level and half plane directivity of the array are -16dB and 14.5dB respectively.

<table>
<thead>
<tr>
<th>n</th>
<th>$\frac{x_n}{\lambda_0}$</th>
<th>$\phi_n^\circ$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.60</td>
<td>40.4</td>
</tr>
<tr>
<td>2</td>
<td>2.77</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>1.90</td>
<td>11.6</td>
</tr>
<tr>
<td>4</td>
<td>1.08</td>
<td>15.5</td>
</tr>
<tr>
<td>5</td>
<td>0.35</td>
<td>13.7</td>
</tr>
</tbody>
</table>

Table 3.1: The element positions and phases of the 10 element active reflect array derived using the MDE algorithm ($[(x_n, y_n), \phi_n]$=position and phase of the $n^{th}$ element).

3.2.2 Configuration of the active reflect-array

Based on the synthesized element positions described in the previous section, two arrays were fabricated on a TLC-30 substrate with $\epsilon_r = 3.0$ and a thickness of 0.5mm. One of the fabricated arrays is the actual active reflect-array and the other is a passive reflect array for the comparison in performance, see Figure 3.9. For the passive reflect array the unit cell is simply the dual polarized microstrip-T coupled patch antenna and the for the active reflect-array the unit cell is based on the active reflect-antenna element described in section 3.1. The gate bias of the common source configuration is applied zero, see Figure 3.1, by grounding the bias pad. The configuration of the active power combining reflect-array along with the feed horn is shown in Figure 3.10. The sections of the transmission lines between the dual polarized ports of each antenna element is calculated based on the feed position such that the phase of the reflected wave at each element correspond to the element phases described in Table 3.1. The feed is located offset 20º at a distance of 150mm from the array in its near field but at the far field of individual elements. However the collector is located at the far field of the array. As shown in
3.2.3 Results and discussion

For the proper working of the power combining active reflect-array, it is very important to ensure that the active stage associated with each active reflect-antenna element delivers the appropriate gain. If not the overall gain of the power combining array can be adversely affected. Since the gate bias of each amplifying stage is selected to be zero, the gain of the amplifier sandwiched in each active reflect-antenna element was expected to vary. However the degree of variation in the gain was unknown in the beginning due to the unit to unit variation in the $I_{DSS}$ of the PHEMPT, ATF-36077 (Agilent) from
3.2. The power combining active reflect-array

Figure 3.10: Active reflect-array with the feed horn.

Figure 3.11: Fault diagnosis of the active reflect-array by achieving the required total bias current in 10 steps.
Chapter 3. An unequally spaced power combining active reflect-array

15 to 45mA according to the data sheet of the transistor. An amplifier was therefore separately fabricated and the measurement of its gain was carried out for different $I_{DS}$ at a fixed $V_{DS}$ of 1.5V. It was noted that the gain of the amplifier showed only about .6dB variation for different $I_{DS}$ within in 15mA and 45mA respectively which is not therefore a matter of significant concern. However it is also very important to make sure that the each transistor in the array is not faulty. For the fault diagnosis of the active reflect-array the number of transistors biased is progressively increased in steps of one and each time noting the collected power as well as the total bias current. The results are summarized in Figure 3.11. It can be seen from Figure 3.11 that the total bias current increased each time an extra transistor is biased showing that the transistors are in good order. The slope of the relative received power in Figure 3.11 is also found to vary. Another important aspect of the power combiner is its stability. For the verification of the stability at a given frequency we increased the input power from a small value and noted the collected power at each step. The power was insufficient to reach the compression characteristics of the amplifier, due to the limitation in maximum available power from the synthesizer as well as the low effective area of the linear array. The relationship between the output and input power was found to be linear showing the stability of the active power combining reflect-array.

The effective isotropic radiated power (EIRP) of the power combining active reflect-array and the passive reflect-array are compared in Figure 3.12 for different frequencies. It can be seen that from Figure 3.12 that at the design frequency of 10GHz the EIRP of the power combining active reflect array showed a 6dB improvement over the passive reflect-array. However it is to be noted that the theoretical improvement should be equal to the amplifier gain which is approximately 8.5dB in the measurement. This decrease in the added power may be attributed to the variation in the gain of the individual amplifiers, influence of the lumped components on the pattern and alignment errors. Therefore for the proper working of the active reflect-array it is important to give most consideration to these individual sources of errors.

Figure 3.13 shows the gain pattern of the active and passive reflect-array. It can be seen from the patterns that the broad side power of the passive reflect-array is lower by about 6dB compared to that of the active power combining reflect-array. Also it can be seen from the patterns that the sidelobe levels of both active and passive reflect-array has significantly risen compared to the simulated ones, see section 3.2.1. The increase in sidelobe level may
3.2. The power combining active reflect-array

Figure 3.12: Effective isotropic radiated power (EIRP): solid line, power combining reflect-array and dotted line, passive reflect-array.

Figure 3.13: Gain pattern of the active reflect-array (solid) and the passive reflect array (dotted).

be attributed to the unwanted scattering from the finite ground plane edges of the active and passive reflect-arrays and also due to the mutual coupling
between the elements. In addition to these sources of errors, any improper alignment of feed and the feed phase center might have also significantly contributed to the increase in the measured sidelobe level.
This chapter summarizes the investigations on antennas and modelling techniques useful for the design of active antennas. A novel transmission line model for the determination of electric field in H-slots located in the ground plane of a microstripline is introduced. The validity of the model is compared with experiment. The electric field model is then applied to develop a computationally efficient software for the analysis of I-slot, H-slot and micromachined antennas. The analysis of I and H-slot antennas has been carried out using a spectral domain method combined with the developed H-slot electric field model. It is shown that the transverse length of the H-slot antenna is smaller than that of an I-slot antenna for the same resonant frequency. Further it is shown that different impedance characteristics can be achieved by varying the dimension of the side arms of the H-slot. These properties are desirable characteristics for the design of active antennas. We further propose an efficient modelling technique for silicon micromachined antennas. For the modelling, we used a transmission line model of a microtrip patch combined with the developed H-slot electric field model. For dual polarized active antenna applications such as retro-directive and active reflect-antenna applications, we introduce a novel microstrip-T coupled patch antenna. The antenna provides direct 50Ω matching, improved bandwidth and excellent RF and DC isolation between the ports. We then study a dielectric resonator (DR) antenna with slotline feed as an element for the design of active and passive reflect-arrays. It is noted that the slotline feed helps us in achieving
increased inter-element spacing and decreased cross-polarization for broadband DR based reflect-arrays.

4.1 I and H-slot antennas

The geometry of H-slot antenna located in the ground plane of the microstripline is shown in Figure 4.1. With $L_h = W_a$ in Figure 4.1, the H-slot antenna becomes an I-slot antenna. For the analysis of I/H-slot antennas we determine the field representation in the slots using a transmission line model combined with an Artificial Neural Network (ANN) for increased accuracy. Once the I/H-slot field is determined we apply a spectral domain method for its impedance characteristics.

4.1.1 Field distribution in H-slots using a transmission line model

Consider the transmission line model of the H-slot antenna [30], see Figure 4.2. As can be seen from Figure 4.2, each arm of the H-slot is modelled as a section of transmission line. The sections of transmission lines in the transmission line model are related to the physical dimensions of the slot as, $L_1 = (L_a - 2W_a)/2$ and $L_2 = (L_h - W_a)/2$. From Figure 4.2, the impedance
4.1. I and H-slot antennas

presented by the side arm to the main arm of the H-slot can be written as,

\[ Z_2 \simeq 2Z_1 \simeq 2jZ_c\tan(\beta_aL_2) \quad (4.1) \]

where, \( Z_c \) and \( \beta_a \) respectively are the slotline characteristic impedance and phase constant calculated using the transverse resonance technique described in [46]. Using basic transmission line theory, the voltage distribution along the main arm can be written as,

\[ V_y = \frac{I_{Z_2}}{2}(Z_2 + Z_c)\left( e^{j\beta_a(L_1-|y|)} + \Gamma_{Z_2}e^{-j\beta_a(L_1-|y|)} \right) \quad (4.2) \]

where \( I_{Z_2} \) is the current in \( Z_2 \). \( \Gamma_{Z_2} \), the reflection coefficient due to \( Z_2 \) is given by,

\[ \Gamma_{Z_2} = \frac{2j\tan(\beta_aL_2) - 1}{2j\tan(\beta_aL_2) + 1} \quad (4.3) \]

Using (4.2), the normalised voltage along the main arm of the H-slot can be written as,

\[ V'_y = \left| \frac{e^{2j\beta_a(L_1-|y|)} + \Gamma_{Z_2}e^{-2j\beta_aL_1}}{e^{2j\beta_aL_1} + \Gamma_{Z_2}} \right| \quad (4.4) \]

Assuming \( V_0 \) be the voltage at the center of the slot, the electric field variation becomes,

\[ e_x^H(y) = \frac{V_0}{W_a} \left| \frac{e^{2j\beta_a(L_1-|y|)} + \Gamma_{Z_2}e^{-2j\beta_aL_1}}{e^{2j\beta_aL_1} + \Gamma_{Z_2}} \right| \quad (4.5) \]
Chapter 4. Antennas and CAD modelling useful for the design of active antennas

For rectangular or I-slot, \( Z_2 = 0 \) and (4.5) becomes the piece wise sinusoidal (PWS) function as,

\[
e_I(y) = \frac{V_0 \sin(\beta_a (L_1 - |y|))}{W_a \sin(\beta_a L_1)}
\]

(4.6)

Using some algebraic manipulation, it can be seen that (4.5) describing the H-slot field is same as a the PWS function describing the I-slot field in (4.6) but with \( L_1 \) in (4.6) replaced with \( L_1^H \) given by,

\[
L_1^H = L_1 - \frac{1}{2\beta_a} \angle [-\Gamma Z_2]
\]

(4.7)

It is to be noted that the transmission line model in Figure 4.2 do not take into account the stray inductance associated with the short in the side arms and the stray capacitance associated with the open ends of the main arm. On the other hand cases such as when the slot is resonant, these parameters play a significant role in accurately predicting the resonant frequency and resonant resistance. The Artificial Neural Network (ANN) approach to model the capacitance associated with the open end and the inductance associated with a short ended slotline is described in the next section.

4.1.2 Artificial Neural Network (ANN) approach for slot field accuracy improvement

Artificial Neural Networks (ANN) have the ability to model multi-input multi-output microwave devices and components very efficiently [47][48]. In our application we attempt to improve the modelling of slotline junction and shorted ends associated with the H-slot. The motivation for pursuing the ANN approach for modelling the slot field is the non-availability of general closed form expressions for the capacitive/inductive behavior of slotline junctions and shorted ends. On the other hand it has been verified both experimentally and theoretically that the stray capacitances and inductances associated with slotline junctions, open and short ends are significant [49][50].

ANN essentially learn and generalize from a set of training data by adjusting its weights. ANN learning and generalization may be compared with the microwave network-analyzer calibration in the lab. A properly trained ANN has the special ability that it can accurately predict different independent quantities in parallel by processing the set of input data. The application of
4.1. I and H-slot antennas

Figure 4.3: Single artificial neural network (ANN) which is a multilayer perceptron (MLP) used to predict the H-slot shorted slotline inductance $L_s$, junction capacitance $C_j$, characteristic impedance $Z_c$ and the effective dielectric constant $\epsilon_e$.

ANN consists of training the neural net with a set of training data, testing the net again with a set of independent data. The failure of ANN during testing implies more training of the ANN. The trained neural network is further used as a model in an application. The ANN we used is a multi-layer perceptron (MLP), see Figure 4.3 [51]. With the frequency as the input the single neural network is capable of predicting the short end inductance $L_s$, junction capacitance $C_j$, characteristic impedance $Z_c$ as well as the effective dielectric constant $\epsilon_e$. The details on ANN and its applications is beyond the scope but can be found in an excellent reference [51]. The training method
Chapter 4. Antennas and CAD modelling useful for the design of active antennas

We employ for the ANN is an error backpropagation algorithm with a learning rate of 0.0002 and a number of iterations of 10000. In order to generate the training data we simulated using Momentum [52] a shorted slotline and a slotline junction as shown in Figure 4.4. The end inductance can be computed from the input impedance at port $P_1$, see Figure 4.4a, de-embedded to the end of the shorted slotline. The open end capacitance is computed from the open end reactance using the layout shown in Figure 4.4b with $P_1$ as the input port and $P_2$ and $P_3$ matched. The open end reactance is the difference between the input-impedance at $P_1$ de-embedded to the end of the line and $2Z_c$ where $Z_c$ is the characteristic impedance of the slotline. The parameters $L_s$ and $C_j$ appear in the improved transmission line model as shown in Figure 4.5. The electric field distribution can then be computed for the improved transmission line model by first finding $\Gamma_{Z_c}$ and then using (4.5).

![Improved transmission line model of the H-slot.](image)

**Figure 4.5: Improved transmission line model of the H-slot.**

### 4.1.3 Computationally efficient Spectral Domain Method for H-slot impedance

It is shown in [53] that a rectangular or I-slot similar to the H-slot shown in Figure 4.1 can be represented by a network model, see Figure 4.6. Using the reciprocity theorem, the components of the network-model in Figure 4.6 are proved in [53] as,

$$N = \sqrt{Z_{cm}} \int_{S_n} e_x^2(x, y) h_y(x, y) ds$$

(4.8)
4.1. I and H-slot antennas

where, $e_{a_x}^{a}$ is the aperture electric field, $h_y$, $Z_{cm}$ are the modal magnetic field and characteristic impedance of the microstripline respectively and $S_n$ represents the slot surface. Using the Green’s function component $G_{yy}^{HM}(x, y; x_0, y_0)$, see appendix-A, the aperture admittance can be written as,

$$
Y^e = \int_{S_n} \int_{S_n} e_{a_x}^{a}(x, y) G_{yy}^{HM}(x, y; x_0, y_0) \cdot e_{a_x}^{a}(x_0, y_0) \, ds \, ds_0 \quad (4.9)
$$

The method used in [53] to solve the above integrals involves expanding the unknown aperture field $e_x$ as a combination of piecewise sinusoidal (PWS) basis functions and forming a matrix equation. However we use the I/H-slot field determined using the transmission line model directly in the above equations and express the above integrals in spectral domain form.

![Figure 4.6: Network model of a rectangular slot with microstrip feed.](image)

Using transmission line model it was shown that both I-slot and H-slot field follow the piecewise sinusoidal (PWS) form as,

$$
e_{a_x}^{a}(x, y) = \frac{V_0}{W_a} \frac{\sin(\beta_a(L_i - |y|))}{\sin(\beta_a L_i)}, \quad [||x| \leq W_a/2, |y| \leq L_i] \quad (4.10)
$$

It was also shown in section-4.1.1 that for H-slot the length $L_i$ is given by (4.7) and for the I-slot $L_i = L_1$. Using this slot-fields the equations (4.8) and (4.9) transform into the following spectral domain form as was shown for a PWS basis function in [53],

$$
N = \frac{1}{2\pi} \int_{k_y = -\infty}^{\infty} \tilde{F}_u(W_f, k_y) \tilde{F}_p(\beta_a, k_y) \tilde{G}_{yx}^{HJ}(-\beta_m, k_y) \cos(k_y y_d) \, dk_y \quad (4.11)
$$
\[ Y^e = \frac{1}{4\pi^2} \int_{k_x=-\infty}^{\infty} \int_{k_y=-\infty}^{\infty} \left[ \tilde{F}_u(W_a, k_x) \tilde{F}_p(\beta_a, k_y) \right]^2 \tilde{G}_{yy}^{HM} \, dk_x \, dk_y \quad (4.12) \]

where, \( \beta_m \) is the propagation constant for the microstripline and \( \tilde{q} \) indicate the spectral domain representation of quantity \( q \) and \( y_d \) is the slot displacement from the feed center. The functions \( \tilde{F}_u \) and \( \tilde{F}_p \) are described as,

\[
\tilde{F}_u(w, k_i) = \frac{\sin(k_i w/2)}{k_i w/2} \quad (4.13)
\]

\[
\tilde{F}_p(k_i, k_j) = \frac{2k_i [\cos(k_j L_i) - \cos(k_i L_i)]}{\sin(k_i L_i/2)(k_i^2 - k_j^2)} \quad (4.14)
\]

Thus the method we follow is formulated similar to the spectral domain method in [53] for I-slots but with a single basis function describing the accurate representation of the slot electric field. The advantage of the method we propose is that the computation of the slot field is based on simple closed form expressions thereby saving valuable computational time. Also the method we follow is easy to apply for the H-slots. It is to be noted that the solution of the integrals in (4.11) and (4.12) is fraught with difficulties such as singularities. A note on the integration method is also included in appendix-A.

### 4.1.4 Results and discussion

We compare the transmission line model approach for determining the field distribution in an H-slot with the measured results reported in [54]. The dimensions of the H-slot, see Figure 4.1, are \( W_a = 1 \, mm, L_a = 11 \, mm \) and \( L_h = 5.5 \, mm \). The substrate is 1.59mm thick with an \( \epsilon_r \) of 2.2. The measured results carried out at 5GHz is compared with two different models, TLM\(_1\) and TLM\(_2\), see Figure 4.7. Transmission line model TLM\(_1\) do not consider the junction capacitances associated with the H-slot, see Figure 4.2, whereas the transmission line model TLM\(_2\), see Figure 4.5 uses the junction capacitance. Due to the rounded slot ends used in [54], the end inductances were not considered in the model. It can be seen from Figure 4.7, the measured results agree well with TLM\(_2\) compared to TLM\(_1\) in the neighborhood of field maximum where the accuracy of the field is more important in the analysis. It is to be noted that with \( L_h = W_a \), see Figure 4.1, the H-slot becomes a rectangular or I-slot. To test the program developed we compare the calculated impedance characteristics with the measurements reported in [53] for I-slot. The dimensions of the H-slot for this case, see Figure 4.1, are
4.1. I and H-slot antennas

Figure 4.7: Comparison of theoretical and measured results for electric field distribution in H-slot.

Figure 4.8: Comparison of calculated and measured results of normalised I-slot impedance, $Z = r + jx$, solid-line: calculated $r$, *: measured $r$, dashed-line: calculated $x$, ■: measured $x$.

$L_h = W_a = 0.7\, \text{mm}$ and $L_a = 40.2\, \text{mm}$. The substrate used is 1.59mm thick.
Chapter 4. Antennas and CAD modelling useful for the design of active antennas

Figure 4.9: Comparison of Artificial Neural Network generated output and that of Momentum simulator for various H-slot parameters.

with an $\epsilon_r$ of 2.2. The calculated and measured impedance characteristics are shown in Figure 4.8. It can be seen from Figure 4.8 that the calculated and measured results agree very well. The slight deviation of the calculated results from the measured ones may be due to the losses in the substrate. Next we trained and tested two different ANN according to the method described in section 4.1.2 for the slotline characteristic impedance $Z_c$, effective dielectric constant $\epsilon_e$, H-slot junction capacitance $C_j$ and end inductance $L_s$. The width of the slotline was 0.7mm and the substrate thickness and $\epsilon_r$ were 1.59mm and 2.2 respectively. The first ANN has 4 hidden neurons and the second one 8 hidden neurons. The ANN generated outputs and the output from the Momentum simulator [52] for all the four parameters
4.1. I and H-slot antennas

Figure 4.10: Comparison of the H-slot impedance characteristics derived using the proposed spectral domain - transmission line model based on ANN [SD-TLM (ANN)] with commercial simulators.

are shown in Figure 4.9. It can be seen from Figure 4.9, the complexity of ANN in terms of more hidden neurons did not show any notable improvement in the performance of the ANN. Hence we retained 4 hidden neurons for ANN used for the modelling. It can also be seen from Figure 4.9 there is a good agreement between the Momentum simulator and the output from both ANN. We further used the trained ANN with 4 hidden neurons for the H-slot impedance computations. The H-slot dimensions are $L_h = 10mm$, $L_a = 41.6mm$. The substrate parameters and slot width are same as the I-slot case considered in Figure 4.8. The impedance characteristics derived using the combination of spectral domain (SD), transmission line model (TLM) and ANN, [SD-TLM(ANN)], is compared with that of two commercial simulators in Figure 4.10. The commercial simulators employed in the study are HFSS [55] and Momentum [52]. It can be seen from Figure 4.10 that there is a good agreement between all the three methods. The resonant frequency for SD-TLM(ANN) and HFSS were approximately 2.2GHz and Momentum showed slightly higher resonant frequency at 2.3GHz. It is to be noted that Momentum results also showed a similar deviation for the I-slot considered.
Chapter 4. Antennas and CAD modelling useful for the design of active antennas

in [53] and in Figure 4.8. It is to be noted that the both the Momentum and HFSS simulations took more than two hours where as the SD-TLM(ANN) took only about 4 minutes for the entire frequency sweep.

4.2 A micromachined H-slot coupled patch antenna

The micro-machined antenna realised on silicon wafer is shown in Figure 4.11. The antenna consists of two high resistivity silicon wafers one on each side of a ground plane. The radiating patch is etched on the top wafer and the microstripline is etched on the bottom wafer. The microstripline couples to the radiating patch through an aperture. To increase the coupling and to reduce the back-lobe from the aperture, an H-slot is used as shown in Figure 4.12. Part of the silicon in the top wafer is etched away to form an air filled cavity. As shown in Figure 4.11, the top side of the cavity is separated from the metallization forming the patch by a thin silicon membrane. This procedure in fabrication helps in achieving better performance of the antenna since the effective $\epsilon_r$ of the top substrate is reduced. Also the thin membrane provides mechanical support for the metallization forming the patch. If the processing for the top wafer uses wet etching, then the cavity will get a pyramidal shape where the angle between the side walls and the bottom surface is defined by the crystal planes. In the antenna considered dry etching has been used which gives the cavity a rectangular shape. The detailed processing steps followed in the fabrication of the antenna is discussed in [56].

Figure 4.11: Micro-machined H-slot coupled microstrip antenna.
4.2. A micromachined H-slot coupled patch antenna

![Figure 4.12: Top view of micro-machined H-slot coupled antenna.](image)

4.2.1 Transmission line model of the micro-machined antenna

The transmission line analysis derived for the micro-machined aperture coupled antenna follows the modelling of an aperture coupled microstrip antenna on a uniform substrate discussed in [57]. Essentially, [57] incorporates the transmission line model of an edge coupled single layer microstrip antenna [58] and an interesting and simple physics based model for the slot coupling. In order to fully appreciate the transmission line analysis of micromachined antenna, it is important look into the various aspects of [57]. This is because the aperture coupled microstrip antenna in [57] is similar to Figure 4.11 except for the uniform substrate instead of the micromachined cavity in Figure 4.11 and a rectangular or I-slot of length \( L_a \) instead of the H-slot in Figure 4.12. To derive the transmission line model for the aperture coupled patch antenna, the steps followed are,

- Radiating part of the antenna is modelled by two equivalent slots as described in [58]. The slot impedances are represented as \( Y = G + jB \) at the edges of the patch.

- The aperture impedance, \( Y_{ap} \) is represented as a parallel combination of two shorted slotlines of lengths \( L_a/2 \) and the slotline wave number
$\beta_a$ is evaluated using a modification of Cohen’s method [46] taking into account the presence of the conducting patch [59].

- The discontinuity of voltages in the patch and the microstrip line feed at the aperture are represented by two transformers of turns ratio $N_1$ and $N_2$.

Thus the transmission line model of the aperture coupled patch can be represented as shown in Figure 4.13. It is to be noted that maximum time for the computations is devoted to the evaluation of slotline parameters which involves the application of the transverse resonance technique. In the transmission line model shown in Figure 4.13, $L_1$ and $L_2$ is computed using the effective length of the patch and the feed position. The transformer ratios $N_1$ and $N_2$ are given by [57],

$$N_1 = \frac{L_a}{W_{\text{eff}}^m} \quad (4.15)$$

where, $W_{\text{eff}}^m$ is the effective width of the patch.

$$N_2 = \int_{x=-W_a/2}^{W_a/2} \int_{y=-W_{\text{eff}}^m/2}^{W_{\text{eff}}^m/2} h_\nu \frac{E_a}{V_0} \, dx \, dy \quad (4.16)$$

where, $W_{\text{eff}}^m$ and $h_\nu$ are the effective width and TEM modal magnetic field component of the microstrip line respectively. $E_a$ represent the aperture field.
4.2. A micromachined H-slot coupled patch antenna

The $y$ directed TEM modal magnetic field in the microstripline can be written as,

$$h_y^m = \frac{1}{\sqrt{W_{m}^{eff}H_m}} \hat{y} \quad (4.17)$$

where, $H_m$ is the height of microstripline substrate. For the usual case when $L_a > W_{m}^{eff}$, see Figure 4.13, $N_2$ can be computed using (4.6) for the aperture field as,

$$N_2 = \frac{2}{\sqrt{W_{m}^{eff}H_m}} \frac{\cos(\beta_a(L_a - W_{m}^{eff})/2) - \cos(\beta_aL_a/2)}{\beta_a \sin(\beta_aL_a/2)} \quad (4.18)$$

Finally the input impedance of the antenna at the reference plane which is the center of the slot can be written as,

$$Z_{in} = \frac{N_2^2}{N_1^2 Y_p + Y_{ap}} - jZ_{0M} \cot(\beta^m L_s) \quad (4.19)$$

In (4.18) and (4.19) $L_s$, $\beta^m$, $Z_{0M}$ are the length, phase constant and characteristic impedance of the stub respectively. $Y_p$ is the total patch impedance. $Y_p$ can be calculated as series combination of input impedances due to load $Y = G + jB$ at the ends of transmission line sections $L_1$ and $L_2$ (see Figure 4.13). In order to derive the transmission line model of the micromachined antenna, we introduced the following modifications [30, 32].

- Modifications in terms of the effective dielectric constant of the patch side to account for the micromachined cavity.
- A modified field distribution to account for the H-slot [30] instead of I-slot used in [57].

**Effective dielectric constant of patch substrate**

The most crucial part in the analysis of the micro-machined antenna is the derivation for the expression of effective dielectric constant of the top substrate forming the cavity. In [60] a capacitor model is used to derive the equivalent dielectric constant of the micromachined cavity as,

$$\epsilon_c = \frac{\epsilon_r}{1 + (\epsilon_r - 1)\frac{b}{\pi b}}. \quad (4.20)$$
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In the above equation, $b$ is the height of air cavity and $a$ is the thickness of the membrane supporting the patch. We follow a more accurate way of finding the equivalent dielectric constant of the cavity using the results for suspended-substrate microstriplines derived in [61][62]. Consider the suspended microstripline shown in Figure 4.14 where the substrate of dielectric constant $\epsilon_r$ and thickness $a$ is suspended below the microstrip. The remaining region between the substrate and ground plane is assumed to be air having a thickness $b$. By the least square curve fitting method together with the computationally intensive spectral domain analysis, it has been shown in [61][62] that the expression for effective dielectric constant ($\epsilon_e$) of the substrate can be written as,

$$\epsilon_e = \left[ 1 + \frac{a}{b} \left( a_1 - b_1 \ln \frac{W}{b} \right) \left( \frac{1}{\sqrt{\epsilon_r}} - 1 \right) \right]^{-2}$$  \hspace{1cm} (4.21)

where,

$$a_1 = \left( 0.8621 - 0.1251 \ln \frac{a}{b} \right)^4$$  \hspace{1cm} (4.22)

$$b_1 = \left( 0.4986 - 0.1397 \ln \frac{a}{b} \right)^4$$  \hspace{1cm} (4.23)

The above expressions has been used as the starting value in the following way to derive the final expressions of effective dielectric constant.

1) Substitute $a = H - h_{air}$ and $b = h_{air}$ in the above equations where $h_{air}$ is the height of the air cavity shown in Figure 4.11.
4.2. A micromachined H-slot coupled patch antenna

2) The quasi-static expression for the effective dielectric constant of the microstripline [63] is then used to find the dielectric constant equivalent $\epsilon_e$ of the uniform cavity of height $H$ bounded by a strip of width $W$. The following equation can then be solved to find $\epsilon_e$,

$$
\epsilon_e - \left( \frac{\epsilon_e + 1}{2} + \frac{\epsilon_e - 1}{2} \frac{1}{\sqrt{1 + 12H/W}} \right) = 0 \quad (4.24)
$$

Thus by changing the dielectric constant of the uniform substrate in (4.15)-(4.19) to $\epsilon_e$, we can analyze the micromachined aperture coupled patch.

**Self impedance of the H-slot**

It is to be noted that, for the derivation of the aperture impedance $Y_{ap}$ in [57], the aperture is assumed as a parallel combination of two slotted lines. The H-slot used in the micro-machined antenna can be considered as a combination of six different slotted lines, each representing one arm of the H-slot. With $L_1 = (L_a - 2W_a)/2$ and $L_2 = (L_h - W_a)/2$, the model of the H-slot is shown in Figure 4.2 can be used to find the H-slot admittance as,

$$
Y_{ap}^h = -j \frac{2 \tan(\beta_a (L_a - W_a)/2) - \cot(\beta_a L_h/2)}{2 + \cot(\beta_a L_h/2) \tan(\beta_a (L_a - W_a)/2)} \quad (4.25)
$$

**Modification of the slot electric field**

Since the H-slot causes a different electric field distribution compared to I-slot, the expression (4.5) derived in section-4.1.1 is used. However, it is to be noted that $\beta_a$ consider the effect of conducting microstrip in the vicinity of the slot[59]. The aperture field can then be written as,

$$
E_a = \frac{V_0}{W_a} \left| \frac{e^{2j\beta_h L_1} - |y|}{e^{2j\beta_h L_1} + |y|} \right| \quad (4.26)
$$

where $L_1 = (L_a - 2W_a)/2$ and $L_2 = (L_h - W_a)/2$ and $\Gamma_h$ is given by,

$$
\Gamma_h = \frac{2j \tan(\beta_h L_2) - 1}{2j \tan(\beta_h L_2) + 1} \quad (4.27)
$$

By substituting (4.26) in (4.16) the transformation ratio $N_2$ given in (4.16) can be numerically solved.
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4.2.2 Results and discussion

The micromachined antenna was fabricated using high resistivity silicon wafers having a thickness of 254\,\mu m. The substrate carrying the transmission lines and the slot was thinned down to a thickness of 100\,\mu m to improve the coupling between the feed-line and the slot, and to facilitate the implementation of via-holes in the substrate. As shown in Figure 4.11 the patch is supported by a thin silicon membrane. The remaining silicon on the top wafer is etched away using dry etching forming a 204\,\mu m deep air cavity, and a 50\,\mu m thick membrane for the microstrip patch to rest on. The transmission line and the ground plane were fabricated by first gold evaporation and then electro-plating gold to a final thickness of 1\,\mu m. The patch antenna element was formed by evaporation of 1\,mm aluminium to the patch substrate. The patch substrate was glued on to the lower substrate using silver epoxy. The antenna microstrip feed-line is connected to a probe pad where via-holes through the silicon wafer are used to provide ground connections for standard CPW probes. The method proposed for the calculation of the effective dielectric constant is first compared with the HFSS calculations for a suspended substrate microstripline having a width comparable to the width of a 60GHz patch antenna, in this case 1700\,\mu m. The other dimensions for the antenna are the same as given in the previous section. Figure 4.15 shows the variation of effective dielectric constant and the error introduced as a function of the frequency band of interest. It can be concluded from the Figure that the error in the evaluation of the effective dielectric constant of the suspended patch substrate for a wide micro strip line is negligible. Figure 4.16 shows the simulated and measured return loss characteristics of the antenna. The width and length of the patch are both 1500\,\mu m. The H-slot dimensions $L_a$, $L_h$ and $W_a$, see Figure 4.11 and Figure 4.12, are 900\,\mu m, 600\,\mu m and 120\,\mu m respectively. The micromachined suspended patch substrate has a thickness of 50\,mm and the air cavity walls are separated by a distance of 3000\,\mu m, see Figure 4.11. The measurements were done using a wafer probing system as described in [64] together with an Anritsu 360B network analyzer and a V-band frequency extender module. The setup was calibrated using a SLOT-CPW calibration substrate. It can be seen in Figure 4.16. that there is a close agreement between the measured return loss and the simulated results. It is to be noted that parameters such as the thickness of ground plane and losses has been assumed to be zero in the present analysis. As a future work improvements of the model incorporating the above parameters may be
4.2. A micromachined H-slot coupled patch antenna

Figure 4.15: Calculated effective dielectric constant and the relative error compared with that of the HFSS simulator.

Figure 4.16: Theoretical and measured return loss characteristics of the micromachined antenna.
Simulations for the entire frequency sweep with 168 frequency point took only about one minute on a personal computer. This can be considered extremely fast considering the time taken by commercial simulators employing the full wave electromagnetic methods for the same problem.

### 4.3 A dual polarized microstrip-T coupled patch antenna

Dual or circularly polarized antennas, as far as active integrated antennas are concerned, are those which are compact and single layered, although many different antenna configurations has been studied previously, for instance [65][66] and [67]. The dual polarized microstrip-T coupled patch antenna (MTCP) we propose [25] consists of two electromagnetically coupled T junctions of microstriplines on the adjacent sides of a square microstrip patch as shown in Figure 4.17. The microstrip-T junctions acts as impedance transformer, transforming the high radiation resistance of the patch to the desired low impedance. The small width $t$ of the microstrip-T causes the directions of currents in the two arms of the microstrip-T close to the patch to be in opposite directions and opposite to the exited $TM_{10}$ or $TM_{01}$ mode depending upon the exited port, see Figure 4.17. Therefore the normal patch radiation pattern is not affected significantly by the microstrip-T. For a given
4.3. A dual polarized microstrip-T coupled patch antenna

patch dimension, the parameters in attaining matching at the ports are the gap $S$ and the length of the microstrip-T arm $L$.

4.3.1 Advantages of the MTCP in active-antenna applications

Retro-directive and active reflect-arrays [68][69] are typical active-antenna applications which require a dual polarized antenna. As can be seen from Figure 4.17, the microstrip-Ts have inherent DC isolation between them, therefore DC blocking capacitors can be avoided in a graceful way for the dual polarized active antenna applications. By adjusting $S$ and $L$, see Figure 4.17, it is possible to achieve 50Ω input impedance as well as complex input impedance, for instance in matching an active device. These properties makes the proposed antenna suitable for active antenna applications.

4.3.2 Results and discussion

![Figure 4.18: Comparison of the measured and simulated return loss characteristics of the microstrip-T coupled patch antenna.](image)

A MTCP antenna was fabricated on a 0.5mm thick TLC-30 substrate ($\varepsilon_r = 3.0$, $\tan\delta = 0.003$). The width of the patch is $W = 8.18$mm and for the
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Figure 4.19: Comparison of the measured and simulated isolation characteristics of the MTCP antenna.

microstrip-T, the parameters are \( T = 1.28\,\text{mm}, \ t = 0.3\,\text{mm}, \ L = 7.98\,\text{mm} \) and \( S = 0.1\,\text{mm}, \) see Figure 4.17. Figure 4.18 and Figure 4.19 respectively shows the measured and simulated return loss and isolation characteristics of the antenna for a 50\( \Omega \) reference. The simulator used is the Momentum simulator. It can be seen from the comparison that, the measured return loss of 27dB agree well with the simulated results. It can also be seen from the return-loss characteristics that the achieved 10dB bandwidth is about 2.1\%, which exceeds the bandwidth achieved using usual matching techniques such as quarter-wave transformer on a similar substrate. From the measured isolation characteristics shown in Figure 4.19, it can be seen that the measured isolation at the tuning frequency is about 32dB and better than 27dB for the entire below 10dB return loss range. The measured E-plane and H-plane radiation patterns of the antenna are shown in Figure 4.20 and Figure 4.21 respectively. It can be seen from the measured E and H-plane co- and cross-polarization patterns that they are very similar to a square patch antenna. The ripple in the E-plane pattern is accounted for by the finite ground plane effect, as has also been observed in [70]. Figure 4.22 shows the measured broadside gain of the MTCP antenna for a set of frequencies. It can be seen that the maximum broad side gain attained at the tuning frequency is about 6dB. Figure 4.23 shows the impedance characteristics of the MTCP
4.3. A dual polarized microstrip-T coupled patch antenna

de-embedded to the end of the feed. It can be seen from Figure 4.23 that the resonance resistance attained is about 50Ω.

**Figure 4.20**: Measured E-plane pattern of the MTCP antenna, solid, copolarization, dashed, cross-polarization.

**Figure 4.21**: Measured E-plane pattern of the MTCP antenna, solid, copolarization, dashed, cross-polarization.
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4.4 A dielectric resonator antenna with a slot-line feed

Dielectric resonator antennas have received large attention due to their small size, increased impedance bandwidths, high radiation efficiency and compatibility with microwave integrated circuits. Using different shapes of the dielectric resonators and their different operating modes, it is possible to attain...
different radiation patterns. Different shapes studied include dielectric resonators which are of cylindrical [71], rectangular [72], hemispherical [73] and ring [74] shape. Several feeding techniques for dielectric resonators (DR) has also been reported in literature. They include the feeding of the DR using a conducting probe [71][72], aperture [75], microstripline [76] and coplanar waveguide [77]. The slotline fed dielectric resonator antenna presented in this section is depicted in Figure 4.24. The microstripline-slotline transition helps us in testing the antenna characteristics through a 50Ω port at the microstripline. The theoretical resonant frequency of the operating $HE_{11δ}$ is given by the following expression.

$$f_0 = \frac{c}{2\pi\epsilon_r} \sqrt{\left(\frac{1.841}{a}\right)^2 + \left(\frac{\pi}{2d}\right)^2}$$

(4.28)

where, $a$ and $d$ are the radius and height of the cylindrical dielectric resonator respectively and $c$ is the velocity of light in free space. However the following empirically derived formula has gained a better acceptance for more accurate prediction of the resonant frequency [78][79].

$$f_0 = \frac{6.328c}{2\pi a(\epsilon_r + 2)} \left(0.27 + 0.36\frac{a}{2d} + \left(0.02\frac{a}{2d}\right)^2\right)$$

(4.29)

### 4.4.1 The dielectric resonator antenna with slotline stubs as a reflect-array element

The application of the proposed DR antenna with slotline stubs as a reflect-array element is depicted in Figure 4.25. From Figure 4.25 we can see that the phase change introduced by the slotline stubs on the reflected wave is proportional to $L_1, L_2$. Now consider a conventional reflect-array element based
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Figure 4.25: Dielectric resonator antenna with slotline stubs for reflect-array applications.

on microstrip stubs described in [36]. In order to provide large phase changes microstrip stubs in [36] has to be longer. To keep the inter-element spacing of the reflect-array, it is a common practice that bends are introduced in the stubs. It has been found that this method increases the cross-polarization level of the array. However since the two slotlines are in series in Figure 4.25, the individual lengths of the stubs are less than that occupied by a single transmission line. This helps us in releasing more inter-element spacing for reflect-array applications. Another advantage of the proposed method is the case when $L_1 = L_2$. In this case the radiation from the stubs due to their orthogonal directions of the field vectors, see Figure 4.25, get cancelled in the far field. Thus the proposed configuration has negligible effect on the cross-polarization levels of the array.

4.4.2 Results and discussion

For a resonant frequency of 10GHz the radius of the DR was calculated using equation (4.29). The $\epsilon_r$ and height of the resonator are 10 and 2.95mm respectively. The substrate is 0.8mm thick with an $\epsilon_r$ of about 2.2. The radius of the DR was calculated to be 5.33mm using (4.29). However, for the same DR data (4.28) yielded a resonant frequency of 9.58GHz. The microstripline-slotline transition was separately designed using the Momentum simulator [52] with the objective of having least insertion loss. The minimum insertion loss of about 0.33dB was obtained at 10GHz with almost same value through out the frequency band of interest. The stub lengths for microstripline $L_m$ and slotline $L_s$, see Figure 4.24, are 2.25mm and 3mm and their corresponding widths are 2.5mm and 0.3mm respectively. The overlap
distance between the slotline and DR $p$, see Figure 4.24, is 3mm. Figure

![Graph showing return loss characteristics of the DR antenna with slotline feed.](image)

*Figure 4.26:* Return loss characteristics of the DR antenna with slotline feed.

<table>
<thead>
<tr>
<th>frequency, GHz</th>
<th>Gain, dB</th>
</tr>
</thead>
<tbody>
<tr>
<td>9.0</td>
<td>-1.1</td>
</tr>
<tr>
<td>9.5</td>
<td>4.23</td>
</tr>
<tr>
<td>10</td>
<td>5.79</td>
</tr>
<tr>
<td>10.5</td>
<td>5.25</td>
</tr>
<tr>
<td>11</td>
<td>4.05</td>
</tr>
<tr>
<td>11.5</td>
<td>4.30</td>
</tr>
</tbody>
</table>

Table 4.1: Broad side gain of the DR antenna.

4.26 shows the comparison of simulated results using HFSS [55] for two independent positions, $HFSS_{1,2}$, of the slotline-microstrip transition and the measured results for the first position. The motivation for two independent positions in the simulations was to study the influence of slotline fields on the antenna characteristics. From the comparison the HFSS simulations for two independent positions we can see that there is a small influence of the section of the slotline between the transition and edge of DR on the antenna return loss. However it was found that the radiation patterns remained same
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for these two cases. As can be seen from Figure 4.26, the measured bandwidth is comparatively larger compared to the simulated ones. The E-plane

![Figure 4.27: Measured E-plane pattern of the DR antenna with slotline feed, thick-line, co-polarization and thin-line, cross-polarization.](image)

and H-plane pattern of the DR antenna are shown in Figure 4.27 and Figure 4.28 respectively. It can be seen from the patterns that they are similar to those reported in [76] for the microstrip line feed. The broad side gain of the DR antenna measured at different frequencies is tabulated in Table-4.1. It can be seen from the gain characteristics that the maximum gain of the DR antenna is achieved at the design frequency.

![Figure 4.28: Measured H-plane pattern of the DR antenna with slotline feed, thick-line, co-polarization and thin-line, cross-polarization.](image)
Chapter 5

Elements of RF integration techniques

In this chapter, we analyse and design a slot matched microwave amplifier. Using the simulations and measurements we show that the slot matched amplifier exhibits improved gain and noise figure characteristics compared to that of stub matched amplifier. Besides, the slot matched amplifier has a decreased layout size compared to stub matched design. We then propose a new way of using the non-radiative dielectric NRD waveguide as a plug between two substrates for multilayered RF integration. The NRD coupling is studied using a transmission line model. The calculated and experimental characteristics of the NRD plugs under different feed configurations are compared. It is seen that NRD plugs has the potential to release space between two substrates where high power amplifiers and other discrete components can be sandwiched.

5.1 Low noise amplifier design based on slot matching

It was seen in chapter 4 that a slot in the ground plane of a microstrip line can be represented as a series impedance, see Figure 4.6. By adjusting the resistive part of the slot impedance to a small value we can implement a slot matching circuit, for instance in amplifier designs. The low resistance of the slot ensures minimum radiation losses. This is the principle of slot matching. It should be noted that since the equivalent impedance of the slot is in series with the microstrip line, the principle of matching is using the series react-
tance matching technique, which is impossible with the usual transmission line stubs such as microstrip lines. We then employ the non-resonant slot matching technique to match the input and output of a microwave transistor for low noise.

5.1.1 A simple method for the analysis of non-resonant slots

In our analysis, we assume that the non-resonant slots employed in slot-matching is a combination of slot-lines. The examples of such slots include I-slot and H-slot outlined in Chapter 4. The non-resonant slot in the ground plane of the microstripline and its equivalent circuit is shown in Figure 5.1. The reactance of the slot transferred to a microstripline feed can be easily derived by finding the self-reactance of the slot. The method followed is using the slotline-microstripline impedance transformation formula given in [80] which is essentially based on the theory in [81]. Referring to Figure 5.1, the self-reactance of the slot $X_s$ can be expressed as a parallel combination of $X_1$ and $X_2$ as,

$$X_s = \frac{X_1 X_2}{X_1 + X_2}$$  \hspace{1cm} (5.1)

The transformed reactance $X_m$ which in series with the microstripline is given by the following expression.

$$X_m = N_{ms}^2 X_s$$  \hspace{1cm} (5.2)
5.1. Low noise amplifier design based on slot matching

$N_{ms}$ in the above equation represents the slotline-microstripline impedance transformer ratio given in [80],

$$N_{ms} = \cos q_0 - \cot q_0^a \sin q_0$$  \hspace{1cm} (5.3)

where,

$$q_0 = \frac{2\pi h \sqrt{\varepsilon_r - \varepsilon_e}}{\lambda_0}$$  \hspace{1cm} (5.4)

$$q_0^a = q_0 + \tan^{-1} \frac{\sqrt{\varepsilon_r - \varepsilon_e}}{\sqrt{\varepsilon_e - 1}}$$  \hspace{1cm} (5.5)

where, $\varepsilon_r$ and $h$ are dielectric constant and thickness of the substrate and $\varepsilon_e$ is the effective dielectric constant of the slotline computed using [46].

5.1.2 Amplifier design using non-resonant slots

The series slot matching is implemented for the output and input of a low noise amplifier operating at a frequency of 1GHz. The transistor used is ATF-10136 [82] in a common source configuration. In the design, we have considered L-slot and dog bone slots for matching the input and output respectively. The layout representation of the transistor amplifier is shown in Figure 5.2. The optimum dimensions of the slot has been derived from the performance criteria of the amplifier, which is a gain of 18dB at noise figure of 1.5dB. The characterization of the slots was done using a method of moments based CAD program [52]. The following steps has been used for the designing the slot matched amplifier.

- The amplifier input and output reflection coefficients were first determined for the assumed gain and noise figure.

- The planar simulator [52] is then used to design the slots as per the requirements of the amplifier.

- The scattering matrices of the slots are then implemented in an active circuit simulator.
Figure 5.2: Layout representation of the common source LNA (nonlinear components are the slots used for matching) width of slots=0.6mm, substrate:-Teflon, $\varepsilon_r=2.53$, thickness = 0.8mm.

Figure 5.3: Series slot impedance for the input, $Z_1$ and output, $Z_2$ of the transistor as a function of frequency

5.1.3 Results and discussion

Figure 5.3 shows the comparison of results of the slot impedances used for input and output matching using the theory outlined in section 5.1.1 and
5.1. Low noise amplifier design based on slot matching

the Momentum [52] simulator. It can be seen from Figure 5.3 that the Momentum simulator and the calculations agree well. This means that the non-resonant slots for matching can be efficiently designed using the theory outlined in section 5.1.1. It can also be seen from the Momentum results that the resistance is very small at an operating frequency of 1GHz for both the slots. For the input match, since the transverse length of slot is much less than the resonant length, the resonant resistance $R_1$ is almost zero, see Figure 5.3. This ensures minimum radiation losses from the circuit. To compare the performance of the slot based amplifier, a shunt stub matched amplifier was designed for the same specifications as above. Figure 5.4 shows the comparison for the performance of LNA designed using slot matching and shunt stub matching. It can be seen from Figure 5.4 that there is close agreement between simulated and experimental results. It can also be seen

![Graph 1](graph1.png)

*Figure 5.4: Performance characteristics of LNA: solid line: slot amplifier (simulated), dashed line: stub amplifier (simulated), ◦ ◦: stub amplifier (measured) and □ □: slot amplifier (measured).*

that the bandwidth and noise-figure characteristics is improved for the slot amplifier compared to the stub amplifier. It is also worth while here to point out that overall size of the circuit is approximately half that of the
traditional microstrip stub amplifier. Further size reduction of slots in the slot matched amplifier may be possible by reducing the resonant length of slots by dielectric loading.

5.2 Nonradiative dielectric (NRD) waveguides and applications

In this section applications of nonradiative dielectric (NRD) waveguides [33] for multilayer integration technology are investigated. The interest in NRD waveguides stems from their suitability for implementation as interconnects in multilayered integrated circuits and from the advantages such as low cost fabrication and loss characteristics. The low loss at millimeter-wave of the NRD waveguide makes it particularly suitable for emerging millimeter-wave applications. As the name implies, the non-radiative nature of NRD is also highly advantageous in many applications. The NRD waveguide has previously been used in the design of a feed network for slot antennas in [83], as a leaky wave feed for planar antennas in [84] and in the design of a millimeter-wave oscillator in [85]. One of the key problems in the design of circuits involving NRD is the coupling of electromagnetic energy in and out of the NRD waveguide. In [33] a rectangular waveguide has been used as the feed for the NRD guide. Since most applications nowadays is based on planar technology, the rectangular waveguide feed as used in [33] may tend to increase the overall size of the system whereby losing advantages of planar technology such as small size. In [86] a stripline has been used as the NRD guide feed, where the feed structure also suffers from drawbacks such as large size and complexity in design. The hybrid integration technology proposed in [87] where the NRD guide serves as an interconnect between two different layers with energy coupled via an aperture is an attractive option in this context. Also, since the interconnect is three dimensional, considerable space is released for mounting active components between different layers. To analyse the NRD connectors we introduced a transmission line model in [34]. The method proposed speeds up the design phase of NRD guide based systems as the commercial simulators can be very inefficient for multilayer designs. In the present analysis we consider only the dominant NRD mode, however further improvement in the accuracy of the model can be achieved by extending the general framework proposed in this chapter to higher order
5.2. Nonradiative dielectric (NRD) waveguides and applications

In practice the initial design achieved using the transmission line model can be further refined using a full-wave simulator rather than using the fullwave simulator from the beginning, thereby saving time in the design.

5.2.1 Dielectric connectors based on NRD waveguides

The dielectric connectors studied in this chapter is depicted in Figure 5.5 and 5.6. The difference between Figure 5.5 and Figure 5.6 is that in Figure 5.6 the slots lie one over the other so that the effective length of the guide is reduced. Since the microwave and millimeter-wave integrated circuits on single substrates are becoming more and more complex, the NRD guides can serve as effective building blocks for integrating two planar integrated circuits one on each side of the NRD. In the above process the NRD guide also fulfills the requirements of connectors such as minimum radiation from the transitions and less losses. It has also been proved that compared with other transmission lines such as coplanar lines and microstrip lines, radiation losses are small from junctions and curved portions of the NRD guide [33]. The possible modes of the NRD are the longitudinal section magnetic and longitudinal section electric modes. The fundamental criterion to be satisfied for nonradiative operation of the NRD guide is that distance between the ground planes is less than half the free space wavelength (\(\lambda_0/2\)). Therefore the NRD guide has to be operated between the launch of the fundamental mode and higher modes with the constraint that the distance between the

![Figure 5.5: NRD waveguide based multilayer structure.](image)
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ground planes, \(a\), has to be less than \(\lambda_0/2\). A diagram showing the possible

\[ a, b \text{ combinations for operation as mentioned above is called the operational}\]

\[ \text{diagram. Figure 5.7 shows the operational diagram of an NRD guide based} \]

\[ \text{on silicon. The normalised height } a_\lambda (\text{normalised with respect to wavelength}) \]
is shown on the x-axis and the product of normalised width \( b \) and \( \sqrt{\epsilon_r - 1} \) is shown on the y-axis. Each point on a particular modal curve on the operational diagram denotes a particular width-height combination to launch the mode. As usual with any waveguide, we have to limit the operation of the waveguide to its fundamental mode. To meet this objective in the case of the NRD guide, its operational diagram is very convenient as it simplifies the understanding of the otherwise complex propagation characteristics. Thus we can see from Figure 5.7 that, for single mode operation the width-height combination needed is confined between the upper portion of the fundamental mode curve \( LSM_{11} \) and the lower portions of higher order \( LSM_{12} \) and \( LSM_{21} \) mode. The propagation constant of the fundamental symmetric mode, \( LSM_{11} \) and the second higher order symmetric mode, \( LSM_{21} \) satisfies the following eigen system,

\[
\beta_{yn} \tan \left( \frac{\beta_{yn} b}{2} \right) = \epsilon_r \zeta_n
\]

where,

\[
\zeta_n^2 = k_0^2 (\epsilon_r - 1) - \beta_{yn}^2 \\
\beta_{mn}^2 = k_0^2 \epsilon_r - \left( m\pi / a \right)^2 - \beta_{yn}^2
\]

where, \( \beta_{yn} \) is the \( y \)-directed propagation constant in the dielectric region and \( \zeta_n \) is the corresponding attenuation constant in the air region. For propagation the phase constant \( \beta_{mn} \) should be real and positive. It should be noted that \( \beta_{yn} b / 2 \) is between \( (n - 1)\pi / 2 \) and \( n\pi / 2 \) for the \( mn^{th} \) mode. Similarly, for the evaluation of the propagation constant of the antisymmetric mode \( LSM_{12} \) we have to solve the following eigen system.

\[
\frac{\beta_{yn}}{\tan \left( \frac{\beta_{yn} b}{2} \right)} = -\epsilon_r \zeta_n
\]

To prevent the launch of the higher order \( LSM_{21} \) mode, which results in a smaller frequency range for single mode operation, \( \epsilon_r \) should be low. Thus the propagation characteristics are enhanced by less costly low dielectric constant materials. Figure 5.8 shows the operational diagram for NRD guide with teflon (\( \epsilon_r = 2.04 \)) as the dielectric material. It can be seen that \( LSM_{21} \) is absent throughout the operating region where \( a_3 < 0.5 \) resulting in a higher operational bandwidth. Thus we can conclude that the operational diagram
is indispensable for selecting a material for implementing the NRD guide. Of the losses in the NRD guide, the dielectric losses are more predominant as most energy of the wave is concentrated in the dielectric region. Hence ideally loss tangent should be as small as possible. Conduction losses for the dominant \( LSM_{11} \) is very small since the electric field is predominantly parallel to the conducting plates.

### 5.2.2 Development of the transmission line model

The transmission line model of the NRD guide based multilayer structure (see Figure 5.5) is shown in Figure 5.9. As can be seen in the transmission line model, the slots have been efficiently described by two transformers and the self-impedance across the slot. For the development of the transmission line model of the structure, it is assumed that the dominant \( LSM_{11} \) is the propagating mode of the NRD guide. The transformation ratios of the slots, \( N_1 \) and \( N_2 \), can be found as the ratio of the modal voltage induced in the microstrip line and the NRD respectively to the slot voltage respectively. \( jX \) in Figure 5.9 account for stored energy at the slot due to higher order modes of the NRD guide. The following electric field representation has been
assumed in the slot for deriving the transformer ratios.

\[ \vec{E}_s = \frac{V_0 \sin(k_a \left( \frac{L_s}{2} - |y| \right))}{W_s \sin(\frac{k_0 L_s}{2})} \hat{y} \]  

(5.9)

Where, \( V_0 \) represents the voltage at the center of the slot. \( W_s \) and \( L_s \) are the width and length of the slot. The slot wave number \( (k_a) \) which is different from the free space wave number \( (k_0) \) can be derived applying the Cohen’s method [46] modified to account for the presence of ground plane at the distance \( H_N \) from the slot [59]. Then transformer ratio from the slot to the NRD guide and from slot to the microstrip line can be found from the induced voltages as,

\[ N_g = \frac{V_g}{V_0} \]  

(5.10)

where, \( g=1 \) and \( 2 \) for the microstrip line and the NRD guide respectively. \( V_g \) represents the modal voltage induced in guide ’\( g \)’ which can be found by the following equation as,

\[ V_g = \int \int_S \vec{n} \times \vec{E}_s \cdot \vec{h}_w \, dS \]  

(5.11)
From the representation of the $LSM_{11}$ fields \cite{33} in the case of the NRD guide and the representation of quasi TEM fields for the microstrip line \cite{88} the normalised modal magnetic field $\hat{h}_w$ can be determined satisfying the following normalization criteria,

$$\int \int_{S_g} \hat{e}_N \cdot \hat{e}_N \, dS_g = 1 \quad (5.12)$$

Here $\hat{e}_N$ represents the orthogonal electric modal vector in the coupled guide which is the NRD or the microstrip line. $S_g$ denotes the guide cross-section.

The transformation ratios can thus be derived as,

$$N_1 = \frac{\cos (k_a L_s/2 - k_a W_{eff}/2) - \cos (k_a L_s/2)}{W_s k_a \sqrt{W_{eff} h_M \sin (k_a L_s/2)}} \quad (5.13)$$

$$N_2 = \frac{4 \pi k_a \sin (W_s \beta_{y1}/2) \cos (\beta_{11} L_s/2) - \cos (k_a L_s/2))}{T a W_s \beta_{11} \beta_{y1} (\beta_{11}^2 - k_a^2 \cos (\beta_{y1}/2) \sin (k_a L_s/2)} \quad (5.14)$$

As shown in Figure 1, $h_M$ represents the height of the microstrip line substrate and $b$ and $a$ are the width and height of NRD guide. $W_{eff}$ is the effective width of the microstrip line. $T$, which essentially is a normalization term can be derived as,

$$T = \sqrt{\frac{a (b \beta_{y1} + \sin (b \beta_{y1}))}{4 \beta_{y1} \cos^2 (b \beta_{y1}/2)} + \frac{\epsilon_r^2 a}{2 \zeta_1}} \quad (5.15)$$

$\beta_{y1}$ and $\zeta_1$ represents the transverse wave numbers of the fundamental NRD mode satisfying the eigen system,

$$\beta_{y1} \tan \left( \frac{b \beta_{y1}}{2} \right) = \epsilon_r \zeta_1 \quad (5.16)$$

$\beta_{11}$, the fundamental $LSM_{11}$ mode propagation constant is given by,

$$\beta_{11}^2 = \epsilon_r k_0^2 - \zeta_1^2 \quad (5.17)$$

Open ends of the NRD guide in the present analysis has been modelled as a perfect open, however for an improved transmission line model, the open ends may be analysed using the modal analysis as a transition from the $LSM_{11}$ mode to the parallel plate mode. For simplicity the higher order modes which account for the $jX$ in the transmission line model (see Figure
5.2. Nonradiative dielectric (NRD) waveguides and applications

5.9) are neglected in the analysis. Finally, the input impedance $Z_i$ of the structure can be written as,

$$ Z_i = \left( \frac{Z_S^N \parallel Z_{SS}}{N_2^2} \parallel Z_{ap} \right) N_1^2 + Z_S^M $$ (5.18)

where,

$$ Z_j \parallel Z_k = \frac{Z_j Z_k}{Z_j + Z_k} $$ (5.19)

$Z_{ap}$ is the slot self impedance across the transformers and $Z_{SS}$ is a function of the distance between the slots $L_{SS}$. From Figure 5.9) $Z_{SS}$ can be written as,

$$ Z_{SS} = Z_0^N \frac{Z_T + j Z_0^N \tan(\beta_{11} L_{SS})}{Z_0^N + j Z_T \tan(\beta_{11} L_{SS})} $$ (5.20)

where, $Z_T$ is given by,

$$ Z_T = \left( \frac{Z_S^M + Z_0^M}{N_2^2} \parallel Z_{ap} \right) N_1^2 \parallel Z_S^N $$ (5.21)

$Z_S^K, K = M, N$ represents the impedances provided by the microstripline stub and the NRD stub respectively at the center of the slot and $Z_0^K, K = M, N$ are their characteristic impedances. The slot self-impedance $Y_{ap}$ is found as the equivalent impedance of two parallel shorted slotlines with shorts at $\pm \frac{L_s}{2}$ from the center of the slot.

5.2.3 Results and discussion

The application of the NRD as shown in Figure 5.5 has been discussed in [87]. However the experimental analysis in this chapter focuses on NRD plugs as depicted in Figure 5.6. The transmission line model of the NRD plug interconnect is essentially the same as the one for the geometry in Figure 5.5 with the distance between the slots taken to be zero. The dimensions of the NRD guide are (Figure 5.5 and 5.6) $b = 12\text{mm}, a = 12.7\text{mm}$. The NRD guide has been realized using substrate with a dielectric constant close to that of silicon ($\epsilon_r = 12$). The microstrip line has been implemented on Teflon substrate, $\epsilon_r = 2.52$. The NRD stub and microstrip line stubs are given by $L_N = 6\text{mm}$ and $L_M = 8\text{mm}$. Slot dimensions are $W_s = 1.5\text{mm}$ and $L_s = 8\text{mm}$ respectively. The coupling, $S_{21}$ between the microstrip line and the NRD guide has been calculated using transmission line model and compared with
experiments (see Figure 5.10). Since NRD guide is at cutoff below 4.9GHz we are not able to apply the transmission line model for the structure below 4.9GHz, however we can see from the experiment that the coupling cannot be neglected below 4.9GHz. To study the behavior we have presented the results using TM parallel plate modes in connector assuming NRD is infinitely long along the slot and open $\pm b/2$ from the center of slot. It can be seen that the behavior of the calculated results agree well with experimental one. To

![Figure 5.10: Results for the NRD interconnect seen in Figure 5.6.](image)

efficiently utilize space, particularly between the ground planes, a new way of using the dielectric interconnect is introduced, where, a tail-ended tapered microstrip line has been used as the feed for the dielectric interconnect as shown in Figure 5.11. For this structure, the tapered line functions as an impedance transformer. The modes in the dielectric connector is not nonradiative due to the absence of the second ground plane, However there is a strong tendency for waves to propagate forward as in tapered line antennas thereby causing spurious radiation. The action of the tail is to efficiently prevent this radiation while at the same time providing the fringing fields necessary for coupling to the dielectric connector. The optimum position of the open end of the tail is at the connector end due to the fact that the maximum fringing field of open-ended microstrip line occurs at the air boundaries.
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Figure 5.11: Microstrip-dielectric connector transition, Dielectric connector, W=12mm, H=12mm, L=16mm, Microstrip stub, line and substrate dimensions as same as in Figure 3. Taper: b=2.26mm, B=10mm, Length =60mm, Tail: d=6.5mm, c=2.6mm.

Figure 5.12: Dielectric connector fed by a tapered line with tail.

In Figure 5.12, results of the tail ended tapered line fed connector is shown for various positions of the open end of the microstrip line. The signs + and - denotes the position of open end of the tail is outside and inside dielectric
connector respectively. It can be seen that the optimum position of the tail is when $D_x = 0$, where the fringing fields are maximum and connector distance from fringing fields of open end is minimum. For $D_x < 0$ the high $\epsilon_r$ of the connector prevents fringing fields to be excited from tail, thus the coupling is less. For $D_x > 0$, the distance from the connector end prevents the fringing fields to couple to the NRD.
Appendix A

Spectral domain Green’s functions for the analysis of slots

Let \( \tilde{G}(k_x, k_y) \) be the Fourier transform of a given Green’s function component \( G(x, y) \). Then the spectral domain Greens function \( \tilde{G}(k_x, k_y) \) and \( G(x, y) \) are related by,

\[
G(x, y) = \frac{1}{4\pi^2} \int_{k_x=-\infty}^{\infty} \int_{k_y=-\infty}^{\infty} \tilde{G}(k_x, k_y) e^{jk_x(x-x_0)} e^{jk_y(y-y_0)} dk_x dk_y \quad (A.1)
\]

Green’s function \( G_{HJ}^{yx} \) is defined as magnetic field component \( H_y \) at \((x, y, 0)\) due to a \( \hat{x} \) directed electric current element at \((x_0, y_0, d)\) where \( d \) is the thickness of the substrate of dielectric constant \( \epsilon_r \). \( \tilde{G}_{HJ}^{yx} \) can be derived using (A.1) as,

\[
\tilde{G}_{HJ}^{yx} = -\frac{j \epsilon_r (\epsilon_r - 1) \sin k_1 d}{T_e T_m} \left[ \tilde{G}_{HJ}^{(1)} + \tilde{G}_{HJ}^{(2)} + \tilde{G}_{HJ}^{(3)} \right] \quad (A.2)
\]

Green’s function \( G_{HM}^{yy} \) is defined as magnetic field component \( H_y \) at \((x, y, 0)\) due to a \( \hat{y} \) directed magnetic current element (or \( \hat{x} \) directed electric current element in a slot) at \((x_0, y_0, d)\). \( \tilde{G}_{HM}^{yy} \) can be derived using (A.1) as,

\[
\tilde{G}_{HM}^{yy} = \frac{j}{k_0 \eta_0} \left[ \tilde{G}_{HM}^{(1)} + \tilde{G}_{HM}^{(2)} + \tilde{G}_{HM}^{(3)} \right] \quad (A.3)
\]

where,

\[
\tilde{G}_{HM}^{(1)} = \frac{j (k_1 \cos k_1 d + j k_2 \epsilon_r \sin k_1 d) (\epsilon_r k_0^2 - k_0^2)}{k_1 T_m} \quad (A.4)
\]

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\[ \tilde{G}^{HM}_{yy} = -\frac{j k_x^2 k_1 (\varepsilon_r - 1)}{T_e T_m} \quad (A.5) \]
\[ \tilde{G}^{HM}_{yy} = -\frac{(k_0^2 - k_0^2)}{jk_2} \quad (A.6) \]

where,

\[ T_e = k_1 \cos k_1 d + j k_2 \sin k_1 d \quad (A.7) \]
\[ T_m = \varepsilon_r k_2 \cos k_1 d + j k_1 \sin k_1 d \quad (A.8) \]

\[ \beta^2 = k_x^2 + k_y^2, \quad k_0 = 2\pi/\lambda_0, \quad \eta_0 = \sqrt{\mu_0 \varepsilon_0} \quad (A.9) \]

\[ k_1^2 = \varepsilon_r k_0^2 - \beta^2, \quad k_2^2 = k_0^2 - \beta^2 \quad (A.10) \]

For \( \beta^2 > \varepsilon_r k_0^2 \) and \( \beta^2 > k_0^2 \) then,

\[ k_1 = -j \sqrt{\beta^2 - \varepsilon_r k_0^2}, \quad k_2 = -j \sqrt{\beta^2 - k_0^2} \quad (A.11) \]

**Evaluation of integrals involving spectral domain Green’s functions**

Consider the integral,

\[ I = \int_{k_x = -\infty}^{\infty} \int_{k_y = -\infty}^{\infty} F(k_x, k_y) \tilde{G}(k_x, k_y) \, dk_x \, dk_y \quad (A.12) \]

Substitute \( k_x = \gamma \cos(\theta) \) and \( k_y = \gamma \sin(\theta) \). Then,

\[ I = \int_{\theta = 0}^{2\pi} \int_{\gamma = 0}^{\infty} F(\gamma, \theta) \tilde{G}(\gamma, \theta) \gamma \, d\gamma \, d\theta \quad (A.13) \]

The following case arise in the spectral domain analysis of slot,

\[ I = \int_{\theta = 0}^{2\pi} \int_{\gamma = 0}^{\infty} \frac{N(\gamma, \theta)}{D(\gamma)} \, d\gamma \, d\theta \quad (A.14) \]

Split the integrand into singular and nonsingular part. Numerical techniques can be applied to solve the non-singular part and analytical techniques can be applied to solve the singular part.

Singularity cases for the spectral domain analysis of slot are as follows,
Case1: Let \( D(\gamma) = \sqrt{k_i^2 - \gamma^2} \) and consider the evaluation of following integral,

\[
I = \int_{\theta=0}^{2\pi} \int_{\gamma=k_i-\delta}^{k_i+\delta} \frac{N(\gamma, \theta)}{\sqrt{k_i^2 - \gamma^2}} d\gamma d\theta \quad (A.15)
\]

Split the integral as,

\[
I = \int_{\theta=0}^{2\pi} \left[ \int_{\gamma=k_i-\delta}^{k_i} \frac{N(\gamma, \theta)}{\sqrt{k_i^2 - \gamma^2}} d\gamma d\theta + \int_{\gamma=k_i}^{k_i+\delta} \frac{jN(\gamma, \theta)}{\sqrt{\gamma^2 - k_i^2}} d\gamma d\theta \right] \quad (A.16)
\]

For removing the singularity, in the first part substitute \( \gamma = k_i \sin(y) \) and in the second part substitute \( \gamma = k_i + \sec(y) \). The resulting integrals are then adaptable for numerical evaluation.

Case2: Let \( D(\gamma) = f(\gamma) \) and let there be a pole \( \gamma_0 \) in the limits of integration, \( \gamma_0 - \delta \) to \( \gamma_0 + \delta \). Let \( \delta \) be very small.

To solve the integral apply Taylor series of \( f(\gamma) \) at \( \gamma_0 \) and retain first non zero term then,

\[
f(\gamma) = (\gamma - \gamma_0)f'(\gamma_0) \quad (A.17)
\]

Use the following results from the theory of complex integration over real axis pole in a half plane,

\[
\int_{\gamma=\gamma_0-\delta}^{\gamma_0+\delta} \frac{F(\gamma)}{(\gamma - \gamma_0)f'(\gamma_0)} d\gamma = -\pi j \frac{F(\gamma_0)}{f'(\gamma_0)} \quad (A.18)
\]
Appendix A Spectral domain Green’s functions for the analysis of slots
Figure B.1: Cross section of NRD guide.

Figure B shows the cross section of NRD guide. Let $\mathbf{F}$ and $\mathbf{A}$ be the electric and magnetic potentials in a the region. Then $\mathbf{E} = -\nabla \times \mathbf{F}$ and $\mathbf{H} = \nabla \times \mathbf{A}$. Then from Maxwell’s equations we can write,

$$\begin{align*}
\mathbf{E} &= -\nabla \times \mathbf{F} - j\omega \mu \mathbf{A} + \frac{1}{j\omega \varepsilon} \nabla (\nabla \cdot \mathbf{A}) \\
\mathbf{H} &= -\nabla \times \mathbf{A} - j\omega \varepsilon \mathbf{F} + \frac{1}{j\omega \mu} \nabla (\nabla \cdot \mathbf{F})
\end{align*}$$

(B.1) (B.2)
Boundary conditions implies that the modes of the NRD guide to be considered are $TM^y$ and $TE^y$ modes. Pioneering paper in this field [33] refers to this modes are longitudinal-section magnetic(LSM) and longitudinal-section electric(LSE) modes.

**LSM modes**

For $TM^y$ or $LSM$ modes $\mathbf{F} = 0$ and $\mathbf{A} = j\omega_0\phi^e\mathbf{\hat{y}}$, where $\phi^e$ is the scalar potential function. Equations B.1 and B.2 reduces to,

$$E = -j\omega_0A + \frac{1}{j\omega}\nabla(\nabla \cdot A) \quad \text{ (B.3)}$$

$$H = \nabla \times A \quad \text{ (B.4)}$$

The field components can be written in terms of scalar potential function as,

$$E_x = \frac{1}{\epsilon_r} \frac{\partial^2 \phi^e}{\partial x \partial y}$$

$$E_y = k_0^2 \phi^e + \frac{1}{\epsilon_r} \frac{\partial^2 \phi^e}{\partial y^2}$$

$$E_z = \frac{1}{\epsilon_r} \frac{\partial^2 \phi^e}{\partial y \partial z} \quad \text{ (B.5)}$$

$$H_x = -j\omega_0 \frac{\partial \phi^e}{\partial z}$$

$$H_y = 0$$

$$H_z = j\omega_0 \frac{\partial \phi^e}{\partial x}$$

where, $k_0 = \omega \sqrt{\mu_0 \epsilon_0}$ is the free space wave number.

**LSM$_{mn}$ symmetric modes**

In this case,

$$\phi^e = A \sin(m\pi x/a) \cos(\beta_y y) \quad |y| < b/2$$

$$= A \cos(\beta_y b/2) \sin(m\pi x/a) e^{\zeta(b/2-|y|)} \quad |y| > b/2 \quad \text{ (B.6)}$$

The field has $\cos(\beta_y y)$ variation along the cross-section hence the name symmetric mode. In the above equation $A$ is an arbitrary constant, $\beta_y$ is the
$y$-directed propagation constant in the dielectric region, and $\zeta$ is the decay constant in the air region. For convenience the common factor $e^{j(\omega t - \beta z)}$ has been suppressed. The propagation parameters are related by,

$$
\beta^2 = k_0^2 \varepsilon_r - (m\pi/a)^2 - \beta_y^2 = k_0^2 - (m\pi/a)^2 + \zeta^2 \quad (B.7)
$$

where, $\beta$ is the propagation constant of the wave.

The various field components are then given upon substitution in (B.5) as,

**Dielectric region, $|y| < b/2$:**

$$
E_x = -A(\beta_y/\varepsilon_r)(m\pi/a) \cos(m\pi x/a) \sin(\beta_y y) \quad (B.8)
$$
$$
E_y = -A(g^2/\varepsilon_r) \sin(m\pi x/a) \cos(\beta_y y) \quad (B.9)
$$
$$
E_z = \pm jA(\beta_y/\varepsilon_r) \beta \sin(m\pi x/a) \sin(\beta_y y) \quad (B.10)
$$
$$
H_x = -A\omega\varepsilon_0 \beta \sin(m\pi x/a) \cos(\beta_y y) \quad (B.11)
$$
$$
H_y = 0 \quad (B.12)
$$
$$
H_z = jA(\omega\varepsilon_0)(m\pi/a) \cos(m\pi x/a) \cos(\beta_y y) \quad (B.13)
$$

**Air region, $|y| > b/2$:**

$$
E_x = \mp A\zeta(m\pi/a) \cos(m\pi x/a) \cos(\beta_y b/2) e^{\zeta(b/2-|y|)} \quad (B.14)
$$
$$
E_y = \pm Ag^2 \sin(m\pi x/a) \cos(\beta_y b/2) e^{\zeta(b/2-|y|)} \quad (B.15)
$$
$$
E_z = \pm jA\beta\zeta \sin(m\pi x/a) \cos(\beta_y b/2) e^{\zeta(b/2-|y|)} \quad (B.16)
$$
$$
H_x = -A\omega\varepsilon_0 \beta \sin(m\pi x/a) \cos(\beta_y b/2) e^{\zeta(b/2-|y|)} \quad (B.17)
$$
$$
H_y = 0 \quad (B.18)
$$
$$
H_z = jA(\omega\varepsilon_0)(m\pi/a) \cos(m\pi x/a) \cos(\beta_y b/2) e^{\zeta(b/2-|y|)} \quad (B.19)
$$

upper and lower signs apply for air region in $\pm \hat{y}$ direction.

$\beta$ is given by,

$$
g^2 = \beta^2 + (m\pi/a)^2 = k_0^2 \varepsilon_r - \beta_y^2 = k_0^2 + \zeta^2 \quad (B.20)
$$

Matching $E_x$ at the air dielectric interface for the $mn^{th}$ mode,

$$
\beta_{yn} \tan \left( \frac{\beta_{yn} b}{2} \right) = \varepsilon_r \zeta_n \quad (B.21)
$$
Appendix B Analysis of non radiative dielectric waveguides

where,

\[ \zeta_n^2 = k_0^2(\epsilon_r - 1) - \beta_{yn}^2 \]
\[ \beta_{mn}^2 = k_0^2\epsilon_r - (m\pi/a)^2 - \beta_{yn}^2 \]  \hspace{1cm} (B.22)

The modes are usually denoted by, \( LSM_{mn} \), \( m=1,2,3... \), \( n=1,3,5... \)

**LSM\(_{mn}\) antisymmetric modes**

In this case,

\[ \phi^e = B \sin(m\pi x/a) \sin(\beta_y y) \quad \text{\( |y| < b/2 \)} \]
\[ = +B \sin(m\pi x/a) \sin(\beta_y b/2) e^{-\zeta(b/2-|y|)} \quad \text{\( y > b/2 \)} \] \hspace{1cm} (B.23)
\[ = -B \sin(m\pi x/a) \sin(\beta_y b/2) e^{\zeta(b/2-|y|)} \quad \text{\( y < -b/2 \)} \]

The field has \( \sin(\beta_y y) \) variation along the cross-section hence the name antisymmetric mode. The various field components are then given upon substitution in (B.5) as,

**Dielectric region, \( |y| < b/2 \):**

\[ E_x = B(\beta_y/\epsilon_r)(m\pi/a) \cos(m\pi x/a) \cos(\beta_y y) \] \hspace{1cm} (B.24)
\[ E_y = B\gamma^2/\epsilon_r \sin(m\pi x/a) \sin(\beta_y y) \] \hspace{1cm} (B.25)
\[ E_z = -jB(\beta_y/\epsilon_r)\beta \sin(m\pi x/a) \cos(\beta_y y) \] \hspace{1cm} (B.26)
\[ H_x = -B\omega\epsilon_0\beta \sin(m\pi x/a) \sin(\beta_y y) \] \hspace{1cm} (B.27)
\[ H_y = 0 \] \hspace{1cm} (B.28)
\[ H_z = jB\omega\epsilon_0(m\pi/a) \cos(m\pi x/a) \sin(\beta_y y) \] \hspace{1cm} (B.29)

**Air region, \( |y| > b/2 \):**

\[ E_x = -B\zeta(m\pi/a) \cos(m\pi x/a) \sin(\beta_y b/2) e^{\zeta(b/2-|y|)} \] \hspace{1cm} (B.30)
\[ E_y = \pm B\gamma^2 \sin(m\pi x/a) \sin(\beta_y b/2) e^{\zeta(b/2-|y|)} \] \hspace{1cm} (B.31)
\[ E_z = jB\beta\zeta \sin(m\pi x/a) \sin(\beta_y b/2) e^{\zeta(b/2-|y|)} \] \hspace{1cm} (B.32)
\[ H_x = \pm\omega\epsilon_0\beta \sin(m\pi x/a) \sin(\beta_y b/2) e^{\zeta(b/2-|y|)} \] \hspace{1cm} (B.33)
\[ H_y = 0 \] \hspace{1cm} (B.34)
\[ H_z = \mp B(\omega\epsilon_0)(m\pi/a) \cos(m\pi x/a) \sin(\beta_y b/2) e^{\zeta(b/2-|y|)} \] \hspace{1cm} (B.35)
upper and lower signs apply for air region in $\pm \hat{y}$ direction.

Matching $E_x$ at the air dielectric interface for the $mn^{th}$ mode,

$$\beta_{yn} \cot \left( \frac{\beta_{yn} b}{2} \right) = -\zeta_n \epsilon_r$$  \hspace{1cm} (B.36)

where,

$$\zeta - n^2 = k_0^2 (\epsilon_r - 1) - \beta_{yn}^2$$  
$$\beta_{mn}^2 = k_0^2 \epsilon_r - (m\pi/a)^2 - \beta_{yn}^2$$  \hspace{1cm} (B.37)

The modes are usually denoted by, $LSM_{mn}$, $m=1,2,3...$, $n=2,4,6...$

**LSE modes**

For $TE^y$ (LSE) modes $A = 0$ and $F = j\omega \mu_0 \phi^h \hat{y}$, where $\phi^h$ is the scalar potential function. Equations B.1 and B.2 reduces to,

$$E = -\nabla \times F$$  \hspace{1cm} (B.38)

$$H = -j\omega \epsilon F + \frac{1}{j\omega \mu_0} \nabla (\nabla \cdot F)$$  \hspace{1cm} (B.39)

The field components can be written in terms of scalar potential function as,

$$E_x = j\omega \mu_0 \frac{\partial \phi^h}{\partial z}$$  \hspace{1cm} (B.40)

$$E_y = 0$$  \hspace{1cm} (B.41)

$$E_z = -j\omega \mu_0 \frac{\partial \phi^h}{\partial x}$$  \hspace{1cm} (B.42)

$$H_x = \frac{\partial^2 \phi^h}{\partial x \partial y}$$  \hspace{1cm} (B.43)

$$H_y = k_0^2 \epsilon_r \phi^h + \frac{\partial^2 \phi^h}{\partial y^2}$$  \hspace{1cm} (B.44)

$$H_z = \frac{\partial^2 \phi^h}{\partial y \partial z}$$  \hspace{1cm} (B.45)

(B.46)
Appendix B Analysis of non radiative dielectric waveguides

$LSE_{mn}$ symmetric modes

In this case,

\[
\phi^e = C \cos(m\pi x/a) \cos(\beta_y y) \quad |y| < b/2
\]

\[
= C \cos(\beta_y b/2) \cos(m\pi x/a) e^{\xi(b/2-|y|)} \quad |y| > b/2
\]

(B.47)

The field has $\cos(\beta_y y)$ variation along the cross-section hence the name symmetric mode. The propagation parameters are related by,

\[
\beta^2 = k_0^2 \epsilon_r - (m\pi/a)^2 - \beta_y^2 = k_0^2 - (m\pi/a)^2 + \zeta^2
\]

(B.48)

The various field components are then given by,

Dielectric region, $|y| < b/2$:

\[
E_x = C \omega \mu_0 \beta \cos(m\pi x/a) \cos(\beta_y y) \quad (B.49)
\]

\[
E_y = 0
\]

\[
E_z = jC \omega \mu_0 (m\pi/a) \sin(m\pi x/a) \cos(\beta_y y) \quad (B.50)
\]

\[
H_x = C \beta_y (m\pi/a) \sin(m\pi x/a) \sin(\beta_y y) \quad (B.51)
\]

\[
H_y = Cy^2 \cos(m\pi x/a) \cos(\beta_y y) \quad (B.52)
\]

\[
H_z = jC \beta_y \beta \cos(m\pi x/a) \sin(\beta_y y) \quad (B.53)
\]

(B.54)

Air region, $|y| > b/2$:

\[
E_x = C \omega \mu_0 \beta \cos(m\pi x/a) \cos(\beta_y b/2) e^{\xi(b/2-|y|)} \quad (B.55)
\]

\[
E_y = 0
\]

\[
E_z = jC(\omega \mu_0)(m\pi/a) \sin(m\pi x/a) \cos(\beta_y b/2) e^{\xi(b/2-|y|)} \quad (B.56)
\]

\[
H_x = \pm C \zeta (m\pi/a) \sin(m\pi x/a) \cos(\beta_y b/2) e^{\xi(b/2-|y|)} \quad (B.57)
\]

\[
H_y = Cy^2 \cos(m\pi x/a) \cos(\beta_y b/2) e^{\xi(b/2-|y|)} \quad (B.58)
\]

\[
E_z = \pm jC \beta \zeta \cos(m\pi x/a) \cos(\beta_y b/2) e^{\xi(b/2-|y|)} \quad (B.59)
\]

\[
= \pm jC \beta \zeta \cos(m\pi x/a) \cos(\beta_y b/2) e^{\xi(b/2-|y|)} \quad (B.60)
\]

$\pm$ apply for air region in $\hat{y}$ direction.

$g$ is given by,

\[
g^2 = \beta^2 + (m\pi/a)^2 = k_0^2 \epsilon_r - \beta^2_y = k_0^2 + \zeta^2
\]

(B.61)
Matching $E_x$ at the air dielectric interface for the $mn^{th}$ mode,

$$\beta_y \tan \left( \frac{\beta_y b}{2} \right) = \zeta_n \tag{B.62}$$

where,

$$\zeta_n^2 = k_0^2(\epsilon_r - 1) - \beta_y^2$$

$$\beta_{mn}^2 = k_0^2\epsilon_r - (m\pi/a)^2 - \beta_y^2 \tag{B.63}$$

The modes are usually denoted by, $LSE_{mn}$, $m=0,1,2...$, $n=1,3,5...$

$LSE_{mn}$ **antisymmetric modes**

In this case,

$$\phi^x = D \cos(m\pi x/a) \sin(\beta_y y) \quad |y| < b/2$$

$$= +D \cos(m\pi x/a) \sin(\beta_y b/2) e^{\zeta y(b/2-|y|)} \quad y > b/2 \tag{B.64}$$

$$= -D \cos(m\pi x/a) \sin(\beta_y b/2) e^{\zeta y(b/2-|y|)} \quad y < -b/2$$

The field has $\sin(\beta_y y)$ variation along the cross-section hence the name antisymmetric mode. The various field components are then given by,

**Dielectric region, $|y| < b/2$:**

$$E_x = D \omega \mu_0 \beta \cos(m\pi x/a) \sin(\beta_y y) \tag{B.65}$$

$$E_y = 0 \tag{B.66}$$

$$E_z = jD(\omega \mu_0)(m\pi/a) \sin(m\pi x/a) \sin(\beta_y y) \tag{B.67}$$

$$H_x = -D\beta_y (m\pi/a) \sin(m\pi x/a) \cos(\beta_y y) \tag{B.68}$$

$$H_y = Dg^2 \cos(m\pi x/a) \sin(\beta_y y) \tag{B.69}$$

$$H_z = -jD\beta \zeta \cos(m\pi x/a) \sin(\beta_y b/2) e^{\zeta y(b/2-|y|)} \tag{B.70}$$

**Air region, $|y| > b/2$:**

$$E_x = \pm D \omega \mu_0 \beta \cos(m\pi x/a) \sin(\beta_y b/2) e^{\zeta y(b/2-|y|)} \tag{B.71}$$

$$E_y = 0 \tag{B.72}$$

$$E_z = \pm jD(\omega \mu_0)(m\pi/a) \sin(m\pi x/a) \sin(\beta_y b/2) e^{\zeta y(b/2-|y|)} \tag{B.73}$$

$$H_x = D\zeta (m\pi/a) \sin(m\pi x/a) \sin(\beta_y b/2) e^{\zeta y(b/2-|y|)} \tag{B.74}$$

$$H_y = \pm Dg^2 \cos(m\pi x/a) \sin(\beta_y b/2) e^{\zeta y(b/2-|y|)} \tag{B.75}$$

$$H_z = jD\beta \zeta \cos(m\pi x/a) \sin(\beta_y b/2) e^{\zeta y(b/2-|y|)} \tag{B.76}$$
± apply for air region in $\pm \hat{y}$ direction.

Matching $H_x$ at the air dielectric interface for the $mn^{th}$ mode,

\[
\beta_{yn} \cot \left( \frac{\beta_{yn} b}{2} \right) = -\zeta_n \tag{B.77}
\]

\[
\zeta_n^2 = k_0^2 (\epsilon_r - 1) - \beta_{yn}^2 \tag{B.78}
\]

\[
\beta_{mn}^2 = k_0^2 \epsilon_r - (m\pi/a)^2 - \beta_{yn}^2
\]

The modes are usually denoted by, $LSE_{mn}$, $m=1,2..., \ n=2,4,6...$
Bibliography


A doctoral dissertation from the Faculty of Science and Technology, Uppsala University, is usually a summary of a number of papers. A few copies of the complete dissertation are kept at major Swedish research libraries, while the summary alone is distributed internationally through the series Comprehensive Summaries of Uppsala Dissertations from the Faculty of Science and Technology. (Prior to October, 1993, the series was published under the title “Comprehensive Summaries of Uppsala Dissertations from the Faculty of Science”.)