Moving Mathematics

Exploring constructivist tools to enhance mathematics learning

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Abstract

The challenges faced by mathematics education reflect the more immense difficulties of the schooling system as a whole. This thesis investigates such challenges in the light of an ethical learning foundation and aims for a transformation through the use of technologies as learning tools.

Interaction design methods are used to craft constructivist learning kits that aim to move mathematics students from passive receivers of knowledge to active learners. The proposed tools modify new technologies by adapting them to teachers’ and learners’ needs to be best suited for mathematics classrooms adoption. Additionally, social, political, and economic issues that may hinder the adoption of constructivist learning are presented and critically discussed.

Finally, this thesis paves the way for future designers who aim to design mathematics educational kits by providing a design framework based on the learning theory and the design process presented in this thesis.

Keywords: constructivism, mathematics education, interaction design, Arduino, tangible user interface, technological learning tools.
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1 Introduction

1.1 Context

Even with all its significance and beauty, mathematics remains one of the least liked school subjects by students (Gadanidis, 1994; Lin, Chen, & Chang, 2015). Efforts from different fields and disciplines have been made to improve that reality, but with little success or little adoption (Fey, 2002). Saymour Papert, a mathematician and computer scientist who dedicated most of his work to enhancing mathematics learning through technology, rooted a movement for transforming mathematical education. Papert (1980), influenced by the constructivist theory, proposed Mathsland. In this imaginary land, mathematics learners engage with the subject deeply in the same way French learners may engage with the language by living in France. By making this a reality, learning mathematics becomes a natural occurrence, moving it from abstract concepts into a lived experience. With the emergence of new technologies such as the Arduino Science Journal mobile app and the Arduino Nano 33 BLE Sense microcontroller board, transforming math class into Mathsland is becoming closer to reality.

The pedagogical model used in this thesis is in line with Paulo Freire’s (1968) ethical guidelines and his proposal of problem-posing education in place of the banking concept of education. He describes the banking concept as a system where “knowledge is a gift bestowed by those who consider themselves knowledgeable upon those whom they consider to know nothing” (p. 70). In contrast, problem-posing education allows people to interact with and perceive reality creatively and critically. Freire’s guidelines are at the heart of all educational tools and activities used or proposed in this project. The user group in focus are students in grades 7-10 who have mathematics as part of their curriculum.

1.2 Aim & Contribution

This project intends to explore the potential and possibility of integrating technological tools to transform mathematics education into a creative, collaborative, and enjoyable experience for both students and teachers. The main contributions of this project can be categorized into two themes; the first relates to mathematics education and the second to the field of interaction design.

Contributions to mathematic educations include enhancing students’ creative process using technological tools, widening their perception of mathematical concepts, and being the steppingstone for modifying those tools to be adopted by mathematics educators. The framework produced in this project will aid the adaptation of new tools and technologies to
classrooms, which helps expand students’ creative toolkits. This integration is especially relevant as statistics estimate that 65% of kids entering schools now are going to end up with jobs that do not exist yet (Leopold, Zahidi, & Ratcheva, 2016), which demands a shift in education to be more accommodating for students’ creativity, enhancing their ability to respond to change. Additionally, the tools and methods produced will support students’ engagement with mathematical topics, move them from passive to active learners, and provide a broader range of perspectives on the studied topics. Finally, the proposed ideas may aid and inspire teachers in designing their activities using similar tools, with no requirement of any prior knowledge in programming. However, further studies on tool adoption by teachers are needed to integrate them in mathematics classrooms.

Contributions to interaction design involve a design framework for creating educational tools for learning mathematics and integrating new tools and technologies that are significant in the interaction designers’ practice. This thesis will explore how to design for enhancing mathematics’ understanding in an engaging, interactive, and collaborative way. It will aid the adaptation of modern technologies to mathematics’ classrooms’ needs and serve students’ experimentations with real-life mathematics. Additionally, tinkering and integrating new technologies such as Arduino Nano 33 BLE Sense, Arduino Science Journal, and p5.ble.js will pave the way for interaction designers, teachers, and makers to build on the presented results.

1.3 Limitations

Teachers’ input as co-creators was not achievable through co-design workshops, as will be explained in section 4.1. Instead, a thorough literature analysis, interviews with mathematics education experts, and teachers were conducted to make the educators’ voice in the design process. This issue is a limitation of the presented outcomes as they were not designed and tested by teachers.

The accessibility and availability of the proposed tools and technologies are subject to consideration as they may require access to the internet, access to a smartphone and/or a web browser, as well as microcontroller board technologies. Such resources are highly accessible in first-world educational systems but are not as available in third-world countries, thus limiting the possible outreach of the project.

Due to Covid-19 related restrictions, most schools were not allowed to host external activities, which excluded the possibility of holding workshops with teachers. Additionally, testing sessions of the prototypes were limited in number, duration, and dates, which eliminated the possibility of testing multiple functions and design iterations.
1.4 Ethical considerations

The nature of this thesis project raises many ethical concerns that were taken into account in a responsible and considerate manner. The main points to be considered are Vetenskapsrådet guidelines for data collection, the risks of using 3rd party applications and technologies, and the effects and requirements on teachers’ roles. Those main points will be expanded on briefly in this section and elaborated along with other aspects throughout the paper. Furthermore, ethical guidelines for the pedagogical model used in this thesis are highlighted as the foundation of the theoretical study in section 2.2.

This thesis project follows the Swedish research council, Vetenskapsrådet’s (2017) eight ethical rules and data handling guidelines. Hence, all interviews, observations, and tests were carried with complete transparency and with people’s consent. Data in the form of pictures, videos, sound recordings, and surveys were collected after they were subjected to the participants’ consent, with the guarantee of anonymity.

Some of the risks of using 3rd party apps and technology can be related to data collection by the providers and the marketing of a specific application. The technologies presented to the participants are the Arduino Science Journal app and Arduino Nano 33 BLE Sense board, both provided without purchase, subscription, or registration demands. Considering that used technologies are subject to Arduino’s privacy policy and not requiring any personal information from participants (Arduino, 2020).

This thesis considers the importance of teachers’ role and relationship to students and does not aim to neglect or replace them. On the contrary, the process seeks to transform the role and relationship between teachers and students into what Paulo Freire (1968) calls “students-teachers and teacher-student.” Freire explains, “The teacher is no longer merely the one-who-teaches, but one who is himself taught in dialogue with the students, who in turn while being taught also teach. They become jointly responsible for a process in which all grow […] People teach each other, mediated by the world […]” (p. 80). This shift can be mediated by technology, where the teachers’ role shifts into guiding students and giving them space and freedom to explore, experiment, and share their learning.

1.5 Structure

This thesis will investigate the challenges faced by mathematics education, the learning theories presented by the constructivist epistemology, and the possible opportunities of integrating digital interactive technology in the mathematics classroom (Chapter 2 & 3). The findings from the investigation will shape a design framework, which may guide designing and adapting technological tools in mathematics education (Chapter 4). Finally, design methodologies (Chapter 5) will guide the design practice (Chapter 6) that aims at contributing to the refinement of the framework (Chapter 7).
1.6 Research question

The initial open question that inspired this thesis is:

*How can we design technological tools that enhance students’ understanding of mathematical topics?*

The findings from the research phase clarified what constitutes rich mathematics' learning, which reframed the initial question into the following one:

*How can we design constructivist learning tools for mathematics education through the use and adaptation of technology?*

The new research question is unpacked into the two sub-questions listed in section 4.3 to guide the design process.

2 Theoretical Background

Due to the large scope and extensive research on these topics, it’s unfeasible to cover all the views and theories. Hence a selection of literature that is most relevant for the project will be made. The selection provides an overview of (1) the main challenges facing mathematics education, (2) clear ethical guidelines for educational approaches, (3) a learning model that is in line with the ethical guidelines, (4) the opportunities for the development of mathematical education. Additionally, interviews with mathematics educators and researchers were conducted, and their expertise on mathematics pedagogy is used in this part.

2.1 Challenges in mathematics education

English (2015) highlights the importance of mathematics, now more than ever, especially when there is an increased need for dealing with uncertainty and data across many disciplines. Nonetheless, studies show that the way mathematics is taught in most school systems is proven to be ineffective and undesirable (Applefield, Huber, & Moallem, 2001; English, 2015; Lin et al., 2015). This issue is not only drifting mathematics away from students’ hearts, but it is also diminishing its role in the education system in comparison to other science-related subjects that proved to be more successful in adopting more engaging ways of learning (English, 2015). While not intending to oversimplify the challenges faced by education in general and mathematics education in particular, isolating the main elements behind those challenges may be helpful for mapping the most relevant design opportunities for the interaction design field. The recognized issues in mathematics education (Nouri, 2012; Smitherman, 2006) are (1) connectedness to the real world, (2) representation in interdisciplinary fields, and (3) the formality of
mathematics. Issues related to curriculum design, teachers’ power in decision-making, and the importance and influences of politics (and not the political) in education are among the most crucial problems and are discussed in sections 2.2 and 2.4. However, such cases extend beyond the scope of this thesis and the skillset of interaction designers; therefore, they will not be deeply tackled in the design process.

2.1.1 Connectedness to the real-world

Abstraction in mathematics and the discipline’s missing link with the real world is one of the most commonly recognized issues (Applefield et al., 2001; Draper, 2002; French, 2004; Hoyles, 2017; Nouri, 2012; Papert, 1986, 1996; Smitherman, 2006; Steiner, 2002). This implies that students rarely get any explanations of why they are learning mathematics concepts, which demotivates them from engaging with the subject. As a result, calculations and procedures become the “visible face of mathematics” (Hoyles, 2017, p. 5), which creates a false understanding of the nature of the subject, its importance, and the diversity of careers for its specialists (ibid).

2.1.2 Representation in interdisciplinary fields

English (2015) argues that mathematics’ misrepresentation in interdisciplinary education is its biggest challenge and reason for its disconnection to the real world. The author emphasizes the need for a better representation of mathematics in STEM (science, technology, engineering, and mathematics) interdisciplinary initiatives, as the appeal of natural sciences is overshadowing mathematics. Furthermore, Hoyles (2017) provides examples of interdisciplinary contexts where “there is an urgent need for more people who are confident to use mathematical skills to solve problems creatively” (p. 7). English (2015) relates the disintegration of mathematics in an interdisciplinary project to the lack of awareness of its possible contributions.

2.1.3 The formality of mathematics

Finally, playfulness is one of the missing critical factors in mathematical education, making the subject too formal and beyond reach (Chronaki, 2019). Chronaki (2019) highlights the harmful effects of the traditional view of mathematics as a serious, non-playful subject. Those ideas based on “idealized norms of childhood and mathematics” (p. 320) induce inequality into mathematics education and categorize it out of arbitrary students’ capabilities. Assumptions such as the human incapability of learning some of “the more advanced” mathematics until reaching higher levels of schooling are “based on extremely weak evidence” (Papert, 1980, P. 7). Papert (1980) argues that the “widespread fear of mathematics” (p. 39) carries a shift from “mathophile to mathophobe” (p. 40), which does not only disturb mathematics learning but learning in general. Even mathematics games tend
to fall into the trap of being unplayful due to their structured and demanding approach, making them less fun for students (Kafai, 1995, P. 65-66).

2.2 Ethical foundation

Education is a term that carries much responsibility; theories and views on what constitutes a good education are of a great variety and subtle inconsistencies. Therefore, it is necessary to adapt an established ethical framework to guide the selection and adaptation of a refined pedagogical model (see Figure 1). The two main points discussed in the section are (1) learners in education and (2) teachers in education.

*Figure 1: a diagram of the preceding sections*
2.2.1 Learners in education

As mentioned earlier in section 1.1, Freire’s (1968) work highlighted the setbacks of narrative and static education that he called the banking concept of education, where the student’s consciousness is referred to as the bank where the teacher deposits their knowledge. In this model, the students are the oppressed party. They are treated as less valuable, passive perceivers of knowledge, ignoring their natural tendency and need to be active learners, as John Dewey explained (1997, Chapter 3). Similarly, Jiddu Krishnamurti (1981, Chapter 5) regarded any form of education that imposes knowledge on students as highly unethical. He argued that education that “teaches the child what to think, and not how to think” (1948) is problematic. Freire’s (1968) problem-posing concept of education embraces critical thinking, which he defined as one that unites people with the world around them, sees reality as a dynamic process, and immerses itself in action. Krishnamurti (1977) refers to that as the freedom of conditioning that is an essential part of authentic learning, as one learns by interacting, experimenting, and observing rather than acquiring information.

2.2.2 Teachers in education

Krishnamurti (1956, Chapter 31) views teachers as guides that aid students in their growth and assure their freedom, without having any hierarchy or superiority. Similarly, both Dewey (1997, Chapter 12) and Freire (1968) emphasize teachers’ role as guides or facilitators that create the atmosphere for students to discover and learn independently. Furthermore, Freire (1968) and Krishnamurti (1979) emphasize the importance of an open dialogue between teachers and students, where both can learn from each other, mediated by their interaction and perception of the world.

2.3 Constructivist epistemology

Considering the research question of this project and the ethical guidelines mentioned in the previous section, the constructivist model proved to be the most appropriate learning theory to proceed with the research. This choice is due to the possibility of variation and iterations on the model, keeping it dynamic and responsive to change, the availability of thorough research of the model in education, and the practical adaptation of the model to mathematics learning. Moreover, constructivist epistemology is the foundation of most active learning pedagogies such as discovery-based, project-based, inquiry-based, problem-based, case-based (Cattaneo, 2017).

The constructivist theory in education was first introduced by Jean Piaget (Silva et al., 2017) to move from instructional learning to one that is constructive, which has been adapted and iterated on ever since (Gadanidis, 1994). There are shared principles across different versions of
constructivism; this section will divide them into three main categories: (1) understanding, (2) collaboration, and (3) learning tasks (Applefield et al., 2001; Gadanidis, 1994).

2.3.1 Understanding

Constructivist methods emphasize understanding to be the outcome of learners’ construction of knowledge (Gadanidis, 1994). By taking an active part in their learning, students are faced with revelations and obstacles that better their understanding of the world. Therefore, the teacher may prepare an environment that students can interact with actively, but they cannot control their understanding. Gadanidis (1994) provides an example of teaching the specific subject of the addition of integers (positive and negative whole numbers). The subject itself is confusing for students as it involves the addition of negative integers with positive ones, which cancels out the positive ones, and the addition of two negative integers that results in a negative sum. Using the instructional way, teachers explain and give students exercises to practice, compared to the constructivist way where teachers involve students in a situation where the understanding of the addition process may occur (ibid). The author provides few examples of constructivist activities for learning the subject; one of them is through introducing colored pieces (blue for positives and green for negative), where every green piece takes out a blue one. Adding 2 + (-2) results in the two green chips canceling out the two blue, leaving no chips (0), whereas adding 1 + (-2) results in a green chip canceling a blue one, leaving one green chip (-1). According to Gadanidis (1994), such activities allow students to “experience the high-level processes of knowledge creation rather than the low-level skills used to complete schoolwork” (p. 94) and develop their mathematics communication through collaborating with their peers.

2.3.2 Collaboration

The constructivist model puts emphasis on collaboration between students and across disciplines. The work of Lev Vygotsky puts a tremendous emphasis on dialogue and social interaction as facilitators of understanding in the constructivist approach (Vygotsky, 1978, as cited in Applefield et al., 2001). Therefore, group activities with students of different cognitive levels are encouraged to exchange knowledge and perspectives. Additionally, interdisciplinary projects are necessary for the integration of mathematics with real-world problems and making it more relatable for learners (Applefield et al., 2001; Cattaneo, 2017; Gadanidis, 1994). The STEM task force report (2014, as cited in English, 2015) argues that the STEM disciplines “cannot and should not be taught in isolation, just as they do not exist in isolation in the real world or the workforce” (p. 9). Collaboration, both social and interdisciplinary, is one of the core foundations of the constructivist model and highlighted across different active pedagogies, promoting it to be
adopted -for example- in Finnish primary and secondary schools (Cattaneo, 2017).

2.3.3 Learning tasks

For the learning tasks to support students’ understanding and collaboration, they require to be carefully designed by teachers. In turn, teachers should be educated and empowered to be able to design the learning activities. This task should be aimed at supporting students’ expression through mathematical constructions, dealing with problems that are relevant for students’ lives, and being inviting for collaboration. This adds a load of responsibilities and eventual pressure on teachers, which might make them feel “less sure of what it is they should teach and less capable of employing methods to teach it” (Draper, 2002, p. 522). This suggests a gap in the constructivist model, according to Draper (2002), as it clearly describes how learners may best come to learn, but not what are the content or methodologies needed to achieve that level of learning. However, the author suggests that mathematics teachers can adapt constructivist methods from the content-area literacy to modify their classroom into a constructionist one. In addition, Perkins (1991) describes two approaches to constructivism BIG “beyond the information given,” and WIG “without the information given.” The two approaches shape the teachers’ responsibilities and the learning tasks differently. Educators typically choose one over the other, which determines the structure and nature of the learning tasks (ibid). Section 2.4.3 will discuss the two approaches in more detail.

2.4 Opportunities

When challenges in mathematics education mentioned in section 2.1 are considered in the light of the ethical foundation in 2.2 and constructivist epistemology in 2.3, many design opportunities emerge. This section will examine how the theory in the previous sections, together with the interviews conducted with teaching professionals, can be used to adopt technologies within mathematics education. Hoyles (2017) states that “any project in school must, in addition to supporting and assessing students’ interactions with technology, address the curriculum and the teacher’s role in using and deploying the technology” (p. 10). Thus, the presentation of opportunities in this section will consider those elements thoroughly, as shown in Figure 2.
2.4.1 Technology

Technology is needed to make the concept of Mathsland (mentioned in section 1.1) a reality (Papert, 1980). The technology in question should be manipulatable, meaning that learners could make their own and adapt it to their needs. Thus, the aim and focus are not on learning the technology itself but on using it purely as a medium for a direct engagement with mathematics. On the contrary, learning the technology itself may also be a way of learning mathematics. Papert’s turtle geometry is an example of where basic programming instructions can be written to control a moving turtle (see Figure 3). Even though learning programming commands are required, the focus on the technology remains within the context of mathematical thinking and not on mastering the programming commands. This distinction is explained further in section 3.1.1.
It is important to note that constructivist learning can be achieved without any use of technology. Chronaki (2019) presents the concept of *bodying mathematics*, where the concept of area is learned by the use of body parts as measuring units. This embodiment of an abstract concept like the area of a two-dimensional figure immerses learners into mathematics and challenges the “proper mathematics” idea. Other studies demonstrate interdisciplinary problem-solving mathematics’ learning, using constructivist methods without any technological tools (Applefield et al., 2001; English, 2015). Though, even in related learning activities, technology is proven to be a valuable catalyst in facilitating learning (Borba et al., 2016; Perkins, 1991).

Perkins (1991) identifies five facets of learning environments: information banks, symbol pads, construction kits, phenomenaria, and task managers. Information banks (textbook, teachers), symbol pads (notebooks, worksheets), and task managers (teacher, written instructions) are present in most traditional classrooms, while construction kits and phenomenaria are less commonly used (Perkins, 1991). While technological tools can contribute to the betterment of each of the mentioned facets, this thesis is explicitly concerned with construction kits and phenomenaria. Construction kits are building blocks that students can use to construct more complex structures; Lego is a classic example, but programming commands are also a form of construction kits. Phenomenaria are simulations for presenting phenomena that can be observed and manipulated. Identified instances of phenomenaria include simulating a phenomenon in physics using lab apparatus, modeling environments using the visual programming language using *Scratch*, or using the *Arduino Science Kit* to observe and interact with kinetic motion.
Construction kits and phenomenaria technologies will be identified and expanded on in chapter 3.

2.4.2 Curriculum

The curriculum of a class or course describes the learning outcomes, activities, and objectives to be covered. Curriculums have a major role in shaping students’ experience of learning a subject, through their openness to incorporate technological tools and interdisciplinary activities (Fey, 2002). Mathematics topics related to functions (Borba, 2001), data (English, 2015), and geometry (French, 2004), among many others, have been successfully adapted to become interdisciplinary activities. These topics are appropriate for this project, considering the opportunities provided by emerging technological tools, interaction designers’ skillsets, and the mathematics curriculum of the target group (10th-grade student). Additionally, integration of mathematical topics (multitopic) such as functions, data, and geometry with one another and together with other disciplines (interdisciplinary) can be an excellent opportunity for untangling the challenges in mathematical education mentioned in section 2.1. This incorporation is to be done within a scenario that brings the different topics together, making a coherent story for the learner, accompanied by a discussion about the data gathered and a reflection of its uses (A. Chronaki, personal communication, March 02, 2021).

2.4.3 Teachers

Due to the inability to include teachers’ input through co-design workshops – as will be reflected on in section 4.1 - experts’ input and literature will be relied on to determine how teachers may deploy technology in mathematics classrooms. In addition, interviews with two mathematics’ teachers in section 6.1 will refine the outcomes of this section.

To understand how teachers may use and deploy technology in a constructivist manner, a clarification of the two approaches BIG **beyond the information given** and WIG **without the information given** is needed. In the BIG approach, learners are initially given information about what is to be learned before their engagement, allowing them to apply the information and reflecting on their understanding (Perkins, 1991). The WIG approach engages learners directly with the phenomenon by withholding direct information, encouraging them to discover and reflect on their experience (Perkins, 1991), which is more in line with Freire’s (1968) problem-posing approach (p. 79-80). While Perkins (1991) emphasizes the use of a balance of both methods, he states that education without a WIG approach is not successful in engaging students with the “process of discovery and idea construction” (p. 20). The WIG approach motivates dialogues between students and teachers instead of making it barely a process of transferring information. Therefore, based on Freire’s (p. 79-80) guidelines and the constructivist epistemology presented
in sections 2.2 and 2.3, the WIG approach is considered to be better suited than the BIG one.

Mathematics teachers can implement the WIG approach by using construction kits and phenomenaria as mediums. While learners are not given any information in an instructional format, they are supported in various ways by both teachers and the tools that carry their discovery process further (Perkins, 1991). In this case, the teacher’s role is of a guide that aids students with their process by scaffolding it when needed, without offering direct answers. This, of course, might increase the responsibilities and workload of teachers managing a few groups of students. Using well-designed construction kits and phenomenaria could save teachers the time wasted on planning and designing experiments (G. Organtini, personal communication, March 18, 2021).

Giovanni Organtini, a professor of physics at the University of Rome, has held various workshops where he introduced physics teachers to using Arduino and smartphones to design science experiments. While the teachers designed experiments with physics education in mind, Organtini mentioned that many of the experiments proved highly efficient in teaching mathematics concepts such as functions and calculus. Organtini provided pictures, videos, and both written and oral descriptions of the teachers’ designs, which offered valuable insights into teachers’ use and implementation of such technologies. The resulting experiments are identified as phenomenaria that were designed by teachers, using construction kits. This approach opens up more opportunities for interaction designers to mediate the adoption of construction kits to the needs of teachers. It is important to note that the mentioned workshops used earlier models of Arduino microcontroller boards such as the Arduino UNO (dates 2010), which required an introduction to programming with the Arduino programming environment and programming language using analog data. This thesis proposes the use of newer models such as Arduino Nano 33 BLE Sense, which can simplify the process by reducing the time required for learning the technologies (G. Organtini, personal communication, March 18, 2021).

3 Related Design

By researching relevant, interactive technologies that are considered construction kits and/or phenomenaria, various options were found. Blikstein (2013) identified three categories using “selective exposure,” which he describes as the designer’s choices of hiding or exposing aspects of the technology based on their theoretical or pedagogical emphasis. While Blikstein’s chosen categories are solely focused on construction kits made with physical computing units, his selective approach can be used to
recategorize technologies relevant to this thesis. Therefore, the new
categories presented below modify Blikstein’s original categories by adding
phenomenaria and digital technologies as possibilities.

The three categories which will be further explored in the subsequent sections are: programmables, sensor data technologies, and tangible user interfaces. Each one of them can function both separately and with the other forming a unit.

### 3.1 Programmables

This category consists of technological interactions that require programming, which includes both devices that have their digital logic hidden (Logo Turtle, Cricket blocks) and those exposing it for direct manipulation (Arduino). An essential aspect of this category is the compliance of the programming language used within educational contexts. Programming languages that are not purposely designed for the use of children and/or novice learners (see Figure 4) are not as effective for their educational adoption (Blikstein, 2013; Papert, 1980). Focusing on the technical aspects, such as electric connections and the use of electronic components and understanding the syntax of complex programming languages such as C, embedded the aim of “exposing kids to powerful ideas” (Blikstein, 2013, p. 181; Papert, 1980). On the other hand, hiding the complex technicalities and using programming languages such as Logo and Scratch makes such technological devices more accessible for novices (Blikstein, 2013). In turn, this creates a challenge when adapting new technology like the Arduino platform and boards to be used in schools. Along with this challenge comes opportunities that will be presented in the proceeding sections.

![Arduino C](image)

![Cricket Logo](image)

Figure 4: a comparison between Logo vs. Arduino programming commands to turn an LED on and off (Blikstein, 2013).
Resnick et al. (1998) developed Programmable Bricks (Crickets) by adding computation to LEGO bricks, as a continuation of a commercialized LEGO robotic construction kit, see Figure 5. The bricks have both input and output ports where sensors and actuators can be added without any previous knowledge of wiring/electronics. Learners can program the bricks using Logo, a programming language developed by Cynthia Solomon, Seymour Papert, and Wally Feurzeig at MIT (Abelson, Goodman, & Rudolph, 1974), focusing on teaching mathematical thinking. Learners can build creatures out of Crickets, which can be programmed to perform specific movements or even communicate with each other. By doing so, planning and designing movements requires learners to use mathematical thinking and teach them “principles about communication” (Resnick et al., 1998, p. 4) and interaction. This gives an advantage to Crickets in comparison to Papert’s Logo Turtle by allowing for open-ended experiments because of its modularity and reconfigurability. Furthermore, learners use Crickets to design their scientific instruments and to hold their experiments, which according to Resnick et al. (1998), will promote, “[learners] not only become more motivated in science activities but develop critical capacities in evaluating scientific measurements and knowledge, make stronger connections to the scientific concepts underlying their investigations, and develop deeper understandings of the relationship between science and technology” (p. 4).

Figure 5: Resnick et al. (1998) Crickets.
3.2 Sensor data technologies

This category includes the adaptation of new technologies mentioned in the previous sections to school students’ use. The devices featured in this category allow learners and teachers to construct different scientific experiments using sensors to input and data visualizations (graphs) as an output. The advantage of this category is its simplicity and novice friendliness, which allow learners to construct, observe, record, and discuss scientific experiments, making the process of setting up an experiment a lot more efficient for teachers (G. Organtini, personal communication, March 18, 2021). Borba (2001) and Urban-Woldron (2015) used proximity sensors that measure the distance from the sensor to objects in front of it, together with calculators that graph the data. The activities were tested with 8th graders in Brazil (Borba, 2001) and 13-17 years old students in 10 high schools over 15 years (Urban-Woldron, 2015). Both authors concluded that using movement to produce actual data supported students in being more active learners and improved their understanding of data, graphs, and mathematical functions.

In addition, Blikstein (2013) explains the benefits of altering technologies like Arduino microcontroller boards to schools’ use: “they had to run on all platforms, and were designed to be open source from the ground up. Not only did this generate an unprecedented amount of collective expertise, it also brought commercial vendors into the fold, ensuring the wide availability of these devices.” (p. 181).

An example of sensor data technologies is the Arduino Science Kit Physics Lab. The kit consists of building parts and various sensors that can be used together to assemble a set of different scientific experiments, with the possibility of displaying, recording, and analyzing the measured data using the Arduino Science Journal app, see Figure 6. The kit, as described on Arduino’s website, “challenges students to explore and explain the physics behind amusement park rides, make their hypotheses, validate their assumptions using real-world data, and learn about the concepts such as electromagnetism, thermodynamics, kinetics, and kinematics” (Arduino, 2021). The kit’s design inspired the work of this thesis as it combines constructionist elements with a focus on classroom adaptation, which has the potential of interdisciplinary integration.
3.3 Tangible user interfaces

The concept of a tangible user interface (TUI) was first introduced by Hiroshi Ishii and Brygg Ullmer (1997) to coupling physical objects to digital elements, where one can manipulate digital information using physical tools. The use of TUIs in learning contexts has many possible benefits; Marshall (2007) identifies the following: enhancing cognitive and motor skills, encouraging collaboration and communication, engaging through playful learning, and increasing accessibility to younger children, people with learning disabilities, and novices. In addition, TUIs help transform abstract mathematical concepts into more understandable, concrete ones (Urrutia, Loyola, & Marín, 2019).

Guerrero et al. (2016) provide two examples of using TUIs to teach 3D geometry (Figure 7), using the open-source virtual environment OpenSim. The first design is FlyStick, a stick with gyroscope, accelerometer, and force sensors that allow students to generate conic curves (circle, parabola, ellipse, and hyperbola) in the virtual environment. The second design is the PrimBox, which allows students to construct geometrical shapes in the virtual world by following oral instructions from their peers who see a real-world model of the shape. The authors studied the outcomes of using the proposed designs on 60 high school students in Spain; 30 were part of a control group by learning through traditional teaching methods. An assessment of students’ performance (on a 0-100 scale) was carried out based on student’s appropriate use of geometrical language and their understanding of the orientation and arrangement of shapes. The results showed a significant decrease in performance of the control group after two weeks, while the group who used the TUIs sustained a consistent performance.
With the development of new devices such as the *Arduino Nano 33 BLE Sense* and the existence of open-source manipulable virtual environments such as *OpenSims* and *Scratch*, tangible user interfaces encompass valuable possibilities in mathematics education. A valuable advantage of integrating the two spheres is linking the outputs from Arduino microcontroller boards to inputs in digital environments, which simplifies the process of controlling physical elements and provides learners with the freedom to explore scenarios with no physical consequences.

### 4 Reflection & way forward

#### 4.1 Reflection on generative design research

To achieve a better understanding of curriculum and interdisciplinary integration and to be able to design for mathematics classrooms, teachers should be involved in the design process. This approach is based on the societal and sustainable values of co-creation as presented by Sanders and Stappers (2012). The authors describe generative design research as a method where co-creation is used as a mindset in the discovery phase of design and throughout the process. The method assumes that all people are creative, which is in line with the idea mentioned by Ezio and Coad (2015) that everyone can become a designer.

Generative design research would be an appropriate approach to include teachers as the experts in their fields, allowing them to partake in the design of the most applicable model of a constructivist mathematics’ classroom. This inclusion is in line with the ethical consideration towards the teachers’ role - as explained in section 2.4.3, the pedagogical value of the proposed designs, and their suitability in an educational context. Hence, a plan for holding generative design workshops with mathematics teachers was set to be executed in the discovery part of the research. However, due to teachers’ time availability and schedules’ clashes, the workshops did not attract enough
participants. This is a clear limitation of the project as the lack of teachers’ input at an early stage in the design process does not exactly comply with Sanders and Stappers’ view on co-creation.

A deeper analysis of the above-mentioned ethical concerns challenges the initial hypothesis proposed by the research question. This thesis started with the intention of exploring how technological tools can enhance students’ understanding of mathematical topics. At the same time, the inability to collect teachers’ input invited a reconsider this hypothesis. The effort put in reaching out to teachers to include them in the design process was, however, not neglectable. The efforts included contacting six international schools in Sweden, as well as mathematics teachers-students programs at Malmö university, and creating and publishing a co-design event on specialized Facebook groups. A total of four mathematics teachers showed interest through the Facebook group and teachers-students program, but only two could attend the workshop at the same time. Out of the six schools contacted, three showed a willingness to include their students in the design process, while none of them could dedicate any teachers’ time for the co-design workshop.

This issue raises the question of whether this thesis's problem is of a technical or social nature. While there are recognized opportunities in employing technology for the betterment of mathematical understanding, implementing that technology in education raises a new challenge. This is a much larger social and economic issue, described by Kemmis (2006) as the conflict between education and schooling. Education strives to embrace the individual’s self-worth and aspirations as part of the collective’s development, whereas schooling is concerned with preparing individuals to fit into the established societal and economic structures. Schooling has been the more applied approach, wounding teachers, the education career, and students (Kemmis, 2006). This raises a responsibility for designers to create tools and processes that benefit education rather than schooling. Hence, designing advanced educational technologies is not valuable if such devices are not modified for teachers’ and students’ adoption.

4.2 Design framework

The use of frameworks has proven to be significant in the advancements and development of any field, including interaction design (Sedig & Parsons, 2013). Schön (1992) describes designers as reflective practitioners, using their practice to generate theory, where technology can be used as a stimulator of reflection. Hence, this thesis uses this description to draw on the educational and ethical theories presented in chapter 2, together with design practice, to craft the proposed framework.
Sedig and Parsons’s (2013) development of a design framework that is focused on systematic thinking about interaction design inspired the outlining of the proposed framework’s characteristics. To ensure a valuable contribution to the literature and the interaction design field, the proposed framework will have the following characteristics:

1. To unify separate ideas and models on ethical education, learning, and technology into one model.
2. To be applicable for specific uses in mathematics education and adaptable to different contexts, subjects, and technologies.
3. To be generative, inspiring designers in their creative process and creating future theoretical and practical projects.

The characterization is only meant to be used as guidance, with the awareness that it cannot be inclusive to all possible tasks and circumstances (Sedig & Parsons, 2013). Thus, the intended nature of the framework is malleable and dynamic, using it as a starting point, but keeping it alive by constantly modifying it.

4.3 Reframing & exploration statements

The combined outcome of the theoretical research, the study of related designs, and the interviews with educational experts, is nothing less than a reframing of the original research question. The question that this thesis started as:

*How can we design technological tools that enhance students’ understanding of mathematical topics?*

Presents two critical points, the first being the “design of technological tools,” and the second “understanding of mathematical topics.” The targeted technology, as discussed in section 2.4.1, is in the form of construction kits and phenomenaria. Thus, the research part concluded that the specific technology itself is not to be the focus of this thesis, instead, the way that technology is used and implemented is more essential. Additionally, an understanding of mathematical topics is redefined through constructivist theory, using the WIG approach as the engagement with mathematics in a real-world application. Therefore, the initial question is reframed to be:

*How can we design constructivist learning tools for mathematics education through the use and adaptation of technology?*

To answer this question through the design process, exploration sub-questions will be presented. Those sub-questions are means of testing the opportunities provided by merging the constructivist approach, mathematical education, and new technologies. The following sub-questions will guide the design process:

- How might we modify new technological tools to be used as construction kits and/or phenomenaria in mathematical education?
• How might we design for constructivist WIG learning using tools that can be easily modifiable and adopted by mathematics teachers?

5 Methodology

5.1 The double Diamond

The thesis structure follows the double diamond model as described by the British Design Council (2021), which is used as a guideline for setting up the different phases of work and their respective timelines, with the knowledge of the possibility of an overlap between the phases. The four phases in the double diamond are listed with a description of each phase and its respective stages in the thesis, see Figure 8.

Discover: the first phase opens up the project by researching theoretical knowledge, expert perspectives, and related designs connected to the topic.

Define: the second phase uses the results from the first phase to narrow down the research by choosing the most appropriate methodology and concluding the first half of the project with reflections and propositions.

Develop: the third phase uses the propositions from the first diamond, together with insights from educators, to expand the ideation process.

Deliver: the fourth and last phase uses the outcomes of the ideation sessions from the third phase to build, test, and evaluate prototypes, as well as presenting the results.

Figure 8: the double diamond model.
5.2 Research through design

This thesis follows the research through design (RtD) method described by Zimmerman, Forlizzi, and Evenson (2007). The model provided by the authors is an appropriate guide for interaction design research that aims to “[transform] the world from its current state to a preferred state” (p. 1), which resonates with the aim of this thesis of transforming mathematics education. The work of this thesis shares the authors’ definition of design research as “an inquiry focused on producing a contribution of knowledge” (ibid, p. 2). Gaver (2012) views RtD as a generative method, concerned with “creating what might be” rather than forming statements of “what is.” In addition, Gaver suggests the use of annotated portfolios as theoretical contributions to RtD, where artifacts inform and inspire the design process. The tasks and relationships between interaction design researchers, human interaction designers, and the role of artifacts are shown in Figure 9.

![Figure 9: the exchange between interaction designers and other disciplines as illustrated by Zimmerman et al. (2007).](image)

Lastly, this thesis is evaluated through the four lenses process, invention, relevance, and extensibility as described by Zimmerman et al. (2007):

**Process:** methods chosen are justified, and the technique used is reproducible.

**Invention:** contributions are made through a clear expression of proposed designs.

**Relevance:** the anticipated state of the world and its applicability are stated clearly.

**Extensibility:** the ability for future projects to build on the existing work.
5.3 Interaction design research

This thesis supports Löwgren’s (2007) argument that interaction design leads to scientific knowledge. Two of the research strategies in the aforementioned publication are used actively in the work leading to this text; the author describes them as (1) to explore the potential of the design idea, material, or technology and (2) the inclusion of future users in the design process as the experts in their field through a participatory approach.

The two strategies intertwine and complement each other in refining the outcomes of this thesis. More detailed methods will be used to expand on each of the strategies. Section 5.4 will focus on the first strategy, while section 5.5 will elaborate on the second one.

5.4 Prototyping

Prototypes are defined differently across design and research fields; this thesis uses the definition of prototypes as “tools for traversing a design space where all possible design alternatives and their rationales can be explored” (Lim, Stolterman, & Tenenberg, 2008, p. 2). Prototypes are formed by the context and behavior they are occurring in (Lim et al., 2008), which makes them dynamic and multipurpose based on the way they are presented. Houde and Hill (1997) view prototypes as being born or preexisting in any medium, functioning as a representation of design ideas.

![Figure 10: the three dimensions of a prototype (adapted from Houde & Hill, 1997).](image)

Prototypes made in this thesis are assessed through the model’s dimensions described by Houde and Hill (1997). The three dimensions are role, look and feel, and implementation, as shown in Figure 10. Role indicates the usefulness of an artifact in the user’s life, look and feel the sensory outcome experienced by the user when using the artifact, and implementation is the
artifact’s performance of its function. The three dimensions guided the process of creation and analysis of the prototypes made in this thesis.

5.5 Participatory action research

Given the nature of the project and its relationship with deeper social, cultural, economic, and ethical issues regarding education, participatory action research (PAR) is a crucial method to adopt in this thesis. This is carried through the process of coupling reflection with action, which together give birth to creativity (Baum, MacDougall, & Smith, 2006). As PAR “draws heavily on Paulo Freire’s epistemology” (ibid, p. 856), it acts as a methodology for applying Freire's guidelines to the design research and process of this thesis.

Designers’ role in a PAR approach is critical and active in the social, cultural, environmental, and political dimensions. Kemmis (2006, p. 471) quality criteria of educational PAR projects include three main points that can be summarized in the following quotes:

1. “addressing important problems in thought and action, theory and practice.”
2. “that the projects will cross the boundaries between the school and the world beyond it to explore themes and issues of interest both inside and outside the school.”
3. “it will inform wise and prudent collective action by a range of those involved in and affected by the practice, in the interests of transforming the collectively constructed social, cultural–discursive and material–economic fields that shape, structure, and support existing practice.”

This thesis considers participants’ needs and carries the design process to transform their current reality into their preferred future. Besides taking the role of design researcher, my part extends to being a participant in this study. This role is determined by having experienced the same environment as the studied group, meaning undertaking the same course of mathematics, in the same system, school, and by the same teacher. Additionally, an understanding of some of the difficulties faced by students and teachers was gained by tutoring at the mathematics degree under study for six years.

5.6 Interviews

In-depth interviews were held to understand the teachers’ experience when teaching mathematics using technological tools. This is part of the ethnographic research approach for gaining an in-depth qualitative understanding of peoples’ experiences (Muratovski, 2016).

Two interviews were held, one over the Internet and the other physically, taking into consideration the comfort of the interviewees. The interviewees teach at different levels of mathematics, both teaching an international
curriculum in Sweden. The interviews were conducted with the aim of understanding teachers’ perspectives and experience of constructivist methods of teaching mathematics and the use of technology in the classroom. The data resulting from the interviews is described, categorized, and interpreted in section 6.1.

6 Design Process

In this section, insights from mathematics teachers are presented, the plan for the prototyping and testing is set, the prototypes are introduced and tested. Finally, the results of the design process are presented in the evaluation section, see Figure 11.

Figure 11: a diagram of the design process chapter.
6.1 Mathematics teachers interviews

Interviews with two mathematics teachers were held before proceeding into the design process. The teachers are referred to as Jenny and Eric, which are not their real names, to grant them anonymity.

6.1.1 Jenny

Jenny is a high school math teacher who teaches both basic and advanced levels of mathematics in an international school in Sweden. While she holds a strong interest in constructivist methods, she primarily uses instructional ways for teaching. In her classroom, she only uses constructivist activities that she sets up every few weeks, usually containing interdisciplinary problem-solving activities that students explore in groups.

Jenny shared her experience of a “teaching Python for mathematics’ teachers” event that she attended together with many other teachers. The event’s aim was for mathematics teachers to learn basic Python programming skills to design activities in the classroom. She enjoyed the event and picked up on programming, but she was frustrated for not integrating what she learned in her classroom easily. Even though she had some basic knowledge in programming, it was still difficult for her to use what she learned effectively. She noticed that it was even more challenging for her colleagues who did not program before to gain any valuable skills to apply in the classroom. She reflected that learning programming requires a longer time to be used by teachers efficiently and that not all teachers are keen on learning and adapting it. In addition, even if Jenny had the necessary programming skills, designing any learning activities for students requires an amount of time she does not have. Though, when Jenny managed to include an exercise in her classroom that students could manipulate using basic programming skills, she noticed a divide in the class between students who utterly enjoyed it and those who were overwhelmed by it.

Jenny felt sad after noticing her students’ levels in mathematics declining after the breakout of Covid-19 and moving to digital learning. Besides the lack of group work, Jenny noticed that some subjects were suffering difficulties to learn digitally. She recognized that functions had always been one of the more difficult subjects for students to grasp, but teaching it online made it even more difficult. She thinks that technology can help students understand topics like functions, geometry, and “thinking math,” referring to the logical reasoning of a problem, in a better way.

6.1.2 Eric

Eric is a chemist and a high school math teacher who teaches advanced chemistry and entry-level mathematics in an international school in Sweden. Eric likes to use constructivist methods as much as possible in his mathematics and chemistry classes. Though, he often needs to rely on instructional strategies when teaching mathematics compared to chemistry.
When Eric teaches chemistry, he finds it easier to use constructivist methods to plan and hold activities aligned with the curriculum. It is common for students to plan and execute chemical experiments that most often fail, which is seen as a valuable learning experience by Eric. Additionally, one usually encounters students experimenting with explosive, foamy, and colorful materials in Eric’s classes. He sees a great value in letting students have the freedom to explore chemicals in a fun and entertaining way. However, Eric has a more challenging time using similar methods when he teaches mathematics. He contributes that to the way mathematics’ curriculum is designed and its disintegration from other fields. When asked about the value of technology in assisting constructivist activities in math, he was not sure of its effectiveness. Eric sees most contemporary mathematics’ technological tools as more supportive of instructional methods, such as practicing calculations or graphing functions, as the assessment of the subject requires two aspects besides reasoning: precision and efficiency in calculations. However, he sees a potential value of using technology as a facilitator for more constructivist approaches.

6.1.3 Insights

The interviews with Jenny and Eric presented the practical challenges that mathematics’ teachers face when implementing constructivist methods and their views and experiences with using technological tools. Among the challenges, it was harder to implement constructivist approaches when teaching the mathematics curriculum compared to the science curriculum (chemistry). The identified causes of such difficulties relate to the bond of mathematics concepts in the real world with other disciplines, yet mathematics is taught separately. In addition, a requirement for teachers and students to learn the technology creates a divide between those who are experienced/interested and those who are less involved. Another insight on the adoption of technology was that teachers found it difficult to design meaningful activities with their limited time and expertise in technology. Finally, teachers showed a strong interest in adopting a constructivist approach for teaching mathematics, even with all the difficulties of such a move.

6.2 Planning

Tests of the prototypes were arranged to be held with 10th-grade students enrolled in the International Baccalaureate Diploma (IB) in a Swedish school. There were four testing sessions in total; the tangible kits were tested in the first two (1.5 hours and 1 hour), the TUI in the third (1.5), and presentations during the last one (1 hour). The workshops were held in the students’ mathematics classroom, in the presence of their teacher and an assisting photographer.
Before proceeding in the ideation and prototyping phases, the level and topics of mathematics that the students are learning were identified through a conversation with their teacher and a guidebook provided by the IB program.

To answer the questions presented in section 4.3, a selection of both an interdisciplinary field and topics in mathematics must be made. The choice is inspired by the literature, related design projects, mathematics’ teachers’ interviews, teacher’s workshops shared by Organtini (The professor of physics at the University of Rome, introduced in section 2.4.3), and my own experiences with the topics. The chosen topics based on this selection are mathematics functions and geometry and the interdisciplinary subject of motion in physics.

6.2.1 Motion & movement

The topic of motion provides a link between mathematics and the real world. The focus of using the topic is not learning it (even though this is a likely outcome) but using motion as a carrier for understanding functions in real-world applications. The concept of mathematics functions is difficult to perceive in reality but is present all around us. For example, walking can be a function of the distance walked in relation to the time taken. Hence, an example of a link between functions and motion can be the input value of displacement (distance with a direction) over time, which is a function of velocity. With sensors, students can explore different data inputs provided by a specific movement and their respective graph representation.

There are different types of motions in physics; each can be performed through various movements. Considering the duration of the workshops with students, five types of motions are chosen to be explored, as shown in Figure 12. The motions are:

**Rectilinear:** a movement along a straight path. Example: a train driving on a straight track.

**Curvilinear:** a movement along a curved path. Example: a car turning along a curved road.

**Circular:** a movement along a circle’s circumference. Example: a satellite orbiting the earth.

**Rotational:** a movement in a circular motion around a fixed point (center of rotation). Example: earth rotating around its axis.

**Oscillatory:** a movement to and from a fixed point (rest point). Example: the movement of a pendulum in a clock.
Some movements that can generate each of the motions are identified to build on in the prototyping phase. The prototypes aim to give students enough tools to perform different movements relating to a motion they will explore.

6.2.2 The task

A total of 30 students participated in the workshops, divided into six groups, each group consisting of 5 students. Each group was assigned one of the motions, two of the groups were given the same motion (oscillatory). The task for each group was to explore the motion assigned to them through performing movement and recording different data points. The technological instrument, an Arduino Nano 33 BLE Sense, was provided hidden in prototypes that allowed for measurement of acceleration (on three-axis), angular acceleration (on three-axis), proximity, light intensity, color values, temperature, among a few other values. Reporting of results were to be held through presentations where student shared:

1. The movements they chose.
2. How they explored the movements and their relation to the motion.
3. The data they recorded and the reasoning behind their choices.
4. The insights and unexpected results they found from the data.
5. Other real-life scenarios where they could identify their movements.
6. Other valuable contexts for collecting and using similar data inputs.

6.2.3 Intended learning outcomes

As explained in section 2.4.3, the WIG (without information given) approach was chosen, meaning that the prototypes were designed with intended learning outcomes, without giving direct information to students. The prototypes aimed to function as both construction kits and phenomenaria
that created the possibility of fulfilling the intended learning outcomes, but without limiting it to this objective. My role in the testing was of a guide who assisted students when needed, through the process of scaffolding, supported by their teacher. The assessment of the prototypes in achieving the learning outcomes was evaluated through discussions with students and their teacher and their reflections through presentations. Additional evaluation of each of the prototypes' look and feel, functionality, and role were held through observations, a survey, and semi-structured interviews with two students.

This section presents some of the intended learning outcomes that the prototypes aimed to achieve in the topics of mathematics functions and geometry.

The study of functions includes the understanding of mathematical expression that contains a dependent variable (f(x) or y) and an independent variable (x), and their respective graph representation (linear, quadratic, exponential, and trigonometric) by successfully relating the expression to its graph and the ability to draw the graph using the expression. Those are among the primary learning outcomes of functions, which are a requirement for the mathematics level studied by the user group and were covered as possible learning outcomes by the prototypes. Additionally, domains (all possible values of x), ranges (all possible values of y), transformations (moving a graph on a two-dimensional coordinate system), and inverse function (inverting a function through interchanging its variables) are among the possible learning outcomes but considered a more advanced level of mathematics and are not among the learning requirement for the user group.

In geometry, examples of the learning requirements for the user group are concerned with 2-dimensional coordinate systems (coordinates of a point in space, distances between two points, transformation) and the study of shapes (their qualities, areas, angles). The possible learning outcomes in geometry extend way beyond the user group’s mathematics level, dealing even with university-level mathematics concerning 3-dimensional geometry. The potential learning outcomes include but are not limited to: understanding three-dimensional spaces (representations of points, shapes, transformation of shapes, 3-d space perception), 3-d vector geometry (direction and magnitudes along x, y, and z axis), and perception of 3-d shapes (rotation, volume, and dimensions).

### 6.3 Prototyping

In this phase, two prototypes were made and tested over three sessions. The first prototype is a set of tangible kits that students could use as tools to perform and explore various movements, related to one type of motion. The technology used in the tangible kits is the Arduino Nano 33 BLE Sense and the Arduino Science Journal app, which will be explained in more detail in section 6.3.1. The second prototype is a tangible user interface built after the first testing of the tangible kits.
6.3.1 Arduino Nano 33 BLE Sense & Science Journal

The Arduino Nano 33 BLE Sense is a microcontroller board equipped with various built-in sensors on the board itself, as shown in Figure 13. Additionally, the board has a Bluetooth Low Energy chip that allows for a battery-efficient wireless transfer of data to other devices. Additional sensors and actuators can be connected to the Arduino board and programmed using the Arduino IDE (integrated development environment). However, the board can be pre-programmed to send the data from its built-in sensors directly to the application – the Science Journal app or other- used by the user in a phone or tablet, which will allow for receiving and displaying the data directly without any programming.

![Arduino Nano 33 BLE Sense](image)

Figure 13: the Arduino Nano 33 BLE Sense. (Arduino’s courtesy).

The Arduino Science Journal app, an application that can be downloaded on Android, IOS, and AppGallery. The app can read data from the built-in sensors in the user’s device and display them as graphic graphs. Users can record and analyze data over time and add notes and images of their observations. The app supports Bluetooth connection with the Arduino Nano 33 BLE Sense board, which allows users to display, record, and analyze data coming from the board quickly and straightforwardly.

6.3.2 Tangible kits

To build the tangible tools, a few guiding points gathered from literature and interviews with teachers should be taken into considerations:

1. they can act as construction kits, phenomenaria, or both.
2. they should not require any technical or programming skills to operate them.
3. they should be easily constructed by kids and teachers.
Following these points, the idea of providing students with basic construction kits that they could assemble themselves seemed most applicable. In that case, the students could use the construction kits to create phenomenaria that simulate movements, similar to the method used by Organtini (G. Organtini, personal communication, March 18, 2021). Kits that can be supplied to students include springs, wheels, magnets, fidget spinners, sponges, threads, cardboard, glue guns, Velcro strips, hooks, and cutting knives (see Figure 14). The Arduino Nano 33 BLE Sense board and a power bank can be secured in a box, where students can use as the basis of their constructions. In this way, students can ideate on movements that they want to explore and construct the tools they need to achieve such movements. A test run of using the materials as construction kits revealed some issues regarding time consumption, the fragility of materials, and the safety of the Arduino board and power bank.

![Figure 14: initial sketches of the kits that students could assemble themselves.](image)

Therefore, a decision was made to prepare tangible units that can be joined and manipulated by students, ensuring the time efficiency of the short workshops. The units could still be used as construction kits that could be put together to act as phenomenaria, simulating movements (see Figure 15). The units consist of one central box that contains a secured power bank on the inside and a secured Arduino Nano 33 BLE Sense board on top, and extra attachables that can be linked to the central box via Velcro strips. This guarantees the safety factor of any potential hazards caused by dropping the power bank. Houde and Hill’s (1997) model was used to create the prototype by determining its role, look & feel, and implementation.
6.3.2.1 Role

The role of the tangible kits is to enable students to perform movements relating to the motion they are exploring. This is to be done without any required knowledge of electronics or programming. Hence, Molex connectors were used on the central boxes, allowing students to move the Arduino board from one position to another securely and intuitively. The only connection that needs to be made is connecting the Arduino board to power via a USB-B cable. Connecting the board with the Arduino Science Journal app was also made easy by uploading the required code on the board and naming the board according to the group’s number. In this way, students could find a board with their group number on the app and connect to it directly. This configuration job can be done easily by teachers or by assigning it to students who are keener on programming.

Connecting the central box to other tangible attachments is made simple by adding Velcro strips on different sides of the central box so students can attach it easily in the direction they need.

6.3.2.2 Look & Feel

The look and feel of the tangible kits aimed to communicate their functionality while leaving the freedom for students to use them in other ways. The kits needed to have a finished look so students could trust using them to perform dynamic movements while keeping a low fidelity feel to invite manipulation. The size and variety of the prototypes should encourage the collaboration of multiple students (5) altogether or in smaller groups of 2-3. Additionally, a feeling of continuity between the tangible prototypes and the Arduino Science Journal app should be present for students to draw a clear connection between the two.
6.3.2.3 Implementation

The tangible kits were made using accessible and novice-friendly tools to ensure the ability to reconstruct such kits by teachers and students. This guaranteed that the kits produced could be easily integrated into the classroom, that teachers and students do not need prior training or accessibility to advanced machinery to reproduce them. All of the tangible units were made with white foam boards, which is a light material that can be cut and glued easily to create more complex forms. Cutting knives and glue guns were used to assemble the pieces, and additional tools such as fidget spinners, springs, hooks, and sponges were used in the construction of the kits.

The kits needed to allow students to perform the movements mentioned in section 6.2.1; thus, besides the central unit (Figure 17), the following attachable units (Figure 16) were constructed:

- **Speedy**: pushing and turning.
- **Foamy**: dropping and throwing.
- **Spinny**: spinning and rotating.
- **Twirly**: rotating and twirling.
- **Swingy**: swinging and oscillating.
- **Bouncy**: oscillating.

![Image of tangible kits attachables](image)

*Figure 16: the tangible kits' attachables, 2-3 units were made of each attachable depending on the number of groups that need it to perform their assigned motion.*
6.3.3 First workshop

The students were briefly introduced to the project, the tools (tangible kits, Arduino microcontroller boards, and Arduino Science Journal), a brief of their tasks, and their groups. A clarification that the assessment of tasks would be held through presentations, where students shared their findings with their peers and us, noting that there were no expected wrong or right outcomes. Observations and discussions with students were used for evaluating the prototypes at this stage. An additional survey and interviews with two of the students and their teacher were used for assessing both prototypes in more detail, as presented in section 6.4.

A general insight from the session is the success of the tangible kits as tools to encourage collaborative use. Almost all group members were engaged in testing movements, recording data, and discussing strategies and results (see Figure 18). Some groups were using systematic approaches of testing, while others were trying random movements. There was a trend among a few groups of using some of the tools differently to their intended function, which was an aim of the kits. Those groups asked for additional materials such as Velcro strips and tape, which were provided to them. Students were highly motivated and engaged with the kits, which could be due to novelty factors, as will be discussed further in section 6.4.

A shared point of frustration for many groups was the understanding and analysis of data. As my role in the testing session was to act as a guide with the teacher’s help, support was issued to the students through questions about data patterns and outliers, the qualities and types of graphs, comparisons of different sensor data resulting from the same movement, and quality of experiments. This process proved to be extremely helpful for students, as there were noticeable reflections resulting from the scaffolding.
Students had diverse interests when using the tangible kits. Some were interested in creating shapes and patterns with data through movements; others were trying movements that involved more active moves; one group was focused on experimenting with different attributes of one movement, another group of students was focused on analyzing data patterns. Students were asking their peers in the group questions about data, graphs, and execution of experiments and asking me and their teacher a few questions. Answering questions by one teacher was comfortably manageable but providing support by one teacher/guide proved to be extremely hard for such a large group of students. This is to be discussed further in section 6.4.3.

6.3.4 Tangible user interface

The tangible user interface works together with the tangible kits, providing a manipulative digital representation of data. This focus of the prototype was linking sensor input data with digital outputs, as described in section 3.1.3. The prototype aims were the following: uncovering the possibilities of multitopic learning, examining the effects of using TUI on students’ understanding of the topic learned (functions and geometry), exploring the potentials of TUI to be modified and adopted by teachers.

The first attempts at designing the TUI were by linking the Arduino Nano 33 BLE Sense with Scratch and OpenSim via the BLE connection. Both virtual environments are easy to manipulate and build with, which creates an opportunity for teachers’ adoption and implementation in mathematics education. By linking such virtual environments to the physical realm via BLE connection, opportunities for physical experimentation with digital outcomes become an accessible possibility. As mentioned previously in section 3.1.3, some of the advantages of such fusion are the shift of using digital actuators as simulations of physical ones, the freedom of carrying experiments that might have dangerous outcomes if held physically, greater collaboration and engagement between students who are in the same physical spaces or separate ones.
Due to technical difficulties and the limited time frame of the project and testing sessions, the connection between the virtual environments and the Arduino Nano 33 BLE Sense was unattainable. An alternative solution was achieved using the p5.ble.js library to test the concept of such integration, considering that the library is newly developed, making such exploration a valuable asset for future designers and p5/Arduino communities. Additionally, this contributes to the interaction design practice, as it paves the way for future tinkering with the library and easy implementation of BLE data into digital prototyping.

6.3.4.1 Role

The role of the TUI prototype was to enable students to interact with geometric concepts digitally via physical movement using tangible kits, as well as examining the potential of adoption and operation of tangible environments by teachers. Hence, connecting and manipulating the digital environment should be intuitive and familiar. This was tested by adding a side menu to manage a 3-d geometrical shape located in the center of the interface, see Figure 19. The shape and its qualities could be easily changed via drop-down lists in the control menu. Few geometric shapes were added to familiarize students with the shapes and their representation in 3-d space. Connecting the Arduino board to the interface is easily achieved by pressing a “connect” button that lists available Bluetooth devices, where the groups’ boards are displayed with the group number (e.g., Group 5).

6.3.4.2 Look & feel

The interface’s look and feel aimed to create a feeling of familiarity and control for students and teachers. This was achieved by establishing a distinction between the control menu and controlled environment and a clear hierarchical order between controlled elements. The colors were used in accordance with the tangible and the Arduino Science Journal app to maintain continuity.

6.3.4.3 Implementation

The students were provided with lists of shapes and three qualities that they could control on the three-axis, using sensor data from movements. The three controllable qualities are rotation, stretch (volume/size of the shape), and translation (movement along the three-axis). A student could choose a quality to control on a specific axis, and then they could link it with the data from a particular sensor. The sensor data provided in the lists were the ones that students most used in the first test round. In the program, all data received from the sensors are mapped into a specific range. This is to maintain a functioning visual and present the idea of domains and ranges in functions.
6.3.5 Second workshop

The students were provided with a link to access the interface and were briefly introduced to its functions (see Figure 20). The engagement with the interface dropped drastically after about 15 minutes for most students. Questions related to the task were dominant across all groups; students seemed to have fun interacting with the interface but were frustrated with how it could be related to their task. Therefore, a clarification was made of how the interface could be used as part of the task to visualize the data or link to concepts in real life. That was helpful for some of the more involved students, though there was still a feeling of frustration among many groups. By holding discussions with groups individually, few insights were gathered. Part of students’ frustration was linked to the implementation and look & feel of the prototype, which had to do with providing many control points simultaneously. This proved to be overwhelming for most students, and in many cases, affected the role of the prototype, as it did not present a clear link between the data and the quality controlled by that data.
Other points of frustration were associated with explaining the task and a feeling of failure due to comparison. Those points will be expanded on further in section 6.4.

6.4 Evaluation

The outcomes of the workshops are evaluated in this section through observations of the testing sessions presented in sections 6.3.3 and 6.3.5, interviews with two students, an interview with the mathematics teacher, a survey with the participation of 16 students, and students’ presentations. In the survey conducted, students were given questions and a scale of 1 (strongly disagree) to 5 (strongly agree); hence the percentage of students with a neutral response of 3 will be divided into 50% agreeing and 50% disagreeing. Interviews with two students who were participants in the testing will be referred to as Isak and Laura.

6.4.1 Prototypes as educational tools

The assessment of the prototypes will be performed using the following levels: understanding of use and role as educational tools (constructivist kits and phenomenaria).

The insights from the survey of students’ understanding of the tools show that 87.5% understood how to use the tangible kits, while 56.3% were neutral about understating the tangible user interface (the other 31.3% understood it and 12.4% did not). This confirms the insights from observations presented in section 6.3.5 of some of the flaws regarding the implementation of the
protoype itself. Questions were asked to Isak and Laura to determine the issues they faced using the TUI, and both students expressed difficulty with the number of control attributes. Isak said, “it wasn’t really difficult…it was just confusing using all the values. The shape was doing a lot of weird movements, and we didn’t know where it was coming from…like from what value”. An overwhelming feeling of dealing with all the attributes accompanied Laura as well. Understanding how to use the tangible kits was generally easier, though Isak expressed some difficulty using the Arduino Science Journal app to connect and add experiments initially.

Students partly used tangible kits as construction kits by building more complex structures (Perkins, 1991). Though, both the tangible kits and TUI were mainly used as phenomenaria by using the prototypes to simulate movements, changing attributes, and observing the outcomes. When asked about the TUI prototype, Laura and Isak expressed similar feelings. Isak expressed a lot of interest and joy in using the TUI; he said that it gave him a whole different view of the data he previously perceived and recorded as graphs. This was present in students’ presentations as few groups used the TUI as a way to visualize and explain the data gathered from their movements. Before using the prototypes, Isak’s experience of graphs was limited to “math’s textbook and stock market,” he enjoyed seeing how he can make his graphs with movements, and he was thrilled to control the shapes on screen. This is an excellent example of linking abstract mathematical concepts to real-life applications.

In addition, the prototypes engaged students with each other in a playful experience. One student left the following comment in the survey “working with physical things [tangible kits] is more interesting than working with books and websites only.” There was a common playful attitude towards using the prototypes, which was a motivating factor for engagement and collaboration. Examples of that include a group of students trying to accomplish movements that draw a symbol or write a phrase using the plotted graphs, and another group trying to make the TUI shapes behave in funny ways. This balance between serious and playful approaches to learning is proven to be necessary (Kafai, 1995, P. 326-328). Additionally, the playful approach enforced the collaborative spirit between students, encouraging them to participate in experiments, as it released the pressure of mathematical performance. Isak’s favorite part of the workshops was the group work; he tremendously enjoyed carrying tests and discussions with his groupmates. The prototypes catalyzed collaboration between students, manifested in a common arrangement of 3 students performing tests and the other two observing and communicating data with the team.
6.4.2 Learning outcomes

This section will assess whether any of the intended learning outcomes presented in section 6.2.3 were achieved. Additionally, the connectedness of mathematics to the real world and its representation within interdisciplinary activities will be evaluated.

Most students conveyed a robust understanding of the relationship between dependent and independent variables, transformations, graph (linear, sinus, and exponential) representations, domains and ranges, vector geometry, translations in 3d-space, and qualities of 3d shapes. However, understanding such concepts was missing a straightforward way of articulating them because of the missing vocabulary. According to Perkins (1991), correct verbalization is provided by a guiding teacher in the WIG approach after the students achieve an understanding. This is to be discussed further in the proceeding section as one of the teacher’s roles. In a conversation with the class’s mathematics teacher, she expressed amazement that the students express understanding of concepts in higher levels of mathematics. Though, 75% of students participating in the survey did not think that they improved in mathematics. When the teacher asked one of the students if he enjoyed mathematics class, he said, “I had a lot of fun, but it’s not mathematics,” and another student left a comment in the survey saying, “thanks for not forcing us to have real math.” This notion of mathematics definitions and capability imposed on teachers and students is strictly critiqued by Papert (1980, p. 6-9), which will be discussed further in section 7.1.

Laura expressed much enthusiasm when asked about the TUI prototype; she said that the visuals helped her make meaning of the graphs in reality. This was a common thread that was observed from students’ presentations. One group compared oscillating movements to heartbeat and breathing, using the TUI as a visualization. Another group analyzed the data graphs and related them to real-life applications, where the sensors they used could be beneficial. However, a concrete representation of mathematics in interdisciplinary activities proved to be challenging. From observing students’ presentations and discussing their approaches to understanding movements with their teacher, there seemed to be a missing element in some of the groups. Most students succeeded in planning and executing experiments and presenting interesting observations, yet around half missed interpreting and analyzing their findings. This required a more active involvement as part of the teacher’s role as a guide, which will be discussed further in the next section 6.4.3.

6.4.3 Tools for constructivist teachers

The tools were designed to support teachers with their guiding role, without demanding more of their time and adding to their tasks. The prototypes proved to be successful in engaging 30 students, with a number of questions that were manageable by one teacher. However, using the WIG approach,
teachers need to be more active in discussion with students to maintain a scaffolding process (Perkins, 1991), helping them articulate what they understand from their experiments in mathematical terms. The limited time frame of the sessions and the scope of the topics were among the contributions to the inability of carrying such discussions, as the time frame of covering such topics is much longer. Thus, a longer testing period of the prototypes is necessary to study their effects on teachers’ responsibilities. An insight in this regard, though, is that the design of the task proved to be of help for the scaffolding process, as the teacher could have small discussions with students after presenting their findings.

Another aspect of the prototypes is their ease of use and modification by teachers to use as tools for designing their own activities. Sections 2.4.3 and 6.1 provided insights from literature, teachers’ design workshop held by Organtini, and interviews with Eric and Jenny, which together offered well-rounded guidance for designing for teachers’ use. Testing the usability of the tools by teachers requires further testing and is beyond the scope of this thesis. However, the simplicity of the tools proved useful for students’ modification, which required no technical expertise for teachers’ adoption. The students’ mathematics teacher was observing the students’ use of the tools and found them fairly simple to use and adopt in the classroom. However, she explained that designing pedagogical activities with the tools may demand time that she does not usually have; this issue is discussed further in section 7.1. Still, she expressed appreciation for the use of such tools in the mathematics classroom; she said, “it makes me happy seeing students having fun and working together in my classroom; I would rather do this all day long instead of [instructional] teaching.”

6.4.4 Assessment & curriculum

One of the surprising results of the workshops, or what Kemmis (2006) calls “unwelcomed truth,” was the students’ response to the question about their preference of learning mathematics using similar tools and approaches; 50% of students disagreed (31% of which strongly disagreed). This is a clear conflict with the students’ response to whether they enjoyed using the tools, resulting in a 78% agreement (37.5% of which strongly agreed). In the interview with Laura, she conveyed her strong wishes of using similar tools for learning mathematics “at least once or twice a year,” when she was asked, “why not the whole year?” she answered, “because we still need to pass.” Like many other educational institutions, the IB Diploma program assesses students primarily on their performance in standardized tests. This assessment model evaluates all students’ performance with test scores, not accounting for their differences and diminishing their creativity (Gao, 2014). Rather than designing technologies to support this system, we need to question it ethically and challenge its validity. Some effects of this assessment model on students were noticed during the presentations. Students from the second group were highly discouraged when it was their turn to present; they
were anxiously chatting and asked to present later. When asked for reasons as they seemed to be prepared, they expressed dissatisfaction with their experiments' outcomes compared to the first group's results. One group member explained, "our shape didn’t act in the same way. There must be something wrong", noting that the two groups performed totally different movements. The group was assured by their teacher and me that there were no expected outcomes, and especially no "right" or "wrong" results. Still, in their presentation, the group was doubting their experiments which were, in fact, quite outstanding.

The curriculum design is one of the concerns that can be overlooked. The absence of an interdisciplinary approach only adds to the difficulties of using constructive tools for mathematics education. As discussed in the theoretical part and tested in the design process, the connection between mathematics and the real world cannot be achieved separately from other fields. Nevertheless, using technology alone to link math and other disciplines is not ideal, as it unnecessarily adds extra responsibilities to mathematics teachers. Again, this issue extends beyond the adoption of one pedagogical model over another. As Freire (1968, p. 109-115) explains, the curriculum needs to be dynamic, adapting to students’ view of the world, investigated and iterated on by an interdisciplinary team of teachers.

7 Discussion

7.1 Issues beyond technology

The challenges facing mathematical education are way beyond technological solutions alone. The educational system as a whole needs to go through levels of reformation guided by participatory action research to move its approach from schooling to education (Kemmis, 2006). As long as the approach of mathematical education is following a schooling strategy, there remains a great difficulty in implementing constructivist learning and teaching.

The schooling approach does not provide teachers with enough flexibility over the curriculum and assessment they teach. Kemmis (2006) argues, “teachers are in the front line; they see wounds inflicted on their students; despite this knowledge, they feel powerless to stop the governmental erosion of the good for their students and the good for humankind” (p. 463). This was evident from the interviews with mathematics teachers and reflections from the workshops. Thus, implementing constructivist technology in mathematics education will remain difficult if the curriculum, assessment, and educational system do not support constructivist learning. This impacted the technology designed for mathematical education to be focused on feeding the schooling system, leaving most tools used for learning mathematics
focused on the efficiency in using memory and performing calculations (Nicaise & Barnes, 1996).

Furthermore, there is a pressure imposed on mathematics education, which has been elevated to a position of superiority over subjects that are “less valuable.” This hierarchy is formed out of the needs of the work industries, such as requirements of reading and calculating abilities (Gao, 2014) or the more advanced engineering and computing skills, and has, in turn, pushed creative subjects down the list, which affected a potential creative use of mathematics as well. This approach even defeats the purpose of preparing children for the workplace, as it is clear that the base of innovation demands creative solutions more than anything else (Gao, 2014). The hierarchy of subjects generated what Chronaki describes as “hierarchical distribution of knowledge/power” (2019, p. 320), defining a “proper mathematics” that only certain types of students can acquire. This notion resulted in “mathophobia” strictly labeling students as mathematically capable or incapable, which as a consequence “contaminates peoples' images of themselves as learners” (Papert, 1980, p. 8). An observed outcome of this phenomena is a resistance to learning the subject using alternative ways primarily, as they contradict the idea of mathematics being a serious subject for the selected few.

Opportunities of transforming this state are available to be explored by designers, guided by the ethical framework described by Freire (1968), using participatory action research as a methodology and technology as a tool.

### 7.2 Technology as a facilitator of change

Freire’s approach towards the schooling system is to “change it completely and radically and give it birth from a body that doesn’t correspond anymore” (Freire & Papert, 1995). This is what Kemmis (2006) refers to as becoming critical. The role of technology in facilitating such a shift from schooling into education still holds significance. Though, designing technology for education remains a critical matter, as it may benefit the unethical operation of schooling instead of the education offered (Kemmis, 2006), if not steered by clear ethical and methodological guidelines. Thus, the responsibility of interaction designers when designing for educational context is to ensure correct deployment of technology as tools for improving the quality of education in schools and not the schooling process.

Designing and implementing constructivist tools for mathematics education proved to be a difficult task for not only designers but for teachers like Eric and Jenny. Hence, improving the quality of mathematics education needs to be a collaborative process, where designers work with teachers on providing constructivist tools that empower them in driving the change from within.
7.3 Design Framework

Emerging from the research and design process of this thesis is a design framework for mathematics constructivist learning tools. The framework is produced with the intention of benefiting future designers who share a similar interest in designing technological tools for the betterment of mathematics education. The most valuable guiding principles are presented as follows:

**Tools** must be newbie-friendly, allowing novice students and teachers to manipulate them and use them intuitively and simply. Tools can either act as construction kits, phenomenaria, or a combination of both. In addition, the tools should possess the following qualities:

**For students:**

- *Adaptive:* students favor approaching the same problem/topic in diverse ways; hence the tools should enable adaptive use and can be adopted in multiple ways based on students’ preferences, interests, and needs.
- *Playful:* can be used playfully, allowing and welcoming mistakes and creative uses.
- *Collaborative:* allow for multiple uses by more than one student, giving all users the equal opportunity of engagement and learning.
- *Meaningful:* can be used for purposeful activities, allowing students to experiment on and explore their interests, communicating their relevance in real life through interdisciplinary integrations.

**For teachers:**

- *Manipulatable:* easily manipulated and adjusted by teachers, allowing them to build learning tasks or phenomenaria in a simple and time-efficient way.
- *Supportive:* encourage reflections and support teachers’ scaffolding process.
- *Multitopic:* allow for use across mathematical topics and not limited to one.
- *Shareable:* easily shareable and configurable by teachers.

7.4 Future work

This thesis opens up the opportunity of designing for constructivist mathematical education, while leaving areas for further research and explorations.

One of the essential areas to expand on from this research is the usability and adoption of constructivist technologies by mathematics teachers. As the scope and challenges of the thesis did not allow for further exploration of
teachers’ use of the technology, this remains a significant topic to uncover. Future design work may use the outcomes of this thesis to design and test constructivist tools with and for mathematics teachers.

Testing of the learning outcomes of using constructivist technological tools in mathematics is another aspect to be explored by future projects. Longer studies on the benefits of understanding mathematical topics using such tools are needed to examine their long-term benefits, and to present more concrete suggestions of the adoption of such tools in mathematics education.

8 Conclusion

This thesis started as an exploration for designing technological tools that enhance students’ mathematical understanding.

To carry this exploration, the challenges in mathematics education were identified, the values of the constructivist epistemology were presented, and the opportunities that arise from linking the two through technology were explored. Thus, the findings from the discovery phase directed the exploration towards designing constructivist learning tools as facilitators of mathematics’ understanding through the use and adaptation of technology.

Two types of constructivist learning tools are recognized: construction kits and phenomenaria. The two categories can be used separately to enhance understanding through action (construction kits) or perception (phenomenaria) and can be combined together. The tools produced and tested in this project were tangible kits and tangible user interface, which were a combination of both categories, leaning more towards phenomenaria. The tools were successful in facilitating constructive learning through interdisciplinary activities between motion in physics, and functions and geometry in mathematics. Additionally, the tools engaged students with mathematical data, functions, and geometry in both playful and meaningful manners. This was achieved through connecting mathematics concepts to the real world through students’ use of their bodies and the tangible kits to perform movements. Students could clearly relate the graphs and shapes visualizations based on their physical actions, which helped view abstract mathematics concepts in real applications.

One of the main findings is that the tools acted as an excellent catalyst for collaboration between students. A group of 30 students were fully engaged and included in the process of executing experiments, observing outcomes, and analyzing the results. In addition, the tools facilitated communication and group discussions, where students carried experiments, discussed findings, analyzed outcomes, and proposed new approaches collaboratively.
Further, the playful quality of the tools and activity encouraged students to participate, not feeling less mathematically capable to take part.

This thesis identifies ways to integrate technology into mathematics education for the benefit of students and teachers, using Freire’s (1968) problem-posing education in contrast to the oppressive banking education. Designing educational tools and technologies holds a responsibility to benefit education rather than schooling systems through following the ethical and pedagogical guidelines discussed in this thesis. This requires designers to be critical; by presenting unwelcomed truth and problems and taking an active role in social, political, cultural, and economic issues. Additionally, the designers’ role is not as experts but as facilitators of change through the target audience’s involvement, by designing for and with them. Thus, the work of this thesis criticizes aspects in the current schooling system such as assessment methods, curriculum design and identifies them as obstacles for students, teachers, and education.
9 References


Freire, P., Papert, S. (1995). Diálogo entre Paulo Freire e Seymour Papert. [Video]. https://www.youtube.com/watch?v=41bUEyS0sFg


Appendices

1. The code
https://github.com/LiamAljundi/moving-mathematics

2. A live version of the TUI
https://liamaljundi.github.io/moving-mathematics/

3. Videos of the prototypes and workshops.
https://drive.google.com/drive/folders/190gIKv914Rvsy269-AIkxxPFzILUogYo?usp=sharing

4. The complete tangible kits, which were divided into 6 kits.
5. Consent form

Consent Form

Researcher: Liam Aljundi

Malmö University
Interaction Design BSc programme (IDK18)
Thesis project

I am willing to participate in the testing of Moving Mathematics thesis project, and also participate in a digital survey post-testing. I am aware that the researcher from Malmö University is aiming to test tangible and digital educational tools that aim to enhance the understanding of mathematical topics through interactive and manipulative applications, as well as providing teachers with a framework for developing their own tools and exercises.

I understand that I will take part in a total of 4 workshops, which will be documented by the researcher, through images and text notes. I understand that I will be asked about my emotional and practical experience, understanding of the content and format of the workshop and the tangible and digital tools that I interacted with. I understand also that the researcher may ask to contact me in the future for short interviews or permissions, and that I am not obliged to participate or approve of such requests.

If I do not like any question or activity, I do not feel pressured to partake.

I understand that all my data will be treated anonymously and will only be used in this research project. It will not be used in any other contexts without my permission.

Name:

Date:

Signature: