

ALFRED ROSENBLATT (1880–1947). POLISH–PERUVIAN MATHEMATICIAN

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Abstract. Alfred Rosenblatt (1880–1947) was a Polish mathematician born into a Jewish family in Krakow (Kraków, Poland). He studied in Vienna, Krakow, Göttingen, and worked at the Jagiellonian University in Krakow (1910–1936) and at the University of San Marcos in Lima, Peru (1936–1947). During the Second World War, Rosenblatt accepted Peruvian citizenship. His work was important for the development of mathematics in Peru, including the foundation of the National Academy of Exact Sciences, Physics and Natural Sciences in Lima. He is mentioned among the four mathematicians of the twentieth century most important for Peru (F. Villarreal, G. García Díaz, A. Rosenblatt and J. Tola Pasquel). He spent the first half of 1947 on a scholarship at the Institute for Advanced Study in Princeton and had several lectures at other universities in the USA.

Rosenblatt published almost three hundred scientific papers in various fields of pure and applied mathematics, including ordinary and partial differential equations, algebraic geometry, theory of analytic functions, probability, mathematical physics, three-body problem, hydrodynamics

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and other applications of mathematics. About 180 papers were published in the years of his work in Poland and about 120 in the years he worked in Peru. His publications are in Polish, German, French, Italian, Spanish and English. Rosenblatt participated actively in four International Congresses of Mathematicians: Cambridge (1912), Strasbourg (1920), Bologna (1928), Zurich (1932). He presented three talks in Bologna and one in Zurich.

We describe Alfred Rosenblatt's life and important parts of his work in detail. We have made an effort to see all his papers, so as not to miss any of his achievements in mathematics and applications, including papers and information written in Spanish; e.g., [Ro11], [Ro13]–[Ro16] and [Ro20]. We have already written three articles, two in Polish [Ro8], [Ro9] and one in Russian [Ro12], to introduce him to Polish and Russian mathematicians. Now we want to do the same for a wider range of scientists with this article in English. Some information on Rosenblatt can also be found in [Ro1]–[Ro6], [Ro10] and [Ro17]–[Ro19].

1. Rosenblatt's biography. We have divided Alfred Rosenblatt's life and scientific activity into a period of work in Poland and Peru. Then, the third section discusses Rosenblatt's activity in mathematical life, collects information about his participation in congresses and conferences, and examples of his recognition.

1.1. Polish period 1880–1936. Alfred Rosenblatt was born on 22 June 1880 in Krakow (Kraków in Polish), the son of Józef Michał Rosenblatt, professor of law at the Jagiellonian University, and Klara Koppelman. He had four sisters — Eugenia, Paulina, Karola and Helena. The family was Jewish.

He graduated from the St. Anna High School (now Nowodworski Liceum) in Krakow, taking his final examination on 7 June 1898. In the period 1899–1902, he studied at the Faculty of Mechanical Engineering of the Vienna University of Technology, and finished the course without obtaining a diploma in engineering. Probably the profession of engineer did not attract him. Then, in the period 1902/1903–1906/1907, he studied mathematics and physics at the Faculty of Philosophy of the Jagiellonian University (Uniwersytet Jagielloński in Polish=UJ), including work in physical and chemical laboratories. In the academic year 1904–1905, he took a break because of mental illness.

On the basis of the dissertation *O funkcjach całkowitych przestępnych* [On transcendental entire functions] defended on 28 February 1908, he became a doctor of philosophy of the Jagiellonian University; his formal supervisor was Stanisław Zaremba (1863–1942). In the years 1908–1910, Rosenblatt studied at the University of Göttingen, where among his teachers were Felix Klein (1849–1925), David Hilbert (1862–1943) and Edmund Landau (1877–1938). Rosenblatt attended Klein's mathematical seminar on applications of mathematics and his lecture course on the theory of invariants (cf. [60]).

On 11 June 1913, Rosenblatt obtained habilitation (*veniam legendi*) at the Jagiellonian University on the basis of the thesis *Badania nad pewnymi klasami powierzchni algebraicznych nieregularnych i nad biracjonalnymi przekształceniami nie zmieniającymi tych powierzchni* [The study of certain classes of irregular algebraic surfaces and birational transformations not changing these surfaces], which, in 1912, was published in Polish [R9] in one hundred pages in *Rozprawy Akademii Umiejętności* [Dissertations of the Academy of Learning]; its shortened French version was published, in the same year, in the *Bull. Internat. Acad. Cracovie Cl. Sci. Math. Natur. Ser. A Sci. Math.* [Bulletin

of the Academy of Learning] (see [R9]). The title of his habilitation lecture, delivered on 3 March 1913, was *O całkach periodycznych problemu trzech ciał* [On periodic integrals of the three-body problem].



Fig. 1. Rosenblatt in 1927 (part of Figure 3) and his signature

Seven years after, on 26 June 1919, he became a privatdocent of the UJ. This position enabled him to conduct lectures; unfortunately, it was not associated with a permanent academic posts and salary.

Poland's independence in November 1918 made possible to establish national mathematical societies, and on 2 April 1919, the Mathematical Society constituted itself in Krakow. Alfred Rosenblatt was among the sixteen founding members of the Society, and at its second meeting on 30 April 1919, he gave a lecture *Z rachunku warjacyjnego* [On the calculus of variations].

On 9 September 1920, Rosenblatt was appointed titular extraordinary professor of mathematics. This nomination provided only the right to use the title of a professor, but was not associated with a chair or salary. He still ran lectures on commission. Of course, he entertained hopes of receiving a chair. In 1923, Hugo Steinhaus (1887–1972) proposed him as the second candidate for an open chair of mathematics at the University of Poznań (he suggested Kazimierz Kuratowski as the first candidate). In the opinion from 31 January 1924, Steinhaus wrote:

Prof. Rosenblatt's papers concern various areas of mathematics and are scattered across various national and international journals. These papers demonstrate a great mathematical education and understanding of difficult sections e.g. the theory of algebraic surfaces or the calculus of variations.

The job was finally offered to Antoni Łomnicki (1881–1941), who in May 1924 refused to take the post because of the lack of an adequate apartment. In the same

year, for a chair at the Department of Mathematics of the Stefan Batory University in Vilnius, Steinhaus proposed Rosenblatt as the first candidate, followed by Kazimierz Kuratowski (1896–1980), Aleksander Rajchman (1890–1940), Bronisław Knaster (1893–1990) and Stanisław Saks (1897–1942). The chair, however, was given to Stefan Kempisty (1892–1940).

On 29 June 1924, Rosenblatt married Paula Unger (born 8 July 1885 in Biała, now Bielsko-Biała and died in March 1959 in Lima), and the wedding took place in Wieliczka. The marriage was childless.



Fig. 2. Alfred Rosenblatt and his wife Paula. Archives of the Jagiellonian University, sign. S II 619

When in 1926 a chair in mathematics at the General Faculty of the Technical University of Lvov was being filled (ultimately by Kazimierz Kuratowski), Samuel Dickstein (1851–1939) pointed to “Mr. Dr. Alfred Rosenblatt” as a candidate in a letter dated 19 April 1926.

In 1928, a chair in mathematics was reopened at the University of Poznań, and again Steinhaus proposed Rosenblatt as the second candidate (as the first, he suggested Stanisław Saks) but the position was actually given to Mieczysław Biernacki (1891–1959). Opinion of Stanisław Zaremba (1863–1942) of 28 January 1929 was decisive.

Rosenblatt desperately tried to find for himself a chair of mathematics, and on 23 May 1930, he informed Dickstein (cf. [Ro4]):

Simultaneously with a letter from you I received a letter of professor J. Rey Pastor¹ from Buenos-Aires in which he reports on my appointment in La Plata (Argentina, near Buenos Aires, not Lima in Peru!). We will soon receive an official notification of the terms, etc. I am getting the chair of mathematics

¹Julio Rey Pastor (1881–1962), Spanish mathematician and historian of science, working at the Complutense University in Madrid and the University of Buenos Aires from 1921.

*there with the help of Enriques, Severi, Levi-Civita², and Einstein³, to whom, in the last instance, the dean from La Plata turned for an opinion about me. (...) I enjoy the recognition that I have abroad, an evidence of which, moreover, I just had in Liege and Paris. Einstein also told me: **es wundert mich dass Sie die Polen ziehen lassen, sie haben ja so wenig Leute** [I am surprised that Poland allows so many to go, they have so few people].*

A month later, that is, on 22 June 1930, Rosenblatt reported (cf. [Ro4]):

Honorable Professor! I report to you that I received from prof. Rey Pastor in Buenos Aires a letter asking for a quick arrival in Argentina because they are waiting for me. (...) I received a cordial letter from prof. Levi-Civita, in which he writes “thanks to your scholarly merits you obtained quite a senior position, despite the indifference and even hostility of your colleagues. I regret these relations, etc.” (...) I am looking forward to new fields of activity.

In the academic year 1930/1931, Rosenblatt was supposed to lecture in mathematics at the University of La Plata in Argentina. He obtained a paid leave from the Jagiellonian University and a business passport for this trip. He also got an offer to take a chair in Argentina, which he accepted, but because of the military coup in Argentina in September 1930 he could not go there. Thus he remained in Krakow as lecturer (*adiunkt* in Polish).

On 20 March 1930, members of the Polish Academy of Learning (*Polska Akademia Umiejętności*=PAU, now called Polish Academy of Arts and Sciences) in Krakow, Stanisław Zaremba, mathematician, Władysław Natanson (1864–1947) and Czesław Białobrzeski (1878–1953) both physicists, and Karol Dziewoński (1876–1943), chemist, proposed the candidacy of Alfred Rosenblatt as a corresponding member of the Third Department of the PAU. To the application was attached a list of papers, written personally by Rosenblatt, which included 30 papers in mathematical analysis, 32 in geometry, and 27 in theoretical mechanics. Candidacy was withdrawn as a result of the protest of Waclaw Sierpiński (cf. [Ro3]).

Lectures and seminars taught by Rosenblatt at the UJ⁴, were, among others, the following: the theory of planar algebraic curves, on the concept of curve and surface, theory of ordinary differential equations, theory of quadratic surfaces, analytic geometry,

²Federigo Enriques (1871–1946), member of the Italian school of algebraic geometry, professor at the University of Rome La Sapienza; Francesco Severi (1879–1961), leader of the Italian school of algebraic geometry, professor at the University of Rome, together with F. Enriques, he won the *Bordin prize* from the French Academy of Sciences; Tullio Levi-Civita (1873–1941), Italian mathematician known for his work in differential calculus and relativity theory, professor at the University of Padua (1902–1918) and he taught at the University of Rome (1918–1938).

³Albert Einstein (1879–1955), German-born theoretical physicist, Nobel Prize winner in 1921, professor at the German Charles-Ferdinand University in Prague (1911–1912), at ETH Zurich (1912–1914), member of Kaiser Wilhelm Institute for Physics with the right to teach at the University of Berlin as Academy member (1914–1933), and professor at the Institute for Advanced Study (1933–1955).

⁴The information is taken from the *List of lectures held at the Jagiellonian University* in the years 1910–1935, Rosenblatt’s biography from 15 May 1920, and reports of activity of the Dr. W. Kretkowski Fund (cf. [Ro7]).



Fig. 3. First Polish Mathematical Congress, 7–10 September 1927, Lvov. Second from the right: Alfred Rosenblatt. Also the first row, from the left: Maksymilian Tytus Huber, Leon Lichtenstein, Samuel Dickstein, Kazimierz Bartel, Waław Sierpiński, Wiktor Staniewicz, Jan Łukasiewicz, Kazimierz Kuratowski and Stefan Banach. Photo held by the authors

higher algebra and number theory, theory of analytic functions, the theory of probability with applications, Hilbert spaces, mathematical foundations of modern quantum theory and the theory of algebraic curves.

In the years 1926–1936, he traveled to give lectures in Rome (1926 — a talk on algebraic geometry), Bologna (1928 — ICM), Stockholm (1930 — Congress of Applied Mechanics), Liege (1930), Paris (1931 — three talks in hydrodynamics at the Sorbonne), Zurich (1932 — ICM), Sofia (1932 — two talks), Athens and Belgrade.

In 1933, Rosenblatt visited the *Institut de Mécanique des Fluides de l'Université de Paris* (Institute of Fluid Mechanics at the Paris University), whose director was Henri Villat (1879–1972), a specialist in the field of hydrodynamics. The results of Rosenblatt's research had been recognized by mathematicians, and Villat suggested to him the publication of a monograph [R43], and, two years later, of another monograph [R45] on solutions of equations in hydrodynamics. It is a pity that a research center in the field of hydrodynamics was not created in Krakow, since Rosenblatt's work and his recognition abroad offered a good reason for a chair, which could be opened at the UJ.

A further opportunity for a chair of mathematics turned up in 1936, at the Faculty of Chemistry of Warsaw University of Technology. The position was released by Franciszek Leja (1885–1979), who took over a chair of mathematics at the Jagiellonian University. Steinhaus proposed Rosenblatt as the second candidate (the first one was Bronisław Knaster), but the chair was actually offered to and accepted by Władysław Nikliborc (1899–1948).

1.2. Peruvian period 1936–1947. Rosenblatt ultimately received a chair at the University of San Marcos in Lima (Universidad Mayor de San Marcos, now Universidad Nacional Mayor de San Marcos=UNMSM; in English: The National University of San Marcos), where, thanks to his efforts, a whole generation of mathematicians appeared, of whom the most prominent was his doctoral student José Tola Pasquel, founder of the modern Peruvian school of mathematics. Rosenblatt's contribution to mathematics, in particular the mathematics in Peru, was described in the Spanish language in some papers that are inaccessible in Europe. Rosenblatt is listed among the four mathematicians of the twentieth-century most important for Peru: Federico Villarreal (1850–1923), Godofredo García Díaz (1888–1970), Alfred Rosenblatt (1880–1947) and José Tola Pasquel (1914–1999) (see [Ro14], [Ro16, pp. 57–61] and [Ro6, p. 3]). Unfortunately, the young generation of mathematicians in Peru, like those in Poland, do not know who Alfred Rosenblatt was.

So how did Rosenblatt find himself in Peru in 1936? Dr. Godofredo García⁵, who at the time was Dean of the Faculty of Exact Sciences at the University of San Marcos in Lima, had been to Poland during one of his trips to Europe. Perhaps he was at the Second Congress of the Polish Mathematical Society in Vilnius (in September 1931) with a talk in theoretical mechanics? It was then that he had established contacts with professors Sierpiński and Rosenblatt (if not scientific friendships). Certainly, García and Rosenblatt had met in person at the International Congress of Mathematicians in Zurich in August 1932. In mid-1930s Rosenblatt had written to him about risks many Poles of Jewish origin, including him and his family, face due to persecution. García, taking advantage of his scientific and administrative position in Peru, had invited Rosenblatt as lecturer to the USM in Lima. Rosenblatt accepted the invitation, and in the academic year 1936/1937 embarked upon year-long courses at the USM.

Rosenblatt, together with his wife, arrived in Lima on 10 August 1936; from the first September 1936, he became a professor at the university, and this is how he found himself in Peru. Note that the University of San Marcos in Lima is the oldest university in the Americas and one of the oldest in the world, since it was founded as early as 1553. Of course, it is the most important institution of higher education in Peru. Rosenblatt's arrival was the beginning of a new era in the history of mathematics in Peru.

We note two important facts related to his arrival and stay in Peru. Rosenblatt's contract in Peru included taking over the Chair of Astronomy and Geodesy after captain José R. Gálvez⁶, who died on 18 March 1936.

⁵Godofredo García Díaz (1888–1970), Peruvian engineer and mathematician; 1906–1909 studied at the Faculty of Sciences of the University of San Marcos (USM) and at the School of Engineers of Peru, now called the National Engineering University (1908–1910), 1911 graduated in civil engineering, 1912 PhD at USM, from 1919 worked at USM (1928–1940 dean and 1941–1943 rector). In 1938, together with Rosenblatt and other mathematicians from USM, founded the National Academy of Exact, Physical and Natural Sciences of Peru, of which he was president from 1960 until his death in 1970. He also directed the journal *Actas de la Academia*. He was interested in applications of mathematics. From 1946 foreign member of PAU.

⁶José Roberto Gálvez (1880–1936), Peruvian astronomer, professor at the USM.

A confirmation of this fact is found in Rosenblatt's letters to Oswald Veblen⁷, in which the institution header reads:

Universidad Mayor de San Marcos. Facultad de Ciencias Biologicas, Fisicas y Matemáticas. Gabinete de Astronomía y Geodesia.

The chronicle of the University of San Marcos from March 1937 states: *Dr. Alfred Rosenblatt, Catedrático de Astronomía y Geodesia. The Chair of Mathematical Physics and Probability was occupied by Cristóbal de Losada y Puga (1894–1961).* On 17 September 1936, Rosenblatt wrote to Oswald Veblen (cf. [Ro5]):

I am here in Lima invited by the University for 2 years to lecture on mathematical astronomy. I am lecturing about Celestial Mechanics. It would be for this reason very agreeable to me, if I could study your last important paper on the tensorial calculus. You have been so kind to send me to Cracow your important papers published previously.

The dean of our Faculty, Professor Godofredo García would be very obliged to you, if you would be so kind to send to our Faculty your important papers on geometry. He will send you the Journal of our Faculty the "Revista de Ciencias".

The other piece of information is the fact that Rosenblatt, who had already made significant scholarly achievements prior to 1936, due to formal requirements, had to present the paper *Sobre la representación conforme de dominios planos limitados variables* [On the conformal representation of plane domains with variable boundaries] as a doctoral thesis at the University of San Marcos. Surprisingly, the title of Doctor of Philosophy from the UJ in Krakow would not be recognized at the University of San Marcos in Lima as a doctorate in mathematics. His new PhD thesis was published in December of 1936 in [R50]. The promotion ceremony was held on the 21 October 1936, and was attended by the Rector Dr. Alfredo Solís y Muro (1872–1969) — the future Prime Minister of Peru (1939–1944). The profile and scholarly achievements of Rosenblatt were presented by the Dean of the Faculty of Natural Sciences, Dr. Godofredo García. A report related to this promotion can be found in [Ro11], which also carries a photo from the ceremony. Let us mention that at the time in Peru the "Bachiller" was the first scientific degree, the next being the doctorate. It was rare to be granted a doctorate.

Already a chaired professor (*catedrático principal*) on the 23 of December of the same year, Rosenblatt participated in the session of the Faculty of Natural Sciences at which Professor Émile Picard (1856–1941) was an elected honorary professor of the university. In December of the following year, he participated in the ceremony of awarding an honorary doctorate (*doctor honoris causa*) to Paul Montel (1876–1975).

⁷Oswald Veblen (1880–1960) was an American geometer and topologist of Norwegian origin. In 1932 he organized the Institute for Advanced Study in Princeton and chose for the first members of the Institute James Waddell Alexander (1888–1971), Albert Einstein (1879–1955), Marston Morse (1892–1977), John von Neumann (1903–1957) and Hermann Weyl (1885–1955). He was president of the AMS (1923–1924) and member of many academies, including the Polish Academy of Learning in 1946 and the Academy of Sciences of Peru.

In 1938, Godofredo García along with Alfred Rosenblatt and several other mathematicians from the University of San Marcos supported the creation of the *Academia Nacional de Ciencias Físicas y Naturales de Lima* [National Academy of Exact, Physical and Natural Sciences in Lima], now called the *Academia Nacional de Ciencias del Perú* [National Academy of Sciences], which was established on 6 August 1938. Godofredo García was elected its first president and remained in this position until his death in 1970. The journal *Actas de la Academia Nacional de Ciencias, Físicas y Naturales de Lima* began to be published on 23 October 1939. The Government of Peru issued a decree on the official nature of the National Academy. The establishment of the Academy was the most significant event for scientific development in Peru in the 20th century.

Many papers of Rosenblatt and García were published in the journal of the Academy as well as in the *Revista de Ciencias de la Facultad de Ciencias Biológicas, Físicas Matemáticas de la Universidad Mayor de San Marcos* (founded in 1897), which significantly raised the rank of these journals. Thanks to Rosenblatt's and García's contacts, some famous mathematicians published their papers in the Revista, including R. P. Agnew, H. Bateman, S. Bergman, K. Borsuk, G. D. Birkhoff and his son Garrett Birkhoff, L. M. Blumenthal, S. Bochner, N. Dunford, M. Eidelheit, L. Godeaux, S. Gołąb, D. H. Hyers, V. Jarník, M. Kac, J. Karamata, Đ. Kurepa, J. P. La Salle, T. Levi-Civita, S. Lubelski, E. J. McShane, K. Menger, A. D. Michal, R. von Mises, D. S. Mitri-nović, W. Morgan, O. Ore, J. Rey Pastor, L. A. Santaló, W. Sierpiński, W. Ślebodziński, M. H. Stone, A. E. Taylor, T. Y. Thomas, E. C. Titchmarsh, K. F. Vetulani, H. Weyl, A. Wintner and S. K. Zaremba.

Many mathematicians came to the University of San Marcos in Lima to give lectures; e.g., Tullio Levi-Civita (1873–1941) arrived on the 4 August 1937, and presented two talks, *A new elementary approach to relativity theory* and *Trigonometry of a small curvilinear triangle*, and had a course of nine lectures on *The relativistic two-body problem, its solution in the first approximation and possible astronomical control* (see [Ro20, p. 115]). A speech preceding the lectures on Levi-Civita's scholarly achievements was given by G. García, and later it was translated into Polish by Rosenblatt and published in *Wiado-mości Matematyczne* [Mathematical News]. George David Birkhoff (1884–1944) was in Lima in April–May 1942 with lectures on the problem of the n -body, the concept of time and gravity, and contemporary logic and mathematics (the summaries were published in Spanish in the 44th issue of the *Revista de Ciencias*, Lima, 1944).

In 1939, the Academy of Sciences of Mexico elected Rosenblatt and Peruvian pathologist and dermatologist Pedro Weiss (1893–1985) as corresponding members in appreciation of their scholarly achievements.

After 10 September 1939, the grave news from Europe of the outbreak of World War II reached Lima. It was very hard for Rosenblatt. He decided to accepted Peruvian citizenship.

In a letter to Veblen from the 4 August 1940, Rosenblatt wrote on behalf of Prof. García and himself about Polish mathematicians (cf. [Ro5]):

You know that we have many first-class mathematicians in Poland who are the pride of this country (Banach, Saks, Schauder, Steinhaus, Mazur etc.)

and we hope it will be perhaps possible to contract one of them. On the other side, what we need are zoologists, embryologists, biologists etc. who will find here an eldorado for their scientific work and benefit this country.

Ortiz thus presented the importance of Rosenblatt's arrival to Peru (see [Ro15, p. 22]):

The arrival of Rosenblatt was crucial for the development of mathematics in Peru. From that moment on a new stage in the history of mathematics in our country began. He introduced new teaching methods of modern mathematics and as a professor at the University of San Marcos (the only university in the country then having Pure Mathematics as a subject of study) new courses, where complex variables, complex analysis, topology, differential equations, algebra, modern differential geometry and many other subjects were taught. He also conducted seminars and shaped a new generation of mathematicians, which, although sparse, was enough to open new horizons for further conquests and goals.

In subsequent years of World War II, Rosenblatt actively participated in the scientific life by publishing about 60 papers in the years 1940–1945. After 24 May 1940, on which day Lima was hit by a strong earthquake leaving most of the city in rubble, he took up the subject of seismic waves and published six works on it.

In 1946, Rosenblatt published two papers in English in *Revista de Ciencias*, but he had said that *one changes a language, but not loyalty to friends*.

The December 1946 issue of *Revista de Ciencias* was awaited with anticipation due to the work of Stanisław Krystyn Zaremba (1903–1990) with the poignant ending: *The Polish Army, Middle-East* [The Polish Army in the Middle-East was an important part of the Polish effort during World War II]. When Rosenblatt received a copy, he burst into tears (see [Ro20, p. 133]).

At the University of San Marcos, among Rosenblatt's students were Godofredo García Díaz, José Tola Pasquel, Flavio Vega Villanueva (1915–2011), José Ampuero (1922–1998) and Tomás Nuñez Bazalar. Actually, García only collaborated with Rosenblatt; he was a pupil of Federico Villarreal and defended a doctorate under his direction in 1912. He considered himself Rosenblatt's disciple, however, because he wrote with him six articles and one book.

The most prominent pupil of Rosenblatt was José Tola Pasquel, who defended his doctorate entitled *Sobre la equivalencia de las dos formas de continuidad de operaciones con sucesiones y por vecindad en espacios topológicos* [On the equivalence of the two forms of continuity of operations by sequences and by neighborhoods in topological spaces]⁸ on 13 November 1941; Rosenblatt was the supervisor. In 1945 Tola was appointed professor and director of the School of the Institute of Physical and Mathematical Sciences (EIC-FYM), Faculty of Science at UNMSM (1945–1961). He was also Dean and Rector of the

⁸In some other places the title of José Tola Pasquel PhD thesis is a little different *La equivalencia de las formas de continuidad de operaciones por sucesiones y por entornos en espacios topológicos* [The equivalence of sequential and neighborhood system forms of continuity of operations in topological spaces].

Pontifical Catholic University in Lima (PUCP) in the period 1977–1989. He published 21 papers and 18 books. Tola is called the founder of modern mathematics in Peru.

A. Ortiz states firmly (see [Ro16, p. 62]):

Mathematics in Peru in the years 1870–1950 means chronologically three mathematicians: Federico Villarreal (1850–1923), Alfred Rosenblatt (1880–1947) and Godofredo García (1888–1970).

Rosenblatt was a member of the Polish Mathematical Society (since 1919), the Polish Physical Society, the American Mathematical Society (since 1914), the Edinburgh Mathematical Society (since 1914), the German Mathematical Society, the French Mathematical Society, the Royal Czech Scientific Society, the Peruvian Scientific Society, the Peruvian Geographic Society, the Mexican Academy of Sciences (corresponding member since 1939), the Royal Scientific Society in Liège (corresponding member), the Hellenic Mathematical Society (honorary member), the president of the section of Exact Sciences of the National Academy of Sciences, Physical and Natural Sciences in Lima (1941–1944), and member of the Editorial Board of the Brazilian journal *Summa Brasiliensis Mathematicae* published in Rio de Janeiro since 1945.

We believe Rosenblatt was awarded the French Palm of Public Enlightenment and the medal of the University of Liège for his paper [R41] from 1930, published in 1931. We have not found, however, any documents confirming the receipt of these awards.

In the academic year 1946/1947 Rosenblatt had a series of eight lectures at the Institute of International Education in New York. Unfortunately, we know neither the title nor the content of the lectures.

For over half a year (from 1st January to 30th June 1947), he was on a scholarship at the *Institute for Advanced Study* in Princeton. This was possible, because Albert Einstein (1933–1955), who had previously recommended Rosenblatt for a contract in Argentina in 1930, worked there. We should also have in mind that Einstein had received an honorary doctorate from the University of San Marcos in Lima. Rosenblatt’s stay in the United States was described in [Ro20, p. 133].

On 26 February 1947, Rosenblatt had his first talk at Princeton University, and was introduced by Solomon Lefschetz. In a weekly Bulletin of Princeton University (Vol. 26, No. 23 from February 22, 1947) we read that on Wednesday February 26, 1947, at 16.45, at a meeting of the Club of Mathematics, Professor Alfred Rosenblatt would give a talk *The equation of the hyperbolic horn with elliptic sections*. The second talk on *The existence and uniqueness of solutions of ordinary differential equations* took place on the following day at the University of Pennsylvania. Another talk, on March 6, 1947 at Harvard University was entitled *Some inequalities for Green’s function in the plane*, and on March 10 at the University of Chicago he gave a talk *Simple integrity problems with variable limits in exceptional cases*. Finally, the fifth talk *Gravitational waves in two dimensions* took place on March 11, 1947, at the University of Illinois at Urbana-Champaign. Rosenblatt was praised by eminent professors who were among the listeners. He was invited to lecture at Columbia University, Brown University, and Toronto University, but for health reasons he was not able to accept the invitations. He returned to Peru on 15 April 1947.

Rosenblatt twice came down with bronchial pneumonia. The first time he recovered, but the second time he did not. He died on 8 July 1947, at the Clinic of Delgado de Miraflores in Lima (see [Ro13, p. 47]), and was buried in Lima's Jewish cemetery. After his death five obituaries appeared.

On 20 July 1947, Paula Rosenblatt wrote to Oswald Veblen (cf. [Ro5]):

Dear Professor Veblen.

I feel the sad duty to inform You about the death of my husband Alfred Rosenblatt, which occurred on July 8. Since his return from the United States he could not recover from pneumonia and inspite of the efforts of the best physicians he suffered a relapse, which caused his death.

I wish to express my deep gratitude to You for Your thoughtfulness and assistance when he was ill in New York. If there is any information You desired in regard of his work, please write to me and I will be glad to serve You. With best regards, I am truly Yours, Paula Rosenblatt, Miraflores, Apt. 3050; Lima, Perú.



Fig. 4. The tombstone of Alfred and Paula Rosenblatt at the Jewish cemetery in Lima. Photo by Alejandro Ortiz Fernández

1.3. Rosenblatt's mathematical activity and recognition. Alfred Rosenblatt took an active part in mathematical life. Every four years The International Congress of Mathematicians (ICM), which is the most important and prestigious mathematical conference

in the world, is held. It is considered a great honor to be invited to deliver a plenary lecture at the Congress (at each Congress there are about 20 such lectures) or a section lecture (currently there are about two hundred section lectures, and before the war there were about a hundred). Rosenblatt took part in four Congresses, and at two of them he delivered four sectional lectures. Namely, he participated in the 5th ICM in Cambridge (22–29 August 1912), and Congress of ICM in Strasbourg (22–30 September 1920). At the ICM in Bologna (3–10 September 1928), he had three lectures in two sections: Section II, Geometry *Varietà algebriche a tre e più dimensioni* [Algebraic varieties of dimension three and higher] and a communication *Sopra la varietà algebrica a tre dimensioni fra i cui caratteri intercedono certe disuguaglianze* [On some three-dimensional algebraic varieties with certain inequalities between their characteristics] and in Section IIIB, Hydrodynamics, Plasticity, Equations of Mathematical Physics, a communication *Sopra certi moti permanenti dei liquidi viscosi incompressibili* [On some movements of stationary incompressible viscous fluids]. In addition, he chaired Section IIIB. He was present at the Congress of the ICM in Zurich (4–12 September 1932), where he had lecture in Section VIB, Mechanics and Mathematical Physics, *Sur les ondes de gravité* [The gravitational waves].

He also participated in the Third International Congress of Applied Mechanics in Stockholm (24–29 August 1930) with the lecture *Sur certains mouvements stationnaires des fluides visqueux incompressibles* [On some movements of stationary viscous incompressible fluids], the German Congress of Mathematicians in Prague (17–24 September 1929) and the Second Congress of Romanian Mathematicians in Turnu-Severin (5–9 May 1932).

Rosenblatt was also present at the Second Congress of Mathematicians from Slavic Countries in Prague (23–28 September 1934) with a lecture *Sur les équations aux dérivées partielles du type parabolique à deux variables indépendantes* [On partial differential equations of parabolic type with two independent variables].

In Poland (or Polish territory), he participated in the 11th Congress of Polish Physicians and Naturalists in Krakow (18–21 July 1911) with the lecture *Teoria powierzchni algebraicznych* [The theory of algebraic surfaces], in which he discussed the development of this theory up to 1911 (a 143-page extended version of this lecture was published in the journal *Prace Matematyczno-Fizyczne* [Mathematical and Physical Papers] [R8]). At the Polish Congress of Mathematics in Lvov (7–10 September 1927), he delivered three lectures: *O utworach trzechwymiarowych, których przestrzenie styczne spełniają pewne warunki różniczkowe* [On three-dimensional manifolds, whose tangent spaces satisfy some differential conditions], *Twierdzenie Kutty i Żukowskiego w aerodynamice* [Theorem of Kutta and Zhukovsky in aerodynamics] and *O regularyzacji problemu trzech ciał* [The regularization of the three-body problem]. In the Second Congress of Polish Mathematicians in Vilnius (23–26 September 1931) he had two lectures: *O istnieniu i jedności całek równań różniczkowych* [On the existence and uniqueness of integrals of differential equations] and *Zagadnienie turbulencji* [Problem of turbulence].

In recognition of his services for mathematics in Peru, the authorities of the Facultad de Ciencias University of San Marcos in Lima gave his name to the faculty library:

Biblioteca de Matemática “Alfred Rosenblatt”–UNMSM. They considered it a good way to preserve the memory of a great figure of science, a good teacher and a leading representative of Polish-Peruvian cooperation in research. In Lima, there is also *The Alfred Rosenblatt Center for Educational Research and Mathematics* (CIEMAR — Centro de Investigación en Educación y Matemática “Alfred Rosenblatt”) and the Alfred Rosenblatt street in the district of Santiago de Surco, the zone “Huertos de San Antonio”. In this zone, there is also a street named for Godofredo García and streets named for other well-known Peruvian scientists.



Fig. 5. Alfred Rosenblatt street in Lima. Photo by Anna Maria Haase Goldkuhle and her husband Holger Valqui

Tomasz Unger⁹, describes the figure of Rosenblatt [Ro18]:

Paula Unger (Alfred’s wife) was the sister of my father, Gerard Unger, who died in 1957, aged 54. She died in Lima in 1959, aged 74.

I knew Alfred Rosenblatt quite well, but I was too young to understand his work. I remember he spoke and wrote his notes in Greek, and occasionally

⁹Tomasz (Tomás) Unger was born in Krakow on 8 June 1930. He is a well-known scientist, journalist and writer living in Lima (Peru). He authored many books on cars and science. His father, a Polish engineer and director of Braun Boveri, Gerard Unger (1903–1957), arrived in Peru on business with his family in 1937, and the outbreak of World War II retained him in the country. During the war he was honorary consul of Poland in Peru (on behalf Sikorski’s government in London). Gerard lectured at the Technical University in Lima and was dean of the Faculty of Mechanical and Electrical Engineering. He died in a car accident at the age of 54. Gerard was a brother of Paula Unger, Alfred’s wife. After Rosenblatt’s death, Paula lived with Gerard’s family. For more on Thomas, one can read at: https://es.wikipedia.org/wiki/Tomás_Unger.

talked classic Greek and Latin with my mother. He collected butterflies and seemed to know quite a bit about them. He even traveled to the jungle to collect them, quite a hard trip in those times.

Among his best students who visited him I remember Francisco Miró-Quesada Cantuarias (a known philosopher, still alive, over 90) and José Tola Pasquel. I also remember two American mathematicians who during the war came to Lima to visit him: George Birkhoff and Marshall Stone.

Birkhoff was tall and I remember Rosenblatt saying that his theory on esthetics, whatever it was, was a load of BS. Stone was younger, always unkempt and careless about his clothes, according to my mother.

Rosenblatt had a vast library, over 2,000 books in several languages. When he died his wife donated all his books, notebooks (there were over 100 of them) and papers to the Universidad Mayor de San Marcos, where they were lost, or at least not accounted for, as far as I know.

About Rosenblatt's death I remember this: right after the war he went to Princeton invited by his friend Mr. Norbert Wiener. He travelled to New York by boat (Grace Line). He returned by the same means and brought me an autographed photograph of A. Einstein, who was his friend. When he returned he was in very poor health (mentally), was hospitalized, and died of pneumonia, aged 67.

I am sorry to say there are no photographs of Rosenblatt that I know of. As he was 50 years older than me, and I am now 82, you can imagine there are no people left, except Francisco Miró-Quesada, who knew him or could tell us something about him.

All I can say is that he was more than a little eccentric, but highly respected. He spoke fluent German, French, Spanish, Latin, classic Greek and of course Polish. He would read me a book (Voltaire's history of Carol of Sweden) in Polish. When I picked up the book to read on my own, it turned out to be in French. He had this amazing gift for languages. I was the only child he had ever dealt with and he treated me as an adult, except when we reproduced famous battles with my toy soldiers and he behaved like a kid.

Among the papers that were given to the University of S.M. was his lifetime correspondence with Poincaré and Levi-Civita, among other bundles of letters. It is a pity that nobody knows where they all ended up.

Władysław Ślebodziński¹⁰ remembered his older colleague, Alfred Rosenblatt, in these words (cf. [Ro17, p. 18]):

Rosenblatt was a very talented mathematician, dedicated entirely to science, which was possible for him due to his material conditions, freeing him from

¹⁰Władysław Ślebodziński (1884–1972) studied at the Jagiellonian University in the years 1903–1908. The book by Sierpiński referred to in the text was *Zarys Teorii Mnogości* [An Outline of Set Theory], Wydawnictwo Kasy im. Mianowskiego, Warszawa 1912. This book was based on the 1909 lectures at the University in Lvov.

the need to apply for a teaching job. He towered over us with his knowledge of many directions of research in mathematics of that time. The collaboration with him over the last two years of my university studies brought me a lot of benefits. It was through him that I learned about set theory and its problems, and seven years before the publication of prof. W. Sierpiński's book too!

Michael Helfgott, at the conference in Kristiansand (1988), presented a lecture on the importance of providing historical information when teaching calculus, and gave as an example of Professor Rosenblatt's teaching in Peru (cf. [33, p. 141]):

The arrival at San Marcos, in 1936, of the Polish mathematician Alfred Rosenblatt radically changed Peruvian mathematics. Before him, Peruvian mathematics was isolated from the twentieth century European developments. In the ten years of tenacious work, until his unexpected death in 1946¹¹, he trained a whole generation of young mathematicians who, in turn, prepared others for graduate work abroad. The influence of this one outstanding scholar is remarkable.

Professor Rosenblatt brought with him from Europe the latest information on functional analysis, topology and other mathematical areas that was not available in Peru at that time. Besides producing over 225 papers [Ro13], he taught many important mathematical topics.

It is pertinent to mention the book [R74] written in collaboration with Godofredo García, a Peruvian physicist. It is a work filled with historical references and acute observations.

Let us conclude this section with a quotation from a letter by Alejandro Ortiz Fernández dated 7 May 2013:

Peru needs to be proud of Alfred Rosenblatt, a great mathematician !!!

2. Rosenblatt's scientific achievements. Alfred Rosenblatt published six books and about three hundred papers in Polish, German, French, Italian, Spanish and English on ordinary and partial differential equations (more than 70 papers), algebraic geometry (over 30 papers), real and complex analysis (over 30 papers), calculus of variations (over 15 papers), probability (7 papers), elementary geometry (2 papers), theoretical physics and applications of mathematics, including hydrodynamics, viscous incompressible motion, the problem of three bodies, celestial mechanics, lubrication theory, aerodynamics, theory of elasticity, theory of gravity, geometrical optics, mathematical genetics, bacteriology and music scales (over 80 papers). He wrote also 7 papers on astronomy and 8 papers on the history of mathematics. Rosenblatt was the first mathematician in Poland to work creatively in algebraic geometry.

Approximately one hundred eighty papers were published in the years of his activity in Poland (1907–1936), and about one hundred twenty in the years of his activity in Peru (1936–1947), with his last productions appearing posthumously in 1949.

¹¹Of course, it should be 1947.

Among his co-authors were Stanisław Turski¹², with whom he published 3 joint papers on functions of complex variables in the years 1935–1936, and Godofredo García, with whom he published 6 joint papers in the years 1937–1939, including 2 papers in 1937 on the problem of three bodies and 4 papers in the years 1938–1939 on Stokes’ formula in gravity theory.

Now let us turn to some details of Rosenblatt’s achievements in selected branches of mathematics.

2.1. Textbooks and monographs. Rosenblatt authored four textbooks [R27], [R28], [R33], [R74] (the last one jointly with G. García) and two monographs: [R43] and [R45]. The textbooks [R27] and [R28] on analytic geometry in the plane and in the space had a very limited distribution outside the Jagiellonian University. The third book [R33], on analytic geometry in the plane, had a larger readership, and Alexander Wundheiler in his review of the book had this to say (cf. [79, p. 99]):

It is a very conscientious exposition; the author goes to great lengths to ground the basic concepts. (...) The appearance of Prof. Rosenblatt’s book is to be welcome with appreciation. It is possessed of a distinct individuality, which ensures its prominent place in the literature.

The fourth book on algebraic analysis [R74], written jointly with G. García, was reviewed by S. H. Gould in Mathematical Reviews (MR0085307), where we read:

This is an introduction to the real number system. There are many quotations from Greek mathematicians and also from the moderns: Dedekind, Cantor, Weierstrass, Peano, Russell, and many others. The philological and historical setting makes a very pleasant impression. The chapter headings are: real numbers, natural numbers, elements of sets of points on a straight line, sequences and series, powers and logarithms. Though written in an unhurried way, the book contains a great deal of mathematical information.

Rosenblatt’s two monographs [R43] and [R45] can be seen as a summary of his work in the domain of hydrodynamics of incompressible viscous fluids. Both written in French, they contain Rosenblatt’s lectures at Sorbonne University. They were reviewed in *Jahrbuch über die Fortschritte der Mathematik* (JFM 59.1457.01, JFM 61.0920.03) by Prof. Georg Hamel from Berlin.

In the monograph [R43] *On some movements of incompressible viscous liquids* from 1933, Rosenblatt examines the movement of fluids in viscous spirals and related flows.

¹²Stanisław Turski (1906–1986), Polish mathematician, 1924–1928 studied physics, astronomy and mathematics at the Jagiellonian University (UJ), 1935 PhD at UJ, 1939–1941 prisoner at the Nazi German concentration camps at Sachsenhausen and Dachau, 1947–1952 Member of Parliament, 1946–1949 rector of the Gdańsk University of Technology, 1949–1951 professor of the Warsaw University of Technology, 1952–1969 rector of the University of Warsaw (UW), 1978 retired. He was loyal to the government of the postwar period. He was interested in the applications of mathematics to various technical problems and was a pioneer of the development of computer science and computer technology.

In the introduction he informs the reader that this subject was of interest to physicists in Poland such as Rudzki and Smoluchowski. Also Natanson, Zaremba and Zakrzewski¹³ developed ideas on the relaxation of fluids that go back to Poisson. Then he studies the free radiation and neighborhood movements. Also the case of Faxén and Olsson¹⁴, who discovered in 1927 that some spiral movement is governed by elliptic functions, is recognized as an interesting application of the Picard set. The monograph is divided into four chapters: I. Equations of movement of viscous liquids (pp. 3–9), II. Study of radial movement and logarithmic spirals (pp. 10–26), III. Neighborhood movements of radial movements (pp. 27–37), IV. Other movements of the planes (pp. 38–40). On page 41 the conclusions follow.

The monograph [R45] *Exact solutions of the equations of motion of viscous fluids* from 1935 contains a presentation of his and other contemporary mathematicians' and physicists' results in the domain of the exact solutions of the equations of motion of homogeneous viscous fluids. The problem of finding the exact solutions of these equations is important, because the most frequently used linear approximations are in many cases not justified for physical and mathematical reasons. This book is divided into four chapters. In Chapter I (pages 2–14), we find concise information on the principles of hydrodynamics of viscous fluids. We can find here the basic equations of hydrodynamics and their various formulations in two and three space dimensions. In Chapter II (pages 14–33), various kinds of plane motions, potential and non-potential, are presented. He deals with special solutions in two dimensions such as Poiseuille flow (laminar flow) and the circular Couette flow¹⁵. Eleven subsections are devoted to logarithmic spiral motions, circular and radial motions, Hamel's motions in spirals in the unbounded plane, Oseen's movements in the unlimited plane, laminar motions, nonstationary motions of Boussinesq¹⁶ and

¹³Maurycy Pius Rudzki (1862–1916), Polish geophysicist and geographer, professor of mathematical geophysics and meteorology at the Jagiellonian University; Marian Smoluchowski (1872–1917), Polish physicist, pioneer of statistical physics and avid mountaineer, professor at the University of Lvov and the Jagiellonian University; Władysław Natanson (1864–1937), Polish physicist, professor of mathematical physics at the Jagiellonian University (1922/23 rector of UJ); Stanisław Zaremba (1863–1942), Polish mathematician, professor at the Jagiellonian University; Ignacy Zakrzewski (1860–1932), Polish physicist, professor at the University of Lvov.

¹⁴Olov Hilding Faxén (1892–1970), Swedish physicist who received his doctorate in 1921 at Uppsala University, professor at the Chalmers Institute of Technology (1930–1935) and at the Royal Institute of Technology in Stockholm (1935–1958), in fluid dynamics there are Faxén's laws; Olof Olsson (1861–1938), Swedish physicist, associate professor at Örebro.

¹⁵Jean Léonard Marie Poiseuille (1797–1869), French physicist and physiologist who formulated a mathematical expression for the flow rate for laminar (nonturbulent) flow of fluids in circular tubes. Discovered independently by Gothilf Hagen (1797–1884), a German hydraulic engineer; this relation is also known as the Hagen–Poiseuille equation; Maurice Marie Alfred Couette (1858–1943), French physicist known for his studies of fluidity.

¹⁶Georg Karl Wilhelm Hamel (1877–1954), German mathematician with interests in mechanics, foundations of mathematics and function theory; Carl Wilhelm Oseen (1879–1944), Swedish theoretical physicist in Uppsala and Director of the Nobel Institute for Theoretical Physics in Stockholm; Joseph Valentin Boussinesq (1842–1929), French mathematician and physicist who made significant contributions to the theory of hydrodynamics, vibration, light, and heat.

Villat, Taylor's vortices, and Oseen's time-dependent motions¹⁷. In contrary to Rosenblatt's original papers, the calculations are here performed here up to the final results and discussed. Chapter III concerns the movements in the space (pages 34–45). Known exact simple symmetrical solutions in three dimensions are reviewed. This chapter contains six subsections on motions symmetrical about an axis in the planes passing through this axis, general motions symmetrical about an axis, Cisotti's viscous rotations, the tension within a fluid whose motion is symmetrical about an axis, Caldonazzo's helical motions and Oseen's motions in space¹⁸.

The stability of such solutions with respect to finite perturbations is considered in Chapter IV. It does not deal with the general theory of existence of solutions subject to general boundary conditions, for which the author refers to Oseen and Villat. This last chapter is divided into three parts: neighborhoods of radial motions (pp. 45–52), neighborhoods of laminar motions (pp. 52–58), neighborhoods of Couette's circular motions (pp. 58–61). Rosenblatt also gives an account of the research by Hamel, Sommerfeld, Taylor¹⁹, and himself. He cites seven papers of his own from the years 1928–1932. In each case, Rosenblatt presents a derivation of stability conditions. From the preface, we read:

I plan to make this work useful by exposing the state of the art of the integration of the exact equations describing the motion of an incompressible viscous homogeneous fluid. (...) Upon recalling, in the first Chapter, the classical results of the founders of hydrodynamics of viscous fluids, I give some complementary formulae related to stresses that are necessary for the sequel. Then I present the solutions that are known for planar motion as well as in three dimensions. The recent important discoveries of Hamel, Oseen, Cisotti, Crudelli, Caldonazzo etc. have vastly increased the primitive field of exact solutions limited to Poiseuille motion, the circular motion of Couette and laminar motion. In addition, I present some new motions that I have studied in recent works.

In Chapter IV I study the “neighbouring” (small perturbations) motion of the above mentioned motions. To my knowledge, nobody except Næther have tried to study this question exactly. Other authors consider only the approximation of first order and apply the linear theory without asking themselves the relation between these solutions and the exact solutions.

I have obtained by myself some exact results concerning the neighbouring motion of radial motion, the motion of Poiseuille and the motion neighbouring of laminar motion of which I expose some parts.

¹⁷Henry Villat (1879–1972), French mathematician, professor of fluid mechanics at the University of Paris since 1927, member of the French Academy of Sciences from 1932, and its president from 1948, foreign member of PAU from 1929; Geoffrey Ingram Taylor (1886–1975), British physicist and mathematician, and a major figure in fluid dynamics and wave theory.

¹⁸Umberto Cisotti (1882–1946), Italian mathematician and physicist; Bruto Caldonazzo (1882–1960), Italian mathematician and physicist.

¹⁹Arnold Johannes Wilhelm Sommerfeld (1868–1951), German theoretical physicist, a pioneer of atomic and quantum physics.

I have not presented the general theory of existence of solutions satisfying certain conditions on the boundary. We know that it is the Italian geometers on one hand, and the brilliant school of Oseen on the other hand who have obtained brilliant results on this subject. I can only refer to the excellent books of Oseen: Hydrodynamik and Villat: Leçons sur l'hydrodynamique.

Rosenblatt's book [R45] is cited in the books on hydrodynamics written by Birkhoff [8, p. 181], Hamel [28, p. 170], Lagerstrom [45, pp. 64, 266], and Slezkin [75, p. 146], and in several papers.

2.2. Ordinary and partial differential equations

Classical Cauchy problem and Rosenblatt condition. Rosenblatt published many papers on ordinary and partial differential equations of the first and second order. His most important contribution was probably to the uniqueness question of the classical Cauchy problem in which we look for a function $y = y(x)$ that satisfies the differential equation of the first order and the initial condition

$$y' = f(x, y), \quad y(x_0) = y_0, \quad (1)$$

where the function f is at least continuous in the domain $D \subset \mathbb{R}^2$ with $(x_0, y_0) \in D$. By a *solution* of the problem (1) in the interval I containing x_0 we understand the existence of a function $y = y(x)$ such that $y(x_0) = y_0$, $(x, y(x)) \in D$ for $x \in I$, and $y'(x)$ exists and is continuous for $x \in I$ with $y'(x) = f(x, y(x))$. If I is a closed interval then at the ends we require the existence of one-sided derivatives. The continuity assumption on f is quite natural. In this Cauchy problem, there are two basic questions: the existence and the uniqueness of such a function $y = y(x)$.

A theorem on the existence and uniqueness was proved independently by Émile Picard (1856–1941) in 1886 and Ernst Lindelöf (1870–1946) in 1894: If a function f is continuous in the rectangle $P = [x_0 - a, x_0 + a] \times [y_0 - b, y_0 + b]$ and satisfies the Lipschitz condition in P , i.e., there exists a constant $L > 0$ such that

$$|f(x, y_1) - f(x, y_2)| \leq L|y_1 - y_2| \quad \text{for all } (x, y_1), (x, y_2) \in P, \quad (2)$$

then the Cauchy problem (1) has a unique solution on the interval $I = [x_0 - \alpha, x_0 + \alpha]$, where

$$\alpha < \min \left\{ a, \frac{b}{M}, \frac{1}{L} \right\}, \quad M = \sup_{(x, y) \in P} |f(x, y)|,$$

that is, there exists a function $y = y(x)$ of the class $C^1(I)$, whose graph lies in the rectangle P and which satisfies conditions (1). The function $y = y(x)$ is unique in the sense that if some function $z = z(x)$ also has the above properties in some interval $[x_0 - d, x_0 + d]$, then in a common part of the interval I the identity $y(x) = z(x)$ holds.

Standard proof proceeds by considering a Picard sequence of successive approximations

$$y_0(x) = y_0, \quad y_n(x) = y_0 + \int_{x_0}^x f(t, y_{n-1}(t)) dt, \quad n \in \mathbb{N}, \quad (3)$$

for which we have the estimate

$$|y_k(x) - y_{k-1}(x)| \leq M \frac{L^{k-1}|x - x_0|^k}{k!}, \quad k \in \mathbb{N},$$

and hence the series $y_0(x) + \sum_{k=1}^{\infty} [y_k(x) - y_{k-1}(x)]$ is absolutely convergent on I , means the partial sums $y_0(x) + \sum_{k=1}^n [y_k(x) - y_{k-1}(x)] = y_n(x)$ are uniformly convergent on I to a certain continuous function y on I . On the other hand, $f(t, y_{n-1}(t)) \rightarrow f(t, y(t))$ uniformly on I , whence

$$y_n(x) = y_0 + \int_{x_0}^x f(t, y_{n-1}(t)) dt \rightarrow y_0 + \int_{x_0}^x f(t, y(t)) dt, \quad \text{for } x \in I,$$

and so $y(x) = y_0 + \int_{x_0}^x f(t, y(t)) dt$ on I , and which is a required solution. The uniqueness follows from the fact that if z is a solution of the Cauchy problem (1), then

$$|z(x) - y_n(x)| \leq \int_{x_0}^x |f(t, z(t)) - f(t, y_{n-1}(t))| dt \leq \dots \leq M \frac{L^n |x - x_0|^{n+1}}{(n+1)!} \rightarrow 0$$

as $n \rightarrow \infty$, that is, $z(x) = y(x)$ on some interval.

The proof of this theorem is recently a classical example of how to use the Banach fixed point theorem on the closed subspace $C_0(I)$ of the space of continuous functions $C(I)$, defined by

$$C_0(I) = \{y \in C(I) : y_0 - M\beta \leq y(x) \leq y_0 + M\beta \text{ for all } x \in I\}$$

and the nonlinear operator $Ty(x) = y_0 + \int_{x_0}^x f(t, y(t)) dt$.

In 1886, Giuseppe Peano (1858–1932), using a sequence of successive approximations, presented a proof of the existence of a solution of the Cauchy problem (1) without its uniqueness. Peano's theorem about the existence, instead of the Lipschitz condition (2), assumes only the continuity of f in P , and as result gives only the existence of a function y satisfying (1) on an interval $J = [x_0 - \beta, x_0 + \beta]$, where $\beta < \min\{a, b/M\}$ and $M = \sup_{(x,y) \in P} |f(x, y)|$. In this case we might, of course, have one or more solutions as is seen below.

EXAMPLE. For the Cauchy problem $y' = 2\sqrt{|y|}$, $y(0) = 0$ we see that the function $f(x, y) = 2\sqrt{|y|}$ is continuous in \mathbb{R}^2 and does not satisfy the Lipschitz condition in a neighborhood of zero, because

$$\lim_{y \rightarrow 0} \frac{|f(x, y) - f(x, 0)|}{|y|} = \lim_{y \rightarrow 0} \frac{\sqrt{|y|}}{|y|} = \lim_{y \rightarrow 0} \frac{1}{\sqrt{|y|}} = \infty.$$

On the other hand, it is easy to see that the functions

$$y_{ab}(x) = \begin{cases} -(x+b)^2 & \text{if } x \leq -b, \\ 0 & \text{if } -b \leq x \leq a, \\ (x-a)^2 & \text{if } x \geq a, \end{cases}$$

are solutions of the Cauchy problem for any $a, b \geq 0$.

The proof of the Peano theorem recently is one classical application of the Schauder fixed point theorem in Banach spaces (1930) to the nonlinear operator $Ty(x) = y_0 + \int_{x_0}^x f(t, y(t)) dt$ in the Banach space $C(J)$. A larger interval of existence than

$J = [x_0 - \beta, x_0 + \beta]$ can be obtained by taking a different metric (the Bielecki metric) on $C(J)$. With this metric, the Cauchy problem can be solved in a strip $[x_0 - a, x_0 + a] \times \mathbb{R}$. Note that the set of all Lipschitz functions is of the first Baire category in the space of continuous functions, but the set of functions f for which the Cauchy problem (1) has a unique solution is of the second Baire category (Orlicz 1932). This means that, paradoxically, the term “most of solutions” (in the sense of Baire category) of the Cauchy problem (1) is unambiguous.

Another condition for the uniqueness of a solution of the Cauchy problem (1) was introduced by William Fogg Osgood (1864–1943) in 1898: f is continuous on P and satisfies the *Osgood condition*:

$$|f(x, y_1) - f(x, y_2)| \leq g(|y_1 - y_2|), \quad (4)$$

where $g: [0, \infty) \rightarrow [0, \infty)$ is continuous, $g(0) = 0$, $g(u) > 0$ for $0 < u < u_0$ and $\int_0^{u_0} \frac{1}{g(u)} du = \infty$. As g we can take functions $g(u) = Lu$, $Lu \ln(1/u)$, $Lu \ln(1/u) \ln \ln(1/u)$.

In his 1909 paper [R1], Rosenblatt weakened the Lipschitz condition (2) to the *Rosenblatt condition* obtaining the uniqueness of solution of the Cauchy problem (1).

THEOREM 1 (Rosenblatt 1909). *If f is continuous in $[x_0 - a, x_0 + a] \times [y_0 - b, y_0 + b]$ and there are constants $L > 0$, $0 < m < 1$ or $0 < L < 1$ when $m = 1$, such that*

$$|f(x, y_1) - f(x, y_2)| \leq \frac{L}{|x - x_0|^m} |y_1 - y_2| \quad \text{for any } x \neq x_0, \quad (5)$$

then the Cauchy problem $y'(x) = f(x, y(x))$, $y(0) = y_0$ has a unique solution.

Rosenblatt observed in [R1] that for the Picard sequence, thanks to the assumption (5) instead of (2), we obtain the estimates

$$|y_k(x) - y_{k-1}(x)| \leq \frac{L^k M}{(2 - m)^{k-1}} |x - x_0|^{k(1-m)+m},$$

for all $k \in \mathbb{N}$. Therefore, if either $0 < m < 1$ and $L > 0$ or $m = 1$, then we have convergence for $a < 1/L$ in the first case and for $L < 1$ in the other. Hence, $y_n \rightarrow y$ and y is a solution of (1) on $[x_0 - \delta, x_0 + \delta]$. This solution is unique. Why? If z is a solution of (1) on some interval $[x_0 - d, x_0 + d]$, then in the common part $[x_0 - \delta, x_0 + \delta] \cap [x_0 - d, x_0 + d]$ we obtain

$$|z(x) - y_n(x)| \leq \frac{ML^n}{(2 - m)^{k-1}} |x - x_0|^{k(1-m)+m} \rightarrow 0, \quad \text{as } n \rightarrow \infty,$$

whenever either $0 < m < 1$, $L > 0$ (for $a < 1/L$) or $m = 1$ and $L < 1$. ■

In Rosenblatt’s paper [R1] the case $m = 1$ and $L = 1$ is not treated. It was done in 1926 by Mitio Nagumo (1905–1995), who introduced the *Nagumo condition* for a continuous function f in P :

$$|f(x, y_1) - f(x, y_2)| |x - x_0| \leq |y_1 - y_2| \quad \text{for any } (x, y_1), (x, y_2) \in P, \quad (6)$$

Actually, in Nagumo’s paper, there was originally the condition

$$|f(x, y_1) - f(x, y_2)| |x - x_0| \leq L |y_1 - y_2| \quad \text{for any } (x, y_1), (x, y_2) \in P \text{ with } 0 < L \leq 1,$$

but Perron (1928) proved that the constant $L = 1$ is the best possible for the validity of the uniqueness result. Let us note that in 1927 Müller [59] gave an example of a continuous function f satisfying the Nagumo condition, which means that the Cauchy problem has a unique solution, but for which Picard's method of successive approximations does not converge to a solution. Moreover, convergence of Picard's successive approximations for any initial function is not sufficient for the uniqueness of a solution of the Cauchy problem.

The Rosenblatt–Nagumo condition was generalized by many mathematicians, for example, Tonelli (1925), Montel (1926), and Kamke (1930), who introduced the condition

$$|f(x, y_1) - f(x, y_2)| \leq \omega(|x - x_0|, |y_1 - y_2|)$$

in an open neighborhood of the point $P_0 = (x_0, y_0)$, where the function $\omega: [0, a) \times [0, \infty) \rightarrow [0, \infty)$ is continuous and the problem $y' = \omega(t, y)$, $y(0) = 0$, has a unique solution. Krasnosel'skii and Krein (1956) used the Rosenblatt condition with $L > 0$, $m = 1$ and also added the Hölder condition on f in the second variable (cf. [42]).

Further generalizations of the Rosenblatt condition can be found in the book by Agarwal and Lakshmikantham [1] from 1993. Rosenblatt's paper [R1] is cited in several classical books on differential equations (in chronological order): Kamke [36, p. 58], Sansone [71], Walter [78, p. 339], Schäfke and Schmidt [72, p. 157], Flett [23, p. 348], Agarwal and Lakshmikantham [1, p. 309], Hartman [31, p. 601], in the historical book of Dobrovolskii [20, pp. 51 and 450], and in several papers, for example (in alphabetical order) [9, pp. 29–31], [10, p. 1475], [32, p. 633], [42, p. 213], [49, p. 730], [57, p. 607], [58, p. 331], [59, p. 627], [61, p. 202], [73] and [84, p. 91].

Rosenblatt's idea to generalize the Lipschitz condition was improved by Nagumo (1926) and Kamke (1930), and of the initial name, the *Rosenblatt–Nagumo Theorem* (the *Rosenblatt–Nagumo condition*) or the *Rosenblatt–Nagumo–Kamke Theorem* now remains only the Nagumo theorem or perhaps the Nagumo–Kamke Theorem. Rosenblatt's name has disappeared although the first idea was, published already in 1909 in [R1]. It is a great pity that the mathematicians from Krakow (T. Ważewski, M. Krzyżański, J. Szarski and A. Pelczar) preferred to speak of the Kamke or Nagumo uniqueness criterion without mentioning an earlier result of Rosenblatt (see [65, pp. 136–138] and [64], where Rosenblatt name can be found on pages 75, 91, 113–114, and three of his papers from 1930, 1931 and 1932 are cited, but not the paper [R1] from 1909!).

Also Kamke, in his book [37], which is an extensive reference work on differential equations, referred only to himself (but in this book on page 278 there is reference to Rosenblatt's paper [R44] in connection to the differential equation $y'' = f(x, y, y')$). In more recent papers, however, sometimes the name of Rosenblatt reappears, as for example in [10, pp. 1469, 1472], where there is talk of the Perron–Rosenblatt uniqueness theorem.

Systems of differential equations. From the twenties to late forties, in several of his papers, Rosenblatt used his idea about the assumptions on functions amenable to Picard's method of successive approximations to obtain a unique solution of the system of differential equations

$$y' = f(x, y, z), \quad z' = g(x, y, z) \tag{7}$$

with the initial conditions $y(0) = z(0) = 0$, where f, g are continuous functions on $G = [0, a] \times [-b, b] \times [-b, b]$ satisfying

$$\begin{aligned} |f(x, y_1, z_1) - f(x, y_2, z_2)| &\leq A|z_1 - z_2|/x^\alpha, \\ |g(x, y_1, z_1) - g(x, y_2, z_2)| &\leq B|y_1 - y_2|/x^\beta, \end{aligned} \quad (8)$$

for some particular values of $A, B, \alpha, \beta > 0$. Picard's iterations take the form of two sequences

$$y_n(x) = \int_{x_0}^x f(t, y_{n-1}(t), z_{n-1}(t)) dt, \quad z_n(x) = \int_{x_0}^x g(t, y_{n-1}(t), z_{n-1}(t)) dt, \quad n \in \mathbb{N}.$$

In a paper from 1946, Rosenblatt investigated the Cauchy problem for a system of two partial differential equations of the first order with two unknown functions $z = z(x, y)$, $u = u(x, y)$ and the initial conditions $z(0, y) = u(0, y) = 0$, which solve the system

$$z_x = f(x, y, z, u, z_y, u_y), \quad u_x = g(x, y, z, u, z_y, u_y),$$

under the corresponding conditions (8) on f, g .

Picard's method for higher order differential equations. In 1933, Rosenblatt [R44] used Picard's method to the initial value problem

$$y'' = f(x, y, y'), \quad y(x_0) = y_0, \quad y'(x_0) = y_1 \quad (9)$$

with Picard's iterations $y_0(x) = y_0 + y_1(x - x_0)$ and

$$y_n(x) = y_0 + y_1(x - x_0) + \int_{x_0}^x \left[\int_{x_0}^s f(t, x_{n-1}(t), x'_{n-1}(t)) dt \right] ds, \quad n \in \mathbb{N},$$

showing that under suitable assumptions on f , the iterations are convergent to the unique solution of (9). This time Kamke, in his book [37, p. 278], put in a reference to Rosenblatt's result from [R44].

In [R44] Rosenblatt also noticed that the Dirichlet problem $y'' = f(x, y, y')$, $y(a) = y(b) = 0$, can be studied for singular nonlinearities other than L^p -Caratheodory functions f (cf. [17, p. 94] and [18, pp. 5, 426]).

Picard's method in other differential equations. In many papers Rosenblatt applied Picard's method of successive approximations to prove the existence and uniqueness results, for example, for the following differential equations: $z_x = f(x, y, z, z_y)$ (a paper from 1932), the parabolic equation $z_{xx} - z_y = f(x, y, z)$ (two papers from 1927 and 1934), $z_x = f(x, y, z, z_y)$ (at least eight papers from 1928–1932), the elliptic equation $\Delta u = F(x, y, u, z_x, z_y)$ (in at least four papers from 1933–1935) and the biharmonic equation $\Delta \Delta u = F(x, y, u, \dots, u_{yy})$ (at least three papers from 1933–1935). Some of these Rosenblatt results were cited in the book by Miranda [56].

The Briot–Bouquet differential equation is a complex differential equation of the form

$$z^m w' = az + bw + f(z, w), \quad (10)$$

where $w = w(z)$, $m \in \mathbb{N}$, f is analytic at $(0, 0) \in \mathbb{C}^2$, $f(0, 0) = 0$ and $f'_w(0, 0) \neq 0$.

The theorem of Briot and Bouquet (1856) states that the differential equation

$$zw'(z) = az + bw(z) + \phi(z, w(z)), \quad \text{where} \quad \phi(z, w) = \sum_{m+n>1} a_{mn} z^m w(z)^n, \quad (11)$$

$w(0) = 0$, has, on some open disc at the center $z = 0$, a unique solution $w(z) = \sum_{k=1}^{\infty} c_k z^k$, if the power series $\phi(z, w)$ converges on a neighborhood of $(z, w) = (0, 0)$, and a, b are two constants with b not a positive integer. Cauchy's theorem on the existence and uniqueness cannot be directly applied here.

Rosenblatt began his research with a paper in 1908, where he considered the equation $w' = f(z, w)$, assuming that the analytic function $f(z, w)$ is a quotient of two polynomials or two power series convergent at $z = w = 0$, that is,

$$w' = \frac{\sum_{m,n} a_{mn} z^m w(z)^n}{\sum_{p,q} b_{pq} z^p w(z)^q}. \quad (12)$$

Using the Briot–Bouquet method of successive approximations for complex variables, he proved, under some assumptions on the coefficients a_{mn} and b_{pq} , the existence of solutions of the form

$$w(z) = v(z) z^{\mu} (\log z)^k, \quad (13)$$

in which μ is a positive number, k a rational number, and v an analytic function convergent to the value $v_0 \neq 0$ as $z \rightarrow 0$. Subsequently, he published several papers in 1909, 1914, 1916, 1939, 1941 and 1946, where he extended this result to a system of two differential equations of type (12). Moreover, using the method of successive approximations, he investigated the equation (10) with $a \neq 0$, and under some restriction on function $f(z, w)$, he obtained an analytic representation of the solutions, whose existence had been investigated for $a = 0$ by Bendixson. In 1941, Rosenblatt proved the existence of solutions of the form

$$w(z) = v(z) z^{\mu} e^{-k(\log 1/z)^m}, \quad 0 < m < 1, \quad (14)$$

to systems of equations (12). Rosenblatt's results were cited in [21, pp. 21, 36, 48, 62].

The van der Pol equation. The classical van der Pol equation arose in 1922 in the study of circuits containing vacuum tubes, and it was originally proposed by Dutch electrical engineer and physicist Balthasar van der Pol (1889–1959). The homogeneous van der Pol equation is given by

$$y'' - \mu(1 - y^2)y' + y = 0, \quad (15)$$

where $\mu = \text{const} > 0$ is a scalar parameter indicating the nonlinearity and the strength of damping. For small μ , the auto-oscillations of the oscillator are close to the simple harmonic oscillations with period 2π and specified amplitude. The homogeneous van der Pol equation comes also from the differentiation and substitution $z = y'$ in the homogeneous Rayleigh differential equation $y'' - \mu(1 - \frac{1}{3}y'^2)y' + y = 0$, and it is an important special case of the homogeneous Liénard equation $y'' + f(y)y' + y = 0$. The non-homogeneous van der Pol equation

$$y'' - \mu(1 - y^2)y' + y = E_0 + E \sin \omega x,$$

describes the behavior of the van der Pol oscillator when acted upon by a periodic external disturbance. The most important in this context is the study of frequency capture (the existence of periodic oscillations), beats (the possibility of almost-periodic oscillations) and chaotic behavior.

Rosenblatt wrote three papers [R70], [R72] and [R73] on the non-homogeneous generalized van der Pol equation. Using the method of the small parameter, in [R70] and [R72] he found a sufficient condition for the differential equation

$$y'' + \mu(A + Cy^2)y' + \omega_n^2 y + By^2 + Dy^3 = E \cos \omega x + F \sin \omega x, \quad (16)$$

with $\omega = 2\omega_n$ to have a periodic solution of period $2\pi/\omega_n = 4\pi/\omega$. His condition consists of two inequalities $4AC\omega_n^2 + M < 0$ and $A^2\omega_n^2(\omega_n^2 - \omega^2)M < Mk^2B^2 + 2k^2A^2C^2\omega_n^4$, where $M = 2k^2(C^2\omega_n^2 + 9D^2)/(\omega_n^2 - \omega^2)^2$ and $k^2 = E^2 + F^2$, and the requirement that a certain determinant should not vanish.

In [R73], Rosenblatt investigated the van der Pol equation

$$y'' + \mu(A + Cy^2)y' + \omega_n^2 y = E \cos \omega x + F \sin \omega x, \quad A < 0, C > 0, \quad (17)$$

for small forcing term μ when the frequency of the forcing term is an integer multiple of the natural frequency of the system, i.e., $\omega = n\omega_n$, $n > 1$. The substitution of $x = x_0 + \mu x_1 + \mu^2 x_2 + \dots$ in the equation determines the x_k successively, each up to an additive term of the form $a_k \sin x + b_k \cos x$.

2.3. Algebraic geometry. Rosenblatt published 31 papers devoted to algebraic geometry (in several languages): seventeen papers in the years 1911–1915, one paper in 1919, and thirteen in the years 1923–1932.

In 1911, at the Congress of Polish Physicians and Naturalists, Rosenblatt, then aged 31, presented a report concerning contemporary achievements of the theory of algebraic surfaces. The next year, he published *Progress in the theory of algebraic surfaces* [R8] as a result of his studies on the subject²⁰. Rosenblatt's paper contains a self-contained historical study. In algebraic geometry, Rosenblatt was interested in topological properties of algebraic curves, the theory of algebraic surfaces and three and more dimensional algebraic surfaces, and all his results were contributions to the “classification” type theorems. In his notes he described the basic classification tools in the following way [Ro1]:

The basic invariants of birational transformations of algebraic surfaces are:

- 1) *the geometric genus of a surface p_g ; 2) the arithmetic genus of a surface p_a ;*
- and 3) the linear genus of a surface $p^{(1)}$.*

Let us sketch the history of the research on the classification of algebraic surfaces. It began with Max Noether's systematic study of algebraic surfaces, and J. J. Gray [26, p. 816] characterized it thus:

In the wake of work by Cayley and Clebsch, Noether defined what became known as the arithmetic genus of a surface of degree n in his [1871]. It was a number, p_a , obtained by counting coefficients, which was related to the dimension of the space of adjoint surfaces of order $n - 4$ passing $(i - 1)$ -times through each i -fold curve of F and $(k - 2)$ -times through each k -fold point. Zeuthen had shown it was a birational invariant, and so one which would survive attempts to resolve the singularities. In his [1875], Noether defined the

²⁰This almost 150-page paper is much more extensive than Baker's *On some recent advances in the theory of algebraic surfaces* [3]. Baker's article is well known while Rosenblatt's is completely unknown, because it was published in Polish.

*surface genus, later called the geometric genus, p_g , as the actual number of linearly independent surfaces of degree $n-4$ adjoint to a surface F of degree n . He called the genus of the intersection of F with an adjoint surface the linear genus and showed that surfaces with small values of these genera yielded immediately to classification, as surfaces defined by a polynomial equation of a certain degree with such-and-such double curves and multiple points*²¹.

In the paper [R11] Rosenblatt presented two inequalities determining the connection between genera of algebraic surfaces. He was inspired by Max Noether who showed that the linear genus of the surface has upper bound and by Castelnuovo who studied algebraic surfaces whose genera satisfied inequality (now called Castelnuovo inequality)

$$p_g \geq 2(p_a + 2). \quad (18)$$

Castelnuovo showed that such a surface has an irrational pencil of curves, Rosenblatt showed that under the assumption that genus $p^{(1)} > 1$ the inequality

$$p_g \leq 4(p_a + 5) \quad (19)$$

holds. Rosenblatt also established a new upper limit for the linear genus:

$$p^{(1)} \leq 16p_a + 27. \quad (20)$$

Guido Castelnuovo (1865–1952), at the turn of the 20th century, had proven important parts of the classification of classical algebraic surfaces. Rosenblatt’s habilitation thesis (published as *Studies on certain classes of irregular algebraic surfaces and birational transformations not changing these surfaces* [R9]) was devoted to the problems considered by Castelnuovo. In 1914, Federigo Enriques (1871–1946) extended the “classification project” to complex projective surfaces. In 1921, Castelnuovo and Enriques published an extensive review of the achievements of the theory of algebraic surfaces [12]. In this work, three results by Rosenblatt (published in the papers: [R7], [R8], [R11]–[R14], [R17]) are cited. The result published in the series of papers [R13], [R14], [R16], [R19] improved earlier results obtained by Castelnuovo and Severi. In [R14], Rosenblatt presented a classification-type observation based on the Castelnuovo inequality:

THEOREM 2 (Rosenblatt 1913). *For any algebraic surface with geometric genus p_g and arithmetic genus p_a if*

$$p_g > 2(p_a + 2), \quad (21)$$

then the surface is a ruled or an elliptic surface.

Rosenblatt writes about his results as follows ([Ro3]):

A closer examination of the method by which Castelnuovo showed the above led me to the result that if inequality (18) is satisfied then the surface possesses an irrational pencil of curves of hyperbolic of genus 2 (...). A surface for which inequality (18) is replaced with a strict inequality “>” possesses an irrational pencil of rational curves (a ruled surface) or elliptic curves. There are no other surfaces.

²¹In the above Gray’s text [1871] means paper [62] and [1875] means paper [63].

This seems to be the most important of Rosenblatt's results in algebraic geometry since it was cited in Oscar Zariski's monograph *Algebraic Surfaces* from 1935 (see [81, p. 195]; see also [5], [12], [13, pp. 10, 44], [16, pp. 1, 20] and [76]). Several of Rosenblatt's papers were cited in Encyclopedia of Mathematical Sciences [12], [55] and in the survey on algebraic geometry [76].

The classification for the complex projective space was improved by Enriques in 1939 and was extended to non-algebraic surfaces by Kunihiko Kodaira (1915–1997) in the 1960s. Now the classification of nonsingular compact complex surfaces is called also the Enriques–Kodaira classification.

Rosenblatt also pointed out some gaps in Severi's papers [R7], [R9]. Severi's answer is contained in his paper *New contributions to the theory of continuous systems of curves belonging to an algebraic surface* [74].

Rosenblatt was interested in the theory of algebraic surfaces of dimension 3 and more. He examined surfaces satisfying Comessatti's inequality $p_g \leq 3(p_g - p_a - 3)$ in a series of papers [R29], [R30], [R31], and presented his results in 1927 at the Second Meeting of the Polish Mathematical Society in Lvov in the talk *O utworach trzechwymiarowych, których przestrzenie styczne spełniają pewne warunki różniczkowe* [On 3-manifolds with tangent spaces satisfying some differential conditions] and a year later in Bologna, during the International Congress of Mathematicians, in a short talk in the geometry section (in Italian), entitled *Sopra le varietà algebriche a tre dimensioni fra i cui caratteri intercedono certe disuguaglianze* [On three-dimensional manifolds satisfying certain algebraic inequalities] [R39]). The meeting of this session of the Bologna Congress was attended, among others, by: Gino Fano (1871–1952), Guido Fubini (1879–1943), Francesco Severi (1879–1961) and Oscar Zariski (1899–1986). This classification-type result extends to 3-dimensional algebraic surfaces. Let us quote Leonard Roth [69, p. 39] to present the role of its result:

Returning now to the case $d = 2$, we observe that an irregular non-scrollar surface for which $p_g = 0$ necessarily has arithmetic genus $p_a = -1$ (Enriques); moreover, it can be shown that, in addition to the irrational (elliptic) pencil which the surface contains, there is a second pencil, which is rational and free from base points. Comessatti [15] has shown that the surfaces for which $p_g \geq 2(p_a + 2)$, and which, by Castelnuovo's theorem, contain an irrational pencil, contain a second pencil without base points. Beginning with a system of partial differential equations, Comessatti transforms the problem into one of differential line geometry in higher space, and thus arrives at a classification of the surfaces in question [...] Rosenblatt [R39] has applied analogous methods to the threefolds for which $p_g \leq 3(q - 3)$, but has obtained only partial results.

This “partial result” (see also Lefschetz [51, p. 55], Roth [70, p. 276] and Beauville [5, p. 343]) terminated Rosenblatt's research in algebraic geometry. Rosenblatt's results in the field, because of many changes in research methods and significant subsequent progress, are of no great importance today but they are of historical significance.

2.4. Analytic functions

Univalent functions in the unit disc. Let \mathcal{U} denote the class of all functions that are analytic and *univalent* (or one-to-one) in the unit disc $\mathbb{D} = \{z \in \mathbb{C}: |z| < 1\}$ (i.e., if $z_1, z_2 \in \mathbb{D}, z_1 \neq z_2$ implies $g(z_1) \neq g(z_2)$). From the Riemann theorem on conformal mapping we know that any simply connected proper subset of the complex plane is the image of the unit disc \mathbb{D} by an analytic function $g(z)$ such that $g'(0) = c_1 \neq 0$.

By \mathcal{S} we denote the class²² of functions from \mathcal{U} normalized by $f(0) = 0$ and $f'(0) = 1$, i.e.,

$$\mathcal{S} = \{f \in \mathcal{U}: f(0) = 0 \text{ and } f'(0) = 1\}.$$

Each $f \in \mathcal{S}$ has a power series expansion

$$f(z) = z + a_2 z^2 + a_3 z^3 + \dots \quad (22)$$

The most important element of \mathcal{S} is the Koebe function given by

$$k(z) = \frac{z}{(1-z)^2} = z + 2z^2 + 3z^3 + \dots, \quad |z| < 1.$$

The class of maps \mathcal{S} is not closed under addition and multiplication. The class of maps \mathcal{S} is preserved, however, under a number of transformations; for examples, conjugation $g(z) = \overline{f(\bar{z})}$, rotation $g(z) = e^{-i\theta} f(e^{i\theta} z)$ for every $\theta \in \mathbb{R}$, dilation $g(z) = r^{-1} f(rz)$ for every $0 < r < 1$, square-root transformation $g(z) = \sqrt{f(z^2)}$.

In 1916, Ludwig Bieberbach (1886–1982) proved that for any function from \mathcal{S} and for its expansion series the estimate $|a_2| \leq 2$ is true, and the equality $|a_2| = 2$ holds if and only if f is a rotation of the Koebe function. Bieberbach also conjectured that

$$|a_n| \leq n, \quad n = 2, 3, \dots \quad (23)$$

The Bieberbach conjecture attracted the attention of many mathematicians, including Rosenblatt. In a joint paper with Turski [R46] they proved, in 1935, that $|a_4| < 4.50345$, $|a_5| < 6.30956$, and three years later Rosenblatt, after several improvements published in different papers, in [R54] came to the estimates: $|a_4| < 4.2858$, $|a_5| < 5.9158$, $|a_6| \leq 8.3158$. Rosenblatt's estimates were later used by Chernoff [14, p. 472] in his investigations of complex solutions of some partial differential equations. Rosenblatt's proofs of the above estimates used the transformation

$$g(z) = f(z^{-2})^{-1/2} = z - \frac{c_1}{z} + \frac{c_3}{z^3} + \dots$$

and the fact the class \mathcal{S} is preserved under square-root transformation.

These Rosenblatt calculations, however, are currently only mathematical folklore. After Bieberbach's result $|a_2| \leq 2$, Löwner, in 1923, proved that $|a_3| \leq 3$. Then there were several proofs of the Bieberbach conjecture for certain higher values of $n \in \mathbb{N}$; in particular, Garabedian and Schiffer (1955) proved $|a_4| \leq 4$, Ozawa (1969) and Pederson (1968) proved $|a_6| \leq 6$, and Pederson and Schiffer (1972) proved $|a_5| \leq 5$.

In 1984, Louis de Branges proved the Bieberbach conjecture $|a_n| \leq n$ for all natural $n \geq 2$ and that $a_n = n$ only for the Koebe type functions $k_\theta(z) = z/(1 - e^{i\theta}z)^2$ with

²²Instead of “univalent” the German word “schlicht” is used, and so one sometimes speaks of “schlicht functions” — also in the English language literature. That is why the letter \mathcal{S} is traditionally used.

θ any real number. A historical survey is given by Koepf [40], and the first book which includes the proof of the Bieberbach conjecture was written by S. Gong in 1989 in Chinese (English translation [25]).

Many authors were concerned with the following classes of functions that are analytic in the unit disc \mathbb{D} : \mathcal{B} , the class of bounded univalent functions; \mathcal{B}_0 , the class of univalent functions that map \mathbb{D} onto domains of finite area, and \mathcal{C} , the class of functions f given by series $f(z) = \sum_{n=0}^{\infty} a_n z^n$ with $\sum_{n=1}^{\infty} n|a_n|^2 < +\infty$. The inclusion relations

$$\mathcal{B} \subset \mathcal{B}_0 \subset \mathcal{C} \quad (24)$$

hold between these classes. For $f \in \mathcal{C}$ and $0 < r < 1$ let

$$M(r, f) = \max\{|f(z)| : |z| = r\}, \quad \text{and} \quad \lambda(r, f) = \sum_{n=1}^{\infty} |a_n| r^n.$$

The inequality $M(r, f) \leq |a_0| + \lambda(r, f)$ is clearly true.

In 1914, Gronwall [27] proved two estimates for $f \in \mathcal{B}_0$ and $|z| = r < 1$:

$$|f(z)| \leq \sqrt{\frac{A}{\pi}} \sqrt{\log \frac{1}{1-r^2}} \quad \text{and} \quad |f'(z)| \leq \sqrt{\frac{A}{\pi}} \frac{1}{1-r^2}, \quad (25)$$

with the smallest possible upper boundaries. In fact, by the Cauchy–Schwarz inequality

$$|f(z)|^2 \leq \left(\sum_{n=1}^{\infty} |a_n| r^n \right)^2 \leq \sum_{n=1}^{\infty} n|a_n|^2 \sum_{n=1}^{\infty} \frac{r^{2n}}{n} = \frac{A}{\pi} \sum_{n=1}^{\infty} \frac{r^{2n}}{n} = \frac{A}{\pi} \log \frac{1}{1-r^2},$$

and, similarly,

$$|f'(z)|^2 \leq \left(\sum_{n=1}^{\infty} n|a_n| r^{n-1} \right)^2 \leq \sum_{n=1}^{\infty} n|a_n|^2 \sum_{n=1}^{\infty} n r^{2n-2} = \frac{A}{\pi} \frac{1}{(1-r^2)^2}.$$

In 1917, Rosenblatt [R24] explained how from Courant's result (1912) on harmonic functions we can get weaker estimates

$$|f(z)| \leq \sqrt{\frac{A}{\pi}} \log \frac{1}{1-r} \quad \text{and} \quad |f'(z)| \leq \sqrt{\frac{A}{\pi}} \frac{1}{1-r}. \quad (26)$$

Since

$$\sqrt{\log \frac{1}{1-r^2}} < \frac{1}{2} \log \frac{1+r}{1-r} < \log \frac{1}{1-r} \quad \text{for } 0 < r < 1,$$

it follows that Gronwall's inequalities (25) are stronger than estimates (26).

Then, Rosenblatt in [R24] carefully examined Gronwall's proofs and supplemented them a bit.

THEOREM 3 (Rosenblatt 1917). *If $f \in \mathcal{B}_0$, then*

$$M(r, f) = o\left(\sqrt{\log \frac{1}{1-r^2}}\right) \quad \text{and} \quad M(r, f') = o\left(\frac{1}{1-r^2}\right) \quad \text{as } r \rightarrow 1^-. \quad (27)$$

Rosenblatt's method of the proof was, really, a proof that if $f \in \mathcal{C}$, then the first limit in (27) is still 0. In 1954, F. R. Keogh showed by examples that the exponent $1/2$ in the first estimate of (27) cannot in general be replaced by any smaller number.

Rosenblatt's results were extended and improved in [34], [39] and cited in [2], [52]. Further investigations of the behavior of $\lambda(r, f) = o((\log \frac{1}{1-r})^\alpha)$ as $r \rightarrow 1^-$ were investigated in [34] and [39].

In 1943, Rosenblatt [R61] reminded again proofs of (27) and gave some estimates for the partial sums of the coefficients and for the majorant $|a_0| + \lambda(r, f)$ of bounded analytic functions $f(z)$ with $|f(z)| \leq 1$ for $z \in \mathbb{D}$; he also develops analogous inequalities for analytic functions $f(z)$ with bounded first derivatives $|f'(z)| \leq 1$.

Uniqueness result. Riemann’s uniqueness theorem states that if f is an analytic function in the region G and $f^{-1}(0)$ has a limit point in G , then $f \equiv 0$. In his paper [R69, Theorem 1, pp. 40–44], Rosenblatt proved a result which is cited in the book [11, Exercise 5.66, p. 189], with a proof given by S. Saeki.

THEOREM 4 (Rosenblatt 1945). *Let $\theta, \alpha \in [0, 2\pi)$ and let f be an analytic function in $|z| < 1$. Suppose that $|f(re^{i\theta})|$ is constant for $r \in [0, 1)$, and that also $|f(re^{i\alpha})|$ is constant for such r . If $(\theta - \alpha)/\pi$ is irrational, then f is constant.*

Conformal self-maps on the unit disk. In 1917 Pick [67] and independently Rosenblatt [R23] proved the result, which is cited in the book by R. Kühnau [43, Theorem 5.31 on p. 75].

THEOREM 5 (Pick 1917, Rosenblatt 1917). *Let $0 < a < 1$. Then for any conformal mapping $f: \mathbb{D} \rightarrow \mathbb{D}$ such that $f(0) = 0$ and $|f'(0)| = a$ the image $f(\mathbb{D})$ contains a circle with a center at 0 and a radius equal to $(1 - \sqrt{1 - a})^2/a$. In addition, if the image $f(\mathbb{D})$ is a unit disc with a thrown piece of radius (i.e., a segment of the form $se^{it}: s \in [r, 1)$ for a certain $r \in (0, 1)$), then the image $f(\mathbb{D})$ does not contain a larger circle with a center in 0.*

Conformal representation of plane domains. In Rosenblatt’s times a method of solving plane boundary value problems by use of conformal maps was being developed and it was important to provide methods to compute Riemann maps of plane domains, i.e., suitably normalized conformal homeomorphisms of a plane domain onto the unit disk of the complex plane. In 1916, Leon Lichtenstein (1878–1933) considered the problem for a Jordan domain enclosed by a (sufficiently regular) Jordan curve and had proposed an integral equation in order to determine a “boundary correspondence” function associated with the curve and which would determine the boundary values of the Riemann map of the Jordan domain.

Let us mention some papers that have investigated the dependence of the conformal representation upon the domain: Radó [68], work of Kantorovich (1923), as reported in the book Kantorovich and Krylov [38], Rosenblatt [R48], [R50]–[R52], Rosenblatt and Turski [R47], [R49], Zeitlin [82], Yoshikawa [80].

For example in 1936, Rosenblatt [R50] considered a problem of writing an integral representation formula for the Riemann map of a parameter-dependent plane Jordan domain close to the unit circle and to prove a related continuity result upon the parameter. He considers the case of two families.

The first family consists of curves which are perturbations of circle along the normal axes to the circle. Namely, the first family is of the form

$$x(\theta, \lambda) = e^{i\theta} [1 + \varphi(\theta, \lambda)],$$

for all $\theta \in [0, 2\pi]$, when the parameter λ ranges in an interval $[-\lambda_0, \lambda_0]$ for a $\lambda_0 > 0$. The real-valued function φ is continuous, has a Hölder continuous derivative with respect to θ for all $\lambda \in [-\lambda_0, \lambda_0]$ and satisfies due periodicity conditions at the end points 0 and 2π

so that the curve $x(\cdot, \lambda)$ is closed and has a continuous tangent vector at all points. Moreover, $x(\cdot, \lambda)$ is assumed to be a Jordan curve and to enclose a Jordan domain D_λ which contains 0 for all values of $\lambda \in [-\lambda_0, \lambda_0]$. If $\lambda = 0$, then $\varphi(\theta, 0) = 0$ for all $\theta \in [0, 2\pi]$ and D_0 coincides with the unit circle.

The second family is of the form

$$y(\theta, \lambda) = e^{i\theta} + \varphi_1(\theta, \lambda) + i\varphi_2(\theta, \lambda),$$

for all $\theta \in [0, 2\pi]$, when the parameter λ ranges in an interval $[-\lambda_0, \lambda_0]$ for $\lambda_0 > 0$. The real-valued functions φ_1, φ_2 are continuous, have a Hölder continuous derivative with respect to θ for all $\lambda \in [-\lambda_0, \lambda_0]$ and satisfy the due periodicity conditions at the endpoints 0 and 2π so that the curve $y(\cdot, \lambda)$ is closed and has a continuous tangent vector at all points. Moreover, $y(\cdot, \lambda)$ is assumed to be a Jordan curve and to enclose a Jordan domain D_λ which contains 0 for all values of $\lambda \in [-\lambda_0, \lambda_0]$. If $\lambda = 0$, then $\varphi_1(\theta, 0) = \varphi_2(\theta, 0) = 0$ for all $\theta \in [0, 2\pi]$ and D_0 coincides with the unit circle.

Rosenblatt proved some representation formulas, and his results from [R48], [R50]–[R52] were later developed by Zeitlin [82] and Yoshikawa [80]. The monograph of Gaier [24], in which seven papers of Rosenblatt are cited, is a good reference for these results. In Kythe's monograph [44, p. 7], we read that

The small parameters method developed by Kantorovich (1933) was later used by Rosenblatt and Turski (1936) (cf. [R47], [R49]) for conformal mapping of a special type of region. Rosenblatt (1943) ([R63], [R64]) constructed the conformal maps of regions onto the unit disk by Kantorovich method of small parameters and applied it to the dynamic problems of airfoils.

Some other extensions of Rosenblatt's results were considered by Lanza de Cristoforis (cf. [47] and [48]).

Green's function for bounded domains in the plane and in the space. Let D be a finite domain in \mathbb{R}^3 , bounded by a closed surface S , with Green's function $G(P_1, P_2)$. In 1935, Rosenblatt showed that if the surface S has a continuous curvature, then

$$G(P_1, P_2) < K \frac{d_1 d_2}{r^3},$$

where K is a positive constant depending on S and $d_i = \text{dist}(P_i, S)$, $i = 1, 2$, $r = \text{dist}(P_1, P_2)$. In 1939, Keldych and Lavrentieff established the same result for an arbitrary Liapounoff surface. Using a method of Keldych and Lavrentieff, in 1941 Rosenblatt [R57] improved their result to much broader class of surfaces, for example, for surfaces S with bounded curvature.

In 1944, Rosenblatt [R65] presented a result on analogous bounds for bounded domains in the plane. Let P_1, P_2 be points of a bounded simply connected plane domain D and $d_i = \text{dist}(P_i, \partial D)$, $i = 1, 2$, $r = \text{dist}(P_1, P_2)$. If $f(z)$ is an analytic function mapping D onto the interior of the unit circle and if there exist positive numbers α, β satisfying $\alpha \leq |f(z)| \leq \beta$, then Green's function $G(P_1, P_2)$ for D satisfies

$$G(P_1, P_2) \leq \frac{1}{2} \log \left(1 + \frac{4\beta^2 d_1 d_2}{\alpha^2 r^2} \right).$$

2.5. Calculus of variations. Rosenblatt himself, in his self-characterization [Ro1] from 1920, put the calculus of variations as the second in order of importance, after differential equations, of his research interests.

Rosenblatt's first result in this subject was a generalization of the fundamental lemma of the calculus of variations proved by du Bois-Reymond. It is interesting to note that a proof had been attempted in 1854 by Stegmann before du Bois-Reymond proved it in 1879²³.

In [R2] Rosenblatt (see also [R53] for a complete proof), using the theorem that any continuous function can be approximated by polynomials, proved the fundamental lemma of the variational calculus in the following form:

If f is a continuous function on the interval $[a, b]$ and the integral $\int_a^b f(x)h(x) dx = 0$ for all polynomials h that vanish at a and b , then f is identically equal to zero.

In du Bois-Reymond's theorem, h belongs to a larger class of functions $D_0 = \{h \in C^1[a, b] : h(a) = h(b) = 0\}$.

The calculus of variations is a subfield of mathematical analysis that dealing with optimizing functionals, which are mappings from a set of functions to the real numbers. The functionals are often expressed as definite integrals involving functions and their derivatives:

$$I[y] = \int_a^b f(x, y, y') dx = \int_a^b f(x, y(x), y'(x)) dx. \quad (28)$$

A necessary condition for an extremum is the well known Euler–Lagrange differential equation $f_y - \frac{d}{dx}f_{y'} = 0$ or written out

$$f_y - f_{y'x} - f_{y'y'}y' - f_{y'y'}y'' = 0,$$

where f is assumed to be twice differentiable. Solving for y'' , we obtain

$$y'' = \frac{1}{f_{y'y'}}(f_y - f_{y'x} - f_{y'y'}y').$$

Then (x_0, y_0, y'_0) is a regular lineal element if $f_{y'y'}(x_0, y_0, y'_0) \neq 0$ and a singular lineal element if $f_{y'y'}(x_0, y_0, y'_0) = 0$.

Necessary conditions of the second order are the Legendre condition $f_{y'y'} \geq 0$ (or $f_{y'y'} \leq 0$), the Jacobi condition $a^* \geq b$, where a^* is a conjugate point to a , the Weierstrass condition that excess-function $E(x, y, y', k) \geq 0$ for all $x \in [a, b], k \in \mathbb{R}$, where

$$E(x, y, y', k) = f(x, y, k) - f(x, y, y') - (k - y')f_{y'}(x, y, y'),$$

and the Bolza condition. Necessary conditions of Legendre, Jacobi, Weierstrass and Bolza for integrals are not sufficient. Already in 1902, Oskar Bolza presented an example to minimize integral

$$\int_0^1 [a(y')^2 - 4by(y')^3 + 2bx(y')^4] dx, \quad \text{where } a, b > 0 \text{ and } y(0) = y(1) = 0.$$

Then, in 1909, independently Rosenblatt [R3] presented an example

$$\int_0^{x_1} [a(y')^2 + 3by^2(y')^4 - 4bxy(y')^5 + bx^2(y')^6] dx, \quad \text{where } a, b > 0, \quad (29)$$

²³Friedrich Ludwig Stegmann (1813–1891), Emil du Bois-Reymond (1818–1896).

and Hans Hahn (1909) also had his example

$$\int_0^1 [(y')^2 - (y - ax)(y - bx)(y')^4] dx, \quad \text{where } a, b > 0, a \neq b.$$

Rosenblatt in paper [R4], was analyzing again example (29) and a new example, informing about Hahn's above example,

$$\int_0^1 (y')^2 [(y')^4 (y^2 - px)(y^2 - qx) + a] dx, \quad \text{where } p, q, a > 0. \quad (30)$$

In [R4], Rosenblatt observed that we cannot expect sufficient and at the same time necessary analogous conditions to the classical ones. All these examples were mentioned in the French Encyclopedia [50, p. 60].

In papers [R4, R6], Rosenblatt also considered the Lagrange rule in the isoperimetric problem; i.e., find for the functional (28) an extremal under assumption that integral

$$K[y] = \int_a^b g(x, y, y') dx = \int_a^b g(x, y(x), y'(x)) dx \quad (31)$$

is constant. In [R4] he examined the validity of Lagrange's rule when an extremal of the isoperimetric problem is also an extremal of the isoperimetric integral K . In [R4, p. 561] and [R6, pp. 59–60] he gave an example that Lagrange's rule could be false by taking an integral $I = \int_0^1 [a(y')^2 + y] dx$, and as K the integral from (30).

In [R26], Rosenblatt explores the problem of the minimum of the integral (28) in the singular case assuming that the Euler–Lagrange equation $\frac{d}{dx} f_{y'} - f_y = 0$ is linear and analytic, which allows us to apply Fuchs theory of expansions in singular points. He proved that in the case when the Jacobi criterion is true, the Weierstrass and Hilbert methods give a proof of a strong extremum for (28).

In [R20], [R25] and in four articles published in Spanish in Peruvian journals (1938, 1939), he studied the minimum of the integral in the plane

$$J = \int_{t_1}^{t_2} F(x, x', y, y') dt = \int_{t_0}^{t_1} F(x(t), x'(t), y(t), y'(t)) dt. \quad (32)$$

In [R71], which is the last paper of Rosenblatt on calculus of variations, he considers an extremal curve E between two points P_1, P_2 in the plane lying on transversal curves C_1, C_2 , respectively. He demonstrates a reciprocity theorem between the families of extremals E_1 and E_2 that pass through P_1 and P_2 . This theorem is then generalized to the case of an extremal in space, to the case of transversal surfaces as well as transversal curves and finally to the case in which the extremals are restricted to lie on a given surface.

2.6. Probability theory. Rosenblatt published some observations in probability theory and statistics in seven papers: in 1940 (four papers), 1941, 1944, and 1945. Especially frequently cited is his paper [R56], written in connection with Pólya's urn model (cf. [53, p. 281]).

Laws of large numbers and small numbers. Rosenblatt wrote two papers [R55], [R56] in 1940, and one paper [R68] in 1945, on the law of large numbers in the theory of probability.

In the paper from 1945, he was interested in laws of large numbers and small numbers for Bernoulli trials. Repeated independent trials are called *Bernoulli trials* if there are only two possibilities of outcome for each trial and their probabilities remain the same throughout the trials. If $b(k; n, p)$ is the probability that n Bernoulli trials with probabilities p , $0 < p < 1$, for success and $q = 1 - p$ for failure result in k successes and $n - k$ failures, then $b(k; n, p) = \binom{n}{k} p^k q^{n-k}$. If we denote by S_n the number of successes in n trials, then $\Pr\{S_n = k\} = b(k; n, p)$. The function S_n is a random variable, and $b(k; n, p)$ is the “distribution” of this random variable (the binomial distribution) since $\sum_{k=0}^n b(k; n, p) = (q + p)^n = 1$. If $r > np$, then

$$\Pr\{S_n > r\} = \sum_{i=1}^{\infty} b(k+i; n, p) \leq b(r; n, p) \frac{rq}{r - np} \leq \frac{1}{r - np} \cdot \frac{rq}{r - np} = \frac{rq}{(r - np)^2}$$

and so $\Pr\{S_n \leq r\} \leq \frac{(n-r)p}{(r-np)^2}$ if $r < np$.

In [R68], Rosenblatt calculated an explicit estimate for $\varepsilon = \varepsilon(n)$ such that the inequality $\Pr\{|S_n/n - p| < (1 + \varepsilon)((2pq \log \log n)/n)^{1/2}\} > 1 - \varepsilon$ holds for all n , where S_n is the number of successes in the first n trials of an infinite succession of Bernoulli trials. Similar estimates are obtained for the strong law of large numbers.

In the paper [R55] Rosenblatt matched a generalization of the *law of small numbers* (i.e., *Bortkiewicz’s law*), obtained by Pólya and Eggenberger, to plague prevalence in São Paulo, Brazil. This matching, used without reliability tests, turned out not to be the best possible one.

In 1898, Władysław Bortkiewicz (1868–1931) published the book *Das Gesetz der kleinen Zahlen* (*The Law of Small Numbers*) about the Poisson distribution. He described a number of observations on the frequency of occurrence of rare events that appear to follow a Poisson distribution. The Poisson distribution is sometimes called the law of small numbers, because it is the distribution of the probability of occurrence of an event that happens rarely but has many possibilities of happening.

Pólya’s urn model. Urn models have been among the most popular probabilistic schemes and have received considerable attention in the literature (see, for example, [35], [53]). The Pólya urn, which was introduced by Eggenberger and Pólya (1923), was originally applied to problems dealing with the spread of contagious diseases. Later the model was applied in a variety of different areas.

We describe briefly the Pólya urn scheme. From an urn containing $W_0 > 0$ balls labeled 1 (white) and $R_0 > 0$ balls labeled 2 (red), a ball is drawn, its label (color) is noted and the ball is returned to the urn along with additional balls depending on the label of the ball drawn; if a ball labeled i ($i = 1, 2$) is drawn, a_{ij} balls labeled j ($j = 1, 2$) are added. This scheme is characterized by the following 2×2 addition matrix of integers, $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ whose rows are indexed by the label selected and whose columns are indexed by the label of the balls added.

Let W_n and R_n be, respectively, the number of times a white or a red ball is picked in n draws. The object of interest is usually the long-term composition of the urn. After n draws from the urn, one is interested in the exact or asymptotic number of balls of a particular color or the number of times a ball of that color has been selected.

Several of Pólya's urn models have been studied by many authors with various addition matrices. In one of the earliest studies, Eggenberger and Pólya (1923) were concerned with the special case $a_{11} = a_{22}$, $a_{12} = a_{21} = 0$; a detailed discussion can be found in [35]. By the mid-20th century, Bernstein (1940), Savkevich (1940) and Friedman (1949) considered a generalization $a_{11} = a_{22}$, $a_{12} = a_{21}$.

Rosenblatt [R56] studied the case $a_{12} = a_{21} = 0$, $a_{11} \neq a_{22}$ and proved an interesting result in this diagonal urn.

THEOREM 6 (Rosenblatt 1940). *Let $a_{11} > a_{22} > 0$ and consider a generalized Pólya's urn with reinforcement matrix $\begin{bmatrix} a_{11} & 0 \\ 0 & a_{22} \end{bmatrix}$. Then R_n/W_n^ρ , where $\rho := a_{11}/a_{22}$, converges almost surely to a nonzero finite limit.*

Rosenblatt's theorem is formulated and proved in the review article [66, p. 21]. His paper is cited in the books [35, pp. 190, 399], [53, p. 281], and in several papers, for example, [4, p. 182], [22, pp. 389, 398], [41, p. 173] and [54, pp. 148, 162].

2.7. Series. Rosenblatt published three papers on classical series of numbers: [R15], [R18] and [R67]. He investigated in them the Cauchy multiplication of series, the Dirichlet multiplication of Dirichlet series and a generalization of the classical Tauberian theorem. The first two papers were cited in two classical books by Hardy–Riesz [30, p. 77] and Hardy [29, p. 391].

Cauchy multiplication of series. The Cauchy product of two infinite series $\sum_{n=0}^{\infty} a_n$ and $\sum_{n=0}^{\infty} b_n$ is the series

$$\sum_{n=0}^{\infty} c_n, \quad \text{where} \quad c_n = \sum_{k=0}^n a_k b_{n-k} = a_0 b_n + a_1 b_{n-1} + \dots + a_n b_0.$$

This definition is suggested by a consideration of power series. The Cauchy product of two convergent series need not converge, but if the series $\sum_{n=0}^{\infty} a_n = A$ and $\sum_{n=0}^{\infty} b_n = B$ are absolutely convergent, then the series $\sum_{n=0}^{\infty} c_n = C$ is convergent, and $C = AB$. Franciszek (Franz) Mertens (1840–1927), in 1875, proved that it is enough that one of the series be absolutely convergent for the result to hold. On the other hand, it has been known since 1825, thanks to N. H. Abel, that if A, B and C are all convergent, then $C = AB$. In 1890, E. Cesàro generalized Abel's result proving that if A is (C, r) -summable and B is (C, s) -summable with $r > -1$, $s > -1$, then C is $(C, r + s + 1)$ -summable and $C = AB$. This result is sharp (see [29, p. 229]).

Recall that for the series $\sum_{n=0}^{\infty} a_n$ the $(C, 0)$ -method is just the ordinary summation, that is, $\lim_{n \rightarrow \infty} S_n^{(0)} = A$, where $S_n^{(0)} = \sum_{k=0}^n a_k$. The Cesàro (C, r) -summation, $r \in \mathbb{N}$, with Cesàro sum $A \in \mathbb{R}$ means that

$$\lim_{n \rightarrow \infty} S_n^{(r)} / \binom{n+r}{r} = A, \quad \text{where} \quad S_n^{(r)} = \sum_{k=0}^n S_k^{(r-1)} \quad (r = 1, 2, \dots, n = 0, 1, 2, \dots),$$

and we write briefly $\sum_{n=0}^{\infty} a_n = A(C, r)$. Note that (cf. [29, p. 96])

$$S_n^{(r)} = \sum_{k=0}^n \binom{n+r-k}{r} a_k \quad \text{and} \quad \lim_{n \rightarrow \infty} S_n^{(r)} / \binom{n+r}{r} = \lim_{n \rightarrow \infty} \frac{r! S_n^{(r)}}{n^r}.$$

If $s > r > -1$ and $\sum_{n=0}^{\infty} a_n = A(C, r)$, then $\sum_{n=0}^{\infty} a_n = A(C, s)$ (cf. [29, Theorem 43, p. 100]).

In 1908, G. H. Hardy observed that if A and B are convergent and the coefficients are of order $1/n$, that is, $a_n = O(1/n)$, $b_n = O(1/n)$, then C is convergent and $C = AB$. K. Knopp (1907) and S. Chapman (1911) generalized this result, and Rosenblatt in [R15] was able to generalize the results of Hardy, Knopp and Chapman. He showed that the assumptions on the coefficients can be replaced by

$$n\Psi(n)a_n = O(1), \quad \frac{n}{\Psi(n)}b_n = O(1), \quad (33)$$

where Ψ is any function of the form

$$\Psi(n) = (\log n)^\alpha (\log \log n)^\beta (\log \log \log n)^\gamma \dots, \quad \alpha, \beta, \gamma, \dots \in \mathbb{R}.$$

In the same paper, Rosenblatt generalized Hardy's other result connected with a Cesàro type summability.

THEOREM 7 (Rosenblatt 1913). *Let $r > 0$, $s > 0$. If A is (C, r) -summable and B is (C, s) -summable, and for the partial sums*

$$A_n^{(r-1)} = O(n^{r-1}), \quad B_n^{(s-1)} = O(n^{s-1}), \quad (34)$$

then C is $(C, r+s)$ -summable.

This result is cited in Hardy's classical book [29, p. 245].

Dirichlet multiplication of Dirichlet series. A general Dirichlet series is a series of the form

$$\sum_{n=0}^{\infty} a_n e^{-\lambda_n z}, \quad \text{where } a_n, z \in \mathbb{C} \quad \text{and} \quad 0 \leq \lambda_0 < \lambda_1 < \dots, \quad \lambda_n \rightarrow \infty.$$

If $\lambda_n = \ln(n+1)$ then "ordinary Dirichlet series" $\sum_{n=1}^{\infty} \frac{a_n}{n^z}$ becomes for $a_n = 1$ just the Riemann zeta function. If $\lambda_n = n$, the Dirichlet series is a power series in the variable e^{-z} .

The product in the sense of Dirichlet of two Dirichlet series $S_1 = \sum_{m=0}^{\infty} a_m e^{-\lambda_m z}$ and $S_2 = \sum_{n=0}^{\infty} b_n e^{-\lambda_n z}$ is a series

$$S = \sum_{p=0}^{\infty} c_p e^{-\mu_p z}, \quad \text{where} \quad c_p = \sum_{\lambda_m + \lambda_n = \mu_{mn}} a_m b_n$$

and the numbers μ_{mn} are arranged in a non-decreasing order with $mn = p$.

In [R18] Rosenblatt proved a result on the product imposing fewer restrictions on the indices than Hardy in his 1912 paper: If

$$\frac{\lambda_m a_m}{\lambda_m - \lambda_{m-1}} = O(1), \quad \frac{\lambda_n b_n}{\lambda_n - \lambda_{n-1}} = O(1),$$

then the sum S exists and is equal to the product $S_1 \cdot S_2$.

This result of Rosenblatt from [R18], can be found cited in the book by G. H. Hardy and M. Riesz [30, Theorem 58, p. 66]), and Rosenblatt's proof from [R18] was simplified by Landau [46].

Tauberian theorem. A Tauberian theorem is a theorem that deduces convergence of a series from the properties of the function it defines and any kind of auxiliary hypotheses which prevent the general term of the series from converging to zero too slowly. The name goes back to Alfred Tauber (1866–1942), who was the first to prove a theorem of this type for the Abel summation method. It is known that Cesàro summability of a series $\sum_{n=1}^{\infty} a_n$ implies Abel summability, and in 1897 Tauber proved that, under an additional condition on (a_n) , Abel summability implies the usual convergence of the series.

THEOREM (Tauber 1897). *If $\sum_{n=1}^{\infty} a_n$ is Abel summable to A and $a_n = o(1/n)$ as $n \rightarrow \infty$, that is, $\lim_{x \rightarrow 1^-} \sum_{n=1}^{\infty} a_n x^n = A$ and $\lim_{n \rightarrow \infty} n a_n = 0$, then $\sum_{n=1}^{\infty} a_n$ converges and $\sum_{n=1}^{\infty} a_n = A$.*

Rosenblatt [R67] proved that instead of the assumption $\lim_{n \rightarrow \infty} n a_n = 0$ we may have a weaker assumption

$$\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n k a_k = 0, \quad (35)$$

and the Tauberian theorem still holds. Moreover, for $a_k = (-1)^k / \log k$ with $k \geq 3$ the assumption (35) is satisfied, but not the original Tauberian assumption. Next, Rosenblatt generalized his first observation (see [R67, Theorem 2]). Let

$$a_n^{(1)} = \frac{1}{n(n+1)} \sum_{k=1}^n k a_k \quad \text{and} \quad a_n^{(m)} = \frac{1}{n(n+1)} \sum_{k=1}^n k a_k^{(m-1)}, \quad m = 2, 3, \dots$$

THEOREM 8 (Rosenblatt 1945). *Let $m \in \mathbb{N}$ be fixed. If $\lim_{x \rightarrow 1^-} \sum_{n=1}^{\infty} a_n^{(m)} x^n = A$ and $a_n^{(m)} = o(1/n)$ as $n \rightarrow \infty$, then $\lim_{x \rightarrow 1^-} \sum_{n=1}^{\infty} a_n^{(l)} x^n = A$ for all $l = 1, 2, \dots, m$, and both $\sum_{n=1}^{\infty} a_n^{(m)} = A$ and $\sum_{n=1}^{\infty} a_n^{(m-1)} = A$.*

This result is mentioned in Zeller–Beekmann’s book [83, p. 112].

2.8. Applications of mathematics and astronomy. Rosenblatt published also many papers on applications of mathematics, such as theoretical physics, including hydrodynamics, lubrication theory, theory of elasticity, aerodynamics, gravitation, electricity, optics, genetics, music scales and the three-body problem.

Rosenblatt wrote his first paper in theoretical physics *On the general thermoelastic problem* [R5] in Göttingen in 1909. In it, he considered an isotropic elastic medium subject to mechanical and thermal actions. Equations of motion and of thermal conductivity for such a medium constitute a system of four linear partial differential equations.

In the years 1926–1932, he worked intensively in the field of hydrodynamics. He was interested in the movement of flat and spatial viscous fluids, the issue of the stability of laminar movements, all considerations related to the admissibility of linear approximation in the hydrodynamics of viscous fluids. He also examined the scope of applicability of the Kutta–Zhukovsky theorem (on hydrodynamics of perfect fluids).

He studied the behavior of incompressible viscous liquids, including irrotational movement, movement of two tangential layers, Hamel radial movements, determined finite laminar movement disturbances, small vibrations damped between two rotating concentric cylinders, etc. (see, for example, [R36], [R40], [R42] and monographs [R43], [R45]).

A detailed description of Rosenblatt's research in the field of hydrodynamics can be found in the Średniawa paper [77, pp. 130–133].

Rosenblatt was also working on the important astronomical (gravitational) problem of the three-body problem. In the paper [R34] he investigated the case of plane motion of the three-body problem when the total relative angular momentum of them vanishes. Applying the appropriate chosen change of coordinates he reduces the equations of motion to canonical form (see also [R38]).

In [R35], in the same year, he looked to Levi-Civita's research of the three-body problem with non-vanishing total relative angular momentum and derived, thanks to skillful change of variables, the equations obtained previously by Levi-Civita, but in a simpler way. Moreover, Rosenblatt investigated the triple collision and excluded it on the basis of the Levi-Civita method.

In aerodynamics, Rosenblatt applied the Kutta–Zhukovsky mappings to the case when the body has edges. He was also interested in geometrical optics, the theory of elasticity, gravity theory, seismology, lubrication theory, sub-resonance phenomena, cosmology, mathematical genetics, history of exact sciences, even bacteriology.

We can repeat here the opinion of Średniawa about Rosenblatt's results [77, p. 133]:

In theoretical physics Rosenblatt solved the problems in a way satisfactory for mathematicians, but which could not satisfy the physicist, because the author, having obtained the solution of the problem in the form of the system equations, did not solve them, even in approximate way. This lack of explicit solutions did not allow to find their physical interpretation. Rosenblatt's papers were written in a very concise manner. The monography [R45] can be very helpful in studying them, because it collected main Rosenblatt's results, and moreover the calculations, which were unfinished in his original papers were brought to the form useful for physicists.

Rosenblatt's work in the domain of hydrodynamics of incompressible viscous liquids was appreciated by his mathematicians contemporaries. Henry Villat, leader of theoretical research on fluid mechanics, invited Rosenblatt to Paris to give talks there on the solutions of the equations of hydrodynamics, and asked Rosenblatt to write monographs [R43], [R45]. The content of these monographs we discussed in Section 2.1.

Finally, we find Rosenblatt's information about astronomical events in seven publications (cf. [Ro9, p. 33]).

2.9. History of mathematics and reviews. As for the history of mathematics, Rosenblatt wrote articles on the following: Henri Poincaré (1854–1912) [R10], Maurycy Rudzki (1862–1916) as mathematician [R21], Franciszek Mertens (1840–1927) [R37], Émile Picard (1856–1941) [R58], Henri Lebesgue (1875–1941) [R59], and Vito Volterra (1860–1940) [R60], as well as an article of more than thirty pages on Copernicus's place in the history of science [R62]. The paper [R57] is dedicated to the memory of George Green (1793–1841) on the centenary of his death.

Rosenblatt delivered many talks at meetings the Polish Mathematical Society (PTM) in Krakow, and at meetings of the Jagiellonian University Students' Maths and Physics

Society in the years 1906–1909 and PTM Congresses in the years 1916–1935. He participated in the Philosophical Society Seminar in Krakow, where, for example, he spoke on *The concept of group and some recent research on space* (18 June 1909) and he gave the lecture *Is the world infinite? From the book of Genesis to the expanding world of Einstein, Abbé Lemaitre and Eddington* (11 May 1933).

Moreover, in the years 1913–1925 Rosenblatt wrote 133 reviews of papers by many mathematicians, including approximately 15 reviews of papers by S. Mazurkiewicz and W. Sierpiński for *Jahrbuch über die Fortschritte der Mathematik*.

While in Krakow, he wrote reviews of the books by Rudzki, *Gwiazdy i budowa wszechświata* [Stars and structure of the universe, 1912], Witkowski, *Zasady fizyki* [Principles of physics, 1912], Zaremba, *Arytmetyka teoretyczna* [Theoretical Arithmetic, 1912], Rudzki, *Zasady meteorologii* [Principles of meteorology, 1921], Hoborski, *Higher Mathematics*, in two volumes, in French (1923), and Sierpiński, *Zarys teorii mnogości* [Outline of set theory, 1929] and published the reviews in French in the journal *Scientia*, which gave the books good publicity.

In addition, he published a review in Polish of the book written in Italian by Gino Loria, *Pagine di storia della scienza* [Pages of the history of science], from which we learn that the history of science was a mandatory subject in a new type of secondary school in Italy and that Loria, an excellent historian of science, wrote a wonderful textbook in Italian for the teaching of this subject (cf. [R32]). Moreover, he published reviews in French of the book written in German by Aurel Wintner, *Spektraltheorie der unendlichen Matrizen. Einführung in den analytischen Apparat der Quantenmechanik. Mit einer Einleitung von L. Lichtenstein* [Spectral theory of infinite matrices. Introduction to analytical methods of quantum mechanics, with an introduction by L. Lichtenstein], Leipzig 1929, and he published it in the *Annales de la Société Polonaise de Mathématique* 8 (1929), Kraków 1930, 320–322.

In [R22], Rosenblatt expressed his opinion on the impact of war on civilization:

The assertion that war has a civilizing significance seems to us to be a fantastic paradox. (...) Many branches of science receive their greatest advancement through wars. (...) The great progress of modern technique stands in close connection with military technique. (...) war seems to be the educator of nations. (...) One year of war puts more geography and statistics into people's heads than thirty years of peace. (...) Perhaps the greatest war the world has ever seen will also be of the greatest and most important civilizing significance.

While working in Lima, Rosenblatt wrote in Spanish and published in the *Revista de Ciencias* reviews of the following books published in Spanish: Hugo Vieweger, Alfred Holz and Paul Killman *Arithmetic, algebra, planimetria, trigonometry* (1941), Raimundo Götze *Elements of mathematical physics* (1941), G. Mahler, *Plane geometry* (1941) and Cristobal de Losada y Puga, *Course of mathematical analysis*, Vol. I (1946). He strongly criticized the first three books, stating even that Götze's book should be eliminated from libraries in view of the numerous erroneous concepts. In addition, he wrote that Mahler failed to distinguish between axioms and postulates as well as between definitions and theorems.

On 13 April 1942, in the National Academy of Sciences in Lima, Rosenblatt paid tribute to Tullio Levi-Civita, who died on 12th December 1941, and to Émile Picard, who died on 11th December 1941, presenting a history of their lives and scientific achievements. He also wrote a paper on the life and scientific achievements of Picard [R58], Henri Lebesgue, deceased on 26th July 1941 in Paris [R59], and Vito Volterra, deceased 11th October 1940 in Rome [R60]. All these papers are cited, along with information on these mathematicians, by the popular Scottish website, dedicated to the history of mathematics (The MacTutor History of Mathematics archive):

<http://www-history.mcs.st-and.ac.uk/BiogIndex.html>.

In 1943, Rosenblatt, apart from publishing many mathematical papers, participated in the Copernican celebrations. On 24 May 1943 the 400th anniversary of the death of Nicolaus Copernicus (1473–1543) was commemorated at the National Academy of Sciences in Lima. The anniversary ceremonies were not only the celebration of the memory of Copernicus, but also a tribute of the Peruvian society and its intellectual spheres to Poland. Rosenblatt presented the scientific genius of Copernicus and his astronomical system. An extended version of the talk was published in a 34-page article [R62], which was reprinted in a separate publication by the Academy.

Rosenblatt had a large library with over 2,000 books in many languages. When he died, his wife donated all the books and notes (more than 100 notebooks), as well as his papers to the University of San Marcos in Lima. Three volumes of his works are located in the Main Library of the Universidad Nacional Mayor de San Marcos in Lima: *Alfred Rosenblatt, Obras*, Tomo I, 70 pages (ref. Q7.R84, T. I 22261), Tomo II, 170 pages (ref. Q7.R84, T. II 22262), Tomo III, 249 pages (ref. Q7.R84, T. III 22263). Courtesy of Professor Alejandro Ortiz Fernández from Lima, who photocopied all the volumes and sent them to us, we had an opportunity to see Rosenblatt's papers to which access is difficult in Europe. The collection, however, still lacks many papers by Rosenblatt.

It turns out that some archive materials connected with Rosenblatt were not properly stored. Among the documents donated to the University of San Marcos in Lima there was Rosenblatt's entire correspondence, including that with Poincaré and Levi-Civita. It is a pity that no one knows where all these materials are currently located.

3. Publications on Alfred Rosenblatt. The authors of the present paper held talks on Alfred Rosenblatt's life and selected scientific achievements at some conferences dedicated to the history of Polish mathematics in 2012, 2013, and 2015, and at international conferences in 2015 and 2016. In the articles [Ro8] and [Ro9], we collected information related to Rosenblatt's life, and assessed his papers and achievements in mathematics and applications of mathematics. We managed to include the information written in Spanish, as in [Ro13], [Ro20], [Ro14] and [Ro15]. A shortened version of the paper [Ro8] was translated into Russian and published in [Ro12]. An inventory of the titles of Rosenblatt's complete production, which at the moment counts 292 published articles and 7 books as well as his talks and lectures, has been published in [Ro9]. Unfortunately, we still do not possess copies of all his papers.

A more comprehensive overview of Rosenblatt's scientific achievements would require more space since only the list of his published papers alone occupies a dozen pages (as can be checked in [Ro9]). Maybe in the future someone will be able to describe his entire output in more detail, for example, in a monograph.

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Rosenblatt's books, monographs and selected publications

- [R1] *Über die Existenz von Integralen gewöhnlicher Differentialgleichungen* [On the existence of integrals of ordinary differential equations], Arkiv für Mat., Astron. och Fysik 5 (1909), Nr. 2, 4 pp. (in German).
- [R2] *Über das Fundamentallemma der Variationsrechnung* [On the fundamental lemma of the calculus of variations], Arch. der Math. u. Phys. (3) 15 (1909), 284 (in German).
- [R3] *Über die notwendigen Bedingungen des Extremums eines einfachen Integrales in gewöhnlicher Darstellung* [On the necessary conditions for the extremum of a simple integral in ordinary presentation], Arch. der Math. u. Phys. (3) 15 (1909), 284–285 (in German).
- [R4] *Über zwei Fragen der Theorie des Extremums eines einfachen Integrales* [About two questions of the theory of extremum of a simple integral], Math. Ann. 68 (1910), 552–564 (in German).
- [R5] *Über das allgemeine thermoelastische Problem* [On a general thermoelastic problem], Rend. Circolo Math. di Palermo (5) 29 (1910), 324–328 (in German).

- [R6] *Reguła Lagrange’a w zagadnieniu izoperymetrycznem dla całek pojedynczych* [The Lagrange rule in the isoperimetric problem for single integrals], *Prace Mat.-Fiz.* 21 (1910), 55–60 (in Polish).
- [R7] *Sur les surfaces algébriques admettant une série discontinue de transformations birationnelles* [On algebraic surfaces admitting a discontinuous series of birational transformations], *C. R. Acad. Sci. Paris* 153 (1911), 1460–1461 (in French).
- [R8] *Postępy Teorii powierzchni algebraicznych* [Progresses in the theory of algebraic surfaces], *Prace Mat.-Fiz.* 23 (1912), 51–192 (in Polish).
- [R9] *Badania nad pewnymi klasami powierzchni algebraicznych nieregularnych i nad biracjonalnymi przekształceniami nie zmieniającymi tych powierzchni* [Research on certain classes of irregular algebraic surfaces and birational transformations that do not change these surfaces], *Rozprawy Akademii Umiejętności, Wydział Mat.-Przyrodniczy, Kraków* 52 (1912), 100 pp. (habilitation in Polish); published also as: *Sur certaines classes de surfaces algébriques irrégulières et sur les transformations birationnelles de ces surfaces en elles-mêmes* [On certain classes of irregular algebraic surfaces and on the birational transformations of these surfaces in themselves], *Bull. Internat. Acad. Cracovie Cl. Sci. Math. Natur. Ser. A Sci. Math.* 1912, 761–810 (in French).
- [R10] *Henryk Poincaré 1854–1912*, *Czas R.* 45, nr 325, Kraków, 19 July 1912, p. 3 (in Polish).
- [R11] *Sur quelques inégalités dans la théorie des surfaces algébriques* [On some inequalities in the theory of algebraic surfaces], *C. R. Acad. Sci. Paris* 154 (1912), 1494–1495 (in French).
- [R12] *Algebraische Flächen mit diskontinuierlich unendlich vielen birationalen Transformationen in sich* [Algebraic surfaces with discontinuous infinitely many birational transformations in themselves], *Rend. Circ. Mat. Palermo* 33 (1912), 212–216 (in German).
- [R13] *Sur les surfaces algébriques irrégulières de genre linéaire $p^{(1)} > 1$* [On irregular algebraic surfaces of linear genus $p^{(1)} > 1$], *Prace Mat.-Fiz.* 23 (1912), 17–24 (in French).
- [R14] *Sur les surfaces irrégulières dont les genres satisfont à l’inégalité $p_g \geq 2(p_a + 2)$* [On irregular surfaces whose genera satisfy the inequality $p_g \geq 2(p_a + 2)$], *Rend. Circ. Mat. Palermo* 35 (1913), 237–244 (in French).
- [R15] *Über die Multiplikation der unendlichen Reihen* [On multiplication of infinite series], *Bull. Internat. Acad. Cracovie Cl. Sci. Math. Natur. Ser. A Sci. Math. = Krakau Anz. (A)* 1913, 603–631 (in German).
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- [R17] *Sur les surfaces algébriques qui possèdent un faisceau irrationnel de courbes de genre 2* [On algebraic surfaces which possess an irrational pencil of curves of genus 2], *C. R. Acad. Sci. Paris* 156 (1913), 290–292 (in French).
- [R18] *Über einen Satz des Herrn Hardy* [On a theorem of Mr. Hardy], *Jber. Deutsch. Math.-Verein.* 23 (1914), 80–84 (in German).
- [R19] *Sur les surfaces algébriques qui possèdent un faisceau irrationnel de courbes hyperelliptiques de genre deux* [On algebraic surfaces with an irrational pencil of hyperelliptic curves of genus two], *Prace Mat.-Fiz.* 26 (1915), 1–9 (in French).
- [R20] *Über einen Satz der geometrischen Optik und dessen Verallgemeinerung in der Variationsrechnung* [On the theorem of geometrical optics and its generalization in the calculus of variations], *Jber. Deutsch. Math.-Verein.* 24 (1915), no. 10–12, 363–372 (in German).
- [R21] *Maurycy Rudzki jako matematyk* [Maurycy Rudzki as mathematician], *Kosmos* 41 (1916), 119–130 (in Polish).

- [R22] *The civilizing influences of war*, Current History 5 (1916), no. 1, 103–105; originally published in German: *Die zivilisatorische Bedeutung des Krieges*, Nord und Süd 157 (1916), 320–323.
- [R23] *Sur la représentation conforme de domaines limités sur le cercle de rayon un* [On the conformal representation of bounded domains on unit disc], Bull. Internat. Acad. Cracovie Cl. Sci. Math. Natur. Ser. A Sci. Math., Juillet 1917, Cracovie 1917, 184–193 (in French).
- [R24] *Sur la représentation conforme du cercle de convergence d’une série de puissances* [On the conformal representation of the circle of convergence of a power series], Bull. Internat. Acad. Cracovie Cl. Sci. Math. Natur. Ser. A Sci. Math., Nov.–Dec. 1916, Cracovie 1917, 575–585 (in French).
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- [R26] *Sur les problèmes irréguliers du calcul des variations* [On irregular problems of the variational calculus], Prace Mat.-Fiz. 30 (1919), 175–191 (in French).
- [R27] *Geometria analityczna. Cz. I. Na płaszczyźnie* [Analytic geometry. Vol. I. In the plane], the Jagiellonian University Students’ Maths and Physics Society, Kraków 1923, 594 pp. (in Polish).
- [R28] *Geometria analityczna. Cz. II: Przestrzenna* [Analytic geometry. Vol. II. In the space], the Jagiellonian University Students’ Maths and Physics Society, Kraków 1923, 695 pp. (in Polish).
- [R29] *Sur les variétés algébriques à trois dimensions dont les genres satisfont l’inégalité $P_g \leq 3(p_g - p_a - 3)$* [On three-dimensional algebraic varieties whose genera satisfy the inequality $P_g \leq 3(p_g - p_a - 3)$], Prace Mat.-Fiz. 33 (1924), no. 1, 115–124 (in French).
- [R30] *Sur les variétés algébriques à trois dimensions dont les genres satisfont à l’inégalité $P_g \leq 3(p_g - p_a - 3)$* [On three-dimensional algebraic varieties whose genera satisfy the inequality $P_g \leq 3(p_g - p_a - 3)$], C. R. Acad. Sci. Paris 178 (1924), 2222–2224 (in French).
- [R31] *Sur les variétés à trois dimensions dont les espaces tangents satisfont à certaines conditions différentielles* [On three-dimensional varieties whose tangent spaces satisfy certain differential conditions], C. R. Acad. Sci. Paris 179 (1924), 810–812 (in French).
- [R32] Review of the book “Gino Loria, *Pagine di storia della scienza*”, Biblioteca Paravia “*Storia e pensiero*”, 1924, xv+145 pp. [“Gino Loria, Pages of the history of science”, Paravia Library, “History and thought”], Wiadom. Mat. 28 (1924–1925), 47–52 (in Polish).
- [R33] *Geometria analityczna na płaszczyźnie* [Analytic geometry in the plane], The Polish Academy of Learning, Kraków 1926, xv+442 pp. (in Polish).
- [R34] *Sur le cas de la collision générale dans le problème des trois corps* [On the case of general collision in the problem of three bodies], Rendiconti Accad. d. L. Roma (6) 3 (1926), 69–75 (in French).
- [R35] *Sur la régularisation du problème plan des trois corps* [On the regularization of the plane problem of three bodies], Rendiconti Accad. d. L. Roma (6) 3 (1926), 271–274 (in French).
- [R36] *Sur certains mouvements des liquides visqueux* [On certain movements of viscous liquids], Bulletin Sci. Math. (2) 51 (1927), 14–32 (in French).
- [R37] *Działalność naukowa Franciszka Mertensa* [Scientific activity of Franciszek Mertens], Wiadom. Mat. 30 (1927), 79–85 (in Polish).
- [R38] *Sur la régularisation du problème plan des trois corps* [On the regularization of the plane problem of the three bodies], Bull. Sci. Math. (2) 52 (1928), 195–202 (in French).

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- [R41] *Sur la variété de Grassmann qui représente les espaces linéaires à k dimensions contenus dans un espace linéaire à r dimensions* [On the Grassmann manifold which represents the linear spaces with k dimensions contained in a linear space with r dimensions], Mémoires Soc. Roy. Sci. Liège (3) 16 (1931), 1–36 (in French).
- [R42] *Sur certains mouvements plans des liquides visqueux* [On certain plane movements of viscous liquids], Bull. Sci. Math. (2) 55 (1931), 175–192 (in French).
- [R43] *Sur certains mouvements des liquides visqueux incompressibles* [On certain movements of incompressible viscous liquids], Institut de Mécanique des Fluides de l'Université de Paris, Gauthier-Villars, Paris 1933, 41 pp. (in French).
- [R44] *Sur les théorèmes de M. Picard dans la théorie des problèmes aux limites des équations différentielles ordinaires non linéaires* [On the theorems of M. Picard in the theory of boundary problems of ordinary nonlinear differential equations], Bull. Sci. Math. (2) 57 (1933), 100–106 (in French).
- [R45] *Solutions exactes des équations du mouvement des liquides visqueux* [Exact solutions of equations of viscous fluid motion], Mémorial Sci. Math. 72, Gauthier-Villars, Paris 1935, 66 pp. (in French).
- [R46] (with S. Turski) *Sur les coefficients des séries de puissances univalentes dans le cercle unité* [On the coefficients of the series of univalent powers in the unit circle], C. R. Acad. Sci. Paris 200 (1935), 1270–1272 (in French).
- [R47] (with S. Turski), *Sur la représentation conforme de domaines plans* [On the conformal representation of plane domains], C. R. Acad. Sci. Paris 202 (1936), 899–901 (in French).
- [R48] *Sur la représentation conforme de domaines plans* [Conformal representation of plane domains], C. R. Acad. Sci. Paris 202 (1936), 1398–1400 (in French).
- [R49] (with S. Turski), *Sur la représentation conforme de domaines plans* [Conformal representation of plane domains], Bull. Sci. Math. (2) 60 (1936), 309–320 (in French).
- [R50] *Sobre la representación conforme de dominios planos limitados variables* [On the conformal representation of plane domains with limited variables], Revista Ci., Lima 38 (1936), 75–102 (in Spanish).
- [R51] *Sur la représentation conforme de domaines bornés limités par des courbes générales* [On conformal representation of domains bounded by general curves], C. R. Acad. Sci. Paris 202 (1936), 1832–1834 (in French).
- [R52] *Sur la représentation conforme de domaines plans* [Conformal representation of plane domains], Bull. Soc. Math. Grèce 17 (1936), 26–49 (in French).
- [R53] *Sobre el lema fundamental del Cálculo de las Variaciones para las integrales simples* [On the fundamental lemma of the Calculus of Variations for simple integrals], Revista Ci., Lima 39 (1937), no. 421, 57–65 (in Spanish).
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- [R57] *Sobre la función de Green de dominios acotados en el espacio euclidiano de tres dimensiones, George Green 14/7/1793–31/5/1841 in memoriam* [On Green's function of bounded domains in Euclidean space of three dimensions, George Green 14/7/1793–31/5/1841 in memoriam], *Revista Ci., Lima* 43 (1941), 291–318; shorter version in *Actas Acad. Ci. Lima* 4 (1941), 42–52 (in Spanish).
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- [R59] *Henri Lebesgue*, *Revista Ci., Lima* 44 (1942), 357–364 (in Spanish).
- [R60] *Vito Volterra*, *Revista Ci., Lima* 44 (1942), 423–442 (in Spanish).
- [R61] *Sobre las series de potencias en el círculo unitario* [On the power series in the unit circle], *Revista Ci., Lima* 45 (1943), 195–225 (in Spanish).
- [R62] *La posición de Copérnico en la historia de la ciencia* [Position of Copernicus in the history of science], *Revista Ci., Lima* 45 (1943), 409–442 = *Actas Acad. Ci. Lima* 6 (1943), 165–198 (in Spanish).
- [R63] *Sobre el método del Sr. L. Kantorovitch en la teoría del mapeo conforme y sobre la aplicación de ese método a la aerodinámica* [On Mr. L. Kantorovitch's method in the theory of conformal mapping and on the application of that method to aerodynamics], *Actas Acad. Ci. Lima* 6 (1943), 199–219 (in Spanish).
- [R64] *Algunas aplicaciones del método de Kantorovitch de mapeo conforme de dominios planos a aerodinámica* [Some applications of Kantorovitch's method of conformal mapping of plane domains to aerodynamics], *Actas Acad. Ci. Lima* 6 (1943), 236–249 (in Spanish).
- [R65] *Sobre la función de Green en dominios planos* [On Green's function for plane domains], *Revista Ci., Lima* 46 (1944), no. 449, 473–493 (in Spanish).
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- [R68] *Sobre la ley fuerte de los grandes números* [On the strong law of large numbers], *Actas Acad. Ci. Lima* 8 (1945), 7–26 (in Spanish).
- [R69] *Sobre el módulo de las funciones analíticas en el círculo unitario* [On the modulus of functions analytic in the unit circle], *Actas Acad. Ci. Lima* 8 (1945), 27–44 (in Spanish).
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