The valuation of residential rental options with moving thresholds and transaction costs of moving

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Abstract
This paper develops a formula for pricing a residential option with respect to a tenant’s so called outside option. Two parameters are introduced: The tenant’s transaction cost of moving and moving threshold. The derived formula is then used to compute numerical examples of the option price for different parameter values. The pricing formula developed in this paper provides a useful way of conceptualizing and quantifying tenants’ transaction cost of moving, and moving threshold when they are considering to buy a residential option to hedge against strong market rent increases. The numerical examples show that the value of the option increases with higher transaction cost of moving, and decreases with higher moving threshold. With a low moving threshold (i.e. a liquidity constrained household) and/or a high transaction cost of moving, the rental option may have a high value for this household. This value might therefore be higher than the option value calculated the standard way, i.e. without the transaction cost of moving, and moving threshold parameters. The opposite situation occurs for a household with a high moving threshold and/or a low transaction cost of moving.

Keywords: rent regulation · rent control · rental option · real options · transaction costs

JEL Classification R20 · R31 · R38 · D18
1 Introduction

1.1 Background

Various versions of rent control (or rent regulation) systems exist in several housing markets worldwide (see e.g. Lind 2001, 2015; Haffner et al. 2008). Although they have different features, a common characteristic of such rent control policies is that they lack any mechanism that will result in equality between the controlled long run rent levels and the market rent levels. Both economists and landlords usually argue that this discrepancy caused by rent control policies create drawbacks and inefficiencies in housing markets. For instance, there is a widespread agreement that rent control systems discourage new construction, cause abandonment, retard maintenance, reduce mobility, generate mismatch between housing units and tenants, create black markets, exacerbate discrimination in rental housing, encourage the conversion of rental to owner-occupied housing, and generally short-circuit the market mechanism for housing (Arnott 1995; Jenkins 2009). Still, proponents of rent control argue that the distributional benefits and positive welfare effects of rent control, outweigh the inefficiencies and distortions, especially since tenants typically have lower incomes and wealth than other households that own their homes (see Micheli and Schimdt 2015 and references therein).

One reason not to introduce a rental market that only relies on general contract law, is that sitting tenants may have high transaction costs of moving, but also weak bargaining power. Therefore, in order to mitigate the negative effects of regulated rents, as well as to avoid undesirable effects that may occur if there exist no specific rules for the rental market at all besides general contract law, it is of interest to find other ways of protecting at least sitting tenants against major increases in market rents. Ellingsen and Englund (2003) argue that equity and efficiency goals that can be obtained through traditional rent control can be obtained more efficiently either through voluntary contracting or through some other cheaper intervention. Indeed, it is of particular interest to discuss the implementation of “market solutions” based on individual and voluntary contracting when risk reducing devices on the market are both more developed and well known, compared to the time period when rent control policies were introduced. Therefore, it is of interest to consider a regulatory reform that aims at finding some kind of market solution that may serve as a substitute for a regulatory system.

Lind (1999a, b) propose the use of more sophisticated lease agreements and in particular some type of “rental option” contracts, where tenants pay an insurance premium to obtain a protection against high rent increases. Such a market based risk mitigating instrument can indeed be an attractive alternative to traditional rent regulation policies. The proposed option policy may also increase the possibility to find acceptance for letting rents in vacant apartments be set so they reflect supply and demand. There also exist other arguments in favour of market solutions. For instance, with market solutions, adaptation to individual desires can be obtained in a better way. Furthermore, the political risk might be reduced, i.e. the risk that the protection given by rent regulation suddenly disappears due to a change in the political majority.

1.2 Some basic properties of the proposed rental option policy

The aim of the rental option policy presented in this paper is to provide sitting tenants with protection against sharp increases in market rent, for instance due to increases in demand.
The insurance policy may be viewed as a call option. That is, a tenant who owns this insurance has a right to reside in the current apartment after the next rent review paying a rent that is the lower of the market rent and the predetermined rent. In other words, the owner of this option will only exercise it if the market rent exceeds the predetermined rent at the time of the rent review. On the other hand, the tenant will not exercise this option, if the market rent ends up below the strike price, since he or she can pay the lower market rent for the following rental period.\footnote{For this right, the tenant must pay an option premium.} The main points of the proposed option policy are:

- When leasing to a new tenant, rents are based on prevailing market rent levels.
- The landlord is the insurer.
- Tenants who have not purchased any insurance contract, are supposed to pay rents equal to market rents whenever rents are being reviewed.
- For an insured tenant, the new rent will be set to the market rent or the predetermined rent, whichever is lower.

The proposed rental option can constitute an important instrument in a partly deregulated rental system. By going from a regulated rental market to a partly deregulated market might increase the acceptance for rental market deregulation policies.

To clarify the setup, consider the following example. A tenant signs a new rental contract today at market rent. The rental contract states that the rent will be reviewed five years from today for yet another five years. Until that day, the rent will be adjusted with respect to yearly changes in some index, e.g. the consumer price index.\footnote{At the same time as the new rental contract is signed, he or she can also buy a rental option with a maturity date (i.e. exercise date) that is equal to the date for the next rent review. Then, at the time of the rent review, the rent will either be set to the strike rent, or the market rent, whichever is lower. The reviewed rent might again be adjusted according to the changes in some underlying index for yet another five years. Thus the sitting tenant will enjoy a protection against strong market rent increases for a period of ten years in this example, and only face rent increases due to changes in some non-market rent index such as the CPI.} The rental contract states that the rent will be reviewed five years from today for yet another five years. Until that day, the rent will be adjusted with respect to yearly changes in some index, e.g. the consumer price index.\footnote{Indeed, with an indexed rent tied to the CPI, the yearly changes in the indexed rent should be sufficiently foreseeable given a stable interest rate environment.}

Several researchers have studied the problem of deriving appropriate pricing models for different kinds of commercial leases with embedded options (see Amédée-Manesme et al. 2015 and references therein). In particular, so-called upward-only adjusting leases have received much attention among scholars (see for instance Ward and French 1997; Baum et al. 1998; Booth and Walsh 2001a, b; Ambrose et al. 2002; Clapham 2004). But also turnover (or overage or percentage) leases and leases with renewal options have been analyzed in the literature (see e.g. Buetow and Albert 1998; Hendershott 2002; Hendershott and Ward 2003; Clapham 2003; Vimpari and Junnila 2017).

While a commercial renewal option might not only give protection against strong rent increases, but also gives assurance that the lessee can stay in the rented unit for another lease period, the sole purpose of this paper’s proposed rental option is to provide the owner of an residential option an insurance against strong rent increases.

\footnote{We always assume that a tenant has a right to legal protection of tenancy, in the sense that he or she always has a right to reside in the rented apartment as long as he or she “behaves well”.}
1.3 A tenant’s moving threshold and transaction cost of moving

The model developed in this paper explicitly considers the fact that a tenant might choose to exercise his or her right to terminate a current rental agreement. For instance, if the market rent at the time for the rent review ends up above an uninsured tenant’s reservation price for the current apartment, the tenant might choose to terminate the rental contract in order to move to a cheaper apartment. This right, henceforth called the “outside option”, might indeed affect the value of a rental option. Therefore, it is of interest to see how the inclusion of this outside option in a pricing model yields different option prices compared to the “standard” plain vanilla case where the monetary effects of the outside option is not taken into account.

In this paper, we choose to model the impact of the outside option by introducing two parameters: a tenant’s moving threshold (or reservation price) and the same tenant’s total transaction cost of moving to another housing unit. The total transaction cost should be interpreted in a broad sense; it includes both costs for new housing and transaction costs. By introducing these two parameters, we develop a pricing model which implements important outside option considerations that a tenant might care about when he or she will decide upon buying a rental option offered by a landlord, where a tenant’s moving threshold and total transaction costs enter the pricing formula as parameters. The derived pricing formula gives an explicit solution of how to quantify a tenants’ transaction cost of moving and moving threshold when they are considering to buy a residential option to hedge against strong market rent increases.

The rest of the paper is organized as follows. Section 2 presents some basic facts regarding the assumed stochastic process governing the market rent. Section 3 derives a “standard” pricing formula for the residential rental option where a tenant’s outside option is not considered. In section 4, a pricing model for the residential rental option is derived where a tenant’s outside option is taken into consideration by explicitly including a moving threshold and a relocation cost parameter in the pricing model. In Section 5, numerical results are presented and discussed. Finally, section 6 concludes this study and proposes guidelines for future studies and policy implications.
2 The rental process assumption

The real spot market rent at time \( t \geq 0 \) is denoted \( R_t \). This is the constant rent paid at time \( t \) and represents the total rent for the interval \( [t, t + L] \). We assume that \( (R_t) \) follows a geometric Brownian motion (GBM), with constant expected drift rate \( \mu \) and constant volatility \( \sigma \):

\[
dR_t = \mu R_t dt + \sigma R_t dW_t.
\]

(1)

Here \( W \) is a one-dimensional Wiener process. Several papers model \( (R_t) \) in this way (e.g. Grenadier 1996; Booth and Walsh 2001a, b; Clapham 2003, 2004; Bayer et al. 2010), while Grenadier (1995) models the market rent endogenously, determined by current demand and supply.

While the assumed “real-world” or actual rent process (1) above is governed by the objective probability measure or the \( P \)-measure, what really matters in risk neutral valuation is the dynamics under the risk neutral, or \( Q \)-probability, measure (Björk 2004). Here, we denote the risk neutral (or the risk-adjusted) rental process by \( (\tilde{R}_t) \), and assume that it has dynamics

\[
d\tilde{R}_t = \alpha \tilde{R}_t dt + \sigma \tilde{R}_t dV_t,
\]

(2)

where \( \alpha \) denotes the expected risk neutral drift and \( V \) is a Wiener process under \( Q \). By using the Girsanov theorem (Björk 2004), one can show that

\[
\alpha = \mu - \lambda \sigma,
\]

where \( \lambda \) is the market price of risk, and \(-\lambda\) is the “Girsanov kernel” (Björk 2004). To find the market price of risk, several methods are available.

Note that in the original Black and Scholes (1973) option pricing formula for a European call option, the risk-neutral drift of a stock price is simply given by the risk-free interest rate \( r \). By assuming that the risk neutral drift is given by \( r \), as for instance Ward and French (1997) do, the only parameter that has to be estimated in order to use the pricing formula is the volatility, which clearly simplifies the use of the option pricing formula.

Several papers, for instance Grenadier (1995), Booth and Walsh (2001a, b), and Hendershott and Ward (2003) discuss how the risk neutral drift \( \alpha \) can be estimated using the continuous time CAPM. In short, the expected risk neutral drift can be calculated by reducing the expected actual growth rate of rents with the required risk premium on an investment in an asset whose return has the stochastic properties of rents.

3 The standard option pricing model

Here we develop the option pricing model in which we do not model a tenant’s outside option, i.e. by not explicitly modelling the impact of a tenant’s moving threshold and transactions costs. We refer to this as the “standard option”.

The rental option can be considered to be an instrument that provides insurance (or a hedge) against high future rents. For tractability, the option pricing formula will be developed in a two-period setting. At time \( t = 0 \), a tenant signs a new rental contract. The rent for the first rental period is the market rent level given by \( R_0 \). At time \( T_1 \), the rent will be reviewed. The reviewed rent for the second rental period (time \( T_1 \) to \( T_2 \)) will be set to the market rent for the
second period, \( R_1 \), if the tenant has not bought a rental option at time \( t = 0 \). However, if the tenant has bought a rental option, the rent for the second period will either be set to the strike rent \( K \) or to the market rent \( R_1 \), whichever is lower, i.e. the reviewed rent will equal \( \min(R_1, K) \). In this standard model, the tenant is assumed to stay in the same apartment during time \( T_2 \) whether he or she has acquired an option or not.

We assume that the rent level is adjusted annually according to the change in the consumer price index (CPI) until next rent review. With 100% CPI adjusted rents until next rent review, and the rent level during will stay constant in real terms during each of the two lease periods. Figure 1 below shows the two possible rent payment outcomes for this standard model.

![Diagram](image)

**Fig 1.** The two rent payment outcomes for the standard two-period model, in which a tenant is supposed to stay in the same apartment in both rental periods, no matter if the tenant owns a rental option or not.

In order to derive the value of the option based on this two-period setting, we first consider the case when a tenant chooses to not buy an option. While \( R_0 \) is paid at time \( t = 0 \) and therefore is known when the rental contract is written, \( R_1 \) is a random variable which is realized at time \( T_1 \). Thus, the expected present value of rental payments for this tenant is given by

\[
R_0 + e^{-r T_1} E^Q[R_1] = R_0 + e^{(\alpha - r)T_1} \tag{3}
\]

A tenant who wants to hedge against the rent risk can buy a rental option at time \( t = 0 \) with predetermined price \( K \) and time of maturity \( T_1 \). This option gives the insured tenant the right to pay a rent for the second period that is the lower of \( K \) and \( R_1 \). In this case, the rent for the second period is given by the random variable \( \min(R_1, K) \) and hence, the expected present value of total costs now becomes

\[
F(s) + R_0 + e^{-r T_1} E^Q[\min(R_1, K)] \tag{4}
\]

The maximum price a tenant is willing to pay for the option could be derived from indifference pricing. Thus by equating (3) with (4) and solving for \( F(s) \), we get the following expression for the standard option price:

\[
F(s) = e^{-r T_1} E^Q[R_1 - \min(R_1, K)].
\]

Since

\[
R_1 - \min(R_1, K) = \max(R_1 - K, 0), \tag{5}
\]

The price \( F(s) \) can be expressed as the risk neutral valuation formula.
\[
F(s) = e^{-rT}E^Q[\max(R_1 - K, 0)].
\]

Since rents are quoted as per square meter per year and usually paid monthly or quarterly in advance during a several years’ lease period, the payoff function (5) should be premultiplied by the space of the apartment of interest, \(S\), and an appropriate annuity factor, \(A\), respectively, in order to calculate the total option value.\(^3\) In this case the payoff of the option at maturity is given by

\[
S \cdot A \cdot \max(R_1 - K, 0).
\]

However we are mainly concerned about computing and comparing prices per square meter, and therefore it is appropriate to set \(S\) to unity henceforth.

Assuming that the risk neutral market rent process \((R_t)\) is given by the rent process (2), we conclude that with a fixed predetermined rent \(K\), where \(K\) is contractually set at time \(t = 0\), the corresponding call option pricing formula is almost identical with the well-known Black-Scholes formula for the price of a European call option on a non-dividend-paying stock. For our purposes the closed-form pricing formula for a rent option with predetermined strike price \(K\) is given by

\[
F(s) = e^{-rT}A(R_0e^{\alpha T}N(d_1) - KN(d_2)),
\]

where \(N\) denotes the cumulative distribution function of a standard normal distribution, and \(d_1\) and \(d_2\) are given by

\[
d_1 = \frac{\ln(R_0/K) + (\alpha + \sigma^2/2)T}{\sigma \sqrt{T}}
\]

and

\[
d_2 = \frac{\ln(R_0/K) + (\alpha - \sigma^2/2)T}{\sigma \sqrt{T}} = d_1 - \sigma \sqrt{T}
\]

respectively. The expected value of \(R_T\) under the risk neutral measure \(Q\) is

\[
E^Q[R_T] = R_0e^{\alpha T},
\]

and from this result a possibly more intuitively appealing pricing formula can be obtained:\(^4\)

\[
F(s) = e^{-rT}A(E^Q[R_T]N(d_1) - KN(d_2)).
\]

\(^3\) We assume that the annuity amount \(R_T - K\) will remain constant in real terms during the new \(T\) year lease period (or that any real changes are of negligible size).

\(^4\) This formula is used in Hull (2003), chapter 28, example 28.1, which in fact is a real option example considering a lease option.
4 The option pricing model with the outside option

As mentioned above, the two-period model above does not take into account the possibility that a tenant may use his or her right to terminate the current rental agreement; what we in this paper call the outside option. To model the impact of the outside option, we might stipulate that there exist two kinds of tenants: those that are liquidity constrained, and those that are not. Households that are not liquidity constrained are supposed not to face any risk of being forced to move due to a sharp increase in rents. Thus richer, non-liquidity constrained, households that do not speculate only give up a possibility to make a good gain if they do not buy an option. But liquidity constrained and uninsured households do not only give up potential gains, these households also run the risk of having to move to a cheaper apartment if rents reach a certain level, henceforth called the moving threshold. Furthermore, if the rent level at the time of the rent review exceeds the moving threshold, which means that the uninsured tenants must move, the household will also face transaction costs due to the move. This means that liquidity constrained households may be extra eager to acquire a rental option, especially if the moving threshold is low and the tenant’s transaction costs are high.

Below we develop a pricing formula for the rental option when the effect of the outside option is taken into account. This is done by introducing two new parameters: the moving threshold parameter $B$, and the transaction costs parameter $M$. For tractability, we assume that a tenant has a right to terminate the rental contract at the time of the rent review, $T_1$, i.e. when the tenant has knowledge about the size of the rent $R_1$ for the second rental period.

The definition of the moving threshold $B$ is straightforward: If the market rent for the second period ends up below an uninsured and liquidity constrained tenant’s moving threshold, i.e. if $R_1 < B$, he or she will stay in the apartment in the second rental period for sure. On the other hand, if $R_1 \geq B$ at the time of the rent review, this tenant will move to another housing unit due to affordability problems. In this latter case, the tenant will face a transaction cost of size $M$. The definition of the transaction cost parameter in this model is much more subtle and complicated than the definition of the moving threshold. In short, $M$ represents all costs that a tenant that moves will have to pay for the second rental period. In other words, the size of $M$ is the sum of the rent for the new apartment, direct or tangible transaction costs and indirect or intangible transaction costs (see separate discussion below). In the analysis below, both $B$ and $M$ are for tractability assumed to be exogenously given parameters, known at time $t = 0$.

Figure 2 depicts the possible outcomes for this two-period model.

![Fig 2. Possible outcomes for the two-period model where an uninsured will exercise his or her outside option if the market rent level exceeds the tenants moving threshold at the time of the rent review.](image-url)
4.1 The components of the transaction cost parameter

It is a difficult task to define the transaction cost in such a way that its numerical value can be interpreted in an intuitive way in the model. Usually transaction cost refers to costs that can be related to a move, e.g. search costs, direct costs of moving, new social investments and a multiple of other factors (see e.g. Lind 1994). In order to simplify the model, $M$ will here be defined as the sum of the quality adjusted cost/rent of the new housing unit plus the “ordinary” transaction costs. Furthermore, this total transaction cost $M$ is supposed to be evenly distributed during the second rental period. This allows us to compare for instance the rent for the second period for the current apartment with the total transaction costs in a straightforward way in case the tenant moves.

In his analysis of default options embedded in percentage retail leases, Sing (2003) argues that relocation costs for a retail tenant may include tangible items like costs related to reinstating the existing store, fitting-up the new store, search cost, mover costs and commission fee, as well as intangible items like loss of goodwill associated with the existing store location and disruption of business. For the purpose of modelling the default option, Sing (2003) assumes that the relocation cost is a one-time lump sum fixed expense, incurring at the time the default option is exercised.

The same types of costs arise for a household that relocates. The tangible costs are roughly the same as for the retail lease. For the household the intangible items concern various types of utility losses that occur during the (initial) period after the move, e.g. because it takes time to find new friends and learn about the characteristics of the local neighbourhood (best shops, best parks, etc.).

These transaction costs, especially the intangible costs, can be expected to differ much between different types of households. The cost can be very low for a young person with minimal stuff to move and a social network not related to the local neighbourhood. For a family with children that goes to a local day-care centre or a local school, and a strong local network, the cost can be much higher. All figures presented below should be seen as examples of transaction cost estimations, and the various prices for the residential option reflect the prices that a family with different transaction costs would be willing to pay.

4.2 The model

Now consider the pricing of a rental option when a tenant faces a moving threshold. As in the first model, the tenant signs a new rental contract at time $t = 0$, and the tenant’s intention is to stay in the apartment until time $T_2$. At time $T_1$, the rent will be reviewed.

Again, the tenant chooses between two main strategies: buy or not buy an option with predetermined rent $K < B$. If the tenant buys an option at time $t = 0$, he or she will for sure stay in the same apartment until time $T_2$. That is, if $R_1$ ends up above $K$, the insured tenant will exercise his or her option. But now, the situation becomes a little bit more complicated if the tenant chooses to not buy an option, compared with model one. If the market rent for the second period at time of the rent review, $R_1$ ends up below the moving threshold $B$, then the tenant will stay in the apartment until time $T_2$ for sure. But if $R_1$ ends up above the moving threshold, the tenant will move at time $T_1$, and as a consequence, the tenant will face a transaction cost of size $M$ (which, as explained above, also includes the cost for the new housing unit).
How will this affect the size of the option premium a liquidity constrained tenant may want to pay at time \( t = 0 \)? If \( M \) is high, then the tenant may be willing to pay an extra option premium in addition to the standard option price \( F(s) \). But if the tenant expects that \( R_{1,2} \) ends up above his or her transaction cost \( M \), the tenant may be willing to pay an amount that is below the standard option price \( F(s) \). Thus, the option value \( F(o) \) may either be greater or lower than \( F(s) \), depending on the relationship between \( M \) and some expected value of \( R_i \) (we will below derive this result).

Consider first the case when the tenant does not buy an option. Then the expected present value of costs for the two periods is given by

\[
R_0 + e^{-rT_1}E^Q[R_1 \mathbf{1}(R_1 < B) + M \mathbf{1}(R_1 \geq B)]
\]  

(8)

where \( \mathbf{1}(R_1 < B) \) is the indicator function assigning 1 to the case \( R_1 < B \) and 0 otherwise. Similarly, \( \mathbf{1}(R_1 \geq B) \) is the indicator function assigning 1 to the case \( R_1 \geq B \) and 0 otherwise.\(^5\)

In other words, if the market rent for the second rental period, \( R_1 \) is below the moving threshold \( B \) at the time of the rent review, the tenant will stay in the apartment he or she currently lives in. But if \( R_1 \) exceeds \( B \) at the time of the rent review, the tenant will move, paying a transaction cost of size \( M \) (which also includes the cost for the new housing unit, see above).\(^6\)

We let \( F(o) \) denote the price a tenant who considers the level of a moving threshold in the pricing of the option is willing to pay. Again, assume that the tenant at time \( t = 0 \) buys a rental option with predetermined rent \( K \) and maturity date \( T_1 \), which implies that the rent for the second period is given by the random variable \( \min(R_1, K) \). A tenant who buys an option will face the costs

\[
F(o) + R_0 + e^{-rT_1}E^Q[\min(R_1, K)].
\]  

(9)

Equating (8) and (9) and solving for \( F(o) \) yields

\[
F(o) = e^{-rT_1}E^Q[R_1 \mathbf{1}(R_1 < B) + M \mathbf{1}(R_1 \geq B) - \min(R_1, K)].
\]  

(10)

Again, note that

\[
\min(R_1, K) = R_1 - \max(R_1 - K, 0).
\]

Therefore (10) can be expressed as

\[
F(o) = e^{-rT_1}E^Q[R_1 \mathbf{1}(R_1 < B) + M \mathbf{1}(R_1 \geq B) - R_1 + \max(R_1 - K, 0)].
\]

We can now observe that the last term on the right hand side is nothing else but the payoff function of the rental option at time \( T_1 \), and therefore its value is given by standard option price \( F(s) \). Now define the extra premium a liquidity constrained household may be willing to pay as

\[
\pi = F(o) - F(s).
\]

\(^5\) More generally, the indicator function of a subset \( A \) of a set \( B \) is the function with domain \( B \), whose value is 1 at each point in \( A \) and 0 at each point not in \( A \).

\(^6\) Naturally, we must assume that \( M > B \), otherwise the tenant will move at any time.
Notice also that
\[ R_1 \mathbf{1}(R_1 < B) - R_1 = -R_1 \mathbf{1}(R_1 \geq B). \]

Hence, we may write
\[ \pi = e^{-rT_1}E^Q[(M - R_1)\mathbf{1}(R_1 \geq B)]. \]

By definition, the expectation of a random variable \( X \) restricted to the event \( A \) with \( P(A) > 0 \), i.e. \( E[X1_A] \), is given by
\[ E[X1_A] = E[X|A]P(A). \]

Note that if the expectation is taken under the risk neutral measure \( Q \), then \( P(A) \) is exchanged with \( Q(A) \). We are now ready to state the following result.
\[ \pi = F(o) - F(s) = e^{-rT_1}E^Q[M - R_1|R_1 \geq B]Q(R_1 \geq B). \]

**Proposition 1**

*Given the two-period model above, the extra option premium a household may be willing to pay for a rental option, when a known constant moving threshold \( B \) and a constant transaction cost \( M \) are taken into consideration, is given by
\[ \pi = F(o) - F(s) = e^{-rT_1}E^Q[M - R_1|R_1 \geq B]Q(R_1 \geq B). \]

Since \( e^{-rT_1} > 0 \) and \( Q(R_1 \geq B) > 0 \), we see that the sign of \( \pi \) is determined by the sign of \( E^Q[M - R_1|R_1 \geq B] \). We also see that the total price of the rental option with strike price \( K \), moving threshold \( B \) and transaction cost \( M \), is simply
\[ F(o) = F(s) + \pi. \]
There is a natural economic interpretation of formula (11): the premium $\pi$ equals the present value of the difference between the transaction cost of moving and the expected market rent for the second period, given that $R_1$ exceeds the moving threshold at time of renewal, multiplied with the value of the.

This result tells us that the size of $F(o)$ is the result of two offsetting effects. Consider for instance an increase of the drift $\alpha$. Recall that $\alpha = \mu - \lambda \sigma$, so a change in $\mu$, $\lambda$ or $\sigma$ will change the drift of $(R_t)$ under $Q$. This will result in a higher expected value of $R_1$. Then first, with a higher expected value of $R_1$, the expected payoff of the standard option, $\max(R_1 - K, 0)$, will increase, so $F(s)$ should increase as well. Second, a higher expected value of $R_1$, given $M$, decreases the value of the extra premium according to formula (11).

To illustrate the discussion above, we consider the following two cases. Recall that we always assume $B > K$. First, if the outcome at the time of the rent review is such that $M > R_1 > B$, then the corresponding total payoff of owning a rental option is given by

$$ (R_1 - K) + (M - R_1) = M - K. $$

That is, an insured tenant will pay $K$ for the second period, instead of moving and paying $M$. Because $M > R_1$, the total payoff in this case is larger than the payoff given by the standard payoff $R_1 - K$, i.e. $M - K > R_1 - K$. In this case, a tenant “would” have been willing to pay an extra premium $\pi$ for the rental option.

On the other hand, the extra premium $\pi$ can be negative, implying that $F(o) < F(s)$. Consider the case $R_1 > M > B$. The total payoff is again given by Equation (X), but now $M < R_1$. Hence, the total payoff in this case is $M - K < R_1 - K$. In this case, a tenant might not be willing to pay an extra premium $\pi$ for the rental option.

The extra premium is zero if $E^Q[M - R|R_1 \geq B] = 0$. In this case the tenant will be indifferent between staying in the current apartment, and moving to another housing unit, implying that $F(o)$ and $F(s)$ will be of equal size.

Note that the rental option will not be exercised if the market rent for the second period ends
up below the strike price $K$, implying that the option will be worthless. Also note that when $R_{1,2}$ is only slightly above $K$, a tenant’s net profit (payoff from option minus option premium paid) may be negative (see numerical examples below).

4.3 Explicit pricing formula for the extra option premium

Given the assumption that the rental process follows a geometric Brownian motion under $Q$, we now obtain the explicit pricing formula for the extra premium assuming that $M$ is a known constant (and including the annuity factor $A$):

$$\pi = e^{-rT_1}A(MN(z_2) - R_0e^{\alpha T_1}N(z_1)),$$  \hspace{1cm} (12)

where

$$z_1 = \frac{\ln(R_0/B) + (\alpha + \sigma^2/2)T}{\sigma\sqrt{T}}$$

and

$$z_2 = \frac{\ln(R_0/K) + (\alpha - \sigma^2/2)T}{\sigma\sqrt{T}} = z_1 - \sigma\sqrt{T}$$

Again, $N$ denotes the cumulative distribution function of a standard normal distribution. As usual, if rental payments of equal size are evenly distributed during the second rental period, it is appropriate to use the annuity factor $A$.

We can observe that the premium is equivalent to a long position in a binary (or digital) cash-or-nothing call with cash payoff $M$ and a short position in a binary asset-or-nothing call. Hence, we can compute the value of the premium given by (12) as the difference between the value of a cash-or-nothing call and an asset-or-nothing call$^7$.

The total price of the rental option, $F(o)$, is given by

$$F(o) = F(s) + \pi,$$  \hspace{1cm} (13)

where $F(s)$ and $\pi$ are given by (7) and (12), respectively.

It might be interesting to see when the option premium $\pi$ will be zero (that is, for which $M = M^*$ is the standard option price $F(s)$ equal to $F(o)$). By setting (12) equal to zero and solving for $M$, we get

$$M^* = R_0e^{\alpha T_1} \frac{N(z_1)}{N(z_2)}.$$  \hspace{1cm} (14)

It can be shown that ratio of the two distribution functions in the previous equation always exceeds one, so for all $M > M^*$, $F(o)$ exceeds $F(s)$, and vice versa.

$^7$ A detailed derivation from the authors is available on request.
4.4 The special case $B = K$

From general option theory, it is well-known that the price of a European call option becomes more valuable as the strike price decreases, holding all other variables affecting the call option price fixed, and vice versa. Thus there exists a tradeoff between paying a low option premium now and enjoying low rents in the future. We cannot expect that a tenant will have a possibility to choose an arbitrary strike price that will fit his or her moving threshold exactly, but for the sake of the analysis, suppose now that the moving threshold $B$ and the predetermined rent $K$ in fact coincides. This means that a tenant will pay the lowest possible premium for obtaining a rental option with the highest possible strike price, given the size of the moving threshold $B$. If the predetermined rent exceeds the moving threshold, the tenant will naturally avoid buying an option.

Consider again the strategy when a tenant chooses to not buy an option. Then the expected present value of costs is given by (8) above, i.e.

$$R_0 + e^{-rT_1}E^Q[R_11(R_1 < B) + M1(R_1 \geq B)].$$

Now observe that the expression within the expectation can be rewritten as

$$\min(R_1, B) + (M - B)1(R_1 \geq B),$$

implying that the expected present value of costs can be expressed as

$$R_0 + e^{-rT_1}[E^Q[\min(R_1, B)] + E^Q[(M - B)|(R_1 \geq N)]].$$

(15)

A tenant who buys an option at time $t = 0$ will face the expected present value of costs given by the formula in Equation (9) above. But since we now consider a predetermined rent $K$ that equals the moving threshold $B$, Equation (9) becomes

$$F(o) + R_0 + e^{-rT_1}E^Q[\min(R_1, B)].$$

(16)

Therefore, by equating Equations (15) and (16) we obtain that the price of the option when $B = K$ is given by the formula

$$F(o) = e^{-rT_1}[E^Q[(M - B)1(R_1 \geq B)] = e^{-rT_1}(M - B)Q(R_1 \geq B).$$

Recall that the expected value of the indicator function above equals the risk neutral probability that the market rent for the second period ends up above the moving threshold, $Q(R_1 \geq B)$. As mentioned above, in order to evaluate this probability, we can assume that the evolution of the market rent follows another process than the geometric Brownian motion given by (2) above. But in order to compare $F(o)$ with the pricing formula for $F(s)$ above, we must assume that rent follows the geometric Brownian motion specified by Equation (2). In this case, $Q(R_1 \geq B) = N(z_2)$. Thus, when $K = B$, and including the annuity factor $A$, the closed-form pricing formula for $F(o)$ when $K = B$ is given by

$$F(o) = e^{-rT_1}A(M - B)N(z_2),$$

(17)

Note that we can directly obtain (17) by summing the formula for $F(s)$, given by (7), with the formula for $\pi_t$ given by (11). To get the desired result, just replace $K$ with $B$ in (7), and set $S$ and $A$ to one.
where \( z_2 \) is again given by

\[
    z_2 = \ln\left(\frac{R_0}{K}\right) + \frac{(\alpha - \sigma^2/2) T}{\sigma \sqrt{T}}.
\]

Of course, Equation (17) will yield the same result as Equation (15) when \( B = K \).

Above, we argued that the transaction cost \( M \) couldn’t be lower than the moving threshold \( B \), otherwise the tenant will always move. In Equation (17), we see directly that when \( B = K \), then, since \( N(z_2) \) cannot be less than zero, \( M \) must be greater than \( B \) in order to obtain a positive option value. Indeed, when \( M = B \), then the option value will have an intrinsic value of zero.

**Numerical results**

This chapter looks at the input parameters (factors) that affect the value of the rental option according to the pricing models discussed above. These parameters are the current rent or rent for the first period \( R_0 \), strike rent \( K \), time to maturity/rent review \( T_1 \), length of lease period \( L \), risk-free interest rate \( r \), Annuity factor \( A \), risk neutral drift rate \( \alpha \), market rental volatility \( \sigma \), moving threshold \( B \), and finally transaction cost \( M \).

A short discussion of each of the parameters that enters the pricing formulas for \( F(s) \) and \( F(o) \) is presented in section 5.1. In section 5.2, the results the simulation of the standard option price \( F(s) \) are presented. Section 5.3 presents how the option price \( F(o) \) changes with different assumptions of \( B \) and \( M \).

All numerical figures and calculations are in real terms. All monetary figures are in SEK per square meter and year.

**5.1 Assumption of the parameters affecting option prices**

We again consider a two-period model as we did in chapter three; at time \( t = 0 \), a tenant both starts to rent a housing unit, and buys a rental option to obtain a hedge against high rental payments for the second lease period. The rent levels for the two periods are given by \( R_0 \) and \( \min(R_1, K) \) respectively. The rent will be reviewed at time \( T_1 \), \( L \) years from \( t = 0 \).

**5.1.1 Time to maturity/rent review and length of lease period**

The maturity date of the option, \( T_1 \), will naturally coincide with the date for the rent review. The value of the option is supposed to increase as \( L \) increases. One obvious reason is that as \( L \) increases, so does also the insured second period, since the length of each of the two lease periods are assumed to be equal. Furthermore, the likelihood for future rent levels to exceed the strike rent is an increasing function of the time to maturity. Here, we consider the lease length of both periods, \( L \), to be five years. Since the tenant is supposed to buy the option at time \( t = 0 \), \( T_1 \) will be equal to \( L \).
A large expected positive yearly payoff \((R_t - K)\) during the second lease period may result in a high option value because the effect of the option will last for \(L = 5\) years. Therefore, it may be necessary to spread out the cost of the option. A natural alternative is to spread out the cost of the option over the first lease period by including the part-payments in the ordinary rental payments. For instance, when the lease length is five years, the cost of the option will be divided into five payments (assuming rents are paid annually in advance as we do in this paper). Since the calculated option prices, \(F(s)\) and \(F(o)\), are present values, the landlord should be compensated for the time value of money. To achieve this, the sizes of the future yearly option payments during the first period are simply compounded using the real interest rate \(r\). The average option payments per year are denoted \(F_{avg}(s)\) and \(F_{avg}(o)\) respectively.

The following simple example clarifies the discussion above. A tenant signs a new rental contract today (at time \(t = 0\)) for a 70 square meter apartment. The rent that amounts to \(R_o = 1,000\) SEK per square meter and year for the first five years, shall be paid annually in advance. The rent will be reviewed after five years for another five-year period. The tenant also buys a rental option with a maturity date that naturally equals the date of the rent review (i.e. five years from \(t = 0\)). Assume furthermore that the price of the rental option \(F(s)\) is set to 140 SEK per square meter. This implies that the total cost of the option amounts to 9,800 SEK \((140 \cdot 70)\). Consequently, the total size of the first part-payment will amount to 1,960 SEK \((9,800/5)\), or 28 SEK per square meter \((140/5)\). This part-payment will be paid with the first rental payment in the beginning of year 1. The second part-payment will come about with the second rental payment and so on. To compensate the landlord for the time value of money, the size of the second part-payment will equal \(28 \cdot (1 + r)\), or \(28 \cdot \exp(r)\) with continuous compounding, and so on, where \(r\) is the interest rate. The average size of the part-payments, \(F_{avg}(s)\) or \(F_{avg}(o)\), is simply the arithmetic mean of the five part-payments. For instance, with \(r = 3\%\), \(F_{avg}(s)\) will equal 29.8 SEK per square meter, assuming continuous compounding. Consequently, the ratio of the average size of the part-payments over the annual rental payment, \(F_{avg}(s)/R_o\), is in this example 3.0%.

5.1.2 Current rent

The value of the option increases as the current rent \(R_o\) increases (given that the exercise price \(K\) is held constant). The reason is simply because the higher the current rent is, the higher is the likelihood that the rental option will be exercised, i.e. that \(R_1 > K\). Furthermore, the likelihood that the payoff \(\max(R_1 - K, 0)\) will be larger will also increase with higher \(R_o\) (given that the exercise price is held constant).

Here, the rent for the first period, \(R_o\), is set to 1,000 SEK per square meter and year, assumed to be paid annually in advance. It is assumed that the rent is adjusted annually according to the change in the consumer price index (CPI) until next rent review. We assume that rents are 100% CPI adjusted. Since all quantities are expressed in real terms, this implies that the rent during each of the two lease periods will stay constant in real terms. This also implies that we can use an annuity factor \(A\) to discount the eventually yearly payoffs of the option during the second lease period to time \(T_1\) (see below).
5.1.3 Strike price

At time \( t = 0 \), a tenant has the opportunity to buy a rental option with strike price \( K \) for the second rental period. Using same reasoning as above, we can establish that the option becomes less valuable as the strike prices increases.

Naturally, there must be some rules regarding strike price alternatives. Firstly, we cannot expect a system where tenants have a possibility to choose any level of the strike rent. Secondly, landlords must offer options with strike prices that can be considered to be within a reasonable range. Thus, there is a need for more or less standardized options. One alternative is to establish that landlords must offer option alternatives with at least three different strike prices. For instance, the three different strike rents offered can be calculated as the current rent grossed up with 2%, 2.5% or 3% annual real growth.

Here, following three different strike rent will be used: 1,100, 1,150, and 1,200. Table 1 below presents the different strike price alternatives and the corresponding yearly real growth rates.

<table>
<thead>
<tr>
<th>( K )</th>
<th>Annual real growth rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,100</td>
<td>1.9%</td>
</tr>
<tr>
<td>1,150</td>
<td>2.8%</td>
</tr>
<tr>
<td>1,200</td>
<td>3.6%</td>
</tr>
</tbody>
</table>

Table 1. Relationship between strike price \( K \) and annual real growth rate in rents, when current rent is 1000.

5.1.4 Risk-free interest rate

The effect of an increase in the real interest rate on the option value may be difficult to infer. An increase in the risk-free real interest rate affects the option value negatively through the discount factor in the pricing formula, all other factors remained fixed. On the other hand, an increase in the real interest rate may reflect some kind of boom or expected boom, and thus, higher expected growth rate in rents, which will have a positive effect on the option price. The real interest rate has varied considerably since the early twentieth century, however there is an ongoing debate and research concerning the determinants of the significant global downward trend in natural or equilibrium real interest rates the last three decades in developed countries, from about 6 percent to about zero percent, or even to negative territory (see e.g. Carvalo et al. 2017; Beyer and Wieland 2019). Here, the real risk-free interest rate is assumed to be 3% throughout all numerical calculations below, which crudely reflects an historical median figure.
5.1.5 Annuity factor

The annuity factor $A$ is used to calculate the value of the yearly payoffs $R_i - K$ that occurs during the second lease period to time $T_1$. Since it is supposed that rents are paid annually in advance, the annuity factor is calculated as

$$A = \sum_{i=0}^{T_1} e^{-r_i}.$$

5.1.6 Risk neutral drift rate

A higher expected actual real world growth rate $\mu$, or drift, in rents will obviously increase the expected rental growth. When risk neutral valuation is applied, what matters is the risk neutral drift $\alpha$. As explained above, the expected risk neutral growth rate in rents can be calculated by reducing the expected actual real growth rate to the risk neutral expected growth rate by incorporating a risk premium of size $\lambda \sigma$, where $\lambda$ is the so called market price of risk.

It is outside the scope of this paper to calculate different values of $\alpha$ based on the continuous time CAPM. Instead, following Clapham (2004), we just suppose that the expected risk neutral drift will range from some more or less arbitrary minimum value to a maximum value, though not greater than the real interest rate $r$. Nevertheless, the higher expected the risk neutral drift, the greater is the expected option value. Here, given that the real interest rate is 3%, we let $\alpha$ range from $0\%$ to $2\%$.

Another expression of the risk neutral drift is the (real) risk-free interest rate minus the yield; $r - d$, where the yield $d$ can be interpreted as the initial yield of for instance a multi-family property. Note that the risk neutral drift may be negative as well (see Clapham 2004) and this might occur if the initial yield $d$ exceeds $r$.

5.1.7 Market rental volatility

The more volatile market rents, denoted by $\sigma$, the higher the chance for the option to have a high value: the downside risk is limited to the option premium paid, while the potential gains from higher rents might be very large. Based on IPD annual rental data set over the period 1976 to 1997, Booth and Walsh (2001b) estimate the volatility to be just over 11%. Sinai and Souleles (2005) found that the standard deviation of real rents in 44 MSAs during the 1990s in the U.S. ranged from 1.4% to 7.2%. Here, to count for a potential high volatility market rent environment (resulting in higher options prices compared to a low volatility environment), we will base our numerical examples of following three volatility inputs: 7.5%, 10% and 15%.

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9 See Hendershott and Ward (2003) for some examples on how risk neutral drift can be calculated using CAPM.
5.1.8 Transaction cost and moving threshold

The transaction cost and moving threshold parameters enter the pricing formula for the rental option, \( F(o) \), which considers the so called “outside option”. According to the two-period model developed in section 3.2 above, an uninsured tenant will at time \( T \), either choose to stay in the current apartment and pay a rent of size \( R \), or choose to move to another apartment, depending on whether \( R \) ends up below the moving threshold or not. In case the tenant chooses to move, a transaction cost of size \( M \) will arise.

As mentioned above, the transaction cost parameter is supposed to include both the rent for the alternative housing unit (with lease period \( L \)) as well as other direct and indirect costs related to a move (see discussion in section 3.2 above). Here, three different levels of the transaction cost will be applied: 1,200, 1,300, and 1,400 (SEK per square meter and year). As usual, the costs are supposed to be paid annually in advance during the second lease period.\(^1\) Naturally, the option price \( F(o) \) increases as \( M \) increases.

![Fig 4. The timing convention with two time periods.](image)

It is as argued above that there might exist large differences in transaction costs between different households group. In order to get some feeling for what are interesting magnitudes it might therefore be more interesting to focus on what a change in \( M \) implies for a change in direct and indirect transactions cost. If we are looking at a household that rents a 70 square metre apartment, an increase in \( M \) with 100 SEK (for instance from 1,200 to 1,300) means over a 5-year period roughly an increase in transactions cost of \( 140 \cdot 70 \cdot 5 = 35,000 \).

To simplify the presentation below, we will concentrate on cases for which the moving threshold \( B \) equals the strike rent (i.e. \( B = K \), see table 1 above). Recall that this will result in the lowest possible option premium. Table 2 summarizes the discussion above.

<table>
<thead>
<tr>
<th>Table 2. Key input parameter values.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Lease period, ( L )</strong></td>
</tr>
<tr>
<td><strong>Time to maturity, ( T )</strong></td>
</tr>
<tr>
<td><strong>Initial/current rent, ( R_0 )</strong></td>
</tr>
<tr>
<td><strong>Strike rent, ( K )</strong></td>
</tr>
<tr>
<td><strong>Interest rate, ( r )</strong></td>
</tr>
<tr>
<td><strong>Drift rate (risk neutral), ( \alpha )</strong></td>
</tr>
<tr>
<td><strong>Volatility, ( \sigma )</strong></td>
</tr>
<tr>
<td><strong>Moving threshold, ( B )</strong></td>
</tr>
<tr>
<td><strong>Transaction cost, ( M )</strong></td>
</tr>
</tbody>
</table>

\(^{1}\) This assumption simplifies the calculations. Naturally, transaction costs, especially direct moving costs arise at the time of a move. Other components of the transaction cost may decline over the years.
5.2 Numerical examples of the standard option price, $F(s)$.

Table 3 presents our numerical examples of the standard option price $F(s)$, given by formula (6) or (7) above, for different drift and volatility environments and various strike price assumptions. This table shows the average yearly payments of the standard option premium in relation to the size of the yearly rental payments during the first rental period, given by the ratio $F_{\text{avg}}(s)/R_0$.

**Table 3.** Average yearly part-payments of option premium in relation to yearly rental payments during the first rental period; $F_{\text{avg}}(s)/R_0$. The length of the two lease periods equals time to maturity; $L = T_1 = 5$ years. The rent for the first lease period is $R_0 = 1,000$. Strike rent is $K$. Real risk-free interest is $r = 3\%$. Real spot market rent follows a geometric Brownian motion with constant risk neutral drift $\alpha$ and volatility $\sigma$.

<table>
<thead>
<tr>
<th>$K$</th>
<th>$\sigma = 7.5%$</th>
<th>$\sigma = 10%$</th>
<th>$\sigma = 15%$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,100</td>
<td>2.7%</td>
<td>4.5%</td>
<td>8.2%</td>
</tr>
<tr>
<td>1,150</td>
<td>1.7%</td>
<td>3.3%</td>
<td>6.9%</td>
</tr>
<tr>
<td>1,200</td>
<td>1.1%</td>
<td>2.5%</td>
<td>5.8%</td>
</tr>
</tbody>
</table>

Panel A: $\alpha = 0\%$

<table>
<thead>
<tr>
<th>$K$</th>
<th>$\sigma = 7.5%$</th>
<th>$\sigma = 10%$</th>
<th>$\sigma = 15%$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,100</td>
<td>4.3%</td>
<td>6.3%</td>
<td>10.4%</td>
</tr>
<tr>
<td>1,150</td>
<td>3.0%</td>
<td>4.9%</td>
<td>8.8%</td>
</tr>
<tr>
<td>1,200</td>
<td>2.0%</td>
<td>3.7%</td>
<td>7.5%</td>
</tr>
</tbody>
</table>

Panel B: $\alpha = 1\%$

<table>
<thead>
<tr>
<th>$K$</th>
<th>$\sigma = 7.5%$</th>
<th>$\sigma = 10%$</th>
<th>$\sigma = 15%$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,100</td>
<td>6.6%</td>
<td>8.7%</td>
<td>12.9%</td>
</tr>
<tr>
<td>1,150</td>
<td>4.7%</td>
<td>6.9%</td>
<td>11.1%</td>
</tr>
<tr>
<td>1,200</td>
<td>3.3%</td>
<td>5.4%</td>
<td>9.5%</td>
</tr>
</tbody>
</table>

Panel C: $\alpha = 2\%$

As expected, we can see that the ratio $F_{\text{avg}}(s)/R_0$ increases with higher values of the expected risk neutral drift $\alpha$ and higher values of the volatility $\sigma$, but decreases with higher strike prices $K$. Note that the share of the average yearly option (insurance) premiums in relation to yearly rental payments seem to be of reasonable size (say less than 5% or 6% of rental payments), as long as $\alpha$ does not exceed 1% and $\sigma$ does not exceed 10%. For larger values of $\alpha$ or $\sigma$, the ratio $F_{\text{avg}}(s)/R_0$ probably becomes too large to be an attractive alternative for a majority of the tenants.

5.3 Numerical examples with the outside option, $F(o)$

The analysis in chapter three shows that there may be some theoretical justification for the idea that the existence of an outside option might affect the price of a rental option. In order to compute numerical examples of the rental option price $F(o)$, that is, the option price that includes the effect of the outside option, formula (13) will be applied. This pricing formula contains the moving threshold parameter $B$, and the transaction cost parameter $M$. 
In order to keep the analysis as simple as possible, we consider the special case when the moving threshold equals the strike rent \( K = B \). As explained above, this implies that the option price will be lowest possible with respect to \( K \) (recall that a tenant will not sign an option contract if \( K > B \)).

5.3.1 Base results

Table 4 presents the numerical results for a base case scenario \((K = B = 1,150; \alpha = 1\%)\) under different volatility assumptions and where the transaction cost \( M \) varies from 1,200 to 1,400. For comparison, we also present average option expenses based on the calculations of \( F(s) \) in table 3 above (within parenthesis). The size of \( M^* \), i.e. the size of the transaction cost that will make the standard option price \( F(s) \) to equal \( F(o) \), is also presented (see formula 14). Appendix A presents sensitivity analysis for other strike price alternatives and drift assumptions.

Table 4. Average yearly part-payments of option premium in relation to yearly rental payments during the first rental period; \( F_{avg}(o) / R_0 \) and \( F_{avg}(s) / R_0 \) respectively (latter within parenthesis). The length of the two lease periods equals time to maturity; \( L = T_1 = 5 \) years. The rent for the first lease period is \( R_0 = 1,000 \). Strike rent \( K = B = 1,150 \). Real risk-free interest is \( r = 3\% \). Spot rent follows a geometric Brownian motion with risk neutral drift \( \alpha = 1\% \) and volatility \( \sigma \). When \( M = M^* \), \( F(s) = F(o) \), i.e. the premium \( \pi = 0 \).

<table>
<thead>
<tr>
<th>( K = B )</th>
<th>( M )</th>
<th>( F_{avg}(o)/R_0 ) (Average per year)</th>
<th>( F_{avg}(s)/R_0 ) from table 2 above</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha = 1% )</td>
<td>( \sigma = 7.5% )</td>
<td>(3.0%)</td>
<td>(4.9%)</td>
</tr>
<tr>
<td>1,150</td>
<td>1,200</td>
<td>1.2%</td>
<td>1.3%</td>
</tr>
<tr>
<td>1,150</td>
<td>1,300</td>
<td>3.5%</td>
<td>3.9%</td>
</tr>
<tr>
<td>1,150</td>
<td>1,400</td>
<td>5.8%</td>
<td>6.6%</td>
</tr>
<tr>
<td>( M^* = 1,280 )</td>
<td>( M^* = 1,340 )</td>
<td>( M^* = 1,460 )</td>
<td></td>
</tr>
</tbody>
</table>

The numerical results show that the as \( M \) increases, \( F(o) \) also increases, which is quite intuitive. What may be less clear-cut is the relationship between \( F(s) \) and \( F(o) \). For instance, consider the case when the volatility is 7.5\%. Then as already shown above in table 3, the ratio \( F_{avg}(s)/R_0 \) amounts to 3.0\%. Furthermore, when a household’s transaction cost amounts to \( M^* = 1,280 \), then \( F(s) \) and \( F(o) \) will be equal (i.e. \( F_{avg}(s)/R_0 = F_{avg}(o)/R_0 \)). For all other values of \( M \), the extra premium \( \pi = F(o) - F(s) \) will either be positive (when \( M > 1,280 \)), or negative (when \( M < 1,280 \)). Thus the implementation of the outside option by introducing the moving threshold parameter \( B \) and the transaction cost parameter \( M \) in the option pricing model (compare figure 1 and 2) reveals rather interesting price effects; given that the moving threshold equals the strike price \( B = K \), the option price determined the “standard way”, \( F(s) \), may either be lower or higher than the option price that takes into account the outside option \( F(o) \), depending on the size of a household’s transaction cost.

The numerical results suggest that it may be important for a household to take into consideration the size of the household’s moving threshold and transaction cost in order to form an opinion of what a reasonable option price might be. In particular, if rental option prices are determined the “standard way” by the insurer (i.e. the landlord), then the standard option price \( F(s) \) may either be lower or higher than a household’s willingness to pay for the
option $F(o)$, depending on the household’s estimation of the size of its moving threshold and transaction cost.

It is not likely that a household will have the opportunity to choose a strike rent $K$ that will perfectly fit the household’s moving threshold $B$. Instead it is more likely that there will be some standard alternatives offered. For instance, consider the case in which a household’s moving threshold amounts to 1,150, but that only two strike price alternatives are available: $K = 1,100$ and $K = 1,200$. Since the higher strike price alternative is not an interesting alternative (a household will not buy an insurance where $K > B$), the only available option alternative is the one with strike price 1,100. This means that the option price will be higher, holding all other factors constant, since the option price increases with lower strike prices. As mentioned above, the minimum option price will be obtained when $K = B$.

Table 5 differs from table 4 in only one way: the strike price is now 1,100 SEK per square foot and year and not 1,150. First, as expected when the strike price becomes lower, we can see that the standard option price increases (as usual presented as the ratio $F_{avg}(s) / R_0$; see values within parenthesis). For instance, while the price ratio $F_{avg}(s) / R_{0,1}$ amounts to 3.0% when $K = 1,150$ and $\sigma = 7.5\%$, the price ratio is now 4.3% due to a lower strike price. Consequently, $F(o)$ and therefore each value of the price ratio $F_{avg}(o)/R_{0,1}$ in table 5 is higher than each of the corresponding values in table 4. To conclude, we may say that it might be important for a landlord to offer a minimum number of different standard strike price alternatives with, say at most 100 SEK between each alternative.

Table 5. Average yearly part-payments of option premium in relation to yearly rental payments during the first rental period; $F_{avg}(o)/R_0$ and $F_{avg}(s)/R_0$ respectively (latter within parenthesis). The length of the two lease periods equals time to maturity; $L = T_1 = 5$ years. The rent for the first lease period is $R_0 = 1,000$. Strike rent $K = 1,100$. Moving threshold $B = 1,150$. Real risk-free interest is $r = 3\%$. Spot rent follows a geometric Brownian motion with risk neutral drift $\alpha = 1\%$ and volatility $\sigma$.

<table>
<thead>
<tr>
<th>$K (B)$</th>
<th>$M$</th>
<th>$F_{avg}(o)/R_{0,1}$ (Average per year)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>within parenthesis: $F_{avg}(s)/R_0$ from table 2 above</td>
</tr>
<tr>
<td>$\alpha = 1%$</td>
<td>$\sigma = 7.5%$</td>
<td>$\sigma = 10%$</td>
</tr>
<tr>
<td>1,100 (1,150)</td>
<td>1,200</td>
<td>(4.3%)</td>
</tr>
<tr>
<td>1,100 (1,150)</td>
<td>1,300</td>
<td>2.5%</td>
</tr>
<tr>
<td>1,100 (1,150)</td>
<td>1,400</td>
<td>4.8%</td>
</tr>
<tr>
<td>1,100 (1,150)</td>
<td>1,400</td>
<td>7.1%</td>
</tr>
</tbody>
</table>
6 Conclusion and policy and research implications

The rental option vehicle discussed in this paper is supposed to be a “market based” risk mitigating instrument that gives a protection similar to rent regulation, in a sense that it provides sitting and insured tenants with protection against sharp increases in market rent.

This paper proposes a formula for pricing a residential option with respect to a tenant’s so called outside option. The pricing formula developed in this paper provides a useful way of conceptualizing and quantifying tenants’ tenant’s transaction cost of moving and moving threshold when they are considering to buy a residential option to hedge against strong market rent increases. As could be expected, the proposed option valuation model predicts that the value of an option increases with higher transaction cost of moving, and decreases with higher moving threshold. These findings imply households to reflect over what could be a reasonable level of its moving threshold and transaction cost of moving. With a low moving threshold (i.e. a liquidity constrained household) and/or a high transaction cost of moving, the rental option may have a high value for this household. This value might therefore be higher than the option value calculated the standard way. Naturally, the opposite situation occurs for a household with a high moving threshold and/or a low transaction cost of moving.

The proposed rental option can also be combined with some minor elements of rent regulation. Indeed, by going from a regulated rental market to a partly deregulated market, at least initially, might increase the acceptance for deregulation policies. For instance, a partly deregulated policy regime might comprise a rule that obliges landlords to offer tenants a rental option. Such an obligations should in order to meet its intention, be combined with a ceiling for the option premium. Furthermore, the landlord might under a partly deregulated system also be obliged to offer tenants a certain number of insurance period durations, e.g. 5 or 10 years. If rent regulation is replaced with a system where it is mandatory for the landlord to offer a rental option, and where a ceiling of say 4% is introduced for the price for the option, the calculation above can also be used to find “reasonable” terms for the option. Using the results presented in table 4, it can be argued that such an option would protect against real rent increases higher than roughly 3% per year. A lower strike rent would mean a subsidy from the landlord to the tenant, a subsidy that might be motivated taking into account the advantages for the landlord of a deregulated market.
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Appendix A

The three tables presented in this appendix show how option prices, (presented as average yearly part-payments of the option premium in relation to yearly rental payments during the first rental period; \( F_{avg}(o)/R_{0,1} \) and \( F_{avg}(s)/R_{0,1} \) respectively), change with different parameter assumptions. All monetary figures are in SEK per square metre and year. Table A1 shows results for zero drift assumption \((\alpha = 0\%)\), table A2 for \(\alpha = 1\%)\, and finally table A3 for \(\alpha = 2\%)\.

As we did in section 4.2, we assume for simplicity that the household has an opportunity to buy a rental option with a strike price \(K\) that equals the household’s moving threshold \(B\).

As expected, the expenses for the option premium increases with higher rental drift assumptions. For example, when the risk neutral drift increases from 0% to 1% (assuming \(K = B = 1,100, M = 1,200, \sigma =7.5\%)\), the option payment ratio \(F_{avg}(o)/R_{0,1}\) increases from 2.2% to 3.1%. We also notice the relatively large decline in the expense ratio that follows an increase in the strike price \(K\) (and consequently the moving threshold \(B\)).

Table A1. Average yearly part-payments of option premium in relation to yearly rental payments during the first rental period; \(F_{avg}(o)/R_{0,1}\) and \(F_{avg}(s)/R_{0,1}\) respectively (latter within parenthesis). The length of the two lease periods equals time to maturity; \(L = T_1 = 5\) years. The rent for the first lease period is \(R_{0,1} = 1,000\). Strike rent is \(K\). The moving threshold \(B\) is for simplicity set to equal \(K\). Real risk-free interest is \(r = 3\%)\. Real spot rent follows a geometric Brownian motion with risk neutral drift \(\alpha = 0\%)\ and volatility \(\sigma\).

<table>
<thead>
<tr>
<th>(K = B)</th>
<th>(M)</th>
<th>(F_{avg}(o)/R_{0,1}) (Average per year)</th>
<th>Within parenthesis: (F_{avg}(s)/R_{0,1})</th>
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</thead>
<tbody>
<tr>
<td>(\alpha = 0%)\</td>
<td>(\sigma = 7.5%)\</td>
<td>(2.7%)</td>
<td>(4.5%)</td>
</tr>
<tr>
<td>1,100</td>
<td>1,200</td>
<td>2.2%</td>
<td>2.6%</td>
</tr>
<tr>
<td>1,100</td>
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<tr>
<td>1,100</td>
<td>1,400</td>
<td>6.7%</td>
<td>7.6%</td>
</tr>
<tr>
<td>(\alpha = 15%)\</td>
<td>(1.7%)</td>
<td>(3.3%)</td>
<td>(6.9%)</td>
</tr>
<tr>
<td>1,500</td>
<td>1,200</td>
<td>0.8%</td>
<td>1.0%</td>
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<td>1,500</td>
<td>1,300</td>
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<td>3.0%</td>
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<td>1,500</td>
<td>1,400</td>
<td>3.9%</td>
<td>5.0%</td>
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<tr>
<td>(\alpha = 15%)\</td>
<td>(1.1%)</td>
<td>(2.5%)</td>
<td>(5.8%)</td>
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<tr>
<td>2,000</td>
<td>1,200</td>
<td>0.0%</td>
<td>0.0%</td>
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<td>2,000</td>
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<td>1.0%</td>
<td>1.5%</td>
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<tr>
<td>2,000</td>
<td>1,400</td>
<td>2.1%</td>
<td>3.1%</td>
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</table>
Table A2. Average yearly part-payments of option premium in relation to yearly rental payments during the first rental period; $F_{avg}(o)/R_{o,1}$ and $F_{avg}(s)/R_{o,1}$ respectively (latter within parenthesis). The length of the two lease periods equals time to maturity; $L = T_i = 5$ years. The rent for the first lease period is $R_{o,1} = 1,000$. Strike rent is $K$. The moving threshold $B$ is for simplicity set to equal $K$. Real risk-free interest is $r = 3\%$. Real spot rent follows a geometric Brownian motion with risk neutral drift $\alpha = 1\%$ and volatility $\sigma$.

<table>
<thead>
<tr>
<th>$K = B$</th>
<th>$M$</th>
<th>$F_{avg}(o)/R_{o,1}$ (Average per year)</th>
<th>Within parenthesis: $F_{avg}(s)/R_{o,1}$</th>
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</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$\alpha = 1%$</td>
<td>$\sigma = 7.5%$</td>
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<td></td>
<td></td>
<td></td>
<td>$\sigma = 10%$</td>
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<td></td>
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<td>$\sigma = 15%$</td>
</tr>
<tr>
<td>1,100</td>
<td>1,200</td>
<td>(4.3%)</td>
<td>(6.3%)</td>
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<td>(7.5%)</td>
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| Table A3. Average yearly part-payments of option premium in relation to yearly rental payments during the first rental period; $F_{avg}(o)/R_{o,1}$ and $F_{avg}(s)/R_{o,1}$ respectively (latter within parenthesis). The length of the two lease periods equals time to maturity; $L = T_i = 5$ years. The rent for the first lease period is $R_{o,1} = 1,000$. Strike rent is $K$. The moving threshold $B$ is for simplicity set to equal $K$. Real risk-free interest is $r = 3\%$. Real spot rent follows a geometric Brownian motion with risk neutral drift $\alpha = 2\%$ and volatility $\sigma$.

<table>
<thead>
<tr>
<th>$K = B$</th>
<th>$M$</th>
<th>$F_{avg}(o)/R_{o,1}$ (Average per year)</th>
<th>Within parenthesis: $F_{avg}(s)/R_{o,1}$</th>
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<tbody>
<tr>
<td></td>
<td></td>
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<td>$\sigma = 7.5%$</td>
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</table>