Modelling of cavity partial discharges
at variable applied frequency

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Abstract

The presence of partial discharges (PD) in high voltage components is generally a sign of defects and degradation in the electrical insulation. To diagnose the condition of high voltage insulation, PD measurements is commonly used. The Variable Frequency Phase Resolved PD Analysis (VF-PRPDA) technique measures PD at variable frequency of the applied voltage. With this technique, the frequency dependence of PD can be utilized to extract more information about the insulation defects than is possible from traditional PD measurements at a single applied frequency.

In this thesis the PD process in a disc-shaped cavity is measured and modelled at variable frequency (0.01 – 100 Hz) of the applied voltage. The aim is to interpret the PD frequency dependence in terms of physical conditions at the cavity. The measurements show that the PD process in the cavity is frequency dependent. The PD phase and magnitude distributions, as well as the number of PDs per voltage cycle, change with the varying frequency. Moreover, the PD frequency dependence changes with the applied voltage amplitude, the size of the cavity and the location of the cavity (insulated or electrode bounded).

A physical model is presented and used to dynamically simulate the sequence of PDs in the cavity at different applied frequencies. The simulations show that essential features in the measured PD patterns can be reproduced. The PD frequency dependence is interpreted as a variation in influence on the PD activity from the statistical time lag of PD and the charge transport in the cavity surface, at different applied frequencies. The simulation results also show that certain cavity parameters, like the cavity surface conductivity and the rate of electron emission from the cavity surface, change with the time between consecutive PDs, and accordingly with the applied frequency. This effect also contributes to the PD frequency dependence.
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Cecilia Forssén
Stockholm, May 2008
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Chapter 1

Introduction

1.1 Background

The presence of partial discharges (PD) in high voltage components is generally a sign of defects and degradation in the electrical insulation. Partial discharges are localized electrical discharges that bridge only part of the insulation between electrodes. In solid insulation, PD at defects cause local degradation of the insulation material which eventually may lead to breakdown [1].

Breakdown of the insulation in high voltage components can cause failure of the whole component. For components like power cables, power generators and high voltage machines, failures are often costly and cause large disturbances. Insulation diagnostics is a common tool to examine the insulation in high voltage components and diagnose its condition. The diagnose can be used to plan for maintenance or replacements of the components. In this way failures can be avoided and money can be saved.

Partial discharge measurements have been used in insulation diagnostics for a long time. A common electric PD measurement method is the Phase Resolved Partial Discharge Analysis (PRPDA) technique [2]. With this method the PDs are analyzed with respect to the phase of the applied voltage. The results can be used to recognize the insulation defects that cause the discharges [3]. Usually PRPDA measurements are done with an applied voltage with frequency 50 (60) Hz.

A possible further development of the PRPDA technique is to vary the frequency of the applied voltage. This is done in the Variable-Frequency
PRPDA (VF-PRPDA) technique [4, 5]. The benefit of varying the applied frequency is that the local conditions at defects in the insulation alter with the varying frequency. Such local conditions are the electric field distribution and the influence of certain characteristic times on the PD process at the defects. As a result of the change in local conditions, also the PD process alters with the varying frequency. This PD frequency dependence can be utilized to extract more information about the defects than is possible from traditional PRPDA measurements. As an example, Figure 1.1 shows results from VF-PRPDA measurements on two in-service aged stator bars from a hydro-power generator. At applied frequency 50 Hz, the measured total charge per voltage cycle is the same for the two stator bars, indicating they have similar insulation conditions. However, at lower applied frequencies, there is a large difference in the measurement results, pointing to quiet different insulation conditions in the two bars. This demonstrates that PD measurements at variable applied frequency may contain more information than measurements at a single applied frequency.

Figure 1.1: Results from VF-PRPDA measurements on two in-service aged stator bars (epoxy-mica insulation) from a hydro-power generator (from Paper I).
1.2 Aim of work

An additional advantage of the VF-PRPDA technique is the possibility to measure PD at low frequency, thereby reducing the power need of the voltage supply. This is especially important for highly capacitive test objects like power cables and generators. Other PD measurement methods that utilize the benefit of reduced power need at low frequency are the very-low frequency method, where PD is measured at 0.1 Hz, and the damped AC method, where the test object is stressed by a damped AC voltage with a frequency somewhere in the range 20 – 1000 Hz [6, 7].

For interpretation of VF-PRPDA measurements it is crucial to know how the varying applied frequency influences the PD process at the defects in the insulation. Such knowledge is also useful for analysis of PD measurements at 0.1 Hz and with the damped AC method. A number of earlier experimental works have studied PD in cavities at different applied frequencies [8–14]. Variations in PD magnitude as well as in the number of PDs per voltage cycle and the apparent charge per voltage cycle are reported for measurements in the frequency range 0.1 – 100 Hz. There are also experimental investigations showing differences between PD measurements at 50 Hz and PD measurements at 0.1 Hz or with the damped AC method [15]. However, in [7] similar results are reported for PD measurements with the damped AC method and at the power frequency. Earlier modelling works on cavity PD at different applied frequencies are presented in [14–18] and in Paper II. In these it is suggested that the frequency dependence of the PD process in a cavity can be described by use of certain characteristic times related to the statistical time lag of PD and to the charge transport on the cavity surface and in the solid insulation.

1.2 Aim of work

In this work the PD process in a cavity is measured and modelled at variable frequency of the applied voltage. The aim is to present a description of the PD frequency dependence based on the physical conditions at the cavity. The overall goal is to be able to interpret the results from VF-PRPDA measurements in terms of physical characteristics of the PD sources.

This work is a continuation of an earlier project at KTH Electrical Engineering (division of Electromagnetic Engineering) where a Variable-Frequency PRPDA measurement system was developed by Hans Edin,
Uno Gäfvert and Juleigh Giddens [4,5]. The author has earlier presented a Licentiate thesis on measurements and modelling of cavity PD at variable applied frequency [18].

1.3 Main contributions

The main contribution of this work is the development of a physical model of PD in a cavity that is able to reproduce essential features of measured PD patterns at different applied frequencies. The model is used to interpret the results from VF-PRPDA measurements in terms of physical conditions at the cavity.

In the model the discharge process in the cavity is modelled dynamically and the apparent charge is calculated by time integration of the current through the electrode. This is a new modelling approach that was first introduced in Paper II. It gives a charge consistent model without need for λ-functions [19] and analytical estimations of the apparent charge [20].

The time dependent electric field distribution in the test object is calculated by use of the finite element method (FEM). This method has not been used for simulating the sequence of PDs in a cavity before. One benefit of using FEM is its ability to handle complex geometries.

The simulation results presented in this work point out that certain cavity parameters, like the cavity surface conductivity and the rate of electron emission from the cavity surface, change with the time between consecutive PDs, and accordingly with the applied frequency. Hence the constant characteristic times used in [14–18] to describe the PD frequency dependence are actually not constant but may vary with the applied frequency.

In addition, the simulation results indicates that the decay of surface charge in a cavity, through conduction on the cavity surface, should be modelled with a surface conductivity that depends on the amount of charge on the surface. Earlier models of PD in a cavity based on [20] use constant cavity surface conductivity.
1.4 Author’s contributions

The author is responsible for Papers III – VI. In Paper I the author participated in the measurements and performed part of the data analysis. In Paper II the model and simulation program were developed by Prof. Uno Gäfvert (ABB Corporate Research, Västerås, Sweden) and the author only contributed to a minor part by running simulations.

The work has been supervised by Dr. Hans Edin (KTH Electrical Engineering). Prof. Uno Gäfvert has contributed with many valuable comments and ideas.

1.5 Thesis outline

This thesis is based on Papers I – VI. The papers are appended at the end of the book and their content is summarized in Chapter 7.

The thesis also contains an extended summary of the papers. Chapter 1 gives the background to the work and a literature review on PD frequency dependence. Chapter 2 is an introduction to partial discharges in cavities. Chapter 3 discusses modelling of cavity PD in general with references to the models presented in Paper II and Paper VI. Chapter 4 is based on Paper V and describes the phase resolved PD measurements at variable applied frequency and the main measurement results. Chapter 5 describes measurements of PD inception voltage and is based on Paper III. Chapter 6 summarizes the content of Paper VI and describes the model of PD in a cavity at variable applied frequency and the main simulation results. Chapter 7 gives a summary of the Papers I – VI. Finally, in Chapter 8 conclusions from the work are drawn and in Chapter 9 future work is suggested.
Chapter 2

Partial discharges in cavities

2.1 Introduction

This work concentrates on PD in cavities. A cavity is a gas-filled void in a solid insulation material. Cavities appear due to manufacturing errors or due to aging of the insulation material [1]. A cavity is a weak point of the insulation since it has generally lower permittivity and lower electric breakdown strength than the surrounding solid insulation. This causes local electric field enhancement in the cavity and, at high applied electric fields, PDs in the cavity. Partial discharges in a cavity degrade the insulation material through a combination of chemical, mechanical, thermal and radiative processes [21]. Especially, the cavity surface is eroded and solid discharge by-products form on the surface. This causes local electric field enhancements and accordingly concentration of the PDs. This can lead to inception of electrical trees and eventually to breakdown of the insulation [1, 22].

There are two necessary conditions for a PD to start in a cavity: the electric field must exceed a critical value and there must be an initial free electron available to start an electron avalanche. If the electric field is below the critical value, the electron generation is too small to make the discharge self-sustained. The breakdown field of dry air at $20^\circ$C and 1 bar is about 4.7 kV for 1 mm electrode separation. Figure 2.1 shows a schematic picture of a PD in a cavity. The PD ionizes the gas
in the cavity and the resulting charge moves in the electric field and gets trapped in charge traps at the cavity surfaces. The charge build-up at the cavity surfaces opposes the applied electric field and eventually lead to extinction of the discharge.

The charge that a PD generates in a cavity is called the physical charge and the portion of the cavity surface that the PD affects is called the discharge area. The charge that is measured in a (VF-)PRPDA measurement is the charge change at the electrodes of the test object. This is called the apparent charge or the PD magnitude [23].

Figure 2.1: Schematic picture of a PD in a cavity. Here $E_{\text{applied}}$ is the applied electric field and $q_{\text{physical}}$ is the physical charge.

2.2 Statistical time lag

Initial free electrons in a cavity with ongoing PD activity are mainly generated through surface emission from the cavity walls [20, 24]. Electrons are released by the electric field from shallow traps in the cavity surface, and also due to ion and photon impact. These processes can approximately be described with the Richardson-Schottky law for field enhanced thermionic emission [20]. The emission of electrons increases with the electric field and with the amount of electrons in shallow traps in the surface. In virgin cavities that have not yet experienced PDs, ini-
2.3 Surface charge decay

Partial free electrons are mainly generated by radiative gas ionization due to background radiation [20]. In this case the emission of electrons is approximately constant.

If there is a lack of free electrons in a cavity, the electric field in the cavity can exceed its critical value for PD without starting any discharge. The average waiting time for a free electron to appear (from that the field condition for PD is fulfilled) is called the statistical time lag ($\tau_{\text{stat}}$). At sinusoidal applied electric field the effect of the statistical time lag is to shift PDs forward in phase to larger temporal values of the applied field. This results in larger PD magnitudes. The statistical time lag decreases with increasing electron emission in a cavity. As a rough estimation, the statistical time lag for a cavity with ongoing PD activity at 50 Hz applied frequency is in the milli-second range [25].

2.3 Surface charge decay

The charge that is trapped at the cavity surface decays with time. This is mainly due to surface conduction and recombination, but also diffusion into deeper traps in the surface and conduction in the solid insulation may contribute [20]. The decay of surface charge generally reduces both the electric field in the cavity and the electron emission from the cavity surface. In the case of charge diffusion from shallow traps into deeper traps in the cavity surface, the electron emission decreases since electrons in deeper traps are less easily emitted than electrons in shallow traps. However, the charge in deeper traps still contributes to the electric field in the cavity.

2.4 Ageing

Partial discharge activity in a cavity causes degradation of the cavity [22]. This is mainly manifested as a reduction in the gas pressure in the cavity and a change in the properties of the cavity surface due to the formation of a layer of discharge by-products [22,26,27]. Especially the conductivity of the cavity surface is seen to increase with the time of PD exposure [28,29]. Furthermore the statistical time lag can be expected to decrease due to an increased amount of shallow electron traps [29]. At the same time as the cavity is degraded by the PDs, the change in the cavity properties also affects the PD activity [22,30]. The change in properties of the cavity surface during ageing can lead to a transition between different discharge
mechanisms (streamer-like, Townsend-like and pitting discharges) [29]. Changes in measured PD activity in cavities with time of PD exposure are reported in [26, 27, 30–33].

2.5 Frequency dependence

As suggested in Paper II and in [14–18], the frequency dependence of cavity PD can be described by use of certain characteristic times. These characteristic times are related to the statistical time lag of PD and to charge transport on the cavity surface and in the solid insulation. Simulations have shown that the mutual relation between these characteristic times, and their relation to the period time of the applied voltage ($T$), influence the PD frequency dependence (Paper II and [18]). In this thesis mainly three characteristic times are discussed: the statistical time lag ($\tau_{\text{stat}}$), the characteristic time for decay of surface charge in the cavity ($\tau_{\text{decay}}$) and the characteristic time for charge diffusion from shallow traps into deeper traps in the cavity surface ($\tau_{\text{trap}}$). In Paper II characteristic times for conduction in the bulk insulation and on the cavity surface are also considered.

If the statistical time lag is much shorter than the period time of the applied voltage ($\tau_{\text{stat}} \ll T$), it does not influence the PD process in a cavity. But if the statistical time lag is in the same range as the period time ($\tau_{\text{stat}} \approx T$), PDs are shifted forward in phase and occur at higher temporal values of the applied field, due to lack of free electrons. This can be called a statistical effect and results in fewer PDs per voltage cycle and larger PD magnitudes. The statistical effect is intensified with increasing applied frequency, due to the shortening period time, and therefore causes PD frequency dependence.

Surface charge generated by PDs in a cavity decay with time mainly due to conduction and recombination on the cavity surface. This process can be assigned a characteristic time $\tau_{\text{decay}}$, which depends on the geometry and conductivity of the cavity surface. If the surface charge decay is much slower than the rate of change of the applied voltage ($\tau_{\text{decay}} \gg T$), it does not influence the PD process in the cavity. However, if there is significant surface charge decay in the cavity ($\tau_{\text{decay}} \approx T$), the field in the cavity, and consequently the number of PDs per voltage cycle, decreases. This effect is stronger at lower applied frequencies, due to the longer period time, and hence gives rise to PD frequency dependence.

The diffusion of charge from shallow traps into deeper traps in the
2.5 Frequency dependence

cavity surface can be assigned a characteristic time constant $\tau_{\text{trap}}$. If there is a significant transport of surface charge into deeper traps in the cavity surface ($\tau_{\text{trap}} \approx T$) this will reduce the surface emission of electrons and consequently increase the statistical time lag. This effect is intensified with decreasing applied frequency and therefore causes PD frequency dependence.

The characteristic times $\tau_{\text{stat}}$, $\tau_{\text{decay}}$ and $\tau_{\text{trap}}$ are sensitive to the cavity surface conductivity and the surface emission of electrons. These properties of the cavity surface are known to change with the presence of PDs in a cavity [28, 29]. Since the time between consecutive PDs can differ greatly between different applied frequencies, the cavity surface properties, and consequently the characteristic times, can also change with the applied frequency. This is an additional source of PD frequency dependence.

The duration of a PD is short (nano-second range [23, 29]) in comparison to the period time of common applied voltages. Therefore the change in the temporal value of the applied voltage during the discharge process is insignificant and is not expected to cause any PD frequency dependence.
Chapter 3

Modelling of PD in cavities

This is a survey of modelling of PD in cavities at AC applied voltage. References are given to the models presented in Paper II and Paper VI (the latter is also described in Chapter 6). The main focus here is on models that describe the sequence of PDs in the cavity on a time scale comparable with the period time of the applied voltage. From such models the PDs are simulated one after another and the results are usually presented as a PD pattern. There are also other approaches to modelling of PD in cavities. In [34] a PD model is presented in the form of a closed mathematical description based on a stochastic framework. The output of the model is the probability density for PD as a function of the apparent charge and the time of occurrence of the PDs. No simulations are needed. In [35] a model is presented that describes the difference in applied voltage between subsequent PDs in a cavity. This relates to PD measurements with the pulse sequence analysis method in which the difference in time, phase or applied voltage between subsequent PDs is analyzed [36].

The main challenge in modelling of PD in cavities is that many physical parameters needed in a model are hard to determine. Especially the parameters related to the cavity surface are often unknown. Another difficulty in PD modelling is the long simulation times. A simulation of the PD activity in a cavity must extend over several periods of the applied voltage to gain reasonable statistics. At the same time, the time step in
the simulation must be short in comparison to the period time to resolve the PD process.

In [20] it is suggested that a PD model can be subdivided into five parts: classification and characterization of the defect, local electric field enhancement at the defect, generation of initial electrons, discharge process and, finally, charge. The following presentation is based on this subdivision. The classification and characterization of a defect is based on its size and location, and on the nature of the boundaries limiting the PDs at the defect. Partial discharges in cavities are limited by the cavity wall, which can be an insulating surface (insulated cavity) or an conducting surface (electrode bounded cavity).

### 3.1 Local electric field enhancement

The local electric field in a cavity is composed of two parts: the background field due to the applied voltage, and the local field due to space and/or surface charge left by previous PDs in the cavity. The choice of method to calculate the electric field in the cavity divides PD models into different groups. Most common is to use an electric circuit model based on the abc-model [37]. Figure 3.1a shows a schematic picture of a cavity and the corresponding abc-model. The capacitance $C_a$ represents the cavity; $C'_b$ and $C''_b$ represent the capacitance of the bulk material in series with the cavity; and $C'_a$ and $C''_a$ represent the capacitance of the bulk material in parallel with the cavity. The electrodes are connected to the terminals $A$ and $B$. In Figure 3.1b this model is reduced by putting $C_b = \frac{C'_b C''_b}{C'_b + C''_b}$ and $C_a = C'_a + C''_a$. Here $U_a$ is the applied voltage and $U_c$ is the voltage over the cavity.

The abc-model is widely used, either in its original three-capacitance form [38–40], or in modified forms including more circuit components [35, 41]. The model presented in Paper II is an extension of the abc-model including the resistance of the bulk material, the cavity surface and also the discharge in the cavity.

Since the abc-model is only an equivalent circuit model, its operation may be different from the PD processes in an actual cavity. As pointed out in [42], the concept of capacitance is not well suited to describe a cavity. Especially, it does not account for the facts that a real cavity wall is not an equi-potential surface and that there can be space and/or surface charge in the cavity.
3.1 Local electric field enhancement

Figure 3.1: The abc-model of an insulating material containing a cavity: (a) full model, (b) reduced model.

An alternative to the abc-model is to calculate the electric field in the cavity analytically as described by Niemeyer [20]. Here the Poisson’s equation is solved for the cavity geometry and the field enhancement in the cavity is averaged to give a field enhancement factor. This factor, together with the current applied field, is then used to approximate the field enhancement due to the background field. The field enhancement due to space and/or surface charge is estimated in a similar way. This technique is used by many authors [14, 16, 33, 43, 44].

Yet another approach is to calculate the local electric field in the cavity numerically by use of some field calculation method. In [45] the finite difference method is used to solve the Poisson’s equation for the electric field distribution in the test object. In Paper VI the finite element method (FEM) is used to calculate the time dependent electric field distribution.
3 Modelling of PD in cavities

in the test object from (6.1) and (6.2). The FEM is suited for solving partial differential equations over complex geometries.

### 3.2 Generation of initial electrons

The generation of initial free electrons in a cavity is commonly modelled with a generation rate representing the number of free electrons generated in the cavity per unit time. The electron generation rate can be expressed as a sum of two terms: one representing the generation due to background radiation, and one representing the generation due to field emission from the cavity surface [20]. The latter is commonly expressed as a function that increases with the electric field and with the amount of charge in shallow traps in the cavity surface [14, 16, 20, 33].

In the model presented in Paper VI the electron generation rate increases with increasing electric field but is independent of the amount of charge in shallow traps in the cavity surface. This leads to that the electron generation rate has to be changed manually at some applied frequencies in the simulations. The reason for not introducing a dependency on trapped charge in the electron generation rate is that the model cannot distinguish between surface charge generated by PDs in the cavity, and charge induced at the cavity surface by the electric field. This is a weakness of the model.

It is common to model the probability \( P \) for PD in a time interval \( \delta t \) as \( P = N_e \delta t \). Here \( N_e \) is the electron generation intensity and it is assumed that the electric field condition for PD is fulfilled. The occurrence of a PD is then simulated by a Monte Carlo procedure in the following way: A random number \( R \) (uniformly distributed in \([0,1]\)) is generated in each time step in the simulation and is compared to \( P \). If \( P > R \) there is PD in the current time step, otherwise it is not.

In the model presented in Paper VI a different method is used to simulate the occurrence of PD. Each time the electric field condition for PD is fulfilled, the distribution function \( F(t) \) for PD is calculated (see Appendix B). The time point of PD is then simulated from \( F(t) \) by use of a random number \( R \) (uniformly distributed in \([0,1]\)). The advantage of this event-controlled modelling technique is that the calculation of the electric field distribution in the test object does not need to be interrupted in each time step. This shortens the simulation time.
3.3 Discharge process

In principle it is possible to model the actual discharge process in a cavity in detail [46–48]. However, this is generally not done in PD models since detailed modelling of each discharge (with time scale in the nano-second range) in a simulation over several voltage periods (with time scale in the milli-second to minute range) would yield very long simulation times. In addition, for interpretation of results from phase-resolved PD measurements, a detailed modelling of the discharge process is generally not needed.

Instead, it is common to model the discharge process with an instantaneous drop in the voltage over the cavity [20]. The size of the voltage drop is determined from the critical voltage for PD, the time lag and the critical voltage for extinction of PD. The voltage drop results in an instantaneous change in the charge on the cavity surface. For models based on the abc-model, the charge of the cavity capacitance $C_c$ is changed instantaneously [38].

Another alternative is to model the discharge process dynamically by charge transport inside the cavity. This approach makes the model charge consistent and is used in the models presented in this work. In Paper II the discharge process is modelled dynamically with a streamer resistance that depends on the voltage over and current through the cavity. In Paper VI the discharge process is modelled by increasing the conductivity inside the cavity. When modelling the discharge process dynamically the time step in the simulation must be much shorter during discharge than otherwise to resolve the discharge process. This gives a numerically stiff problem and can cause long simulation times.

Finally, it is common to assume that a PD in a cavity affects the whole cavity. An exception is presented in [45] where the propagation of each PD on the cavity surfaces is explicitly modelled with a method based on the stochastic dielectric breakdown model presented in [49].

3.4 Charge

The last part of a PD model describes the physical charge, the apparent charge and the decay of surface charge in the cavity.
3.4.1 Physical and apparent charge

In models based on the abc-model, the physical charge is expressed as $q_{\text{phys}} = C_c \Delta U_c$ where $\Delta U_c$ is the voltage drop over the cavity due to the PD. The relation between physical charge and apparent charge is derived from the circuit [38].

In models based on Niemeyer [20], the physical charge is expressed as $q_{\text{phys}} = g \Delta U$. Here $g$ is a constant that corresponds to capacitance but is adapted to cavities with spherical or ellipsoidal geometry [20]. The relation between physical charge and apparent charge is given by the $\lambda$ function, as describe in [19].

In case the discharge process is modelled dynamically, the physical charge and apparent charge are simply calculated by integrating the current through the cavity and through the electrode surface, respectively.

In [45] the propagation of each PD on the cavity surfaces is modelled and the apparent charge is given as the difference in induced charge at the electrodes before and after a PD. This gives a coupling between the apparent charge and the discharge area. Finally, there are also models where the physical charge is modelled from a statistical distribution [44].

3.4.2 Decay of surface charge

In models based on Niemeyer [20], the decay of surface charge in a cavity is modelled as exponential decay of the number of surface charges. Usually only decay through conduction and recombination on the cavity surface is considered and the characteristic decay time constant decreases with increasing surface conductivity. However, there are also models that include surface charge decay through diffusion from shallow traps into deeper traps in the cavity surface [14, 15, 50].

In models where the electric field distribution in the test object is calculated numerically, the decay of surface charge in a cavity can actually be modelled through conduction on the cavity surface. This is done in the model presented in Paper VI. Here the cavity surface conductivity is modelled as a function of the surface charge. Also in [51], surface charge decay is in principle modelled as conduction on the cavity surface, although the electric field distribution and the current on the cavity surface are not calculated dynamically.

In the model presented in Paper II, which is a modified form of the abc-model, conduction on the cavity surface is modelled with a resistance $R_p$ in parallel with the cavity capacitance $C_c$. This gives a time
constant $\tau_{\text{cavity}} = R_p C_c$. Similarly, conduction in the bulk material is modelled with a resistance $R_s$ in series with $C_c$, which gives a time constant $\tau_{\text{material}} = R_s C_c$. Hence, in principle, surface charge in the cavity can decay through both conduction on the cavity surface and conduction in the bulk insulation. However, since the resistances $R_p$ and $R_s$ are constant, these conduction processes are active also in absence of surface charge generated by PDs in the cavity. Therefore the model presented in Paper II is not well suited to describe surface charge decay. It is more capable of modelling screening of a cavity due to conduction on its own aged surface, and charge build-up at a delamination blocking the conduction through the bulk insulation. This is also what is considered in Paper II and the time constants $\tau_{\text{cavity}}$ and $\tau_{\text{material}}$ are accordingly chosen.
Chapter 4

Variable-Frequency Phase Resolved PD Analysis

This Chapter is based on Paper V and describes the phase resolved PD measurements at variable applied frequency.

4.1 Measurement method

The PD measurements at variable applied frequency in this work are performed with the Variable-Frequency PRPDA (VF-PRPDA) technique [4,5]. The VF-PRPDA technique is based on the Phase Resolved Partial Discharge Analysis (PRPDA) technique [2] with the addition that the frequency of the applied voltage is varied. In the PRPDA technique the apparent charge and the phase position relative the applied voltage is recorded for each detected PD. The recorded values are sorted into phase and charge channels and are stored in a matrix (see schematic illustration in Figure 4.1a). The columns of the matrix represent the phase channels, the rows represent the charge channels and the elements represent the number of detected PDs with a certain combination of phase and charge. The phase resolution is set by the number of phase channels and the charge resolution is set by the resolution of the A/D converter in the measurement system.
Variable-Frequency Phase Resolved PD Analysis

Figure 4.1: (a) Schematic illustration of PRPDA result matrix. The elements represent the number of detected PDs. (b) Example of PD pattern. The color scale represents the number of detected PDs. The unbroken line gives a phase reference to the applied voltage.

The resulting matrix from a PRPDA measurement can be displayed as a PD pattern. An example of a PD pattern is shown in Figure 4.1b. The x-axis in the PD pattern represents phase, the y-axis represents apparent charge and the color scale represents the number of detected PDs with a certain combination of phase and charge. In addition, results from PRPDA measurements can also be displayed as phase and charge distributions. Phase distributions show the PD activity as a function of phase without respect to apparent charge (for example total number of PDs at each phase position). Charge distributions show the PD activity as function of apparent charge irrespective of phase (for example total number of PDs at each charge level).

The results of VF-PRPDA measurements can be displayed in the same way as PRPDA results. This gives one PD pattern (or phase or magnitude distribution) for each applied frequency in the VF-PRPDA measurement. Another way to display VF-PRPDA data is by use of integral parameters where the detected PDs at each individual applied frequency are summed up. Examples of integral parameters are the total number of PDs per voltage cycle or the average apparent charge. Integral
4.2 Measurement system

The phase resolved PD measurement system used in this study is described in detail in [4, 5]. It is based on the commercial PD measurement system ICM (Insulation Condition Monitoring) [52] which is modified to synchronize between the phase resolved PD acquisition and the applied voltage in the frequency range 0.001 – 400 Hz. In this work the applied frequencies is restricted to the range 0.01 – 100 Hz. The upper frequency limit was set by the loading of the voltage supply in the measurement system and the lower frequency limit was set to keep the measurement time down.

A schematic picture of the measurement system is shown in Figure 4.2 and a photo is shown in Figure 4.3. The system comprises a high-voltage supply \( V \), a high-voltage filter \( Z_f \), a coupling capacitance \( C_k \), a detection impedance \( C \), \( R \) and \( L \), a pre-amplifier, the ICM system and a personal computer. The high voltage is supplied from a computer generated low-voltage signal amplified by a high-voltage amplifier. The high voltage amplifier has a maximal output of 20 kV and variable frequency in the range 0 - 1000 Hz. The high voltage filter reduces noise, preferably the switching frequency of the amplifier. It also acts as a security disconection between the high-voltage amplifier and the test object. The coupling capacitance \( C_k \) is 200 pF and acts as a stable voltage source during partial
discharge in the test object. Current is driven from $C_k$ to the test object during the short time duration of a discharge. The coupling capacitance also contributes to the high-voltage filter. The detection impedance includes $L$ (3.9 mH), $R$ (1 kΩ), $C$ (4.7 – 33 nF) and the capacitance of the connecting cables (about 200 pF). A PD in the test object gives rise to a voltage pulse over the detection impedance and the time dependent voltage $V_m(t)$ is measured. The measured signal is amplified by a pre-amplifier and sent to the ICM system. For each detected PD pulse the measurement system determines the phase position relative the applied voltage and the apparent charge. The apparent charge is the charge transmitted from the coupling capacitance to the test object during a partial discharge [23].

The measurement system has 256 phase channels and 256 charge channels and its bandwidth is 40 - 800 kHz. After each detected PD pulse a dead time is set during which no further pulses are detected. This is to avoid detecting the same PD pulse more than once. In this work the dead time was set to 50 µs. Furthermore a discrimination level is set to reduce noise in the measurements. Detected PDs with apparent charge below this level are disregarded.

The measurement system is calibrated by connecting a step voltage $\delta V_{cal}$ in series with a capacitance $C_{cal}$ over the test object, thus injecting a charge $q_{cal} = C_{cal}\delta V_{cal}$ to the electrodes. The connections for calibration are indicated with red in Figure 4.2. In this work the calibration impulse
4.2 Measurement system

Figure 4.3: Photo of phase resolved PD measurement system.

generator CAL1D from [52] is used. It has charge value in the range 10 pC to 1 nC.

Special attention was paid to the settings of the PD signal amplifiers, that is the pre-amplifier shown in Figure 4.2 and the main amplifier inside the ICM. It was observed that for low gains and steep input pulses the amplification factor was slightly different for positive and negative pulses. To avoid this problem a high gain of the amplifiers was used. In addition $C$ and $R$ in the detection impedance were increased to reduce the amplitude and increase the rise time of the input pulses.

Another difficulty related to the amplifier settings was that the scatter in PD magnitude often differed between different applied frequencies. Hence the amplification had to be adjusted manually at each frequency to resolve the PD activity. Finally, the spread in PD magnitude was sometimes larger than the dynamic range of the amplifiers so that all PDs could not be detected. To overcome this problem each frequency was measured at both a low and a high amplification consecutively. The two measurement results were then merged into one by taking the small PDs from the high-amplification measurement and the larger PDs from the low-amplification measurement. This problem arose for test objects with large cavities (diameter 10 mm) and large electrodes (cylindrical
electrodes, see Section 4.4) where many PDs occurred simultaneously. The measurement system incorrectly interpreted the simultaneous PDs as one and added their magnitudes. This caused the large spread in measured PD magnitude.

4.3 Materials

Polycarbonate is used as insulation material for all specimens in this study. Polycarbonate is an amorphous polymer with relative permittivity ($\varepsilon_r'$) equal to 3. The choice of polycarbonate as insulation material was made early in this work with the intention to use a transparent and easily worked material with good PD resistance. It was observed that the PD activity in cavities in polycarbonate reached a quasi-static state after a reasonable time of conditioning and that this state was maintained for a time sufficiently long to allow for measurements. Figure 4.4 shows measured relative permittivity ($\varepsilon_r'$) and dielectric loss factor ($\varepsilon_r''$) for polycarbonate in the frequency range used for the PD measurements (0.01 – 100 Hz). The relative permittivity is nearly frequency independent and the loss factor is below 6 · 10^{-3}. This is desirable since otherwise the applied electric field distribution in the test object would change with frequency. Attempts to influence the PD activity in the test object by illumination with UV light failed since no changes were observed, probably due to too low light intensity.

4.4 Specimens

All specimens in this study are disc-shaped cavities in polycarbonate. They are made by pressing together three plates of polycarbonate with a drilled hole in one of the plates. Placing the hole plate between the other two plates gives an insulated cavity; placing it on top of the other plates gives an electrode bounded cavity. The reason for using cylindrical cavities is the simple manufacturing process. The polycarbonate plates and especially the drilled hole are inspected for irregularities before assembly. The protection plastic on the polycarbonate plates is removed just before measurements to avoid surface contamination. Cleaning of the plates is omitted since no significant influence on the measurement results was observed for tests with iso-propanol cleaning. The specimens are conditioned before measurements to reach a quasi-static PD activity in the cavity. Each measurement is repeated on at least two similar
specimens to check the repeatability.

Two different electrode types are used: cylindrical and spherical (see schematic figures in Figure 4.5 and photos in Figure 4.6). Cylindrical electrodes are used to study how the applied voltage amplitude, cavity size and cavity location (insulated or electrode bounded) influence the PD frequency dependence. Spherical electrodes are used for comparisons between measurements and simulations. The spherical electrode geometry was introduced to concentrate the discharges to the cavity center. This gives an axi-symmetric electric potential distribution in the test object and makes a two-dimensional model geometry possible. In addition, the cavity diameter is chosen large compared to the electrode diameter to avoid interaction of the discharges with the cavity wall.
4.5 Measurement procedure

The PD activity in a cavity may change with time due to among other things changes in the gas pressure in the cavity and in the properties of the cavity walls [22, 26, 27]. If these changes are significant during the
4.6 Main results

Measurements on test objects with cylindrical electrodes show that the PD activity in the cavity is dependent on the frequency of the applied voltage. The PD frequency dependence changes with increasing amplitude of the applied voltage. At lower voltage amplitude the PD activity in the cavity shows a statistical effect with increasing PD magnitudes at increasing applied frequency. Figure 4.7 shows the PD patterns at applied frequency 0.01 Hz and 100 Hz from measurements on an insulated cavity with diameter 4 mm. The maximum PD magnitude at 0.01 Hz and 100 Hz is about 400 pC and 900 pC, respectively, whereas the min-
imum PD magnitude is about the same. Hence there is a wider spread in PD magnitude at 100 Hz, which is interpreted as an influence of the statistical time lag.

At higher applied voltage amplitudes the PD activity in the cavity has a different frequency dependence. There is no statistical effect. Instead the PD activity is influenced by decay of surface charge in the cavity. Figure 4.8 shows PD patterns at 0.01 Hz and 100 Hz from measurements at voltage amplitude 10 kV. The PD magnitude is about the same at both frequencies but the number of PDs per voltage cycle is lower at 0.01 Hz than at 100 Hz, which is interpreted as an effect of surface charge decay.

The change in PD frequency dependence with increasing amplitude of the applied voltage can be explained with an enhanced emission of electrons from the cavity surface. The electron emission increases with increasing electric field [20]. As a consequence the statistical time lag shortens and its influence on the PD activity diminishes. As the statistical effect disappears the dominating influence on the PD frequency dependence instead comes from the decay of surface charge in the cavity.

The PD activity changes more with the varying frequency in an electrode bounded cavity than in an insulated cavity. It is supposed that the surface charge decay is more intense in an electrode bounded cavity since charge can recombine or move readily in the electrode. Figure 4.9 shows PD patterns at 0.01 Hz and 100 Hz from measurements at voltage amplitude 10 kV on an electrode bounded cavity with diameter 4 mm. At 0.01 Hz the PD activity is almost extinguished due to enhanced surface charge decay at this low frequency.
4.6 Main results

Figure 4.6: Photo of test object with (a) cylindrical electrodes and (b) spherical electrodes.
Figure 4.7: Measurement at applied voltage amplitude 8 kV and frequency (a) 0.01 Hz and (b) 100 Hz on an insulated cavity with diameter 4 mm. Cylindrical electrodes. The broken lines mark the discrimination level in the measurement system.

Figure 4.8: Measurement at applied voltage amplitude 10 kV and frequency (a) 0.01 Hz and (b) 100 Hz on an insulated cavity with diameter 4 mm. Cylindrical electrodes.
4.6 Main results

Figure 4.9: Measurement at applied voltage amplitude 10 kV and frequency (a) 0.01 Hz and (b) 100 Hz on an electrode bounded cavity (placed against upper electrode) with diameter 4 mm. Cylindrical electrodes.
Chapter 5

PD inception voltage

This Chapter is based on Paper III and describes the measurements of PD inception voltage (PDIV).

5.1 Measurement method

Partial discharge inception voltage is the lowest applied voltage level at which PDs appear in a test object. It is measured by increasing the applied voltage from a low level until PDs are detected. The definition of PDIV is not unambiguous. Sometimes PDIV is defined as the voltage level at which the first PD appears. However, in practical applications it is usually the occurrence of continuous PD activity that is of interest. The PDIV is then defined as the voltage level at which a minimal number of PDs above a certain apparent charge level are detected per unit time. It has been shown [54] that the measured PDIV increases with increasing ramp rate of the applied voltage. This comes since at higher ramp rates the temporal value of the applied voltage increases more during the statistical time lag. In [54] it is also reported that PDIV measurements are sensitive to changes in the statistical time lag caused by irradiation. In [55] the PDIV for a test object with a disc-shaped insulated cavity in polyethylene (placed in a dark box) is measured at different applied frequencies. No clear frequency behaviour of the PDIV is seen.

The aim of the PDIV measurements in this work is to study how the critical voltage for PD in a cavity is influenced by the frequency of the applied voltage. Therefore PDIV is defined as the amplitude of
the applied voltage at which the first PD is detected in the test object. All specimens are conditioned before the PDIV measurements to reach a quasi-static state in the PD activity. The intention in this work is to measure PDIV for an “active” cavity with ongoing PD activity. Therefore the specimens are pre-excited at each applied frequency. This means that a voltage with amplitude 10 kV and the current frequency is applied for 30 min prior to the PDIV measurement at each new frequency. For comparison, PDIV is also measured without pre-excitation. In this case the specimens are left without voltage supply for 14 hours before the first PDIV measurement and then for an additional 30 min prior to the PDIV measurements at each new applied frequency.

The PDIV is measured with a sinusoidal applied voltage with frequency in the range 0.1 – 100 Hz and with linearly increasing amplitude. The amplitude is increased from zero at ramp rate 0.1 kV/s, which was the lowest ramp rate allowed by the voltage supply equipment used. The PDIV is calculated as the product of the ramp rate and the time to the first PD. The PDIV measurements are repeated ten times at each applied frequency. The results are presented as average values with error bars indicating 90 % confidence intervals (assuming normal distribution). Pauses between consecutive measurements at one frequency and between pre-excitation and PDIV measurements are avoided. During a pause in the measurements surface charge in the cavity can decay. This can alter the properties of the cavity (mainly the statistical time lag) and affect the PDIV measurement results.

5.2 Main results

Figure 5.1 shows measured PDIV as function of applied frequency for a test object with an insulated disc-shaped cavity and cylindrical electrodes. The PDIV is measured with three different measurement procedures on the same specimen. In two cases the specimen is pre-excited at each new applied frequency and the PDIV is measured with either decreasing (blue curve in Figure 5.1) or increasing (green curve in Figure 5.1) applied frequency. In the third case the specimen is not pre-excited and the PDIV is measured at decreasing applied frequency (red curve in Figure 5.1).

In the frequency range 0.1 – 10 Hz the measured PDIV is approximately constant and similar for the three measurement procedures. At 100 Hz there is an increase in the measured PDIV, especially for mea-
5.2 Main results

Figure 5.1: Measured PD inception voltage for a test object with an insulated cavity with diameter 10 mm and cylindrical electrodes. The PDIV is measured with pre-excitation and decreasing applied frequency (blue), with pre-excitation and increasing applied frequency (green), without pre-excitation and at decreasing applied frequency (red).

Measurements with pre-excitation. The result is the same for decreasing and increasing applied frequency. This increase may be explained by a larger influence of the statistical time lag during the measurement at 100 Hz than at lower applied frequencies. However, since it is mainly for measurements with pre-excitation that the PDIV increases, there might also be a change in some of the cavity properties during the pre-excitation at 100 Hz that affects the measured PDIV.

In [56] it is shown that the measured PDIV of insulated disc-shaped cavities obeys Paschen’s law (for gaseous breakdown between plane metal electrodes) [23]. If Paschen’s law is used to estimate the PDIV of the test object used in this work, a value of 7.7 kV is obtained. This is higher than the measured PDIV as shown in Figure 5.1. This may be due to the pre-excitation in the PDIV measurements presented here that probably causes a larger influence of surface charge on the measured PDIV.
Chapter 6

Model of PD in a cavity

This Chapter presents the model of PD in a cavity and is based on Paper VI. The model describes PD in a test object with an insulated cavity and spherical electrodes. It is used to dynamically simulate the sequence of PDs at different applied frequencies. The model consists of four main parts: the calculation of the time-dependent electric potential distribution in the test object (Section 6.1); the electron generation in the cavity and determination of time for next PD (Section 6.2); the discharge process (Section 6.3); and the apparent charge and surface charge decay (Section 6.4). Output from the model is the voltage over the cavity centre at each time step, the total current through the test object at each time step, and the point of time and apparent charge for each PD. Figure 6.1 shows a flowchart of the model and Table 6.1 gives a list of symbols used in the model.

The measurement circuit contains a filter impedance and a detection impedance [4]. The voltage drops over these are small in comparison to the voltage drop over the test object and are neglected in the model. The filter impedance also introduces a small phase shift (< 1 degree) between the applied voltage and the voltage over the test object. This phase shift is adjusted for in the software of the measurement system and do not need to be accounted for in the model.
Table 6.1: List of symbols used in the model.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V$</td>
<td>Electric potential</td>
</tr>
<tr>
<td>$U_{\text{cav}}$</td>
<td>Voltage over cavity centre</td>
</tr>
<tr>
<td>$U_{\text{crit}}$</td>
<td>Critical voltage for PD</td>
</tr>
<tr>
<td>$U_{\text{ext}}$</td>
<td>Critical voltage for extinction of PD</td>
</tr>
<tr>
<td>IncTol, FutTol, ExtTol</td>
<td>Tolerance for the FEM-solver stop conditions $</td>
</tr>
<tr>
<td>$N_{\text{e}}$</td>
<td>Electron generation rate</td>
</tr>
<tr>
<td>$F$</td>
<td>Distribution function for PD</td>
</tr>
<tr>
<td>$R$</td>
<td>Random number</td>
</tr>
<tr>
<td>$r_{\text{cyl}}$</td>
<td>Radius of discharging cylinder</td>
</tr>
<tr>
<td>$V_{\text{surf}}$</td>
<td>Electric potential on cavity surface</td>
</tr>
<tr>
<td>$\sigma_{\text{surf}}$</td>
<td>Conductivity of cavity surface</td>
</tr>
<tr>
<td>$\sigma_{\text{SurfLow}}$</td>
<td>Cavity surface conductivity not during surface charge decay</td>
</tr>
<tr>
<td>$\sigma_{\text{SurfHigh}}$</td>
<td>Cavity surface conductivity during surface charge decay</td>
</tr>
<tr>
<td>$Q_{\text{crit}}$</td>
<td>Critical charge level for surface charge decay</td>
</tr>
</tbody>
</table>
6.1 Electric potential

It is assumed that all PDs occur at the centre of the cavity where the applied electric field is strongest. Hence the electric potential distribution
in the test object is axi-symmetric. The potential distribution is governed by (6.1) where $V$ is electric potential, $\sigma$ is conductivity, $\varepsilon_0$ is vacuum permittivity and $\varepsilon_r$ is relative permittivity. The first term in (6.1) relates to transport current and the second term relates to displacement current. A derivation of (6.1) is given in Appendix A.

$$\nabla \cdot \left( -\sigma \nabla V - \frac{\partial}{\partial t} (\varepsilon_0 \varepsilon_r \nabla V) \right) = 0 \quad (6.1)$$

It is further assumed that charge can move on the cavity surface due to conduction. Therefore condition (6.2) is imposed on the cavity surface where $V_{\text{surf}}$ is the potential on the surface and $\sigma_{\text{surf}}$ is the conductivity of the surface.

$$\begin{cases} 
\nabla \cdot (-\sigma_{\text{surf}} \nabla V_{\text{surf}}) = 0 \\
V = V_{\text{surf}} \text{ on cavity surface} 
\end{cases} \quad (6.2)$$

The potential distribution in the test object is calculated in each time step from (6.1) and (6.2) by use of the finite element method (FEM). The FEM is suited for solving partial differential equations over complex geometries. The choice of FEM as solution method in this project was facilitated by recent developments in commercial FEM based programs [57].

### 6.2 Electron generation

It is assumed that the generation of free electrons in the cavity is dominated by surface emission from the cavity walls. The electron generation rate $N_e$ (number of electrons generated per time unit) is modelled as

$$N_e(t) = N_{e0} \exp \left( \frac{|U_{\text{cav}}(t)|}{U_{\text{crit}}} \right) \quad (6.3)$$

where $U_{\text{cav}}$ is voltage over the cavity centre, $U_{\text{crit}}$ is critical voltage for PD and $N_{e0}$ is a constant. Hence the electron generation rate increases with increasing electric field. The simple expression (6.3) for the electron generation rate is inspired from the Richardson-Schottky law [20] and is used since a number of the material parameters in the Richardson-Schottky law are hard to determine. Hence the aim of (6.3) is to model a reasonable voltage dependence in the electron generation rate instead of using a detailed physical model with many unknown parameters.
6.3 Discharge process

The probability that a free electron is generated in the cavity in the time interval \([t, t + \delta t]\) is assumed to be \(N_c(t)\delta t\), provided that \(|U_{cav}| > U_{crit}\). The corresponding distribution function for PD is

\[
F(t) = 1 - \exp\left(-\int_0^t N_c(t')dt'\right)
\]  

(6.4)

A derivation of (6.4) is given in Appendix B. The time point of PD in the cavity is simulated from this distribution function by use of a random number \(R\) uniformly distributed in \([0, 1]\). This is done in the following way: Each time there is a possibility of PD (that is \(|U_{cav}|\) exceeds \(U_{crit}\)) the future values of \(U_{cav}\) (provided there is no PD) are calculated until \(|U_{cav}|\) drops below \(U_{crit}\) again. This corresponds to step a and b in Figure 6.1. The future values of \(|U_{cav}|\) are then used to calculate the future values of \(N_c\) and accordingly to calculate \(F\) (step c and d in Figure 6.1). If \(F\) is always less than \(R\) there is no PD (return from step e to step a in Figure 6.1)). Otherwise a PD occur at the point of time when \(F\) exceeds \(R\) (continue from step e to step f in Figure 6.1)). This event-controlled modelling technique gives much shorter simulation times than if the FEM-solver is interrupted in each time step to check whether or not there is PD.

6.3 Discharge process

A discharge in the cavity is modelled dynamically by increasing the conductivity in a cylinder (radius \(r_{cyl}\)) centred inside the cavity and extending from the lower to the upper cavity wall (step g in Figure 6.1). Increasing the conductivity in the cylinder results in a current flow through the cavity and a corresponding decrease in \(U_{cav}\) (step h in Figure 6.1). As \(U_{cav}\) drops below a certain level called \(U_{ext}\), the conductivity of the cylinder is decreased and the discharge extinguishes (step i in Figure 6.1). It is assumed that all PDs in the cavity affect the same area on the cavity surface and hence the radius of the discharging cylinder is constant.

Figure 6.2a shows an example of the simulated voltage over the cavity centre. In this case there is no influence from the statistical time lag on the PD process and a PD starts immediately each time the critical voltage level for PD is reached. The PDs continue until the voltage over the cavity centre drops below \(U_{ext}\), which is here set to 10 V. In contrast, Figure 6.2b shows the case when the statistical time lag has a significant
influence on the PD process. Here PDs are shifted forward in time and occur at higher voltages over the cavity.

6.4 Charge

The dynamical modelling of the discharge process makes the model charge consistent. Therefore there is no need for $\lambda$-functions and analytical estimations of the apparent charge [19], which is otherwise common in PD models since the discharge process is usually modelled as an instantaneous change in the charge on the cavity surface [20]. In the model presented here, the apparent charge is calculated numerically by time integration of the total current through the test object. The total current is calculated by integrating the current density over the surface of the lower electrode.

The decay of surface charge in the cavity is modelled as conduction on the cavity surface and is governed by (6.2). In addition, the cavity surface conductivity ($\sigma_{\text{surf}}$) is modelled as dependent on the amount of charge present on the cavity surface. The reason for this is discussed in Section 6.6 in connection to the simulation results. After a PD in the cavity $\sigma_{\text{surf}}$ is set to a high value ($\sigma_{\text{SurfHigh}}$). This value is maintained until the total amount of charge $Q$ (without sign) on the upper horizontal cavity surface drops below a critical level called $Q_{\text{crit}}$. Then $\sigma_{\text{surf}}$ is changed back to its initial low value ($\sigma_{\text{SurfLow}}$). The conductivity is the same over the whole cavity surface.

6.5 Simulations

The FEM calculations of the potential distribution in the test object are performed with Comsol Multiphysics® 3.3a [57]. The Comsol Multiphysics® model is described in detail in Appendix C. Since the potential distribution is axi-symmetric a 2D model geometry and mesh is used (see Figure 6.3). The mesh is refined inside the cavity and at the electrode surfaces where the potential is of most interest. To avoid initial transients in the potential distribution, the initial conditions are chosen as the solution after one period of applied voltage without any PD.

The parameter values used in the simulations are shown in Table 6.2 and the specific choices of parameter values are discussed in detail in Paper VI. To reach agreement with measurements at different applied frequencies, mainly three simulation parameters were adjusted. These are the cavity surface conductivity during surface charge decay ($\sigma_{\text{SurfHigh}}$),
the electron generation rate \( N_{e0} \), and the critical voltage for extinction of PD \( U_{ext} \). Variations in these parameters influence essential features in the simulated PD patterns, like the distribution in phase and magnitude of the PDs, and the number of PDs per voltage cycle. Generally, the value of \( \sigma_{\text{SurfHigh}} \) determines the number of PDs per voltage cycle and the earliest possible PD phase position during each voltage half cycle. This comes since the value of \( \sigma_{\text{SurfHigh}} \) influences how fast surface charge in the cavity decays. The value of \( N_{e0} \) mainly determines the spread in phase and magnitude of the PDs since \( N_{e0} \) influences the statistical time lag. Finally, the value of \( U_{ext} \) generally determines the minimal PD magnitude. However, this is a simplified picture since the distribution in phase and magnitude of the PDs, and the number of PDs per voltage cycle, are not independent of each other. As an example, the spread in phase and magnitude of the PDs (which is mainly determined from \( N_{e0} \)) influences the amount of charge on the cavity surface and hence also the number of PDs per voltage cycle.
Figure 6.2: Simulated voltage over the cavity centre (unbroken line) for the cases when the influence of the statistical time lag on the PD process is (a) negligible and (b) significant. The dotted line gives a phase reference to the applied voltage and the dashed lines mark the critical voltage level for PD. Critical level for extinction of PD $(U_{ext})$ is set to 10 V.
Figure 6.3: Model geometry and mesh (symmetry axis $r = 0$). The mesh has 4160 triangular elements.
6.6 Main results

The PD activity in a test object containing an insulated disc-shaped cavity and spherical electrodes is measured and simulated at applied frequency in the range 0.01 – 100 Hz. The measurement and simulation results are shown in Figure 6.4. A detailed discussion of the results is given in Paper VI.

The measurement results show that the PD activity is frequency dependent. The spread in phase and magnitude of the PDs differ greatly between different frequencies. Moreover the number of PDs per voltage cycle is higher at 100 Hz than at lower frequencies.

The measured PD frequency dependence is interpreted by use of the simulation results. The PD frequency dependence can be described as a variation in influence on the PD activity from the statistical time lag and the charge transport in the cavity surface at different applied frequencies. There is a statistical effect in the frequency range 0.1 – 10 Hz, which is intensified with increasing frequency. It results in larger PD magnitudes at higher frequency. In addition, there is a reduction in surface charge decay with increasing frequency in the range 0.1 – 100 Hz. This causes PDs to occur earlier in phase at higher frequency since more charge remain on the cavity surface, which enhances the electric field in the cavity. Finally, the transport of charge from shallow traps into deeper traps in the cavity surface increases at the lowest frequency (0.01 Hz). Since electrons in deeper traps are less easily emitted than electron in shallow traps, this causes a reduction in the electron emission from the cavity surface, which results in a statistical effect.

The simulation results show that the cavity surface conductivity, the emission of electrons from the cavity surface and the PD extinction voltage change with the frequency of the applied voltage. This effect contributes to the PD frequency dependence. It is interpreted as a consequence of the difference in time between consecutive PDs at different applied frequencies. The presence of PDs in a cavity is known to change the properties of the cavity surface [28, 29].

The simulation results also show that, when modelling the decay of surface charge in the cavity as conduction on the cavity surface, the cavity surface conductivity cannot be constant. Instead the surface conductivity must be higher when there is charge from a PD present on the surface than otherwise. Especially, to reproduce the measured PD pattern at 0.1 Hz, approximately all surface charge must decay between consecutive PDs. To obtain this, the cavity surface conductivity must exceed
### Table 6.2: Parameter values used in the simulations.

<table>
<thead>
<tr>
<th>Name</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Applied voltage amplitude</td>
<td>11 kV</td>
</tr>
<tr>
<td>Frequency of applied voltage ((f))</td>
<td>0.01 – 100 Hz</td>
</tr>
<tr>
<td>Periods of applied voltage at each frequency</td>
<td>100 ((f \leq 0.1 \text{ Hz}))</td>
</tr>
<tr>
<td></td>
<td>500 ((f \geq 1 \text{ Hz}))</td>
</tr>
<tr>
<td>Time step (not during PD)</td>
<td>((1/(2000f))) s</td>
</tr>
<tr>
<td>Time step during PD</td>
<td>1 ns</td>
</tr>
<tr>
<td>(U_{\text{crit}})</td>
<td>2.72 kV</td>
</tr>
<tr>
<td>(U_{\text{ext}})</td>
<td>10 V ((f \leq 10 \text{ Hz}))</td>
</tr>
<tr>
<td></td>
<td>2.4 kV ((f = 100 \text{ Hz}))</td>
</tr>
<tr>
<td>(N_{e0})</td>
<td>0.02 s(^{-1}) ((f = 0.01 \text{ Hz}))</td>
</tr>
<tr>
<td></td>
<td>100 s(^{-1}) ((0.1 \leq f \leq 10 \text{ Hz}))</td>
</tr>
<tr>
<td></td>
<td>500 s(^{-1}) ((f = 100 \text{ Hz}))</td>
</tr>
<tr>
<td>Permittivity of polycarbonate</td>
<td>3.0</td>
</tr>
<tr>
<td>Conductivity of polycarbonate</td>
<td>(10^{-15}) S/m</td>
</tr>
<tr>
<td>Permittivity of epoxy</td>
<td>5.2</td>
</tr>
<tr>
<td>Conductivity of epoxy</td>
<td>(10^{-15}) S/m</td>
</tr>
<tr>
<td>Permittivity of cavity</td>
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</tr>
</tbody>
</table>

*Continued on next page.*
Continuation from previous page.

<table>
<thead>
<tr>
<th>Name</th>
<th>Value</th>
</tr>
</thead>
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<td>2.8 mm</td>
</tr>
<tr>
<td>Conductivity of cavity (not during PD)</td>
<td>0</td>
</tr>
<tr>
<td>Conductivity of cavity during PD</td>
<td>$10^{-4}$ S/m ($r \leq r_{\text{cyl}}$)</td>
</tr>
<tr>
<td></td>
<td>0 ($r &gt; r_{\text{cyl}}$)</td>
</tr>
<tr>
<td>$\sigma_{\text{SurfLow}}$</td>
<td>$10^{-15}$ S</td>
</tr>
<tr>
<td>$\sigma_{\text{SurfHigh}}$</td>
<td>$3 \cdot 10^{-15}$ S ($f = 0.01$ Hz)</td>
</tr>
<tr>
<td></td>
<td>$2 \cdot 10^{-11}$ S ($f \geq 0.1$ Hz)</td>
</tr>
<tr>
<td>$Q_{\text{crit}}$</td>
<td>10 pC</td>
</tr>
<tr>
<td>IncTol, FutTol, ExtTol</td>
<td>1 %</td>
</tr>
</tbody>
</table>

a minimal level. However, if the surface conductivity is set above this minimal level throughout the whole simulation, no PDs occur at all since conduction on the cavity surface reduces the voltage over the cavity to below the critical voltage for PD. This is the reason for modelling the cavity surface conductivity ($\sigma_{\text{surf}}$) as dependent on the amount of charge present on the cavity surface. There are experimental studies by other authors [58] showing that the mobility of surface charge on insulating surfaces increases at high charge densities.
Continued on next page.
Continuation from previous page.

Figure 6.4: Measurement (a – e) and simulation (f – j) at applied voltage amplitude 11 kV and frequency 0.01 – 100 Hz. The test object contains an insulated disc-shaped cavity and spherical electrodes. Number of voltage periods 100 ($f \leq 0.1$ Hz) or 500 ($f \geq 1$ Hz). The y-axis scaling is the same for all plots. The broken lines in (a – e) mark the discrimination level in the measurement system.
Chapter 7

Summary of papers

Paper I

This paper illustrates the PD frequency dependence through measurements on a real high-voltage component. The VF-PRPDA technique is used to measure PD at variable applied frequency (0.01 – 100 Hz) in in-service aged stator bars from a hydro-power generator. The apparent charge at each applied frequency is studied and two types of PD frequency dependence are observed: with decreasing applied frequency the total amount of apparent charge increases in some of the stator bars whereas it decreases in others. No major differences in the PD phase distribution are seen for the two types of PD frequency dependence.

In the paper it was suggested that the two types of PD frequency dependence are due to charge transport in the bulk insulation and on the surfaces of cavities, respectively. Later on in this project it was noticed that the PD activity in a cavity can change drastically during the first time after voltage application, supposedly due to changes in the cavity surface properties caused by the discharges (see Section 2.4). Since the stator bars in this paper were not conditioned before measurements, this effect may contribute to the observed PD frequency dependence.

Paper II

This paper presents the first model of PD in a cavity at variable applied frequency in this project. The electric potential distribution is modelled
with an electric network model which is a further development of the abc-model (see Section 3.1). The PD current path is modelled by a voltage and current dependent streamer resistance. The model is used to dynamically simulate the sequence of PDs in the cavity at applied frequencies in the range $0.01 - 1000 \, \text{Hz}$. The dynamic modelling of the discharge process makes the model charge consistent, unlike earlier models of the sequence of PDs in a cavity (see Section 3.3). The apparent charge is calculated by integration of the current through the electrode.

The simulation results show that the frequency dependence of the PD activity is influenced by the mutual relation between three characteristic times and their relation to the period time of the applied voltage. The characteristic times are the statistical time lag $\tau_{\text{stat}}$ and the two dielectric time constants $\tau_{\text{cavity}}$ and $\tau_{\text{material}}$. The time constant $\tau_{\text{cavity}}$ is related to charge transport on the cavity surface and $\tau_{\text{material}}$ is related to charge transport in the bulk insulation.

**Paper III**

In this paper the partial discharge inception voltage (PDIV) is measured at applied frequency $0.1 - 100 \, \text{Hz}$ for a test object with an insulated disc-shaped cavity and cylindrical electrodes. It is studied how different types of pre-excitation of the test object influences the measured PDIV. The PDIV is seen to increase at the highest applied frequency, especially for measurements with pre-excitation. It is also briefly investigated how the PD activity in the cavity changes with time after voltage application.

**Paper IV**

This paper presents a model of the PD activity in a test object with an insulated disc-shaped cavity and spherical electrodes. The sequence of PDs in the cavity is simulated dynamically at two different applied frequencies ($0.02 \, \text{Hz}$ and $100 \, \text{Hz}$). The simulation results are compared with measurements. In the test object the flat cavity walls are covered with copper foil so that each PD affects the whole cavity. The copper foil is introduced to simplify the modelling. In the model each discharge is assumed to affect the whole cavity. This is a preliminary model which was further developed in Paper VI.
Paper V

This paper studies how the applied voltage amplitude, cavity size and cavity location (insulated or electrode bounded) influence the frequency dependence of the PD process in a cavity. Partial discharges in a test object with a disc-shaped cavity and cylindrical electrodes are measured at variable frequency (0.01 – 100 Hz) of the applied voltage. The PD activity in the cavity is seen to depend on the applied frequency. Moreover, the PD frequency dependence changes with the applied voltage amplitude, the cavity diameter, and the cavity location. It is suggested that the PD frequency dependence is governed by the statistical time lag of PD and the surface charge decay in the cavity.

Paper VI

In this paper, the PD activity in a test object containing a disc-shaped insulated cavity and spherical electrodes is measured and simulated at applied frequency in the range 0.01 – 100 Hz. The measurement results show that the PD activity is frequency dependent. Both the distribution in PD phase and magnitude, as well as the number of PDs per voltage cycle, change with the applied frequency.

A final model of PD in a cavity is presented and used to dynamically simulate the sequence of PDs in the cavity at different applied frequencies. The simulation results are used to interpret the measured PD frequency dependence. The PD frequency dependence can be described as a variation in influence on the PD activity from the statistical time lag and the charge transport in the cavity surface at different applied frequencies. The simulation results also show that the cavity surface conductivity, the emission of electrons from the cavity surface and the PD extinction voltage can change with the frequency of the applied voltage. This is interpreted as an effect of the difference in time between consecutive PDs at different applied frequencies.
Chapter 8

Conclusions

Partial discharge activity in a test object containing a disc-shaped cavity in polycarbonate is measured for applied frequencies in the range 0.01 – 100 Hz by use of the VF-PRPDA technique. It is shown that the PD process in the cavity is frequency dependent. Moreover the frequency dependence changes with the applied voltage amplitude and the size of the cavity. A difference in frequency dependence is also seen between insulated and electrode bounded cavities.

The PD activity in the test object is seen to change drastically during the first 1.5 hours after voltage application. In a PD measurement at variable applied frequency, this can be misinterpreted as PD frequency dependence. Therefore it is important to condition the test object before VF-PRPDA measurements. It is also observed that the foregoing applied frequencies can influence the PD process at the current applied frequency. This memory effect can be avoided by pre-exciting the test object prior to measurement at each new applied frequency.

A physical model of PD in a cavity is developed and used to dynamically simulate the sequence of PDs in the test object at different applied frequencies. The time dependent electric field distribution in the test object is calculated by use of the finite element method. The discharge process in the cavity is modelled dynamically and the apparent charge is calculated by time integration of the current through the electrode. Hence there is no need for $\lambda$-functions and analytical estimations of the apparent charge in the model.
Simulations show that essential features in the measured PD patterns at different applied frequencies can be reproduced. The PD frequency dependence is interpreted as a variation in influence on the PD activity from the statistical time lag, and the charge transport in the cavity surface, at different applied frequencies.

The simulation results also show that certain cavity parameters, like the cavity surface conductivity and the rate of electron emission from the cavity surface, change with the applied frequency. This is likely due to the difference in time between consecutive PDs at different applied frequencies. This effect contributes to the PD frequency dependence.
Chapter 9

Future work

In this study PD in a test object containing a disc-shaped cavity in polycarbonate is measured at variable applied frequency in the range 0.01 – 100 Hz. It would be interesting to measure the PD frequency dependence also for other cavity geometries and other insulation materials. In addition, increasing the applied frequency above 100 Hz may give further information about the PD frequency dependence.

The PD measurements at variable applied frequency in this work are performed with the VF-PRPDA technique. It would be interesting to measure PD at different applied frequencies also with other PD measuring techniques. Discharge area could be studied optically and time-resolved PD measurements could reveal changes in PD mechanism. Furthermore, measurements of the surface conductivity and electron emission rate of a cavity surface at varying applied frequency would be instructive.

The presented model of PD in a test object containing an insulated disc-shaped cavity in polycarbonate can easily be modified to model other cavity geometries and other insulation materials. However, only test objects with axi-symmetric electric field distribution can be modelled. Hence, either each PD must affect the whole cavity or the PDs must be concentrated to the cavity centre.

The model could also be further developed. In the model it is assumed that all PDs in the cavity affect the same area on the cavity surface and therefore the radius of the discharging cylinder ($r_{cyl}$) is constant. By
instead modelling $r_{\text{cyl}}$ with a suitable function, the model could be extended to allow for variations in discharge area.

Other developments would be to couple the electron generation rate in the model to the amount of charge in shallow traps in the cavity surface and to improve the modelling of the cavity surface conductivity. This however requires a refined model that distinguishes between surface charge originating from PDs and charge induced at the cavity surface by the applied electric field.

Finally, the model could be extended to three dimensions (3D) to model PDs at different locations in a cavity or the interaction between PDs in different cavities. However, this would drastically increase the degrees of freedom in the electric potential problem and cause longer simulation times.
Appendix A

Electric potential distribution

Here the model equation (6.1) for the electric potential distribution in the test object is derived. The governing equations for the electric potential distribution are (A.1) and the equation of current continuity (A.2)

\[ \nabla \cdot \vec{D} = \rho_f \]  
(A.1)

\[ \nabla \cdot \vec{J}_f + \frac{\partial \rho_f}{\partial t} = 0 \]  
(A.2)

Here $\vec{D}$ is the electric displacement field, $\rho_f$ is the free charge density and $\vec{J}_f$ is the free current density. The dielectric material is assumed to be linear, hence

\[ \nabla \cdot \vec{D} = \nabla \cdot (\varepsilon_0 \varepsilon_r \vec{E}) = -\nabla \cdot (\varepsilon_0 \varepsilon_r \nabla V) \]  
(A.3)

where $\varepsilon_0$ is the vacuum permittivity, $\varepsilon_r$ is the relative permittivity, $\vec{E}$ is the electric field and $V$ is the electric potential. With $\vec{J}_f = \sigma \vec{E} = -\sigma \nabla V$ equation (A.2) can be rewritten as

\[ \nabla \cdot (-\sigma \nabla V) + \frac{\partial}{\partial t} \left( -\nabla \cdot (\varepsilon_0 \varepsilon_r \nabla V) \right) = 0 \]  
(A.4)

where $\sigma$ is the electric conductivity. The dielectric material is assumed to be non-dispersive with an instantaneous polarization for applied fre-
quences in the range used in this work (0.01 Hz – 100 Hz). Hence (A.4) can be rewritten as (A.5), which is similar to (6.1).

\[ \nabla \cdot \left( -\sigma \nabla V - \frac{\partial}{\partial t} (\varepsilon_0 \varepsilon_r \nabla V) \right) = 0 \]  

(A.5)
Appendix B

Distribution function for PD

Here the model equation (6.4) for the PD distribution function is derived. The electron generation in a cavity is a stochastic process. Therefore the waiting time $\tau$ for an initial free electron to appear (from that the electric field condition for PD is fulfilled) is a stochastic variable. The expectation of $\tau$ is the statistical time lag $\tau_{\text{stat}}$.

The inception of a PD in a cavity can be considered as a discrete stochastic process with two states and continuous time $t$. The process is denoted with the stochastic variable $X(t)$ and is valid for time $t > 0$. The states are called “no PD” and “PD” and are denoted with $X = 0$ and $X = 1$, respectively. The process starts with $t = 0$ and $X(0) = 0$. After some time a PD occurs and the process converts to $X = 1$, without changing more after that. Consider a small time interval $[t', t' + \delta t]$ and assume that $X(t') = 0$, that is no PD has occurred up to time $t'$. The probability for $X(t' + \delta t) = 1$, that is for a PD occurring in $[t', t' + \delta t]$, is then assumed to be

$$P\left(X(t' + \delta t) = 1 \bigg| X(t') = 0\right) = N_e(t')\delta t \quad \text{(B.1)}$$

The electron generation rate $N_e(t)$ is the number of electrons that are generated in the cavity per unit time. It is a time-dependent non-negative intensity function. With these assumptions the inception of a PD in a cavity can be regarded as a lifetime process with intensity function $N_e(t)$ [59]. From this follows that the probability density for inception of
a PD, provided that the electric field exceeds the critical level for PD, is

\[ f(t) = N_e(t) \exp \left( - \int_0^t N_e(t')dt' \right) \]  

(B.2)

The corresponding distribution function is

\[ F(t) = 1 - \exp \left( - \int_0^t N_e(t')dt' \right) \]  

(B.3)

which is similar to (6.4).
Appendix C

Comsol Multiphysics® model

Here the Comsol Multiphysics® model is described in detail. This model is used in the PD model to calculate the time-dependent electric potential distribution in the test object. The PD model is described in Chapter 6.

The Comsol Multiphysics® model is a multi-physics model with two application modes: the Meridional Electric Currents application mode and the Weak Form Boundary application mode. Figure C.1 shows the two-dimensional (2D) axi-symmetric model geometry. The Meridional Electric Currents mode has dependent variable $V$ and is used to solve (6.1) on the full 2D model geometry. Table C.1 show the boundary settings for the Meridional Electric Currents mode. Here $\vec{n}$ is the normal vector to a boundary, $\vec{J}$ is the total current density, $U_a$ is the applied voltage amplitude, $f$ is the applied voltage frequency, $t$ is time, and $\vec{J}_1$ and $\vec{J}_2$ is the total current density at each side of a boundary, respectively.

The Weak Form Boundary mode has dependent variable $u$ and is used to solve (6.2) on the one-dimensional (1D) cavity surface, which corresponds to boundary 12 – 14 in Figure C.1. The advantage of using the Weak Form Boundary mode, instead of modelling the cavity surface as a thin layer in 2D, is the reduced size of the mesh. For use in the Weak Form Boundary mode, (6.2) must be written on weak form. This is done in (C.1) for boundary 12. Here $u_{\text{test}}$ is a suitable test function and the integration extends over the whole boundary.

\[ \int_a^b \frac{\partial}{\partial r} \left( -\sigma_{\text{surf}} \frac{\partial u}{\partial r} \right) u_{\text{test}} r \, dr = 0 \]  

After integration by parts (C.1) is expressed as
If the test function \( u_{\text{test}} \) is chosen such that \( u_{\text{test}}(a) = u_{\text{test}}(b) = 0 \), (C.2) can be rewritten as

\[
\int_a^b \sigma_{\text{surf}} \frac{\partial u}{\partial r} \frac{\partial u_{\text{test}}}{\partial r} r \, dr = 0 \quad (C.3)
\]
With a similar reasoning as above the weak form of (6.2) for boundary 13 and 14 can be expressed as (C.4) and (C.5), respectively.

\[ \int_{c}^{b} \sigma_{\text{surf}} \frac{\partial u}{\partial z} \frac{\partial u_{\text{test}}}{\partial z} \, dz = 0 \quad (C.4) \]

\[ \int_{d}^{c} \sigma_{\text{surf}} \frac{\partial u}{\partial r} \frac{\partial u_{\text{test}}}{\partial r} r \, dr = 0 \quad (C.5) \]

The boundary settings for the Weak Form Boundary mode are shown in Table C.2. Here the Comsol Multiphysics® variables \( uT\mathbf{r} \) and \( uTz \) are tangential derivative variables who represent the components of the tangential projection of the gradient of \( u \) on the boundary. The operator \( test \) creates the test function for the variable that it operates on. Table C.3 shows the point settings for the Weak Form Boundary mode.

The two application modes in the model are coupled through the boundary condition on boundary 12 – 14 in the Meridional Electric Currents application mode. To calculate the solution for \( V \) and \( u \), the time-dependent solver \( femtime \) is used with stop conditions related to the voltage over the cavity centre. Extrusion coupling variables are used to access the value of \( V \) at the points \( a \) and \( d \). For calculation of the total current through the test object an integration coupling variable is used to integrate the expression C.6 over boundary 6.

\[ 2\pi r \left( \mathbf{n} \cdot \mathbf{J} \right) \quad (C.6) \]

For calculation of the total charge (\( Q \)) on the upper cavity surface another integration coupling variable is used to integrate the expression C.7 over boundary 12. Here \( Dz \) is the component in the z-direction of the displacement field. The operators \( up \) and \( down \) evaluates an expression on either side of a boundary, respectively.

\[ 2\pi r \left( up(Dz) - down(Dz) \right) \quad (C.7) \]
Table C.1: Boundary settings for the Meridional Electric Currents application mode.

<table>
<thead>
<tr>
<th>Boundary</th>
<th>Condition</th>
</tr>
</thead>
<tbody>
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<td>1 – 5, 7 – 9</td>
<td>Electric insulation ( \vec{n} \cdot \vec{J} = 0 )</td>
</tr>
<tr>
<td>10</td>
<td>Electric potential ( V = U_a \cdot \sin(2\pi f t) )</td>
</tr>
<tr>
<td>6</td>
<td>Ground ( V = 0 )</td>
</tr>
<tr>
<td>12 – 14</td>
<td>Electric potential ( V = u )</td>
</tr>
<tr>
<td>11, 15</td>
<td>Continuity ( \vec{n} \cdot (\vec{J}_1 - \vec{J}_2) = 0 )</td>
</tr>
</tbody>
</table>

Table C.2: Boundary settings for the Weak Form Boundary application mode.

<table>
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<tr>
<th>Boundary</th>
<th>Weak term</th>
</tr>
</thead>
<tbody>
<tr>
<td>12, 14</td>
<td>(-r\sigma_{\text{surf}} \cdot uT_r \cdot \text{test}(uT_r))</td>
</tr>
<tr>
<td>13</td>
<td>(-\sigma_{\text{surf}} \cdot uT_z \cdot \text{test}(uT_z))</td>
</tr>
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</table>

Table C.3: Point settings for the Weak Form Boundary application mode.

<table>
<thead>
<tr>
<th>Point</th>
<th>Weak term</th>
</tr>
</thead>
<tbody>
<tr>
<td>a, d</td>
<td>( 0 ) (Continuity, ( \frac{\partial u}{\partial r} = 0 ))</td>
</tr>
<tr>
<td>b, c</td>
<td>( 0 ) (Insulation, ( \frac{\partial u}{\partial r} = 0 ))</td>
</tr>
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</table>
Bibliography


