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The book has not been updated since first published in 1997.

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Years of joint efforts between universities, research institutes and industry in the Nordic countries have been spent on research into different aspects of sandwich constructions. However, it is generally agreed that a major obstacle for further growth of the structural concept of sandwich construction is not only the lack of specialist knowledge requiring increased means for research but perhaps even more the need for the broad mass of engineers and designers in the industry to learn more about the fundamentals and get insight into a wider spectrum of the concept. Most engineers are very skilled and experienced in the design and materials selection process of metal and composite structures, but lack the fundamental theory, experience and confidence in sandwich design. The fact that many materials used in sandwich structures, especially the core materials, are new and little known of by many people, just increases the suspiciousness. Although there are still many areas in which considerable research is needed, it is a general feeling that if a majority of engineers and designers were to be given the opportunity to acquire basic knowledge and confidence in the sandwich structural concept, one could anticipate an even faster growing number of applications within the coming years.

As of today there is no open literature giving an overview of the analysis methods, material characteristics, design philosophies, and state-of-the-art research results. The existing literature is either highly academic or scattered in short overviews in various textbooks and journals. The objective of this book is to give an overview of typical material properties, summarise the existing theories and design methods, plus cover new research results in one single text. This is to give a comprehensive overview of all aspects of the concept. The text is designed so that hopefully anyone will be able understand and use the content. It gives no long derivations, but only the actual result, the regime in which the results may be used, and how to use it. It is to be seen as an easy access design tool and knowledge base rather than a textbook.

The initiation of this book originates from a wish to summarise existing knowledge and creating a suitable forum to present, in an engineering manner, results from the research efforts spent within the framework of the Nordic Industrial Fund (NI). Most of the contents of each chapter in this text originates from one or several NI sponsored research- and technical dissemination projects. These are:
The Handbook of Sandwich Construction

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The authors would very much appreciate to be contacted if; errors are found, clarifications are needed, parts of the text are confusing, parts are conceptually wrong, there exists places in the text where information is redundant, there is a lack of information, information is not up-to-date, or if someone wants to add information to the text.

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<th>Description</th>
<th>Unit</th>
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<td>$C$</td>
<td>Compliance</td>
<td>mm/N</td>
</tr>
<tr>
<td>$C_{E}$, $C_{G}$, $C_{t}$</td>
<td>Constants defining core material properties</td>
<td></td>
</tr>
<tr>
<td>$D$</td>
<td>Flexural rigidity (bending stiffness)</td>
<td>Nmm</td>
</tr>
<tr>
<td>$D_{x}, D_{y}, D_{z}$</td>
<td>Flexural rigidity of components in a sandwich</td>
<td>Nmm</td>
</tr>
<tr>
<td>$E$</td>
<td>Young's modulus of elasticity</td>
<td>N/mm² (MPa)</td>
</tr>
<tr>
<td>$G$</td>
<td>Shear modulus</td>
<td>N/mm²</td>
</tr>
<tr>
<td>$G$</td>
<td>Energy release rate</td>
<td>N/mm²</td>
</tr>
<tr>
<td>$K$</td>
<td>Buckling coefficient</td>
<td>–</td>
</tr>
<tr>
<td>$K_{I}$, $K_{II}$</td>
<td>Stress intensity factors in mode I and mode II</td>
<td>MPa√m</td>
</tr>
<tr>
<td>$K_{s}$, $K_{z}$</td>
<td>Foundation moduli</td>
<td>N/mm³</td>
</tr>
<tr>
<td>$L$</td>
<td>Length of beam</td>
<td>mm</td>
</tr>
<tr>
<td>$M$</td>
<td>Bending moment</td>
<td>N/mm</td>
</tr>
<tr>
<td>$N$</td>
<td>Normal force</td>
<td>N/mm</td>
</tr>
<tr>
<td>$P$</td>
<td>Load (line load)</td>
<td>N/mm</td>
</tr>
<tr>
<td>$Q$</td>
<td>Point load</td>
<td>N</td>
</tr>
<tr>
<td>$Q$</td>
<td>Generalised stress intensity factor in chapter 14</td>
<td>Nmm²</td>
</tr>
<tr>
<td>$R$</td>
<td>Reaction force or initial radius of shell</td>
<td>N/mm or mm</td>
</tr>
<tr>
<td>$R$</td>
<td>Rotary inertia</td>
<td>kg</td>
</tr>
<tr>
<td>$S$</td>
<td>Shear stiffness</td>
<td>N/mm</td>
</tr>
<tr>
<td>$T$</td>
<td>Transverse force</td>
<td>N/mm</td>
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<tr>
<td>$T_1$, $T_2$, $T_m$</td>
<td>Temperatures</td>
<td>K</td>
</tr>
<tr>
<td>$U$</td>
<td>Strain energy, potential energy</td>
<td>N/mm</td>
</tr>
<tr>
<td>$W$</td>
<td>Weight</td>
<td>kg</td>
</tr>
<tr>
<td>$Z$</td>
<td>Curvature parameter</td>
<td>–</td>
</tr>
<tr>
<td>$A$, $B$, $D$</td>
<td>Extension, coupling and bending stiffness matrices</td>
<td></td>
</tr>
<tr>
<td>$a$, $b$</td>
<td>Sides of rectangular panel (width of beam)</td>
<td>mm</td>
</tr>
<tr>
<td>$a$</td>
<td>Coefficient of heat transfer</td>
<td>W/m²K</td>
</tr>
<tr>
<td>$d$</td>
<td>Distance between centroids of the sandwich</td>
<td>mm</td>
</tr>
<tr>
<td>$e$</td>
<td>Distance defining position of neutral axis</td>
<td>mm</td>
</tr>
<tr>
<td>$k$</td>
<td>Thermal transmittance</td>
<td>W/m²K</td>
</tr>
<tr>
<td>$k_p$, $k_s$</td>
<td>Deformation coefficients for beams and panels</td>
<td>–</td>
</tr>
<tr>
<td>$k_r$, $k_m$</td>
<td>Loading and geometry constants</td>
<td>–</td>
</tr>
<tr>
<td>$m$, $n$</td>
<td>Integers – summation, modes, or exponents</td>
<td>–</td>
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<tr>
<td>$p_x$, $p_z$</td>
<td>Surface loads</td>
<td>N/mm²</td>
</tr>
<tr>
<td>$q_x$, $q_z$</td>
<td>Interface stress components in chapter 8</td>
<td>N/mm²</td>
</tr>
<tr>
<td>$q$</td>
<td>Distributed load (load/surface area) or heat flux</td>
<td>N/mm² or W/m²</td>
</tr>
<tr>
<td>$r$, $\phi$, $z$</td>
<td>Cylindrical coordinate system</td>
<td>mm, –, mm</td>
</tr>
<tr>
<td>$t$</td>
<td>Component thicknesses</td>
<td>mm</td>
</tr>
<tr>
<td>$u$, $v$, $w$</td>
<td>Deformation components in x, y, and z-directions</td>
<td>mm</td>
</tr>
</tbody>
</table>
Deformations due to bending and shear, respectively \( \text{mm} \)

Cartesian coordinate system \( \text{mm} \)

Decay length \( \text{mm} \)

Local \( z \)-axis for a single face \( \text{mm} \)

<table>
<thead>
<tr>
<th>Greek symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha )</td>
<td>Coefficient of thermal expansion</td>
<td>K(^{-1})</td>
</tr>
<tr>
<td>( \varepsilon )</td>
<td>Strain</td>
<td>–</td>
</tr>
<tr>
<td>( \kappa )</td>
<td>Curvature</td>
<td>mm(^{-1})</td>
</tr>
<tr>
<td>( \gamma, \gamma_0 )</td>
<td>Transverse and in-plane shear strain</td>
<td>–</td>
</tr>
<tr>
<td>( \eta )</td>
<td>Bending efficiency factor</td>
<td>–</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>Wave length or thermal conductivity</td>
<td>mm or W/mK</td>
</tr>
<tr>
<td>( \nu )</td>
<td>Poisson's ratio</td>
<td>–</td>
</tr>
<tr>
<td>( \theta )</td>
<td>Shear factor or nodal rotation in FEM</td>
<td>–</td>
</tr>
<tr>
<td>( \rho )</td>
<td>Density (mass/unit volume)</td>
<td>kg/m(^3)</td>
</tr>
<tr>
<td>( \rho^* )</td>
<td>Mass per unit surface area</td>
<td>kg/m(^2)</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>Direct stress</td>
<td>N/mm(^2) (MPa)</td>
</tr>
<tr>
<td>( \tau )</td>
<td>Shear stress</td>
<td>N/mm(^2) (MPa)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Subscript</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x, y, z )</td>
<td>Property refers to Cartesian direction</td>
</tr>
<tr>
<td>( r, \varphi, z )</td>
<td>Property refers to cylindrical direction</td>
</tr>
<tr>
<td>( c )</td>
<td>Property refers to the core</td>
</tr>
<tr>
<td>( f )</td>
<td>Property refers to the face</td>
</tr>
<tr>
<td>( 1, 2 \text{ or } f_1, f_2 )</td>
<td>Property refers to dissimilar faces</td>
</tr>
<tr>
<td>( L, R )</td>
<td>Left and right, respectively</td>
</tr>
<tr>
<td>( I, II, I_c, II_c )</td>
<td>Mode I and mode II, ( c ) indicates critical value</td>
</tr>
<tr>
<td>( \text{elastic, plastic} )</td>
<td>Elastic or plastic part of e.g. strain</td>
</tr>
<tr>
<td>( \text{max, min} )</td>
<td>Maximum or minimum value</td>
</tr>
<tr>
<td>( \tan )</td>
<td>Short for tangent modulus</td>
</tr>
<tr>
<td>( \text{tot} )</td>
<td>Total</td>
</tr>
<tr>
<td>( b )</td>
<td>Refers to a pure bending case</td>
</tr>
<tr>
<td>( r )</td>
<td>Short for reduced modulus</td>
</tr>
<tr>
<td>( s )</td>
<td>Refers to a pure shear case</td>
</tr>
<tr>
<td>( y )</td>
<td>Yield point</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Superscript</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(^\wedge)</td>
<td>Ultimate value, e.g., ( \sigma ) is ultimate or allowable stress</td>
</tr>
<tr>
<td>(^-)</td>
<td>Maximum value, e.g., ( W ) is maximum deflection</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Operators</th>
<th>Description</th>
</tr>
</thead>
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<tr>
<td>( d/dx_i )</td>
<td>Ordinary differential with respect to variable ( x_i )</td>
</tr>
<tr>
<td>( \partial / \partial x_i )</td>
<td>Partial differential with respect to variable ( x_i )</td>
</tr>
<tr>
<td>( \Delta )</td>
<td>Laplace operator</td>
</tr>
</tbody>
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CHAPTER 1

INTRODUCTION

1.1 Historical Background

John Montagu (1718-1792), 4th Earl of Sandwich and British first lord of the Admiralty during the American revolution, gave his name to - either due to lack of time or a healthy appetite - a lunch sandwiched between two meetings, referring to the act of making time or room for something [1]. Another tale states that John Montagu, who was a devoted billiard-player, invented to eat at the billiard-table [2] - in 1762 he spent 24 hours at the table without any other food than his sandwiches [3]. The archetypal sandwich consists of two slices of bread with meat in between. The sandwich construction discussed in this text is built in a manner similar to the edible to sandwich but instead of bread two thin sheets of high-performing material, and in between these the filling is replaced with a low density material.

Historically, the advantages of the concept of using two co-operating faces separated by a distance is thought to have been first discussed by a Frenchman, Duleau, in 1820, and later by Fairbairn [4]. Although it was not until 110 years later that the concept was first applied commercially. In World War One sandwich panels of asbestos faces with a fibreboard core were used and prior to World War Two some use was made of sandwich panels in small planes. However, it was the invention and widespread acceptance of structural adhesives in England and the United States in the 1930's that allowed the application of bonded sandwich composites. The Mosquito aircraft, produced in England during World War Two, saw mass production of sandwich laminates for the first time, utilising veneer faces with a balsa core [5]. This was however primarily due to a shortage of other materials as opposed to an appreciation of the structural efficiency of the concept. It was towards the completion of World War Two, in the late 1940's, that some of the first theoretical works on sandwich constructions were published. Since these early days the technology of sandwich laminates has progressed significantly and today far more comprehensive use of the advantages of sandwich laminates is being made.

Development of core materials has continued from the 1940's through to today in an effort to reduce the weight of the sandwich laminate. Balsa, the first core material to be used, is still in use where weight is not critical such as in cruising yachts and launches. Although heavy it still generally offers advantages over single skin designs. The late 1940's and 1950's saw the advent of honeycomb core materials, developed primarily for the aerospace industry. Honeycomb cores currently offer the greatest shear strength and stiffness to weight ratios but require care in ensuring adequate bonding to the faces. The core materials have been produced in various forms and
developed for a range of applications, generally utilising a hexagonal cell shape for optimum efficiency. The continued high cost of honeycomb cores has restricted their application predominantly to the aerospace industry. The late 1950's and early 1960's brought about the advent of the poly(vinyl chloride) (PVC) and polyurethane (PUR) core materials commonly used today in low and medium cost applications. Although PVC foams were developed in Germany in the early 1940's they were not utilised commercially until 15 years later due to the softness of these early cores. In the last twenty years few new cores have been developed with research oriented around facing materials and core bonding techniques. The next generation of core materials under development are cellular thermoplastic cores where properties can be tailored by orienting the cell structure.

Research into the theoretical analysis of sandwich constructions began following World War Two with several papers being published between 1945 and 1955 on the strength and stability of sandwich beams, columns and plates. An extensive and very complete listing of these early works, through to 1960, is published by Plantema [6] in his book on the topic. The theoretical research has generally preceded the practical application of sandwich constructions, from the 1940's right through to modern times. In the early years this was primarily due to the practical difficulties such as bonding the facings to the core, providing cores of sufficient stiffness and establishing reliable repair and inspection procedures. More recently, similar problems are still restricting the use of sandwich composites in areas such as primary aircraft structure. A considerable portion of the early research into sandwich constructions was undertaken by the Forest Product Laboratories of the United States Forest Service, e.g. [7]. These works are of both an experimental and theoretical nature and this laboratory has produced several significant works. These papers present analytical solutions to various beam and panel bending problems, using some degree of simplification to solve the various problems, and are referred to in several places in this text. Also during this period Reissner [8] published his well known theory on sandwich plates which derives the differential equation for deflection of a sandwich panel. Other significant early works on sandwich panels are those of Libove and Batdorf [9], Hoff [10] and Mindlin [11]. Libove and Batdorf [9] derived differential equations for the deflection and shear forces in orthotropic panels with thin faces, Hoff [10] considers the strain energy of a sandwich panel in terms of the transverse and in-plane deflections and derives the governing equation for an isotropic sandwich panel but with respect to thick faces, and Mindlin [6] derived the governing equation of motion for an isotropic plate accounting for both transverse shear deflections and rotary inertia. These theories form the basis of the two important texts on sandwich constructions published in the 1960's by Plantema [6] and Allen [12].

In the last twenty years the emphasis in theoretical research has shifted to optimisation of laminates, with finite element analysis used as a design tool for the panel analysis problems. As a result little further work has been conducted into the theoretical analysis of sandwich panels, principally due to the difficulty of obtaining more exact solutions in deriving and solving the differential equations for deflection of sandwich panels The errors in the current approximations used are often negligible for practical composite laminates but require consideration. Finite element techniques utilising especially designed sandwich elements also allow accurate analysis of sandwich design problems. These are generally more accurate than many of the existing analytical solutions which require several approximations and the use of finite difference methods to solve the differential equation. Research into sandwich constructions over the last two decades
INTRODUCTION

has revolved primarily around the areas of impact resistance, fatigue and fracture analysis with these areas being of major concern to the aerospace industry. This research is now allowing the introduction of composite materials in aircraft primary structure. The theoretical analysis of sandwich beams and plates has received little attention with the aerospace industry content to use finite element analysis for design purposes.

1.2 Definition of a Sandwich Element

A sandwich consists of three main parts as illustrated in Fig.1.1. Two thin, stiff, and strong faces are separated by a thick, light, and weaker core. The faces are adhesively bonded to the core to obtain a load transfer between the components.

![Figure 1.1 Schematic of a structural sandwich panel.](image)

The ASTM defines a sandwich structure as follows:

*A structural sandwich is a special form of a laminated composite comprising of a combination of different materials that are bonded to each other so as to utilise the properties of each separate component to the structural advantage of the whole assembly.*

The modus operandi of a sandwich is much the same as that of an I-beam, which is an efficient structural shape because as much as possible of the material is placed in the flanges situated farthest from the centre of bending or neutral axis. Only enough material is left in the connecting web to make the flanges act in concert and to resist shear and buckling. In a sandwich the faces take the place of the flanges and the core takes the place of the web. The difference is that the core of a sandwich is of a different material from the faces and it is spread out as a continuous support for the faces rather than concentrated in a narrow web. The faces will act together to form an
efficient stress couple or resisting moment counteracting the external bending moment. The core resists shear and stabilise the faces against buckling or wrinkling. The bond between the faces and the core must be strong enough to resist the shear and tensile stresses set up between them. The adhesive that bonds the faces to the core is thus of critical importance.

In the type of sandwich elements considered in this text the faces usually consist of thin and, in a classical meaning, high performing material while the core material is a thick, light but relatively low performing. The choice of constituents depends mainly on the specific application and the design criteria set up by it. The design of a structural sandwich will not be one of geometry only but an integrated process of geometrical design and materials selection.

1.3 Advantages and Disadvantages
A sandwich element provides the opportunity, through efficient design, to utilise each material component to its ultimate limit. The most obvious advantages gained by this assembly is a very high stiffness-to-weight ratio and also a high bending strength-to-weight ratio. The way that a sandwich enhances the flexural rigidity of a structure without adding substantial weight has made the concept even more advantageous since the introduction of composite materials. These material generally offer at least the same or even higher strengths as metals such as aluminium or steel, but their moduli are often much lower giving poor stiffness performance. By using sandwiched composites this problem can easily be overcome. The continuous support of the face sheet, unlike a stiffened structure, implies that surfaces remain flat even under quite high compressive stress without buckling. This is important in e.g. aircraft structures in which control surfaces preferably should remain smooth even under loading. Sandwich structures have in several applications shown to have superior fatigue strength, although this is matter that needs further investigation. The same thing goes for acoustical insulation. The absorption of mechanical energy can in some deformation modes be multiplied compared with monocoque structures due to an imposed shorter mode of buckling waves. The use of cellular core materials means that no additional thermal insulation needs to be added to the structure, thus ensuring a low structural weight, since most cellular cores have a very low thermal conductivity. This is one of the so called integrated function that in most cases comes "for free" with the concept. Sandwich elements can be manufactured in large sheets, giving large smooth areas without need for connections like rivet and bolts. This means that less parts are needed and the assembly of the structure is simplified, which in consequence saves money. When using fibre composite faces, even large structures can be manufactured in more or less one piece, thus reducing assembly costs and ensuring smooth and continuous load paths without disturbing stress concentrations.

There are of course a number of disadvantages with using sandwich constructions. One major obstacle is the cause of this text; the lack of general knowledge, among engineers and designers in the industry, about the concept and the materials used. This problem is easily overcome by education, both on under-graduate and graduate level. "It is never too late to learn something new". Since the concept is quite new to many applications, there is strong need for research and development in some areas. A main obstacle is that the used manufacturing methods are in infancy, requiring much labour and that they to a very little degree are automated. This fact also makes quality control a difficult task. Some suggested solutions for automated quality control is given within this text. Many materials used in sandwich structures are relatively new and the designer will therefore have little experience with the materials, and a limited access to material data. This
makes the designer cautious and conservative thus designing the structure heavier than needed, which contradict the primary aim of using a sandwich design, namely to save weight. A major area in which there is need for further research is that of fatigue of sandwich materials and structures. This involves not only un-notched fatigue data like S-N curves, but also the more complicated questions of the fatigue life when accounting for a damage and damage growth. This has further implications; damage can be created either in the manufacturing process, so called manufacture induced flaws, or during in-service. In the former group we have different types of disbond, voids in the core material, and delaminations, and in the latter group, damage caused by over-loading, fatigue or impact. Some information about the effect of damage on the residual static strength is given in this handbook, whereas the fatigue life reduction due to damage is a part that hopefully will covered in later editions.

1.4 Applications
Below are photographs and illustrations of some applications of sandwich constructions. It is not always easy to just look at a structure and realise it is a sandwich design. Some further applications are given in relation to their respective manufacturing method in chapter 15, and not shown again in this section. As seen, sandwich constructions are used in almost every industrial sector ranging from building to aerospace applications.

The GRP terrain vehicle shown in Fig.15.12 uses sandwich in parts of the structure to obtain higher stiffness and strength and integrated thermal insulation. Low structural weight is a feature of the vehicle in order to be able to operate in e.g. deep snow conditions. By reducing the structural weight of a truck structure the pay-load can be increased. By using a sandwich design, as the one shown in Fig.1.2, high stiffness and strength is combined with integrated high thermal insulation.

![Figure 1.2 Sandwich truck structure; low weight, integrated manufacturing and thermal insulation. Courtesy of Specialkarosser AB, Ätran, Sweden.](image-url)
A similar application to the truck structure is the sandwich container shown in Fig.1.3. Here, a low weight structure is combined with high thermal insulation for the transportation of cold goods, e.g. fruit or other types of food.

![Figure 1.3 Sandwich container; low weight, integrated manufacturing and thermal insulation. Courtesy of Swecom Reefer AB, Uddevalla, Sweden.](image)

There is much research and development into using sandwich design for other type of transportation applications, including cars, subway cars and trains. One such example is shown in Fig.15.16. The aim is to reduce the weight, and hence the emissions, but also to integrate details for reduced manufacturing costs. Possibilities of having integrated functions of thermal and acoustic insulation is of course also a possible advantage. In the case of the Danish IC3 train, Fig.1.4, parts of the structure uses sandwich design, including flooring, interior and exterior panels.

![Figure 1.4 Parts of the IC3 train is made in sandwich design. Courtesy of Divinycell International AB, Laholm, Sweden.](image)

There is a variety of pleasure boats and ships made in sandwich design. In pleasure boats, decks and hull are commonly made in a sandwich design. Examples of this are shown in Fig.1.5 and 1.6.
Even larger ships utilise GRP-sandwich design; examples of this are SMYGE, shown in Fig.1.5, and the new monohull navy ship YS2000, shown in Fig.1. The choice of sandwich design for these structures comes from the urge to use non-magnetic materials combined with high energy absorption capability and low structural weight.
Newly designed large and fast ferries with aluminium hulls, Fig.1.8, utilise sandwich design for the superstructure, front, car decks, etc. in order to save weight. Smaller version of the so-called Surface Effect Ships (SES) have sometimes an all-sandwich structure.

![Image of Stena Line fast ferry](image1)

Figure 1.8 Superstructure, front, bilge keel, bow tip and car deck of Stena Line fast ferry in GRP sandwich for low weight. Courtesy of Finnyards Oy, Rauma, Finland.

A rather new sandwich application is the wellhead protection structure for North Sea oil pumps, as shown in Fig.1.9. These are placed on top of oil pumps on the sea floor to protect them from being damaged by foreign objects. The main feature of the protection is of course high impact strength but also low corrosion and low weight are important characteristics.

![Image of sub-sea wellhead protection](image2)

Figure 1.9 Sub-sea wellhead protection in GRP-sandwich for maximum energy absorption properties combined with no corrosion and low weight. Courtesy of Brødrene Aa, Hyen, Norway.

In civil engineering applications sandwich panels have long been used for low weight and thermal insulation. One such example is the Stockholm Globe Arena, shown in Fig.15.9, which utilises sandwich elements assembled to a load carrying aluminium frame work. Other examples are the wind-mill housing shown in Fig.1.10 and the garage door in Fig.1.11.
In aerospace, sandwich construction has been used for a long time and applications include wings, doors, control surfaces, radomes, tailplanes, stabilisers, etc. for both military and civil aircraft. The Swedish SAAB JAS 39 Gripen, shown in Fig. 15.7, where several parts are made in a sandwich design. Space structures, such as antennas and solar panels are often sandwich design.
References


As outlined in previous sections, a sandwich consists of three or more constituents; the faces, the core and the adhesive joints. In general, the faces may be of different materials and even the two adhesive joints may be made of different adhesives, all depending on the requirements on the structure and the manufacturing process. The choice of materials is vast and since the introduction of fibre composites the choice of face materials has increased to an almost infinite number of different materials, all with different properties. Even the number of available cores has increased dramatically in recent years since the introduction of more and more competitive cellular plastics. Hence, the design of sandwich structures is just as much a materials selection problem as a sizing problem. The vast number of material choices may appear as an additional complexity but is really one of the main features of using sandwich constructions; the materials best suited for a specific application may be utilised and some drawbacks can be overcome by geometrical sizing. For example, some reinforced plastics lack the advantage of the high stiffness of metals, but by increasing the core thickness a feasible rigidity may still be obtained. Materials are often chosen on grounds that are not purely mechanical but rather for reasons such as, environmental resistance, surface finish, the use of a specific manufacturing method, cost, wear resistance etc.

The following section is not intended to be complete in any way but is rather an introduction to the materials commonly used in load carrying sandwich structures. All data presented are typical values collected from different sources and no guarantee is given for their correctness. Material data have a habit of showing quite large scatter, especially data for fibre composites, since they are very dependent on the manufacturing process. However, the numbers given should be fairly typical for the materials described.

2.1 Face Materials
Quoting Allen [1], "Almost any structural material which is available in the form of thin sheet may be used to form the faces of a sandwich panel", gives a good view of the variety available in materials selection. This is a prime feature of the concept, the designer has the opportunity through efficient design to utilise each material component to its ultimate limit. The properties of primary interest for the faces are;
• High stiffness giving high flexural rigidity
• High tensile and compressive strength
• Impact resistance
• Surface finish
• Environmental resistance (chemical, UV, heat, etc.)
• Wear resistance

The commonly used face materials can be divided into two main groups; metallic and non-metallic materials. The former group contains steel, stainless steel and aluminium alloys. There is a vast variety of alloys with different strength properties whereas the stiffness variation is very limited. The larger of the two groups is the latter, including materials such as plywood, cement, veneer, reinforced plastic, and fibre composites.

The most important of the non-metallic materials are fibre composites which since their introduction has had a major impact on the use of sandwich constructions. The reason for this is that most composites offer strength properties similar to or even higher than those of metals, although their stiffness is often magnitudes lower (see Table 2.4). Thus, in order to achieve high rigidity, composites are more often sandwiched with a light core. Another important reason is that the manufacturing of sandwich composites are much easier than the manufacturing of metal face sandwich structures. Another feature of composites is their anisotropic behaviour, i.e., they have different properties in different directions. This is an initial complexity often viewed of as an obstacle by engineers but is in reality an advantage as it offers the opportunity to tailor properties in conjunction with the applied loads. For example, one can place enough amount of fibres in one direction to carry the load in that particular direction, and a different amount in another direction. Hence, not only the material components are stressed to their ultimate limit but the component itself may be utilised in a more optimised way.

2.1.1 Metallic face materials
Metals need no further explanation in terms of composition or behaviour. There are a host of steel, aluminium, and titanium alloys and the actual choice depends on the application considered. The advantages by using metal faces are; high stiffness and strength, low cost, good surface finish, and high impact resistance. The drawbacks are; high density and difficulty in manufacturing of sandwich structures. The latter restraint is imposed by the difficulty to manufacture sandwich panels with double curvature using sheet metal.

Density and Young's modulus of metals are very little affected by alloy composition, whereas the strength varies very much between different alloys. There is such a variety that only some are listed below. Much more detailed information can be found in various textbooks, and from materials manufacturers and suppliers, including fatigue and creep properties, heat and chemical resistance, etc.
### Materials and Material Properties

<table>
<thead>
<tr>
<th>Material</th>
<th>$\rho$ (kg/m$^3$)</th>
<th>$E$ (GPa)</th>
<th>$\nu$</th>
<th>$\alpha$ ($^{\circ}$C$\cdot$10$^{-6}$)</th>
<th>$\sigma$ (MPa)</th>
<th>$\lambda$ (W/m°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mild steel</td>
<td>7800</td>
<td>206</td>
<td>0.29</td>
<td>13</td>
<td>360 (1)</td>
<td>46</td>
</tr>
<tr>
<td>Steel - cold rolled HS</td>
<td>7800</td>
<td>206</td>
<td>0.3</td>
<td>11</td>
<td>800 (1)</td>
<td></td>
</tr>
<tr>
<td>Stainless SIS2301</td>
<td>7700</td>
<td>196</td>
<td>0.29</td>
<td>11</td>
<td>250</td>
<td>110</td>
</tr>
<tr>
<td>Stainless steel (18-8)</td>
<td>7900</td>
<td>196</td>
<td>0.29</td>
<td>18</td>
<td>200</td>
<td>110</td>
</tr>
<tr>
<td>Stainless steel (17-7)</td>
<td>7900</td>
<td>199</td>
<td>0.25</td>
<td>-</td>
<td>1000</td>
<td>110</td>
</tr>
<tr>
<td>Aluminium 2024-T4</td>
<td>2700</td>
<td>73</td>
<td>0.3</td>
<td>23</td>
<td>300</td>
<td>140-190</td>
</tr>
<tr>
<td>Aluminium 5052</td>
<td>2700</td>
<td>70</td>
<td>0.3</td>
<td>-</td>
<td>170</td>
<td>140-190</td>
</tr>
<tr>
<td>Aluminium 6061</td>
<td>2700</td>
<td>69</td>
<td>0.3</td>
<td>-</td>
<td>240</td>
<td>140-190</td>
</tr>
<tr>
<td>Aluminium 7075-T6</td>
<td>2700</td>
<td>70</td>
<td>0.3</td>
<td>23</td>
<td>470</td>
<td>140-190</td>
</tr>
<tr>
<td>Titanium annealed</td>
<td>4500</td>
<td>108</td>
<td>0.25</td>
<td>9</td>
<td>550</td>
<td></td>
</tr>
<tr>
<td>Titanium heat treated</td>
<td>4500</td>
<td>108</td>
<td>0.25</td>
<td>9</td>
<td>980</td>
<td></td>
</tr>
</tbody>
</table>

(1) yield strength or strain, variation between different alloys.

Table 2.1 Typical mechanical properties of some metal face materials [2].

#### 2.1.2 Non-metallic face materials

A brief introduction to the material constituents of fibre composites, which are the most widely used type of non-metallic face materials in sandwich constructions.

Glass fibres: The most commonly used reinforcement material in fibre composites used in load bearing sandwich constructions is the so called E-glass fibre. It has good mechanical properties and environmental resistance, but it’s competitiveness comes primarily from the relative low price. There are other types of glass reinforcements like S- and R-glass with slightly better mechanical properties but their price are significantly higher at present time. The main ingredient in glass is SiO$_2$, about 50 - 70% [3], but other metal oxides are often added such as Al$_2$O$_3$, Fe$_2$O$_3$ and CaO. The main drawback with glass reinforcement is that the elastic modulus is fairly low and that the density is higher than for other reinforcements.

Aramid fibres: This type of fibre reinforcement is perhaps more well known under its trade name Kevlar® but today there exists many aramid fibres from several manufacturers. It is made from aromatic polyamid, has low density, high modulus and high strength. The fibres are extremely wear resistance, in fact making aramid fibre laminates difficult to machine. It is the fibre with the highest strength-to-weight ratio in tension of all known fibres but with a much poorer strength in compression.

Carbon fibres: Carbon fibres are built-up by long carbon-carbon molecular chains yielding very stiff fibres. The trends have driven development of carbon fibres in two directions; high-strength (HS) fibres with very high tensile strength and a fairly high strain to failure (1-1.5%) and high modulus (HM) fibre with very high stiffness. Especially the latter has found their use in advanced aerospace applications where the use of light weight materials with high stiffness is essential. Carbon fibres have a low coefficient of thermal expansion, good friction properties, good X-ray penetration and is non-magnetic. The main drawback is the high cost and that all carbon composites are relatively brittle.

For fibres commonly used in composites, there are great variations in mechanical data between different products. In Table 2.2, a list of typical data is presented.
### Table 2.2  Typical mechanical properties of some fibres [3,4,5].

<table>
<thead>
<tr>
<th>Material</th>
<th>( \rho )</th>
<th>( E )</th>
<th>( \alpha )</th>
<th>( \sigma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>E-glass fibre</td>
<td>2600</td>
<td>70</td>
<td>5.4</td>
<td>2500</td>
</tr>
<tr>
<td>S-glass</td>
<td>2600</td>
<td>85</td>
<td>-</td>
<td>4800</td>
</tr>
<tr>
<td>Aramid (Kevlar 49)</td>
<td>1400</td>
<td>125</td>
<td>-</td>
<td>2200</td>
</tr>
<tr>
<td>Carbon HS</td>
<td>1750</td>
<td>220</td>
<td>-</td>
<td>3000</td>
</tr>
<tr>
<td>Carbon HM</td>
<td>1950</td>
<td>400</td>
<td>-</td>
<td>2500</td>
</tr>
<tr>
<td>Boron</td>
<td>2600</td>
<td>400</td>
<td>-</td>
<td>3400</td>
</tr>
</tbody>
</table>

### Table 2.3  Typical mechanical properties of some matrix materials [3,7].

<table>
<thead>
<tr>
<th>Material</th>
<th>( \rho )</th>
<th>( E )</th>
<th>( \alpha )</th>
<th>( \sigma )</th>
<th>( T_{\text{max}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Iso-polyester</td>
<td>1100</td>
<td>4</td>
<td>-</td>
<td>40</td>
<td>-</td>
</tr>
<tr>
<td>Orto-polyester</td>
<td>1100</td>
<td>4</td>
<td>-</td>
<td>40</td>
<td>-</td>
</tr>
<tr>
<td>Vinylester</td>
<td>1100</td>
<td>8</td>
<td>-</td>
<td>50</td>
<td>-</td>
</tr>
<tr>
<td>Epoxy</td>
<td>1100</td>
<td>12</td>
<td>-</td>
<td>60</td>
<td>-</td>
</tr>
<tr>
<td>PEEK</td>
<td>1310</td>
<td>3.8</td>
<td>40-47</td>
<td>80</td>
<td>334</td>
</tr>
<tr>
<td>PPS</td>
<td>1400</td>
<td>4</td>
<td>49</td>
<td>65</td>
<td>285</td>
</tr>
<tr>
<td>PP</td>
<td>900</td>
<td>1.7</td>
<td>80-100</td>
<td>35</td>
<td>170</td>
</tr>
<tr>
<td>Polyimide (Nylon 6)</td>
<td>1070</td>
<td>1.0</td>
<td>10-80</td>
<td>60</td>
<td>210</td>
</tr>
</tbody>
</table>

\(^{(1)}\) maximum allowed temperature for thermosets, melting temperature for thermoplastics.

### Fibre composites: The fibres and the matrix is mixed to form a composite. This can be achieved in an almost infinite number of ways using different material combinations and manufacturing techniques. The feature is, however, to utilise the strength and stiffness of the fibres to the benefit of the structural component. Thus, it is advantageous to have a high fraction of fibres, oriented in the directions the loads act and use a matrix with good resistance to the environment in which the composite is to be working. Composites have the advantage of being lighter than metals, equally strong or even stronger but will in most cases have a much lower stiffness. There is an enormous...
amount of research going on concerning fibrous composites at present time dealing with everything from micro mechanics to manufacturing technology, and the amount of published research work is steadily increasing. This text is not intended to, and cannot, cover every aspect of composites and their use in conjunction with sandwich constructions in load carrying structures. However, some data are given in Tables 2.4 and 2.5 reviewing mechanical properties of a variety of material systems. It must be remembered, however, that the properties of a composite depends not only upon the characteristics of the constituent but also in a very high degree upon the manufacturing method used. Thus, the mechanical properties of fibre composites show very large scatter.

<table>
<thead>
<tr>
<th>Material</th>
<th>$v_f$</th>
<th>$\rho$</th>
<th>$E_x$ (1)</th>
<th>$E_y$ (1)</th>
<th>$G_{xy}$ (1)</th>
<th>$\nu_{xy}$ (1)</th>
<th>$\alpha$ $/{}^\circ\text{C}10^6$</th>
<th>$\lambda$ W/m°C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wood [2,8,9]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pine</td>
<td>-</td>
<td>520</td>
<td>12</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>4</td>
<td>-</td>
</tr>
<tr>
<td>Fir plywood</td>
<td>-</td>
<td>580</td>
<td>12.4</td>
<td>12.4</td>
<td>-</td>
<td>0.1</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Pine plywood</td>
<td>-</td>
<td>580</td>
<td>12.4</td>
<td>12.4</td>
<td>-</td>
<td>0.1</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Unidirectional prepreg [4]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T300/5208 Carbon/Epoxy</td>
<td>70</td>
<td>1600</td>
<td>180</td>
<td>10</td>
<td>7.2</td>
<td>0.28</td>
<td>5-11</td>
<td>0.3-0.35</td>
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<tr>
<td>B4/5505 Boron/Epoxy</td>
<td>50</td>
<td>2000</td>
<td>204</td>
<td>19</td>
<td>5.6</td>
<td>0.23</td>
<td>5-11</td>
<td>0.3-0.35</td>
</tr>
<tr>
<td>AS/3501 Carbon/Epoxy</td>
<td>66</td>
<td>1600</td>
<td>138</td>
<td>9.0</td>
<td>7.1</td>
<td>0.26</td>
<td>5-11</td>
<td>0.3-0.35</td>
</tr>
<tr>
<td>Scotchply 1001 Glass/Epoxy</td>
<td>45</td>
<td>1800</td>
<td>39</td>
<td>8.3</td>
<td>4.1</td>
<td>0.26</td>
<td>5-11</td>
<td>0.3-0.35</td>
</tr>
<tr>
<td>Kevlar 49/Epoxy</td>
<td>60</td>
<td>1500</td>
<td>76</td>
<td>5.5</td>
<td>2.3</td>
<td>0.34</td>
<td>5-11</td>
<td>0.3-0.35</td>
</tr>
<tr>
<td>Unidirectional hand lay-up [9]</td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>E-glass/polyester</td>
<td>36</td>
<td>1700</td>
<td>26</td>
<td>6</td>
<td>-</td>
<td>-</td>
<td>5-14</td>
<td>0.3-0.35</td>
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<tr>
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<td>1800</td>
<td>34</td>
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<td>5-14</td>
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<td>1300</td>
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<tr>
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<td></td>
</tr>
<tr>
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<td>1900</td>
<td>22</td>
<td>22</td>
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<td>34</td>
<td>1700</td>
<td>15</td>
<td>15</td>
<td>-</td>
<td>-</td>
<td>9-11</td>
<td>0.3-0.35</td>
</tr>
<tr>
<td>E-glass woven roving</td>
<td>28</td>
<td>1600</td>
<td>11</td>
<td>11</td>
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<td>-</td>
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<td>0.3-0.35</td>
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<td>-</td>
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<td>0.3-0.35</td>
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<td>19</td>
<td>1500</td>
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<td>-</td>
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<td>1500</td>
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<td>-</td>
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<tr>
<td>SMC polyester</td>
<td>20</td>
<td>1800</td>
<td>9</td>
<td>9</td>
<td>-</td>
<td>-</td>
<td>20-35</td>
<td>0.2-0.25</td>
</tr>
</tbody>
</table>

(x) in fibre direction, (y) transverse fibre direction

Table 2.4 Typical elastic properties of wood and fibre composites.

Wood: Wood is by nature an anisotropic material due to its build-up of cellulose fibres. It can thus be considered as a composite with a dominant fibre direction (direction of growth). Like all unidirectional composites the mechanical properties in the fibre direction are good whereas properties are very poor in the perpendicular directions. The mechanical properties vary highly
between different wood, from hard and stiff timber such as oak to very soft wood like balsa. Plywood is thus a way of overcoming the low perpendicular strength by assembling a number of thin wood layers in different directions to form a laminate, much in the same way a composite laminate is built. Wood is cheap and light but lacks the high stiffness and strength properties of its fibre reinforced plastic counterparts.

<table>
<thead>
<tr>
<th>Material</th>
<th>tension</th>
<th>compression</th>
<th>in-plane</th>
<th>inter-laminar</th>
<th>at v_f (%)</th>
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<tbody>
<tr>
<td></td>
<td>$\sigma_1$ MPa</td>
<td>$\sigma_2$ MPa</td>
<td>$\sigma_1$ MPa</td>
<td>$\sigma_2$ MPa</td>
<td>$\tau_{12}$ MPa</td>
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<td>46</td>
<td>7</td>
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<td>-</td>
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<td>18</td>
<td>-</td>
<td>-</td>
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<td>53</td>
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<td><em>Plytron</em></td>
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<td><em>Unidirectional hand lay-up [10]</em></td>
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<td>550</td>
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<td>350</td>
<td>100</td>
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<tr>
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<td>270</td>
<td>230</td>
<td>230</td>
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<td>E-glass woven roving</td>
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<td>200</td>
<td>160</td>
<td>160</td>
<td>70</td>
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<td>S-glass weave</td>
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<td>350</td>
<td>300</td>
<td>300</td>
<td>70</td>
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<tr>
<td>Kevlar weave</td>
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<td>380</td>
<td>140</td>
<td>140</td>
<td>70</td>
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<td>425</td>
<td>280</td>
<td>280</td>
<td>70</td>
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<td><em>Random hand lay-up [12]</em></td>
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<td></td>
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<tr>
<td>E-glass CSM-mat</td>
<td>90</td>
<td>90</td>
<td>120</td>
<td>120</td>
<td>-</td>
</tr>
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<td>E-glass spray-up</td>
<td>65</td>
<td>65</td>
<td>95</td>
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<td>70</td>
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<tr>
<td><em>Moulding compound [12]</em></td>
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<td></td>
<td></td>
<td></td>
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<tr>
<td>SMC polyester</td>
<td>60</td>
<td>60</td>
<td>250</td>
<td>250</td>
<td>70</td>
</tr>
</tbody>
</table>

Table 2.5 Typical strength properties of wood and fibre composites.
2.1.3 Estimation of fibre composite properties

There are a number of more or less advanced methods for the calculation and design of fibre composite materials. Of these, classical lamination theory is the most widely used [5]. For more practical purposes engineers frequently use the so called rule-of-mixtures, e.g. [12] to design and analyse more simple lay-ups of reinforcement. The simple formulae provided by this rule seem to yield good enough results for most engineering purposes, especially for stiffness properties. A good assessment of strength characteristics is often more complicated.

This section will only deal with estimation of properties for composite materials, since properties of metals are found from testing or handbook data. It starts by showing how the properties of the composite can be approximately calculated by using the properties of the material constituents on a microscopic level. The main purpose is to find approximate properties of the lamina (one layer) as function of the materials building up the lamina. The properties of the lamina can then be used to calculate the properties of the laminate (a stack of lamina) which is to be used as a face sheet of the sandwich.

A lamina is defined as a thin orthotropic layer of a composite material. It is defined as the smallest element on a macroscopic level to be used to build up a composite laminate. The reasons for using the orthotropic approximation for the lamina are several; the lamina often consists of a of unidirectional, bi-directional or random layer of reinforcement. These can with good approximation be treated as orthotropic layers. It is also a great simplification to use an orthotropic lamina instead of a general anisotropic one for computational purposes.

![Figure 2.1 Example of a lamina - a unidirectional layer of fibres and matrix.](image)

Rule-of-mixtures

(i) Determination of $E_i$

The average modulus in the fibre direction of a uni-directional composite lamina $E_i$ is according to the rule-of-mixtures

$$E_i = E_f v_f + E_m v_m$$

(2.1)

where $v_f$ and $v_m$ are the volume fractions ($v_i = V_i/V$), $E_f$ is the fibre modulus and $E_m$ the matrix modulus. In more general terms this equation can be written as

$$E_i = \sum_i E_i v_i$$

(2.2)
where the summation is taken over all constituents. This approximation is called the parallel model, and the result a Voigt material which constitutes an upper bound of the elastic modulus any composite can have. The stress in the composite is thus a volume weighted average.

\[ \sigma_i = \sum \sigma_i v_i \]  

(2.3)

(ii) Determination of \( E_2 \)
The apparent Young's modulus in the direction transverse to the fibres can be assessed in a similar way.

\[ \frac{1}{E_2} = \frac{v_f}{E_f} + \frac{v_m}{E_m} \quad \text{or more generally} \quad \frac{1}{E_2} = \sum \frac{v_i}{E_i} \]  

(2.4)

This approximation is called the serial model, and the result a Reuss material which constitutes a lower bound of the elastic modulus any composite can have.

(iii) Determination of \( \nu_{12} \)
The Poisson ratio \( \nu_{12} = -\varepsilon_2/\varepsilon_1 \) by definition (rule-of-thumb: index 1 - direction of load, index 2 - direction of deformation). If we denote \( v_f \) the fibre Poisson ratio and \( v_m \) the matrix Poisson ratio we have as for the serial model that,

\[ \nu_{12} = v_f v_f + v_m v_m \quad \text{or more generally} \quad \nu_{12} = \sum v_i v_i \]  

(2.5)

(iv) Determination of \( G_{12} \)
The in-plane shear modulus \( G_{12} \) is found by assuming that the shear stress acting on the fibres and the matrix are equal. If \( G_f \) is the fibre shear modulus and \( G_m \) the matrix shear modulus, the average shear modulus of the lamina is

\[ \frac{1}{G_{12}} = \frac{v_f}{G_f} + \frac{v_m}{G_m} \quad \text{or more generally} \quad \frac{1}{G_{12}} = \sum \frac{v_i}{G_i} \]  

(2.6)

“Practical” rule-of-mixtures
It is known from experience that the Voigt material model agrees well with experiments whereas the Ruess model show very poor agreement, especially for unidirectional composites. The tensile modulus may be predicted for a unidirectional composite using the rules-of-mixture but in practice the composite laminate may be built up of fabrics, e.g., weaves, roving, mats and/or in combination with unidirectional layers. The rule-of-mixtures in the fibre direction may be modified to suit this purpose by simply introducing reinforcement efficiency factors. The mixture rule along the fibres may now take the form

\[ E_i = \sum \alpha_i v_i E_i \]  

(2.7)

In practice, this means that we only consider the amount of fibres in the laminate acting in the studied direction. For example, a bi-directional symmetric weave has only half of its fibres in the 1-direction and the other half in its 2-direction. Thus, the efficiency factor should be 0.5. Hence, the efficiency factor \( \alpha \) is simply the fraction of the fibres acting in the studied direction. The values of \( \alpha \) are for different reinforcements.
MATERIALS AND MATERIAL PROPERTIES

1 for unidirectional fibres and the matrix
0.5 for bi-directional symmetric
0.375 for random in-plane
0.2 for random in space

For any other configuration use the definition above, e.g., a weave with 70% of the fibres in its 1-direction and 30% in its 2-direction, \( \alpha = 0.7 \).

**Conversion weight fraction/volume fraction**

Most calculations on composites are carried out on the basis of volume fractions. However, in production it is normal to work in terms of weight fractions. It is therefore necessary to convert from one to the other. The following formulae are often useful.

The total weight is:

\[
W = \sum_i W_i = \sum_i \rho_i V_i = V \sum_i \rho_i v_i \quad (2.8a)
\]

The weight fraction is:

\[
w_i = \frac{W_i}{W} = \frac{\rho_i V_i}{V \sum_i \rho_i v_i} = \frac{\rho_i v_i}{\sum_i \rho_i v_i} \quad \text{with} \quad \sum_i w_i = 1 \quad (2.8b)
\]

The total volume is:

\[
V = \sum_i V_i = \sum_i \frac{W_i}{\rho_i} = W \sum_i \frac{w_i}{\rho_i} \quad (2.8c)
\]

The volume fraction is:

\[
v_i = \frac{V_i}{V} = \frac{W_i / \rho_i}{W \sum_i w_i / \rho_i} = \frac{w_i / \rho_i}{\sum_i w_i / \rho_i} \quad \text{with} \quad \sum_i v_i = 1 \quad (2.8d)
\]

**Thickness prediction**

The simplest way to estimate a composite thickness is by using the rule-of-mixtures once again. The thickness of 1 m² of a material weighing 1 kg and with a density \( \rho \) (kg/m³) is simply \( 1/\rho \) (m).

If a laminate consists of \( W_i \) (kg) of material \( i \) then the total volume equals \( \Sigma W_i / \rho_i \). Hence, the thickness may be predicted using

\[
t = \sum_i \frac{W_i^*}{\rho_i} \quad \text{where} \quad W^* \text{ is the weight per unit area} \quad (2.9)
\]

Since the weight fraction, \( w_i \), usually is known, the total weight of the components \( W_i^* \) (weight of reinforcement per unit area) and \( W_m^* \) (weight of matrix per unit area), are readily available and the thickness may therefore commonly be estimated. In practice, composite laminates always contain voids of unwanted particles or air bubbles which will increase the thickness. Anyone who has ever made a laminate will also know that the thickness will vary over the laminate, especially if it is made by hand lay-up.

**Stiffness properties of the lamina**

Since the lamina layer is thin a state of plane stress prevails and we have

\[
\sigma_3 = \tau_{23} = \tau_{31} = 0
\]

Hooke's law for such a layer is then
The inverse of the compliance matrix is easily found as

\[
\begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\tau_{12}
\end{bmatrix} =
\begin{bmatrix}
1 & -\nu_{21} & 0 \\
\nu_{12} & 1 & 0 \\
0 & 0 & 1/G_{12}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_1 \\
\varepsilon_2 \\
\gamma_{12}
\end{bmatrix}
\]

The inverse of the compliance matrix is easily found as

\[
\begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\tau_{12}
\end{bmatrix} =
\begin{bmatrix}
1 & -\nu_{21} & 0 \\
\nu_{12} & 1 & 0 \\
0 & 0 & 1/G_{12}(1-\nu_{12}\nu_{21})
\end{bmatrix}
\begin{bmatrix}
\varepsilon_1 \\
\varepsilon_2 \\
\gamma_{12}
\end{bmatrix}
\]

or in a shorter form: \( \boldsymbol{\sigma} = \boldsymbol{Q} \boldsymbol{\varepsilon} \) where index \( l \) stands for lamina.

In the manufacturing of composite laminates several lamina are assembled on top of each other but with different orientation to the global coordinate system. Thus, in order to describe the behaviour of the laminate one must know the behaviour of each lamina described in the global coordinate system \((x, y, z)\) and not in its local coordinate system \((1, 2, 3)\). The stiffness of the lamina in the global coordinate system \(x-y\) is found by using the transformation matrix.

Assuming that the local coordinate system, \(1-2\), is rotated and angle \( \theta \) to the global coordinate system, \(x-y\). The transformation matrix is then written as

\[
\begin{bmatrix}
\sigma_x \\
\sigma_y \\
\tau_{xy}
\end{bmatrix} =
\begin{bmatrix}
c^2 & s & 2sc \\
s^2 & c^2 & -2sc \\
-sc & sc & c^2-s^2
\end{bmatrix}
\begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\tau_{12}
\end{bmatrix}
\]

or \( \boldsymbol{\sigma} = \boldsymbol{T}^{\top} \boldsymbol{\sigma} \) (2.12a)

where \( c = \cos \theta \) and \( s = \sin \theta \). Alternatively we can write

\[
\begin{bmatrix}
\sigma_x \\
\sigma_y \\
\tau_{xy}
\end{bmatrix} =
\begin{bmatrix}
c^2 & s & -2sc \\
s^2 & c^2 & 2sc \\
-sc & sc & c^2-s^2
\end{bmatrix}
\begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\tau_{12}
\end{bmatrix}
\]

or \( \boldsymbol{\sigma} = \boldsymbol{T} \boldsymbol{\sigma} \) (2.12b)

where \( \boldsymbol{T} \) is the transformation matrix. For the strains we can in the same way write

\[
\begin{bmatrix}
\varepsilon_1 \\
\varepsilon_2 \\
\gamma_{12}
\end{bmatrix} =
\begin{bmatrix}
c^2 & s & sc \\
s^2 & c^2 & -sc \\
-2sc & 2sc & c^2-s^2
\end{bmatrix}
\begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\gamma_{xy}
\end{bmatrix}
\]

or \( \boldsymbol{\varepsilon} = \boldsymbol{T} \boldsymbol{\varepsilon} \) (2.12c)

and similarly

\[
\begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\gamma_{xy}
\end{bmatrix} =
\begin{bmatrix}
c^2 & s & -sc \\
s^2 & c^2 & sc \\
2sc & -2sc & c^2-s^2
\end{bmatrix}
\begin{bmatrix}
\varepsilon_1 \\
\varepsilon_2 \\
\gamma_{12}
\end{bmatrix}
\]

or \( \boldsymbol{\varepsilon} = (\boldsymbol{T}^{\top}) \boldsymbol{\varepsilon} \) (2.12d)

Now we can write the transformed stiffness matrix as

\[
\boldsymbol{Q} = \boldsymbol{T} \boldsymbol{Q}_l \boldsymbol{T}^{\top}
\]

(2.13)

Hence, the stress-strain relation in the global coordinate system can be written
\[
\begin{pmatrix}
\sigma_x \\
\sigma_y \\
\tau_{xy}
\end{pmatrix} =
\begin{bmatrix}
Q_{11} & Q_{12} & Q_{16} \\
Q_{12} & Q_{22} & Q_{26} \\
Q_{16} & Q_{26} & Q_{66}
\end{bmatrix}
\begin{pmatrix}
\varepsilon_x \\
\varepsilon_y \\
\gamma_{xy}
\end{pmatrix}
\]
or
\[
\sigma = Q \varepsilon
\]
whose components written out in full reads
\[
Q_{11} = Q_{l11} \cos^4 \theta + 2(Q_{l12} + 2Q_{l66}) \cos^2 \theta \sin^2 \theta + Q_{l22} \sin^4 \theta
\]
\[
Q_{12} = (Q_{l11} + Q_{l22} - 4Q_{l66}) \cos^3 \theta \sin \theta + Q_{l12} (\sin^4 \theta + \cos^4 \theta)
\]
\[
Q_{22} = Q_{l11} \sin^4 \theta + 2(Q_{l12} + 2Q_{l66}) \cos^2 \theta \sin^2 \theta + Q_{l22} \cos^4 \theta
\]
\[
Q_{16} = (Q_{l11} - Q_{l12} - 2Q_{l66}) \sin \theta \cos^3 \theta + (Q_{l12} - Q_{l22} + 2Q_{l66}) \sin^3 \theta \cos \theta
\]
\[
Q_{26} = (Q_{l11} - Q_{l12} - 2Q_{l66}) \sin^3 \theta \cos^3 \theta + (Q_{l12} - Q_{l22} + 2Q_{l66}) \sin \theta \cos^3 \theta
\]
\[
Q_{66} = (Q_{l11} + Q_{l22} - 2Q_{l12} - 2Q_{l66}) \sin^2 \theta \cos^2 \theta + Q_{l12} (\sin^4 \theta + \cos^4 \theta)
\]

As an alternative to the foregoing we can write all this in compliances. Now, if
\[
S = (T^{-1})^T S T^{-1}
\]
which is the transformed compliance matrix. Its components are
\[
S_{11} = S_{l11} \cos^4 \theta + (2S_{l12} + S_{l66}) \cos^2 \theta \sin^2 \theta + S_{l22} \sin^4 \theta
\]
\[
S_{12} = (S_{l11} + S_{l22} - S_{l66}) \cos^3 \theta \sin \theta + S_{l12} (\sin^4 \theta + \cos^4 \theta)
\]
\[
S_{22} = S_{l11} \sin^4 \theta + (2S_{l12} + S_{l66}) \cos^2 \theta \sin^2 \theta + S_{l22} \cos^4 \theta
\]
\[
S_{16} = (2S_{l11} - 2S_{l12} - S_{l66}) \sin \theta \cos^3 \theta - (2S_{l22} - 2S_{l12} - S_{l66}) \sin^3 \theta \cos \theta
\]
\[
S_{26} = (2S_{l11} - 2S_{l12} - S_{l66}) \sin^3 \theta \cos^3 \theta - (2S_{l22} - 2S_{l12} - S_{l66}) \sin \theta \cos^3 \theta
\]
\[
S_{66} = 2(2S_{l11} + 2S_{l22} - 4S_{l12} - S_{l66}) \sin^2 \theta \cos^2 \theta + S_{l12} (\sin^4 \theta + \cos^4 \theta)
\]

**Stiffness properties of the laminate**

A *laminate* is a stack of arbitrary oriented laminae assembled to form a plate or a shell. The *lamina* was considered being thin hence having rigidities in its plane only, i.e., no bending stiffness. The laminate on the other hand has a finite thickness and thus a flexural rigidity.
Denote
\[ N = \begin{bmatrix} N_x, N_y, N_{xy} \end{bmatrix} \quad \text{and} \quad M = \begin{bmatrix} M_x, M_y, M_{xy} \end{bmatrix} \]

\[ \sigma_0 = \begin{bmatrix} \sigma_{x1}, \sigma_{y1}, \tau_{xy1} \end{bmatrix}, \quad \varepsilon_0 = \begin{bmatrix} \varepsilon_{x0}, \varepsilon_{y0}, \gamma_{xy0} \end{bmatrix} \quad \text{and} \quad \kappa = \begin{bmatrix} \kappa_x, \kappa_y, \kappa_{xy} \end{bmatrix} \]

Assume that the thickness and position of each lamina in the stack are known, as shown in Fig.2.3.

![Figure 2.3 Geometry of an n-layered laminate. (Note that the total thickness \( t = 2z_0 \).)](image)

The relation between forces and bending moments to strains and curvatures can be written
\[
\begin{bmatrix} N \\ M \end{bmatrix} = \sum_{i=1}^{n} \int_{z_{i-1}}^{z_i} \begin{bmatrix} Q_i \\ Q_i \end{bmatrix} \begin{bmatrix} \varepsilon_0 \\ \kappa \end{bmatrix} dz = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{bmatrix} \varepsilon_0 \\ \kappa \end{bmatrix}
\]

(2.15)

where
\[
[A, B, D] = \sum_{i=1}^{n} Q_i \left[ (z_i - z_{i-1}), \frac{1}{2}(z_i^2 - z_{i-1}^2), \frac{1}{3}(z_i^3 - z_{i-1}^3) \right]
\]

(2.16)

\( A \) is called the extensional stiffness matrix, \( B \) the extension-bending coupling matrix, and \( D \) the bending stiffness matrix.

By computing the stiffness matrix components we can estimate the desired properties for the face sheet. Assume that the face sheet laminate is orthotropic (which is true in most practical cases), we can then get the Poisson's ratios as
\[
\nu_{21} = \frac{A_{12}}{A_{11}} \quad \text{and} \quad \nu_{12} = \frac{A_{12}}{A_{22}}
\]

(2.17)

The average moduli are then
\[
E_x = \frac{A_{11}(1 - \nu_{12} \nu_{21})}{t}, \quad E_y = \frac{A_{22}(1 - \nu_{12} \nu_{21})}{t} \quad \text{and} \quad G_{xy} = \frac{A_{66}}{t}
\]

(2.18)

where \( t \) is the laminate thickness. For symmetrical laminates we have that \( B = 0 \) and for orthotropic ones also that \( D_{16} = D_{26} = 0 \). The local bending stiffnesses of the face sheet laminate are then simply
\[
D_{fx} = \frac{D_{11}}{1 - \nu_{12} \nu_{21}}, \quad D_{fy} = \frac{D_{22}}{1 - \nu_{12} \nu_{21}} \quad \text{and} \quad D_{fxy} = 2D_{66}
\]

(2.19)
**Strength predictions**

There exists an almost infinite number of fracture criteria for composite laminates, ranging from very simple estimates based on the rule-of-mixtures, to quite complex multi-dimensional criteria aiming at incorporating every possible effect of different stresses and failure modes. These will not be covered in this text but some of the most common criteria are given in references [4,5].

### 2.2 Core Materials

This material component is perhaps the most important of all even though it might not appear so at first glance. It is also the material component of which the engineer commonly has the least knowledge. The cores used in load carrying sandwich constructions can divided into four main groups; corrugated, honeycomb, balsa wood and foams, schematically illustrated in Fig.2.4. First of all the core should have low density in order to add as little as possible to the total weight of the sandwich. Even though the transverse forces creating normal stresses $\sigma_z$ in the core usually are low, even a small decrease in core thickness would create a large decrease in the flexural rigidity and hence Young's modulus perpendicular to the faces should be high. The core is mainly subjected to shear and the core shear strains produces global deformations and core shear stresses. Thus, a core must be chosen that won't fail under the applied transverse load and with a shear modulus high enough to give the required shear stiffness. The critical wrinkling load depends both on the Young's modulus and the shear modulus of the core. Other functions of the sandwich such as thermal and acoustical insulation depends mainly in the core material and its thickness. The properties of primary interest for the core may be summarised as;

- Density
- Shear modulus
- Shear strength
- Stiffness perpendicular to the faces
- Thermal insulation
- Acoustic insulation

![Figure 2.4 Main groups of core materials.](image)

Following is a description of the commonly used core materials in load carrying sandwich constructions and their typical mechanical and physical properties as collected from different sources. The data is far from complete but is aimed as a list of data for typical materials. For more information the reader is referred to manufactures data sheets. Material data are constantly being changed due to improvements, new manufacturing methods, new materials, new test methods etc., and the numbers should hence be used only as guide-lines and for purpose of comparing.
2.2.1 Balsa wood

Balsa was the first material used as cores in load carrying sandwich structures. Balsa is a wood with a high-aspect-ratio closed-cell structure. The fibres or grains are oriented in the direction of growth (Axial) producing cells with a typical length of 0.5-1 mm and with a diameter of about 0.05 mm, thus giving the cell ratio of approximately 1:25. The properties of balsa are therefore very high in the direction of growth (A) but much lower in the others.

![Figure 2.5 Cross-section of a tree defining coordinate system of wood products.](image)

Balsa exists in different qualities with densities in the regime 100 to 300 kg/m³. Balsa is very sensitive to humidity with the properties rapidly declining with the water content. To overcome all the above problem balsa in most commonly utilised in its "end-grain" shape. This means that the balsa wood is cut up in cubic pieces and bonded together so that a block is produce where the fibre direction is perpendicular to the plane if the block. With this procedure several advantages are gained; the fibres and hence the principal direction of stiffness is perpendicular to the faces. Humidity is primarily spread along the fibres and hence a damage would only cause localised humidity damage. The drawback is that all the small balsa blocks have different densities and the design limit must hence be taken from the piece having the lowest properties. End-grain balsa is only available in a limited number of densities. The mechanical properties are, even though restricted to the minimum density, quite good and higher than for most cellular plastics.

<table>
<thead>
<tr>
<th>Density (kg/m³)</th>
<th>λ (W/m °C)</th>
<th>σₐ(¹) (MPa)</th>
<th>σₐ(²) (MPa)</th>
<th>Eₐ(²) (MPa)</th>
<th>Г (MPa)</th>
<th>G (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>96</td>
<td>0.0509</td>
<td>6.9</td>
<td>6.5</td>
<td>2240</td>
<td>1.85</td>
<td>108</td>
</tr>
<tr>
<td>110</td>
<td>0.0548</td>
<td>9.1</td>
<td>8.2</td>
<td>2740</td>
<td>2.17</td>
<td>120</td>
</tr>
<tr>
<td>130</td>
<td>0.0588</td>
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<td>10.0</td>
<td>3260</td>
<td>2.49</td>
<td>134</td>
</tr>
<tr>
<td>150</td>
<td>0.0649</td>
<td>13.1</td>
<td>12.9</td>
<td>4070</td>
<td>2.98</td>
<td>159</td>
</tr>
<tr>
<td>180</td>
<td>0.0710</td>
<td>15.7</td>
<td>16.0</td>
<td>4930</td>
<td>3.46</td>
<td>188</td>
</tr>
<tr>
<td>250</td>
<td>0.0890</td>
<td>23.7</td>
<td>26.6</td>
<td>7720</td>
<td>4.94</td>
<td>312</td>
</tr>
</tbody>
</table>

(¹) tension in the axial A-direction, (²) compression in A-direction.

*Table 2.6 Typical average mechanical and physical of balsa wood [13,14].*

As mentioned above, balsa is highly anisotropic and its Poisson ratios are given as [15]

\[ ν_{pt} = 0.66, \ ν_{pa} = 0.02, \ ν_{tp} = 0.24, \ ν_{ta} = 0.01, \ ν_{ap} = 0.23, \text{ and } ν_{at} = 0.49 \]

The coefficients of thermal expansion are approximately [14]
\[ \alpha_T = 10.5 \cdot 10^{-6}, \quad \alpha_R = 7 \cdot 10^{-6}, \quad \text{and} \quad \alpha_A = 1.7 \cdot 10^{-6} \]

### 2.2.2 Honeycomb cores

Core materials of honeycomb type have been developed and used primarily in aero/space applications. However, cheap honeycomb materials made from impregnated paper are also used in building applications. Honeycomb materials can be manufactured in a variety of cell shapes such as square, rectangular, triangular or corrugated but the most commonly used shape is the hexagonal shape shown in Fig. 2.7a. Others are the square (2.7b), the over-expanded hexagonal (2.7c) and the so called "flex-core" (2.7d). The two latter configurations are primarily used when the core needs to be curved in the manufacturing of the sandwich element. Over-expanded hexagonal and flex-core shapes reduces the anticlastic bending and cell wall buckling when curved. There are other cell shapes used such as rectangular, and reinforced hexagonal.

![Commonly used cell configurations for honeycomb core materials.](image)

The most commonly used honeycombs are made of aluminium or impregnated glass or aramid fibre mats, such as Nomex®. Due to the manufacturing methods involved most honeycombs have not only different out-of-plane properties but also the in-plane properties are different from each other. This is easily seen as both the corrugation and the expansion process produces double cell walls in one direction and single walls in the other. Over-expanded cells also create additional anisotropy. There are three principal directions to which material properties of most honeycombs are referred; the width (\(W\)), the length (\(L\)) and the transverse (\(T\)) directions (see Fig. 2.4).

The manufacture of honeycombs is performed in two different ways, illustrated in Fig. 2.8. The corrugating process implies that pre-corrugated metal sheets are stacked into blocks and bonded together. When the adhesive has cured, blocks with the required thickness can be cut from the stack. This process is commonly used in the manufacture of high-density metal honeycombs.

The expansion process begins with the stacking of thin plane sheets of the web material on which adhesive node lines have been printed. By stacking many thin layers in this way a more or less solid block is made. When the adhesive has cured the block may be expanded by pulling in the \(W\)-direction until a desired cell shape has been achieved. Metal honeycombs are cut into the desired thickness (\(T\)) prior to expansion, and when expanded they retain their shape since the material yields plastically. Non-metallic materials, such as impregnated fibre mats or paper, are heat treated after expansion to retain their shape, after which the block is dipped in resin, which is cured in an oven. After this process is completed the core is sliced.
Table 2.7  Typical mechanical properties of some Kraft paper honeycomb cores [17]. Properties vary with core thickness according to Fig. 2.9.

Kraft paper honeycombs are manufactured by impregnating paper with some resin to make it stiff, strong and water resistant. This provides a cheap, but still mechanically good sandwich core. Some manufactures can even fill the cell of Kraft paper honeycomb with a light weight foam (usually PUR or phenolic) for improved thermal insulation.
Aluminium honeycomb has been extensively used in aero/space applications during the past decades. They are commonly made of the aluminium alloys 5052, 5056 and 2024. 5052 is a general purpose alloy, 5056 a high strength version of 5052 and 2024 a heat treated aluminium alloy with good properties even at elevated temperature. The 5052 and 5056 alloy honeycombs can be used

---

**Table 2.8** Typical mechanical properties of some aluminium hexagonal honeycomb cores [18,19].

Properties vary with core thickness according to Fig.2.9.

Aluminium honeycomb has been extensively used in aero/space applications during the past decades. They are commonly made of the aluminium alloys 5052, 5056 and 2024. 5052 is a general purpose alloy, 5056 a high strength version of 5052 and 2024 a heat treated aluminium alloy with good properties even at elevated temperature. The 5052 and 5056 alloy honeycombs can be used
in environments up to 180°C and the 2024 up to 210°C. The cell-wall foil of the honeycombs can be perforated with small holes so that curing of the adhesive may be vented.

<table>
<thead>
<tr>
<th>density</th>
<th>cell size</th>
<th>( E_T )</th>
<th>( \sigma_T )</th>
<th>Length direction</th>
<th>Width direction</th>
</tr>
</thead>
<tbody>
<tr>
<td>kg/m(^3)</td>
<td>mm (^{(1)})</td>
<td>MPa</td>
<td>MPa</td>
<td>( G_{LT} )</td>
<td>( \tau_{LT} )</td>
</tr>
</tbody>
</table>

**Aluminium 5052 flex-core honeycomb [18,19]**

<p>| | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
<th></th>
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</thead>
<tbody>
<tr>
<td>34</td>
<td>7.6</td>
<td>440</td>
<td>1.3</td>
<td>120</td>
<td>0.62</td>
<td>68</td>
</tr>
<tr>
<td>50</td>
<td>7.6</td>
<td>860</td>
<td>2.5</td>
<td>220</td>
<td>1.2</td>
<td>89</td>
</tr>
<tr>
<td>66</td>
<td>7.6</td>
<td>1270</td>
<td>3.8</td>
<td>310</td>
<td>1.8</td>
<td>110</td>
</tr>
<tr>
<td>91</td>
<td>7.6</td>
<td>2000</td>
<td>6.4</td>
<td>460</td>
<td>2.7</td>
<td>150</td>
</tr>
<tr>
<td>69</td>
<td>3.8</td>
<td>1340</td>
<td>4.4</td>
<td>310</td>
<td>1.9</td>
<td>120</td>
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<tr>
<td>104</td>
<td>3.8</td>
<td>2130</td>
<td>6.8</td>
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</tr>
<tr>
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<td>2750</td>
<td>12</td>
<td>670</td>
<td>4.2</td>
<td>210</td>
</tr>
</tbody>
</table>

**Aluminium 5056 flex-core honeycomb [18,19]**

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<table>
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<tr>
<th></th>
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<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>34</td>
<td>7.6</td>
<td>440</td>
<td>1.7</td>
<td>120</td>
<td>0.72</td>
<td>68</td>
</tr>
<tr>
<td>50</td>
<td>7.6</td>
<td>860</td>
<td>3.3</td>
<td>220</td>
<td>1.5</td>
<td>89</td>
</tr>
<tr>
<td>66</td>
<td>7.6</td>
<td>1270</td>
<td>4.6</td>
<td>310</td>
<td>2.3</td>
<td>110</td>
</tr>
<tr>
<td>69</td>
<td>3.8</td>
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<td>6.1</td>
<td>320</td>
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<td>120</td>
</tr>
<tr>
<td>104</td>
<td>3.8</td>
<td>2130</td>
<td>10</td>
<td>500</td>
<td>4.4</td>
<td>160</td>
</tr>
<tr>
<td>128</td>
<td>3.8</td>
<td>2820</td>
<td>13</td>
<td>680</td>
<td>5.0</td>
<td>220</td>
</tr>
</tbody>
</table>

(1) average length, (2) Compression.

Table 2.9  Typical mechanical properties of some aluminium flex-core honeycomb cores [18,19]. Properties vary with core thickness according to Fig.2.9.

<table>
<thead>
<tr>
<th>density</th>
<th>cell size</th>
<th>( E_T ) (^{(1)})</th>
<th>( \sigma_T ) (^{(1)})</th>
<th>Length direction</th>
<th>Width direction</th>
</tr>
</thead>
<tbody>
<tr>
<td>kg/m(^3)</td>
<td>mm</td>
<td>MPa</td>
<td>MPa</td>
<td>( G_{LT} )</td>
<td>( \tau_{LT} )</td>
</tr>
</tbody>
</table>

**Hexagonal cells - bonded [13]**

<p>| | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
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<th></th>
<th></th>
<th></th>
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</tr>
</thead>
<tbody>
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<td>123</td>
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<td>-</td>
<td>2.6</td>
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<td>2.2</td>
<td>200</td>
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<tr>
<td>149</td>
<td>6.4</td>
<td>-</td>
<td>4.5</td>
<td>1040</td>
<td>2.5</td>
<td>600</td>
</tr>
</tbody>
</table>

**Square cells - welded**

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<table>
<thead>
<tr>
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<th></th>
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<th></th>
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</thead>
<tbody>
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<td>-</td>
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<td>390</td>
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<tr>
<td>152</td>
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<td>-</td>
<td>8.0</td>
<td>530</td>
<td>2.8</td>
<td>340</td>
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<td>128</td>
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<td>186</td>
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<td>-</td>
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<td>-</td>
<td>-</td>
<td>440</td>
</tr>
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<td>125</td>
<td>6.4</td>
<td>-</td>
<td>4.8</td>
<td>640</td>
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<td>520</td>
</tr>
<tr>
<td>139</td>
<td>6.4</td>
<td>-</td>
<td>6.6</td>
<td>1030</td>
<td>3.4</td>
<td>520</td>
</tr>
</tbody>
</table>

(1) compression, (2) corrugated foil.

Table 2.10  Typical mechanical properties of some stainless steel honeycomb cores [13]. Properties vary with core thickness according to Fig.2.9.

Non-metallic honeycombs, like fibre-reinforced plastic honeycombs are manufactured by simply dipping and thus wetting a prefabricated cell-shaped fabric in a bath of resin. Different honeycombs are available with glass, aramid or even carbon fibre fabric reinforcement. The
matrices in which the fabrics are dipped are usually phenolics, heat resistant phenolics, polyimide or polyester. The phenolic impregnated types have a maximum working temperature up to 180°C, the polyimide ones can be used up to 250°C and the polyester impregnated up to 80°C. A well-known type of fibre-impregnated honeycomb is made of Nomex® paper, which is a aramid fibre based fabric expanded in much the same way as aluminium honeycomb before coated with resin. It is widely used because of its high toughness and damage resistance and since it has almost as high mechanical properties as aluminium honeycomb. Nomex® honeycomb can be used up 180°C at which its strength still is about 75% of its room temperature value [16].

<table>
<thead>
<tr>
<th>density kg/m³</th>
<th>cell size mm</th>
<th>$E_T$ (1) MPa</th>
<th>$\sigma_T$ (1) MPa</th>
<th>$G_{LT}$ MPa</th>
<th>$\tau_{LT}$ MPa</th>
<th>$G_{TW}$ MPa</th>
<th>$\tau_{TW}$ MPa</th>
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<td>hexagonal glass/phenolic</td>
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<td>390</td>
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<td>96</td>
<td>1.9</td>
<td>48</td>
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<td>88</td>
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<td>650</td>
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<td>89</td>
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<td></td>
<td>128</td>
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<td>1130</td>
<td>10</td>
<td>234</td>
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<td>130</td>
</tr>
<tr>
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<td>190</td>
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<td></td>
<td>72</td>
<td>9.6</td>
<td>440</td>
<td>4.7</td>
<td>96</td>
<td>2.0</td>
<td>55</td>
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<td></td>
<td>96</td>
<td>9.6</td>
<td>680</td>
<td>6.8</td>
<td>170</td>
<td>3.4</td>
<td>82</td>
</tr>
<tr>
<td>flex-core glass/phenolic</td>
<td>56</td>
<td>0.6 (2)</td>
<td>250</td>
<td>2.7</td>
<td>110</td>
<td>1.1</td>
<td>55</td>
</tr>
<tr>
<td></td>
<td>72</td>
<td>0.6 (2)</td>
<td>330</td>
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<td>170</td>
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<td>690</td>
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<td>6.7</td>
<td>900</td>
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<td>340</td>
</tr>
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</table>

(1) compression, (2) average cell length.

Table 2.11 Typical mechanical properties of some glass and carbon reinforced honeycombs [18,19]. Properties vary with core thickness according to Fig.2.9.
<table>
<thead>
<tr>
<th>density $\text{kg/m}^3$</th>
<th>cell size $\text{mm}$</th>
<th>$E_x^{(1)}$ MPa</th>
<th>$\sigma_x^{(1)}$ MPa</th>
<th>$G_{LT}$ MPa</th>
<th>$\tau_{LT}$ MPa</th>
<th>$G_{TW}$ MPa</th>
<th>$\tau_{TW}$ MPa</th>
</tr>
</thead>
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<td><strong>hexagonal</strong></td>
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<td></td>
<td></td>
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<td></td>
</tr>
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<td>27</td>
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<td>0.38</td>
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<td>-</td>
<td>2.5</td>
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<td>-</td>
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<td>0.42</td>
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<td>0.26</td>
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<td>0.72</td>
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<td>0.40</td>
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<td>0.72</td>
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<td>0.66</td>
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<td>0.52</td>
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<td>0.26</td>
</tr>
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<td>32</td>
<td>9.6</td>
<td>-</td>
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<td>27</td>
<td>0.66</td>
<td>17</td>
<td>0.38</td>
</tr>
<tr>
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<td>-</td>
<td>1.8</td>
<td>36</td>
<td>1.0</td>
<td>20</td>
<td>0.58</td>
</tr>
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<td>-</td>
<td>0.95</td>
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<td>0.56</td>
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<td>0.26</td>
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<td>12.8</td>
<td>-</td>
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<td>0.62</td>
<td>16</td>
<td>0.56</td>
<td>11</td>
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<td></td>
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<tr>
<td>40</td>
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<td>27</td>
<td>0.90</td>
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<td>0.56</td>
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<td>8.7 $^{(2)}$</td>
<td>-</td>
<td>2.7</td>
<td>39</td>
<td>1.4</td>
<td>19</td>
<td>0.69</td>
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<td>72</td>
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<td>-</td>
<td>4.3</td>
<td>50</td>
<td>2.0</td>
<td>25</td>
<td>1.2</td>
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<tr>
<td>56</td>
<td>6.1 $^{(1)}$</td>
<td>-</td>
<td>2.4</td>
<td>52</td>
<td>1.1</td>
<td>21</td>
<td>0.55</td>
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<td>72</td>
<td>6.1 $^{(2)}$</td>
<td>-</td>
<td>4.0</td>
<td>76</td>
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<td>48</td>
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<td>6.1 $^{(1)}$</td>
<td>-</td>
<td>5.5</td>
<td>96</td>
<td>2.7</td>
<td>55</td>
<td>1.6</td>
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</tr>
<tr>
<td>29</td>
<td>4.8</td>
<td>-</td>
<td>0.76</td>
<td>14</td>
<td>0.41</td>
<td>21</td>
<td>0.41</td>
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<td>-</td>
<td>2.5</td>
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<td>-</td>
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<td>0.91</td>
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<td>48</td>
<td>6.4</td>
<td>-</td>
<td>2.4</td>
<td>21</td>
<td>0.76</td>
<td>41</td>
<td>0.79</td>
</tr>
</tbody>
</table>

$^{(1)}$ compression, $^{(2)}$ average cell length.

Table 2.12 Typical average mechanical properties of some Nomex® honeycomb cores [18,19,16]. Properties vary with core thickness according to Fig.2.9.
Poisson ratios for honeycombs are given in [15] where it is seen that they depend strongly on cell geometry. In practice, this value is of less importance when considering sandwich panels since the global ratio of the panel in much depends on the Poisson ratio of the faces. All numbers in tables 2.7-12 are valid for a core thickness of 12.7 mm (1/2 inch). For other thicknesses than that strengths and moduli should be multiplied with a thickness correction factor given in Fig.2.9 [8,18].

![Figure 2.9 Thickness correction factor for metallic (solid line), non-metallic (dashed line), and paper honeycomb (dash-dotted line) core materials [8,18].](image)

Honeycombs have excellent mechanical properties, very high stiffness perpendicular to the faces and the highest shear stiffness and strength to weight ratios of all available core materials. The main drawbacks are high cost and difficult handling during lay up of sandwich element and that they may not be used with wet lay-up manufacturing. Plane panels are easily manufactured but a pre-set curvatures could be quite difficult to achieve due to Poisson effects when the core is forced to curve. This can overcome by using another cell shape [20]. An example of this is by varying the hexagonal cell shape a core can be made that has negative Poisson ratio, thus when curved in one direction the secondary curvature in the other direction is of the same sign (see eq.(9.2)). Hence, a core can be manufactured to fit a specific double or even single curvature.

![Figure 2.10 Example of honeycomb core with negative Poisson ratio.](image)

Thus, if the in-plane Poisson ratio’s $\nu_{xy}$ or $\nu_{yx}$ equals zero a sheet of core can be bent into a cylinder applying a single bending moment. If this Poisson’s ratio is made positive, as for hexagonal shape honeycombs, the bending will be *anticlastic* (different signs on $\kappa_x$ and $\kappa_y$) making the sheet difficult to fit the shape a doubly curved mould, but if the Poisson ratio is negative, as for the shape Fig.2.10, the reaction to a single bending moment will be *synclastic* (same signs on $\kappa_x$ and $\kappa_y$) and the core sheet will be easier to handle.
The thermal conductivity of honeycombs differ very much between different materials; metals giving a high value since the thermal resistance will be very. Non-metallic honeycombs, on the other hand, offer a much higher thermal resistance since the cell walls themselves have a low conductivity and the air between is a good thermal insulator. It is found that for metallic honeycombs density is a much more important factor than cell shape whereas the other way around is more true for non-metallic honeycombs. The values of the thermal conductivity $\lambda$ for different honeycombs are summarised in Table 12.13.

<table>
<thead>
<tr>
<th>Metallic honeycombs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density (kg/m$^3$)</td>
</tr>
<tr>
<td>$\lambda$ (W/m°C)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Non-metallic honeycombs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cell size (mm)</td>
</tr>
<tr>
<td>$\lambda$ (W/m°C)</td>
</tr>
</tbody>
</table>

Table 12.13 Thermal conductivity $\lambda$ of honeycombs at room temperature [18,19].

For details concerning the variation of thermal conductivity as function of mean temperature, the reader is referred to [18,19], but in short one can summarise that $\lambda$ varies linearly with temperature so that it reduces to half that value at 220°C and increases to twice the value of Table 2.14 at −130°C.

### 2.2.3 Cellular foams

The relatively recent development of high density and high quality cellular foams has had a major impact on the use of the sandwich concept. Cellular foams do not offer the same high stiffness and strength-to-weight ratios as honeycombs but have other very important advantages. Firstly, cellular foams are in general less expensive than honeycombs but more importantly, they are a solid on a macroscopic level making the manufacturing of sandwich elements much easier; the foam surface is easy to bond to, surface preparation and shaping is simple and connections of core blocks are easily performed by adhesive bonding. In addition to this, cellular foams offer high thermal insulation, acoustical damping, and the closed cell structure of most foams ensures that the structure will become buoyant and thus water penetration is of little problem. There exist a variety of foams, all with different advantages and disadvantages. Some of these are briefly described below.

**Polyurethane foam** (PUR): The urethane polymer is formed through the reaction between isocyanate and polyol, and tri-chlor-flour-methane or carbon dioxide is used as a blowing agent and is vaporised by the heat released by the exothermal reaction. PUR foams are produced in many variations from soft foam with more or less open cells to rigid types with predominantly closed cells and in densities from 30 to 500 kg/m$^3$. They can be made fire resistant by using additives containing phosphorus. Because of the high molecular weight PUR foams have low thermal conductivity and diffusion coefficients giving it very good insulation properties. Rigid PUR foams generally have quite brittle cell walls and hence the PUR core has low toughness, and low ultimate elongation. The mechanical properties are lower than for most other cellular plastic cores. However, PUR foams are probably the cheapest of all available core materials. Hence, the primary use of PUR is for insulation purposes or in less critical load bearing elements. Another advantage
is that PUR foam can not only be produced in finite size blocks but can also be foamed in-situ thus giving an integrated manufacturing in conjunction with the manufacturing of sandwich elements.

### Table 2.14  Typical mechanical and physical properties of PUR foam core materials [3].

<table>
<thead>
<tr>
<th>density kg/m³</th>
<th>α 10⁻⁶/°C</th>
<th>λ (1) W/m °C</th>
<th>T_max °C</th>
<th>σ_z (2) MPa</th>
<th>σ_z (3) MPa</th>
<th>E_z (3) MPa</th>
<th>τxz (4) MPa</th>
<th>Gxz (4) MPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>100</td>
<td>0.025</td>
<td>100</td>
<td>0.3</td>
<td>0.2</td>
<td>10</td>
<td>0.2</td>
<td>3</td>
</tr>
<tr>
<td>40</td>
<td>100</td>
<td>0.025</td>
<td>100</td>
<td>0.35</td>
<td>0.3</td>
<td>12</td>
<td>0.25</td>
<td>4</td>
</tr>
<tr>
<td>50</td>
<td>100</td>
<td>0.025</td>
<td>100</td>
<td>0.4</td>
<td>0.35</td>
<td>-</td>
<td>0.3</td>
<td>-</td>
</tr>
</tbody>
</table>

(1) room temperature, (2) tension and (3) compression perpendicular to the plane of the block, (4) out-of-plane

### Table 2.15  Typical mechanical and physical properties of extruded PS foam core materials [21,3].

<table>
<thead>
<tr>
<th>density kg/m³</th>
<th>α 10⁻⁶/°C</th>
<th>λ (1) W/m °C</th>
<th>T_max °C</th>
<th>σ_z (2) MPa</th>
<th>σ_z (3) MPa</th>
<th>E_z (3) MPa</th>
<th>τxz (4) MPa</th>
<th>Gxz (4) MPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>70</td>
<td>0.028</td>
<td>75</td>
<td>0.5</td>
<td>0.3</td>
<td>8-20</td>
<td>0.25</td>
<td>4.5</td>
</tr>
<tr>
<td>45</td>
<td>70</td>
<td>0.025</td>
<td>75</td>
<td>1.0</td>
<td>0.75</td>
<td>25-33</td>
<td>0.60</td>
<td>6.0</td>
</tr>
<tr>
<td>60</td>
<td>70</td>
<td>0.035</td>
<td>75</td>
<td>1.2</td>
<td>0.9</td>
<td>60</td>
<td>0.60</td>
<td>20</td>
</tr>
</tbody>
</table>

(1) at room temperature, (2) tension and (3) compression perpendicular to the plane of the block, (4) out-of-plane property.

**Polystyrene foam** (PS): Polystyrene foam is produced either by extrusion or by expansion in closed moulds. In both cases the plastic is mixed with the blowing agent which then expands at elevated temperature. A major obstacle was that CFC was used as a blowing agent, but recently PS foams have been expanded without use of environmentally dangerous CFC-gases. PS has closed cells and is available in densities ranging from 15 to 300 kg/m³. PS foam has quite good mechanical and thermal insulation properties, and it is cheap. A drawback is its sensitivity to solvents, particularly styrene, and hence ester-based matrices cannot be used as adhesives but rather epoxy or polyurethane. PS is primarily used as a thermal insulation material but lately it has also been used in load carrying structures such as refrigerated tanks and containers.

**Polyvinylchloride foam** (PVC): PVC foam exist in two different forms; one purely thermoplastic also called linear PVC foam, and one cross-linked iso-cyanate modified type. The linear PVC has high ductility, quite good mechanical properties but softens at elevated temperatures. The cross-linked PVC is more rigid, has higher strength and stiffness, is less heat sensitive, but more brittle. Still, even the cross-linked PVC has an ultimate elongation of about 10% in tension which is much higher than any PUR foam. PVC foam is available in finite size blocks of densities from 30 to 400 kg/m³. The mechanical properties of PVC are much better than those of both PUR and PS, but it is also more expensive than those. It is a non-flammable foam but when burnt a hydrochloric acid gas is released. PVC foams are used in almost every type of sandwich application varying from pure insulation applications to aero/space structures and is hence the most widely used of all foams and perhaps of all core materials. The temperature resistance is, however, usually restricted to below 100 °C so that PVC normally cannot be used in an autoclave manufacturing process. PVC has about 95% closed cells for the lower densities and almost entirely closed cells for the higher, which is much appreciated in applications where water absorption is a problem.
Table 2.16 Typical mechanical and physical properties of PVC foam core materials [22,23].

<table>
<thead>
<tr>
<th>density (kg/m³)</th>
<th>α (10⁶/°C)</th>
<th>λ (W/m °C)</th>
<th>T_max (°C)</th>
<th>σ_z (2) (MPa)</th>
<th>σ_z (3) (MPa)</th>
<th>E_z (3) (MPa)</th>
<th>τ_xz (4) (MPa)</th>
<th>Gxz (4) (MPa)</th>
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</thead>
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<tr>
<td>30</td>
<td>35</td>
<td>0.022</td>
<td>80</td>
<td>0.9</td>
<td>0.30</td>
<td>20</td>
<td>0.35</td>
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<tr>
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<td>0.024</td>
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<td>1.6</td>
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<tr>
<td>80</td>
<td>35</td>
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<td>1.20</td>
<td>85</td>
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<td>175</td>
<td>2.0</td>
<td>52</td>
</tr>
<tr>
<td>160</td>
<td>35</td>
<td>0.038</td>
<td>80</td>
<td>5.1</td>
<td>3.40</td>
<td>230</td>
<td>2.6</td>
<td>66</td>
</tr>
<tr>
<td>200</td>
<td>35</td>
<td>0.043</td>
<td>80</td>
<td>6.4</td>
<td>4.40</td>
<td>310</td>
<td>3.3</td>
<td>85</td>
</tr>
<tr>
<td>250</td>
<td>35</td>
<td>0.048</td>
<td>80</td>
<td>8.8</td>
<td>5.80</td>
<td>400</td>
<td>4.5</td>
<td>108</td>
</tr>
</tbody>
</table>

(1) at room temperature, (2) tension and (3) compression perpendicular to the plane of the block, (4) out-of-plane property, (5) linear PVC.

The thermal conductivity varies with temperature; For the PMI foam it varies from approximately 0.015 W/m°C at −160 °C to 0.048 at +140 °C, for the PVC foam λ approximately equals 0.022 W/m°C at −10°C and increases linearly to about 0.05 at +37 °C. For details see references [21,22,24].

Poisson’s ratios are not given for any material due to lack of reliable data. However, some unpublished results indicate that the Poisson’s ratio for most foams are in the regime 0.2 - 0.4, although varying in different directions due to anisotropy. Most foam cores are only moderately anisotropic with fairly equal properties in the plane of the block (xy-plane). Note that the elastic modulus E is given for the out-of-plane direction (z) which is the direction of interest for cores of sandwich structures.
2.2.4 Estimation of properties

There is an excellent book written by Gibson and Ashby [15] on the topic of the behaviour of cellular materials, covering both honeycombs and foams. The theoretical background is based on the fact that the primary mode of deformation of cellular materials originates from cell wall bending rather than tension or compression. Formulae for the prediction of almost any mechanical property are derived and compared with experimental results. The derived theories agree well with experimental results. However, the derivation is based on that the properties of the unfoamed material are known and sometimes that micro geometry such as cell wall thickness, cell shapes etc. are known. This is usually not the case and the engineer is primarily interested in macro-mechanical properties. Hence, for details refer to reference [15]. The properties of primary interest are the out-of-plane deformation and stress. These are given in [15] from a theoretical reasoning and from [13] based on testing.

**Honeycomb Cores:** Starting with honeycomb cores, the out-of-plane properties can be estimated as follows:

The two Poisson’s ratios $\nu_{TL}$ and $\nu_{TW}$ must be equal to that of the solid from which the cell walls are built up, that is

$$\nu_{TL} = \nu_{TW} = \nu_s$$  \hspace{1cm} (2.20)

where index $s$ refers to the solid material, e.g. aluminium, steel, or reinforced phenolic resin. By means of the reciprocity theorem, this implies that

$$\nu_{LT} = \frac{E_L}{E_T} \nu_{TL} \approx 0 \text{ and } \nu_{WT} = \frac{E_W}{E_T} \nu_{TW} \approx 0$$  \hspace{1cm} (2.21)

since the Young’s modulus is much higher in the $T$-direction than in the plane of the honeycomb. This modulus is easily estimated by [13]

$$E_T = \frac{\rho}{\rho_s} E_s$$  \hspace{1cm} (2.22)

where $\rho$ is the density of the honeycomb and $\rho_s$ is the density of the cell wall material. Gibson and Ashby [15] derived upper and lower bounds for the out-of-plane shear moduli for honeycombs as function of cell geometry (for details see [15]) which for regular hexagons with all cell wall having equal thickness reduces to

$$G_{LT} = G_{WT} = 1.15 \left( \frac{t}{s} \right) G_s$$  \hspace{1cm} (2.23)

where $t$ is the cell wall thickness and $s$ is the diameter of a circle inscribed in the hexagonal cell (see Fig.3.11). In practice, however, honeycombs have double cell walls in the $L$-direction (see Fig.2.7) due to their manufacturing, and the shear modulus estimation is then modified to [13]

$$G_{LT} = \frac{4t}{3s} G_s \text{ and } G_{WT} = \frac{16t}{30s} G_s$$  \hspace{1cm} (2.24)

For square cells the out-of-plane shear moduli are simply [13]
The strength in the \( T \)-direction can be estimated by [15, 13]

\[
\sigma_T = \frac{\rho}{\rho_s} \sigma_S
\]  

(2.26)

**Cellular Foams:** Most foams are available in a wide range of densities and the first design problem is to choose the right density. It is seen from most material data of foams and honeycombs that the mechanical properties vary with the density of the material. Thus, in general terms we can write

\[
E_c = C_E \rho^k, \quad G_c = C_G \rho^l, \quad \sigma_c = C_o \rho^m, \quad \text{and} \quad \sigma_c = C_r \rho^n
\]  

(2.27)

Some of the constants and exponents may of course be different for the ultimate strength in tension and compression, respectively. In [15] it is suggested that \( k = 2 \) for open cell foams. Curves can, if needed, easily be extracted from the data given in this section. Remember, however, that development of materials is a continuous effort by material manufacturers and changes in material data thus occur frequently. The formulae derived by Gibson and Ashby [15] assume knowledge about the properties of the unfoamed polymer. The analysis also assumes some other sometimes not accessible knowledge as the portion of open and closed cells, cell wall thickness, and cell geometry. If some, or all, of these properties are known most mechanical properties can be predicted by formulae given by Gibson and Ashby [15]. However, as mentioned, foam core manufacturers support extensive material testing of their products and supply comprehensive data sheet in which almost all mechanical and physical properties may be found.

### 2.2.5 Fatigue properties

Repetitive stresses or deformations often cause damage or failure even if these are well below their allowable values for static strength. This phenomena is usually called fatigue and occurs in, for example, slamming of boat hulls, vibration of non-moving parts in vehicles and aircrafts, or any other type of repetitive loading of structures. Fatigue is generally said to cause major part of all structural failures, but for sandwich structures this is not true; Sandwich constructions have gained reputation for being a very good concept in avoiding fatigue failures. One reason may be that the faces may fail in local instability at loads lower than their fatigue limit and the core is designed with a high margin of safety due to lack of knowledge about its fatigue properties. It is vital to realise that the constituents in a sandwich are subjected to different kinds of loading; the faces exhibit almost entirely membrane tension/compression and the core pure shear. Fatigue of the faces could then easily be included in the design process since fatigue properties commonly are known, especially for metals, and the loading situation simple. The core, on the other hand, exhibits a more complex loading situation and fatigue data are almost non-existent. Repetitive shear stresses may also cause fatigue failure of the adhesive joints. Remembering that most adhesives and cellular foam cores are highly visco-elastic makes the problem of fatigue even more complex.

Some limited data are available and presented below as non-dimensionalised shear fatigue data for some sandwich core materials. Here \( \tau_{\text{max}} \) means the maximum alternating stress, \( \tau_c \), the static shear strength of the core as given in previous sections, and \( R \) the ratio of the minimum alternating stress...
MATERIALS AND MATERIAL PROPERTIES

to the maximum, i.e., \( R = \frac{\tau_{\text{min}}}{\tau_{\text{max}}} \). Thus, the \( y \)-axis in the following graphs is the maximum alternating shear stress in percentage of the ultimate shear strength.

Werren [25] performed fatigue tests on sandwich panels using a block shear specimen, of the same type as the ASTM C-273 test specimen for sandwich core materials. The reason for using sandwich panels instead of a core material solely was to include the adhesive joint. Test were performed on several materials out of which some are presented below. The test data are for fatigue failure in the core only, since all test giving fatigue failure in the adhesive joint between the face and the core material were excluded. The tests were performed at a test frequency of 15 Hz and at the stress ratio \( R = 0.10 \).

In Fig.2.11 is an \( S-N \) curve is for end-grain balsa is presented. Note once again that the data are for one balsa density only and should therefore be used with care. One might assume that things like temperature and humidity content, that influences static properties of balsa, may have an even greater effect on the fatigue life. The balsa core failed by cracks forming and extending along the fibres in the \( A \)-direction.

![Figure 2.11  Shear fatigue properties of a 103 kg/m\(^3\) end-grain balsa at R = 0.1 [25].](image)

Next follows an \( S-N \) curve for an impregnated paper hexagonal honeycomb, originating from the same source. The curve refers to both \( TW \) and \( TL \)-planes and is for one density only, namely 80 kg/m\(^3\).
Hexcel [26] has performed shear fatigue tests on some of their honeycomb products using a similar methodology as described above. The test frequency in this program was, however, 20 Hz and the tests were only performed in the $TL$-plane. However, the tests on glass/phenolic and paper honeycombs performed by Werren [25] indicate that the fatigue life is very much similar in the $TW$-plane as in the $TL$-plane. Figs. 2.13-15 shows the results of this program.

Figure 2.12  Shear fatigue properties of a paper honeycomb at $R = 0.1$ [25].

Figure 2.13  Shear fatigue properties in the $TL$-plane of a 72 kg/m$^3$ aluminium 5052 (CRIII-1/8-5052-4.5) hexagonal honeycomb at $R = 0.1$ [26].
Brittle foil honeycombs tend to fail by cracking rather than buckling and do not perform as well in fatigue as more ductile foils. Perforated foil cores tend to promote fatigue cracks and premature failure [13]. Since the data given for these materials are for a very limited set of material types and densities they can only serve as an indication. As mentioned above, different aluminium alloy honeycombs could exhibit very different fatigue properties due to different ductility, the use of different adhesives could most likely have the same effect. The same is valid for different fibre reinforced honeycombs.
Figure 2.16 Shear fatigue properties of a 50 kg/m³ PUR foam [28]. The dashed line is for $R = 0.5$, and the solid line is for $R = 0.05$.

In [27], a comprehensive testing programme dealing with fatigue properties of PVC, PS and PUR foam core materials have been performed from which results are shown in Figs.2.16-18. The tests were conducted using a sandwich beam in four-point bending, designed so that when loaded the failure mode is core shear fracture. The four-point bending configuration offers a special feature, namely that between the outer and inner supports the transverse force is constant and hence the core shear stress is constant (see chapter 11).

Figure 2.17 Shear fatigue properties of a 40 kg/m³ PS foam [28]. Data for Styrofoam RTM®. Dashed line is for $R = 0.5$ and solid line for $R = 0.05$.

It was found [27] that the core actually fails in shear in a region mid-way between the outer and inner supports, thus very little influenced by local effects around the supports. This specimen is found to be advantageous to the block shear specimen since it includes no geometrical
irregularities which create stress concentrations, i.e., the stress field is smooth and very close to constant in the area between the inner and outer supports. All tests were performed at a rate of 5 Hz except for the $R = -1$ tests which were performed at 1.5 Hz. It has been established that those frequencies can be used without any global heating of the specimen due to hysteresis, plastic flow, or friction [27].

![Graph showing shear fatigue properties](image1)

**Figure 2.18** Shear fatigue properties of a 130 kg/m$^3$ PVC [22]. Data for Divinycell® H130. The dashed line is for $R = 0.5$, the solid for $R = 0.05$, and the dash-dotted for $R = -1$.

Brittle foams like PUR fail in shear causing a $45^\circ$ shear cracks extending from one face to the other. In more ductile cores, like PS and PVC, micro cracking can be seen forming mid-way between the outer and inner supports, in the middle of the core. These cracks eventually grow together to a macro-crack which causes the final failure.

![Photograph of fracture zone](image2)

**Figure 2.19** Photograph of fracture zone in a fatigue test specimen. Specimen with PVC foam core.
2.2.6 Fracture Toughness

Fracture toughness values of core materials are not commonly found in the manufacturer data sheets but must usually be obtained from tests. Some relevant test methods for this purpose are given in Chapter 11. Some numbers are given in Table 2.18 which were obtained from ref. [31].

<table>
<thead>
<tr>
<th>Core Material</th>
<th>$K_{ic}$ (MPa$\sqrt{m}$)</th>
<th>$K_{IIC}$ (with ENF) (MPa$\sqrt{m}$)</th>
<th>$K_{IIC}$ (with CSB) (MPa$\sqrt{m}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PVC 60 kg/m$^3$</td>
<td>0.12</td>
<td>0.16</td>
<td>-</td>
</tr>
<tr>
<td>PVC 80 kg/m$^3$</td>
<td>0.18</td>
<td>0.19</td>
<td>-</td>
</tr>
<tr>
<td>PVC 100 kg/m$^3$</td>
<td>0.21</td>
<td>0.21</td>
<td>0.48</td>
</tr>
<tr>
<td>PVC 200 kg/m$^3$</td>
<td>0.45</td>
<td>0.50</td>
<td>-</td>
</tr>
<tr>
<td>PMI 51 kg/m$^3$</td>
<td>0.08</td>
<td>0.13</td>
<td>0.27</td>
</tr>
</tbody>
</table>

Table 2.19 Fracture toughness data some foam core materials [31].

The difference between the values given in Table 2.19 is; $K_{ic}$ is for mode I loading and mode I propagation, $K_{IIC}$ (with end-notch flexure - ENF specimen) is for mode II loading but where the crack may propagate in mode I and finally $K_{IIC}$ (with cracked sandwich beam - CSB specimen) is with a specimen where both the loading is mode II and the following crack propagation is in and along the face/core interface also in mode II. For more information, see Chapter 11.

2.2.7 Fabrication and customisation of core materials

Before the sandwich may be produced the core material block or sheet must usually be fabricated or customised into the right shape. For in-situ foamed core there is obviously no need for such operations but for all prefabricated cores it must be done. The procedures to fabricate cores differ between different materials and types and detailed information about tooling, cutting speeds, etc. can often be obtained from the material manufacturer. In some cases, the manufacturers even provide handbooks or information about how to proceed with the fabrication, e.g. [22,24]. The way to fabricate honeycombs differs considerably from that of foam cores and a rather comprehensive description can be found in e.g. [13].

**Metal honeycombs**

Aluminium honeycomb is obtained in the form of blocks or slices of the correct thickness. The block can be cut or sliced on a metal cutting band saw - high cutting speeds and fine-tooth blades are desirable. Special sawing devices also exist for quality production of contoured cores. Steel honeycombs may be cut with a band-saw or a disk-cutting device. For final finishing some other method must often be used to get the correct tolerances.

**Non-metal honeycombs**

Usually a band saw can be used to cut most non-metal honeycombs, although special blades must be used. Some special features of this are mentioned in [13]. More caution must be taken when customising Nomex since it is made from an aramid-based fibre (of the same type as Kevlar) and is thus more difficult to machine than glass or carbon fibre composites. Again, the best solution is to get specific information from the supplier or manufacturer.
Plastic foams

Plastic foams may be machined in numerous ways; Sawing, cutting, sanding, milling, planing, drilling, turning or even by thermoforming. Sawing is usually done by cross-cut, circular or band sawing using similar tools as in wood or plastics processing. The cutting speed must, however, be carefully adjusted; too low speeds lead to rough surfaces, too high speeds to overheating and burning of the core. Each material and thickness require different types of blades and cutting speed for optimal performance.

Thin sheets or blocks can be cut with an ordinary thin knife. Thicker sheets can be scored and then broken, e.g. at the edge of a table or similar. Surprisingly clean surfaces can obtained with this simple technique. For higher quality cutting a machine must be used and again the cutting speed must be carefully adjusted for proper performance.

A sheet of foam core can be shaped by sanding. This can be done by an abrasive belt or even by hand. However, the resulting surface is different from a one which is cut and consists of deformed cells with a distinct direction. Hence, a sanded surface is poorer to bond to than a cut surface.

Standard milling machines and heads can be used to mill most foam core material although specially designed heads often give a better performance. A two-knife head is often to prefer from a four-knife head [22]. Planning can also be done using standard machinery for wood. Turning may be a little bit more difficult but the rules of milling usually apply.

Holes can be drilled using standard types of drills. Some caution must taken to adjust the drill and feed speed for good performance.

Thermoforming is in short; heating up the material so that it softens and then gently changes its shape by "plastic deformation" in a mould using external pressure and/vacuum. Thermosetting foams can also be thermoformed, although much care must taken. The material can only be heated up to a certain temperature without being degraded. Only quite thin sheets of material can successfully be thermoformed, thick blocks are very difficult and require a long time for preheating and moulding. The same applies for thermoplastic foams; if the temperature is too high, the foam simply melts, if it is too low the foam will fracture during the forming process.

For small series, wooden moulds can be used but for series production metal moulds are to prefer. Large single curved products are best formed in male moulds and for smaller radii female moulds perform better [22]. Heating is done prior to the forming in an oven, preferably with circulating hot air. IR-heaters can also be used. Pre-heating times and temperatures are usually given by the material manufacturers. The pressure can be applied either by another mould half (small and/or many products) or by a vacuum bag (prototypes, small series).

Some thermosetting foams and most thermoplastic foams can also be shaped in heated mould tools. A piece of foam, which is cut slightly larger than the finished part, is placed in a heated closed mould and formed in a pressing operation. During the operation, the surface of the core part will collapse or be compressed whereas the inner structure usually is held intact. The procedure is similar to ordinary pressing of composite laminates.
2.3 Adhesives - Description and Properties
First some basic features of the adhesive when utilised in sandwich structures are discussed. The requirements of the adhesive are somewhat different from normal use in that bonding sandwich structures involves bonding two very dissimilar constituents, one solid compact component to a softer cellular one. This fact implies that attention must be taken to some aspects in choosing the adhesive.

2.3.1 Requirements on the adhesive
This is a summary of some of the requirements for the adhesives and adherends used for bonding of sandwich panels [8].

Surface preparation
Metal and composite face material surfaces are to be prepared before bonding in the same manner as when bonding e.g., metal-to-metal or composite-to-composite. This usually involves cleaning, either mechanically or chemically, and sometimes priming. Bonding to metal surfaces is usually greatly improved by pre-treatment with a wash primer. The core, on the other hand, may be more difficult to clean but the same requirements are valid for all cores, they should be clean from particles, grease, oil and other substances that may influence the bond. Dust can be removed by vacuum or blowing with oil-free compressed air. Grease and oil can be removed from metal honeycomb using by liquid immersion, e.g., trichloroethylene. Foams and balsa are more difficult when it comes to removing grease and oil. Best is to ensure that the core is not exposed to such substances before bonding. In some cases the core has to be prepared by a sawing or sanding operation which removes the surface layer and hence leaves a clean surfaces for bonding. However, when sawing and sanding foams one has to make sure that the tooling is sharp, so that it cuts through all cell walls properly. Inadequate tools will damage the surface cells leaving partially attached cell walls like lids covering the cells. If so, the adhesive will bond to the lids rather than penetrating down into the surface cells, thus creating a poor bond.

Solvents
Some core materials are highly sensitive to certain solvents. For example, polystyrene foams are sensitive to styrene which means that polyester or vinylester resins cannot be used as adhesives since they contain styrene. Epoxies and polyurethanes may well be used on PS cores. Similar combinations may need to be investigated prior to the choice of adhesive.

Curing vapours
Some adhesives, like phenolics, give off vapour when curing. Since the bond line is situated in a closed space between the face and the core this could lead to several problems such as
(i) Internal pressure build-up preventing the surfaces to attach during curing resulting in disbonds.
(ii) The pressure could damage the core.
(iii) The core material could move during curing resulting in unusable parts.
(iv) Corrosion of faces or core due to chemical action of the vapour.
**Bonding pressure**
Some adhesives require a bonding pressure to prevent creation of pores in the adhesive during cure. In these cases care must be taken so that the core itself will not fail due to compression when the bonding pressure is applied.

**Adhesive viscosity**
When bonding to honeycomb cores the adhesive must have exactly the right combination of surface wetting and controlled flow so that the adhesive does not flow into the cell [19]. It is favourable if the adhesive does flow to a certain extent down the cell wall thus increasing the area of contact with the core. In the case of foam or balsa core sandwich the viscosity must be low enough to enable the adhesive to properly fill the surface cells leaving as little trapped air as possible. Low viscosity has in this case another advantage in that when bonding pressure is applied, the adhesive will flow from rich areas to more dry, or even totally dry areas smoothing out the bond line thickness. On the other hand, if the viscosity is too low there is a possibility that the bonding pressure will squeeze out the adhesive from between the face and the core, once again leaving a too thin bond line.

**Bond thickness**
The amount of adhesive applied must be large enough to ensure that both surfaces are properly wet and that no dry areas exist. Each type of core material will require different amounts depending on cell size, cell shapes, type of adhesive etc. Core material manufactures can normally provide such information. However, the bond line should not be thicker than necessary since that adds weight to the part.

**Strength**
The adhesive joint must be able to transfer the design loads, that is, have the desired tensile and shear strengths. Even if the bond has the required static strength it may still cause premature failure due to fatigue. Most adhesives have varying strength with temperature. The adhesive used must therefore have an adequate strength in the temperature range of its final environment.

**Thermal stresses**
A frequent cause of debonding failures are thermal stresses. If one face is heated, e.g., by sunlight or any other heat source it will deform due to thermal expansion. Since most core materials are very good insulators there will be a high thermal gradient over the bond line. This means that high shear stresses may develop at the interface (the bond line) between the face and the core, and initiate debonding. If the structure is to be used in such an environment, and the face and core materials have very different thermal expansions, it is necessary to use a ductile adhesive, i.e., one with a high strain to failure.

**Toughness**
When talking about adhesives, toughness usually refer to the resistance of the adhesive to permit interface crack formation and growth under impact loading. The toughness in this context depends on several parameters such as adhesive ductility, bond line thickness, surface preparation, face material, core material, core cell size etc. [8]. There are, however, toughened adhesives on the market for improved impact resistance. These are often ordinary resins in which elastomer particles like rubber have been added.
**Viscoelastic properties**

Most adhesives have a significant viscoelastic behaviour, that is, their strength and stiffness depend not only on temperature but also on loading rate. Commonly, adhesives lose stiffness and become more ductile as the loading rate decrease, mainly due to stress relaxation or creep. Highly viscoelastic adhesives may be advantageous for example when there are high thermal gradients.

**Curing shrinkage**

Some adhesive resins, like polyesters, exhibit a significant volume change when curing. In fact, as much as 7% decrease in volume from its uncured to its fully cured state is common. Problems occur when these are bonded to fairly stiff core materials like high density foams, where the shrinkage will create high interface shear stresses. The problem is even more significant when a wet laminate is laid up directly onto the core, in which case the resin itself acts as adhesive. The implication is clearly seen if only one face is laid up on a core which causes the entire panel to curve when cured. The magnitude of these interface shear stresses can be so high that they severely decrease the strength of adhesive joint [32].

**Curing exotherm**

Most thermoset adhesives exhibit an exothermal curing, that is, the curing process gives off heat. This is seldom a problem since the adhesive layer is thin and spread over a large area. However, if an entire wet laminate is cured in one piece on the core the exotherm can, since heat only can flow from the laminate in one direction due to the high thermal insulation of the core, create temperatures so high it will damage not only the core but also the laminate.

### 2.3.2 Adhesives and their properties

There exists a variety of adhesives, far to many to mention all within the scope of this text. Most are for special purposes, e.g., special PUR for bonding to stainless steel, toughened epoxy for aluminium in high temperature applications, etc. This text will therefore only give an introduction to the most commonly used adhesives, and the main features of each group. The choice of adhesive is primarily focused on finding an adhesive that satisfies the mechanical requirements of the structure of providing a good bond between the material components in the environment that the structure is to work, and considerations like fatigue, heat resistance, strength, ageing and creep are of primary interest. Fortunately, there are a wide variety of adhesives on the market which satisfactory meet the requirements of a mechanically good bond between almost every plausible combination of sandwich materials. Secondly, the adhesive must also meet the requirements of the environment in which it is supposed to be used. Thus, issues like health considerations, manufacturing technique, curing time, curing temperature, special tooling requirements etc., will just as much decide the choice of adhesive system for the particular application and manufacturing environment, and such requirements will commonly have the greatest influence on the choice.

**Epoxy resins**

Epoxies are low temperature curing resins, normally between 20 and 90°C, but some formulations are made for high temperature curing (130-220°C). They have the advantage of being used without solvents and curing without creating volatile by-products and have thus a low volume shrinkage. The absence of solvents makes epoxies usable with almost every type of core material. Epoxies are available as paste, powder, films, or as solid adhesives. They generally have quite good
mechanical properties with a shear strength in room temperature of about 20-25 MPa [13]. The bond to metals is greatly improved by pre-treating of the metal surfaces with a primer. A major drawback with epoxy resins are that it may cause serious health problems.

**Modified epoxies**
Toughened epoxies are similar as common epoxies but mixed with synthetic rubber, like polysulfide elastomers, which greatly improves the peel resistance. The greater the portion of elastomers the greater ductility but the creep tendencies increase correspondingly as well and the heat resistance decreases. Other modifications are the inclusion of Nylon to improve filleting and control flow [8]. These types are however, sensitive to humidity. By mixing the epoxy with nitrile instead of Nylon the same advantages are gained but with maintained resistance to humidity. These are the most common of the toughened thermoset adhesives and are usually limited to approximately 150°C service temperature. The shear strength of toughened epoxies approach values of about 35 MPa. Toughened epoxy adhesive films are the most common material used when bonding honeycomb sandwich parts.

**Phenolics**
Phenolic adhesives have an excellent strength, high-temperature mechanical properties and durability. The main drawbacks are that they give off some water when curing making venting essential. The viscosity is also quite high and adhesive films must thus be used. These characteristics have limited the use of phenolics to mainly the process of making honeycombs, where venting is no obstacle and a high temperature bond is required. The out-gassing makes phenolics unsuitable for use in bonding sandwich constructions [8], apart from applications where venting is possible. Phenolic adhesives are often modified with synthetic rubbers to improve their toughness.

**Polyurethanes**
Polyurethane (PUR) adhesives are probably the most widely used adhesive for bonding sandwich elements. This is since they provide excellent adhesion to most materials. They can be used as paste or liquid in a wide range of viscosities, may have long or short cure times and can be made fire-retardant and water resistant [33]. PUR adhesives contain virtually no solvents and are thus environmentally friendly and the least toxic of the resins mentioned herein. There exist two different types of PUR-adhesives; one-component moisture-cured and two-component systems. One-component PUR adhesives are in short pre-reacted two-component adhesives which continue to cure when exposed to moisture. Moisture necessary for curing is simply provided by spraying water on the surfaces prior to bonding. Onset of curing vary between minutes and several hours depending on the choice of adhesive. Two-component PUR adhesives consist of various polyols, water scavengers, catalysts, fire retardants, fillers, etc. The curing agent is usually a polymeric methylene-di-phenyl-diisocyanate, which is the least volatile of all isocyanates. The pot-life when mixed can be made to vary between 5 minutes and several hours, and the consistency from liquid to paste. PUR adhesives can be applied by spraying, rolling or even by brushing. Curing must take place under pressure, preferably mechanical pressure but vacuum is commonly used in the making of sandwich elements. Heat drastically decreases the cure time. PUR adhesives are mainly used in bonding of foam or balsa core sandwich structures.
Urethane acrylates
Urethane acrylate is a resin which is compatible with polyesters and vinylesters. In fact, acrylates are so compatible that they can be incorporated in e.g. a wet polyester laminate. Urethane acrylates are very tough, and exhibit almost no curing volume shrinkage. A way to drastically increase the face-to-core bond in foam core GRP-sandwich structures is to use urethane acrylate resin for the first reinforcing layer [34], that closest to the core. The rest of the laminate can then be laminated wet, using for example polyester resin on top of the acrylate layer and still provide a perfect interlaminar bond.

Polyester and vinylester resins
Polyesters and vinylesters are the most commonly used matrix materials for reinforced plastic composites outside the aerospace industry. Prefabricated laminates can be bonded to e.g. foam or balsa cores using the same resin as in the laminate, and it will in most cases prove to be an adequate bond. Usually, however, the laminate is built-up directly onto the core and hence the first layer of the laminate is laid wet onto the core and bonds directly to it. It is essential in the above procedure that enough resin is used to fill all surface cells otherwise leaving dry areas in the bond line. A problem with these resins are their curing volume shrinkage creating sometimes very high interface shear stresses. A way of decreasing the effect of shrinkage is to prime the core surface by applying a thin layer of resin to it which only fills the surface cells, and which is allowed to cure before the rest of the laminate is applied.

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[24] Rohacell and Rohacell WF, Technical Data Sheet, Röhm, Darmstadt, Germany

[26] Hexcel Corporation, Dublin, California, USA.


[28] Dow Europe, Middlesex, UK


Chapter 3

Beam Analysis

In this section formulae and relations will be given for the analysis and design of sandwich beams. All derivations are omitted and only the final result is presented, its limitations, if it is exact or approximate, its accuracy and how to use the formula. The theory is in much the same as ordinary engineering beam theory with the addition of shear stresses and transverse shear deformations. This theory is often referred to as the Timoshenko beam theory. For simplicity reasons, all beams are assumed to have unit width, and thus, all loads, bending moments, stiffnesses etc., are also given per unit width. The theory given here is only a summary of what is thoroughly described in the books by Allen [1] and Plantema [2].

3.1 Definitions and Sign Convention

The sign conventions used in this section are as indicated in Fig.3.1. Forces and stresses are positive when in the direction of a positive coordinate when acting on a surface with a positive normal vector, e.g., $\sigma_x$ is positive when acting on the right surface in Fig.3.1 and in the direction of positive $x$-coordinate. Bending moments are positive when creating a positive deformation.

![Figure 3.1 Sign convention used for stresses, forces and bending moments.](image)

$u$ - deformation in global $x$-direction

$w$ - deformation in global $z$-direction

The strains are then

$$\varepsilon_x = \frac{\partial u}{\partial x} \quad \text{and} \quad \gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}$$

where the curvature $\kappa_x$ is the inverse of the radius of curvature $R_x$.  

$$
\varepsilon_x = \frac{\partial u}{\partial x} = \frac{z}{R} = -z \frac{\partial^2 w}{\partial x^2} \quad \text{and} \quad \gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \quad (3.1)
$$
3.2 Flexural Rigidity

(i) Symmetrical sandwich

For a symmetrical sandwich cross-section, that is when the faces are of the same material and of equal thickness as in Fig.3.2, we have that

\[
D = \int Ez^2 \text{d}z = \frac{E_f t_f^3}{6} + 2E_f t_f \left[ \frac{d}{2} \right]^2 + \frac{E_c t_c^3}{6} = \frac{E_f t_f}{12} + \frac{E_f t_f d^2}{2} + \frac{E_c t_c^3}{12} = 2D_f + D_o + D_c \tag{3.2}
\]

where
- \(2D_f\) is the bending stiffness of the faces about their individual neutral axes
- \(D_o\) the bending stiffness of the faces about the middle axis
- \(D_c\) the bending stiffness of the core

and \(d\) is the distance between the centroids of the faces, i.e., \(d = t_f + t_c\)

Thin face approximation: \[\frac{2D_f}{D_o} < 0.01 \text{ if } 3 \left( \frac{d}{t_f} \right)^2 > 100 \text{ or } \frac{d}{t_f} > 5.77 \tag{3.3}\]

Weak core approximation: \[\frac{D_c}{D_o} < 0.01 \text{ if } \frac{6E_f t_f d^2}{E_c t_c^3} > 100 \tag{3.4}\]

If both the above relations are satisfied then we can approximately write

\[D = D_o = \frac{E_f t_f d^2}{2} \tag{3.5}\]
(ii) Non-symmetrical sandwich (Dissimilar faces)

A non-symmetrical sandwich cross-section is one that has dissimilar faces, that is, of different materials and/or of different thickness as in Fig.3.3.

Before one can calculate the flexural rigidity of this cross-section, the position of the neutral axis must be found. It is given by the coordinate system for which the first moment of area is zero when integrated over the entire cross-section. The distance $e$ from the middle axis of the lower face to the neutral axis is then calculated from

$$E_1 t_1 \left( \frac{t_1}{2} + t_c + \frac{t_2}{2} \right) + E_2 t_c \left( \frac{t_2}{2} + \frac{t_3}{2} \right) = e \left[ E_1 t_1 + E_2 t_c + E_3 t_2 \right]$$

The flexural rigidity is then

$$D = E_1 t_1^3 + E_2 t_2^3 + \frac{E_1 t_1 t_2 E_2 t_2 d^2}{E_1 t_1 + E_2 t_2}$$

(3.6)

where $d = t_1/2 + t_c + t_2/2$ (distance between centre lines of the faces). The sum of the two first terms may be denoted $2D_f$, the sum of the third and sixth term $D_c$ and the fourth and fifth $D_0$.

If the core is weak, $E_c << E_\mu$ the bending stiffness appears as (weak core approximation)

$$D = \frac{E_1 t_1^3}{12} + \frac{E_2 t_2^3}{12} + \frac{E_1 t_1 E_2 t_2 d^2}{E_1 t_1 + E_2 t_2}$$

(3.7)

and if the both the core is weak, $E_c << E_\mu$ and the faces are thin, $t_1, t_2 << t_c$ (thin face approximation), then

$$D = D_0 = \frac{E_1 t_1 E_2 t_2 d^2}{E_1 t_1 + E_2 t_2}$$

(3.8)
3.3 Stresses and Strains

(i) Direct stresses and strains

The direct stresses in the faces and core due to bending equal

\[ \varepsilon_x = \frac{M_x z}{D}, \quad \text{and thus} \quad \sigma_f = \frac{M_x z E_f}{D} \]  

(3.9)

thus giving tensile stress in one face and compressive stress in the other. The coordinate \( z \) is defined as in Fig.3.2 and 3.3 and must be measured within the face and the face modulus should that of the face considered.

\[ \varepsilon_c = \frac{M_x z}{D}, \quad \text{and thus} \quad \sigma_c = \frac{M_x z E_c}{D} \]  

(3.10)

where \( z \) must be measured within the core. Hence, the direct stress varies linearly within each material constituent, but there is a jump in the stress at the face/core interface. By assuming the faces to be thin (\( t_1 \) and \( t_2 << t_c \)) and the core to be weak (\( E_c << E_f \)) we can instead write

\[ \sigma_{f1} = - \frac{M_x (d - e) E_f}{D_0} \approx \pm \frac{M_x}{t_1 d} \quad \text{and} \quad \sigma_{f2} = - \frac{M_x E_f}{D_0} \approx \pm \frac{M_x}{t_2 d} \]  

(3.11)

with positive stress in lower face and negative in the upper, if the bending moment is positive. The direct stress and strain due to an in-plane load is simply

\[ \varepsilon_s = \frac{M_x z}{D}, \quad \text{and thus} \quad \sigma_f = \varepsilon_s E_1, \quad \sigma_c = \varepsilon_s E_2, \quad \text{and} \quad \sigma_c = \varepsilon_s E_c \]  

(3.12)

where \( \varepsilon_s \) is the strain in the neutral axis. The strains and stresses due to bending and in-plane loads can then be superimposed.

(ii) Shear stresses

The shear stress in the faces and the core of a symmetrical sandwich is

\[ \tau_f(z) = \frac{T_x}{(D_0 + 2D_f)} \left( \frac{t_f^2}{4} + t_f + t_f^2 - z^2 \right), \quad \text{and} \quad \tau_c(z) = \frac{T_x}{D} \left[ \frac{E_f t_f d}{2} + E_c \left( \frac{t_c^2}{4} - z^2 \right) \right] \]  

(3.13)

The maximum shear stress appears in the neutral axis, i.e., for \( z = 0 \), and the minimum core shear stress in the face/core interface.

\[ \tau_{c,max} (z = 0) = \frac{T_x}{D} \left( \frac{E_f t_f d}{2} + \frac{E_c t_c^2}{8} \right), \quad \text{and} \quad \tau_{c,min} = \tau_{f,max} = \frac{T_x}{D} \left( \frac{E_f t_f d}{2} \right) \]  

(3.14)

The ratio between the maximum and minimum shear stress is less than 1 percent if

\[ \frac{4E_f t_f d}{E_c t_c^2} > 100 \]  

(3.15)

The shear stresses in a non-symmetrical sandwich can be written as
For $z < 0$ (face 1 and core part between neutral axis and face 1)

$$\tau_{f1}(z) = \frac{T_x E_i}{D} \left[ \left( d - e + \frac{t_1}{2} \right)^2 - z^2 \right]$$

$$\tau_c(z) = \frac{T_x}{D} \left[ E_i t_1 (d - e) + \frac{E_c}{2} \left( d - e - \frac{t_1}{2} \right)^2 - z^2 \right]$$

For $z > 0$ (face 2 and core part between neutral axis and face 2)

$$\tau_{f2}(z) = \frac{T_x E_i E_2}{D} \left[ \left( e + \frac{t_2}{2} \right)^2 - z^2 \right], \quad \tau_c(z) = \frac{T_x}{D} \left[ E_2 t_2 e + \frac{E_c}{2} \left( e - \frac{t_2}{2} \right)^2 - z^2 \right]$$

When assuming a weak core the maximum core shear stress can be written

$$\tau_{c,\text{max}} = \frac{T_x E_i t_1 E_2 t_2 d}{D E_i t_1 + E_2 t_2}$$

and when also assuming the face to be thin, it further simplifies to

$$\tau_{c,\text{max}} = \frac{T_x}{d}$$

(iii) Approximations

The conclusion of the above formulae summarises the modus operandi or the principal load carrying and stress distributions in a structural sandwich construction to; **The faces carry bending moments as tensile and compressive stresses and the core carry transverse forces as shear stresses.**

The stress distributions for the different degrees of approximation can also be graphically represented by plotting the above equations as function $z$. By taking a symmetrical sandwich we get the relation plotted in Fig.3.4.

![Figure 3.4 Direct and shear stresses for different levels of approximations.](image-url)
3.4 Transverse Shear Deflections and Shear Stiffness

When a structural element is subjected to shear forces it will deform, without volume change however, according to Fig.3.5. This deformation can be divided into two different parts, transverse (middle) and in-plane (far right) shear deformation.

\[
\gamma \cdot \text{dx}
\]

![Figure 3.5 Deformation of a structural element subjected to shear forces.](image)

The transverse deformation of an element is according to Fig.3.5 equal to \( \gamma \cdot \text{dx} \) (here \( \gamma_0 \)) and in order to find the total deformation an integration is performed, e.g. over the length of a beam. But, to do this the shear strain must be known and therefore also the shear stiffness. For a sandwich cross-section, the shear stiffness must be computed by using an energy balance equation. The shear stiffness, \( S \), is found by calculating the average shear angle of the cross-section, \( dw/dx \), as

\[
\frac{1}{2} T \frac{dw}{dx} = \frac{1}{2} \int \tau_{xz}(z) \gamma_{xz}(z) \, dz \\
\frac{d^2 w}{dx^2} = \frac{1}{S} \frac{dT}{dx}
\]

(3.20)

Using the approximations for a sandwich with thin faces, \( t_f \ll t_c \), weak core, \( E_c \ll E_f \), and assuming the shear modulus of the faces to be large, it is seen that \( \tau_{xz} = T/d \) so that

\[
S = \frac{G \cdot d^2}{t_c}
\]

(3.21)

This expression can be shown to agree very well the exact one as calculated by the integration above. In fact, for dimensions and materials used in most sandwich applications, the difference between the exact value and the approximate is usually less than 1 percent. Shear deformation can be transverse (symmetrical loading) or in-plane, or both (anti-symmetrical loading). Now study Fig.3.6 which assumes shear deformation only in the core and that this deformation is linear, i.e., \( E_c \ll E_f \) from above, giving a constant core shear stress and a constant shear strain.

![Figure 3.6 Shear deformation of a sandwich element.](image)
In Fig.3.6, an element is shown the deformation is transverse and in-plane; the in-plane part is denoted \( \gamma_0 \) and the total shear of the core is denoted by \( \gamma \). By the geometry it is seen that

\[
\frac{dw_s}{dx} d = (\gamma - \gamma_0) t_c \Rightarrow \frac{dw_s}{dx} = \frac{T_x}{S} - \frac{\gamma_0 t_c}{d}
\]

(3.22)

since \( \gamma = \tau_c/G_c = T_x/G_c d \) and hence \( S \) equals \( G_c d^2/t_c \) as also found above using the same approximations. Now, the deformation is readily found by an integration as

\[
w_s = \int \left( \frac{T_x}{S} - \frac{\gamma_0 t_c}{d} \right) dx = \frac{M_x}{S} - \frac{\gamma_0 t_c x}{d} + \text{constant}
\]

(3.23)

leaving two unknown constants to be found from the boundary conditions. In most cases the loading is symmetrical making \( \gamma_0 = 0 \).

### 3.5 Partial Deflections

For any type of sandwich structure the deformation always consists of two parts

(i) deformations due to bending moments - bending - \( w_b \)

(ii) deformations due to transverse forces - shear - \( w_s \)

![Figure 3.7 A cantilever sandwich beam illustrating bending, shear, and total superimposed deformation.](image)

Shear deformations are usually neglected in classical analysis of structures with homogeneous cross-sections unless the studied member has a very short span, because the shear part, \( w_s \), is usually only fractions of the bending part, \( w_b \). But, for short beams or cross-sections with low shear stiffness this deformation component must be included and for sandwich beams the latter is commonly true. For a sandwich with thin faces the two deformation parts may be superimposed as \( w = w_b + w_s \), see Fig.3.7.
3.6 Inertia Forces
(i) Vertical inertia
Vertical inertia is the body force acting on the element when subjected to an acceleration $\frac{\partial^2 w}{\partial t^2}$ and is

$$f = \int \rho \frac{\partial^2 w}{\partial t^2} dz = -\rho^* \frac{\partial^2 w}{\partial t^2}, \text{ and thus } \rho^* = \int \rho dz = \rho_1 l_1 + \rho_2 l_2 + \rho_c l_c$$

(3.24)

where $\rho^*$ is the mass per unit length. This mass may of course be a function of $x$ if the beam has a varying cross-section. Hence, an acceleration in positive $w$-direction (downwards) creates a body force with opposite direction (upwards).

(ii) Rotary inertia
The cross-section rotates when bending, and this rotation equals $dw_b/dx$. If one assumes the faces to be thin which implies that the shear strain is constant over the cross-section, we can write the $x$-displacement of a particle as

$$u = -z \frac{\partial w_b}{\partial x}, \text{ since } \varepsilon_x = -z \frac{\partial^2 w_b}{\partial x^2}$$

If subjected to an acceleration the bending moment inertia will then be

$$M_b = -\int \rho x^2 \frac{d^2 u}{dt^2} dz = \int \rho x^2 \frac{d^3 w_b}{dx dt^2} dz = R \frac{d^3 w_b}{dx dt^2}$$

(3.25)

$M_b$ is here defined positive in the same direction as $M_x$ and $R$ is the rotary inertia. That is, a positive angular acceleration creates a positive bending moment. The way to compute $R$ is exactly the same as calculating the flexural rigidity $D$ but with $\rho$ substituted for $E$. Hence,

$$R = \frac{\rho_1 l_1}{12} + \frac{\rho_2 l_2}{12} + \frac{\rho_c l_c}{12} + \rho_1 (d-e)^2 + \rho_2 (d-e)^2 + \rho_c \left( \frac{l_c}{2} - e \right)^2$$

(3.26)

Note that the contribution from the core (term 3 and 6) in this expression may be significant since the ratio of $\rho_c/\rho_c$ usually is lower than $E/\varepsilon_c$.

3.7 Equilibrium Equations
The equations of equilibrium takes the following form

Vertical:

$$\frac{\partial T_x}{\partial x} + q + N_x \frac{\partial^2 w}{\partial x^2} - \rho^* \frac{\partial^2 w}{\partial t^2} = 0$$

(3.27)

Moment:

$$-T_x + R \frac{\partial^3 w_b}{\partial x \partial t^2} + \frac{\partial M_x}{\partial x} = 0$$

(3.28)

where the terms containing $\rho^*$ and $R$ originate from motion inertia and may be omitted if only static or quasi static loading is considered. If the load is static we then have
\( \frac{\partial T}{\partial x} = -q \), and \( T = \frac{\partial M}{\partial x} \) \hspace{1cm} (3.29)

### 3.8 Governing Beam Equations

(i) **Static beam equation - thin faces**

The governing beam equation for static loading, i.e., static transverse and in-plane loading only, and sandwiches with thin faces is

\[
D \frac{d^4w}{dx^4} = \left( 1 - \frac{D}{S} \frac{d^2}{dx^2} \right) \left( q + N_x \frac{d^2w}{dx^2} \right) \hspace{1cm} (3.30)
\]

(ii) **Static beam equation in partial deflections - thin faces**

This equation is given in the total deflection field and may be rewritten in terms of partial deflection to by using the relation

\[
S \frac{d^2w}{dx^2} = -D \frac{d^4w_b}{dx^4} \hspace{1cm} (3.31)
\]

giving the two differential equations

\[
D \frac{d^4w_b}{dx^4} = q + N_x \frac{d^2w}{dx^2} \quad \text{and} \quad -S \frac{d^2w}{dx^2} = q + N_x \frac{d^2w}{dx^2} \hspace{1cm} (3.32)
\]

where \( q \) is the applied transverse pressure and \( N_x \) the in-plane direct load.

(iii) **Governing beam equation of motion - thin faces**

\[
D \frac{\partial^4 w}{\partial x^4} - \left[ q + N_x \frac{\partial^2 w}{\partial x^2} - \rho \frac{\partial^2 w}{\partial t^2} \right] + \frac{D}{S} \frac{\partial^2}{\partial x^2} \left[ q - N_x \frac{\partial^2 w}{\partial x^2} - \rho \frac{\partial^2 w}{\partial t^2} \right] - \frac{R}{S} \frac{\partial^2}{\partial x^2} \left[ q + N_x \frac{\partial^2 w}{\partial x^2} - \rho \frac{\partial^2 w}{\partial t^2} \right] - \frac{R}{\partial x^2} \frac{\partial^4 w}{\partial x^2 \partial t^2} = 0 \hspace{1cm} (3.33)
\]

It is seen in the above expression that the first two terms represent the ordinary differential equation for a beam when neglecting the transverse shear deformation. The third term is the shear deformation which together with the first two terms equals the above expression. The fourth term is a combination of shear and rotary inertia and last the fifth term which represent the rotary inertia. One very important observation to make is that all units must be consistent, e.g., SI-units, or the coefficients will have different magnitudes. A good advice is to use kilograms, metres, seconds and Newtons only, giving that all terms have the dimension kg/ms² (or N/m²). This is the same equation as given by Timoshenko [3] but written in a slightly different notation.

For free undamped vibration, i.e., the applied loads are zero the governing equation reduced to

\[
D \frac{\partial^4 w}{\partial x^4} + \rho \frac{\partial^2 w}{\partial t^2} - \frac{D}{S} \frac{\partial^2 w}{\partial x^2 \partial t} \left( D \frac{\partial^4 w}{\partial x^2 \partial t^2} - R \frac{\partial^4 w}{\partial x^2 \partial t^2} \right) - R \frac{\partial^4 w}{\partial x^2 \partial t^2} = 0 \hspace{1cm} (3.34)
\]
It should be noted, however, that in most cases the rotary inertia term is quite small and can be omitted without making too much error.

(iv) Static beam equation - thick faces
If the faces of the sandwich has significant thickness, i.e., the sum of the flexural rigidities of the faces $2D_f$ is not negligible compared with $D_0$, then the following equation can be used

$$2D_f \frac{d^6w}{dx^6} - \frac{(2D_f + D_0)S}{D_0} \frac{d^4w}{dx^4} = \left( \frac{d^2}{dx^2} - \frac{S}{D_0} \right) \left( q + N_x \frac{d^2w}{dx^2} \right)$$  \hspace{1cm} (3.35)

It is seen that this expression equals the governing equation of (i) if $2D_f$ is taken as zero. If the faces are dissimilar then $2D_f$ should be taken as the sum of the flexural rigidities of the faces, i.e., $2D_f = D_1 + D_2$. The thickness of the faces only makes a significant difference to the behaviour of the beam locally at for example clamped edges or in the vicinity of point loads. The total deformation of the beam is very little affected by these local anomalies. Thus, consideration of the face thickness must be taken in order to properly account for the local effect in the vicinity of supports, point loads or clamped edges. Otherwise, if the global deformation is sought after only, it is more convenient to use the thin face governing equation but with $D = D_0 + 2D_f$.

(v) Strain energy expressions
The internal strain energy in the sandwich beams due to deflections field $w$ can be written as

$$U = \frac{1}{2} \int_0^L \left( D_0 + D_f \left( \frac{d^2w}{dx^2} \right)^2 + 2D_f \left( \frac{d^2w}{dx^2} \right)^2 + S \left( \frac{dw}{dx} \right)^2 \right) dx$$

$$\approx \frac{1}{2} \int_0^L D \left( \frac{d^2w}{dx^2} \right)^2 + S \left( \frac{dw}{dx} \right)^2 \right) dx$$  \hspace{1cm} (3.36)

The second term in this expression could be neglected for sandwiches with thin faces and weak core. The potential energy of the in-plane forces, $N_x$, is

$$U_2 = \frac{1}{2} N_x \int_0^L \left( \frac{dw}{dx} \right)^2 dx$$  \hspace{1cm} (3.37)

and the potential energy of the transverse force, $q$, equals

$$U_3 = -\int_0^L q(x)wdx$$  \hspace{1cm} (3.38)

and finally, the kinetic energy of the beam is

$$U_4 = \frac{1}{2} \int_0^L \left[ R \left( \frac{\partial^2 w}{\partial x^2} \right)^2 + \rho \left( \frac{\partial w}{\partial t} \right)^2 \right] dx$$  \hspace{1cm} (3.39)

The total energy of the system can then be written as

$$U = U_1 + U_2 + U_3 - U_4$$  \hspace{1cm} (3.40)
3.9 Rigid Core

If the core has considerable stiffness, i.e., $E_c$ is not much lower than $E_f$, the formulae above can still be used if the core shear modulus $G_c$ is substituted with $G^*$ as

$$ G^* = \frac{G_c}{1 + \frac{E_t t^2_c}{6E_f t d}} \quad (3.41) $$

This procedure yields the correct deflections and face stresses but not the correct core shear stress. However, correct values can be achieved using the formulae in 3.3.

3.10 General Buckling

(i) Buckling of column - thin faces

In the buckling analysis of sandwich columns, transverse shear deformations must be accounted for. This decreases the buckling load compared with the ordinary Euler buckling cases. The critical buckling load can approximately be written as

$$ \frac{1}{P_{cr}} = \frac{1}{P_b} + \frac{1}{P_s} \quad (3.42) $$

where $P_b$ is the buckling load in pure bending and $P_s$ in pure shear. These are given by

$$ P_b = \frac{n^2 \pi^2 D}{(\beta L)^2} \quad \text{and} \quad P_s = S \quad (3.43) $$

where the factor $\beta$ depends on the boundary conditions as given in Fig.3.8. The buckling load can also be written

$$ P = \frac{n^2 \pi^2 D / (\beta L)^2}{1 + n^2 \pi^2 D / S(\beta L)^2} \quad (3.44) $$

Thus, when the column is long and/or has infinite shear stiffness (pure bending)

$$ \lim_{S \to \infty \text{ and/or } L \to \infty} P_{cr} = \frac{n^2 \pi^2 D}{L^2} = P_b $$

If, on the other hand, the beam is very short and/or is weak in shear the limit will be
(ii) Buckling of column - thick faces
When accounting for the thickness of the faces the above formula takes a slightly different form since it must be derived from another governing equation. The result can be written

\[
P = \frac{2n^4 \pi^4 D_f D_0}{S(\beta L)^4} + \frac{n^2 \pi^2 D_0}{(\beta L)^2} + \frac{2}{S(\beta L)^2}
\]

(3.45)

In this case there is another limit when the column is very short or weak in shear, namely

\[
\lim_{S \to 0 \text{ and/or } L \to 0} P_{cr} = S = P
\]

(iii) Buckling stress exceeding elastic limit
If the stress in the faces computed using the above formulae exceeds the limit of proportionality, yield stress or an offset strength, the elastic modulus of the faces must be replaced by a reduced modulus in order to yield an accurate buckling load. Thus, if \( \sigma_f > \sigma_y \) then \( E_f \) should be substituted with \( E_r \) according to

\[
E_r = \frac{2E_f E_{tan}}{E_f + E_{tan}}
\]

(3.46)

where \( E_{tan} \) is the tangent modulus at the given point on the stress-strain relation according to Fig.3.9 and \( \sigma_y \) the yield strength.

In the strain regime considered most core materials can be assumed linear elastic. The internal load of the column hence equals

\[
P = 2t_f \sigma_f + t_c E_c \varepsilon_f
\]

(3.47)

Since the buckling load is a function of \( E_{tan} \) it is also a function of \( \varepsilon_f \). Thus, at some point on the stress-strain curve the internal load, as given above, equals the buckling as calculated in (i) or (iii) above but with \( E \) substituted for \( E_r \). The instability load is then found as the point on the stress-strain curve where the buckling load equals the internal load \( P \).
3.11 Local Buckling of Sandwich Beams

Local buckling is a form of local instability of the face sheet. It can be seen as buckling of a thin strip of material (the face) supported by a continuous or discontinuous elastic medium, the core. Schematically it looks like in Fig.3.10.

This kind of instability can occur whenever a face sheet is in compression, that is, not only when the in-plane load resultant is negative but also in pure bending since then one face exhibits membrane tension and the other membrane compression.

(i) Local buckling formulae

The face is assumed to deform as

\[ w_x = W \sin \frac{\pi x}{L} \]

where \( L \) is wavelength of the buckle which can be estimated by

\[ L = \pi \left( \frac{4D_f}{E_c G_c} \right)^{1/6} \]  \hspace{1cm} (3.48)

The actual instability load can be derived in several ways. The mode of buckling changes from symmetrical to unsymmetrical depending on the core thickness and the actual buckling load then depends on the ratio \( t/t_c \). The exact solution to this problem has been found to yield non-conservative results mainly due the fact that initial imperfections has a quite large effect on the local buckling load. For practical purposes the following formula has shown to yield very good results.
\[ \sigma_f = 0.5 \sqrt{E_f E_c G_c} \]  
\[ (3.49) \]

In the case of an isotropic plate, simply substitute \( E_f \) for \( E_f / (1 - \nu_f^2) \) and \( E_c \) for \( E_c / (1 - \nu_c^2) \).

(ii) Buckling beyond the elastic limit

Just as in the general buckling case above the buckling may exceed the elastic limit of the face sheet. A similar kind of modulus reduction can then be made. Thus, when \( \sigma_f > \sigma_c \), substitute \( E_f \) with

\[ E_f = \frac{4E_f E_{\text{tan}}}{(\sqrt{E_f} + \sqrt{E_{\text{tan}}})^2} \]
\[ (3.50) \]

The same kind of trial-and-error process as in the general buckling case then follows. A point on the stress-strain relation of the face sheet (see schematic in Fig. 3.9) gives a tangent modulus. This modulus gives a reduced modulus as calculated above, and by inserting into the buckling stress formula an value for the critical stress is obtained. Somewhere on the stress-strain curve these stress values are equal and that is defined as the actual buckling stress.

(iii) Effect of initial irregularities

The actual wrinkling load is very much affected by any initial waviness of the face. Some design guidelines to account for such effects are given in [1,2] where it is found that the reduction in wrinkling strength can be significant for very high waviness. However, waviness in the faces is usually carefully avoided, not only because of a reduction in the wrinkling stress, but mainly because a smooth surface is a sought after characteristic when using a sandwich design. In practical cases, initial irregularities are likely to reduce the wrinkling strength to about 80\% of the theoretical [2].

(iv) Wrinkling with honeycomb cores

A honeycomb core is highly anisotropic with very high modulus perpendicular to the faces and very low in the plane of the panel. Sandwich panels with honeycomb cores may exhibits local buckling in the same manner as described above, particularly when the cell size of the honeycomb is small. However, since in a honeycomb or corrugated sandwich panel, a large part of the face sheet is un-supported, buckling may occur in this region locally with a forced wave length equal to the size of that un-supported region.

For square cell honeycomb this buckling stress can be estimated by

\[ \sigma_{f,cr} = 2.5E_f \left( \frac{t_f}{a} \right)^2 \text{ for } \nu_f = 0.3 \]
\[ (3.51) \]
where $a$ is length of the side of the cell. An empirical formula proposed for hexagonal honeycombs with constant cell size is [4]

$$\sigma_f = \frac{2E_f}{1 - V_f^2} \left( \frac{t_f}{s} \right)^2$$

(3.52)

where $s$ is the radius of a circle inscribed in the hexagonal cell, as shown in Fig. 3.11. If this stress exceed the elastic limit of the face, the same approach as outlined in (ii) may be used.

(v) Wrinkling with corrugated cores

For corrugated cores, such as one shown above, inter cellular buckling could occur in the plate inscribed by the corrugation. This can be predicted using buckling theory of an ordinary homogeneous plate which is

$$\sigma_{cr} = \frac{k \pi^2 E_f}{12(1 - V_f^2) t_f^2}$$

(3.53)

where $k$ is the buckling coefficient that depends on size of the plate, i.e., $l/b$ ($b$ - width) and the restraint, i.e., the edge conditions and the number wavelengths in the buckling mode.

3.12 Torsion

The rate of twist of a beam subjected to a torque $M_{tx}$ is written

$$\frac{d\phi}{dx} = \frac{M_{tx}}{GJ}$$

(3.54)
where $GJ$ is the torsional stiffness and $\varphi$ the twist. Providing the faces are isotropic and the beam is wide ($b$ is much larger than the thickness of the sandwich) the torsional stiffness may be written

$$GJ = \frac{8}{3} \left[ \left( 1 + \frac{t_c}{2t_f} \right)^3 + \left( \frac{G_{sf}}{G_f} - 1 \right) \left( \frac{t_c}{2t_f} \right)^3 \right] G_f bt_f^3$$  \hspace{1cm} (3.55)

where $b$ is the width of the beam (span in the $y$-direction). When the core shear modulus is sufficiently small compared to the shear modulus of the faces (reasonable assumption in almost all practical cases) the beam acts as two faces twisting independently of each other reducing the torsional stiffness to the well-known relation

$$GJ = \frac{8}{3} \left[ \left( 1 + \frac{t_c}{2t_f} \right)^3 - \left( \frac{t_c}{2t_f} \right)^3 \right] G_f bt_f^3 = \frac{2D_0b}{1 + \nu}$$  \hspace{1cm} (3.56)

which can be seen is the same as the twisting stiffness $D_{xy}$ defined in section 4.

3.13 Examples

This section gives formulae for calculation of deformations, both bending and shear deformations, transverse forces and bending moments. From the latter, stresses can easily be calculated using the relations given in section 3.3. All relations assume sandwich beams with thin faces, so that partial deflections may be superimposed, that is, $w = w_b + w_s$.

3.13.1 Cantilever Beam

(i) Point load

\[x < aL: \quad T_b(x) = P, \quad M_b(x) = -P(aL-x), \quad w_b(x) = \frac{PL^3}{6D} \left[ 3a \left( \frac{x}{L} \right)^2 - \left( \frac{x}{L} \right)^3 \right], \text{ and } w_s(x) = \frac{Px}{S}\]

\[x > aL: \quad T_s(x) = 0, \quad M_s(x) = 0, \quad w_b(x) = \frac{PL^3}{6D} \left[ 2a^3 + 3a^2 \left( \frac{x}{L} - a \right) \right], \text{ and } w_s(x) = \frac{PaL}{S}\]
(ii) Uniform load

\[ T_s(x) = Q \left(1 - \frac{x}{L}\right), \quad M_s(x) = -\frac{Q}{2L}(L - x)^2 \]

\[ w_s(x) = \frac{qL^4}{24D} \left[ \left(\frac{x}{L}\right)^4 - 4\left(\frac{x}{L}\right)^3 + 6\left(\frac{x}{L}\right)^2 \right], \quad \text{and} \quad w_s(x) = \frac{qx}{2S}(2L - x) \]

where the total distributed load \( Q = qL \).

(iii) Hydrostatic pressure

\[ T_s(x) = \frac{q_{\max} L}{2} \left(1 - \left(\frac{x}{L}\right)^2\right), \quad M_s(x) = \frac{q_{\max} L^2}{6} \left[3\left(\frac{x}{L}\right) - 2 - \left(\frac{x}{L}\right)^3 \right] \]

\[ w_s(x) = \frac{q_{\max} L^4}{120D} \left[\left(\frac{x}{L}\right)^5 - 10\left(\frac{x}{L}\right)^3 + 20\left(\frac{x}{L}\right)^2 \right], \quad \text{and} \quad w_s(x) = \frac{q_{\max} Lx}{6S} \left[3 - \left(\frac{x}{L}\right)^2 \right] \]

where the total load \( Q = \frac{q_{\max} L}{2} \) and the distributed load \( q(x) = \frac{q_{\max} x}{L} \)

\[ T_s(x) = \frac{q_{\max} L}{2} \left(1 - \frac{x}{L}\right)^2, \quad M_s(x) = -\frac{q_{\max} L^2}{6} \left[1 - \left(\frac{x}{L}\right)^3 \right] \]

\[ w_s(x) = \frac{q_{\max} L^4}{120D} \left[10\left(\frac{x}{L}\right)^2 - 10\left(\frac{x}{L}\right)^3 + 5\left(\frac{x}{L}\right)^4 - \left(\frac{x}{L}\right)^5 \right], \quad \text{and} \]

\[ w_s(x) = \frac{q_{\max} Lx}{6S} \left[3 - 3\left(\frac{x}{L}\right) + \left(\frac{x}{L}\right)^2 \right] \]

where the total load \( Q = \frac{q_{\max} L}{2} \) and the distributed load \( q(x) = \frac{q_{\max} (L - x)}{L} \)
3.13.2 Beam on two supports subjected to point load

(i) Simply supported edges

This problem is solved by simply superimposing the deformations due to bending and shear. One can also arrive at the shear deformation by using the relation between $w_s$ and $w_b$ given in section 3.8 to obtain the shear deformation. The results are:

$$ R_L = P b, \quad R_R = P a \quad \text{and} \quad M_L = M_R = 0 $$

$$ x \leq aL \quad w(x) = w_b(x) + w_s(x) = \frac{P L^3}{6 D} b \left( \frac{x^3}{L^3} - \frac{x^3}{L^3} \right) + \frac{P b x}{S} $$

$$ T_s(x) = R_L \quad \text{and} \quad M_s(x) = R_L x $$

$$ x \geq aL \quad w(x) = w_b(x) + w_s(x) = \frac{P L^3}{6 D} a \left[ \left( \frac{L-x}{L} \right)^3 - \left( \frac{L-x}{L} \right)^3 \right] + \frac{P a (L-x)}{S} $$

$$ T_s(x) = - R_R \quad \text{and} \quad M_s(x) = R_R (L-x) $$

This simple superposition can be performed since the two extreme cases, $S$ infinite or $D$ infinite, yields the same reactions forces, and hence the deformations occurring in the different modes are independent of each other.

(ii) One edge simply supported, the other clamped.

This case is not as simple as it looks at first sight. The clamping moment at the edge will depend on the shear stiffness, $S$, giving the ordinary moment $PL/2b(3b−b^2−2)$ if $S$ is infinite and $M_r = 0$ if $D$ is infinite. This fact yields an inconsistency; the transverse shear deformation will not be continuous under the load if the loads at the supports are calculated using beam bending theory and equilibrium equations since due to a clamping moment $M_r \neq 0$, they will differ from the above ($bP$ and $aP$, respectively) and the shear deformation calculated by using support forces different from that would yield that the shear deformation $w_s(L) \neq 0$. The shear deformations following the transverse forces computed from the pure bending case are; the left reaction force times $aL/S$ and the right reaction force times $bL/S$. It is now clearly seen that these two are unequal unless $M_r = 0$, leading to an incompatibility at the edge. The way to treat this is by neglecting that $w_b$ and $w_s$.
equals zero at one of edges and instead concentrating on the fact that their sum equals zero at the edges. In fact, the concept of partial deflections can still be used here but the clamping moment will depend on the shear stiffness which in consequence means that \( w_s \) and \( w_b \) are dependent, i.e. only their sum will yield the correct deformation field. Now, relax the boundary condition \( w_b(0) = w_s(0) = 0 \) and use instead \( w_b(0) + w_s(0) = 0 \), and compute the corresponding deformation fields due to bending and shear, respectively. Thus, treat the beam as a cantilever but ensure that the deflection of the free edge (at \( x = 0 \)) is zero. Both partial deflection will depend on the reactions \( R_s \) and \( R_b \) which now can be solved using the boundary condition stated above. The equations required are the deformation fields as function of the reactions. One soon arrives at

\[
x \leq aL \quad T(x) = R_L \quad \text{and} \quad M(x) = R_L x
\]

\[
w_b(x) = \frac{PL^3}{6D} \left[ 3b^2 \left( 1 - \frac{x}{L} \right) - b^3 \right] - \frac{R_L L^3}{6D} \left[ \left( \frac{x}{L} \right)^3 - 3 \left( \frac{x}{L} \right) + 2 \right] , \quad \text{and}
\]

\[
w_s(x) = w_s(0) + \frac{R_L x}{S} = \frac{1}{S} [R_b bL - R_s (aL - x)]
\]

\[
x \geq aL \quad T(x) = -R_R \quad \text{and} \quad M(x) = R_s x - P(x - aL) = PaL - R_R x
\]

\[
w_b(x) = \frac{PL^3}{6D} \left[ \left( \frac{x}{L} - a \right)^3 - 3b^2 \left( \frac{x}{L} - a \right) + 2b^3 \right] - \frac{R_L L^3}{6D} \left[ \left( \frac{x}{L} \right)^3 - 3 \left( \frac{x}{L} \right) + 2 \right]
\]

\[
w_s(x) = w_s(0) + \frac{R_L aL}{S} - \frac{R_s (x - aL)}{S} = 0
\]

\[
\Rightarrow w_s(0) = \frac{R_b bL}{S} - \frac{R_s aL}{S} \quad \text{and} \quad w_s(x) = \frac{R_R}{S} (L - x)
\]

Using these equations along with two independent equations of equilibrium gives after some manipulation and denoting the shear factor \( \theta = D/L^2S \)

\[
R_L = \frac{P[6 \theta b + (3b^2 - b^3)]}{2(1 + 3 \theta)} , \quad R_R = \frac{P[6 \theta a + (2 - 3b^2 + b^3)]}{2(1 + 3 \theta)} , \quad M_R = \frac{PbL(3b - b^2 - 2)}{2(1 + 3 \theta)}
\]

It is seen that \( w_s(L) = w_s(0) = 0 \) but that \( w_b(0) \) and \( w_b(L) \) are dependent, but it can be shown that \( w_b(0) + w_s(0) = 0 \). It is also seen that when \( S \) approaches infinity, the pure bending case (\( \theta = 0 \)), then

\[
R_L = \frac{P}{2} (3b^2 - b^3) , \quad R_R = \frac{P}{2} (2 - 3b^2 + b^3) , \quad M_R = \frac{PbL}{2} (3b - b^2 - 2)
\]

For the case of pure shear (\( \theta = \infty \)) one arrives at

\[
R_L = Pb , \quad R_R = Pa \quad \text{and} \quad M_R = 0
\]
which is the same as for the simply supported case above. It is easy to show that the total
deformation at $x = 0$ actually equals zero, though the individual deflections do not. A *hyperstatic*
beam usually exhibits the behaviour that the reaction forces depend on the deformation field, as
opposed to any *isostatic* beam.

(iii) *Both edges clamped*

![Diagram of a beam with both edges clamped](image)

The same thing as above must also here be done, that is, e.g., use the boundary condition $w_b(0) + w_s(0) = 0$ and $dw_i/dx = 0$, along with the equations of equilibrium. The deformations due to bending
and shear can directly be written as function of the reactions forces and clamping moments and are

$$ w_b(x) = \frac{PL^3}{6D} \left[ 3b^2 \left( 1 - \frac{x}{L} \right) - b^3 \right] - \frac{R_p L^3}{6D} \left[ \left( \frac{x}{L} \right)^3 - 3 \left( \frac{x}{L} \right) + 2 \right] - \frac{M_L L^2}{2D} \left[ \left( \frac{x}{L} \right)^2 - 2 \left( \frac{x}{L} \right) + 1 \right] $$

$$ w_s(x) = \frac{1}{S} \left[ R_b bL - R_L (aL - x) \right] $$

$$ x \leq aL \quad T_i(x) = R_L \quad \text{and} \quad M(x) = M_L + R_L x $$

$$ x \geq aL \quad T_i(x) = -R_R \quad \text{and} \quad M_s(x) = M_R + R_R (L - x) $$

$$ w_b(x) = \frac{PL^3}{6D} \left[ \left( \frac{x}{L} - a \right)^3 - 3b^3 \left( \frac{x}{L} - a \right) + 2b^3 \right] - \frac{R_i L^3}{6D} \left[ \left( \frac{x}{L} \right)^3 - 3 \left( \frac{x}{L} \right) + 2 \right] $$

$$ - \frac{M_L L^2}{2D} \left[ \left( \frac{x}{L} \right)^2 - 2 \left( \frac{x}{L} \right) + 1 \right] \quad \text{and} \quad w_s(x) = \frac{1}{S} R_R (L - x) $$

The boundary conditions stated above yields two equations which along with two independent
equations of equilibrium gives the four unknowns as

$$ R_L = \frac{P[b^2(1+2a) + 12b\theta]}{1 + 12\theta}, \quad R_R = \frac{P[a^2(1+2b) + 12a\theta]}{1 + 12\theta} $$

$$ M_L = - \frac{PbL(6\theta + b)}{1 + 12\theta}, \quad M_R = - \frac{PbL(6\theta + a)}{1 + 12\theta} $$

We can now observe that the limits for these equation equals the special cases infinite shear
stiffness and infinite bending stiffness, respectively. Thus,

Pure bending ($\theta = 0$): $R_L = Pb^2(1+2a), \quad R_R = Pa^2(1+2b), \quad M_L = -PbL^2, \quad M_R = -Pb^2b$
Pure shear ($\theta = \infty$): $R_L = Pb$, $R_R = Pa$, $M_L = M_R = -\frac{PL}{2}ab$

Hence, the clamping moments $M_L$ and $M_R$ will be equal if $a = b$ and/or $\theta = \infty$, otherwise unequal. The same thing goes for the support forces $R_L$ and $R_R$. If $a = b$, then all forces and moments will equal those of the pure bending case, invariably of the shear stiffness. The interesting thing with this case is that the bending moments in the general case, $a \neq b$, changes from the ordinary clamping moments for $\theta = 0$ to be equal as the shear stiffness decreases.

3.13.3 Beam on two supports subjected to a uniformly distributed load

(i) Simply supported edges.

This problem is once again solved by simply superimposing the deformations due to bending and shear. The results are;

$$T_s(x) = R_L - qx \text{ and } M_s(x) = R_Lx - \frac{qx^2}{2}$$

$$w_s(x) = \frac{qL^4}{24D} \left[ \left( \frac{x}{L} \right)^4 - 2\left( \frac{x}{L} \right)^3 + \left( \frac{x}{L} \right) \right] \text{ and } w_s(x) = \frac{q}{2S} \left[ Lx - x^2 \right]$$

$$R_L = R_R = \frac{Q}{2} = \frac{qL}{2} \text{ and } M_L = M_R = 0$$

(ii) One edge simply supported, the other clamped.

This problem is solved in the same way as outlined for point load above, that is, use the boundary condition $w_s(0) + w_s(L) = 0$ along with two independent equations of equilibrium to solve the three unknowns $R_L$, $R_R$ and $M_R$. The results are;

$$T_s(x) = R_L - qx \text{ and } M_s(x) = R_Lx - \frac{qx^2}{2}$$

$$w_s(x) = \frac{qL^4}{24D} \left[ \left( \frac{x}{L} \right)^4 - 4\left( \frac{x}{L} \right)^3 + 3 \right] - \frac{R_L}{6D} \left[ \left( \frac{x}{L} \right)^3 - 3\left( \frac{x}{L} \right) + 2 \right]$$
By these equation the reactions and the clamping moment can be derived to

\[ R_L = \frac{qL}{8} \left( \frac{3 + 12\theta}{1 + 3\theta} \right), \quad R_R = \frac{qL}{8} \left( \frac{5 + 12\theta}{1 + 3\theta} \right), \quad M_R = -\frac{qL^2}{8(1 + 3\theta)} \]

As seen from the above results, the reactions are the same as the ordinary bending case when \( \theta = 0 \) and \( R_L = R_R = qL/2 \) and \( M_R = 0 \) when \( \theta = \infty \). This is similar to the point load case.

(iii) Both edges clamped.

It is quite quickly realised in this case that due to symmetry the reaction forces are equal and so are the clamping moments. By assuming otherwise and performing an analysis similar to the above one reaches the following solution

\[ T_s(x) = \frac{q}{2}(L - 2x), \quad M_s(x) = \frac{q}{2}\left(Lx - x^2 - \frac{L^3}{6}\right) \]

\[ w_s(x) = \frac{qL^4}{24D}\left[\left(\frac{x}{L}\right)^4 - 2\left(\frac{x}{L}\right)^3 + \left(\frac{x}{L}\right)^2\right] \quad \text{and} \quad w_s(x) = \frac{q}{2S}\left(Lx - x^2\right) \]

which is the same solution as given by superimposing the partial deflections. The reaction forces and clamping moments are

\[ R_L = R_R = \frac{qL}{2}, \quad M_L = M_R = -\frac{qL^2}{12} \]

3.13.4 Beam on two supports subjected to hydrostatic pressure

(i) Simply supported edges.

Denote \( q(L) = q_{max} \) and thus total load \( Q = q_{max}L/2 \) and \( q(x) = q_{max}x/L \).

\[ T_s(x) = \frac{q_{max}L}{6}\left[1 - 3\left(\frac{x}{L}\right)^2\right] \quad \text{and} \quad M_s(x) = \frac{q_{max}Lx}{6}\left[1 - \left(\frac{x}{L}\right)^2\right] \]
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\[ w_h(x) = \frac{q_{\text{max}} L^4}{360 D} \left[ 3 \left( \frac{x}{L} \right)^5 - 10 \left( \frac{x}{L} \right)^3 + 7 \left( \frac{x}{L} \right) \right] \quad \text{and} \quad w_s(x) = \frac{q_{\text{max}} L x}{6 S} \left[ 1 - \left( \frac{x}{L} \right)^2 \right] \]

\[ R_L = \frac{q_{\text{max}} L}{6} \quad \text{and} \quad R_R = \frac{q_{\text{max}} L}{3} \]

(ii) One edge simply supported, the other clamped.

\[ T_s(x) = R_L - \frac{q_{\text{max}} x^2}{2L} \quad \text{and} \quad M_s(x) = R_L x - \frac{q_{\text{max}} x^3}{6L} \]

\[ w_h(x) = \frac{q_{\text{max}} L^4}{120 D} \left[ \frac{1}{6} \left( \frac{x}{L} \right)^5 - 5 \left( \frac{x}{L} \right)^3 + 4 \right] - \frac{R_L L^3}{6D} \left[ \frac{1}{3} \left( \frac{x}{L} \right)^3 - 3 \left( \frac{x}{L} \right) + 2 \right] \]

\[ w_s(x) = \frac{1}{S} \left[ R_L (x - L) - \frac{q_{\text{max}} x^3}{6L} (x^3 - L^3) \right] \]

\[ R_L = \frac{q_{\text{max}} L}{10} \left( \frac{1 + 5\theta}{1 + 3\theta} \right), \quad R_R = \frac{q_{\text{max}} L}{10} \left( \frac{4 + 10\theta}{1 + 3\theta} \right), \quad \text{and} \quad M_R = -\frac{q_{\text{max}} L}{15(1 + 3\theta)} \]

\[ T_s(x) = R_L - \frac{q_{\text{max}} x^2}{2} \left( 2 - \frac{x}{L} \right) \quad \text{and} \quad M_s(x) = R_L x - \frac{q_{\text{max}} x^2}{6} \left( 3 - \frac{x}{L} \right) \]

\[ w_h(x) = \frac{q_{\text{max}} L^4}{120 D} \left[ - \frac{1}{6} \left( \frac{x}{L} \right)^5 + \frac{5}{6} \left( \frac{x}{L} \right)^4 - 15 \left( \frac{x}{L} \right)^2 + 11 \right] - \frac{R_L L^3}{6D} \left[ - \frac{1}{3} \left( \frac{x}{L} \right)^3 - 3 \left( \frac{x}{L} \right) + 2 \right] \]

\[ w_s(x) = \frac{1}{S} \left[ R_L (x - L) - \frac{q_{\text{max}} x^2}{2} + \frac{q_{\text{max}} L^2}{3} + \frac{q_{\text{max}} x^3}{6L} \right] \]

\[ R_L = \frac{q_{\text{max}} L}{40} \left( \frac{11 + 40\theta}{1 + 3\theta} \right), \quad R_R = \frac{q_{\text{max}} L}{40} \left( \frac{9 + 20\theta}{1 + 3\theta} \right), \quad \text{and} \quad M_R = -\frac{7q_{\text{max}} L^2}{120(1 + 3\theta)} \]
(iii) Both edges clamped.

\[ T_s(x) = R_L - \frac{q_{\text{max}} x^2}{2L} \quad \text{and} \quad M_s(x) = R_L x + M_L - \frac{q_{\text{max}} x^3}{6L} \]

\[ w_s(x) = \frac{q_{\text{max}} L^4}{120D} \left[ \left( \frac{x}{L} \right)^5 - 5 \left( \frac{x}{L} \right)^4 + 4 \right] - \frac{R_L L^2}{6D} \left[ \left( \frac{x}{L} \right)^3 - 3 \left( \frac{x}{L} \right)^2 + 2 \right] \]

\[ - \frac{M_L L^2}{2D} \left[ \left( \frac{x}{L} \right)^2 - 2 \left( \frac{x}{L} \right) + 1 \right] \]

\[ w_s(x) = \frac{1}{S} \left[ R_L (x - L) - \frac{q_{\text{max}}}{6L} \left( x^3 - L^3 \right) \right] \]

\[ R_L = \frac{q_{\text{max}} L}{20} \left( \frac{3 + 40\theta}{1 + 12\theta} \right), \quad R_R = \frac{q_{\text{max}} L}{20} \left( \frac{7 + 80\theta}{1 + 12\theta} \right), \]

\[ M_L = -\frac{q_{\text{max}} L^2}{30} \left( \frac{1 + 15\theta}{1 + 12\theta} \right), \quad \text{and} \quad M_R = -\frac{q_{\text{max}} L^2}{20} \left( \frac{1 + 10\theta}{1 + 12\theta} \right) \]

### 3.13.5 Example of hyperstatic beam calculation

Consider a design case of a sandwich beam with one edge simply supported and the other clamped, subjected to a uniform pressure loading, as solved in example 3.13.3 (ii). To illustrate the implication of hyperstatic beams lets assume all the materials and cross-section geometry are given and the length of the beam is the sought design value. Take the following example:

- \( t_f = 0.43 \text{ mm}, \ E_f = 210000 \text{ MPa (steel)}, \ \sigma_{f,cr} = 96 \text{ MPa} \)
- \( t_c = 100 \text{ mm}, \ G_c = 3 \text{ MPa (low density PUR)}, \ \tau_{c,cr} = 0.1 \text{ MPa} \)
- uniform pressure \( q = 1.2 \text{ kPa} \)
- Maximum allowed deflection in the middle of the beam \( \Delta = L/150. \)

Giving

\[ D = 451.500.000 \text{ Nmm}^2 \text{ and } S = 300 \text{ N} \]

per unit width. Hence, there are two strength and one stiffness requirement on the beam. From example 3.7.5 (ii) in conjunction with eq.(2.19) it follows
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I: \[ w(x = L/2) = \frac{qL^4}{192D} \left(1 + 31.5\theta + 72\theta^2\right) < \frac{L}{150} \]

II: \[ \tau_{c,cr} > R_{c} = \frac{qL^5 + 12\theta}{8d^2 \left(1 + 3\theta\right)} \]

III: \[ \sigma_{f,cr} > \frac{qL^2}{8t_f d \left(1 + 3\theta\right)} \]

Since the shear factor \(\theta\) is a function of beam length \(L\), the above equations become fairly difficult to solve. However, by trial-and-error it is easy to find an approximate value of \(L\) for each design constraint. By inserting the values of \(D\) and \(S\) from above in these equations, rearranging the above equations for \(L\) as function of \(\theta\), \(L\) can be calculated for different choices of \(\theta(L)\). Take for example the third constraint above.

\[ L = \sqrt{\frac{\sigma_{f,cr} 16D(1 + 3\theta)}{qdE_f}} \]

Inserting \(L = 5000 \text{ mm} \Rightarrow L = 5700 \text{ mm}\)

\(L = 5700 \text{ mm} \Rightarrow L = 5600 \text{ mm}\)

\(L = 5600 \text{ mm} \Rightarrow L = 5609 \text{ mm} \) etc.

By doing this for the three constraints the results are

I: \(L = 6075 \text{ mm}\)

II: \(L = 13400 \text{ mm}\)

III: \(L = 5610 \text{ mm}\)

Hence, the beam length should not exceed 5610 mm. The same kind of approach has to take place whatever property is to be calculated, since thicknesses, materials and length all depend on \(\theta\). Hence, in whatever design situation, the calculation leads to a trial-and-error operation.

3.13.6 Free vibration of beams

(i) Cantilever sandwich beam

For the cantilever sandwich beam, as in 3.13.1, the assumed spatial/time displacement function is

\[ w(x, t) = \frac{w}{L} \left[ \cos \left(\frac{(2m-1)\pi x}{2L}\right) - 1 \right] \sin \omega t \]

which is approximate, but satisfies the boundary conditions \(w = \partial w/\partial x = 0\) at \(x = 0\). The error done in this assumption is that really only the bending slope is supposed to be zero at the clamped edge. However, this fact can be shown to have little influence on the total deflection field away from the edge as long as the faces do not have considerable thickness. By inserting this assumption in the governing differential equation and omitting the rotary inertia term, one arrives at the natural frequency for a cantilever sandwich beam accounting for transverse shear deformation but not for rotary inertia as
The rotary inertia will have very little effect on sandwich beams, since they usually have a quite low shear stiffness and a high flexural rigidity means that terms in the governing equation containing $D/S$ will be much higher than terms containing $R/\rho^*$, where the latter thus will be negligible. When also omitting the effect of transverse shear deformation, that is assuming $S$ is infinitely large, one arrives at the natural frequency for an ordinary beam as

$$\omega_m = (2m-1)^2 \pi^2 \frac{D}{2 \rho^* L^4 [1 + (2m-1)^2 \pi^2 \theta]}$$

(ii) Beam on two edge supports with simple support
A displacement function that satisfies the boundary conditions of the beam in 3.13.2 (i) is

$$w(x,t) = (W_b + W_s) \sin \frac{m \pi x}{L} \sin \omega t$$

In fact, this assumption satisfies not only the kinematic boundary conditions $w(0) = w(L) = 0$, and the natural boundary conditions $M_x(0) = M_x(L) = 0$, but also the differential equation itself. The solution in this case is therefore exact. By inserting the above assumption in the governing differential equation and omitting the rotary inertia by letting $R = 0$, we get

$$\omega_m = m^2 \pi^2 \frac{D}{\sqrt{\rho^* L^4 [1 + m^2 \pi^2 \theta]}}$$

which for a shear stiff beam ($S$ very large) reduces to

$$\omega_m = m^2 \pi^2 \frac{D}{\rho^* L^4}$$

(iii) Beam on with two clamped edges
In this case one cannot find a simple displacement function that both satisfies the boundary conditions and is a solution to the governing equation. An approximate value for the first mode of vibration can however be found by using Rayleigh-Ritz method. A displacement function that satisfies the boundary conditions of the beam in 3.13.2 (iii) is

$$w(x,t) = (W_b + W_s) \sin \frac{\pi x}{L} \sin \omega t$$

where once again the error is made that also the slope of $w_s$ is zero at the clamped edges. By inserting this assumption in the energy equations, integrating over the entire length of the beam, letting the strain energy equal the kinetic energy one arrives at an equation in $W_b$ and $W_s$. After
differentiation with respect to these two unknowns and finding the stationary value of the equation gives the natural frequency as

$$\omega_1 = 4\pi^2 \frac{D}{\sqrt{3\rho^* L^4 (1 + 4\pi^2 \theta)}}$$

This value of the natural frequency will be slightly higher than the exact value. By comparing, it was found that the lowest natural frequency calculated by the above equation is about 2-15 percent higher than the value computed using the FE-method [6] for shear factors $\theta$ ranging from 0 to 1.

References


The following section gives an overview of the governing and fundamental equations for the bending, buckling, and vibration of sandwich panels, and is merely a brief summary of the basic plate small-deformation bending analysis by Timoshenko and Woinowsky-Krieger [1] which is extended to account for transverse shear deformation following the work by Libove and Batdorf [2]. Partial deflections are then again introduced in a similar manner as for the beams. The analysis assume that the transverse normal stiffness of the core is infinite thus keeping the distance between the centroids of the faces, $d$, constant, also called antiplane core [3]. The theory is developed for orthotropic plates with the $x$- and $y$-axes being the principal axes of orthotropy, and for which the properties are constant throughout the panel. This means that the properties of the panel are fully described by the seven constants, the flexural rigidities $D_x$, $D_y$, the twisting stiffness $D_{xy}$, the Poisson ratios $\nu_{yx}$ and $\nu_{xy}$, and the shear stiffnesses $S_x$, $S_y$.

4.1 Definitions and Sign Convention
First define the coordinate system and the positive directions for loads and bending moments as in Fig. 4.1.

The deformations are defined as:  
$u$ - deformation in the $x$-direction  
$v$ - deformation in the $y$-direction  
$w$ - deformation in the $z$-direction

The strain-displacement relation assumes strains to be much smaller than unity so that [1]

$$\varepsilon_x = \frac{\partial u}{\partial x}, \quad \varepsilon_y = \frac{\partial v}{\partial y}, \quad \text{and} \quad \gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x},$$ (4.1)

The curvature is defined as the inverse of the radius of curvature and is written in terms of the deflection field as

$$\kappa_x = -\frac{\partial^2 w}{\partial x^2}, \quad \kappa_y = -\frac{\partial^2 w}{\partial y^2}, \quad \text{and} \quad \kappa_{xy} = -\frac{\partial^2 w}{\partial x \partial y},$$ (4.2)

where the first two are the curvatures in $x$- and $y$-directions whereas the third usually is referred to as the twist. The Poisson' ratios are defined as
\begin{align*}
\nu_{xy} &= -\frac{\partial^2 w / \partial x^2}{\partial^2 w / \partial \xi^2} \quad \text{and} \quad \nu_{yx} = -\frac{\partial^2 w / \partial \xi^2}{\partial^2 w / \partial y^2} \\
(4.3)
\end{align*}

(rule-of-thumb to remember indices: first index corresponds to loading and the second to the strain, i.e. \( \nu_{xy} \) is found by applying a curvature in the \( x \)-direction and measuring the responding curvature in the \( y \)-direction.)

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure4.1}
\caption{Sign convention used for the sandwich panel.}
\end{figure}

4.2 Cross-section Properties

As for the beam, \( D_x \) is the relation between the bending moment \( M_x \) and the corresponding curvature \( \partial^2 w / \partial \xi^2 \) when applied to a thin strip of the panel and can thus be calculated in the same manner as for the beam in sections 3.2 and 3.3.

\begin{align*}
D_x &= \int z_x^2 E_x dz_x \approx \frac{E_x t_1 E_x t_2}{E_x t_1 + E_x t_2} d^2 \\
(4.4)
\end{align*}

for thin faces and weak core. 1 and 2 refer to the upper and lower faces, respectively, as defined in Fig.4.2.

Since in a general case the position of the neutral axes in \( x \)- and \( y \)-direction may differ (if the faces have different orthotropy), \( z_x \) is the out-of-plane coordinate in the \( xz \)-plane and \( z_y \) the out-of-plane coordinate in the \( yz \)-plane. The position of the neutral axis must however be found both in the \( x \)- and the \( y \)-direction as outlined in section 3.3 but according to Fig.4.2. Similarly

\begin{align*}
D_y &= \int z_y^2 E_y dz_y \approx \frac{E_y t_1 E_y t_2}{E_y t_1 + E_y t_2} d^2 \\
(4.5)
\end{align*}

where the following relation must be satisfied
\[
\frac{V_{xy}}{D_x} = \frac{V_{yx}}{D_y}
\]  
(4.6)

![Diagram of cross-section and position of neutral axis in x- and y-directions.](image)

Figure 4.2 Schematic of the cross-section and position of the neutral axis in x- and y-directions.

For exact expressions for the flexural rigidity refer to section 3.2 and perform the calculation in both x- and y-directions. An exact expression for the torsional stiffness \(D_{xy}\) is a bit more difficult to assess, but through its definition one can write

\[
D_{xy} = \int 2z^2 G_{xy} dz \approx \frac{2G_{xy} t_1 G_{xy} t_2 d^2}{G_{xy} t_1 + G_{xy} t_2}
\]  
(4.7)

assuming thin faces and weak core [3]. The shear stiffnesses \(S_x\) and \(S_y\) are computed in the same manner as for the beam, or approximately [3,4]

\[
S_x = \frac{G_{xy} d^2}{t_c} \quad \text{and} \quad S_y = \frac{G_{xy} d^2}{t_c}
\]  
(4.8)

The cross-section properties referring to motion, panel density \(\rho^*\) and rotary inertia \(R\) are easily calculated for a sandwich as

\[
\rho^* = \rho_1 t_1 + \rho_c t_c + \rho_2 t_2
\]  
(4.9)

where \(\rho_1\) is the density of the material component. Hence, \(\rho^*\) can also be interpreted as the panel surface weight. The way to compute \(R\) is exactly the same as calculating the flexural rigidity \(D\) but with \(\rho\) substituted for \(E\). Hence, from above

\[
R_x = \frac{\rho_1 t_1^3}{12} + \frac{\rho_c t_c^3}{12} + \frac{\rho_2 t_2^3}{12} + \rho_1 t_1 (d - e_x)^2 + \rho_c t_c \left( \frac{t_c + t_f}{2} - e_x \right)^2 + \rho_2 t_2 e_x^2
\]  
(4.10)

and in the \(y\)-direction similarly but with \(e_x\) substituted by \(e_y\). Note that the contribution from the core (term 3 and 6) in this expression may be significant since the ratio of \(\rho/\rho_c\) usually is lower than \(E/E_c\). One very important observation to make is that all units must be consistent, e.g., SI-units, or the coefficients will have different magnitudes. A good advice is to use kilograms, metres, seconds and Newtons only, so that all terms have the dimension kg/ms² (or N/m²).
In practice, the vertical inertia will make significant contribution to the behaviour of the panel, whereas the rotary inertia will have a very little effect, even less than for a homogeneous cross-section.

### 4.3 Stresses and Strains

The stresses and strains in the panel can be found once the deflection field, and the transverse force and bending moment distributions are computed using the above. However, for a general orthotropic cross-section the neutral axes in the $x$- and $y$-directions will be in different positions giving rise to some complexity when calculating the stresses and strains. This adds severe complexity since the definition of the middle plane no longer is well defined. The fact that different neutral axes makes the definition of the middle plane difficult has not be treated in any of the open literature. Without getting into too much detail of the stress analysis one can derive the following formulae which can be used with good accuracy.

#### (i) Direct stresses

For sandwiches with thin unequal faces and weak core we have pure membrane face stresses equaling

$$
\sigma_x = \frac{M_x}{D_x} \quad \text{and} \quad \sigma_y = \frac{M_y}{D_y}
$$

which reduces to

$$
(4.11)
$$

$$
\sigma_{f11} \approx -\frac{M_x E_{y1} E_{z1} t_x d}{D_y (E_{x1} t_1 + E_{x2} t_2)} \quad \text{and} \quad \sigma_{f12} \approx \frac{M_y E_{x1} E_{z1} t_y d}{D_x (E_{x1} t_1 + E_{x2} t_2)}
$$

$$
\sigma_{f21} \approx -\frac{M_x E_{y1} E_{z2} t_x d}{D_y (E_{y1} t_1 + E_{y2} t_2)} \quad \text{and} \quad \sigma_{f22} \approx \frac{M_y E_{x1} E_{z2} t_y d}{D_x (E_{y1} t_1 + E_{y2} t_2)}
$$

which when the faces are thin (not necessarily equal) reduces to [3]

$$
\sigma_{f1x} \approx -\frac{M_x}{t_1 d} \quad \sigma_{f2x} \approx \frac{M_y}{t_2 d} \quad \sigma_{f1y} \approx -\frac{M_y}{t_1 d} \quad \sigma_{f2y} \approx \frac{M_y}{t_2 d} \quad \text{and} \quad \sigma_c \approx 0
$$

(4.12)

The sign of the stresses will differ between the two faces so that for a positive bending moment the stress in lower face (positive $z$-coordinate) will exhibit a tensile (positive) stress whereas the upper face will be in compression. The strains are according to the generalised Hooke's law

$$
\varepsilon_x = \frac{\sigma_x}{E_x} - \nu_{yx} \frac{\sigma_y}{E_y}, \quad \varepsilon_y = \frac{\sigma_y}{E_y} - \nu_{xy} \frac{\sigma_x}{E_x}
$$

(4.13)

or rewritten

$$
\sigma_x = \frac{E_x}{1 - \nu_{yx} \nu_{yx}} \left( \varepsilon_x + \nu_{yx} \varepsilon_y \right) \quad \text{and} \quad \sigma_y = \frac{E_y}{1 - \nu_{xy} \nu_{yx}} \left( \varepsilon_y + \nu_{xy} \varepsilon_x \right)
$$

(4.14)

The stresses and strains appearing due to in-plane loads are calculated in the same manner as for the beam, as
\[ \varepsilon_{x0} = \frac{N_x}{E_x t_1 + E_w t_e + E_y t_2} = \frac{N_x}{A_x} \quad \text{and} \quad \varepsilon_{y0} = \frac{N_y}{E_y t_1 + E_w t_e + E_y t_2} = \frac{N_y}{A_y} \]  

(4.15)

and the corresponding stresses are calculated using Hooke's law. The strains and stresses due to bending and in-plane loads can then be superimposed.

(ii) In-plane shear stress:
The in-plane shear stress and strain are, however, often of secondary importance for the performance of the panel or beam. However, for sandwiches with thin faces and weak core we have

\[ \gamma_{xy} = \frac{2M_{xy} z}{D_y} \quad \text{and} \quad \tau_{xy} = G_y \gamma_{xy} \] which reduces to

\[ \gamma_{xy} \approx \frac{M_{xy}}{G_y t_f d} \quad \text{and thus} \quad \tau_{fxy} \approx \pm \frac{M_{xy}}{t_f d}, \quad \text{and} \quad \tau_{cxy} \approx 0 \]  

(4.16)

(iii) Core shear stress:
The exact stresses are assessed in the same way as for the beam and can be computed using the equations of section 3.3 in the \( x \)- and \( y \)-directions, respectively. The thin face, weak core approximations leads to

\[ \tau_{cxz} = \frac{T_x}{d}, \quad \tau_{cyz} = \frac{T_y}{d}, \quad \gamma_{cxz} = \frac{\tau_{cxz}}{G_{cx}}, \quad \text{and} \quad \gamma_{cyz} = \frac{\tau_{cyz}}{G_{cy}} \]  

(4.17)

4.4 Thermal Stresses and Strain
A temperature change in a material induces strains proportional to the temperature change. Since a sandwich in a general case is anisotropic, these strains may cause bending and warping as an effect. The thermoelastic stress-strain relation may be written

\[ \varepsilon_i = \sum_j S_{ij} \sigma_j + \delta_{ij} \alpha \Delta T \] or inverted to

\[ \sigma_i = \sum_j C_{ij} \left( \varepsilon_j - \delta_{ij} \alpha \Delta T \right), \quad i = 1...6 \]  

(4.18)

where \( \alpha \) are the coefficients of thermal expansion, \( \delta_{ij} \) the Kronecker delta function (which equals 1 if \( i = j \) but is otherwise zero), \( S_{ij} \) the material compliance matrix, and \( C_{ij} \) the material stiffness matrix. Note that the coefficients of thermal expansion only affects extensional strains, not the shear strains. Thus, if the axes of symmetry of the plate coincides with the axes of orthotropy, thermal strains will not cause the plate to twist or induce any twisting moments.

In the case of a sandwich, both the faces and the core may in general be treated as orthotropic and faces are usually thin, implying than only in-plane characteristics are of interest, thus reducing the face stiffness matrices to only 4 independent constants. If strains are prevented from being developed, e.g., a plate is fixed within a stiff frame, stresses develop. If, on the other hand, the structure is free to move, the thermal strains will induce some thermal deformation field.
The above characteristics do have some important effects on sandwich plates: Consider a sandwich plate with dissimilar faces, also having dissimilar coefficients of thermal expansion, \( \alpha_{1x}, \alpha_{1y}, \alpha_{2x}, \) and \( \alpha_{2y}, \) and antiplane core (\( E_c = 0 \)). Further assume that the temperature and temperature change are constant over the face, but not necessarily the same in both faces. If transverse deformation is free to take place, the plate will curve if the temperature is changed on one side of the plate (typically refrigerated tanks, truck-bodies, train-cars or building panels), or even if the temperature is changed over the entire body. The latter case is the most general so assume a temperature change \( \Delta T_1 \) on the side of face 1 and \( \Delta T_2 \) on the side of face 2.

(i) Deformation constrained

In an orthotropic plate, for which the deformations are constrained, stresses will develop. For face 1 the stresses will, according to generalised Hooke’s law, be (plane stress assumed)

\[
\sigma_{1x}^{(T)} = -\frac{E_x \Delta T_1}{1 - \nu_{1xy} \nu_{1yx}} \left[ \alpha_{1x} + \nu_{1yx} \alpha_{1y} \right] \quad \text{and} \quad \sigma_{1y}^{(T)} = -\frac{E_x \Delta T_1}{1 - \nu_{1xy} \nu_{1yx}} \left[ \alpha_{1y} + \nu_{1yx} \alpha_{1x} \right] \quad (4.19)
\]

and similarly for face 2. These stresses act as force-couples equivalent of a bending moment and an in-plane load. The latter is simply the face stresses times the face thickness on which they act, i.e.

\[
N_x^{(T)} = \sigma_{1x}^{(T)} t_1 + \sigma_{2x}^{(T)} t_2 \quad \text{and} \quad N_y^{(T)} = \sigma_{1y}^{(T)} t_1 + \sigma_{2y}^{(T)} t_2 \quad (4.20)
\]

Since the face strains in the general case are different (\( \alpha_{1x} \neq \alpha_{2x}, \alpha_{1y} \neq \alpha_{2y}, \) and/or \( \Delta T_1 \neq \Delta T_2 \)) the asymmetry implies the presence of bending moments which derive to

\[
M_x^{(T)} = \frac{D_x \Delta T_1}{(1 - \nu_{1xy} \nu_{1yx})} d \left[ \alpha_{1x} + \nu_{1yx} \alpha_{1y} \right] - \frac{D_x \Delta T_2}{(1 - \nu_{2xy} \nu_{2yx})} d \left[ \alpha_{2x} + \nu_{2yx} \alpha_{2y} \right] \quad (4.21a)
\]

\[
M_y^{(T)} = \frac{D_y \Delta T_1}{(1 - \nu_{1xy} \nu_{1yx})} d \left[ \alpha_{1y} + \nu_{1yx} \alpha_{1x} \right] - \frac{D_y \Delta T_2}{(1 - \nu_{2xy} \nu_{2yx})} d \left[ \alpha_{2y} + \nu_{2yx} \alpha_{2x} \right] \quad (4.21b)
\]

or if \( \nu_{1xy} = \nu_{2xy} = \nu_{xy} \) and \( \nu_{1yx} = \nu_{2yx} = \nu_{yx} \) it simplifies to

\[
M_x^{(T)} = -\frac{D_x}{(1 - \nu_{xy})} d \left[ \alpha_{2x} \Delta T_2 - \alpha_{1x} \Delta T_1 + \nu_{xy} \left( \alpha_{2y} \Delta T_2 - \alpha_{1y} \Delta T_1 \right) \right] \quad (4.21b)
\]

\[
M_y^{(T)} = -\frac{D_y}{(1 - \nu_{xy})} d \left[ \alpha_{2y} \Delta T_2 - \alpha_{1y} \Delta T_1 + \nu_{xy} \left( \alpha_{2x} \Delta T_2 - \alpha_{1x} \Delta T_1 \right) \right] \quad (4.22)
\]

The expressions simplify considerably if an isotropic plate (all \( \alpha \) are equal) is considered and the bending moments can then be written

\[
M_x^{(T)} = M_y^{(T)} = -\frac{D \alpha}{(1 - \nu)} d \left[ \Delta T_2 - \Delta T_1 \right]
\]

Thus, if the deformation due to thermal strains is prevented, forces and bending moments are introduced into the structure which has to be accounted for.
(ii) Deformation unconstrained
Another case is that when a plate or structure is free to deform as the thermal strains develop. Contrary to the constrained case the plate will now exhibit in-plane deformations and curvatures in stead of in-plane loads and bending moments. Since only direct strains act in the faces there will be no global transverse shear presence and the curvatures may be defined as in chapter 4 but without respect to in-plane shear. The thermal strains are then

$$\varepsilon_x^{(T)} = \varepsilon_{x_0}^{(T)} + z_x \kappa_x^{(T)} \quad \text{and} \quad \varepsilon_y^{(T)} = \varepsilon_{y_0}^{(T)} + z_y \kappa_y^{(T)} \quad (4.23)$$

The in-plane deformation at the neutral axis depends on the average face strain and is given by

$$\varepsilon_{x_0}^{(T)} = \frac{1}{d} \left[ \alpha_{x_2} \Delta T_2 (d - e_x) + \alpha_{x_1} \Delta T_1 e_x \right] \approx \frac{\alpha_{x_2} \Delta T_2 E_{x_2} t_2 + \alpha_{x_1} \Delta T_1 E_{x_1} t_1}{E_{x_1} t_1 + E_{x_2} t_2} \quad (4.24)$$

$$\varepsilon_{y_0}^{(T)} = \frac{1}{d} \left[ \alpha_{y_2} \Delta T_2 (d - e_y) + \alpha_{y_1} \Delta T_1 e_y \right] \approx \frac{\alpha_{y_2} \Delta T_2 E_{y_2} t_2 + \alpha_{y_1} \Delta T_1 E_{y_1} t_1}{E_{y_1} t_1 + E_{y_2} t_2} \quad (4.25)$$

where $e_x$ is the distance from the neutral axis in the $x$-direction to face 2 and $e_y$ the distance in the $y$-direction. The strains in the faces may also be written as function of the curvatures as

$$\varepsilon_{x_1}^{(T)} = \varepsilon_{x_0}^{(T)} + (d - e) \kappa_x = - \alpha_{x_1} \Delta T_1$$

$$\varepsilon_{x_2}^{(T)} = \varepsilon_{x_0}^{(T)} + e \kappa_x = - \alpha_{x_2} \Delta T_2$$

and by subtracting the first with the second one arrives at

$$\kappa_x^{(T)} = \frac{1}{d} \left( \alpha_{x_2} \Delta T_2 - \alpha_{x_1} \Delta T_1 \right) \quad \text{and similarly} \quad \kappa_y^{(T)} = \frac{1}{d} \left( \alpha_{y_2} \Delta T_2 - \alpha_{y_1} \Delta T_1 \right) \quad (4.26)$$

For an isotropic sandwich (all $\alpha$, $E$ and $\nu$ are equal) these expressions reduce to

$$\varepsilon_{x_0}^{(T)} = \varepsilon_{y_0}^{(T)} = \frac{E \alpha}{t_1 + t_2} \left[ t_1 \Delta T_1 + t_2 \Delta T_2 \right] \quad \text{and} \quad \kappa_x^{(T)} = \kappa_y^{(T)} = \frac{\alpha}{d} \left( \Delta T_2 - \Delta T_1 \right) \quad (4.27)$$

and now it is clearly seen that if the both faces exhibit the same change in temperature, the isotropic plate only extends in its plane and no curvature will develop.

4.5 Equilibrium Equations
(i) Vertical equilibrium:
There are three vertical force equilibrium equations, one in each of the $x$-, $y$-, and $z$-directions,

$$\frac{\partial N_x}{\partial x} + \frac{\partial N_y}{\partial y} = 0, \quad \frac{\partial N_y}{\partial y} + \frac{\partial N_{xy}}{\partial x} = 0, \quad \text{and} \quad \frac{\partial T_x}{\partial x} + \frac{\partial T_y}{\partial y} + q^* - \rho \cdot \frac{\partial^2 w}{\partial x^2} = 0 \quad (4.28)$$

Note that in a general case, however seldom in practice, $\rho^*$ may be a function $\rho(x,y)$ if the panel thickness for example varies. $q^*$ is just short for the applied loads, and is
\( q^* = q + N_x \frac{\partial^2 w}{\partial x^2} + 2N_{xy} \frac{\partial^2 w}{\partial x \partial y} + N_y \frac{\partial^2 w}{\partial y^2} \) \hspace{3cm} (4.29)

(ii) Equilibrium of bending moments

In the same manner as above, there are three bending moment equilibrium equations, for bending about the \( y \)-, \( x \)-, and \( z \)-directions as

\[
T_x - \frac{\partial M_x}{\partial x} - \frac{\partial M_{xy}}{\partial y} - R \frac{\partial^2 w_b}{\partial x \partial^2} = 0, \quad T_y - \frac{\partial M_y}{\partial y} - \frac{\partial M_{yx}}{\partial x} - R \frac{\partial^2 w_b}{\partial y \partial^2} = 0, \quad \text{and} \quad N_{xy} = N_{yx} \hspace{2cm} (4.30)
\]

4.6 Partial Deflections

Partial deflection similar to those used for the beam can also be introduced for the panel. They are the deflections due to bending and shear and are defined by assuming only one mode of deformation at the time, as done above. For a sandwich panel with dissimilar faces, this is only approximate but can be justified by the simplifications of the formulae. Assume that we can separate the displacement fields due to bending \( w_b \) and that to transverse shear \( w_s \), and then simply superimpose the two as

\[ w = w_b + w_s \hspace{3cm} (4.31) \]

In doing so, we can see that the rotations defined in 4.1 get a more physical interpretation. Bending causes the cross-section to rotate, whereas shearing is a sliding and thus not add to any rotation. Under these assumptions we can write

\[
M_x = \int [\sigma_x zdz = - \frac{D_x}{1 - V_{xy} V_{yx}} \left\{ \frac{\partial^2 w_b}{\partial x^2} + V_{yx} \frac{\partial^2 w_b}{\partial y^2} \right\}] \hspace{2cm} (4.32)
\]

\[
M_y = \int [\sigma_y zdz = - \frac{D_y}{1 - V_{xy} V_{yx}} \left\{ \frac{\partial^2 w_b}{\partial y^2} + V_{yx} \frac{\partial^2 w_b}{\partial x^2} \right\}] \hspace{2cm} (4.32)
\]

\[
M_{xy} = \int \tau_{xy} zdz = -D_{xy} \frac{\partial^2 w_b}{\partial x \partial y}, \quad \frac{\partial T_x}{\partial x} = S_x \frac{\partial^2 w_s}{\partial x^2}, \quad \text{and} \quad \frac{\partial T_y}{\partial y} = S_y \frac{\partial^2 w_s}{\partial y^2}\hspace{2cm} (4.32)
\]

It is now seen that the partial deflection \( w_b \) represents the classical plate bending deformation and since the shear deflection does not rotate the cross-section, all bending moments will depend on solely on \( w_b \), and all transverse forces on \( w_s \).

4.7 Governing Plate Equations

(i) Governing plate equations - thin faces

One can write the governing equation in terms of transverse force and bending moment fields by substituting the equations of equilibrium, and one then arrives at

\[
\frac{\partial^2 M_x}{\partial x^2} + 2 \frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} = -q^* + \rho \frac{\partial^2 w}{\partial t^2} - \left( R_x \frac{\partial^2 w_b}{\partial x^2} + R_y \frac{\partial^2 w_b}{\partial y^2} \right) \hspace{3cm} (4.33)
\]
and the bending moments can be written in terms of the displacement field and the transverse forces as

\[
M_x = -\frac{D_x}{1 - v_{xy} v_{yx}} \left[ \frac{\partial}{\partial x} \left( \frac{\partial w - T_x}{S_x} \right) + v_{yx} \frac{\partial}{\partial y} \left( \frac{\partial w - T_y}{S_y} \right) \right] = 0
\]  
(4.34)

\[
M_y = -\frac{D_y}{1 - v_{xy} v_{yx}} \left[ \frac{\partial}{\partial y} \left( \frac{\partial w - T_y}{S_y} \right) + v_{xy} \frac{\partial}{\partial x} \left( \frac{\partial w - T_x}{S_x} \right) \right] = 0
\]

\[
M_{xy} = -\frac{D_{xy}}{2} \left[ \frac{\partial}{\partial x} \left( \frac{\partial w - T_y}{S_y} \right) + \frac{\partial}{\partial y} \left( \frac{\partial w - T_x}{S_x} \right) \right] = 0
\]

It will also prove useful for the solution of these equations to eliminate the bending moments from the above equations, since many solutions contain assumed shapes not only of the deflection \( w \) but also of the transverse forces \( T \). In doing so, the assumed fields may be solved as function of the applied loading. The result of this substitution is

\[
\frac{D_x}{1 - v_{xy} v_{yx}} \frac{\partial^3 w}{\partial x^3} + \left( \frac{v_{yx} D_x}{1 - v_{xy} v_{yx}} + D_{sy} \right) \frac{\partial^3 w}{\partial x \partial y^2} - \frac{D_x}{S_x (1 - v_{xy} v_{yx})} \frac{\partial^2 T_x}{\partial x^2} - \frac{v_{yx} D_x}{S_y (1 - v_{xy} v_{yx})} = \frac{D_x}{2 S_y} \frac{\partial^2 T_x}{\partial y^2} + T_x = R_x \left( \frac{\partial^3 w}{\partial x^3} - \frac{1}{S_x} \frac{\partial^2 T_x}{\partial x^2} \right)
\]  
(4.35)

\[
\frac{D_y}{1 - v_{xy} v_{yx}} \frac{\partial^3 w}{\partial y^3} + \left( \frac{v_{xy} D_y}{1 - v_{xy} v_{yx}} + D_{sx} \right) \frac{\partial^3 w}{\partial x^2 \partial y} - \frac{D_y}{S_y (1 - v_{xy} v_{yx})} \frac{\partial^2 T_y}{\partial y^2} - \frac{v_{xy} D_y}{S_x (1 - v_{xy} v_{yx})} = \frac{D_y}{2 S_y} \frac{\partial^2 T_y}{\partial x^2} + T_y = R_y \left( \frac{\partial^3 w}{\partial y^3} - \frac{1}{S_y} \frac{\partial^2 T_y}{\partial y^2} \right)
\]

Along with the vertical equilibrium equation in \( z \)-direction, these three equations can be solved simultaneously for the three unknowns \( w \), \( T_x \) and \( T_y \) and are valid for a general orthotropic sandwich panel.

(ii) Static plate equations in partial deflections - thin faces
The above equations are very complex and for static loading it is often easier to rewrite them in terms of partial deflections.

\[
\frac{D_x}{1 - v_{xy} v_{yx}} \frac{\partial^4 w_h}{\partial x^4} + \left[ \frac{v_{yx} D_x}{1 - v_{xy} v_{yx}} + 2 D_{sy} \right] \frac{\partial^4 w_h}{\partial x^2 \partial y^2} + \frac{D_y}{1 - v_{xy} v_{yx}} \frac{\partial^4 w_h}{\partial y^4} = q
\]  
(4.36)

which, when omitting dynamic terms, equals the differential equation in pure bending of an ordinary orthotropic panel [4]. When rewritten, the above can be made to look like
\[ S_x \frac{\partial^2 w_x}{\partial x^2} + S_y \frac{\partial^2 w_y}{\partial y^2} = -q^* \]  

(4.37)

by using the relation between \( w_b \) and \( w_s \), which for an orthotropic sandwich panel looks like

\[ S_x \frac{\partial^2 w_x}{\partial x^2} = \frac{D_x}{1 - \nu_{yx} \nu_{xy}} \left[ \frac{\partial^4 w_b}{\partial x^4} + \nu_{yx} \frac{\partial^4 w_b}{\partial x^2 \partial y^2} \right] - D_{yx} \frac{\partial^2 w_b}{\partial x^2 \partial y^2} \]  

(4.38)

\[ S_y \frac{\partial^2 w_y}{\partial y^2} = \frac{D_y}{1 - \nu_{yx} \nu_{xy}} \left[ \frac{\partial^4 w_b}{\partial y^4} + \nu_{xy} \frac{\partial^4 w_b}{\partial x^2 \partial y^2} \right] - D_{yx} \frac{\partial^2 w_b}{\partial x^2 \partial y^2} \]  

(iii) Isotropic panel - thin faces

For isotropic sandwich panels we have

\[ D_x = D_y = D, \quad D_{yx} = D/(1 + \nu), \quad \nu_{yx} = \nu_{xy} = \nu, \quad S_x = S_y = S, \quad \text{and} \quad R = R = R \]  

(4.39)

By using this in the above the governing differential equations reduces to

\[ \frac{D}{1 - \nu^2} \Delta^2 w - \left( 1 - \frac{DA}{S(1 - \nu^2)} + \frac{R \partial^2}{S \partial^2} \right) \left[ q^* - \rho^* \frac{\partial^2 w}{\partial t^2} \right] - R \frac{\partial^2 w}{\partial t^2} \Delta w = 0 \]  

(4.40)

which is the well-known Mindlin plate equation [5]. Is also the two-dimensional form of the so-called Timoshenko beam equation [6] given in section 3.8. By accounting for transverse shear but omitting the rotary inertia by letting \( R = 0 \), which can be done for most material combinations and sizes used in practical applications, it reduces to

\[ \frac{D}{1 - \nu^2} \Delta^2 w = \left( 1 - \frac{DA}{S(1 - \nu^2)} \right) \left[ q^* - \rho^* \frac{\partial^2 w}{\partial t^2} \right] \]  

(4.41)

If the transverse shear also is omitted by letting \( S \) approach infinity it reduces to the ordinary plate equation

\[ \frac{D}{1 - \nu^2} \Delta^2 w + \rho^* \frac{\partial^2 w}{\partial t^2} = q^* \]  

(4.42)

By instead omitting the vertical inertia we get

\[ \frac{D}{1 - \nu^2} \Delta^2 w = \left( 1 - \frac{DA}{S(1 - \nu^2)} \right) q^* \]  

(4.43)

using the relation between the partial deflection

\[ \Delta w_s = - \frac{D}{S(1 - \nu^2)} \Delta^2 w_b \]  

(4.44)

This equation can be rewritten to two separate equation in \( w_b \) and \( w_s \) as

\[ \frac{D}{1 - \nu^2} \Delta^2 w_b = q^* \quad \text{and} \quad -S\Delta w_s = q^* \]  

(4.45)
Panel Analysis

$w_s$ can be interpreted as the deflection of a panel having finite shear stiffness $S$ and infinite bending stiffness $D$. As pointed out by Plantema [4], $w_s$ and $w_b$ will not generally vanish separately on the boundary but only their sum will equal zero. Only when $\Delta w_b$ is constant (or zero) along the entire circumference of the panel, $w_s$ and $w_b$ can be chosen to vanish separately.

(iv) Isotropic panel - thick faces - static governing equation

Hoff [8] used variational principles to derive the governing differential equations for sandwich panels with thick faces, that is, the strain energy stored in the faces due to bending about their individual neutral axes were included. However, by adopting the same approach as when deriving the differential equation for sandwich beams with thick faces the derivation becomes simple and straightforward. The result will be the same in either case and is

$$\frac{2D_f}{1-\nu^2} \Delta^3 w - \frac{D}{D_0 + D_c} S \Delta^2 w = \left[ \Delta - \frac{S(1-\nu^2)}{D_0 + D_c} \right] q^*$$

(4.46)

By assuming thin faces, $D_f = 0$, this equation reduces to the above. If the faces are dissimilar then $2D_f$ should be taken as the sum of the flexural rigidities of the faces, i.e., $2D_f = D_{f1} + D_{f2}$. The thickness of the faces only makes a significant difference to the behaviour of the beam locally at for example clamped edges or in the vicinity of point loads. Thus, consideration of the face thickness must be taken in order to properly account for local effects in the vicinity of supports, point loads or clamped edges. Otherwise, if the global deformation is sought after only, it is more convenient to use the thin face governing equation but with $D = D_0 + 2D_f$.

(v) Energy expressions

The strain energy of a sandwich panel can be expressed in several ways; in terms of loads as

$$U_1 = \frac{1}{2} \int \int \left[ \frac{M_x^2}{D_x} \left( \frac{v_{xy}}{D_x} + \frac{v_{yx}}{D_y} \right) M_x M_y + \frac{M_x^2}{D_x} + \frac{2}{S_y} \frac{T_x^2}{S_x} \right] dx dy$$

(4.47)

or the strain energy can be expressed in terms of the total deflections and transverse shear angles as

$$U_1 = \frac{1}{2} \int \int \left[ \frac{D_x}{1-\nu_{xy}v_{xy}} \left( \frac{\partial}{\partial x} \left( \frac{\partial w}{\partial x} - \frac{T_x}{S_x} \right) \right)^2 + \frac{D_y}{1-\nu_{xy}v_{xy}} \left( \frac{\partial}{\partial y} \left( \frac{\partial w}{\partial y} - \frac{T_y}{S_y} \right) \right)^2 + \frac{v_{xy}D_x}{1-\nu_{xy}v_{xy}} \left( \frac{\partial}{\partial x} \left( \frac{\partial w}{\partial y} - \frac{T_x}{S_x} \right) \right) \left( \frac{\partial}{\partial y} \left( \frac{\partial w}{\partial x} - \frac{T_y}{S_y} \right) \right) + \frac{D_{xy}}{2} \left( \frac{\partial}{\partial x} \left( \frac{\partial w}{\partial y} - \frac{T_x}{S_x} \right) + \frac{\partial}{\partial y} \left( \frac{\partial w}{\partial x} - \frac{T_y}{S_y} \right) \right)^2 \right] dx dy$$

(4.48)

Finally, one can express the energy in terms of partial deflections.
\[ U_1 = \frac{1}{2} \iint \left[ \frac{D_x}{1 - \nu_y \nu_x} \left( \frac{\partial^2 w_b}{\partial \xi^2} \right)^2 + \nu_y D_x + \frac{D_y}{1 - \nu_y \nu_x} \left( \frac{\partial^2 w_b}{\partial \eta^2} \right)^2 \right] d\xi d\eta + \frac{D_y}{1 - \nu_y \nu_x} \left( \frac{\partial^2 w_b}{\partial \eta^2} \right)^2 + 2D_{xy} \left( \frac{\partial^2 w_b}{\partial \xi \partial \eta} \right)^2 + S_x \left( \frac{\partial w_x}{\partial \xi} \right)^2 + S_y \left( \frac{\partial w_y}{\partial \eta} \right)^2 \right] d\xi d\eta \] (4.49)

The potential energy of the external forces of the panel is independent of the panel’s internal structure and depends only on the displacement of the middle surface. Therefore, the potential energy expression for the general panel is the same as that for an homogeneous isotropic panel, that is, the potential energy of the forces \( N_x, N_y \) and \( N_{xy} \). If the potential of the transverse load \( q \) is added to that the resulting expression will be

\[ U_2 = \frac{1}{2} \iint \left[ -2qw + N_x \left( \frac{\partial w}{\partial \xi} \right)^2 + 2N_{xy} \left( \frac{\partial w}{\partial \xi} \right) \left( \frac{\partial w}{\partial \eta} \right) + N_y \left( \frac{\partial w}{\partial \eta} \right)^2 \right] d\xi d\eta \] (4.50)

The kinetic energy of the panel can be written as

\[ U_3 = \frac{1}{2} \iint \left[ \rho^* \left( \frac{\partial \ddot{w}}{\partial \xi} \right)^2 + R_x \left( \frac{\partial^2 w_b}{\partial \xi \partial \tau} \right)^2 + R_y \left( \frac{\partial^2 w_b}{\partial \eta \partial \tau} \right)^2 \right] d\xi d\eta \] (4.51)

The potential energy of the entire system is then, since the strain energy must equal the kinetic,

\[ U = U_1 + U_2 - U_3 \] (4.52)

### 4.8 Boundary Conditions

The boundary conditions for the in practice most common types of edge supports are; complete freedom, simple support and clamped.

(i) **Free edge**

The boundary condition for a free unloaded edge can be expressed as

\[ x = 0, a \quad M_x = M_{xy} = T_x = 0 \]  \hspace{1cm} (4.53)
\[ y = 0, b \quad M_y = M_{xy} = T_y = 0 \]

These are, however, three boundary conditions and only two may be specified at each edge to solve the governing equation. According to classical Kirchhoff theory the two latter conditions in eq.(4.53) may be rewritten so that the resultant vertical force is set to zero at the edge. One can specify that for an edge parallel to the \( y \)-axis \((x = 0 \) or \( a \)) we have that \( M_x = 0 \) so that \( T_x - \partial M_{xy} / \partial y = 0 \). Thus, eq.(4.53) is equivalent with

\[ x = 0, a \quad M_x = 0 \text{ and } T_x - \frac{\partial M_{xy}}{\partial y} = 0 \]  \hspace{1cm} (4.54)
\[ y = 0, b \quad M_y = 0 \text{ and } T_y - \frac{\partial M_{xy}}{\partial \xi} = 0 \]
(ii) Simply supported edge
The principal boundary conditions for a simply supported edge are that deflections and bending moments $M_x$ or $M_y$ are zero. Now, there are two different conditions that may be imposed as the third boundary condition. One may set the twisting moment $M_{xy}$ to zero thus permitting the edge to shear, e.g. on an edge parallel to the $x$-axis $\gamma_{xz} \neq 0$. This is a rather unrealistic condition since in most practical cases, there will be an edge stiffener, or some symmetry constraint preventing such shearing deformation. If so, the boundary condition should rather be allowing the existence of a twisting moment but restricting the shear of the edge. The former condition, that the twisting moment along the edges are zero (shearing permitted) is called soft boundary condition, and the latter, preventing shear deformations, is called hard boundary condition. These are then written as

**Soft boundary:**

\[
\begin{align*}
x = 0, a \quad w &= M_x = M_{xy} = 0 \\
y = 0, b \quad w &= M_y = M_{xy} = 0
\end{align*}
\] (4.55)

In this case, since both $\partial M_x / \partial x$ and $\partial M_{xy} / \partial y$ are zero along a free edge parallel to the $y$-axis the resultant force (the vertical force acting on the support) equals the transverse force $T_x$. For the same reason no resulting force in at the corners appear in this case.

**Hard boundary:**

\[
\begin{align*}
x = 0, a \quad w &= M_x = \gamma_{xz} = 0 \\
y = 0, b \quad w &= M_y = \gamma_{xz} = 0
\end{align*}
\] (4.56)

where $\gamma_{xz}$ also can be written as $\partial w / \partial z$ and $\gamma_{yz}$ as $\partial w / \partial y$, since $\partial w / \partial x = \partial w / \partial y = 0$ on the boundaries.

(iii) Clamped edge
The principal boundary conditions characterising a clamped edge are zero displacements of the middle surface and zero rotation of the cross-section at the boundary. The latter requirement that the cross-section remains parallel to the $z$-axis is satisfied by letting $\partial w / \partial x = T_x / S_x$ (note this is no rotation of the cross-section but a sliding). This assumes thin faces since the local rotation $T_x / S_x$ of the faces can not take place unless the faces are membranes. If now $S_x$ approaches infinity then this condition reduces to the ordinary boundary condition $\partial w / \partial x = 0$. Just as in the simply supported case there are two ways to describe the third boundary condition. Thus,

**Soft boundary:**

\[
\begin{align*}
x = 0, a \quad w &= 0, \quad \frac{\partial w}{\partial x} - \frac{T_x}{S_x} = 0 \text{ or } \frac{\partial w_y}{\partial x} = 0 \text{ and } M_{xy} = 0 \\
y = 0, b \quad w &= 0, \quad \frac{\partial w}{\partial y} - \frac{T_y}{S_y} = 0 \text{ or } \frac{\partial w_y}{\partial y} = 0 \text{ and } M_{xy} = 0
\end{align*}
\] (4.57)
Hard boundary:

\[ x = 0, a \quad w = 0, \quad \frac{\partial w}{\partial x} = 0 \quad \text{or} \quad \frac{\partial w_b}{\partial x} = 0 \quad \text{and} \quad \gamma_{yz} = 0 \]  

(4.58)

\[ y = 0, b \quad w = 0, \quad \frac{\partial w}{\partial y} = 0 \quad \text{or} \quad \frac{\partial w_b}{\partial y} = 0 \quad \text{and} \quad \gamma_{xz} = 0 \]

However, if the faces are considered thick, then indeed the rotation of the entire cross-section is prevented leading to the first of the above condition is changed to

\[ w = 0 \quad \text{and} \quad \frac{\partial w}{\partial x} = 0 \]  

(4.59)

which once again is the same as for the ordinary homogeneous plate. The equivalent resultant vertical forces at the edges and in the corners appear in the same way as in the simply supported case for both soft and hard boundary conditions.

4.9 Rotary Symmetric Plates

In this section solutions to rotational symmetric panel problems will be given. The solutions assume isotropic sandwich panels with thin faces, \( t_f << t_c \) and weak cores, \( E_c << E_f \) so that the concept of partial deflections can be used. The circular sandwich panel subjected to a rotational symmetric transverse load will be considered. If the deflection is expressed in terms of \( w_b \) and \( w_s \), then they will both be constant along the circumference of the panel and the problem will become one-dimensional as a beam. It will also imply that shear forces, moments and deflections are identical to those of an ordinary circular plate, except for the additional term \( w_s \). Only the shear deflection, which is invariable with the boundary condition under these assumptions, and transverse force is given. The corresponding bending deformation can be found in e.g. [1] or [9].

Consider a circular sandwich plate as illustrated below. It has radius \( R \) and is subjected to a uniform transverse load \( q \) on an eccentric circular area between radii \( \rho_1 \) and \( \rho_2 \).

![Circular sandwich plate subjected to a uniform load](image)

Figure 4.3  Circular sandwich plate subjected to a uniform load \( q \) between \( \rho_1 \) and \( \rho_2 \).

Assume the plate to be simply supported along the outer radius \( R \). The supporting transverse load equals

\[ T_r(r) = \frac{q \pi (\rho_1 - \rho_2)^2}{2 \pi R} \]  

(4.60)
Note also that the transverse shear deformation $w_s$ is independent of the boundary condition at $r = R$. Now, the transverse shear load $T_r$ and the corresponding shear deformation can directly be written as function of $r$ (note that due to the choice of coordinate system the deflection will measured from $r = 0$ with positive displacement downward). There are three distinct regions to be analysed

(a) \( r \leq \rho_1 \): \( T_r = 0 \), and thus \( w_s^{(a)}(r) = w_s(\rho_1) \) \hspace{1cm} (4.61)

(b) \( \rho_1 \leq r \leq \rho_2 \): \( T_r = -\frac{q\pi(r-\rho_1)^2}{2\pi r} = -\frac{q(r^2-2r\rho_1+\rho_1^2)}{2r} \)

and

\[
w_s^{(b)}(r) = \int_{\rho_1}^{\rho_2} \frac{T_r}{S} dr = -\frac{q(r^2-\rho_1^2)}{4S} + \frac{q\rho_1(r-\rho_1)}{S} - \frac{q\rho_1^2}{2S} \ln \frac{r}{\rho_1} + w_s(\rho_2)
\]

(c) \( r \geq \rho_2 \): \( T_r = -\frac{q(\rho_2-\rho_1)^2}{2r} \)

\[
w_s^{(c)}(r) = \int_{\rho_2}^{\infty} \frac{T_r}{S} dr = -\frac{q(\rho_2-\rho_1)^2}{2S} \ln \frac{r}{\rho_2} + w_s(\rho_2)
\]

Now some examples can be solved using these expressions. Only the transverse shear deformation field is given herein, since the corresponding bending deformation can be found in almost any handbook, e.g. [9].

(i) Uniform load over the entire plate, \( \rho_1 = 0, \rho_2 = R \)

\[
w_s(r) = \frac{q(R^2-r^2)}{4S}
\]

(ii) Uniform load over the central part of the plate, \( \rho_1 = 0 \)

\[
r \geq \rho_2: \quad w_s = \frac{q\rho_2^2}{2S} \ln \frac{R}{r}
\]

\[
r \leq \rho_2: \quad w_s = \frac{q(\rho_2^2-r^2)}{4S} + \frac{q\rho_2^2}{2S} \ln \frac{R}{\rho_2}
\]
(iii) Uniform load but over central part of the plate, $\rho_2 = R$

\[
\begin{align*}
  r \leq \rho_1: \quad w_x &= \frac{q(R^2 - \rho_1^2)}{4S} + \frac{q\rho_1(R - \rho_1)}{S} - \frac{q\rho_1^2}{2S} \ln \frac{R}{\rho_1} \\
  r \geq \rho_1: \quad w_x &= \frac{q(R^2 - r^2)}{4S} + \frac{q\rho_1(R - r)}{S} - \frac{q\rho_1^2}{2S} \ln \frac{R}{r}
\end{align*}
\]

(iv) Annular line load, $\rho_1 = \rho_2 = \rho$

\[
\begin{align*}
  r \leq \rho: \quad w_x &= \frac{P}{2\pi S} \ln \frac{R}{\rho} \\
  r \geq \rho: \quad w_x &= \frac{P}{2\pi S} \ln \frac{R}{r}
\end{align*}
\]
4.10 Bending of Rectangular Panels

This section gives solutions to some problems of sandwich panels subjected to transverse loading. In each case it is stipulated under which assumptions the solution is valid, e.g., thin or thick faces, isotropic or orthotropic, and whether the solution is exact or approximate. In no case is the derivation of the solution given, only the final result. The results are presented as closed form solutions where possible, in tables, graphs or combinations thereof. The deflection field or the maximum deflection is given, and partial deflections when possible. The maximum appearing transverse forces and bending moments are also presented for the exact cases, from which all stress components can be calculated using the formulae in section 4.3. In the approximate solutions no good results can be achieved in terms of forces and moments, and thus, none are given, but it is discussed how they could be assessed.

4.10.1 Isotropic, simply supported panel with thin faces - uniformly distributed load

Consider a rectangular simply supported sandwich plate with sides \( a \) and \( b \) as shown in Fig.4.4.

![Figure 4.4 Rectangular simply supported plate.](image)

The boundary conditions for a simply supported panel are

\[
\begin{align*}
    w &= 0, M_x = 0 \text{ at } x = 0 \text{ and } x = a \\
    w &= 0, M_y = 0 \text{ at } y = 0 \text{ and } y = b
\end{align*}
\]

The exact solution to the deflection of the panel is

\[
w = \frac{16qb^4(1-v^2)}{\pi^6D} \sum_{n=1,3,5,\ldots}^{\infty} \sum_{m=1,3,5,\ldots}^{\infty} \frac{1 + \pi^2 \theta \left( \frac{mb}{a} \right)^2 + n^2}{mn \left( \frac{mb}{a} \right)^2 + n^2} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \quad (4.62)
\]

where the shear factor \( \theta = D/b^2S(1-v^2) \). The partial deflections can be extracted as

\[
w_b = \frac{16qb^4(1-v^2)}{\pi^6D} \sum_{n=1,3,5,\ldots}^{\infty} \sum_{m=1,3,5,\ldots}^{\infty} \frac{\sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b}}{mn \left( \frac{mb}{a} \right)^2 + n^2} \quad (4.63)
\]
The bending moments and transverse forces are calculated by using the equations in section 4.4 by using the partial deflections. The series converge rather quickly for the deflections and bending moments but require more terms to provide accurate numbers for the transverse forces. The maximum deflection and bending moments appear in the middle of the panel at \((x,y) = (a/2, b/2)\) and the maximum transverse forces \(T_x\) at \((0, b/2)\) and \((a, b/2)\) and \(T_y\) at \((a/2, 0)\) and \((a/2, b)\), i.e., in the middle of the sides. By summation using terms of \(m\) and \(n\) up to 27, following data is obtained.

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<th>(a/b)</th>
<th>(Dw_b/\pi^4b^2(1-\nu^2))</th>
<th>(Sw_b/\pi^4b^2)</th>
<th>(M_x/\pi^4b^2)</th>
<th>(M_y/\pi^4b^2)</th>
<th>(T_x/\pi^4b)</th>
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<td>0.0493</td>
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<td>0.365</td>
<td>0.435</td>
</tr>
<tr>
<td>1.8</td>
<td>0.00931</td>
<td>0.1098</td>
<td>0.0479</td>
<td>0.0948</td>
<td>0.368</td>
<td>0.452</td>
</tr>
<tr>
<td>2.0</td>
<td>0.01013</td>
<td>0.1139</td>
<td>0.0464</td>
<td>0.1017</td>
<td>0.370</td>
<td>0.465</td>
</tr>
<tr>
<td>3.0</td>
<td>0.01223</td>
<td>0.1227</td>
<td>0.0404</td>
<td>0.1189</td>
<td>0.372</td>
<td>0.493</td>
</tr>
<tr>
<td>4.0</td>
<td>0.01282</td>
<td>0.1245</td>
<td>0.0384</td>
<td>0.1235</td>
<td>0.372</td>
<td>0.498</td>
</tr>
<tr>
<td>5.0</td>
<td>0.01297</td>
<td>0.1249</td>
<td>0.0375</td>
<td>0.1246</td>
<td>0.372</td>
<td>0.500</td>
</tr>
<tr>
<td>(\infty)</td>
<td>0.01302</td>
<td>0.1250</td>
<td>0.0375</td>
<td>0.1250</td>
<td>0.372</td>
<td>0.500</td>
</tr>
</tbody>
</table>

Table 4.1 Maximum bending and shear deformations, bending moments and transverse forces for a simply supported rectangular sandwich panel.

For \(a/b\) ratios less than unity all properties are easily found by interchanging \(a\) and \(b\) so that

\[
\begin{align*}
    w_h(b/a) &= w_h(a/b)
    \left(\frac{b}{a}\right)^4, \\
    w_s(b/a) &= w_s(a/b)
    \left(\frac{b}{a}\right)^2, \\
    M_x(b/a) &= M_x(a/b)
    \left(\frac{b}{a}\right)^2, \\
    M_y(b/a) &= M_y(a/b)
    \left(\frac{b}{a}\right)^2, \\
    T_x(b/a) &= T_x(a/b)
    \left(\frac{b}{a}\right), \\
    T_y(b/a) &= T_y(a/b)
    \left(\frac{b}{a}\right)
\end{align*}
\]

4.10.2 Isotropic, simply supported panel with thick faces - uniformly distributed load

The solution to this problem is very much the same as in the thin face case, but using the governing equation accounting for thick faces given in section 4.4. The exact deflection field can be written as

\[
w = \sum_{m=1,3,5,} \sum_{n=1,3,5,} \frac{16q}{\pi^2K_{mn}} \left[ 1 + \frac{D}{S(1-\nu^2)} \left( \frac{m\pi}{a} \right)^2 + \left( \frac{n\pi}{b} \right)^2 \right] \sin \frac{m\pi}{a} \sin \frac{n\pi}{b} (4.65)
\]

where the denominator equals
One can no longer express the deflection and the forces and bending moments in non-dimensional terms in such a simple way as for the thin face case. This is due to that the denominator \( K_{mn} \) this time includes terms of panel lengths in different powers, i.e., \( a^4 \) and \( a^6 \). It is worth noticing though, that when the shear stiffness \( S \) is large, the panel deflection equals those computed using the thin face theory, but using \( D = 2D_f + D_0 \). One can also see that when \( S \) approaches zero, i.e., no shear rigidity, the deformation equals that of the two faces bending independently of each other (that is, using the thin face theory, but now with \( D = 2D_f \)). This behaviour can also be explained by the following reasoning: if the shear stiffness is very low then the total deformation would equal \( w_s \) (pure shear) providing the faces are considered thin. However, if the faces have a bending stiffness of their own they cannot undergo this shear deformation but will stiffen the panel. At the limit, \( S \) approaches zero, the shear deformation cannot equal infinity but is restricted by the bending of the two faces, which now bend independently of each other. At the other limit, \( D_0 \) is very large, and \( D_f = 0 \), then \( w_s \) equals that given in Table 4.1.

\[
K_{mn} = \frac{2D_fD_0}{S(1-v^2)^2} \left[ \left( \frac{m \pi}{a} \right)^2 + \left( \frac{n \pi}{b} \right)^2 \right]^3 + \frac{2D_f + D_0}{1-v^2} \left[ \left( \frac{m \pi}{a} \right)^2 + \left( \frac{n \pi}{b} \right)^2 \right] \]  

Figure 4.5 Effect of thick faces on the deformation of a square simply supported sandwich plate subjected to a uniform load. The lines are for \( \pi^2 \theta = 0, 0.01, 0.1, 1, 10 \) and 100.

In order to clearly show these effects Fig.4.5 has been prepared so that the ratio \( w/w^* \) is plotted versus \( D_f/D_0 \) for different values of the shear factor \( \theta = D_0/\alpha^2 S \). Now, \( w \) is the deflection computed using the thick face theory and \( w^* \) is the deflection computed using the thin face theory given in Table 4.1, but with \( D = 2D_f + D_0 \).

As seen in Fig.4.5, the effect of the bending stiffness of the faces is very small, since \( \theta \) usually is in the order of 0.1 or less and \( D_f/D_0 \) usually is in the order of 0.01 or less for sandwiches used in practical applications. The approximation that \( D_f \) can be neglected in the calculation of the total flexural rigidity derived in section 2, stated that \( 2D_f/D_0 < 0.01 \) if \( d/t_f > 5.77 \). Hence, this approximation could once again be used since, as seen in Fig.4.5, the influence of thick faces is negligible for \( D_f/D_0 < 0.01 \) and \( \theta \) smaller than 0.1. One can therefore conclude that thick faces must only be considered when the panel is weak in shear, i.e. \( \theta \) is small, otherwise the deflection can just as well be calculated using the thin face theory but accounting for the thickness of the faces when calculating the flexural rigidity \( D \).
The way to use Fig. 4.5 is to compute the deflection using the thin face theory in Table 4.1, then account for the thickness of the faces by multiplying with the factor \( \frac{w}{w^*} \) from the figure for a given \( \theta \) and \( D/D_0 \) ratio.

The bending moments and transverse forces are more difficult to assess in this problem. However, it is likely that distribution of the moments and transverse forces are more or less the same as in the thin face case, at least for reasonably small shear factors \( \theta \). However, additional stresses may occur in the faces due to the local bending of the faces taking place.

### 4.10.3 Isotropic, simply supported panel with thick faces - point load

This time we have the same panel as above but now subjected to a point load, acting at a point \((x', y')\). To be general the solution is given assuming thick faces but can easily be reduced to a thin face solution by letting \( D_f = 0 \). The exact solution is

\[
w = \sum_{n=1,3,5,\ldots}^{\infty} \sum_{m=1,3,5,\ldots}^{\infty} 4P \sin \frac{m\pi x'}{a} \sin \frac{n\pi y'}{b} \left[ 1 + \frac{D_0}{S(1-\nu^2)} \left( \frac{m\pi}{a} \right)^2 + \left( \frac{n\pi}{b} \right)^2 \right] \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \quad (4.67)
\]

with \( K_{mn} \) the same as in 4.10.2. If the point load is applied in the middle of the panel, we have that \((x', y') = (a/2, b/2)\). Table 4.2 gives the non-dimensional deflections for the case of a point load in the middle of the panel. These are computed for \( D_f/D_0 = 0 \) meaning that partial deflection can be used, and the deflection can be computed independently for the bending and shear modes.

<table>
<thead>
<tr>
<th>( a/b )</th>
<th>1.0</th>
<th>1.2</th>
<th>1.4</th>
<th>1.6</th>
<th>1.8</th>
<th>2.0</th>
<th>3.0</th>
<th>5.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{Dw_0}{Qb^2(1-\nu^2)} )</td>
<td>0.0116</td>
<td>0.0135</td>
<td>0.0148</td>
<td>0.0157</td>
<td>0.0162</td>
<td>0.0165</td>
<td>0.0169</td>
<td>0.0169</td>
</tr>
<tr>
<td>( \frac{Sw_2}{Q} )</td>
<td>0.613</td>
<td>0.609</td>
<td>0.601</td>
<td>0.592</td>
<td>0.581</td>
<td>0.571</td>
<td>0.524</td>
<td>0.456</td>
</tr>
</tbody>
</table>

Table 4.2 Maximum bending and shear deformations for a simply supported rectangular sandwich plate.

The bending moments and shear forces are not given since the computation of those using the present model would yield dubious results. The maximum transverse forces and bending moments appear in the vicinity of the load point and since a point load in reality never exists but is distributed over a small area an exact value cannot be calculated using the present methodology. The maximum bending moment has been computed for the pure bending case in [1] which might give some lead even in this case. In order to assess the forces and moments at the load point a three-dimensional elasticity approach must be used. In fact, a concentrated load will in practice be applied to only one face, whereas the other will be loaded over a much larger area since the load must be transferred through the core. Hence, the analysis must account for the transverse stress \( \sigma_z \) in the core.

As for the uniformly distributed load case, factors for the influence of thick faces have been prepared and are shown in Fig. 4.6. These are to be used in the same manner as outlined above,
that is, compute the deflection $w^*$ assuming thin faces using Table 4.1 and $D = 2D_f + D_0$, and then find the true deflection using Fig. 4.6.

![Figure 4.6](image-url)

Figure 4.6 Effect of thick faces on the deformation of a square simply supported sandwich plate subjected to a point load. The lines are for $\pi^2 \theta = 0, 0.01, 0.1, 1, 10$ and 100.

It is seen in Fig. 4.6 that the influence of the thickness of the faces is much greater for the point load case than for the uniformly distributed load case.

**4.10.4 Orthotropic, simply supported panel with thin faces - uniformly distributed load**

In this case one must use the full solution in order to assess all variables. This is a quite lengthy and complex derivation and even the results given below are fairly complex [10]. Anyway, the solution is exact for an orthotropic rectangular sandwich panel and the deflection field and the transverse force fields can be written as

$$w = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} - \frac{W_{mn} q_{mn}}{Z_{mn}} \sin \frac{m \pi x}{a} \sin \frac{n \pi y}{b}$$

(4.68)

$$T_x = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{X_{mn} q_{mn}}{Z_{mn}} \cos \frac{m \pi x}{a} \sin \frac{n \pi y}{b}$$

$$T_y = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{Y_{mn} q_{mn}}{Z_{mn}} \sin \frac{m \pi x}{a} \cos \frac{n \pi y}{b}$$

where
\[ W_{mn} = -\frac{1}{S_x S_y} \left( \frac{1}{2} D_{xy} \left( \frac{m\pi}{a} \right)^4 - \frac{D_x}{1 - \nu_{xy} \nu_{yx}} \left( \frac{n\pi}{b} \right)^2 \right) \left( \frac{m\pi}{a} \right)^2 \left( \frac{n\pi}{b} \right)^2 \nu_{yx} D_y + \nu_{yx} D_x \]
\[ + \left( \frac{n\pi}{b} \right)^2 \frac{D_y}{S_y (1 - \nu_{xy} \nu_{yx})} \]  
\[ + \frac{n\pi}{b} \left( \frac{D_y}{S_y (1 - \nu_{xy} \nu_{yx})} \right) - \frac{1}{2} D_{xy} \left( \frac{1}{S_y} \left( \frac{m\pi}{a} \right)^2 + \frac{1}{S_x} \left( \frac{n\pi}{b} \right)^2 \right) - 1 \] (4.69)

\[ X_{mn} S_y = \frac{1}{2} \left( \frac{m\pi}{a} \right)^5 D_{xy} \left( \frac{D_x D_y}{1 - \nu_{xy} \nu_{yx}} + \frac{D_x D_y}{1 - \nu_{xy} \nu_{yx}} \right)
\[ + \frac{1}{2} \left( \frac{m\pi}{a} \right) \left( \frac{n\pi}{b} \right)^4 D_{xy} + S_y \left( \frac{m\pi}{a} \right) \left( \frac{n\pi}{b} \right)^4 \nu_{yx} D_x \]  

\[ Y_{mn} S_x = -\frac{1}{2} \left( \frac{n\pi}{b} \right)^5 D_{xy} \left( \frac{D_x D_y}{1 - \nu_{xy} \nu_{yx}} - \frac{D_x D_y}{1 - \nu_{xy} \nu_{yx}} \right)
\[ - \frac{1}{2} \left( \frac{m\pi}{a} \right) \left( \frac{n\pi}{b} \right)^4 D_{xy} - S_x \left( \frac{n\pi}{b} \right)^4 \nu_{yx} D_y \]  

\[ Z_{mn} = \left( \frac{m\pi}{a} \right) X_{mn} - \left( \frac{n\pi}{b} \right) Y_{mn} \]

and the load factor \( q_{mn} \) equals
\[ q_{mn} = \frac{16q}{mn \pi^2} \text{ for odd values of } n \text{ and } m, \text{ otherwise } q_{mn} = 0 \] (4.70)

for a uniformly distributed load, and
\[ q_{mn} = \frac{4P}{ab} \sin \frac{m\pi x'}{a} \sin \frac{n\pi y'}{b} \text{ for odd values of } n \text{ and } m, \text{ otherwise } q_{mn} = 0 \] (4.71)

for a point load acting at \((x,y) = (x',y')\). By using these relations it is possible to write the bending moments as
\[ M_x = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{-D_x}{1 - \nu_{xy} \nu_{yx}} \left[ W_{mn} \left( \frac{m\pi}{a} \right)^2 + \nu_{yx} \left( \frac{n\pi}{b} \right)^2 \right] + \left( \frac{m\pi}{a} \right) \frac{X_{mn}}{S_x} - \nu_{yx} \left( \frac{n\pi}{b} \right) \frac{Y_{mn}}{S_y} \]
\[ \times \frac{q_{mn}}{Z_{mn}} \frac{\sin \frac{m\pi x}{a}}{a} \sin \frac{n\pi y}{b} \] (4.72)
These rather complex equations can now be coded and solved numerically. A full solution in the form of graphs is not possible due to the number of variables. Anyway, below there are some results derived using the above equations, prepared in order to show the trends of the different parameters involved for an orthotropic sandwich panel in comparison with an isotropic one. As in Table 4.1 only the maximum values are given, that is the deformation and bending moments at the centre of the panel and the maximum transverse forces appearing mid-way between the corners.

In the following graphs the maximum deflection, bending moments and transverse forces are plotted in terms of the factor $\beta$ defined as

$$w = \beta \frac{qb^4}{D_y}, \quad M_y = \beta \sigma_q b^2, \quad T_y = \beta \sigma_q b, \quad \text{and} \quad T_x = \beta \sigma_q b$$

The Poisson ratios $\nu_{xy}$ and $\nu_{yx}$ are taken as

$$\nu_{xy} = 0.25 \sqrt{\frac{D_y}{D_x}}, \quad \text{and} \quad \nu_{yx} = 0.25 \sqrt{\frac{D_x}{D_y}}$$

so that the product $\nu_{xy} \nu_{yx}$ is kept constant. This also means that in the isotropic case $D_x = D_y = 1.25 D_{xy}$. The set of graphs are also derived using $m = n = 27$, which gives fairly good accuracy for the deflection but less good for the bending moments and transverse forces and as long as $a/b$ is close to unity.

The first set of graphs show the influence of the torsional stiffness $D_{xy}$, and the ratio of the flexural stiffness $D_x/D_y$ on the response of a square orthotropic plate subjected to a uniformly distributed load $q$. The values of $D_{xy} = 0.1 D_y$ and $1.2 D_y$ could be seen as extreme values and most practical should lie within that regime. The curves are for the shear factor equalling zero (pure bending) and $\theta_y = 1.0$ which implies a fairly large amount of shear.

As seen in Fig.4.7, a high torsional stiffness stiffens the plate even though the flexural rigidities are kept constant. Different values of the torsional stiffness $D_{xy}$ than $D(1+\nu)$ can be obtained using fibrous composite faces. It is also seen that increasing the rigidity in one direction not only decreases the deformation but also changes the bending moments and transverse forces. In fact, increasing the rigidity in one direction influences the moments and forces in the same manner as decreasing the plate span in that direction, e.g., increasing $D_x$ increases $M_x$ and $T_x$, whereas $M_y$ and $T_y$ are decreasing relatively, which is what happens if the span in the $x$-direction, $a$, is decreased. It is also seen that though changing the shear factor $\theta$ has a considerable influence on the deflection, it very little affects the maximum values of the transverse forces and the bending moments.
A second set of graph that follows in Fig.4.8 are for the same assumptions as given above, but now the properties are plotted for different panel aspect ratios, \(b/a\). In order to do so the ratio \(D_x/D_y\) have been set to 0.8 (which yields the isotropic case for \(D_x = D_y\)). Maximum bending moments and transverse forces (\(\beta_2 - \beta_5\)) are only plotted for \(\theta = 0\) (pure bending) and \(\theta = 1.0\). An increased shear factor of course strongly influences the deflection as the transverse shear part increases, but the relative change between the cases \(\theta = 0\) and \(\theta = 1.0\) is very much the same irrespective of the panel aspect ratio \(a/b\). \(M_x\) and \(T_x\) are as seen almost unaffected by both the change in \(a/b\) or the shear factor \(\theta\), whereas for \(M_y\) and \(T_y\) the relative change for different \(\theta\)-values is about the same for different panel aspect ratios.

In the third set of graphs of Fig.4.9, the faces are considered isotropic with flexural rigidity \(D\) but now the core is orthotropic. The graphs show the deformation, bending moments and transverse forces as function of the of the shear parameter \(\theta\) in the \(y\)-direction (\(\theta_y = D/b^2S_y\)) for different values of orthotropy, i.e. \(S_x/S_y\). Some foam cores are slightly orthotropic with a \(S_x/S_y\) ratio close to unity whereas most honeycomb cores have a \(S_x/S_y\) ratio in the order of 0.4 - 0.7. In this case, the deflection increase linearly with \(\theta\) irrespective of shear stiffness ratio \(S_x/S_y\). As also seen the shear stiffness orthotropy influences the internal force fields. This illustrates one weakness with the concept of partial deflections under which assumption the bending moments only depends on the bending of the plate and the transverse forces only on the shearing of the plate. This implies that the shear stiffness orthotropy will not influence the bending moment fields. The same thing can be seen in Fig.4.7 where a bending orthotropy obviously influence the transverse force fields even for the case of pure bending.

It is similarly seen in the graphs of Fig.4.9, that the shear orthotropy influence is similar to those when only the faces where orthotropic; increasing the stiffness in the \(x\)-direction has the same effect as decreasing the length of span in the \(x\)-direction, \(a\). It is also seen that the average shear stiffness used gives a quite good prediction of the deformation even for quite highly orthotropic cores with a divergence from the true value of less than 15 percent. The same results as the above can be found in [9] for \(S_x/S_y = 1\) and 0.4.

To be comprehensive similar set of graphs should be produced for every value of the panel aspect ratio \(b/a\) which in that case must include inverse values of \(S_x/S_y\) to give the full picture. This cannot be done within the scope of this text but two such sets, for \(b/a = 1.4\) and 2.0, are prepared in Fig.4.10 in order to show the trends of each parameter. Once again, the central plate deflection is very much affected by the orthotropy of the core in this case. It is also seen that the maximum bending moments and transverse are affected in a quite similar way as in for the square plate. It is interesting to note that \(T_y\) seem to stay almost unaffected by both the shear factor or the panel aspect ratio and only takes a strong influence from the shear stiffness ratio \(S_x/S_y\).
Figure 4.7 Normalised maximum deflection, bending moments, and transverse forces for a square, simply supported plate with orthotropic faces and isotropic core subjected to uniform pressure. Solid lines are for $\pi^2 \theta_y = 1.0$ and dashed lines for $\pi^2 \theta_y = 0$ (pure bending). The lines are for $D_{xy}/D_y = 0.1, 0.8$ (isotropic case) and 1.2.
Figure 4.8 Normalised maximum deflection, bending moments, and transverse forces for orthotropic, simply supported rectangular plates with $D_{xy}/D_y = 0.8$ and isotropic core subjected to uniform pressure. Solid lines are for $\pi^2 \theta_y = 1.0$ and dashed lines for $\pi^2 \theta_y = 0$ (pure bending). The lines are for $a/b = 1.0, 1.4, 1.8$ and 3.0.
Figure 4.9 Normalised maximum deflection, bending moments, and transverse forces for square simply supported plates with isotropic faces and an orthotropic core subjected to uniform pressure. The lines are for $S_x/S_y = 0.4$, 0.7, 1.0, 1.43, and 2.5.
Figure 4.10  Normalised maximum deflection, bending moments, and transverse forces for rectangular simply supported plates with isotropic faces and an orthotropic core subjected to uniform pressure. Solid lines are for $a/b = 1.4$ and dashed lines for $a/b = 2.0$. The lines are for $S_x/S_y = 0.4$, 1.0, and 2.5.
As mentioned earlier, in order to give all the information similar set of graphs must be drawn for each panel aspect ratio. In reality, however, the case may very well be that both the faces and the core are orthotropic and, in such case, the only way of achieving good numbers for the deflection, bending moments and transverse forces is actually to perform the summation outlined above. This can be done by coding the formulae in a PC-computer or equivalent and since the series converge rather rapidly this requires very little computing time.

To summarise the behaviour of orthotropic sandwich panels the figure below shows what happens if some of the stiffnesses are changed compared with an isotropic panel. The effect of increasing one stiffness has the same effect as changing the panel aspect ratio.

![Diagram showing the effect of increasing one stiffness relative to the others in an orthotropic sandwich panel.](image)

**Figure 4.11** The effect of increasing one stiffness relative to the others in an orthotropic sandwich panel.

### 4.10.5 Approximate solutions to panels with thin faces - uniformly distributed load

The following solutions are derived using Ritz’ method, which in short is based on assuming a deflected shape of the panel, and using that calculating the total energy of the panel, that is, strain energy and potential of the applied loads. This total energy is then minimised giving an approximate deflection field. The following solutions all have symmetrical boundary conditions and the assumed deflected shape and the mid-point deflection of the panel are given. The solutions only gives relatively good results within certain limits of panel aspect ratio and orthotropy.

The errors between the exact solution of section 4.10.5 and the approximate solution for an orthotropic sandwich panel with simply supported are surprisingly small even for quite large values of \( \frac{b}{a} \) and \( \frac{D_x}{D_y} \), and even in extreme cases the difference is less than 30%, with the approximate value always higher than the exact value. It is also worth noticing that the bending deflection \( w_b \) is much more accurate than that due to shear, \( w_s \). By comparing the exact solution to the approximate in the simply supported case it is found that \( w_b \) is less than 10% higher than the exact value when approximately

\[
0.33 < \frac{b}{a} \left( \frac{D_x}{D_y} \right)^{\frac{1}{3}} < 3 \quad \text{and/or} \quad 0.12 < \frac{b}{a} \left( \frac{D_a}{D_y} \right) < 8
\]  

(4.73)

For the shear deflection on the other hand there is a more systematic error in the solution so that \( w_s \) is 11.5% higher than the exact solution already for an isotropic core and a square panel. However, the error is between 11.5 and 20% if

\[
0.33 < \frac{b}{a} \left( \frac{S_x}{S_y} \right)^{\frac{1}{2}} < 3
\]

(4.74)
These are all approximate limits and variations of several parameters can of course yield higher errors. Still, for engineering purposes errors in the order of such magnitudes may be acceptable, especially if known beforehand. In practice, the degree of orthotropy is commonly quite low, as well as the panel aspect ratio, thus increasing the possible use of the approximate formulae.

A way of estimating the deflection for cases which lie outside this range, e.g. for large or small \(b/a\) ratios is the following: For a large \(b/a\) ratio one can argue that the line at \(y = b/2\) is more or less unaffected by the boundary conditions at \(y = 0, b\), and one can thus approximate the deflection in the middle of panel by a beam of length \(a\) and width \(b\). Formulae for cases with large or small ratios \(a/b\), \(D_x/D_y\) and \(S_x/S_y\) are also given in each case.

(i) Simply supported edges

An approximate closed form solution to the deflection field of a simply supported sandwich panel, as illustrated in Fig.4.4 is

\[
\bar{w} = \bar{w} \sin \frac{\pi x}{a} \sin \frac{\pi y}{b} \\
\bar{w} = \frac{16qb^4(1 - \nu_{xy}v_{yx})}{\pi^6 D_x \left( \frac{b}{a} \right)^4 + 2\left[ D_x v_{xx} + (1 - \nu_{xy}v_{yx})D_{xy} \left( \frac{b}{a} \right)^2 + D_y \right]} + \frac{16qb^2}{\pi^4 S_x \left( \frac{b}{a} \right)^2 + S_y}
\]

which for an isotropic plate reduces to

\[
\bar{w} = \frac{16qb^4(1 - \nu^2)}{\pi^6 D_x \left( \frac{b}{a} \right)^4 + 1} + \frac{16qb^2}{\pi^4 S_x \left( \frac{b}{a} \right)^2 + 1}
\]

As seen, this is the same as taking the first term in the exact solution. The exact solution converges very rapidly for the deflection component but more slowly for the bending moments and the transverse forces. Outside the range of validity, one can sometimes use the beam approximations to obtain a better deflection value. These are the so called beam approximations and are given as

\[
w = \bar{w} \sin \frac{\pi x}{a}, \quad \bar{w} = \frac{4qa^4(1 - \nu_{xy}v_{yx})}{\pi^2 D_x} + \frac{4qa^2}{\pi^2 S_x}, \quad \text{when} \quad \frac{b}{a} \left( \frac{D_x}{D_y} \right)^{\frac{1}{4}} >> 1 \quad \text{and} \quad \text{or} \quad \frac{b}{a} \left( \frac{S_x}{S_y} \right)^{\frac{1}{3}} >> 1
\]

\[
w = \bar{w} \sin \frac{\pi y}{b}, \quad \bar{w} = \frac{4qb^4(1 - \nu_{xy}v_{yx})}{\pi^2 D_y} + \frac{4qb^2}{\pi^2 S_y}, \quad \text{when} \quad \frac{b}{a} \left( \frac{D_x}{D_y} \right)^{\frac{1}{4}} << 1 \quad \text{and} \quad \text{or} \quad \frac{b}{a} \left( \frac{S_x}{S_y} \right)^{\frac{1}{3}} << 1
\]
(ii) Two sides clamped, the other two simply supported

First study a plate that has its sides along the x-axis clamped and the two other simply supported, that is, a plate with the boundary conditions

\[ w = 0, \quad M_x = 0 \quad \text{for} \quad x = 0, a \]

\[ w = 0, \quad \frac{\partial w}{\partial y} = 0 \quad \text{for} \quad y = 0, b \]

The deflected shape in this case is (approximate but satisfies the boundary conditions)

\[ w = (W_h + W_i) \sin \frac{\pi x}{a} \sin^2 \frac{\pi y}{b} = \overline{w} \sin \frac{\pi x}{a} \sin^2 \frac{\pi y}{b} \]

\[ \overline{w} = \frac{16qb^4(1 - \nu_{xy} \nu_{yx})}{\pi^5 \left[ 3D_x \left( \frac{b}{a} \right)^4 + 8 \left( \frac{b}{a} \right)^2 + 16 \right]} + \frac{16qb^2}{\pi^3 \left[ 3S_y \left( \frac{b}{a} \right)^2 + 4S_y \right]} \]

which for an isotropic plate reduces to

\[ \overline{w} = \frac{16qb^4(1 - \nu^2)}{\pi^5 D \left[ 3 \left( \frac{b}{a} \right)^4 + 8 \left( \frac{b}{a} \right)^2 + 16 \right]} + \frac{16qb^2}{\pi^3 S \left[ 3 \left( \frac{b}{a} \right)^2 + 4 \right]} \]

Outside this range of validity, one can use the beam approximations, given as

\[ w = \overline{w} \sin \frac{\pi x}{a}, \quad \overline{w} = \frac{4qa^4(1 - \nu_{xy} \nu_{yx})}{\pi^5 D_x} + \frac{4qa^2}{\pi^3 S_x}, \quad \text{when} \quad \frac{b \left( \frac{D_x}{D_y} \right)^{\frac{1}{2}}}{a} >> 1 \quad \text{and / or} \quad \frac{b \left( \frac{S_x}{S_y} \right)^{\frac{1}{2}}}{a} >> 1 \]

\[ w = \overline{w} \sin^2 \frac{\pi y}{b}, \quad \overline{w} = \frac{qb^4(1 - \nu_{xy} \nu_{yx})}{4\pi^2 D_y} + \frac{qb^2}{\pi^2 S_y}, \quad \text{when} \quad \frac{b \left( \frac{D_x}{D_y} \right)^{\frac{1}{2}}}{a} << 1 \quad \text{and / or} \quad \frac{b \left( \frac{S_x}{S_y} \right)^{\frac{1}{2}}}{a} << 1 \]

If we now shift the boundary conditions to as illustrated in Fig.4.13 we get
Figure 4.13 Rectangular sandwich plate with two edges clamped and two simply supported.

\[ w = 0, \quad \frac{\partial w}{\partial x} = 0 \quad \text{for} \quad x = 0, a \]

\[ w = 0, \quad M_y = 0 \quad \text{for} \quad y = 0, b \]

The assumed deflected shape in this case can be chosen as

\[ w = (W_x + W_y) \sin^2 \frac{\pi x}{a} \sin \frac{\pi y}{b} = \bar{w} \sin^2 \frac{\pi x}{a} \sin \frac{\pi y}{b} \]

\[ \bar{w} = \frac{16qb^4(1 - \nu_{xy}\nu_{yx})}{\pi^4 \left[ 16D_x \left( \frac{b}{a} \right)^4 + 8 \left[ D_y \nu_{yx} + (1 - \nu_{xy}\nu_{yx})D_{xy} \right] \left( \frac{b}{a} \right)^2 + 3D_y \right]} + \frac{16qb^2}{\pi^3 \left[ 4S_x \left( \frac{b}{a} \right)^2 + 3S_y \right]} \]

which for an isotropic plate reduces to

\[ \bar{w} = \frac{16qb^3(1 - \nu^2)}{\pi^5 D \left[ 16 \left( \frac{b}{a} \right)^4 + 8 \left( \frac{b}{a} \right)^2 + 3 \right]} + \frac{16qb^2}{\pi^3 S \left[ 4 \left( \frac{b}{a} \right)^2 + 3 \right]} \]

Outside this range we have

\[ w = \bar{w} \sin^2 \frac{\pi x}{a}, \quad \bar{w} = \frac{qa^4(1 - \nu_{xy}\nu_{yx})}{4\pi^4 D_y} + \frac{qa^2}{\pi^2 S_y}, \quad \text{when} \quad \frac{b}{a} \left( \frac{D_x}{D_y} \right)^{\frac{1}{3}} \gg 1 \quad \text{and/or} \quad \frac{b}{a} \left( \frac{S_x}{S_y} \right)^{\frac{1}{3}} \gg 1 \]

\[ w = \bar{w} \sin \frac{\pi y}{b}, \quad \bar{w} = \frac{4qb^4(1 - \nu_{xy}\nu_{yx})}{\pi^5 D_y} + 4qb^2 \frac{\pi^4 S_y}{\pi^3}, \quad \text{when} \quad \frac{b}{a} \left( \frac{D_x}{D_y} \right)^{\frac{4}{3}} \ll 1 \quad \text{and/or} \quad \frac{b}{a} \left( \frac{S_x}{S_y} \right)^{\frac{1}{3}} \ll 1 \]
(ii) All edges clamped

Figure 4.14 Rectangular sandwich plate with all edges clamped.

\[ w = 0, \quad \frac{\partial w}{\partial x} = 0 \quad \text{for} \quad x = 0, a \]

\[ w = 0, \quad \frac{\partial w}{\partial y} = 0 \quad \text{for} \quad y = 0, b \]

The assumed deflection field for the panel can approximately be written

\[ w = (W_b + W_s) \sin^2 \frac{\pi x}{a} \sin^2 \frac{\pi y}{b} = \bar{w} \sin^2 \frac{\pi x}{a} \sin^2 \frac{\pi y}{b} \]

\[ \bar{w} = \frac{q b^4 (1 - \nu_{xy} \nu_{yx})}{\pi^4 D_s \left( \frac{a}{b} \right)^4 + 2 \left[ D_s \nu_{yx} + (1 - \nu_{xy} \nu_{yx}) D_{xy} \right] \left( \frac{b}{a} \right)^2 + 3 D_y} + \frac{4q b^2}{3 \pi^2 \left[ S_x \left( \frac{b}{a} \right)^2 + S_y \right]} \]

which for an isotropic panel reduces to

\[ \bar{w} = \frac{q b^4 (1 - \nu^2)}{\pi^4 D_s \left( \frac{b}{a} \right)^4 + 2 \left( \frac{b}{a} \right)^2 + 3} + \frac{4q b^2}{3 \pi^2 S \left( \frac{b}{a} \right)^2 + 1} \]

Outside this range of validity we have

\[ w = \bar{w} \sin^2 \frac{\pi x}{a}, \quad \bar{w} = \frac{q a^4 (1 - \nu_{xy} \nu_{yx})}{4 \pi^4 D_s} + \frac{q a^2}{\pi^2 S}, \quad \text{when} \quad \frac{b}{a} \left( \frac{D_x}{D_y} \right)^{\frac{1}{2}} \gg 1 \quad \text{and/or} \quad \frac{b}{a} \left( \frac{S_x}{S_y} \right)^{\frac{1}{2}} \gg 1 \]

\[ w = \bar{w} \sin^2 \frac{\pi y}{b}, \quad \bar{w} = \frac{q b^4 (1 - \nu_{xy} \nu_{yx})}{4 \pi^4 D_y} + \frac{q b^2}{\pi^2 S}, \quad \text{when} \quad \frac{b}{a} \left( \frac{D_x}{D_y} \right)^{\frac{1}{2}} \ll 1 \quad \text{and/or} \quad \frac{b}{a} \left( \frac{S_x}{S_y} \right)^{\frac{1}{2}} \ll 1 \]
4.11 Buckling of Rectangular Panels

This section will give formulae for the calculation of the instability load for sandwich panels subjected to uniaxial compressive loads. The cases are the same as in the previous section, isotropic panel with thin and thick faces, orthotropic and approximate solutions for panels with various boundary conditions. In all cases the assumed shape of the buckled panel is sinusoidal. The governing equations used are the same as in the transverse load cases, that is the equations given in section 4.4, but now with \( q = 0 \) and \( N_x = -P_x, N_y = -P_y \) whereas the shear force \( N_s \) in all cases is taken as zero.

4.11.1 Buckling of a simply supported, isotropic panel - thin faces

Study a rectangular sandwich panel with sides \( a \) and \( b \), subjected to in-plane compressive forces \( P_x \) and \( P_y \) according to Fig. 4.15.

![Figure 4.15 Sandwich plate subjected to biaxial compression.](image)

By assuming the sinusoidal deflection field

\[
w = \overline{w} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b}
\]

and inserting into the governing equation we get the general form of the stability criterion as

\[
\frac{D}{1-\nu^2} \left[ \left( \frac{m\pi}{a} \right)^2 + \left( \frac{n\pi}{b} \right)^2 \right] = \left[ P_x \left( \frac{m\pi}{a} \right)^2 + P_y \left( \frac{n\pi}{b} \right)^2 \right] \left[ 1 + \frac{D}{S(1-\nu^2)} \left( \frac{m\pi}{a} \right)^2 + \left( \frac{n\pi}{b} \right)^2 \right]
\]

(4.75)

for biaxial loading for which its minimum with respect to the wave form \( m \) and \( n \) is sought for a given set of \( N_x \) and \( P_y \). To further simplify and take an example assume that the compression is uniaxial with \( P_x = P \) and \( P_y = 0 \). The minimum buckling load is given by \( n = 1 \) [4] and the buckling load can then be written

\[
K = \frac{(1-\nu^2)b^2}{\pi^2 D} P = \left( \frac{mb}{a} + \frac{a}{mb} \right)^2 \left( 1 + \pi^2 \theta \left( \frac{mb}{a} \right)^2 + 1 \right)^{-1}
\]

where \( \theta \) is the shear factor which in the isotropic equals \( D/b^2S(1-\nu^2) \). \( K \) is the so called buckling coefficient, which is a non-dimensional property. The same expressions are found in [4] and [3] but written with a slightly different notation. \( K \) is now plotted versus the panel aspect ratio \( a/b \) in Fig.4.16.
As seen in Fig.4.16, the critical buckling load has a finite value even for $a/b$ equal to zero. Recall the buckling of a sandwich column which approaches a finite value, the shear buckling load equal to the shear stiffness, when the column length approached zero. Even here, as seen in the graph above, the buckling coefficient approaches $1/\pi \theta$, or $P_x$ approaches $S$ when the panel aspect ratio $a/b$ is small. Hence, in analogy with the bending case, the panel buckling problem can be approximated by a beam with length $a$ when the panel aspect ratio is small. The number of waves, $m$, in the buckling mode depends on the panel aspect ratio as well as the shear factor $\theta$. For the case of pure bending ($S$ equals infinity) we have that

$$K = \frac{(1 - \nu^2)b^2}{\pi^2 D} P_h = \left(\frac{mb}{a} + \frac{a}{mb}\right)^2$$

which is the buckling coefficient for an ordinary isotropic plate. At the other limit, pure shear ($D$ equals infinity) we have

$$K = \frac{(1 - \nu^2)b^2}{\pi^2 D} P_s = \frac{1}{\pi^2 \theta} \left(1 + \left(\frac{a}{mb}\right)^2\right), \text{ or } P_s = S \left(1 + \left(\frac{a}{mb}\right)^2\right)$$
4.11.2 Buckling of a simply supported, isotropic panel - thick faces

In order to account for the bending stiffness of the faces about their individual neutral axes in the buckling of a sandwich column one instead uses the differential equation accounting for thick faces, as given in section 4.4. The result is

\[
\frac{2D_f}{1-\nu^2}\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right] + \left(\frac{D_0 + 2D_f}{D_0}\right)S\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]^2 = \left[\frac{P_x}{a} + P_y\left(\frac{n\pi}{b}\right)^2\right]\left(1-\nu^2\right) + \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 = 0
\]

(4.76)

Once again assuming uniaxial compression, i.e., \(P_x = P, P_y = 0\) and \(n = 1\), the expression for the critical load becomes

\[
K = \frac{(1-\nu^2)b^2}{\pi^2D}P = \left(\frac{mb}{a} + \frac{a}{mb}\right)^2\left[\frac{2D_f}{D_0}\left(1 + \pi^2\theta\left(\frac{mb}{a}\right)^2 + 1\right)\right]^{-1}
\]

where \(\theta\) is as before. This is the expression derived by Hoff [3] and as can be seen it is very similar to the expression for thin faces but with the term \(2D/D_0\) added to. Hence, if the faces are thin \((D_f = 0)\) this expression converges to that derived above. In perfect analogy with the beam case, the incorporation of the flexural rigidity of the faces leads to an infinite buckling load as the side length \(a\) approaches zero. By plotting \(K\) vs. \(a/b\) in the same manner as for the thin face case above, this is clearly seen; when the side length \(a\) is large, the buckling load is similar to that of the thin face solution but with \(D = D_0 + 2D_f\), and as the side length \(a\) approaches zero the buckling equals that of the two faces buckling independently of each other, and hence approaches infinity. The buckling coefficients for some values of the shear parameter \(\theta\) has been plotted below for both \(D_f = 0\) and for \(2D/D_0 = 0.1\), where the latter can be seen as a limiting case for practical purposes.

![Figure 4.17 Buckling coefficients for a simply supported sandwich plate under uniaxial compression. Dashed lines are for \(D_f = 0\) (\(\pi^2\theta = 0.05, 0.2, 0.4, 1.0\)) and full lines for \(2D/D_0 = 0.1\) (\(\pi^2\theta = 0.05, 0.2, 0.4, 1.0\)).](image-url)
4.11.3 Buckling of an simply supported, orthotropic panel - thin faces

The solution to the buckling load for an orthotropic simply-supported sandwich panel can be derived using the same approach as in the lateral bending case described in section 4.9.4. Following once again the work by Robinsson [10], assuming uniaxial buckling \((P_y = 0)\) and using the following notation

\[
\theta_x = \frac{D_x}{a^2 S_x (1 - \nu_{xy} \nu_{yx})} \quad \text{and} \quad \theta_y = \frac{D_y}{b^2 S_y (1 - \nu_{xy} \nu_{yx})}
\]

(4.77)

and by identifying that the minimum buckling load follows by \(n = 1\), i.e., one wave in the direction perpendicular to the load. The result is

\[
\frac{b^2 (1 - \nu_{xy} \nu_{yx}) P_x}{\pi^2 D_y} \left[ \frac{\pi^4 \theta_x^2 S_y}{S_x} \left( 1 - \nu_{xy} D_y \right) \right] \frac{D_x D_{xy}}{2D^2_y} \left( \frac{mb}{a} \right)^4 + \frac{D_x - \nu_{xy} D_{xy}}{D_y} \left( \frac{mb}{a} \right)^2 + \frac{D_{xy}}{2D_y} \]

\[
+ \pi^2 \theta_y \left\{ \frac{\left( \frac{mb}{a} \right)^2}{D_x D_y} \left( \frac{D_x S_y}{D_y S_x} + 2 \frac{D_{xy}}{D_y} - \frac{D_{xy}}{2D_x} \right) \right\} + \left( 1 + \frac{D_{xy}}{2D_x} \left( \frac{D_x}{D_y} - \frac{\nu_{xy} S_y}{S_x} \right) \right) + 1
\]

\[
= \pi^2 \theta_x \left[ 1 - \frac{D_y}{D_x} \nu_{xy} \right] \left( \frac{D_x D_{xy}}{2D^2_y} \left( \frac{mb}{a} \right)^4 + \frac{D_x - \nu_{xy} D_{xy}}{D_y} \left( \frac{mb}{a} \right)^2 + \frac{D_{xy}}{2D_y} \right] \left( \frac{mb}{a} \right)^2 + \frac{D_y}{D_x} \left( \frac{mb}{a} \right)^4 + 2 \left( \frac{D_{xy}}{D_y} - \frac{D_y}{D_x} \nu_{xy} \right) \left( \frac{mb}{a} \right)^2 + 1
\]

By plotting the in-plane compressive load \(P_x\) versus \(a/b\) for various integers of \(m\) using this rather messy equation the same kind of buckling curves can be produced as in the isotropic case. It is easily shown that the limiting value of the buckling load \(P_x\) when the ratio \(a/b\) approaches zero equals \(S_x\), which is not surprising since this can approximated by a beam of length \(a\) subjected to an in-plane compressive load \(P_x\). It is convenient to introduce the buckling coefficient

\[
K = \frac{b^2 (1 - \nu_{xy} \nu_{yx}) P_x}{\pi^2 D_x}
\]

(4.78)

giving the value of \(K\) for \(a/b = 0\) equal to \(1/\pi^2 \theta_x\). To illustrate the influence of transverse properties on the buckling of a simply-supported orthotropic sandwich panel subjected to uniaxial compression \(P_x\), \(K\) have been plotted in Fig.4.18. These are prepared by keeping the properties in the \(x\)-direction constant whilst changing \(D_{xy}, D_y\) and \(S_y\), respectively. Due to the number of variables in doing so, the graphs are for one value of the shear factor in the \(x\)-direction, \(\pi^2 \theta_x = 0.1\). In the same manner, graphs can easily be drawn for other values of \(\theta_x\). In order to keep \(1-\nu_{xy} \nu_{yx}\) constant, the Poisson ratios has be chosen in the same way as in section 4.9.

As can be seen in the graphs of Fig.4.18, the critical buckling load decreases with decreasing torsional stiffness \(D_{xy}\), it also decreases if the flexural rigidity and/or the shear stiffness in the perpendicular direction \(D_y\) and \(S_y\), respectively, decreases. This is in analogy with the panel bending case. In fact, the critical buckling load depends on the same parameters as the deflection
due to transverse loading, i.e., panel geometry and stiffness. To get a comprehensive picture of the buckling of orthotropic sandwich panels, the same set of graphs as above must be produced for every value of the shear factor $\theta_x$, and in the general case both the faces and the core could be orthotropic, yielding an almost infinite number of graphs. However, the above equations may look complex but are quite easily solved once the geometry and properties of a panel are known.

(a) $K$ versus $a/b$ for different $D_{xy}$. All curves are for $D_x = D_y$, $\nu_{xy} = \nu_{yx} = 0.25$ and isotropic core. Solid lines are for $\pi^2 \theta_x = \pi^2 \theta_y = 0.1$ and dashed lines for $\pi^2 \theta = 0$.

(b) $K$ versus $a/b$ for different $D_y$. All curves are for $D_{xy} = 0.8D_x$, $\nu_{xy} = 0.25 \sqrt{D_x/D_y}$ and isotropic core. Solid lines are for $\pi^2 \theta_y = 0.1$ and dashed lines for $\pi^2 \theta = 0$.
4.11.4 Approximate buckling formulae for orthotropic panels with various boundary conditions - thin faces

The solutions presented in this section are derived using the Ritz method, in the same manner as for the approximate formulae given for the bending cases in section 4.9.5. The methodology was first given by March [11] and then by March and Ericksen [12]. The expressions given in [12] are valid for orthotropic sandwich panels with unequal thick faces and are hence of great generality. These formulae are used in [13] where a very large set of buckling coefficient curves are given for various combinations of orthotropy. A simpler way of approaching the problem than done by March [11] is to assume the concept of partial deflections to be applicable, that is

\[
\frac{1}{P} = \frac{1}{P_b} + \frac{1}{P_s}, \text{ with } w = w_b + w_s
\]

(4.79)

By using energy principles, like in the bending cases of section 4.9.5 together with the partial deflections we can derive the following:

(i) Simply supported edges

As above we have that for uniaxial buckling (\( P_y = 0 \)) that the minimum buckling load is given by \( n = 1 \), that is, only one wave length in the buckling mode in the \( y \)-direction. If one assumes the deflected shape as

\[
w = w_b + w_s = (\bar{w}_b + \bar{w}_s) \sin \frac{m \pi x}{a} \sin \frac{n \pi y}{b}
\]

Then the buckling load can be written

\[
\frac{1}{P} = \frac{b^2 (1 - v_{xy} v_{yx})}{\pi^2 \left[ D_x \left( \frac{mb}{a} \right)^2 + D_y \left( \frac{a}{mb} \right)^2 + 2 v_{xy} D_x + D_y (1 - v_{xy} v_{yx}) \right]} + \frac{1}{S_x + S_y \left( \frac{a}{mb} \right)^2}
\]
and the buckling coefficient as

\[
K = \left[ 1 + \frac{S_x}{S_y} \left( \frac{a}{mb} \right)^2 \right] \left[ \left( \frac{mb}{a} \right)^2 + \frac{D_y}{D_x} \left( \frac{a}{mb} \right)^2 + 2 \left( \frac{\nu_{yx} + \frac{D_y(1 - \nu_{xy} \nu_{yx})}{D_x}}{D_x} \right) \right]
\]

For an isotropic plate this relation reduces to

\[
K = \left[ 1 + \left( \frac{a}{mb} \right)^2 \right] \left[ \left( \frac{mb}{a} \right)^2 + \left( \frac{a}{mb} \right)^2 + 2 \right]
\]

which can be rewritten to exactly the same relation as derived in section 4.11.1 and has the same graphical shape as in Fig.4.16.

(ii) Loaded edges simply supported, the other clamped

![Figure 4.19 Rectangular orthotropic sandwich with loaded edges simply supported and the others clamped.](image)

\[
w = w_b + w_x = (\bar{w}_b + \bar{w}_x) \sin \frac{m \pi x}{a} \sin^2 \frac{\pi y}{b}
\]

\[
\frac{1}{P} = \frac{3b^2(1 - \nu_{xy} \nu_{yx})}{\pi^2 \left[ 3D_x \left( \frac{mb}{a} \right)^2 + 16D_y \left( \frac{a}{mb} \right)^2 + \frac{8}{3} \left( \frac{\nu_{yx}D_x + D_y(1 - \nu_{xy} \nu_{yx})}{D_x} \right) \right]} + \frac{1}{S_x + \frac{4S_y}{3} \left( \frac{a}{mb} \right)^2}
\]

\[
K = \left[ 1 + \frac{4S_x}{3S_y} \left( \frac{a}{mb} \right)^2 \right] \left[ \left( \frac{mb}{a} \right)^2 + \frac{16D_y}{3D_x} \left( \frac{a}{mb} \right)^2 + \frac{8}{3} \left( \frac{\nu_{yx} + \frac{D_y(1 - \nu_{xy} \nu_{yx})}{D_x}}{D_x} \right) \right]
\]

which for an isotropic plate reduces to (with \( \theta = \theta_y \))
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\[
K = \left[ 1 + \frac{4}{3} \left( \frac{a}{mb} \right)^2 \right] \left[ \frac{(mb)^2}{a} + \frac{16}{3} \left( \frac{a}{mb} \right)^2 + \frac{8}{3} \right] + \pi^2\theta \left[ \frac{(mb)^2}{a} + \frac{16}{3} \left( \frac{a}{mb} \right)^2 + \frac{8}{3} \right]
\]

This equation is graphically represented in Fig. 4.20.

![Figure 4.20 Buckling coefficients \(K\) for an isotropic panel with thin faces subjected to uniaxial compression, having loaded edges simply supported and the others clamped. The lines are for \(\pi^2\theta = 0, 0.05, 0.1, 0.2\) and 0.4.](image)

_(iii) Loaded edges clamped, the other simply supported_

\[
w = w_b + w_x = (\bar{w}_b + \bar{w}_x) \sin \frac{\pi x}{a} \sin \frac{m\pi x}{a} \sin \frac{\pi y}{b}
\]

Due to the assumed deflection field, the solution must be obtained for \(m = 1\) and for \(m > 1\) as:
For $m = 1$:

\[
1 \frac{P}{b^2} = \frac{4b^2(1 - \nu_{xy}\nu_{yx})}{\pi^2 \left[ 16D_y \left( \frac{b}{a} \right)^2 + 3D_y \left( \frac{a}{b} \right)^2 + 8\nu_{yx} D_x + D_{xy} (1 - \nu_{xy}\nu_{yx}) \right]} + \frac{1}{S_x + \frac{3S_y}{4} \left( \frac{a}{b} \right)^2} \]

\[
K = \left[ 1 + \frac{3S_x}{4S_y} \left( \frac{a}{b} \right)^2 \right] + \pi^2 \phi \left( \frac{a}{b} \right)^2 \left[ 16 \left( \frac{b}{a} \right)^2 + 3 \left( \frac{a}{b} \right)^2 + \frac{8}{3} \left( \frac{b}{a} \right)^2 \right]
\]

which for an isotropic plate reduces to (with $\theta = \theta_y$)

\[
K = \left[ 1 + \frac{3\left( \frac{a}{b} \right)^2}{4\left( \frac{a}{b} \right)^2} \right] + \pi^2 \phi \left( \frac{a}{b} \right)^2 \left[ 16 \left( \frac{b}{a} \right)^2 + \frac{8}{3} \left( \frac{b}{a} \right)^2 \right]
\]

For $m > 1$:

\[
1 \frac{P}{b^2} = \frac{b^2 A_m (1 - \nu_{xy}\nu_{yx})}{\pi^2 \left[ D_y B_m \left( \frac{b}{a} \right)^2 + D_x \left( \frac{a}{b} \right)^2 + 2A_m \nu_{yx} D_x + D_{xy} (1 - \nu_{xy}\nu_{yx}) \right]} + \frac{A_m}{S_x A_m + S_y \left( \frac{a}{b} \right)^2} \]

where $A_m = (1+m^2)$, and $B_m = (m^4+6m^2+1)$. This gives

\[
K = \frac{1}{A_m} \left[ A_m + \frac{S_x}{S_y} \left( \frac{a}{b} \right)^2 \right] + \pi^2 \phi \left( \frac{a}{b} \right)^2 \left[ B_m \left( \frac{b}{a} \right)^2 + D_y \left( \frac{b}{a} \right)^2 + 2A_m \left( \nu_{yx} + \frac{D_{xy} (1 - \nu_{xy}\nu_{yx})}{D_x} \right) \right]
\]

For an isotropic plate this reduces to (with $\theta = \theta_y$)

\[
K = \frac{1}{A_m} \left[ A_m + \left( \frac{a}{b} \right)^2 \right] + \pi^2 \phi \left( \frac{b}{a} \right)^2 \left[ B_m \left( \frac{b}{a} \right)^2 + \left( \frac{a}{b} \right)^2 + 2A_m \right]
\]

This equation is graphically represented in Fig.4.22.
Figure 4.22 Buckling coefficients $K$ for an isotropic panel with thin faces subjected to uniaxial compression, having loaded edges simply supported and the others clamped. The lines are for $\pi^2 \theta = 0$, 0.05, 0.1, 0.2 and 0.4.

(iv) All edges clamped

Figure 4.23 Rectangular orthotropic sandwich with loaded edges simply supported and the others clamped.

\[ w = w_b + w_x = (\overline{w}_b + \overline{w}_x) \sin \frac{\pi x}{a} \sin \frac{m \pi x}{a} \sin^2 \frac{\pi y}{b} \]

Due to the assumed deflection field, the solution must be obtained for $m = 1$ and for $m > 1$ as:

**For $m = 1$:**

\[
\frac{1}{P} = \frac{b^2 (1 - \nu_{xy} \nu_{yx})}{\pi^2 \left[ 4D_x \left( \frac{b}{a} \right)^2 + 4D_y \left( \frac{a}{b} \right)^2 + \frac{8}{3} \nu_{yx} D_x + D_{xy} (1 - \nu_{xy} \nu_{yx}) \right]} + \frac{1}{S_x + S_y \left( \frac{a}{b} \right)^2}
\]

\[
K = \frac{\left[ 1 + \frac{S_x}{S_y} \left( \frac{a}{b} \right)^2 \right]}{\left[ 1 + \frac{S_y}{S_x} \left( \frac{a}{b} \right)^2 \right]} \left[ 4 \left( \frac{b}{a} \right)^2 + \frac{4D_x \left( \frac{a}{b} \right)^2}{D_x} + \frac{8}{3} \nu_{yx} D_x + \frac{D_{xy} (1 - \nu_{xy} \nu_{yx})}{D_x} \right]
\]

\[
\left[ 1 + \frac{S_y}{S_x} \left( \frac{a}{b} \right)^2 \right] + \pi^2 \theta \left( \frac{a}{b} \right)^2 \left[ 4 \left( \frac{b}{a} \right)^2 + \frac{4D_y \left( \frac{a}{b} \right)^2}{D_y} + \frac{8}{3} \nu_{yx} D_y + \frac{D_{xy} (1 - \nu_{xy} \nu_{yx})}{D_y} \right]
\]
which for an isotropic plate reduces to (with \( \theta = \theta_j \))

\[
K = \frac{\left[ 1 + \left( \frac{a}{b} \right)^2 \right] \left[ 4 \left( \frac{b}{a} \right)^2 + 4 \left( \frac{a}{b} \right)^2 + \frac{8}{3} \right]}{\left[ 1 + \left( \frac{a}{b} \right)^2 \right] + \pi^2 \theta \left[ 4 \left( \frac{b}{a} \right)^2 + 4 \left( \frac{a}{b} \right)^2 + \frac{8}{3} \right]}
\]

For \( m > 1 \):

\[
\frac{1}{P} = \frac{3b^2 A_m (1 - \nu_{xy} \nu_{yx})}{\pi^2 \left[ 3D_y B_m \left( \frac{b}{a} \right)^2 + 16 D_y \left( \frac{a}{b} \right)^2 \right] + 8 A_m \left[ \nu_{yx} D_x + D_y \left( 1 - \nu_{xy} \nu_{yx} \right) \right]} + \frac{A_m}{S \left( \frac{4S_x}{3} \left( \frac{a}{b} \right)^2 \right)}
\]

where \( A_m = (1 + m^2) \), and \( B_m = (m^2 + 6m^2 + 1) \). This gives

\[
K = \frac{1}{A_m} \left[ \frac{4S_y}{3S_y} \left( \frac{a}{b} \right)^2 \right] + \pi^2 \theta \left[ \frac{a}{b} \right]^2 \left[ \frac{B_m}{a} \right] + \pi^2 \theta \left[ \frac{a}{b} \right]^2 \left[ \frac{B_m}{a} \right] + \frac{8 A_m}{3} \left( \nu_{yx} + \frac{D_y \left( 1 - \nu_{xy} \nu_{yx} \right)}{D_x} \right) \]

which for an isotropic plate reduces to (with \( \theta = \theta_j \))

\[
K = \frac{1}{A_m} \left[ \frac{4\left( \frac{a}{b} \right)^2}{3} \right] + \pi^2 \theta \left[ \frac{4\left( \frac{a}{b} \right)^2}{3} \right] + \frac{8 A_m}{3}
\]

This equation is graphically represented in Fig.4.24.

Figure 4.24 Buckling coefficients \( K \) for an isotropic sandwich plate with thin faces having all edges clamped and subjected to uniaxial compression. The lines are for \( \pi^2 \theta = 0, 0.05, 0.1, 0.2 \) and 0.4.
The chosen deflection functions satisfy the boundary condition but not the governing differential equation of eq.(4.36) leads to that the calculated buckling loads are approximate (except for the simply supported case). Thurston [14] solved the problem of a clamped isotropic sandwich plate exactly by using double trigonometric cosine series. By comparison of the clamped panels, the approximate results calculated by the above formulae yields buckling loads that are 5 to 10 % too high. Though approximate, this solution has the advantage of being much more general than any other known solution procedure in that it handles sandwich panels with different boundary conditions, having orthotropic core and orthotropic faces.

In [13] a solution is presented based on the same assumptions as the above but incorporating the effect of thick and dissimilar faces. Even if the derived formulae are complex in the number of parameters used it still constitutes a fairly simple closed form solution easily transformed into computer code. The derived equations have also been plotted for not less than 125 cases which would at least approximately cover almost every plausible practical case. Thus, the approximate buckling formulae derived here, which only assumed thin faces, can be found in [13] although written in slightly different parameters and some of the 125 buckling curves given are also plotted assuming thin faces. However, since the buckling load assuming thin faces fairly easily can be found through the closed form solution no other buckling curves than for the special case of an isotropic panel has given in this text.

4.11.5 Shear Buckling
Kuenzi and Ericksen [16] solved the problem of rectangular isotropic sandwich plates, simply supported or clamped, subjected to shear loads as illustrated in Fig.4.25.

\[ K = \frac{K_0}{1 + \pi^2 \theta \left( K_0 - 1 - \frac{b^2}{a^2} \right)} \quad \text{for} \quad 0 \leq \pi^2 \theta \leq 1 + \frac{b^2}{a^2} \]  

(4.80a)

The problem is solved for a number of cases using Ritz's method where different deflection assumptions are adopted for the various boundary conditions and values of the shear factor \( \theta (\theta = D/b^2 S) \). The analyses are rather lengthy and are therefore omitted here but Kuenzi and Ericksen [16] suggest some approximate design formulae which are very simple to use. The formulae suggested proved to be slightly conservative. Providing \( a \) is larger than \( b \) the formula for simply supported edges reads (with \( K = P_{xy}(1-\nu)c/b^2D) \)
and for all edges clamped

\[
K = \frac{K_0}{1 + \pi^2 \theta \left( K_0 - \frac{4}{3} \left(1 + \frac{b^2}{a^2}\right) \right)} \text{ for } 0 \leq \pi^2 \theta \leq \frac{4}{3} \left(1 + \frac{b^2}{a^2}\right) \tag{4.80b}
\]

where \(K_0\) is the value of the buckling coefficient for an ordinary plate with \(\theta = 0\). These can be written approximately \[4\]

Simply supported:

\[
K_0 = \frac{16}{3} + 4 \frac{b^2}{a^2} \tag{4.81a}
\]

Clamped edges:

\[
K_0 = 9 + \frac{17}{3} \frac{b^2}{a^2} \tag{4.81b}
\]

For \(\theta\) larger than the values indicated in eqs. (4.80) then \(K\) takes the value \(1/\pi^2 \theta\), the same limiting value as in the previous buckling analysis. A closed form solution to buckling of infinitely long plates is given in \[4\].

### 4.12 Combined Buckling and Transverse load

The governing equations of section 4.4 are derived so that in the general case transverse as well as in-plane loading may be considered simultaneously. Doing so may, however, be fairly difficult and only a few solutions of combined loading situations are known, e.g., \[10\]. However, in the case of an isotropic sandwich panel subjected to transverse loading \(q\) and in-plane compressive loads \(P_x\) and \(P_y\), the governing equations can be used to arrive at an expression that can be used in the general case.

\[
w_{mn} = \frac{w_0}{1 - P / P_{mn}} \tag{4.82}
\]

where \(w_{mn}\) is the coefficient \(mn\) in the series expansion of the solution to the transverse load response and \(P_{mn}\) is the buckling load in the \(mn\)th mode. Hence, the effect of an in-plane compressive load \(P\) can be added in the equations for the deflection of rectangular panels simply by substituting \(w_{mn}\) by the expression given above. The complexity introduced is that the value of the buckling load in the \(mn\)th mode must be calculated for every step in summation of these equations. A practical way over overcoming this can be used if the panel aspect ratio is close to unity and \(P/P_{mn}\) is small \[3\]; one can then use only the first term in the series, i.e., only \(P_{11}\) must be known. This yields fairly accurate results in terms of deflection and bending moments, whereas the transverse forces are not.
4.13 Free Vibration of Rectangular Plates

4.13.1 Simply supported isotropic plate with thin faces
The governing equation of motion for free undamped vibration is given in section 4.4. Assuming free harmonic excitations the deflection can be written as

\[ w(x, y, t) = (A \cos \omega t + B \sin \omega t) \Phi(x, y) = \Psi(t) \Phi(x, y) \]

where \( \phi(x, y) \) is the static spatial displacement function and \( \omega \) the natural frequency. A non-trivial solution implies that \( \Phi \) and \( \Psi \) are non-zero and we get the governing equation for harmonic excitations as

\[
\frac{D}{1 - v^2} \left[ \left( \frac{m\pi}{a} \right)^2 + \left( \frac{n\pi}{b} \right)^2 \right] - \rho^* \omega^2 - \frac{\rho^* \omega^4}{1 - v^2} - \frac{R^* \omega^2}{S(1 - v^2)} \left[ \left( \frac{m\pi}{a} \right)^2 + \left( \frac{n\pi}{b} \right)^2 \right] = 0
\]

By omitting the effect of the rotary inertia (\( R = 0 \)) the eigen-frequency can be written

\[
\omega_{mn} = \pi^2 \left[ \frac{m^2}{b^2} + \left( \frac{na}{b} \right)^2 \right] \frac{D}{\rho^* a^4 (1 - v^2)} \left[ \frac{1}{1 + \pi^2 \theta \left( \frac{mb}{a} \right)^2 + n^2} \right], \text{ with } \theta = \frac{D}{S(1 - v^2) b^2}
\]

This can be seen as the ordinary plate theory expression divided by a correction term (the denominator) that depends on the amount of shear deformation. Once again, the portion of shear deformation is seen to increase as the size of the panel decrease. It is important to notice that the correction also depends on the mode of vibration \( n \), giving a small correction in the first eigen-frequency but larger and larger corrections for higher modes. By omitting both the rotary inertia and the transverse shear (\( S = 0 \)) the this expression equals that of the engineering plate theory

\[
\omega_{mn} = \pi^2 \left[ m^2 + \left( \frac{na}{b} \right)^2 \right] \frac{D}{\rho^* a^4 (1 - v^2)}
\]

4.13.2 Simply supported isotropic plate with thick faces
In the case of thick faces the same approach and assumptions can be used but now using the governing equation in 4.13.1. This results in an expression for the natural frequencies as

\[
\omega_{mn} = \pi^2 \left[ \left( \frac{mb}{a} \right)^2 + n^2 \right] \frac{2D/D_0}{S\rho^* b^4 (1 - v^2) + \rho^* b^4 (1 - v^2)} \frac{D}{1 + \pi^2 \theta \left( \frac{mb}{a} \right)^2 + n^2}
\]
It is seen that for thin faces, $D_f = 0$, the expression equals that for thin faces. For a very low shear stiffness, the natural frequencies will equal that of the two faces vibrating independently of each other, i.e.

$$\omega_{mn} = \sqrt{\frac{2D_f \pi^2 \left(\frac{mb}{a}\right)^2 + n^2}{\rho^* b^2 (1 - \nu^2)}}$$

References


The design of a sandwich element is often an integrated process of sizing and materials selection in order to get not only a feasible design but also in some way an optimum design with respect to an objective, such as weight, strength, or stiffness. With the introduction of fibrous composites, the choice of face materials has become almost infinite in terms of mechanical properties. Core materials, especially foams, are now also available in wide ranges of densities and properties. All material systems have some advantages and some disadvantages implying that the choice of materials are given by the objectives of the specific application and cannot be stated in general terms. These objectives are often practical, e.g., chemical or heat resistance, surface wear resistance, thermal insulation or related to the manufacturing process. Therefore, materials are often, in the practical case, already predefined by the service or manufacturing requirements of the structure. However, some material related properties can still be considered as variables even if the material itself is predefined, e.g., the density of a specific core material. However, most material properties, as well as thicknesses, do not vary continuously but in discrete steps, for example, number of plies of a composite laminate, available thicknesses of sheet metal or of core materials. Anyway, it is convenient to have methods for design which can give an indication of a suitable start point for the design process giving approximate thicknesses and maybe even materials selection.

The constraints in the design process are several; stiffness, strength, manufacturing constraints, environment, machining, cost etc. From a mechanical point of view, stiffness and strength are the properties that can easily be formulated. For a sandwich, the stiffness is estimated by some of the expressions given in chapters 3 and 4, where the deformation is given as function of the applied load. The strength, on the other hand, depend on the failure mode, which depend on the internal structure of the sandwich as well as the applied loading.

Methods are also described for optimum design of sandwich constructions. In this context it means finding sizes and materials to give a minimum weight, maximum strength or stiffness or minimum cost with respect to one or several constraints. The constraints could be stiffness or strength in different failure modes or a combination thereof. Simple methods of that kind will also serve as a good starting point for the design process. Since there generally are many different constraints on a structure an optimum may be hard to find with respect to all constraints without using general mathematical programming techniques. Such are often both complex and costly. Only considering the most important constraints and using a simple optimisation technique could prove useful in the
design process. The methodologies outlined in this chapter may therefore rather be called "intelligent sizing" than optimisation.

5.1 Failure of Beams and Panels
Sandwich panels can fail in several ways, each mode giving one constraint on the load bearing capacity of the sandwich. Depending on the geometry of the sandwich and the loading, different failure modes become critical and set the limits for the performance of the structure. The most important failure modes have already been treated in the analysis but will be highlighted again due to the importance of recognising them. Other failure modes that appear are of a more practical significance. The most common failure modes are schematically illustrated in figure 5.1.

![Figure 5.1 Failure modes. (a) Face yielding/fracture, (b) core shear failure, (c and d) face wrinkling, (e) general buckling, (f) shear crimping, (g) face dimpling and (h) local indentation.](image)

There are quite a number of failure criteria for composite laminate face materials varying from fairly simple to extremely complex criteria including allowables for several different modes of failure. Such will not be discussed here, but only the criteria most commonly used by engineers in design, sizing and analysis.

(a) Yielding or fracture of the face in tension or compression:
Depending on the materials used and on the chosen fracture criterion one will consider the face or the core to have failed either if yielding occurs or if the component has actually fractured. Hence, for every material component there will be a maximum allowed stress, whether this stress is a yield or a fracture stress. The criterion for failure is then when the maximum stress in the component reaches this allowable. The maximum principal stress in the faces is according to Mohr's circle of stress

$$\bar{\sigma}_f = \frac{\sigma_{f_1} + \sigma_{f_2}}{2} + \left(\frac{\sigma_{f_1} - \sigma_{f_2}}{2}\right)^2 + \tau_{fxy}^2 \right)^{1/2}$$

(5.1)

For most loading situations, $\tau_{fxy} << \sigma_f$ as mentioned in chapters 3 and 4. In fact, the direct stresses in the faces are usually orders of magnitude higher than the shear stresses in the core and faces. The direct face stress is calculated using the formulae given in chapters 3.3 and 4.3.
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For a metal face the maximum face stress should perhaps, on the other hand, rather be estimated by e.g. von Mises yield criterion. In other cases, like when using fibre composite faces, the failure criterion must take another form. There are an almost infinite number of failure criteria for fibre composites. Once the stress components are known, which in this case can be calculated using the formulae given in chapters 4.3, the load failure can be estimated using e.g., the maximum-stress-criterion, the maximum-strain-criterion or the Tsai-Hill criterion. Similar criteria could be stated for the core, but such failure criteria are seldom used since most core materials have a higher yield and fracture strain than the faces implying that tensile or compressive failure (or yielding) will occur in the faces long before any allowable stresses are reached to the core. Note also that the allowables may well differ between tensile and compressive modes, so that e.g. even if the compressive face has a nominal lower stress level it may be the first to fail if the compressive strength is lower than the tensile. Hence, the criterion must be used twice, once for each face.

(b) Core shear failure
As pointed out in chapter 3, the core material is mainly subjected to shear since it carries almost the entire transverse force. However, the direct stresses in the core could be of the same order of magnitude as the shear stresses. The maximum core shear stress can be written

\[ \tau_{\text{cxz}} = \left( \frac{\sigma_{\text{cx}}}{2} + \tau_{\text{cxz}}^2 \right) < \hat{\tau}_{\text{cx}} \text{ and } \tau_{\text{cyz}} = \left( \frac{\sigma_{\text{cy}}}{2} + \tau_{\text{cyz}}^2 \right) < \hat{\tau}_{\text{cy}} \] (5.2)

which is used as the fracture criterion. And, in fact, in many practical cases the direct stress is much lower than the shear stress reducing the maximum shear stress to approximately equal \( \tau_c \). This allowable could now also be either a yield or a fracture stress. The core shear stress is calculated by formulae in chapters 3.3 and 4.3. This shear stress produces a tensile stress, equal to \( \hat{\tau}_c \), at a 45 degree angle from the x-direction which causes cracks inclined 45 degrees. Such cracks are typical for a pure shear failure and are also called shear cracks.

(c and d) Face wrinkling
Face wrinkling can in practical cases occur in a sandwich either when subjected to an in-plane compressive buckling or in the compressive face during bending, or in a combination of those. The critical face stress is in either case calculated using the formulae given in chapter 3.11. The actual failure can occur in two ways; (i) a wrinkle that becomes unstable causes an indentation in the core if the compressive strength of the core is lower than the tensile strength of the core and the adhesive joint or (ii), the wrinkle causes a tensile fracture if the tensile strength of the core or the adhesive joint is lower than the compressive strength of the core. This formulation is rather vague but is usually correct. Whatever case does not really affect the actual wrinkling stress, but in fact, a poor adhesive joint will undoubtedly reduce the wrinkling stress of the sandwich.

For a panel, the same formulae as for beams may used if the loading is uniaxial but now all elastic moduli \( E \) must be substituted with \( E/(1-\nu^2) \), for both the faces and the core. If the loading is biaxial one can still use the one-dimensional formulae given in section 3. However, one should then calculate the critical buckling stress in both \( x \) - and \( y \)-directions and use the lowest of the two as the design load.
(e) General buckling
Although buckling itself sometimes doesn't damage a structure, it must still be avoided since a structure which has buckled may have lost its capability of fulfilling its purpose. The actual buckling load may also be the ultimate load bearing capacity of the sandwich since in its buckled shape it may not sustain any more load. Therefore, it is often stated that buckling may not occur and the buckling load hence becomes an allowable. In chapter 3.10, the critical buckling load was given for various sandwich column cases, with thin or thick faces, and these formulae may well serve as constraints when designing sandwich beams or struts. For panels, formulae and design graphs are given in chapter 4.11.

In practice, a buckled sandwich usually retains its original state if unloaded, providing the faces has not yielded, and hence buckling has not damaged the structure. If, however, the buckling load equals the maximum load that can be applied it means that in a dead load situation the structure has actually failed. If the deformation is controlled, on the other hand, the load will drop after buckling, and if the loading continues the deformation will increase until failure. This final failure can occur in several ways; (i) the face on the compressive side fail in compression, (ii) the face on the compressive side fail by wrinkling (see case c and d), or (iii) the failure by core shear fracture. The latter case occurs since, as the deformation increases so does the transverse force and eventually the transverse force has grown in some point to a level that will cause core shear fracture as stated above. This kind of failure mode appears just as the shear crimping in Fig.5.1g.

(f) Shear crimping
The shear crimping failure is actually the same as the limit of the general buckling mode considering thin faces, i.e., when the critical load equals $P_S = S (S$ shear stiffnes). The failure itself looks like illustrated in Fig.5.1g and is shear instability failure. The critical face stress is hence

$$\sigma_f = \frac{S}{2t_f}$$  \hspace{1cm} (5.3)

As mentioned above, a failure of this kind is more likely to happen as a result of large out-of-plane deformations in a post-buckled state when transverse forces build up due to the deformation. The failure will then appear where this transverse force has a maximum, e.g., at the edges for a simply supported column, and at $L/4$ for a clamped column.

(g) Face dimpling
As given in chapter 3.11, another instability phenomena that may occur in sandwich structures with honeycomb or corrugated cores is dimpling or inter cellular buckling. Some dimpling formulae are given in chapter 3.11 for cases of square and hexagonal honeycombs and for corrugated cores. These may be used for panels as well but $E$ should be replaced with $E/(1-\nu^2)$, for both the faces and the core.
(h) Core indentation
Indentation of the core occurs at concentrated loads, such as fitting, corners, or joints. Practically they can be avoided by applying the load over a sufficiently large area. What actually happens when point loads are applied is that the face will act as a plate on an elastic support. The face will bend independently of the opposite face and if the deformation and thus the elastic stress supplied by the core exceed the compressive strength of the core, the core will fail. In practice, there are many ways to enhance the local strength of a sandwich to avoid indentation, but these will discussed in chapter 8.

(i) Vibration
In some cases there are constraints on the minimum allowed natural frequency. In moving structures there is often an imposed movement within a given frequency range. It is then preferable to avoid having a natural frequency of vibration for the structural member lying within that range. Examples of calculations of natural frequencies for beams are given in chapter 3.10 and for panels in chapter 4.12.

(j) Debonding
Debonding means that the adhesive joint bonding of the face to the core fails. This can occur due to overloading. The shear stress in the bond line is almost as high as in the middle of the core, and if the adhesive joint has less strength than the core it will fail prior to the core. This, however, should be avoided by choosing an adhesive and a manufacturing methods prevents the above from happening. The bond line will also be subjected to high stresses if there is a thermal field with high gradient acting on the face. Since the core usually is a very good thermal insulator, whereas the face usually is not, especially if a metal face is used, then if the face is subjected to a temperature change, sunshine for example, the thermal gradient will be very high in the interface causing high thermal stresses in the bond line. The adhesive joint may also fail due to fatigue, impact, ageing or numerous other causes. The main problem with debonding failures are that they are sub-surface, making them difficult to detect and can therefore grow into critical size before detected.

The effect of disbonds will be further discussed in chapter 14, along with methods of predicting the loss in strength due to disbonds and other types of damage.

(k) Fatigue
Fatigue is generally said to cause more than 90% of all structural failures. It can be accounted for if relevant material data is available. Since the face acts in almost direct tension or compression one can account for fatigue by estimating the number load cycles the structure will have, the load spectrum (maximum and minimum loads) by taking the allowable face stress $\sigma_f$ as the material fatigue stress at the given number of load cycles and stress ratio, providing the information is available. For most metals that is the case, whereas other materials, such as most fibre composites, such data is often lacking. A conservative way is the to use the fatigue limit under which the material can undergo an infinite number of load cycles without exhibiting any damage.

For the core the reasoning is similar; substitute the allowable shear stress $\tau_c$ with its fatigue limit. Data for some core materials are given in chapter 2.4. Here, the same problem occurs; there is
clear lack of data, with only limited test results for some core materials covering a few densities, cell sizes, and stress ratios. Hopefully, more data of this kind will be available in the future.

(l) Impact damage
Impact occurs for example when someone drops a tool on the structure, when a boat hits the quay, or when an aircraft flies through a hail storm. An impact may create visible damage like making a dent in the face, causing "whitening effects" on a laminate or it may not leave any visible marks at all. Damage does not always occur in the direct vicinity of the actual impact but can cause debonding, core fracture, delamination or wrinkling damage far away from the point of impact. The resistance to impact is not fully understood but it does depend on the face material, the core and the geometry of the structure. It does also depend upon the size of the impactor and the speed at which the impact occurs.

5.2 Failure Mode Maps
This section will describe a technique to design sandwich structures in a way that will improve the performance of the sandwich from the view that no single component is over-designed with respect to the other components. The technique also allows the designer to choose the anticipated failure mode, or making two different failure modes equally likely to occur. This can be advantageous in some cases where certain failure modes should be avoided.

For a general load case one can express the maximum bending moment and the maximum transverse load as function of the applied load as

\[ M_{\text{max}} = k_M PL \] and \[ T_{\text{max}} = k_T P \] or as \[ M_{\text{max}} = k_M qa^2 \] and \[ T_{\text{max}} = k_T qa \] (5.4)

The constants \( k_M \) and \( k_T \) depend on the loading. For a panel we have that the apparent stress levels differ in different directions, due to orthotropy or panel aspect ratio, so that the values of \( k_M \) and \( k_T \) are different in different directions. For example, for a cantilever beam loaded with a point load at its free edge, \( k_M = 1 \) and \( k_T = 1 \) whereas for a square simply supported isotropic panel we have that \( k_M = 0.0479 \) and \( k_T = 0.338 \) (see chapter 4.9). Each of the above failure mode formulae contains three sets of variables \([1]\); those relating to the loading configuration (\( k_M \) and \( k_T \), and \( L \)), to the material properties (\( E_f, \sigma_f, E_c, G_c, \tau_c \)) and to the design (\( t_f, t_c \) and maybe even \( \rho_c \)). Now it remains to determine how the failure mode depends on the design for a given combination of materials. The design at which two failure modes occur simultaneously is found by equating the formulae for these two modes, and thereby one can establish the transition between the two modes.

First define the principal failure modes: Face yield or face fracture and core shear are independent modes and of great importance to a structure. If we leave out the case of in-plane compressive loads causing buckling or shear crimping and look only at beams and panels subjected to transverse loads then the third failure of importance would be face wrinkling. Depending on the core material type we might have to consider wrinkling or face dimpling. As done in \([1]\), let us use the former. Since tensile/compressive yield of the core seldom is of any practical importance that as well is omitted. The variables used are, apart from face and core thicknesses, also the core density. One may use core density as a variable providing the core properties can be related to its density. One such type of relationship that was proposed in chapter 2 is
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\[ E_c = C_E \rho_c^n, \quad G_c = C_G \rho_c^n, \quad \text{and} \quad \tau_c = C_\tau \rho_c^n \]  \hfill (5.5)

and other properties like the core tensile and compressive strength can be written in a similar manner. The constants relating the properties to the density can be extracted from tests but commonly only a curve fit to data from the material supplier should be satisfactory. We can then rewrite the equations for the three studied failure modes, face yield, core shear and face wrinkling to given the critical load as function of the sought variables \( t_f, d \) and \( \rho_c \). For simplicity reasons, let's study a beam. The formulae are then

**Face yield:**

\[ P = \frac{\hat{\sigma}_f t_f d}{k_M L} \]  \hfill (5.6)

**Core shear:**

\[ P = \frac{C \rho_c^n d}{k_T} \]  \hfill (5.7)

**Wrinkling:**

\[ P = \frac{t_f d}{2k_M L} \sqrt{E_f C_E C_G \rho_c^n} \]  \hfill (5.8)

The transition between the failure modes is now easily found by equating the loads given above, two by two, giving three transition lines in the failure mode map. Performing these exercises one arrives at the following relations between the core density and the ratio \((t_f/L)\)

Transition between face yield and core shear

\[ \log \rho_c = \frac{1}{m} \log \frac{\hat{\sigma}_f k_T}{k_M C_\tau} \left( \frac{t_f}{L} \right) \]  \hfill (5.9)

Transition between wrinkling and core shear

\[ \log \rho_c = \frac{3}{3m-2n} \log \frac{k_T}{2k_M C_\tau} \sqrt{E_f C_E C_G} \left( \frac{t_f}{L} \right) \]  \hfill (5.10)

Transition between wrinkling and face yield

\[ \log \rho_c = \frac{3}{2n} \log \frac{2\hat{\sigma}_f}{\sqrt{E_f C_E C_G}} \]  \hfill (5.11)

To illustrate this lets take an example; a beam in three-point bending, thus, \( k_M = 1/4 \) and \( k_T = 1/2 \), having aluminium faces with an ultimate strength of 150 MPa. Assuming a linear relation between the core properties and the core density \((n = m = 1)\), and that \( C_E = 1, C_G = 0.40 \) and \( C_\tau = 0.015 \), gives the failure mode map graphically presented in Fig.5.2.
For a panel the procedure will be exactly the same with the exception that the criteria must used twice in all cases, once in each of the $x$- and $y$-directions. For example, face tensile or compressive failure may occur at different load levels in $x$- and $y$-directions. The same thing goes for core shear failure if the core is orthotropic. This implies that it some cases it might be beneficial to produce one failure mode map for each direction.

5.3 Other Design Constraints

The above constitute the design constraints on a structure but often there are other constraints that appear in an application that are not directly of a mechanical nature but rather of a practical one. Depending on the application they may of course differ but some common constraints are

(i) minimum core thickness and specific core materials for thermal insulation purposes.
(ii) minimum face thickness, and a combination of face and core materials for given impact resistance.
(iii) specific face material and face thickness for surface wear resistance.
(iv) specific face material for surface finish.
(v) specific face material for environmental resistance.
(vi) maximum total thickness by a volume requirement.
(vii) fire resistance.

Furthermore, there are always some other requirements on the materials depending on the manufacturing or assembly method, working environment, waste, recycling, to only mention some.
5.4 Design Procedures

The most common and essential constraint of a structure is, apart from strength requirements, its stiffness. This constraint is often stated as a maximum deflection at some location of the structure. The compliance, the inverse of the stiffness can, assuming thin faces and weak core, in general be written as

\[ C = \frac{\Delta}{P} = k_b \frac{L^3}{D} + k_s \frac{L}{S} \]  

or for panel as

\[ C = \frac{\Delta}{q} = k_b \frac{a^4}{D} + k_s \frac{a^2}{S} \]  

(5.12)

where \( \Delta \) is the deflection (on which there is a constraint), and \( k_b \) and \( k_s \) are constants relating to the geometry and loading. For example, for a beam in three-point bending \( k_b = 1/48 \) and \( k_s = 1/8 \), and for a square simply supported isotropic panel, \( k_b = 0.00406 \) and \( k_s = 0.0737 \) (see chapter 4.9). Strength and stability constraints are usually those given by the different failure modes described in chapter 5.1.

Since the main feature of using sandwich construction is for weight saving purposes (or increased stiffness for a given weight) the objective is usually one that will minimise the total weight of the structure given one or several constraints. The total weight of the sandwich per unit area is

\[ W = \rho_{ct} + 2 \rho_{ft} \approx \rho_{cd} + 2t_f \rho_f \]  

(5.13)

The variables in the design process are apart from sizes, i.e., \( t_f \) and \( t_c \), maybe also materials. A variable that easily can be introduced is the core density by letting the core properties vary with its density. This variation is supposed to be continuous and was given in chapter 5.3. Other properties like the core tensile and compressive strength can be written in a similar manner. The constants relating the properties to the density can be extracted from tests but commonly only a curve fit to data from the material supplier should be satisfactory. The exponent \( n \) is proposed to equal 2 for cellular foams [2] but tests on other materials have shown that the elastic properties vary linearly with density, within a density regime. In reality, the choices of core material density is not continuous but discrete. However, this assumption is a great simplification in the analysis and the results should rather be interpreted as a first good approximation to serve as a starting point in the design process.

5.4.1 Determination of thicknesses

Consider a beam or a panel subjected to a transverse load and find the core thickness required to carry this load. Assume the materials are known and that the face thickness is already given. The structural member must fulfil all constraint given to it. The ultimate face stress is not only given by the yield or fracture strength of materials but may instead be limited by the wrinkling or, if a honeycomb core is used, dimpling strength of the face in compression. Hence,

\[ \min \left\{ \sigma_f, 0.5 \sqrt{E_f E_s G_s}, \frac{2E_f}{1-v_f^2} \left( \frac{t_f}{s} \right)^2 \right\} \leq \sigma_f = \frac{|M|_{\text{max}}}{t_f d} \]  

(5.14)

where \( \sigma_f \) is the lower of the tensile and compressive yield/fracture strengths. The local buckling stress in the above expression may be substituted for an expression valid for biaxial loading, if that is the case. The maximum shear stress in the core must be lower than the ultimate shear strength. Thus,
and the stiffness must have a certain value according to the above. In the case of a panel, the same thing must be performed in both x- and y-directions. Hence, there are three constraints to be considered when sizing the sandwich. If the face thickness is known then these equation give three values of the thickness \( d \), and the largest of these gives the design value.

For example, consider a simply supported symmetrical sandwich beam with uniformly distributed load, \( q \), where the maximum deflection must be less than \( \Delta \). In this case, the maximum bending moment is \( qL^2/8 \) and the maximum transverse force equals \( qL/2 \). The requirements for the minimum allowed thickness \( d \) is then the maximum of

\[
\frac{qL^2}{8t_f\hat{\sigma}_f}, \frac{qL}{2\hat{\tau}_f}, \frac{qL^2}{16G_cA\left[1 + \sqrt{1 + \frac{20\Delta G_c^2}{3qE_f t_f}}\right]}
\]

(5.16)

If the core thickness was known beforehand, the face thickness can be calculated in a similar manner. For a sandwich panel, the sizes can be found in the same way, it is all a matter of calculating the deflection as function of the geometry and using formulae valid for sandwich plates for the strength constraints, e.g., wrinkling formulae for biaxial loading as in chapter 3.11. If the sandwich is subjected to in-plane compressive loads then the constraints will be buckling constraints. The face strength \( \hat{\sigma}_f \) must now though be taken as the compressive strength only. However, this strength should now be compared with the applied stress \( P/2t_f \), and hence this constraint only influences the choice of face thickness. The overall buckling constraints can be found in chapter 3.10 and 4.10. For a sandwich column we can write this as

\[
d \geq \frac{P}{2G_c}\left[1 + \sqrt{1 + \frac{8\beta^2 L^2 G_c^2}{\pi^2 E_f t_f P}}\right]
\]

(5.17)

taking \( n = 1 \) as the lowest buckling load and \( \beta \) is the edge-constraint factor defined in chapter 3.10.

Once again, the same procedure can be used for panel buckling problems.

A way of improving a design is to recognise that a design that allows for simultaneous failure of the face and the core has a potential of being closer to the optimum design, that is, to find a design that stresses all components to their limits at maximum load. For example, to design for simultaneous face yield and core shear fracture, or face wrinkling and core shear failure. It is merely a matter of recognising the important failure modes that may appear and finding the sizes and even materials that allow for simultaneous failure in these modes. This is done in the same way as when constructing the failure mode maps given in chapter 5.2.

In the simplest possible way the design of a sandwich can be described as follows;

(i) The core thickness is determined by the fact that it should not fail under the applied shear stress. That is
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\[ d \approx t_e \geq \frac{[T]_{\text{max}}}{\hat{\tau}_c}, \text{ where } \hat{\tau}_c \text{ may vary in different directions if the core is orthotropic.} \]

(ii) The face thickness is then determined by that the faces should be able to carry the applied bending moment, or

\[ t_f \geq \frac{[M]_{\text{max}}}{\hat{\sigma}_f d}, \text{ where } \hat{\sigma}_f \text{ also may vary in different directions if the faces are orthotropic.} \]

(iii) The calculated component thicknesses should then be checked so that

\[ w \leq \Delta, \text{ where } w \text{ is the maximum deflection and } \Delta \text{ is the allowed deflection.} \]

(iv) Other constraints, such as local buckling, dimpling etc., must subsequently be checked, and, if violated, sizes or materials must be modified until all is satisfied.

If any constraint is violated then either \( t_f \) or \( t_c \), or both, must be increased until the necessary stiffness is obtained. A design of this kind is obviously simple and may in most cases serve as an initial design. This design could then be changed, allowing for dissimilar faces, orthotropic material components and a full set of constraints, in the pursuit of improved performance.

5.4.2 Sizing routine

In the more general case of a panel with dissimilar faces the same procedure can still be performed. Now, however, the face stress and strength in general vary between the two faces and in different directions so that

\[ \overline{\sigma}_{fx1} \neq \overline{\sigma}_{fx2} \neq \overline{\sigma}_{fy1} \neq \overline{\sigma}_{fy2} \text{ and also } \hat{\sigma}_{fx1} \neq \hat{\sigma}_{fx2} \neq \hat{\sigma}_{fy1} \neq \hat{\sigma}_{fy2} \]

The core shear stresses and strengths generally vary between different directions so that

\[ \overline{\tau}_{cx} \neq \overline{\tau}_{cy} \text{ and also } \hat{\tau}_{cx} \neq \hat{\tau}_{cy} \]

The maximum appearing stresses are calculated using the formulae in chapter 4.4. The deflection and buckling load are functions of the orthotropic material properties, and calculated using some of the expression given in chapters 4.9 and 4.10, depending on boundary conditions etc. All constraints must be checked and it is of course the constraint giving the largest of value of the sought parameter, \( t_c \) or \( t_f \), that must be chosen. This also gives and indication in what property improvements are mostly needed.

A simple way of improvement is then to tailor the properties so that all constraints give similar values of that parameter. For example, when going through all the calculations one finds that face 1 will fail in tension prematurely in the \( y \)-direction, one can then improve the tensile strength for face 1 in \( y \)-direction by for example adding a layer of unidirectional reinforcement. Remember that the choice of materials also offers an opportunity to tailor the behaviour; an overly performing core can be substituted with a lighter one and still satisfy all the requirements. When that is done all the calculations must be performed all over again, since now the stiffnesses have changed, thus also the maximum deflections, the stress and strain field, and the component strengths. Now, maybe another failure mode is critical or some other components are overly strong so that some thicknesses must be reduced or another material is better suited. Dealing with this many parameters

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as in the case of an orthotropic panel, the procedure will be one of trial-and-error, incrementally varying core and face properties until a satisfactory results is obtained. The routine can be schematically represented as follows;

1. Preliminary design

2. Calculate stiffness and strength properties \( D, S, \bar{s} \), buckling loads, etc.

3. Calculate the structural response \( w, \bar{\omega}, \sigma \), etc.

4. Check all contraints: \( \bar{s} < \bar{\sigma}, w < \Delta \), etc.

5. Are all the constraints satisfied?
   Can changes be made to improve the design?

   6. YES
      Modify component sizes, properties and/or materials and continue from 2

   6. NO
      Design Ok!

Figure 5.3 Schematic of the design procedure for sandwich structures.

In words the routine is to start with a preliminary design, something feasible achieved by some simple sizing rules and using materials that the designer assumes appropriate. Stiffness properties are then calculated using formulae given in chapters 3.2, 3.4 and 4.2. Strength properties can be achieved in several ways, from data manufactures data sheet, tests, or calculated by some proposed formulae as for example for fibre composites. The structural analysis is then performed, e.g. using the proposed formulae in chapter 3.10 and 4.10-12, finite element analysis or any other tool that would give the deflection, transverse force and bending moment values. Once that is done the constraints must checked, that is, that the maximum deflection of the structure is less than the allowable, the stresses nowhere in the structure exceed the strength, that buckling does not occur, no wrinkling, no dimpling, or that any other constraint is violated. Even if all constraints are satisfied there might be room for improvements, some components may be over-sized, weight-savings might be possible. As long as the designer believes that the structure may be improved, modifications can be implemented, properties up-dated and the calculations performed again.

5.4.3 Single parameter optimum
This section describes the simplest possible type of optimum design criteria, that is, e.g., finding minimum weight of the sandwich using only one constraint [3-4]. Here, the constraints used will be a given flexural rigidity, flexural strength or dimpling strength. As will be seen in the following section, using just one constraint is far from adequate but since the analysis and result are so simple they may serve as a first choice in a design process. The results in this section are valid for both beams and panels.
(i) **Flexural rigidity**

For a minimum weight sandwich of a specified flexural rigidity, the weight of the core should be twice that of the faces combined (similar or dissimilar faces), or

\[ W_c = 2W_f \]  

(5.18)

One must remember that the flexural rigidity is only one of two stiffness parameters governing the stiffness of a sandwich.

(ii) **Flexural strength**

For a minimum weight sandwich of specified flexural rigidity the weight of the core should be equal to that of the faces combined.

\[ W_c = W_f \]  

(5.19)

This, however, only considers the bending moment capacity of the sandwich and not the load bearing of transverse forces. As above, assuming dissimilar faces gives the same result.

(iii) **Face dimpling**

The optimum face thickness to avoid face dimpling is to choose a core that weighs one-third of the combined weight of the faces, i.e.

\[ 3W_c = W_f \]  

(5.20)

5.4.4 **Minimum weight for given stiffness**

The above analysis only considered one constraint at the time but also only the flexural rigidity instead of the total stiffness. The constraint is more often on the deflection of the structure than on the flexural rigidity. This section will describe methods to find the optimum face and core thicknesses for a given stiffness, providing the materials are predetermined. It will also be assumed that the core properties can be varied by choosing different densities. The formulae given in this section are derived for a sandwich beam but can be rewritten to be valid for a panel case. As a first case it might be easy enough to use the beam formulae on the panel problem by simply substituting \( P \) for \( qa \) and \( L \) for \( a \), and performing the analysis in both \( x \)- and \( y \)-directions.

(i) **Core properties predetermined**

If the core material and its density is predetermined then \( \rho_c \) cannot be used as a variable in the optimisation which is equivalent to letting \( n = 0 \). Performing the analysis as above, that is, solving \( t_f \) as function of \( d \) gives in the beam case

\[ t_f = \frac{2k_fL^2}{E_f} \left[ \frac{Cd^2}{L} - \frac{k_fd}{G_c} \right]^{-1} \]  

(5.21)

and substituting into the weight equation gives the total weight as function of \( d \) alone

\[ W = \frac{4\rho_c k_b L^2}{E_f} \left[ \frac{Cd^2}{L} - \frac{k_fd}{G_c} \right]^{-1} + \rho_c d \]  

(5.22)
Differentiating this equation yields a fourth degree equation in $d$ to which zeros must be found. This is a bit awkward and a solution is easier found by means of plotting $W$ versus $d$, and finding the sought minimum graphically.

The problem of optimum sizes for a column subjected to compressive loads is given in the same manner since the constraint is a given buckling load. The overall buckling depends on the two stiffness parameters $D$ and $S$ just as the compliance $C$. A solution may therefore be found in the same manner as outlined above for the bending case. This problem has been solved for a plate buckling case by Kuenzi [3].

(ii) Core properties varying with density

In this case there are three variables, two thicknesses and the core density. By using the definition of the core material properties and inserting in the compliance equation and writing the equations in the same form as in [5] one arrives at the following relation for the core density

$$\rho_c = \left( \frac{k_s E_f}{C_G \Delta E_f d^2 - 2PLk_b} \right)^{\frac{1}{n}} \tag{5.23}$$

The total weight per unit area can now be written as function of $t_f$ and $d$ as

$$W = 2\rho t_f + t_c \left( \frac{k_s E_f}{C_G \Delta E_f d^2 - 2PLk_b} \right)^{\frac{1}{n}} \approx 2\rho t_f + \frac{1}{A} \left( \frac{At_f d^2}{n} \right)^{\frac{n+1}{n}} \frac{1}{(Bt_f d^2 - 2PLk_b)^{\frac{1}{n}}} \tag{5.24}$$

where

$$A = \left( \frac{k_s E_f PL}{k_b C_G} \right)^{\frac{1}{n}} \text{ and } B = \frac{\Delta E_f}{k_b} \tag{5.25}$$

In order to find an optimum, the partial derivatives $\partial W/\partial t_f$ and $\partial W/\partial d$ should be set to zero, giving the two equation for the unknowns $t_f$ and $d$.

Taking $n = 1$ in the above expression, that is, linearly varying core properties with density, yields no optimum solution. However, the weight decreases asymptotically to a finite value when the core thickness is increased while the face thickness and core density simultaneously are decreased. This is, however, in reality no optimum. By letting the face thickness be predetermined leads to the same result as above, infinitely thick and infinitely light core yields a minimum weight. Hence, for these cases the optimum design is by choosing as thick and as light core as possible in conjunction with other constraints on the structure. This leads automatically to a given minimum face thickness. By instead predetermining the core thickness $d$, which often is stipulated in practical cases due to requirements on the space or thermal insulation, it is possible to find an optimum face thickness and to that a corresponding core density by plotting $W$ versus $t_f$ and graphically determine the optimum face thickness.

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Following the derivation by Gibson [5] assuming \( n = 2 \), which is the theoretical value for foams [2], an optimum can be found from the conditions \( \frac{\partial W}{\partial t_f} = \frac{\partial W}{\partial d} = 0 \). After some algebraic exercise one arrives at the optimum thicknesses [5]

\[
d = 4.34 L \left[ \frac{\rho_f k_f}{E_f} \right] \frac{P C_c k_c^2}{k_i^2 \Delta^2} \]^{\frac{1}{3}} \quad \text{and} \quad t_f = \frac{6k_c P L^3}{\Delta E_f d^2} \tag{5.26}
\]

Substitution back into expression above gives the optimum core density. Test with different material and thickness combinations performed in [5] showed that designs using the above sizing rules give structures with optimum stiffness to weight ratios.

5.4.5 Minimum weight for given strength

If one now instead is aiming at finding a minimum weight design for a given strength it is convenient to use the failure mode maps described in chapter 5.2. One approach is to use each of the failure modes as constraints and optimise the structure with respect to this constraint alone, using the methodology described above for the stiffness problem. One may then compare the results for the different "optima" and on the basis thereof choose a minimum weight design. The chosen design should then be plotted on the failure mode map to ensure that it will fail in the given mode. Another approach, used in [6], is to observe that in a minimum weight design, the faces and core should fail at the same load. This is same reasoning as considering that an optimum design is one in which all constituents are stressed to their ultimate limit. If considering only the three dominant failure modes, face yield/fracture, face wrinkling and core shear, an optimum sizing technique could be as follows. Assuming that the design applies either simultaneous face yield and core shear or simultaneous face wrinkling and core shear. Using the methodology in [6] yields the following.

(i) Simultaneous face yield - core shear

Assume that we want a design for simultaneous face yield and core shear failure which has a minimum weight. We can express the face thickness governing the face yield in a symmetric sandwich beam and the core density governing core shear fracture as

\[
t_f = \frac{k_f L}{\sigma_f P d} \quad \text{and} \quad \rho_c = \left( \frac{k_f P}{C_c d} \right)^{\frac{1}{m}} \tag{5.27}
\]

which of course also could be written for a sandwich with dissimilar faces. Substituting these into the weight equation, differentiating with respect to the thickness \( d \) and finding the zeros gives the optimum thickness in the beam case as

\[
d = \left[ \frac{2 \rho_f P L k_f}{\hat{\sigma}_f} \left( \frac{k_f P}{C_c} \right)^{\frac{1}{m}} m \right]^{\frac{m}{m-1}} \tag{5.28}
\]

The optimum face thickness and core density are found by back substituting the value of \( d \) to find \( t_f \) and \( \rho_c \). For a panel, on the other hand, the problem is a bit more difficult; \( k_f, k_r \) and \( C_r \) will differ in the \( x \)- and \( y \)-direction hence giving two values of \( t_f \) and \( \rho_f \). The procedure must thus be performed.
twice, once in each principal direction and the largest of the obtained $d$ must be taken. This may then provide aid in altering the design by e.g. improving the strength of a laminate face sheet in a given direction, choosing another core with another set of $C$, so that optimisation in both directions yields the same value of $d$.

(ii) Simultaneous face wrinkling - core shear

The face thickness for which wrinkling occurs at load $P$ as

$$t_f = \frac{2PLk_M}{d}(E_fC_fC_c\rho_f^\frac{2n}{3})^\frac{1}{3}$$

along with the core density for which core shear failure occurs at load $P$, one can write the weight as function of $d$ alone. Differentiating with respect to $d$ and finding the zero to this equation leads to the optimum

$$d\left(\frac{2n}{3m-2n+1}\right)^{\frac{2n}{3m-2n}} = \left(\frac{k_IP}{C_u}\right)^{\frac{2n}{3m-2n}} \frac{3m-3}{3m-2n} \left(\frac{E_fC_fC_c}{4\rho_fP\kappa_M}\right)^{\frac{1}{3}}$$

The analysis derived above can easily be modified for use on panels by simply changing the loading parameters $k_M$ and $k_T$. Even other load cases may be considered in similar analyses. Flügge [7] considered overall buckling and face wrinkling and derived minimum weight design thickness $t_f$ and $\rho_f$. The number of combinations of different failure modes and other constraints are vast and will not be given here. The methodology outlined here could be used for any other constraints given on the structure.

5.4.6 General optimisation techniques

In the general case there may be a large number of different types of constrains on a structure and the aim may not always be to minimise the structural weight. The target of the optimisation problem is called the objective function and may be, e.g., minimum weight, minimum cost, maximum stiffness. It may also involve procedures to minimise maxima, e.g. minimise the maximum stress. For a sandwich a problem statement could look like

**Objective function:**

Minimise $W = 2\rho_f t_f + \rho_c t_c$

**subjected to constraints:**

- Tensile face stress: $\sigma_f \leq \bar{\sigma}_f$
- Compressive face stress: $\min\left\{\bar{\sigma}_f, 0.5 \sqrt{E_fE_cG_c}, \frac{2E_f}{1-v_f^2} \left(\frac{t_f}{L}\right)^2\right\} \leq \sigma_f = \frac{|M|_{\text{max}}}{t_f d}$
- Core shear stress: $|\bar{\tau}_c| \leq \bar{\tau}_c$
- Deflection: $w \leq \Delta$
**Design Procedures**

*Limits on design variables*

\[ t_f^L \leq t_f \leq t_f^U \]

\[ t_c^L \leq t_c \leq t_c^U \]

\[ \rho_c^L \leq \rho_c \leq \rho_c^U \]

where \( U \) and \( L \) means upper and lower limits, respectively. Ringertz [8] used analytical expressions for the objective function as well as all constraints and their derivatives and used mathematical programming techniques to optimise sandwich beams with different boundary conditions subjected to different load cases. Such methods proved not only as accurate but much faster numerically than performing the optimisation using multi-purpose optimisation code [9,10]. However, multi-purpose optimisation programs, such as the OASIS-system [10], based on an iterative process with finite element calculations and the use of numerical derivatives, can be used to find optimum design of both large and complex sandwich structures [11].

**References**


CHAPTER 6

ANALYSIS AND DESIGN
BY FINITE ELEMENTS

This chapter covers the basic aspects of stress analysis and design of sandwich constructions by the finite element method (FEM). A thorough knowledge of FEM is required as well as a fundamental understanding of the mechanical behaviour of sandwich structures, before the user is advised to use FE programs for sandwich analysis and design.

6.1 General Remarks

The finite element method is the most general and versatile engineering tool for analysis and design of structures [1]. Commercially available FE programs such as NASTRAN, ANSYS, ABAQUS, NISA etc. can cope with very complex problems. The key to a successful use of FEM is, however, that all of the following types of errors are kept under control:

1. **Modelling error.** The word "modelling" refers to the mathematical modelling of the real physical problem. Hence, the user must have a basic understanding of the physics of the problem to be solved. Adequate constitutive models for the materials have to be employed (linear elastic; elasto-plastic; visco-elastic; visco-plastic; rate dependent etc.), and an appropriate level of analysis must be selected (small or large displacements; small or large strains; static or dynamic analysis etc.). Apart from engineering experience, skill, and intuition of the user, a good way of certifying the FE-data is to compare computer results with analytical ones.

2. **Discretization errors.** Due to the approximate nature of FEM, errors will emerge in the results due to the use of a finite number of elements. The fineness of the FE mesh is crucial to the outcome of the analysis. In areas of high stress gradients, smaller elements are required than in areas of little variation. Since a too fine mesh might be too costly to run through the computer, the user is encountered with an optimisation problem on how to choose the FE mesh. Methods of automatic remeshing by the FE program during computations are being developed but are still not generally available in commercial codes ("adaptive" FE analysis). Therefore, experience and skill of the user and numerical and/or analytical verifications will be required to ensure reliable results. This means that the same problem should be analysed by two or more different meshes with successively finer elements, and that comparisons with closed-form analytical solutions shall be carried out if possible.
Another error source is the solution of non-linear problems as a set of linear ones, often with iterative corrections of the equilibrium equations [1]. This class of errors is addressed in section 6.7.

3. **Numerical errors.** When the computer solves the sets of equations that result from the FE discretization, numerical handling (truncation) errors may emerge. A good way of checking such errors is to investigate if the entire structure and parts of the structure are kept in equilibrium under applied loads and calculated support forces and/or section forces. If so, the solution is probably OK from a numerical point of view. If not, higher computer accuracy or remodelling is required [1].

### 6.2 Special Considerations for Sandwich Structures

Apart from the general comments on FE analysis given in section 6.1, special care has to be taken when analysing sandwich constructions due to the inhomogeneous and often anisotropic build-up of sandwich. In general, the following items will have to be kept in mind when analysing sandwich constructions with the finite element method:

1. **Core shear.** The shear deformation of the core (see chapter 3) has to be considered in most cases, be it static or dynamic; beam or shell; linear or non-linear analysis. General finite elements do not normally include shear deformation, since it usually is negligible for metallic structures. The user has to select elements that are adequate for sandwich analysis and design. Specific problems may occur for cores with very low shear modulus, e.g. low density PUR cores. If the FE formulation of the beam, plate or shell element is incorrect (which it might be also for commercial codes), the shear stiffness of the faces might dominate over the shear stiffness of the core even if the faces are very thin. The user is advised to check this by comparing with analytical solutions, experiments or full 3D FE analyses.

2. **Anisotropy.** The anisotropy of composite face materials has to be accounted for (even in the case of a $0^\circ/90^\circ$ fabric due to different material properties in the $45^\circ$ directions). Honeycomb cores have different shear moduli in different directions and this property has also to be included in the analysis.

3. **Local effects.** Due to the low stiffness and strength in the thickness direction of the sandwich, local load introductions, corners and joints must be checked by 2D or 3D analyses in a far larger extent than what is the case for metal structures. By the same reason curved panels with small radii of curvature (less than 10 times the sandwich thickness) will have to be analysed in 2D or 3D to account for the transverse normal stresses not included in shell elements.

Further precautions will have to be taken at dynamic, buckling and non-linear analyses, see sections 6.5 and 6.7.

### 6.3 Linear Analysis

This section covers linear static sandwich analysis by FEM. For such an analysis to be relevant, the physical problem to be solved must behave linearly (linearly elastic response of the material, and small deflections), and the loading must be applied slowly enough for inertia forces to be negligible.
(i) Beam analysis
For a structural component to be defined as a beam, the width of its cross-section shall be considerably smaller than the height. Sandwich beams are not frequently used as structural elements. However, beam theory is quite useful since it can be applied to sandwich panels subjected to cylindrical bending (see chapter 6.4). In this case, the apparent Young’s modulus $E' = E/(1-\nu^2)$ should be used for isotropic faces, and a corresponding modification should be done for anisotropic faces. Hence, the current description of sandwich beam elements is limited to plane beams.

The two most common beam elements are the linear two-node and the quadratic three-node elements, see Fig.6.1. Of these, the three-node element is the most efficient one [2]. Before the three-node elements are assembled to a beam structure, the interior node is usually condensed. To account for shear deformations, the beam rotation degrees of freedom $\theta$ must be independent of the slope $w'$ of the beam. The in-plane degrees of freedom $u$ are essential in case of axial loading and/or unequal faces of the sandwich.

For beams with constant cross-section and concentrated loads only, these beam elements will give exact results according to the sandwich beam theory if the element mesh is constructed so that loads and supports are situated at element interconnections only. For beams with uniformly distributed loads, two to three three-node elements or three to four two-node elements will usually give accurate results (errors within a few percent) [2], see also sample problem on the next page.

As a sample problem, a case analysed in [2] is given below: A sandwich beam, clamped at one end and simply supported at the other, is loaded by a point load. Material and geometric data are given in Fig.6.2. The boundary conditions are $w(0) = w(L) = \theta(0) = 0$.

Figure 6.1  Sandwich beam elements. (a) A two-node element and (b) a three-node element
Shear strains in the faces and normal strains in the core are neglected. Analytical solutions for midpoint deflection and support force at the roller support are:

\[
 w = \left( \frac{2P - 5R}{48D} \right) \frac{L^3}{2} + \left( \frac{P - R}{2} \right) L \quad \text{with} \quad R = \frac{P(5 + 24\theta)}{2(8 + 24\theta)}
\]

as given in section 3.12. The numerical values obtained using two three-node beam elements were identical to the analytical solution:

\[
 w = 55.436 \text{ mm}, \quad \text{and} \quad R = 163.202 \text{ N}
\]

(ii) Plate analysis

Analogously to the beam elements above, the four-node and the eight-node plate elements are most common. Fig.6.3 shows the eight-node element [3]. The four-node elements has nodes only at its corners. Also here, the quadratic eight-node element is superior. Both types of elements may take general quadrilateral shapes as is normal for isoparametric elements (the eight-node element can also have curved boundaries). The comment on the in-plane degrees of freedom in the previous section is equally valid for the plate elements.
For uniformly loaded panels, a mesh of two to three elements in each direction is usually sufficient, see the sample problem below: A simply supported square sandwich plate with uniform load is considered [3], Fig.6.4.

One quarter of the plate (due to double symmetry) is analysed using four different meshes as shown in Fig.6.5. Here, $NA$ indicates the number of active variables in the analyses. Due to the non-existence of in-plane loads, and a symmetric sandwich, the in-plane variables $u$ are inactive (set to zero).

The effect on maximum deformation and maximum stresses of using eight-node plate elements with two different shear moduli of the core (to illustrate influence of shear deformations) are given in Tables 6.1 and 6.2 for the four different meshes.

<table>
<thead>
<tr>
<th>Mesh</th>
<th>$G_c = 22$ MPa</th>
<th>$G_c = 220$ MPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>1x1</td>
<td>20.356</td>
<td>14.365</td>
</tr>
<tr>
<td>2x2</td>
<td>21.243</td>
<td>15.156</td>
</tr>
<tr>
<td>4x4</td>
<td>21.262</td>
<td>15.165</td>
</tr>
<tr>
<td>6x6</td>
<td>21.263</td>
<td>15.166</td>
</tr>
<tr>
<td>exact</td>
<td>21.3</td>
<td>15.2</td>
</tr>
</tbody>
</table>

Table 6.1 Midpoint deflection (mm) for simply supported plate
Mesh | normal stress in face (MPa) | shear stress in core (MPa)
--- | --- | ---
1x1 | 347 | 0.675
2x2 | 333 | 0.673
4x4 | 333 | 0.679
6x6 | 333 | 0.681
exact | 333 | 0.684

Table 6.2 Maximum stresses in simply supported plate

(iii) Shell analysis
Since plate elements often are obtained by degenerating shell elements [4,5], most of what was mentioned above for plates is also valid. The specific issue that arise is the curvature(s) of the shell middle surface. If the smallest radius of curvature is in the order of magnitude of the thickness of the shell (less than ten times), then local effects such as normal stresses in the core have to be considered in the analysis.

(iv) Application examples
Three examples of linear analysis of structures are given in this section. In Fig.6.6, the element subdivision [3] of a refrigerated sandwich superstructure for trucks is shown. The panels are modelled by plane eight-node sandwich plate elements, the supporting beams by three-node beam elements, and stiffeners and reinforcement profiles (e.g. L-bars at roof/wall and wall/floor connections) by three-node stringer (rod) elements. Sub-structuring [1] is employed to make the analysis rational, and to take advantage of symmetry (side walls equal).

The load cases include mechanical loading introduced by deep frozen beef sides hanging from the roof and thermal loading from a temperature gradient between inner and outer walls. The
mechanical loads are: vertical load with a dynamic magnification factor; horizontal load from 0.5g retardation, and vertical load with the truck supported on two diagonally situated wheels.

The refrigerated sandwich container tank [6] in Fig.6.7 is divided into plane, singly curved, and doubly curved sandwich shell elements, and the frame is represented by two-node beam elements. Loading include internal pressure, lifting, stacking, and thermal gradient.

In Fig.6.8, a balcony with sandwich floor and railing, and single laminate walls, is shown [3]. Eight-node plate elements are used for both the floor and the walls. A separate 2D plane strain model (inserted in the upper left corner) is employed for obtaining critical stresses (mainly delamination stresses imposed by bending) where the sandwich is transferred to a single skin.
### 6.4 Local Effects: Joints and Load Introduction

Solutions obtained by beam, plate or shell elements usually only give normal stresses in the faces (in the plane of the face), and shear stresses in the core. In many cases, other stress components may exist in the sandwich. In corners and joints, normal stresses in the core and delamination stresses in the faces and between the faces and the core may arise, see Fig. 6.8. Likewise, these types of stresses may be found in strongly curved parts and at load introductions such as supports and inserts. To trace these stress components, full 3D (or 2D) analyses has to be performed. If the aim of the analysis is to estimate the order of magnitude of the stresses, to compare different designs, or to determine local stiffnesses, then linear elastic analyses might suffice. On the other hand if the aim is to estimate the load carrying capacity of the sandwich with respect to local effect, then a non-linear analysis has to be performed and proper failure criteria applied. It should be kept in mind, however, that the latter type of analysis requires very high efforts with regard to costs and competence, and that accurate predictions of the failure load cannot always be obtained.

As a further example of an analysis of local effects, a bulkhead joint according to Fig. 6.9 [7] is given. Assuming that the structure has a considerable length perpendicular to the plane shown, and that loads and boundary conditions are uniform, a 2D model can be used for the stress analysis. Due to symmetry, only half of the joint is modelled, Fig. 6.10. Eight-node plane stress/plane strain isoparametric elements [1] are employed for laminates as well as core. Displacements and stresses are calculated for compression and tension loading, and failure in laminates and core are estimated by an appropriate failure criterion, the Tsai-Hill fracture criterion (unequal tensile and compressive strengths) for the faces and maximum principal stress for the core. It should, again, be emphasised that the failure load predictions only can be used for comparing competitive designs and not for getting absolute values. Interface stresses between core and laminates might be critical.

![Figure 6.9 Dimensions and loading of bulkhead configuration](image-url)
6.5 Dynamic Problems and Buckling
The primary issue that has to be addressed when studying dynamics and buckling of sandwich structures is, as before, the shear deformation of the core. This lowers buckling loads and eigenfrequencies compared to when shear is not significant, e.g. homogeneous metal structures. The influence is specially pronounced at higher eigenmodes since the length scale then is decreased. Hence, analysing sandwich constructions without proper representation of shear might results in severe and unconservative errors. Test problems for vibration and buckling of sandwich beams and shells are given in [2] and [8]. At very high eigenfrequencies, which can be of interest in acoustic problems, local bending and even local shear of the faces can occur.

6.6 Optimum Design
Rational design of structures can be achieved by mathematical optimisation techniques [9]. The designer defines the objective (function) of the design, the constraints, and the design variables. Iteratively and using either analytical [10] or numerical methods, the computer arrives at an optimum design. Very powerful use of optimisation in sandwich design is possible when the finite element method is utilised for the analysis of the basic mechanical problem [11,12].

6.7 Nonlinear Analysis
For many years, the finite element method has been established as the leading tool for analysis and design of complex structures. Nonlinear finite element analyses are being used increasingly in the design of structures. In linear finite element analysis one assumes that the displacements of the finite element assemblage are infinitely small, the materials are linearly elastic and that the boundary conditions remain unchanged during the application of loads. In the most general nonlinear analyses, none of these assumptions are valid.
The advantages of linear stress analyses, as compared to nonlinear analyses, are obvious; direct solutions may be obtained without costly load incrementations and equilibrium iterations, solutions for different load cases can be superimposed to find their combined effect and the number of material constants needed in the analyses are kept to a minimum. However, one may identify a series of reasons that can justify a relatively costly and complicated nonlinear analysis:

- In recent years there has been a shift in design requirements towards high performance and more efficient components in advanced applications (e.g. automotive industry, aerospace, nuclear industries). This calls for sophisticated nonlinear finite element analyses.

- Even if a linear approximation is adequate for the service states of a structure, it may be of interest to perform a nonlinear analysis for higher loads to investigate the behaviour of the structure after "first failure" in order to document that this does not cause a total collapse of the construction.

- Use in assessing existing structures whose integrity may be in doubt due to visible damage, special load not envisaged in the design stage or concern over material degradation.

- Help to establish the causes of a structural failure.

- Use in research: establish simple algorithms for analysis and design, to help understand basic structural behaviour and to test the validity of proposed material models.

- Use in the simulation of material processing and manufacturing, e.g. metal and glass forming or casting processing.

- Investigation of highly nonlinear structural behaviour such as collision incidents.

- When highly non-linear materials are used, such as PVC foam core or putty.

Nonlinear effects are often classified into geometric- (kinematic) and material nonlinear effects. Nonlinear geometric effects arises when the deformations of the structure are sufficiently large to cause stress redistributions, while nonlinear material effects are of importance when the materials in question are stressed beyond their linear limit. In most analyses, the nonlinear material behaviour is assumed to be time-independent. This is an idealisation of real behaviour, but will in many cases be adequate. Time dependence can manifest itself in many ways: the stress at a given strain usually increases with the imposed strain-rate (strain-rate dependency), the strain will increase if the stress is kept at a constant level for a substantial period of time (creep) and the stress will usually decrease if the strain is kept at a constant level for a substantial period of time (relaxation). In addition to the nonlinear effects mentioned above, a finite element analysis may take into account effects due to changing boundary conditions which arises particularly in the analysis of contact problems.

Nonlinear analyses of sandwich structures does in essence not differ from other structures, but due to its composite nature, it does pose some special difficulties to the design engineer. Some of the difficulties arise due to the nonlinear behaviour of anisotropic materials. Nonlinear material
behaviour in a sandwich member will also alter the stress distribution through the thickness of the sandwich. Furthermore, due to its composite nature, there exists a number of potential failure modes particular for sandwich member such as wrinkling and debonding of the face layers. In addition, load introductions and joints result in complex three dimensional stress distributions which may call for further and even more detailed nonlinear analysis.

An engineer who wants to perform a nonlinear finite element analysis of a given sandwich structure must find an answer to a series of questions regarding the degree of detail of the numerical model. Some of these are:

- Should the local bending stiffness of the face layers be included?
- Will transverse normal strain affect the solution?
- How detailed should the anisotropic material characteristics be modelled?
- What failure criteria should be applied for the materials used?
- If nonlinear material behaviour is significant, what is the stress distribution through the thickness of the sandwich layers?
- How "large" is the linear domain of the structure in question?
- Is it sufficient to assume perfect bonding between core and faces or should interlayer slip and possible debonding be modelled?
- What buckling modes can be expected and at what load level?

### 6.7.1 Nonlinear core materials

The most commonly used nonlinear material model is the so-called elasto-plastic material theory, which is based on three sets of descriptions:

- An explicit relationship between stress and strain which describes the material behaviour under elastic conditions, i.e. prior to the onset of plastic deformations.
- A yield criterion that indicates the stress level at which plastic flow starts.
- A flow rule describing the relation between stress and strain for post-yield behaviour.

In the elastic range, the stress-strain relation is described by the standard constitutive equation:

\[
\sigma = C \varepsilon
\]  

(6.1)

where \( C \) is the elastic stiffness matrix which in this case constants only constants, up to twenty-one independent constants for a generally anisotropic material but only two for an isotropic material. There exist a number of yield criteria, of which the most commonly used are the Tresca
and von Mises laws which are depicted geometrically in two dimensions in Fig.6.11. (In three dimensions, these yield functions extend to yield surfaces.)

These two yield criteria closely approximate plasticity behaviour in metals. The Tresca yield criterion states that yielding begins when

$$\sigma_e(\kappa) = \sigma_1 - \sigma_3$$

where $\sigma_1$ and $\sigma_3$ are the largest and smallest principal stresses, respectively (in three dimensions), and $\sigma_e$ is a material parameter that must be experimentally determined, and may be a function of the hardening parameter $\kappa$. As illustrated in Fig.6.11, von Mises law is a quadratic yield criterion:

$$\sigma_e(\kappa) = \sigma_x^2 + \sigma_y^2 + \sigma_z^2 - \sigma_x\sigma_y - \sigma_y\sigma_z - \sigma_z\sigma_x + 3\tau_{xy}^2 + 3\tau_{yz}^2 + 3\tau_{zx}^2$$

As indicated in eqs.(6.2) and (6.3), the yield criteria may, through $\kappa$, be dependent on the current degree of plastic strain. The most commonly used relation is termed strain hardening, and two different types of hardening are shown in Fig.6.11: If the subsequent yield surfaces are a uniform expansion of the original yield curve, the hardening model is said to be isotropic, while if the yield surface translates in stress space as a rigid body, kinematic hardening is said to take place.

After initial yielding, any increment of stress will cause a change in strain which is assumed to be divisible into an elastic and a plastic part. The elastic strain increment is related to the stress increment through an incremental version of eq.(6.1). There exists a number of theories that relate the plastic strain increment to the strain increment. The simplest form is the associated flow theory, which is often used for metals. This assumes that the plastic strain increment is proportional to a vector normal to the yield surface at the stress point under consideration.

Many core materials, such as honeycomb, balsa wood and expanded foams are anisotropic, and the nonlinear behaviour of such materials is not yet well known. However, commonly sandwich constructions are designed in such a way that shear dominates the stress state in the core. Hence, making sure that the material characteristics in shear are properly accounted for will usually be the most important task when modelling the nonlinear behaviour of the core. As a consequence of this, one must renounce on accuracy of the part of the material law that describes the relation between normal strains and normal stresses. Rothschild [13], has shown that it is possible to perform...
reliable nonlinear analyses with finite element programs which use plasticity models for isotropic materials. The basic problem to solve is then to map the nonlinear \( \tau - \gamma \) relation from shear tests performed on the core material to a \( \sigma - \varepsilon \) relation used in the finite element analyses, as illustrated in Fig.6.12.

\[
\begin{align*}
\sigma &= \frac{\sqrt{3} \tau}{2G(1 + v)} + \frac{1}{\sqrt{3}} \left( \frac{\gamma - \tau}{G} \right) \\
\varepsilon &= \frac{\sqrt{3} \tau}{2G(1 + v)} + \frac{1}{\sqrt{3}} \left( \frac{\gamma - \tau}{G} \right)
\end{align*}
\]

(6.4)

By employing these expressions it is shown [13] that this procedure can work well both for two- and three dimensional problems, i.e. when the converted \( \sigma - \varepsilon \) relation is used as input, the finite element program reproduces quite accurately the original \( \tau - \gamma \) relation.

To the author's knowledge, no nonlinear analysis has been performed on honeycomb and/or balsa wood core materials. However, it is reasonable to assume that this mapping procedure developed in [13] would yield good results for these types of core materials as well.

Another effect that one should be aware of is that when rigid cellular foams are manufactured the curing process results in a varying density through the thickness of the core. The density may be 10% lower in the middle of the layer than close to the surfaces. The material characteristics of expanded foams are highly dependent on the density of the foam. This means that the core will be considerably weaker close to its middle plane that in the areas closer to the faces. The consequence is that when the linear limit of the core material is reached, a plastic zone will develop close to the mid-plane of the core. Experiments and analyses, [14], have shown that this density variation significantly effects the nonlinear behaviour of sandwich beams.

6.7.2 Nonlinear face materials

The face layers in a sandwich construction are often made of metal sheets or fibre composite laminates. For metals there exists a number of material laws that take nonlinearity into account; e.g. nonlinear hyper-elastic materials, nonlinear hypo-elastic materials, elastic-plastic materials without hardening and elastic-plastic materials with kinematic or isotropic hardening. The laws
that governs the behaviour of these materials are discussed thoroughly in several text books and have been employed for several years such that engineers have gained experience in the use of these in numerical analyses.

Composite laminates which are made up of a number of laminae of fibre reinforced plastics are anisotropic, and theories that describe the nonlinear behaviour of these materials are not as well established as those for isotropic materials. Reinforced plastics are in general brittle in their behaviour, and accurate results can in most cases be obtained by assuming that the laminate behaves linearly until failure. When it comes to the strength characteristics of reinforced plastics, a somewhat empirical approach is often adopted, i.e. to compare actual failure envelopes in stress space with theoretical failure envelopes. These failure envelope differs little from the concept of yield surfaces for ductile materials. The failure criteria for an orthotropic material will be given in terms of strength characteristics in the principal material directions. The best approach is by laminate theory, which defines failure criteria for the individual plies and uses this as a basic building block. Failure criteria may be given by maximum stress, maximum strain, or quadratic failure criteria such as the Tsai-Hill or Tsai-Wu tensor theory. In stress space, the maximum stress criterion describes a boxy envelope, the maximum strain criterion will be rectangular in strain space and show the form of a trapezoid when transformed into stress space, while the failure envelope for quadratic criteria will be an elongated ellipsoid. These three types of failure criteria are depicted schematically in Fig. 6.13. A three dimensional plot would be needed to include the shear component in the failure criteria which then would extend to a failure surface.

![Figure 6.13 Failure criteria for single plies.](image)

Once a failure envelope in the orthotropic axes of a ply is determined, the off-axis failure envelope can be generated by coordinate transformation of stresses or strains. Each individual ply in the laminate will produce one failure envelope, which are put on top of each other as indicated in Fig. 6.14 for a maximum stress criteria. From this, a ply-by-ply strength analysis can be performed from which the first-ply-failure and the last-ply failure strength can be determined: The ply with the lowest strength in the load direction will rupture first, followed by a degradation in stiffness and eventually when the last ply fails, a total collapse is obtained.
6.7.3 Large deflection plate analysis

The main objective in a finite element analysis is to solve the equilibrium equation, i.e. to find a configuration of the structure where there is a balance between the externally applied loads and the so-called internal forces. The deformed configuration, and thus the internal forces of the structure is predicted from the structures current stiffness and the applied load increment. At each load increment corrective updates of the displacements (on the basis of the unbalanced forces) are repeated until a desired accuracy of the results are obtained, see refs.[1,15].

When a nonlinear finite element analysis is to be performed it is of great help to know approximately the behaviour of the structure in question. Important information in this context is the load at which nonlinear effects becomes significant. If material nonlinearities are of interest, one should try to estimate the stress distribution in the sandwich members and compare these to the stress-strain curves for the materials used. It is then possible to approximate the load level at which the elastic limits are exceeded. It is in general more difficult to predict the onset of geometric nonlinearities. However, it is generally recognised that linear bending theory for homogeneous beams and plates is only valid when the transverse deflections are small compared to the thickness of the member. This is also true for sandwich plates if the shear modulus of the core material is of the same order of magnitude as the Young's modulus of the face layers. However, as the transverse shear deformability of the core becomes more pronounced, the range of linearity will decrease.

Allen [16], states that in the limit (when the shear modulus of the core approached zero), linear theory is valid only when the transverse deflection of the sandwich member is small compared to the thickness of the faces which normally is much smaller than the total thickness of the sandwich. This means that nonlinear kinematic effects will in general occur at smaller deflections than in the case of homogeneous beams and plates. It is also for the analyst to investigate the potential buckling modes and estimate the critical loads associated with these before a complete, nonlinear analysis is performed. When it comes to overall buckling of sandwich plates and beams, one should keep in mind that transverse shear deformability must be accounted for. Furthermore, the compressed face layers in a sandwich may exhibit local instability, so-called wrinkling. Both global and local buckling of sandwich members are treated in detail elsewhere in this handbook, see chapter 3 and 4.
Rothschild [13], performed a detailed study of the nonlinear behaviour of clamped sandwich plates subjected to uniform pressure load. The plates considered represented a part of a large continuous sandwich ship hull supported by bulk heads and stringers, and was analysed by use of the two nonlinear finite element codes ABAQUS and FENRIS. The results were compared with measured results from experiments. The plates had glass fibre reinforced plastics in the faces and a rigid PVC foam in the core. The faces were modelled by using membrane elements (thus neglecting their bending stiffness), and solid elements were used to model the core. Much useful experience was extracted during this study, and the reader is referred to ref.[13] for more details. The main conclusions from this study can be summarised as follows:

- A finite element mesh of 28 (7 by 4) eight noded solid elements combined with 56 (7 by 4 for each face) four noded membrane element for one quarter of the plate was sufficient to obtained a converged solution.

- Geometrically nonlinear effects became significant (3-5% deviation from the linear solution) when the total deflection was equal to the total thickness of the sandwich.

- For this particular sandwich configuration, the nonlinearity due to plasticity in the foam core layer occurred at a much earlier stage than the geometrically nonlinear effects.

The results from an analysis of a four-point bend test, as shown in Fig.6.15, is presented to illustrate some nonlinear effects. Due to symmetry, only one half of the beam was modelled. The material characteristics of the faces is realistic for a glass fibre reinforced plastic while the data for the core material was taken from shear tests of an expanded PVC. The model was run using two different assumption about the core material: one linear elastic model and one elasto-plastic model with the same shear modulus in the linear range and a maximum capacity of approximately 3.5 MPa. Also, two different element meshes were used to illustrate the accuracy of the model. It is seen from Fig.6.15 that the two material models have the same initial stiffness, but while the one with the nonlinear core material loses almost all its stiffness at a deflection of less than 10% of the thickness of the core. The stiffness of the model using linear core material increases when the so-called "hammock" effect starts. The coarse models computes a deflection which is approximately 10% lower than the finer meshes in the linear range. This discrepancy between the meshes is approximately constant when the linear material model is used, but decreases when using the nonlinear material model. It is a well known fact that load introductions and joints result in a highly localised denting of the face layers in a sandwich construction which again may introduce substantial normal stresses and thus a complex three dimensional stress distribution in the core. These areas may require detailed three dimensional nonlinear analysis. If the object of the analysis was to obtain detailed information on the stress distribution in the beam (e.g. close to the load introduction point or near the clamped edge) a much finer mesh should be used than illustrated herein.
6.7.4 Miscellaneous
The theories developed to describe the behaviour of sandwich members normally assume a perfect bonding between the core layer and the faces. Nevertheless, interlayer slip may occur because of the finite bonding stiffness and this may have a significant effect e.g. in places where a high interlayer shear stress occurs due to load introduction. The engineer must in such cases consider the necessity of using a finite element model which takes into account this deformability of the adhesive. If the stress or strain at the interlayer exceeds the strength criteria of the adhesive, the bonding between core and face will fracture, and the sandwich member will obviously no longer behave according to sandwich theory. If a finite element model is meant to represent the post
buckling of a delaminated sandwich shell, one must obviously have a model which is capable of
handling such a response and possibly also consider crack growth.

Another commonly neglected effect in sandwich theories is the core strain transverse to the plate
or shell plane. This effect is usually negligible for flat sandwich plates [17,18], but its importance
increases as the curvature of the shell increases. In numerical analyses where large displacements
are included, neglecting this effect may introduce significant inaccuracies.

One of the features of sandwich construction is the opportunity, through efficient structural design,
of utilising each materials to the practical limit of their possibilities. This is reflected in the well-
known assumption of stress distribution through the thickness of a sandwich panel; The overall
moment is carried by large normal stresses in the thin face layers and the transverse shear stress is
approximately uniformly distributed over the thickness of the core. A consequence of such an
efficient load carrying member, is that when the linear limit of one of the materials is reached, the
cross section has only a small residual capacity. Hence, one should expect that a sandwich cross
section in it self will behave more brittle than the less "efficient" homogeneous cross section. One
should also note that the stress distribution described above is based on linear elastic material
behaviour. If nonlinear material behaviour is taken into account one must also reconsider the stress
distribution through the thickness of the sandwich member. McGeorge and Echtermeyer [14] has
shown that if the sandwich faces are relatively thick, these layers may carry a substantial
proportion of the transverse shear stress if the applied load corresponds to a core shear stress which
exceeds the linear limit.

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ANALYSIS AND DESIGN BY FINITE ELEMENTS


DESIGN OF CURVED BEAMS AND PANELS

Curved sandwich panels and beams under loading have a deformation and stress field that differs from plane panels and straight beams. The difference, of course, depends on the loading and geometry of the panel or beam itself. In certain applications curved panels may be stronger than plane, but as is often the case with beams, they also well be weaker. This chapter will state a few formulas and their application range to make it possible to utilise curved panels and beams to their limit. Two principal load cases are treated. One is curved panels loaded in axial compression, and the other is curved beams loaded in pure bending.

Curved sandwich panels are of great interest for applications in high speed boats, containers, tanks, aircraft, etc. to obtain more optimum design solutions. A condition is that suitable methods for design are available. There exist some papers on curved panels subjected to in-plane compression and transverse loading, but very few on curved sandwich panels in bending.

7.1 Curved Sandwich Panels in Compression
The theoretical compressive buckling load of sandwich panels with isotropic core and faces is presented. The panels are simply supported along the edges. The formulae are derived for panels with constant curvature and small thickness compared to the radius of curvature, $R$, and the axial and circumferential dimensions. The flexural stiffness of the faces is neglected in section 7.1.1 and 7.1.2, but included in section 7.1.3. Local buckling of the faces is considered in section 7.1.4. Data and equations Stein and Mayers [1] are used.

7.1.1 Isotropic core
The buckling coefficient of a thin curved sandwich plate with isotropic core can be obtained from equation (7.1). The buckling coefficient must be minimised with respect to the number of half waves, $m$ and $n$, into which the plate or cylinder buckles in axial and circumferential direction, respectively. $a$ is axial length of plate or cylinder and $b$ circumference of cylinder or circumferential width of plate as illustrated in Fig.7.1.
where $a$ is the axial and $b$ the circumferential length,

$$K_x = \frac{P^2 b^2 (1 - \nu^2)}{D \pi^2}, \quad \theta_b = \frac{D \pi^2}{S (1 - \nu^2) b^2} \quad \text{and} \quad Z_b = \frac{2 t_f b^4 E_f (1 - \nu^2)}{R^2 D} \quad (7.2)$$

$D$ is the flexural rigidity and $S$ the shear stiffness, defined in chapters 3 and 4. Values of some of the used parameters are tabulated in Table 7.1 for materials and geometries common to sandwich applications.

For the special case of an infinitely long curved plate $(a, m)$, eq.(7.1) can be reduced to

$$K_{x_0} \approx \frac{4}{(1 + \theta_b)^2} + Z_b^2 \frac{1 - \theta_b}{\pi^2} \quad \text{when} \quad \frac{Z_b}{\pi^2} < \frac{4 \sqrt{1 - \theta_b}}{(1 + \theta_b)^2}, \quad (7.3a)$$

$$K_{x_0} \approx \frac{Z_b}{\pi^2} \left(2 - \frac{Z_b \theta_b}{\sqrt{1 - \theta_b}}\right) \quad \text{when} \quad \frac{4 \sqrt{1 - \theta_b}}{(1 + \theta_b)^2} < \frac{Z_b}{\pi^2} < \frac{4 \sqrt{1 - \theta_b}}{\theta_b}, \quad (7.3b)$$

$$K_{x_0} = \frac{1}{\theta_b} \quad \text{when} \quad \frac{Z_b}{\pi^2} \geq \frac{4 \sqrt{1 - \theta_b}}{\theta_b} \quad (7.3c)$$

Eqs.(7.3) are not exact but are quite accurate for the indicated curvature parameter ranges. In Fig.7.2 buckling coefficients for an infinitely long curved panel is plotted versus the shear factor $\theta_b$. 

Figure 7.1 Sandwich panel with radius of curvature $R$ under uniaxial compression.
Figure 7.2 Compressive buckling coefficients for simply supported infinitely long curved isotropic sandwich plates.

Figure 7.3 Sandwich cylinder under axial compression
For the special case of a cylinder, as illustrated in Fig. 7.3, \( b = 2R \) and the equation can be rewritten in terms of \( a \) instead of \( b \). The resulting equation is

\[
K_{xc} = \frac{1}{m^2} + \theta_a \left[ 1 + \left( \frac{na}{mb} \right)^2 \right] + \frac{Z_b^2}{\pi^2 m^2} \left( \frac{1}{1 + \left( \frac{na}{mb} \right)^2} \right) ^2
\]

(7.4a)

with

\[
K_{xc} = \frac{P a^2 (1 - \nu^2)}{D \pi^2}, \quad \theta_a = \frac{D \pi^2}{S (1 - \nu^2) a^2} \quad \text{and} \quad Z_a^2 = \frac{2 t f a^4 E_f (1 - \nu^2)}{R^2 D}
\]

(7.4b)

When applied to a cylinder the value of \( n \) must be set to 0 or even integers greater than or equal to 4. The minimisation is performed analytically with respect to \( m \) and \( n \) and the resulting equations are:

\[
K_{xc} = \frac{1}{1 + \theta_a} + \frac{Z_a^2}{\pi^2} \quad \text{when} \quad \frac{Z_a^2}{\pi^2} \leq \frac{1}{1 + \theta_a},
\]

(7.5a)

Figure 7.4 Compressive buckling coefficients for simply supported sandwich cylinders.
DESIGN OF CURVED BEAMS AND PANELS

\[ K_{xa} = \frac{Z_a}{\pi^2} \left( 2 - \frac{Z_a}{\pi^2} \theta_a \right) \quad \text{when} \quad \frac{1}{1 + \theta_a} \leq \frac{Z_a}{\pi^2} \leq \frac{1}{\theta_a}, \]  

(7.5b)

\[ K_{xa} = \frac{1}{\theta_a} \quad \text{when} \quad \frac{Z_a}{\pi^2} \geq \frac{1}{\theta_a}, \]  

(7.5c)

Buckling coefficients for simply supported sandwich cylinders are plotted versus the shear factor \( \theta \) in Fig. 7.4.

### 7.1.2 Effect of thick dissimilar faces

In the preceding chapter, the face flexural stiffness was not taken into consideration. The flexural stiffness can have an important effect on the buckling load when the core is weak in shear. The exact equation will be stated, followed by approximate formulas which are illustrated in diagrams. The approximate formulas are fairly exact in the parameter ranges stated in the diagrams. The general equation for the buckling load coefficient is [2]

\[
K_b = \left( \frac{a}{mb} \right)^2 + \frac{Z_b}{\pi^2} \left( \frac{a}{mb} \right)^2 + \left( 1 + \frac{na}{mb} \right)^2 \left( 1 + \frac{na}{mb} \right)^2 \]  

(7.6)

where

\[ K_b = \frac{P_b^2(B_{f1} + B_{f2})}{\pi^2 d^2 B_{f1} B_{f2}}, \]  

\[ Z_b = \frac{b^4(B_{f1} + B_{f2})^2(1 - \nu^2)}{R^2 d^2 B_{f1} B_{f2}} , \]  

\[ \nu_s = \frac{\pi^2 d \nu}{b^2} = \frac{\pi^2 t B_{f1} B_{f2}}{b^2 G_{f1} (B_{f1} + B_{f2})} \]  

\[ d \nu = \frac{c B_{f1} B_{f2}}{G_{f1} (B_{f1} + B_{f2})}, \]  

\[ \Lambda = \frac{D_{f1} + D_{f2}}{d \nu} = \frac{(B_{f1} + B_{f2})(D_{f1} + D_{f2})}{d^2 B_{f1} B_{f2}}, \]  

and

\[ d \nu = \frac{d \nu B_{f1} B_{f2}}{B_{f1} + B_{f2}}. \]  

(7.7)

where \( \nu \) is the Poisson ratio of the panel, and \( B \) is extensional stiffness of the faces as

\[ B_{f1} = \frac{E_{f1} t_{f1}}{1 - \nu_{f1}^2} \quad \text{and} \quad B_{f2} = \frac{E_{f2} t_{f2}}{1 - \nu_{f2}^2}. \]  

(7.8)

If the flexural stiffness is neglected \( (D_f = 0) \) this equation is identical to the one stated in section 7.1.1, except that the curvature parameter is more general and that sandwich asymmetry is taken into account. For a symmetrical sandwich \( Z_b \) from eq.(7.2) is identical to that in eq.(7.7) and \( b \).

The equation is quite general for all ranges of parameters and is applicable for both curved plates and cylinders. The buckling coefficient is once again for a shell of finite length obtained by minimising \( K \) with respect to \( m \) and \( n \). For an infinitely long shell the minimisation should be carried out with respect to \( n \) and the buckle -wavelength ratio \( mb/na \).

For low shear stiffness and/or an infinitely long plate the buckling coefficient expression reduces to
\[
K_b \approx \frac{1}{\psi_b} + \frac{2Z_b \sqrt{\Lambda}}{\pi^2} \text{ when } > 0 \text{ and } \frac{Z_b}{\pi^2} \geq \frac{1 - \psi_b}{\psi_b} \tag{7.9}
\]

Plots of eq.(7.9) are illustrated in Figs.7.5-10, for different values of \(\psi_b\). The buckling coefficient for a cylinder is

\[
K_a \approx \frac{1}{\psi_a} + \frac{2Z_a \sqrt{\Lambda}}{\pi^2} \text{ when } > 0 \text{ and } \frac{Z_a}{\pi^2} \geq \frac{1}{\psi_a} \tag{7.10}
\]

where

\[
K_a = \frac{P_a a^2 (B_{f1} + B_{f2})}{\pi^2 d^2 B_{f1} B_{f2}}, \ Z_a^2 = \frac{a^4 (B_{f1} + B_{f2})^2 (1 - \nu^2)}{R^2 d^2 B_{f1} B_{f2}}, \ \text{and} \ \psi_a = \frac{\pi^2 t_c B_{f1} B_{f2}}{a^2 G \psi (B_{f1} + B_{f2})} \tag{7.11}
\]
Figure 7.6 \( \psi_b = 0.1 \)

Figure 7.7 \( \psi_b = 0.2 \)
Figure 7.8 $\psi_b = 0.5$

Figure 7.9 $\psi_b = 0.9$
7.1.4 Wrinkling of curved panels
Some tests have been conducted on curved panels in compression along the axis of curvature, see [2]. The panels were designed to fail in wrinkling after the well known formula given in section 3 for plane panels. The panels were curved with a radius of curvature about three times core thickness and had plane ends. The edges of the plane sections were free. The tests showed that these panels buckled at the same loading as similar plane panels with equal boundary conditions. It is therefore recommended to design curved and plane panels in the same way when wrinkling is concerned.

7.2 Curved Sandwich Beams in Bending
For straight sandwich beams the compressive and tensile forces from the bending moment are taken up by the faces, while transverse forces are taken up by the core as shear stress. The load distribution is different for curved sandwich beams, and the core in a curved sandwich beam under bending carries considerable normal stresses. These stresses can be tensile or compressive depending on the direction of the of the bending moment, and lead to stress concentrations in the intersection between the straight and curved parts.

The normal stresses in curved sandwich beams in bending can be a limiting design factor. Predictions of these stresses can be obtained by calculation or testing. Testing is expensive and used for verification only. Simple manual calculations are useful, but can yield uncertain results for curved sandwich structures. The Finite Element Method (FEM) is flexible enough to make good predictions of the special effects encountered with curved sandwich beams in bending.

Today the faces are often made of composite materials and the core of polymeric (plastic) foams, for which precise material properties are difficult to obtain. The material data are often non-linear
and the load response of the structure is also often non-linear. This makes it difficult to predict the exact failure modes and the failure loads. Non-linear FEM calculations and testing is necessary to evaluate design guidelines.

7.2.1 Stress and deformation
As mentioned earlier, when curved sandwich beams are loaded in bending, the core exhibits normal stresses, and stress concentrations occur near or at the intersections between the curved and straight parts of the beam. Testing and FEM analyses has shown that at this intersection the faces are bent in an s-shape. This initiates a pre-buckling state of the faces. Thus, the faces may fail prematurely in local buckling at a lower load than predicted.
7.2.2 Hand calculations on curved sandwich beams

It is possible to calculate by hand the stresses in a curved sandwich beam under pure bending load under the usual assumption of thin faces and small deformations. In the curved and straight parts of the beam the in plane stress is

\[
\sigma_f = \frac{M}{t_f d}
\]  

(7.12)

where \(d\) is the distance between the centre lines of the faces (see chapter 3.2), \(t_f\) is the face thickness and \(M\) is the bending moment per unit run.

The radial normal stress in the core (perpendicular to the faces) is [4]
where \( R \) is the inner radius of curvature of the beam. Tests and FEM has shown that there are stress concentrations at the intersections between different radiiuses of curvature. Up to 15% higher stresses have been observed at the intersection between straight and curved parts of sandwich beams. One should take this into consideration in design. No approximation formulas can be given for this case however.

### 7.2.3 Strength of curved sandwich beams in bending

The bending strength of curved sandwich beams compared to similar straight beams is given in Fig. 7.15. The compared beams have the same materials and geometry except from curvature. A bending efficiency factor \( \eta_b \) is introduced to give a relationship between the bending strength of the two types of beams. In fact \( \eta_b \) shows when the failure mode switches from face failure to core failure. \( \eta_b \) equals unity when the failure mode is face failure and is less when the core fails first.

The direction of the bending moment is important since many core materials fail at different stress levels in compression and tension.

\[
\eta_b = \frac{M_k}{M_r} = \frac{\sigma_{r,max} R}{\sigma_{f,t,f}}, (\eta_b \in (0,1))
\]  

(7.14)

where \( M_k = \sigma_{r,max} dbR \) is the maximum bending moment for curved beam

\( M_r = \sigma_{f,t,bd} \) is the maximum bending moment for straight beam
References


Sandwich structures are notoriously sensitive to failure by the application of highly localised external lateral loads such as point loads, line loads or concentrated surface loads of high intensity. This pronounced sensitivity towards the application of localised external loads is due to the inducing of significant local deflections of the loaded face into the core material of the sandwich panel, thus causing high local stress concentrations. Actually, the stress state corresponding to the ideal load transfer mechanism of sandwich panels, where the core material is loaded in a stress state of pure shear and the faces are loaded in a membrane state of stress, can be completely destroyed, and instead a complex multiaxial state of stress is obtained in the near vicinity of the external load application. The result of this may be a premature failure.

The local bending problem is illustrated in Fig.8.1 showing the simplest possible case, i.e., a symmetrically supported and symmetrically loaded sandwich beam in three point bending. The deformed beam configuration, shown in Fig.8.1, consists of two contributing parts:

(i) Overall bending of the sandwich beam due to the overall bending and shearing.
(ii) Local bending of the loaded face about its own neutral axis.

Figure 8.1 Undeformed and deformed sandwich beam in 3-point bending.
8.1 Theoretical Background - Elastic Foundation Analogy

Quite an impressive amount of work has been done on the subject of analysis of sandwich beams and panels, e.g. [1]-[3], including explanation and prediction of the different types of possible active failure modes. However, the problem of indentation, or local bending, has only been treated in a few references, [4]-[12]. The theoretical investigations [4]-[10], all treats the problem assuming linear elasticity of the constituent materials, and none of the cited references give an explicit description of the onset and development of indentation failure. Thus, the results obtained do not reflect the actual sequence of events experienced by the constituent materials during the development of failure, which of course is by nature a strongly non-linear phenomenon (geometrical and material non-linearity). Linear elastic analysis, however, does have the potential of giving valuable information about the parameters controlling the onset of failure.

The purpose of the present chapter is to introduce a simple method of analysing the stress field in the near vicinity of localised loads applied to the faces of sandwich beams. The mathematical details are not included in the present text, which instead concentrates on explaining the physics of the local bending problem and the solution of engineering design problems by use of graphical design-charts. The theory behind the method presented is described in details in refs. [8]-[10] and [13].

Considering the problem of a sandwich beam subjected to arbitrary localised lateral loading (or a sandwich plate in cylindrical bending), two solution parts are sought: 1. local bending of the loaded face, and 2. an overall solution which could be derived by use of classical sandwich beam theory (or a simple finite element solution). Considering now the local solution part, it seems reasonable to consider the relative deflection of the loaded face of a sandwich beam subjected to arbitrary localised loading, against the not loaded face, as being governed by some appropriate elastic foundation model. The simplest possible elastic foundation model is the well known Winkler foundation model [14-15], which assume that the supporting medium (the core) can be modelled as continuously distributed linear tension/compression springs. The Winkler foundation model, however, suffers a serious drawback in the present context, since it does not account for the possible existence of shearing interactions between the loaded face and the core material. This feature of the Winkler foundation model suggests that it becomes inadequate for deformations of short wave-length, in which the shearing deformations becomes important. Actually, the build up of interface shearing stresses adjacent to the area of external load application, are attributed significant influence (in certain cases) on the onset and development of failure of sandwich panels subjected to strongly localised loads.

The method of analysis presented in this chapter of the handbook is essentially similar to the Winkler foundation approach, except that the elastic foundation model introduced herein is a two-parameter elastic foundation model. Thus, a model, which accounts for the possible existence of shearing interaction between the loaded face and the core material, is applied in the local bending analysis (see refs. [8]-[10] and [13] for details). The local bending problem considered, i.e., the loaded face of a sandwich beam (subjected to arbitrary surface loads $p_x(x)$, $p_y(x)$) and the supporting core material, is illustrated in Fig.8.2.
According to the two-parameter elastic foundation model, the elastic response of the supporting medium can be expressed in the following form, relating deflections of the loaded face to the interface stress components measured per unit length of the considered face-beam

\[ q_z(x) = -K_z w(x), \quad q_x(x) = K_x u(x, t_f/2) \]  

(8.1)

where \( q_z(x), q_x(x) \) are the foundation shear and transverse normal stress resultants per unit length (referred to as stress distribution functions); \( K_z, K_x \) are the shearing and transverse foundation moduli of the supporting medium; \( u(x, t_f/2) \) is the longitudinal displacement of the lower fibre of the loaded face; and \( w(x) \) is the lateral displacement of the loaded face.

The elastic foundation moduli \( K_z, K_x \) are suggested related to the geometrical and material properties of the loaded face and the core material through the following expressions (assuming isotropic or nearly isotropic core material behaviour):

\[ K_z = 0.28 E_f \frac{E_c}{D_f}, \quad K_x = \frac{K_z}{2(1 + \nu_c)} \]  

(8.2)

By use of eqs.(8.2) together with classical beam theory it can be shown that the elastic response functions \( q_z(x), q_x(x) \) are governed by two ordinary non-homogeneous constant coefficient differential equations of sixth and seventh order respectively. The derivation of these equations will not be given here (the details are given in [13]), but the solution to the problem, i.e., the two stress distribution functions can be expressed as

\[ q_z(x) = A_0 + A_1 \cosh(\phi_1 x) + A_2 \sinh(\phi_1 x) + A_3 \cosh(\xi x) \cos(\eta x) + A_4 \sinh(\xi x) \cos(\eta x) + A_5 \sinh(\xi x) \sin(\eta x) + A_6 \cosh(\xi x) \sin(\eta x) \]  

(8.3)

\( A_j (j=0,...,6) \) are seven integration constants which have to be determined from the boundary conditions for the elastically supported loaded face. \( \phi_1, \xi, \eta \) are coefficients which can be expressed in terms of the elastic and geometrical properties of the constituent materials. The presentation given herein will not deal with the actual determination of the integration constants of \( A_j \) for a specific problem; instead the reader is referred to [13] for details.
The interface stress components $\sigma_{\text{int,local}}$, $\tau_{\text{int,local}}$ (transverse normal and shear stress components at the interface between the loaded face and the core material) induced by local bending can be expressed in terms of $q_z$, $q_x$ (per unit width):

$$\sigma_{\text{int,local}}(x) = q_z(x), \quad \tau_{\text{int,local}}(x) = q_x(x)$$

(8.4)

The important thing at this stage, is that a local bending solution has been derived and that the information obtainable from this solution is the local deflection of the loaded face, the local bending stresses in the loaded face, and the stress components at the interface between the loaded face and the core material. From this, it is clear that the local solution does not give any information about the decay of the local bending effects down through the core material.

A complete solution (denoted by superscript "total") containing the local bending solution parts (denoted by subscript "local") as well as the overall bending and shearing solution parts (denoted by subscript "overall") can be found by superposition of the approximate local bending solution (which can be derived from the interface stress distribution functions given by eqs.(8.3)) and an overall classical sandwich beam theory as derived by refs. [1]-[3], or as given elsewhere in this handbook. The complete solution can be represented in the form:

$$w_{\text{total}}(x) = w_{\text{overall}}(x) + w_{\text{local}}(x)$$

$$\sigma_{f,\text{total}}(x,z) = \sigma_{f,\text{overall}}(x,z) + \sigma_{f,\text{local}}(x,z)$$

$$\tau_{\text{int,total}}(x) = \tau_{\text{int,overall}}(x) + \tau_{\text{int,local}}(x)$$

$$\sigma_{\text{int,local}}(x) = \sigma_{\text{int,local}}(x)$$

(8.5)

An important question is the range of applicability of the method. Some of the most important questions in this context are:

(i) Is the assumption of "constant-value" foundation moduli $K_z$, $K_x$ generally justified?
(ii) Is the assumption of linear elastic material properties realistic?
(iii) Can the elastic foundation analogy be used for modelling all types of core-materials in use for structural sandwich constructions (polymeric foam, balsa and honeycomb)?

An elaborate discussion of these questions is given in [13], and only the most basic answers will be given in this section of the handbook (in very abbreviated form).

Unfortunately the answer to the first question posed is negative, i.e., it is not possible to select constant values of $K_z$, $K_x$ which are appropriate for deformations of any wave-length. The reason for this is that the shearing deformations of the core material becomes very influential for deformations with short wave-length. For practical sandwich panels, however, the bounds imposed by the vaguely formulated concept of "deformations of short wave-length" are not likely to be active, since the typical face thicknesses ($0.5 \text{ mm} < t_f < 10.0 \text{ mm}$), and the typical modular ratios ($25 < E/E_c < 1500$), will ascertain sufficiently large deflection wave-lengths to ensure the justification of the simple elastic foundation approach.
Without going into details about the answer to the second question posed, it can be said that the constituent materials of typical structural sandwich structures may well behave far more non-linear than is usually expected. The non-linear behaviour observable typically includes viscous, plastic as well as hygrothermal effects. However, the service conditions under which structural sandwich panels are employed, are usually sought to be well within the safe domain specified by the proportional limit of the constituent materials.

Finally, the answer to the third question posed is, that the proposed elastic foundation model suggested will usually be able to model most polymeric foams and balsa cores satisfactory from an engineering design point of view. The term "usually" is used because successful application of the elastic foundation approach is limited to cases where the micro structure of the foam and balsa core materials considered is at least one order of magnitude smaller than any of the characteristic dimensions (face thickness, core thickness, length, width) of the sandwich beam or plate under consideration. With respect to honeycomb cores, successful application of the elastic foundation approach is more questionable, but no more attention will be focused on this subject in this presentation. Instead the reader is referred to ref. [13] for a more rigorous discussion of the possible application of the proposed method for analysing honeycomb-cored sandwich panels.

8.2 Design with Respect to Localised Loads

8.2.1 Application of Method for Solving Engineering Design Problems

In order to illustrate the applicability of the theory introduced in the preceding chapter an approach is suggested for using the local bending analysis results for adjusting calculations based on classical sandwich beam theory. To convince the reader that such an approach can be justified from a physical point of view, attention is focused on the structure of the local solutions obtained from the elastic foundation formulation.

The component parts of the derived solutions (see eqs.(8.3)) consists of products of simple trigonometric and hyperbolic functions, thus implying that the solutions exhibits a wavy harmonic nature as well as an exponential (increase or decay) dependency of the longitudinal coordinate. As it is, the exponential terms appearing in the solutions ensures a very steep decay of the solutions with increasing longitudinal coordinate. Thus, it is observed, that the very nature of the elastic foundation solutions ensures that the local bending effects can be looked upon as local disturbances of the "ideal" stress and deformation states predicted from classical sandwich theory. The following steps are suggested for the solution procedure:

(i) The considered sandwich beam (or plate) problem is analysed by classical sandwich theory or by a rough finite element model using sandwich plate or beam elements. Thus, an overall solution is obtained.

(ii) The areas of the considered sandwich beam (or plate), where localised loads are introduced, are analysed by use of the elastic foundation formulation. The superposition of the local bending solution and the overall solution can be carried out if the following conditions are satisfied:

a. The local bending effects induced by each of the localised loads applied upon the sandwich beam (or plate) are not allowed to interfere with the bending effects induced by the other
localised loads or to interfere with boundaries (if any) of the considered sandwich beam. The meaning of this is that the local bending solutions should exhibit a sufficiently steep decay, so that the local bending solutions or edge effects will not influence each other. If this condition is not fulfilled the suggested simple approach (superposition) will not be valid. To determine the extension of the zone, in which the local bending effects are of importance, the so called decay length \( x^* \) is defined by (see ref. [13] for details):

\[
\frac{\lambda}{2} = \pi \sqrt{\frac{4D_f}{K_z}} \Rightarrow x^* \approx 2.66 t_f \sqrt{\frac{E_f}{E_c}}
\]  

(8.6)

where \( \lambda \) is the wave-length of the elastic deformations. If the distance from the point of introduction of some localised load to other local disturbances (external loads or boundaries) exceeds the decay zone distance \( x^* \), then the superposition suggested can be carried out without any problems, as the local bending effects do not influence each other significantly.

b. If the results obtained by the suggested superposition should be capable of giving a fairly accurate description of the state of stress present in the lower (not loaded) face of the considered sandwich beam (or plate), the thickness of the core material \( t_c \) (see Fig.8.1) should be larger than the decay zone distance \( x^* \). If this condition is fulfilled, no significant local bending effects are transmitted from the loaded face to the not loaded face (through the core material), and the overall solution obtained by classical sandwich theory or a simple FEM-solution will give adequately accurate results. If the condition is not fulfilled, however, the results obtained for the lower face are not very good, but as the results for the loaded face and the core-material adjacent to the loaded face are very good anyway (ref. [12]), this is no serious problem from a practical point of view, since the loaded face is the most severely loaded (and therefore the most interesting) part of the structure.

### 8.2.2 Example

The actual solution of a problem is illustrated by considering the simple case of a unit width sandwich beam in 3-point bending (point load). The geometrical notations for the problem are adopted from Fig.8.1 and the geometry, material data and the point load are as follows:

**GEOMETRY:**

\( L = 500.0 \text{ mm} \quad t_f = 4.0 \text{ mm} \quad t_c = 50.0 \text{ mm} \).

**FACES:**

\( E_f = 15.0 \text{ GPa} \) (corresponding to 50 vol. % E-glass/epoxy).

**CORE:**

\( E_c = 0.1 \text{ GPa} \quad \nu_c = 0.35; \) (PVC-foam, \( \rho_c = 100.0 \text{ kg/m}^3 \)).

**POINT LOAD:**

\( P = 100.0 \text{ N/mm (per unit width).} \)

Following the suggested two-step procedure the first task is to analyse the considered sandwich beam using classical sandwich beam theory. When this is accomplished the local bending analysis can be carried out, and in order to check whether the suggested method of superposition is valid, the decay zone distance \( x^* \) is evaluated using the approximate expression given by eq.(8.4). \( x^* \) is found to be \( x^* \approx 57 \text{ mm} \), and, as \( x^* \approx 57 \text{ mm} < L = 500 \text{ mm} \), it is concluded that the method will give sufficiently accurate results.
Figure 8.3 Lateral deflection of sandwich beam in 3-point bending. Only one half of the sandwich beam is considered ($0 \leq x \leq L$) due to the symmetry of the problem.

Figure 8.4 Normal stress distribution in upper and lower fibres of the loaded face.
Fig. 8.3 shows the lateral deflection $w_{\text{overall}}$ obtained by the classical sandwich beam theory, and the lateral deflection of the loaded face $w_{\text{total}}$ obtained by superposition of the overall and local solution parts. It is observed that the midpoint ($x = 0$) deflection of the sandwich beam is about 35 mm, and it is clear that the local bending effects really are very localised, as the two curves shown are identical except very near the point of load application ($x = 0$).

Figure 8.4 shows the distribution of longitudinal normal stresses $\sigma_f$ in the upper and lower fibre of the loaded face, respectively. From Fig. 8.4 it is observed that the local bending effects are of significant importance near the point of external load application ($x = 0$), as severe stress concentrations are induced in this area.

The curve paths in the right hand side of Fig. 8.4, which are straight lines, corresponds to the classical sandwich beam solution, and by extrapolating these lines to their intersection with the ordinate axis the peak stresses obtained from classical sandwich theory are found to be: upper boundary: $\sigma_f \approx -120$ MPa; lower boundary: $\sigma_f \approx -105$ MPa. Due to the local bending of the loaded face, the true stress state is very much different from that obtained by classical sandwich theory. At the upper boundary of the loaded face a compressive state of stress is present, and the peak value encountered ($x = 0$) is about -290 MPa. The peak stresses found in the upper fibre are about 2.4 times larger than predicted by classical sandwich beam theory. At the lower boundary of the loaded face a tensile stress is present in the regions very near the centre of the beam span, whereas the stress state is compressive some distance away from $x = 0$; say $x > 5-10$ mm. The change of sign in the longitudinal normal stress at the lower boundary of the loaded face can be attributed to the significant local bending contribution. The peak tensile stress in the lower fibre of the loaded face is $\sigma_f \approx 62$ MPa.

Fig. 8.5 shows the distribution of transverse normal and shear stresses, $\sigma_{\text{int}}$ and $\tau_{\text{int}}$ at the interface between the loaded face and the core material. The overall tendency observed is that severe stress concentrations are present at the centre ($x = 0$) and in the zones very near the centre. The wavy harmonic pattern as well as the characteristic decay of $\sigma_{\text{int}}$ with increasing value of $x$ is clearly observed, and the decay zone distance $x^*$ determined earlier to $x^* \approx 57$ mm corresponds to the positive (tensile) peak $\sigma_{\text{int}}$-value ($\sigma_{\text{int}} \approx 0.12$ MPa), which is only about 4% of the overall peak value encountered at $x = 0$ ($\sigma_{\text{int}} \approx -2.75$ MPa). For $x > x^*$ the $\sigma_{\text{int}}$-values fades out and goes to zero. Thus, the presence of transverse normal stresses is truly a local phenomenon.

Fig. 8.5 also shows the longitudinal distribution of the total interfacial shear stress $\tau_{\text{int}}$, and as expected (due to the symmetry) no shear stresses are present at the centre of the beam span. The overall tendency is that the shear stress component $\tau_{\text{int}}$ builds up and attains its maximum, $\tau_{\text{int}} \approx 1.0$ MPa, adjacent to the centre of the beam span after which the $\tau_{\text{int}}$-value approaches a constant value, $\tau_{\text{int}} \approx 0.9$ MPa, which equals the constant shear stress predicted to be present in the sandwich beam by classical sandwich beam theory.

For the considered example it is seen from Fig. 8.5, that the peak interfacial shear stresses induced by local bending are not very significant in magnitude compared to the shear stresses predicted by the classical sandwich beam theory. Thus, it is concluded the shearing interaction between the loaded face and the core is not important in the present example.
8.2.3 Parametric Effects

The local bending effects induced in sandwich panels by localised loading are influenced by especially two parameters: the modular ratio \( E_f/E_c \), and the face thickness \( t_f \). Other parameters, such as \( L \) and \( t_c \) in the earlier given example also influence the stress distribution, but they primarily exert influence on the overall bending and shearing of the sandwich panel considered, i.e., their influence is included in the description supplied by classical sandwich theory.

In order to illustrate the influence of the modular ratio \( E_f/E_c \) and the face-thickness \( t_f \) on the interfacial stress components, some results obtained from a brief parametric study will be presented. The problem considered, once again, is a sandwich beam in three point bending as in Fig.8.1, but this time only the loaded face is considered, as only the local bending effects are of interest. The base line geometry and the material parameters, from which all variations are made, are assumed to be as quoted in the preceding section.

Starting with the effect of altering the modular ratio \( E/E_c \), the distribution of the interfacial transverse normal stress component \( \sigma_{int} \) is shown in Fig.8.6 for three different values of \( E/E_c \). \( E_c \) is kept fixed to \( E_c = 0.1 \) GPa and the face thickness is fixed to \( t_f = 4.0 \) mm. From Fig.8.6 it is observed that the lower the value of the modular ratio \( E/E_c \), the higher the peak value of \( \sigma_{int} \) (at \( x = 0 \)). Furthermore, it is observed that the wave-length of the elastic response increases significantly as \( E/E_c \) is increased. This observation is in close agreement with the relation in eq.(8.6) from which an estimate of the wave-length of the elastic line (\( \lambda \)) can be expressed in terms of \( E/E_c \) and \( t_f \).
Figure 8.6 $\sigma_{int}$ vs. $x$ for three different values of $E_f/E_c$ ($E_c = 0.1$ GPa, $t_f = 4.0$ mm).

Figure 8.7 $\tau_{int}$ vs. $x$ for three different values of $E_f/E_c$ ($E_c = 0.1$ GPa, $t_f = 4.0$ mm).
Fig. 8.7 shows the longitudinal distribution of the interfacial shear stress component $\tau_{int}$ induced by local bending, for the same three different modular ratios used in Fig. 8.7. It is also observed that the peak value of $\tau_{int}$ decreases dramatically as $E_f/E_c$ is increased. Furthermore, it is seen that the wave-length of the shearing response increases with increasing $E_f/E_c$-values.

Having discussed the effects of altering the modular ratio $E/E_c$, the next issue is to evaluate the influence of altering the thickness $t_f$ of loaded face. Fig. 8.8 shows the longitudinal distribution of the interface transverse normal stresses $\sigma_{int}$ for three different thicknesses of the loaded face ($t_f = 1.0, 4.0$ and $16.0$ mm), and Fig. 8.9 shows the distribution of interface shear stresses $\tau_{int}$ for the same three $t_f$-values. For the calculations leading to the results shown in Figs. 8.8 and 8.9 the modular ratio has been set to $E/E_c = 150.0$ ($E_c = 0.1$ GPa).

The overall tendencies shown by Figs. 8.8 and 8.9 are similar to those shown by Figs. 8.6 and 8.7 (obtained by altering the modular ratio). Thus, it is recognised that the peak values of $\sigma_{int}$ and $\tau_{int}$, respectively, decreases significantly as the thickness $t_f$ of the loaded face is increased. Furthermore, it is observed that the wave-length of the elastic response functions increases with increasing $t_f$-values, just as predicted by eq. (8.6).
Figure 8.9 \( \tau_{\text{int}} \) vs. \( x \) for different values of \( t_f \) (\( E_f/E_c = 150.0, \ E_c = 0.1 \ \text{GPa} \)).

Figs. 8.6-8.9 illustrates that the local bending effects are strongly influenced by the modular ratio \( E_f/E_c \) as well as the thickness \( t_f \) of the loaded face. The results shown, which represents the interfacial stress distributions, indicate that the local bending phenomenon really is a very local phenomenon, and also reveals that it is possible to extract the most important results by plotting the peak values of \( \sigma_{\text{int}} \) and \( \tau_{\text{int}} \), as well as the maximum bending stress \( \sigma_f \) in the loaded face, versus \( E_f/E_c \) and \( t_f \).

With the aid of such graphs ("design-charts"), representing locally induced peak stresses as functions of the modular ratio and the face-thickness, it will be possible to adjust engineering design calculations, based on classical sandwich theory (analytical or FEM calculations), with respect to local bending ("indentation") with very little computational efforts involved.

8.3 Graphical Design-Charts

Graphical "design-charts" of the type described above can be used for estimating the severity of the stress concentration induced by local bending, and among the type of questions which can be answered are:

(i) Considering a sandwich with given face and core materials with given strengths, given face and core thicknesses; what magnitude of external load could be applied?

(ii) Considering a sandwich with given face and core materials with given strengths; what face-thickness is required to prevent failure of the loaded face or the core?

(iii) Considering a sandwich with given face and core materials with given thicknesses and strengths, given total external load; over how large an area should the external load be distributed to prevent the maximum allowable stresses (core and face) to be exceeded?
In the following sections, "design-charts" are presented for three different types of "unit" loads:

8.3.1 point load; \( P_0 \)
8.3.2 uniformly distributed load; \( p_0 \)
8.3.3 bending moment load; \( M_0 \)

The three cases, for which the peak stresses are presented, are all analysed assuming that the faces are loaded symmetrically about \( x = 0 \), and that the faces extends to infinity on each side of \( x = 0 \). Thus, it is assumed that solutions obtained (and thereby the obtained peak stresses) do not exhibit any interference with the boundaries of the faces, or with other localised loads.

8.3.1 Unit point load; \( P_0 \)

The simplest possible example of local bending, is that of a face subjected to a unit point load \( P_0 = 1.0 \) N/mm (force per unit width), as shown in Fig.8.10.

The locally induced peak interface transverse normal stress \( \sigma_{\text{int}} \) (compressive) and peak shear stress \( \tau_{\text{int}} \), as well as the maximum longitudinal bending stress \( \sigma_f \) (absolute value), have been calculated for various combinations of modular ratios \( E_f/E_c \) and face-thicknesses \( t_f \). The obtained results are shown in Figs.8.11, 8.12 and 8.13.

Fig.8.11 shows the peak value of the interfacial transverse normal stress component \( \sigma_{\text{int}} \) (compressive stress at \( x = 0 \)), as function of the modular ratio \( E_f/E_c \) and the face thickness \( t_f \). The peak value of the interface shear stress occurs at some position adjacent to the point of load application at \( x = 0 \).

Fig.8.12 shows the peak value of the interfacial shear stress component \( \tau_{\text{int,peak}} \) induced by local bending, as function of the modular ratio \( E_f/E_c \) and the face thickness \( t_f \). The peak value of the interface shear stress occurs at some position adjacent to the point of load application at \( x = 0 \).
Figure 8.11 $\sigma_{\text{int,peak}}$ (compressive) vs. modular ratio $E_f/E_c$ and face thickness $t_f$.

Figure 8.12 $\tau_{\text{int,peak}}$ (local bending) vs. modular ratio $E_f/E_c$ and face thickness $t_f$. 
The distance from $x = 0$ to the point of peak shear stress is highly dependent on the parameters $E/E_c$ and $t_f$, as indicated in Figs. 8.7 and 8.9. However, this distance is usually not very long, and its magnitude can be approximated by $\lambda/8$, where $\lambda$ is the wave-length of the elastic response as defined by eq. (8.6). Thus, $\tau_{\text{int,peak}}$ is located at $x \approx \lambda/8$ where:

$$\frac{\lambda}{8} \approx 0.67 \, t_f \sqrt[3]{\frac{E_f}{E_c}}$$

(8.7)

The overall tendencies observed in Fig. 8.12 are similar to those observed from Fig. 8.11, i.e., the $\tau_{\text{int,peak}}$ - values strongly depend on $E/E_c$ and $t_f$ in the way that very large values are obtained for combined small values of $E/E_c$ and $t_f$, and that $\tau_{\text{int,peak}}$ decreases rapidly as $E/E_c$ and $t_f$ are increased. It can be shown that $\tau_{\text{int,peak}}$ approaches zero asymptotically as $E/E_c$ and $t_f$ goes to infinity.

Fig. 8.13 shows the maximum longitudinal normal stress $\sigma_{f,\text{max}}$, representing the absolute values of the longitudinal normal stresses in the outer fibres ($z = t_f/2$, $-t_f/2$) of the loaded face at $x = 0$ induced by local bending in the loaded face, versus $E/E_c$ and $t_f$. The tendency observed from Fig. 8.13 is to some extent quite different from the tendencies observed from Figs. 8.11 and 8.12, and it is seen that $\sigma_{f,\text{max}}$ increases with increasing values of $E/E_c$, meaning that the more flexible the core material compared to the face material, the more severe bending stresses are induced in the loaded face. This phenomenon can be attributed to the circumstance, that the more flexible the core material compared to the face material, the less resistance against the local bending of the face will be exerted by the core material. Thus, the local curvature of the face will increase with decreasing value of the core modulus, causing the locally induced bending stresses to increase.
Concerning the effects of altering $t_f$ on the other hand, it is seen that $\sigma_{f,max}$ decreases significantly with increasing values of $t_f$. This result is similar to the results observed from Figs. 8.11 and 8.12, and is easily explained as the flexural rigidity of the face is proportional to the cube of $t_f$.

To conclude this section, it is repeated that the curves shown in Figs. 8.11, 8.12 and 8.13 can be used as graphical "design-charts", from which the peak interface transverse normal and shear stresses, as well as the maximum bending stress, can be found as functions of $E_f/E_c$ and $t_f$. These values can then be superimposed to the corresponding stresses obtained from classical sandwich theory, and a fair estimate of the severity of the stress concentrations induced by local bending, as well as overall bending and shearing, is obtained. Only three different $t_f$-values are represented in each of the quoted figures, but if information about the stresses for other face thicknesses is required, approximate results can be found from linear interpolation (or extrapolation) between the curves included in the figures.

### 8.3.2 Uniformly distributed load: $p_0$

This section describes the results obtained by analysing the case of an elastically supported face (extending to infinity in both directions) of a sandwich beam subjected to a partially distributed uniform load of intensity $p_0$. This load case is illustrated in Fig. 8.14.

![Figure 8.14 Elastically supported face (extending to infinity on both sides of $x = 0$) of sandwich beam subjected to partially distributed uniform load $p_0$.](image)

The magnitude of the distributed load $p_0$ is specified such that the total load $P$ (force per unit width) equals the unit load ($P = 1.0$ N/mm)

$$p_0 = \frac{1}{2\delta}$$  \hspace{1cm} (8.8)

Comparing this case with the case of a point load presented in the preceding section, it is clear that the results should be identical for small values of $\delta$, i.e., when the total load $P$ is distributed over a small area. Therefore, it is also clear that it is necessary to include $\delta$ as a parameter in the parametric study in order to evaluate the effect of distributing the external load over various areas. Thus, the locally induced peak interface transverse normal stress $\sigma_{int}$ (compressive), peak shear stress $\tau_{int}$, as well as the maximum longitudinal normal stress $\sigma_f$ (absolute value), have been calculated for various combinations of $E_f/E_c$, $t_f$ and $\delta$. The results are shown in Figs. 8.15, 8.16 and 8.17.
Fig. 8.15 shows the peak value of $\sigma_{\text{int}}$ (located at $x = 0$) as a function of the modular ratio $E/E_c$, the face-thickness ($t_f = 0.5$, 2.0 and 4.0 mm) and the load distribution parameter $\delta$ ($\delta = 1.0$ and 4.0 mm). It is observed, as expected, that the peak values of $\sigma_{\text{int}}$ are strongly dependent on $E/E_c$ as well as $t_f$ and, furthermore, that also $\delta$ is an important parameter. Comparing the results obtained for $\delta = 1.0$ mm with the results for a unit point load shown in Fig. 8.11 (for $t_f = 0.5$ and 2.0 mm), it is noticed that the $\sigma_{\text{int,peak}}$ values are almost similar, i.e., by distributing the unit load over the rather small distance $2\delta = 2.0$ mm, the $\sigma_{\text{int,peak}}$ values are not reduced significantly compared to the unit point load case shown in Fig. 8.11.

Considering the results obtained for $\delta = 4.0$ mm in Fig. 8.15, the results are somewhat different; for $t_f = 0.5$ mm (very thin face) it is seen that the $\sigma_{\text{int,peak}}$ values are reduced significantly (especially for low values of $E/E_c$), while no significant effects are observed for larger $t_f$-values. Thus, it is recognised that the influence on the local bending effects of distributing the external load over some area is especially important for very thin faces, and that the effect of distributing the load over larger areas tends to diminish as the face-thickness $t_f$ is increased, i.e., as the flexural rigidity of the loaded face is increased.

Figure 8.15 $\sigma_{\text{int,peak}}$ (compressive) vs. modular ratio $E/E_c$, face thickness $t_f$ and load distribution parameter $\delta$. 

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Fig. 8.16 shows the peak values of the interfacial shear stress $\tau_{\text{int,peak}}$, as function of $E_f/E_c$, $t_f$ ($t_f = 0.5, 2.0, 4.0$ mm) and $\delta$ ($\delta = 1.0, 4.0$ mm), and it is seen that the tendencies, concerning the influence of altering the load distribution parameter $\delta$ observed from Fig.8.16 are very similar to the tendencies observed from Fig.8.15. Thus, the conclusions drawn from Fig.8.15 goes for the results shown in Fig.8.16 as well, except that the peak values of the interface shear stresses are located some distance away from $x = 0$ (the actual distance is determined by $E_f/E_c$, $t_f$ as well as $\delta$, i.e., the relation in eq.(8.7) will only give a reasonable estimate of this distance for small values of $\delta$).

Comparing Fig.8.16 and Fig.8.12 for a unit point load, a very close match of the overall tendencies is observed, meaning that very large values of $\tau_{\text{int,peak}}$ are obtained for low values of $E_f/E_c$ and $t_f$, and that $\tau_{\text{int,peak}}$ decreases rapidly as $E_f/E_c$ and $t_f$ are increased.

Fig.8.17 shows the maximum longitudinal normal $\sigma_{f,\text{max}}$, as a function of $E_f/E_c$, $t_f$ and $\delta$. The values of $\sigma_{f,\text{max}}$ represents the absolute values of the bending stresses present in the outer fibres ($z = t_f/2, -t_f/2$) of the face at $x = 0$. Considering the results shown in Fig.8.17, it is clear that the overall results are different from the overall results shown in Figs.8.15 and 8.16 in the way that the $\sigma_{f,\text{max}}$ values tends to increase with increasing values of $E_f/E_c$, i.e., the more flexible the core material compared to the face material, the more severe local bending stresses are induced in the loaded face. The explanation for this phenomenon was given in the discussion of the results shown in Fig.8.13 ($\sigma_{f,\text{max}}$ vs. $E_f/E_c$, $t_f$ for a unit point load).
Considering the effect of altering $t_f$, it is observed that $\sigma_{f,max}$ decreases rapidly as $t_f$ is increased (i.e., as the flexural rigidity of the face is increased), and this result is similar to the results shown in Figs.8.15 and 8.16 (and Figs.8.11-8.13 as well).

The last item investigated, in the parametric study shown in Fig.8.17, is the effect of altering the load distribution parameter $\delta$, and it is seen that the overall results are similar to the results observed from Figs.8.15 and 8.16. Thus, it is recognised again that the stress reducing effect of distributing the external load over some area (as opposed to a point load) is most efficient for very thin faces, and that the effect tends to diminish as the face thickness $t_f$ is increased.

Concluding this section, it is repeated that the curves shown in Figs.8.15, 8.16 and 8.17 can be used as graphical "design-charts" from which the peak interface stresses, as well as the maximum bending stress (all stresses induced by local bending), can be found for given modular ratios $E_f/E_c$, given face-thicknesses $t_f$ and given load distribution parameters $\delta$. If information about the locally induced stresses are needed for other $t_f$-values than the ones included in Figs.8.15, 8.16 and 8.17 ($t_f = 0.5, 2.0$ and $4.0$ mm) approximate results can be found by linear interpolation (or extrapolation).

8.3.4 Unit bending moment Load; $M_0$

The last example to be analysed is the very simple case of an elastically supported face of a sandwich beam subjected to a unit bending moment $M_0$ per unit width acting about the neutral axis of the face, as shown in Fig.8.18.
Even though, the unit bending moment load case is very simple, it is also rather important, as it is seen in many practical examples. Thus, it can be observed (for instance in connection with fasteners and inserts) that concentrated bending moments are acting along with other loads such as point loads, distributed loads or axial loads.

![Figure 8.18 Elastically supported face (extending to infinity on both sides of x = 0) of sandwich beam subjected to unit bending moment load $M_0$.](image)

The parametric study on the bending moment load case is once again based on altering the modular ratio $E_f/E_c$ and the face thickness and the results obtained are shown in Figs.8.19 and 8.20. Fig.8.19 shows the peak transverse normal stress component $\sigma_{\text{int,peak}}$ versus $E_f/E_c$ and $t_f$, and Fig.8.20 shows the peak interface shear stress $\tau_{\text{int,peak}}$ versus $E_f/E_c$ and $t_f$.

The variation of the maximum longitudinal normal stress component $\sigma_{f,max}$ is not included in the parametric study, as it is given exclusively by the relation:

$$\sigma_{f,max} = \left| \frac{M_0 E_f t_f}{2 D_f} \right|$$  \hspace{1cm} (8.9)

By inserting the expression for $D_f$ into eq.(8.9) the following expression is obtained

$$\sigma_{f,max} = \left| \frac{6 M_0}{t_f^2} \right|$$  \hspace{1cm} (8.10)

Since $M_0$ is assumed to be constant ($M_0 = 1.0 \text{ Nmm/mm}$), it is concluded that $\sigma_{f,max}$ is inversely proportional to the square of $t_f$, and consequently $\sigma_{f,max}$ will attain constant values for constant values of $t_f$.

The peak transverse normal stress component $\sigma_{\text{int,peak}}$ shown in Fig.8.19 versus $E_f/E_c$ and $t_f$ is not located at $x = 0$ for the concentrated moment load case but rather some distance away. The actual location where $\sigma_{\text{int,peak}}$ will occur, is determined by $E_f/E_c$ and $t_f$, and the distance can be estimated by $\lambda/8$ (see eq.(8.7)). The distribution of the interfacial stress $\sigma_{\text{int}}$ will be skew-symmetric about $x = 0$ (odd function), and for $M_0$ positive as shown in Fig.8.18 $\sigma_{\text{int}}$ will be positive (tensile) for positive $x$-values and negative (compressive) for negative $x$-values.
As expected from the results presented in the preceding chapters, Fig.8.19 show that $\sigma_{int,peak}$ depends very strongly on both the modular ratio $E_f/E_c$ as well as the face thickness $t_f$. This dependency is shown by the fact that very large values of $\sigma_{int,peak}$ are obtained for low values of $E_f/E_c$, and this is especially the case if $E_f/E_c$ and $t_f$ attain low values at the same time (corresponding to deformations of very short wave-length). Again it is emphasised that the actual values of $\sigma_{int,peak}$ found for combinations of very low $E_f/E_c$ - and $t_f$-values are questionable, as the elastic foundation approach is incapable of describing the physics of the local bending problem for deformations of very short wave-lengths (see ref. [13] for details).

Only three $t_f$ values ($t_f = 0.5, 2.0$ and $8.0$ mm) are included in the parametric study presented in Fig.8.19, but if $\sigma_{int,peak}$ - values are requested for other values of $t_f$, it is possible to obtain very good approximate results by linear interpolation (or extrapolation) between the curves included in Fig.8.19. The distribution of the interfacial shear stress $\tau_{int}$ is symmetric about $x = 0$ (even function) for the concentrated moment load case, and the peak value of $\tau_{int}$ is located precisely at $x = 0$. For $M_0$ positive as shown in Fig.8.18, $\tau_{int}$ will attain negative values for all $x$. Considering the results presented in Fig.8.20, it is seen that the peak interface shear stress component $\tau_{int,peak}$ is also very strongly dependent of $E_f/E_c$ as well as $t_f$, and it is observed that this tendency is even more pronounced than for $\sigma_{int,peak}$ as observed from Fig.8.19.

For large values of $t_f$ (especially for $t_f = 8.0$ mm) it is observed that almost no interface shear stresses are induced locally, and it can be shown that the obtained peak values of $\tau_{int,peak}$ are inversely proportional to the square of $t_f$. As expected, $\tau_{int,peak}$ decreases rapidly as $E_f/E_c$ is increased, and in fact it turns out that $\tau_{int,peak}$ approaches zero asymptotically as $E_f/E_c$ goes to
infinity. Thus, it is concluded that the peak interface shear stresses $\tau_{\text{int,peak}}$ induced by local bending diminishes for large values of $E_f/E_c$.

Summing up the results presented for the concentrated moment load case, it is repeated that only the interface transverse normal and shear stresses need be represented in the graphical design-charts (Figs.8.19 and 8.20) as functions of $E_f/E_c$ and $t_f$. This is due to the fact that the maximum longitudinal normal stress $\sigma_{f,\text{max}}$ induced by local bending, is independent of $E_f/E_c$, proportional to the specified bending moment $M_0$ and, lastly, inversely proportional to $(t_f)^2$ as specified by eq.(8.10).

For a given problem, characterised by specified values of $E_f/E_c$ and $t_f$, $\sigma_{\text{int,peak}}$, $\tau_{\text{int,peak}}$ can be found from Fig.8.19 and 8.20, and if the peak interface stresses are requested for other than the three $t_f$-values ($t_f = 0.5$, 2.0 and 8.0 mm) included in the design-charts, this can be achieved by proper interpolation or extrapolation between the curves corresponding to constant $t_f$-values. For the sake of clarity, it is repeated that proper interpolation/extrapolation for the concentrated bending load case should be carried out according to the following proportionalities:

$$\sigma_{\text{int,peak}} \propto t_f^{-1}, \quad \tau_{\text{int,peak}} \propto t_f^{-2}$$

(8.11)
8.4 Examples
In order to exemplify how the suggested approximate solution procedure can be used, two examples of sandwich beams subjected to concentrated lateral loads are presented. The sandwich beam theory from e.g. ref.[1], more refined sandwich theories, or a finite element analysis, could form the basis of the overall bending and shearing analysis if considered necessary. Only the calculation of the stresses is included in the presentation, as the induced stresses (local as well as overall) are of primary interest when the effect of local bending is to be estimated. However, the displacements of the loaded face can be estimated equally simple by following the guidelines stated by the first of eqs.(8.5).

8.4.1 Example I
The first example treated is the case of a sandwich beam simply supported at both ends. The total length of the sandwich beam is \( L_1 + L_2 \), the faces are of equal thickness \( t_f \), and the beam is subjected to a point load \( P \) at \( x = L_1 \) (the origin of the \( x \)-axis is assumed to be placed at the left end of the sandwich beam), as illustrated in Fig.8.21. The point load \( P \) is assumed to be distributed uniformly over the width of the sandwich beam.

The geometrical, material and external load data are assumed to be:

- **Geometry:** \( L_1 = 250.0 \text{ mm}; \quad L_2 = 750.0 \text{ mm}; \quad t_f = 2.0 \text{ mm}; \quad t_c = 40.0 \text{ mm} \)
- **Faces:** \( E_f = 10.0 \text{ GPa}. \)
- **Core:** \( E_c = 60.0 \text{ MPa}; \quad \nu_c = 0.35. \)
- **Point load:** \( P = 1500.0 \text{ N}. \)

The calculations are performed in two steps:

1. **Overall bending and shearing analysis**
2. **Local bending analysis** (based on the graphical design-charts; unit point load case).

\( (i) \) Overall bending and shearing analysis
As only the stresses are of interest in the present case, the only thing needed, is expressions for the distribution of shear forces \( T \) and bending moments \( M \) in the sandwich beam. From beam theory the following results are obtained:

\[
0 \leq x \leq L_1: \quad Q(x) = -\frac{P^* L_2}{L_1 + L_2}, \quad M(x) = -\frac{P^* L_2}{L_1 + L_2} x^* \\
L_1 \leq x \leq L_1 + L_2: \quad Q(x) = \frac{P^* L_1}{L_1 + L_2}, \quad M(x) = -\frac{P^* L_1 (L_1 + L_2 - x)}{L_1 + L_2} \tag{8.12}
\]
where $P^*$ is the external load per unit width; $P^* = P/b = 50.0 \text{ N/mm}$. The bending moment attain its peak value at $x = L_1$ (just beneath the point load), and the shear force attain its peak value in the left part of the beam ($0 \leq x \leq L_1; T = \text{constant}$). By inserting the values of $L_1$, $L_2$ and $P^*$ in the two first of eqs.(8.12), the following peak values of $M(x)$ and $T(x)$ are obtained:

$$M_{\text{peak}} = -9375.0 \text{ Nmm/m}, \quad T_{\text{peak}} = -37.5 \text{ N/mm}$$

The peak bending stress $\sigma_f$ (upper fibre of the loaded face) and the peak core shear stress $\tau_c$ (constant through the core thickness) can be determined to:

$$\left[\sigma_{f,\text{overall}}\right]_{\text{peak}} \approx -116.9 \text{ MPa}, \quad \left[\tau_{c,\text{overall}}\right]_{\text{peak}} \approx 0.9 \text{ MPa}$$

(ii) Local bending analysis

The first thing to be done is to find out whether the suggested approach can be used without making any serious errors concerning interference of the local bending effects with the free boundaries of the considered sandwich beam. To do this, the decay zone distance $x^*$ is calculated by use of eq.(8.6):

$$x^* \approx 2.66 t_f \sqrt{\frac{E_f}{E_c}} \approx 29.3 \text{ mm}$$

Since $x^* < L_1$ and $L_2$, no interference problems will be encountered, i.e., the superposition of the overall solution and the local solution will give sufficiently accurate results. For the considered beam that has $E_f/E_c = 166.67$, and $t_f = 2.0 \text{ mm}$, the peak interface stress components ($\sigma_{\text{int,peak}}$, $\tau_{\text{int,peak}}$), and the maximum bending stress induced by local bending ($\sigma_{f,\text{max}}$), are found from the graphical design-charts Figs.8.11, 8.12 and 8.13:

$$\left[\sigma_{\text{int,local}}\right]_{\text{peak}} \approx P^*(-0.05 \text{ MPa}) = -2.50 \text{ MPa}$$

$$\left[\tau_{\text{int,local}}\right]_{\text{peak}} \approx P^*(0.0013 \text{ MPa}) = 0.07 \text{ MPa}$$

$$\left[\sigma_{f,\text{local}}\right]_{\text{peak}} \approx P^*(3.5 \text{ MPa}) = 175.0 \text{ MPa}$$

Thus, all the information necessary to estimate the severity of the stresses in the considered sandwich beam is now available, and the peak values of the stresses, induced by overall as well as local bending and shearing, are found by use of eqs.(8.5):

$$\left[\sigma_{\text{int,total}}\right]_{\text{peak}} = \left[\sigma_{\text{int,local}}\right]_{\text{peak}} \approx -2.5 \text{ MPa}$$

$$\left[\tau_{\text{int,total}}\right]_{\text{peak}} = \left[\tau_{c,\text{overall}}\right]_{\text{peak}} + \left[\tau_{\text{int,local}}\right]_{\text{peak}} \approx -0.9 - 0.07 \approx -1.0 \text{ MPa}$$

$$\left[\sigma_{f,\text{total}}\right]_{\text{peak}} = \left[\sigma_{f,\text{overall}}\right]_{\text{peak}} + \left[\sigma_{f,\text{local}}\right]_{\text{peak}} \approx -116.9 - 175.0 \approx -292.0 \text{ MPa}$$

The negative sign on the local contribution to the interface shear stress is caused by the fact that the shear stress is negative on the left hand side of the point of load application (the locally
induced shear stress distribution is skew-symmetric about \( x = 0 \). The peak value of the total interface shear stress is located on the left hand side \( (0 \leq x \leq L_1) \) of, and adjacent to, the point of load application, and the actual distance from the point of load application to the location of the peak interface shear stress can be estimated by \( \lambda/8 \approx 7.3 \text{ mm} \) (see eq.(8.7)). The negative sign on the local bending stress contribution is caused by the fact, that the peak bending stresses will be located in the upper fibre of the face where the stress state is compressive. In fact, the stress in the lower fibre is tensile and equals \( [\sigma_{f,\text{total}}]_{\text{peak, lower face}} \approx 68.8 \text{ MPa} \).

Comparison between the peak stresses found from classical sandwich beam theory and the peak stresses found by superposition of the overall and local solutions show that severe stress concentrations are induced by the local bending effects. This is especially the case for the bending stresses \( (\sigma_f) \) in the loaded face, where the peak stress is increased by a factor of nearly 2.5. The increase of the peak interface shear stress due to local bending is not very large for the present example (about 10\%), and this is attributed to the fact that the actual \( E/E_c \)- and \( t_f \)-values ensures that no significant interface shearing interaction is present. Considering, at last, the peak interface transverse normal stress, it is observed that it cannot be ignored, as it is about 1.5 times larger than the core shear stress \( (\tau_c) \) predicted by classical sandwich theory.

From the results it is seen that the local bending effects do contribute significantly to the stress state, and if a real sandwich beam with properties as quoted was subjected to a point load of the magnitude in the example, it would undoubtedly experience a complete structural failure.

### 8.4.2 Example II

The second example is the case of a simply supported sandwich beam identical to the one analysed in example I but with the external load \( P \) distributed over an area of the sandwich beam, as shown in Fig.8.22 (the total load \( P \) is distributed over an area \( 2\delta \times b \)). The load distribution parameter \( \delta \) is specified to \( \delta = 4.0 \text{ mm} \), and the geometrical, material and load properties are as quoted for example I. Thus, the load intensity \( p \) is found to be:

\[
p = \frac{P}{2\delta} = \left\{ P = 1500.0 \text{ N} \right\} = \frac{1500}{24.0} = 187.5 \text{ N/m}
\]

![Figure 8.22 Simply supported sandwich beam subjected to distributed load \( p \).](image-url)
(i) Overall bending and shearing analysis

At first, the distribution of the shear force $T$ and the bending moment $M$ are written (it is assumed that the origin of the $x$-axis is placed at the left end of the considered beam):

$$0 \leq x \leq L_1 - \delta \quad T(x) = -\frac{P^*L_2}{L_1 + L_2}, \quad M(x) = -\frac{P^*L_2}{L_1 + L_2} x$$

$$L_1 - \delta \leq x \leq L_1 + \delta \quad T(x) = -\frac{P^*L_2}{L_1 + L_2} + \frac{P^*}{2\delta}(x - (L_1 - \delta))$$

$$M(x) = -\frac{P^*L_2}{L_1 + L_2} x + \frac{P^*}{4\delta}(x - (L_1 - \delta))^2$$

$$L_1 + \delta \leq x \leq L_1 + L_2 \quad T(x) = \frac{P^*L_1}{L_1 + L_2}, \quad M(x) = \frac{P^*L_1}{L_1 + L_2} x - \frac{P^*}{L_1 + L_2}$$

where

$$P^* \text{ in eqs.(8.14) represents the total external load per unit width; } P^* = P/b = 50.0 \text{ N/mm. The peak value of the bending moment } M(x) \text{ occurs where } T(x) \text{ is zero, and from the expressions (8.14) the following is derived:}$$

$$M_{\text{peak}} = -\frac{P^* L_2}{(L_1 + L_2)} \left(\frac{L_1 + 2L_2 - \delta}{L_1 + L_2}\right) \text{ for } x = \frac{L_1^2 + L_2^2}{L_1 + L_2} - \frac{\delta}{L_1 + L_2}$$

The peak value of $T(x)$ occurs in the left part of the beam ($0 \leq x \leq L_1$; $T = \text{constant}$ according to the first of eqs.(8.14)). By inserting the actual values of $L_1$, $L_2$ and $P^*$ in the first of eqs.(8.14) and in eq.(8.15), the following results are obtained.

$$M_{\text{peak}} = -9337.5 \text{ MPa at } x = 252.0 \text{ mm}$$

$$T_{\text{peak}} = -37.5 \text{ mm for } 0 \leq x \leq L_1$$

The peak bending stress $\sigma_f$ (upper fibre of the loaded face) and the peak core shear stress $\tau_c$ are found to be

$$\left[\sigma_{f,\text{overall}}\right]_{\text{peak}} \approx -116.4 \text{ mm}, \quad \left[\tau_{c,\text{overall}}\right]_{\text{peak}} \approx -0.9 \text{ mm}$$

(ii) Local bending analysis

As was the case for example I, the first thing to be done is to find out whether the suggested approximate solution procedure is applicable. This was investigated for the sandwich beam analysed in example I, and since the sandwich beam of example II is the same, it is concluded that no interference problems will be encountered.

For the considered sandwich beam ($E/E_c = 166.67$; $t_f = 2.0 \text{ mm and } \delta = 4.0 \text{ mm}$), the locally induced interface stress components, as well as the maximum bending stress, are found from the graphical design charts Figs.8.15, 8.16, 8.17:
\[
\begin{align*}
\left[\sigma_{\text{int,local}}\right]_{\text{peak}} &\approx P^* (-0.05 \text{ MPa}) = -2.50 \text{ MPa} \\
\left[\tau_{\text{int,local}}\right]_{\text{peak}} &\approx P^* (0.0012 \text{ MPa}) = 0.06 \text{ MPa} \\
\left[\sigma_{f,local}\right]_{\text{peak}} &\approx P^* (2.1 \text{ MPa}) = 105.0 \text{ MPa}
\end{align*}
\]

All the information necessary is now available, and the peak values of the stresses in the considered sandwich beam can be evaluated by use of eqs.(8.5):

\[
\begin{align*}
\left[\sigma_{\text{int,total}}\right]_{\text{peak}} &= \left[\sigma_{\text{int,local}}\right]_{\text{peak}} \approx -2.5 \text{ MPa} \\
\left[\tau_{\text{int,local}}\right]_{\text{peak}} &= \left[\tau_{t,overall}\right]_{\text{peak}} + \left[\tau_{\text{int,local}}\right]_{\text{peak}} \approx -0.9 - 0.06 \approx 1.0 \text{ MPa} \\
\left[\sigma_{f,overall}\right]_{\text{peak}} &= \left[\sigma_{f,local}\right]_{\text{peak}} \approx -116.4 - 105.0 \approx -221.4 \text{ MPa}
\end{align*}
\]

The negative signs on the local interface shear and bending stress components are introduced for the same reasons given in example I. Comparison between the peak stresses, found by overall bending and shearing analysis, and by local bending analysis, reveals that severe stress concentrations are induced by the local bending effects in this case as well.

Looking back to the results of example I, it is seen that the distribution of the total load \(P\) over the distance \(2\delta = 8.0 \text{ mm}\) has reduced the peak bending stress component in the loaded face with nearly 25\% from \(-291.9 \text{ MPa}\) (example I: point load) to \(-221.4 \text{ MPa}\) (example II: uniformly distributed load). Comparing the interface stress components, as obtained from the two examples, it is observed that no significant stress reduction has been obtained by distributing the load \(P\) over the small distance \(2\delta = 8.0 \text{ mm}\).

### 8.5 Concluding Remarks

A method of approximate analysis of the local stress and displacement fields in the near vicinity of strongly localised external loads applied to sandwich beams (or plates in cylindrical bending) has been introduced. The formulation presented is based on the assumption, that the deflection of the loaded face against the other face can be modelled properly by use of some elastic foundation model. Furthermore, it is assumed that the faces as well as the core material of the considered sandwich beams behaves linearly elastic.

The principle of the solution procedure is very simple: the local stress and displacement fields derived by application of an elastic foundation model are superposed to the overall stress and displacement fields obtained by use of, for instance, classical sandwich beam theory, and an approximate complete solution, which includes overall bending and shearing effects (classical sandwich theory or FEM solution) as well as local bending effects, is obtained.
Solutions to specific sandwich beam problems have been discussed, and three important load cases have been analysed: 1. the point load case; 2. the uniformly distributed load case; and 3. the concentrated bending moment load case. The results obtained for the three loaded cases have been presented in the form of graphical design-charts from which the peak interface (interface between loaded face and core) transverse normal stress, the peak interface shear stress, and the peak bending stress in the loaded face, can be found for specified values of the modular ratio $E_f/E_c$, the face-thickness $t_f$, and the external load (type and magnitude).

As mentioned only three load cases have been included, but if local bending solutions are required for other load cases, elementary solutions based on the classical Winkler foundation model can be found (and used successfully, except for deformations with short wave-length) in references such as Hétenyi [14] and Roark & Young [16].

The approximate solution procedure presented is only strictly valid for sandwich beams and sandwich plates in cylindrical bending. Plate analysis in general, however, cannot be accomplished directly with the solutions presented so far, but general sandwich plate solutions, based on an extended version of the two-parameter elastic foundation approach employed herein, will be presented in the near future.

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References


CHAPTER 9

INSERTS IN SANDWICH PANELS

In this chapter, different insert types and concepts are outlined, along with load carrying mechanisms between insert and the components of the sandwich panel. Finite-element models are used to clarify the application of the principles described, and the state of stress near the insert. At the end of the chapter is a list of recommended literature and reports of test results.

Due to the variability of the different physical elements, the states of stress and strain caused by an insert will typically change significantly from case to case.

In most cases, appropriate calculations must be performed.

This chapter will not contain many ready-to-use formulas and rules of thumb. Instead, the aim is to describe the structural principles of inserts, to facilitate the necessary calculations. Above all, a thorough understanding of the load carrying mechanisms is useful. Sandwich panels, while possessing many desirable qualities, require a well-designed application of loads. Contrary to a steel plate, which is often quite tolerant to the occasional welded-on “afterthought”, a sandwich panel can be unforgiving to such violation of its basic structural integrity.

9.1 The Purpose of Inserts

Loads and reactions on a structural panel may in general be applied to the edges or somewhere on the surface of the panel. Usually, the edges are supported and the load is applied to the surface. Whatever the case, any localised load directly applied to a sandwich panel will cause deformations rather unlike those in a similar steel or aluminium panel.

An insert is a local change in stiffness and strength of the sandwich panel, the purpose of which is to distribute a localised load in an appropriate manner to the sandwich panel.

Sandwich plates are basically unsuited for carrying localised loads, for the simple reason that the core does not have the stiffness necessary to distribute the forces effectively. Therefore, a sandwich structure should ideally be constructed so that inserts are avoided. As this would clearly be impractical, it is important to choose an appropriate type of insert. This chapter will in particular treat circular inserts. Other types of inserts falling within the definition above will be mentioned in passing.
9.1.1 Region of influence

Consider first the case of a concentrated load on the surface of a panel. At any closed section surrounding the load, the conditions of equilibrium must be satisfied (assuming a static situation). The closed path is denoted $\Pi$, and the concentrated load is in this case a transverse force $Q$, see Fig.9.1. This means that by integrating the transverse force components $T$ along the curve, the result must be $-Q$.

A circular path given by the radius $r$ is particularly convenient.

![Figure 9.1](image1)

Since the length of $\Pi$ increases by $r$, the reaction $T$ must decrease by $r$. In the cases of a concentrated force and a concentrated moment on a plate, the dependency is as outlined below.

![Figure 9.2](image2)

Force $F$: $T \approx \frac{1}{r}$ (total shear component $T$ proportional to $1/r$)

Moment $M$: $T \approx \frac{1}{r^2}$ (total shear component $T$ proportional to $1/r^2$)

The $1/r$ or $1/r^2$ dependency indicates that the heavily disturbed zone is small, and a calculation model needs not generally include the whole panel.

The above considerations are true for thin plates. However, a sandwich plate is not necessarily very thin compared to the length or width. Thus, a significant amount of moment $M$ can be transferred through the face sheets as in-plane tension/compression, particularly for small $r$ (until the radius is sufficiently large compared to the thickness). This is shown in Fig.9.3.
The region considered for calculation purposes must be estimated with this in mind. For a sandwich plate, \( T \) itself will also be distributed between face-plates and core. The distribution will, as hinted above, depend upon the parameters of the actual situation (geometry, materials and loads).

### 9.2 The Elements Involved

Apart from the basic structural element, the sandwich panel, two more elements are of interest: the insert and sometimes a fixing agent (adhesive or potting compound which holds the insert in place). Inserts, sandwich panel and fixing agent are briefly described below.

#### 9.2.1 Different types of inserts

Fig. 9.4 shows some different types of inserts, divided into 4 categories.

The categories 1 and 4 may not exactly qualify as inserts in the strict sense of the word. They are included because they are also used for applying loads to the panel surface.
1. Self-tapping screws and rivets are “low-grade” inserts. They may be used for attaching light equipment to panels at points where these panels are not otherwise subjected to any significant loads.

2. Through-the-thickness inserts can directly transfer shear force to the core and moments and in-plane forces to the face sheets. They may be used for applying significant loads to the sandwich panels.

3. The partial insert is directly attached to one face sheet and the core. It is used where through-the-thickness inserts are undesirable (in ship-building, it may be necessary to leave at least one face sheet undamaged). It should, like wood screws and rivets, be avoided for transfer of moment and in panels subjected to significant shear force.

4. The adhesively bonded cylinder relies on the adhesive joint for transfer of forces and moments. Thus, for transfer of in-plane forces, it is not fail-safe in the same manner as the partial insert. However, it is useful in panels subjected to a high, uniform shear load, since it does not disrupt the core.

This chapter is concerned primarily with cylindrical inserts made from metal (steel). Since the modulus of elasticity of the insert is thus considerably higher than that of FRP face-sheets and much higher than the core material, calculations may usually be made under the assumption of infinitely rigid inserts, to within a fair degree of precision.

**9.2.2 Sandwich panel**

In its pure form, a sandwich panel consists of two face sheets of negligible individual bending stiffness compared to the bending stiffness of the whole sandwich panel, but possessing large extensional stiffness, held apart by a core of comparatively small extensional stiffness and large shear-stiffness.

The sandwich panel then carries loads in the following manner:
- In-plane loads (loads directly applied along the panel-plane and moments in the form of force-pairs) are carried by the face sheets
- Transverse loads are carried by the core

The idealised and the actual distribution of loads are outlined in Fig.9.5.

![Figure 9.5 Sandwich panel, parameters and load-carrying mechanisms. (1a), (1b), (1c): actual load distribution of M, N and T, respectively. (2a), (2b), (2c): simplified load distribution of M, N and T, respectively](image)
This reflects the situation in a non-disturbed portion of a sandwich panel. An insert should ideally be designed so as to reach these distributions. Differences between the idealised and the actual distributions are caused mainly by the bending and shear stiffness of the face sheets.

The face sheets are assumed isotropic\(^1\) and do in fact have some individual bending stiffness, \(D_i\)

\[
D_i = \frac{E_i t_i^3}{12(1 - \nu_i^2)}
\]

- \(E_i\): Young’s modulus, \(i\)’th face sheet
- \(t_i\): Thickness, \(i\)’th face sheet
- \(\nu_i\): Poisson’s ratio, \(i\)’th face sheet

The stiffness of the face sheets are of particular importance near a concentrated load, where they will contribute to the transfer of shear force, and also bend, even though they are supported by the core. Choosing a thicker face sheet may lessen the local bending.

An example: A steel face sheet (thickness 1 mm, \(E = 210,000\) MPa) will have roughly the same extensional stiffness and weight as a 3 times thicker aluminium face sheet (thickness 3 mm, \(E = 70,000\) MPa), thus giving the same overall sandwich panel bending stiffness. But the individual bending stiffness of the aluminium face sheet will be 9 times that of the steel face sheet. Thereby, the local compression/extension of the core may be significantly reduced.

The core material may typically be honeycomb, which is heavily orthotropic, or polymer foam, which is isotropic. These are shown in Fig.9.6.

![Figure 9.6 Sandwich “cut-outs” with honeycomb core (left) and foam core (right).](image)

The honeycomb core cannot transfer in-plane stresses; it expands or collapses like the bellows of an accordion. It is thereby described well by the antiplane assumption for transversely

---

\(^1\) The assumption of isotropic face sheets may seem unreasonable, but in reality, the application of a concentrated load onto a heavily anisotropic face sheet (such as FRP) should be avoided. Instead, the panel is usually reinforced by extra fibre plies at different angles, rendering the face sheet(s) quasi-isotropic.
compressible cores\textsuperscript{2}. Equilibrium of a differential core element shows that the out-of-plane shear stresses must in that case be constant across the height of the core. A honeycomb will thus cause a very close approximation to the idealised distribution of shear.

In case of a disbond between core and face sheet, which may be caused by a poor choice of insert (or a badly produced panel, or impact), the shear stress is nil at the debonded interface. Since the shear stress is distributed evenly across the height of an antiplane core, the core shear stress is consequently nil and the sandwich panel is thereby locally reduced to two independent face sheets.

\subsection*{9.2.3 Fixing agent}
An insert is held in place by a \textit{fixing agent} which is considered isotropic; it may be a potting compound or adhesive. The stiffness of the fixing agent is usually about ten times that of the core. Therefore, the fixing agent is assumed infinitely rigid relative to the core-insert interface i.e. it is considered part of the insert. The assumptions at the insert/face-sheet interface vary, depending on the exact conditions. This is described in chapter 9.3.1.

\section*{9.3 Load Transfer between the Elements}
As indicated above, the insert is characterised by the connections to the various parts of the sandwich panel. Therefore, a study of the load carrying mechanisms at the interfaces is appropriate. This will show which stresses should be included in a calculation (for instance, a finite-element calculation) at varying insert geometries. Throughout this section, it is worth remembering the foundation principle of any combination structure or composite element, which states that:

\textit{The stiffness of a combination structure is obtained as long as the individual components restrict each other’s free deflection.}\textsuperscript{3}

The cost of increasing the stiffness by use of the combination principle is usually stress concentrations in one form or another.

\subsection*{9.3.1 Face sheets / insert interface}
In Fig.9.7, an interface element at the edge of an insert is shown. In general, the force resultants $N_{\varphi}, N_{r}, T_{r} [\text{N/m}]$ and moment resultants $M_{r}, M_{r\varphi} [\text{Nm/m}]$ are all present at any such sections of a plate, according to common plate theory. However, the choice of insert will decide which ones are significant.

\textsuperscript{2} Antiplane assumption: The core is incapable of transferring in-plane stresses. Choosing a cylindrical coordinate system with transverse direction $z$, this means that stresses $\sigma_{r}, \sigma_{\varphi}$ and $\tau_{r\varphi}$ are zero.

\textsuperscript{3} For example, the deflection of a loose stack of two face sheets and a core, subjected to a transverse load and supported at the edges, is vastly larger than the deflection of an intact sandwich panel subjected to the same load and boundary conditions.
The interface position is chosen according to Fig. 9.8 for some different inserts. Note that in cases (b) and (c), the interface is chosen at the edge of the flared part.

Case (a): The adhesive or potting compound that holds the insert in place is usually much softer and weaker than both insert and face plate. Since the thickness of the face layers is small, the stress variations across the thickness will usually be insignificant in terms of interface stiffness, so this eliminates the moments $M_r, M_r\varphi$ as contributing to the overall stiffness. In this case, the adhesive layer acts as a hinge between the face-sheet and the insert in case of a transverse load. However, the moment stresses may easily be large enough to break the adhesive itself, causing a circular crack in the face-sheet/insert interface. This will further reduce the transferable reactions to compressive $N_r$. If the insert is subjected to an in-plane force or (in the case of through-the-thickness inserts) moment-vector, this is often sufficient.

Cases (b) and (c): The function of the flared disc, apart from giving a bigger bonded surface and increasing the radius (due to the $1/r$-dependence) is to allow the transfer of moment and significant transverse force $T$ by the face sheets, thus stiffening the joint and minimising any stress concentrations in the core. While the face-sheet bending moment will certainly contribute to the stiffness, it may cause large bending stresses in the face sheets (because the deformation of the face sheets is often largely dictated by the core). The obvious solution to this is either locally increasing the thickness of the face sheets or reducing the stiffness of the adhesive layer underneath the flared disc, so that the moment does not become critical.

Case (d): The screw or rivet is often chosen for being convenient and cheap. Since the radius $r$ of the “insert” is very small, and because of the geometric discontinuities arising in the core, a screw or rivet cannot transfer as much load as the larger inserts. In general, $T_r$ is reliable in this case; both screws and rivets will grip the face sheet from underneath. Only the compressive part of $N_r$ should be considered, since neither screws nor rivets are normally fixed by use of adhesive. Moments on...
single screws should be avoided. Instead, a moment load can be distributed to several screws, the distance between these giving a much larger moment arm.

### 9.3.2 Core/insert interface

In the core, an insert is usually held in place by a adhesive or potting compound. Structurally, the adhesive should be considered part of the insert, since it is generally 5-10 times as stiff as the core. With a proper bond, all stress components are present at any point in the interface, as outlined in Fig.9.9.

![Interface elements between insert and core](image)

The core is the weaker part and in the case of partial inserts in particular, stress concentrations in the core tends to cause cracks. In an isotropic core material, these grow through the core perpendicular to the direction of largest principal stress. Screws and rivets penetrating the face sheet are a special case, since they are not normally connected to the core by adhesive. They may cause serious discontinuities in the core.

### 9.4 Passive Insert

An insert is a disruption of the basic structure, and will cause stress concentrations even if no load is applied to the insert itself. This is referred to as the passive insert. Points of particular interest are mentioned hereafter.

#### 9.4.1 Holes in face sheets

Most of the insert types mentioned in this chapter involve drilling holes in the face-sheets. This causes a concentration of the in-plane stresses. The concentration factor will be 3 in the case of isotropic face sheets in compression or tension. The adhesive connection between insert and face sheet is not necessarily reliable, as mentioned in section 9.3.1, so generally, the unreduced factor applies (again, note that the stress concentration will be different for anisotropic face sheets, where the adhesive layer has significant relative stiffness, and for bending).

If the insert has flared ends bonded onto the face sheets, the relevant concentration factor is reduced. The local offset of bending stiffness may cause significant peel stresses between face sheet and insert flare, depending on the actual parameters.
9.4.2 Inserts in panels subjected to shear

![Figure 9.10 Rhombic shape of shear-subjected sandwich panel.](image)

In shear, the face sheets of a sandwich panel will move in-plane relative to each other, causing very large deformations compared to for example a steel plate (the assumption of negligible shear strain are not valid for sandwich plates). An undisturbed part of the panel will change from rectangular to rhombic, as shown in Fig.9.10.

Depending upon the insert, the relative movement of the face-sheets will be prevented to a certain extent, the total shear stiffness of the panel increased by the insert. In Fig.9.11, some different cases are outlined.

![Figure 9.11 Different passive inserts in shear-subjected panel. Points of severe stress-concentration are marked by circles. (a) Very soft insert or diaphragm, (b) stiff through-thickness insert with flared ends, (c) stiff simple cylindrical through-thickness insert and (d) partial insert](image)

A soft insert or diaphragm will stay at right angles to the face sheets, getting a “stretched S”-shape. This causes severe bending in the insert near the face sheets, and thereby large stresses in the insert/face-sheet interface. The stiff through-thickness inserts will not change shape. This gives rise to anti-symmetric deflection of the face sheets. If the insert is simple cylindrical, the sudden, sharp deflection will cause stress concentrations in the core and the core/face-sheet interface. This is avoided by the use of flared ends; instead, the face sheets will bend, causing large peel-stresses between flared edge and face sheet. A partial insert should be avoided in places of large shear deformation. The stress concentration in the core will most likely cause core failure or debonding between core and insert.

9.5 Summary

Based on the circumstances outlined above, the following table summarises the abilities of different inserts to carry different loads. It is assumed that forces are applied so that no rotation will occur (that is, at the apparent centre of stiffness).
### Table 9.1 Summary of insert performances.

<table>
<thead>
<tr>
<th>Insert Type</th>
<th>Transverse force</th>
<th>In-plane force</th>
<th>Moment</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Partial</td>
<td>+</td>
<td>+</td>
<td>++</td>
<td>Use where one face sheet must be kept intact. Avoid moment load. Do not use in panels subjected to much shear. Beware of stress concentration in core.</td>
</tr>
<tr>
<td>Through-thickness</td>
<td>+ +</td>
<td>+ + +</td>
<td>+ +</td>
<td>Use for heavy loads. Note stress-concentrations in core and in core/face-sheet interface.</td>
</tr>
<tr>
<td>Flared top</td>
<td>+</td>
<td>+ +</td>
<td>+</td>
<td>Use for improved stiffness and pull-out strength. Beware of bending stress in face sheets and peel-stresses underneath flared edge.</td>
</tr>
</tbody>
</table>

### 9.6 Calculation Examples

This section illustrates the different principles described. Four different inserts are investigated, using finite element models. In each case, a transverse force $F$ is applied to an insert in a sandwich panel.

The panel has the following properties:

- **Face sheets**: Aluminium ($E_f = 70000$ MPa, $\nu_f = 0.30$), thickness $t_f = 2$ mm
- **Core**: Divinycell H130 PVC foam ($E_c = 140$ MPa, $\nu_c = 0.32$), thickness $t_c = 30$ mm

At first, the modelling radius $r$ must be estimated. In this case, the important local effects are assumed to be described well within a radius of 100 mm, roughly 3 times the height of the core.

Now, the structural boundary conditions must be decided. By having assumed infinitely rigid inserts, we can conveniently choose to “invert” the loading by supporting the model at the insert and applying an equivalent load to the boundary at the radius $r$. This has two advantages:

1. We don’t have to actually model the insert. Instead we impose suitable constraints at the interface between sandwich panel and insert.
2: The problems of assuming certain support-conditions at a hypothetical radius \( r \) can be circumvented. By choosing the radius \( r \), we assume that at this point, the local effects have “died out”, and the transverse force is distributed evenly across the height of the core. In this case, it means distributing shear stress \( \tau = -Q/(2\pi r t_c) \) on the core at the radius \( r \). Setting \( Q \) at 1000 N, \( \tau \) becomes 0.0531 MPa. In Fig.9.12, the choice of element modelling radius and boundary conditions is outlined.

![Figure 9.12](image)

Figure 9.12  **Left:** Sandwich panel, simply supported at the edges, with insert subjected to transverse force. **Right:** Cut-out, radius \( r = 100 \text{ mm} \), with equivalent shear stress distributed evenly on core and supported at the insert.

Note that the centre support shown in Fig.9.12 is symbolic; the actual support will be determined by the insert shape. Finally, since model and load are in this case axi-symmetric, the FE-model is essentially plane. This reduces model complexity and calculation time significantly.

Initially, however, it is worthwhile to clarify the primary load-carrying components in the insert/sandwich-panel interface for the pullout load. These are shown in Fig.9.13 for three\(^4\) of the four cases considered in this section.

![Figure 9.13](image)

Figure 9.13  **Primary interface reactions for cases.** (a) Self-tapping screw or rivet, (b) partial insert and (c) through-the-thickness insert (simple cylindrical insert)

The initial rough estimate of load-carrying capacity may be:

**Case a:** \( Q_{\text{max}} = \pi D \bar{T} \)

---

\(^4\) The fourth case, the through-the-thickness insert with flared edges, is not shown in figure 9.13. The basic interface reactions are somewhat like case (c) (simple cylindrical through-the-thickness insert).
where $D$: screw/rivet minimum diameter
$T$: allowable face-sheet shear resultant

**Case b:** $Q_{\text{max}} = \pi (r^2 \sigma^E + 2rh \tau^E)$

where $r$: insert radius
$h$: insert height
$\sigma^E$: allowable core material tensile stress
$\tau^E$: allowable core material shear stress (roughly $\frac{1}{2} \sigma_{\text{all}}$)

**Case c:** $Q_{\text{max}} = 2\pi rh \tau$

These are very rough estimates, assuming uniform distribution of stress on the relevant surfaces and disregarding any stress concentrations. The calculated load carrying capacity may therefore have to be reduced significantly.

### 9.6.1 Self-tapping screw or rivet

![Image of sandwich panel with rivet](image)

Figure 9.14 Actual geometry (left) and modelling (right) of sandwich panel with rivet in upper face sheet. The load $\tau$ is equivalent to 1000 N transverse to the panel-plane, applied to the insert.

Since a rivet or screw is in this case assumed to transfer transverse force $T$ and nothing else, the equivalent support is as shown (transverse movement at inner edge of upper face sheet restricted). Also, at the centre-line, radial displacements must be 0 (this is not shown).

The shear stress distribution across the thickness of the panel indicates which parts carries what. Two sections have been chosen, one at a radius 5 mm larger than the face-sheet/insert interface and one at radius $r = 80$ mm. For ease of comparison, the shear stresses $\tau_{rz}$ have in each case been multiplied by the relevant radius. In this case, the radii are therefore 10 and 80 mm, and the shear distribution is as shown in Fig.9.15.
The maximum face sheet bending stress \( \sigma_r \) at \( r = 10 \) mm is 22 MPa. This rather large value is not surprising, considering the small radius.

Several pullout tests of self-tapping screws in FRP-face sheets have been performed. The main parameters were as shown in Fig.9.16.

The screws had core diameters 2.0, 3.0 and 4.8 mm. The face sheets consisted of 2-6 layers of combi-mat (chopped-strand + woven) 800T2/300, with about 60\% glass fibre and a “hand lay-up” quality. One layer is roughly 1.1 mm thick, and the total face sheet thickness is thus about 2-7 mm. It should be noted that in most cases, the pullout caused local interface debonding between face-sheet and core, and local face-sheet delamination.
Table 9.2 Pullout force for different screw inserts

<table>
<thead>
<tr>
<th>Screw core diameter Ø_s [mm]</th>
<th>2.0</th>
<th>3.0</th>
<th>4.8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drilling diameter Ø_d [mm]</td>
<td>2.3</td>
<td>3.5</td>
<td>4.8</td>
</tr>
<tr>
<td>Pullout force [kN], 2 layer face-sheet</td>
<td>0.78</td>
<td>1.97</td>
<td>2.64</td>
</tr>
<tr>
<td>Pullout force [kN], 4 layer face-sheet</td>
<td>1.19</td>
<td>2.99</td>
<td>4.89</td>
</tr>
<tr>
<td>Pullout force [kN], 6 layer face-sheet</td>
<td>1.70</td>
<td>3.61</td>
<td>5.63</td>
</tr>
</tbody>
</table>

The dependence upon drill diameter was found to be insignificant at small variations

\[ \phi_d = \phi_s (-0.2) - (+0.5) \text{ mm} \]

The drill diameter should be a bit larger than the screw core diameter for ease of mounting.

9.6.2 Partial insert

Since the insert does not have a flared top, no significant face sheet bending moment \( M_r \) can contribute to the transference of load, and only the force-components \( T \) and \( N_r \) are present at the face-sheet/insert interface. In this case, the shear distribution is evaluated at radii 15 and 80 mm, see Fig.9.18.
The max. bending stress $\sigma_r$ in the upper face sheet at $r = 15$ mm is 12 MPa. The transverse tensile stress $\sigma_z$ is shown in Fig.9.19 as a function of the radius, from $r = 0$ to $r = 100$ mm. The section chosen lies 17 mm underneath the upper face-sheet/core interface, that is, 2 mm underneath the bottom of the insert.

The insert, has a radius of 10 mm. From Fig.9.19, it is seen that the mean transverse tensile stress from $r = 0$ to 10 mm is about 0.7 MPa, which means that the force $\pi r^2 \sigma_z,\text{mean} = 220$ N is transferred at the bottom of the insert. This is about $1/4$ of the total 1000 N, and the rest is thus transferred as shear stress acting upon the cylindrical surface. The stress concentration at the insert bottom causes fracture. A test specimen for pullout tests is shown in Fig.9.20, after the insert has been pulled out.

The fracture sequence is:

a) Linear or roughly linear force-displacement curve until fracture in interface at bottom of insert. The resulting conical crack immediately propagates most of the way to the upper face sheet

b) Propagation of conical crack, accompanied by acoustic emission. Crack starts at face-sheet/insert interface and propagates towards bottom crack

c) Insert detached from panel, held only by friction.
The core fracture mode is similar to that of a punching problem, where a transversely loaded thick plate is supported by columns. A few pullout tests of different partial inserts were performed for determining the fracture sequence, in each case characterised by the transverse force at point \( a \) as described above.

The sandwich panel used had the following main characteristics:
- Face sheets: Aluminium, thickness 1.5 mm
- Core: Divinycell H130, thickness 30 mm

Three partial insert shapes were tested, all with an interface area of 1000 mm\(^2\), but with varying diameters: \( \varnothing_i = 12, 16, 20 \) mm.

The parameters are summarised in Fig.9.21.

The insert heights \( h_i \) were adjusted to reach the desired interface area. The max.-force pullout values were as shown in table 9.3.

<table>
<thead>
<tr>
<th>Insert diameter ( \varnothing_i ) [mm]</th>
<th>Max. force [kN]</th>
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</thead>
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<tr>
<td>12</td>
<td>3.28</td>
</tr>
<tr>
<td>16</td>
<td>2.93</td>
</tr>
<tr>
<td>20</td>
<td>3.40</td>
</tr>
</tbody>
</table>

Table 9.3 Results of pullout tests.

Fatigue tests of similar partial inserts show that the mode of initial fracture is the same as that of static fracture. By rounding the core end of the partial insert, as shown above, the stress concentration decreases somewhat compared to a sharp edge, giving a better lifetime expectancy.
9.6.3 Through-the-thickness insert

Figure 9.22  Actual geometry (left) and modelling (right) of through-the-thickness insert. The load $\tau$ is equivalent to 1000 N transverse to the panel-plane, applied to the insert.

Conditions at the face-sheet/insert interface are similar to those for the partial insert (section 9.6.2), and the shear distribution is again evaluated at radii 15 and 80 mm, see Fig.9.23.

At $r = 15$ mm, the max. face sheet bending stress $\sigma_r$ is 10 Mpa.
9.6.4 Through-the-thickness insert with flared ends

The flared edges have diameter 40 mm, i.e. twice the diameter of the cylindrical part of the insert. The load $\tau$ is equivalent to 1000 N transverse to the panel-plane, applied to the insert.

The shear distribution is in this case evaluated at radii 25 and 80 mm, see Fig.9.25.

At $r = 25$ mm, the max. face sheet bending stress $\sigma_r$ is 19 MPa. This is nearly as much as the face sheet bending stress at $r = 10$ mm in the case of a rivet/screw, described in section 9.6.1.
9.7 References


Large and/or complicated sandwich structures are often manufactured by shaping pre-fabricated sandwich panels and then joining them. For example, the bulkheads in sandwich-ships are fitted to the hull in this fashion. Joints between sandwich panels are practically unavoidable in most sandwich structures. This may be considered unfortunate, since it will inevitably compromise the basic principles of sandwich construction. This chapter will deal with methods of stiffening/strengthening the joints, so that the composite action is improved. For estimating local effects at unreinforced joints, a Winkler-foundation model is presented. The application of this model is described for V- and T-joints. Results from investigations of T-joints are briefly mentioned.

The primary joint geometry may be defined in the following manner: Two panels, each defined by a middle surface, joined at the intersection curve of said middle surfaces. Examples of mid-surface intersections are shown in Fig.10.1.

Figure 10.1 Panel mid-surface intersections. The panels may in general, be plane or curved, and intersect each other at an angle $\neq 0^\circ, 180^\circ$.

This chapter will treat only plane panels, and cases where the intersection line is long compared to the thickness of the individual sandwich panels. This means that problems of stress/strain analysis are reduced to cases of plane strain.
10.1 Basic Types
Some basic joint-types are of particular importance. These are shown in Fig.10.2.

![Diagram of basic joint-types: T-joint: Bulkheads, stiffeners, walls, L-joint: Deck-hull, wall-roof, and V-joint: Hull-bottom]

10.2 Load Transfer
The purpose of the joint is to transfer reactions between the panels joined. The reactions for the T-, L- and V-joints are shown in Fig.10.3.

![Diagram of reactions in the joints]

Basically, sandwich panels should act in the following manner:
- The face-sheets act as membranes and transfer in-plane force $N$ directly and moment $M$ as force couples.
- The core transfers the transverse reaction $T$

This is clearly violated in all the joint-types shown in Figs.10.2 and 10.3, the general problem being that in-plane reactions on one panel are out-of-plane reactions on the other. This means that the stiffness/geometry is “discontinuous” relative to the direction of the reactions.

In the following sections, the T-, L- and V-joints will be treated separately. The more important effects of the stiffness-discontinuities are outlined. Suggestions for reinforcement of the individual joints are presented.
10.3 T-joints

The T-joint is used wherever one sandwich panel is stiffened transversely by another (sandwich-) panel. In larger sandwich constructions, such as ships, several kilometres of T-joints are not uncommon. Thus, T-joints have been subjected to extensive research in an effort to develop a cheap, reliable design.

Transverse stiffening of ship hulls is basically a matter of transferring force from the hull to a bulkhead, as shown by the component $N$ at the T-joint in Fig. 10.3. Since the bulkhead is usually perpendicular to the hull, the large stiffness of the bulkhead face-sheets will cause a reaction as two parallel line-loads on the hull inner face-sheet. This is shown in Fig. 10.4.

The rather direct transfer of force between bulkhead face-sheets and hull core may cause severe bending of the hull inner face sheet and compression of the hull core. Furthermore, when only a thin layer of adhesive bonds the bulkhead to the hull (as is assumed to be the case in Fig. 10.4), only a negligible amount of tensile force $P$ may be transferred, because of the large stress-concentration. In a fast boat moving at slamming\(^1\) speed, the dynamics are such that tensile force may be present and non-negligible. Thus, a T-joint should distribute the loads so that interface stresses between the panels are sufficiently low. Fig. 10.5 shows some of the methods used.

Fig. 10.5 shows several components that may be used in combination or individually. The gap-filler serves two purposes: It fills the gap between the hull and the bulkhead, and, importantly, distributes the force between the bulkhead face sheets and the hull panel. The bonded tapered laminate is normally used in combination with a fillet and a gap-filler. This distributes the force and enables the joint to transfer moment (whether this is desirable or not). For producing a cheap joint, a putty-fillet (made from Crestomer, Divilette or some similar filler) may be used alone. A joint of that type should not be subjected to fatigue loads. Reinforcement of the hull inner face-sheet increases the bending stiffness, thus distributing the load transverse to the hull panel on a larger portion of the core. Similarly, the local high density core is able to withstand a higher local stress arising from the bulk-head.

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\(^1\) Slamming: when the bow of a boat rises on one wave and slams back onto the ridge of the next wave.
10.4 L-joints
The L-joint is used for joining the roof and sides of lightweight containers and trailers\(^2\), for joining a sandwich ship’s deck and hull and similar. If the joint-interface is primarily subjected to a limited amount of compression, and insignificant bending, the joint needs not be very elaborate. However, if the bending moment is significant, problems may arise, as illustrated in Fig.10.6.

\(^2\) For refrigerated trailers and containers, a sandwich construction with foam core is often used, due to the high thermal insulation of the foam.
since the horizontal panel is cut off flush with the upright, the outermost portion of the horizontal panel core possesses little shear stiffness. This means that the upper face sheet of the horizontal panel cannot contribute significantly to the stiffness. As indicated in Fig.10.6, the combination of circumstances causes compression/extension of the horizontal panel core, and local bending of the lower face sheet. Fig.10.7 shows some ways to reinforce the L-joint.

The fillet, used in combination with the bonded tapered laminate, distributes interface reactions over a larger area. The reinforcement of the horizontal panel inner face-sheet increases the face-sheet bending stiffness, thus distributing the reaction from the upright face-sheets to a larger portion of the horizontal panel core. Using a high density core in the edge of the horizontal panel serves two purposes: It is able to withstand greater stress arising from the reactions from the upright face-sheets, and it has greater shear stiffness to better transfer shear stresses to the upper face-sheet. The bulk laminate may be used to improve shear transfer between the face-sheets of the horizontal panel, similar to using a high density core. It may have to be very thick (and heavy) to be sufficiently stiff. An edge laminate, used in combination with a high density core and/or bulk laminate, increases the bending stiffness. It is often used in any case, since it seals the edge neatly.

Some other ways to make an L-joint are shown in Fig.10.8. These may be more demanding in terms of precision and preparation, since the panels may have to be manufactured specially in order to accommodate the edges.
The welded-edge joint can only transfer small bending moments.

10.5 V-joints

The V-joint is used at the bottom of boat-hulls and for assembling other similarly symmetric halves (half-shells). It has a superficial similarity to the butt-joint, where two core-plates are joined to form a continuous plate. The difference is that in the V-joint, the planes of the plates are joined at some angle to each other. The resulting discontinuity may cause problems, as illustrated in Fig.10.9.

The V-joint is deliberately shown as a two-part core with continuous face-sheets. Boat-hulls are often made by attaching core materials to an upside-down framework. The core is then coated with several layers of fibre to form the outer face-sheet. When the resin is cured, the shell is removed from the framework and turned around so that the inner face-sheet may be applied. Since the hull-bottom is mostly V-shaped, and since the core material usually has a given thickness, the resulting shape is as shown.

The components $N_x$ from the left and right parts of the V-joint are equal and opposite, due to symmetry. The components $N_y$, however, will cause severe transverse compression/extension of the comparatively soft core.
An improved design should aim to increase the rigidity of the whole joint. Some of the means by which this may be obtained are shown in Fig.10.10.

Figure 10.10 Reinforcement of V-joint. The parts are: (1) sandwich panel, (2) reinforced inner face sheet, (3) fillet core, (4) fillet face-sheet, (5) higher density core, and (6) reinforced outer face sheet.

One or more of the methods shown can be used (apart from the fillet parts, 3 and 4, which should be used in combination). Reinforcement of the face sheets is done by applying a strip of extra laminae. This increases the bending stiffness of the face sheets locally, thereby distributing the vertical force-component more evenly on the core. When reinforcing the outer face sheet in this manner, due consideration should be given to the required shape (i.e. the shape of a boat keel may affect stability). The core in the immediate vicinity of the V-joint may have a higher density, enabling it to accommodate the local stress-concentration. The fillet reinforcement transforms the joint-region to a multi-skin, multi-core sandwich. It may be difficult to manufacture, since the fillet core must be shaped to fit properly in the V.

10.6 Localised Deflection

Usually, elaborate reinforcement of a joint is undesirable. If the joint must be very stiff, reinforcement is generally necessary, but if not, a simple joint may be considered. In many cases, the simpler joints will cause line loads transverse to one or both the panels joined.

When a sandwich panel is subjected to a line load, localised bending of one face-sheet in particular will take place. In the case of the T-joint, for example, the two face-sheets of the bulkhead will act roughly as two parallel line loads acting upon the hull inner face-sheet. Similar considerations are valid for the other joint types.

Localised bending of face sheets in compression may initiate buckling of that face sheet, whereupon the entire bending stiffness of the panel is easily lost. The load-case outlined in Fig.10.11 is thus of particular importance: a single line load acting upon a long sandwich panel.
Figure 10.11 Line load (extending out of the plane of the paper) on a sandwich panel. (a) Overall deflection, as calculated using ordinary sandwich beam theory, (b) local deflection of upper face-sheet, and (c) total deflection. The total deflection (c) is estimated by adding the overall deflection (a) and the local contribution (b) due to localised bending of the upper face-sheet.

The simple superposition outlined in Fig.10.11 is an approximate method, but will often give a good indication of the local core compression and the inner face-sheet bending stresses near the load.

10.6.1 Deflections
For calculating the local deflection, a one-parameter elastic foundation model (Winkler foundation model) is convenient, see Fig.10.12.

Figure 10.12 Face-sheet with bending stiffness $D$, placed upon an elastic foundation with stiffness $k$, and subjected to a line load $F$.

The deflection is in this case given\(^3\) by:

$$y(x) = \frac{Pe^{-\beta x}}{8D\beta^3} \left[ \cos(\beta x) + \sin(\beta x) \right]$$  \hspace{1cm} (10.1)

where  \hspace{1cm} $P$ [N/mm]: Load per width (line load extending perpendicular to section)

---

\(^3\) The equation can be found by combining elementary cases from “Formulas for Stress and Strain” or similar literature.
JOINTS BETWEEN SANDWICH PANELS

\[ D \text{ [Nmm]} : \text{Face-sheet bending stiffness, } D = \frac{Et^3_f}{12(1-\nu^2)} \]

\( E, t, \nu \) are the modulus of elasticity, the thickness and the Poisson’s ratio for the face sheet, respectively, for an isotropic face-sheet

\[ \beta \text{ [mm}^{-1}] : \text{Parameter of relative stiffness, } \beta = \frac{k}{4D} \]

\( k \text{ [N/mm}^3] : \text{Foundation stiffness} \)

The foundation stiffness \( k \) may be estimated by \( E_c/t_c \), where \( E_c \) is the modulus of elasticity of the sandwich panel core, and \( t_c \) is the thickness of the core. Note that the expression is for \( x \geq 0 \). Since the case is clearly symmetric, however, deflections for negative \( x \) may be found by taking the deflection at the corresponding positive \( x \), i.e. \( y(-x) = y(x) \).

The wavelength of the deflection is given by

\[ \lambda = \frac{2\pi}{\beta} \quad (10.2) \]

Although the above expression for deflection is based upon a panel extending infinitely in the ±\( x \)-directions, the decay is so rapid that the calculation is usually valid if there is no free sandwich panel edge within a distance of approximately \( \lambda \) or \( 3t_c^4 \), whichever is largest.

Application for T-joint

The simple case of a single line load may be combined by superposition. Fig.10.13 shows a few cases for the T-joint.

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Figure 10.13 The local deflection \( y \) of the upper face-sheet is the sum of the individual deflections, \( y_1 \) and \( y_2 \), at each point \( x \). Left to right: Left: Thin plate transferring a single force to the sandwich panel, essentially this is a single line load. Middle: Bulkhead transferring force to hull. Bulkhead face sheets 1 and 2 act as parallel line loads with magnitude \( P/2 \), placed a distance \( c \) from each other. Right: Bulkhead transferring a moment to the hull. The moment \( M \) acts as two opposite line loads \( P = M/t_c \).

---

\(^4\) The number \( 3t_c \) may have to be varied, depending on the relation between core shear stiffness and face sheet extensional stiffness. For soft cores, the number should be larger.
Application for V-joint
The V-joints may also be considered in this manner. At the concave notch (the inner face-sheet) in particular, the sharp angle in the panel may cause something similar to a line load, as shown in Fig.10.14.

From the geometry, $P_y$ is given by $P_y = -2M\sin(\alpha)/t_c$. Note that the core thickness used for calculating the foundation modulus $k$ should be based upon the thickness in the direction of the force $P_y$. The foundation stiffness for the V-joint is then given by $k = E_c \cos(\alpha)/t_c$. This means that the stiffness is less than for a plane panel.

10.6.2 Face-sheet bending stress
By taking the second derivative of the expression for deflection, the approximate curvature of the face sheet is found.

$$y''(x) = \frac{P}{4D\beta} \cdot e^{-\beta x} \cdot (\sin(\beta x) - \cos(\beta x))$$  \hspace{1cm} (10.3)

The face sheet bending stress for isotropic face sheets is then

$$\sigma_s = \frac{y''E}{1-\nu^2} \cdot \frac{t_c + t_f}{2} \hspace{1cm} \text{(10.4)}$$

10.7 Calculation example, T-joint
This example considers the case of a sandwich ship hull, subjected to a load $q$ from one side and supported by a number of bulkheads, see Fig.10.15. The thickness of the sandwich panels is assumed to be small, compared to the span $l$. All loads and geometric entities are expressed per width (perpendicular to the plane of the paper), assuming plane strain.
Figure 10.15  **Left:** Long sandwich panel supported by bulkheads and subjected to uniform pressure $q$.
**Right:** Cut-out of representative section. $l/2$ marks the “free length” of the hull panel, where the bending moment is zero. The deflection $y(x)$ of the hull is shown below the cut-out.

At the bulkhead, the reactions at relevant sections will be considered, as shown in Fig.10.16. Initially, the shear force and bending moment in the hull panel should be controlled.

**Shear**

The nominal shear stress in the hull panel at the bulkhead is

$$\tau = \frac{T}{t_{c1}} = \frac{ql}{2t_{c1}} \quad (10.5)$$

assuming that the shear is carried by the core alone. This should be considerably smaller than the allowable core shear stress, since the bulkhead will cause stress concentrations.
**Bending Moment**

The tension/compression stress in the hull face-sheets due to the panel moment are probably about 1-2 orders of magnitude larger than the tension or compression stresses in the bulkhead face-sheets. For a known free length $l_f$, the moment in the hull panel at the bulkhead is

$$M = q l_f (\frac{l}{4} - \frac{l_f}{8})$$  \hspace{1cm} (10.6)

For uniform load $q$, $l_f / 2$ is approximately $0.21l$. Assuming thin face sheets carrying the entire moment as a force-pair, the moment alone causes the compression/extension stress

$$\sigma_M = \pm \frac{M}{t_c l_f}$$  \hspace{1cm} (10.7)

**Local bending of hull upper face-sheet**

The force $2T = ql$ is supported by the bulkhead, where each of the bulkhead face sheets, being much stiffer than the core, acts on the hull upper face sheet with the force $P \equiv -|T|$, $P$ measured positive upwards. This is then treated by superposing two forces $P$ at a distance $t_c$, as shown in Fig.10.13.

**10.8 Calculation example, L-joint**

This example is included to demonstrate the stiffness-increasing effect of modifying an L-joint, by adding three different reinforcements one by one. The calculations are done using the finite-element method. The geometry and loading is as shown in Fig.10.17.

The load $\tau = 0.1$ MPa is applied across the height of the core, at end (a) as shown in Fig.10.17.

The basic panel features are:
- Panel face-sheets: thickness $t_f = 4$ mm, GFRP, $E_f = 10\,000$ MPa, $\nu = 0.30$
- Panel core: thickness $t_c = 40$ mm, Divinycell H130, $E_c = 140$ MPa, $\nu = 0.32$

The reinforcements are:
1. High density core material, $E = 300$ MPa replaces original core at outer 90 mm of horizontal panel
2. Divinycell H130 fillet, covered by GFRP bonded tape laminate (thickness and stiffness as panel face sheets), added at inner corner
3. GFRP bonded tape laminate (thickness and stiffness as panel face sheets) added on outer corner
Figure 10.17  L-joint, geometry and loading. 1, 2 and 3 represents the different reinforcements.

Plane strain is assumed in the FE-model. It should be noted, that the geometry shown in Fig.10.17 is simplified. Gap-fillers, thick adhesive-layers and common imperfections should be included if the local stresses and strains are investigated. For the purpose of investigating the stiffness, however, a coarser model may be used. For comparison of the effects of increasing the stiffness, the vertical deflection $y$ is taken at the end $a$ of the horizontal panel (at the end where the shear stress $\tau$ is applied). The deflections for the four different cases is as shown in the diagram below.

Clearly, the reinforcements increases the stiffness significantly. Whether the joint should actually be stiffened, and how much, is usually a matter of economics.
10.9 T-joints - Tests and Observations
The T-joint has been subjected to much research, as mentioned previously. This section will treat some primary results from tests and calculations, based mainly upon the works of C. Kildegaard and C. Burchardt. Three joint configurations have been considered. These are shown in Fig.10.18.

![Figure 10.18 T-joints - (a) circular fillet, filler between intersecting panels, (b) triangular fillet, filler between intersecting panels and (c) circular fillet, bulkhead face-sheets cut off](image)

The face-sheets and the bonded tape laminate (covering the fillet) are CFRP, the core and fillet are made from PVC-foam (Divinycell), and the filler is a polyester compound.

For testing the joints, the most common method is to support a bulkhead panel at the ends and apply a tensile force to the bulkhead. This, and the resulting fracture modes for joints (a), (b) and (c), is shown in Fig.10.19.

![Figure 10.19 Load case (left) and fracture modes (dashed lines). 1 denotes initiation of cracks.](image)

Common to the fractures is that cracks tend to initiate in the same place, as marked by “1” in Fig.10.19. The stress concentration in this place is caused by the stiffness discontinuity and the difficulty of avoiding air bubbles when manufacturing the joint. The types (a) and (b) were investigated by C. Kildegaard.

**Case (a):** For the circular fillet, the joint strength clearly depends upon the fillet radius; larger fillet gives greater strength. The stress concentration at the point where the bonded tape laminate meets the hull upper face-sheet will eventually initiate a crack.

**Case (b):** In the case of a triangular fillet, the flat sides of the bonded tape laminate means that significant transverse (vertical) force can be transferred between the bulkhead and the hull upper face-sheet. This causes a large stress concentration at the point where the bonded tape laminate touches the hull upper face-sheet. Different fillet sizes were tested, with side lengths ranging from ½ to 2 times the thickness of the cores. No significant difference in strength was detected for
different triangular fillet sizes. The circular fillet will in general be stronger than the triangular fillet.

Case (c): This type was investigated by C. Burchardt and others. The main difference from type a is that the bulkhead face-sheets and the bonded tape laminate form a continuous sheet. Burchardt concluded, among other things, that the thickness of the bonded tape laminate is not of primary importance; it should merely delay fracture from points #2. The fillet material carries the actual load.

10.9.1 Test results
This section contains a summary of T-joint test results, as investigated by C. Kildegaard. The three joint-configurations tested, and the overall geometry, are shown in Fig.10.20. Note that a simple fillet, using a polyester filler, is included.

![Diagram of T-joint test results]

- Panel core and foam fillet: Divinycell H100, Thickness $t_c = 60$ mm
- Face-sheets: GFRP, three layers combi-mat, 900 g/m² (600 g/m² 0°/90° woven mat and 300 g/m² chopped strand mat) and one layer 450 g/m² chopped strand mat
- Bonded tape laminate: six layers 450 g/m² chopped strand mat
- Face-sheet and bonded tape laminate matrix: Isopolyester
- Filler: Polyester filler

The following table contains the joint-configuration parameters, test number, initial fracture force and final fracture force. The fracture forces are given per joint-length (perpendicular to the plane of the paper) in Table 10.2.
The type-II joints tend to break suddenly, and the initial and the final fractures are coincident.

References


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<th>Final fracture force [N/mm]</th>
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<td>Δ60-1</td>
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<td>146.7</td>
</tr>
<tr>
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<td>R60-1</td>
<td>-</td>
<td>162.0</td>
</tr>
<tr>
<td>II</td>
<td>60</td>
<td>R60-2</td>
<td>-</td>
<td>142.7</td>
</tr>
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<td>120</td>
<td>R120-1</td>
<td>-</td>
<td>193.3</td>
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<tr>
<td>II</td>
<td>120</td>
<td>R120-2</td>
<td>-</td>
<td>200.0</td>
</tr>
<tr>
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<td>15</td>
<td>F1</td>
<td>134.7</td>
<td>134.7</td>
</tr>
<tr>
<td>III</td>
<td>15</td>
<td>F2</td>
<td>112.0</td>
<td>137.3</td>
</tr>
</tbody>
</table>

Table 10.2 Test results from testing of different T-joints.
Testing may be one of the most important parts in the design and verification process of a structure or structural component. In the early design stages, basic material properties must be obtained for use in simple hand-calculations or as input to finite element programs. The material properties can be extracted in principal in three different ways: from books or other reference, from theoretical calculations or estimations, or from testing. Handbook data may be difficult to use since it is seldom likely that others have used exactly the same materials and processes as the one to used in the particular design case. For example, physical and mechanical properties of composite materials are strongly influenced by fibre content, lay-up sequence, manufacturing processes, etc. Theoretical estimation are sometimes a good tool in the early design stages to obtain approximate mechanical properties. However, at later stages more accurate data might be a necessity and then one must rely on testing.

During the course of the design process, testing is sometimes necessary of a sub-structure for verification of a specific mechanical or physical behaviour. Calculations can take us very far these days in terms of simulating a physical behaviour, but finally one must actually test if the analytical or numerical simulation was correct and testing of the full-scale structure or sub-structure becomes necessary.

This chapter first describes testing of face materials and core materials alone for the extraction of physical and mechanical properties, and then follows a section on testing of sandwich components. All tests methods described give a brief introduction to why and how the test is used, in most a schematic of the test specimen and/or rigging is included and some formulae, or similar, are given for how the specific sought after property(-ies) is obtained. After each test method follows a list of references to standards, books and papers where more information can be found.

11.1 General Information on Testing of Materials

If proper protective equipment is not used during composite or sandwich specimen manufacturing, systemic toxic effects can occur from overexposure to many of the components used. The toxicity may occur from inhalation and/or dermal exposure. The reactivity of the resins is essential for their curing properties, and contributes to their acute toxicity. Thus, eye and skin irritation, irritation of the mucous membranes of the nose and throat, skin sensitation, and respiratory sensitation can occur in composites workers where adequate industrial hygiene practices are not employed. Also,
mechanical irritation of the eyes and mucous membranes may be caused by exposure to fibres and dust where composites are worked.

Examples of factors that influence the measured properties are material, methods of material preparation, specimen lay-up stacking sequence, specimen preparation, specimen conditioning, environment of testing, specimen alignment and gripping, speed of testing, time and temperature, void content, and volume percent of reinforcement.

The control of fibre alignment during panel manufacturing is critical. Improper fibre alignment will reduce the measured properties and it will also increase the coefficient of variation.

Specimen preparation is extremely important. Take precaution to avoid notches, undercuts, rough or uneven surfaces, or delaminations due to inappropriate machining methods. Marks left from coarse machining operations shall be carefully removed with a fine file or abrasive and then smoothed with fine abrasive paper. The methods and techniques for machining composites are related to the type of composite, i.e., the thermoplastic or the thermoset matrix, and reinforcement type such as continuous or discontinuous and inorganic versus organic reinforcement. The machining techniques for glass reinforced laminates are well established. Standard metal and wood-working machining equipment can be used with some modification for increasing spindle speeds along with reduced feeds. Standard cutting tools are suitable only for short production runs. Therefore, presently-used tools are tungsten carbide or diamond tipped. Although the machining of thermoplastics is a well known art, it is complicated by the large property differences among the available materials. As a complement to the traditional machining techniques, non-conventional machining methods, e.g. laser beam machining, water jet or electrical discharge machining, can be used. The advantage of using some of these noncontact tool cutting methods are that the rate of tool wear is small and surface damage due to mechanical action of the tool will be negligible. It should be pointed out that these methods may not produce shape changes of a work piece, as is possible with traditional machining methods. However, some non-conventional approaches have proved to be more effective for secondary processes.

Some extra precaution must be taken when machining sandwich components since the properties of the materials involved so greatly differ. Cutting and drilling and other machining operations create heat due to the abrasive nature of metals and composites. Most core materials have a rather low heat resistance and if extreme care is not taken, machining can severely damage the core. Most core materials also have a high thermal insulation capability making the cutting or drilling even more complicated since heat is not transferred away as fast as when machining only metals or composites.

If bonded resistance strain gauges are used the specimen shall be carefully prepared before bonding. Reinforcing fibres should not be exposed or damaged during the surface preparation. Bonding strain gauges directly to a core material is an art in itself. A strain gauge bonded with e.g. epoxy resin to a foam core actually creates a stress concentration since even the thin layer of adhesive and the strain gauge itself is much stiffer than the core.

End tabs are strongly recommended when testing specimens using grips, both in compression and in tension, due to the large stress concentrations in the gripping area. Hence, designing mechanical
test specimens, especially those using end tabs, remains to a large extent an art rather than a science, with no industry consensus on how to approach the engineering of the gripping interface. Each major composite testing laboratory has developed gripping methods for the specific material systems and environments commonly encountered within that laboratory. Comparison of these methods shows them to differ widely, making it extremely difficult to recommend a universally useful approach or set of approaches.

In general, the physical and electrical properties of plastics are influenced by temperature and relative humidity in a manner that materially affects test results. In order that reliable comparisons may be made of different materials and between different laboratories, it is necessary to standardise the humidity, as well as the temperature, to which specimens of these materials are subjected prior to and during testing. The specimens are conditioned through exposure of specified temperature and humidity during a specified time following a procedure for conditioning prior to test. The purpose of the conditioning is to bring the material into equilibrium with normal or average room conditions or to obtain reproducible results, regardless of previous history of exposure. But the purpose can also be to subject the material to abnormal conditions of temperature or humidity in order to predict its in-service conditions.

Finally, it is of great concern to make a good report of the test series with a complete description of the test methods used. The report should contain a description of the equipment and the techniques used for each series of tests, a description of the test specimens and a report of test results, which summarises the results of each test series. It should also contain detailed descriptions of types of specimen fractures and other, from a material testing point of view, interesting details, e.g., illustrated with a photograph.

Referenced Documents


ASTM D 3544-76 Standard Guide for Reporting Test Methods and Results on High Modulus Fibers.
11.2 Face Materials

11.2.1 Tensile tests

Aim and Purpose of Test Methods

These test methods are designed to produce tensile property data for material specifications, research and development, quality assurance, and structural design and analysis. The testing procedures are similar between the different test methods but there are differences in the specimen geometries. Some of the test methods use a dumbbell shaped specimen while others use a rectangular coupon and one method uses a notched specimen to determine the notch sensitivity.

Specimen types, Dimensions and Manufacturing

Three different kinds of specimen geometries used in tensile testing are shown in Fig. 11.1. The specimens are prepared by machining operations from panels.

![Tensile Specimens Diagram](image)

Figure 11.1 Different shapes of tensile specimens: The dumbbell shaped specimen (a) the rectangular coupon without tabs (b), and with tabs (c) and the notched specimen (d).

Tabs are strongly recommended especially when testing unidirectional (or strongly unidirectionally dominated) laminates to failure in the fibre direction. Tabs may also be required when testing unidirectional materials in the matrix direction to prevent gripping damage. The selection of a tab configuration that can successfully produce a gauge section tensile failure is dependent upon the specimen material, the specimen ply orientation and the type of grips being used. The most consistently used bonded tab material has been continuous E-glass fibre-reinforced polymer matrix materials in a [0/90]_ns laminate configuration. The tab material is commonly applied at 45° to the loading direction to provide a soft interface. Incorporated steel tabs and tabs made of the same material being tested are other configurations that may be successful. Tabs need not always be bonded to the specimen. Friction tabs, essentially non-bonded tabs held in place by the pressure of the grip, and often used with emery cloth or some other light abrasive between the tab and the specimen, have been successfully employed in some applications.
Apparatus
The testing machine shall have the capability to control and regulate the velocity of the movable head and the loadsensing device of the testing machine shall be capable of indicating the total load being carried by the test specimen. It is highly desirable that the grips for holding the test specimen in the testing machine are of a rotationally self-aligning type to minimise bending stresses in the specimen. Strain data shall be determined by means of either a strain transducer (strain gauge) or an extensometer. Attachment of the strain indicating device to the specimen shall not cause damage to the specimen surface. If Poisson’s ratio is to be determined the specimen shall be instrumented to measure strain in both lateral and longitudinal directions.

Testing Procedure
Before testing, measure the specimen dimensions and then calculate the cross-section area in the gauge length. Place the specimen from the material tested in the grips of the mechanical testing machine, taking care to align the long axis of the gripped specimen with the test direction. The holes in the notched specimens may be filled with rivets or bolts. Apply load to the specimen at the specified rate in tension until failure while recording data. The ultimate strength of the material can be determined from the maximum load carried prior to failure as

\[ \sigma_{cr} = \frac{P_{cr}}{tw} \]  \hspace{1cm} (11.1)

Where \( \sigma_{cr} \) is the tensile stress at the failure load, \( P_{cr} \), and \( w \) is the width of the specimen at the test section and \( t \) is the specimen thickness. If the specimen strain is monitored with strain or displacement transducers then the stress-strain response of the material can be determined, from which the ultimate tensile strain, tensile modulus of elasticity, Poisson’s ratio, and transition strain can be derived.

Referenced Documents
ASTM D 3039-93 Test Method for Tensile Properties of Fibre-Resin Composites
CRAG Method 300 Method of Test for the Longitudinal Tensile Strength and Modulus of Unidirectional Fibre Reinforced Plastics
CRAG Method 301 Method of Test for the Transverse Tensile Strength and Modulus of Unidirectional Fibre Reinforced Plastics
CRAG Method 302 Method of Test for the Tensile Strength and Modulus of Multidirectional Fibre Reinforced Plastics
CRAG Method 303  Method of Test for the Notched Tensile Strength of Multidirectional Fibre 
Reinforced Plastics

ISO/DIS 294-2  Plastics--Injection Moulding of Test Specimens of Thermoplastic Materials-  
-Part 2: Small Tensile Bars (Revision in part of ISO 294: 1995).


Films and Sheets.

ISO/DIS 527-4  Plastics--Determination of Tensile Properties--Part 4: Test Conditions for  

ISO/DIS 527-5  Plastics--Determination of Tensile Properties--Part 4: Test Conditions for  


ISO 6252: 1992  Plastics--Determination of Environmental Stress Cracking (ESC)--  
Constant-Tensile-Stress Method

Tensile Vibration--Non-Resonance Method

ISO/DIS 6721-9  Plastics--Determination of Dynamic Mechanical Properties--Part 9:  
Tensile Vibration--Sonic-Puls Propagation Method


EN 61  Glass Fibre Reinforced Plastics – Determination of Tensile Properties.

EN 2561  Fibre Reinforced Plastics – Uni-Directional Laminates –Tensile Test  
Parallel to the Fibre Direction.

* ASTM D638-93 and ISO 527-1966 are technically equivalent.
11.2.2 Compressive tests

Aim and Purpose of Test Methods
These test methods cover the determination of the mechanical properties of reinforced rigid plastics, including resin-matrix composites reinforced by oriented continuous high-modulus fibres, when loaded in compression. There are some different methods in measuring the mechanical properties in compression, some using coupon shaped, or prism specimens loaded in compression, using some more or less complicated fixture to prevent buckling, and one test method using a sandwich beam loaded in a four-point bending rig.

Specimens Loaded in Compression

Specimen types, Dimensions and Manufacturing:
The test specimens used together with some supporting fixture are prepared by machining operations from sheets or panels of the material to be tested. Great care shall be taken in machining the ends so that smooth, flat, parallel surfaces and sharp, clean edges perpendicular to the long axis of the specimen, are obtained. Three different geometries of compression test specimen, with and without tabs, are shown in figure 11.2, where the specimen with a hole in the middle is used to determine the notch sensitivity of in compression of the material. The prism specimen, shown in figure 11.5, is used for testing "thick" laminates.

![Figure 11.2](image)

Figure 11.2 Different shapes of compressive test specimens: The specimen without tabs and with a waist (a), and the notched specimen in (c) is used with a support jig (Figure 11.3), and the slender coupon with tabs (b) is used with a fixture (Figure 11.4).
**Apparatus:**
A fixture (figure 11.3 and figure 11.5) or a support jig (figure 11.4) is used with a testing machine that must be capable of control of constant-rate-of-crosshead movement and have a mechanism capable of showing the total compressive load carried by the test specimen. The fixture or support jig, respectively, is used to prevent the specimen to buckle. A compressometer or strain gauges is needed for determining the distance between two fixed points on the test specimen at any time during the test. It is desirable that the compressometer automatically record this distance as a function of the load on the test specimen.

![Figure 11.3 Schematic view of fixture used together with the specimen type in figure 11.2 (b).](image1)

![Figure 11.4 Schematic view of the support jig used together with the specimen type in figure 11.2 (a).](image2)

![Figure 11.5 A square cross-section prism specimen, with collars to prevent brooming, is used for thick laminate testing.](image3)

**Testing Procedure:**
The specimen is mounted in the specially designed fixture or support jig, depending on which specimen used. When testing the notch sensitivity, the hole in the specimen can be filled with a rivet or bolt. The assembly is then placed between the loading plates of the testing machine. The specimen is then loaded to failure to obtain the ultimate compressive strength. The compressive strength is then calculated as

\[
\sigma_{cr} = \frac{P_{cr}}{tw}
\]

(11.2)
Where $P_{cr}$ is the failure load, $w$ is the width of the specimen in the gauge area and $t$ is the specimen thickness. Load strain curves must be obtained during the test if the compressive modulus is desired.

**Sandwich Beam Specimen Loaded in Four-Point Bending**

In this test method, a sandwich beam is used for measuring the mechanical properties in compression.

**Specimen types, Dimensions and Manufacturing:**

The test skin is bonded on top of a heavy density core, e.g. 368 kg/m$^3$ hexagonal aluminium honeycomb core, using a structural adhesive. On the opposite side the skin shall be approximately twice as thick as the test skin. The test skin shall be a laminate with plies only in the 0$^\circ$ direction and with the 0$^\circ$ orientation in the longer direction. Bond the test laminate and the opposite skin to the core and then cut the panel into test specimens as shown in figure 11.6.

![Figure 11.6](image)

Figure 11.6. The four-point bending sandwich beam used as a compressive test specimen.

**Apparatus:**

A four-point bending rig as shown in figure 11.7 is used with a testing machine with the same capabilities as described above.
Testing Procedure:
Strains are measured using two axial strain gauges which are applied as shown in figure 11.6. The specimen is placed in the four-point bending rig with rubber load pads placed between the specimen and the contact points and the strain gauges is connected to some strain recording equipment. The specimen is then loaded to failure at a constant crosshead speed and the load and strain data is constantly recorded. The ultimate compressive strength is then calculated as

$$\sigma_{cr} = \frac{P_{cr} l_m d}{2 W t_1} \text{ with } d = t_c + \frac{t_1 + t_2}{2}$$

(11.3)

Where $P_{cr}$ is the load at failure, $W$ the width and $t_1$ the thickness of the investigated composite facing, $t_2$ the thickness of the opposite facing, $t_c$ is the core thickness and $l_m$ is the length of the moment arm (Figure 11.6).

Referenced Documents
ASTM D 3410-87 Test Method for Compressive Properties of Unidirectional or Crossply Fibre-Resin Composites.
CRAG Method 400 Method of Test for Longitudinal Compression Strength and Modulus of Unidirectional Fibre Reinforced Plastics
CRAG Method 401 Method of Test for Longitudinal Compression Strength and Modulus of Multidirectional Fibre Reinforced Plastics
11.2.3 In-plane shear tests

**Aim and Purpose of Test Methods**

The in-plane shear is the shear associated with shear forces applied to the edges of the laminate so that the resulting shear deformations occur in the plane of the laminate rather than through the thickness. Some different test methods for measuring the in-plane shear strength and stiffness are described. These methods are using different techniques and specimens for measuring the different shear properties. One method uses a v-notched rectangular specimen that is loaded in shear through a special fixture. Another method uses a coupon with two notches in the plane that is mounted in a fixture and then loaded in compression. To use a tensile test method is another possibility and then a method using small plates loaded in shear using two different kinds of special fixtures is presented. All these methods are valid for tests of reinforced plastics, including high-modulus composites.
V-notched beam method
With this test method it is possible to determine both the in-plane and interlaminar shear properties of a composite laminate, depending upon the loading direction relative to the material coordinate system. The measured laminates shall be composed only of unidirectional fibrous laminae loaded in 0° or 90° direction, or laminates containing equal number of plies of unidirectional fibrous laminae oriented 0° and 90° in a balanced and symmetric stacking sequence. The laminates can also be composed of woven-fabric filamentry laminae loaded in 0° or 90° direction, or short fibre reinforced composites with a majority of the fibres being randomly distributed.

Figure 11.8 Schematic view of V-notched beam specimen with test fixture.

Specimen type, Dimensions and Manufacturing:
The specimen is a rectangular flat strip with symmetrically centrally located v-notches as, shown in figure 11.8. The notches shall be carefully machined to an angle of 90°. The specimens may be cut from a laminate to measure the shear properties in the plane, or through the thickness of a thick laminate (about 20 mm) to measure the interlaminar shear properties. Use of tabs is recommended when testing laminates that are less than 2.5 mm thick. Tabs, locally bonded to both faces of the specimen away from the test region, strengthen and stabilise the specimen by locally increasing the thickness in the gripping region, as shown in figure 11.9.

Apparatus:
The testing machine must be capable of control of constant-rate-of-crosshead movement and have a mechanism capable of showing the total load carried by the test specimen. The lower half of the test fixture is attached to one of the testing machine heads and the other head shall be attached to the upper half of the fixture using an adapter which, if required, may be capable of relieving minor misalignments between the heads. A minimum of two strain gauges, centred about the loading axis in the gauge section of the specimen, as shown in figure 11.9, shall be used to measure the strain.
If specimen twisting is of concern, then strain gauge elements should be mounted on both sides of the specimen.

![Strain gauge location on the V-notched beam specimen.](image)

**Figure 11.9** Strain gauge location on the V-notched beam specimen.

**Testing Procedure:**
The specimen is inserted into the fixture with the notch located along the line-of-action of loading by means of an alignment tool that references the fixture. The two halves of the fixture are compressed by the testing machine while monitoring the load. The shear response of the material is measured by the strain gauges which are mounted on the specimen. The ultimate shear strength, which is the shear stress at maximum load prior to failure, is calculated as

\[
\tau_{cr} = \frac{P_{cr}}{t \cdot h}
\]

Where \( P_{cr} \) is the failure load, \( t \) is the thickness in the gauge section and \( h \) is the thickness at the notch of the specimen as shown in figure 11.9.

**In-plane plate method**
In these methods the in-plane shear properties are determined by imposing edgewise loads on the specimen using either a fixture consisting of two pairs of rails tensile loaded, or a fixture consisting of three pairs of rails, or a quadratic frame, in tension or compression loading.

**Specimen type, Dimensions and Manufacturing:**
Three different specimens are used in three different methods. The geometries of the specimens are shown in figure 11.10 and 11.11. The specimens are cut from composite laminates with a recommended thickness between 1.3 and 3.2 mm. The straight edges of the specimen may have coarse tool marks from the machining operation; however, the holes shall be drilled and reamed if minor delamination occurs. The holes may be of oversize to the bolts, although press fit bolts have been used with success, particularly with tabbed specimens. Note that while the sample outer dimensions are uniform, many variations of hole patterns and tabbed edges have been used.
Apparatus:
The test fixtures, schematically shown in figure 11.10 and 11.11, are used with a testing machine with capabilities as described in the previous section.

Testing Procedure:
The specimen is bolted to the test fixture. A tensile load is then applied to the rail which induces an in-plane shear load on the specimen. To measure the shear modulus a strain gauge is bonded at the centre of the specimen at $45^\circ$ to the specimen longitudinal axis as shown in figure 11.10. The specimen is then loaded to failure. Record the load and strain continuously during the test. Also record the failure load and observe the mode of failure, which usually is from buckling out of plane. When using the fixture with two pair of rails, the ultimate shear strength is calculated as

\[
\tau_{cr} = \frac{P_{cr}}{l \cdot t}
\]  

and when using the fixture with three pair of rails, the ultimate shear strength is calculated as
\[ \tau_{cr} = \frac{P_{cr}}{2l \cdot t} \]  \hspace{1cm} (11.6)

Where \( P_{cr} \) is the failure load, \( l \) is the total length, and \( t \) is the thickness of the specimen. The shear modulus is calculated from

\[ G = \frac{\Delta P/\Delta \delta}{2l \cdot t} \]  \hspace{1cm} (11.7)

for +45\(^o\) or -45\(^o\) strain gauge, when using the fixture with two pair of rails. When using the fixture with three pair of rails, the shear modulus is calculated from

\[ G = \frac{\Delta P/\Delta \delta}{4l \cdot t} \]  \hspace{1cm} (11.8)

\( \Delta P/\Delta \delta \) equals the slope of the plot of load as function of deformation within the linear portion of the curve.

**Compressive method**

In this test method, the in-plane shear strength is measured by applying a compressive load to a notched specimen of uniform width. This test method is useful in testing laminates having randomly dispersed fibre reinforcement as well as laminates with parallel-fibre, or nonparallel-fibre, reinforcement.

**Specimen type, Dimensions and Manufacturing:**

A rectangular coupon, of the material tested, of about 80 by 13 mm is cut from a laminate of thickness between 2.5 and 7 mm (ASTM D 3846). Then two parallel cuts, one of each opposite face of the specimen with a depth of half the specimen thickness and about 6.4 mm apart, shall be sawed across the entire width of the specimen as shown in figure 11.12.

![Figure 11.12](image)

Figure 11.12 The specimen geometry (left) and the support jig used in the testing procedure (right).
Apparatus:
A supporting jig (figure 11.12), used to prevent the specimen to buckle, is used with a testing machine with capabilities as described in previous section.

Testing Procedure:
The specimen is mounted in the supporting jig so that it is flush with the base and centred and then the assembly is placed in a compression tool in the testing machine. Record the maximum load, which usually is the load at the moment of rupture, carried by the specimen during the test. Then the length of the failed (sheared) area is determined by measurement of this surface with respect to either half of the ruptured specimen. This technique affords the most accurate determination of the length of the sheared plane defined by the separation of the notches machined in the specimen. The in-plane shear strength, $t_{cr}$, is calculated by dividing the maximum shear load carried by the specimen by the product of the width of the specimen and the length of the failed area.

Tensile method
Test methods for determining the in-plane shear stress-strain response of polymer matrix composites often require expensive and complex test specimens or specialised test fixtures. The test method presented here has the advantage of utilising the relatively inexpensive straight-sided coupon and conventional test equipment.

Specimen type, Dimensions and Manufacturing:
A rectangular test specimen, as shown in figure 11.13, is cut from a balanced, symmetric polymer matrix composite laminate which is composed of only +45 and -45 plies. Tabs are strongly recommended.

![Figure 11.13 The specimen without (top) and with tabs.](image)

Apparatus:
The testing machine used, shall have the capabilities as described in previous section. It is desirable that the grips for holding the test specimen in the testing machine are of a rotationally self-aligning type to minimise bending stresses in the specimen. Load strain data shall be determined by means of either a strain gauge or an extensometer in both longitudinal and lateral directions.
Testing Procedure:
Before testing, measure the specimen width and thickness and then calculate the cross-section area. Place the specimen from the material tested in the grips of a mechanical testing machine, taking care to align the long axis of the gripped specimen with the test direction. Apply load to the specimen at the specified rate in tension until failure while recording load and both longitudinal and lateral strains. The maximum load carried by the specimen during the test and the longitudinal and transverse strains at the moment of rupture is also recorded. Calculate the unidirectional shear strength using the equation

\[ \tau_{cr} = \frac{P_{cr}}{2w \cdot t} \]  

(11.9)

Where \( P_{cr} \) is the failure load, \( w \) is the width, and \( t \) is the thickness of the specimen.

The shear strain at load, \( P \), is given by

\[ \gamma_{12} = \varepsilon_0 - \varepsilon_{90} \]  

(11.10)

where \( \varepsilon_0 \) is the longitudinal strain and \( \varepsilon_{90} \) is the transverse strain. Make a shear stress-strain plot from the measured data. The unidirectional shear modulus can then be calculated as

\[ G_{12} = \frac{\Delta \tau_{12}}{\Delta \gamma_{12}} \]  

(11.11)

Where \( \Delta \tau_{12}/\Delta \gamma_{12} \) equals the slope of the plot of the unidirectional shear stress-strain curve within the linear portion of the curve.

Referenced Documents
ASTM D 3518-76 Practice for In-plane Shear Stress-Strain Response of Unidirectional Reinforced Plastics.

ASTM D 3846-79 Test Method for In-Plane Shear Strength of Reinforced Plastics.


CRAG Method 101 Method of Test for In-Plane Shear Strength and Modulus of Fibre Reinforced Plastics

ISO/DIS 14129 Fibre-Reinforced Plastic Composites--Determination of In-Plane Shear Modulus and Strength by Plus or Minus 45 Degrees Tension Test Method.
11.2.4 Interlaminar shear tests
Aim and Purpose of Test Methods
Only one test method for measuring the interlaminar shear strength are described here, but the V-notched specimen described in section 11.2.3 can also be used to determine interlaminar shear properties. These test methods are applicable for research and development programs concerned by interply strength but they are also useful for quality control and specification purpose.

Short beam method
This test method covers the determination of the interlaminar shear strength of parallel fibre reinforced plastics. The specimen is a short beam in the form of a segment cut from a ring type sample or a flat laminate. The apparent shear strength obtained in this method should not be used as a design criteria, but can be utilised for comparative testing of composite materials.

Specimen type, Dimensions and Manufacturing:
This specimen is a short beam (figure 11.14) cut from a ring-type specimen, or a flat laminate, of parallel fibre reinforced plastic up to 6.4 mm in thickness (recommendation in ASTM D 2344). The specimen size is not limited but a specified span to depth ratio of 5, when reinforcement with Young’s modulus \( <10 \times 10^9 \) is used, or 4, when reinforcement with Young’s modulus \( >10 \times 10^9 \) is used, is recommended.

Figure 11.14  Schematic view of the two different kinds of short beam specimens.

Apparatus:
A three-point bending rig with supports and loading nose with specified diameters is used in a testing machine that must be capable of control of constant-rate-of-crosshead movement and have a mechanism capable of showing the total load carried by the test specimen.

Testing Procedure:
Measure the thickness and width of the specimen and place it in the test fixture as shown in figure 11.14. Be careful to align the specimen so that its midpoint is centred and its long axis is perpendicular to the cylindrical axis or under the loading nose. Apply the load to the specimen at
a specified crosshead rate and record the load to break the specimen. The apparent shear strength is then calculated as

\[ \tau_{cr} = \frac{3P_{cr}}{4wt} \]  \hspace{1cm} (11.12)

Where \( P_{cr} \) is the breaking load, \( w \) the width, and \( t \) the thickness of the specimen.

**Referenced Documents**

ASTM D 2344-84  Test Method for Apparent Interlaminar Shear Strength of Parallel Fibre Composites by Short Beam Method.

CRAG Method 100  Method of Test for Interlaminar Shear Strength of Fibre Reinforced Plastics


ISO/DIS 14130  Fibre-Reinforced Plastic Composites--Determination of Apparent Interlaminar Shear Strength by Short-Beam Method.

### 11.2.5 Flexural tests

**Aim and Purpose of Test Methods**

These test methods cover the determination of flexural properties of unreinforced and reinforced plastics, including high-modulus composites. Two test methods are described; The three-point loading system, utilising centre loading on a simply supported beam, is designed principally for materials that break at comparatively small deflections. Materials that do not fail at the point of maximum stress under the three-point bending test should be tested by the four-point loading test method. The four-point loading system, utilising two load points equally spaced from their adjacent support points, is designed particularly for those materials that undergo large deflections during testing. The basic difference between the two test methods is in the location of the maximum bending moment and axial fibre stresses. Comparative tests may be run according to either test method or procedure, provided that the test method or procedure is found satisfactory for the material tested. A third test method, the cantilever beam method, is well suited for determining the relative flexibility of materials over a wide range but particularly useful for materials too flexible to be tested by the above test methods. This test method will not be described here.

**Three-point loading method**

**Specimen type, Dimensions and Manufacturing:**

The test specimens are cut from laminated plates according to a specified span-to-depth ratio. The support span-to-depth ratio shall be chosen such that failure occur in the outer fibres of the
specimens, due only to the bending moment. Three recommended support span-to-depth ratios are 16, 32 and 40 to 1.

Apparatus:
A properly calibrated testing machine that can be operated at constant rates of crosshead motion over the range indicated shall be used. It shall be equipped with a deflection-measuring device and a load measuring system with good accuracy. The loading nose and supports in the three-point bending rig shall have cylindrical surfaces (Figure 11.15). In order to avoid excessive indentation, or failure due to stress concentration directly under the loading nose, the radius of the nose and the supports shall be at least 3 mm and up to 1.5 times the specimen depth.

Testing Procedure:
Align the loading nose and the supports so that the axes of the cylindrical surfaces are parallel and the loading nose is midway between the supports. Centre the specimen on the supports, with the long axis of the specimen perpendicular to the loading nose and supports. Apply the load at a specified crosshead rate, and take simultaneous load-deflection data. The specimen is deflected until rupture occurs in the outer fibres or until maximum fibre strain is reached, whichever occurs first. When laminated materials exhibit low compressive strength perpendicular to the laminations, they shall be loaded with a large radius loading nose to prevent premature failure to the outer fibres. For some highly anisotropic composites, shear deformation can significantly influence modulus measurements, even at span-to-depth ratios as high as 40:1. Hence, for these materials, an increase of span-to-depth ratio to 60:1 is recommended to eliminate shear effects when modulus data are required. The flexural strength is equal to the maximum stress in the outer fibres at the moment of fracture and is calculated as

$$\sigma_{cr} = \frac{3P_{cr}L}{2wt^2}$$  \hspace{1cm} (11.13)

Where $P_{cr}$ is the load at the moment of fracture, $L$ is the support span, $w$ the width and $t$ the thickness of the beam tested.

Four-point loading method
Specimen type, Dimensions and Manufacturing:
The test specimens are cut from laminated plates according to the same specifications as in the previous section.
Apparatus:
A testing machine, as described in the previous section, shall be used with a four-point bending rig with the same specifications, prior to loading noses and support design, as the three-point bending rig described in the previous section.

Testing Procedure:
Align the loading noses and the supports so that the axes of the cylindrical surfaces are parallel and the load span is either one-third or one-half of the support span. Centre the specimen on the supports, with the long axis of the specimen perpendicular to the loading nose and supports. The loading nose assembly shall be of the type which will not rotate. Apply the load at a specified crosshead rate, and take simultaneous load-deflection data. The specimen is deflected until rupture occurs in the outer fibres or until maximum fibre strain is reached, whichever occurs first. When the load span, \( l \), is one third of the support span (Figure 11.16), the flexural strength, which is equal to the maximum stress in the outer fibres at the moment of fracture, is calculated as

\[
\sigma_{cr} = \frac{P_{cr} L}{wt^2}
\]  

(11.14)

Where \( P_{cr} \) is the load at the moment of fracture, \( L \) is the support span, \( w \) the width and \( t \) the thickness of the beam tested. If the load span is one half of the support span the flexural strength is calculated as

\[
\sigma_{cr} = \frac{3P_{cr} L}{4wt^2}
\]  

(11.15)

Referenced Documents


CRAG Method 200 Method of Test for Flexural Strength and Modulus of Fibre Reinforced Plastics

11.2.6 Impact test

**Aim and Purpose of Test Methods**
Some different types of impact tests are presented here; Test methods for impact resistance of flat specimens, test methods for impact resistance of notched and unnotched beam specimens and test method for tensile-impact energy. The first test method covers the determination of the relative ranking of materials according to the energy required to crack or break flat specimens made of rigid, reinforced or unreinforced, plastics. These tests are made under various specified conditions of impact of a free falling tup or weight, or of a striker impacted by a falling weight. The second test method covers the determination of the resistance to breakage by flexural shock of plastics, as indicated by the energy extracted from "standardised" pendulum type hammers, mounted in "standardised" machines, in breaking standard specimens with one pendulum swing. The methods using unnotched specimens are to prefer while testing reinforced plastics through the notch may mask the effects of orientation. The third test method covers the determination of the energy required to rupture specific tension-impact specimens of plastic in tension by a single swing of a standard calibrated pendulum under a set of specified conditions.

**Impact Resistance of Flat Specimens**
Plastics are viscoelastic and therefore may be sensitive to changes in velocity of weights falling on their surfaces. However, the velocity of a free falling object is dependent on the square root of the drop height. It has been found that the mean failure energy a laminate is constant at drop heights between 0.3 and 1.4 m. This suggests that a constant weight-variable height method will give the same result as a constant height-variable weight method. On the other hand, different materials respond differently to changes in the velocity of impact. These types of test methods covers the determination of the relative ranking of materials according to the energy required to crack or break flat specimens under various specified conditions of impact of a free falling tup or weight, or of a striker impacted of a falling weight. Because of the nature of impact testing, the selection of a test method and striker must be somewhat arbitrary. While any one of the striker geometries may be selected, knowledge of the final or intended end-use application should be considered.

**Specimen Type, Dimensions and Manufacturing**
These test methods uses a support plate with a circular hole and the diameter or width of the flat test specimens used shall be at least 25 mm greater than the hole in the support and or, as another recommendation says, the specimens shall be large enough so that they can be clamped firmly if clamping is desirable. Select a suitable method for making the specimens that will minimise the effect of specimen preparation on the impact resistance of the material. The specimens shall also be carefully examined visually to ensure that samples are free from cracks or other obvious imperfections, unless the imperfections constitute a variable in the study.

**Apparatus**
The testing machine consists, principally, of a suitable base to withstand the impact shock, steel rod impact mass, a hardened steel striker having a round nose with a specified diameter, a guide tube in which the impact mass or striker slides (Figure 11.17). A bracket is used to hold the tube in a vertical position by attaching it to the base. On the base the specimen-support plate with a hole of a specified diameter is mounted. The difference between the test methods is whether the specimen is hit by a free falling dart, or if it is impacted by a striker hit by a falling weight. It is
also some differences in the diameter of the hole in the specimen-support plate and weather the specimen shall be clamped or not. The shape and weight of the striker varies also between different test methods.

![Schematic view over two different types of impact test machines](image)

**Figure 11.17** Schematic view over two different types of impact test machines, the left one using a free falling dart and the right one using a striker hit by a falling weight.

**Testing Procedure**

There are basically two different types of test methods and they differ, as mentioned, in the design of the testing machine and method of holding and striking the specimens. In one of the test methods, a free falling dart is allowed to strike a supported specimen directly. Either a dart having a fixed weight may be dropped from various heights, or a dart having an adjustable weight may be dropped from a fixed height. In the other type of test method, a weight falls through a guide tube and strikes an impactor resting on top of a supported specimen. The fixed weight is dropped from various heights.

**Impact Resistance of Beam Specimens**

This test method covers the determination of the resistance to breakage by flexural shock, of a notched or unnotched beam specimen, as indicated by the energy extracted from standardised pendulum-type hammers in breaking specified specimens with one pendulum swing. The notch in the notched specimen serve to concentrate the stress, minimise the plastic deformation, and direct the fracture to the part of the specimen behind the notch. Scatter in energy-to-break is thus reduced.
However, because of differences in the elastic and visco-elastic properties of plastics, response to a given notch varies among materials. A measure of plastic "notch sensitivity" may be obtained with this method by comparing energies to break specimens with identical notches.

**Specimen Type, Dimensions and Manufacturing**
Mainly two different types of specimens exist; The unnotched beam and the notched beam (Figure 11.18). It is recommended that the specimens are cut from panels in both lengthwise and crosswise directions and the test specimen width shall be the same as the thickness of the plate. Test specimens can also be prepared by other means, e.g. injection moulding, but comparisons between specimens prepared in different ways should be avoided. Though, in some test methods, the specimen is tested as a cantilever beam, the longitudinal edge faces must be parallel within very small tolerances. It is also advisable to avoid very thin specimens since they usually twist in the clamping vise. Thick test specimens (greater than 6.35 mm is the limit in e.g. ASTM D 4812) shall also be avoided. The practice of cementing, bolting, clamping or otherwise combining thinner specimens to form a composite test specimen should be avoided since the test results may be seriously affected by interface effects. Notching shall be done in a milling machine or other suitable machine tool and it is recommended to have provision for cooling the specimen with gas or liquid coolant. Each test specimen shall be free of twist, deformation, scratches, or any other obvious imperfection.

![Figure 11.18](image)

**Apparatus**
The testing machine consist of a massive base on which a device for holding the specimen is mounted and to which a pendulum type hammer is connected, through a rigid frame and anti-friction bearings, having an initial energy suitable for use with the particular specimen to be tested, plus a pendulum holding and releasing mechanism and a pointer and dial mechanism for indicating the excess energy remaining in the pendulum after breaking the specimen. The pendulum shall consist of a single or multi membered arm with a bearing on one end and a head, containing the
striking nose, on the other. The striking nose of the pendulum shall be hardened steel and have a cylindrical surface with a specified radius of curvature (Figure 11.19.).

![Figure 11.19 Schematic view over different types of beam impact test machines. (a) is showing a cantilever beam test (e.g. Izod test) and (b) is showing a test of a simple supported beam (Charpy test).](image)

**Testing Procedure**

Two different types of test methods according to test specimens are described; Tests made with an unnotched beam (e.g. ASTM D 4812) and tests made with a notched beam (e.g. ASTM D 256).

Two basically different test methods are used when testing the notched specimens. These test methods differ some as to design of the machine and specimen and method of holding and striking the specimen. In the Izod type test method the specimen is supported as a cantilever beam and is broken by a single swing of the pendulum with the line of initial contact at a fixed distance from the specimen clamp and from the centreline of the notch and on the same face as the notch (Figure 11.18(a)). With this test method it is possible to include a determination of the energy expanded in tossing a portion of the specimen. The value reported is called the "estimated net Izod strength". It is also possible to provide a measure of the notch sensitivity of the material. The stress-concentration at the notch increases with decreasing radius and, for a given system, greater stress-concentration results in higher local rates-of-strain. Since the effect of strain-rate on energy-to-break varies among materials, a measure of this may be obtained by testing specimens with different notch radii. To get an indication of the unnotched impact strength a reversed notch test is used, which means that the test specimen is reversed in the vise of the testing machine 180° to the usual striking position, such that the striker of the apparatus impacts the specimen on the face opposite the notch (Figure 11.18(b)). In the Charpy type test method the specimen is supported as a horizontal simple beam and is broken by a single swing of the pendulum with the impact line midway between the supports an directly opposite the notch (Figure 11.18(c)). The test of the unnotched specimen is basically similar to the Izod type test method and it is especially useful for reinforced materials where a notch may mask the effects of orientation.
Tensile-Impact Energy Test
Tensile impact energy is the energy required to break a standard tension-impact specimen in tension by a single swing of a standard calibrated pendulum under a set of standard conditions. This test method covers the determination of the energy required to rupture standard tension-impact specimens of plastic.

Specimen Type, Dimensions and Manufacturing
The specimen shall be sanded, machined or die cut to one of the specified specimen geometry. There are two different geometries, short and long specimen type respectively, as shown in figure 11.20. For bolting the specimen in the grips of the testing machine, a hole is drilled or, as an alternative method of bolting, a slot is made (figure 11.20(c)) which makes the insertion easier.

![Diagram of specimen geometries](image)

**Figure 11.20** The two different geometries of the tensile-impact specimen, the short type (a) and the long type (b). (c) shows a slotted specimen.

Apparatus
A pendulum type testing machine, of the same type as shown schematically in figure 11.19, is used. This machine is basically built up in the same way as the pendulum machine described in the previous section. The pendulum is holding the head, in which the greatest mass is concentrated. The crosshead clamp (Figure 11.21) should be rigid and light in weight and it shall be supported by the pendulum so that the test region of the specimen is not under stress until the moment of impact, when the specimen shall be subjected to a pure tensile force. Serrated jaws in the clamps are recommended to prevent slipping. A means shall be provided for measuring the energy absorbed from the pendulum by measuring on a suitable scale the height of the pendulum swing after the break.

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Testing Procedure
The energy utilised in this test method is delivered by a single swing of a calibrated pendulum of a standardised tension impact machine. The energy to fracture by shock in tension is determined by the kinetic energy extracted from the pendulum of the impact machine in the process of breaking the specimen. One end of the specimen is mounted in the pendulum. The other end of the specimen is gripped by a crosshead that travels with the pendulum until the instant of impact and the instant of maximum pendulum kinetic energy, when the crosshead is arrested.

Referenced Documents


ASTM D 3029-90  Test Methods for Impact Resistance of Rigid Plastic Sheeting or Parts by Means of a Tup (Falling Weight).

ASTM D 4812-93  Test Method for Unnotched Cantilever Beam Impact Strength of Plastic

ASTM D 5420-93  Test Methods for Impact Resistance of Flat, Rigid Plastic Specimen by Means of a Striker Impacted by a Falling Weight (Gardener Impact).

11.2.7 Compression after impact
Aim and Purpose of Test Methods
This test method shall be used to determine the residual compressive strength of fibre reinforced plastics after impact damage. The impacting shall be performed in accordance with some suitable method described in the previous section.
Specimen type, Dimensions and Manufacturing
It is recommended that the test laminate should be quasi-isotropic and the thickness is recommended to approximately 3 mm. Compression specimens, shown in figure 11.22, shall be cut from the impacted laminate ensuring that the damaged area lies in the centre of the specimen. End tabs are recommended.

Figure 11.22  Example of specimen for determination of the residual compression strength after impact.

Apparatus
A suitable impact test set-up, for impact tests on flat specimens, of the type described in section 11.2.6, is required for the initial impacting of the test panel. Furthermore, a support jig, to prevent the specimen to buckle, (figure 11.23) and a testing machine that must be capable of control of constant-rate-of-crosshead movement and have a mechanism capable of showing the total compressive load carried by the test specimen are required. A compressometer is needed for determining the distance between two fixed points on the test specimen at any time during the test. It is desirable that the compressometer automatically record this distance as a function of the load on the test specimen.

Figure 11.23  Example of supporting jig (anti-buckling guide).
Testing Procedure

The test specimen shall be mounted in the supporting jig and carefully aligned in the test machine. The specimen is then loaded to failure to obtain the residual compressive strength after impact. The residual compressive strength is then calculated as

\[ \sigma_{cr} = \frac{P_{cr}}{tw} \]  

based on gross area, where \( P_{cr} \) is the failure load, \( w \) is the width of the specimen in the gauge area and \( t \) is the specimen thickness.

Referenced Documents

CRAG Method 403 Method of Test for Residual Compression Strength After Impact of Multidirectional Fibre Reinforced Plastics.
11.2.8 Fatigue tests

Aim and Purpose of Test Methods

These test methods describe the applicability of some various test specimens for the determination of fatigue properties of reinforced resin composites. The obtained fatigue data is for use in material specifications, research, and development, and as a guide for design and selection of materials for service under conditions of repeated loads. Both load controlled and strain, or deformation controlled systems exist.

Specimen type, Dimensions and Manufacturing

Examples of specimen geometries used in different fatigue test methods are shown in Figs. 11.24 and 11.25. When performing axial testing (tension/compression), the tensile coupon in Fig. 11.24 is used together with a support that prevents buckling if compressive load is used. The specimens are similar to the ones used for static testing and are prepared by machining operations from panels.

![Fatigue Specimen with Tabs](image)

Figure 11.24 The geometry of a fatigue specimen with tabs used for axial tension/compression testing. When compressive loads are present, a support to prevent buckling is mounted on the specimen (Fig. 11.25(a)).

Tabs are strongly recommended especially in axial testing of unidirectional (or strongly unidirectional dominated) laminates in the fibre direction, or when tested in the matrix direction to prevent gripping damage. The selection of tab configurations for tensile specimens are discussed in section 11.2.1. If bonded resistance strain gauge is used the specimen shall be carefully prepared before bonding. Reinforcing fibres should not be exposed or damaged during the surface preparation.
Figure 11.25 A fatigue specimen used for axial testing with a support to prevent buckling (a). Schematic view of three-point bending (b) and short beam tests (c) for flexural, and interlaminar shear fatigue testing, respectively.

Apparatus
Either a nonresonant mechanical, hydraulic or magnetic system, or a resonant type of tension testing machine, using forced vibrations exited by a magnetic or centrifugal force, is used. The specimens may be instrumented with longitudinal or lateral, or both, element strain gauges. The gauges, surface preparation, and bonding agents should be chosen to provide adequate performance on the material tested. In some cases it may be preferable to use an extensometer for strain control. Suitable automatic strain-recording equipment shall be employed. Fatigue tests are commonly run at high cyclic rates which can cause heating of the specimen and therefore affect the fatigue life of the specimen. If specimen heating is likely to occur or when there is any doubt, the specimen temperature should be monitored during the cycling. The use of radiation thermometer, adhering thermocouples to the specimen or employing temperature sensitive waxes or crayons are some possible ways in measuring the temperature. When performing interlaminar shear, or flexural fatigue tests, a three-point bending rig of the type used for static testing, but also equipped with opposed rollers (Fig.11.25(b) and (c)), shall be used. In designing a three-point bending rig for fatigue testing great care must be taken to ensure that the rollers can pivot to obviate any clamping moment due to angular deflection.

Testing Procedure
The specimen is carefully aligned when mounted in the bending rig or in the grips of the testing machine. Then there are different ways in loading the specimen. Either the specimen is cycled
between two tensile loads at a specified rate, or between a tensile and a compressive load, or the specimen is cycled between two different strain limits at a specified rate. If creep effects are to be monitored (load controlled testing), load-strain curves must be obtained during the test.

**Referenced Documents**

- ASTM D 3479-76 Test Method for Tension-Tension Fatigue of Oriented Fibre, Resin Matrix Composites.

**11.2.9 Tests for dynamic mechanical properties**

**Aim and Purpose of Test Methods**

These test methods provide a simple means of characterising the thermomechanical behaviour of thermoplastic and thermosetting resins and composite systems using a very small amount of material tested using nonresonant forced-vibration techniques. The test methods are valid for a wide range of frequencies, typically from 0.01 to 100 Hz. Plots of the elastic loss, and complex moduli and tan delta (the ratio of the loss modulus to the storage modulus), as a function of frequency, time, or temperature are indicative of significant transitions in the thermomechanical performance of the polymeric material system. Mainly five different test methods are described here; The tensile test, where a test specimen of rectangular cross-section is gripped longitudinally between two clamps and tested in dynamic tensile; The compression test, where a cylindrical specimen is placed between two flat plates or parallel discs and tested in dynamic compression; The torsion test, where a rectangular test specimen is gripped longitudinally between two clamps and tested in dynamic torsion; The dual cantilever beam test, where a specimen of rectangular cross section is gripped between two clamps and tested as a beam in dynamic linear displacement or bending, with the displacement strain or deformation applied at the centre of the dual-cantilever beam; The three-point bending test, where a specimen of rectangular cross-section is tested in flexure as a beam resting on two supports and loaded by means of a loading nose midway between the supports. All these different tests are performed at either isothermal conditions or with a linear temperature increase.

**Specimen type, Dimensions and Manufacturing**

The test specimens may be cut from sheets, plates, or moulded shapes, or may be moulded to the desired finished dimensions. For the torsion, tension three-point bend and cantilever beam tests specimens with rectangular cross section with somewhat different dimensions are used. In the compression test a cylindrical test specimen is used. Typical dimensions of the test specimens, as recommended by ASTM, are shown in Fig.11.26. Other dimensions can be used but should be clearly specified.
Apparatus
The function of the apparatus is to hold a test specimen so that the material acts as the elastic and dissipative element in a mechanically driven linear displacement, or torsional system. Dynamic mechanical instruments operate at a forced, constant amplitude, and at either a fixed frequency, or variable frequencies. The testing machine shall be equipped with a fixed member with one grip, or flat plate, (compression test) and a moveable member with a second grip, or flat plate, is required to hold the test specimen. Devices for applying force-displacement as well as devices for determining experimental parameters such as force, deformation, frequency and temperature is also required. The loading nose and the supports in the rig, used for the three-point bending test, shall have cylindrical surfaces having a sufficient radius to avoid excessive indentation or failure due to stress concentrations directly under the loading nose (Fig.11.27). To maintain a constant specimen environment an oven with a sufficiently stable temperature controller is recommended.
Testing Procedure
Measure the specimen dimensions and mount the test specimen between the fixed and moveable members. When performing the three-point bending test, the test specimen is carefully centred on the supports, with the long axis of the specimen perpendicular to the loading nose and supports. Pre-load the specimen and monitor the normal force to ensure adequate pre-loading. Measure the separation between the moveable and stationary members. When the cantilever beam test is performed, the length of the specimen between the grips is measured and then the specimen is pre-loaded so that a positive deflection is maintained during testing unless the test mode allows a zero-displacement measurement. Select the desired frequency, or frequencies, and the amplitude for the dynamic linear displacement. Temperature increases should be controlled to 1 to 2 °C/min for linear increases or 2 to 5 °C/min with a minimum of 1-min of thermal-soak time for step increases. The elastic or loss moduli, or both, of the polymeric material system are measured while varying time, temperature of the specimen or frequency, or both, of the oscillation. Plots of the elastic and loss moduli are indicative of viscoelastic characteristics of the specimen.

Referenced Documents
11.2.10 Fracture toughness tests

Aim and Purpose of Test Methods

The susceptibility to delamination is one of the major weaknesses of many advanced laminated composite structures. Knowledge of a laminated composite material’s resistance to interlaminar fracture is useful for product development and material selection. A measurement of the interlaminar fracture toughness, independent of specimen geometry or method of load introduction, is useful for establishing design allowables used in damage tolerance analyses of composite structures made from these materials. Most of the interlaminar fracture toughness tests presented here are limited to unidirectional laminates where the crack propagates between the plies along the fibre direction. In multidirectional laminates, the crack may have a tendency to branch through the neighbouring plies invalidating the coplanar assumption in fracture analysis.

Mode I Interlaminar Fracture Toughness Test: DCB Specimen

This method describes the determination of the opening mode (Mode I) interlaminar fracture toughness, $K_{IC}$, and critical energy release rate, $G_{IC}$, of unidirectional fibre-reinforced polymer matrix composites using the double cantilever beam (DCB) specimen. This method is limited to use with composites consisting of unidirectional carbon fibre tape laminates with brittle and tough single-phase polymer matrices. This limited scope reflect the experience gained in round robin testing. This test method may be proved useful for other types and classes of composite materials, however certain interference have been noted.

Specimen Type, Dimensions and Manufacturing

The DCB consists of a rectangular, uniform thickness, unidirectional laminated composite specimen, cut from a laminate containing a nonadhesive insert on the midplane which serves as a delamination indicator. Therefore the test laminates must contain an even number of plies, and it is recommended to be unidirectional, with delamination growth occurring in the zero degree direction. The nonadhesive insert, e.g. a 0.025 mm Teflon film, shall be inserted at the midplane.


of the laminate during lay-up to form an initiation site for the delamination. Opening forces are applied to the DCB specimen by means of hinges or loading blocks bonded to one end of the specimen (Fig. 11.28).

![Figure 11.28. The DCB specimen with hinges (left) and with loading blocks (right). a is the delamination length.](image)

**Apparatus**

The testing machine shall have the capability to control and regulate the velocity of the movable head and the loadsensing device of the testing machine shall be capable of indicating the total load being carried by the test specimen. To apply the load to the specimen the testing machine shall be equipped with grips to hold the loading hinges, or pins to hold the loading blocks, that are bonded to the specimen. To estimate the opening displacement either the crosshead separation, or an external properly calibrated gauge or transducer attached to the specimen, is used. Load and opening displacement data shall be recorded and stored and then post processed, or directly plotted in a X-Y plotter, or similar device. A travelling optical microscope shall be used to observe the delamination front as it extends along one edge during the test. This device shall be capable of pinpointing the delamination front with an accuracy of at least ±0.5 mm.

**Testing Procedure**

Coat both edges of the specimen just ahead of the insert with a thin layer of water-based typewriter correction fluid, or equivalent, and mark with thin vertical lines to aid in visual detection of delamination onset. Mount the load blocks or hinges on the specimen in the grips of the loading machine, making sure the specimen is aligned and centred. Set the optical microscope (or an equivalent magnifying device) in a position to observe the motion of the delamination front and measure the delamination length on one side of the specimen. The specimen is loaded continuously and the load versus opening displacement is plotted. Mark the location of the delamination front on the plot of load versus opening displacement as the delamination grows. An alternative way is to load the specimen until the crack extended about 10 mm, and stop the machine (Fig. 11.29). Measure the actual crack length and unload the specimen. Also mark the crack length on the recorder chart to help in later identification. Repeat this procedure until the crack is approximately 150 mm in length.
The critical energy release rate, $G_{IC}$, can then be calculated as

$$G_{IC} = \frac{P_c^2 a^2}{wEI}$$  \hspace{1cm} (11.17)

Where $P_c$ is the critical load, $a$ is the delamination length and $w$ is the width of the specimen. The interlaminar fracture toughness, $K_{IC}$, can then be evaluated from the relationship

$$G_{IC} = \frac{K_{IC}^2 (1 - \nu^2)}{E}$$ \hspace{1cm} (11.18)

in plane strain.

**Mode II Interlaminar Fracture Toughness Test: ENF Specimen**

The purpose of the End Notched Flexure (ENF) specimen is to determine the critical strain energy release rate in pure mode II loading. The test principle, three-point bending, is shown in Fig. 11.30. This specimen has been found to produce shear loading at the crack tip without introducing excessive friction between the crack surfaces.

**Specimen Type, Dimensions and Manufacturing**

The ENF specimen is cut from a similar laminate as the DCB specimen, described in the previous section. It is recommended that the test span shall be 100 mm and therefore the specimen length should be equal to span length plus 10 mm.
Figure 11.30 Schematic view over the ENF specimen, \( a \) is the delamination length, with the three-point bending test set-up.

**Apparatus**
The testing machine shall have the capabilities as described in the previous section. The loading nose and the supports on the three-point bending fixture shall have cylindrical surfaces having a sufficient radius to avoid excessive indentation or failure due to stress concentrations directly under the loading nose. The displacement at the mid point may be recorded by an extensometer or similar device.

**Testing Procedure**
The crack should be carefully wedged open and extended about 2 mm beyond the insert in order to achieve a natural starter crack. The specimen shall then be placed in the three-point bending fixture, properly aligned, so that the initial crack length is about 25 mm (Fig.11.30). The specimen is loaded continuously and the load versus displacement is plotted. It has been observed that the crack generally propagates to the central loading point in an unstable manner. This means that only one value of the critical energy release rate, \( G_{IIc} \), is obtained for each specimen. An expression for \( G_{IIc} \) is derived from the elastic beam theory;

\[
G_{IIc} = \frac{9P_c^2Ca^2}{2l(2L^3 + 3a^3)}
\]

(11.19)

Where \( P_c \) is the critical load, and

\[
C = \frac{2L^3 + 3a^3}{8Elt^3}
\]

(11.20)

is the compliance, \( a \) is the crack length, \( l \) the length and \( t \) half the total thickness of the specimen, and \( L \) is the span between the central loading pin and the outer support pins.

**Mode II Interlaminar Fracture Toughness Test: CDD Test**
The Curvature-Driven Delamination (CDD) test for mode II provides a direct, steady-state measurement of the pure mode II delamination toughness without the compliance calculations inherent in other delamination test protocols. The CDD test measures the toughness at controlled crack growth rates and the results correlates well with the results of the more established ENF test, described in the previous section, but is not subjected to the geometric instabilities of that test. Changes in the delamination mechanism are easily observed by the direct, continuous measurement of the toughness in this test.
Specimen Type, Dimensions and Manufacturing
The specimen used in the CDD test is a ENF type specimen, described in the previous section, with the difference that the CDD specimen is longer.

Apparatus
The testing machine shall have the capabilities as described earlier. A fixture, schematically shown in Fig.11.31, shall be used to apply the deflection to the specimen. The deflection is determined by the set screw A. The loading pins shall be supported by roller bearings and the centre pin shall be supported by three additional large bearings to reduce the friction. To apply the load, the testing machine shall be equipped with a grip to hold the specimen when it is pushed through the bending fixture. Load and displacement data shall be recorded and stored and then post processed, or directly plotted in a X-Y plotter, or similar device.

![Figure 11.31 Schematic view over the CDD test concept.](image)

Testing Procedure
The specimen is mounted in the fixture in the testing machine and then the crack is grown beyond the resin rich region at the insert before the data is collected. The concept behind the CDD fixture is that the sample can be continually loaded into the fixture at a controlled rate, with the force being a direct measurement of the toughness. A small load ring is inserted at the set screw to measure the deflection force. The sample is driven into the fixture by the piston of the testing machine through a pin loading of the sample end. The pin loading does not allow a torque to be applied to the sample end. The crack growth will be stable with constant values of the critical energy release rate, $G_{\text{IC}}$, simply calculated as

$$G_{\text{IC}} = \frac{P_C}{B}$$

(11.21)

where $B$ is the width of the specimen and $P_C$ is the critical load.
Mixed Mode Interlaminar Fracture Toughness: CLS Specimen

The Cracked Lap Shear (CLS) specimen was originally designed for shear dominated fracture investigations of adhesive joints and developed as a Mode II specimen for composites. However, this specimen does not produce a pure mode II loading at the crack tip. Consequently, the CLS specimen is a mixed mode specimen.

Specimen Type, Dimensions and Manufacturing

The CLS specimen is manufactured in a similar way as the above specimens. Then the specimen is machined to its final shape shown in Fig. 11.32 and it is also recommended that end tabs are used. Before testing, the crack should be carefully wedged open and extended about 10 mm beyond the insert in order to achieve a natural starter crack.

![CLS Specimen Diagram](image)

Figure 11.32  Schematic view over the CLS specimen where a is the delamination length.

Apparatus

The testing machine shall have the capabilities as described above. To apply the load to the specimen the testing machine shall be equipped with grips to hold the specimen. Load and displacement data shall be recorded and stored and then post processed, or directly plotted in a X-Y plotter, or similar device. A travelling optical microscope is preferred over a precision dial calliper to measure the crack length as the delamination front extends along one edge during the test, since it may be difficult to visually identify the crack front.
Testing Procedure

Determine the initial crack length, $a$, from the cut line to the tip of the starter crack. Mount the specimen in the grips. The specimen is loaded continuously and the load versus displacement is plotted. Load the specimen until visible delamination growth occurs, detected on either side of the specimen. Delamination onset may also be detected from a slight deviation in the load/displacement curve. The specimen is then unloaded to zero load and the crack length is measured. The above procedure is then repeated for several crack lengths up to approximately 150 mm. The equation for the critical strain energy release rate, $G_c$, which contains contributions from both Mode I and Mode II, is obtained from the compliance versus crack length data as:

$$G_c = \frac{P_c^2 (t_1 - t_2)}{2w^3 E t_1 t_2}$$  (11.22)

Where $P_c$ is the critical load and $t_1, t_2$ and $w$ are shown in Fig.11.32.

Mixed Mode Interlaminar Fracture Toughness: MMB Test

The Mixed Mode Bending test maintains a constant mode mix (within certain limits) while being simple in practice and requiring only simple data analysis.

Specimen Type, Dimensions and Manufacturing

The specimen used is the DCB-specimen described earlier in this chapter.

Apparatus

The testing machine shall have the capabilities as described above. To apply the load to the specimen the MMB fixture, schematically shown in Fig.11.33, is used. The translation and rotation of the end of the loading arm shall be monitored with some suitable device. Load and displacement data shall be recorded and stored and then post processed, or directly plotted in a X-Y plotter, or similar device. A travelling optical microscope is recommended to measure the crack length as the delamination front extends along one edge during the test. Since it may be difficult to visually identify the crack tip the edge of the specimen can be coated with water-based typewriter correction fluid.

![Figure 11.33  Schematic view over the MMB test set-up.](image)

Testing Procedure

The $G_I/G_{II}$ ratio is chosen by varying the load point location along the lever, i.e., by varying the distance $c$ of Fig.11.33. To achieve pure mode I or mode II loading, respectively, the DCB test and the ENF test is used. The test is configured by adjusting the fixture to provide the desired mode...
mix ratio and then the specimen is loaded at a constant displacement rate. To make it easier to measure the crack growth a grid of lines spaced 1 mm apart should be imprinted on the side of the specimen. Delamination growth initiation is detected using the microscope. Load and deflection data shall be recorded for every mm of crack growth thereafter till the crack has grown 20 mm. This test set-up can also be used in fatigue testing. The following expressions for the strain energy release rate can be obtained using beam theory:

\[
G_{IC} = \frac{3P_C^2(3c - L)^2}{4E_{11}w^2t^4L^2} \left[ a^2 + \frac{2a}{\lambda} + \frac{1}{\lambda^2} + \frac{t^2 E_{11}}{10G_{13}} \right]
\]  

(11.23)

\[
G_{IIc} = \frac{9P_C^2(c - L)^2}{16E_{11}w^2t^4L^2} \left[ a^2 + \frac{t^2 E_{11}}{5G_{13}} \right]
\]  

(11.24)

In these expressions, \(P_C\) is the peak load, \(c\) and \(L\) are dimensions from the loading fixture as shown in Fig.11.33, \(w\) and \(t\) are specimen width and half thickness, \(a\) is the delamination length, \(E_{11}\) and \(G_{13}\) are the axial tensile and shear moduli of the specimen material, and \(\lambda\) is the elastic foundation parameter defined by

\[
\lambda = \frac{6E_{22}}{t^4 E_{11}}
\]  

(11.25)

(see chapter 2.1.3). The above expressions are valid only if the loading is such that the crack actually opens. This condition is expressed approximately by requiring \(c\) to be greater than \(L/3\). It is evident from the above expressions that the mode mix ratio is a function of crack length, specimen dimensions and material properties in addition to the fixture geometric parameters \(c\) and \(L\). However, these dependencies have been shown to be relatively weak. Thus a simplified form of the expressions for \(G_I\) and \(G_{II}\) can be used to obtain the mode mix ratio as

\[
\frac{G_I}{G_{II}} = \frac{4}{3} \left[ \frac{3c - L}{c + L} \right]^2
\]  

(11.26)

The Arcan Specimen

The Arcan fixture and specimen geometry, shown in Fig.11.34, were developed in an attempt to produce uniform plane stress in the test section. When the specimen is loaded in the \(y\)-direction, a state of pure shear is introduced in the test section, AB. By varying the angle \(\alpha\) a combined stress state is achieved in the test section. The use of this fixture is extended to produce fracture mechanics data by replacing the characteristic V-notched Arcan specimen by a single edge notch specimen as shown in Fig.11.35. By varying the angle, \(\alpha\), from 0° to 90°, pure mode II, mixed mode and pure mode I data may be obtained. A disadvantage of this test specimen is that, due to the small test section, only one fracture toughness value is obtained per specimen. This fact renders the test less useful for large scale testing.
Specimen Type, Dimensions and Manufacturing
The Arcan fracture specimen should be machined to its final shape either by surface grinding or by hand in a disc sander. The length should be 33 mm, the depth 13 mm, and the height 12.3 mm. Then it is bonded to the Arcan halves (Fig. 11.35) using some suitable fixture holding the halves properly. Just as for the above specimen types, the crack should be located in the centre of the specimen thickness. Before testing, the crack should be extended a very small amount in order to achieve a natural starter crack. Due to the small test section and the unstable configuration, precracking is difficult to achieve. Extreme care has to be taken in the handling of the specimen in the Arcan halves to prevent premature damage of the specimen.
Apparatus
The testing machine shall have the capabilities as described earlier. To apply the load through the Arcan test fixture to the specimen the testing machine shall be equipped with holders with pins to hold the specimen assembly. Load and displacement data shall be recorded and stored and then post processed, or directly plotted in a X-Y plotter, or similar device.

Testing Procedure
Before mounting the fixture in the testing machine, determine the initial crack length, \( a \), on both sides of the specimen. Mount the fixture in the desired angle in the test machine with pins through the holes on the fixture. The specimen is loaded continuously and the load versus displacement is plotted. Load the specimen until fracture and determine the critical load, \( P_C \), for crack propagation.

The critical strain energy release rate, \( G \), is calculated from the critical load via the stress intensity factors \( K_I \) and \( K_{II} \) as

\[
G_{IC} = K_{IC}^2 \left( \frac{S_{11} S_{22}}{2} \right)^{1/2} \left[ \left( \frac{S_{22}}{S_{11}} \right)^{1/2} + \frac{2S_{12} + S_{66}}{2S_{11}} \right]^{1/2}
\]  

(11.27)

\[
G_{IIIC} = K_{IIIC}^2 \sqrt{2} \left[ \left( \frac{S_{22}}{S_{11}} \right)^{1/2} + \frac{2S_{12} + S_{66}}{2S_{11}} \right]^{1/2}
\]  

(11.28)

where \( S_{ij} \) are the elements of the compliance matrix for a transversely isotropic material (see e.g. [2.14]) and \( K_{IC} \) and \( K_{IIIC} \) are obtained from the following expressions

\[
K_{IC} = \sigma_C \sqrt{\pi a} \cdot f_i(a/c)
\]  

(11.29)

\[
K_{IIIC} = \tau_C \sqrt{\pi a} \cdot f_{II}(a/c)
\]  

(11.30)

where \( f_i \) and \( f_{II} \) are correction factors for finite crack length to specimen ratio given in [2] as

\[
f_i(a/c) = 1.12 - 0.231(a/c) + 10.55(a/c)^2 - 21.27(a/c)^3
\]  

(11.31)

\[
f_{II}(a/c) = \frac{1.122 - 0.561(a/c) + 0.085(a/c)^2 + 0.180(a/c)^3}{\left(1 - (a/c)\right)^{1/2}}
\]  

(11.32)

and \( \sigma_C \) and \( \tau_C \) are critical stresses, obtained as

\[
\sigma_C = \sigma_{AC} \sin \alpha
\]  

(11.33)

\[
\tau_C = \sigma_{AC} \cos \alpha
\]  

(11.34)

where \( \sigma_{AC} \) is defined as the applied critical load, \( P_C \), divided by the cross-sectional area of the specimen.
The Edge Delamination Test
The edge delamination test (EDT) was developed to determine interlaminar fracture toughness of laminated composites. Laminate tensile coupons are designed to delaminate at the edges by choosing a lay-up such that a large interlaminar normal stress is obtained at the free edges because of a large Poisson ratio mismatch between the plies in the laminate. The application of fracture mechanics concepts for this specimen is somewhat cumbersome since no crack exists initially. More about this method can be read in [L.A. Carlsson, R.B. Pipes, ”Experimental Characterization of Advanced Composite Materials”].

Referenced Documents

CRAG Method 600  Method of Test for Interlaminar Fracture Toughness of Fibre Reinforced Plastics.


11.2.11 High strain rates
Aim and Purpose of Test Methods
To improve dynamic analysis capabilities, experimental data must be obtained to characterise the high strain rate response of fibre reinforced composites. This information can be used to develop the dynamic material models required in finite element codes for accurate structural modelling and design. It is very difficult to get accurate results from these tests because of, e.g., stress wave reflections in the specimen and the test equipment, end effects, edge effects, and gripping will affect the results.
Specimen Type, Dimensions and Manufacturing
The specimens shall be carefully machined from samples of the laminates investigated. Great care should be taken of the parallelity and flatness of the ends of the compression specimen. The compression test specimen can be a cube with side length of about 7 mm. Other examples of geometries of suitable test specimen for testing at high strain rates are shown in Fig.11.36. Many test methods uses quite small specimens (around 20 mm in length) which makes them difficult to manufacture within desired tolerances.

Apparatus
The tests are performed using a suitable hydraulic testing machine that shall have the capability to control and regulate the velocity of the movable head up to a testing speed, of at least, 500 mm/min. For higher strain rates, either a pendulum type testing machine as described in section 11.2.6, or a high energy drop tower type testing machine (Fig.11.37) is used. The loadsensing device of the testing machine shall be capable of indicating the total load being carried by the test specimen. To apply the load to the specimen, suitable grips and/or fixtures, depending on which property that is to be tested (compression, tensile, or shear), shall be used. To estimate displacement either the crosshead separation, or an external properly calibrated extensometer, or strain gauge, attached to
the specimen, is used. It is also recommended to use some high speed data acquisition system to collect desired data.

![Diagram of drop tower for compression impact](image)

**Figure 11.37** Schematic view of drop tower for compression impact.

**Testing Procedure**
The compression test should be performed using some suitable alignment fixture and the tensile specimen in Fig.11.36(a) is tested using ordinary clamping grips, or modified with a drilled hole and a pin holding the specimen (Described in section 11.2.6). In the shear test, a specimen is supported in the lower half of a fixture. The top half is then forced through the unsupported section of the specimen to produce shear (Fig.11.36(c)). The test assembly of the desired method is mounted in a suitable testing machine, depending on the strain rate used in the tests, and the high speed data acquisition system is connected.

**Referenced Documents**

11.2.12 Creep tests

Aim and Purpose of Test Methods
Data from creep and creep-rupture tests are necessary to predict the creep modulus and strength of materials under long-term loads and to predict dimensional changes that may occur as a result of such loads. Data from these test methods can be used to compare materials, in the design of fabricated parts, to characterise plastics for long-term performance under constant load, and under certain conditions, for specification purposes. These test methods cover the determination of tensile and compressive creep and creep-rupture of plastics under specified conditions. Hence, the test methods consist of measuring the extension or compression as function of time and time-to-rupture, or failure of a specimen subjected to constant tensile or compressive load under specified conditions. This test method is not specified for reinforced plastics in the referenced document (ASTM D2990-93a) but it seems reasonable that the test methods are valid for these materials too.

Tensile Creep

Specimen Type, Dimensions and Manufacturing
The test specimen for tensile creep measurement shall be of the dumbbell shaped type, used in standard tensile tests, shown in Fig.11.38(d) and the specimen types in Fig.11.38(a), (b) and (c) are recommended for creep rupture testing.

Apparatus
The grips and gripping technique shall be designed to minimise eccentric loading of the specimen. The loading system must be so designed that the load applied and maintained on the specimen have very good accuracy with the desired load and the loading mechanism must allow reproductively rapid and smooth loading. The extension shall preferably be measured directly on the specimen, rather than by grip separation, but great care must be taken that any device used not will influence on the specimen behaviour.

Testing Procedure
Mount the properly conditioned specimen in the grips and, if necessary, mount a properly conditioned control specimen alongside the test specimen in the same manner. Attach the deformation measuring devices to the specimen or, if these are optical, install ready for measurements. Apply the full load rapidly and smoothly to the specimen and start the timing at the onset of loading. Measure the extension of tension of the specimen in accordance to a specified time schedule. The readings should increase in frequency if discontinuities in the creep strain versus time plot are suspected. If some environmental agent is used, make sure to apply it to the entire gauge length of the specimen and that the equipment used not affect on the applied load.

**Compressive Creep**

*Specimen Type, Dimensions and Manufacturing*

The standard test specimen for unconfined compressive creep test shall be in the form of a cylinder or a prism with a square cross section. Alternatively, slender bars of square cross section can be used.

*Apparatus*

Parallel anvils shall be used to apply the load to the unconfined-type specimen. One of the anvils of the machine shall preferably be self aligning and shall be arranged so that the specimen is accurately centred and the resultant of the load is through its centre. When testing slender specimens, a fixture with a guide tube shall be used to prevent buckling (Fig. 11.39). The compression can be measured directly from anvil displacement, but preferably it will be measured directly on the specimen. Great care must be taken that any device used for measuring the compression not will influence on the specimen behaviour. The loading system is similar to that described in the previous section.

![Figure 11.39 Schematic view of test fixture with guide tube for compressive creep testing with slender specimens.](image)

*Testing Procedure*

The testing procedure for the compressive creep testing is similar to that for tensile creep testing, described above.

**Flexure Creep**
Specimen Type, Dimensions and Manufacturing
The test specimens for flexural creep measurements shall be rectangular bars with the recommended (preferred in ASTM D2990) sizes of 63.5 by 12.7 by 3.18 mm or 127 by 12.7 by 6.4 mm. Close tolerances of specimen and span dimensions are not critical as long as actual dimensions are used in calculating loads.

Apparatus:
A rigid test rack (Fig.11.40) shall be used to provide support to the specimen at both ends with a span equal to about 16 times the thickness of the specimen. A stirrup shall be used which fits over the test specimen from which the desired load may be suspended to provide flexural loading at mid-span. The deflection of the specimen at midspan shall be measured using e.g. a dial gauge. The loading system is similar to that described in the tensile creep section above.

Testing Procedure
The testing procedure for the flexural creep testing is similar to that for tensile creep testing.

Referenced Documents
ASTM D 2990-77 Test Methods for Tensile, Compressive and Flexural Creep and Creep-Rupture of Plastics

11.2.13 Fibre volume fraction test
Aim and Purpose of Test Methods
The mechanical properties of structures made with advanced composites can be sensitive to difference in the fibre/resin ratio of the material used. Depending on the material system and the application, keeping fibre/resin ratio within specification in the order of a few percent can be critical. The test methods presented here covers the determination of the resin content for fibre reinforced plastics and prepregs using some different methods.

Ignition Loss Method
This test method is used to obtain the ignition loss of a cured reinforced resin sample. If only glass fabric or filament is used as the reinforcement of an organic resin that is completely decomposed to volatile materials under the conditions of this test and the small amount of volatiles (water,
residual solvent) that may be present is ignored, the ignition loss can be considered to be the resin content of the sample. The test method does not provide a measure of resin content for samples containing reinforcing materials that loose weight under the conditions of the test or containing resins that do not decompose to volatile materials released by ignition. Although there is a possibility to use this kind of test method for carbon fibre reinforced materials if great care is taken to heating time and temperature.

**Specimen Type, Dimensions and Manufacturing**
The specimen should weigh approximately 5g with a maximum size of 2.5 by 2.5 cm by thickness. A minimum of three specimens shall be tested of each sample.

**Apparatus**
A platinum or porcelain crucible of approximately 30 ml capacity, a digital scale with very high accuracy and an electric muffle furnace, capable of maintaining a temperature of at least 600°C is required for this test.

**Testing Procedure**
Heat a crucible at 500 to 600°C for about 10 min, cool it to room temperature in a desiccator and then weigh the crucible. Place the specimen in the crucible and weigh. Heat the crucible and specimen so that the specimen burns until only ash and carbon remain when the burning ceases. Then the crucible and residue is heated in the muffle furnace at about 565°C until all carbonaceous material has disappeared. Cool the crucible to room temperature in a desiccator and weigh the crucible and residue. If carbon fibre reinforced materials is investigated the specimen shall be heated to maximum 460°C. The fibre weight fraction, \( w_f \), is calculated as

\[
W_f = \frac{W_1 - W_2}{W_1}
\]

where \( W_1 \) is the weight of the specimen and \( W_2 \) is the weight of the residue.

**Mechanical and Ultrasonic Methods**
This test method describes measurement procedures for determining physical properties of polymer matrix thermoset prepreg such as resin content, fibre volume, fibre areal weight, and thickness per ply. It provides a systematic process for measuring these physical properties using a combination of mechanical and ultrasonic methods. The consistent measurement of ultrasonic properties of thermoset prepreg requires a reproducible configuration. The determination of an optimal consolidation point achieves this reproducibility. The thickness and ultrasonic wave propagation characteristics of prepreg depend on the fibre and matrix of which it is composed. Therefore, a material-specific calibration database must be prepared for each fibre/matrix combination.

**Specimen Type, Dimensions and Manufacturing**
The test specimens shall be cut from locations on the prepreg to be representative of the material. A prepreg roll is typically sampled at the beginning and the end of the roll. Woven prepreg materials shall be cut and folded into a multi-ply specimen. Prepreg with unidirectional fibres and woven prepreg with directionality of fibres shall be cut and laid up cross-plied.
Apparatus
An experimental configuration composed of a mechanical loading apparatus, an ultrasonic pulser-receiver, a digitiser and a computer for data acquisition is required in this test procedure. The mechanical apparatus for applying load to the specimen shall include a displacement gauge and holders for the ultrasonic transducers above and below the specimen.

Testing Procedure
Determine the optimal consolidation point and the thickness per ply for each prepreg specimen measured. Use an optimum pressure as recommended from the supplier or selected from a series of consolidation experiments. Bring the ultrasonic heads into contact, without a sample, using sufficient couplant for good ultrasonic coupling to establish a reference waveform. Remove the couplant from the ultrasonic heads and place the prepreg specimen between them and apply the optimum pressure to the sample for a defined period of time. During the deformation period, transmit a series of ultrasonic pulses through the specimen and record the waveforms and determine also the time-of-flight through the specimen for each waveform. Measure and record the thickness of the specimen simultaneously with each ultrasonic pulse. Derive the resin content of the test specimens from the material-specific calibration database.

Matrix Digestion Method
The technique used in this test method is based on the digestion of the matrix resin by liquid media, which do not attack the fibres excessively.

Specimen Type, Dimensions and Manufacturing
The specimen may be of any convenient shape, weighing at least 0.30g, as long as it will fit into the container used in the test.

Apparatus
To perform these three somewhat different test methods described here, a sintered-glass filter crucible, a crucible holder, an analytic balance, a drying oven, a borosilicate glass vacuum filter flask, a vacuum source, a desiccator and a water reflux condenser is required. Furthermore, for the first test method a water or oil bath of about 75°C is needed, for the second method a borosilicate glass beaker is needed and for the third method one requires Erlenmeyer flasks of two different sizes.

Testing Procedure
The first procedure is used for composite with epoxy resin matrices. The density of each specimen are determined before each specimen is placed in a separate flask containing 70% nitric acid. Then the flasks are fitted with the reflux condensers and placed in a hot water or oil bath (about 75°C). When the digestion is complete, filter the contents of each flask onto a tared sintered-glass filter under vacuum. Then wash fibres in distilled water followed by acetone and then heat the filter and specimen in an oven at 100°C for 1 hour to remove residual water and acetone and, finally, weigh the filter and specimen.

The second procedure is used for composite with polyimide or phenolic resin matrices. Here, each specimen is placed in a beaker of concentrated sulphuric acid which is heated until the sulphuric acid begins to fume. After the solution has become dark, hydrogen peroxide solution is added.
When the oxidation of the resin matrix is complete, the fibres float to the top and the solution below the fibres become clear. Then the fibre residue is washed, dried and weighed as described above.

The third procedure is especially applicable to anhydride-cured epoxy resin matrices reinforced with aramide or carbon fibres. A solution of potassium hydroxide in ethylene glycol is prepared in Erlenmeyer flasks and a specimen is placed in each flask. Heat the solution and let it boil gently until the resin is completely digested. Complete digestion is approached when the fibres separate and appear to float freely in the solution. Then, filter the contents of each flask as described earlier and wash the fibre residue in dimethylformamide then in distilled water and finally in acetone. Then the fibre residue is dried and weighed as described above.

**Matrix Dissolution Method**

In this test method the determination of the resin solids content of epoxy-matrix prepreg is measured by dissolving the matrix with a solvent.

**Specimen Type, Dimensions and Manufacturing**

The specimen size shall be, at minimum, 80 by 80 mm or an equivalent area and the specimen thickness shall be the thickness of the prepreg.

**Apparatus**

An analytical balance with good accuracy, a timer, a fume exhaust hood, borosilicate beakers, weighing dishes or fritted glass crucibles and an electric hot plate are required to perform this test.

**Testing Procedure**

Weigh each specimen and then place each specimen in a separate beaker of boiling solvent known to dissolve completely the resin matrix. Decant the solution and rinse the fibre material remaining in methyl ethyl ketone (MEK). Decant the MEK and repeat for a total of two rinses. Dry the washed fibres and weigh them.

**Solvent Extraction Method**

This test method covers the determination of the resin content of carbon and graphite prepregs by Soxhlet extraction. Special provisions are included to determine the resin content of filled systems. Although most glass, quartz, and high-silica prepreg resin contents may be determined by this test method, a simpler, equally accurate burn-of method is preferred.

**Specimen Type, Dimensions and Manufacturing**

A representative bulk sample of prepreg is selected, weighing approximately 10 to 15 g. Cut the sample in approximately 12.5 mm squares but take care in cutting of sample to prevent loss of resin which will tend to flake off of some prepregs.
**Apparatus**
To perform this test a standard analytical balance, some different kinds of bottles and flasks, filtering crucibles with holders, a desiccator, borosilicate extraction thimbles and paper thimbles, an oven, and a muffle furnace capable of at least 525°C is required. Furthermore a Soxhlet extraction assembly consisting of a hot plate, borosilicate flask, borosilicate extraction chamber, and a borosilicate condensing chamber is required. The solvent needed are ethyl alcohol and dimethylformamide (DMF) which have been found to provide accurate results. However, for referee testing, reagent grade solvents must be utilised.

**Testing Procedure**
In order to determine the true dry resin content, the volatile content is first determined. Condition and position a thimble within the Soxhlet extraction assembly and add the solvent. Turn on the hot plate and the cooling water for the condenser and adjust the apparatus to effect 3 to 10 reflux changes per hour. Remove thimble and sample from the extraction assembly, drain off the solvent, dry and weigh. If filler has appeared in the extract, the filler content has to be determine.

**Referenced Documents**
ASTM C 613-67 Test Method for Resin Content of Carbon and Graphite Prepregs by Solvent Extraction


ASTM D 3171-76 Test Method for Fibre Content of Resin-Matrix Composites by Matrix Digestion.


CRAG Method 1000 Methods of Assessment of Fibre Volume Fraction of Fibre Reinforced Plastics.

(ASTM D 792-91 Test Method for Density and Specific Gravity (Relative Density) of Plastics by Displacement)

(ASTM D 1505-85 Test Method for Density of Plastics by Density-Gradient Technique)

**11.2.14 Void content tests**
**Aim and Purpose of Test Methods**
The void content of a composite may significantly affect some of its mechanical properties. Higher void contents usually mean lower fatigue resistance, greater susceptibility to water penetration and weathering, and increased variation or scatter in strength properties. The test method presented here, covering the void content of reinforced plastics or composites, is applicable to composites...
for which the effect of ignition on the materials are known. The knowledge of void content is desirable for estimation of quality of composites. A good composite have 1% voids or less, while poorly made composite can have a much higher void content. Finite values under 1% should be recognised as representing a laminate density quality, but true void content level must be established by complementary tests or background experience, or both. Ultrasonic scanning methods [e.g. CRAG Method 1001] will not be described here, but can also be used to determine the void content.

**Specimen type, Dimensions and Manufacturing**
The volume of each test specimen should not be less than 2 cm$^2$ and have smooth edges and surfaces as free as possible from geometric irregularities.

**Apparatus**
The specimen dimensional measurements should be made using a micrometer with very good accuracy. A platinum or porcelain crucible of approximately 30 ml capacity, a digital scale with very high accuracy and an electric muffle furnace, capable of maintaining a temperature of at least 600°C is required for this test.

**Testing Procedure**
The densities of the resin, the reinforcement and the composites are measured separately. Then the resin content is measured (Section 11.2.13) and a theoretical composite density is calculated as follows.

\[
\rho_t = \frac{100}{\frac{R}{D} + \frac{r}{d}}
\]

(11.36)

where $\rho_t$ is the theoretical density, $R$ is the resin content and $r$ is the reinforcement content in weight percent, $D$ is the density of the resin, and $d$ is the density of the reinforcement. This is compared to the measured composite density and the difference in densities indicates the void content and is calculated as

\[
\rho_v = \frac{100(\rho_t - \rho_m)}{\rho_t}
\]

(11.37)

where $\rho_m$ is the measured density.

**Referenced Documents**

CRAG Method 1001 Methods of Assessment of Void Volume Fraction of Fibre Reinforced Plastics by Ultrasonic Scanning.
11.2.15 Degree of cure

Aim and Purpose of Test Methods
These methods provide means of characterising the cure behaviour, the degree of cure, and gel time of thermosetting resins. One of the methods referred to here uses dynamic mechanical testing for determining the cure characteristics by measuring the elastic and loss moduli as a function of temperature or time, or both. A second method uses a microscope to determine the gel time of a prepreg. The data obtained may be used for quality control, research and development, and establishment for optimum processing conditions.

Specimen type, Dimensions and Manufacturing
Due to the various geometries that might be used for dynamic mechanical curing of thermosetting resins/composites, specimen size is not fixed by this practice. Cure rates may be influenced by specimen thickness, so equal volumes of material should be used for any series of comparisons. When a prepreg is investigated the test specimen shall consist of material cut to approximately 6 mm in square.

Apparatus
The function of the apparatus is to hold a neat resin or uncured supported composite formulation or coated substrate of known volume and dimension. The material acts as the elastic and dissipative element in a mechanically driven oscillatory shear or dynamic compression system. The apparatus shall consist of a plate and a cone having a known cone angle or two parallel plates with polished or serrated surfaces. A clamping arrangement that permits gripping of the composite specimen is required. An oven with a device for controlling the temperature, a device for applying continuous oscillatory deformations to the specimen and devices for determining dependent and independent experimental parameters, such as force, frequency, and temperature is also required. A hot plate, some suitable timer, a microscope, and microscope coverglasses is required for the second test method determining the gel time of an epoxy prepreg.

Testing Procedure
The sample of thermosetting liquid or resin impregnated substrate is placed in mechanical oscillation at fixed or natural resonant frequencies at either isothermal conditions, with a linear temperature increase or a time-temperature relation simulating a processing condition. The elastic or loss modulus, or both, of the composite specimen are measured in shear as a function of time. The point in time when tan delta (the ratio of the imaginary part to the real part of the complex modulus) is maximum, and the elastic modulus levels after an increase, is calculated as the gel time of the resin under the condition of the test.

To investigate the gel time of an epoxy prepreg, a prepreg specimen is placed between the microscope coverglasses on a hot plate to a some recommended test temperature. Pressure is applied to the specimen through the coverglass with a wooden probe. The wooden probe is then used to form a bead of resin at the edge of the specimen. The time from the application of the heat until the resin ceases to form strings by contact with the probe is noted as the gel time.

Referenced Documents
ASTM D 3532-76 Test Method for Gel Time of Carbon Fibre-Epoxy Prepreg.
11.2.16 Other tests

Aging Procedures

Aim and Purpose of Test Methods
Aging procedures can be performed in some different ways, e.g. outdoor weathering or by exposing a specimen for hot air for extended periods of time. These types of test procedures may be used as a guide to evaluate the stability of plastic materials when exposed to heat ageing, or outdoors to the varied influences which comprise weather, and to compare ageing characteristics of materials as measured by the changes in some property of interest.

Specimen type, Dimensions and Manufacturing
Either panels, from which different specimens can be cut for tests of whatever property to be determined, or test specimens for the specific property to be determined can be used for this test procedure.

Apparatus
Different types of racks with sample holders, a pyranometer, and an ultraviolet radiometer is required to perform the outdoor weathering procedure. To perform heat ageing, an air flow oven is required

Testing Procedure
When heat ageing tests are performed at a single temperature, all materials must be exposed at the same time in the same device. Use a sufficient number of replicates of each material for each exposure time so that results of tests used to characterise the material property can be compared by analysis of variance or similar statistical data analysis procedure. When testing at a series of temperatures in order to determine the relationship between defined property change and temperature, use a minimum of four exposure temperatures.

When performing outdoor weathering tests, weathering racks shall be located in cleared areas, preferably at a suitable number of climatologically different sites representing the variable conditions under which the plastic product or material will be used. Mount the test samples in the holders and carefully mark the samples with some identifying number or symbol. It is convenient to group in one holder, samples to be removed from exposure at the same time. The exposure time can be from one week to several years.

Referenced Documents
ASTM D 1435-85 Standard Practice for Outdoor Weathering of Plastics.
Moisture and Water Absorption Properties

Aim and Purpose of Test Methods

This procedure is designed to produce moisture diffusion material property data that may be used to determine approximate exposure times used in procedures for conditioning material specimens prior to other types of testing, or for making qualitative decisions on material selection or performance under environmental exposure to various forms of moisture.

Specimen Type, Dimensions and Manufacturing

The test specimen for moisture diffusivity constant determination shall consist of either a nominally square plate or curved panel with dimensions that satisfy the relation \( t/a = 100 \), where \( a \) = side length and \( t \) = thickness, or a 100 mm square plate with stainless steel foil bonded to the edges such that moisture absorption through the edges is essentially eliminated. The coupon size and shape for the other test procedures presented here is normally that required for subsequent material evaluation following conditioning. When the coupon is of such type or geometry that the moisture change in the material cannot be properly measured by weighing the specimen itself, a traveller coupon of the same material and thickness, and of appropriate size shall be used to determine moisture equilibrium for the specimens being conditioned.

Apparatus

To perform these test methods one requires an analytical balance, an air circulating oven or vacuum drying chamber, and a conditioning chamber. Furthermore, a temperature and vapour-level controlled vapour exposure chamber for absorption by vapour exposure is required. For absorption by liquid immersion, a temperature controlled liquid bath is used.

Testing Procedure

This is a gravimetric test method that monitors the change over time to the average moisture content of a material specimen by measuring the total mass change of a coupon that is exposed on two sides to a specified environment. The procedure covers the determination of the two Fickian moisture diffusion material properties, the moisture diffusivity constant and the moisture equilibrium content. The first part of the test procedure covers the conditioning of a material coupon to an essentially moisture free condition. A general test coupon is maintained in an air-circulating oven at a prescribed elevated temperature environment until effective moisture equilibrium is reached. Then, the percent moisture mass gain versus time is monitored for the material specimen that is maintained in a steady-state environment at a known temperature and moisture exposure level until the material reaches effective moisture equilibrium. From this data the moisture equilibrium content and the one dimensional moisture absorption rate of the coupon may be determined and the through-the-thickness moisture diffusivity constant calculated. Immersion in a liquid bath should be used to simulate vapour exposure only when apparent absorption properties are desired for qualitative purposes.

The test procedure can also be used for general moisture conditioning of material coupons prior to other types of testing. However, any periodic monitoring requirement is not necessary then.

Referenced Documents

Volatile Content

Aim and Purpose of Test Methods
Knowledge of the volatile content is useful in developing optimum manufacturing processes. Quite similar methods are used to determine the volatile content for different resins and prepregs. These test methods does not identify the different volatile components.

Specimen Type, Dimensions and Manufacturing
While investigating prepregs, the specimen size shall be, at a minimum, 80 by 80 mm or, for materials such as narrow tapes, an equivalent area. When pure resin is investigated approximately 10g of sample resin is spread at the bottom of a weighing vessel and the mass of the specimen is carefully determined.

Apparatus
To perform these tests an analytical balance, some suitable cutting device, a timer, a sample container or a portable rack with hooks from which the specimens may be suspended (depending on if pure resin or prepregs are investigated), and an oven of circulating-air type or forced-ventilation type are required.

Testing Procedure
While investigating prepregs, the specimen is carefully weighed and then exposed to elevated temperature in a circulating air oven to remove the volatiles. The exposed samples are reweighed and the percent change in weight expressed as volatile content. When pure resin is investigated, a known quantity of powdered resin is heated to constant mass. The calculated mass loss defines quantitatively the volatile matter present in the sample.

Referenced Documents
ASTM D 3030-84 Test Method for Volatile Matter (Including Water) of Vinyl Chloride Resins.
ASTM D 3530-90 Test Method for Volatiles Content of Epoxy Matrix Prepreg.

Coefficient of Linear Thermal Expansion

Aim and Purpose of Test Methods
The thermal expansion of a reinforced plastic is composed of a reversible component on which are superimposed changes in length due to changes in moisture content, curing, loss of plastizers or solvents, release of stresses, phase changes and other factors. The intention should be to determine the coefficient of linear thermal expansion under the exclusion of these factors as far as possible. Because of the directional influence of the reinforcement, their interaction with each other and the matrix, it is necessary to ensure sufficient specimens are evaluated to accurately determine the property in the specified direction of the material. The method presented here covers the determination of the coefficient of linear thermal expansion for plastic materials having coefficients of expansion greater than $10^{-6}/^\circ C$ by use of a vitreous silica dilatometer. The nature of most plastics and the construction of the dilatometer make -30 to +30°C a convenient
temperature range for linear thermal expansion measurements of plastics. This range covers the
temperatures in which plastics are most commonly used. For materials having very low coefficient
of expansion, interferometer or capacitance techniques are recommended.

**Specimen type, Dimensions and Manufacturing**
The test specimen may be prepared by any suitable machining operation under conditions which
give a minimum of strain or anisotropy. The cross section of the specimen may be round, square,
or rectangular and the length shall be between 50 and 125 mm and it should fit easily into the
measurement system of the dilatometer without excessive clearance on the one hand or friction on
the other. To eliminate the influence of moisture content the test specimen must be pre-dried to a
constant weight. Cut the ends of the specimen flat and perpendicular to the length axis of the
specimen and protect the ends against indentations by means of flat, thin steel plates bonded or
otherwise firmly attached to them before the specimen is placed in the dilatometer.

---

**Apparatus**
A suitable fused-quarts-tube dilatometer, schematically shown in Fig.11.41, is required.
Furthermore, some suitable device for measuring the changes in length, a scale or calliper for
measuring the initial length of the specimen, and a controlled temperature environment to control
the temperature of the specimen is also required.

**Testing Procedure**
The specimen must be carefully aligned in the dilatometer tube to prevent friction. The measuring
device which is firmly attached to the outer tube is in contact with the top of the inner tube and
indicates variations in the length of the specimen with changes in temperature. Temperature
changes are brought about by immersing the outer tube in a liquid bath. The top of the specimen
shall be at least 50 mm below the level of the bath, and the open end of the dilatometer shall be at
least 50 mm above the liquid level.

**Referenced Documents**
CRAG Method 801  Method of Test for the Determination of the Coefficient of Linear Thermal
Expansion of Fibre Reinforced Plastics.
ASTM D 696-91  Test Method for Coefficient of Linear Thermal Expansion of Plastics Between -30 °C and 30 °C

**Not Yet Referenced Documents**

<table>
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<td>ASTM D 543-87</td>
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11.3 Core Materials

11.3.1 Tensile tests

Aim and Purpose of Test Methods
These test methods covers the determination of the tensile properties of rigid cellular plastics in
the form of test specimens of standard shape under defined conditions of temperature, humidity,
and testing machine speed. There are types of test specimen that may be preferred in those cases
where enough sample material exists to form the necessary specimen. Another type of specimen
may be used when only smaller specimens are available, as in, e.g., a sandwich construction.

![Figure 11.42 Different types of tensile test specimens. Shape (a) (d) and (e) may be preferred when
enough sample material exists, shape (b) and (c) may be preferred when only smaller samples of
material are available.](image)

Specimen type, Dimensions and Manufacturing
A cylinder with a diameter of approximately 34 mm is cut out of the core material with a hole saw.
Thereafter the middle of the specimen is cut down on a turning lathe forming a waist with a
diameter and a gage length of approximately 25 mm. As shown in Fig.11.42 there are two kinds
of pulley shaped specimens. One type is used in a specially designed grip (Fig.11.43(b)), while the
other type is carefully centred and bonded to a pair of loading blocks, as shown in Fig.11.43(a).
The specimen length must be at least 50 mm when using this tensile test method in order to avoid
end-effects and to get a suitable gauge length. If only smaller specimens are available, a test
specimen with a circular or square cross-section and a cross-section area of at least 645 mm² may
be used (Fig.11.42(b) and (c)). When testing materials that are suspected to be anisotropic, it is
recommended to prepare duplicate sets of tension test specimens having their long axes
respectively parallel and normal to the suspected direction of anisotropy.
Apparatus
A testing machine that is capable of control of constant-rate-of-crosshead movement and having a mechanism capable of showing the total tensile load carried by the test specimen is required. Furthermore the grips for holding the test specimen shall be of a self aligning type. That is, they must be attached to the fixed and moveable members in such a way that they will move freely into alignment as soon as any load is applied, so that the long axis of the test specimen will coincide with the direction of the applied pull through the centre line of the grip assembly. Universal-type joints immediately above and below the specimen holder are recommended (As the assembly in Fig.11.43(a)). If measurement of the extension is desired, a suitable extensometer is used.

![Image of test specimen and loading assembly](image)

Figure 11.43 Two different pulley shaped tensile test specimen with their loading assemblies. (a) shows a modified type of ASTM 1623 bonded to loading blocks with a universal joints assembly. (b) shows a cut-through of a ASTM 1623 with grip assembly.

Testing Procedure
It is recommended to test at least five specimens. If a specimen fail at some obvious flaw the specimen shall be discarded and a retest shall be made, unless such flaws constitute a variable that is to be studied. To start with, the cross-sectional dimensions of the test section shall be measured at several points and the minimum value shall be recorded. After the testing equipment is calibrated, the specimen is placed in the assembly and the entire assembly is aligned properly. Determine and record the load at the moment of rupture. If an extensometer is used, a complete stress-strain curve may be obtained thereby. Also determine and record the extension at the moment of rupture of the specimen. The tensile strength is calculated by dividing the fracture load by the original minimum cross-sectional area of the specimen. The modulus of elasticity is calculated by extending the initial linear portion of the load-extension curve and dividing the
difference in stress corresponding to any segment of section on this straight line by the corresponding difference in strain.

**Referenced Documents**


**11.3.2 Compressive tests**

**Aim and Purpose of Test Methods**

The compressive properties of sandwich cores are usually determined, for design purposes, in a direction normal to the plane of facings as the core would be placed in a structural sandwich construction. These test procedures pertain to compression in this direction in particular, but also can be applied with possible minor variations to determining compressive properties in other directions. Two test are described where one provides complete deformation data, and from a complete load–deformation curve it is possible to compute the compressive strength at any load and to compute the effective modulus of elasticity of the core as a unit. An alternative method measures crushing strength only and is often proposed as an acceptance test for honeycomb cores. Considerable time is saved in specimen preparation, and once the results are related to the actual compressive strength, as obtained from the former test procedure, the ram crushing test can be used for any core thickness as a rapid quality control or acceptance test.

**Specimen type, Dimensions and Manufacturing**

The test specimens shall be of square or circular cross section having areas not exceeding 100 cm², but some minimum areas are also recommended for different types of core materials. For continuous cores, such as balsa, foamed rubbers, and foamed resins having small pores (less than 6 mm in diameter), the minimum cross section area shall be about 625 mm². Specimens from open-celled or gridded type cores having cells larger than 12 mm in diameter, shall have a minimum cross section area of 2500 mm², or large enough to include at least one complete cell. The height of the specimens shall be 100 to 200 mm if possible, but not greater than four times the width or diameter. A practical minimum height of 12 mm is necessary if strains are to be measured. It is of great concern that the loaded ends are parallel to each other and perpendicular to the sides of the specimen. In order to avoid local crushing at the ends of some cores, particularly honeycomb and
gridded cores, it is desirable to reinforce the ends with a thin layer of resin or another suitable material. When testing the crushing strength, the test specimen need only to be a slice of core material about 12 mm thick and at least 250 by 250 mm in cross section. No end coatings or castings shall be used here.

**Apparatus**

A Spherical bearing block, of the suspended, self aligning type (Fig.11.44) is used together with a testing machine that is capable of control of constant-rate-of-crosshead movement and having a mechanism capable of showing the total compressive load carried by the test specimen. To measure the strain, a extensometer is needed. It shall be light in weight and require a minimum of force to operate. For extremely soft cores, a suitable means for measuring strain consists of filar microscopes focused on points of fine needles inserted in the core. A compression ram, at least 65 mm in diameter and large enough, for cores having large cells, to include at least one cell, is needed for the crushing strength test.

![Figure 11.44 Schematic view of compression test with self aligning spherical bearing block.](image)

**Testing Procedure**

Apply the load to the specimen through the spherical loading block in such a manner that the block distributes the load as uniformly as possible over the entire loading surface of the specimen. The load is applied at a constant rate of movement of the movable head of the testing machine. Data for stress–strain curves may be recorded to determine the modulus of elasticity, which can be calculated as follows;

\[
E_c = \frac{P}{A\varepsilon}
\]  

(11.38)
where $E_c$ is the modulus of elasticity in compression, $P$ is the load, $A$ is the specimen cross section area, and $\varepsilon$ is the strain at load $P$. Measure the strain by means of an extensometer attached to the central portion of the length of the specimen in such a way that it will not damage the specimen nor affect the test results.

When performing the crushing strength test, support the test slice over its entire area on the lower platen of the testing machine and press the compression ram into the slice. Apply the load at a constant rate of movement of the ram. Several tests can be made on one core slice. The compressive strength is calculated by dividing the maximum load by the cross section area.

**Referenced Documents**


### 11.3.3 Shear tests

**Aim and Purpose of Test Methods**

This test method provides information on the load–deformation behaviour of sandwich constructions or cores when loaded in shear parallel to the plane of the facings. From a complete load–deformation curve it is possible to compute shear stress of the sandwich or core at any load and to compute an effective shear modulus of the sandwich as a unit or shear modulus of the core. From tests of sandwich construction it is possible to compute the core shear modulus from the effective sandwich shear modulus if the shear modulus of the facing material is known. By measuring the strains in the core, it is possible to obtain core shear modulus directly from the test of a complete sandwich construction. The test may also be conducted on core materials in which case facings are not bonded to the core and loading plates are bonded directly to the core. The test does not produce pure shear, but the specimen length is prescribed so that secondary stresses have a minimum effect.

**Specimen type, Dimensions and Manufacturing**
The specimen shall have a thickness equal to the thickness of the sandwich, a width not less than twice the thickness, and a length not less than twelve times the thickness. Support the specimen by means of steel plates bonded to the facings, or directly to the core. If the core or facing materials show directional characteristics with respect to shear strength, separate tests shall be made in such a manner as to develop shearing stresses in each of the principal directions.

**Apparatus**
The test assembly is mounted with a spherical bearing block, or universal joint (Fig.11.45) in a testing machine that is capable of control of constant-rate-of-crosshead movement and having a mechanism capable of showing the total shear load carried by the test specimen. To measure the deformation, an optical-lever system, dial gage, or any other suitable means is needed.

**Testing Procedure**
Apply the load to the ends of the rigid plates in tension or compression through spherical bearing blocks or a universal joint so as to distribute the load uniformly across the width of the specimen. The load shall be applied at a constant rate of movement of the movable head of the testing machine. Obtain sufficient readings, preferably at equal strain increments, to define the shape of the stress–strain diagram. The deformations may be read by means of an optical-lever system, dial gage, or any other suitable means. It is suitable to record data for load–deformation curves, which may be used to determine the effective shear modulus of the sandwich. The shear stress, \( \tau \), is calculated as follows;

\[
\tau = \frac{P}{Lw} \quad (11.39)
\]

where \( P \) is the load on the specimen, \( L \) is the specimen length, and \( w \) is the specimen width. The shear strain is obtained as \( \gamma = \Delta / t_c \) and the shear modulus of the core is calculated as

\[
G_c = \frac{\tau}{\gamma} = \frac{Pt_c}{\Delta Lw} \quad (11.40)
\]

where \( \Delta \) is the displacement or movement of one loading plate of the specimen with respect to the other and \( t_c \) is the thickness of the core specimen.
Figure 11.45 Schematic view of shear test.

**Referenced Documents**


ISO 1922: 1981  *Cellular Plastics--Determination of Shear Strength of Rigid Materials*

**11.3.4 Flexural tests**

The flexure tests can only be used on foam core materials, and are performed in the same way as described in Chapter 11.2.5. Since foam core materials usually are sensitive to surface damages,
great care should be taken to avoid excessive indentations at the load points. This can be done using supports and loading noses with cylindrical surfaces with large radius.

**Referenced Documents**

ISO 1209-1: 1990  *Cellular Plastics, Rigid--Flexural Tests--Part 1: Bending Test*


**11.3.5 Fatigue tests (block shear)**

**Aim and Purpose of Test Methods**
If the facings of a sandwich construction are designed so that they are elastically stable, the most critical stress to which the core is subjected is shear. The effect of repeated shear stresses on the core material is therefore important. This test method covers the determination of the effect of repeated shear loads on sandwich core materials.

**Specimen type, Dimensions and Manufacturing**
The specimen shall have a thickness equal to the thickness of the sandwich, a width not less than twice the thickness, and a length not less than twelve times the thickness. Support the specimen by means of steel plates bonded directly on to the core. If the core materials show directional characteristics with respect to shear strength, separate tests shall be made in such a manner as to develop shearing stresses in each of the principal directions. Some specimens are required for static control tests, and a minimum of five stress levels with a similar number of specimens is tested for each stress level.

**Apparatus**
A standard constant load fatigue testing machine capable of applying a direct stress to the specimen and with a counter shall be used. Test fixtures and a tension testing machine, as described in chapter 11.3.3, is also required, for static control tests.

**Testing Procedure**
Prior to the fatigue test, test a minimum of five test specimens statically in a tension test machine in accordance with a test method described in chapter 11.3.3. Use the average result from the static tests as a 100% level of the fatigue test. The stress levels, defined as the maximum repeated stress to which the specimen is subjected, to be tested shall be agreed upon by the parties involved in the testing. The stress ratio recommended for standard tests is minimum to maximum load of 0.1. After testing has begun, check the loading frequently unless the machine is equipped with automatic load maintainers. Continue testing until failure has occurred or a specified number of cycles has been reached without failure. Be careful to conduct the tests under the same temperature and humidity conditions.

**Referenced Documents**


ISO 3385: 1989  *Flexible Cellular Polymeric Materials--Determination of Fatigue by Constant-Load Pounding*
11.3.6 Fracture toughness tests
Aim and Purpose of Test Methods
The property $K_{IC}$ determined by these test methods characterises the resistance of a material to fracture in a neutral environment in the presence of a sharp crack under severe tensile constraint, such that the state of stress near the crack front approaches plane strain, and the crack-tip plastic region is small compared with the crack size and specimen dimensions in the constraint dimension. In a similar way the property $K_{IIC}$ characterises the resistance of a material to fracture in a neutral environment in the presence of a sharp crack under severe shear constraint. Cyclic loads can cause crack extension at $K$ values less than the critical $K$ value (e.g. $K_{IIC}$). Crack extension under cyclic load will be increased by the presence of an aggressive environment and therefore, application of $K_{IC}$ and $K_{IIC}$ in the design of service components should be made considering differences that may exist between laboratory tests and field conditions.

Mode I: The Single-Edge-Notch Bending Specimen (SENB)
The SENB specimen which is used to extract the stress intensity in mode I has several advantages: the specimen is easy to manufacture and the test is simple to perform in any quasi static testing machine only with the use of a simple bending rig. The SENB-specimen is mainly to be used with isotropic- or moderately anisotropic materials like most foams.

![Figure 11.46 The SENB specimen for measuring the Mode I fracture toughness.](image)

Specimen type, Dimensions and Manufacturing
The SENB-specimen is cut out as a rectangular piece of core material. The length, $L$, should be at least four times the height of the specimen and the height should be about twice the thickness. A crack is sawed using a thin sawblade leaving the last 2-4 mm which is cut with a razor blade. The crack should be in the order of $0.2h < a < 0.8h$, where $a$ is the crack length and $h$ is the specimen height (Fig. 11.46).
Apparatus
A properly calibrated testing machine that can be operated at constant rates of crosshead motion over the range indicated shall be used. It shall be equipped with a deflection-measuring device and a load measuring system with good accuracy. The loading nose and supports in the three-point bending rig shall have cylindrical surfaces. In order to avoid excessive indentation, or failure due to stress concentration directly under the loading nose, the radius of the nose and the supports shall be at least 3 mm and up to 1.5 times the specimen depth.

Testing Procedure
The specimen is mounted with a short overhang on each side in the three-point bending rig and the length between the supports should be four times the height, as shown in Fig.11.46. Apply the load at a specified crosshead rate, and take simultaneous load-deflection data. The fracture toughness can be written as

$$K_{IC} = \frac{6P_C}{h} \sqrt{\pi a} \cdot F\left(\frac{a}{h}\right)$$

(11.41)

where $P_C$ is the critical load, i.e. the load when fracture occur, and $F(a/h)$ is a function determined by the crack length and the specimen geometry and is commonly called the finite width correction factor. The following approximate formula is valid for any $a/b$ within 0.5%, if the length of the specimen is $4h$.

$$F\left(\frac{a}{h}\right) = \frac{1}{\sqrt{\pi}} \left(1.99 - \frac{a}{h}\left(1 - \frac{a}{h}\right)\left(2.15 - 3.93\frac{a}{h} + 2.7\left(\frac{a}{h}\right)^2\right)\right)$$

$$\left(1 + 2\frac{a}{h}\left(1 - \frac{a}{h}\right)^{3/2}\right)$$

(11.42)

The finite width correction factor $F(a/h)$ calculated with, e.g., $a/h = 0.5$, which is a commonly used crack length to specimen height ratio, is $F(a/h)=1.416$.

Mode I: The Compact Tension Specimen (CT)
The standard compact tension specimen is a single edge-notched plate loaded in tension. This specimen is also quite easy to manufacture and the test is simple to perform.

Specimen type, Dimensions and Manufacturing
The specimen thickness should be identical with the material sheet thickness and the specimen width, $W$, shall be twice the thickness. A crack is sawed using a thin sawblade leaving the last 2-4 mm which is cut with a razor blade. The crack length, $a$, should be selected such that $0.45 < a/W < 0.55$. The specimen geometry is shown in Fig.11.47.
Apparatus
The specimen is loaded through loading clevises mounted in a testing machine of the same type as described in the previous section.

Testing Procedure
When assembling the clevises and their attachments to the tensile machine great care should be taken to minimise eccentricity of loading due to misalignments external to the clevises. Load the compact tension specimen at some specified rate. The mode I fracture toughness, $K_{IC}$, is calculated from

$$K_{IC} = \frac{P_c}{B\sqrt{W}} f\left(\frac{a}{W}\right)$$  \hspace{1cm} (11.43)

where $a$, $B$, and $W$ are shown in Fig.11.47, $P_c$ is the critical load and $f(a/W)$ is the finite width correction factor, which is calculated as follows:

$$f\left(\frac{a}{W}\right) = \frac{\left(2 + \left(\frac{a}{W}\right)\right)(0.866 + 4.64\left(\frac{a}{W}\right) - 13.32\left(\frac{a}{W}\right)^2 + 14.72\left(\frac{a}{W}\right)^3 - 5.6\left(\frac{a}{W}\right)^4)}{\left(1 - \left(\frac{a}{W}\right)\right)^{\frac{3}{2}}}$$  \hspace{1cm} (11.44)

The expression for $f(a/W)$ is valid for $0.2 < a/W < 0.8$.

Mode II: The End-Notch Flexure Specimen (ENF)
In a sandwich construction, the core material carries transverse loads and transfer loads between the faces as shear stresses. A crack in the core situated along or perpendicular to the faces will therefore be subjected to a state of almost pure shear. The end-notch flexure specimen (ENF) is used to extract the toughness of a crack in the core subjected to only mode II deformation. In similarity with the SENB specimen, the crack is situated at the edge and in the middle of the core but in the direction parallel to the faces. Another difference is that thin face materials are attached on each side of the core to prevent the specimen from compressive failure under the load point or tensile failure at the bottom middle point of the specimen prior to crack extension. This specimen should also mainly be used with isotropic or moderately anisotropic materials like foams.
Specimen type, Dimensions and Manufacturing
The ENF specimen is manufactured by using two equally thick pieces of core material from the same block. The pieces are bonded together with a thin Teflon film placed between the core pieces where the crack is to be simulated (Fig.11.48). Some overhang of the specimen must be allowed for, thus requiring the Teflon film to be somewhat longer than the simulated crack length. The thickness of the specimen should be at least $h/2$, where $h$ is the specimen height.

Apparatus
The testing facilities used are similar to the ones described in the SENB section presented above.

Testing Procedure
The specimen should be mounted with a short overhang on each side in the three-point bending rig with the length between the supports is four times the height. Apply the load at a specified crosshead rate, and take simultaneous load-deflection data. The crack will propagate at an initial direction about 80 degrees from the direction of the simulated crack (Fig.11.48).

In Mode II one can write the stress intensity and the energy release rate in a similar way to the Mode I case, that is

$$ K_{II} = \tau_\infty \sqrt{\pi a} g \quad \text{and} \quad G_{II} = \frac{K_{II}^2(1-\nu^2)}{E} $$

(11.45)

where $g$ is the finite width correction factor, $E$ Young’s modulus, $\nu$ Poisson’s ratio and $\tau_\infty$ is the remote shear stress, i.e. the stress that would act on the crack surface if the crack was absent. To calculate the finite width correction factor from $K_{II} = \tau_\infty \sqrt{\pi a} g$, one has to calculate the shear stress, $\tau_\infty$, first. This is done in chapter 3 using formulae 3.13-14 and 3.16-18. The finite width correction factor, $g$, is given from finite element calculations and some values are found in [Schubert et.al.] and [Grenestedt et.al.]

Referenced Documents
11.3.7 High strain rates

Aim and Purpose of Test Methods
Special transient load cases of sandwich structures puts high demands on the properties of the constituent materials. When the materials in a sandwich structure are subjected to rapid loading, the behaviour of the materials at high strain rates becomes an important factor. This is particularly in focus for core materials, since most commercially available foam cores are relatively brittle. A method for tensile testing at high strain rates of foam core material is presented in this section.

Specimen type, Dimensions and Manufacturing
A specimen, 200 by 25 mm, is cut from blocks of the investigated core material and a waist of 15 mm thickness is formed as shown in Fig.11.49. The specimen thickness should be 10 mm and the gauge length 100 mm.

Figure 11.49 A commonly used specimen shape for determining the tensile properties at high strain rates.

Apparatus
For strain rates up to approximately 500 mm/sec, the tests are performed using some suitable mechanical testing machine that shall have the capability to control and regulate the velocity of the movable head up to a testing speed, of at least, 500 mm/min. For higher strain rates, a pendulum type testing machine as described in section 11.2.6 is used. The load sensing device of the testing machine shall be capable of indicating the total load being carried by the test specimen. To apply the load to the specimen, some suitable light weight grips shall be used. To measure the strain, an external properly calibrated extensometer, preferably a laser extensometer, though a strain gauge bonded to the specimen would affect the results, is used. A laser extensometer contains two laser sources producing two laser beams through optic cables which are attached perpendicularly across the specimen at either end of the gauge length. The laser beams are then received by two sensors, recording the position of the beams. An oscilloscope and a suitable high speed data acquisition system, to collect desired data, are also required.
Testing Procedure
When performing a high strain rate test, it is important that the masses introduced into the system are as low as possible. These would otherwise affect the test results by generating inertia forces. The test set-up must however be rigid enough to withstand the applied loads without flexing and hence introducing errors in the results. The test set-up is schematically shown in Fig.11.50. A series of tests using different strain rates, to compare how the strain rate affect different material properties, should be made. The desired strain rate is obtained by choosing different initial heights, from were the pendulum movement starts. The energy of the pendulum is transferred to the specimen by letting the pendulum hit a pin between two very stiff and light rods (Thin carbon fibre laminates are recommended) which are attached to one end of the specimen. The load is measured by a load cell mounted between the other specimen end and a rigid base, and the load, strain, and time history is recorded by the oscilloscope and data acquisition equipment. The Young’s modulus is calculated from the stress-strain ratio at a strain between, e.g., 0.5 and 1.5 percent which should produce a reliable tangent value based on the conditions in the beginning of the test. The energy absorbed per unit volume in the gauge length can be calculated by adding the product of the stress and the increment in strain between each sampling during the entire cycle.

Referenced Documents
11.3.8 Creep tests
The creep tests for foam core materials are performed in a similar way as for face materials, described in chapter 11.2.12. Great care should be taken to avoid excessive indentations at the load points when performing flexure creep tests. This can be done using supports and loading noses with large radius cylindrical surfaces.

Referenced Documents
ISO 10066: 1991 Flexible Cellular Polymeric Materials--Determination of Creep in Compression

11.3.9 Density tests
Aim and Purpose of Test Methods
This test method is used to determine the density of core materials used in structural sandwich constructions.

Specimen type, Dimensions and Manufacturing
The test specimen shall be in the form of a piece about 75 mm in length by about 75 mm in width by the thickness of the sandwich core material. It is recommended that at least three specimens are tested.

Apparatus
An analytical balance or scale with good accuracy, a drying oven, a micrometer capable of measuring accurately 0.01 mm and a dessicator is required for this test.

Testing Procedure
Subject the test specimens to a specified condition e.g. atmospheric conditions, some specified temperature in a drying oven, or some condition agreed upon by the purchaser and the seller. After conditioning the specimens are, if necessary, cooled at room temperature in a desiccator. Weigh the specimens and determine the dimensions and then calculate the density.

Referenced Documents
11.3.10 Tests of thermal transmission properties

Aim and Purpose of Test Methods

The test method described here is known as the guarded hot plate method and it covers the measurement of the steady-state heat flux through flat specimens. Since heat flux and its uncertainty may be dependent upon environmental and apparatus test conditions, as well as intrinsic characteristics of the specimen, the report from the testing must include thorough description of the specimen and of the conditions. Also, since this test method is applicable to a wide range of specimen characteristics, test conditions, and apparatus design it is impractical to give an all-inclusive statement of the precision and bias of the test method. Thermal transmission properties can be calculated based on the heat flux measurements conducted by this test method.

Specimen type, Dimensions and Manufacturing

The specimen used in the guarded hot plate test shall be sized to cover the entire metered region and the specimen surfaces shall be flat and parallel. Furthermore, the specimen thickness that can be measured to a given accuracy is dependent on several parameters, including the size of the apparatus, thermal resistance of the material, and the accuracy desired. To maintain edge heat losses to below about 0.5 %, for a guard width that is about one-half the linear dimension of the metered region, the recommended maximum thickness of the specimen is one-third the maximum linear dimension of the metered region.

Apparatus

The general arrangement of the mechanical components of the guarded hot plate apparatus are illustrated in Fig.11.51. This system consists of a guarded heating unit, two auxiliary heating plates, two cooling units, secondary guarding in the form of edge insulation, and a temperature controlled secondary guard. The environmental chamber may not be necessary if atmospheric air is used for fill gas. Two essentially identical specimens are placed on either side of the guarded
heating unit. The opposite faces of the specimens are in contact with the auxiliary heating units which are adjusted so they are at the same temperature. The purpose of these three isothermal units is to create an accurately measurable steady-state heat flux unidirectionally through the two specimens. The purpose of the secondary guard is and edge insulation is to further reduce radial heat flow and the cooling units are isothermal heat sinks to remove the energy generated by the heating units. A suitable temperature sensor-readout system possessing an accuracy consistent with the required error analysis shall be used for measurement and control of the temperatures within the system.

**Testing Procedure**

After installation of the specimen and installation of appropriate secondary guarding and an environmental chamber (if necessary), the thermal test procedure begins. The various heating and cooling units are placed into operation to achieve the test temperature conditions. The time required to achieve thermal steady-state of the system varies considerably with the characteristics of the apparatus design, the specimen to be measured, and the test conditions. Generally, since this method is applicable to low conductance specimens, the setting time is on the order of hours. After achievement of the desired steady-state, three successive repeat data acquisition runs shall be completed. These runs shall be conducted at intervals of no less than 30 min.

The primary data required for this test method include electrical power, surface temperatures, area and thickness of the specimen. Of these, only the thickness is generally a directly measured quantity. The others are either calculated from other more fundamental measurements or are converted by an electrical device. The heat flux to be reported is that which passes through each specimen. In the method described, only half of the power generated by the heater flows through each specimen. The power, $W$, is determined from electromotive force (emf), $D$, and current, $Z$, measurement, and is calculated as

$$W = DZ$$  \hspace{1cm} (11.46)

The thermal conductance, thermal resistance, thermal conductivity, and thermal resistivity can also be calculated from the measured data.

**Referenced Documents**


11.3.11 Coefficient of thermal expansion
The test method for measuring the coefficient of thermal expansion for foam core materials is similar to the described in chapter 11.2.16.

Referenced Documents
ASTM D 696-91 Test Method for Coefficient of Linear Thermal Expansion of Plastics Between -30°C and +30°C

ISO 4897: 1985 Cellular Plastics -- Determination of the Coefficient of Linear Thermal Expansion of Rigid Materials at Sub-Ambient Temperatures

11.3.12 Water absorption tests
Aim and Purpose of Test Methods
The moisture content of most core materials is related to such properties as electrical and mechanical properties. Also important is the amount of weight the structure may gain by the core absorbing water. This test method covers the determination of the relative rate of water absorption by various types of structural core materials, such as honeycomb, foam, and balsa wood, when immersed or in a high relative humidity environment

Specimen type, Dimensions and Manufacturing
Machine, saw, or shear the test specimens from the core sample so as to have smooth surfaces that are free from cracks. The specimens shall be about 76 by 76 by 13 mm thick and the thickness of the specimen shall be in the same direction as the core thickness when used in a sandwich panel.

Apparatus
An analytical scale with good accuracy, a circulating air oven capable of maintaining uniform temperatures with good accuracy, and a humidity chamber capable of maintaining uniform temperatures and uniform relative humidity is required. One test procedure also require a underwater weighing rig. Only distilled water or deionized water shall be used.

Testing Procedure
One test procedure is the immersion where a specimen is completely immersed in a container of 23°C water. Materials that float should be held under water by a loose net. After 24 hours, the specimen is removed from the water and surface water is wiped off with a dry cloth and then the specimen is weighed and the weight is recorded. For materials that tend to collect water on the surfaces or trap water in corners, the specimen is dipped in alcohol which then is allowed to evaporate before the specimen is weighed.

In another test the specimen is exposed for elevated temperature and humidity in a humidity chamber. The condition recommended is about 70°C and 85% relative humidity for 30 days. However other temperatures, relative humidities, and length of time can be used. When the specimen is taken out of the chamber it shall be allowed to cool to room temperature before weighing.

The aim of a third testing procedure is to determine maximum percent weight gain. Again a specimen is completely immersed in a container of water of approximately 23°C. In the same way
as above, materials that float should be held under water by a loose net, but this time for 48 hours. Then the specimen is removed from the container and the surface water is removed, and when the specimen is weighed it is put back in the water again. This process is repeated until the weight gain after one 48 hours interval is less than 2% of the entire weight gain of all the previous intervals.

There is also a method for correction when surface water on the specimens presents a problem. Weigh a control sample, dip it quickly in water, then follow the same procedure used on the actual specimen to determine the weight gain. Subtract the surface water weight gain to correct the actual wet specimen weight.

In a testing procedure only valid for rigid cellular plastics, the specimen is placed in an underwater weighing rig and then immersed in an immersion tank. After 96 hours immersion time, the specimen is weighed by means of the weighing rig.

**Referenced Documents**

ASTM C 272-91 Test Method for Water Absorption of Core Materials for Structural Sandwich Constructions


ISO 2896: 1987 Cellular Plastics, Rigid--Determination of Water Absorption

**11.3.13 Vapour transmission**

**Aim and Purpose of Test Methods**

The purpose of these tests is to obtain, by means of simple apparatus, reliable values of water vapour transfer through permeable and semipermeable materials. Two basic methods, the desiccant method and the water method, are provided for the measurement of permeance, and two variations include service conditions with one side wetted and service conditions with low humidity on one side and high humidity on the other. A permeance value obtained under one set of test conditions may not indicate the value under a different set of conditions. For this reason, the test conditions should be selected that most closely approach the conditions of use.

**Specimen type, Dimensions and Manufacturing**

The specimen shall be of the same size as the open “mouth” of the dish containing the desiccant, or water, and the thickness shall be the same as the thickness of the material used.

**Apparatus**

The test dish required can be of any suitable shape and it shall be made of a noncorroding material, impermeable to water or water vapour. A large, shallow dish is preferred (Fig.11.52), but its size and weight are limited when an analytical balance is chosen to detect small changes in weight. The mouth of the dish shall be as large as practical and the desiccant or water area shall not be less than the mouth area. For the desiccant method, anhydrous calcium chloride in form of small lumps and free from sieve is recommended as desiccant, and distilled water shall be used for the water method. The room or cabinet where the assembled test dish are to be placed shall have a controllable temperature and relative humidity.
Testing Procedure
When performing the desiccant method, the dish is filled with desiccant within about 6 mm of the test specimen, leaving enough space so that shaking of the dish, which must be done before each weighing, will mix the desiccant. Attach the specimen to the dish, by sealing and clamping, and place it in the controlled chamber, with specimen up, weighing it at once. Weigh the dish assembly periodically, often enough to provide at least eight or ten points during the test, to obtain a weight versus time curve.

When performing the water method, is filled with distilled water to a level about 20 mm from the test specimen. The air space thus allowed has a small water vapour resistance, but is necessary in order to reduce the risk of water touching the specimen when handling the dish assembly. Then the test is performed in a similar way as described above. The Water Vapour Transmission (WVT) is calculated as

\[ WVT = \frac{\Delta W}{tA} \]  

where \( \Delta W \) is the weight change, \( t \) is the time, and \( A \) is the test area, i.e., the area of the “mouth” of the test dish.

Referenced Documents


SS 02 15 82 Water Vapour Permeability

11.3.14 Other tests
Cell Size of Rigid Cellular Plastics
Aim and Purpose of Test Methods
Several physical properties of rigid cellular plastics are dependent on cell size and cell orientation. Measuring water absorption and open-cell content requires knowledge of surface cell volume, which uses cell size values in the calculation. This test method covers the determination of the cell
size of rigid cellular plastics by counting the number of cell wall intersections in a specified distance.

**Specimen type, Dimensions and Manufacturing**
The specimen shall be about 51 by 51 mm by thickness of the material and shall be cut from the sample in the area to be tested.

**Apparatus**
A cutting blade apparatus capable of slicing very thin specimens for cell size viewing is required. For the cell size viewing a conventional 35-mm slide projector that accepts standard 51 by 51 mm slides can be used as a cell size projector. A cell size scale slide assembly, consisting of slide glass hinged by take along one edge, between a calibrated scale printed on a thin plastic sheet is placed, is also required.

**Testing Procedure**
Prepare the cell size viewing specimen by cutting a thin slice from one of the cube surfaces of the specimen. Slice thickness should be as thin as practicable so that a shadowgraph will not be occluded by overlapping cell walls. Insert the thin sliced foam specimen into the sell size slide assembly. Position the zero on the grid line at the top of the area to be measured. Insert the slide assembly into the projector and focus the projector on a screen so that a sharp image shadowgraph results. Determine the average cell size by counting the number of cells that intersect the 30 mm straight line projected with the specimen and then divide the length of line by the number of cells.

**Referenced Documents**

### Delamination Strength of Honeycomb Type Core Material

**Aim and Purpose of Test Methods**
This test method covers determination of the delamination strength of the node-to-node bond of honeycomb core materials. The test is useful in determining whether cores can be handled during cutting and machining without delaminating.

**Specimen type, Dimensions and Manufacturing**
The test specimen shall be about 130 mm wide and 260 mm long with a test section out side the grips of about 200 mm. The specimen width shall be parallel to the node-to-node bond areas. To prevent crushing of the specimen, while using clamping type of grips, the specimen ends can be reinforced by filling with plaster or with a dip coating of resin.

**Apparatus**
A tension testing device, equipped with some suitable grips and capable of slow, uniform head motion, that will indicate load at failure is required for the testing.

**Testing Procedure**
Apply tensile load normal to the ribbon so as to produce a constant rate of grip separation, and record the ultimate load developed. Calculate the delamination strength of the core as

\[
F_u = \frac{P}{wt}
\]
Where $F_u$ is the delamination strength, $P$ the ultimate tensile load, $w$ the width and $t$ the thickness of the specimen.

**Referenced Documents**
ASTM C 363-57 Test Method for Delamination Strength of Honeycomb Type Core Material

**Measurement of Thickness of Sandwich Cores**

**Aim and Purpose of Test Methods**
This test provides information regarding the variation in thickness of flat sandwich core materials, and provides a basis for obtaining an average thickness dimension. The test methods are designed for measuring thickness of a core as it is produced and are not intended for use in determining dimensions of core specimens for other tests. Normally a close tolerance is desirable for core thickness so that sandwich may be manufactured without core crushing.

**Specimen type, Dimensions and Manufacturing**
The test specimen shall be flat but otherwise may be any length, width, and thickness consistent with the limits of the measuring apparatus.

**Apparatus**
A roller-type thickness tester can be used, consisting of a flat table with a rigid yoke framework attached (Fig. 11.53). Two rollers shall be mounted on this yoke, one fixed in position and one movable in the vertical direction. The vertical movement of the upper roller is translated to a dial gage that registers the amount of variation in thickness.

An alternative apparatus to use is the disc-type thickness tester consisting of a flat table with a rigid yoke framework attached, where a presser disk, movable in a vertical direction, is mounted on the yoke. The vertical movement of the disc is translated to a dial gage that registers the amount of variation in thickness.

![Figure 11.53 Schematic view of roller-type thickness tester.](image)

**Testing Procedure**
Place a spacer bar of thickness equal to the desired nominal core thickness in between the rollers, or beneath the disc if the disc-type thickness tester is used, to calibrate the dial gage. Remove the spacer bar and insert the core material to be measured. Move the core through the rollers back and forth and observe the dial gage readings. If a disc-type thickness tester is used the core can be moved in a saw tooth pattern along the length of the specimen.

**Referenced Documents**
Open Cell Content

Aim and Purpose of Test Methods
Cellular plastics are composed of the membranes or walls of polymer separating small cavities or cells. These cells may be interconnecting (open cells), non-connecting (closed cells), or any combination of these types. This test method determines numerical values for open cells. It is a porosity determination, measuring the accessible cellular volume of a material.

Specimen type, Dimensions and Manufacturing
The test specimen shall be a cube, machined or sawed from the sample so as to have smooth surfaces, having a nominal dimension of 25 by 25 by 25 mm.

Apparatus
An air pycnometer, a cutting device capable of producing smooth cuts, and a dial micrometer measuring device are required for this test.

Testing Procedure
The test method is based on a determination of porosity in which the accessible cellular volume of a cellular plastic is determined by application of Boyle’s law, which states that the decrease in volume of a confined gas results in a proportionate increase in pressure. The apparatus consists of two cylinders of equal volume with an accessible chamber provided in one of the cylinders for insertion of the test specimen. Pistons in both cylinders permit volume changes. The pressures are increased equally by decreasing both volumes when a specimen is present in the specimen chamber. The volume change for the specimen cylinder is smaller than for the empty reference chamber, and corresponds to the displacement volume of the specimen. The difference between this volume and the geometric volume of the specimen is a measure of the open cell volume.

Referenced Documents
ASTM D 2856-87 Test Method for Open Cell Content of Rigid Cellular Plastics by the Air Pycnometer.


Thermal and Humid Ageing

Aim and Purpose of Test Methods
Because of the wide variety of potential uses of rigid cellular plastics, artificial exposure to estimate the effective behaviour of these materials must be based, to great extent, on the intended application. The conditions recommended here have been widely used in artificially exposing rigid cellular plastics and in determining the effects of various temperatures and humidities on these materials. The results from this test are not suitable for predicting end-use product performance or characteristics, nor are they adequate for engineering or design calculations.

Specimen type, Dimensions and Manufacturing
Test specimens shall be sawed or machined from the sample so as to have smooth edges free from cracks. The test specimen shall have nominal dimensions of 100 by 100 by 25 mm. Materials with and intended for use with natural or laminated skin surfaces shall be tested with this skin intact.
Apparatus
To perform this test a balance, an oven of circulating-air type, a cold box, a dial gage, and a humidity oven is required.

Testing Procedure
Determine the dimensions and mass of the specimen and make a visual examination of the specimen. Expose the specimens at some specified condition 24 hours, 168 hours and 336 hours. Intermediate observations are recommended to serve as a guide to full-term performance. After exposure, allow the specimen to come to room temperature before measuring specimen dimensions, visual examination and carrying out tests for whatever properties are to be evaluated.

Referenced Documents
ASTM D 2126-87  Test Method for Response of Rigid Cellular Plastics to Thermal and Humid Ageing.


11.4 Sandwich Constructions
(including core/face adhesive joint)

11.4.1 Tensile tests
Aim and Purpose of Test Methods
This test method covers the determination of the strength in tension, flatwise, of the core, or of the bond between core and facings, of an assembled sandwich panel. The test consists of subjecting a sandwich construction to tensile load normal to the plane of the sandwich, such loads being transmitted to the sandwich through thick loading blocks bonded to the sandwich facings.

Specimen type, Dimensions and Manufacturing
The test specimen shall be square or round and equal in thickness to the thickness of the structural sandwich panel. For continuous cores, such as balsa, foamed rubber and foamed resins having small pores (less than 6 mm), the minimum facing area of the specimen shall be approximately 625 mm². If the core is of an open-celled or gridded type having cells greater than 6 mm and up to 12 mm, the minimum area shall be 250 mm², and if the cells are greater than 12 mm the area shall be at least 250 mm², or large enough to include at least one complete cell. If it is of interest to determine the strength of the bonding between the core and the face on both sides of a sandwich sample, a cone formed specimen (Fig.11.54(b)) or a somewhat more complicated specimen, shown in Fig.11.54(c), can be used. A loading fixture shall be bonded to the facing of the test specimen by any suitable method, which shall not appreciably affect the existing bond between the facing and the core.

![Figure 11.54](image-url)

Figure 11.54 Different tensile test specimens with universal-type joints assembly. (a) show an ordinary ASTM C 297 tensile test specimen while (b) and (c) show specimen modified at KTH Stockholm for testing of the bonding strength in both sides of a sandwich panel.
Apparatus
A testing machine that is capable of control of constant-rate-of-crosshead movement and having a mechanism capable of showing the total tensile load carried by the test specimen is required. The loading fixtures (Fig.11.54) shall be self aligning and shall not apply eccentric loads and the loading blocks shall be sufficiently stiff to keep the bonded facings essentially flat under load. Loading blocks not thinner than the cross-sectional dimension of the specimen have been found to perform satisfactorily.

Testing Procedure
Apply tensile load at a constant rate of movement of the moveable head of the testing machine and record the load. The flatwise tensile strength is then calculated as "maximum load"/"gross cross-sectional area". For open celled or gridded type cores, such as honeycomb where failure occurs in the bond between core and facing, the strength per unit length of adhesive fillets is calculated as "flatwise tensile strength"/"fillet length per unit core area". Fillet length per unit core area can be found by consideration of the core cell geometry; for core with hexagonal or square cells it has been found that fillet length per unit core area equals four divided by the cell size.

Referenced Documents
ASTM C 297-61 Test Method for Tensile Strength of Flat Sandwich Constructions in Flatwise Plane

11.4.2 Out-of-plane shear tests
Aim and Purpose of Test Methods
This test method provides information on the load–deformation behaviour of sandwich constructions or cores when loaded in shear parallel to the plane of the facings. From a complete load–deformation curve it is possible to compute shear stress of the sandwich or core at any load and to compute an effective shear modulus of the sandwich as a unit or shear modulus of the core. The test does not produce pure shear, but the specimen length is prescribed so that secondary stresses have a minimum effect.

Specimen type, Dimensions and Manufacturing
The specimen shall have a thickness equal to the thickness of the sandwich, a width not less than twice the thickness, and a length not less than twelve times the thickness. Support the specimen by means of steel plates bonded to the facings, or directly to the core. If the core or facing materials show directional characteristics with respect to shear strength, separate tests shall be made in such a manner as to develop shearing stresses in each of the principal directions.

Apparatus
The test assembly is mounted with a spherical bearing block, or universal joint (Fig.11.55) in a testing machine that is capable of control of constant-rate-of-crosshead movement and having a mechanism capable of showing the total shear load carried by the test specimen. To measure the deformation, an optical-lever system, dial gage, or any other suitable means is needed.
Testing Procedure
Apply the load to the ends of the rigid plates in tension or compression through spherical bearing blocks or a universal joint so as to distribute the load uniformly across the width of the specimen. The load shall be applied at a constant rate of movement of the movable head of the testing machine. Obtain sufficient readings, preferably at equal strain increments, to define the shape of the stress–strain diagram. The deformations may be read by means of an optical-lever system, dial gage, or any other suitable means. It is suitable to record data for load–deformation curves, which may be used to determine the effective shear modulus of the sandwich. The shear stress of the sandwich specimen, \( \tau \), is calculated as follows;

\[
\tau = \frac{P}{Lw}
\]  

(11.49)

where \( P \) is the load on the specimen, \( L \) is the specimen length, and \( w \) is the width. It should be noted that the effective shear modulus obtained by this method is that of a combination of all the material between the loading plates acting as a unit. The theoretical relationship of this modulus, \( G \), to the modulus of the facings, \( G_f \), and the modulus of the core, \( G_c \), is expressed as

\[
G = \frac{G_f h}{\left[ t_c + \left( h - t_c \right) \frac{G_c}{G_f} \right]}
\]  

(11.50)

where \( t_c \) is the thickness of the core and \( h \) is the total thickness of the specimen. The shear modulus of the sandwich specimen is calculated as follows

\[
G = \frac{\tau}{\gamma} = \frac{Pt}{\Delta Lw}
\]  

(11.51)

where \( \gamma = \Delta/\ell \) is the shear strain and \( \Delta \) is the displacement or movement of one loading plate of the specimen with respect to the other. In most sandwich constructions, \( G_f \) is so large compared with \( G_c \), and \( t \) and \( c \) differ by so little, that \( G \) is essentially the same as \( G_c \).
11.4.3 In-plane shear test

Aim and Purpose of Test Methods
In this section two quite similar test methods to measure the in-plane shear properties, using a quadratic panel type specimen are described. The two different test specimens are schematically shown in Fig.11.56 and the different test set-ups is shown in Fig.11.57.

Specimen type, Dimensions and Manufacturing
The specimen shown in Fig.11.56(a), is manufactured to its shape by wet lay-up, compression moulding or any other suitable method. Tabs shall be bonded to the specimen were the loading frame is to be mounted. The other type of specimen is simply cut from a sandwich panel of the material to be measured. The specimen size is recommended to be about 900 by 900 mm. Stiffeners are bonded to the specimen edges as shown in Fig.11.56(b) and a loading frame is mounted with bolts through the specimen and stiffeners. This test method is suitable while testing large configurations such as specimens cut out from ship panels.

Referenced Documents
ASTM C 273-61 Test Method for Shear Properties in Flatwise Plane of Flat Sandwich Constructions or Sandwich Cores
Figure 11.56  Schematic of sandwich in-plane shear test specimen. The dashed lines in (a) indicates the loading frame.

**Apparatus**

A testing machine that is capable of control of constant-rate-of-crosshead movement and having a mechanism capable of showing the total shear load carried by the test specimen is required. When testing the larger specimens, separate hydraulic cylinders with some suitable load control system may be preferred. Furthermore, testing fixtures with a loading mechanism, as shown in Fig.11.57, and strain gauges, or any other suitable means to measure the deformation, is needed.

Figure 11.57  Schematic of the sandwich in-plane shear tests.
Testing Procedure
Care must be taken to ensure that the loading frames are rigidly bonded to the specimen. The test assembly shown in Fig. 11.57(a) is mounted in the testing machine. The strain can be measured using strain gauges bonded to the specimen. When testing larger specimens with the set-up shown in Fig. 11.57(b), the displacement can be measured by using a dial gauge and the strain by using strain gauges. The load and displacement data should be recorded for post processing. The shear modulus, for a sandwich panel with equal face thickness, is calculated from

\[ P = \int_0^a \tau_c t_c \, dx + 2 \int_0^a \tau_f t_f \, dx \]  \hspace{1cm} (11.52)

Where index \( c \) denotes the core and index \( f \) denotes the face. Because the shear load bearing capacity in the core is only a few percent of the total shear load, this part is not counted for. Then the above expression can be reduced and rewritten as

\[ G = \frac{1}{2t_f a} \frac{dP}{d\gamma} \]  \hspace{1cm} (11.53)

where \( t_f \) is the face thickness, \( a \) is the specimen side length and \( dp/d\gamma \) is measured from the slope in the load strain diagram.

Referenced Documents


11.4.4 Flexure tests
Aim and Purpose of Test Methods
The three-point and four-point bend test methods are used for determination of properties of flat sandwich constructions subjected to flatwise flexure in such a manner that the applied moments produce curvature of the plane of a sheet of the sandwich construction.

Flexure tests on flat sandwich construction may be conducted to determine flexural and shear stiffness of the construction, shear modulus and shear strength of the core, or compressive or tensile strength of the facings. Tests to evaluate core shear strength may also evaluate bonds between core and facings inasmuch as core shear stress values may be lower than actual core shear strength, thus indicating that failure initiated in the bond. By appropriate design of the beam, it is possible to ensure that the beam will fail due to fracture of the core material due to shear. Proper design of test specimen to determining compressive or tensile strength of the facings is obtained by a reverse of considerations for determining core shear strength.

In the three-point bend test, shown in Fig. 11.58, the loads should be applied over some larger surface to prevent local indentation of the faces. When using the four-point bend test the magnitude
of the local load decreases, which gives a lower risk of getting local failure due to indentation of the face sheet. These tests can be used to test the shear strength of the core in sandwich constructions, both in static loading and in fatigue (Section 11.4.10).

Figure 11.58 Three-point bending test.

**Specimen type, Dimensions and Manufacturing**

Sandwich specimens of any configuration may be used; having cellular foam core, honeycomb or balsa cores, with metal or composite faces of equal or dissimilar thickness. The faces should, however, be thin. The specimen is a flat sandwich beam with dimensions according to the test rig. The width of the beam should be at least the same as the total height of the beam, preferably wider. In order to get a constant shear stress field in the core, as thin faces as possible should be used. However, they must be thick enough so that the face stress is so low that face failure is prevented. Thin faces are usually defined by the relation

\[
\frac{t_c + t_f}{t_f} \geq 5.7
\]  

(11.54)

The specimens can either be manufactured for the testing purpose only using any available method or taken directly from the manufacturing. In either case it is important that the bonding between the faces and the core is good enough to prevent the specimen from premature failure in the interface.

**Apparatus**

Any form of deflection testing machine capable of operation at a constant rate of motion of the movable head is required. In the three-point bending rig, the loading nose and supports shall have cylindrical surfaces. In order to avoid excessive indentation, or failure due to stress concentration directly under the loading nose, the radius of the nose and the supports shall be at least 3 mm and up to 1.5 times the specimen width. An example of a four-point bending test rig is shown schematically in Fig.11.59. Through the design the load supports are allowed to rotate around the neutral axis of the beam by means of ball bearings. This is done in order to minimise the stress concentrations near the load introductions. Furthermore, the supports are movable in the direction of the beam (horizontal direction) to enable various settings of \(L_1\) and \(L_2\). The supports are covered
with rubber pads in order to smooth out the load transfer. Furthermore, the outer load arms are allowed to move horizontally thus preventing any membrane forces to build-up.

![Diagram of four-point bending test assembly](image)

**Figure 11.59** Four-point bending test assembly.

**Testing Procedure**

The test rig is mounted in a static testing machine and the test specimen is placed in the rig. Static tests may be performed at any rate, but for quasi-static testing a machine cross-head displacement of 1-10 mm/s is suitable. The tests are monitored using both data acquisition and visual inspection. The maximum bending moment and transverse force appearing in the beam during a three-point bending test are

\[
M_{\text{max}} = \frac{PL}{4} \quad \text{and} \quad T_{\text{max}} = \frac{P}{2} 
\]

(11.55)

The deflection under the point load is

\[
w = \frac{PL^3}{48D} + \frac{PL}{4S} 
\]

(11.56)

From this equation the bending stiffness, \(D\), and the shear stiffness, \(S\), can be determined by testing two identical beams at two different spans. In the region between the inner and outer supports in a four-point bend test, the transverse force is constant, which in turn means that the shear stress in the core is constant over a fairly long part of the beam. Between the inner supports, the bending moment is constant while in the same zone the transverse force is identically equal to zero. The deformation of the load points, assuming thin faces, is equal to the crosshead displacement of the testing machine and is given by

\[
w\left(\frac{L_2 - L_1}{2}\right) = \frac{P(L_2 - L_1)^2}{24D}\left(L_2 + 2L_1\right) + \frac{P(L_2 - L_1)}{2S} 
\]

(11.57)

The maximum deflection in the middle of the beam can similarly be written
\[ w \left( \frac{L_2}{2} \right) = \frac{P(L_2 - L_1)(2L_2^2 + 2L_1L_2 - L_1^2)}{24D} + \frac{P(L_2 - L_1)}{2S} \] (11.58)

The maximum direct stress in the faces appear between the inner supports and the maximum transverse shear stress appear between the outer and inner support and can, in the case of thin equal faces, be written as

\[ \sigma_f = \pm \frac{M}{t_f d} = \pm \frac{P(L_2 - L_1)}{2t_f d} \quad \text{and} \quad \tau_c = \frac{P}{d} \] (11.59)

Though the transverse forces are zero and the bending moment is constant in the mid-section, the bending stiffness can easily be measured in this section since the curvature derives only from the bending of the beam. A dial gauge is mounted in a rig which is placed on top of the four-point bend specimen, to measure the displacement as a function of the applied load, in the region between the inner supports (Fig. 11.60). Since the bending moment is constant the bending stiffness is given by

\[ w = \frac{Me^2}{8D} = \frac{P(L_2 - L_1)c^2}{16D} \] (11.60)

where \( w \) is the displacement measured by the dial gauge and \( c \) is the distance between the supports of the rig as shown in Fig.11.60.

![Diagram](image)

Figure 11.60 Measurement of the bending stiffness in the four-point bending test specimen.

**Referenced Documents**

ASTM C 393-62 Test Method for Flexural Properties of Flat Sandwich Constructions

M. Burman and D. Zenkert, "Fatigue of Foam Core Part 1: Undamaged Specimens", Accepted for publication in *International Journal of Fatigue*.

11.4.5 Impact tests on sandwich panels

Aim and Purpose of Test Methods
Impact tests are performed to measure the impact resistance of a sandwich construction. The impact event involves relatively high contact forces acting on a small area over a period of small duration. The composite generally absorb energy through fracture mechanisms such as delaminations, matrix and fibre cracks and it is, in several investigations, described how the delaminations and fibre breakage can reduce the strength. The impact tests are therefore often completed with compression after impact tests and/or in-plane shear tests, to determine the residual strength of the panel.

Specimen type, Dimensions and Manufacturing
Specimens in form of quadratic panels, e.g. of the type shown in Fig.11.56(a), which may then be tested in shear to measure the residual shear strength, or as rectangular or quadratic specimens, cut from sandwich panels and which then can be tested in compression after impact.

Apparatus
The impact tests are conducted in a drop weight impact tester (of the type schematically shown in Fig.11.17) with an anti-rebound device to prevent multiple impact. The indentor shall consist of a steel hemispherical tup and the applied impact energy must be controllable. An impact support fixture made of a steel plate with a cut-out and guiding pins, or equivalent to position the specimen centrally on the cut-out, is required. The specimen should be held in position by rubber-tipped clamps or equivalent devices. In order to measure the impact force the impactor can be provided with a contact force transducer and the impactor velocity at the moment of contact beginning can be measured using a photo-electric sensor.

Testing Procedure
The specimen is carefully clamped at all edges and the impactor is dropped and a impact force versus time curve can be recorded using the equipment described above. Hence, the contact force, \( F(t) \), is measured by a load cell, mounted on the impactor, the impactor acceleration is obtained as

\[
a(t) = \frac{F(t) - P}{m}
\]

where \( F \) is the contact force, \( m \) is the impactor mass and \( P \) is the weight of the falling impactor. By numerical integration, the velocity during impact is determined as

\[
v(t) = v_0 - \int_0^t \frac{F(t) - P}{m} \, dt
\]

where \( v_0 \) is the velocity at the contact beginning \((t=0)\), to which correspond the impactor kinetic energy \( u_0 \). This is the velocity measured by the photo-electric sensor. By further integration, the displacement of the impactor during the impact is determined as

\[
x(t) = x_0 + \int_0^t \left[ v_0 - \int_0^t \frac{F(t) - P}{m} \, dt \right] \, dt
\]

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where $x_o=0$. The energy exchanged at time $t$, due to work done by the contact force during the loading and unloading time, is defined as

$$e(t) = \int_0^t F(t) \left[ v(t) - \int_0^t \frac{F(t) - P}{m} dt \right] dt$$

At time $t=t^*$, when the contact ends (i.e., the impactor leaves the plate after the rebound or passes through the laminate), the above defined energy results equal to $e^*$ and because no more energy exchange occurs this value remains constant for $t>t^*$. Thus, in case of perfectly elastic rebound,

$$u_o = e_{\text{max}} \quad e_{\text{max}} > e^* = 0$$

otherwise, when a damage occurs, a part of the energy transferred to the laminate is spent in damage phenomena.

**Referenced Documents**


**11.4.6 Compression and buckling tests**

**Undamaged Panel**

**Aim and Purpose of Test Methods**

The edgewise compressive strength of short specimens of sandwich construction provides a basis for judging the load-carrying capacity of the construction in terms of developed facing stresses as compared to the yield stress of the facings. The sandwich column, no matter how short, usually is subjected to a buckling type of failure unless the facings are so short that they themselves are in the short column class. The failure of the facings manifests itself by wrinkling of the facing, in which case the core deforms to the wavy shape of the facings; by dimpling of the facings into the cells of honeycomb or gridded type cores; or by bending of the sandwich, resulting in crimping near the ends due to shear failure of the core or perhaps failure in the facing-to-core bond.

**Specimen type, Dimensions and Manufacturing**

The test specimens shall be rectangular in cross section and the width of the specimens shall be at least 50 mm but not less than twice the total thickness, nor less than two complete cells for large-gridded type cores. The unsupported length (parallel to the direction of applied load) shall be not greater than twelve times the total thickness. Take care in preparing the test specimens to ensure smooth end surfaces and that the ends are parallel to each other and at right angles to the length of the specimens. The test specimens shall be laterally supported adjacent to the loaded ends on the facings of the sandwich to prevent early buckling failure due to separation of the facings from the core at the point of contact with the loading plates. This may be done by fitting the specimen snugly in round steel bars slotted axially to their diameter (Fig.11.61), where such bars shall have a diameter not less than the thickness of the sandwich plus 6 mm, or using clamps made of rectangular steel bars fastened together so as to clamp the specimen lightly between them. End
support may also be obtained by casting the ends of the specimens in a resin or other suitable moulding material. The bearing ends of the facings should be ground flush with the hardened moulding material.

![Figure 11.61 Schematic of a sandwich compression test assembly.](image)

**Apparatus**
The test assembly is mounted with a spherical bearing block, of the suspended, self-aligning type, in a universal testing machine that is capable of control of constant-rate-of-crosshead movement and having a mechanism capable of showing the total compression load carried by the test specimen. To measure the strain, strain gauges, having a gauge length not greater than two thirds of the unsupported length of the specimens to be tested, shall be used.

**Testing Procedure**
Apply the load through a suitable apparatus designed to distribute the load properly to each facing and uniformly in each facing (e.g. as schematically shown in Figure 4.6.1). The load may be considered to be distributed properly if the strains measured on each facing are within 5% of each other in the early stages of loading. Apply the load through a constant rate of movement of the moveable head and measure the strain with the strain gauge. Be careful to observe and describe the type of failure, i.e., whether the facing wrinkled, dimpled or crimped; whether the specimen was bent before failure occurred and whether the core and bond failed. The compression modulus is calculated as

$$E = \frac{\Delta P}{A \Delta \varepsilon}$$  \hspace{1cm} (11.65)

where $\Delta P$ and $\Delta \varepsilon$ are the changes in load and strain, respectively, between two fixed points on the load-strain curve, and $A$ is the nominal cross-sectional area of the sum of the two faces.

**Referenced Documents**
ASTM C 364-61 Test Method for Edgewise Compressive Strength of Flat Sandwich Constructions
CompressIon After Impact (CAI)

Aim and Purpose of Test Methods

Weight and cost reduction of sandwich structures would be achieved if components of this type could be designed with damage tolerance considered. The test method described here tests sandwich constructions, with an artificial impact and debonding damage, in compression. One of the most critical loading constraints for sandwich panels is in compression, particularly when defects and damage are present. The specimen and testing geometry referred to here is based upon procedures described in the Boeing Specification Support Standard BSS 7260 and ACOTEG/TP10 which are valid for compression after impact testing of FRP laminates.

Specimen type, Dimensions and Manufacturing

The specimen used in the method referred to here uses a specimen 150 by 100 mm side lengths, but there are other similar test methods that uses larger specimen configurations. The specimen is impacted, in accordance to the method described in section 11.4.6, and the specimen ends shall be carefully machined parallel. Strain gauges shall be bonded to both sides of the specimen in a way so that the alignment of the specimen can be checked when loaded.

Apparatus

The testing equipment is similar to the described above in the compression test section. A suitable test fixture is shown in Fig.11.62.

Testing Procedure

The specimen shall be carefully aligned in the compression test fixture ensuring that the specimen is held perpendicular to the base plate. Prior to testing the specimen is initially loaded to ensure that the applied load is uniform. If the strain across the width or thickness vary by more than ±10% the specimen should be unloaded and checked for alignment. When the strains are uniform the loading continues until failure. The failure point is defined as the first significant drop in applied
load. During the test the strains and displacement, respectively, versus applied load are recorded. The ultimate compression strain is calculated as the summation of the ultimate compression strain from each strain gauge divided by the number of gauges. The ultimate compression strength is given by the ultimate compression load divided by the nominal cross-sectional area of the skins.

**Referenced Documents**

**11.4.7 Rectangular panel subjected to uniform pressure**

**Aim and Purpose of Test Methods**
The sandwich panel test is used for determining the strength or stiffness of entire sandwich panels under a uniformly distributed transverse static load. The panel is simply supported along all edges. The test rig is designed to distribute the load evenly over the surface of the panel independent of the panel's deflection. The sandwich can be designed to different purposes. The stiffness of the panel can be measured if the sandwich is designed to withstand the applied loads, the failure load can be measured if the sandwich is designed to break under the load applied during testing.

**Specimen Type, Dimensions and Manufacturing**
Sandwich specimens of any configuration may be used; having cellular foam core, honeycomb or balsa cores, with metal or composites face of equal or dissimilar thickness. The test rig described here is designed to accept flat, square sandwich panels with side length 850 mm. The distance between the supports is 800 mm. Rectangular panels with the longest side 850 mm can be tested using an auxiliary support in the test rig. In this case the panel should be 50 mm wider than the distance between the supports. Other dimensions may of course be used with another rig. The sandwich should be designed so that compressive failure of the lower face is avoided since this could damage the rubber bladder.

**Apparatus**
The plate bending rig consists of a lower frame, an upper support frame and a water filled rubber bladder, with dimensions according to Fig.11.63 and 11.64. The rubber bladder is made out of 2 mm EPDM Ethylene Propylene rubber and has a sealed filler hole for water on the underside. The supports on the support frame are made out of split cylindrical rods which are bolted on to the frame.


**Testing Procedure**

The rubber bladder should be filled with some specified amount of water during the testing. The lower frame and the bladder is placed on the lower table of a compression testing machine and the support frame is attached to the upper table of the machine. The test panel is placed on top of the bladder. Pressure is applied by raising the lower table with the panel until the desired load is reached.

A number of different parameters can be measured, e.g. transverse deflection, tensile strain or shear strain. The upper table provides a solid reference to which dial gauges can be attached using magnetic bases (Fig.11.65). The test can be carried out with the dial gauges in different positions on the top surface of the panel. The shear strain in the core can be measured using shear strain gauges built into the sandwich and ordinary strain gauges can be attached to the top surface of the sandwich (Fig.11.66). Wiring of gauges and sensors is done through holes in the sides of the support frame.

When testing panels with imperfections (e.g. impact damages) in the lower face the bladder can be protected from sharp edges or fibres using a plastic film or a rubber sheet.
11.4.8 Fracture toughness tests

In the interface between two different materials in a multi-material body, the singularity at the crack-tip differs from that in a homogenous medium. The stress intensification around the crack-tip arises both from material and geometrical discontinuities, whereas in a homogenous body stress intensification arise only from geometrical discontinuities. As a result of this, a single mode loading induces both an opening mode ($K_I$) and a shearing mode ($K_{II}$).

**Mode I (DCB)**

**Aim and Purpose of Test Methods**

This method describes the determination of the opening mode (Mode I) interlaminar fracture toughness, $K_{IC}$, and critical energy release rate, $G_{IC}$, of a sandwich construction with an interface disbond between the core and the face, using a modified Double Cantilever Beam (DCB) specimen. This test method may be proved useful for different types and classes of sandwich constructions, however, great care should be taken during the tests and the report should include thorough description of the constituents of the specimen and the specimen behaviour during the test.

**Specimen Type, Dimensions and Manufacturing**
This modified DCB specimen consists of a rectangular, uniform thickness piece cut from a sandwich panel, with only one face, containing a nonadhesive insert between the core and the face laminate which serves as a delamination indicator. The nonadhesive insert, e.g. a 0.025 mm Teflon film, shall be inserted at the interface between the core and the face during lay-up, or bonding, of the face onto the core, to form an initiation site for the delamination. Opening forces are applied to the DCB specimen by means of hinges or loading blocks bonded to one end of the specimen (Fig.11.67).

![Diagram of modified DCB specimen with hinges](image)

Figure 11.67 The modified DCB specimen with hinges. $a$ is the delamination length.

**Apparatus**

The testing machine shall have the capability to control and regulate the velocity of the movable head and the load sensing device of the testing machine shall be capable of indicating the total load being carried by the test specimen. To apply the load to the specimen the testing machine shall be equipped with grips to hold the loading hinges, or pins to hold the loading blocks, that are bonded to the specimen. To estimate the opening displacement either the crosshead separation, or an external properly calibrated gage or transducer attached to the specimen, is used. Load and opening displacement data shall be recorded and stored and then post processed, or directly plotted in a X-Y plotter, or similar device. A travelling optical microscope shall be used to observe the delamination front as it extends along one edge during the test. This device shall be capable of pinpointing the delamination front with an accuracy of at least ±0.5 mm.

**Testing Procedure**

Mount the load blocks or hinges on the specimen in the grips of the loading machine, making sure the specimen is aligned and centred. Set the optical microscope (or an equivalent magnifying device) in a position to observe the motion of the delamination front and measure the delamination length on one side of the specimen. To be able to register the length of the crack, the control program of the testing machine should be such that the test can be paused, the crack length registered and then the test is continued and the load versus opening displacement is plotted. Mark the location of the delamination front on the plot of load versus opening displacement as the delamination grows. It may be difficult to register the crack length at specified intervals, though
the crack often tend to grow in steps. With a simple analysis based on a beam on an elastic foundation, the compliance, \( \frac{d}{P} \), will be a third degree polynomial of the crack length \( a \).

\[
\frac{d}{P} = c_0 + c_1 a + c_2 a^2 + c_3 a^3
\]  

(11.66)

where \( d \) is the displacement, \( P \) the applied force and \( c_{0-3} \) are functions of geometry and elastic stiffness. In the following a third degree polynomial can be fit to the obtained experimental data, i.e. the coefficients \( c_{0-3} \) are determined with a least squares fit. The critical energy release rate can be determined as

\[
G_c = \frac{P_c^2}{2w} \frac{\partial (d/P)}{\partial a}
\]

(11.67)

Where \( P_C \) is the critical load \( a \) is the delamination length and \( w \) is the width of the specimen. The energy release rate is not pure mode I due to that the crack propagates in a non-homogeneous area between two different materials. The interlaminar fracture toughness, \( K_C \), can then be evaluated from the relationship

\[
G_c = \frac{K_C^2 (1 - v^2)}{E}
\]

(11.68)

in plane strain.

**Referenced Documents**


**Mode II: The Cracked Sandwich Beam (CSB)**

**Aim and Purpose of Test Methods**

In the interface between two different materials in a multi-material body, the singularity at the crack-tip differs from that in a homogenous medium. The stress intensification around the crack-tip arises both from material and geometrical discontinuities, whereas in a homogenous body stress intensification arise only from geometrical discontinuities. As a result of this, a single mode loading induces both an opening mode \( (K_I) \) and a shearing mode \( (K_{II}) \). A specimen proposed for the measuring of the toughness of interfacial cracks is the cracked sandwich beam (CSB) specimen. This specimen may well be used for all combinations of materials as long as the stress intensity factor can be accurately calculated.

For the CSB specimen, as opposed to the ENF specimen (described in chapter 11.3.6), the crack is forced to propagate along the interface and not into the core material. Thus, if the specimen is turned upside down, the crack will grow upwards, i.e. into the core material in a similar way to the ENF specimen. The measured fracture toughness is then totally different and similar to that of the ENF specimen. The advantage is that this specimen may be used with honeycomb cores.
Specimen type, Dimensions and Manufacturing

The specimen is manufactured in a way similar to the ENF specimen. A thin Teflon film is placed onto the core to prevent bonding in the area of the simulated crack. Thereafter, the faces are attached to the core by adhesive bonding or by laminating directly onto the core. The specimen is machined to the right dimensions including a bit of overhang needed for the testing. It is recommended that the thickness should equal the height of the specimen and the crack length is measured from the support to the crack front as shown in Fig. 11.68.

Apparatus

A properly calibrated testing machine that can be operated at constant rates of crosshead motion over the range indicated shall be used. It shall be equipped with a deflection-measuring device and a load measuring system with good accuracy. The loading nose and supports in the three-point bending rig shall have cylindrical surfaces. In order to avoid excessive indentation, or failure due to stress concentration directly under the loading nose, the radius of the nose and the supports shall be at least 3 mm and up to 1.5 times the specimen width.

Testing Procedure

The specimen is mounted with a short overhang on each side in a three point bending rig and tested under a constant rate of deflection until fracture. The stress intensity is almost pure mode II and is expressed just as for the ENF specimen in chapter 11.3.6. There is another, analytical, way to calculate the stress intensity. In this method, the derivation of the energy release rate, $G_{II}$, is based on the rate of change in beam compliance with crack extension. The compliance of the CSB specimen is defined as the displacement, $\delta$, at the central loading pin divided by the applied load, $P$. The beam displacement will be derived based on laminated beam theory including shear deformation. The starting point of the analysis is the plane stress laminated plate constitutive relations

$$
\begin{pmatrix}
N \\
M
\end{pmatrix}
= 
\begin{bmatrix}
A & B \\
B & D
\end{bmatrix}
\begin{pmatrix}
\varepsilon_0 \\
\kappa
\end{pmatrix}
$$

(11.69)

where $[N]$ and $[M]$ are the stress and moment resultants, $[A]$, $[B]$ and $[D]$ are the extensional-, coupling- and bending stiffness matrices, respectively, and $[\varepsilon_0]$ and $[\kappa]$ are the mid-plane strains and curvatures, respectively. Inversion of Equation (11.69) yields,

$$
\begin{pmatrix}
\varepsilon_0 \\
\kappa
\end{pmatrix}
= 
\begin{bmatrix}
A' & B' \\
B' & D'
\end{bmatrix}
\begin{pmatrix}
N \\
M
\end{pmatrix}
$$

(11.70)
where \([A^\prime], [B^\prime], [H^\prime]\) and \([D^\prime]\) are defined in [R. M. Jones]. For the sandwich beam a beam theory formulation is developed and \([D^\prime]\) are defined in [R. M. Jones]. For the sandwich beam a beam theory formulation is developed by assuming \([N]=0\) and \(M_y=M_{xy}=0\).

![Figure 11.69 The geometry and loading of the Cracked Sandwich Beam.](image)

In this analysis the sections BC and CD of the beam (Fig.11.69) are assumed to be identical in properties and of symmetric and balanced lay-up. The solution of each region of the beam is achieved by combining equations containing the midplane curvature, the bending stiffness and the moment-curvature relation of the beam. The energy release rate, \(G_{II}\), is obtained by differentiation of the beam compliance,

\[
C = \frac{l^3 (D^\prime_{11})_{BC}}{6b} + \frac{l (A^\ast_{55})_{BC}}{2bk} + \frac{a^3 [(D^\prime_{11})_{AB} - (D^\prime_{11})_{BC}]}{12b} + \frac{a [(A^\ast_{55})_{AB} - (A^\ast_{55})_{BC}]}{4bk} \tag{11.71}
\]

with respect to crack length times \(P^2/2b\)

\[
G_{II} = \frac{P^2}{2b} \frac{dC}{da} = \frac{P^2}{8b^2} \left[ a^2 [(D^\ast_{11})_{AB} - (D^\ast_{11})_{BC}] + \frac{a [(A^\ast_{55})_{AB} - (A^\ast_{55})_{BC}]}{k} \right] \tag{11.72}
\]

\((D^\ast_{11})\) and \((A^\ast_{55})\) are the flexural and shear compliances, respectively, for the different parts of the beam, \(b\) and \(l\) is the width and the length of the beam, respectively and \(k\) is a shear correction factor introduced by Reissner and Mindlin [L. A. Carlsson]. Examination of this equation yields that the energy release rate is increasing with increased difference in flexural rigidity between the cracked and uncracked regions, and the shear deformation contribution is proportional to the difference in shear stiffness between the cracked and uncracked regions.

![Figure 11.70 The load transformation in the cracked part of the specimen.](image)
In region AB the load must be transferred from the lower part of the beam to the upper face sheet through compressive surface tractions over the crack interface. This load transfer is approximated by application of two concentrated loads \( P_1 \) and \( P_2 \) applied at and above the load support as illustrated in Fig.11.70. Static equilibrium requires,

\[
P_1 + P_2 = P/2
\]

From this equation and by compatibility of the ends displacements shear deformation theory applied on each beam (upper and lower part) in Fig.11.70 yields the following load-shearing relation,

\[
2P_2 = \frac{a^3(D^*_1)_{1} + d(A^*_5)_{1}}{3} - \frac{3}{k} \left[ \frac{a^3(D^*_1)_{2} + d(A^*_5)_{2}}{3} \right]
\]

The effective bending and shear coefficients for region AB, \( (D^*_1)_{AB} \) and \( (A^*_5)_{AB} \), may be obtained by application of \( P/2 \) to a beam of length \( a \), with the effective bending and shear properties and by setting the end displacements equal to that produced by \( P_1 \) acting on the upper part of the beam (top face), or equivalently \( P_2 \) acting on the lower part of the beam (core and lower face), in Fig.11.70. This analysis yields

\[
(D^*_{11})_{AB} = \frac{2P}{P} (D^*_1)_{2}
\]

\[
(A^*_{55})_{AB} = \frac{2P}{P} (A^*_5)_{2}
\]

This set of equations define a solution to the CSB geometry. The reader is referred to [L. A. Carlsson] for further details.

**Referenced Documents**


**11.4.9 Fatigue tests**

**Core Shear**

Aim and Purpose of Test Methods

The four-point bend (section 11.4.4) test can also be used to test the shear strength of the core in sandwich constructions in fatigue. The four-point bending beam offers special features for this particular purpose. Firstly, the local load intensity from the supports are lower than in the similar three-point bend test thus reducing the risk of local indentation failure of the beam. Secondly, there are no stress concentration in the core, such as those in the block shear, e.g., ASTM C394-62,
which may cause premature failure. Most importantly, in the region between the outer and inner support there is a region with constant transverse force giving a fairly large part of the core material subjected to an almost constant shear stress. By appropriate design of the beam, as given herein, it is possible to ensure that the beam will fail due to fracture of the core material due to shear, both for static and fatigue loading. The region between the inner supports is well suited for membrane tension or compression strength tests of the faces. It is also possible to design the beam to get an almost constant core shear stress (by using thin and stiff faces) and yet having a low enough face stress in the middle section to safely avoid face compressive, tensile or local buckling failure and hence get core failure in shear between the inner and outer supports. The four-point rig is schematically shown in Fig.11.71.

**Specimen type, Dimensions and Manufacturing**

Sandwich specimens of any configuration may be used; having cellular foam core, honeycomb or balsa cores, with metal or composite faces of equal or dissimilar thickness. The faces must, however, be thin. The specimen is a flat sandwich beam with dimensions according to the test rig. The width of the beam should be at least the same as the total height of the beam, preferably wider. In order to get a constant shear stress field in the core, as thin faces as possible should be used. However, they must be thick enough so that the face stress is so low that face failure is prevented. Thin faces are usually defined by the relation

\[
\frac{t_f + t_c}{t_f} \geq 5.7
\]  

(11.77)

The specimens can either be manufactured for the testing purpose only using any available method or taken directly from the manufacturing. In either case it is important that the bonding between the faces and the core is good enough to prevent the specimen from premature failure in the interface.

![Figure 11.71 Four-point bending rig.](image-url)
Apparatus

The test rig is shown schematically in Fig. 11.71. Through the design the load supports are allowed to rotate around the neutral axis of the beam by means of ball bearings. This is done in order to minimise the stress concentrations near the load introductions. Furthermore, the supports are movable in the direction of the beam (horizontal direction) to enable various settings of $L_1$ and $L_2$. The supports are covered with rubber pads to smooth out the load transfer. Furthermore, the outer load arms are allowed to move horizontally thus preventing any membrane forces to occur.

Testing Procedure

The test rig is mounted in a hydraulic fatigue testing machine according to Fig. 11.71. Place the test specimen in the rig and tighten the adjusting screws. Specimens with very thin faces require more caution when tightening since a too high torque induces stresses in the core which may cause premature failure. If the adjustment torque is too loose there will be gap and problems will occur during testing. Testing in fatigue also require caution in choosing the right testing frequency. Frequencies around 1-3 Hz have been found to be suitable for foam cores and honeycombs. Too high testing frequencies will raise the temperature in the core due to hysterisis which will influence the fatigue life. Lower $R$-values and higher relative loads also require lower test frequencies. Based on tests with foam core materials it has been found suitable to choose a shear strain rate of approximately 10%/s in order to get good reproducible results. The frequency, $f$, may then be adjusted according to

$$\frac{dy}{dt} = \Delta \gamma \cdot f = \frac{P}{G_c (t_f + t_c)} (1 - R) \cdot f$$

(11.78)

where $P$ is the applied load, $G_c$ is the shear modulus of the core, $t_f$ is the face thickness and $t_c$ is the core thickness. The fatigue tests are monitored using both data acquisition and visual inspection. The stiffness is continuously monitored and any growth of the damages is checked visually at regular intervals. There could be problems of load or displacement control of the test since quite low load levels are required. A way to solve this is to conduct the tests by a so called cyclic indirect method. This means that the test is run in displacement control but the response from the load cell is continuously registered and after a specified number of cycles an average of the load deviation is fed back to the displacement control which is then adjusted to give the desired load level.

There are several fatigue failure criteria that can be used for this test; when the damage is so extensive that cracks run from one face to the other, first visual crack formation or when the stiffness has degraded a to a certain level. It is likely that all these will give similar results since the initiation of failure in fatigue takes the major part of the total life. After first visual damage formation, the growth is usually rather rapid. The face and core stresses is calculated according to

$$\sigma_f = \pm \frac{M}{t_f d} = \pm \frac{P(L_2 - L_1)}{2t_f d}$$

(11.79)

and

$$\tau_c = \frac{P}{d}$$

(11.80)
respectively, which is directly related to the applied load. The core stress, $\tau_c$, is then accompanied with the number of cycles to failure for the specific test specimen and is then presented in an $S/N$-diagram, or similar.

**Referenced Documents**

ASTM C 393-62  Test Method for Flexural Properties of Flat Sandwich Constructions

M. Burman and D. Zenkert, "Fatigue of Foam Core Part 1: Undamaged Specimens”, Accepted for publication, International Journal of Fatigue.

M. Burman and D. Zenkert, "Fatigue of Foam Core Part 2: Effect of Initial Damages”, Accepted for publication, International Journal of Fatigue.

**11.4.10 Other tests**

**Flexural-Creep Test**

**Aim and Purpose of Test Methods.**

The determination of creep rate provides information on the load behaviour of sandwich constructions under constant load. The test method covers the determination of the creep characteristics and creep rate of sandwich constructions loaded in flexure. Deflection data obtained from this test method can be plotted against time, and a creep rate determined. By using standard specimens and constant loading, the test method may also be used to evaluate creep behaviour of adhesives for use in sandwich structures.

**Specimen type, Dimensions and Manufacturing.**

The test specimen shall be rectangular in cross-section with the depth of the specimen equal to the thickness of the sandwich constriction, and the width shall be not less than twice the total thickness, not less than three times the dimension of a core cell, nor greater than one half the span length. The specimen length shall be equal to the span length plus 5 cm or plus one half of the sandwich thickness whichever is the greater.

![Figure 11.72 Schematic of flexural creep test set-up.](image)

**Apparatus.**

The load shall be applied, through a four-point bending rig with steel bars or pipes having a diameter not less than one half of the core thickness and not greater than one and a half of the sandwich thickness, by means of a weight (Fig.11.72).
Testing Procedure.
Attach the weight tray to the lever arm and support it temporarily so that no load is applied to the specimen. Remove the temporary support and apply the load slowly. Measure the deflections by means of any suitable equipment. Read the initial deflection and record it and then take deflection readings at sufficient time intervals to define completely a creep curve with the deflection plotted as the ordinate and time as abscissa.

Referenced Documents
ASTM C 480-62 Test Method for Flexural-Creep of Sandwich Constructions

Ageing Test
Aim and Purpose of Test Methods.
This test method covers the determination of the resistance of sandwich panels to severe exposure conditions as measured by the change in selected properties of the material after exposure. The exposure cycle to which the specimen is subjected is an arbitrary test having no correlation with natural weathering conditions.

Testing Procedure.
Before the ageing procedure, tests for selected properties are made on specimens of the investigated material. After exposure to some specified ageing conditions, the same tests for selected properties are made and then the results are compared. In the ageing procedure the test specimen will be subjected to a specified number of complete cycles of laboratory ageing. An example of a laboratory ageing cycle is: Totally immerse the specimen horizontally in water at about 50 °C for 1 hour, spray the specimen with steam and water vapour at approximately 93 °C for 3 hours, store the specimen at -12 °C for 20 hours, heat the specimen at about 100 °C in dry air for three hours, spray again with steam and water vapour at approximately 93 °C for 3 hours and then, finally, heat the specimen at about 100 °C in dry air for 18 hours. The material shall be frequently inspected during the ageing cycles for any signs of delamination or other disintegrations.

Referenced Documents
ASTM C 481-62 Test Method for Laboratory Ageing of Sandwich Constructions

Sandwich Peel Test
Aim and Purpose of Test Methods.
These test methods are intended for determining the comparative peel resistance of adhesive bonds between facings and cores of sandwich constructions, when tested under specified test conditions. One method, the climbing drum peel method, is most applicable when the facing being peeled are relatively thin. The peeling torque calculated from this test includes both the forces required to peel the adhesive bond, and to bend the facing. The other methods are not dependent of specimen having very thin faces. One of them is very similar to the DCB mode I fracture toughness test and the other test uses air pressure to delaminate the face from the core.
Climbing Drum Peel Test

Specimen type, Dimensions and Manufacturing.
The climbing drum peel test specimen shall conform to the general form of the specimen shown in Fig.11.73(a). Recommended dimensions of the specimen is 76 mm wide by at least 305 mm long, including about a 25 mm overhang of one facing at each end. The thickness of the core is not important, except in the sense that the sandwich specimen shall not bend while the facing is being peeled. For sandwich constructions based on orthotropic cores (e.g. honeycomb cores), specimens shall be peeled both in the length direction and the width direction, as determined from the configuration of the cell structure.

Apparatus.
The peeling apparatus consists of a flanged drum, flexible loading straps or cables, and suitable clamps for holding the test specimen (Fig.11.73(a)). A testing machine, capable of applying tensile load at constant rate of crosshead movement is also required, as well as some data acquisition equipment to record load versus crosshead movement.

Testing Procedure.
When performing the climbing drum peel test, the face in the lower end of the specimen is securely clamped to the drum and the other face, in the upper end, is attached in the top clamp, and the loading bar to the moveable head of the testing machine (Fig.11.73(a)). Apply a tensile load, at a constant crosshead rate, to the test assembly and determine the peel resistance over at least 150 mm of the bond. It is preferred that some autographic recording of the load versus crosshead movement or load versus distance peeled be made during the test. Determine the average peeling load from the autographic curve. Calculate the average peel torque as

$$ T = \frac{(r_o - r_i)(F_p - F_o)}{W} $$

(11.81)

where $r_o$ is the radius of the flange (including one half of the thickness of the loading straps), $r_i$ is the radius of the drum plus one half of the thickness of the facing being peeled, $F_p - F_o$ is the average load required to bend and peel the facing, $F_o$ is the load required to overcome the resisting torque (calculated as a correction factor), and $W$ is the width of the specimen.

DCB-Type Peel Test

Specimen type, Dimensions and Manufacturing.
The DCB-type specimen is shown in Fig.11.73(b). The core of the specimen is milled down, leaving an overhang of the face where the load is applied. Also in this method, the thickness of the core is not important, though the specimen is bonded to a stiff steel plate to prevent the sandwich specimen to bend while the facing is being peeled. For sandwich constructions based on orthotropic cores (e.g. honeycomb cores), specimens shall be peeled both in the length direction and the width direction, as determined from the configuration of the cell structure.

Apparatus.
A testing machine with the same capabilities as described above, and some suitable arrangement for load introduction (e.g. hinges with a pulling rod mounted in a grip in the testing machine) is required.
Testing Procedure.
This peel test is performed similar to the DCB fracture toughness, described in chapter 11.4.8. The test set-up is schematically shown in Fig.11.73(b). The load-displacement is measured and integrated to give a value of the energy needed to peel off the face sheet.

![Diagram of peel test set-up](image)

**Figure 11.73** Schematics of different peel tests: (a) shows a climbing drum peel test assembly, (b) shows a “DCB-type” peel test and (c) shows a peel test that uses air pressure to peel off the face sheet rapidly.

**Peel Test Using Air Pressure**
**Specimen type, Dimensions and Manufacturing.**
The specimen shown in Fig.11.73(c) is used for peel testing with air pressure. It is a panel with a circular initial delamination in the middle. A thin, circular sheet of Teflon film is placed on the core to create the initial delamination and a pipe, to which the air pressure is applied, is mounted...
through one face and the core and connected to the disbond between the core and the opposite face.

**Apparatus.**
This test method requires an air pressure system with a reduction valve to control the applied pressure.

**Testing Procedure.**
This test method can be used to find out what happens when a thin face sheet is peeled off rapidly. The test set-up is schematically shown in Fig.11.73(c). Air pressure is applied to the unbounded area through a hose connected to the pipe mounted in the specimen. The applied pressure is controlled by the reduction valve and the pressure when the face sheet begins to peel off, or is "blown off" in an unstable manner, is recorded. This critical value of the pressure can then be used to estimate the energy required to peel off the face.

**Referenced Documents**
ASTM D 1781-93  Test Method for Climbing Drum Peel for Adhesives


**Tests of Thermal Transmission Properties**
**Aim and Purpose of Test Methods**
The test method described here is known as the guarded hot box method and it covers the measurement of the steady-state thermal transfer properties of panels. This test method is primarily designed for the temperatures encountered in normal building use, however, it is recognised that the method may find application in conditions that are outside this normal range. Since heat flux and its uncertainty may be dependent upon environmental and apparatus test conditions, as well as intrinsic characteristics of the specimen, the report from the testing must include thorough description of the specimen and of the conditions. Also, since this test method is applicable to a wide range of specimen characteristics, test conditions, and apparatus design it is impractical to give an all-inclusive statement of the precision and bias of the test method. Thermal transmission properties can be calculated based on the heat flux measurements conducted by this test method.

**Specimen type, Dimensions and Manufacturing**
The specimen used in the guarded hot box test shall be representative of the construction to be investigated. Furthermore, temperature and other sensors may be installed throughout the interior and on the surface of the specimen during manufacturing. When the test panel is installed in the test box, its edges shall be insulated to prevent edge effect from overtaxing the guarding effect of the guard area of the panel.
Apparatus
The general arrangement of the components of the guarded hot box apparatus are illustrated in Fig. 11.74. The size of the metering box is largely governed by the metering area required and it also determines the minimum size of the other elements. The metering box walls shall have a very good and uniform thermal resistance. An arrangement to assure an even, gentle movement of air over the metering area of the panel is also required. Electric heaters which are mounted in a housing with walls of high resistance and a low emittance outsidesurfacing to minimise radiation heat transfer to the metering box walls can be used. To this arrangement the air is continuously circulated by a small fan. To obtain reliable test results, accurate temperature control equipment must be utilised. Furthermore, to provide the metering box walls to serve as a heat flux transducer, a means of detecting the temperature difference across the metering box walls or the heat flux through the metering box walls (e.g. by applying a number of differential thermocouples connected in a series to the inside and outside surfaces of the metering box walls to form a thermopile) shall be provided. It is recommended that the guard box be large enough so that there is a clear distance between its inner wall and the nearest surface of the metering box of not less than the thickness of the thickest panel to be tested, but in no case less than 150 mm. Also the guard box shall have the same type of walls, heating arrangements and temperature control as described above. The size of the cold box is governed by the size of the test panel or by the arrangement of boxes used. The cold box should be heavily insulated to reduce the required capacity of the refrigerating equipment and it shall also be equipped with a suitable air circulation arrangement and a temperature control device. To measure the surface temperatures on the investigated specimen, a temperature measuring equipment containing of thermocouples, or other temperature sensors, and some suitable measuring instruments are required.

Testing Procedure
The various heating and cooling units are placed into operation to achieve the test temperature conditions. The time required to achieve thermal steady-state of the system varies considerably with the characteristics of the apparatus design, the specimen to be measured, and the test conditions. Generally, since this method is applicable to low conductance specimens, the setting
time is on the order of hours. Impose steady-state conditions for at least four hours prior to final
data collection. During this period, data shall be collected at intervals of one hour or less. When
these conditions have been satisfied, the test period is continued by at least eight hours.
The thermal conductance, $C$, thermal resistance, $R$, and the thermal conductivity, $\lambda$, can then be
calculated as follows from the measured data.

\[ C = \frac{q}{\Delta T} \]  (11.82)

\[ R = \frac{\Delta T}{q} \]  (11.83)

where $q$ is the rate of heat flux through the specimen per unit area and $\Delta T$ is the temperature
difference between the specimen surfaces.

\[ \lambda = \frac{Q}{A} \frac{t}{\Delta T} \]  (flat specimen)  (11.84)

where $Q$ is the rate of heat flux through the specimen area, $A$ the specimen area normal to the heat
flux direction and $t$ is the thickness of the specimen.

**Referenced Documents**

11.5 Miscellaneous

11.5.1 Shear Strain Gauges

In a pure shear strain field as exists in the core, according to
\[ \tau_{xz} = \frac{T_x}{(t_f + t_c)} \quad \text{and} \quad \tau_{yz} = \frac{T_y}{(t_f + t_c)} \]  
(11.85)

the principal strain directions are at 45° to the coordinate axes. A strain gauge mounted in this direction will give a measure of the shear strain. So-called shear plugs can be used for this purpose. A strain gauge is mounted in a piece of core material of the same kind used in the sandwich core. The strain gauge is bonded in place between two blocks of core material at an angle of 45° and at a depth corresponding to the centre of the core (Fig. 11.75). The block is then turned in a lathe to a diameter of 30 mm.

![Diagram](image)

Figure 11.75 The block of core material with the strain gauge attached and the finished shear plug, ready to be mounted in the sandwich. \( t_c \) is the core thickness.

The plugs are cut to fit the core thickness and mounted in the sandwich while only one face is bonded to the core. A 30 mm diameter hole is drilled through the core all the way down to the opposite face using a knife drill to produce a smooth surface with open cells in the hole. A small hole shall be drilled in the face for the connectors. The shear plug is then primed and mounted using some suitable adhesive, with the gauge oriented correctly in the sandwich, i.e. so that the gauge would be subjected to tensile load when the global load is applied.

Calibration of Shear Plugs

Two sandwich beams with built-in shear plugs can be used to calibrate the shear gauges. The sandwich beams shall have the same cross-section and the same materials as the measured panel. The data from the calibration are used when interpreting the results from the shear plugs when the sandwich panel is tested in the plate bending rig (chapter 11.4.7). Calibration of the shear plugs should be carried out in a universal testing machine using a four-point bending rig. The shear strain gauges are mounted in the part of the beam where the shear force is constant, i.e. between an inner and an outer support and are calibrated up to the maximum shear stress in the core during the panel test. The level shall be within the elastic region of the core material.
The readings from the shear strain gauges shall show a good agreement among the different individual gauges. This reflects a high level of accuracy in the manufacturing of the shear plugs. The readings from the strain gauges shall be plotted versus the load to show that the behaviour of the curves are nearly linear and that a slight hysteresis can be detected between loading and unloading. The shear stress in the core is calculated according to eq.(11.85).

Referenced Documents
Per Wennhage and Dan Zenkert, "Testing of Sandwich Panels Under Uniform Pressure", Department of Aeronautics, Division of Lightweight Structures, Royal Institute of Technology, S-100 44 Stockholm, Sweden.

11.5.2. Vibration Test
The requirement for reliable mechanical performance data is obvious in all structural design tasks. Material tests are also regularly needed as stage in manufacturing and quality control. Regardless of the use of the information there is often a non trivial choice to be made of an adequate testing method. As already seen, there are a number of standards for measurement of the elastic constants of fibre composites. Although there exists well documented standard test procedures, such as those from e.g. ASTM, DIN, or ISO, the results are always to some extent depending on the person executing the tests. The full procedure ranging from extracting and measuring the specimens to interpretations of the final averaged results and deviations involve many stages in which subjectivity may be affecting the final results. Another major drawback of the static test methods is that they involve many samples and special test fixtures and are therefore slow and expensive to perform. Thus, methods for the determination of anisotropic elastic material properties of plates using their natural frequencies has been developed (and verified). The elastic properties can be estimated through minimisation of the disparity between the measured resonances and the eigenfrequencies obtained by, e.g., Finite Element (FE) analysis using the elastic properties as parameters. An advantage with the FE-based method is that more complex plate shapes can be analysed than for similar methods based on finite series displacement field assumptions, which is the base in earlier work. These methods are usually generally based on Rayleigh type model description.

The novel method of identifying material properties has several appealing advantages:

• The identified properties are automatically averaged over the whole specimen in contrast to most standard tests where strain gauges or clip gauges are used.
• All properties are in many cases possible to measure on one single specimen.
• The method allows, within limits, the specimen to be tailored in size and shape to be sensitive to a specific parameter.
• The need to accomplish very accurate cutting, both with respect to specimen side alignment and fibre orientation, of the specimen is dramatically reduced.
• The method is fast and requires significantly less expensive equipment.
• The simplicity of the experiments implies repeatability and low influence of the skill of the experimentalist.
• Plate stiffness are measured during dynamic loading and low strain levels.
• The method is non destructive and handles fairly complex specimen shapes. Hence it is possible to measure the properties of the very plate later to be used in other experiments.

Experimentally, the plate natural frequencies are determined by the impulse technique. The suspended specimen is brought into vibration by excitation through a hammer blow. The decaying
response is measured using either microphone or accelerometer. A simple acoustic condenser microphone have proved satisfactory. The advantage of using a non-contact microphone is that no extra mass is added to the specimen and that the location of the sensor is easily altered. A robust steel stand, firmly clamped onto a vibration-isolated table and a cross-bar, with two stiff cantilever beams clamped to each end, clamped on the stand is suitable as a holder of the specimen. Then the composite plate specimen is hung in the cantilever beams using thin nylon filaments or adhesively bonded light, elastic rubber bands, in order to simulate the completely-free boundary condition as accurately as possible. In another method of suspension that can be used the specimen is placed on a cotton bed. A scheme of the experimental set up is shown in Fig.11.76. A blow from a hand-held hammer or an instrumented impact hammer is used to bring the specimen into vibration. The excitation signal is not needed in some type of experiments where the only needed frequency response function characteristics are the locations of the resonances in the frequency domain. Hence, this fact neutralises the need for an instrumented impact hammer. The objective is to excite the specimen with a force pulse containing a continuous excitation spectrum over a broad frequency range. The characteristic of the impactor tip has been found to affect the excitation spectrum, hence, different types of tips can be tested and used depending on the type of specimen. The resonances can be identified directly from the Fast Fourier Transformed (FFT) response signal. The acquisition system can be based on a standard personal computer with some suitable software and some suitable signal conditioning board along with a data acquisition board. Typically, a series of measurements have to be performed to capture a subsequent series of resonances during which both impact location and microphone locations are altered.

![Figure 11.76 Scheme of the experimental set-up. The suspended plate specimen with microphone and hammer connected to the signal analyser. The modal analysis program run on the PC which is connected to the analyser](image_url)

The objective with the FE calculation is exclusively to calculate a number of eigenfrequencies of the plate. The choice of most adequate plate model is not obvious, though in plate problems, this choice is often done based on the problem thickness-to-size ratio. It is stated however that thickness is not a strict geometric notion in plate (or shell) problems. Whether or not a plate is "thin" depends on the goal of computation. A specific plate can be regarded as "thin" if only the first few eigenfrequencies are of interest but "thick" if the moments, shear forces or higher frequencies are computed. The fundamental problem is to find the set of stiffnesses, $D$, that in some sense comply "best" with the measured resonances, $\hat{f}$, and the calculated eigenfrequencies, $f(D)$. The perhaps most commonly used approach, is to solve variations of the unconstrained least square formulation, a weighted least squares formulation or a minimisation algorithm method of
moving asymptotes. The different causes for uncertainties and errors in the process can be divided into three main types, experimental, modelling and numerical. A complicating factor however is that some of the contributing error causes are difficult to quantify.

**Referenced Documents**


An inherent feature of sandwich constructions is, except their very high stiffness and strength to weight ratios, their thermal properties, an integrated function which depending on the choice of materials often yields a structure with very good thermal insulation properties. This is a most important feature giving the sandwich concept a very strong position in the design of structures such as containers, tanks and other transportation structures where thermal insulation is necessary.

This chapter is in no way intended to be complete but rather a brief introduction to the parameters used and some formulae for the calculation of thermal properties of layered materials such as sandwich cross-sections.

For more information on the fundamentals of thermal characteristics of foams and honeycombs, the reader is referred to the book on cellular solids by Gibson and Ashby [1].

12.1 Definitions
Below follows a short description of the parameters used in the context of thermal characteristics.

*Coefficient of thermal expansion*, $\alpha$, is the constitutive relation between a temperature change and the strains that developed. Dimension K$^{-1}$ (Kelvin).

*Thermal conductivity*, denoted $\lambda$, is the heat flow rate that passes perpendicular through a cube of 1 m$^3$ between two opposite sides at a temperature difference between the two surfaces of 1 Kelvin (1 °C). $\lambda$ is measured in W/mK.

*Thermal resistance*, denoted $R$, is the resistance to heat transfer through a section of unit area. $R$ is measured in m$^2$K/W.

*Coefficient of heat transfer*, $a$, is the heat flow rate that is transferred between a surface of 1 m$^2$ and the air at a temperature difference of 1 Kelvin (1 °C), and is measured in W/m$^2$K.

*Thermal transmittance*, $k$, is the heat flow rate that passes perpendicular through a wall of known structure with an area of 1 m$^2$ at a temperature difference of 1 Kelvin (1 °C), and is measured in W/m$^2$K.

*Heat flux*, $q$, heat rate per unit area, measured in W/m$^2$.

*Mean temperature*, $T_m$, average temperature between inside and outside of the structure, and is measured in Kelvin (K).
12.2 Estimation of Thermal Insulation

To estimate the thermal insulation of a sandwich panel one can start by studying a sandwich cross-section illustrated in Fig. 12.1, having temperature $T_1$ on the inside and $T_2$ on the outside.

![Figure 12.1 Schematic of the temperature field through a sandwich cross-section.](image)

The thermal conductivity of a material depends on the actual temperature; for the face sheets 1 and 2 we can simply use the surface temperatures $T_{f1}$ and $T_{f2}$ since the faces usually are fairly thin. For the core, however, we need the average temperature. Without making too much error, we can assume that the core temperature varies linearly between $T_{f1}$ and $T_{f2}$ and thus

$$T_m = \frac{T_{f1} + T_{f2}}{2} \quad (12.1)$$

and a value $\lambda_c$ should be found for that particular temperature. The heat loss per unit area through a cross-section with different temperatures on each side, $T_1$ and $T_2$, as shown in Fig. 12.1 equals the thermal transmittance times the temperature difference, i.e.,

$$q = k(T_1 - T_2) = \frac{T_1 - T_2}{R} \quad (12.2)$$

with $q$ positive in the positive $z$-direction as indicated in Fig. 12.1. In order to continue and estimate the value of $k$ we must know something about the physics of heat transfer. There are three main causes of heat transfer that must be included in the analysis: conduction, convection and radiation. The latter, radiation, is often of minor importance and is therefore usually omitted in structural analysis. In special cases, however, such as in space applications, it may be more important.

(i) Conduction

For an isotropic body with varying temperature heat flows in the direction of the gradient of the temperature field, that is, towards and in the direction of the lower temperature. This phenomena is associated with molecular movement, which increases with increasing temperature, and when molecules with high energies (high velocity, high "temperature") collide with neighbouring slower moving molecules, the energy is transferred. This occurs in solids, as well as fluid and gas phases of all materials.
In this case we can approximate the flow to one dimension, from one side of the panel to the other. If the z-coordinate is as defined in Fig.12.1 we can then write the flow using Fourier's law \( q = -\lambda \frac{dT}{dz} \) or if integrated \( T_1 - T_2 = \frac{qT}{\lambda} \) (12.3)

(ii) Convection
Convection occurs between different media where one media must be a fluid or a gas that may move over the more solid surface. Consider the example of the sandwich in Fig.12.1 where the face material and the surrounding air or fluid have different temperatures. In a region near the face there will exist a thermal boundary layer in which there must be temperature gradient. The heating or cooling of the air or fluid in the boundary layer causes it to move due to the temperature difference to the surrounding fluid. This motion causes new fluid to enter the boundary layer which in turn is heated, or cooled, and a heat transfer process is going. The heat flux depends on the temperature difference between the media and the convection heat transfer coefficient, \( a \), between just those two media at the specific temperature. In certain applications, the first media may in steady be water, a boat for example, and in such a case, an a heat transfer coefficient \( a \) must be used for that particular material combination. The heat flux in this case is written

\[ q = a \Delta T \] (12.4)

where \( \Delta T \) is the difference in temperature between the face sheet and the surrounding air temperature. \( a \) it the convection heat transfer coefficient which usually depends on several parameters and may therefore be difficult to obtain.

(iii) Radiation
Thermal radiation is energy emitted by matter that is at a finite temperature. All materials, regardless of form emits energy. The emission is due to changes in the electron configurations or the constituent atoms or molecules. Radiation transfers energy regardless of the surrounding medium, as conduction and convection, and is, in fact, most efficient in vacuum. The radiation heat flux is given by Stefan-Boltzmann's law

\[ q = \sigma T^4 \] (12.5)

where \( T \) is the absolute temperature (in Kelvin) and \( \sigma \) the so called Stefan-Boltzmann constant (\( \sigma = 5.67 \times 10^{-8} \) W/m²K⁴). Such a surface is called an ideal radiator or a blackbody. For any other type of surface the radiation is less than for a blackbody an may be written

\[ q = \varepsilon \sigma T^4 \] (12.6)

where \( \varepsilon \) is a radiative property called the emissivity, and has values \( 0 < \varepsilon < 1 \). Conversely, if a surface is exposed to radiation, a portion of the energy will be absorbed by the surface proportional to the radiation through

\[ q_{abs} = \gamma q_{rad} \] (12.7)

where \( q_{rad} \) is the radiated energy, \( q_{abs} \) the absorbed energy and \( \gamma \) the so called absorptivity, which has values \( 0 < \gamma < 1 \).
Going back to the sandwich cross-section in Fig. 12.1 and assembling the total heat flow assuming steady-state conditions and one-dimensional flow only. By omitting the radiation part since it is negligible in most circumstances and considering conduction and convection only, one can write the thermal resistance $R$ of each heat transfer mode as $q/\Delta T$. Then expresses the total flux at the temperature difference divided by the total resistance and the result is

$$ q = \frac{T_1 - T_2}{\sum R_i} = (T_1 - T_2) \left[ \frac{1}{a_1} + \frac{t_1}{\lambda_1} + \frac{t_c}{\lambda_c} + \frac{t_2}{\lambda_2} + \frac{1}{a_2} \right]^{-1} \quad (12.8) $$

Hence, the thermal transmittance of the entire cross-section is

$$ k = \left[ \frac{1}{a_1} + \frac{t_1}{\lambda_1} + \frac{t_c}{\lambda_c} + \frac{t_2}{\lambda_2} + \frac{1}{a_2} \right]^{-1} \quad (12.9) $$

where indices 1 and 2 refer to the faces and $c$ to the core. Now the energy loss is given from eq.(12.2). Considering that $\alpha$ has values in the order of 2 to 25 for free convection of gases and even higher for fluids. For forced convection, wind or fluid flow for example, the convection heat transfer coefficient increases even further. Common face materials, like metals of fibre composites, has thermal conductivity values of 40 - 200 W/m°C for metals and in the order of 0.2-0.4 for composites, whereas common core materials have $\lambda$-values such as 0.07 for balsa wood, 0.1 for non-metallic honeycombs and 0.03 for most foams. Considering that the core layer is often much thicker than the faces we can simplify the thermal transmittance equation to include the core layer only as

$$ q \approx \frac{(T_1 - T_2)\lambda_c}{t_c} \quad (12.10) $$

Now it is clearly seen that to obtain good thermal insulation, that is, to prevent any heat loss by minimising the heat flux, it is advantageous to choose a thick core with a low thermal conductivity.

**References**


The demand for constructions with low weight in combination with low life-cycle-cost, especially in the transportation-sector, require increased knowledge about the design variables and the underlying theory and/or empirical knowledge. There are a number of ways to increase the knowledge, however, a common denominator for all methods are the desire to reduce the weight while maintaining or even increasing the load bearing capability. Among the methods frequently used for improvements are: increased material characterisation, improved theoretical analysis, increased numerical analysis, improved manufacturing techniques. The latter include the utilisation of non-destructive testing (NDT) techniques which are aimed at confirming the structural integrity and thus clearing the part for further assembly or final delivery to the customer.

Sandwich structures has the potential to meet the requirements of low life-cycle-cost. However, they present difficult problems in that they are produced in a process involving many manufacturing variables and thus providing opportunities for introducing defects. Some of the difficulties arise due to the fact that a large number of constructions are produced in the same operation as the material. The anomalies that might occur in sandwich structures originates from two sources: first, material defects and property variations which cause defects when the material is consolidated. Second, defects caused by deviation from the manufacturing processes or the service specifications. Undetected or unintentional deviation from the material specification and/or the manufacturing process is possible. Hence, there can be no guarantee that two different components of the same kind are identical. Thus, establishing strictly controlled procedures for quality assurance (QA) throughout each step of the manufacturing process is most important.

As the theory and the numerical methods improve the requirements on the NDT methods increase. Therefore, a wide array of methods has been suggested for various applications. Furthermore, the size and complexity of sandwich structures may vary widely, from small light-weight aerospace applications to large ship hulls for naval mine hunters or surface effect ships. Therefore, the requirements for QA and NDT will also vary widely. Traditionally, the highest requirements is requested in the aerospace industry, however, the tendency is that the methods are getting increasingly complex, specialised and usually very time consuming and as a result very costly. In conclusion, other branches cannot cope with such very high costs while maintaining competitiveness, thus, efforts are being made to find new methods which can solve this problem.
This chapter will consider, in general terms, the importance of QA and especially NDT on sandwich structures.

### 13.1 Why NDT

The overall objective with NDT is to provide relevant information in order to facilitate reliable evaluation of the quality and structural integrity of the manufactured structure. This may be achieved in two ways; either by monitoring and controlling the manufacturing process or by using non-destructive inspection techniques to assess specific defects. The first, the indirect method, is aimed at assessing the mechanical and physical properties of the materials used in the manufacturing process. The other is methods used to determine the structural integrity of the final component or structure.

Producers of sandwich structures are most concerned with the quality of the fabricated part, hence, those are interested in delivering a sound and fit product to the customer. Companies operating constructions or independent vendors for structural assurance are concerned with the integrity of the construction. Those apply inspections for these main purposes:

1. **QA** - involves the whole manufacturing process, from control of the constituent to the finished component or whole structure.
2. **Maintenance** - the construction needs periodical inspection to ensure the soundness or safe and proper function.
3. **Repair** - after accidents or owing to maintenance, the repair needs to be inspected and confirmed before the component is taken into service again.
4. **Renewal** - some part of the structure might not need replacement, therefore, the inspector can decide which parts to replace.

### 13.2 Notes on Basics of NDT Techniques

The basics of all NDT techniques are that they all rely on the measurement of some physical quantity which reflects the current state of some property of the material under inspection. One fundamental principle used is that the object under test is subjected to a disturbance from its equilibrium and the reaction is measured. The principle is based on the comparison of reactions, one of which is known to be the correct response describing the material property of interest.

#### 13.2.1 General examples

Consider material which, owing to the internal structure, dissipates an introduced disturbance rather quickly. The principle of measuring the response of a disturbance in those materials would not be successful. However, a technique based on another principle would probably solve the inspection problem at hand. Further, if the defects constitutes a small density variation from the host material an NDT technique based on the measurement of density may run into problems whereas other techniques might have a solution to that particular flaw type.

By these two examples, the importance of understanding the basic physical principal which the NDT technique is based on is emphasised. With this knowledge it is easier to comprehend limitations and feasibilities for the application of various techniques. Hence, a cost-effective
strategy to find solutions of inspection-problems are more likely obtained with the potentials of the NDT technique in mind.

13.2.2 Safety margins
When designing load-carrying structural parts different types of safety-margins (SM) are applied depending upon how vital the part is for the whole construction. The SM is mainly for static loading and are used to safe-guard against scatter in the materials used and for deviations in the geometry of the structure which can occur during fabrication. Additional factors, knock-down-factors (KDF), are applied to structures which might be affected by external factors, for example, environmental exposure. The size of SM and KDF are usually dependent upon the application of the structure and the experience from these applications.

The SM and KDF means increased weight. For example, in the aerospace industry where weight is an important parameter, a large amount of effort is put into weight saving actions. If the production can be reliably controlled and the final components structural integrity be confirmed to a certain degree of confidence, these SM and KDF can probably be reduced.

13.2.3 Considerations when Choosing Method
When deciding what technique is most suitable for a particular inspection it is wise to answer a few questions. In the following a few questions and reasons for them are put forward. The purpose is to provide the reader with a guide on what questions could be asked and to encourage to form your own relevant questions before deciding what method to apply.

- What type of defects are of interest to detect? - some techniques are based on principles which are not suitable for detection of particular flaws in certain materials.

- Must the defect be thoroughly characterised? - various NDT techniques can detect specific flaws while others are better at characterisation of the very same flaws.

- Is it required to cover large areas while inspecting? - finding disbonds in a large structure, like a ship hull, is less time-consuming when using a technique with the capability to locate defects while scanning large areas rather fast. The located defects can, if necessary, be characterised with the use of a technique more suitable for characterisation of the particular defect.

- Are there any safety precautions to be considered? - When using X-ray equipment increased visibility are at times generated by the use of certain penetrating liquids which can be hazardous for human health. Likewise, if X-ray instrumentation are used for field inspections the exposure to X-ray beams can cause health problems.

- Can the method of inspection jeopardise the structural strength? - When applying heat to a sandwich structure caution should be exercised because the adhesive bond in the interface between the face and the core can be severely damaged, causing strength reduction, if over-heated.

- Where is the method going to be applied? - in the laboratory, as production control or for in-service inspections? - some techniques are more suitable for on-line production control than field inspections whereas most techniques can be used in an laboratory environment.
- What accept/reject criterion's does the design offer? - the aerospace applications have rather stringent criterion's whereas other applications might not require likewise stringent criterion's.

- What is indicated by regulations and quality standards as regards to NDT? - some customers require the manufacturer to conform to certain quality standards, like ISO 9000 or AQAP 1, in addition to the specification of the construction. Whereas operators of constructions, for example civil aircraft, are requested to conform to regulations to ensure safe operation. Further, the manufacturer of the aircraft imposes requirements on periodical structural inspection in order to guarantee safe operation.

and finally
- Does the personnel need education and certification on the decided technique to posses the capability of qualified inspections? - education is offered by various organisations on most of the established techniques. Depending on the complexity of the inspections different levels of education has to be passed to be qualified for that particular type of inspection.

13.2.4 Aspects of NDT on sandwich structures
Consider, for a moment, the three constituents of a sandwich structure from a NDT point of view; one will find materials with, in large, very different properties. For example; the faces, metal or composite, are in comparison with the core of high density and very thin and therefore, usually lend themselves to conventional NDT techniques developed for metals. In some cases a few modifications are made to the conventional methods so as to make them more suitable for composites. In the case of the core the situation is the opposite; the material is relatively thick and of low density and the issue of NDT have not been considered until quite recently on these materials. Hence, the NDT techniques and the experience from them is not as extensive on typical core materials as on face materials.

When the component is manufactured from prefabricated constituents they may prior to assembly be inspected by an appropriate method for each components particular material. For example, if an application uses aluminium faces and honeycomb core, the bond line is the only part that needs evaluation after the manufacturing process. However, if the faces are manufactured in an autoclave or by hand-lay-up both the face and the bond line between the face and the core need to be confirmed after final fabrication. Here problems arise, when the faces and core are bonded together the result is a structural element with widely different properties through the cross-section. Thus, yielding difficulties in the inspection of the whole cross-section.

Hence, there are no easy answers to meeting the NDT requirements of sandwich structures. As the materials of the constituents differ extensively these structures poses more difficult problems than met during inspection of each single constituent. However, the large investment in research efforts on NDT techniques will increase and add to the knowledge about the techniques available today. Most of which, due to intrinsic features, are restricted to a limited range of applications.

13.2.5 Defect types
Flaws in sandwich structures can be divided into three groups depending upon where the flaws occur. The groups are:

- Flaws in the faces.
• Flaws in the adhesive layer.
• Flaws in the core.

There is a large number of materials to choose from which all can be applied in all these groups but, generally speaking, the same type of undesired features might occur in most materials during manufacturing or indeed during service.

(i) Manufacturing generated defects
There are two stages of production; the fabrication of the constituent and the manufacturing of the component. For the faces there are two different groups of materials of interest for structural sandwich structures, metals and fibrous composites. The different types of defects that are among the most commonly seen in the composite faces after manufacturing are; voids, porosity, fibre misalignment, wrinkling, pore cure, resin-rich and/or resin-poor areas, see Fig.13.1. Whereas metal faces very rarely show any defects that needs non-destructive inspection to be detected before or after the manufacturing of the component.

A large selection of core materials are used ranging from metals via impregnated paper to polymeric foams. Most of the cores used today for load-carrying sandwich structures are seldom the object for NDT prior to fabrication of the structures. Therefore, some of the mis-features that can be found in these core types are; disbonds, voids and geometrical deviations. Disbonds and voids affect the mechanical properties of the structure which can cause stress concentrations and are therefore potential sites for fracture initiation.
(ii) In-service induced flaws
A defect of most concern when a structure is in service is the impact damage. An impact damage on a composite face usually have a severe negative effect on the load-carrying capability of the face, especially in compression. Thus, the whole sandwich construction experience a drastic reduction in structural strength. Moreover, the structural strength reduction is commonly much larger than what the visual part of the damage suggests. Whereas, the effect of an impact on a metal face is usually an indentation. The depth and extension of the indentation is dependent on the impact energy and the geometry of the object causing the indentation. The structural strength of an metal faced sandwich structure is also reduced, mostly in compression.

Other common causes for in-service defects are over-loading, both static and in fatigue, which can cause defects such as, shear-cracks, disbonds and delaminations. All of these defects have a large influence on the structural strength. Another factor to take into account during service is the environment in which the construction operates. For example, water absorption in honeycomb cores on civil aircraft can at times be a problem.

13.3 NDT Techniques Applicable to Sandwich Structures
In the following a short description of some of the NDT techniques that constitutes plausible choices to be used on sandwich structures will be given.

13.3.1 Acoustic emission
Acoustic Emission (AE) is one of the methods commonly used on fibrous composites for proof testing. However, the method is not in the true sense non-destructive. The idea of the method is as follows. When a material is subjected to a mechanical load the material is increasingly loaded with energy. Once the local ultimate strength is passed a crack occurs in the matrix or in the fibre. This means a release of stored strain energy which dissipates through the material as energy waves, which are detected by transducers. If several transducers are placed on the same object and the geometry and the velocity of sound in the material used is known then it is theoretically possible to determine where the energy originated from.

The degree of sophistication of the instrumentation employed determines how much information that can be extracted from the released strain energy, the events, that is recorded. The number of events, location and severity of the events are among parameters of interest. The use of AE to determine the type of defect or failure that is emitting events is, however, very difficult. The best results are obtained when using AE on a number of the same type of component during test loading or proof loading. Thus, generating know-how on how that particular part behaves under loading.

13.3.2 Holographic methods
The holographic methods are based on the use of a laser. Holography is a form of wave front reconstruction imaging in which two images of the same object are reconstructed and spatially superimposed. Any relative displacement between the object and superimposed image is easily identified. Holographic interferograms measure differences either in shape or relative displacement. These methods are very exact and can measure very small disturbances. There are three different main techniques that are used. They are the double exposure, time average and real time techniques.
13.3.3 Mechanical impedance methods
The general idea with impedance methods is to induce a bending wave, perpendicular to the cross section, into the structure. This is commonly accomplished by a transducer which transforms a electrical oscillation to a mechanical deflection. Once the mechanical deflection is introduced in the structure the idea is to look for areas where the mechanical impedance (bending stiffness perpendicular to the surface) changes relative to the surrounding area. The bending stiffness is usually measured with the same transducer which introduced the mechanical deflection. The impedance is for instance influenced by a flaw parallel to the surface which divides the structure into layers. The impedance of a flawed section is inversely proportional to the amplitude. Thus, a flaw is recognised as a region with lower impedance. These methods are successful in finding cracks that are parallel to the surface. The size of the crack that can be detected depends on the frequency of the introduced deflection. This means that a rather large frequency interval has to be run through when a structure is tested. Further, the results depend very much on the contact stiffness between the transducer and the structure.

13.3.4 Radiography
Radiography is a penetrating electromagnetic beam which is directed perpendicular towards the object under test. The beam energy is partially absorbed in the object and the rest passes through. The absorption of energy is dependent on the material composition, the thickness and the density of the material. The energy that passes through the specimen is visualised with any suitable medium, usually a special film.

The type and the quality of the beam, the geometry of exposure, the visualisation media and the evaluation technique yields variations in the results. The parameters that are involved in the general application are the contrast of the object, the contrast of the visualising media, distortion and the ability to visualise. The ability to visualise is measured directly while the other parameters are determined by comparing with an indicator that is exposed simultaneously with the object. The distortion is related to the technique of exposure and the composition of the material of the specimen.

The contrast of the object is of primary concern. The contrast is the result of the difference in absorption in the tested object. Thus, it is dependent on differences in density or on the composition of the material.

13.3.5 Ultrasonics
A number of ultrasonic methods which were originally developed for homogeneous materials, and well established on those materials, have been applied to sandwich structures. In the following a brief description of the fundamentals of the basic methods will be given. A thorough explanation of the principles is beyond the scope of this presentation, however, the interested reader can find further details in [18,19].

General foundations of ultrasonics
Ultrasonic inspections are carried out by coupling the ultrasonic transducer to the object via a solid or liquid medium because of the large difference in impedance between air and solids. For large components water-jet ultrasonic testing has been developed, which is mainly used for through-transmission inspection using a transmitter and receiver on each side of the part. For smaller parts
immersion testing can be used, which lends itself to both the through-transmission and the pulse-echo techniques.

**Common obstacles encountered on sandwich structures**
The large impedance difference of solids and air causes the inspection of honeycomb sandwich structures to be difficult using ultrasonic pulse-echo because the ultra-sound will not propagate through the cross-section except at the cell walls. Hence, using the pulse-echo technique only the top face and the bond line beneath it can be reliably inspected. Using the through-transmission method both bond lines can be inspected in a single test, however, the result will not distinguish which one of the bond lines that contain a detected defect. Further, owing to the structure of rigid foam cores the ultra-sound will scatter on attempt to penetrate the material. Therefore, using the pulse-echo methods on foam core sandwich panels is only suitable for inspection of the faces and the bond lines, provided that the face material is no obstacle. Whereas, for the trough-transmission method the rigid foam core provides very difficult problems, especially for cores with lower density than 250 kg/m³.

**Manual inspection**
When the inspection is carried out manually, usually with a single transducer for pulse-echo measurements, a gel is commonly used as a coupling media between the transducer and the structure. Using a gel is particularly suitable when performing in-field inspections, however, there exists applications where other coupling media, for example water, are preferable. Manual inspection is time consuming and therefore some work has been done to use roller probes, where the transducer is held inside a rubber wheel, the sound being propagated into the structure via the soft rubber tyre. However, the latter method is not satisfactory for detailed characterisation of defects. Another method to decrease the manual inspection time is to mount transducers in arrays which will scan larger areas simultaneously. However, it requires the component to be inspected to feature surfaces that are either flat or of single curvature.

13.4 NDT Techniques Applied on Sandwich Structures
In the following a short description of the techniques used on sandwich structures reported on in the literature will be given.

13.4.1 Acoustic emission
AE has become very useful for on-line monitoring of tests of composite structures [1,2,3] to prevent premature failure. A few reports on the use of AE on sandwich structures has been published. For example, Sriranga and Samuel [4] used AE to monitor loading of adhesive potted inserts in honeycomb sandwich panels.

13.4.2 Holography
A number of research efforts on holographic non-destructive testing of sandwich structures have been reported in the literature [5-14]. Most of them report test performed on sandwich elements made of honeycomb core with different types of faces applied. Both composite and metallic cores and faces has been used. Core thicknesses of 6, 12 and 20 mm with faces thicknesses varying from 0.125 to 1.5 mm have been used.
Stressing of the component is achieved in a number of ways; by applying heat, mechanical loading, pressurisation and vibrations. All stressing methods are aimed at generating a strain field in the part and if a flaw is present, it would create a localised strain field differing from the surrounding strain field. Thus yielded a disturbed fringe pattern.

The dominating holographic method reported for detection of flaws is interferometry and various systems for recording and analysing the fringe pattern are proposed. The development of these systems working towards true real time evaluation [12] of rather large components under industrial conditions [8]. The area of simultaneous inspection and evaluation varies depending on test-set-up but sizes as large as 500’500 mm² has been reported [9,10]. [13] reported on a system which calculated, by a hybrid stress analysis, the effect of the detected flaw on the structural integrity.

Shearography is another laser-based NDT method which is applicable in ambient light and not sensitive to vibrations it has also been reported [14] to detect interface disbonds. The type of flaws that have been investigated are; disbonds (lack of adhesive film) and delaminations (impact damages). [9] reports successful investigation of simulated circular interface disbonds with diameters varying from 12 to 24 mm under 0.3 mm aluminium faces sheets. detection of delaminations with a smallest dimension of 5 mm was also reported [11].

13.4.3 Radiography
The rapid progress in electronics has facilitated further development of the radiographic methods. Perhaps the most prominent progress is the development of the tomograph which provides the capability to yield information on the absorption coefficient at every point in the part under test. Most of the development of the tomograph has been for medical applications, however, as the cost for aerospace application rise the demand for reliable and thorough inspections of composite components has been a driving force to develop tomography for structural applications. [15,16]. [15] reports on application of industrial computed tomography (CT) to sandwich structures and pulltruded composite parts. Defects such as, delaminations, disbonds and voids were easily detected.

13.4.4 Ultrasonic methods
The most commonly used techniques include; velocity and attenuation measurement but more complicated methods such as acousto-ultrasonics, leaky-lamb waves and back-scattering have also been tested [17]. The feasibility of fatigue and impact damages on foam core aluminium faced sandwich panels using ultrasonics has been investigated by [17]. It was found that disbonds, simulated by gaps in the adhesive layer, could be detected by pulse-echo. Fatigue and impact damages were detected by the use of acousto-ultrasonics in the same type of panels.

13.5 Thermal NDT on Foam Core Sandwich Structures
In the following, the work to find an NDT technique suitable to foam-core sandwich structures that is being carried out at the Department of Aeronautics at the Royal Institute of Technology in Stockholm will be described.
13.5.1 Defect types
Apart from the plausible flaws described elsewhere in this text the adhesive bonds were of fundamental interest in this work. Generally, adhesive bonds are among the most important issues concerning sandwich structures. In the type of sandwich structure of primary interest here there are two types of adhesive bond lines. The first type is in the core which in order to obtain the desired thickness of the core it sometimes is necessary to bond one or more core blocks together. Here, inclusions of air and/or poor bonding might occur in the bond line between the core blocks. The second type is in the interface between the face and the core. It is very difficult, by the means of NDT, to distinguish an adhesive joint with full strength from a joint that is not capable of full strength. Hence, the first step is to focus on locating areas where the adhesive joint is non-existent. From the theory of sandwich structures it is easily concluded that if there exist a disbond somewhere in the structure, locally it will reduce the bending stiffness rather drastically and consequently these type of flaws are important to uncover.

13.5.2 Requirements on the desired NDT technique
It is desired that the NDT method is able to detect all the typical defects and it is considered to be of great advantage if the method had the following qualities:

- Easy to use, both in a laboratory and in-field.
- Possibility to scan large areas fast with maintained high resolution.
- Easy to interpret.

Ideally, the method should have the capability to inspect the structure for defects and make quantitative assessments. If the suggested method would not be capable to characterise the detected defects, a technique capable to characterise the flaws once they were located should be used locally. Two major techniques were suggested: Laser based and Infrared (IR) methods.

Both methods showed promising results in a preliminary study but it was decided that thermography provided the best chance to meet the requirements, especially since the hardware already existed for thermography while laser methods needed further development. Therefore, in the following, the application of thermal NDT on foam-core sandwich structures, especially with composite faces, will be described.

13.5.3 Physics
The underlying physics of the thermal NDT technique is rather straightforward. Consider a homogenous, isotropic body. The body is exposed to thermal heating on one side, see Fig.13.2, for an appropriate amount of time after which the heat source is removed and the body is again exposed to the ambient environment. The heat induced into the body will dissipate through the body homogeneously. However, if there exists an irregularity in the body the heat flux will be disturbed as it passes through the structure. This disturbance depends on the shape of the irregularity, the material, the duration and the amplitude of the heat flux. If this irregularity is large enough it may be detected as a change thermal field as it reaches the surface of the body. The possibility of the detection therefore also depends on the resolution of the temperature measurement and the time when the measurement is being made. Therefore, if an unexpected disturbance is detected it suggests that an unwanted feature caused it. This is schematically illustrated in Fig.13.2.
Considering that the temperature gradient of the thermal field is decreasing as the heat passes through the panel, a disturbance in this field may become difficult to detect. Fig.13.3 shows a temperature measurement on a foam core sandwich panel as function of time. As can be seen, the temperature gradient is very large on the side of the panel where the heat is introduced (curve 1) but decreases drastically as the heat passes through the panel to become very small on the reverse side. In the case of Fig.13.3, the heat only increases a few degrees in the entire history, and in this temperature gradient a disturbance must be found.

The thermal characteristics of the material is governing the chances of detection of a certain defect. For example, the ratio of the conductivity in the glass fibre, parallel to perpendicular, is approximately 1 whereas in aramid or carbon fibre the same ratio is in round numbers 10. Therefore, the effect of using carbon fibre reinforced faces will reduce the thermal signal.
transmitted perpendicular to the face by a factor of 10 as compared to an otherwise equal glass fibre reinforced face.

There are various ways of detecting thermal information. The methods used include thermocouples, liquid crystals and IR-based detectors. Thermocouples does only provide local monitoring of the thermal event whereas the last two does provide area coverage. Thermocouples measure the change in temperature via the change in resistance detected as change in voltage. Liquid crystals are chemical compounds which emits visible light of various colour depending on the temperature of the substance. IR detectors are the most commonly used thermal measurement system and has an advantage which at times can be beneficial, they do not require physical contact with the object whereas the thermocouple and the liquid crystals do so but are less expensive.

**13.5.4 Methodology**

Depending on what object to be inspected and in what environment there exists a number of methods of how to apply thermal NDT. However, in the following only passive transient thermal NDT on foam-core sandwich structures utilising IR-detectors will be considered. In this context passive means that no mechanical load is introduced into the object while transient refers to the induced thermal load and the subsequent thermal behaviour.

Both transmission and reflection methods can be used, see Figs.13.4 and 13.5. The transmission method has the advantage of providing better in-depth resolution whereas the reflection method is faster. As a result, the transmission method proved to be most useful to detect cavities or shear cracks in the core while the reflection methods is most powerful in detecting flaws in the face and the interface between the face and the core, e.g. disbonds and shear cracks which has reached the interface.

**13.5.5 IR detectors**

There exists a large variety of IR-detectors for various applications. However, the commercially available ones are designed to provide the possibility of visualisation of an IR image. This type of detectors are usually connected to a computer for storage and subsequent manipulation of images. Some of these systems does also provide on-line monitoring of the thermal event viewed by the detector. The camera itself is usually the size of an ordinary video camera and even together with the peripheral equipment, it can all be handled by one single operator. The IR detectors are sensitive to electromagnetic radiation, commonly between 2-5 µm or 8-12 µm because in these wave-lengths the ambient air does absorb very little of the radiated energy. The development of
electronics has provided very sensitive IR detectors and 0.1 °C is a common thermal resolution available today.

**In-field inspections**
The reason for in-field inspections are generally scheduled maintenance, the suspicion of a flaw or a structural failure. However, inspection or testing on the site of operation of the construction frequently impose rather difficult conditions such as harsh environment, area of interest not within reach and so forth, which put special demands on the method and the equipment to be used. An obvious requirement for in-field inspections is that the equipment should be portable. It is also advantageous if the equipment allow for recordings for subsequent investigation. In the case of IR-based thermography both the aforementioned qualities are accommodated for while a third positive feature is provided: no physical contact with the object under test is needed.

**Laboratory or production inspections**
In a laboratory or a production facility the circumstances and the environment are usually easier to control and thus, provides means for more accurate testing. Also, the most common situation is that a special place is prepared with specially designed test set-up for testing of that particular part or structure. In connection to NDT in production, routines are usually prepared which to follow when performing the test. If the production is of the type one-of-a-kind, the NDT situation becomes more like the in-field situation.

**13.5.6 Heat sources**
The literature reports on the use of several types of heat sources ranging from photoflashes via IR heaters to heat blankets. The most important feature to be afforded from the heat source is the capability to provide a uniform heat input. In this work, the IR heaters utilising quarts lamps were chosen because of ease of use and the possibility of high energy output. The main advantage of IR heaters utilising quarts lamps are:

1. Radiant heat transfer is rapid with a low loss factor.
2. The heating starts and stops in effect instantly.
3. Rapid response makes it possible to follow complex heating programmes.
4. The simple reflector unit construction facilitates changes in size and shape of the heater installations.

The stationary module is predominantly used as heat source when applying the transmission method whereas the handheld heater is mostly used for the reflection method.

**13.5.6 Calibration**
Calibration is always of paramount importance when performing any type of measurements, and thermal NDT is no exception.

**Thermal detector equipment**
The are several way to check that the thermal detector is measuring the right exact thermal values. Here the best advice is to check your manual or the manufacturer about what procedures to use when performing calibration of the thermal detector system. When the IR detector system is confirmed OK there are a few concerns when preparing the actual measuring. The most important parameter which plays a significant role in measuring with IR detectors is the emissivity of the
surface of the object to be tested. For exact temperature measurements it is of vital importance to get the right value of the emissivity into the system. Here it is strongly advised to inform yourself with the manufacturer on how to proceed with your particular situation.

**Structural integrity**

The concept of NDT states that it is non-destructive, therefore measures must be taken as to ascertain that no damage is caused to the object under test. In all the thermal NDT test performed in the investigations reported on herein calibrations on a small piece of structure of the same type as the real object, using thermocouples, before the real measurements was performed. The distance of the object from the heater, radiant energy output levels and time of exposure was carefully registered. A certain temperature level was chosen which would ensure that no problems with the structural integrity would occur and the corresponding time of exposure was selected for the subsequent tests. The limiting factor with the foam-core was $T_g$ (the glass transition temperature) therefore the temperature was chosen well below $T_g$. Further, the adhesive in the bond line in the interface between the face and the core was also of concern.

### 13.5.7 Material characteristics

The material characteristics of interest when considering thermal NDT are: conductivity, convection, density and specific heat. All these are included in the diffusion equation which describes the thermal behaviour in a media and therefore have major influence on the thermal NDT process. In general the thermal characteristics of materials are rather poorly investigated although there exists ASTM standards defining procedure to obtain the thermal characteristics.

Scatter in material data will affect the possibility to detect flaws negatively in that it can cause spurious contrasts confusing the operator. Depending on the manufacturing process and the quality standards the manufacturer works with there is always scatter around the nominal quality, e.g. $+x \% - y \%$ of nominal density of core material. Therefore, if the manufacturer of the materials used in a structure provides adequate material data it is recommended to check the scatter in the data and in what temperature range these values are valid. The same consideration should be made when fabricating composites which will be subject for thermal NDT. Generally, components fabricated by hand-lay-up will cause a larger scatter in material data than the equivalent component produced in an autoclave.

Numerical calculations can be used on any type of structure to predict the appearance of a thermal pattern generated from a flaw. However, it is a well established fact that numerical calculations will only be as accurate as the data input. Further, the exact data of the environment in which the test is to be performed is rather hard to find thus, yielding another source of uncertainty. Furthermore, numerical calculations on thermal response usually requires 3-D models to take into account the isotropic or anisotropic effects of the material. Therefore, numerical calculations can provide useful information but it is suggested only to be used as a tool for guidance as to how the structure responds to transient thermal loads and thus on the possibility of detection of typical flaws.
13.6 Examples of Thermal NDT measurements

All the thermal measurements performed presented in this section were done with IR detectors sensitive to electromagnetic radiation in the range of 2-5 or 8-12 µm. The former thermoelectrically cooled and the latter with liquid nitrogen. Both systems could resolve temperature differences of 0.1 °C or better at an object temperature of 30 °C. Most of the tests was carried out in an laboratory environment with equipment more suited for stationary tests, however, some test were accomplished in-field with portable systems and very good results were obtained [20]. The IR-heaters used were both a stationary IR-module and a handheld IR-lamp of maximum energy output of 6 kW and 1 kW, respectively.

13.6.1 Cavities

The tests on cavities presented here [21] were done on prefabricated cavities using PVC foam core material with three nominal densities; 60, 130 and 200 kg/m³. Only one thickness, 30 mm, was tested but with varying cavity sizes, 4, 8, 12 and 16 mm, and cavity depths (the depth from the observed surface to the cavity), 2, 6, 10 and 14 mm.

The set-up for all these tests were; the scanner was placed 0.6 m from the test specimen. The area scanned at this distance was approximately 350×350 mm. The heat source was placed on the other side of the specimen, 0.3 m from the specimen, that is, the transmission method was used and the heat source was aimed at the centre of the specimen. Using 6 kW energy output from the heat source the time of exposure, for the different densities, was set to;

<table>
<thead>
<tr>
<th>Density</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>H60</td>
<td>36 sec.</td>
</tr>
<tr>
<td>H130</td>
<td>1 min. 42 sec.</td>
</tr>
<tr>
<td>H200</td>
<td>2 min.</td>
</tr>
</tbody>
</table>

The chosen exposure times left the core temperature well below $T_g$. Approximately the same time interval was needed until actual establishment, from visual impression of the thermal image, of what was detected. One of the results can be viewed in Fig.13.6.
13.6.2 Disbonds
Both interface disbonds and disbonds within the core, both parallel and perpendicular to the plane, were simulated in [22], see Fig.13.7. The simulation was performed by leaving an area without any adhesive. The parallel types constituted circular areas; 50, 100, and 150 mm in diameter, whereas the perpendicular were half the length of the bond line but through the whole thickness left without adhesive. Using this method a small volume which formed an inclusion of air was obtained. The thickness of the inclusion was approximately a cell size and half the cell size of the core material for the core-to-core and core-to-face disbonds, respectively, see Fig.13.8.

![Figure 13.7 Types of simulated disbonds on which thermal NDT has been tested.](image1)

![Figure 13.8 Drawing of simulation of disbonds: core-to-core and core-to-face](image2)

In these test were three nominal densities included; 60, 130, and 200 kg/m³ and the total core thicknesses were 30 mm, excluding a few specimens of mid-plane disbonds with a total core thickness of 60 mm. The faces was fabricated of glass-fibre reinforced vinylester and the thicknesses varied from 1 to 4 mm.
For the core-to-face disbonds the reflection methods was applied and proved capable to detect all defect sizes. The transmission method, using the handheld IR heater, was applied to the core-to-core disbonds. For the core thickness of 30 mm all defect sizes were detected whereas for the 60 mm thickness only the disbond within the 60 kg/m³ material was detected. This was most probably caused by not obtaining sufficient energy input with the handheld heater. The perpendicular disbonds was also successfully detected using both the reflection and the transmission method.

The time span until the core-to-core disbonds were detected (that is, until the disbonds were readily visible in the thermal image) varied, depending upon what core density was exposed, from 30 seconds to 1 minute and 30 seconds which was approximately the same time of exposure that was used on these core densities. The shorter and the longer time interval corresponding to 60 kg/m³ and 200 kg/m³ respectively. Concerning the core-to-face disbonds the thermal event was very rapid when applying the reflection method therefore, the stored images needed subsequent manipulation, for example, finding the best resolution or the best relative grey scale for best visible interpretation.
13.6.3 Shear cracks

Shear cracks occur due to some type of overloading of the structure, caused by either fatigue or static loads. Shear cracks caused by fatigue usually originate from a fatigue crack, at the centre of the core, parallel to the faces which makes them harder to detect than shear cracks obtained by static loads which, in most cases, have propagate from face to face, and sometimes even in the interface. This type of flaw is detrimental to the constructions capability to transfer load between the faces and therefore reduce the bending stiffness rather dramatically.

![Figure 13.11](image)

Test panels with three different types of prefabricated shear cracks, see Fig. 13.11, were produced [23]. All shear cracks were simulated by a Teflon insert which was to ascertain that no mechanical loads would be transferred across the crack. The reason was that after the panels had been subjected to NDT the panels would be cut into beams and subsequently tested in four-point-bending to investigate the reduction in load bearing capability. The geometrical dimensions of the cross section were; \( t_f = 2 \) mm and \( t_c = 30 \) mm. The faces were made from 8 layers of Fiberite MXB 7701/7781 epoxy/fibreglass and the core was a PVC foam of nominal density of 110 kg/m\(^3\).

Both the transmission and the reflection methods were applied on the test panels. Both were capable to detect the simulated flaws. The former method provided more information which enabled on-line characterisation of the flaw as regards to direction and depth. The thermal event was very rapid when applying the reflection method therefore the resulting images needed subsequent manipulation revealing only the direction of the cracks. The time span, until direction and depth could be decided, for the transmission method was about 3 minutes. Whereas for the reflection method it was only a few seconds, not including the manipulation of the images.

![Figure 13.12](image)
The conclusion from this test was that using an handheld IR heater and a portable IR detector it is indeed possible to scan large areas fast for shear cracks on similar types of structures.

### 13.6.4 J-joints

When building sandwich constructions the j-joint, see Fig.13.14 is frequently used for joining the core material. The putty is put in place in a liquid form, however some times the putty does not fill the gap all the way to the bottom, yielding a cavity causing a potential "hot-spot" in the structure.

![Figure 13.14 Schematic drawing of cross-section of J-joint under fabrication. Note the missing putty.](image)

In-field tests were performed on a large ship-hull comprising j-joints. The reflection method was applied from the bottom side. The method proved to be very rapid in detecting cavities from missing putty, even rather small ones. Also in this case it was concluded that using an handheld IR heater and a portable IR detector it is indeed possible to scan large areas fast for cavities from missing putty.
13.7 Future of NDT

Companies start to realise the importance of easy and reliable maintenance as means of competitiveness and it is a well established fact that the design of the structure has major influence on the maintenance. Thus, the application of plausible NDT techniques will be taken into account when designing new constructions. As a result the methods will be suited for in-field inspections and possibly also customised for a particular application in combination with easy-to-follow inspection-trees which will guide the operator through the inspection.

The development of manufacturing methods will be reflected in the requirements on NDT methods. More automated inspections with the objective of minimised test time which will move the emphasis on NDT equipment from cost to speed. For sandwich structures, particularly in the area of in-service inspections, a move towards the requirement of rapid scan of large areas in combination with equipment for localised investigations to characterise the defects is likely.

References


The idea of designing a structure and taking account for pre-existing defects in the form of cracks was established in the early 1970's, when the U.S. Air Force defined damage tolerance requirements for the design of new military aircraft. A crack is allowed to exist, but must be found during a regular inspection before it reaches a critical length and may cause structural failure. To transfer such concepts to sandwich structures one has to have the ability both to detect the defects and to have control over the defect size and at what time or load they start to grow. The inspection part of this problem is covered in chapter 13 of this book. The defect control part of the problem is twofold:

(i) When a defect is found by the inspection method, reliable tools for estimation of its influence on the structural integrity the found defect has must be readily available. The immediate question to be answered is really one of whether the structure can still be used at the service loads it was designed for, or if it will fail.

(ii) The second part is one of design; the inspection method has the capability of finding a defect of a certain size, depending on parameters like materials, thicknesses, defect shape etc. These minimum size defects that the inspection methods can find, must now be allowed to exist in the design without causing premature failure, that is, the defects will not grow under the service loads of the structure. This is achieved by introducing artificial defects in the design calculations, i.e., FE or analytical methods, at a location where they would be most critical.

In both the above cases it is necessary to have access to engineering tools for the estimation of the load for onset of damage growth for different types of damage, and that is what this section deals with. The section only deals with defects referring to the core or the adhesive joints in sandwich panels. Damage types like those caused by foreign object impact that primarily damage the faces and maybe also the core underneath face are to be addressed in a separate section in a later edition of this book.

14.1 Defects
There are two causes for the presence of defects in sandwich structures; manufacture-induced flaws and in-service damage. Flaws of different kind may arise already in the manufacturing of the constituent materials. Examples of such are cracks and voids in metal face sheets, delaminations in composite laminates, and large voids in cellular foam cores. Flaws in the face sheets are not treated in this text since extensive research efforts are spent on these problems and
several textbooks are already in the market, e.g. [1,2]. Unwanted large voids in cellular foam core may act as an initiation point for crack growth and are hence vital to detect. Thus, this should be done prior to the manufacturing of the sandwich component or structure.

More severe problems may occur in the manufacturing of sandwich panels. Since the build-up of sandwich elements are rather complex with several different materials with very different properties which are joined by adhesive bonding, there are many points of plausible manufacturing errors. Poor or even totally missing adhesive bonding may occur due to errors in the manufacturing process and result in disbands either between blocks (core materials must sometimes be bonded together to achieve a required thickness) or in the interface between the core and the face materials, as illustrated in Fig.14.1a and 14.1b. These flaws, act as stress concentrators, or even as sharp cracks. Since most core materials are manufactured in finite size blocks, they must be joint edgewise using so called butt-joint. Foam core must often be cut in even smaller pieces in order to comply with a certain curvature, thus creating very large numbers of butt-joints. These are often bonded using a filler material and the joint is left wide to ensure an easy filling of the joint. Errors in butt-joints occurs as unfilled joint, partly or even totally as shown in Fig.14.1c, thus leaving a part of the structure empty of load-carrying material but also creating wedges acting as stress concentrators.

In-service damage is, on the other hand, caused by foreign object impact, overloading or fatigue. The defects occurring in this category are for example static core shear cracks illustrated in Fig.14.1d, core or interface fatigue crack formation and propagation, which will appear as manufacture-induced cracks in Figs.14.1a and b. Face indentation due to impact is another in-service damage type but is not treated in this section.

![Figure 14.1 Plausible manufacture-induced flaws and in-service damage in sandwich constructions.](image)

The singular stress fields appearing at geometrical and material discontinuities are commonly used for the prediction of damage growth in components and structures. The analysis presented in this section is based on a linear elastic fracture mechanics (LEFM) approach. For a general wedge problem, one can write the displacement field in the vicinity of a singular point, such as a crack tip, as
\[ u_i = \sum_{n=1}^{N} \sum_{j=1}^{3} K_j^{(n)} r^{\lambda(n)} f_{ij}^{(n)}(y, \phi) \]  

(14.1)

where \( K \) are stress intensity factors and \( \lambda(n) \) eigenvalues or singularity values, and \( f_{ij}^{(n)} \) functions of axial (\( y \) - along the edge) and circumferential (\( \phi \) - in the x-z plane) variation in a cylindrical coordinate system. Often there exists only one \( \lambda(n) \) smaller than unity which will consequently govern the stress field in the vicinity of the singular point, that \( \lambda(n) \) is often referred to as the strength of the singularity. Equation (14.1) is then simplified to

\[ u_i = \sum_{j=1}^{3} K_j r^\lambda f_{ij} = K_I r^\lambda f_{Ii} + K_{II} r^\lambda f_{IIi} + K_{III} r^\lambda f_{IIIi} \]  

(14.1')

where \( K \) are defined in the three principal modes I, II and III corresponding to opening, in-plane sliding and out-of-plane sliding of crack surfaces. Similar modes can be defined for general wedge problems. The singularity \( \lambda \) is generally a complex quantity where for a crack problem \( Re(\lambda) = 0.5 \) and in most practical situations \( Im(\lambda) \) is very small (\( Im(\lambda) \) causes an oscillatory behaviour of the stress and strain field in a close vicinity of the singular point). The stress intensity functions can in such cases be approximated by

\[ K_I = \lim_{r \to 0} \sqrt{2\pi r^{1-\lambda}} \sigma(x, y, 0) \]

\[ K_{II} = \lim_{r \to 0} \sqrt{2\pi r^{1-\lambda}} \tau_{x}(x, y, 0) \]

\[ K_{III} = \lim_{r \to 0} \sqrt{2\pi r^{1-\lambda}} \tau_{y}(x, y, 0) \]

14.2 Core Adhesive Disbond
A disbond appearing between two blocks of core material as illustrated in Fig.14.1a, is due to an adhesive bonding manufacturing error.

(i) Fracture mechanics foundations
The disbond creates a crack which may be approximated as a crack in a homogeneous media. If the core can be treated as isotropic, like most foam cores can, this problem can be addressed using standard LEFM techniques. That is, \( \lambda \) is a real number that equals 0.5. Depending on the loading situation, different modes of deformation of the crack surfaces may appear, crack opening (Mode I) or crack sliding (Mode II). This is analytically expressed as

\[ K_I = \sigma \sqrt{2\pi} \text{ and } K_{II} = \tau_{0} \sqrt{2\pi} \]  

(14.2)

where \( \sigma \) is the remote stress and \( \tau_{0} \) the remote shear stress, and \( K \) will hence have the dimension MPa\(\sqrt{m}\). However, the crack is subjected to a state of almost pure shear (see chapters 3 and 4) and thus the deformation mode is purely Mode II, and \( K_I \) is thus close to zero.
(ii) Finite element analysis
A crack problem like this may be assessed by using crack flank displacement data from an FE-analysis. Firstly, the FE-mesh must be rather fine in the vicinity of the crack tip since the stress gradients are very high in that region. Typically the smallest elements close to the tip are in the order of 0.01\(a\), or smaller. Quadratic elements should be used, and if so, the nodes closest to the crack tip node should be moved to quarter point locations, that is, the distance from the node closest to the crack tip should only one fourth of that to the next node along a line from the crack tip node as illustrated in Fig.14.2.

If the this is done then the elements surrounding the crack tip will display the square root singularity as in theory which hence much improves the accuracy of the calculation. There are several methods to extract the stress intensities from the node displacement output of the calculation. One fairly simple, accurate and reliable method is that of Shih et al. [3], which takes advantage of the quarter point nodes. \(K_I\) and \(K_{II}\) are computed as

\[
K_I = \frac{G\sqrt{2\pi}}{(1+\kappa)} \left[\frac{4w_b - w_c}{\sqrt{L}}\right] \quad \text{and} \quad K_{II} = \frac{G\sqrt{2\pi}}{(1+\kappa)} \left[\frac{4u_b - u_c}{\sqrt{L}}\right]
\]

where \(\kappa = 3 - 4\nu\) in plane strain, \(\nu\) the Poisson ratio, and \(L\) the length of the side of the first element along the crack flank (see Fig.14.2). The plane strain energy release rate \(G\) is then

\[
G_I = \frac{K_I^2(1-\nu^2)}{E} \quad \text{and} \quad G_{II} = \frac{K_{II}^2(1-\nu^2)}{E}
\]

(iii) Predictions
The crack tips in this problem is surrounded by one single media, the core, and subjected to a pure Mode II. The local geometry is thus the same as in the End-Notch Flexure (ENF) specimen described in chapter 11 of this text. One may also assume that the crack will start to grow as the stress intensity \(K_{II}\) reaches the value of \(K_{II}^{ENF}\) obtained by this testing procedure. That is, onset of crack propagation when

\[
K_{II} \geq K_{II}^{ENF}
\]

Some values of this fracture toughness for some different core materials are given in chapter 2.
(iv) Case study - Beam with mid-plane disbond

A sandwich beam is studied which has a disbond in the mid-plane of the beam between two blocks of core material, as illustrated in Fig. 14.3. The disbond is situated in a field where the transverse force $T$, and thus also the shear stress, is constant, e.g., between the outer and inner supports in a four-point bend test.

![Figure 14.3 A sandwich beam with mid-plane core disbond in a constant shear field.](image)

This problem has been solved analytically by Zenkert [4] by using energy principles. The solution is omitted here but should be referred to for more accurate calculations. It was found in [4] that the stress intensity factor $K_{II}$ increases with increasing crack length and increasing core modulus, and decreases with increasing face modulus, face thickness and core thickness. For simplicity reasons, only a few examples are given here. In Fig.14.4, where the geometry factor $g$, defined as

$$K_{II} = \tau_0 \sqrt{2\pi g}$$

with

$$\tau_0 = \frac{T}{2D} \left[ \frac{E_f t_f d}{2} + \frac{E_c t_c^2}{8} \right]$$

is plotted versus crack length for the examples given in Table 14.1. $\tau_0$ is the remote shear stress, i.e., the shear stress in a undisturbed mid-plane of the beam and $T$ is the section transverse force per unit width of the beam. In Fig.14.4 the upper curve corresponds to example 1 in Table 14.1 which has low $E_f$, and high $E_c$, $t_f$, and $t_c$, thus representing an upper extreme case. Similarly, example 4 in Table 14.1 has the opposite, thus representing a lower extreme case. Examples 1 and 2 are meant to illustrate some geometries representative to practical geometries. As seen in Fig.14.4, $g$ varies very little for cases where the geometry and materials are used in practice.

<table>
<thead>
<tr>
<th>Example No.</th>
<th>$E_f$ (GPa)</th>
<th>$\nu_f$</th>
<th>$t_f$ (mm)</th>
<th>$E_c$ (MPa)</th>
<th>$\nu_c$</th>
<th>$t_c$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
<td>0.3</td>
<td>0.5</td>
<td>400</td>
<td>0.32</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>12</td>
<td>0.3</td>
<td>2</td>
<td>80</td>
<td>0.32</td>
<td>25</td>
</tr>
<tr>
<td>3</td>
<td>70</td>
<td>0.3</td>
<td>1</td>
<td>100</td>
<td>0.32</td>
<td>30</td>
</tr>
<tr>
<td>4</td>
<td>210</td>
<td>0.3</td>
<td>5</td>
<td>10</td>
<td>0.32</td>
<td>60</td>
</tr>
</tbody>
</table>

Table 14.1 Geometries and material properties of examples plotted in Fig.14.4.
Now, when the crack length \( a \) is known, \( g \) can be estimated by the use of Fig.14.4, which in turn gives the stress intensity according to eq.(14.6). A \( K^\text{ENF}_{\text{lc}} \) is then obtained for the core material in question and the estimated fracture load can be calculated.

### 14.3 Face/Core Interface Disbonds

An interface disbond between the face and the core as illustrated in Fig.14.1b, could either originate from inaccurate manufacturing (an absent or poor bond line) or from impact damage during service.

(i) Fracture mechanics foundations

In the interface disbond between two materials in a multi-material body, the singularity at the crack-tip differs from that in a homogenous medium. The stress intensification around the crack-tip arises both from material and geometrical discontinuities, whereas in a homogenous body stress intensification arise only from geometrical discontinuities. As a result of this, a single mode loading induces both an opening mode \( (K_1) \) and a shearing mode \( (K_\sigma) \). A way to visualise this is to study an interface crack in a tension field with two extremely different materials as illustrated in Fig.14.5. Material 1 is very stiff with a high Young's modulus and material 2 of the opposite character, representing the face and the core respectively. With such combination of materials there is a large difference in deformation and one could see that the growth of the interface crack is more connected with failure in shear rather than in tension.
In similarity with the homogenous case the stresses are inversely proportional to the square root of the radial distance from the crack-tip, but with the distinction that in an interface crack between dissimilar materials, the stresses exhibits an oscillatory behaviour of the type $r^{-1/2} \sin(\epsilon \log r)$, where $r$ is the radial distance from the crack-tip and $\epsilon$ is a function of material constants. This means that there is an infinite change in sign near the crack tip which indicates that the surfaces of the crack wrinkle and overlap and that the strength of the singularity, $\lambda$ in eq.(14.1), is a complex constant equal to $1/2 \pm i\epsilon$. However, in most practical applications, considering that the oscillating region is extremely small compared to the size of the crack, this phenomenon may be ignored and $r^{-1/2}$ ($\lambda = 0.5$) may still be used.

The face-to-core interface crack in a sandwich is mainly subjected to shear, and thereby the crack growth is dominated by the shearing mode as illustrated in Fig.14.6. The shear stress is of opposite sign at the crack-tips and due to the material discontinuity the crack is unsymmetrical with a large contact zone at one of the crack-tips. In problems with multi-material bodies these contact zones are assumed to be frictionless; this is an idealisation and one should take account for this since these zones probably affects the global solution to be conservative. In similarity with the core adhesive disbond, the stress intensity in the crack region can be written as

$$K_I = \sigma_0 \sqrt{2\pi r} \quad \text{and} \quad K_{II} = \tau_0 \sqrt{2\pi r}$$

(14.7)

on a radial distance $r$ from the crack tip. $\sigma_0$ and $\tau_0$ is the tensile stress and the shear stress respectively in an intact interface.
(ii) Finite element analysis

As mentioned above, in a multi-material body stress concentrations occur in both opening and shearing modes. To compute fracture mechanics parameters for this situation from a finite element model, a method developed by Smelser [5] is presented here. It is based on the numerical displacement of the crack flank, which is the relative displacement between two nodes on each side of the crack surface, initially with the same coordinates.

\[ \Delta u_\theta = \frac{\Delta u_2}{\Delta u_1} \]

This method requires that the crack surfaces are traction-free. The node displacement in a complex coordinate system and the complex crack opening are defined as

\[ u = u_r + i u_\theta \text{ and } \Delta u = u_2|_{\theta=-\pi} - u_1|_{\theta=-\pi} \]  \hspace{1cm} (14.8)

respectively. The subscripts refer to materials 1 and 2 in Fig.14.7b. The magnitude of the crack opening can be written as

\[ |\Delta u| = \frac{1}{4\sqrt{2}} \left( \Lambda_1 + \Lambda_2 \right) \frac{k_0}{\lambda_0} \sqrt{r}, \quad \Lambda_i = \frac{4(1-\nu_i)}{\mu_i} \text{ in plane strain} \]  \hspace{1cm} (14.9)

where \( r \) is the radial distance from the crack tip, \( \lambda = \lambda_0 e^{i\delta} \) the strength of the singularity at the crack tip, with \( \lambda_0 = \sqrt{1/4 \pm i\varepsilon^2} \), \( \varepsilon = 1/2 \pi \ln \gamma \), \( \gamma \) the bi-material constant, and \( \delta = \tan^{-1}(2\varepsilon) \). \( k \), defined as the stress intensity factor, has the polar form \( k = k_0 e^{i\beta} = k_1 + ik_2 \). To find the angle \( \beta \) of the complex stress intensity factor, the finite element node displacement data is first used to obtain the angle of displacement between two nodes

\[ \phi = \arctan \left[ \frac{\Delta u_\theta}{\Delta u_r} \right] \]  \hspace{1cm} (14.10)

Thereby \( \beta \) can be calculated from

\[ \beta = \varepsilon \ln r - \phi - \delta - \frac{\pi}{2}, \]  \hspace{1cm} (14.11)

and using the expressions above, the stress intensity factors, \( K_1 \) and \( K_{ip} \), can be calculated for each node pair on the crack flank. In the vicinity of the crack tip \( K \) is constant and independent of \( r \) and
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a plot of these quantities, taken from a set of node-pairs closest to the crack tip, versus each other gives the mean value for the stress intensity in each mode. When the stress intensities are found, the energy release rate, $G$, can (in plane strain) be computed as

$$G = \left[ \frac{\kappa_f + 1}{G_f} + \frac{\kappa_e + 1}{G_e} \right] \frac{K_I^2 + K_{II}^2}{16} = G_f + G_{II}$$

(14.12)

where $\kappa = 3 - 4\nu$ in plane strain. Due to the extremely high stress gradients in the vicinity of the crack tip the finite element mesh must be rather fine in this area, as a suggestion in the order 1/100 of the crack length or less. Furthermore, it is recommended to move the four nodes closest to the crack tip (actually five, since the crack surface have two nodes on the same coordinate) to quarter point locations and thereby simulate the square root singularity behaviour. It is also advisable to attach each node pair along the crack with a constraint element that prevents negative relative displacement between the two nodes and hence avoid the unrealistic situation of overlapping of the crack surface. Such element has no function as long as the relative displacement is positive or zero.

This method is also applicable on three-dimensional problems by repeatedly, for a number of locations along the crack tip, study intersections of the crack perpendicular to the tip, thus yielding the distribution in stress intensity along the crack tip. This is illustrated in Fig.14.8a-c by a square panel with a quarter-circular disbond located at one corner of the plate. In problems like this the refinement in element size towards the crack tip can not be performed as strictly as for two-dimensional FE-models since the size of the model then becomes too large to handle. Nevertheless, the refinement in the mesh should be made as fine as possible.

Figure 14.8 (a) Sandwich panel with a circular interface disbond in the centre. A quarter of the panel is simulated with a finite element model, (b) and (c) The stress intensity is calculated from the relative node displacement of each intersection c-c, thus yielding the distribution along the crack tip.

(iii) Predictions

The choice of fracture toughness ($K_{IIc}$) test specimen to predict failure depends on the sign of the Mode II stress intensity ($K_{II}$) at the crack tip. Studying a disbond in shear as in Fig.14.6, it is seen that the global shear field implies crack propagation up into material 1 for the left crack tip and down into material 2 for the right crack tip. If a crack is situated in the interface between the face and the core in a sandwich structure, the large difference in material properties brings about two different failure types. The high toughness of the face material forces the crack to propagate along the interface, in the same manner as the Cracked Sandwich Beam (CSB) specimen. If the sign of

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the shear field is reversed and the crack propagates into the core material, the *End-Notch Flexure (ENF)* test specimen should be used. This since the geometry in the vicinity of the crack tip and in the direction of crack propagation corresponds to the *ENF* specimen. The crack starts to propagate when the stress intensity $K_{II}$ reaches the fracture toughness $K_{IC}$, i.e.,

$$K_{II} \geq K_{IC}^{CSB}$$ for crack propagation in the interface

$$K_{II} \geq K_{IC}^{ENF}$$ for crack propagation into the core

Some values of these fracture toughnesses for different core materials are given in chapter 2.

*(iv) Case Study - Beam with interface disbond*

The first object of this case study is a part of a sandwich beam of unit width subjected to a transverse force, $T$ (the shear field is constant over the studied section), and with an interfacial disbond of length $2a$ as illustrated in Fig. 14.9. The stress intensity dominated by Mode II can be calculated from

$$K_{II} = \tau_0 \sqrt{2 \pi a} g(a)$$

in MPa$\sqrt{\text{m}}$, where $\tau_0$ is the remote shear stress, $a$ the crack length and $g(a)$ the geometry factor as a function of the crack length.

![Figure 14.9 A sandwich beam with a face-to-core disbond in a constant shear field.](image)

Table 14.2 shows four different examples of a sandwich beam, with different thicknesses and materials, likewise the beam examples presented in the case study in chapter 14.2. Two representing average values from “realistic” cases (2 and 3) used in practical applications and the other two representing upper (1) and lower (4) extreme, limiting cases to give a hint of the sensitiveness of the geometry factor $g$. 
For a prescribed crack length and for a certain load, the stress intensity in shearing mode, $K_{II}$, can be calculated by using eq.(14.14) and a proper curve in Fig.14.10. Linear elastic fracture mechanics (LEFM) implies that the stress intensity increases linear and proportional to an increased applied load. Thereby the fracture load can be calculated when the fracture toughness ($ENF$ or $CSB$ specimen depending on the load case) is known; 

$$P_{\text{fracture}} = \frac{P_{\text{calc}}K_{lc}^{ENF \text{ or } CSB}}{K_{II}}$$ \hspace{1cm} (14.15)

where $P_{\text{fracture}}$ is the fracture load and $P_{\text{calc}}$ is the load where $K_{II}$ is calculated. A more detailed solution is given in ref.[6].

The second example is a square sandwich panel subjected to an uniformly pressure load with a circular interface disbond at the centre of the panel (Fig.14.11). The size of the panel is 800 by 800 mm. This example, more thoroughly presented in ref.[7], is not as general as the first example, but shows the magnitude of the influence of an interface disbond in a three-dimensional panel problem.
As for the earlier examples in this section, four different material combinations are given to represent some average material combinations and extreme cases. These are the same as in the previous example given in Table 14.2. When the disbond size is known, Fig. 14.12 is used to extract the stress intensity (at the load level $P_{calc} = 1$ MPa). With a proper value on the fracture toughness ($ENF$ or $CSB$ specimen depending which side of the panel the load is applied on), the fracture load can be calculated in the same manner as in the sandwich beam-example.

For all these examples the stress intensity increases when the crack size and core modulus increase, and decreases when the core thickness, the face thickness, and the face modulus increase. In the panel example the variation in stress intensity along the crack front is negligible for small defects ($r < 150$ mm), and varies up to 15 percent for large defects with minima at the diagonals of the panel.

![Figure 14.11](image)

Figure 14.11 A square panel with a circular disbond crack subjected to uniform pressure.

![Figure 14.12](image)

Figure 14.12 The stress intensity in shearing mode $K_\pi$ as a function of the crack length $a$ for four examples of sandwich panels with a circular face-to-core interface disbond at the centre of the panel. $K$ is given in MPa$\sqrt{m}$ for a unit load of 1 MPa.
14.4 Flawed Butt-Joints
The butt-joint bonding blocks of core material edgewise might become flawed during manufacturing, resulting in a joint which is partly or totally unfilled with adhesive. As a worst case a joint without any filler or adhesive, as illustrated in Fig.14.1c, is studied. The analysis assumes that sharp corners, or wedges, are created due to the absence of the filler material and that these corners act as stress concentrators from which fracture is initiated. It should be mentioned that even a perfect butt-joint, that is, the entire joint is filled, creates stress concentrator, however not as severe as if it is unfilled.

(i) Fracture mechanics foundations
In linear elasticity, singular stress and strain fields appear at geometrical and material discontinuities. The singular stress field at the tip of a crack is well known and is widely used for the prediction of fracture and crack growth in components and structures. The bi-material wedge created by the absence of filler material in a butt-joint as shown in Fig.14.1c, thus also creates singular stresses. It is likely that fracture will be initiated from the bi-material wedge corner for that reason, and it is hence justified to chose these points as critical locations for onset of damage growth.

![Figure 14.13 Enlargement of the bi-material wedge in a flawed butt-joint.](image)

A part of the bi-material wedge is shown in more detail in Fig.14.13. The singular stresses appearing at the wedge tip are characterised by the Young's moduli of the materials, $E_f$ and $E_c$, and the Poisson's ratios $\nu_f$ and $\nu_c$. For fracture of a sandwich panel at the wedge of a butt-joint, it is of interest to look at the normal stress component $\sigma_y$ which causes peeling of the adhesive layer. For a one-parameter description in general terms this can be written

\[
\begin{align*}
\sigma_{yy} &= Q_I r^{\lambda - 1} \\
\tau_{xy} &= Q_{II} r^{\lambda - 1}
\end{align*}
\]

(14.16)

where $Q_I$ and $Q_{II}$ are the generalised stress intensity factor and $\lambda$ is the strength of the singularity. $\lambda$ is in general a complex quantity, but for many practical cases it is found to be real valued. $\lambda$ can be calculated either from the studies by Hein and Erdogan [8] or Bogy [9, or by estimating the stress variation along a radius from FE-calculations.
(ii) Finite element analysis

One can determine the parameters of eq.(14.16) by means of finite element calculations by plotting

\[
\sigma_{yy}(r, \varphi = 0) = Q_2 r^{\lambda - 1} \rightarrow \log \sigma_{yy} = (\lambda - 1) \log (Q_2 r)
\]

(14.17a)

\[
\tau_{xy}(r, \varphi = 0) = Q_2 r^{\lambda - 1} \rightarrow \log \tau_{xy} = (\lambda - 1) \log (Q_2 r)
\]

(14.17b)

for a large number of \(r\)-values. Too large \(r\)-values should be omitted since the zone in which the singular field is dominating may be rather small. The derivatives of eq.(14.17) are now

\[
\frac{d\{ \log \sigma_{yy} \}}{d\{ \log (r) \}} = \frac{d\{ \log \tau_{xy} \}}{d\{ \log (r) \}} = (\lambda - 1)
\]

(14.18)

from which \(l\) can be determined. Since the stress gradient is very large in the vicinity of the wedge, due to the singularity, the FE-mesh must be refined in that area. Remember also that the singular stress field only governs the total stress field in the vicinity of the wedge, therefore, \(Q\) can only be evaluated for nodes close to the corner node. Thus, as many nodes as possible (at least 10) should be in that area (within 1 mm from the corner node). \(Q\) can then be plotted versus radial distance \(r\). It is usually found that \(Q\) is fairly constant for several nodes; for small \(r\) (e.g. the first element) the stress gradient deteriorates the accuracy and for large \(r\) the singular field no longer governs, and thus, in the region between evaluation according to eq.(14.16) is plausible. When \(Q_2\) is calculated for a number of positions along the edge so that a plot of \(Q\) versus \(r\) can be made, the extrapolated value for \(r = 0\) is used as \(Q_2\). \(Q\) will have the slightly awkward dimension \(\text{Nmm}^{-3/2}\) (compare with \(\text{Nmm}^{-3/2}\) for the crack problem where \(\lambda = 0.5\)).

(iii) Predictions

In analogy with classical fracture mechanics, under small scale yielding conditions, one can assume that initiation of fracture occurs as soon as a critical value of the generalised stress intensity factor \(Q_z\) is reached. This fracture toughness, now denoted \(Q_{crp}\), is similar to the ordinary fracture toughness values \(K_c\) or \(K_{IC}\) but is valid for the specific strength of singularity \(\lambda\) for this particular wedge case. Some values of \(Q_{crp}\) for some specific cases and materials are given in [10] and [11].

(iv) Case study - Beam with unfilled butt-joint

A beam subjected to a constant shear force field, \(T\), is studied. This could be achieved by placing the defect between an outer and inner support in a four-point bend test. No analytical model has been developed yet for this, or similar, cases. The beam configuration has been analysed using the FE-method for several different geometries [11] in terms of beam geometry, material properties and the maximum stress intensity \(Q\). In these cases two materials were used; Divinycell H100 and Rohacell WF51 (see [11] for details).

A numerical parametric study was performed in order to obtain a relation between geometrical and material parameters of the beam and the calculated stress intensity factor. It was performed keeping the face material constant, while varying the parameters: core thickness (\(t_c\)), face thickness (\(t_f\)), core modulus (\(E_c\)) (isotropic with the same Poisson ratio in all cases) and butt-joint width (\(t_j\)). The aim of this study was to find simple, and for engineering purposes accurate, formulae for the estimation of the stress intensity factors. These could then be used to analyse more complex
problems providing that these have the same local geometry and that the remote stress field easily
can be estimated.

**Configuration with H100 core:**

\[ Q_I = 0.2553 t_f^{-0.7214} t_c^{-0.2427} E_c^{0.1922} t_j^{0.0896} \]  \hspace{2cm} (14.19a)

\[ Q_{II} = 0.0597 t_f^{-0.7925} t_c^{-0.2012} E_c^{0.1927} t_j^{0.0898} \]  \hspace{2cm} (14.19b)

for \( 1 \leq t_f \leq 5, 15 \leq t_c \leq 50, 85 \leq E_c \leq 200, 1 \leq t_j \leq 15 \) and face material of type (A)

**Configuration with WF51 core:**

\[ Q_I = 0.1427 t_f^{-0.4141} t_c^{-0.1120} E_c^{0.1633} t_j^{0.2683} \]  \hspace{2cm} (14.20a)

\[ Q_{II} = 0.0384 t_f^{-0.2947} t_c^{-0.0060} E_c^{0.1251} t_j^{0.3531} \]  \hspace{2cm} (14.20b)

for \( 0.96 \leq t_f \leq 4, 10 \leq t_c \leq 40, 60 \leq E_c \leq 130, 1 \leq t_j \leq 8 \) and face material of type (B)

These approximate formulae give surprisingly good correlation with the FE-results. Observe,
however, that they have limited use, i.e., they are only valid for the given type of face material and
only within certain limits of the variables \( t_f, t_c, E_c \) and \( t_j \).

Once \( Q \) is estimated by extrapolations of the above results or from FE-calculations, the fracture
load is found from

\[ P_{fracture} = \frac{Q_{cr}}{Q_I} \]  \hspace{2cm} (14.19)

The final fracture will be one that grows along the face/core interface, initiated in two opposite
corners of the butt-joint.

Fracture toughness data are essential in the analysis of cracks, disbonds or wedges in any type of
material. The *toughness* is a material constant if measured and used correctly and describes at what
load a discontinuity, like a crack will start to grow. Information on how to extract these properties
are given in section 11 on testing of sandwich materials.

**References**


 Quadratic Isoparametric Elements”, *International Journal of Fracture*, Vol 12, 1976, p 647-
651.


Techniques to manufacture sandwich components for structural applications are summarised and discussed in terms of processing steps, characteristics of both technique and manufactured component, and application examples. The emphasis is on the commercially most common manufacturing techniques, though less common processing routes are briefly discussed as well. The intentions of this review are two-fold: First, the paper aims to provide a means of comparing available manufacture techniques in terms of feasibility for a specific application; second, the paper aims to provide a baseline for further research and development efforts in the field of sandwich manufacture. A discussion of recent developments and future trends in terms of both materials and processing routes rounds up the paper.

The structural sandwich concept involves the combining of two thin and stiff faces with a thick and relatively weak core. By sandwiching the core between the two faces and integrally bonding them together, a structure of superior bending stiffness and low weight is obtained. Since the core often has exceptional insulative properties, the entire sandwich structure may further be characterised by excellent thermal insulation and also acoustic damping at certain frequencies.

Sandwich structures are used in a wide variety of applications, such as in automobiles, refrigerated transportation containers, pleasure boats and commercial vessels, aircraft, building panels, etc. The face materials commonly used include sheet metal and fibre-reinforced polymers, while common core materials are balsa wood, honeycomb, and expanded polymer foam. These materials and material combinations all have their own market share and one or more of advantages such as low cost, high mechanical properties, good thermal and acoustic insulation, fire retardancy, low smoke emission, compliance, ease of machining, ease of forming, etc.

While the sandwich concept is used in an impressive variety of applications, the techniques employed to manufacture components tend to be few and usually involve a large degree of manual labour. This chapter describes and discusses the most common manufacturing techniques in some detail, but also briefly cover some of the more unorthodox techniques that have been documented in the literature. By attempting to compile the state of the art of sandwich manufacture, where comparatively little work has been documented in the open literature, the aim is still to create a

baseline for future research and development efforts into the relatively immature field of sandwich manufacture.

15.1 Face Materials
Face materials used in sandwich applications may, for the purpose of this paper, be divided into fibre-reinforced polymer composites and other materials. Polymer composites are unique as face material in that they under certain conditions may be laminated directly onto the core. Alternatively and regardless of material type, the face may be manufactured in one step and then bonded to the core in another step at a later stage.

In terms of fibre-reinforced polymers, all constitutive materials commonly used in composites applications are used also as sandwich face materials. Reinforcements thus include all kinds of glass, carbon, and aramid; similarly, virtually all types of thermoset and thermoplastic resins are used. When performance so dictates, preimpregnated unidirectional or woven reinforcement (prepreg) is used to obtain high-performance faces and/or to gain some manufacturing advantage. Prepregs are usually composed of glass- or carbon-fibre-reinforced epoxy (EP). When composite faces are laminated directly onto the core in marine and transportation applications, glass reinforcement in the form of random mats (CSM), fabrics, and combinations thereof dominate, while resins used in such applications normally are unsaturated polyesters (UP) and sometimes vinylesters (VE). The norm when laminating faces directly onto the core in aerospace applications is to use epoxy prepregs reinforced with carbon fibres. However, also more advanced marine and transportation applications tend to employ various kinds of prepregs to an increasing degree. Naturally, all types of composite faces may be premanufactured in any conventional composites-manufacturing process, meaning that no material combination is excluded from being used as face material.

In addition to the previously mentioned material forms, faces may readily be premanufactured using moulding compounds in compression moulding. These moulding compounds include sheet-moulding compound (SMC), bulk-moulding compound (BMC) and glass-mat–reinforced thermoplastic (GMT). SMC and BMC are thermoset-based compounds, normally UP, whereas GMT is thermoplastic-based, normally polypropylene (PP). In most cases, the moulding compounds are reinforced with discontinuous and/or randomly oriented glass fibres.

Non-composite face materials are always manufactured in one step and bonded to the core in another. Common examples include wood veneer, sheet metal, and unreinforced polymers, although the latter rarely results in a structurally capable sandwich component. The most common face material in this category by far is sheet metal, which offers good properties at reasonable cost but with a weight penalty; applications include refrigerated transportation containers and construction elements.

15.2 Core Materials
The earliest material used as core in sandwich components was balsa wood, which still is used in some applications although alternative core materials tend to replace balsa in an increasing number of applications.
The most common core materials used in all applications except aerospace are expanded polymer foams, which are often thermoset to achieve reasonably high temperature tolerance, though thermoplastic foams are used as well. Almost any polymer may be expanded, but the most common ones in sandwich applications are polyurethanes (PUR), polystyrenes (PS), polyvinylchlorides (PVC), polymethacrylimides (PMI), polyetherimides (PEI), and polyphenolics (PF). PUR may be in-situ foamed between the faces and thus does not need to be prefoamed into blocks. In-situ foaming thus may eliminate the need to form or machine complex core geometries.

Although some high-performance cores, such as PMI and PEI, are used in aerospace applications, honeycomb cores clearly dominate over alternative materials. Any of several materials may be used to manufacture a honeycomb core: sheet metals; fibre-reinforced polymers; unreinforced polymers; and papers. The most common honeycomb cores are based on aluminium and aramid-fibre paper dipped in phenolic resin.

15.3 Wet Lay-Up
Wet lay-up is one of the oldest but still one of the most commonly used methods to manufacture sandwich components with composite faces. The method is very flexible yet labour-intensive and thus best suited for short production series of especially large structures. The Procedure section commences with descriptions of wet lay-up of single-skin laminates (e.g. [1,2]) followed by a description of how these methodologies also may be used to produce sandwich components (e.g. [1,3-6]).

15.3.1 Procedure
Wet lay-up of laminates may be performed either by hand lay-up or spray-up. A schematic of the hand lay-up process is shown in Fig.15.1. The process uses a one-sided mould, male or female, which is treated with a mould-release agent. Normally, a neat resin layer, a gel coat, is deposited directly onto the mould and is allowed to gel before lamination starts. The gel-coat resin usually is of high-quality and has good environmental resistance, thus allowing the use of a lower-quality, cheaper resin within the actual laminate. The gel-coat also produces a smooth, cosmetically appealing surface that hides the reinforcement structure, which otherwise may be visible on the composite surface.

An appropriate amount of resin is applied and distributed on top of the mould whereupon the dry reinforcement, typically in mat and fabric form, is placed on top, see Fig.15.1. The resin is worked
into the reinforcement (upwards, through the reinforcement) with a hand-held roller, which also compacts the laminate and removes voids. After one reinforcement layer has been satisfactorily impregnated and compacted, the impregnation step is repeated until the desired number of reinforcement layers have been applied or the desired laminate thickness has been reached. The lamination is often finished with a top-coat that is similar to the gel-coat in function and composition.

Spraying of a mixture of resin and discontinuous fibres is an alternative to hand lay-up that reduces the manual work required for impregnation and lay-up. A special gun, which may be hand-held or mounted on a robot, is employed to deposit the fibre-resin mixture onto a one-sided mould, see Fig.15.2. The chopped roving may be combined with fabrics or mats on the mould if the fibre content in the sprayed mixture intentionally is kept low.

![Figure 15.2 Schematic of wet spray-up.](image)

Although much of the manual work of wet lay-up is reduced by employing spray-up, manual rolling still may be required and may be important in the final compaction of the laminate to remove voids and improve impregnation. The use of discontinuous and randomly oriented fibres, as opposed to a certain degree of continuous and oriented fibres in hand lay-up, results in inferior mechanical properties.

The lamination procedure described above may also be used to manufacture faces of sandwich components. In most cases, the faces are laminated directly onto the core, which then acts as the mould. With this procedure, the core is often primed to improve adhesion to the faces. The primer, usually pure resin of the same type as that used in the faces, is applied to the core and is allowed to gel or even cure before lamination commences. Alternatively, if traditional moulds (such as in Figs.15.1 and 15.2) are used, the core may be placed on top of the laminated but not cross-linked laminate or it may be adhesively bonded onto the fully cross-linked laminate as further discussed in the Adhesive Bonding section below. If good surface finish is required, fully closed moulds may be used; a gel coat is then applied onto each mould half and faces laminated on top, whereupon the mould is closed with the core sandwiched in between.

Pleasure boats and larger ships built according to the sandwich concept are manufactured in a fashion somewhat different from that described above (e.g. [1,3-6]). Core planks are nailed onto a male wooden frame using finishing nails, thus crudely forming the hull, see Fig.15.3. For plane
and low-curvature surfaces large, flat core blocks can be used, while sharply curved structures can be formed by using smaller blocks or by grooving the core in one or several directions to allow for bending and shaping. Alternatively, cores bonded to a light-weight fabric may be periodically sawed almost all the way through to the fabric in two perpendicular directions to form small blocks, thus allowing a significant degree of draping. In all these cases, the gaps between the core sections are spackled using a putty that is typically based on the same resin as that used as laminate matrix. When the putty is fully cross-linked and the nails are removed, the structure is carefully sanded and sanding dust removed to provide a smooth surface for subsequent lamination. The core structure is then used as a male mould and the aforementioned lamination takes place directly onto the core. Since the cross-linked resin act as an adhesive, the bonding between the face and the core is sufficient if the lamination is correctly performed. Priming of the core as described above is common. When the lamination is completed and the laminate fully cross-linked, the hull is rotated and the wooden frame removed. The lamination procedure is then repeated on the inside of the hull. Since no exterior mould is used with this manufacturing method, at least the external laminate must be spackled, sanded and polished if a smooth surface with good dimensional tolerances is required. Top coat and/or paint layers also may be applied to protect the laminate as well as for purely cosmetical reasons.

Figure 15.3 Core planks are nailed to a wooden frame, thus creating a male mould onto which the outer face is laminated. Reprinted from [1] with permission from Prof. K.-A. Olsson.

Loading points may be integrated into the core prior to application of faces through introduction of high-strength (i.e. high-density) core or metal inserts. Hull members such as bulk heads, deck sections, etc. are normally prefabricated separately and are then laminated onto the hull.

Regardless of whether a laminate has been hand laid-up or sprayed-up, as well as whether it is part of a sandwich structure or not, it must be consolidated through cross-linking of the resin. Successful cross-linking normally requires precise control of laminate temperature and pressure as functions of time. The simplest cross-linking requirements possible clearly are ambient conditions, i.e. room temperature and no externally applied pressure, which with some resins result in reasonably well-consolidated laminates. Resins that start to cross-link at room temperature heat up as the exothermal cross-linking process progresses, thus further fuelling the reaction, which normally is sufficient to ensure proper cross-linking. However, many resins require an increased temperature for cross-linking to start and often also require that a specific temperature-time relationship is followed throughout cross-linking. Although specific temperature requirements
may be sufficient to properly cross-link the resin, the quality of the laminate is unlikely to end up very good unless externally applied pressure is used to compact the laminate as it sets, i.e. until the cross-linking is more or less complete. Further, vacuum is often used to minimise void creation.

The laminate temperature during cross-linking may be controlled through heating of the mould and/or the surrounding atmosphere. In the simplest case, heating of the surrounding air may be achieved by blowing hot air under a tarpaulin “tent“ covering the laminate. External pressure may be applied using a closed mould, a gas-filled bladder, weights, a hydraulic press etc. To use vacuum, a flexible rubber membrane called a vacuum bag is placed on top of or around the laminate and the edges of the membrane are sealed to prevent leakage, see Fig.15.4. Since the vacuum bag also applies a certain degree of pressure onto the laminate, this may for many applications be sufficient to properly compact the laminate. However, the most refined way to exactly control the cross-linking conditions is to use an autoclave, which is a pressure vessel capable of exactly controlling the temperature and pressure of the internal atmosphere. In addition to controlling pressure of the internal atmosphere, i.e. outside a vacuum bag, the autoclave may be used to draw vacuum under the vacuum bag. In terms of temperature, the autoclave can control both the temperature of the internal atmosphere and of the mould, if required. Nevertheless, it is rare that an autoclave is used to cross-link wet laid-up laminates.

Unsaturated polyesters heavily dominate the wet lay-up processes, but vinylesters are also used to some degree in more demanding applications. Although not very common, epoxies formulated for low-temperature cross-linking may be used as well. Just as clearly as polyesters dominate as resin, glass—and specifically E-glass—dominates as reinforcing fibre. In hand lay-up, woven fabrics, random mats and stitched combinations thereof are used, whereas spray-up usually employs roving that is chopped to lengths in the range 10-40 mm. Both balsa and a variety of expanded foam cores are used; by far the most common is cross-linked PVC.
15.3.2 Characteristics

The wet lay-up methods:

- require small capital investments
- typically use resins that cross-link at room temperature with little or no applied pressure and that are tolerant to variations in processing temperature
- use simple tooling due to modest cross-linking requirements
- are labour intensive
- are very cost-effective for short production series and prototype production
- are suitable for any size structures, notably very large
- bring on worker health concerns due to the active chemistry of the resin (especially spray-up)

Sandwich components manufactured through the wet lay-up methods are characterised by:

- modest mechanical properties (especially with spray-up)
- matrix and void contents in the faces strongly depends on the skill of the workers (high contents with spray-up)
- laminate quality depends on the skill of the workers
- no well-controlled exterior surface if both laminates are laid-up directly onto the core (but one or two controlled surfaces obtainable with use of rigid, closed moulds)

15.3.3 Applications

Due to low capital and high labour costs, wet hand lay-up is used for products manufactured in short or very short series and where the requirements on structural and environmental properties are not excessive, typically meaning low to moderate loads and ambient temperatures. Applications include [1,4,5,8]:

- motor and sailing yachts
- mine-sweepers and high-speed passenger ships
- refrigerated truck and railroad containers
- storage tanks

Particularly interesting applications are ships, since they are the largest composite components built. Mine-sweepers, or mine-counter-measure vessels (MCMV), longer than 50 meters, wider than 10 meters and with displacements up to 400 metric tons have been built [7]. Although sometimes built as single-skin structures, they are more commonly built according to the sandwich concept using foam cores. In this application, composite materials and wet hand lay-up offer substantial advantages; the non-magnetic material is highly relevant so as not to risk detonating magnetically sensitive mines; the foam core sandwich structure is damage tolerant to under-water detonations; wet hand lay-up is highly competitive, since large structures are manufacturable and few look-alike MCMVs are built.

Similarly, surface-effect ships (SES) are capable of transporting up to four hundred passengers at speeds over 50 knots. The SES is a catamaran with curtains between the two keels in bow and stern allowing an air cushion to be maintained between the keels; the air cushion lifts the hull partly out of the water to significantly reduce drag, thus allowing high speeds to be reached. The largest composite sandwich SES built measures approximately 40 by 15 meters [7]. Fig.15.5 shows the Swedish stealth vessel SMYGE, which is a 30 m long SES prototype craft built according to
the sandwich concept using aramid- and glass-fibre-reinforced vinylester/polyester in the faces and PVC foam cores. SMYGE has a displacement of 140 metric tons and permits speeds up to 50 knots in calm waters. In SES applications composites help achieve a low enough weight to suspend an entire ship on an air cushion and hand lay-up is once again highly competitive for size and series-size reasons.

Figure 15.5 The Swedish SES SMYGE is built using stealth technology. Courtesy of the Swedish Materiel Defense Administration (FMV).

While MCMVs and SESs are spectacularly successful case studies of wet hand lay-up of sandwich structures, they are nevertheless relatively rare. In contrast, the same techniques are in wide-spread use in manufacture of yachts and various tanks and containers.

The main differences in applications between hand laid-up and sprayed-up composites are due to the differences in labour costs and mechanical properties. The lower labour cost of spray-up implies that longer series are economically feasible and the inferior mechanical properties achieved mean that commodity-type products are more common. Sprayed-up sandwich components include:

- Small pleasure boats
- Storage containers and tanks

15.4 Prepreg Lay-Up

Popular as wet lay-up is, it is at best limited to moderately loaded structures due to the materials used, the cross-linking conditions employed, and the manner in which the impregnation is accomplished. Both single-skin laminates and sandwich structures for more advanced structures, e.g. for competition yachts and in aerospace applications, tend to be laid up using prepregs. The use of prepregs ensures that the reinforcement is well impregnated and resins used in prepregs also tend to have better properties than the ones available for wet lay-up. However, prepreg resins typically require well-controlled cross-linking conditions, meaning that the temperature must be increased above room temperature. For consolidation, a vacuum bag is likely required and in advanced applications also an autoclave.
15.4.1 Procedure
Manufacturing of sandwich components with faces made from prepregs may be accomplished in two overall manners. Under certain conditions, the laminate may be laid directly onto the core in a fashion similar to that used in wet hand lay-up (e.g. [9-11]). Alternatively, previously manufactured single-skin laminates may be adhesively bonded to the core in a separate process (see further the Adhesive Bonding section below). Following the previous section, also this section commences with a description of how a single-skin laminate is manufactured (e.g. [12]), whereupon descriptions of how sandwich components with faces made from prepregs may be laid-up directly onto the core.

To manufacture a single-skin laminate from prepregs, a one-sided mould is first treated with a mould-release agent, whereupon prepreg sheets are placed one at a time on top of one another, carefully ensuring that no voids or other contaminants (such as prepreg backing paper) are entrapped and that the sheets are thoroughly tacked to each other. Lay-up of prepregs is most often performed manually but may be automated. Automated tape lay-up combining the cutting, lay-up and compaction processes is becoming an accepted process in the aerospace industry to manufacture unidirectionally reinforced flat or nearly flat components (e.g. wing skins) [13,14]. In high-performance applications, the lay-up is performed in specially conditioned rooms to further minimise contaminants in the laminate.

Since almost all resins used in prepregs require controlled temperature and pressure to varying degrees to achieve intended properties, cross-linking usually occurs under a vacuum bag and with heat applied; in high-performance applications autoclaves are used [15]. To prepare the laid-up prepreg stack for cross-linking, it is covered with a perforated separator, a bleeder ply, a second separator, a barrier, a breather ply, and a vacuum bag (see Fig.15.6). The separator ensures that the part can be released, while the bleeder ply absorbs excess resin squeezed out of the prepreg stack. The barrier prevents the resin from diffusing into the breather, which ensures that the vacuum pressure of the vacuum bag is evenly applied. In several types of applications moulds are complemented with so-called caul plates which may be elastomeric or rigid, normally of metal. Elastomeric caul plates may be added on top of the prepreg stack to improve the surface finish of the part by ensuring more even pressure. Cast or moulded elastic caul plates may also be used to eliminate bridging over concave areas through application of localised pressure. Rigid caul plates are used to allow precise geometrical control at edges, holes, flanges, etc. by not allowing resin bleeding and thus tapering of the part.

![Schematic of vacuum-bag assembly.](image-url)
After the vacuum-bag assembly is completed, the consolidation process starts with evacuation of the bag; vacuum may or may not be maintained throughout the moulding operation. Pressure is then applied and the temperature gradually increased to the specified resin cross-linking temperature, which is maintained for a significant amount of time. After cross-linking has been completed, vacuum and pressure are released and the temperature gradually lowered. Note that the processing conditions vary with material system but, in general, a typical processing sequence for an epoxy prepreg system may be:

- apply release agent to mould
- arrange vacuum-bag assembly on mould, e.g. as illustrated in Fig.15.6
- place in autoclave and apply vacuum
- apply specified pressure and release vacuum
- increase temperature to specified temperature at specified rate
- maintain temperature and pressure for specified time
- cool at specified rate

It is often possible to lay prepregs directly onto the core and cross-link the face laminate already in place in a process similar to that common in wet lay-up. In aerospace applications, both direct lay-up onto the core [9,11,16] and separate face manufacture with subsequent bonding [9,16-18] are used. Prepreg lay-up directly onto the core is of increasing interest also in the ship-building industry [10] to improve properties and to reduce the worker health hazards associated with wet lay-up. Also with prepreg lay-up, balsa and foam cores may need to be primed and/or a separate adhesive film added to achieve sufficient face-core bonding.

Since the aim of using prepregs instead of wet lay-up usually is to achieve improved composite properties, higher-performance materials are more common. Thus, carbon and aramid fibres are commonplace, although S-glass is also used; the fibres are continuous and oriented in weaves or merely aligned. In most cases the matrix is an epoxy, especially for aerospace and other advanced applications. With such high-performance prepregs, it is natural to use high-performance cores with high temperature tolerance, e.g. honeycombs and PMI and PEI foams. Recent developments in prepregs with low-temperature cross-linking polyesters and epoxies, typically glass- or perhaps carbon-reinforced, appear to be attractive as direct replacements of wet lay-up polyesters; in such applications the likely cores are expanded foams, e.g. PUR and PVC, and balsa.

15.4.2 Characteristics
The prepreg lay-up method:

- requires medium capital investments, high if autoclave is used
- uses resins that require increased temperature, vacuum, and often externally applied pressure to cross-link as intended and that are fairly intolerant to variations in processing conditions
- is labour intensive
- is suitable for short production series
- is suitable for structures of any size

Sandwich components manufactured through prepreg lay-up are characterised by:

- good to excellent mechanical properties
- low void contents in the laminate faces
MANUFACTURING

- consistent laminate quality
- no well-controlled exterior surface if both laminates are laid-up directly onto the core (but one or two controlled surfaces obtainable with use of rigid, closed moulds)

15.4.3 Applications
Prepreg lay-up is most suitable for manufacturing of products in short series where the need for good or exceptionally good properties can motivate the higher costs. Lay-up of prepregs directly onto the core is common in the aerospace industry, where applications include [9,16,18-20]:
- vertical and horizontal stabilisers
- control surfaces
- landing-gear doors
- floors
- rotor blades and servo flaps

As an example of high-performance applications, Fig.15.7 shows the Swedish military aircraft JAS 39 Gripen where the canard wing, the vertical stabiliser and access doors are sandwich elements manufactured from carbon-fibre-reinforced epoxy prepregs and aluminium honeycomb cores.

![Figure 15.7 The Swedish military aircraft JAS 39 Gripen. Courtesy of SAAB Military Aircraft. Photo by A. Nylén.](image)

As previously mentioned, lay-up of prepregs directly onto the core is of significant interest in the ship-building industry to improve both properties and work environment when compared to the wet lay-up processes. Nevertheless, this practice is still typically limited to competition yachts and the established wet hand lay-up technique still dominates this industry.

15.5 Adhesive Bonding
The two previous sections have concentrated on methods to manufacture sandwich components in a one-step procedure. Although clearly desirable from an economical perspective, it is not always possible or desirable to do so. In such cases the faces and the core may instead be bonded to each other in a separate manufacturing step [9,17,18,21-25], which for example is the case when the faces are not polymeric in origin where there is no alternative to a separate bonding step.
15.5.1 Procedure
Conceptually, adhesive bonding of faces and core is quite simple and independent of face and core materials, see Fig.15.8. Adhesive layers are interleaved between the faces and the core and the whole stack is subjected to increased temperature and pressure as required by the adhesive resin, whereupon the sandwich is cooled. For high-performance applications the bonding process likely takes place using a vacuum bag and an autoclave, whereas for less demanding applications it may be sufficient to use a vacuum bag and/or weights or a hydraulic press. Since there should be little or no resin bleeding if the bonding is correctly performed, the vacuum-bagging arrangement is simplified when compared to laminate manufacture.

![Figure 15.8 Schematic of adhesive bonding of sandwich structures.](image)

It is normally necessary to prepare the surfaces to be bonded in order to achieve a good enough bond. Unless already done, foam cores are typically sintered and all loose particles removed; they may also be primed. Laminates intended for bonding are often manufactured with a peel ply, which is removed right before bonding to leave a clean and somewhat rugged surface. The surface still should be abraded to ensure proper adhesion. Metal faces should be abraded and chemically treated to promote adhesion.

The processing conditions vary with material system but in general a typical processing sequence for bonding of composite laminates to a honeycomb core using an epoxy adhesive may be:

- remove peel plies, abrade surfaces, and wash with solvent
- apply adhesive film onto faces and place on core
- arrange vacuum-bag assembly on mould
- place in autoclave and apply vacuum
- apply specified pressure and release vacuum
- increase temperature to specified temperature at specified rate
- maintain temperature and pressure for a specified period of time
- cool at specified rate
Face materials used may be composite laminates manufactured through prepreg lay-up or any other composites manufacturing technique capable of producing the required face geometry, or sheet metal. In advanced applications, the faces tend to be fibre-reinforced epoxies and the core Nomex or aluminium honeycomb or high-performance and high-temperature–tolerant expanded foams, e.g. PMI or PEI. Metal-faced sandwich structures typically have foam cores, such as PUR and PVC. The adhesive is used in film or liquid form depending on application and is usually epoxy or PUR. While thermoset adhesives dominate, thermoplastic (“hot-melt”) adhesives are used as well. Adhesives are discussed in references [21-23,25,26].

15.5.2 Characteristics
Adhesive bonding:

- requires small to medium capital investments, high if autoclave is used
- typically uses adhesive resins that require increased temperature and externally applied pressure to achieve intended properties
- is labour intensive
- is suitable for short production series
- is suitable for small to medium-sized structures

Sandwich components manufactured through adhesive bonding are characterised by:

- good to excellent mechanical properties
- well-controlled surfaces (assuming the faces have at least one good surface)
- potentially having partially failed bonds in curved sections due to geometric mismatch between preformed faces and core

15.5.3 Applications
Adhesive bonding of faces and core is rather common in the aerospace industry and the applications are the same as for direct prepreg lay-up [9,16-20,25]. The most common applications of metal-faces sandwich structures are various shipping containers for refrigerated goods, aircraft cargo containers, and construction elements. An interesting application is the external structure of the Stockholm Globe Arena shown in Fig.15.9 which is made of sandwich panels with aluminium faces bonded to the core material.

![Figure 15.9](image_url)
15.6 Liquid Moulding
Several related liquid-moulding processes are used to manufacture sandwich components. They all have in common that the reinforcement is first placed in the mould whereupon the liquid resin is infused into the reinforcement fabric through the difference in pressure. The liquid-moulding processes used include:

- resin transfer moulding (RTM) [27-33]
- structural reaction injection moulding (SRIM) [27-29]
- vacuum-injection moulding [27,29,34,35]

Liquid moulding has received much interest in recent years due to its capability of producing geometrically complex structures in an economical fashion, without creating an unhealthy work environment since the processes use closed moulds. Especially RTM is becoming increasingly popular in the automobile industry to manufacture components for vehicles produced in short series.

15.6.1 Procedure
The initial procedure description below is generic and covers all three aforementioned liquid-moulding processes, whereupon the distinguishing differences are pointed out.

The reinforcement, in the form of fabrics and mats or preforms, is placed in the mould together with the core, normally by hand. However, not only cores, but also inserts and fasteners are easily integrated into the reinforcement or the core before impregnation. After the mould is closed, the resin is introduced into the mould to impregnate the reinforcement using pressure and/or vacuum. The resin is often heated to lower the viscosity and thus facilitate impregnation. The resin infusion is stopped when the resin front has reached all the ventilation holes in the mould and the resin starts to leak out. The resins used may cross-link at ambient temperature or, alternatively, the mould may be heated. The cross-linking reaction should not begin until the mould is nearly filled, as gelation of the resin will prevent it from impregnating the reinforcement completely, thus creating dry spots and voids.

The processes of RTM, SRIM, and vacuum-injection moulding are distinguished from one another primarily by the type of resins, moulds, and impregnation technique used, see Table 15.1. In vacuum-injection moulding conventional preformulated resins, similar to those used in the wet lay-up processes, are used. In contrast, SRIM employs highly reactive resins that are mixed right before injection. Although the resins used in RTM also are similar to those used in vacuum-injection moulding, they may either be preformulated or mixed right before injection. In RTM and SRIM rigid, matching moulds are used, whereas vacuum-injection moulding employs a one-sided mould, often a marginally modified version of a wet lay-up mould, covered by a vacuum bag. The mould halves are closed using clamps or a press, depending on part size and injection pressure. In RTM and SRIM the resin is injected into the mould under pressure, in RTM sometimes assisted by drawing vacuum at the ventilation ports, whilst in vacuum-injection moulding the sole force driving the impregnation is vacuum drawn from under the vacuum bag. Fig.15.10 illustrates RTM or SRIM and Fig.15.11 vacuum-injection moulding.
The differences in resin reactivity between RTM and vacuum-injection moulding on the one hand and SRIM on the other translate into two major differences: with the former the mould fill times range from a few minutes to a few hours; in the latter case enabling filling of large parts before the increasing resin viscosity prohibits further impregnation. In contrast, the fill times in SRIM are usually less than a minute due to the much higher resin reactivity, meaning that only smaller parts than with the other two processes are manufacturable. On the other hand, the cross-linking times are in the range of a few minutes to a few hours for RTM and a few hours with vacuum-assisted moulding, whilst in SRIM parts can be demoulded in a matter of a few of minutes. The higher injection rates of SRIM increase the potential problem of so-called fibre washout, which is when the reinforcement is moved by the advancing resin front. The manufacture of large, complex parts with high fibre volume fractions thus is more difficult with SRIM than with RTM.

In SRIM, the two-component resin is mixed right before injection in an impingement-mixing nozzle using dedicated pumps. In RTM injection may be achieved with a dedicated pump or a simple pressure pot, which is a closed vessel containing the resin; in the latter case the resin is forced out of the pot through injection of pressurised air. With dedicated pumps the resin is mixed right before injection also in RTM. In all three processes multiple inlet ports may be necessary for large components. An alternative way to facilitate complete wetout of large components, especially in RTM, is to inject the resin into a channel running around the entire exterior of the part, thus allowing the resin to impregnate the reinforcement from all sides concurrently. Vacuum is drawn or air allowed to escape from the centre of the part in one or more locations, so the impregnation occurs from the perimeter and into the centre of the part.
<table>
<thead>
<tr>
<th>Resin formulation</th>
<th>RTM</th>
<th>SRIM</th>
<th>Vacuum-Injection Moulding</th>
</tr>
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<tr>
<td></td>
<td>Preformulated or mixed right before injection</td>
<td>Mixed right before injection</td>
<td>Preformulated</td>
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<tr>
<td>Resin reactivity</td>
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<td>High</td>
<td>Low</td>
</tr>
<tr>
<td>Mould type</td>
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<td>Matching, rigid</td>
<td>Open-faced; vacuum bag</td>
</tr>
<tr>
<td>Mould material</td>
<td>Composite, metal</td>
<td>Composite, metal</td>
<td>Composite, metal; vacuum bag</td>
</tr>
<tr>
<td>Impregnation</td>
<td>Pressurised injection, may be complemented by vacuum</td>
<td>Pressurised injection</td>
<td>Vacuum</td>
</tr>
<tr>
<td>Cycle time</td>
<td>Long</td>
<td>Short</td>
<td>Long</td>
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<tr>
<td>Part size</td>
<td>Small to large</td>
<td>Small to medium-sized</td>
<td>Small to large</td>
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Table 15.1 Comparison of liquid-moulding processes.

Of the several resin systems that perform adequately in RTM and vacuum-injection moulding, polyesters are most often used, although vinylesters, epoxies, and several others may be used as well. In SRIM polyurethanes clearly dominate usage due to the flexibility in altering the resin reactivity to achieve very rapid cross-linking. Reinforcements may be of virtually any type, although E-glass dominates for cost reasons. The reinforcement form is often mats and fabrics, although rovings may well be braided onto a foam core acting as a mandrel. High-volume production of geometrically complex parts generally requires preforming of the reinforcement (e.g. [36,37]). Either chopped reinforcement and a binder are sprayed onto a screen mould thus forming the desired geometry or a reinforcement mat held together with a thermoplastic binder is heated and compression moulded into the desired geometry. Using such reinforcement preforming techniques allows the entire manufacturing process to be highly automated and for example automobile parts can be manufactured in a matter of minutes. However, the use of randomly oriented, chopped fibres lowers the structural performance of the composite; if needed, fabrics therefore may be added locally to reinforce a preform. All kinds of foam cores are used; the most common ones being PVC, PUR, and PMI.

15.6.2 Characteristics

The RTM method:

- requires relatively small capital investments (for short series and simple parts)
- uses resins that cross-link at room-temperature or at elevated temperature
- uses simple tooling for moderately long production series and low injection pressures; long series, high injection pressures, and high-class surfaces require metal tooling
- may be semi-automated, thus suitable for moderate to long production series
- is cost effective for short and medium-sized production series
- yields moderate to long cycle times
- allows manufacture of large, highly integrated structures
- may cause reinforcement movement during injection
- causes low emission of volatiles due to closed moulds
Sandwich components manufactured through RTM are characterised by:

- good mechanical properties
- good control over reinforcement orientation
- allowing very high fibre fractions
- potentially having resin- and void-rich areas
- having well-controlled surfaces that may be of high class

**The SRIM method:**

- requires relatively large capital investments
- uses resins that cross-link at elevated temperature
- uses simple tooling for moderately long production series and low injection pressures; long series, high injection pressures, and high-class surfaces require metal tooling
- may be fully automated, thus suitable for long production series
- is cost effective for long production series
- yields short cycle times
- allows manufacture of small to medium-sized, highly integrated structures
- may cause reinforcement movement during injection
- causes low emission of volatiles due to closed moulds

Sandwich components manufactured through SRIM are characterised by:

- good mechanical properties
- good control over reinforcement orientation
- potentially having resin- and void-rich areas
- having well-controlled surfaces that may be of high class

**Vacuum-injection moulding:**

- requires small capital investments
- uses resins that cross-link at room-temperature or at elevated temperature
- uses very simple tooling
- is labour-intensive
- is cost effective for short production series
- yields long cycle times
- allows manufacture of large, highly integrated structures
- causes low emission of volatiles due to closed moulds

Sandwich components manufactured through vacuum-injection moulding are characterised by:

- good to modest mechanical properties
- good control over reinforcement orientation
- potentially having resin- and void-rich areas
- having one well-controlled surface
15.6.3 Applications

RTM and SRIM have become very popular in the automobile industry [35,37-40]; RTM for short to medium-sized production series and SRIM for long series. The major reasons for this interest is that complex parts may be economically produced in one step using low-cost moulds and that the parts may have high-class surfaces. Applications include almost any conceivable, but most often exterior panels of semi-exclusive cars. However, the interest in manufacturing structural parts, such as the entire base plate, is increasing. Vacuum-injection moulding is suitable for large products manufactured in short series, such as in ship building, offshore, exterior vehicle panels, etc. (e.g. [34,35]).

An interesting application where the advantages of RTM have been utilised to a significant degree is in the manufacture of the entire self-supporting body of the Swedish all-terrain military caterpillar vehicle no. 206 shown in Fig.15.12a (e.g. [8,31]). The vehicle is unique in that large composite-faced sandwich structures are manufactured entirely through RTM using E-glass and polyester. The panels (shown in Fig.15.12b) are sandwich elements with PVC foam cores in roof, floor and sides and PUR foam cores in doors. The panels are bonded together using an epoxy adhesive to form a self-supporting structure. The composite body weighs approximately 300 kilograms. The size of the floor panel is 1.9 by 3 meters and the height of the composite structure 2 meters.

Similarly, the self-supporting driver’s cab of a Swiss locomotive is manufactured entirely through RTM [32]. The cab is prefabricated in four individual sections—roof, front and two side panels—that are bonded together using an epoxy adhesive. The sections are sandwich panels with glass-fibre-reinforced polyester faces and PUR foam cores. To bond the driver’s cab module to the steel locomotive body a highly elastic rubber-modified PUR adhesive is used. A similar application is the self-supporting front cab of an Italian high-speed train where the panels are vacuum-injection moulded using glass-fibre reinforcement—stacked continuous strand mats and woven fabrics—and a low-viscosity polyester resin system on PUR foam cores [34].

Another interesting RTM case study is the manufacture of composite propeller blades for a variety of regional aircraft and hovercraft, such as Fokker F50, Saab SF340, and Bell LCAC hovercraft (see reference [33]). The basic construction of the blade is shown in Fig.15.13. Preformed layers of carbon fibres and fabric held together with a binder are stacked and inserted into the blade
mould where a PUR core is foamed in-situ. The braided glass-fibre skin is draped over the foam-filled preform whereupon metal fittings are added together with leading edge reinforcement layers and an aluminium braid for lightning protection. The complete assembly is then placed in the RTM blade mould, which is heated to reduce the epoxy resin viscosity during vacuum-assisted injection.

![Rotor blade construction](image)

Figure 15.13  Rotor blade construction. Reprinted from [33] with kind permission from Butterworth-Heinemann journals, Elsevier Science Ltd, The Boulevard, Langford Lane, Kidlington OX5 1GB, UK.

Other high-performance applications include an 8.5 meter long hollow wind turbine blade with an internal axial stiffening web separating the hollow chambers and an 18 meter long hollow sailing boom with an internal furling (sail-rolling) mechanism [41]. Both the wind turbine blade and the sailing boom are manufactured through RTM using a “smart core” technique, which is a lost-core method said to offer advantages over conventional RTM [41]. Materials used are carbon fibre reinforcement, low-viscosity epoxy and thermoformed PVC core.

15.7 Continuous Lamination
From an economical perspective a continuous manufacturing process naturally is preferable. A suitable way to manufacture continuous sandwich panels is using a double-belt press, as shown in Fig.15.14. With a double-belt press it is possible to both heat and cool the material while at the same time subjecting it to a specified pressure profile, thus making it a potentially useful device to impregnate and/or laminate composites.

![Continuous lamination using a double-belt press](image)

Figure 15.14  Schematic of continuous lamination using a double-belt press.
When using a double-belt press to manufacture sandwich components, the face sheets are likely coiled up in very long lengths. Two rolls of face sheets are first uncoiled and guided in between the belts of the press. The core is then, in any of a number of fashions, inserted between the face sheets, possibly together with adhesive layers. The faces and the core are then bonded to one another through concurrent application of heat and pressure, whereupon the sandwich is cooled under pressure to consolidate it [42,43].

Face materials may be sheet metal, unreinforced polymers, and composite laminates or prepregs. To obtain a truly continuous core it may prove convenient to in-situ foam it between the faces through injection and subsequent expansion of for example PUR [44-46]. An alternative route is to sandwich a thermoplastic polymer film containing a foaming agent between the faces; as soon as the double-belt press melts the polymer film, the foaming agent is free to expand, thus filling the gap between the faces with a foam [47]. It is naturally possible to insert discrete blocks of wood or prefoamed core between the faces, although this procedure brings on concerns of core-core disbonds.

A slightly different approach to continuous sandwich manufacture involves a vertical facility similar in concept to a double-belt press [48]. In this process, reinforcement fabrics are simultaneously impregnated and laminated onto core blocks in a continuous process. The materials used were glass fabrics and mats, polyester resin, and polymer-foam cores.

Since there is little conceptual difference between the continuous processes described above and the most well-known continuous composites-manufacturing technique of them all—pultrusion—it should not be surprising to find that pultrusion of sandwich panels has been tried [49]. Just as in the vertical semi-continuous process described above, such manufacture would involve concurrent manufacture of faces and sandwich in the same process.

All continuous processes have in common that they can only with the greatest technical difficulty produce anything but constant cross-section components. With the exception of pultrusion, even a deviation from flat, constant thickness components requires complex technical solutions.

15.8 Other Processes
The manufacturing techniques described above are merely the more common ones used to date. As so often holds true with composites, there is nothing preventing an innovative mind from inventing new or combining/modifying established techniques to enable manufacturing of components. Unconventional processes that have been tried in sandwich manufacturing include compression moulding, filament winding and various in-situ foaming techniques. These processes are briefly discussed in the following.

Compression moulding of structural sandwich components is similar to the compression moulding of single-skin laminates (e.g. [50]). The process is schematically shown in Fig.15.15. When using thermoplastic-based faces, e.g. GMT, the material is heated in an oven to a temperature exceeding the softening point of the matrix and thereafter placed in a cooled mould with the core sandwiched in-between. The mould closes very fast and the material is forced to conform to the mould before it consolidates, whereupon the component is ejected. Rapid closing is essential to achieve high surface finish. The choice of core material is important to ensure that it has a compression strength
adequate to withstand the moulding pressure (between 0.2 and 4 MPa) which is important for dimensional stability and surface finish. Also essential is good bonding of the faces to the core. Thermoformability is advantageous, especially for complex-shaped parts; a thermoplastic foam core then may be appropriate since it easily may be reshaped and compacted. Improved bonding between faces and core and improved dimensional tolerance may be obtained if the thermoformable core is slightly over-dimensioned since the increased pressure caused by core compaction may reduce surface irregularities. The use of thermoplastic cores may further enhance bonding since heating of the face sheets will cause the core surface to melt. Although the compression-moulding process described above applies to thermoplastic materials, it is naturally also possible to use thermoset-based face materials, such as SMC and BMC [9,35,40,51].

Similarly, the self-supporting driver’s cab of a Swiss locomotive is manufactured entirely through RTM [32]. The cab is prefabricated in four individual sections—roof, front and two side panels—that are bonded together using an epoxy adhesive. The sections are sandwich panels with glass-fibre-reinforced polyester faces and PUR foam cores. To bond the driver’s cab module to the steel locomotive body a highly elastic rubber-modified PUR adhesive is used. A similar application is the self-supporting front cab of an Italian high-speed train where the panels are vacuum-injection moulded using glass-fibre reinforcement—stacked continuous strand mats and woven fabrics—and a low-viscosity polyester resin system on PUR foam cores [34].

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Semi-structural applications of compression-moulded sandwich components found in the automotive industry include compression moulded bumper beams with PP foam core and glass-mat-reinforced PP faces [52,53], a compression moulded bonnet of a passenger-car with SMC faces and a foam core [40], partition walls of glass-mat-reinforced PP faces combined with a styrene-acrylonitrile (SAN) foam core in large trucks [52], trunk-floors of passenger cars [52] and folding rear seat backs of a passenger car manufactured using glass-mat-reinforced PP faces and expanded PP foam core [54]. The major reasons why glass-mat-reinforced thermoplastics are gaining interest in the automotive industry [51,52,54,55] are that they look, handle and are processed in a manner similar to sheet metal and have the advantages of being lightweight and having high chemical and corrosion resistance.

In filament winding of sandwich components the inner face would likely be wound first and the winding stopped to apply a flexible or prefoamed core onto the inner face. The winding would then resume to apply the outer face. A particularly interesting application of filament wound sandwich structures is the self-supporting inner body of a railway passenger-car [56-58]. The body is wound in three sections in the manner described above and windows and doors are cut out, whereupon the modules are assembled inside a metal outer body. The body sections are mounted through the roof and slid in to position whereupon the roof section is put in place as illustrated in Fig.15.16. The interior of the car is either integrated in the wound structure or pre-assembled.

A similar technique has been used to manufacture prototypes of self-supporting external structures of complete railway cabs mounted on conventional steel chassis. The outer shell-structure is manufactured through filament winding using PMI cores and the windows and doors are cut out. Thermal insulation, wiring and ventilation ducts are added to the wound structure before it is completely cross-linked. Another example of a railway application is a covered hopper car [59] where the car body is filament wound using E-glass-reinforced polyester. Bulkheads and side walls are manufactured using E-glass-reinforced polyester faces and balsa wood cores. Hat section
stiffeners and wide flange beams are prefabricated using hand lay-up and pultrusion, respectively. For joints a combination of adhesive bonding using an acrylic adhesive system and bolting is employed. A slightly different technique has been employed in manufacturing a box girder [60]. The preformed core, fitted on a pipe, serves as mandrel. The face is wound in two steps: the inner laminate layer is wound directly onto the core and the outer is wound after unidirectional prepreg sheets have been laid up on the flanges to further stiffen the structure.

A technique to fabricate an all-thermoplastic sandwich structure employing a thermoplastic foaming hot-melt adhesive as core material has been developed [61,62]. The foaming hot-melt is sandwiched between the face sheets in a “precompact” and placed in either a hot platen press or in a shaped mould. The sandwich structure forms as heat is applied and the thermoplastic hot-melt foams. The fully thermoplastic character of the sandwich system allows finishing treatment, thermoforming, such as edge forming and folding [61-64]. All-thermoplastic sandwich structures are gaining interest partly due to the possibility of producing shaped articles from flat sheets using local heating and ordinary sheet metal forming techniques [65-67].

Methods to manufacture net-shaped all-thermoplastic structures in-situ, i.e. in one step, have also been developed [68,69]. Generally in these processes, the core materials (powdered thermoplastic resins, foaming agents and fillers) are mixed and sandwiched between preformed fabric—either commingled, cowoven or unimpregnated—placed in a closed mould and subjected to heat. The face sheets are impregnated with resin during the process as the core foams. These processes potentially offer advantages over other manufacturing techniques in that core and face sheet prefabrication, bonding and machining are eliminated.

15.9 Outlook
The development of structural sandwich components in terms of manufacturing science and constitutive materials primarily does not appear to be performance-driven, since high-performing sandwich components indeed already are manufacturable—albeit at very high cost. Rather, the development appears to be driven by desires to lower component price, improve recyclability, and improve work environment. As obvious from the project descriptions above, today’s prevailing manufacturing techniques—with the exceptions of double–belt–press laminating and to some degree SRIM which both have limited application areas—tend to involve a large degree of manual labour and long cycle times. The labour intensity translates into high marginal costs commonly dwarfing other costs, and most manufacturing techniques thus can only be economically justified in relatively short production series. Further, the long cycle times are unacceptable in many industries, most notably the automotive industry. It is therefore hardly surprising that few structural sandwich components have been manufactured in long series.

Much recent work has been dedicated towards automating techniques to manufacture structural sandwich components of complex geometry with the overall aim of satisfying the requirements of the automobile industry in terms of both processing rate and part cost (e.g. [70]). The ever more important issue of recycling is having an increasing impact on the constituent materials used in sandwich applications. The perhaps most obvious solution to the recyclability issue is to use thermoplastic resins only and to utilise the same resin in both faces and core [52,53,70-72]. Although less straightforward in concept, work is underway to develop methods to recycle thermoset-based mixed-material sandwich components, e.g. the combination of glass/polyester
faces and PVC cores so common in the ship-building industry [73]. Work-environment issues in composites manufacturing tend to be synonymous with solvent (mainly styrene) emissions and (epoxy-related) allergies. Approaches to reduce the solvent problem are to use closed-mould processes [29], for example liquid moulding, or to use prepregs; the latter approach has been the subject of much interest as direct replacement for wet-lay up of laminates in for example the ship-building industry. A route to complete elimination of both solvent emissions and allergies is to entirely abandon thermosets for thermoplastics, which is an approach potentially solving all three issues of improved productivity/reduced cost, recyclability and improved work environment simultaneously [47,53,61-64,68-71].

There is little doubt that the manufacturing science of sandwich components will develop further to enable low-cost, long-series manufacture of structurally capable sandwich components. To a not insignificant degree this development will be aided by new material systems and forms, such as for example three-dimensional reinforcing fabrics that integrate faces and core and thus have the potential of significantly improving structural integrity [74,75]. There should be equally little doubt that the issues of recyclability and work environment will be satisfactorily solved. One solution to these problems is the aforementioned use of thermoplastics in favour of thermosets, while other possible solutions ought to be within reach through enhancements in constituent materials and manufacturing techniques.

References


[40] “Qualité optimisée et économie d’utilisation“, technical information brochure, Renault, Amiens, France.


