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TEACHING FOR STRUCTURE AND GENERALITY: ASSESSING CHANGES IN TEACHERS MEDIATING PRIMARY MATHEMATICS

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From a sociocultural perspective that a teacher’s use of mediational means is central to student learning, this paper presents an analysis of six teachers and their mediating, across a two-three year time gap. Drawing on the Mediating Primary Mathematics framework – developed to examine the type and quality of mediational means – we propose two composite assessments of quality of mediation – extent and depth –that indicate the extent to which teaching addresses mathematical structure and generality. The findings reveal a range of differences in these two assessments for each of the six teachers, but that all six teachers were more coherent in their use of mediational means in the later lesson than in the earlier one. These findings have implications for other schooling systems and researchers seeking to improve the quality of mathematics instruction.

INTRODUCTION

The multiplicity of factors that shape learning make it difficult to track forward in any direct way from teaching actions to learning outcomes. Yet studying changes in teaching that might better support mathematical learning remains, in many ways, a holy grail of mathematics education research, especially as research overviews continue to point to the quality of teaching as among the most influential factors impacting on learning outcomes (Coe Aloisi, Higgins & Major, 2014). Working, as we do, in disadvantaged contexts in a developing country, classrooms often provide the only sites of access to mathematical learning, and so understanding and improving the quality of teaching is important. With models of effective teaching from the ‘global north’ often assuming a baseline of quality in teaching yet to be established in less advantaged contexts, these models fail to capture nuanced features that mark significant changes in pedagogy. This paper presents findings from a context-sensitive framework for assessing quality of teaching – the Mediating Primary Mathematics (MPM) framework – that allows us to identify and track changes in teaching. The framework may thus be helpful to other researchers working in similar contexts of poor performance and imperatives to improve the quality of mathematics teaching.

BACKGROUND

Students’ dependence on counting strategies not only well into the upper primary years but also into secondary schools has been widely written about in South Africa (Fleisch,
This continued dependence is fostered, on the one hand, by pedagogic practices in which numerical tasks are recurrently treated from scratch and counting is taken, by teachers and students alike, as the primary means of working out answers, and, on the other hand, by a marked absence of didactical practices oriented towards building a base of known results from which other results can be derived. In our work, we are keen to change this dependence on counting and to encourage teachers to begin to understand mathematics as a scientific discipline (Vygotsky, 1978) and adopt instructional practices that focus on mathematical structure and generality.

In addressing teaching for structure, we do not expect teachers to meet Freudenthal’s (1983) high bar requirement of structure being the total network of basic and derived properties and actions that can be associated with an initial relationship like \( a + b = c \). Instead, we take a more pedagogical position that learners’ awareness of mathematical structure is as a network of basic and derived general relationships that expands over time and follow Brown's (2011) position that concepts are structures that emerge through noticing similarities and differences across tasks together with reasons that might underlie these relationships. Such awareness of structure is central to learners’ engagement with mathematical generality, which we take to underpin the power of mathematics. We, thus, see teaching towards (awareness of) structure and generality as key in teaching in South Africa moving from the limitations noted above, but given the wide gap between this vision and current practices there was a need for a tool that can provide nuanced assessments of pedagogy that nudge in that direction.

To this end we developed the Mediating Primary Mathematics (MPM) framework for analyzing the quality of instruction in mathematics, the detailed background to which is written about elsewhere (Venkat & Askew, 2018). Here we present findings from the application of the framework and analyse earlier and later teaching of six teachers from six primary schools. This analysis allows us to characterize differences in the nature of mathematics teaching across a variety of topics and suggests that the differences can be interpreted as improvements in instruction in the direction of more attention to structure and generality. Key questions addressed are:

Using the MPM framework, what differences, over time, in attention to mathematical structure and generality can be seen in these teachers’ practices?
Can these differences be interpreted as showing improvements in teaching?

THEORETICAL BACKGROUND: TEACHING AS MEDIATION

Prior writing details the theoretical and empirical concerns underpinning our work (Venkat & Askew, 2018); we therefore only overview that detail here. Our work is based in a sociocultural view of instruction as mediational and directed towards learning which focuses on mathematical structure and generality. Building on the classic subject–mediational means–object triad, we find it helpful to consider four, interconnected, strands of teacher mediation:

- tasks and associated examples;
- artefacts (physical equipment and manipulatives);
inscriptions (images, diagrams, written words and symbols produced within a lesson);
teacher talk/gesture (divided into three sub-strands):
  o methods offered or accepted for generating/validating answers,
  o mathematical connections made or accepted, and
  o student-level connections offered and developed.
Tasks, and the examples accompanying these, form the base for mediation, which is enacted through coordinated use of artefacts, inscriptions and talk/gesture. These mediating means provided theoretical lenses for analysing lessons and led to the development of the MPM framework. Working with recordings of a number of baseline lessons in 2011/12 we parsed each lesson into a sequence of episodes and then analyzed the bundle of mediating means used within each episode, with particular attention to whether or not a teacher’s mediation was likely to move students from a focus on simply succeeding at the task at hand, to beginning to develop a sense of mathematical structure and generality. Empirical contrasts from episodes observed were used, in conversation with the theoretical base, to construct levels of mediation within each of the strands. Iteratively moving between the framework and applying it to lesson episodes, led to a fine-tuning of the framework into its current form (Figure 1). With regard to instructional goals, Askew (2015, 2019) has examined ways in which teachers have used all four mediating means in ways that may or may not support moves to structure and generality. Here the framework is used to examine two aspects of the quality of mediation; extent and depth.

**DATA SOURCES**

Working with ten previously disadvantaged primary schools, we gathered baseline videos of mathematics lessons in 2011/12 with Grade 2 and 3 teachers. Returning to these schools to video lessons in 2014 there were six teachers from six different schools for whom we had both the earlier baseline and later lesson video-data: these six teachers form the sample reported on here. Of these six teachers, four taught in ‘suburban’ schools and two in ‘township’ schools. All of the teachers had taught in primary schools for more than five years, some for much longer. While all the schools served historically disadvantaged student populations, the township schools had larger school rolls and classes, and smaller classrooms than their suburban counterparts. Between 2011 and 2014 the project team worked on professional development in these six schools, more broadly focusing on supporting teachers in attending to the structures and generalities underpinning ‘number sense’, hence our interest in whether we could determine changes in teaching practices across the years.

**METHODOLOGY FOR ANALYZING THE QUALITY OF INSTRUCTION**

Analyzing the quality of instruction using the MPM framework required an approach that enabled us to look both within and across a lesson’s episodes, to build a picture of
<table>
<thead>
<tr>
<th>MEDIATING TASKS.EXAMPLES (Listed)</th>
<th>MEDIATING ARTEFACTS</th>
<th>MEDIATING INSCRIPTIONS</th>
<th>MEDIATING TALK &amp; GESTURE</th>
</tr>
</thead>
<tbody>
<tr>
<td>MEDIATING ARTEFACTS</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>0</strong></td>
<td><strong>1</strong></td>
<td><strong>2</strong></td>
<td><strong>3</strong></td>
</tr>
<tr>
<td>No artefacts or artefacts that are problematic/ inappropriate</td>
<td>Unstructured artefacts used in unstructured ways (Bags of counters/tally marks)</td>
<td>Structured artefacts used in unstructured ways (Abaci, 100 squares, etc, used with unit counting, and without reference to structural properties)</td>
<td>Structured artefacts used in structured ways / unstructured artefacts used in structured ways (Abacus, 100 square/place value blocks/cards, number lines, etc used with reference to structure/relations)</td>
</tr>
<tr>
<td><strong>MEDIATING INSCRIPTIONS</strong></td>
<td><strong>0</strong></td>
<td><strong>1</strong></td>
<td><strong>2</strong></td>
</tr>
<tr>
<td>No inscriptions or inscriptions that are problematic/ incorrect</td>
<td>Inscriptions that only record tasks or responses</td>
<td>Unstructured inscriptions (e.g. tally marks)</td>
<td>Structured inscriptions (e.g. tables of ordered bonds; structured/empty number lines; inscriptions underpinned by relations)</td>
</tr>
<tr>
<td><strong>MEDIATING TALK &amp; GESTURE</strong></td>
<td><strong>0</strong></td>
<td><strong>1</strong></td>
<td><strong>2</strong></td>
</tr>
<tr>
<td>Method for generating/ validating solutions</td>
<td><strong>0</strong></td>
<td><strong>1</strong></td>
<td><strong>2</strong></td>
</tr>
<tr>
<td>No method or problematic generation/validation (e.g. mixing of knowns and unknowns)</td>
<td>Singular method/validation (provides a method that generates the immediate answer; enables production of answers in the immediate example space)</td>
<td>Localized method/validation (provides a method that can generate answers beyond the particular example space)</td>
<td>Generalized method or validation (provides a strategy/method that can be generalized to both other example spaces AND without restriction to a particular artefact / inscription)</td>
</tr>
<tr>
<td><strong>Building mathematical connections</strong></td>
<td><strong>0</strong></td>
<td><strong>1</strong></td>
<td><strong>2</strong></td>
</tr>
<tr>
<td>Disconnected and/or incoherent treatment of examples OR oral recitation with no additional teacher talk</td>
<td>Every example treated from scratch</td>
<td>Teacher talk connects between examples or artefacts/inscriptions or episodes</td>
<td>Teacher talk makes vertical and horizontal (or multiple) connections between examples/ artefacts/ inscriptions/ episodes</td>
</tr>
<tr>
<td><strong>Building learning connections: explanations and evaluations - of errors/ for efficiency/ with rationales</strong></td>
<td><strong>0</strong></td>
<td><strong>1</strong></td>
<td><strong>2</strong></td>
</tr>
<tr>
<td>Pull-back to naïve methods OR No evaluation of offers (correct or incorrect)</td>
<td>Accepts/evaluates offers Accepts strategies or offers a strategy OR Notes or questions incorrect offer</td>
<td>Advances or verifies offers. Builds on, acknowledges or offers a more sophisticated strategy OR addresses errors/misconceptions through some elaboration, e.g. ‘Can it be ----?’ ‘Would – this be correct, or this?’ Non-example offers</td>
<td>Advances and explains offers. Explains strategic choices for efficiency moves OR provides rationales in response to student offers related to common misconceptions OR provides rationale in anticipation of a common misconception</td>
</tr>
</tbody>
</table>

Figure 1: The MPM framework (Venkat & Askew, 2018, p.90)
instructional quality. To this end we overlaid onto the MPM framework the constructs of ‘extent’ and ‘depth’ of mediation for structure and generality. The ‘extent’ indicator provides an assessment of the extent of mediation for structure and generality across the lesson episodes, while the ‘depth’ indicator provides insight into the ways in which mediational means were deployed coherently and responsively within episodes, in relation both to the mathematical object of learning and to student offers.

The research team initially parsed lessons into episodes on the basis of tasks set and their related example spaces. Within episodes, we noted whether or not students made incorrect, and/or overtly inefficient, offers, as the absence of incorrect/inefficient offers suggested such episodes functioned largely as rehearsal or revision of prior learning. Low-level mediation in such episodes (no explicit instruction or simply recording or acknowledging results) was, thus, interpreted in relation to this inference of rehearsal/revision, and such episodes were not included in the lesson coding. Similarly, smoothly run chorused counting episodes and individual seatwork episodes that did not involve whole-class or small-group instruction were also omitted from the analysis. In contrast, evidence of incorrect/overtly inefficient answers indicated a need for responsive instruction that built connections with learning: to explain or remediate in some way, so these episodes were the focus of the quantitative coding reported on here.

Teachers’ work in each episode was given a 0-3 score for each of the mediating strands, creating a mediation ‘map’ of the lesson. Initial coding was done in groups: individual team members coded teachers’ earlier and later lesson videos, convened to discuss and agree on framework level summaries capturing the essence of mediation. Agreed MPM ‘maps’ of the earlier and later lessons were thus created for each teacher. For each of these maps, extent and depth judgments were made as two distinct but complementary ways of assessing mediating for structure and generality. Quantitatively, each cell of the MPM map could attain a maximum score of 3: multiplying this by the number of coded episodes and the teacher’s actual scores across the episodes, produced an assessment of the extent of mediation across the strands of artefacts, inscriptions and the three aspects of talk/gesture. For example, if a lesson had four coded instructional episodes, there was a maximum score of 12 available for, say, use of artefacts. The summed coding score (S) for use of artefacts in relation to this total provided an S/12 fraction for artefact-based mediation. Given that the talk/gesture strands focused on three different dimensions of mediation, we produced proportional summaries for each of these strands in each lesson. However, artefact and inscription scores were averaged together to produce a single indication of the extent of use of the more structured, relatively ‘permanent’ mediational forms in each lesson. Comparisons of proportions for earlier and later lessons within each strand provides a lens for considering differences in the extent of mediation for structure and generality across teachers across years. Given the exploratory nature of this analysis, we set a relatively high bar (20% point difference) for considering difference in extent of mediation to be substantive. Depth arises from a ‘vertical’ reading of the coordination and coherence of mediation within episodes. For example, in a lesson with, say, six coded episodes, we looked
vertically for the number of episodes containing higher codes (2 or 3) across two or more of the artefact/inscriptions and talk strands, producing another proportional score.

This coding for both extent and depth allowed us to see differences between earlier and later teaching in terms of the teacher’s mediation for structure and generality. Table 2 shows the 2012 and 2014 scores for one of our teachers Ms M. In the findings and analyses that follows, our focus is on patterns of shift towards greater attention to mathematical structure and generality that help us to understand the kinds of mediation underlying the differences in early number learning that we noted at the outset.

<table>
<thead>
<tr>
<th></th>
<th>EXTENT</th>
<th>DEPTH</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2012 (6 episodes)</td>
<td>2014 (5 episodes)</td>
</tr>
<tr>
<td>Artifacts &amp; Inscriptions</td>
<td>4/18 &amp; 5/18 (25%)</td>
<td>0/15 &amp; 9/15 (30%)</td>
</tr>
<tr>
<td>Talk/gesture: generating solutions</td>
<td>3/18 (17%)</td>
<td>11/15 (73%)</td>
</tr>
<tr>
<td>Talk/gesture: mathematical connections</td>
<td>8/18 (44%)</td>
<td>13/15 (87%)</td>
</tr>
<tr>
<td>Talk/gesture: learning connections</td>
<td>5/18 (28%)</td>
<td>13/15 (87%)</td>
</tr>
</tbody>
</table>

|                      | 2012 (6 episodes)             | 2014 (5 episodes)             |
| Higher level mediation in two or more strands | 1/6 episodes (17%) | 5/5 episodes (100%) |

Table 1: Scores of extent and depth in Ms M’s lessons

**FINDINGS AND ANALYSIS**

Figure 2 presents the differences in the extent scores for each of the six teachers across the four strands of mediation. As can be seen, for two of the teachers, Ms M and Ms S, there was a substantially greater extent of mediation in their later lesson compared to their earlier lesson, with Ms M demonstrating the greatest difference. For Ms Mp there is some change from the early lesson to the later, but it is not substantial as for Ms M and Ms S. For Ms R and Ms D, the profile of extent is similar across both years, whilst Ms B’s extent of mediation in 2014 looks somewhat weaker than in her 2011 lesson.
Figure 2: Differences in extent of mediation across teachers and years.

Using the 20% point difference, Ms M’s teaching indicated differences meeting this criterion in 3 of the 4 categories (that is, across all of the talk/gesture strands). Ms S met this criterion in 2: arefacts/inscriptions and talk/gesture for mathematical connections with Ms Mp also meeting this criterion on talk/gesture for mathematical connections. The three other teachers did not cross this threshold on any of the strands.

Table 2 presents the assessment of depth for the teachers in each of the years. This shows that for all of the teachers, within the 2014 lesson, there were higher levels of mediational means being coordinated within the teaching episodes. All episodes in Ms M’s 2014 lesson showed a coordinated depth of higher-level mediation for structure in two or more strands across all 5 episodes, with Ms S and Ms Mp achieving this in at least 50% of their episodes in 2014. The other teachers demonstrated less difference but given the lack of any coordination of mediating means in the earlier lesson, what increased scores they did achieve indicate a difference in their teaching.

<table>
<thead>
<tr>
<th></th>
<th>Ms M</th>
<th>Ms S</th>
<th>Ms Mp</th>
<th>Ms R</th>
<th>Ms D</th>
<th>Ms B</th>
</tr>
</thead>
<tbody>
<tr>
<td>2012</td>
<td>17%</td>
<td>50%</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2014</td>
<td>100%</td>
<td>60%</td>
<td>50%</td>
<td>40%</td>
<td>40%</td>
<td>29%</td>
</tr>
</tbody>
</table>

Table 2: Proportion of episodes with coordinated mediation at level two or three

DISCUSSION

On the empirical side, we see broad differences in terms of extent and depth of mediation for mathematical structure and generality employed across the 12 lessons within this analysis, with more extensive differences across some teachers than others. The theoretical derivation of the framework from Vygotskian notions of structured networks of scientific concepts allows the patterned direction of differences – higher scores in the later lessons – to be interpreted in terms of improved extent of mediation for structure and generality. The variation in the extent of mediation ranges from extensive change in extent across multiple strands – as in the case of Ms M – to lack of substantial change in extent of mediation within any of the strands – as in the case of Ms B. As Figure 2 shows, these six teachers essentially fell into two groups, those teachers who expanded their work with structured mediation, and those who have begun to introduce some structured mediation but not to the same extent.

A key commonality is the broad increase in depth of mediation for structure across five of the six teachers. This points to more episodes exhibiting a coordinated orchestration across mediational means. As our coding for depth involved looking for higher level mediation scores (2 or 3) that point to moves beyond simply a coherent lesson towards connection and generality, the increases in the depth indicator suggest, therefore, that connected teaching was more in view in the latter lessons than in the former ones. Of particular note is that substantial moves on this indicator were seen among teachers working from a low base of mediation for structure and generality, suggesting that a
focus on helping them to coordinate actions across mediational means may be a particularly useful direction to follow in primary mathematics teacher development.

Our sense is that the central commitment to mathematical goals of structure and generality are both important and pragmatic in a context where much recent writing has noted that the lack of attention to specialized knowledge may be part of the problem in relation to improving disciplinary instruction (Hugo & Wedekind, 2013). The explicitness of mathematical goals in the MPM framework – structure and generality – coupled with an openness on the pedagogic forms in which these goals are couched, means that the MPM framework provides a lens for exploring primary mathematics teaching in ways that are sensitive to the ground as well as being mathematically ambitious in seeking to expand the spaces of productive mathematical working.

References


