Basic Creep of Young Concrete-Sensitivity in the Evaluation Method

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Preface

This master thesis was performed at the Department of Civil & Architectural Engineering, at KTH, The Royal Institute of Technology in Stockholm. The research within this thesis is in collaboration with Betong- och Stålteknik AB, Luleå University of Technology and KTH. The research started in January 2019 and was ended in May 2019. The idea of this thesis comes from Anders Hösthagen PhD student within Structural and Fire Engineering at the Department of Civil, Environmental and Natural Resources Engineering at Luleå University and Technology and Dr. Carsten Vogt.

First, we want to show our gratitude to our supervisor during the writing of this thesis, Professor Johan Silfwerbrand at KTH, for his continuous support, guidance and knowledge. We also want to thank our supervisor Anders Hösthagen for continuous providence of support, guidance and knowledge within this field. We also want to show our gratitude to Professor Jan-Erik Jonasson, Professor Mats Emborg and Associate Professor Martin Nilsson who have given us knowledgeable input to this work.

Stockholm, May 2019

Benar Ekm at and Natalea Hermes
Abstract

Creep is defined as deformation that takes place under constant load after an initial elastic response. This thesis focuses on a material property problem area that concerns stress analysis. Focus is on stress development considering creep deformations occurring when a concrete structure is under load, i.e. stress analysis with viscoelastic properties of the material. From laboratory tests, both elastic modulus and deformations over time are estimated in an evaluation process. Usually, deformations of moist sealed samples are denoted basic creep. At Luleå Technical University creep measurements are evaluated according to the theory and methodology in Larson and Jonasson (2003a, 2003b). The model is denoted Linear Logarithmic Model, used for moist sealed concrete samples. This thesis involves an investigation of the evaluation procedure for basic creep performed in Thysell laboratory at LTU, to examine how sensitive the evaluation process is for the outcome from stress calculations. The calculations are performed in the Finite Element Method software ConTeSt Pro.

The aim of the thesis is to analyze the sensitivity of evaluation of basic creep and of the Linear Logarithmic Model (LLM) by making changes in the evaluation process to see how different parameters sets effect calculated stresses/strains during through crack analysis. The changes are solely done in the relaxation spectra.

The purpose is also to analyze how sensitive the changes made in the evaluation process are when typical cases are studied. The typical cases are defined with a structure of a newly cast wall on a mature slab, where various thickness of the wall during different temperature conditions are analyzed. The temperature conditions are named standard, winter and summer. With this, concrete is tested and evaluated to yield two material parameter sets useful for temperature - and stress calculations for young concrete.

The material parameter sets were analyzed and their creep values were converted into relaxation values, i.e. relaxation spectra, according to Maxwell-chain formulation for LLM. ConTeSt calculations generate temperature development for the walls and slabs. Colour maps and values of the strain ratio for each studied case are also obtained.

It can be established that the evaluation process of basic creep is sensitive. A conclusion to be drawn is that small changes in the relaxation spectra, gives changes in the results of stress calculations for the typical cases. As soon as we deviate from the temperature development for the test performed in the laboratory, either by changing the thickness of the wall or by testing different temperature conditions we get a different temperature development than the tested one. With the deviation in the calculated temperature development compared to the measured one, a difference in the calculated strain ratios for the two different material parameter sets created are found.

The main discovery in this work is that when a geometry of the wall that is identical to the geometry of the concrete tested at the laboratory is analyzed, a small deviation in the calculations of strain is obtained. This is expected since the temperature development in the created wall will follow the temperature development of the tested concrete. When differing from this geometry and temperature case, differences in calculated strain ratios are observed.

Keywords: thermal cracking, young concrete, creep, Linear Logarithmic Model, temperature development, average strain, sensitivity.
Sammanfattning


Syftet med detta arbete är att analysera känsligheten i utvärdering av krypning för fuktöverförande betongprover och för den Linjära Logaritmiska modellen genom att göra ändringar i utvärderingsprocessen för att se hur olika materialparametersuppsättningar påverkar beräknade spänningar under analys av genomgående sprickor. Ändringar görs endast i relaxationsspektra.

Syftet är också att analysera hur känsliga förändringarna i utvärderingsprocessen är när olika typer studeras. Typen definieras av ny gjuten vägg på en mogen betongplatta, där olika väggtycklenar under olika temperaturförhållanden analyseras. Temperaturförhållandena benämns standard, vinter och sommar. Därvid testas och utvärderas betongen för att ge två materialparameteruppsättningar som är användbara för temperatur- och spänningsberäkningar för ung betong.

Materialparameteruppsättningarna analyserades och deras krypvärden konverterades till relaxationsspektra, så kallade relaxationsspektra, genom att använda Maxwell element för LLM. Från ConTeSt beräkningar erhålls värden för temperaturutveckling i vägg samt platta. Värmeutvecklingskarta tillsammans med värden på töjningskvoten för väggna under varje studerat temperaturfall erhålls också från ConTeSt programmet.


Den huvudsakliga upptäckten i detta arbete är att när den beräknade geometrin på väggen är identisk med den geometri som används för testet i laboratoriet, erhålls små variationer i de beräknade töjningskvoten. Detta är förväntat eftersom temperaturutvecklingen i den beräknade väggen är densamma som för betongen i testet i laboratoriet. När man avviker...
från den geometri eller de temperaturförhållandena som är identiska till dem i laboratoriet så erhålls större avvikelser i värden för den beräknade töjningskvoten.

Nyckelord: Temperatursprickor, ung betong, krypning, Linjär Logaritmisk Modell, temperaturutveckling, töjning, känslighet.
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1. Introduction

1.1 General background of the thesis

The knowledge of the factors impacting the risks of thermal cracking in concrete is of great importance. Improved understanding about the problem and the affecting parameters gains the building process and lowers the financial costs. In the last decennium more thorough and extensive research has been implemented by many companies and universities in the field. One of the major research areas is at Luleå University of Technology, the Division of Structural Engineering, where there is a large focus on thermal cracking of young concrete (Nilsson, 2000).

Thermal cracking of young concrete in civil engineering structures, such as bridges or tunnels, especially those exposed to freezing or chlorides should be avoided. Note that the definition of young concrete varies in the literature, therefore in this thesis young concrete is defined as concrete structures during the hydration phase when the chemical reactions between water and cement generate heat that leads to large thermal deformations. Any type of cracking may lead to an increased moister or water intrusion which can lead to consequences such as freezing causing spallation or increased propagation of chlorides into the concrete causing corrosion of reinforcement. This initiates and increases the degradation process of the concrete which leads to a shorter service life.

The temperature in the newly cast concrete varies within the cross section during the hydration process. Usually the temperature development is larger in centre parts than in surface layers leading to larger deformations in the center parts which creates tensile stresses in the surface area. If the stresses are larger than the concrete tensile strength, surface cracks will occur. The surface area in turn counteracts the expansion in the center parts and cause compressive stresses in the inner parts when the maturing concrete increase in temperature. After the concrete reaches its temperature maximum, the concrete starts to contract. The central parts are often more expanded than the surface parts, and the differential contracting movement causes tensile stresses on the center parts. If the stresses are larger than the actual tensile strength, cracks can occur. Because of the same reasons described above, the temperature induced movements in the center parts are larger than in the surface area and as a result the surface cracks have a tendency to close. The surface area counteracts these movements and cause tensile stresses in the center parts. Once more if the stresses are larger than the actual tensile strength, cracks can occur. The difference between cracks occurring during the expansion phase and cracks occurring during contraction phase is that the ones taking place during contraction phase are through cracks and are often more sever in its nature than surface cracks (Nilsson, 2000).

Concrete cast on mature concrete can also be exposed to the corresponding stress situation as described above. The volume of the newly cast concrete increases when the temperature in the young concrete increases. The increase in volume causes tensile stresses in the adjacent older concrete which restricts the expansion and causes compressive stresses in the young concrete. When the temperature in the young concrete decreases it will start to contract which leads to compressive stresses in the older member, which again restricts the deformations and this time causes tensile stresses in the young concrete which can result in through cracks (Nilsson, 2000).

This thesis focuses on a material property problem area that concerns stress analysis, concentrating on stress development considering creep deformations occurring when a
hydrating concrete structure is under load, i.e. stress analysis with viscoelastic properties of the material. From laboratory tests, both elastic modulus and deformations over time are estimated in an evaluation process. Creep is defined as deformation that takes place under constant load after an initial elastic response. Usually, deformations of moist sealed samples are denoted basic creep. When the sample is not moist sealed then the deformations increase due to both basic creep and a drying out process, denoted drying creep.

At Luleå Technical University creep measurements are evaluated according to the theory and methodology described in Larson and Jonasson (2003a, 2003b). The model is denoted Linear Logarithmic Model which is a model only used for moist sealed concrete samples and known to be relatively easy.

There is a need to estimate the sensitivity of the evaluation of basic creep and its impact when studied for different structural cases and temperature circumstances. This is done by performing an alteration in the evaluation process of the creep for a tested concrete mix. This results in two different material parameter sets with which thermal through cracking analysis is performed.

To analyze the risk of through cracking within a concrete structure, stress/strain analysis at early ages can be calculated with the aid of computers and customized software. This thesis involves an investigation of the evaluation procedure for basic creep performed in Thysell laboratory at LTU, to examine how sensitive the evaluation process is for the outcome of stress calculations. The calculations are performed in the Finite Element Method software ConTeSt. Impact on the final results is carried out.

1.2 Cracks in concrete and reasons to restrict them

Any cracks that occur at early ages will result in negative effect on durability, function, maintenance but also on the surrounding environment of the concrete structure. When analyzing the functionality of a reinforced concrete with regard to cracking, it is found that the functionality is dependent on the crack widths and the type of damage attack Sweden has requirements in the design process regulating the maximum crack width at different environmental conditions. The philosophy in the national regulations is to aim for a “crack free condition” (Hösthagen, 2017).

There are several factors that cause thermal cracking of concrete at early ages when analyzing and estimating the crack risk; variable temperature, maturity, mechanical properties, moisture development, thermal properties, and restraint. The types of damage that connect to thermal cracks at early ages in reinforced concrete can be divided into the following groups (Hösthagen, 2017):

- **Lowering the bearing capacity of the concrete.** Cracks that cause collapse shortly after construction are excluded here, since this usually is caused by fatal mistake made either in the design process or in the work on site. But if the bearing capacity is lowered due to advanced durability attack and no repair is performed in time, the consequences might be a partial or total collapse of the structure.

- **Corrosion of reinforcement bars.** Cracks in concrete might strongly increase the corrosion initiation time which means that cracks are very important with respect to corrosion. This
is controlled in regulations by restrictions of the calculated stress ratios with respect to early age cracking depending on the exposure classes for the situation.

- **Degradation mechanisms of concrete.** The most important durability attacks in Sweden are corrosion of reinforcement, frost attack and chemical attack.

- **Through flow of gases and liquids.** This is significant already for small cracks. The flow of gases within a crack might be dangerous for people if the concrete structure is a shelter from the dangerous gas and gases can also dissolve the concrete paste and harm the construction.

- **Appearance of the concrete surface.** One reason to bad appearance is when repairs of cracks are made visible, for example when the injected crack has a sharp contrast to the general concrete surface.

Measures against undesired cracking during concrete hardening are dimensioned using crack risk analyses often based on Finite Element Modelling. Early age cracks of concrete can be avoided by for instance decreasing temperature differences and air temperature within the casting stage and between newly cast concrete. Temperature differences can be decreased by choosing concrete with low heat development, this by choosing slow curing cement (Emborg et al, 1997). There are many other possible measures against undesired cracking during concrete hardening.

1.3 Aim of the thesis and research questions

The aim of the thesis is to analyze the sensitivity of evaluation of basic creep by making changes in the evaluation process and of the Linear Logarithmic Model to see how different parameters sets effects calculated stresses/strains during through crack analysis. The changes are solely done in the relaxation spectra.

The purpose is also to analyze how sensitive the changes made in the evaluation process are when typical cases are studied. The typical cases are defined with a structure of a newly cast wall on a mature slab, where various thickness of the wall during different temperature conditions are analyzed. The temperature conditions are named standard, winter and summer.

With this, concrete is tested and evaluated to yield two material parameter sets useful for temperature and stress calculations for young concrete. If there are variations in the calculated strains/stresses when using these different material parameter sets, it indicates that the evaluation process for the basic creep is sensitive to some extent.

The research performed within this thesis is to follow up within the scientific work performed by Anders Hösthagen, carried out at the department of civil, environmental and natural resource engineering at division of structural and fire engineering in Luleå University of Technology in Sweden.
The research questions within this study are described as follows:

- How sensitive is the evaluation of performed tests of basic creep for concrete structures?
- How sensitive is the evaluation of basic creep for thermal through crack analysis for concrete structures?

1.4 Choice of method

Several combined research methods are chosen to address the aim and the research questions within this thesis.

A literature study was performed in order to understand basic creep of concrete and how to estimate the sensitivity level of the evaluation of the basic creep. The study helped determine which creep model to use during the evaluation process. The creep model, Linear Logarithmic model (LLM) was chosen after comparison with other methods. This, due to the fact that its formulation demonstrates very good agreement directly with experimental creep data and indirectly with measured thermal stresses. LLM formulation has also the best correlation with experimental data when linked to other commonly used creep models such as double power law (DPL) or triple power law (TPL).

The chosen method for the evaluation process contains laboratory tests combined with FE-simulations performed with ConTeSt Pro. The software ConTeSt is used to perform stress/strain calculations on the concrete samples. Using the method of laboratory experiment makes it possible for parts of the experiments presented by Hösthagen to be repeated in this thesis, which can increase the validity of the theories that are being tested.

1.5 Limitations

The study of this thesis is restricted by several limitations presented below:

- Only three typical cases have been studied.
- Only different thickness of the concrete wall is studied, and not different thicknesses of the concrete slab.
- Not more than two material parameter sets are established during the evaluation of the basic creep.
- The performed laboratory tests have been carried out for one concrete mixture containing structural engineering cement with fly ash.
- The effect of drying shrinkage is neglected to decrease the scope of this thesis.
- The used method for thermal crack risk estimation is based on history independent formulation, and the formulation is achieved in this thesis by using Maxwell-chain model instead of Kelvin-chain model. This because Maxwell-chain model can be used in the software, ConTeSt.
• Only one of several existing creep models are used. The creep model is the Linear Logarithmic Model.

### 1.6 Outline of the thesis

This thesis consists of six chapters, their contents are briefly described below.

*Chapter 1* introduces the subject matter and the research questions and aims of the thesis.

*Chapter 2* presents thermal crack risks and characteristics of concrete with focus on the viscoelastic behaviour and basic creep. The process of evaluation of material properties for young hardening concrete are discussed together with computerized methods of stress and heat calculations.

*Chapter 3* provides information about methods used for determinations and evaluations of basic creep together with calculations of stress/strains for typical concrete structures.

*Chapter 4* presents the outcome of the methods used for determination of basic creep and the heat and stress calculations compared to measured values of the concrete structure and for the typical studied cases.

*Chapter 5* discusses the obtained results and the sensitivity of the evaluations process of basic creep.

*Chapter 6* presents the conclusions of the most accurate results, answers to the research questions and also suggestions for future research within this field of research.

*Appendix A* shows all the results of the thesis.
2. Theory

2.1 Thermal crack risks

In newly cast concrete, the risk of thermal cracks is usually stated in relations of strain, stress or temperature whereby the failure criteria are related to a tensile failure strain, a tensile strength, or a failure temperature respectively. The temperature situation in a newly cast concrete element is the overall cause of thermal cracking. There is an exothermic chemical reaction between water and cement which generates heat that causes the temperature to arise, see Figure 2.1. When the rate of the hydration process becomes slower the temperature in the concrete falls towards the level of the environment. The temperature of the concrete element variates both over the cross Section and in time (Larson, 2000).

The newly cast concrete element undergoes an initial expansion due to temperature rise, see Figure 2.1. Thereafter as the temperature falls the concrete element contracts. For concrete that experiences basic shrinkage (autogenous shrinkage), the initial expansion is reduced while deformations during contraction is increased, this is due to that these deformations are added to the thermal dilution. If the concrete element is restrained, compressive stresses occur during the expansion time. As the element successively contracts, the compressive stresses decrease and a stress-free phase occurs. When the concrete continues to contract, tensile stresses will increase. Micro cracks start to develop at high tensile stresses, which indicates that the stress does not increase in the same frequency as the change of restrained deformations. This means that non-linear stress strain behaviour has occurred. In time, the concrete element may go to failure, whereby cracks that are often exterior, originate through the element. The magnitude of the restraint stresses is dependent on the viscoelastic behaviour of the concrete which depends on how far the hardening process has reached (Larson, 2000).

![Diagram](image)

**Figure 2.1.** Demonstration of generalized temperature, strain and stress development in a newly cast concrete element: a) concrete element, b) temperature, c) volumetric strain, d) stress development at end restraint (Larson, 2000).
2.1.1 Types of cracks

There is no unified scientific definition of types of thermal cracks at early ages. According to Hösthagen (2017) for a “typical engineering structure” where the newly cast concrete body can be defined by two larger spatial dimensions, and one significant smaller third dimension, the thickness, two types of cracks can be distinguished and significantly defined as:

- Early surface cracks
- Later through cracks

The early surface cracks occur during the expansion phase which can be one or two days after casting. The time of the expansion phase might be significantly longer for thicker bodies. Cracks forming in the heating phase normally tend to “close” in the cooling phase. That is why the effect of these cracks on function, static capacity and durability can be discussed. The term close indicates that the concrete goes from a tensile to a compression state and due to “interlocking” there are probably still slits from the concrete surface into some depth of the concrete. These slits can act as channels to transport harmful liquids and gases into the concrete. From a phenomenological point of view the concrete may be viewed as “cracked”. It is noted that surface cracks can develop to through cracks, which would not occur otherwise. Hence, crack risks regarding early surface cracks are in the regulations considered as harmful as later through cracks with respect to the ratio of tensile stress to the momentary tensile strength (Hösthagen, 2017).

Later through cracks are associated with the average volume decrease due to both temperature decrease and homogenous basic shrinkage in the critical contraction phase. Characteristically, through cracks may develop in the entire cross section as a result of restraint from the adjacent structural concrete or subgrade. This type of cracks may appear weeks or months after a section has been poured, depending on dimensions and other prevailing conditions. The through cracks occur from the point of zero stress, shortly after the temperature maximum, and continue until cracking occurs or the maximum ratio of tensile stress to tensile strength is reached. Cracks that occur during the cooling phase have a tendency to remain open permanently. With this, surface cracks are considered less critical than through cracks (Hösthagen, 2017).

2.1.2 Mechanisms of thermal cracking

The mechanism behind the occurrence of surface cracks is the fact that the internal of the concrete element is getting warmer than the surface. At the same time the surface is getting cooled by the surroundings. With this, the internal of the element is more probable to expand than the surface. At this stage the force equilibrium over the thickness causes tensile stresses at the surface and compression in the interior. Early surface cracks will arise if the surface stress is larger than the momentary tensile strength. The tensile stresses decrease and usually convert to compression, as the range of the temperature gradient slows down and decreases. This is a consequence of changed material properties in a hydrating concrete body (Hösthagen, 2017).

In Figure 2.2, it is demonstrated how later through cracks are formed at the time of full restraint. The loading is viewed as the average temperature contraction over the cross Section as well as the basic shrinkage. The loading of the newly cast concrete is considered as a formal homogenous contraction during the critical contraction phase, this is only effective with respect to valuation of risk of later through cracks. In Figure 2.2a, the concrete has just been poured.
into the form and the hydration has not started yet. Full restraint is reached along the length of the axis of the concrete body. Figure 2.2b shows that when the hydration starts the temperature increases and the concrete body expands in every direction. Because of the full restraint the thermal dilation is hindered which compresses the concrete. At this stage the young concrete has low strength and the compression yields both elastic and plastic deformations. Figure 2.2c, as the concrete reaches its maximum temperature it starts to cool off. Due to negative thermal dilation and basic shrinkage the compression is reducing until it reaches a stress-free state occasionally after the temperature peak. After this, the stress inside the concrete increases and later through cracks might appear if it exceeds the tensile strength, see Figure 2.2d. As shown in Figure 2.2e, if the tensile strength is not reached, the relaxation of the concrete starts. Over time, the effects of the relaxation become more significant, which decrease the tensile stress (Hösthagen, 2017).

The balance between tensile stress and tensile strength is fundamental in the formation of cracking in concrete. Different factors influence the strength and the stress. It is illustrated in Figure 2.3 that the restraint has a significant impact on the stress and thereby on the formation of later through cracks (Hösthagen, 2017).

![Figure 2.2. Mechanism of probable presence of through cracks at full restraint at occasion of thermal dilation and basic shrinkage (Hösthagen, 2017).](image)
According to Emborg and Bernander (1994) four main factors may be identified at an analysis of the stress development in a newly cast concrete element:

- The temperature development in the concrete element
- The degree of restraint that the element is subjected to
- The mechanical behaviour of the young concrete
- The temperature of adjacent structures

Figure 2.4 shows a scheme over the factors that have an impact on thermal cracking in a newly cast concrete element whereby the temperature mostly depends on the sizes and geometry of the element, the cement type, cement content and the thermal properties of the concrete. The temperature development is affected by the conditions on site when concreting, the geometry of the adjoining structures and the conditions of the environment. The degree of restraint is dependent on the position in the structure and the performance of the adjoining structures. When the concrete element is totally prevented to deform, full or total restraint appears. No restraint occurs if the element can deform freely, which means that no stress arises (Larson, 2000). According to Larson (2000) the following factors may affect the degree of restraint in a young cast element:

- The geometry of the cast element
- The adhesion in the casting joint among the newly cast element and the adjoining structure
- The stiffness and geometry of the adjoining structure
- The stiffness and flexibility of the ground
Larson (2000) presents the properties describing the mechanical behaviour of the concrete that are of great importance in thermal stress analysis as following:

- The maturity development
- The shrinkage
- The thermal dilation
- The viscoelastic behaviour
- The non-linear stress-strain behaviour at high tensile stresses

To restrict thermal through cracking it is vital to reduce temperature differences and with that, the deformations between the newly cast concrete element and the adjoining structures. Larson (2000) presents the following actions to be taken into account at construction sites:

- Optimization of the concrete
- Cooling the fresh concrete before casting
- Cooling the hardening concrete
- Heating and / or insulation of the adjoining structures
- Direct reduction of restraint

![Figure 2.4. Scheme over estimation of early age thermal cracking considering influencing factors (Larson, 2000).](image)
2.1.3 Methods for thermal crack risk estimations

There are different methods for thermal crack risk estimations. These methods can be divided into groups depending on how the basic modelling is performed. Basic modeling that is used in more complex methods can be divided into history independent differential types of models or history dependent types where the entire loading history has to be known (Larson, 2000).

There are several methods for thermal crack risk estimation and in this part of the thesis, different methods will be briefly presented.

Method based on temperature formulation: This method uses only temperature to express the risk of thermal cracking whereby a loading temperature during the contraction phase is compared with a critical failure temperature (Larson, 2000).

Method based on strain formulation: This method expresses the risk of thermal cracking in terms of strain, whereby the elastic part of a restrained deformation is compared to a critical failure strain. The method is originated in 1946 by Löfquist (Larson, 2000).

Method based on stress formulation: This method presents the risk of thermal cracking in terms of stress, whereby the maximum tensile stress is compared to the tensile strength of the concrete. The formulation of the maximum stress is based on the Age Adjusted Effective Modulus method which was originally described by Trots in 1967 and has been further developed by Bazant in 1972 (Larson, 2000).

Simplified direct methods: There are basic formulations that make it possible to estimate the risk of cracking. The evaluation of the simplified methods are based on results from complex differential methods. A reference concrete is used for calculations, whereby the concrete behaviour has been modelled. The calculations are computed for stresses and all calculations are linear. Most of the simplified methods are realized in the decisive point of the structure, which indicates direct use of the constitutive relation without any structural analysis (Larson, 2000).

Method based on history integral: Another method that can be used is based on history integral. If history dependent integrals for calculation of creep effects are to be used, the whole history of the entire stress or strain development must be identified. If the principle of superposition is presumed to be valid, compliance functions and relaxations functions are correlated to each other. Another formulation that is a method based on history integral is RIM, relaxation integral method. In this method no structural analysis is needed because that the stress calculations are done in one point of the structure and only the constitutive relation needs to be considered (Larson, 2000).

Method based on history independent differential formulation: When performing structural analysis that include creep effects, it is beneficial to use methods that do not demand storage of the whole stress or strain history. This is most important for analysis with large degrees of freedom. History independent formulations may be achieved by using Maxwell- or Kelvin-chain model. In this thesis the Maxwell chain model is used, mainly because that the computer program ConTeSt Pro, which is used to extrapolate the test results from the reference concrete, utilizes the Maxwell-chain model (Larson, 2000). The fundamental features of the Maxwell-chain formulations are given in Section 2.2.5.2.
2.2 Characteristics of concrete

2.2.1 General

Concrete is a composite material which is made up of aggregates, cement, water and admixtures. The cement paste, by going through a chemical reaction between cement and water, binds together the aggregate particles. By adjusting the proportions of the different materials, a wide series of strength properties can be achieved (Burström, 2006). At the point when the cement paste becomes stiff and binds the ballast to the concrete, the strength of the concrete starts to grow. This is performed by the hydration process. This process is exothermic, which means that heat is generated into the construction during the hardening of concrete. As long as water is available, the cement particles will continue hydrating. The longer the distance between the cement particles, the more space is available for water pores (Harrison, 2003). Thereof, the strength development of the young concrete decreases with high water-to-cement ratio (vct). Water-to-cement ratio is the ratio between amount of water and cement. The water-to-cement ratio will change when using additives in the concrete mixture and the strength development is therefore considered by the equivalent water-to-cement ratio, $vct_{ekv}$, see Eq. 2.1. (Betong handbok, 1994).

$$vct_{ekv} = \frac{W}{C+k\cdot D}$$  \hspace{1cm} (2.1)

where
- $W$ = amount of water, kg
- $C$ = amount of cement, kg
- $k$ = the effectivity factor considering the strength development of the additives in relation to the strength development of the cement
- $D$ = amount of admixtures
- $C + k \cdot D$ = alternative binders

The water-to-cement ratio is given for the cement paste, which is the concrete part that consists of cement and water reaction. The aggregates consist of stone in different fractions. The concrete mixture consists of an interaction between the cement paste and the aggregates, where the cement paste surrounds the aggregate material and binds them together (Eriksson, 2017).

When constructing massive concrete infrastructures such as bridges, foundations and tunnels structural engineering cement is used. In this thesis the structural engineering cement with fly ash, FA, is used and analyzed. The reason for using this type of cement is that the cement is grinded down in thicker fractions, which reduces the hydration and heat development. Minimizing the risk of attack from sulphates or sea water and the risk of harmful aggregates reactions and better frost resistance is other advantages of the cement type FA compared to the traditional cement (Almgren et al, 2013).
2.2.2 Hydration process

Several chemical reactions start when cement and water interact, this process is as earlier mentioned called hydration process. The smaller proportion of cement in cement paste, the bigger distance between the cement particles, and vice versa, as shown in Figure 2.5. This means that when the cement paste and the pores of the cement paste expands, the smaller the size of the capillary pores becomes and so does the porosity of concrete with decreased vct (Almgren et al., 2013).

![Figure 2.5. Illustration of cement paste development with high vct (at left) and low vct (at right) (Almgren et al., 2013).](image)

From the beginning the connection between the cement particles is weak, but as soon as the cement reacts with water in several chemical reactions, the particles grow together and become stronger. Thereafter, the hydration process will take place as long as water is available. How the hydration process develops is decided by several parameters, but the main difference is the speed of the chemical reactions, and how long time it takes for the concrete to cure. Other central parameters impacting the development of the hydration process are the chemical composition of cement, the particle size of cement, water-to-cement ratio, the temperature during casting and additives. The final degree of hydration is in practice not exceeding 70-80 % (Eriksson, 2017).

The type and amount of cement, aggregates and the water-to-cement ratio are factors that are directly connected to the heat development of the concrete that is determined by the evaluation of semi adiabatic calorimetric measurements. To take the heat loss into account in the test set-up, a correction factor, $\eta$, was introduced. The heat of hydration and thermal properties are determined by applying several processes. The total heat of hydration by cement weight at a certain time, $q_{cem}(t)$, calculated from measured temperatures, using the semi-adiabatic calorimetric set-up can be described as follows (Fjellström, 2013)

$$ q_{cem}(t) = \frac{\rho_c}{C_c} \left( \eta \cdot (T_c(t) - T_{air}) + a \cdot \int_0^t (T_c(t) - T_{air}) \cdot dt \right) $$

(2.2)

where

- $q_{cem}(t)$ = heat energy by cement weight, [J/kg]
- $\rho_c$ = concrete density, [kg/m$^3$]
- $C_c$ = heat capacity by weight of concrete, [J/kg °C]
- $C$ = cement content, [kg/m$^3$]
- $\eta$ = correction factor with respect to heat stored in the test set-up, values for $\eta$ see (Fjellström, 2013)
- $T_c(t)$ = measured temperature in the concrete specimen, [°C]
- $T_{air}$ = ambient temperature, [°C]
- $a$ = cooling factor, [l/s]
The heat loss to the surroundings of the semi-adiabatic test set-up is determined by the cooling factor, a. The cooling factor will vary between different tests and must be accurately predicted for each single test (Fjellström, 2013).

According to (Hösthagen, 2017), the generated heat per concrete volume, \( Q_h(t) \) is relevant in similar heat calculations, and is expressed by

\[
Q_h(t) = \frac{dq_{cem}(t)}{dt} \cdot C
\]

(2.3)

where

\( Q_h(t) \) = generated heat per concrete volume, [W/m³]

Using Eq 2.2 the evaluated heat energy development can be approximated for computer calculations with the following equation (Hösthagen, 2017)

\[
q_{cem}(t) = \exp \left( - \left( \ln \left( 1 + \frac{t_e}{t_1} \right) \right)^{-\kappa_1} \right) \cdot q_u
\]

(2.4)

where

\( q_u \) = total heat energy by cement weight, formally after infinite time [J/kg]

\( \kappa_1 \) = free model parameter to get the acceptable fit with the test data [-]

\( t_1 \) = free model parameter to get the acceptable fit with the test data [s]

2.2.3 Temperature development

In a newly cast concrete, the temperature increases rapidly after the exothermic hydration process that is described in Section 2.2.1. By the initial heat the hydration process goes faster, which results in higher maximum temperature (Eriksson, 2017). The moisture is difficult to measure with adequate precision and is therefore neglected in the evaluation models (Hösthagen, 2017).

According to Emborg et al.,(1997) there are several other parameters that have great impact on the temperature development, such as the additives, heat properties of the concrete, size and geometry of the structure and many more.

The maximum temperature in a structure is normally reached 25-30 hours after a finished casting (Emborg, 1989). However, this varies with the size of the structure, but also the use of additives that changes the process of the hydration. For massive structures, a large amount of exothermic reactions is taking place. This will result in a huge temperature development, especially in the inner parts of the structure. The outer parts of the concrete construction cool down rapidly, which means an uneven temperature and stress distribution in the construction (Emborg et a., 1997). Figure 2.6 illustrate how the temperature gradient changes through the cross-Section. Normally, the mean temperature of the cross-Section is considered when calculating the temperature development.
2.2.4 Strength development

At the point when the cement paste becomes stiff and binds the aggregates to the concrete, the strength of the concrete starts to grow. The compressive strength development, $f_{cc}(t)$, of the concrete will be most developed between 24 and 36 hours after casting as a result of the high temperature in the concrete. Depending on the temperature conditions, the concrete will mature with changing speed. To be able to calculate the maturity of the concrete, the equivalent time of maturity, $t_e$, at 20°C is introduced (Almgren et al., 2013).

Even if the temperature and the strength of the concrete increase simultaneously, it is important to mention that the cracks in the structure are also occurring due to stresses exceeding the strength. It is therefore important to take into account the E-modulus, coefficients of thermal expansion and the viscoelastic properties (Emborg et al., 1997).

Since 1900, tests of how to determine the strength development of concrete, have been performed (McDaniel, 1915). The aim of the strength tests performed at LTU is to establish the reference strength development function. The temperature dependent maturity function, $\beta_T$, is the first part of the tests to develop, and the second function is the equivalent time of maturity, $t_e$. The reference strength development can with these two functions be established (Hösthagen, 2017).

To determine the strength growth, the compressive strength development, $f_{cc}(t)$, needs to be examined by allowing concrete specimens to cure in different water baths at different temperature conditions and loading each of them until they fail. More about the test procedure is described in Section 2.3. The obtained results from the strength tests are used to establish the parameters for the equivalent time of maturity, $t_e$(Hösthagen, 2017).

The reference strength development is referred to the compressive strength at 20°C, without taking into account the effects of high curing temperature, and is defined for three stages, by Eq. 2.5, according to Hösthagen (2017). The three stages are

Stage 1: fresh concrete ($0 \leq t_e < t_S$)

Stage 2: between initial and final setting ($t_S \leq t_e < t_A$)

Stage 3: hardening concrete ($t_e \geq t_A$)
The expression for stage 3 in Eq. 2.5 is based on a formula in EN 1991-1-1:2004 (Euro Code 2) and is modified to fulfil the condition \( f_{cc}^{ref}(t_A) = f_A \) and \( t^* \) is calculated by the following formula

\[
t^* = \frac{672 - \delta_c t_A}{1 - \delta_c}
\]  

(2.6)

with

\[
\delta_c = \left(1 - \frac{1}{s} \cdot \ln \frac{f_A}{f_{cc,28}}\right)^{1/n_{cc,28}}
\]

where

\( t^* \) is calculated by Eq. 2.6, but has no physical meaning, h

\( t_e \) = equivalent time calculated by Eq. 2.7, h

\( t_s \) = equivalent time at initial setting, where the concrete starts to transform from a “liquid” to a “solid” state, h

\( t_A \) = equivalent time at final setting, where the concrete surface no longer can be troweled, modelled by the time when the strength reaches \( f_A \), h

\( f_A \) = concrete strength at final setting, usually chosen to be strength level 0.5 MPa

\( s \) = parameter influencing the curve shape in time of the hardening concrete, -

\( n_{cc,28} \) = parameter influencing the curve shape in time of the hardening concrete, -

\( f_{cc,28} \) = 28 days strength of concrete, Pa

Hösthagen (2017) describes the expression of the equivalent time of maturity, \( t_e \), with the following formula

\[
t_e = \beta_\Delta \cdot \int_0^t \beta_T \cdot dt + \Delta t^0_e
\]

(2.7)

where

\( \beta_\Delta \) = possible adjustment parameter due to admixture changes, normally \( \beta_\Delta = 1 \), -

\( \Delta t^0_e \) = possible adjustment parameter due to admixture changes, normally \( \Delta t^0_e = 0 \), h

\( \beta_T \) = temperature dependent maturity function expressed by Eq. 2.8, (Hösthagen, 2017) -

\[
\beta_T = \exp \left( \frac{\Theta}{T + 273} - \frac{1}{T + 273} \right)
\]

(2.8)

where

\( T \) = concrete temperature, °C
$\Theta = \text{the thermal activation energy function, described by}$

$\Theta = \Theta_{ref} \cdot \left(\frac{30}{T + 10}\right)^{\kappa_3}$

where

$\Theta_{ref} = \text{reference maturity parameter, formally activation energy divided by general universal gas constant, determined from strength growth tested at variable temperatures, K}$

$\kappa_3 = \text{parameter reflecting the variation of the activation energy by temperatures, determined from strength growth tested at variable temperatures, -}$

By using the temperature dependent maturity function, $\beta_T$, the conversion of real time into equivalent time of maturity $t_e$ is made possible. Both functions are determined by regression analyses. The temperature dependent maturity function describes the rate of cement reaction to the chosen reference temperature, 20°C (Hösthagen, 2017).

The stress at full restraint is measured, with a temperature load corresponding to the mean temperature of a 700 mm thick wall. The main function of this restraint test is to make the concrete specimen undergo tensile strength failure. How the tensile strength of concrete, $f_{ct}$, is related to the compressive strength is described by Hösthagen (2017) as

$f_{ct} = \left(\frac{f_{cc}}{f_{cc}^{ref}}\right)^{\beta_1} \cdot f_{ct}^{ref}$

(2.9)

where

$f_{ct} = \text{compression strength, Pa-}$

$\beta_1 = \text{connection parameter tensile-compression strength according to Eurocode 1992-1-1, -}$

$f_{cc}^{ref} = \text{reference compressive strength, Pa}$

$f_{ct}^{ref} = \text{reference tensile strength, Pa}$

2.2.5 Basic shrinkage and free thermal dilation

For a hydrating concrete, the deformation from the free deformations tests has to be divided into thermal dilation and basic shrinkage. Thermal dilation is defined as the free deformation of the concrete specimen, where no load is applied to the specimen, caused by variation in concrete temperature. Basic shrinkage is the free contraction of a concrete specimen during the hydration process at moist sealed conditions, caused by self-desiccation.

At LTU, there are two different methods, I and II, for evaluation of basic shrinkage that could be used (Fjellström, 2013). In Method I, the basic shrinkage is described as a function of equivalent time, while in Method II it is described as a function of equivalent time and temperature.

Two specimens for the free deformation tests are used, where one of them have a temperature of 20 °C, see specimen A in Figure 2.7, and the second one is placed in a temperature water. The temperature water simulates a temperature development in a real structure, which is referred to a 0.7 m thick wall, see specimen B in Figure 2.8. In order to obtain valid basic
shrinkage for any temperature curve, it is essential to withdraw the thermal dilation from the measured deformation, which is more necessary for the case with heated specimen, see Figure 2.8. The thermal dilation is approximately zero for Figure 2.7.

![Figure 2.7. Measured basic shrinkage for a specimen A at a temperature of 20°C (Fjellström, 2013).](image)

![Figure 2.8. Measured deformation from thermal dilation and basic shrinkage for a specimen B at a temperature of 20°C (Fjellström, 2013).](image)

The sum of the measured deformation from specimen A and B, can as mentioned by Fjellström (2013) and Hösthagen (2017), be formulated by the following equation

\[ \varepsilon_{\text{free}} = \varepsilon_T + \varepsilon_{SH}^0 \]  

(2.10)

where

- \( \varepsilon_{\text{free}} \) = measured combined free deformation, -
- \( \varepsilon_T \) = thermal dilation, -
- \( \varepsilon_{SH}^0 \) = basic shrinkage, -

The thermal dilation is expressed by

\[ \varepsilon_T = \alpha_T \cdot \Delta T_c(t) \]

where

- \( \alpha_T \) = thermal dilation coefficient, to be determined in the evaluation procedure, °C\(^{-1}\)
- \( \Delta T_c(t) \) = measured temperature change in the concrete, °C

The basic shrinkage is according to Hedlund (2000), expressed by

\[ \varepsilon_{SH}^0 = \beta_{so}(t_e) \cdot \varepsilon_{su} = \exp\left(\frac{t_{sh}}{t_{e-t_s}}^{\eta_{sh}}\right) \cdot \varepsilon_{su} \]  

(2.11)
where
\[ \varepsilon_{su} = \text{reference ultimate shrinkage, to be determined in the evaluation procedure, } \]
\[ t_s = \text{time of initial setting, end of stage 1 in Eq. 2.5 (see Section 2.2.4), s} \]
\[ t_{sh} = \text{time parameter affecting the shrinkage development, to be determined in the evaluation procedure, s} \]
\[ \eta_{sh} = \text{parameter affecting the shrinkage development, to be determined in the evaluation procedure, s} \]

2.2.6 Viscoelastic behaviour and basic creep

Concrete has a viscoelastic behaviour at early ages. When subjecting concrete to a load, an instant deformation occurs, as shown in Figure 2.9. The E-modulus of the concrete can be used to describe the instantaneous deformation. After the instantaneous deformation the concrete will keep deforming over time, even though the load is kept constant. The deformations, after the instantaneous deformation, that happen over time are related to creep. The creep is thus defined as the time dependent deformation under an imposed stress history. The relaxation of concrete however, is defined as stress development due to an imposed strain history. The elastic part, is referring to the instantaneous deformation when the concrete is unloaded. When unloading the concrete, in time, parts of the concrete creep will reverse which often is signified as creep recovery (Larson, 2000).

The creep deformations can be influenced by inner and outer factors. The inner factors are subject to the composition of the concrete, such as water to cement ratio, cement type, aggregate. For instance, a larger quantity of aggregate in relation to the cement paste reduces the creep deformation (Larson, 2000). According to Byfors (1980) the most important outer factors are

- *Duration of loading*. The concrete will continue to creep as long as it is loaded. However, the creep rate will decrease with time.
- **Inner moisture state.** The creep is increased when having a high relative humidity within the concrete. Dry concrete has lower creep.

- **Temperature level.** The creep will increase with a high temperature within the concrete. The high temperature will also accelerate the maturity growth, hence decreasing the creep response.

- **Load level.** The creep is having a linear behaviour and is directly proportional to the load at low loads. The creep is having a progressively increasing behaviour with increasing load.

- **Type of load, compression or tension.** It is not yet clarified if there are any difference between creep in tension or creep in compression.

- **Age of the concreate at loading.** Loading concrete at a young age leads to much larger creep deformation than loading it at older age. The age at loading has a dominating impact on the creep of early aged concrete.

- **Variations in temperature and moisture.** The creep is influenced by variations of temperature or moisture within the concrete. The creep deformations are increased by large variations.

![Figure 2.10. The most important outer factors that influence creep according to Byfors (1980).](image)

### 2.2.6.1 Modeling of creep and relaxation

For modelling elasticity and creep there are two primary ways, the creep coefficient formulation and the creep compliance formulation. When using the creep coefficient, the measured deformation is parted into elastic (instantaneous) deformation and creep. The total deformation can according to Larson (2000) be described as
\[ \varepsilon(t, t_0) = \frac{\sigma(t_0)}{E(t_0)} + \rho(t, t_0) \cdot \frac{\sigma(t_0)}{E(t_0)} \]  

(2.12)

where

\( \sigma(t_0) \) = the stress applied at time \( t_0 \), [Pa].

\( E(t_0) \) = the modulus of elasticity at time \( t_0 \), [Pa].

\( \rho(t, t_0) \) = the creep coefficient or the creep function at time \( t \) for loading at \( t_0 \) [-].

The creep and the E-modulus have to be evaluated from the same test to achieve acceptable accuracy when using the creep coefficient formulation. Both the instantaneous deformation and the creep is included in the creep compliance function, thus a division into separated parts is not needed, see Figure 2.11 (Larson, 2000).

\[ \varepsilon(t, t_0) = J(t, t_0) \cdot \sigma(t_0) \]  

(2.13)

with

\[ J(t, t_0) = \frac{1 + \rho(t, t_0)}{E(t_0)} = \frac{1}{E_{\text{eff}}(t, t_0)} \]

where \( E_{\text{eff}}(t, t_0) \) is the effective modulus, [Pa].

Usually, drying shrinkage and creep take place simultaneously. In view of the various combinations of moisture conditions, loading, temperature and restraint, the following terms are defined according to Nemati (2015):

**Basic creep:** is the creep that occurs under conditions without any drying shrinkage or moisture exchange between concrete and the ambient environment.

**Drying creep:** is the additional creep that occurs when the specimen that is loaded also is drying.

In this work, the focus is on deformations in concrete due to basic creep.
Relaxation defines the time dependent stress-decrease under a constant strain, as shown in Figure 2.12 (Larson 2000).

According to Larson (2000) the relation between the initial stress $\sigma(t_0)$ and time dependent stress $\sigma(t, t_0)$ is defining the relaxation function, giving

$$\psi(t, t_0) = \frac{\sigma(t, t_0)}{\sigma(t_0)} \tag{2.14}$$

Which gives the time dependent stress development as

$$\sigma(t, t_0) = \sigma(t_0) \cdot \psi(t, t_0) = \varepsilon(t_0) \cdot E(t_0) \cdot \psi(t, t_0) \tag{2.15}$$

Which may also be formulated with the relaxation modulus $R(t, t_0)$ like in Figure 2.12 as

$$\sigma(t, t_0) = \varepsilon(t_0) \cdot R(t, t_0) \tag{2.16}$$

where

$$R(t, t_0) = E(t_0) \cdot \psi(t, t_0)$$

2.2.6.2 Maxwell- chain model and RELAX program

2.2.6.2.1 General background

Basic creep can be described by two different values. The first one is creep values, with a variable strain and constant stress, while the second one is relaxation values, where the strain is constant and the time-dependent stress is variable (Jonasson and Westman, 2001). The constitutive relation, which describes the relation between the time-dependent deformation of a material and the stress, can be divided into history dependent differential types and history independent differential types, as mentioned earlier. This type of division of the constitutive relations is performed when using more advanced methods when estimating thermal cracks.
this thesis, when analyzing structures that include the impact of creep, it is beneficial to use methods that do not require the whole stress and strain history. The requirement of a big amount of data storage and longer calculation time, is the main reason for not using history dependent differential method. Thereby, using a method based on history independent formulations may be more useful (Larson, 2000).

Several models can be used to achieve history independent formulations, for instance, Maxwell- or Kelvin-chain models, but also the Burger-model (Bazant and Wittman, 1982). The finite element method ConTeSt Pro is going to be used in this master thesis which utilizes the Maxwell-chain model to describe the viscoelasticity. With this, Maxwell-chain model is used instead of Kelvin-chain model and Burger-model. For the relaxation of the different Maxwell elements when performing analyses with the Maxwell-chain model, the values of \( E_\mu(t) \) and \( \eta_\mu(t) \) are required (Larson, 2000). The required values are further described in Section 3.4.3.

There are several methods which can be used to express the relaxation function from a known creep function. In this thesis, the creep is converted into relaxation by solving the relaxation function \( R(t,t_0) \) from a compliance function \( J(t, t_0) \). This is performed by solving the following equation (Larson, 2000):

\[
\Delta R_j = \frac{\Delta R_1}{J_1} \cdot \left[ \sum_{s=2}^{j-1} \Delta R_s \cdot \left( J_{j,s-1/2} - J_{j-1,2-1/2} \right) + \Delta R_1 \cdot \left( J_{j,0} - J_{j-1,0} \right) \right]
\]

(2.17)

The formulation is implemented in a computer program called RELAX (Westman and Jonasson, 1999), whereby creep data is converted into relaxation values. Larson claims that this method is more precise than other simplified methods described by Wittman (1974) and Trost (1967) when using it due to the conversion of creep into relaxation.

The RELAX program calculates discrete values of relaxation function and discrete values of the age-dependent elastic modulus \( E_\mu(t_0) \), which yields a relaxation spectrum. Creep model derived at American concrete institute model (ACI), linear logarithmic model (LLM), the double power law (DPL), triple power law (TPL) and also engineering method are some examples of creep models out of eight supplemented in the RELAX program. Converting the creep data in the RELAX program directly and not using any creep model is another feasibility. In this study, LLM is chosen to be used as a creep model due to its simplicity and reliability, see further Larson (2000) and more detail further in this thesis.

2.2.6.2.2 The process of conversion of creep values into relaxation data

The conversion of creep values into relaxation values are performed by solving the relaxation function \( R(t,t_0) \) from the compliance function \( J(t, t_0) \). This is done by stating the time \( t \) in discrete times \( t_1, t_2, t_3, ....t_N \) yielding time steps \( \Delta t = t_j - t_{j-1} \), according to Bazant and Wu (1974).

It is important to apply the unit strain \( \varepsilon(t_0) = 1 \), to determine the relaxation stress at the age \( t_0 \). The relaxation function from the first time step \( t_1 - t_0 \), is calculated by the effective E-modulus as described by Jonasson and Westman (2001) shown in Eq 2.18.

\[
R(t_1, t_0) = R_{1,0} = E_{\text{eff}} \cdot \varepsilon_1 = E_{\text{eff}} \cdot 1 = \frac{1}{J(t_1, t_0)} = \frac{1}{J_{1,0}}
\]

(2.18)

To compute the relaxation function for the retained time steps, an adjustment function is used to cover all ages, which is illustrated in Figure 2.13.
Figure 2.13. From a numerical solution based on the principle of superposition, the relaxation function \( R(t, t_0) \) is obtained from creep values (Jonasson and Westman, 2001).

The expression of the strains related to the times \( t_1, t_2, t_3, ..., t_N \) is as following (Jonasson and Westman, 2001):

\[
\begin{align*}
\varepsilon_1 &= 1 = R_{1,0} J_{1,0} = \Delta R_1 J_{1,0} \\
\varepsilon_2 &= 1 = \Delta R_1 J_{2,0} + \Delta R_2 J_{2,1.5} \\
\varepsilon_3 &= 1 = \Delta R_1 J_{3,0} + \Delta R_2 J_{3,1.5} + \Delta R_3 J_{3,2.5} \\
\varepsilon_N &= 1 = \Delta R_1 J_{N,0} + \Delta R_2 J_{N,1.5} + \Delta R_3 J_{N,2.5} + \ldots + \Delta R_N J_{N,N^{1/2}}
\end{align*}
\]

The relaxation increment from time \( t_{j-1} \) to time \( t_j \) is expressed by equation (2.19) by subtracting the strain \( \varepsilon_{j-1} \) from \( \varepsilon_j \) and finally evaluating the relaxation function \( R(t, t_0) \) for the load application age \( t_0 \) (Jonasson and Westman, 2001).

In a relaxation test, tensile stresses cannot develop after a long time in the case when applying an increment of compressive strain, which yields negative relaxation values in the relaxation spectrum. This estimation is an important presumption when a relaxation function may be established.

The consequences of using creep models is that it may cause negative relaxation values when applying loads at early ages and during a long period of time. Obtaining negative relaxation values in the analysis gives an unrealistic overestimation of tensile stresses (Larson, 2000). Hence, it is important to avoid negative relaxation to occur during early ages in combination with long load durations. A way to deal with this problem is to choose a certain age of loading \( (t_0)_{\text{age}} \) and a certain load duration \( (t - t_0)_{\text{age}} \) as limit values (Jonasson and Westman, 2001).

The modification is performed so that all creep curves for \( t_0 < (t_0)_{\text{age}} \) and \( t - t_0 > (t - t_0)_{\text{age}} \) are parallel to that of \( t_0 = (t_0)_{\text{age}} \), as it is shown in Figure 2.14. By the spectras in Figure 2.14, the stress related strain, at time \( t \) after mixing, for uniaxial loading of concrete is expressed (Jonasson and Westman, 2001). This type of adjustment and corrections is made for any type of creep formula. One thing is that this method is not able to eliminate negative relaxation values for very long load durations like more than 30 years, but it still fulfils its purpose since the thesis deals with young concrete at early ages (Jonasson and Westman, 2001).
2.2.6.2.3 Estimation of material parameters

During the hardening process of young concrete, several different ongoing processes have a large impact on the total deformation of the young concrete. By a rheological model, which is illustrated in Figure 2.15, the process could be explained where the total strain consists of three strain parts i.e. load independent strain, $\varepsilon_{vol}$, viscoelastic strain, $\varepsilon_{visc}$, and non linear strain, $\varepsilon_{fract}$. Change in volume due to thermal dilation and shrinkage is the main reason for the load independent strain. The viscoelastic strain is the part that corresponds to elastic deformation and creeps, while nonlinear strain, due to cracking corresponds to fracturing mechanics. Those later two parts are included in the total deformation only when a load is applied (Larson, 2000).

The constitutive relation, which is the relation between the time-dependent strain of a material and the stress, is given on incremental form for a time step $\Delta t$. In the Maxwell-chain model, the incremental formulation of creep analyses is used, concerning the viscoelastic part of the total
strain. In a rheological model, the material that is used as basic units is often a spring and a dashpot, as illustrated in Figure 2.16 (Larson, 2000).

By Hooke’s law the spring unit can be described as following:

\[ \sigma_1 = E \cdot \varepsilon_1 \]  \hspace{1cm} (2.20)

where index I is used to indicate the spring unit.

The time derivative of Eq. 2.20 gives the strain increment expressing the incremental formulation for the spring unit (Larson, 2000):

\[ \dot{\varepsilon}_I = \frac{\dot{\sigma}_I}{E}, \]  \hspace{1cm} (2.21)

where the time derivatives of strain and stress are expressed as \( \dot{\varepsilon}_I = \frac{\delta \varepsilon}{\delta t} \) and \( \dot{\sigma}_I = \frac{\delta \sigma}{\delta t} \).

The viscous behaviour of the dashpot as it is demonstrated in Figure 2.16, is according to Larson (2000) described by

\[ \varepsilon_{II} = \frac{\sigma_{II}}{\eta_v} \cdot t \]  \hspace{1cm} (2.22)

Which by the time derivatives can be defined as

\[ \varepsilon_{II} = \frac{\sigma_{II}}{\eta_v} \]  \hspace{1cm} (2.23)

where the viscosity of the dashpot is described by \( \eta_v \) [Pa s].

According to Figure 2.17, a basic Maxwell unit consists of a spring unit and a dashpot coupled in series.
Figure 2.17. Basic Maxwell unit, a spring unit and a dashpot (Larson, 2000).

For the basic Maxwell unit represented in Figure 2.17, Larson (2000) defines the compatibility equations as following

\[ \varepsilon = \varepsilon_I + \varepsilon_{II} \]  
(2.24a)

or

\[ \dot{\varepsilon} = \dot{\varepsilon}_I + \dot{\varepsilon}_{II} \]  
(2.24b)

for the corresponding time derivatives values of strain,

and

\[ \sigma_I = \sigma_{II} = \sigma \]  
(2.25a)

or

\[ \dot{\sigma}_I = \dot{\sigma}_{II} = \dot{\sigma} \]  
(2.25b)

for the corresponding time derivatives values of stress.

By inserting Eq. 2.21, 2.23 and 2.25 into Eq. 2.24 the following equation is obtained (Larson, 2000):

\[ \frac{\dot{\sigma}}{E} + \frac{\sigma}{\eta_v} = \dot{\varepsilon} \]  
(2.26)

Which could be analytically solved for some special cases, where the term on the right side of the equation, the strain rate, consists of a group of functions. A basic case is when the strain rate is constant, and gives therefore the following solution of the equation (Larson, 2000):

\[ \sigma = \sigma_H + \sigma_P \]  
(2.27)

With a homogenous part and a particular part

\[
\begin{align*}
\sigma_H &= A \cdot \exp(-Bt) \\
\sigma_P &= C
\end{align*}
\]

where the integral constants A, B and C are expressed as follows

\[
\begin{align*}
A &= \sigma_0 - \eta_v \cdot \dot{\varepsilon}_{\text{const}} \\
B &= \frac{E}{\eta_v} [s^{-1}] \\
C &= \eta_v \cdot \dot{\varepsilon}_{\text{const}}
\end{align*}
\]

The integral constant A is determined by implementing the initial conditions which are \( \sigma = \sigma_0 \) for \( t=0 \). The homogenous part, by identification of Eq. 2.26, gives the B constant. The constant C is obtained by the particular part, when the strain rate is constant, \( \dot{\varepsilon} = \dot{\varepsilon}_{\text{const}} \).
By inserting the integral constants into Eq. (2.27), the total solution is expressed as follows (Larson, 2000):

$$\sigma = \sigma_0 \cdot \exp\left(- \frac{E}{\eta_v} \cdot t \right) + \eta_v \cdot \dot{\varepsilon}_{\text{const}} \cdot \left(1 - \exp\left(- \frac{E}{\eta_v} \cdot t \right)\right)$$  (2.28)

Many basic Maxwell units will together create a chain of Maxwell units, as illustrated in Figure 2.18. Nowadays, Prony Series model describes the corresponding general Maxwell-chain model for modeling complex behaviour of materials in relaxation. The Maxwell model was proposed to find a solution to the issue of having zero stress at time infinity in relaxation (Fathi et al, 2010).

![Figure 2.18. Chain of Maxwell units consisting of N Maxwell basic units (Larson, 2000).](image)

For the Maxwell-chain model, the solution from Eq. 2.27 is directly applied to this case as follows (Larson, 2000):

$$\sigma_\mu = \sigma_{\mu_0} \cdot \exp\left(- \frac{E_\mu}{\eta_\mu} \cdot t \right) + \eta_\mu \cdot \dot{\varepsilon}_{\text{const}} \cdot \left(1 - \exp\left(- \frac{E_\mu}{\eta_\mu} \cdot t \right)\right)$$  (2.29)

where

\(\mu\) = arbitrary basic Maxwell unit numbered \(\mu\)

\(\mu_0\) = start of time \(t\) for unit number \(\mu\).

If assuming that Eq. 2.29 is only valid during a time step \(\Delta t\), time \(t\) should, therefore, be exchanged with \(\Delta t\) in Eq. 2.29 and then introducing the expression of constant strain rate

$$\dot{\varepsilon}_{\text{const}} = \frac{\Delta \varepsilon}{\Delta t}$$  (2.30)

and the expression of relaxation time, \(\tau_\mu\), for unit number \(\mu\)

$$\tau_\mu = \frac{\eta_\mu}{E_\mu}$$  (2.31)

where

\(E_\mu(t)\) = stiffness of the \(\mu\):th spring

\(\eta_\mu\) = the viscosity of the \(\mu\):th dashpot.
By rewriting the equation, the viscosity of the \( \mu \):th dashpot, \( \eta_\mu \), could be expressed as

\[
\eta_\mu = E_\mu \cdot \tau_\mu
\]

(2.32)

For the end of time step \( \Delta t \) Eq. 2.28 becomes

\[
\sigma_{\mu1} = \sigma_{\mu0} \cdot \exp\left( -\frac{\Delta t}{\tau_\mu} \right) + E_\mu \cdot \tau_\mu \cdot \frac{\Delta \varepsilon}{\Delta t} \cdot \left( 1 - \exp\left( -\frac{\Delta t}{\tau_\mu} \right) \right)
\]

(2.33)

where

\( \mu_1 \) = the end of time step \( \Delta t \) for unit number \( \mu \).

Applying the condition \( \Delta t = 0 \) into Eq. 2.33 gives

\[
\sigma_{\mu0} = \sigma_{\mu0}
\]

(2.34)

If subtracting Eq. 2.34 from Eq. 2.33 and introducing

\[
\Delta \sigma_\mu = \sigma_{\mu1} - \sigma_{\mu0}
\]

(2.35)

The final expression now becomes

\[
\Delta \sigma_\mu = -\sigma_{\mu0} \cdot \left( 1 - \exp\left( -\frac{\Delta t}{\tau_\mu} \right) \right) + E_\mu \cdot \frac{1-\exp\left( -\frac{\Delta t}{\tau_\mu} \right)}{\frac{\Delta t}{\tau_\mu}} \cdot \Delta \varepsilon
\]

(2.36)

The sum of all basic Maxwell units gives

\[
\Delta \sigma = \sum_{\mu=1}^{N} (\Delta \sigma_\mu)
\]

(2.37)

or

\[
\Delta \sigma = E^* \cdot \Delta \varepsilon - \Delta \sigma^*
\]

(2.38)

with

\[
E^* = \sum_{\mu=1}^{N} E^*_\mu = \sum_{\mu=1}^{N} \left( E_\mu \cdot \frac{1-\exp\left( -\frac{\Delta t}{\tau_\mu} \right)}{\frac{\Delta t}{\tau_\mu}} \right)
\]

(2.39)

and

\[
\Delta \sigma^* = \sum_{\mu=1}^{N} (\Delta \sigma^*_\mu) = \sum_{\mu=1}^{N} \sigma_{\mu0} \cdot \left( 1 - \exp\left( -\frac{\Delta t}{\tau_\mu} \right) \right)
\]

(2.40)

An important notation of Eq. 2.38 is that the equation signifies a quasi-elastic calculation for a time step with a quasi-elastic modulus, \( E^* \) and a fixation stress, \( -\Delta \sigma^* \).

Beyond Eq. 2.36 there is an alternative formulation:

\[
\Delta \sigma^* = E^* \cdot (\Delta \varepsilon - \Delta \varepsilon^*)
\]

(2.41a)

with

\[
\Delta \varepsilon^* = \frac{\Delta \sigma^*}{E^*}
\]

(2.41b)

where
\( \Delta \varepsilon^* \) is a quasi-inelastic strain, which means analogous with thermal dilation and shrinkage.

Using Eq. 2.36 or 2.39 is fully equivalent to use for the solution of the stress increment for a time step. After every calculation of a time step the total stress is updated by

\[
\sigma_{t+\Delta t} = \sigma_t + \Delta \sigma
\]  

(2.42)

Regarding each Maxwell unit, the stress is updated due to Eq. 1.17. These types of stresses are often called hidden stresses.

2.2.6.2.4 Input and output for the program RELAX

In this thesis the version RELAX_m of the program is used and the input and output from RELAX is described from the same version. The unit of times is given in days. The relaxation values and the elastic modulus are given in the unit GPa and the creep compliance is given in the unit 1/GPa (Jonasson and Westman, 2001).

The requirement of input for RELAX are two files, whereby from the first file, the shape of output data is determined and from the second one, creep data or parameter values in chosen creep model are given.

In Figure 2.19, it is illustrated the choice of loading ages and load duration for which relaxation values are to be calculated. In the input file, the first-time span in days \((t - t_0)\) is given, thereafter each decennium in running time is divided into intervals of number (JDEC) splitting the other load durations, as it is shown in Figure 2.19a. The application of the first load age should be larger than the set time of concrete. The time span after \(t_s\) until the first loading age \(\Delta(t_0)_1\) is in the input file given instead of \((t_0)_1\), thus to ensure that \((t_0)_1\) is larger than \((t_s)_1\). The application of the first load age is therefore in the calculation system determined by the following equation (Jonasson and Westman, 2001):

\[
(t_0)_1 = (t_s) + \Delta(t_0)_1
\]  

(2.43)

Choosing \(\Delta(t_0)_1 > 0.1\) day is recommended. In the input file, the value of \((t_s)\) is given which contains values on parameters in chosen creep model. Each decennium in running time after the first loading age is divided into intervals of number \((JA)\), as represented in Figure 2.19.

The input file contains the following:

\[
\begin{align*}
\text{Name} & \\
NDEC & \\
JDEC & \\
M & \\
JA & \\
(t - t_0)_1 & \\
\Delta(t_0)_1 & \\
\text{FileNameCreep} & \\
\end{align*}
\]

Input file*.ire

where

Name = the output file name - Name. The Name may sometimes include also the path where to store the output files.

NDEC = number of decennium in time spans

JDEC = number of steps in each decennium in time spans

M = total number of loading ages in the computed system, for which \(E_\mu\) is determined

JA = number of intervals per decennium in loading age
(t - t_0)_1 = first time span, days
Δ(t_0)_1 = time span after t_s for first loading age, days

Figure 2.19. In case a) the load durations for which relaxation values are calculated for the choice NDEC=7, JDEC=3, and (t - t_0)_1 =0.001. N=NDEC*JDEC+1=22. In case b) it is shown the loading ages in the computed system for which E_µ is determined for the choice M=9, JA=3, and (t_0)_1 = 0.5 (Jonasson and Westman, 2001).

The file name FileNameCreep consists of values for parameters of used creep model. A single figure (KTYPE) is given on the first line in the file FileNameCreep, which creep model is currently used, and it defines in what way the input values are given. There are several creep models in RELAX, and in this case the KTYPE=8 is used, which refers to the creep model Linear Logarithmic model.

Six output files are formed when RELAX_m is implemented. The file names and what it is received from the file is as follows (Jonasson and Westman, 2001):

FileNameCreep.nfo a receipt on what input information RELAX has been given
FileNameCreep.cin creep data used as input for conversion into relaxation data
FileNameCreep.cut creep data according to chosen Maxwell chain for the same loading ages and load durations as in FileNameCreep.cin
FileNameCreep.rin relaxation data computed from input creep data
FileNameCreep.rut relaxation data to be used in the Maxwell chain calculations
FileNameCreep.rel Values of spring constants and loading ages according to chosen Maxwell chain (=relaxation spectra for the chosen “relaxation” times to be used in numerical step-by-step calculations)

The first relaxation time, \( \tau_{r1} \) is set to 5(t - t_0), and thereafter there is relaxation time for each decennium according to \( \tau_{r2} = 10\tau_{r1}, \tau_{r3} = 10\tau_{r2} \) and so on. The number of relaxation times
are dependent on the number of decennium stated in the input file (.ire). There is one for each decennium plus one with the value of infinity. This represent eight Maxwell elements in Maxwell chain-model.

In a similar way, the choice of loading ages is performed, but here the desired number of loading ages are given directly instead of choosing a number of decennium. The youngest age at loading and number of intervals each decennium is divided into values of the loading ages that will be determined. Each decennium was divided into three intervals (JA=3) with the youngest age at loading equal to 0.5 day. Choosing nine loading ages (M=9) gives 10 load application ages in the computed system. According to the choice, it is nine loading ages and an additional one, which represents the concrete setting time. The first loading age is \( t_0 = t_s - 0.001 \) day = 0.209 day. The one representing the setting time of concrete is added to the spectra to secure that \( R=0 \) for \( (t_0)_i = t_s \). By setting all \( E(1, \mu) = 0.01 \) GPa for \( (t_0)_s \) so this could numerically be fulfilled (Jonasson and Westman, 2001).

By the means of the Maxwell chain that has been chosen, the creep data can be calculated back when simulating a step load history. Therefore, the accuracy of the conversion process and the least square approach is possible to check. The input data in accordance with the creep data achieved from the Maxwell-chain model is really good, as it is shown in Figure 2.20.

![Figure 2.20. The output obtained using Maxwell chain model with eight elements in comparison to a) creep values and b) creep values converted into relaxation values (Jonasson and Westman, 2001).](image)

2.2.6.3 Modelling and determination of creep

As mentioned earlier, in this thesis, the creep is modeled with the Linear Logarithmic model developed by Larson and Jonasson (2003a, 2003b), which is a basic creep model primarily aimed for early age determination, see detailed description of LLM further in this thesis. The evaluation procedure of the basic creep is performed by measurements of basic creep. The creep tests are performed in a laboratory and consist of two stages. In the first stage the deformation of a loading concrete sample is expressed by the E-modulus (Fjellström, 2013). The second stage is the deformations of the concrete that continue after the load is kept constant which is time dependent and called long-term creep. The long-term creep behaviour is dependent on the E-modulus with larger creep deformations for lower E-modulus and vice versa (Larson, 2003).

To measure creep, it is needed to determine the deformation for a sample under loading, and an equivalent unloaded sample. The unloaded concrete sample deformation is then subtracted from...
the loaded concrete sample deformation, and according to Fjellström (2013), the following expression is obtained:

\[ \varepsilon = \frac{\Delta l_2 - \Delta l_1}{l_0} \]  

(2.44)

where

\( \varepsilon \) = the total strain, -

\( l_0 \) = initial length of the measuring length for all samples, m

\( \Delta l_1 \) = deformation of the unloaded concrete sample, m

\( \Delta l_2 \) = deformation of the loaded concrete sample, m

As expressed in Fjellström (2013), the total creep compliance is usually defined as a time dependent deformation per unit of applied stress \( \sigma(t) \) at concrete age \( t_0 \) as

\[ J(\Delta t_{\text{load}}, t_0) = \frac{\varepsilon(\Delta t_{\text{load}}, t_0)}{\sigma(t_0)} \]  

(2.45)

where

\( J(\Delta t_{\text{load}}, t_0) \) = the total creep compliance \([\frac{1}{Pa}]\)

\( \Delta t_{\text{load}} = t - t_0 \), which is the load duration [d]

\( t = \) time after mixing [d]

\( t_0 = \) time after loading specified as time after mixing [d]

\( \sigma(t_0) \) = applied stress at age \( t_0 \) [Pa]

\( \varepsilon(\Delta t_{\text{load}}, t_0) \) = total strain at time of interest [-]

The quasi-instantaneous elastic deformation, which is the deformation from start of loading up to a "chosen" elastic load duration formulated by

\[ E(t_0) = J(\Delta t_0, t_0)^{-1} = \left( \frac{f_{cc}(t_e(t_0))}{f_{cc,28}} \right) \eta^E \cdot E_{c,28} \]  

(2.46)

where

\( E(t_0) \) = the elastic modulus for load duration = \( \Delta t_0 \) [Pa]

\( \Delta t_0 = \) time span after loading employed as the definition of elastic modulus, usually 0.001d is used [d]

\( f_{cc}(t_e(t_0)) \) = compressive strength at time \( t_0 \) [Pa]

\( t_e(t_0) \) = equivalent time at loading time \( t_0 \) [d]

\( \eta^E \) = connection parameter between the compressive strength and the time development of elastic modulus [-]

\( E_{c,28} \) = elastic modulus at equivalent time = 28d [Pa]

And the total creep compliance is expressed by
\[ J(\Delta t_{\text{load}}, t_0) = \frac{1}{E(t_0)} + J(\Delta t_{\text{load}}, t_0 + \Delta t_0) \] 

where

\[ J(\Delta t_{\text{load}}, t_0 + \Delta t_0) \] is the creep compliance with respect to the elastic modulus \[ \frac{1}{E(t_0)} \]

The method is used in order to determine the fitting parameters of the creep compliance, which according to Hösthagen (2017) are described by

\[ \Delta J(\Delta t_{\text{load}}, t_0) = a_1(t_0) \cdot \log \left( \frac{\Delta t_{\text{load}}}{\Delta t_0} \right) \text{ For } \Delta t_0 \leq \Delta t_{\text{load}} < \Delta t_1 \] (2.48a)

And

\[ \Delta J(\Delta t_{\text{load}}, t_0) = a_1(t_0) \cdot \log \left( \frac{\Delta t_1}{\Delta t_0} \right) + a_2(t_0) \cdot \log \left( \frac{\Delta t_{\text{load}}}{\Delta t_1} \right) \text{ For } \Delta t_1 \leq \Delta t_{\text{load}} \] (2.48b)

where

\[ \Delta t_1 \] = breakpoint in creep behaviour, d

\[ a_i(t_0) \] = “logarithmic” creep rates for \( i = \{1.2\}, [Pa^{-12}/ \log(\Delta t_{\text{load}})] \)

\[ a_i(t_0) \] = “logarithmic” creep rates for \( i = \{1.2\}, [Pa^{-12}/ \log(\Delta t_{\text{load}})] \)

The values of \( a_i^{\min}, a_i^{\max}, \eta^a_i, \) and \( t_{a_i} \) for \( i = \{1.2\} \) are set by regression analysis.

The total compliance is then defined as

\[ J(\Delta t_{\text{load}}, t_0) = \frac{1}{E(t_0)} + a_1(t_0) \cdot \log \left( \frac{\Delta t_{\text{load}}}{\Delta t_0} \right) \text{ for } \Delta t_0 \leq \Delta t_{\text{load}} < \Delta t_1 \] (2.49a)

And

\[ J(\Delta t_{\text{load}}, t_0) = \frac{1}{E(t_0)} + a_1(t_0) \cdot \log \left( \frac{\Delta t_1}{\Delta t_0} \right) + a_2(t_0) \cdot \log \left( \frac{\Delta t_{\text{load}}}{\Delta t_1} \right) \text{ for } \Delta t_1 \leq \Delta t_{\text{load}} \] (2.49b)

The formulation of the total compliance function \( J \) above, refers to that the creep part of the concrete sample is considered to start after the elastic response. From Figure 2.21 below it is also considered that if the concrete sample is loaded during younger age, the elastic deformation and creep will be larger compared to the concrete sample with older concrete.

\[ J \]

\[ \frac{1}{E(t_0)} \]

\[ \Delta t_0 \]

\[ \Delta t_1 \]

\[ \eta^a_i \]

\[ t_{a_i} \]

\[ \text{logarithmic scale} \]

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2.2.6.3.1 Modelling of creep with Linear Logarithmic model

LLM method is based on piece-wise linear curves in logarithmic time. The model is presented as a flexible and robust formulation that is suited for modeling both young and mature concrete. Young concrete is, in the concept of LLM, used for a period up to roughly 100 days after casting. Because of the robustness, it is possible to make reliable creep modeling with very limited test data. The presence of negative relaxation in linear viscoelastic modeling is very trivial and not worth considering with respect to thermal stresses. With this, the formulation may be used directly in an analysis of thermal stress for young concrete deprived of any adjustments for negative relaxation. The LLM formulation has been used to model the viscoelastic behaviour of the concrete in this work. The choice of LLM is because that the formulation demonstrates very good agreement directly with experimental creep data and indirectly with measured thermal stresses (Larson and Jonasson, 2003 I). According to Larson and Jonasson (2003 I), the LLM formulation has the best correlation with experimental data when likened to other commonly used creep models that have been analyzed in their paper Linear Logarithmic Model for Concrete Creep and presented further in this work.

The total strain $\varepsilon_{\text{tot}}$ is used when concerning mathematical models for creep and shrinkage that develops self-induced stresses during the hydration process of young concrete. For an uniaxial case, this may be considered as the sum of stress dependents and stress independent strains as in the following equation according to Larson and Jonasson (2003, I)

$$\varepsilon_{\text{tot}} = [\varepsilon_{\text{vol}}] + [\varepsilon_{\text{visc}} + \varepsilon_{\text{fract}} + \varepsilon_{\text{rest}}] \quad (2.50)$$

where

- $\varepsilon_{\text{vol}} =$ volumetric strain related to shrinkage and temperature, stress independent [-]
- $\varepsilon_{\text{visc}} =$ strain due to viscoelastic behaviour (creep and relaxation), stress dependent [-]
- $\varepsilon_{\text{fract}} =$ strain related to fracturing mechanics, stress dependent [-]
- $\varepsilon_{\text{rest}} =$ strain related to restraint, stress dependent [-]

It is not necessary for all four types of strains to occur for the same concrete specimen, for example when the strain is not related to restraint then $\varepsilon_{\text{rest}}$ is not necessarily included in the sum of all four above mentioned strains. Larson and Jonasson (2003, I) claim that there are several methods to study creep behaviour in young concrete but that many of these methods are complex and include sets of irrelevant parameters which makes the model difficult to understand and use in a practical context. It is also claimed that some of the other methods have a dependency between the free parameters. This means that the methods give dissimilar solutions with regard to the start position of the free parameters and/or on the quantity of existing test data in the regression procedure (Larson and Jonasson, 2003 I). It is shown that to get reliable stress calculation, an accurate prediction of creep during the hardening period of the concrete is essential (Bosnjak, 2000).

In the past, it has been common to formulate creep functions from the loading age of about 2 days or more and regard them as representatives for young concrete. According to Larson and Jonasson (2003, I), a group of creep formulas is constructed as
\[ J(\Delta t_{\text{load}}, t_0) = \frac{1}{E_0} (1 + F(\Delta t_{\text{load}}, t_0)) \]  

(2.51)

where

\[ E_0 = \text{Constant "infinite" modulus of elasticity which is formally valid for } \Delta t_{\text{load}} = 0. \]

The most known functions \( F(\Delta t_{\text{load}}, t_0) \) are

\[
F(\Delta t_{\text{load}}, t_0) = \begin{cases} 
\text{Double Power Law (Bazant and Osman, 1976)} & \\
\text{Triple Power Law (Bazant and Chern, 1985a)} & \\
\text{Log Double Power Law (Bazant and Chern, 1985b)} & 
\end{cases}
\]

In the original form, these functions can’t reflect very early age concrete behaviour, for example loading ages below two days. However, they have been commonly used by investigators when modeling concrete performance. Linear logarithmic model is an additional type of function than the ones mentioned above. For the Linear logarithmic model, a modulus of elasticity \( E(t_0) \) is formulated based on creep tests. For this, a quasi-instantaneous “elastic” deformation needs to be defined by the choice of an “elastic” time period \( \Delta t_0 \) (Larson and Jonasson 2003 I). As described in Larson and Jonasson (2003, I), this gives

\[ J(\Delta t_{\text{load}}, t_0) = \frac{1}{E(t_0)} + \Delta J(\Delta t_{\text{load}}, t_0) \]  

(2.52)

With

\[ E(t_0) = \frac{1}{J(\Delta t_0, t_0)} \]  

(2.53)

where \( J(\Delta t_{\text{load}}, t_0) \) is the creep part related to the description of the elastic modulus in the previous equation.

When formulating the “elastic” modulus, a time period defining it has to be chosen, the time period may be chosen within \( 0 < 0.001 \leq \text{days} \) according to Neville et al. (1983). In this work, a time period based on the definition from Westman (1999), is chosen to \( \Delta t_0 = 0.001 \text{ days} \).

The linear logarithmic model is a creep formulation fulfilling the need of being simple, robust and easy to understand, where the actual behaviour of the concrete is described by the expressions. The compliance development, \( J(\Delta t_{\text{load}}, t_0) \), in LLM formulation, is described by piece-wise linear curves in the logarithm of time span after loading, \( \Delta t_{\text{load}} \). As described in Larson and Jonasson (2003, I), there are only two linear curves in the most basic application of the linear logarithmic model, presented below

1. “Short-term creep” which is a linear curve with the inclination \( a_1 \) from \( \Delta t_{\text{load}} = \Delta t_0 \) to \( \Delta t_{\text{load}} = \Delta t_1 \).

2. “Long-term creep” which is a linear curve with inclination \( a_2 \) from \( \Delta t_{\text{load}} = \Delta t_1 \) and further on.

The applications mentioned above are illustrated in Figure 2.22.
Figure 2.22 Two linear curves in logarithmic time scale for a detailed loading age $t_0$, illustrating creep development where I denote short-term creep and II denotes long-term creep (Larson and Jonasson 2003 I).

To achieve this kind of creep behaviour with two linear curves for each loading age according to Figure 2.22, some functions and parameters are needed (Larson and Jonasson 2003 I).

The needed functions and parameters are

- $J_0$ or $E = \frac{1}{J_0}$ = the modulus of elasticity
- $\Delta t_0$ = the load duration for a definition of a modulus of elasticity
- $\Delta t_1$ = the limit between short-term and long-term creep
- $a_1$ = the inclination (or “logarithmic” creep rate) of short-term creep
- $a_2$ = the inclination (or “logarithmic” creep rate) of long-term creep

The inclinations of the linear curves are defined in (Larson and Jonasson 2003 I) as

$$a_i = \frac{dJ_i}{d(\log\Delta t_{load})} \quad \text{For } i = 1, 2 \quad (2.54)$$

where $\log$ is a denotation for “$10\log$” and the solution of the integral form of Eq 2.54, with two start values, $J_{i-1}$ and $\Delta t_{i-1}$, and the inclination, $a_i$, is expressed as

$$J = J_{i-1} + a_i \cdot (\log(\Delta t_{load}) - \log(\Delta t_{i-1})) \quad \text{for } i = 1, 2.$$

With this, Larson and Jonasson (2003, I) claim that the increase in creep compliance can be described in Eq 2.55, as

$$\Delta J(\Delta t_{load}, t_0) = \quad (2.55)$$

\[
\begin{align*}
(1) & \quad a_1(t_0) \cdot (log \left( \frac{\Delta t_{load}}{\Delta t_0} \right)) \\
(2) & \quad a_1(t_0) \cdot (log \left( \frac{\Delta t_1}{\Delta t_0} \right)) + a_2(t_0) \cdot (log \left( \frac{\Delta t_{load}}{\Delta t_1} \right))
\end{align*}
\]

(1) For $\Delta t_0 \leq \Delta t_{load} < \Delta t_1$

(2) For $\Delta t_0 \leq \Delta t_{load} < \Delta t_1$
Having fixed values of $\Delta t_0$, $\Delta t_1$ and E or $J_0$, is needed to know the inclinations $a_1$ and $a_2$ which can be modelled as follows

$$a_i(t_0) = a_i^{\text{min}} + (a_i^{\text{max}} - a_i^{\text{min}}) \cdot \exp \left(-\left(\frac{t_0 - t_s}{t_{ai}}\right)^{n_{ai}}\right) \text{ For } i = 1, 2.$$  \hspace{1cm} (2.56)

where

- $n_{ai} = \text{model parameter}$
- $t_{ai} = \text{model parameter}$
- $t_s = \text{apparent setting time when the concrete transforms from nearly liquid phase to solid phase}$

The inclination where $a_1 > a_2$ at a very early age is essential as one of the parts of the technique to decrease the presence of negative relaxation values, see Figure 2.23.

For calculations of thermal stresses some constitutive relations demand relaxation data instead of creep which is why it is vital to check that the used creep model doesn’t give negative relaxation values. To eliminate negative relaxation values it is possible to choose a certain age of loading $(t_o)_{age}$ and a certain load duration $(\Delta t_{load})_{age}$ as limit values, as described in Emborg (1989). It is adjusted so that for $t_0 < (t_0)_{age}$ and $\Delta t_{load} > (\Delta t_{load})_{age}$ the creep curves are parallel to the curves of $t_0 = (t_0)_{age}$. Applying this to creep models will mean that negative relaxation values are avoided in practical applications (Larson and Jonasson 2003 I).

Larson and Jonasson (2003, I) claim that in the linear logarithmic model the adjustment is introduced by adding an additional linear curve in Eq. 5.55 as outlined in Figure 2.24 and can be expressed as

$$\Delta J(\Delta t_{load}, t_0) =$$

$$\begin{align*}
\begin{cases}
    a_1(t_0) \cdot \log \left(\frac{\Delta t_{load}}{\Delta t_0}\right) \quad (1) \\
    a_1(t_0) \cdot \log \left(\frac{\Delta t_1}{\Delta t_0}\right) + a_2(t_0) \cdot \log \left(\frac{\Delta t_{load}}{\Delta t_1}\right) \quad (2) \\
    a_1(t_0) \cdot \log \left(\frac{\Delta t_1}{\Delta t_0}\right) + a_2(t_0) \cdot \log \left(\frac{\Delta t_2}{\Delta t_1}\right) + a_2((t_0)_{age}) \cdot \log \left(\frac{\Delta t_{load}}{\Delta t_2}\right) \quad (3)
\end{cases}
\end{align*}$$

Figure 2.23. Logarithmic creep rate for concrete in relation to age of loading (Larson and Jonasson 2003 I)
(1) For $\Delta t_0 \leq \Delta t_{\text{load}} < \Delta t_1$

(2) For $\Delta t_1 \leq \Delta t_{\text{load}} < \Delta t_2$ and for $\Delta t_{\text{load}} \geq \Delta t_2$ if $t_0 > (t_0)_{\text{age}}$

(3) For $\Delta t_{\text{load}} \geq \Delta t_2$ if $t_0 \leq (t_0)_{\text{age}}$

Figure 2.24. Three curves adjusted for negative relaxation, in logarithmic time scale (Larson and Jonasson 2003 I).

The LLM formulation shows the same or better agreement to experimental data when compared to other models like log power law (LPL) and double power law (DPL). Modeling creep with LLM shows none, or small, negative relaxation, as with the LPL. However, the DPL formulation can end up in accelerating negative relaxation. In all cases, the negative relaxation affects the calculated stress. It is possible to do reliable modeling with LLM using few test data. This is possible because that creep is demonstrated with straight lines in logarithmic time scale and the growth is given by age-related inclinations, which have an easy to comprehend meaning in the material behaviour. The inclinations will successively decrease to constant levels when the concrete hardens, and the inclination will follow the same path with high values at young ages. With this, LLM is exceptionally fit as a prediction formula where general inclination developments can be used for different types of concrete (Larson and Jonasson 2003 I).

2.2.6.4 Estimation of elastic modulus and creep

The behaviour of individual recipes for a group of tested concrete mixtures can be modelled based on the evaluated logarithmic creep rate $a_q$, and the elastic modulus $E (t_0)$ from laboratory results. Figure 2.25 shows the general behaviour for creep rate and elastic modulus.

Figure 2.25. Diagrams of elastic modulus and logarithmic creep rate for concrete in relation to age of loading (Larson and Jonasson 2003 I,II)
From Figure 2.25 it is shown that when the concrete is fresh \((t_0 < t_s)\) there is an “infinite” creep rate. It is shown that the creep rate is decreasing with increasing concrete age. From the Figure is is also seen that the inclinations of \(a_1\) and \(a_2\) are meeting each other for very young concrete. The source of this is that the so-called negative relaxation that has to be avoided for loading of the very young concrete (Larson and Jonasson 2003 I).

Using regression analysis and evaluated logarithmic rates of creep, the following model can be fitted for creep rate according to Fjellström (2013)

\[
a_i(t_0) = a_i^{\text{min}} + (a_i^{\text{max}} - a_i^{\text{min}}) \cdot \exp \left( - \frac{t_0 - t_s}{t_{ai}} \right)^{n_{ai}}
\]

where

\[
a_i(t_0) \quad [\text{Pa}^{-1} \cdot 10^{\log (\Delta t_{load})}] = \text{logarithmic creep rate of the concrete sample}
\]

And

\[
a_i^{\text{min}} \quad [\text{Pa}^{-1} \cdot 10^{\log (\Delta t_{load})}],
\]

\[
a_i^{\text{max}} \quad [\text{Pa}^{-1} \cdot 10^{\log (\Delta t_{load})}],
\]

\[
t_{ai} \quad [\text{d}] \quad \text{and}
\]

\[
n_{ai} \quad [-] = \text{are parameters fitted against evaluated laboratory results using least square method.}
\]

\[
t_s \quad [\text{d}] = \text{evident setting time when the concrete changes from liquid stage to solid stage.}
\]

The elastic modulus is evaluated from the laboratory tests and following equations are used

\[
E(t_0) = E_{ref} \cdot \beta_E(t_0) \quad \text{and}
\]

\[
\beta_E(t_0) = \{\exp[s \cdot (1 - \frac{28 - t_s}{t_0 - t_s})]\}^{0.5}
\]

where

\[
\beta_E(t_0) \quad [-] = \text{the relative time development of elastic modulus}
\]

\[
E_{ref} \quad [\text{Pa}] \quad \text{and} \quad s \quad [-] = \text{are fitting parameters, decided by the least square method.}
\]

The number 28 signifies 28 days, which indicates that \(E_{ref}\) is the modulus of elasticity at 28 days. Creep curves can be recognized for any load period plus any loading age, when the estimated curves of logarithmic creep and elastic modulus are identified with parameters assessed in the equation above (Westman 1999).
2.3 Tests and evaluation processes of material parameters

The laboratory and evaluation process of the material parameters are based on the methods used at LTU. All the dimensions of specimens and used forms, if any, for the different type of tests are presented according to how it is performed in Thysell lab, at LTU.

2.3.1 Strength development

One of many things, that testing of young concrete should include, is the strength growth at variable temperatures. To characterize the strength class, a test of young concrete for strength at 28 days using hardening conditions, is needed. Another important aspect is the strength development with time for some calculations and situations. Temperature estimations and calculations of strength are of great importance in situations of planning actions on site, for instance to avoid early freezing and fulfil requirements of predetermined moisture hardening periods etc. (Hösthagen, 2017).

The reference strength development function is the function that needs to be determined in this test. To achieve this, two other parts are determined, where the first one is the temperature dependent maturity function, and the other one is the equivalent time of maturity.

2.3.1.1 Laboratory process and geometry of specimens

To establish the compressive strength development, $f_{cc}(t)$ several concrete specimens are placed in water baths to cure at different temperature conditions. Later, they are being loaded by compression until they fail. Plastic and cubic forms with the dimensions $100 \times 100 \times 100$ mm is used to cast and cure the concrete specimens (Hösthagen, 2017). Normally, it takes approximately 30 minutes from that the concrete is poured into the forms and vibrated until they are placed in temperature water baths for curing (Fjellström, 2013). At LTU, the different water baths have four curing temperatures; 5°C, 20°C, 35°C and 50°C. The water bath experiment is for the tests in this thesis, made for all the different curing temperatures except for the 5°C. For the concrete specimens with curing temperature 20°C, the strength is tested at 24, 32, 52, 120 and 672 hours after casting. For each strength test, three specimens are used, see Figure 2.26. The strength test for the specimens with curing temperatures 35°C and 50°C, are performed at 8, 24, 32, 52 hours after casting. By using the results from the strength growth tests at different curing temperature, the parameters for the equivalent time of maturity, $t_e$ is determined. By using the equations in Section 2.2.4, the temperature dependent maturity function $\beta_T$ and the equivalent time of maturity $t_e$ are established. The reference strength development, which is referring to the compressive strength at 20°C, and is not taking effects of high curing temperature into consideration is thereby determined.
2.3.1.2 Evaluation process

From the performed strength growth tests, the diagrams for strength growth and the temperature dependent maturity are obtained. The diagram for strength growth will show the recorded temperature development for the different specimens at different water baths. Figure 2.27 shows typical results from the strength tests performed at LTU, originally from Hösthagen (2017). The temperature dependent maturity function and the strength growth from Hösthagen (2017) are also shown in Figure 2.28a), described as a function of equivalent time. In Figure 2.28b), measured strengths as a function of the equivalent time are illustrated, where every point in the diagram gives the average of all tested cubes at each test occasion. The compressive strength of the concrete at 28 days is used as a restraining point at each test. The evaluation of the strength growth is done by observing the strength loss of the concrete at a specific temperature and maturity age, which in this case is taking place at higher temperatures than 35°C. Important to mention is that this observation is not of great importance when analyzing risk of temperature cracking, where the higher temperatures is not maintained for long periods for civil engineering structures.

![Figure 2.27. Diagrams of registered temperatures as a function of time after casting, for two different concrete specimens in a water baths (Hösthagen, 2017).](image)
Figure 2.28. In a) a maturity function is presented as a function of temperature. In diagram b) a reference compressive strength development is shown as a function of equivalent time of maturity. Each point gives a mean value of three tested cubes for all tested specimens at equivalent time of maturity (Hösthagen, 2017).

2.3.2 Heat of hydration

In building engineering, heat of hydration has normally been studied by the evaluation of various versions of adiabatic calorimetric measurements, see Figure 2.29. The theoretical background and the semi adiabatic calorimetric setup are described by the equations in Section 2.2.2.

Figure 2.29. Various test methods with adiabatic calorimetry (Fjellström, 2013).

In the semi adiabatic and true adiabatic setups, the concrete temperature is registered as a function of time. In the true adiabatic setup, there is no heat loss to the environment, in contrast to the semi adiabatic. The semi adiabatic test permits energy losses which are taken into consideration in the calculation process. In this thesis, test results from the semi adiabatic setup are used, therefore the laboratory and evaluation process are described for the semi adiabatic setup (Fjellström, 2013).
2.3.2.1 Laboratory process and geometry of specimens

The semi adiabatic test setup existing at LTU, used for the tests in this thesis, consists of a cylindrical concrete sample cast in a metal bucket with a heating device, all placed in a cellular plastic unit shaped as a cylinder. At the same time, two sizes of the cellular plastics are used for testing. The test starts by logging the temperature of the ambient air. This is measured by gauges placed near the top and bottom at each one of the cellular plastic unit. The concrete temperature is measured by gauges placed in the two specimens (Hösthagen, 2017). In Figures 2.30 and 3.31 the semi adiabatic calorimetric tests are shown.

![Figure 2.30. Cylindrical semi-adiabatic calorimetric tests. Numbers 1, 2 and 3 are temperature sensors placed in the concrete sample, number 4 is placed in the ambient air (Fjellström, 2013).](image)

![Figure 2.31. Two semi adiabatic tests at LTU (Hösthagen, 2017).](image)

The measurement of the concrete temperature starts as soon as the concrete is placed in the metal bucket. Besides measuring the natural temperature development, the cooling evolution after a mandatory heating is performed. A heating device attached to the metal bucket is activated after the temperature in the concrete has cooled down to the ambient temperature. The heating and the cooling processes are documented, see Figure 2.32 (Hösthagen, 2017).
2.3.2.2 Evaluation process

With the information obtained, presented in Figure 2.32a) and b), the heat of hydration $q_{cem}(t)$ can be calculated as a function of equivalent time of maturity. The equation of $q_{cem}(t)$ is described thoroughly in Section 2.2.2, Eq. 2. The equation consists of, among others, a cooling factor, $a$. The cooling factor describes the heat loss to the surroundings of the semi adiabatic test setup. This factor has been observed to vary among different tests and must be correctly documented for each one of the tests. In heat calculations the generated heat per volume concrete $Q_h(t)$ is of importance and it is calculated from the results of the semi adiabatic test results (Hösthagen, 2017). The equation for $Q_h(t)$ and the parameters for the heat of hydration, such as $q_u$, $\kappa_1$ and $t_1$, are described in Section 2.2.2. From the relevant equations and the test results an assessment of the heat of hydration for the tested concrete can be defined, see Figure 2.33.

Figure 2.33. Assessment of heat of hydration, from calculation and from two semi adiabatic tests (Hösthagen, 2017).
2.3.3 Basic shrinkage and free thermal dilation

From tests of free deformations, where at both constant and variable temperature the deformations are unrestrained, the thermal dilation and the basic shrinkage at moisture sealed conditions can be established. The obtained results of the mean temperature development can be referred to a realistic structure (Hösthagen, 2017).

2.3.3.1 Laboratory process and geometry of specimens

To examine basic shrinkage and free thermal dilation a test is performed, consisting of two concrete specimens cast in metal cylindrical forms with the diameter of 80 mm. The two specimens are released from the metal form after a time interval of 5-10 hours, and inside, strain gauges have been mounted. To avoid any loss of moisture to the surrounding, both of the specimens are sealed with plastic. One of the specimens is placed in a laboratory with an environment temperature of 20°C. The other specimen is placed in a water bath with a temperature referring to the calculated temperature development for the reference wall, which is a 0.7 thick wall (Hösthagen, 2017).

2.3.3.2 Evaluation process

A determination of basic shrinkage and free thermal dilation is determined for the evaluation process. The basic shrinkage is referred to the volume change of the concrete. The concrete specimens with moist sealed conditions will undergo a volume change as a function of time. The main part of the volume change for the specimen stored at 20°C is due to basic shrinkage. For the other specimen, placed in a water bath, the volume change is due to both thermal dilation and basic shrinkage (Hösthagen, 2017).

The strain gauges mounted on the concrete, register the volume change of two specimens during a test. The obtained result for the measured strain and calculated total strain are illustrated in Figure 2.34. From the test of the specimen that is placed in room temperature, the parameters $\epsilon_{su}$ and $\tau_{su}$ are obtained for determination of basic shrinkage, see Figure 2.34a). From the regression of the tested specimen, placed in temperature water bath, the thermal expansion coefficient $\alpha_T$ is determined, see Figure 2.34b). The main purpose of the regression analysis is to mimic the calculated total strain to the measured strain. The strain caused by basic shrinkage and the free thermal dilation are calculated as shown in Figure 2.34 (Hösthagen, 2017).

![Figure 2.34](image.png)

*Figure 2.34. a) Regression analysis of measured and calculated strain $\epsilon$ at constant temperature. b) Strain development for a realistic temperature curve, for the reference target of a 0.7 m thick wall (Hösthagen, 2017).*
2.3.4 Basic creep

The used method for measurement of basic creep in this thesis is inspired from the journal of advanced concrete technology “Linear Logarithmic Model for Concrete Creep” by Larson and Jonasson (2003) and a summary of this is presented below.

2.3.4.1 Laboratory process and geometry of specimens

The creep measurement starts with determination of deformation for a specimen under loading, and a corresponding unloaded specimen. The deformation of the unloaded specimen is then subtracted from the loaded specimen deformation.

Young and mature ages of concrete under moist sealed conditions are used to measure creep. The sealing of the concrete is made of plastic and for the same concrete mixture there are seven concrete specimens loaded after different times. The first two specimens are loaded after 1 day, two other specimens are loaded after 5 days, and after 14 days two more specimens are loaded and the seventh one is unloaded, see Figure 2.35.

![Figure 2.35. Concrete samples under moist sealed conditions going through loaded and unloaded deformations (Fjellström, 2013).](image)

At LTU the creep is tested on cylindrical concrete samples with a diameter of 80 mm and a length of 100 mm. The reason to why the concrete samples are sealed is to circumvent moist exchange to the surroundings and to not expect any drying shrinkage or drying creep (Hösthagen, 2017).

For the creep tests the measurements are implemented with two hydraulic test rigs, as shown in Figure 2.36 and 2.37. Two strain gauges of type Schaewitz LVDT010 MHR are located symmetrically on opposite edges of the specimen. The equipment consists of the material invar, since it shows small effects to the temperature change, which is completely compensated for. In each of the creep rigs the temperature is continuously recorded (Fjellström, 2013).
The value of the prechosen load is set to approximately 20% of the compressive strength at the time of loading. This value of the prechosen load is deemed suitable to study a creep, where both the creep and the elastic strain are proportional to the applied load i.e. linear creep (Hösthagen, 2017). For the creep measurements there is always two different conditions, loaded and unloaded sample. The unloaded sample is used to extract information about the sum of basic shrinkage. Basic creep at sealing is obtained from the difference between the loaded and unloaded samples.

2.3.4.2 Evaluation process

Concrete is a changing material where it is hard to obtain characteristic values, due to the fact that different samples of the same concrete mixture do not necessarily behave the same. This is because of natural variation in the concrete mixture and can result in differences in the modulus of elasticity, heat capacity and thermal dilation. With the described test rig, we obtain measured values of deformation and the surrounding temperature of the concrete samples. To further evaluate basic creep, results from maturity tests are needed. After loading the concrete samples, they are continuing to cure and hydrate so that the equivalent time of maturity $t_e$ is also changing. The equivalent time of maturity $t_e$ is associated with the maturity age of a concrete sample that is curing and hydrating in 20°C (Hösthagen, 2017).
A way of visualizing the basic creep is to use logged strain and fitting with Linear Logarithmic Model in logarithmic time scale, as shown by Hösthagen (2017) in Figure 2.38.

![Figure 2.38. Fitting of linear lines in logarithmic timescale to measure initial and basic creep strains. A and B signify two specimens (Hösthagen, 2017).](image)

The elastic modulus is gained from the test for every loaded specimen at the elastic load duration of 0.001 days (about 80 seconds). The choice of the load duration is motivated in Section 3.3. After this, the difference in length of the specimens is considered to be initiated by basic creep. In Figure 2.39 the fitting of the elastic modulus development towards obtained measurements is presented.

![Figure 2.39. Demonstration of the fitting of E (t₀), A and B signify two specimens (Hösthagen, 2017).](image)

From the fitting of Figures 2.38 and 2.39 an overall formulation from the total creep can be determined for any arbitrary time of loading (t₀) and arbitrary time interval of loading (∆t_{load}) done by the methodology presented in Section 2.2.5.3. The creep rate coefficients α₁(t₀), α₂(t₀) and the model parameter ∆t₁ have to be determined by regression analysis, the result is shown in Figure 2.40.
Figure 2.40. Calculated and measured creep rate coefficients $a_1(t_0)$ and $a_2(t_0)$ of the Linear Logarithmic Model. a) Steep behaviour of the calculated $a_2$-function. To avoid negative relaxation, limited values are needed. b) Flatter behaviour of the calculated $a_2$-function. To avoid negative relaxation values no limiting values are needed (Hösthagen, 2017).

Thereafter a computer program RELAX can be used to calculate a relaxation spectrum to create input data to the ConTeSt software. The creep values are used by RELAX to create a relaxation spectrum, see Figure 2.41. How this process works, and how to prevent negative relaxation is explained in Section 2.2.5.2. The calculated data can be used to make it possible for ConTeSt to take the relaxation phenomena into the stress/strain calculation, using a law by Maxwell chain model (Hösthagen, 2017).

The total creep compliance and the quasi-instantaneous elastic deformation with help from the results from this kind of tests are formulated in Section 2.2.5.3.

Figure 2.41. Relaxation spectrum where the curves denote different equivalent time of maturity for the concrete, $t_0$ at the time of applied, $t_0$ presented in hours (Hösthagen, 2017).
2.3.5 Stresses in concrete at full restraint

The test of stress at full restraint consists of specimen undergoing thermal development at full restraint, yielding the stress development.

2.3.5.1 Laboratory process and geometry of specimens

The stress at full restraint is measured with a temperature load corresponding to the average temperature of a 0.7 m thick wall. By an apparatus the free strain is avoided and the applied force is adjusted to zero strain (Hösthagen, 2017).

Regarding the test of stresses in concrete at full restraint, a stress rig and a concrete specimen, with the dimensions $150 \times 150 \times 1000$ mm are used. The concrete specimen is cast into a form work, and after casting the form work is sealed from drying, see Figure 2.42. With an air fan and the temperature reference curve, the temperature in the concrete specimen is controlled. A positive or negative force is acting on a hydraulic servo-cylinder at the end of one of the specimens, this due to temperature change that causes volume change in the specimens. The servo-cylinder generates a resisting force to simulate at full restraint, which means 100% restraint with no external deformation. The force is directly proportional to the stress in the specimen (Hösthagen, 2017).

![Figure 2.42. Test rig at LTU for stress at full restraint (Hösthagen, 2017).](image)

2.3.5.2 Evaluation process

The main purpose of the test rig is letting the concrete specimen undergo tensile strength failure. This is reached either by increasing the force in the hydraulic servo-cylinder or due to the unstrained deformation, see Figure 2.38. At this point the reference compressive strength can be determined, but to fulfill the whole test process and obtain results, the material parameters $\alpha_{ct}$, $\rho_T$ and $\rho_{\varphi}$ needs to be determined. Manually this could be achieved, by using the measured concrete temperature to do stress calculations with the program ConTeSt together with other evaluated parameters from other tests. The measured and calculated stress curves are compared to each other in the evaluation process. If needed the calculated stress could be regulated by varying the material parameters $\alpha_{ct}$, $\rho_T$ and $\rho_{\varphi}$.

If the results still are not accepted a re-evaluation of the creep test and the free deformation test may be needed. The explanation for the re-evaluation is that there is a tendency that this test can give the so called “tentative unlogic” results, which at present have no explanation. At the same time, the measured stress at full restraint is the final combined result of the whole test series and illustrate a direct useful result comparing different recipes. It is therefore necessary
that the measured and calculated stresses always have to meet in a good and acceptable way (Hösthagen, 2017).

It is of great important that the calculated curve corresponds well enough to the measured curve, when analyzing the risk of through cracking. The corresponding part should be similar until reaching the tensile strength and onwards (Hösthagen, 2017).

It can sometimes be observed that the calculated stress is lower than the measured, from the time of casting until about 24 hours after casting. This is a consequence of an implemented “smoothening” in ConTeSt, created to hinder the program from crashing. It is caused by non-linear deformations when calculating the strain at the first-time period after casting (Hösthagen, 2017).

![Figure 2.43. Comparison between measured and calculated stresses from test rig. The test specimen is loaded by a temperature curve corresponding to the reference target, which is a 0.7 m thick wall (Hösthagen, 2017).](image)

2.4 ConTeSt Pro

2.4.1 General

ConTeSt Pro is a software that is developed by JEJMS Concrete together with Luleå Technical University, Cementa AB and PEAB Öst AB. The software is a two-dimensional Finite Element program (FEM) developed to calculate temperature, strength and cracking risk in young concrete. ConTeSt Pro can foresee probable behaviour of the studied structure before the casting (JEJMS Concrete, 2006).

2.4.1.1 Temperature calculations

The temperature calculations performed in ConTeSt are studied for a two-dimensional structure in the xy-plane, see Figure 2.44.
Figure 2.44. The studied surface of a temperature calculation (JEJMS Concrete, 2006).

For this to work properly, it is assumed that the structure is long enough in the z-direction so that the heat flow in the same direction is neglectable. With this, the heat flow in the z-direction can be described by giving the Eq. 2.58, according to (JEJMS Concrete, 2006).

\[ q_z = -k_z \frac{\partial T}{\partial z} \approx 0 \]  

(2.58)

where

- \( q_z \) = heat flow in the z-direction [W/ m²]
- \( k_z \) = heat conductivity for the heat flow in the z-direction [W/mK]
- \( T \) = temperature in the studied structure [°C] or [K]

For the two-dimensional heat flow in the xy-plane, the heat conduction equation for the inner parts of the structure, is as follows (JEJMS Concrete, 2006)

\[
\rho c \frac{\partial T}{\partial t} = \frac{\partial}{\partial x} (k_x \frac{\partial T}{\partial x}) + \frac{\partial}{\partial y} (k_y \frac{\partial T}{\partial y}) + Q_H
\]

where

- \( \rho \) = density of the material [kg/ m³]
- \( c \) = specific heat of the material per mass [J/kgK]
- \( \rho c \) = specific heat of the material per volume [J/ m³K]
- \( k_x \) = heat conductivity for the heat flow in the x-direction [W/mK]
- \( k_y \) = heat conductivity for the heat flow in the y-direction [W/mK]
- \( Q_H \) = generated heat inside the body [W/m³]

It is assumed that the heat conductivity of the material is the same in all directions, meaning \( k_x = k_y = k \), where \( k \) is the material isotopic heat conductivity.
For the external boundary conditions, the heat flow is defined as below, from (JEJMS Concrete, 2006).

\[ q_n = h_{surf} (T_{surf} - T_{env}) - I \]

where

- \( q_n \) = heat flow from the body to the boundary along the perpendicular surface in the xy-plane \([\text{W/m}^2]\)
- \( h_{surf} \) = heat transfer coefficient for the external boundary \([\text{W/m}^2\text{K}]\)
- \( T_{surf} \) = temperature on the surface \([^\circ C\) or \([\text{K}]\)
- \( T_{env} \) = temperature of the environment \([^\circ C\) or \([\text{K}]\). Usually, the temperature of the ambient air.
- \( I \) = heat radiation to the boundary from the surrounding \([\text{W/m}^2]\)

In (JEJMS Concrete, 2006) it is defined how the heat transfer coefficient is described at external boundaries, where the surrounding medium is air, with the following equation

\[ h_{free} = \begin{cases} 5.6 + 3.9v & \text{for } v < 5 \text{ m/s} \\ 7.8v^{0.78} & \text{for } v > 5 \text{ m/s} \end{cases} \]

where

- \( h_{free} \) = heat transfer coefficient for a free surface bounded by air \([\text{W/m}^2\text{K}]\)
- \( v \) = velocity of the air \([\text{m/s}]\)

2.4.1.2 Stress calculations

Stress computations are performed with regard to where the temperature variations have been calculated. The stresses in the z direction (\(\sigma_z\)) are being studied if a 2D surface is represented by a xy-plane. This is presented in Figure 2.44, where it can be seen that the cracks formed by \(\sigma_z\) are perpendicular to the z-direction.

There are two different ways of performing stress calculations, Linear Line analysis, denoted LL, and Plane Surface analysis, denoted PLS. With LL model if the variation in the x-direction is measured, the stresses in the y-direction are averaged. In the same way, if the variation in y-direction is measured, the stresses in the x-direction are averaged. With this, the LL analysis may be viewed as advanced beam analysis for rotation around either the x- or the y-axis. With the PLS analysis the stresses vary according to the boundary conditions in the xy-plane, where the boundary conditions are given by the user. This method gives clearer results where the stress concentrations can be more exact (JEJMS Concrete, 2006).
2.4.2 Calculations in ConTeSt

The main goal when using ConTeSt has been to recreate the conditions that was applied during the casting of the concrete. This was done to reach an as valid calculation result as possible when using the software. The calculations were made to be able to study the impact from the changes in the material parameters, such as relaxation spectra, temperature and stresses.

A way of increasing the validity of the use of the software was through meetings with Anders Hösthagen held several times from January 2019 to May 2019 to teach a grounding in ConTeSt. Anders Hösthagen was also consulted when facing more complex questions about modeling. The ConTeSt Pro User’s manual (JEJMS Concrete, 2006) was used as a support during the modelling phase and literature study phase.

2.4.2.1 Model and geometry

The model and geometry are created in the “Geometry/Time” tab which defines the geometrical description of the block included in the computation. It is possible to draw two, or more, blocks that are physically connected to each other, by having connecting basepoints that are specified in coordinates. A Figure of how a block looks like when drawn in ConTeSt Pro is shown in Figure 2.45.

![Figure 2.45. Connecting basepoints forming the edges of the connecting block (JEJMS Concrete, 2006).](image)

The duration of the computations defines the amount of time that the computation extends to. When deciding the computation time the possible changes in the construction are taken into consideration, for example the time before and after adding the cement (JEJMS Concrete, 2006).

2.4.2.2 Computation mesh

In order to make computations with the program ConTesSt, the program needs to generate a computation mesh that contains of a number of sub-blocks (elements). Depending on how accurate a computation is, the size of the computation elements is changed. To find sufficient accuracy in the computation there should be at least eight elements in the width of the narrowest part of the structure. Blocks that are large and have less impact on the computation may be denoted larger elements, see Figure 2.46. Having a fine mesh would lead to longer duration of computation and larger data files without leading to improvements in the accuracy of the computation. In the connecting borders of blocks concentrations in the computation mesh becomes finer (JEJMS Concrete, 2006).

When editing the element size to obtain the accurate computation mesh three values need to be defined as described in JEJMS Concrete (2006):

1. The element size that indicates the size of the elements in the computation mesh.
2. *Distance from object,* is a value indicating how far out from the actual object this element size ought to be applied.

3. *Change rate* indicates the rate that the element size changes to fit the neighboring element sizes of different objects.

Different standard values are accurate for different types of structures or objects, see Figure 2.46.

<table>
<thead>
<tr>
<th>Object type</th>
<th>Element size</th>
<th>Dist. from object</th>
<th>Change rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Block</td>
<td>0.150</td>
<td>0.200</td>
<td>0.5</td>
</tr>
<tr>
<td>Inner hole</td>
<td>0.030</td>
<td>0.030</td>
<td>0.7</td>
</tr>
<tr>
<td>Inner point</td>
<td>0.020</td>
<td>0.030</td>
<td>0.3</td>
</tr>
<tr>
<td>Boundary (by definition)</td>
<td>0.040</td>
<td>0.080</td>
<td>0.5</td>
</tr>
</tbody>
</table>

*Figure 2.46. Standard element values for varying objects (JEJMS Concrete, 2006).*

The effect of the change rate, distance from object and different element sizes are shown in Figure 2.47, by comparing two blocks.

*Figure 2.47. Demonstration of differences in values on element features (JEJMS Concrete, 2006).*
2.4.2.3 Material parameters

Every block drawn in the program represents a different material. A block might consist of mature concrete, young concrete or of other materials such as gravel, clay or rock. It is therefore possible to give a block certain material features in the program. Besides from choosing the material for a block, the start temperature of the concrete can be changed and determined. It is possible to choose a constant start temperature over the area of the block, or to choose a start temperature that varies linearly in one or two dimensions. Both for concrete and other materials it is possible to edit the density, heat transfer conductivity and heat capacity. External boundaries can be defined as specific materials, such as wood or insulation with the wanted width and material properties. In the program, it is also possible to specify the rate of concrete pour (JEJMS Concrete, 2006). Figure 2.48, shows a dialog box with the mechanical properties of a young concrete. These mechanical properties can be set to wanted values to study how sensitive they are in the calculations. In this thesis, the relaxation spectra in Figure 2.48 will be changed and its impact will be studied.

![Figure 2.48. The Dialog box for a young concrete showing its mechanical properties (JEJMS Concrete, 2006).](image)

2.4.2.4 Temperature parameters

In the ConTeSt program, temperature, wind and external power for the studied block can be regulated. The value that is entered in the field connected with the condition will be valid during the whole computation time. The parameters of temperature, wind and external power can either be set to constant values or values that vary in time (JEJMS Concrete, 2006).

In this thesis, since we are studying a wall created of young concrete on a slab created with a mature concrete, the temperatures for the young and mature concrete are defined. Also, the temperature for the external boundaries that for the young concrete is wood and for the old concrete is a free surface, which represents the ambient air are defined. These temperatures are
changed for two typical cases representing summer and winter cases, this is further described in chapter 3. Aside from the mechanical properties of a material, the heat properties can also be edited and determined as shown in Figure 2.49.

![Figure 2.49. The Dialog box for a young concrete showing its heat properties.](image)

2.4.2.5 Temperature calculations

After the definitions of the parameters in ConTeSt have been completed, the computation of heat can then be performed. The program will calculate the material parameters for each element at each time point. To maintain sufficient accuracy, the time steps are automatically varied for the computation, which can be seen in the computation window in Figure 2.50. The output of the temperature calculations will show up as curve diagrams, and in this case as temperature curves, see Figure 2.51 (JEJMS Concrete, 2006).

![Figure 2.50. The heat computation window when performing the temperature calculations (JEJMS Concrete, 2006).](image)
2.4.2.6 Stress calculations

When performing stress calculations in ConTeSt it is possible to define the stress state for the computation, if using uniaxial stress state or cylindrical stress state. Stress variation line along the various axis can be defined as desired. The structure can be set to full restraint or to free translation without any rotation or with free rotation. When analyzing a cross section according to the beam theory the hypothesis is that plane sections remain plane. This hypothesis is valid for long structures where length to height ratio (resilience) is higher than seven. For many real structures the ratio of length and height is less than seven. In the software the resilience definition can be edited to a constant value between 0 and 1 or by using the length/height-ratio related resilience (JEJMS Concrete, 2006).

An example of data from a stress computation is presented in Figure 2.52, where the strain ratio for a 0.7 m thick wall on slab is shown. The strain ratio is the walls instantaneous strain because of thermal movement, divided by the strain that the wall can withstand before cracking.
3. Method

3.1 General

The method is selected to fulfil the aim of the thesis, which is to analyze the sensitivity of evaluation of basic creep by making changes in the evaluation process and of the Linear Logarithmic Model and to see how different parameters sets effects calculated stresses/strains during through crack analysis. With this, concrete is tested and the results is evaluated so that the material parameter set can be determined as useful for temperature- and stress calculations for through thermal cracking analysis for young concrete. If there are variations in the calculated strain/stresses when using different material parameter sets, it indicates that the evaluation process for the basic creep is relatively sensitive.

The used method is based on equations, definitions and theories described in Chapter 2. Therefore, in this chapter only the differences from the equations stated in Chapter 2 are presented, together with specific values used in the evaluation process in this thesis.

The concrete that is used consists of the structural engineering cement with fly ash, C35/45 S4 16 mm, called Anl Fa in this thesis. The content and amount of the components in the used concrete are presented in Table 3.1.

Table 3.1. The used structural engineering cement, fly ash (FA) with type and content of binders and type and amount in the concrete mixture is presented in this table.

<table>
<thead>
<tr>
<th>Material</th>
<th>kg/m³</th>
<th>Volume</th>
<th>Density</th>
</tr>
</thead>
<tbody>
<tr>
<td>Civil engineering cement FA (Anl FA)</td>
<td>430</td>
<td>142</td>
<td>3,02</td>
</tr>
<tr>
<td>Water</td>
<td>168,6</td>
<td>169</td>
<td>1</td>
</tr>
<tr>
<td>SX-A (floatation)</td>
<td>3,640</td>
<td>3</td>
<td>1,07</td>
</tr>
<tr>
<td>SR-N (floatation)</td>
<td>0,000</td>
<td>0</td>
<td>1,05</td>
</tr>
<tr>
<td>MasterAir105 (air)</td>
<td>0,665</td>
<td>1</td>
<td>1,00</td>
</tr>
<tr>
<td>Aggregates 0-2</td>
<td>307,7</td>
<td>116</td>
<td>2,65</td>
</tr>
<tr>
<td>Aggregates 11-16</td>
<td>587,4</td>
<td>213</td>
<td>2,76</td>
</tr>
<tr>
<td>Aggregates 4-11</td>
<td>266,1</td>
<td>97</td>
<td>2,75</td>
</tr>
<tr>
<td>Aggregates 0-4</td>
<td>594,3</td>
<td>219</td>
<td>2,71</td>
</tr>
<tr>
<td>Pate including air</td>
<td>0,665</td>
<td>1</td>
<td>1,00</td>
</tr>
<tr>
<td>Aggregates</td>
<td>355</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Control of volume 1 m³</td>
<td>1755,4</td>
<td>645</td>
<td></td>
</tr>
<tr>
<td>Ratio between water and cement + fly ash</td>
<td>400</td>
<td>1000</td>
<td></td>
</tr>
</tbody>
</table>

3.2 Determination of basic creep

3.2.1 Determination of elastic modulus

The implemented method started with that the elastic modulus was evaluated from the results of the laboratory tests described in Section 2.3, and the equations from Section 2.2.6.4. The evaluation was made with help from an Excel file that contains all relevant equations and test data from the laboratory test made in Thysell lab, at LTU. There is no standard guideline used at the laboratory at LTU that could be a reference in this report. The method in the laboratory
is however described in Section 2.3. The adjusted elastic modulus is presented as a function of equivalent time, see Figure 3.1 that shows the first stage in the evaluation process.

![Figure 3.1](image1.png)

Figure 3.1. Various values of the E-modulus as a function of the equivalent time, for the cement type Anl FA. Tests A and B denote different specimens loaded at 1,5 and 14 days.

3.2.2 Linear logarithmic model of creep

The valuation of basic creep was made with Linear Logarithmic Model, which is described in the theory Section 2.2.6.3.1. The creep compliance function is defined as the inverse of the earlier determined elastic modulus plus creep part, see Eq 2.51. The creep compliance is adjusted by changing the values of the parameters $a_1$ and $a_2$, where $a_1$ and $a_2$ are the inclination, or “logarithmic” creep rate, of short-term creep and long-term creep, respectively. The adjustment of $a_1$ and $a_2$ is made with respect to the measured values, where the aim is to mimic the measured values when obtaining calculated values. The changes in the inclination parameters was altered until a good enough fit was obtained. These changes were made in the Excel file.

![Figure 3.2](image2.png)

Figure 3.2. Three linear curves describing the modeling of creep tests, where the compliance function is given in logarithmic time scale. The first part of the linear line denotes short term creep, and the second part of the linear line denotes the long-term creep. The inclination of the lines is given by $a_1$ and $a_2$. 
In Figure 3.3, two fits of the obtained creep rate parameters for each loading age was performed, named \( a_1 \) and \( a_2 \) respectively, to study its impact on risk of through thermal cracking when studying calculations of several typical cases, later on. The adjusted values for \( a_1 \) and \( a_2 \) for the two material parameter sets, Fa 1 and Fa 2, are shown in Table 3.2.

![Figure 3.3. Diagram of logarithmic creep rate for Anl FA in relation to age of loading.](image)

**Table 3.2. The adjusted values of the inclinations \( a_1 \) and \( a_2 \) for two material parameter sets, Fa 1 and Fa 2.**

<table>
<thead>
<tr>
<th></th>
<th>Fa1</th>
<th>Fa2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( A_{\min} ) [Pa(^{-12}\log(\Delta t_{\text{load}})])</td>
<td>( A_{\min} ) [Pa(^{-12}\log(\Delta t_{\text{load}})])</td>
</tr>
<tr>
<td></td>
<td>( A_{\max} ) [Pa(^{-12}\log(\Delta t_{\text{load}})])</td>
<td>( A_{\max} ) [Pa(^{-12}\log(\Delta t_{\text{load}})])</td>
</tr>
<tr>
<td>( a_1 )</td>
<td>1.50</td>
<td>1.00</td>
</tr>
<tr>
<td>( a_2 )</td>
<td>6.00</td>
<td>5.00</td>
</tr>
</tbody>
</table>

3.2.3 Determination of relaxation with RELAX program

The creep values obtained from the Excel file and a file with general input are needed as input to the RELAX program. The general file defines the output of the program, such as how many Maxwell elements the model should contain. As an output, the program will generate a relaxation spectrum which defines the relaxation for different maturity and loading ages.

The input file (.txt file) containing creep values is described by six rows as follows, where the fourth and fifth rows are values taken from the Excel file.

KTYPE
\( E_{\text{ref}} \) [GPa], \( t_{\text{ref}} \) [d], \( s \) [-], \( n_E \) [-], \( t_s \) [d]
\( \Delta t_0 \) [d], \( \Delta t_1 \) [d],
\( a_1^{\text{\text{min}}} \) [creep unit], \( a_1^{\text{\text{max}}} \) [creep unit], \( t_{a1} \) [d], \( n_{a1} \) [-],
\( a_2^{\text{\text{min}}} \) [creep unit], \( a_2^{\text{\text{max}}} \) [creep unit], \( t_{a2} \) [d], \( n_{a2} \) [-],
\( (\Delta t_{\text{load}})_{\text{age}} \) [d], \( (\Delta t_0)_{\text{age}} \) [d]
where

\[ K_{\text{TYPE}} = \text{the used relaxation model within RELAX. The KTYPE}=8 \text{ refers to the LLM.} \]

\[ E_{\text{ref}} = \text{Modulus of elasticity at equivalent age, } t_{\text{ref}}. \]

\[ n_{E} = 0.5 \text{ but can be changed if needed.} \]

\[ (\Delta t_{\text{load}})_{\text{age}} \text{ and } (\Delta t_{0})_{\text{age}} = \text{limit values for the load duration and loading ages to avoid negative relaxation.} \]

An example of how one of the input files looks like with numbers is presented in Figure 3.4

<table>
<thead>
<tr>
<th>File</th>
<th>Edit</th>
<th>Format</th>
<th>View</th>
<th>Help</th>
</tr>
</thead>
<tbody>
<tr>
<td>β</td>
<td>34.77</td>
<td>28.0</td>
<td>0.232</td>
<td>0.5</td>
</tr>
<tr>
<td>β</td>
<td>0.001</td>
<td>0.3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>β</td>
<td>1.30e-3</td>
<td>16.63e-3</td>
<td>0.3181</td>
<td>0.2548</td>
</tr>
<tr>
<td>β</td>
<td>4.5035e-3</td>
<td>8.248e-3</td>
<td>22.872</td>
<td>1.315</td>
</tr>
<tr>
<td>β</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Figure 3.4 Input file to the RELAX program.**

With the relaxation data computed from the file (FileNameCreep.rin) and relaxation data according to equations used in Maxwell chain calculations (FileNameCreep.rut), values of spring constants and loading ages according to chosen Maxwell chain are obtained (FileNameCreep.rel), this is equal to relaxation spectra for the chosen relaxation times. See Chapter 2.2.6.2 for equations and description of Maxwell chain model and RELAX program. See Figure 3.5 and 3.6 for FileNameCreep.rin, -rut and -rel.

**Figure 3.5 Diagram showing the results from conversion of creep values into relaxation data according to equations used in Maxwell chain calculations from the .rut-file.**
Figure 3.6 Diagram showing the values of spring constants and loading ages according to eight Maxwell chains from .rel-file, this is equal to the relaxation spectra for the chosen relaxation times.

3.3 Comparisons of measured and calculated stresses

In order to obtain two parameter sets with different relaxation spectra, the process in Sections 3.1 and 3.2 is performed twice. The two different relaxation spectra are put into the Material Editor in ConTeSt. In this material editor, two materials are created with chosen mechanical properties and heat properties, see Section 2.4 for description of how materials are specified in ConTeSt. In Figure 3.8, the two different relaxation spectra and other mechanical properties of the young concrete as seen in the Material editor.

![Material Editor screenshot](image-url)
b) Figure 3.8. The mechanical properties for the two created materials, young concrete with the civil engineering structural cement FA, named a) ANL FA 1 and b) ANL FA 2.

The two parameter sets are used in ConTeSt to calculate stresses/strains obtained by fully hindered temperature movement of a 700 mm thick construction part (i.e. a wall with a thickness of 700 mm). Stresses obtained from the two parameter sets are compared to each other and to a test counterpart, where measured stresses are obtained from a test set up with 100% restraint. ConTeSt is used to create blocks that represent the tested block and its properties in the laboratory setup for stress at full restraint in Luleå technical university. The two created blocks are assigned with the material parameter sets created in the Material Editor. See the block in Figure 3.9. Heat calculations are performed for the created block with ConTeSt and its result is shown in Chapter 4.

Figure 3.9. The block is from ConTeSt representing the tested block in the laboratory at LTU for stress at full restraint.

From the calculations, the ConTeSt program gives outputs that can be combined to create a stress diagram. To study the impact of the two created relaxation spectra, the stress diagrams of material parameter set FA 1 and FA2 are compared to the stress diagram from the measured values in the laboratory at LTU, see Chapter 4 for results.
3.4 Calculations of stresses/strains for typical cases

To analyze the sensitivity of the differences in the relaxation spectra and the different stress results, the material parameter sets, ANL FA 1 and FA 2, are used on structures that represents typical cases. Different models where modelled in ConTeSt with three different conditions, standard, “summer” and “winter” conditions, see Figures 3.12, 3.13 and 3.14. In the typical cases, each model consists of two blocks that are physically connected to each other. One of the blocks represents the ground and the second one represents a wall with different thickness according to the studied case. The ground consists of mature concrete and the wall consists of young concrete encased by wooden formwork.

The conditions for standard, summer and winter are analyzed with different temperature values of the air, young concrete and the wooden formwork, the corresponding values are presented in Table 3.3 by $T_1$, $T_2$, $T_3$ and $T_4$. $T_1$ is the ambient air temperature, $T_2$ is the temperature of the young concrete, $T_3$ is the temperature of the wooden formwork and $T_4$ is the temperature of the slab.

![Figure 3.10. Wall on ground modelled in ConTeSt, where the ambient air temperature, the temperature of the young concrete, the temperature of the formwork and the temperature of the concrete representing a ground slab are represented by $T_1$, $T_2$, $T_3$ and $T_4$ respectively.](image)

**Table 3.3. Values of the temperatures of $T_1$, $T_2$, $T_3$ and $T_4$ in winter and summer case.**

<table>
<thead>
<tr>
<th>Temperature $[^\circ C]$</th>
<th>Standard</th>
<th>Winter</th>
<th>Summer</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_1$</td>
<td>20</td>
<td>5</td>
<td>15</td>
</tr>
<tr>
<td>$T_2$</td>
<td>20</td>
<td>15</td>
<td>20</td>
</tr>
<tr>
<td>$T_3$</td>
<td>20</td>
<td>5</td>
<td>15</td>
</tr>
<tr>
<td>$T_4$</td>
<td>20</td>
<td>5</td>
<td>20</td>
</tr>
</tbody>
</table>
Table 3.4. Thickness of the concrete walls in all typical cases with their corresponding values of element size and specifications of the FEM-mesh used in ConTeSt.

<table>
<thead>
<tr>
<th>Concrete wall thickness [m]</th>
<th>Element size [-]</th>
<th>Dist from object [-]</th>
<th>Change rate [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.2</td>
<td>0.12</td>
<td>0.2</td>
<td>0.5</td>
</tr>
<tr>
<td>0.7</td>
<td>0.08</td>
<td>0.2</td>
<td>0.5</td>
</tr>
<tr>
<td>0.4</td>
<td>0.04</td>
<td>0.2</td>
<td>0.5</td>
</tr>
</tbody>
</table>

3.4.1 Geometry of typical cases

For all cases the length of the structure is set to 8 meters, and the rest of the dimensions are presented in the figures below.

Figure 3.11. Model representing 0.4 m thick wall on ground. Bl:1 is mature concrete and Bl:2 is the young concrete with either material parameter set FA 1 or FA 2.

Figure 3.12. Model representing 0.7 m thick wall on ground. Bl:1 is mature concrete and Bl:2 is the young concrete with either material parameter set FA 1 or FA 2.
3.4.2 Comparisons of calculations with altered parameter set-up

ConTeSt was used to compute heat and stress calculations for the different cases described in Section 3.4.1. To analyze the sensitivity of the different parameter setup, FA 1 and FA 2, the differences of the outputs from the heat and stress calculations are compared. The heat calculation gives the temperature development as a function of time, and the stress calculations shows a strain ratio as a function of time. The results of these calculations and their differences are presented in Chapter 4.
4. Results

4.1 Outcome of compared relaxation spectra

Creep values have been converted into relaxation values according to equations used in Maxwell-chain calculations with eight Maxwell elements for the used model, LLM. As an output it generates a relaxation spectrum which defines the relaxation for different maturity and loading ages. The E-modulus for each spring in the Maxwell chain at different loading ages for the two different material parameter sets FA 1 and FA 2 are shown in the tables below. These are used to create relaxation spectrum, showing relaxation modulus as a function of time after loading.

Table 1. Numerical values of the E-modulus for each spring in the Maxwell chain, its summation and loading ages for the material parameter set FA 1.

<table>
<thead>
<tr>
<th>Loading ages (h)</th>
<th>Maxwell element 1-8</th>
<th>Summation</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.124</td>
<td>0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.08</td>
<td></td>
</tr>
<tr>
<td>0.275</td>
<td>1.2149 0.77079 0.92806 0.70233 0.81 0.58511 0.62672 0.44788 6.08579</td>
<td></td>
</tr>
<tr>
<td>0.39212</td>
<td>2.4534 1.1985 1.3457 1.1193 1.2104 0.89566 0.94791 1.0339 10.20477</td>
<td></td>
</tr>
<tr>
<td>1.24</td>
<td>2.9075 1.2111 2.3031 3.5243 2.6879 2.3029 2.272 4.9334 22.1422</td>
<td></td>
</tr>
<tr>
<td>3.9212</td>
<td>1.7152 0.47191 2.9469 6.0532 3.6095 3.4445 3.2355 10.435 31.91171</td>
<td></td>
</tr>
<tr>
<td>39.212</td>
<td>2.5359 0.34232 4.0332 7.2722 3.673 3.5453 3.2676 17.026 41.69552</td>
<td></td>
</tr>
<tr>
<td>124</td>
<td>2.7456 0.16668 4.1313 7.3745 3.7038 3.4469 3.1699 18.952 43.69068</td>
<td></td>
</tr>
<tr>
<td>392.12</td>
<td>2.865 0.036945 4.1517 7.3681 3.7537 3.3937 3.0938 20.142 44.80495</td>
<td></td>
</tr>
<tr>
<td>1240</td>
<td>2.9356 0.038769 4.1649 7.3597 3.7933 3.3898 3.0535 20.787 45.52257</td>
<td></td>
</tr>
</tbody>
</table>

Table 2. Numerical values of the E-modulus for each spring in the Maxwell chain, its summation and loading ages for the material parameter set FA 2.

<table>
<thead>
<tr>
<th>Loading ages (h)</th>
<th>Maxwell element 1-8</th>
<th>Summation</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.124</td>
<td>0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.08</td>
<td></td>
</tr>
<tr>
<td>0.275</td>
<td>1.1166 0.70883 0.89321 1.0851 1.1564 0.80592 0.81342 0.50004 7.07952</td>
<td></td>
</tr>
<tr>
<td>0.39212</td>
<td>2.2868 1.1345 1.2817 1.5684 1.6226 1.1432 1.1473 0.010788 10.19529</td>
<td></td>
</tr>
<tr>
<td>3.9212</td>
<td>2.3708 0.55136 2.6728 4.7768 3.1419 2.6694 2.4418 12.408 31.03286</td>
<td></td>
</tr>
<tr>
<td>12.4</td>
<td>2.747 0.45871 3.3602 5.6188 3.2489 2.8776 2.5792 16.279 37.16941</td>
<td></td>
</tr>
<tr>
<td>39.212</td>
<td>3.3177 0.57012 4.005 6.3705 3.4216 3.0588 2.7204 17.634 41.09812</td>
<td></td>
</tr>
<tr>
<td>124</td>
<td>3.6957 0.6318 4.4339 6.9015 3.5741 3.1735 2.8108 18.253 43.4743</td>
<td></td>
</tr>
<tr>
<td>392.12</td>
<td>3.9263 0.66857 4.6934 7.2193 3.6841 3.2433 2.857 18.576 44.86797</td>
<td></td>
</tr>
<tr>
<td>1240</td>
<td>4.0621 0.68994 4.8458 7.4024 3.7533 3.291 2.8806 18.746 45.67114</td>
<td></td>
</tr>
</tbody>
</table>
Figure 4.1. Comparison of the relaxation spectra for FA 1 and FA 2. The continuous line denotes the relaxation spectra for FA 1 and the dashed lines denote relaxation spectra for FA 2.

The relaxation spectrum in Figure 4.1 shows how changes in $a_1$ and $a_2$ will affect the relaxation modulus as a function of time after loading. The differences between the two spectra are analyzed further in Chapter 5.

4.2 Outcome of the heat and stress calculations compared to measured stress values for the tested block

The output of the heat calculations made in ConTeSt is presented as temperature development in Figure 4.2. Since the input of temperature parameters for the two material parameter sets, FA 1 and FA 2 are the same, the temperature development will also be the same for both of the sets.

Figure 4.2. Temperature development for both material parameter sets, FA 1 and FA 2, respectively.

The output of the stress calculations made in ConTeSt is presented in Figure 4.3 as two stress curves for the material parameter sets FA 1 and FA 2, compared to the curve for the measured stress from the tested block in the laboratory test.
Figure 4.3. Comparison of the measured stresses for the concrete ANL FA with the calculated stresses for the concrete with material parameter set ANL FA 1 and ANL FA 2.

4.3 Outcome of typical case studies

4.3.1 Results for the different typical cases with different conditions

The typical cases studied in this thesis are based on the typical structure slab-on-wall with different thickness of the wall and different conditions. The results for the reference wall, corresponding to the wall for the tested block in the laboratory test, with the thickness of 0.7 m for the material parameters set FA 1 are only presented in this Section and for the standard conditions only. For the rest of the cases, the results for the different thickness and different conditions for the two material parameter sets FA 1 and FA 2 are presented in Appendix A.

In Figure 4.4, the minimum, average and maximum temperature development for block 2, the wall, and the average temperature development for block 1, the slab, is presented. From the curves, the maximum, minimum and the average values of the temperature in the wall and the average temperature values for the slab can be obtained.

Colour maps of the different studied walls are obtained as a result from ConTeSt to identify the area of the walls where the temperature development is observable, see Figure 4.5. By the colour maps, the calculation of the more observable and specific average value of the strain ratio for each studied wall can be performed and plotted out together with the strain curves for the minimum, average and maximum strain values of the wall, see Figure 4.6. The strain ratio is the walls instantaneous strain because of thermal movement, divided by the strain that the wall can withstand before cracking.
Figure 4.4. Temperature development for the material parameter set FA 1 with the standard conditions for the wall thickness of 0.7 m.

Figure 4.5. Colour map of the strain ratio for the material parameter set FA 1 with the standard conditions for the wall thickness of 0.7 m.

Figure 4.6. Strain ratio for the material parameter set FA 1 with the standard conditions for the wall thickness of 0.7 m.
4.3.2 Numerical results for all analyzed cases

Numerical values of the maximum temperature, maximum strain and average strain as well as their differences for the two material parameters sets FA 1 and FA 2 used for different thickness of the wall and different conditions are presented in the tables below. The differences of average strain values for different thickness of the wall at each condition are calculated and presented in the tables to be able to compare them with the corresponding values for the different conditions.

Table 4.3. Numerical results for the two material parameter sets FA 1 and FA 2 used for different thickness of the wall at standard conditions.

<table>
<thead>
<tr>
<th>Wall thickness [m]</th>
<th>Max temperature [°C]</th>
<th>Max strain</th>
<th>Average strain</th>
<th>Max temperature [°C]</th>
<th>Max strain</th>
<th>Average strain</th>
<th>Differences of average strain values [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.4</td>
<td>47.706</td>
<td>0.754</td>
<td>0.677</td>
<td>47.708</td>
<td>0.734</td>
<td>0.656</td>
<td>2.1</td>
</tr>
<tr>
<td>0.7</td>
<td>53.150</td>
<td>0.751</td>
<td>0.657</td>
<td>53.150</td>
<td>0.748</td>
<td>0.653</td>
<td>0.4</td>
</tr>
<tr>
<td>1.2</td>
<td>57.739</td>
<td>0.692</td>
<td>0.604</td>
<td>57.739</td>
<td>0.701</td>
<td>0.613</td>
<td>0.9</td>
</tr>
</tbody>
</table>

Table 4.4. Numerical results for the two material parameter sets FA 1 and FA 2 used for different thickness of the wall at summer conditions.

<table>
<thead>
<tr>
<th>Wall thickness [m]</th>
<th>Max temperature [°C]</th>
<th>Max strain</th>
<th>Average strain</th>
<th>Max temperature [°C]</th>
<th>Max strain</th>
<th>Average strain</th>
<th>Differences of average strain values [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.4 m</td>
<td>48.442</td>
<td>0.967</td>
<td>0.882</td>
<td>48.442</td>
<td>0.936</td>
<td>0.845</td>
<td>3.7</td>
</tr>
<tr>
<td>0.7 m</td>
<td>55.799</td>
<td>1.004</td>
<td>0.865</td>
<td>55.799</td>
<td>0.992</td>
<td>0.852</td>
<td>1.3</td>
</tr>
<tr>
<td>1.2 m</td>
<td>61.736</td>
<td>0.938</td>
<td>0.802</td>
<td>61.736</td>
<td>0.943</td>
<td>0.805</td>
<td>0.3</td>
</tr>
</tbody>
</table>

Table 4.5. Numerical results for the two material parameter sets FA 1 and FA 2 used for different thickness of the wall at winter conditions.

<table>
<thead>
<tr>
<th>Wall thickness [m]</th>
<th>Max temperature [°C]</th>
<th>Max strain</th>
<th>Average strain</th>
<th>Max temperature [°C]</th>
<th>Max strain</th>
<th>Average strain</th>
<th>Differences of average strain values [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.4 m</td>
<td>36.566</td>
<td>0.903</td>
<td>0.813</td>
<td>36.566</td>
<td>0.868</td>
<td>0.785</td>
<td>2.8</td>
</tr>
<tr>
<td>0.7 m</td>
<td>44.436</td>
<td>0.974</td>
<td>0.868</td>
<td>44.436</td>
<td>0.955</td>
<td>0.849</td>
<td>1.9</td>
</tr>
<tr>
<td>1.2 m</td>
<td>50.821</td>
<td>0.923</td>
<td>0.800</td>
<td>50.821</td>
<td>0.923</td>
<td>0.790</td>
<td>1.0</td>
</tr>
</tbody>
</table>
The average strain for both material parameter sets are plotted as a function of the wall thickness and the results are presented in Figure 4.7, 4.8 and 4.9 for the standard, summer and winter condition. This is performed to be able to observe and compare the different average strain values for the two different material parameter sets for the same wall thickness at different conditions.

Figure 4.7. The average strain as a function of the wall thickness for the standard conditions.

Figure 4.8. The average strain as a function of the wall thickness for the summer conditions.

Figure 4.9. The average strain as a function of the wall thickness for the winter conditions.
5. Discussion and sources of error

5.1 Discussion

During the evaluation process of basic creep, the tests are working well. For the heat calculations the two material parameter sets are identical, hence they generate the same temperature development as a function of time. Since the parameters connected to the viscoelastic properties of the concrete, differences between the two material parameter sets generates differences in the outcome of the stress calculations. The material parameter set that could be considered the most fit is the one that most accurately provides a stress development that mimics the one from the laboratory test. It is observed in Figure 4.3 in Chapter 4 that the stress diagram for material parameter set FA 1 and FA 2 are very similar to each other due to small changes in the material parameter set. With this observation, both material parameter sets could be interpreted as equally accurate.

When analyzing how sensitive the evaluation process is for basic creep and how sensitive it is for the different typical cases, we notice that the tests are agreeing mostly for the cases with standard conditions and the differences in average strain values are not very large. With this, the changes in the relaxation spectra is not very sensitive for the 0.7 m thick wall for the standard conditions. When we deviate from these standard conditions that are the same as in the laboratory environment, we get discrepancies of calculated strain ratios between the different material parameter sets.

When we change the temperature conditions, it yields a different temperature development and equivalent time of maturity for the various geometrical cases. The interesting part of the wall, out of a thermal through crack risk perspective, is the part with the largest value of average strain ratio, which is denoted as the dimensioning part of the wall.

Since we get different equivalent time of maturity for the walls, different parts of the relaxation spectra are valid for each thickness of the wall, which affects the stress calculations. See Figure 5.1 for a schematically presentation of where in the relaxation spectra the different thicknesses of the wall are placed. It is just a principal presentation since the relaxation spectra are changing with respect to the temperature. Hence, the relaxation spectrum shown in Figure 5.1 are solely valid for a reference temperature and will therefore be shifted with a deviating temperature.
Figure 5.1 Comparison of the relaxation spectra for FA 1 and FA 2 for a standard temperature condition and a 0.7 m thick wall. The colourful lines denote the relaxation spectra, where the continuous lines are the relaxation spectra for FA 1 and the dashed lines denote the relaxation spectra for FA 2. The three black continuous lines A, B and C schematically represent the equivalent time of maturity for the thicknesses 1.2 m, 0.7 m and 0.4 m respectively.

Principally for the thinner wall, the equivalent time of maturity of the curing concrete in the relaxation spectra has a flatter line than the thicker wall since that one cures faster. Since the two creep evaluations yielding two relaxation spectra are performed from at test set up representing a 0.7 m wall, the calculated strain ratios for a 0.7 m wall differ only slightly. Especially with temperature conditions similar to the test set up. When studying the thicker wall, 1.2 m, the relaxation spectra are differing more from each other for the two material parameter sets and this should result in larger differences in the calculated strains but that is not the case in the results in this thesis. In the results, smaller strain differences are obtained for the 1.2 m thick wall than for the 0.7 m thick wall which might be due to margins of error since it should be larger differences for the 1.2 m thick wall than for the 0.7 m thick wall. A reason for this could also be that the changes and differences in the relaxation spectra for the two material parameter sets are relatively small and might not give the right effect. The largest differences in average strain are obtained in the 0.4 m wall, which is believed to be accurate, since, for this thickness the relaxation spectra are believed to differ more for the two material parameter sets.

Logically, the thinner the studied walls are, the lesser thermal dilation and thermal load. With this we expect higher stresses for the thicker walls. But in our modelled geometry we have not got larger strains for the thicker walls than for the thinner walls, except for the winter case.

The stresses occur mainly due to differential deformations between the restraining concrete slab and the newly cast concrete wall. With this, a 0.4 m wall on a slab should have less average strain ratio than a 1.2 m wall on a slab with the same conditions. This is valid for the studied walls in winter conditions but not for the walls in the remaining cases, where smaller average strains are obtained for the 1.2 m thick walls compared to 0.4 m thick walls. The reason is that the thicker wall transfers more heat to the adjacent ground slab and makes it expand more than for the thinner case. If the ground slab would have been larger than the modeled one, the ground slab would not have been heated up by the wall to the same extent creating excessive temperature differences. If the ground slab would be approximately 1m thick instead of the
modelled one with a thickness of 0.3m then the 1.2 m wall would have larger average strain than the 0.4 m wall. This is because it would be harder for the wall to expand the ground slab to the same extent and we would see the restraining effects in a larger magnitude.

To summarize there should be smallest differences of average strain values for the 0.7 m thick wall for the standard case since this is the geometry and the conditions that are mimicking the tests performed in the laboratory. This was also shown in the result of this thesis except for the summer case for the 1.2 m thick wall that had even smaller differences of average strain, which could be due to that the warm conditions making it even easier to heat up the ground slab and decrease the differential movements between slab and wall. Secondly the 0.4 m thick wall and the 1.2 m thick wall should have larger differences of average strain in all environmental conditions, which was not fully achieved in the results due to the reasons explained above.

In this work a maximum difference in average strain values of approximately 4 % is obtained. This relatively small magnitude of difference could be because of sources of error or the reasons explained above. If the difference in the average strain between FA 1 and FA 2 would be above 10% then we believe to have an error range that is not exclusively because of sources of error, since the error range then would be too large to be considered only from sources of error.

5.2 Sources of errors

When performing a work of this magnitude, sources of error are important to be considered. In this thesis the evaluation process can be considered as a source of error, since choices have been made in how to evaluate the creep such as using the Linear Logarithmic model or the software program ConTeSt. Another source of error could be the accuracy in the measured values from the laboratory tests since there can be faults in the used equipment at the laboratory or errors in the manually registered measurement values. There are not too much of sources of error in this work and one of the sources mentioned, the sensitivity of the evaluation process, is actually the one that is being examined in the thesis. One other source of error that can be examined is the element sizes of the generated mesh in the finite element method program ConTeSt, since it affects the results due to calculation accuracy. The material properties such as heat and mechanical properties are not considered to be sources of errors since they are fixed values and identical for the two parameter sets.
6. Conclusions and future research

6.1 Conclusions

During the evaluation process of basic creep, the tests in the laboratory and the used programs are working well. A conclusion to be drawn is that changes in the inclination of short-term creep and long-term creep denoted as “logarithmic” creep rates $a_1(t_0)$ and $a_2(t_0)$, hence changes in the relaxation spectra give changes in the results of stress calculations for concrete structures such as the cases used in the report. With this it can be established that the evaluation process of basic creep is relatively sensitive.

The placement of the equivalent time of maturity for the different thicknesses of the walls in the relaxation spectra is the reason to differences in the 1.2 m and 0.4 m thick walls compared to the 0.7 m thick wall. One reason to relatively small differences in average strain values in the 1.2 m is that the thicker wall transfers heat to the adjacent ground slab and makes it expand, which decreases the differential movements between the slab and the wall, yielding a lower restraint which results in lower strain and stresses.

We get a temperature development for the tested concrete in the laboratory and as soon as we deviate from the temperature development for the test, either by changing the thickness of the wall or by testing different temperature conditions we get a different temperature development. With the deviation in the calculated temperature development compared to the measured one, we get a difference in the calculated strain ratios for the two different material parameter sets.

The main discovery in this work is that when we set up a geometry for a 0.7 m wall we get small deviations in the calculations of strain, which is expected since the temperature development in the created geometry of a wall on the slab will follow the temperature development of the concrete tested in the rig at the laboratory. But if we deviate from this temperature case, we experience differences in calculated strain ratios.

6.2 Future research

Suggestions for future research is to use statistical mathematical analyzations so that it is possible to statistically analyze the results so that a margin of error can be obtained.

Other suggestions are to analyze more than two different material parameter sets and examine if a pattern can be detected in the strain or heat calculations with changes in the relaxation spectra. Also, larger changes in the “logarithmic” creep rates $a_1(t_0)$ and $a_2(t_0)$ and relaxation spectra should be evaluated to see if they would result in strain values larger than the margins of error.

Other geometry and thicknesses of the walls could be evaluated to more thoroughly analyze the effect of the geometry on the heat and stress calculations.

To perform tests with temperature reference curves representing, for instance, a 0.4 m and/or 1.2 m thick walls would be interesting in order to examine a temperature-strain relationship.

Another suggestion for future research would be to investigate how much changes in the creep evaluation would result in a difference in average strain above 10%. This is because that 10% is the approximate limit for knowing that it is not only differences due to sources of error.
One last suggestion for future research within this area is to use other creep evaluation methods such as Double Power Law or Triple Power Law and compare the outcome to the results from the Linear Logarithmic Model.
References


A. Appendix

A.1. Outcome of compared relaxation spectra

Figure 1. Relaxation spectra of the material parameter set FA 1

Figure 2. Relaxation spectra of the material parameter set FA 2
A.2. Outcome of the heat and stress calculations compared to measured stress values for the tested block

Figure 3. Comparison of the relaxation spectra for FA 1 and FA 2, the continuous line denotes the relaxation spectra for FA 1 and the dashed lines denote relaxation spectra for FA 2.

Figure 4. Diagram showing same temperature development as output of the heat calculations made in ConTeSt for both material parameter sets, FA 1 respective FA 2. This because of same temperature parameters.
A.3. Outcome of typical case studies

A.3.1 Results for the case with standard conditions

Results for the material parameter set FA 1 with the standard conditions for the wall thickness of 1.2 meters are presented in Figure 6, 7 and 8.

Figure 5. Comparison of the measured stresses for the concrete ANL FA with the calculated stresses for the concrete with material parameter set ANL FA 1 and ANL FA 2.

Figure 6. Temperature development for the material parameter set FA 1 with the standard conditions for the wall thickness of 1.2 m.
Figure 7. Colour map of the strain ratio for the material parameter set FA 1 with the standard conditions for the wall thickness of 1.2 m.

Figure 8. Strain ratio for the material parameter set FA 1 with the standard conditions for the wall thickness of 1.2 m.

Results for the material parameter set FA 1 with the standard conditions for the wall thickness of 0.7 meters are presented in Figure 9, 10 and 11.

Figure 9. Temperature development for the material parameter set FA 1 with the standard conditions for the wall thickness of 0.7 m.
Results for the material parameter set FA 1 with the standard conditions for the wall thickness of 0.4 meters are presented in Figure 12, 13 and 14.

Figure 10. Colour map of the strain ratio for the material parameter set FA 1 with the standard conditions for the wall thickness of 0.7 m.

Figure 11. Strain ratio for the material parameter set FA 1 with the standard conditions for the wall thickness of 0.7 m.

Figure 12. Temperature development for the material parameter set FA 1 with the standard conditions for the wall thickness of 0.4 m
Figure 13. Colour map of the strain ratio for the material parameter set FA 1 with the standard conditions for the wall thickness of 0.4 m.

Figure 14. Strain ratio for the material parameter set FA 1 with the standard conditions for the wall thickness of 0.4 m.

Results for the material parameter set FA 2 with the standard conditions for the wall thickness of 1.2 meters is presented in Figure 15, 16 and 17.

Figure 15. Temperature development for the material parameter set FA 2 with the standard conditions for the wall thickness of 1.2 m.
Figure 16. Colour map of the strain ratio for the material parameter set FA 2 with the standard conditions for the wall thickness of 1.2 m.

Figure 17. Strain ratio for the material parameter set FA 2 with the standard conditions for the wall thickness of 1.2 m.

Results for the material parameter set FA 2 with the standard conditions for the wall thickness of 0.7 meters are presented in Figure 18, 19 and 20.

Figure 18. Temperature development for the material parameter set FA 2 with the standard conditions for the wall thickness of 0.7 m.
Figure 19. Colour map of the strain ratio for the material parameter set FA 2 with the standard conditions for the wall thickness of 0.7 m.

Figure 20. Strain ratio for the material parameter set FA 2 with the standard conditions for the wall thickness of 0.7 m.

Results for the material parameter set FA 2 with the standard conditions for the wall thickness of 0.4 meters are presented in Figure 21, 22 and 23.

Figure 21. Temperature development for the material parameter set FA 2 with the standard conditions for the wall thickness of 0.4 m.
A.3.2 Results for the case with summer conditions

Results for the material parameter set FA1 with the summer conditions for the wall thickness of 1.2 meters are presented in Figure 24, 25 and 26.

Figure 22. Colour map of the strain ratio for the material parameter set FA 2 with the standard conditions for the wall thickness of 0.4 m.

Figure 23. Strain ratio for the material parameter set FA 2 with the standard conditions for the wall thickness of 0.4 m.

Figure 24. Temperature development for the material parameter set FA 1 with summer conditions for the wall thickness of 1.2 m.
Figure 25. Colour map for the strain ratio for the material parameters set FA 1 with the summer conditions for the wall thickness 1.2 m.

Figure 26. Strain ratio for the material parameters set FA 1 with the summer conditions for the wall thickness 1.2 m.

Results for the material parameter set FA1 with the summer conditions for the wall thickness of 0.7 meters are presented in Figure 27, 28 and 29.

Figure 27. Temperature development for the material parameter set FA 1 with summer conditions for the wall thickness of 0.7 m.
Figure 28. Colour map for the strain ratio for the material parameters set FA 1 with the summer conditions for the wall thickness 0.7 m.

Figure 29. Strain ratio for the material parameters set FA 1 with the summer conditions for the wall thickness 0.7 m.

Results for the material parameter set FA1 with the summer conditions for the wall thickness of 0.4 meters are presented in Figure 30, 31 and 32.

Figure 30. Temperature development for the material parameter set FA 1 with summer conditions for the wall thickness of 0.4 m.
Figure 31. Colour map for the strain ratio for the material parameters set FA 1 with the summer conditions for the wall thickness 0.4 m.

Figure 32. Strain ratio for the material parameters set FA 1 with the summer conditions for the wall thickness 0.4 m.

Results for the material parameter set FA2 with the summer conditions for the wall thickness of 1.2 meters are presented in Figure 33, 34 and 35.

Figure 33. Temperature development for the material parameter set FA 2 with summer conditions for the wall thickness of 1.2 m.
Figure 34. Colour map for the strain ratio for the material parameters set FA 2 with the summer conditions for the wall thickness 1.2 m.

Figure 35. Strain ratio for the material parameters set FA 2 with the summer conditions for the wall thickness 1.2 m.

Results for the material parameter set FA2 with the summer conditions for the wall thickness of 0.7 meters is presented in Figure 36, 37 and 38.

Figure 36. Temperature development for the material parameter set FA 2 with summer conditions for the wall thickness of 0.7 m.
Figure 37. Colour map for the strain ratio for the material parameters set FA 2 with the summer conditions for the wall thickness 0.7 m.

Figure 38. Strain ratio for the material parameters set FA 2 with the summer conditions for the wall thickness 0.7 m.

Results for the material parameter set FA2 with the summer conditions for the wall thickness of 0.4 meters is presented in Figure 39, 40 and 41.

Figure 39. Temperature development for the material parameter set FA 2 with summer conditions for the wall thickness of 0.4 m.
A.3.3 Results for the case with winter conditions

Results for the material parameter set FA 1 with the winter conditions for the wall thickness of 1.2 meters is presented in Figure 42, 43 and 44.
Figure 43. Colour map of the strain ratio for the material parameter set FA 1 with the winter conditions for the wall thickness of 1.2 m.

Figure 44. Strain ratio for the material parameter set FA 1 with the winter conditions for the wall thickness of 1.2 m.

Results for the material parameter set FA 1 with the winter conditions for the wall thickness of 0.7 meters is presented in Figure 45, 46 and 47.

Figure 45. Temperature development for the material parameter set FA 1 with the winter conditions for the wall thickness of 0.7 m.
Figure 46. Colour map of the strain ratio for the material parameter set FA 1 with the winter conditions for the wall thickness of 0.7 m.

Figure 47. Strain ratio for the material parameter set FA 1 with the winter conditions for the wall thickness of 0.7 m.

Results for the material parameter set FA 1 with the winter conditions for the wall thickness of 0.4 meters is presented in Figure 48, 49 and 50.

Figure 48. Temperature development for the material parameter set FA 1 with the winter conditions for the wall thickness of 0.4 m.
Figure 49. Colour map of the strain ratio for the material parameter set FA 1 with the winter conditions for the wall thickness of 0.4 m.

Figure 50. Strain ratio for the material parameter set FA 1 with the winter conditions for the wall thickness of 0.4 m.

Results for the material parameter set FA 2 with the winter conditions for the wall thickness of 1.2 meters is presented in figure 51, 52 and 53.

Figure 51. Temperature development for the material parameter set FA 2 with the winter conditions for the wall thickness of 1.2 m.
Figure 52. Colour map of the strain ratio for the material parameter set FA 2 with the winter conditions for the wall thickness of 1.2 m.

Figure 53. Strain ratio for the material parameter set FA 2 with the winter conditions for the wall thickness of 1.2 m.

Results for the material parameter set FA 2 with the winter conditions for the wall thickness of 0.7 meters is presented in Figure 54, 55, and 56.

Figure 54. Temperature development for the material parameter set FA 2 with the winter conditions for the wall thickness of 0.7 m.
Figure 55. Colour map of the strain ratio for the material parameter set FA 2 with the winter conditions for the wall thickness of 0.7 m.

Figure 56. Strain ratio for the material parameter set FA 2 with the winter conditions for the wall thickness of 0.7 m.

Results for the material parameter set FA 2 with the winter conditions for the wall thickness of 0.4 meters is presented in Figure 57, 58 and 59.

Figure 57. Temperature development for the material parameter set FA 2 with the winter conditions for the wall thickness of 0.4 m.
Figure 58. Colour map of the strain ratio for the material parameter set FA 2 with the winter conditions for the wall thickness of 0.4 m.

Figure 59. Strain ratio for the material parameter set FA 2 with the winter conditions for the wall thickness of 0.4 m.
A.4. Numerical results for all analyzed cases

*Table 1. Numerical values of the maximum temperature, maximum strain and average strain as well as their differences for the two material parameter sets FA 1 and FA 2 used for different thickness of the wall at standard conditions.*

<table>
<thead>
<tr>
<th>Wall thickness [m]</th>
<th>Standard conditions</th>
<th>FA1</th>
<th>FA2</th>
<th>Differences of average strain values [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Max temperature [°C]</td>
<td>Max strain</td>
<td>Average strain</td>
</tr>
<tr>
<td>0.4</td>
<td></td>
<td>47.706</td>
<td>0.754</td>
<td>0.677</td>
</tr>
<tr>
<td>0.7</td>
<td></td>
<td>53.150</td>
<td>0.751</td>
<td>0.657</td>
</tr>
<tr>
<td>1.2</td>
<td></td>
<td>57.739</td>
<td>0.692</td>
<td>0.604</td>
</tr>
</tbody>
</table>

*Table 2. Numerical values of the maximum temperature, maximum strain and average strain as well as their differences for the two material parameter sets FA 1 and FA 2 used for different thickness of the wall at summer conditions.*

<table>
<thead>
<tr>
<th>Wall thickness [m]</th>
<th>Summer conditions</th>
<th>FA1</th>
<th>FA2</th>
<th>Differences of average strain values [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Max temperature [°C]</td>
<td>Max strain</td>
<td>Average strain</td>
</tr>
<tr>
<td>0.4 m</td>
<td></td>
<td>48.442</td>
<td>0.967</td>
<td>0.882</td>
</tr>
<tr>
<td>0.7 m</td>
<td></td>
<td>55.799</td>
<td>1.004</td>
<td>0.865</td>
</tr>
<tr>
<td>1.2 m</td>
<td></td>
<td>61.736</td>
<td>0.938</td>
<td>0.802</td>
</tr>
</tbody>
</table>
Table 3. Numerical values of the maximum temperature, maximum strain and average strain as well as their differences for the two material parameter sets FA 1 and FA 2 used for different thickness of the wall at winter conditions.

<table>
<thead>
<tr>
<th>Winter conditions</th>
<th>FA1</th>
<th>FA2</th>
<th>Differences of average strain values [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wall thickness [m]</td>
<td>Max temperature [°C]</td>
<td>Max strain</td>
<td>Average strain</td>
</tr>
<tr>
<td>0.4 m</td>
<td>36.566</td>
<td>0.903</td>
<td>0.813</td>
</tr>
<tr>
<td>0.7 m</td>
<td>44.436</td>
<td>0.974</td>
<td>0.868</td>
</tr>
<tr>
<td>1.2 m</td>
<td>50.821</td>
<td>0.923</td>
<td>0.800</td>
</tr>
</tbody>
</table>

Figure 60. The average strain as a function of the wall thickness for the standard conditions.
Figure 61. The average strain as a function of the wall thickness for the summer conditions.

Figure 62. The average strain as a function of the wall thickness for the winter conditions.