Experimental studies of large particles in Newtonian and non-Newtonian fluids

by

Sagar Zade

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Department of Mechanics
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To all teachers
Experimental studies of large particles in Newtonian and non-Newtonian fluids

Sagar Zade
Linné FLOW Centre, KTH Mechanics, Royal Institute of Technology
SE-100 44 Stockholm, Sweden

Abstract

In everyday human life, laminar flow is arguably an exception whereas turbulent flow is the norm. Yet, the former has been much better understood, naturally since laminar flow renders itself to treatment in a relatively easier fashion compared to turbulence with its chaotic dynamics across multiple scales in space and time. A parallel analogy in terms of sophistication of dynamics can be drawn between single phase and multiphase flows; the latter being the norm yet poorly understood due to numerous complexities arising on account of the huge parameter space involved. It is also remarkable that numerical studies are more prevalent in this field and there is a dearth of experimental results, which are important for both validation purposes and as a beacon to navigate research in practically relevant directions. This work has emerged to address the above issues. The attention has been largely directed towards understanding the flow of spherical particles in a square duct at moderately high concentrations using Particle Image Velocimetry (PIV) with refractive-index-matched (RIM) hydrogel particles. Fluids with Newtonian, viscoelastic and elastoviscoplastic rheology have been investigated due to their presence in natural and industrially relevant flows. Experiments and Direct Numerical Simulations (DNS) with spherical particles in a round pipe with turbulent flow of a Newtonian fluid are also conducted to extend and generalise the observations made in the square duct.

With the ability to optically interrogate the bulk of the flow at high particle concentrations (20% in this work), many interesting measurements are made possible, focussing on the turbulent regime. For the Newtonian fluid, the pressure drop or, equivalently, the energy required to pump the fluid-particle mixture is a complex function of particle size and concentration in the duct. This phenomenon arises due to the particle concentration distribution, with a local maxima at the core and the walls, and its resulting effect on the dominant stresses in the system i.e. the Reynolds shear stress and particle-induced stress. Particles also migrate in a similar fashion in a turbulent flow of viscoelastic suspending fluid but, with a larger tendency to accumulate in the core compared to its Newtonian counterpart at the same Reynolds number leading to a faster rise in total stress with concentration. Finally, for the thick elastoviscoplastic fluid, the single-phase flow is laminar but it exhibits turbulence-like fluctuations when particles are added, which are distributed in exotic configurations depending on the interplay between the viscoelastic forces and the ensuing secondary flows as well as inertial forces. On the other hand, a quantitative comparison between
simulations and experiments for particles transported along the floor of the duct under turbulent conditions has helped in reinforcing confidence in both approaches.

We believe that these results will establish more confidence in the experimental usage of hydrogel particles for studying the flow of moderately dense suspensions. A natural extension would be the investigation of flow geometries more complex than a pipe or a square duct. Our results at higher Reynolds numbers is expected to motivate numerical simulations which are capable of investigating the detailed causes behind these observations, which are still unclear as of now. The information provided about the overall drag and the associated particle concentration and stress distribution will be helpful in painting a unified picture of turbulent suspension dynamics for a comprehensive range of flow rates and particle sizes. Future studies, either experimental or numerical, bearing similarities or deviations from our observations would also be constructive, for e.g. in assessing the sensitivity of the system to parameters that may be overlooked in the present study.

**Key words:** turbulence, finite-size, particle-laden flows, duct flow, pipe flow, viscoelastic fluid, viscoplastic fluid
Experimentella studier av stora partiklar i newtonska och icke-newtonska flider

Sagar Zade
Linné FLOW Centre, KTH Mekanik, Kungliga Tekniska Högskolan
SE-100 44 Stockholm, Sverige

Sammanfattning

Man kan med visst fog hävda att turbulens är normen och laminär strömning mer ovanlig i vårt dagliga liv. Förståelsen av laminär strömning är å andra sidan bättre, vilket är naturligt eftersom det ofta är enklare att studera laminärt flöde än turbulens med sin kaotiska dynamik som sträcker sig över ett stort intervall av tids- och längdskalar. En parallell analogi vad gäller komplexitet och sofistikerad dynamik kan göras mellan enfas- och flerfasströmning. Det senare fallet är vanligast, men det är inte lika väl förstått och beskrivet på grund av att komplexiteten ökar snabbt när nya parametrar såsom partikelstorlek och koncentration införs. Det bör också nämnas att numeriska simuleringar domineras i flerfasforskningen, varför det råder brist på detaljerade experimentella resultat, trots att dessa är viktiga både för validering av simuleringar och för att identifiera relevanta forskningsproblem.

Det här föreliggande avhandlingsarbetet har ambitionen att bidra med experimentella resultat på flerfasströmning. Fokus har huvudsakligen varit att förstå flöden av suspensioner (blandningar) med sfäriska partiklar i kanaler med kvadratiska tvärsnitt och måttligt hög koncentration av partiklar. Partiklarna som använts är brytningsindexmatchade (refraction-index-matching, RIM) och har studerats med PIV (particle image velocimetry). Fluider med newtonsk, viskoelastisk och elastoviskoplastisk reologi har använts. Sådana fluider förekommer i flöden både i naturen och i industriella tillämpningar. Experiment och direkta numeriska simuleringar (DNS) av turbulent strömning med sfäriska partiklar i ett rör med cirkulärt tvärsnitt har också genomförts för att utöka och generalisera observationerna i den kvadratiska kanalen.

Tack vare möjligheten att optiskt studera stora delar av strömningen vid relativt höga partikelkoncentrationer (maximalt 20 procent) har den turbulenta strömningen karaktärisats i detalj. Med newtonsk fluid är tryckfallet eller energin som krävs för att pumpa suspensionen, en komplicerad funktion av partikelstorlek och koncentration. Detta beror på att partikelfördelningen kan ha maxima antingen i mitten av röret/kanalen eller längs väggarna, vilket i sin tur påverkar skjutsfångningarna i systemet (turbulenta Reynolds-spänningar och partikelinducerade spänningar). Suspensioner med partiklar i viskoelastiska fluider uppvisar liknande tendens till migrering, men i detta fall samlas partiklarna i större utsträckning i kanalens mitt jämfört med det newtonska fallet (vid samma värde på det så kallade Reynoldstalet, en parameter som beskriver strömningen). Detta leder till en snabbare ökning av den totala spänningen,

Resultaten som presenteras i denna avhandling styrker tilltron till experimentellt utnyttjande av hydrogelpartiklar för att studera partikelsuspensioner vid måttligt höga koncentrationer. En naturlig fortsättning skulle kunna vara att studera mer komplexa geometrier än raka rör och kanaler med cirkulärt respektive kvadratiskt tvärsnitt. De experimentella resultaten vid högre Reynoldstal motiverar också numeriska simuleringar som kan undersöka den detaljerade fysiken bakom observationerna. Mätningarna av det totala strömningsmotståndet med tillhörande partikel- och spänningsfördelning bidrar till en komplett bild av turbulent suspensionsdynamik för ett stort intervall av strömningshastigheter och partikelstorlekar. Framtida studier, experimentella såväl som numeriska, kan inriktas mot att studera hur systemets känslighet är för viktiga parametrar som inte inkluderats i den här studien.

Nyckelord: turbulens, finit storlek, partikelsuspensioner, kanalflöde, rörföde, viskoelastisk fluid, viskoplastisk fluid

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Preface

This thesis deals with the experimental study of finite-size rigid particles in duct and pipe flows of Newtonian and non-Newtonian suspending fluids. A brief introduction about the relevance of the present work and the description of the experimental facility along with some important outcomes of this work is presented in the first part. The second part contains four articles related to experiments in a square duct. The papers are adjusted to comply with the present thesis format for consistency, but their contents have not been altered as compared with their original counterparts.


September 2019, Stockholm

Sagar Zade
Division of work between authors
The main advisor for the project is Luca Brandt (LB). Fredrik Lundell (FL) acts as co-advisor.

Paper 1. The experimental set-up was designed and build by Sagar Zade (SZ) with the help of FL. Experiments were performed by SZ. The paper was written by SZ with inputs from Pedro Costa (PC), Walter Fornari (WF), FL and LB.

Paper 2. Experiments were performed by SZ. Simulations were performed by WF. The paper was written by SZ with inputs from WF, FL and LB.

Paper 3. Experiments were performed by SZ. The paper was written by SZ with inputs from FL and LB.

Paper 4. Experiments were performed by SZ. The rheological measurements were performed by Tafadzwa John Shamu (TJS) and SZ. The paper was written by SZ with inputs from TJS, FL and LB.

Other publications
The following papers, although related, are not included in this thesis.


Conferences
Part of the work in this thesis has been presented at the following international conferences. The presenting author is underlined.


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Part I

Overview and summary
Chapter 1

Introduction

Multiphase flow is popularly associated with the word *ubiquitous*, meaning omnipresent, in the literature studying them (see Brennen & Brennen 2005, amongst others). It is rightly so since such flows are an integral part of the environment and technology surrounding us. Anything from blood flow to sediment transport, from transportation of crushed coal to combustion of pulverised fuel, from boiling to making paper comes under the purview of multiphase flows, which can be broadly defined as the combined flow of more than one phases of matter. Even if we restrict ourselves to the relatively smaller subset of dispersed two-phase flows, say a suspension of solid particles dispersed inside a continuous fluid media, one may have diverse scenarios arising due to variation in properties of the dispersed particle phase: particle size, density, concentration, shape, stiffness, chemical properties, etc.; and variation in the properties of the continuous fluid phase: fluid rheology can be Newtonian, viscoelastic, viscoplastic, etc.; as well as variation in the flow geometry: unbounded like deep inside an ocean or wall-bounded as in a round pipe. Varying at least one of the above parameters can result in drastically different flow characteristics. The flow physics starts to become further complicated, and interesting, with increasing fluid and particle inertia and the associated non-linearity in the governing equations. This poses significant challenges while treating these multidimensional problems analytically. It is also important to mention the crucial role of particle-particle or particle-wall interactions that are also difficult to model. Hence, to gain fundamental understanding, it is essential to investigate a relatively idealised system with fewer variables to disentangle the multifarious effects. We believe that experiments have a vital role to play in this direction and hence, the present work is undertaken. In the following paragraphs, a general introduction about particle-laden flows is presented. Knowledge of some introductory concepts in fluid mechanics and turbulence is presumed and relevant sources are cited wherever it is felt necessary.

An important parameter that dictates fluid-particle dynamics is the particlescale Reynolds number, $Re_p$, which is the ratio of fluid inertial forces to viscous forces at the particle scale. Based on the specific application, its definition may vary. For e.g., in sedimentation if we assume $U_p$ and $U_f$ to be the particle and fluid velocities respectively, then based on the slip velocity $|U_p - U_f|$, $Re_p$ is
defined as \( Re_{p,\text{slip}} = \rho_f |U_p - U_f| d_p / \mu_f \). Here, \( d_p \) is the particle diameter, and \( \rho_f \) and \( \mu_f \) are the fluid density and dynamic viscosity, respectively. On the other hand, in simple shear flow with shear rate \( \dot{\gamma} \), one can use \( Re_{p,\dot{\gamma}} = \rho_f \dot{\gamma} d_p^2 / \mu_f \). When \( Re_p \) is very small, fluid momentum transfer at the particle scale is governed by viscous diffusion and the flow regime is called Stokesian (see Brady & Bossis 1988), which is typical of particles in microfluidic devices. When \( Re_p \) increases to finite values, typical of environmental flows, it is inertia that predominantly convects momentum at the particle scale and this is reflected in the macroscopic flow properties of the suspension (see Guazzelli & Morris 2011).

The motion of extremely tiny particles (say sub-micron sized) may be affected by the random thermal fluctuations in the fluid molecules causing Brownian diffusion. The Péclet number \( Pe \) measures the relative strength of advection (due to shear) over thermal diffusion. These tiny particles may also be subject to colloidal forces like van der Waals attraction, electrostatic repulsion, etc. In this work, we focus on the transport of non-Brownian and non-colloidal particles i.e. large particles (with large \( Pe \)) whose motion is determined by hydrodynamic forces and inter-particle interactions like collisions. An example of such a system could be the flow of slurry that regularly involves particles that are around \( \sim 1 \) mm in size.

It is practically useful to categorise flow properties in the order of increasing presence of particles. This can be quantified by the value of particle volume fraction \( \phi_v \) and mass fraction \( \phi_m \) (Elghobashi 1994). In the first level characterised by very low \( \phi_v \) and \( \phi_m \), particle dynamics are predominantly governed by the motion of the suspending fluid and there is negligible feedback or momentum transfer to the fluid itself. Hence, the dynamics of the fluid phase remains unaffected/unmodulated by the presence of particles and therefore this regime is called the one-way coupled regime. In such dilute flows, the primary interest is to investigate the passive transport of particles by the fluid.

When \( \phi_m \) increases, whilst restricting \( \phi_v \) to a low value (by increasing the density ratio of particle to fluid: \( \rho_p / \rho_f \)), to such an extent that particles now carry enough momentum to influence the dynamics of the carrier fluid, we reach the second level that is called the two-way coupled regime. In this regime, both particle transport and corresponding flow modulation is of importance. Both the above regimes are collectively referred to as the dilute regime.

Finally, when \( \phi_v \) increases to such an extent that inter-particle interactions become dynamically significant in addition to the above forces, the system conforms to the third level, named as the four-way coupled regime or the dense regime (Stickel & Powell 2005). For extreme values of particle concentrations, we approach the granular regime where particles are mostly in contact with each other with negligible proportion of the interstitial fluid. Under such conditions, the long range hydrodynamic and short range lubrication interactions between particles are of lesser significance compared to forces transmitted through direct contacts between them (Guazzelli & Pouliquen 2018).
1.1. From point-size to finite-size particles

In order to numerically solve for the motion of discrete particles in the fluid, models can range from the computationally inexpensive but idealised point-particle approximation to the computationally expensive but realistic interface resolved simulations. All cases involve solving the governing equation of the carrier fluid phase (Cauchy momentum and continuity equations if a continuum approach is followed) to various degrees of fidelity (either partially or fully-resolved). The fluid flow equations are coupled to the equations of motion describing the particle dynamics along with suitable boundary conditions. The applicability of point-particle simulations rely on the assumption that particles are smaller than the smallest relevant length scales in the flow (Kolmogorov scale $\eta_K$ in turbulence) so that the fluid velocity is uniform at the particle scale. The feedback effect of these particles on the fluid would arise through modelled point-forces that are averaged over the particle. This feedback is applicable only in the two and four-way coupled regime since, in the one-way coupling the particles are merely advected by the flow. The well known Maxey-Riley equation (Maxey & Riley 1983) is often used to simulate the motion of such point-particles in the dilute regime, and has for quite some time been instrumental in furthering our understanding of various problems. Even though this thesis deals with the transport properties of suspensions with finite-size particles i.e. particles larger than the smallest length scales of the flow, it is of practical and scientific significance to appreciate the better-understood physics revealed by point-particle simulations before moving towards the finite-size particle domain.

At a given small value of $\phi_v$ in the one-way coupled regime, the motion of these point-particles is a function of their Stokes number $St = T_p/T_f$; a ratio of the particle relaxation time in the Stokes regime, $T_p = \rho_p d_p^2/(18\mu_f)$, to the time scale of the flow that is of interest, $T_f$ (e.g. large eddy-turn over time, Kolmogorov time scale or the time scale associated with the mean shear $1/\dot{\gamma}$). This implies that particles with a very low $St$ would instantaneously respond
to the changes in the flow velocity and faithfully follow the fluid streamlines like passive tracers or Lagrangian fluid particles. This property makes them ideal candidates for fluid velocity measurements using experimental techniques like Particle Image Velocimetry (PIV). On the other hand, those particles with a very high \( St \), called ballistic particles, would be non-responsive to the flow fluctuations and continue along their initial trajectory under the influence of their own momentum or some external force like gravity. The \( St \) in shear can be trivially shown to be equal to \( Re_p \gamma (2 \rho_p/9 \rho_f) \) and hence, for neutrally-buoyant particles \( (\rho_p = \rho_f) \) they are similar with a constant of order unity.

In a turbulent flow with spatiotemporal distribution of vorticity, heavier than fluid particles (e.g. small solid particles or liquid droplets in a gas) are centrifuged out of a vortex core on account of their finite inertia. For a limited range of \( St \) close to unity (when scaled by the Kolmogorov time-scale), these heavier particles cluster in regions of high strain rate and low vorticity (Wang & Maxey 1993b; Eaton & Fessler 1994; Bec et al. 2007), whereas the reverse is true for lighter than fluid particles (Wang & Maxey 1993a). Maxey (1987) showed that aerosol-like particles under the action of gravity settle faster in homogenous turbulent flows than in a quiescent media due to the above inertial bias. This preferential sweeping even has implications on demixing or segregation of different particle species in flow which contains vortical coherent structures (Squires & Eaton 1991). High localised concentration due to such clustering in turbulent flows is responsible for excessive collision and consequent enlargement of tiny water droplets in clouds which later precipitate as rain (Falkovich et al. 2002). In a different situation, pulverized coal in burners may congregate and burn in fuel-rich regions even when the average stoichiometry is relatively lean.

In in-homogeneous turbulent flows (like a channel flow), inertial point-particles can also be convected from regions of high to low turbulence intensities, a phenomenon known as turbophoresis (Reeks 1983). This happens due to the lower probability of particles in regions of low turbulence intensity to gain enough momentum to escape to regions of high turbulence intensity. In wall-bounded turbulent flows this phenomenon may cause particles to statistically drift towards the wall leading to accumulation there. Near-wall particle accumulation is most pronounced when the particle inertial time-scale nearly matches the circulation time of the near-wall turbulence structures (Soldati & Marchioli 2009). An equilibrium particle concentration is achieved near the wall due to a balance between the turbophoretic drift directed towards the wall and ejection events pushing the particles away from the wall. This interaction also cause particles to preferentially reside in the turbulent low-speed streaks (Kline et al. 1967) in the form of elongated clusters (Sardina et al. 2012).

In the two-way coupling regime, it is observed that point-particles can increase the fraction of turbulent kinetic energy at high wave-numbers and the turbulence is generally attenuated with increasing mass loading (Squires & Eaton 1990; Elghobashi & Truesdell 1993). Particles can directly modulate turbulence through routes like additional dissipation on the particle surface.
or increased fluctuations by vortex shedding (Hetsroni 1989). They can also indirectly alter the turbulence features by disrupting coherent structures. The increase or decrease in the overall turbulence intensity is shown by various studies to depend on some effective non-dimensional number which maybe a function of the $St$, flow Reynolds number $Re$, particle number density (which is the number of particles in a unit volume), ratio of particle size to integral scale of the flow, or the ratio of flow length-scales itself (Gore & Crowe 1989; Poelma et al. 2007; Tanaka & Eaton 2008).

In a canonical wall-bounded turbulent planar channel flow, inertial point-particles may exert a torque opposite to the rotation of the near-wall quasi-streamwise vortices thereby weakening them and damping the low-speed streaks (Dritselis & Vlachos 2008). The final outcome is the modification of the turbulence flow field similar to that achieved by polymeric additives (described later in section 1.7.1). This is characterised by a reduced Reynolds shear stress and reduction in drag compared to single phase flow (Zhao et al. 2010).

Either in the dense regime where particle-particle interactions become important or when the particle size is no longer smaller than the Kolmogorov length scale $\eta_K$, the assumptions made in the point-particle model are no longer valid and resolving forces on the entire fluid-particle interface becomes essential to describe the physics. This would amount to accurately implementing the no-slip and no-penetration condition on the moving boundary of a rigid particle transported in the flow. Despite the need for such interface resolved simulations since a long time, only recently significant progress has been made, especially by the use of Immersed Boundary Method (IBM) (see Prosperetti 2015). These advancements have been made possible due to rapid improvement in computer processing power that can facilitate massive parallel simulations like, for e.g., the study of flow modulation by many freely suspended finite-size non-spherical particles in a turbulent channel by Ardekani & Brandt (2019). Experimental capabilities, on the other hand, have not witnessed a similar surge. Most of the experimental measurements are limited to macroscopic observables like total flow rate and pressure drop without providing detailed information about the microscopic velocity and particle concentration distribution in the bulk of the suspension. Some of the main issues plaguing experiments are noted in Chapter 2. In the present work, an attempt is made to measure these elusive quantities and interpret the macroscopic changes in the light of these measurements.

In the case of turbulent flow of finite-size particle suspensions, the fluid velocity can no longer be assumed to be uniform at the particle scale. Thus, the slip velocity between the two phases, $|U_p - U_f|$, cannot be uniquely defined at the location of a large particle (Bellani & Variano 2012). This is an issue since models for drag force are usually constructed based on the slip velocity. Size effects are also exhibited in the markedly different clustering dynamics of smaller heavier particles in comparison to larger neutrally-buoyant particles, despite both of them having a similar value of conventional Stokes number (Xu &
Bodenschatz 2008). One of the first studies of finite-size effects in wall-bounded flows was performed by Pan & Banerjee (1997) for slightly heavier than fluid particles at very low volume concentrations, denoted as $\phi$ hereafter. Particles were found to enhance the intensity of sweep (inrush of high-speed fluid towards the wall) and ejection (low-speed fluid ejected away from the wall) events in the near-wall region which resulted in an increase of the turbulence shear stress. Even though the Lagrangian acceleration statistics of a neutrally-buoyant solid particle in turbulent flow (this being a direct measure of the forces on the particle) is similar to that of a fluid particle for sizes up to $5\eta_K$ (Voth et al. 2002), a departure from this behaviour is seen for particles larger than $15\eta_K$ (Qureshi et al. 2007) thus, highlighting the importance of finite-size effects.

From the noteworthy works of Ten Cate et al. (2004); Lucci et al. (2010); Yeo et al. (2010), the following general picture emerges with regards to turbulence modulation in homogenous flows: finite-size particles directly influence the turbulent flow structures similar in size or smaller than the particle size. These modifications coupled to the non-linearity of the system affects the overall turbulence cascade, generally causing an increase of the turbulent kinetic energy at scales smaller than the particle size and reduction at larger scales. Similar to the case of a particle in a laminar flow, the local fluid strain rate increases in the vicinity of a particle in relative motion with the fluid, leading to enhanced dissipation which can cause fluid turbulence to decay faster. Regarding particle motion, it can be said that particles approaching each other are slowed down by the repulsive lubrication forces. For sufficiently high approach velocities, the particles physically make a rigid body contact through surface asperities, thus marking a collision event. The same lubrication forces become attractive for separating particles and tend to keep particles together until they are acted upon by strong enough fluid forces or collision forces. Henceforth, this introduction will mostly concern the behaviour of finite-size particles unless specifically stated.

With increasing volume fraction of finite-size particles, the dynamics start to become increasingly affected by short-range hydrodynamic interactions and collisions. In simulations of wall-bounded flows of concentrated suspensions, a step towards disentangling the complex physics is usually taken by considering the exceptional case of neutrally-buoyant particles ($\rho_p = \rho_f$), while remembering that the results cannot be trivially extrapolated to other cases with different densities (Prosperetti 2015). Such density matched particles will be largely dealt with in this work. In nature, wandering zooplankton in ocean can be classified as particles having a finite-size and density similar to the suspending fluid (Byron 2015).

In a turbulent channel flow, Shao et al. (2012) studied the effect of two different sizes of neutrally-buoyant spherical particles up to $\phi \approx 7\%$. They found that particles weaken the intensity of the near-wall large scale streamwise vortices thus, reducing the streamwise velocity fluctuations in a substantial region of the channel. On the other hand, particles also generate small scale
vortices that increase the spanwise and wall-normal fluctuations compared to single phase flow. These opposing effects are directly reflected in the Reynolds shear stress level, which is seen to decrease (for larger particles) or increase (for smaller particles). Interestingly, smaller particles are seen to alter turbulence more severely than larger particles; an effect ascribed to the higher number of smaller particles for the same \( \phi \). A later study by Picano et al. (2015) showed that with increasing \( \phi \), the maximum value of mean streamwise velocity (at the centre-line of the channel) increases and the shape of the velocity profile increasingly resembles a laminar parabolic flow. Up to the maximum \( \phi = 20\% \) with a particle size of \( d_p^+ \sim 20 \), the log law of classical turbulence: \( U^+ = \log(y^+)/\kappa + B \) is found to be valid, but with different values of the von Kármán constant \( \kappa \) and the additive constant \( B \) compared to single phase flow of Newtonian fluid. In the above relation \( U^+ = U/u_\tau \) and \( y^+ = y/\delta \) are the mean streamwise velocity \( U \) and the wall-normal distance \( y \) normalized using the inner scaling for velocity \( u_\tau = \sqrt{\tau_w/\rho_f} \) and length \( \delta = \nu/u_\tau \), respectively.

The parameter \( \tau_w \) is the shear stress at the wall, i.e. the drag forces associated with transporting the suspension. This drag was found to increase monotonically with \( \phi \) compared to single phase flow in the simulations of Picano et al. (2015). As will be discussed later in section 1.5, this overall increase in drag is not associated with an increase in the Reynolds shear stress, which would be expected for single phase flow. In fact, the Reynolds shear stress varies non-monotonically with \( \phi \), showing an increase up to \( \phi = 10\% \) and a decrease thereafter. Instead the increase in drag is attributed to an increase in the stress due to the solid phase, called the particle-induced stress (see section 1.5). The greatest reduction in the Reynolds shear stress at \( \phi = 20\% \) is also followed by a corresponding reduction in the turbulence fluctuations.

In a related study of finite-size effects, Fornari et al. (2016) observed that in the absence of gravity, drag increases more significantly at a fixed mass fraction with increasing volume fraction compared to the case of a fixed volume fraction with increasing mass fraction. Thus, the predominant effect on drag is due to the excluded volume occupied by the particles. Apart from this, Picano et al. (2015) observed that the near-wall flow dynamics was strongly influenced by the high local concentration of particles near the wall, henceforth referred to as the particle-wall layer. In contrast to the fluid phase which is bound to obey the no-slip boundary condition at the wall, finite-size particles can move with a significant slip near the wall. Particles approaching (leaving) the wall layer squeeze (dilate) the fluid between them and the wall, giving rise to enhanced velocity fluctuations very close to the wall. The slip velocity reduces towards the core (central) region of the channel.

1.2. Suspension rheology

A lot of effort has been devoted in understanding the relationship between two macroscopic observables: applied rate of strain and the resulting stress, usually under simple shear flow (to yield the shear viscosity), under the purview of
the field of rheology. The presence of a particle(s) affects the deformation of the surrounding fluid and the apparent viscosity of the particle-fluid mixture now depends on the properties of the dispersed phase as well as the shear rate. The rheology of such a suspension may exhibit non-Newtonian properties like shear-thinning (decrease of apparent viscosity with increasing shear rate), shear-thickening (increase of apparent viscosity with increasing shear rate), structural memory effects like thixotropy (thinning with time at constant applied shear rate) or antithixotropy/rheopexy (thickening with time at constant applied shear rate), yield stress (no strain rate below a critical stress), normal-stress differences, etc. The specific behaviour is a combined effect of hydrodynamic and non-hydrodynamic (inter-particle collisions, surface roughness, electrostatic forces, Brownian motion, etc.) interactions as well as the ensuing particle microstructure i.e. relative particle position and orientation (Brady & Bossis 1985; Stickel & Powell 2005).

At the beginning of the previous century, Einstein (1905) derived an analytical expression to describe the increase in the apparent or effective viscosity $\mu_e$ of a dilute suspension (assuming negligible inter-particle interactions) of rigid mono-disperse neutrally-buoyant spheres dispersed in a viscous Newtonian liquid under the assumption of negligible particle inertia (Stokesian regime). This expression goes as $\mu_e = \mu_f (1 + 2.5 \phi)$. The origin of this increased viscosity lies in the resistance of the rigid particle to deformation under the straining motion of the fluid, quantified by the stresslet, thereby disturbing the neighbouring flow field that leads to enhanced viscous dissipation (Guazzelli & Pouliquen 2018). The analytical treatment was later extended to the case of slightly higher concentrations involving mutual particle interactions by Batchelor (1970); Batchelor & Green (1972). Only semi-empirical relationships like that from Eilers (1941): $\mu_e = \mu_f (1 + 1.25\phi/(1 - \phi/\phi_{Max}))^2$ or Krieger & Dougherty (1959) are available for higher concentrations in the non-inertial regime and these relationships diverge to infinite viscosity (system jams) at a certain high concentration $\phi_{Max}$, called the maximum packing fraction. The value of $\phi_{Max}$ depends on the particle size distribution (Ouchiyama & Tanaka 1981), particle shape, etc.

Under the assumptions of inertia-less flow and no particle friction, the effective viscosity is a function of the particle volume fraction only. The influence of particle friction on the viscosity becomes increasingly important with increasing concentration (Mari et al. 2014). When Brownian diffusion is
1.3. Particle migration

significant (at relatively small $Pe$), the viscosity of a suspension of rigid spheres exhibit a dependence on the shear rate in the form of shear-thinning due to increased orderliness in particle arrangement with shear (Hoffman 1972). When inertia at the particle scale (quantified by the particle Reynolds number, $Re_p$) can no longer be neglected, the suspension shows shear-thickening (Kulkarni & Morris 2008). This shear-thickening behaviour can be quantitatively estimated if one accounts for the change in the particle microstructure: there is an increase in the effective volume fraction in the form of an excluded region in the neighbourhood of a reference particle that is devoid of any particle flux (Picano et al. 2013).

The effect of inertia on suspension rheology was already exposed in the celebrated work of Bagnold (1954), who experimented with neutrally-buoyant particles in a Taylor-Couette set-up at varying shear rates. At small shear rates he observed a viscous Newtonian-like regime (shear stress varied linearly with the applied shear rate) with a viscosity corrected for the bulk particle concentration. Even at these small shear rates, non-Newtonian behaviour was observed in the form of normal or dispersive stress. On the other hand, at higher shear rates he observed a grain-inertia dominated regime, now called the Bagnoldian regime, where the shear stress varied quadratically with the applied shear rate i.e. shear-thickening. A distinction between the above regimes is made using the Bagnold number $Ba = 4Re_p, \dot{\gamma}^{1/2}$ which is the ratio of the inter-particle inertial to viscous stresses, mediated by the average linear particle concentration $\lambda$ that is equal to the ratio of particle diameter to the particle radial separation distance (Hunt et al. 2002).

![Particle migration in a turbulent pipe flow - one possibility](image)

1.3. Particle migration

In wall-bounded flows, particles migrate to specific regions of the flow based on their relative size, inertia, confining geometry, flow field etc. This migration, in-turn, changes the flow field and, ultimately, the drag incurred in transporting the suspension. Migration in particle suspensions sheared in conventional rheometers would generate a heterogenous particle distribution and hence, the macroscopic stress-strain measurements would not reflect the
local stress-strain relationship (Fall et al. 2010) thus, posing difficulties in the formulation of a constitutive law.

The relatively simple case of the motion of a single rigid sphere in Stokes flow is reversible and hence, no migration is observed. However for concentrated or multi-particle suspensions, particles irreversibly migrate from regions of high shear to low shear through a phenomenon named as shear-induced migration (Leighton & Acrivos 1987; Koh et al. 1994; Yeo & Maxey 2011). The above irreversibility under non-inertial, i.e. Stokes flow conditions, arises due to the many short range non-hydrodynamic interactions (e.g. particle friction) occurring in non-dilute systems that generates a normal stress which pushes particles away from each other (Brady & Morris 1997). To predict the particle distribution arising due to such a shear-induced migration, two commonly used models are: (i) the diffusive flux model (Leighton & Acrivos 1987; Phillips et al. 1992) which describes particle diffusion due to gradients in shear and viscosity; and (ii) the suspension balance model (Nott & Brady 1994; Morris & Boulay 1999) where particle migration is modelled through stress gradients inside the particle phase.

Cross-stream migration can also happen due to inertia at the particle scale. Under dilute conditions, the experiments of Segré & Silberberg (1962) showed that a random distribution of neutrally-buoyant particles at the inlet of a tube arranged themselves into a well-defined annular ring of particles with a radius of \(0.6R\), \(R\) being the radius of the tube. This peculiar focussing phenomenon has been termed as the tubular pinch effect and is due to an equilibrium between the shear-gradient lift force (from the curvature in the fluid velocity) that pushes the particle away from the centre and the repulsive lubrication force from the wall that pushes it away from the wall (Ho & Leal 1974).

The above equilibrium location is pushed closer to the wall with increasing flow Reynolds number \(Re\), and even another stable annulus may appear in the inner region of the pipe (Matas et al. 2004b). Particles may also travel in train-like structures in the outer annulus, closer to the wall. This ordered formation is due to flow recirculation zones forming between neighbouring particles (under shear) at finite \(Re_p \sim Re(d_p/D)^2\). The size of these recirculation zones reduces with increased \(Re_p\) and thus, the inter-particle distance also reduces (Matas et al. 2004). The confining geometry also has a decisive role in determining the final equilibrium position. In a rectangular duct, particles can be focussed at well-defined point locations; a feature that can be advantageously used for cell-sorting in micro-fluidic flows (Di Carlo 2009).

Migration is more complex in turbulent flows and the resulting mean particle distribution may depend on additional flow properties like ratio of particle size to the size of turbulent eddies as well as additional mechanisms like turbulent modulation by the particles. In many simulations (Picano et al. 2015; Costa et al. 2016) and our own experiments (Zade et al. 2018), it has been observed that finite-size particles in turbulence migrate towards the wall and form a statistically robust particle-wall layer. This produces a local maxima in mean
1.3. Particle migration

concentration field at the wall, which is then followed by a minima due to
the excluded volume of the first particle layer. The mechanism behind this
migration is different than turphoresis, described earlier in section 1.1, which
also forms a particle-wall layer but is appropriate for dense point-particles.

In the turbulent flow regime, vigorous inter-particle interactions can lead to
collision-driven shear-induced inertial migration. As illustrated in Fornari et al.
(2016), two nearby particles displaced slightly in the wall-normal direction and
moving with slightly different streamwise velocities (due to the shear in the mean
flow) can collide and would be deflected from their positions towards opposite
wall-normal directions. One of them migrates towards the wall whereas the other
goes towards the core. Once a particle reaches the wall, statistically it tends to
remain there due to the dynamically stable location of the wall compared to
other locations in the flow. This can be understood by realising that the wall
is bombarded by particles only from one of the two possible directions, thus
reducing the probability of a destabilising impact from a neighbouring particle.
Perhaps more importantly, the attractive lubrication forces tend to prevent the
departure of a particle away from the wall, once it has arrived there. Hence,
it becomes difficult for particles belonging to the particle-wall layer to escape
from the wall.

Particles migrating towards the core, on the other hand, can either redis-
tribute evenly into a nearly uniform concentration profile away from the wall
(Picano et al. 2015; Costa et al. 2016) or continue migrating towards the centre
to form a peak of high concentration (Ardekani et al. 2018; Lashgari et al. 2016;
Zade et al. 2018). The above two possible outcomes can be qualitatively under-
stood as follows: particles in a region away from the wall are more susceptible
to be redistributed to produce a more uniform concentration profile (i) due to
the dispersive forces in turbulence and (ii) due to the relatively flatter turbulent
mean streamwise velocity profile (i.e. smaller shear and hence smaller difference
in relative velocity between interacting particles). The tendency of particles to
homogeneously redistribute is higher for lower particle concentrations \( \phi \) and
higher bulk flow Reynolds number, \( Re_{Bulk} = \rho_f U_{Bulk} 2H/\mu_f \) (2H being the full
height of the channel, duct or pipe geometry). This effect is most likely because
turbulence structures have enough space and energy to effectively disperse the
particles under such conditions. This tendency also increases with reducing
particle size \( d_p/2H \) at a fixed \( \phi \) and \( Re_{Bulk} \).

Whenever there is a departure from the above conditions (i.e. smaller
particles at lower \( \phi \) at higher \( Re_{Bulk} \)) in turbulent flows, shear-induced inertial
migration would dominate and particles will preferentially migrate towards the
centre of the channel (or pipe) to form a local maxima. Simply put, this means
that at a given \( Re_{Bulk} \) larger particles can preferentially migrate towards the
core at lower \( \phi \) compared to smaller particles. This feature is also seen in the
recent simulations of Wang et al. (2016). Since this thesis largely deals with
experiments in a square duct flow, the specific migration characteristic occurring
in it is described in a separate Chapter 3.
1.4. Transition in suspensions

Single phase flow can exist in the laminar, transition or turbulent states in the order of increasing bulk Reynolds number $Re_{Bulk}$. The transition regime is characterised by intermittent turbulent puffs (at lower $Re_{Bulk}$) and slugs (at higher $Re_{Bulk}$) (Nishi et al. 2008) and thus, a critical $Re_{Bulk}$ can be defined corresponding to the inception of this intermittency. Transition in single phase flow is also sensitive to the magnitude of the initial disturbance and can be delayed to very high $Re_{Bulk}$ if the disturbance level is kept small (Hof et al. 2003).

The influence of neutrally-buoyant spherical particles on the above transition scenario in a pipe flow was experimentally explored by Matas et al. (2003). These authors observed that smaller particles ($d_p/2H \leq 1/65$) lead to a monotonic concentration-dependent and size-independent increase in the critical $Re_{Bulk}$, i.e. the transition is delayed. This was explained because of the increase in the effective viscosity by the particles (as explained earlier in section 1.2).

However, a non-monotonic behaviour is seen for larger particles ($d_p/2H \geq 1/65$): the critical $Re_{Bulk}$ first decreases with concentration before reaching a minima that is lower for larger particle size, and then increases again. This early transition for larger particles is attributed to the increasing velocity fluctuations introduced on the base flow by increasing particle size. This was confirmed from the simulations of Loisel et al. (2013) who showed that larger particles breakdown the growing flow structures, that occur during re-laminarization, into smaller and more numerous structures having stronger vorticity that can now sustain turbulence down to lower $Re_{Bulk}$. In addition, the finite-size particles by virtue of their inertia (i.e. higher $Re_p$) interact with the local shear rate and act as additional sources of perturbation, especially in the near-wall region where they mostly migrate.

Lashgari et al. (2015) found that the initial arrangement of particles also play a role in determining whether turbulence would be sustained by the particles, at least in the dilute regime. These findings were corroborated by Wang et al. (2018) who found that if particles are introduced in the turbulent streaks, they would cause the streaks to grow due to an enhanced lift-up effect, eventually causing them to breakdown and regenerate as in classical wall turbulence (Robinson 1991). On the other hand, randomly seeded particles can cause the flow to become laminar due to the additional dissipation.

In a recent study, Agrawal et al. (2019) reported that for large particles, transition to turbulence occurs via two distinct instabilities based on the particle concentration $\phi$. At low $\phi$, transition occurred in a way similar to that of traditional Newtonian fluids i.e. sharp increase in flow resistance due to intermittent puffs and higher sensitivity to initial disturbance, while the critical $Re_{Bulk}$ decreased with $\phi$. However, at high $\phi$, the fluctuations continuously increased without the emergence of puffs and a smooth transition was observed while the critical $Re_{Bulk}$ further decreased with $\phi$. Both kinds of instabilities
co-existed at intermediate $\phi$ with initiation of puffs being delayed to higher $Re_{\text{Bulk}}$ for higher $\phi$, a phenomenon also reported by Matas et al. (2003).

One may also point out that at higher $\phi$, the definition of a critical $Re_{\text{Bulk}}$ may have to be arbitrarily related to the threshold of velocity fluctuations since these fluctuations continuously grow starting from low speeds (Yu et al. 2013). After transition, in the fully turbulent regime, Agrawal et al. (2019) also observed a non-monotonic variation of drag as a function of $\phi$. In a very similar study, Hogendoorn & Poelma (2018) reached similar conclusions while noting that the turbulent puffs became progressively weaker and the particle induced fluctuations became stronger with increasing $\phi$, thereby modifying the transition scenario significantly compared to single phase flow.

1.5. Division of stresses

![Figure 1.4: For a given bulk Reynolds number $Re$ and particle concentration $\Phi$, which stress dominates? Adapted from Lashgari et al. (2014).](image)

As observed in the experimental studies reviewed above, the transition at higher $\phi$ is no longer abrupt and the dominant particle-induced fluctuations continuously grow with increasing $Re_{\text{Bulk}}$. A similar observation was made in a channel flow by means of simulations by Lashgari et al. (2014). By simulating multiple $\phi$ of finite-sized neutrally-buoyant particles at varying $Re_{\text{Bulk}}$, it was found that compared to single phase flow the drag increases with particle volume fraction $\phi$. More importantly, they identified three different regimes, based on the predominance of either viscous stresses (at low $\phi$ and low $Re_{\text{Bulk}}$), turbulent stresses (at low to moderate $\phi$ and high $Re_{\text{Bulk}}$) or particle-induced stresses (at high $\phi$) in the stress budget.

In a channel (pipe) flow, the total shear stress varies linearly as a function of the wall-normal (radial) position, from zero value at the centre to a maximum value at the wall, known as the wall-shear stress (Pope 2000). In a single phase turbulent flow at any wall-normal position, it can be shown that the total shear stress is given by the sum of the local viscous stress, which is significant only in
the near-wall region, and the local Reynolds shear stress, which is the dominant stress away from the wall. In the presence of particles, an additional stress, referred to as particle-induced stress, also appears.

The contribution of each of the three stresses to the total shear stress can be calculated by integrating the phase-averaged streamwise momentum equation (Zhang & Prosperetti 2010) along the non-homogenous direction (wall-normal direction) of the channel flow (see appendix of Picano et al. 2015, for the derivation). The aforementioned turbulent stresses accounts for the streamwise momentum transfer in the wall-normal direction due to the coherent motion of both the fluid and solid phases. Whereas the particle-induced stress accounts for momentum transfer solely due to the solid phase by virtue of hydrodynamic stresslet (mentioned previously in 1.2), particle acceleration, and inter-particle interactions like collisions, etc. (Batchelor 1970). Lashgari et al. (2014) observed that at substantially high $\phi$, the flow resistance increased with increasing $Re_{Bulk}$ without a proportionate increase of turbulent stress. This additional resistance was thus attributed to particle-induced stress (calculated as the total shear stress minus the viscous and turbulent stress).

Interestingly, despite observing similar bulk flow resistance for systems sharing the similar Bagnold number $Ba$, Lashgari et al. (2016) showed that different physical mechanisms in terms of momentum and phase (solid and fluid) transfer were at play at the microscopic scale: uniform particle distribution was seen in flows dominated by the turbulent stress and particles accumulated in the centre for flows dominated by the particle-induced stress (as already indicated in section 1.3). In all cases, a particle-wall layer was also observed. The high particle concentration in the core damp the turbulent velocity fluctuations in that region to a level similar to laminar flows while the fluctuations are similar to turbulent flows away from the core where the particle concentration is not so high. Similarly, as can be expected, the spatial distribution of these particle-induced stresses is related to the local particle concentration in the flow.

Picano et al. (2015) observed that under fully turbulent conditions, significant particle-induced stress first appear at low $\phi$ inside the particle-wall layer. With increasing $\phi$, the local concentration inside the particle-wall layer increases and so does the value of the particle-induced stress, which is always highest in this layer compared to other regions in the flow. Due to these non-homogeneities in concentration distribution of finite-size particles, the observed increase in the overall drag cannot be simply explained by using the notion of an increased effective suspension viscosity.

Considering the peculiar nature of the particle-wall layer in terms of high local concentration and significant slip velocity compared to the bulk of the suspension, Costa et al. (2016) proposed the decoupling of flow dynamics in these two regions in order to explain the observed increase in drag and the mean streamwise velocity distribution in the channel. This higher relative motion in the near-wall region is reflected in the instantaneous distribution of wall shear
stresses, characterised by stress events both higher and lower than the mean value (Costa et al. 2018). Costa et al. (2016) assumed that the suspension in the bulk, owing to its nearly uniform particle concentration distribution and negligible mean inter-phase slip, can be modelled as an effective single phase turbulent flow with an effective suspension viscosity. Classical scaling laws of wall-bounded turbulence can then be appropriately used in this smaller region away from the wall to describe the observed variation in the mean streamwise velocity. Such a semi-empirical treatment has not yet been extended to other classical geometries, e.g. a pipe flow where particles migrate towards the core at similar $\phi$ and $Re_{Bulk}$ (Ardekani et al. 2018). Our experimental measurements in a pipe flow (discussed in Chapter 4) indeed show that migration towards the core, especially at larger particle sizes would cause an overall drag that cannot be explained by the particle-wall layer theory of Costa et al. (2016).

1.6. Sedimentation

![Figure 1.5: Particle sedimentation in a container. The drafting-kissing-tumbling sequence is illustrated.](image)

Particle sedimentation is another important class of problems where interesting effects manifest due to complex fluid-particle and particle-particle interactions. Calculating the mean sedimentation velocity is one of the more important objectives in these problems. At low particle scale Reynolds number $Re_p$, the terminal velocity of a single isolated particle falling in a quiescent medium can be conveniently estimated by balancing the buoyancy force with the viscous Stokes drag, which is linearly proportional to the particle velocity. At finite $Re_p$, the drag force now also consists of contribution from fluid inertia which can be included by means of some non-linear empirical correlation (Schwarzkopf et al. 2011).

The Galileo number $Ga$, which is the ratio of buoyancy to viscous forces, decides the nature of wake behind a falling particle. In batch sedimentation systems, where particle(s) descend inside a container with a fixed bottom,
increasing the volume fraction $\phi$ of falling particles causes the fluid phase to rise upwards (in order to maintain a null mean velocity of the mixture) and this upward motion in turn hinders the settling velocity causing it to drop below that of a single particle when inertial effects are negligible (Richardson & Zaki 1954). However, in inertial (higher $Ga$) multi-particle systems, the settling velocity may increase due to particles forming clusters (Uhlmann & Doychev 2014). These clusters may result from wake-induced events like drafting-kissing-tumbling (Yin & Koch 2007) happening within a particle-pair. If the suspending phase exhibits turbulence then the significant interaction of the particles with turbulent eddies leads to greater reduction in the particle settling velocity (Fornari et al. 2016b).

1.7. Non-Newtonian suspending fluid

The rheological behaviour of water, the most abundant fluid spread on earth, is Newtonian. This property is shared by many more fluids including air, glycerol, etc. However, there are many commonly used fluids like polymer solutions, paints, yogurt, ketchup, toothpaste, concrete, etc. as well as naturally occurring fluids like blood, lava, mud, etc. that display a non-Newtonian behaviour i.e. the stress is not linearly proportional to the rate of strain and/or it is a function of the history of deformation. These fluids may also exhibit elasticity, a property that is usually attributed to solids. The dispersion of particles in such suspending media, and the flow of the suspension is relatively less known compared to their behaviour in Newtonian fluids.

1.7.1. Viscoelasticity

Dissolving very small amounts, a few parts per million (ppm), of long-chain polymer into a solvent like water leads to remarkable changes in the flow
properties of the resulting solution. Generally, the shear viscosity increases above the value for the solvent but reduces with increasing shear i.e. shear-thinning, whereas the extensional viscosity increases with shear i.e. tension-thickening. Normal stress differences even appear in simple shear flow pointing towards a more complex constitutive law than for Newtonian fluids. These normal stresses can manifest in peculiar phenomenon like rod-climbing, die-swelling, etc. The above non-Newtonian effects are primarily due to the deformation of the polymer molecules, or their aggregates, under flow and the anisotropy in the resultant restoring forces (Barnes et al. 1989).

Under turbulent flow conditions, there is a significant decrease in the friction at the wall, which is famously referred to as the Toms effect (Toms 1948). This drag reduction capability has been successfully used in crude-oil pipelines for increasing the flow rate at fixed pumping costs, the most famous example being the Trans-Alaska Pipeline in 1979 (Burger et al. 1980), in preventing flooding by increasing the discharge of sewage during excessive rainfall (Sellin & Ollis 1980), district heating and cooling (Leca & Leca 1984), etc. Polymer additives are particularly attractive for industrial applications since only minute quantities can have substantial drag-reducing effect.

There have been quite a few explanations for the mechanism behind drag reduction. The high molecular weight polymers swell inside a solvent and form coiled microstructures that have elastic properties, and thus the resulting solution is viscoelastic in its rheology.

When the relaxation time $\lambda$ (the time taken for a microstructure to return from a stretched to an equilibrium state or vice versa) is comparable or larger than the characteristic deformation time of the flow, say $1/\dot{\epsilon}$ where $\dot{\epsilon}$ is the extensional strain rate, these coiled microstructures stretch. This stretching is accompanied by a substantially increase in the extensional viscosity of the solution. The increased extensional viscosity, which mostly occurs in the near-wall region where $\dot{\epsilon}$ is the highest, suppresses turbulent fluctuations. The effectiveness of polymer solutions, thus, depends on the stretching of the molecular aggregates by the stresses in the flow (Gyr & Bewersdorff 2013). It is also observed that turbulence is attenuated at smaller scales due to the increased elastic energy stored in the stretched coils at these small scales, thus interfering with the usual turbulence cascade mechanism (Sreenivasan & White 2000). The non-dimensional Weissenberg number $Wi$ compares the magnitude of elastic forces (expressed as normal stress difference) to viscous forces (shear stress) in the fluid. Another non-dimensional number called the Deborah number $De$, is also used to quantify viscoelastic effects and it is equal to the relaxation time of the polymers to the characteristic time scale of the process (Poole 2012b).

In wall-bounded flows, it has consistently been observed that with increasing drag reduction, there is an increase in the spanwise spacing between the low-speed velocity streaks, and there is a reduction in the number and strength of near-wall vortical structures, while their size increases (White et al. 2004). The wall-normal and spanwise velocity fluctuations are always damped in
comparison to Newtonian fluids. The peak value of the streamwise velocity fluctuations is displaced away from the walls and its magnitude may initially increase but eventually decreases with increasing level of drag reduction. In the low drag reduction regime (< 35–40%; Warholic et al. 1999), the mean turbulent streamwise velocity profile (plotted in wall units) has a slope similar to that of a Newtonian fluid but the thickness of the near-wall viscous buffer region increases with the level of drag reduction.

In the higher drag reduction regime, the slope of the log law increases in proportion to the level of drag reduction. The drag reduction is ultimately bounded by the maximum drag reduction asymptote (Virk 1975) at which, the mean velocity profile appears like a thickened buffer layer (White et al. 2012). It is observed that the Reynolds shear stress monotonically reduces with increasing drag reduction. At the maximum drag reduction point, the Reynolds shear stress reduces to nearly zero but, turbulence is sustained because of the interaction between fluctuating polymer stresses and the fluctuating velocity gradient (Warholic et al. 1999).

Amongst the many proposed mechanisms for regeneration of polymer wall turbulence, Dubief et al. (2004) stated that polymer chains extract energy from the near-wall vortices ($y^+ \geq 20$) as they are pulled around the vortices, and release energy in the high speed streaks that are located just above the viscous sublayer ($y^+ \approx 5$) thus, causing an autonomous regeneration cycle.

1.7.2. Viscoplasticity

If the long chain molecules that are responsible for viscoelastic behaviour, as described above, are cross-linked (either physically or chemically) to form a network, it results in a gel or ‘soft solid’ like substance (Barnes et al. 1989). Viscoplasticity is a characteristic of such polymer gels and is associated with the existence of a critical shear stress (called the yield stress $\tau_y$) below which the material does not flow i.e. the shear rate is zero below the yield stress. Before yielding, the gel has solid like properties: it can sustain shear stress and deform elastically. The yield stress $\tau_y$ has its origin in the microstructure of the material and, thus, can dynamically adjust under the action of stress during flow. These yield stress fluids may also possess complex macroscopic properties because of their different elastic (both before and after yielding) and viscous characteristics (Piau 2007).

A commonly used constitutive law to describe a large family of yield stress fluids is the Herschel-Bulkley model given in shear as: $\dot{\gamma} = 0$ when $\tau \leq \tau_y$; and $\tau = \tau_y + \kappa \dot{\gamma}^n$ when $\tau \geq \tau_y$. Here, the power law exponent $n$ is the flow behaviour index that quantifies the shear-thinning ($n < 1$) or shear-thickening ($n > 1$) behaviour, and $\kappa$ is the consistency parameter. For the special case of $n = 1$, the Herschel-Bulkley model transforms to the simpler Bingham model applicable for fluids that exhibit a Newtonian behaviour (constant viscosity) after yielding. The Bingham number $Bi$ (also referred to as the Oldroyd or Herschel-Bulkley number) quantifies the relative magnitude of plastic (yield stress) to viscous
(shear stress) effects. This model is a mathematical representation of a simple yield stress fluid, for which the shear stress is only a function of the shear rate (e.g. foams, some Carbopol microgels, etc.; Bonn et al. 2017).

However, for a few materials (e.g. colloidal dispersion of hard spheres, laponite clay suspensions), the stress is also a function of the flow history of the sample and the material is called thixotropic. One of the several manifestations of this thixotropic behaviour is in the form of hysteresis in the flow-curve (shear stress plotted as a function of the shear rate). This is due to the finite time associated with the build-up of the broken microstructure under flow (Moller et al. 2009). Thus, this time-dependency is not elastic in nature as observed for previously mentioned viscoelastic fluids but structural in origin. In contrast to simple yield stress fluids that flow continuously and steadily under a homogeneous applied shear rate (e.g. in a narrow gap planar Couette flow), the thixotropic fluids may have a critical shear rate below which they do not flow steadily. This unsteadiness is characterised by the presence of time-dependent locally yielded regions, known as shear-bands, related to the dynamics of ageing and rejuvenation of the microstructure under shear (Ovarlez et al. 2009).

Practically used yield stress fluids may also show an apparent slippage at the wall. This is caused by the presence of a thin highly sheared lubricating layer of lower viscosity (e.g. for a colloidal gel, this lubricating layer is composed of the pure solvent) near the wall. The slip behaviour is prominent for smooth walled geometries at shear stresses that are close to the yield stress and hence, a rough wall is preferred for rheological characterisation of these fluids (Vinogradov et al. 1975).

Laminar flow of simple yield stress fluids in pressure driven flows have been thoroughly investigated and analytical solutions of the flow field are available for a number of simple geometries like a planar channel, round pipe, etc. (Bird et al. 1983). The variation of stress in these flows causes the fluid to yield in regions of higher stress (e.g. near the pipe walls) and move like a solid plug in regions of lower stress (e.g. in the central core region). The yield surface between solid and fluid regions is generally defined by a critical value of the stress invariant (e.g. the von Mises Hencky yield criteria).

There have also been some studies concerning laminar to turbulent transition of yield stress fluids, mostly in the classic pipe geometry, and a few criteria able to predict transition have been proposed (Hanks 1963; Güzel et al. 2009b). In general, it is observed that the presence of a yield stress acts to stabilise the flow, thereby delaying transition (Peixinho et al. 2005). In turbulence, where the Reynolds stresses is sufficient to break the plug, the behaviour of a viscoplastic fluid resembles a simple shear-thinning fluid (Güzel et al. 2009) which is characterized by a reduced wall-normal turbulence intensity. Recently, Rosti et al. (2018) simulated the turbulent channel flow of a viscoplastic fluid with small elasticity and observed that in the turbulent regime, the friction factor decreases with increasing $Bi$, until the flow becomes fully laminar at a certain high $Bi$. 
1.8. Aim of the thesis

From the above introduction, one may infer that even the restricted case of pressure driven flow of suspension of mono-disperse spherical particles inside a relatively simple geometry (e.g. a pipe or a duct) offers fertile grounds to explore physics that will help in understanding the rich field of particle-laden flows a little bit better. The role of experiments is of course vital in this pursuit. Perhaps the most significant outcomes would be to develop the ability to predict the power required to transport a suspension of given particle size and concentration, and to physically relate it to particles migration and its affect on the velocity field. Accordingly, the main motivation of this study has been detailed in the form of the following objectives:

- to build an experimental facility to investigate moderately dense concentration of finite-size spherical particle laden flows; the main flow geometry is a square duct;
- to cross-validate fully resolved DNS simulations and experimental results to establish confidence in both methods;
- to study the influence of variation in size and volume fraction of spherical particles in turbulent flow of a Newtonian fluid; the goal is to quantify the change in drag, particle migration as well as the modulation of the turbulent flow field;
- to extend the above investigation to the flow of complex suspending fluids, which includes (i) turbulent flow of polymeric viscoelastic fluid and (ii) laminar flow of an elastoviscoplastic fluid;
- to predict the friction loss during turbulent transport of spherical particles in another geometry: a round pipe.

Thesis structure. The work is presented in the form of a few chapters related to distinct topics. The present chapter provided an extended introduction to the field of particle-laden flows with an emphasis on suspensions containing spherical particles. The following Chapter 2 will give details about the experimental set-up and the measurement techniques. Chapter 3 will look deeper into the dynamics of flow inside a square duct; the flow geometry that is primarily used in this study. Later, in Chapter 4, new results pertaining to drag in a round pipe carrying particles is presented. This is then followed by Chapter 5 that briefly summarises the main findings of this research. Finally, an outlook is provided for the future of these investigations.
2.1. Some velocity measurement techniques

The most commonly used techniques for velocity measurement in fluid mechanics are optics-based (Tropea & Yarin 2007). Laser Doppler Velocimetry (LDV) or Laser Doppler Anemometry was once a widespread optical technique to study dilute dispersed multiphase flows and has since been largely superseded by Particle Image Velocimetry (PIV) (Balachandar & Eaton 2010). LDV utilises the Doppler shift in the frequency of an incident beam of coherent light, i.e. laser, caused due to scattering by a moving object, to determine the velocity of the moving object. Thus, the velocity of the fluid phase is indirectly measured by tracking the motion of small tracer particles, which owing to their small size and Stokes number $St << 1$, instantaneously and faithfully follow fluid motion at all relevant scales. For particle-laden flows, it is possible to differentiate using LDV between the stronger signal scattered by the larger particles and the weaker (but more frequent) signal scattered by the smaller tracer particles thus, measuring both the velocity of the dispersed phase as well as the modification to the carrier flow (see Kulick et al. 1994). Thus, LDV is non-invasive i.e. the flow is not disturbed by the measurement technique, and it has a high temporal and reasonably good spatial resolution. However, at a given time instance, information is only obtained at a single point in the flow domain. Moreover, with increasing concentration of larger particles, their corresponding scattering would interfere with the signal from the fluid tracers, and the ensuing crosstalk would lead to inaccuracy in the measurement of velocity of both the interacting phases.

The problem of signal distortion and attenuation due to increasing optical turbidity can be largely reduced by matching the refractive indices of the two phases at the wavelength of the measurement light beam, as it was done by Koh et al. (1994) up to a volume fraction of $\phi = 30\%$. However, only the particle velocity and concentration could be measured and it was not possible to measure the fluid velocity at such high concentrations. It is also noteworthy to mention that the volume of their set-up was only 150 milliliters, and the flow was in the Stokes regime driven by small syringe pumps.

We use the same recipe of matching the refractive indices of the fluid and particle phase but, on a much larger scale in order to study turbulent flows. Also,
we use a different technique - PIV combined with Particle Tracking Velocimetry (PTV).

Particle image velocimetry is another non-invasive optical technique and can yield instantaneous fluid velocity information in a larger spatial domain. In this method, instances of the flow field are recorded in time in the form of discrete images. These images consist of light scattered by the small fluid tracer particles, as described before, when the flow is illuminated by a high intensity laser light. Instead of following the motion of individual tracer particles, a group of tracers are tracked inside smaller regions of the image (or flow), known as interrogation areas or windows (IWs).

In two-dimensional PIV, the laser light is limited in thickness to form a thin sheet that illuminates a planar section of the flow domain. From a single point of view, only two components of the in-plane fluid motion can be estimated. Using two points of view, called Stereoscopic PIV, it is possible to measure even the out-of-plane motion, thus yielding all three components of velocity in that plane. Through recent advances in tomographic reconstruction (Discetti & Coletti 2018; Scarano 2012), it is now even possible to calculate the instantaneous velocity distribution inside a three-dimensional volume with modest computational resources.

In order to estimate the average displacement of tracer particles inside an IW, statistical methods like cross-correlation between the intensity of neighbouring IWs (in time) is used; the location of the cross-correlation peak corresponds to the displacement. Typically the size of the IW is chosen to be smaller than the smallest scales of the flow, so that the velocity gradient is nearly zero and all the tracers inside the IW move uniformly. This determines the spatial resolution of the measurement.

The use of advanced algorithms like (i) multi-pass interrogation where the neighbouring IWs are again cross-correlated after offsetting them according to the displacement estimate from a previous cross-correlation; (ii) grid refining where the size of the IW is successively reduced after every cross-correlation step; and (iii) image deformation techniques where the cross-correlation is performed on IWs taking into account the local velocity gradient, have all resulted in significant improvements in the signal-to-noise ratio as well as the dynamic range (ratio of largest to smallest observable length scales). Nowadays, these methods and algorithms are fairly well-developed and are standard in many commercial digital PIV processing softwares. It is advised to refer to Raffel et al. (2018) for in-depth details about the PIV measurement technique. In this work, we have used an in-house PIV algorithm.

Larger particles, when introduced as the dispersed phase scatter more light than the smaller fluid PIV tracers. At very low particle concentration, the two phases can be differentiated by making use of a median filter (see Kiger & Pan 2000) that effectively smears out the smaller particles in to the background, or by using a combination of size and brightness (see Khalitov & Longmire 2002). One of the most challenging problems while using PIV under such dilute
conditions is to accurately measure the fluid velocity very close to the particle surface. This issue calls for a very high resolution (sub-Kolmogorov) imaging in a small region of flow (Tanaka & Eaton 2010). For increasing concentrations of particle suspensions, measuring velocity using PIV relies on the use of refractive-index-matched (RIM) particles, as mentioned previously. However, particles usually available are composed of materials like plastic, metal, glass etc., which, on account of being either completely opaque or semi-transparent, block or distort any light passing through them. The scattering of light by such non-transparent dispersed particles and the accompanying shadow limits their usage to volume fraction that can be even less than 0.5% (Poelma 2017), with the exact value being a function of particle and domain size.

There are indeed some limited options available for matching the refractive indices of the two phases to enable the use of PIV (see Wiederseiner et al. 2011; Dijksman et al. 2012). However, since most of the fluids used in such cases are a mixture of organic fluids (especially when both refractive index and density needs to be matched), they are often difficult to scale-up due to issues related to long-time optical and rheological properties of the suspending solution. The fluid may also react with the dispersed phase, it may exhibit strong sensitivity to temperature, there could be safety concerns during handling and transport through pumps, and very often their high cost prohibits their use (and re-use) in set-ups having large volumes. Thus, there is a need for a particle and fluid combination which is not severely limited by the above constraints. Hydrogel particles in water is one such combination and has been used here.

For completeness in terms of the prominent velocity measurement techniques used in multiphase fluid flows, it is noteworthy to mention velocimetry using ultrasound and Nuclear Magnetic Resonance (NMR). Both these techniques have been adapted from clinical imaging processes, and have a unique practical advantage of measuring through an opaque suspension. This extends their applicability to many more particle-fluid mixtures than for optics-based techniques.

Similar to LDV, ultrasound Doppler velocimetry relies on the frequency shift of the reflected ultrasound waves to measure the velocity of the dispersed phase. In another variant, ultrasound intensity images that are incrementally separated in time can be correlated using PIV algorithms to find the velocity field - a technique known as ultrasound image velocimetry (Poelma 2017). A major drawback of this technique is that it is very difficult to distinguish between the signal originating from the larger particles and fluid tracers.

The physics of NMR based imaging, also known as Magnetic Resonance Imaging (MRI), is rather complicated and the hardware is relatively more expensive. The signal is based on the resonance of MR-sensitive nuclei (say, Hydrogen nuclei in water) under the influence of a strong magnetic field (Elkins & Alley 2007). It has been used to study the concentration and velocity distribution inside concentrated flows at low Reynolds numbers (Hampton et al. 1997; Han et al. 1999). However, differentiating the velocity of the dispersed
and continuous phase as well as measuring velocity fluctuations in turbulent flows is still a challenge.

Other velocimetry techniques that have proved to be useful in certain applications include imaging based on X-Rays (Fouras et al. 2007) and tomographic reconstruction based on electrical impedance (Jaworski & Dyakowski 2001) and optical coherence (Buchsbaum et al. 2015). The advantage (and disadvantage) of a particular technique over others strongly depends on the application at hand.

2.2. The hydrogel particles

The motivation to use hydrogel particles for this work was primarily from the original works like Klein et al. (2012); Byron (2015), amongst others. Hydrogel is a super-absorbent polymer that can absorb water over 100 times its dry mass and, thus, it has a density and refractive-index that is very similar to that of water. This makes them almost invisible to the naked eye once fully swollen particles are submerged inside water. This extreme expansion property of hydrogels stems from the presence of hydrophilic functional groups attached to a polymeric backbone (Ahmed 2015).

Sodium polyacrylate is a popular hydrogel and is also used in this study. This chemical is synthesised by the polymerisation of Sodium acrylate, a salt of acrylic acid CH\(_2\)-CH=COOH. The polymerisation reaction occurs in the presence of an initiator (e.g. potassium persulfate) and a cross-linker (e.g. N,N'-Methylenebisacrylamide). This leads to the formation of long chains of acrylate molecules interconnected by bridges of the cross-linker. In the dry form the polymer is tightly wound and hence, occupies a smaller volume. Once in contact with water, the positive Sodium ions break away exposing the negatively charged Carboxyl groups COO\(^-\) which mutually repel each other, thus, causing the chains to uncoil and swell. The negatively charged Carboxyl group is attracted and attached to the Hydrogen end of the polar water molecule via Hydrogen bonding thus, further expanding the polymer. In short, Sodium ions are replaced by water molecules in a swollen network that is held in shape by the cross-linking molecules. The cross-linker is what allows the structure to retain water under mechanical pressure and adds to the rigidity of the swollen frame.

Commercial utilisation of hydrogel is widespread as a material for liquid absorption in diapers, for water-retention in agricultural soil, as a fire-retardant, for drug delivery, in making contact lenses, etc. By making use of a production technique called suspension polymerisation, also known as inverse-suspension polymerisation, dry spherical beads in controlled sizes can be manufactured in bulk quantities (Ahmed 2015; Xie et al. 2012).

Klein et al. (2012) measured the three-dimensional time-resolved trajectory of a spherical hydrogel particle, including rotation, in a von Kármán turbulent flow field generated between two counter rotating propellers, a set-up nicknamed the ‘French washing machine’. In order to track the complete motion of this
near-invisible particle, it was carefully marked using smaller PIV tracer particles that were impregnated into the body of the hydrogel sphere using a clinical lancet needle. Tracers representing the motion of the surrounding fluid as well as the tracers embedded inside the solid particle were simultaneously tracked using a Lagrangian particle tracking algorithm (Ouellette et al. 2006). The two types of tracer motion were differentiated from each other by realising that the relative separation between the tracers inside the solid body remained fixed while those in the fluid decorrelated quickly.

Fabricating individual particles in such a fashion is useful for studying the dynamics of a few particles in a smaller geometry. However, the volume of our facility is relatively large (around 60 liters) to accommodate essential components like a long duct, tank, pump, etc., which demands particles numbering around 100000, even when repetitions are excluded.

A remedy was provided by Byron (2015) who, instead, fabricated multiple particles of different shapes, including spheres, using injection moulding of a polymer solution that gradually solidifies due to polymerisation to form rigid hydrogel particles. A small amount of PIV tracers was dispersed inside the polymer solution before injecting it into the moulds. These tracer particles were, thus, locked in position once the polymerisation is complete. By tracking the motion of these tracers using standard PIV algorithms, Byron (2015) measured the translational and rotational dynamics of the rigid particles. Stereoscopic PIV was performed in a large tank where nearly homogeneous isotropic turbulence was induced by randomly actuated jets. However, their experiments were restricted to a bulk volume fraction $\phi$ of only 0.1%, partly due to their research objective that precluded particle-particle collisions and, perhaps, also due to the difficulties in manufacturing a very large number of particles with the injection moulding method. Their work primarily used a biodegradable naturally occurring hydrogel - agarose, which is softer and more fragile than synthetic hydrogels but, are closer in density to the suspending fluid - water.

The above studies of Klein et al. (2012) and Byron (2015) guarantee the measurement of complete motion of the particle and fluid phases at lower particle concentrations. Since our work started with the aim to achieve higher bulk
concentrations in turbulent flow, it was only practical to procure these particles commercially. Moreover, damage sustained by particles during the course of experiments would prevent their reusability thus, further strengthening the appeal for outsourcing their production. Sodium polyacrylate based spherical beads were purchased from the Great Lion (Xiamen) Co., Ltd. for around 200 USD per kilogram. They are delivered in the form of an unexpanded and polydispersed batch with diameter ranging from 0.5 – 1.1 mm. They could expand to an equilibrium size between 3 – 6 mm in tap water. Particles are graded into different sizes using a range of sieves, prior to expansion in water (as they become softer and would break while sieving after expansion). The particle size after expansion was determined primarily using a digital imaging system and also from the PIV images of particles in flow.

To precisely determine the density of our particles, we used the following two approaches: (i) measure the volume displaced by a known mass of particles and (ii) measure the terminal settling velocity of a single particle in a long settling column filled with liquid. In the first method using Archimedes’ principle, a known mass of fully expanded particles was put in a water-filled container of uniform diameter. The rise in the level of water due to the particles was measured using a very precise laser distance meter (optoNTDC 1710, Micro-Epsilon Messtechnik GmbH, resolution: 0.5 µm). Even though the weighing scale was precise enough, the higher uncertainty due to the very high sensitivity of the laser distance meter made this procedure less repeatable. In the second method, a single particle with a known diameter was gently dropped in a long and wide cylindrical vertical column filled with water, and the settling velocity is determined (after it has reached steady state). By equating the buoyancy force, \( F_{\text{Buoyancy}} = 4\pi(d_p/2)^3(\rho_p - \rho_f)g/3 \), to the empirical drag force \( F_{\text{Drag}} \) exerted by the fluid on a settling particle: \( F_{\text{Drag}}/((\rho_f U_T^2 A_p) = 12(1+0.15Re_{\rho}^{0.687})/(Re_p) \) (Crowe et al. 2011), which is applicable in the transitional regime: \( 1 < Re_p = \rho_f U_T d_p/\mu_f < 750 \), the unknown particle density \( \rho_p \) can be deduced from the knowledge of the particle diameter \( d_p \) and measured terminal velocity \( U_T \). Here, \( A_p \) is the projected area of the particle in the direction of settling. This indirect method yielded a more repeatable particle to fluid density ratio \( \rho_p/\rho_f \) of 1.0035±0.0003. Even for such small density mismatch, the particle settling velocity is not negligible in comparison to the flow velocities in our experiments. Especially at low bulk Reynolds number \( Re_{2H} = \rho_f U_{\text{bulk}} (2H)/\mu_f \) in water, sedimentation is observed.

The hydrogel particles are not completely rigid and can elastically deform. In this spirit, they have been used at high concentrations as model particles to study the packing characteristics of a deformable media under stress (Mukhopadhyay & Peixinho 2011). A thin hydrogel slice made of 8% polyacrylamide was measured by Byron (2015) to have an elastic modulus of around 10 kPa, at small strains. Commercially obtained spherical hydrogel particles used in the study of Baker & Coletti (2019), on the other hand, had an elastic modulus of around 80 kPa. This value is 2 – 3 orders of magnitude lower than silicone rubber and around 5
orders of magnitude lower than polystyrene, a material whose particles we have later used as model rigid spheres for pressure-drop measurements in a round pipe. The density of polymer cross-linking is the main factor that determines the swelling behaviour and strength of hydrogels. Polymer particles that have a higher cross-linking density would absorb lower amount of water and swell to a lower degree leading to higher strength. On the downside, lower water content also implies a higher mismatch in the refractive-index thus, impeding optical measurements.

The commercial particles used in this study did not display any measurable deformation even at the highest speed and concentration reached in our set-up. This was visually confirmed through time-resolved movies of our flow. Hence, it can be assumed that the shear and normal forces generated in the flow are not sufficient to deform the freely moving particles from their spherical shape.

It can also be said that the normal forces present during the collision of a particle with another particle or with the wall in water is lower compared to the case of collisions in air due to the retardation by the lubricating forces from the surrounding fluid. The coefficient of restitution, which is a measure of the energy lost during a collision, will thus be higher in air (measured to be around 0.9 for a 3 mm particle dropped from a height of 0.1 m). In water, due to the higher viscous dissipation in the fluid, it has been shown in Joseph et al. (2001); Gondret et al. (2002) that the coefficient of restitution is a function of the particle Stokes number \( St = (\rho_p R_{ep})/(9 \rho_f) \), where the particle Reynolds number \( R_{ep} \) is based on the impact velocity. For \( St \leq 10 \) collisions are dominated by viscous effects, and particles do not rebound at all, i.e. the effective coefficient of restitution will be negligibly small.

The hydrogel spheres are always wet, i.e. they are covered by a layer of water that makes them slippery. So they are able to slide with friction coefficients that are smaller than what is observed in regular solid materials like plastic and glass. Also in contrast to hard solid-solid contact, the coefficient of sliding friction for gels has been observed to decrease with increasing normal force (Gong et al. 1997) and increase with increasing sliding velocity, along with being a function of contact area. For our spherical particles, in the limit of negligible deformation as mentioned above, we can assume a single point-of-contact between two spheres or a sphere and the wall. In a recent work by Cuccia (2017), the friction coefficient between a sphere and flat hydrogel surface has been measured to have a value around 0.01, under an applied normal force of 0.1 N, at sliding velocities between 0.001 to 0.1 m/s.

2.3. The fluids

Since the hydrogel particles mostly constitute water, their density is very close to that of water but, not exactly the same which leads to sedimentation at low flow velocities. In order to increase the density of the suspending medium (water) to match the density of the dispersed phase (particles), so as to reach the desired neutral buoyancy point while not substantially changing the refractive
index, water-soluble salts are usually added. However, ions released by salts on dissolving in water can surround the swollen hydrogel and create an osmotic pressure across the hydrogel-water membrane. This causes water to be expelled from the body of the hydrogel and the spherical bead shrinks in size, causing its density to rise relative to the surrounding salty suspending solution.

A slower rate of shrinkage occurs when sugar is dissolved in water as compared to common salt dissolved in water. Thus, it takes a large enough solute molecule, which essentially has a smaller diffusivity, to ensure that the additives increase the density of water without causing hydrogel to shrink. One such possible additive is Polyethylene Glycol (PEG): H-(O-CH₂-CH₂)ₙ-OH, also known as Polyethylene Oxide (PEO). Interestingly, it is the same chemical that is used for preserving the once sunken ship Vasa of Sweden. Spraying PEG prevents cracks and shrinkage in waterlogged wood and is thus, ideal for restoration activities. The polymer is readily soluble in water and forms a clear solution. Addition of small quantities of these linear polymers will increase the viscosity in direct proportion to their molecular weight and concentration (Gonzalez-Tello et al. 1994). If the molecular weight of a given PEG is not substantially high, the increased viscosity remains fairly constant across different shear rates i.e. Newtonian behaviour. For the experiments which demanded comparison of turbulent suspension in Newtonian and non-Newtonian fluids at the same bulk Reynolds number with neutrally-buoyant particles, we used PEG with a molecular weight of 8000 dissolved in water as the Newtonian suspending fluid. Another additive that can yield a clear solution whilst being biodegradable could be Carboxymethyl Cellulose (CMC), which is often used as a thickener in the food industry. A PEG with a higher molecular weight, i.e. possessing longer molecular chains, shows non-Newtonian behaviour such as shear-thinning and viscoelasticity.

For experiments with a viscoelastic fluid, we use an aqueous solution of a high molecular weight Polyanionic polymer (FloPAM AN934SH, SNF, Molecular weight > 15 × 10⁶). This solution is also clear and its density is practically the same as water, at the very small concentration ≈ 250 ppm used in our study. Poole (2016) has measured the intrinsic viscosity (\([\eta]_{int} = \lim_{\text{conc} \to 0} (\eta - \eta_{\text{solvent}})/(\eta_{\text{solvent}} \times \text{conc})\)) of the above polymer to be around 4400 ml/gm, which leads to a critical overlap concentration \((\text{conc}^* \sim 1/|[\eta]_{int}, \text{Graessley 1980})\) of around 225 ppm. Thus, at 250 ppm the solution can be considered to be semi-dilute and hence some interactions between the polymer molecules is expected.

The solution was prepared in steps by dissolving the dry powdered form of the polymer in water, followed by gentle stirring using a rotary mixer for around one day. It was then gradually diluted with water down to the desired concentration inside a larger cylindrical plastic barrel. The solution was occasionally mixed using a submersible pump, to prevent aggregates and make it homogeneous. Before measurements, the final solution is recirculated in the flow loop for a long time to further homogenise it.
The drag reduction is a function of polymer degradation, thus being higher in the beginning of the experiment and decreasing (rather rapidly in the initial period and very slowly later on) to a nearly constant value after a certain time. At this stage, the long chain polymers are degraded enough so that the rheological and flow properties do not change significantly and thus, nearly steady state PIV measurements are possible. It can be noted that the degradation of aqueous FLoPAM solution is slower than the more popular PEO solutions. Also, the larger volume of our set-up reduces the rate of mechanical degradation and, thus, provides enough measurement time.

Degradation of polymers reduces their relaxation time and the shear viscosity and shear-thinning tendency of the solution. A way to achieve steady and higher level of drag reduction would be not to recirculate the polymer solution i.e. a one-pass system. However, at the high flow rates that we intend to achieve, this would require a very large supply of solution and hence, this option was not used.

For experiments with a viscoplastic fluid, we use an aqueous solution of Carbomer powder supplied as Carbopol NF 980 (Lubrizol Corporation, USA), a commercial thickener sold in anhydrous powder form. It consists of cross-linked polyacrylic acid (similar to hydrogel) and requires neutralisation with Sodium Hydroxide (NaOH) for gelation. It is chosen due to its high optical transparency, small thixotropy and material stability i.e. very slow ageing. Under neutral pH, the resin is anionic i.e. the many side chains carry a negative charge, and it has the ability to absorb and retain water and swell so enormously that concentration of even below 0.1% mass is sufficient for the particles to form a percolated network which behaves like a yield stress fluid (Piau 2007).

Similar to FLoPAM, a concentrated solution of Carbopol powder was first prepared by dispersing it gradually in a smaller quantity of water and mixing it with a high shear mixer (Silverson AX5, Silverson Machines, Inc., USA). The dispersion was then left stationary for some time, allowing the air bubbles to evacuate. NaOH was gradually added with a pipette whilst stirring the solution gently at low rotational speeds using a helical mixer (RW 20 DZM-P4, IKA-Labortechnik, IKA-Werke GmbH & Co. KG, Germany). At the end of the neutralisation process, the pH was noted to be ideally within the required range (6.5 – 7.0) so as to ensure maximum swelling and, hence, a high yield stress. The neutralized solution was further mixed in a large cement mixer (1402 HR, AL-KO, Germany) at a low rotational speed for complete homogenisation. This concentrated solution has a very high yield stress and it was nearly impossible to pump in our set-up. Hence, to facilitate pumping it was made thinner by gradually diluting it in the flow loop by mixing with water and recirculating for a few hours to completely homogenise before the start of measurements.

2.4. The square duct facility
The decision to opt for a square duct geometry against the more popular round pipe was spontaneously, and somewhat impulsively, taken by the two
supervisors followed by an obligatory nod from the newly joined PhD student. This happened during one afternoon meeting in the autumn of 2015. The square duct is known to have two non-homogeneous directions in terms of variation in mean flow quantities, instead of only one for the round pipe and plane channel, which are *canonical* geometries in fluid mechanics. This added complexity along with the existence of turbulence driven secondary flow inside the duct, more about it later in Chapter 3, was expected to bring out more *interesting* physics than the round pipe once particles were added. Moreover, duct flow is found in many applications like Heating, Ventilation and Air Conditioning (HVAC), jet engines and channels of nuclear reactors.

The decision to adopt a square duct also resolved the issue of excessive bending of light at the curved fluid-wall and wall-air interface that occurs in a round pipe, thus enabling better measurements, especially in the near-wall region. In the round pipe this would require the use of a pipe with thinner walls and a surrounding rectangular water-box so as to minimise refraction. This advantage was soon overshadowed by the complications involved in constructing a truly ‘square’ duct - the duct had to be built whereas a thin-walled pipe with a tight tolerance can be readily bought. It was also decided to make the duct transparent throughout its length so as to pursue studies related to the development of velocity and concentration profiles along the streamwise direction. Such studies were, unfortunately, not exhaustively conducted during the course of this PhD. However, the limitations on construction due to the designed transparency made the duct susceptible to recurrent water leakages, as will be described below. Fixing these leakages during crucial periods of measurements would prove to be the single most important factor contributing to frustration during this work.

Since there is a dearth of knowledge regarding the streamwise length required for achieving a fully developed state (in terms of pressure gradient, velocity and concentration distribution) of particle-laden turbulent flows, the duct was conceived to have a length as long as what is permitted in the assigned lab space. This length was slightly more than 6 meters and in lieu of the equipment to be built around the duct, it was decided to have a duct which is $L = 5 \text{ m}$ long. Since the smallest hydrogel particles at our disposal were slightly larger than 3 mm in diameter $d_p$, the cross section of the square duct was fixed to $2H \times 2H = 50 \text{ mm} \times 50 \text{ mm}$. This resulted in $2H/d_p \approx 16$ which is in between the oft cited channel flow simulations of Lashgari *et al.* (2014) and Picano *et al.* (2015). This cross section amounted to a streamwise length to duct height ratio of 100, sufficient for achieving a fully developed turbulent flow, at least for single phase flow, up to the investigated $Re_{Bulk}$.

The entire duct is fabricated in-house by gluing several elongated sections of 10 mm thick plexiglass sheets in a staggered fashion as shown in figure 2.2b. The staggered configuration prevented joints from occurring at a single location. This improved the strength of the entire duct. Use of screws for attaching the four sides had to be avoided in order to permit unhindered visual observation
2.4. The square duct facility

Figure 2.2: (a) Cross section of the four plates that make the duct. (b) Perspective showing the staggered assembly.

from all the four sides. The edges of every sheet were machined to have slots (see figure 2.2a). These slots made it possible to accurately position adjacent sheets. Such a design also provided a larger surface area for applying glue between adjacent sheets. While gluing the different sheets, they were rigidly held in confinement inside a wooden frame, which prevented their relative movement during the time when the glue cured. The above exercise was intended to achieve a cross section that had as little deviation from squareness as possible. The final result is a duct with an average side length of 50.7±0.1 mm.

During experiments, fluid often leaked through the various joints, either because of differential thermal expansion of the glue compared to plexiglass and/or due to the fact that plexiglass marginally swells when in constant contact with water (and shrinks when water is removed). This leakage problem was finally overcome by using a combination of a softer and more elastic silicone based glue for sealing the joints under mild expansion-contraction and a harder epoxy based glue for strength. At a few places retaining metal straps were also mounted to reinforce the duct, at the cost of optical access. The duct and related components were carefully positioned on a flat surface that was levelled using a line laser (Bosch GLL380). The entire support structure was built using aluminium profile bars.

The flow loop is closed in the sense that the fluid-particle mixture is recirculated through the loop. The driving force is provided by a Discflo disc pump (Model: 2015-8-2HHD Close coupled, Discflo Corporations, CA, USA with an ABB 1500 RPM motor, 2.2 kW) on the downstream side of
2. Experimental methods

Figure 2.3: Photo of the flow loop.

the duct. Thus, it is a suction type of flow loop, a choice made to eliminate flow disturbances that would have been created in a forced (or blowing type) configuration by the pump installed on the upstream end of the duct. The choice of pump was crucial in order to minimise mechanical damage to particles. The above-mentioned pump is known to be gentle and can handle solids up to a concentration of 40% for particles as large as 6 mm. It has a pumping capacity up to 15 m$^3$/h without pulsations. Essentially, it is a centrifugal pump with discs attached to the rotor instead of an impeller. A similar pump has been previously used to study the laminar-turbulent transition in flow of polymer solution where the degradation of the polymer-chains was to be minimised so as to achieve high drag-reduction for longer time (Draad et al. 1998). The rotational speed of the pump is controlled using a variable frequency drive (ABB ACS580-01-05A-6+J400, 2.2 kW).

Another type of pump which is often seen in literature dealing with transport of large particles is the Moineau progressive cavity pumps. Such pumps are of the positive displacement type and despite their ability to handle particles, can create some pulsations in the flow and, thus, require the use of pulsation dampers. They are also more expensive than the disc pump. They can of course be used in a gravity driven flow loop where the flow in the test section is isolated from the pump as it was done in Matas et al. (2003).

Figure 2.3 shows the actual photo of the set-up. Evidently, a lot of clutter is created due to the many tubes (blue in colour in the photo) connected to the pressure taps and the electrical wirings associated with the data acquisition system. A long round pipe can also be seen above the duct. More information about the pipe flow loop will be given in section 2.6.
Referring to the schematic of the flow loop in figure 2.4, the fluid-particle mixture is introduced inside a conical tank that is open to the atmosphere. The funnel shape of the tank was helpful in directing all particles towards the bottom exit. This prevented any dead zones where particles may accumulate. In practice, initially only the fluid phase (tap water) is introduced and the flow loop is run for some time. During this time, any residual bubbles would rise up in the open tank and exit the system. The pressure-drop with single phase Newtonian flow is measured in the fully developed zone at different flow rates and these observations are checked against the expected empirical values. This is a test to ascertain that all the systems are performing normally.

To minimise the amount of dissolved air in the water (which can form tiny bubbles on the inner walls of the duct and block the laser light during PIV), the water is first heated and allowed to cool to room temperature, before being introduced in the flow loop.

Particles are gently added to the tank only after the above steps are completed. A point which has often been misunderstood in terms of the operation of the loop is the role of gravity. It should be noted that the discharge hose coming from the pump is eventually fully submerged under the free surface level of the tank i.e. the entire loop is one long continuous closed pipe and thus, the pump does not work against gravity to transport the fluid. It only works against the frictional pressure drop in the pipes, bends and valves.

Again referring to figure 2.4, fluid (or fluid-particle mixture) from the conical tank flows through a gradual 90° bend at the exit of the tank. A larger
bend radius and smaller pipe diameter is advantageous to reduce the intensity of secondary Dean vortices (Vester et al. 2016). The bend is followed by a static mixer (Vortab company, CA, USA), in the form of an insertion sleeve, that is mounted inside the round pipe section close to the inlet of the square duct. The mixer is expected to neutralise any secondary flow that may arise from the bend. It consists of a series of radial and inclined transverse tabs projecting from the inner surface of the conduit that promotes cross-stream mixing.

Finally before the flow enters the duct, a small pipe that provides a smooth transition from circular to square cross-section is installed between the mixer and the duct. A similar square to circular transition is installed at the exit of the duct. Air vents are provided towards the upstream and downstream end of the duct to remove any trapped air in these regions. The far end of the duct is also equipped with a screw cap which provides access for cleaning. A very flexible polyurethane hose is connected between the downstream end of the duct and the suction side of the pump to reduce the transmission of vibrations.

After sufficient distance (> 20D) from the discharge end of the pump, an electromagnetic flowmeter (Krohne Optiflux 1000 with IFC 300 signal converter, Krohne Messtechnik GmbH, Germany) is installed to measure the volume flow rate of the particle-fluid mixture. To reduce the risk of faulty readings caused by electrical disturbances from particles in the flow meter, another flow meter with a ceramic electrode was also tested. Interestingly, this flow meter belonged to a new line of products from Krohne and was not yet commercially available. It was generously offered to us by Gustaf Fagerberg AB in order to test its performance with our hydrogel and polystyrene particles (used in the round pipe flow as will be described later).

The change in the mean value and the level of noise with the new flow meter at different particle concentrations (up to \( \phi = 20\% \)) and flow rates indicated a deviation less than 1% between the two flow-meters. It can also be noted from Leeungculsatien & Lucas (2013) that the important aspect for the proper operation of electromagnetic flow meters in multiphase flows is that the mixture is ‘fluid-continuous’ so that it is electrically conducting. Small variations of conductivity has little or no effect (which is particularly true up to \( \phi = 40\% \) of a non-conducting dispersed phase).

To measure the pressure drop, we use a membrane type differential pressure transducer (0–1 kPa, Model: FKC11, Fuji Electric France, S.A.S.) connected between the downstream and upstream position of the duct. At multiple locations along the length of the duct, symmetric pressure taps, 2 mm in diameter, were drilled perpendicular to the top and bottom walls of the duct to measure the streamwise pressure gradient. The pressure tubes (blue in figure 2.3) emerging from these two holes (at a given streamwise location) were joined together into one single tube and then connected to the pressure transducer. Bubbles were carefully removed from the tubes and the transducer by bleeding out water under gravity. The pressure difference under no flow conditions was recorded before and after every measurement to check for any drift. Since the
primary interest was to measure in the fully developed flow region where the pressure gradient has reached a constant value, measurements were only done in the far downstream end of the duct.

Both the flow meter and the pressure transducer generated a HART signal (4-20 mA) that was linearly proportional to the flow rate and the pressure drop respectively. The sensitivity of the flow meter and the pressure transducer was increased by lowering the range of its default operation, so as to obtain an amplified signal. Instead of directly measuring the current, we had to measure the corresponding voltage drop across a precise resistor, due to the limitations of the NI-6215 data acquisition (DAQ) card. This voltage signal was transmitted to the above DAQ card and recorded on a computer using Labview™ software. The pressure transducer was periodically calibrated using a Furness Controls FCO560 calibrator. For the round pipe flow, a pressure transducer with a higher range (0–6 kPa) was used, and it was calibrated using a Druck DPI 610. In order to check the flow quality, we compare the measured non-dimensional flow resistance expressed in terms of the Fanning friction factor $f = \tau_w/(\rho_f U_{Bulk}^2/2)$ as a function of bulk Reynolds number $Re_{2H}$ to the empirical correlation valid for Newtonian turbulent flows (e.g. Duan et al. 2012), i.e. the classical Moody diagram. Here, $\tau_w = (dP/dx)(H/2)$ is the wall shear stress measured from the streamwise pressure gradient $dP/dx$.

The use of a conventional settling chamber and subsequent contraction section to reduce disturbances at the inlet has been avoided since we are dealing with larger particles which may largely sediment, on account of their slightly higher density, in the low speed zones of the settling chamber. Also honeycombs or other flow straighteners would only be restricted to those with a very low solidity so as to allow the passage of particles. Hence, they would be less effective compared to the case of single phase flow and are, therefore, not used. In the absence of a dedicated flow conditioning zone, the inlet region can be considered to be hydrodynamically rough and the induced disturbances are sufficient to trigger transition at a low Reynolds number $Re_{2H}$. It has been shown by Uhlmann et al. (2007) that turbulence can be sustained in a square duct at $Re_{2H} = 2200$, which is close to the critical Reynolds number in a round pipe. The flow is seen to be fully turbulent at $Re_{2H} \approx 4000$ in our set-up. As an additional source of disturbance, a layer of tripping tape is also glued at the inlet of the duct.

By using a three-way valve, the set-up has the provision to direct the flow through a parallel loop that is in turn connected to a mesh bag so as to filter out particles. An ultraviolet water filter is also installed to restrict the growth of organic matter. This comes in handy if the fluid is to be reused for a longer time. An additional valve and the associated pipeline also allowed the drainage of used fluid in to the sewage line.

Cleaning the inner surface of the long duct was indeed a challenging task. The duct is closed from all sides and the available space in the upstream and downstream sides of the duct is too little for inserting any long cleaning device.
Small PIV tracer particles would often stick on the inner surface and this reduced the intensity of the transmitted laser light. The first solution consisted of sliding a small cleaning sponge mounted at the end of a Lego™-based car, which had wheels and could fit inside the duct. This tiny carriage had a magnet fixed on top, and using another attractive magnet from the outside of the duct it was possible to move the car along the length of the duct. Since this approach was slow and the sliding magnet could potentially scratch the outer surface of the duct, another solution was devised; an extendable arm (with small sections that could be screwed together one at a time to form a long stick) is used. This stick, which has a thick cleaning sponge, cut in the shape of the square cross-section, is slid inside the duct and can quickly clean all the four sides simultaneously.

Temperature control was not initially installed in the flow-loop. However, we faced a vexing conundrum for a long period of time, whose source was finally attributed to the difference in temperature between the ambient air and the working fluid; water that was ever so slightly heated by the pump. The problem manifested itself in the form of an asymmetric velocity profile in the laminar flow regime (see figure 2.5). The laminar streamwise velocity field inside a fully developed rectangular duct flow can be represented in the form of a sum of infinite series of trigonometric terms (Spiga & Morino 1994) and the wall-normal velocity is strictly zero. The measured velocity profile, one of the first PIV measurements in the loop, consistently deviated from this symmetric function.
2.4. The square duct facility

The direction of asymmetry was very robust and was unaffected by a change in the conditioning section at the inlet (the flow was better streamlined by a battery of long straws at the inlet) or by flipping the duct by 180° (which was expected to reverse any roughness induced effects that may be more prominent on one wall than the other). The skewed streamwise velocity profile always appeared together with a non-negligible wall-normal velocity profile; which effectively drove the high speed fluid from the centre towards the top wall in the central region of the duct, and from the top wall towards the bottom wall along the side walls of the duct. Thus, a pair of counter-rotating vortices were set-up in the duct as sketched in figure 2.6a. However, the rotational direction of these vortices was opposite to the Dean vortices that can be expected from the bend at the inlet. Doubt was also cast on the Coriolis effect produced by the Earth’s rotation relative to the orientation of the duct (Draad & Nieuwstadt 1998; Owolabi et al. 2016). However, due to the nearly north-south alignment of the duct, the Coriolis force would most likely create a horizontal asymmetry in the velocity contour in contrast to the vertical asymmetry observed in experiments. Another observation was that the above asymmetry in the streamwise velocity vanished in the turbulent regime.

After six months since the first PIV measurement, the asymmetry reversed for the first time when, in a moment of desperation and curiosity, a cold bag of snow was dropped in the tank (courtesy of the snow-storm that hit
Stockholm on the 8th of November 2016). This observation provided a conclusive evidence about the role of temperature in the flow dynamics. The reason for the asymmetry was understood to be the convection currents set-up inside the duct: flowing water is slightly heated due to pumping above the temperature of the ambient air. Thus, the walls of the duct which are facing air on one side are colder than the water flowing inside the duct. Therefore, along the side walls, water cools down and becomes heavier than the water in the bulk and hence moves downwards. Continuity then forces the water in the central region to rise upwards and, thus, compensates for the downward motion along the walls i.e. a secondary flow is set-up which displaces the region of maximum streamwise momentum towards the top wall. The scenario reverses when the water is cooled down below the ambient air temperature (see sketch in figure 2.6b) and the direction of the secondary flow is reversed.

This phenomenon of natural convection occurs when the non-dimensional Rayleigh number \( Ra = g \beta \Delta T (2H)^3 / (\nu \alpha) \) exceeds around \( 10^3 \). Here, \( \beta \) is the volumetric thermal expansion coefficient, \( \Delta T \) is the temperature difference, \( \alpha \) is the thermal diffusivity and \( \nu \) is the kinematic viscosity. In our set-up with water flow, this implies that the temperature between the inner side of the duct and the water should not differ by more than \( 0.5 \times 10^{-3} \, ^\circ C \) ! The allowable difference in temperature between the ambient air and the working fluid is higher than the above value due to heat-transfer across the duct’s wall but, not significantly higher. Hence, a heat-exchanger and thermal insulation around the duct is required if laminar flow of water is to be studied in our set-up.

In turbulent flows, inertial mixing will overcome the effects of natural convection and hence, a control on temperature is relaxed. Nevertheless, we chose to control the temperature so as to prevent significant changes in the viscosity of the solution during the course of measurements. The temperature is maintained at nearly \( 20^\circ C \) by means of an immersed-coil heat-exchanger in the tank, and is monitored using a Fluke digital thermometer with a Type K thermocouple sensor. In studies with elastoviscoplastic fluid, even though we operate in the laminar regime, buoyancy driven secondary flows are not observed due to the high viscosity of the solution resulting in a lower \( Ra \).

2.5. The PIV system

A continuous wave laser (wavelength = 532 nm, power = 2 W, CNI Lasers, China) is used to illuminate the flow. A circular beam of light from the laser is transformed into a thin sheet (thickness \( \sim 1 \, mm \)) using a series of cylindrical lenses: a plano-concave diverging lens is first used to expand the beam vertically, and is followed by a plano-convex converging lens to focus the beam vertically. These two lenses determine the thickness of the sheet. Finally a lens oriented perpendicular to the previous two lenses expands the beam horizontally. This lens controls the width of the sheet. The resultant beam is made vertical by a 45\(^\circ\) mirror that directs the widening sheet of light towards the bottom of the duct, along the streamwise direction. The distance between the first two
2.5. The PIV system

Lenses is adjusted in such a way that the thinnest section of the laser sheet - the waist - occurs close to the midpoint of the square duct.

The laser and optics are collectively mounted on a single mechanical traverse so as to illuminate different spanwise planes in the duct. This traverse along with the optics is in turn mounted on a larger carriage that can slide along the entire length of the duct, thus, enabling PIV measurements at different streamwise locations.

Since the laser is of continuous type, a high-speed camera (Phantom Miro 120, Vision Research, NJ, USA) is used to capture different instances of the flow. This camera can capture up to 800 frames per second (fps) at a maximum resolution of 1920×1200 pixels. Higher fps can be achieved at lower resolutions.

Lenses with a focal length of 35 mm (used at f/2) and 200 mm (used at f/2.8) are used to image the entire duct height (at 1024×1024 pixels that covers a physical space of ≈ 60×60 mm) and the near-wall region (at 1280×800 pixels covering a physical space of ≈ 15×9 mm) respectively. Since the physical distance between the top and bottom walls of the duct is known in an image, it provides an easy reference for calibration. Calibration when performing PIV in the near-wall region is less straightforward since there is no provision to insert a calibration plate inside a water-filled closed duct. Hence, a dummy duct with a smaller length but identical cross-section and wall thickness is built. A calibration plate can be inserted inside this dummy duct. Distances were calibrated using this smaller duct after filling it with the working fluid. It was placed on top of the main duct and a calibration image was acquired after raising the camera vertically. Once this is done, the camera is translated vertically downwards (very carefully so that there is no motion in the other directions).

During a PIV measurement, the camera is periodically signalled to capture PIV sequences (a sequence contains two images) through Labview™ software via the DAQ card. Each PIV sequence is separated from its neighbouring sequence by a time interval that is large enough so that the flow becomes uncorrelated. Once the camera receives a signal from the DAQ card, the Phantom Camera Control software (PCC, Version 2.8.761.0) internally triggers the camera to capture two images (belonging to one PIV sequence) that are separated by a time delay. The delay is chosen such that the maximum pixel displacement between two images, based on the mean velocity, did not exceed a quarter of the size of the interrogation window IW (Raffel et al. 2013). The exposure time is selected such that enough light is captured by the camera sensor while pixel stretching is kept lower than one pixel. Somewhere between 500 to 1000 statistically independent PIV sequences are found to be sufficient to get converged statistics in terms of the Reynolds stresses. Flow rate and pressure data for each case were sampled for the same duration as the PIV measurements.

Images were processed using an in-house, three-step, Fast Fourier Transform (FFT) based, cross-correlation algorithm that has been previously used in
Kawata & Obi (2014); Kawata & Alfredsson (2016). The first step consisted of basic PIV with a larger IW (48 x 48 pixel). This is followed by the discrete-window-shift PIV at the same IW size. Finally, the central-difference-image-correction method (Wereley & Gui 2003) is applied to account for velocity gradients at a smaller IW size (32 x 32 pixel). The maximum pixel displacement is around 12 pixels. Regarding the subpixel interpolation, we use a three-point Gaussian subpixel estimator (Raffel et al. 2013). As noted in Kawata & Obi (2014), the accuracy of the in-house PIV algorithm was estimated using artificial particle images (Okamoto et al. 2000), and uncertainty in evaluation of the particle displacement was approximately 0.2 pixels.

The degree of overlap (for PIV of the full duct height) is around 47% and can be estimated from the fact that the corresponding resolution is 1 mm x 1 mm per IW (32 x 32 pixel). Near-wall measurement are conducted by zooming the camera on a small region close to the wall with a resolution that is $\approx 3$ times higher than for the full duct height. Before measuring the velocity field in particle-laden flow, the results for single phase in terms of mean velocities and their correlation have been satisfactorily validated against previous DNS simulations in a square duct for $Re_{H}$ ranging from 5000 to 10000 from the groups of Uhlmann et al. (2007) and Zhang et al. (2015), as well as from our own group: Fornari et al. (2018a).

Hydrogel particles when added to the flow and visualised in a laser sheet appear like darker blobs surrounded by tiny PIV particles. This is because these particles do not contain significant light scattering elements dispersed inside them. It is thus, quite difficult, if not impossible, to figure out their position, size and displacement from such images. Hence, in order to make them clearly visible, some additive needs to be impregnated inside their bodies. As already mentioned, Byron (2015) accomplished this by fabricating their own particles embedded with PIV tracers.

We also pursued this idea, now on a much larger scale due to our interest in high particle concentrations. Fortunately, we were able to convince a manufacturer in China to try making hydrogel particles, using the same suspension polymerisation technique that is conventionally used for normal hydrogel particles, but now with the addition of a small amounts of PIV tracer particles during the manufacturing process. Conventional micron size PIV seeding particles made of Polyamide or hollow glass were too light to be properly dispersed in the polymer mix. Thus, a heavier powder in the form of Zinc sulphide had to be used. This made the hydrogel particles slightly more heavier than those without tracer particles. Even though these particles worked fairly well at low $\phi$ and could provide valuable information about particle rotation, it was almost impossible to measure the flow beyond 2-3 particle layers. Typical images obtained with such particles are shown in figures 2.7a and 2.7b, for a lower and higher concentration of Zinc sulphide respectively. As evident from the images, the optical turbidity increases substantially beyond a few particle layers. On the other hand, normal hydrogel particles without tracers were much more
2.5. The PIV system

(a) Particles with lower concentration of embedded tracers

(b) Particles with higher concentration of embedded tracers

Figure 2.7: Failed attempts: Images of hydrogel particles with embedded PIV tracers (Zinc sulphide) taken inside a transparent plexiglass box filled with water. This set-up was equipped with a Lego$^\text{TM}$-based stirrer and was often used for testing different kinds of particles on a small scale.

Figure 2.7: Failed attempts: Images of hydrogel particles with embedded PIV tracers (Zinc sulphide) taken inside a transparent plexiglass box filled with water. This set-up was equipped with a Lego$^\text{TM}$-based stirrer and was often used for testing different kinds of particles on a small scale.

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To make the dark hydrogel particles visible in the flow, they are expanded in water mixed with an empirically adjusted small amount (in ppm) of concentrated Rhodamine 6G solution. Particles immediately start swelling inside water and are left submerged for a total duration of around 24 hours before they are used for measurements. During PIV experiments, the Rhodamine dye which is absorbed by the particles along with water is excited by the green laser light and it scatters light at a higher wavelength thus, making the particles appear yellowish orange. Figure 2.8 shows such particles illuminated by the laser sheet. A typical grayscale PIV image obtained with such particles is shown in figure 2.9a. The amount of Rhodamine dye is crucial since an excessive amount of dye can cause the particles to scatter a lot of light and thus undermine the light from the PIV tracers during the short exposure time of the camera sensor.
2. Experimental methods

Figure 2.8: Hydrogel particles glowing in the laser light sheet. The back side of the duct is visible away from the laser light, across several particle layers.

(a) Raw image used in PIV
(b) Raw image used in PTV

Figure 2.9: One of two images of a PIV sequence denoting the same instance in time but differing in contrast and sharpness. The particle size is $2H/d_p = 9$, and the volume fraction is $\phi = 20\%$. The fluid is water flowing at a $Re_{2H} \approx 27000$.

Since we do not use optical wavelength filters to distinguish between light originating from the fluid tracers and the particles, image enhancement will be done in the post-processing steps by utilising the difference in size and intensity between the two kinds of particles. In particular, by adjusting the sensitivity and flare options in the PCC software, potential (large) particles were made
Figure 2.10: Steps in processing PTV image: (a) Minimum intensity background image, (b) Image after subtracting the background, (c) Low-pass filtered image, (d) After eroding and dilating, (e) Final smoothening filter and (f) Image showing the detected particles.

Particles were detected using a circular Hough transform (Yuen et al. 1990), adapted from Matlab™ File Exchange (Peng 2010). Such an object recognition algorithm performs better on an image where the particle edges are clearly defined, i.e. a sharp intensity gradient is required. We tried the simpler thresholding and binarization technique but it did not work satisfactorily on our images that have a broad intensity distribution. Instead, the following recipe,
whose steps are featured in figure 2.10 was adapted for image-processing. Firstly, the local-minimum-intensity background (see figure 2.10a) is subtracted from each image so as to get rid of the high intensity light scattered from the top and bottom walls. These regions often erroneously act as potential candidate points in the Hough transform algorithm. On the resultant image (see figure 2.10b), a pixel-wise adaptive low-pass Wiener filter is used to remove the small bright tracer particles (see figure 2.10c). The size of the filter is kept at a small value to prevent excessive smoothing of the particle edges. Due to the small filter size, a few larger and brighter tracer particles persist in the image. These are then effectively removed by eroding and dilating the image by the same number of pixels (see figure 2.10d). This again guarantees that the larger particles do not change in size compared to the original image. At this stage, the image is almost ready. A final small size median filter is applied to smoothen the image and remove any small inhomogeneities (see figure 2.10e). To prevent false particle detection due to large intensity gradients occurring near the walls, the region beyond the walls that is outside the duct is masked with a zero intensity value, before applying the circular Hough transform. Particles that are detected by the algorithm but, whose average intensity level lies below 70% of the average intensity of all detected particles are ignored. Figure 2.10f highlights the detected particles at a certain time instance $t$.

A similar (slightly higher or lower) number of particles would also be detected for the next time instance $t + \Delta t$ ($\Delta t$ being the time delay between two images in a PIV sequence). From the detected particles in these two images of the PIV sequence, a nearest-neighbour approach is used to determine the translational motion of a particle. The nearest-neighbour approach used for PTV is rather simple and readily works because the average displacement of a particle between $t$ and $t + \Delta t$ is substantially smaller ($< 15$ pixels) compared to the distance between the centres of two neighbouring particles ($> 30$ pixels).
2.5. The PIV system

Figure 2.12: The instantaneous velocity and fluctuations (instantaneous-mean) in the velocity field. For the fluid phase, the fluctuations have been calculated by subtracting the mean fluid velocity field obtained using PIV, whereas for the particles the fluctuations have been calculated by subtracting the mean particle velocity field obtained using PTV.

Accordingly, the Cartesian distances \( \sqrt{(\Delta x_p)^2 + (\Delta y_p)^2} \) between the centre-of-area of a reference particle in the image corresponding to \( t \) and all the particles in image corresponding to \( t + \Delta t \) is calculated. Here, \( x_p \) and \( y_p \) are the streamwise and wall-normal pixel positions of a particle’s centroid. The particle with the minimum distance, lower than a threshold of 20 pixels, is chosen as the match for the reference particle and the corresponding velocity is calculated as \( (\Delta x_p / \Delta t, \Delta y_p / \Delta t) \). The same procedure is repeated for all the remaining particles in image \( t \). Particles in image \( t \) that could find no matching candidate in image \( t + \Delta t \), either due to out-of-plane motion or motion beyond the field of view, are eliminated by the PTV algorithm. Figure 2.11 shows the streamwise pixel position of the particles for the two images (A at \( t \), and B at \( t + \Delta t \)) of a PIV sequence.

For the Eulerian PIV velocity field, we define a mask \( M \) whose size is equal to the size of the velocity matrix. \( M \) assumes the value 1 if the point lies inside any particle and 0, if it lies outside. The particle velocity that is determined using PTV is, to be precise, only defined at its centroid. Nevertheless, all the PIV points lying inside a particle are assigned a value obtained through PTV. This ensures that the velocity field of the particle phase is available on the same grid as that of the fluid phase. This is helpful in phase-ensemble averaging.

Figure 2.12a and 2.12b shows the instantaneous and fluctuating velocity field for both the phases obtained using the combination of PIV and PTV. Owing to the curvature of the particles, it is possible that IWs close to the particle-boundary have a contribution in velocity from the particle phase and...
hence, some particle-fluid averaging may ensue. To circumvent this issue, we also tried calculating the fluid statistics by ignoring one layer of PIV points around the particle but no significant difference was observed in the long time statistics as compared to no-omission. Hence, it can be said that bias introduced by the particles on the fluid phase velocity is small.

The hydrogel particles are almost invisible to the naked eye under ambient lighting. Thus, qualitative visualisation that is easily done for ordinary opaque particles (at dilute concentrations) is hindered for our transparent particles. However, rather accidentally, we once observed that these particles inside the fluid can cast shadows if illuminated by a point source of light in a dark room. This works due to the slight non-uniformity in the refractive index created by the particles inside the suspending medium. This is the operating principle behind optical shadowgraphy, used to visualise temperature difference in air, amongst other things. Such shadowgraphy-based images and videos have helped in qualitatively evaluating the particle distribution along the streamwise length of the duct, at low speeds. Figure 2.13 shows a typical shadowgraph from our experiments.

2.6. The round pipe facility
Our first study with hydrogel particles in turbulent square duct flow revealed a puzzling variation in flow resistance as a function of particle size and volume fraction. This motivated us to check whether such an observation is due to the peculiar geometry of the square duct or if it is robust enough to be observed in an azimuthally symmetric round pipe. Hence, a round pipe was also integrated in the existing flow loop. The pipe has a smaller diameter \((D = 21.7 \text{ mm})\) than the duct \((2H = 50.7 \text{ mm})\). Hence, for the same length, the pipe has a larger length to diameter ratio \(L/D \approx 230\) compared to \(\approx 100\) for the duct; and for a fixed flow rate \(Q\) of a given fluid, the pipe can yield a higher \(Re_{Bulk \propto 1/D}\)
2.6. The round pipe facility

Figure 2.14: Photo of the pipe and the duct taken from the inlet side and pointing along the length towards the exit.

Figure 2.15: Different sizes of (a) spherical and (b) oblate particles used in pipe flow experiments.

than the duct. The same pump and piping system that is connected to the duct is used to operate the pipe, which is mounted on additional supports on top of the existing frame, as seen in the photo in figure 2.14. With the help of a new tank, an additional valve and few other connectors, the flow can be quickly diverted from the duct to the pipe.

Since we are only interested in measuring the pressure drop with particles as a function of flow rate in a pipe, RIM particles were not needed. We
chose STMMA (Styrene-Methyl Methacrylate Copolymer) particles due to their availability in large quantities, different sizes ranging from 0.5 mm to 3.3 mm (see figure 2.15a), and also because their density is only slightly higher than water ($\rho_p = 1.03 \text{ gm/cm}^3$). These beads are used as a raw material for making foam.

To match the density of the particles with the suspending liquid, table sugar at a concentration of Brix (gm of sucrose/100 gm of the solution) = 7.4±0.2 was dissolved in water. The viscosity of sucrose solution as a function of Brix and temperature can be accurately calculated using an empirical relationship (see equation 12 in Longinotti & Corti 2008, and for empirical constants for sucrose, see table 2 in the same reference).

Recently, a numerical study by Ardekani et al. (2017) from our group revealed the possibility to achieve turbulent drag reduction in a channel flow populated with finite-size oblate spheroids. To investigate whether such a practically important phenomenon occurs in a turbulent pipe flow, we performed a few experiments with planar oblate discs. These particles were purchased from a company manufacturing fastening washers, which in the absence of holes can serve as nearly spheroidal oblate particles (see figure 2.15b, material = Nylon PA66, $\rho_p = 1.14 \text{ gm/cm}^3$, Brix for matching density $\approx 32.2$). Two sizes with planar diameter $\times$ height = 1 $\times$ 3 mm (corresponds to the same aspect ratio as studied in Ardekani et al. 2017) and 1 $\times$ 5 mm were investigated.

2.7. Rheological measurements

The rheological characterisation of the fluids used in this work mostly comprised of measuring the shear viscosity as a function of the shear rate, i.e. flow curve. For Newtonian (PEG solution) and viscoelastic fluids (FLoPAM solution), quick ballpark tests were done using a viscometer (Brookfield DV-II + Pro, Brookfield Engineering Laboratories, Inc. MA, USA, with a bob and cup attachment) whereas, for more accurate measurements a temperature controlled rheometer (Kinexus pro+, Malvern Panalytical with plate-plate as well as bob and cup attachments) was used in the controlled shear-rate mode.

Due to instrument inertia, it was quite difficult to estimate the relaxation time of our dilute FLoPAM viscoelastic polymer solution by means of oscillatory strain tests. A more suitable Capillary Breakup Extensional Rheometer (CaBER, see Rodd et al. 2005) would be required for such data. Alternately, guided by the recent work of Owolabi et al. (2017) who used the same flexible polymer in the semi-dilute regime (for a similar range of $Re_2H$ in a square duct flow) as our study, the polymer relaxation time can be estimated from the experimentally measured value of drag reduction.

For the viscoplastic fluid (Carbopol solution), the shear viscosity and yield stress has been measured using a TA AR-2000ex rheometer (TA Instruments, Inc., USA with a vane and cup attachment). Viscoelastic moduli have also been measured under oscillatory strain for the Carbopol solution to quantify its linear
viscoelastic behaviour. This is done using a splined bob and cup attachment in the Kinexus pro+ rheometer.

2.8. Limitations of the present measurement technique

Despite being versatile enough to facilitate measurements in complex fluids at moderately dense volume fractions, there are indeed limits beyond which the effectiveness of the present measurement technique reduces. One of the principle limitations is posed by the deformability of the particles. This problem can becomes acute when the flow velocities are very high (which increases the collision velocity and hence, deformation) and when the concentration increases to a very high value.

Another practical issue during velocimetry is that the fluorescent Rhodamine dye slowly diffuses out of the hydrogel particles, into the surrounding fluid. This effectively makes the particle appear dimmer and smaller. The release of Rhodamine is much faster when Carbopol is used as the suspending medium, thereby reducing the time available for PIV-PTV measurements. It is nevertheless possible to ‘regenerate’ the particles by (again) immersing them in water containing Rhodamine.

It could eventually be observed during pumping these particles that, over long time, they shrink and few of them even break. Hence, a fresh batch of particles is used for every new experiment.

Since these hydrogel particles are largely composed of water, exchange of water with the surrounding complex fluid across the surface of the particle can dilute the suspending fluid and alter its rheological properties. This, again, is a problem that appears prominently at very high particle concentrations.

In our measurements, we have tried to control and minimise these issues wherever possible. One such measure is checking whether the flow characteristics and rheological properties of the suspending fluid is similar before and after the introduction of particles.

Finally, the refractive indices of the dispersed and continuous phases are not exactly matched, albeit very close. Thus, bending of light rays occur at every fluid-particle interface and the compounding effect of these distortions can introduce significant errors, particularly for the case of high concentration of smaller particles in a larger domain. This poses the upper limit for the present measurement technique.
Chapter 3

Square duct flow

3.1. Secondary flow in Newtonian fluid

Turbulent flow carrying large particles inside pipe and duct geometries is widely encountered in many engineering applications. Turbulent duct flow is particularly interesting, even in the single phase case, since the presence of adjoining walls creates a flow field that is inhomogeneous in the wall-normal as well as spanwise directions. This inhomogeneity gives rise to a net cross-stream secondary flow of Prandtl’s second kind (Prandtl 1926), which is superimposed on the primary axial flow.

To note, secondary motion of Prandtl’s first kind is much higher in magnitude (generally 10 to 40% of the bulk velocity). This is encountered in curved passages, where centrifugal forces act at an angle to the streamwise direction. On the other hand, the secondary flow of Prandtl’s second kind is driven by gradients in the turbulent stresses and were, perhaps, first visualised by Nikuradse (1930), a student of Prandtl, who is more famous for finding empirical relationships for turbulent friction factor as a function of Reynolds number for different pipe roughnesses. The magnitude of this secondary flow is smaller than the root-mean-square (rms) velocity of turbulent fluctuations and generally between 1 to 4% of the bulk velocity in most straight ducts with non-circular cross section.

Nevertheless, these secondary motions, in the form of four pairs of counter-rotating vortices i.e. eight vortices in total, located at the duct corners (see figure 3.1 for a schematic), act to transfer fluid momentum from the centre of the duct to its corners, thereby causing a bulging of the mean streamwise velocity contours toward the corners. Also, their effect on the distribution of wall shear stress and heat transfer rates are quite significant (Demuren 1991). Therefore, the simple geometry of a square duct provides an excellent case to test and develop turbulence models that explore secondary flows of the second kind.

To understand the origin of the above mean secondary flow, one can look at the Reynolds averaged equation of the streamwise vorticity $\Omega_x =$
3.1. Secondary flow in Newtonian fluid

Figure 3.1: Schematic of secondary flow of Prandtl’s second kind (closed solid lines with small arrows corresponding to direction of motion). The closed blurred lines in the background are intended to show the iso-contours of streamwise velocity. The coordinate system is also shown.

\[
\frac{\partial U_y}{\partial y} - \frac{\partial U_z}{\partial z}, \text{ which is given as}
\]
\[
U_y \frac{\partial \Omega_x}{\partial y} + U_z \frac{\partial \Omega_x}{\partial z} = \nu \left( \frac{\partial^2 \Omega_x}{\partial y^2} + \frac{\partial^2 \Omega_y}{\partial z^2} \right) + \frac{\partial^2 \langle u_y'^2 \rangle - \langle u_z'^2 \rangle}{\partial y \partial z} - \left( \frac{\partial^2 \langle u_y'u_z'^2 \rangle}{\partial y^2} - \frac{\partial^2 \langle u_y'u_z'^2 \rangle}{\partial z^2} \right)
\]

The coordinate system is sketched in figure 3.1. Here \( U \) and \( u' \) are the mean and fluctuating velocities and \( \langle \rangle \) represents the time averaging. The left hand side of equation 3.1 represent the convection of mean streamwise vorticity by the secondary flow \((U_y, U_z)\). The first term on the right hand side is the viscous diffusion term that tries to reduce the gradients in \( \Omega_z \), whereas the last two terms on the right hand side may either produce or destroy \( \Omega_z \) and hence, secondary flow, based on the location in the flow. The first term amongst them represents the gradient in the cross-stream Reynolds normal stress difference while the second term represents the gradient in the secondary Reynolds shear stress.

Early experiments of Brundrett & Baines (1964) have shown that the gradient in the Reynolds normal stress is mainly responsible for producing streamwise vorticity away from the corners, the convection term transported this vorticity to the walls where it was diffused by the viscosity. Experiments of Gessner (1973) concluded that, instead, the gradient of Reynolds shear stress played the dominant role. A more detailed understanding was made possible by the DNS simulations of Gavrilakis (1992), albeit at a lower \( Re_{2H} \). He observed that the gradient in the Reynolds normal stress is largely responsible for generating streamwise vorticity but, now in the viscous sublayer. In contrast to Brundrett & Baines (1964), the role of the convection term in vorticity transport was found to be rather weak. It was soon followed by another DNS simulation by Huser & Biringen (1993) at a higher \( Re_{2H} \), and they inferred
that the secondary flow is a result of the interaction between ejection events from adjacent walls.

Pinelli et al. (2010) observed that the mean streamwise vorticity is a statistical portrait of the most probable locations of the buffer layer coherent structures. This causes a high speed streak to be invariably present near the duct’s corner leading to a maxima of the wall shear stress in that region. With increasing bulk Reynolds number $Re_{2H}$, the size of the coherent structures reduces (in physical units) and the vortices penetrate the corners thus, approaching the wall (Brundrett & Baines 1964).

In the transition regime from laminar to turbulent flow, DNS simulations of Uhlmann et al. (2007) and experiments of Owolabi et al. (2016) have shown the existence of a secondary flow structure characterised by four vortices, instead of eight in the fully turbulent regime, that switches in direction with time. Launder & Ying (1972) found the friction velocity to be the appropriate parameter that scales the secondary velocity profiles in both smooth and rough ducts, at least for high $Re_{2H}$.

### 3.2. Secondary flow in viscoelastic fluid

Lesser is known about the nature of turbulence driven secondary flow for a viscoelastic fluid in a square duct. The earlier studies from Rudd (1969) and Logan (1972) using LDV revealed the modification of the mean profiles of streamwise velocity, turbulence intensity as well as the primary Reynolds shear stress for low polymer concentration. Escudier & Smith (2001) performed detailed spatial measurements of mean axial and secondary flow velocity as well as turbulence statistics for various polymer solutions in the duct. They found that apart from a reduction in the transverse turbulence intensity, there is also a strong reduction in the secondary flow velocities. In the recent DNS simulations of Shalmardi et al. (2019), it was observed that the counter rotating vortices become larger and their centres are displaced towards the centre of the duct away from the walls; the maximum streamwise vorticity reduced but its integral within a representative octant increased. This was due to the increasing size of the near-wall coherent structures and their direct relation to the location of the secondary flow, as elucidated by Pinelli et al. (2010).

Laminar flow of a viscoelastic fluid in a square duct may also be associated with the presence of a secondary flow in the form of four pair of counter-rotating vortices, as studied by many researchers since Ericksen (1956). In contrast to Reynolds stress gradient driven secondary flow under turbulent conditions, these vortices in the laminar regime now originate due to the viscoelastic forces in non-circular pipes, specifically the imbalance in the second normal stress difference $N_2(\dot{\gamma}) = \tau_{yy} - \tau_{zz}$ (Dodson et al. 1974). Their sense of rotation is generally (with some exceptions, see Yue et al. 2008) opposite to that for turbulence driven vortices, i.e. they drive high speed fluid from the core towards the centre of the wall of the duct and transport the low speed fluid from the...
corners towards the core. Also their strength increases with fluid elasticity (Debbaut & Dooley 1999).

No secondary flow would be observed in a purely viscous or viscoplastic fluid, or for certain special relationships between the second normal stress difference and the fluid viscosity in a viscoelastic fluid (Xue et al. 1995). For the flow of a Bingham fluid in an approximate square duct geometry, Letelier et al. (2018) observed secondary flow in the form of streamwise vortices only when they introduced some elasticity in their simulations (by including higher order terms in Weissenberg number $Wi$). Increasing the Bingham number $Bi$ at the same $Wi$ reduced the intensity of the secondary flow, and displaced the vortices further away from the centre of the duct.

3.3. Particle motion in Newtonian fluid

In the laminar regime, understanding particle migration characteristics in a rectangular geometry is practically useful due to its recent applications in particle focusing and size based filtration using inertia inside microfluidic channels (Di Carlo 2009; Amini et al. 2014). In such applications, the $Re_{2H}$ is typically small so as to preclude turbulence. The cross section of the flow geometry is often square shaped due to the lithography based manufacturing process.

The equilibrium positions to which a single or group of particles migrate is a function of the ratio of particle to duct size (also called confinement), $Re_{2H}$ as well as particle concentration $\phi$. The inhomogeneity in the cross-sectional plane causes certain ‘point-locations’ to be more preferred by the particles, against a ring like configuration in a pipe (Segré & Silberberg 1962).

Abbas et al. (2014) showed that for small but finite $Re_{2H} \sim 1$, shear-induced particle migration drives the particles towards the core, even at very low $\phi < 1\%$ (at $d_p/2H \approx 0.1$). Between $Re_{2H} = 1$–$10$, inertial effects become important and causes the particles to stabilise at a position between the wall and the centre forming an annulus (also see Choi et al. 2011). Using experiments and simulations, Di Carlo et al. (2009) found that, for low $Re_{2H} < 100$, the wall face-centres are the only stable positions and larger particles equilibrate further away from the walls. At around $Re_{2H} = 250$, Miura et al. (2014) found that the corner position becomes stable. Chun & Ladd (2006) had previously shown that at higher $Re_{2H} \sim 500$, the face-centre is no longer a stable position and all the particles migrate towards the corners, while approaching closer to the corners with increasing $Re_{2H}$. Similar behaviour was also exhibited in a multi-particle system at $\phi = 1\%$.

Recently, Kazerooni et al. (2017) investigated the influence of particle volume fraction up to $\phi = 20\%$ and reported the generation of a mean secondary flow associated with the preferential accumulation of particles. The intensity of this secondary motion increases up to a certain volume fraction $\phi$ but, reduces with further increase in $\phi$ since it is damped by the presence of too many particles near the core region. Particle rotation is also said to play an important
role in determining the intensity of the secondary flow as well as the final equilibrium position for these high $\phi$.

Fewer studies exist for particle motion in a turbulent duct flow. One of them is the recent study by Fornari et al. (2018a) at $Re_2H = 5600$, $d_p/2H \approx 0.05$, and $\phi$ up to 20%. These authors found that similar to the case of laminar flows, particles accumulated near the corners in a statistical sense up to $\phi = 10\%$, whereas they moved towards the core for the highest $\phi = 20\%$. Apart from this, they also observed an increase in the intensity of turbulence driven secondary flow up to $\phi = 10\%$, which at further higher $\phi$ reduced due to a reduction in the turbulent kinetic energy.

Differences in density between particles and the solid phase may cause gravitational settling of finite-size particles at the bottom of the duct. In such a situation, Lin et al. (2017) observed that the presence of particles increases the secondary flow circulation which, in-turn, causes particles to accumulate preferentially at the face-centre of the bottom wall.

3.4. Particle motion in viscoelastic fluid

Cross-stream particle migration in laminar viscoelastic fluid flow is quite different to their migration in a Newtonian fluid (Leshansky et al. 2007; D’Avino & Maffettone 2015). Under assumptions of very low inertia, depending on the initial particle position, fluid elasticity drives the particle towards the centre of the duct or towards the corners (Villone et al. 2013). This is due to the non-uniformity in the distribution of the first normal stress difference $N_1(\dot{\gamma}) = \tau_{xx} - \tau_{yy}$ (Ho & Leal 1976).

In a square duct, local minima of $N_1$ occurs at the centre and the corners causing the particles to move in those regions. The resulting equilibrium position may also be affected by shear-thinning (Huang & Joseph 2000), and with higher shear-thinning the tendency of the particles to move to the wall face-centre increases (D’Avino et al. 2012). When a second normal stress difference $N_2$ is also present, then the ensuing secondary flow (mentioned previously) may force a particle to find a stable equilibrium position near the centre of the streamwise vortices (Villone et al. 2013).

Using the synergy between inertia and elasticity, Yang et al. (2011) experimentally managed to focus all the particles towards the centre-line of the duct. Naming the observed phenomenon as elasto-inertial particle focussing, they noted that the wall-repulsion inertial forces pushes the particle away from the walls and the non-negligible elasticity stabilises it in the core. Along similar lines, Lim et al. (2014a) could focus particles at a high throughput rate even at very high $Re \approx 10000$, i.e. high inertia, provided that the elastic normal stresses also increase in proportion to the inertia. As shown by Seo et al. (2014), the complex interplay between elasticity, inertia, shear-thinning and confinement can lead to multiple stable and unstable equilibrium configurations.

The literature concerning square duct particle-laden flow of a viscoplastic fluid is rather scarce. Some recent simulations that treat the particles as a
3.4. Particle motion in viscoelastic fluid

continuous phase have studied particle migration in a round pipe flow. In the Stokes flow regime i.e. negligible particle inertia, Siqueira & de Souza Mendes (2019) used the diffusive flux model (Phillips et al. 1992) to describe the shear-induced particle migration in a pipe flow of a Herschel-Bulkley fluid. Particles were found to concentrate at the boundary of the plug and with increasing particle concentration, the maxima shifted towards the tube-wall due to increasing influence of diffusion due to viscosity gradients. A different variant of the diffusive flux model was used by Lavrenteva & Nir (2016) for a tube flow of a Bingham fluid. They also found the maximum particle concentration near the yield boundary which itself moves to increase the unyielded fraction in the pipe.

Hormozi & Frigaard (2017) provided a general framework to model particle-laden flow of yield stress fluid in fracturing applications using the suspension balance phenomenology (Nott & Brady 1994). In contrast to the above studies, they found particles to concentrate in the plug region of the fracture channel.
Particles in a turbulent pipe flow

Before summarising results from the papers that deal with experiments in a square duct in Chapter 5, the present chapter discusses results from experiments and simulations for particle-laden flow in a round pipe.

4.1. Spherical particles in a pipe

Turbulence induced drag accounts for significant amount of energy losses in pipelines. Low concentration of polymer additives and fibres dispersed in flows have, in most scenarios, resulted in damping of turbulent stresses and reduction in drag. Spherical particles, on the other hand, can show both a decrease (Zhao et al. 2010) and an increase (Picano et al. 2015) in drag, with the latter tendency being predominantly observed in many studies. A systematic investigation of drag due to variation in particle size and concentration at varying flow rates (or Reynolds number $Re_{Bulk} = \rho_f U_{Bulk} D / \mu_f$) is missing.

By means of experimental measurement of pressure drop for suspensions of four different particles sizes in two different circular pipe flow facilities (with equal contribution from Martin Leskovec, KTH), we have tried to obtain the above information in a broad range of pipe-to-particle diameter: $D/d_p \in (7, 82)$. The particle volume fraction $\phi$ is increased up to 20% and $Re_{Bulk} \in (10000, 41000)$. Fully resolved direct numerical simulations using the immersed boundary method have also been performed (by Mehdi Niazi Ardekani, KTH) for a few cases at the lowest $Re_{Bulk} \approx 10000$ to yield useful information in terms of the stress balance, which is essential in the explanation of the non-monotonic behaviour that was first observed in experiments in our square duct (Zade et al. 2018).

Results have been compared with the estimate of drag based on a simple effective suspension viscosity formulation (Eilers 1941), which seems to satisfactorily predict the order of magnitude of the changes. A more refined theory that makes distinction between the dynamically inhomogeneous particle-wall layer region and nearly-homogenous central region in the suspension by Costa et al. (2016) is also tested.

The predictions of this theory seem to better match the experimental observations for a limited range that corresponds to smaller particle sizes ($d_p^+ \leq 40$) and smaller $\phi \leq 10\%$. The overall picture suggests that, in almost
4.1 Spherical particles in a pipe

\[ f = \frac{\tau_w}{(0.5 \rho U_{\text{Bulk}}^2)} \times 10^{-3} \]

\[ Re_{\text{Bulk}} = \frac{U_{\text{Bulk}} D}{\nu} \]

Figure 4.1: Variation of friction factor \( f \) with the bulk Reynolds number \( Re_{\text{Bulk}} \) for all particle sizes \( D/d_p \) and mean volume fractions \( \phi \). The solid line corresponds to an empirical correlation corresponding to single phase flow from Duan et al. (2012).

In all cases, the drag with spherical particles is higher than single phase flow (at the same \( Re_{2H} \)) and it monotonically increases with concentration \( \phi \). The drag increase becomes insignificantly small with increasing \( Re_{\text{Bulk}} \), suggesting that at sufficiently high \( Re_{\text{Bulk}} \) the drag would be similar to single phase flow.

The dependence on particle size is more complex and non-monotonic; the net result being determined by the relative change in turbulent and particle-induced stresses. These stresses are intimately connected to the particle distribution, which itself is a function of the particle size and volume fraction.

Figure 4.1 shows the summary of all the measurements in terms of the commonly used Fanning friction factor \( f \), as a function of \( Re_{\text{Bulk}} \). Larger \( \phi \) yields larger \( f \). The approach of the data points for particle-laden cases towards the single phase flow curve is apparent for increasing \( Re_{\text{Bulk}} \). The above data is replotted in figure 4.2a as a function of a reduced Reynolds number that accounts for the increased suspension viscosity due to the addition of particles. The increased effective viscosity \( \mu_e \) under non-inertial conditions can be accurately described by standard semi-empirical fits like that from Eilers (1941): \( \mu_e/\mu_f = (1 + 5\phi/4(1 - \phi/\phi_{\text{Max}}))^2 \). The parameter \( \phi_{\text{Max}} \) is fixed to a value of 65% (approximately corresponding to random close packing) and for \( \phi < 20\% \), small variations in \( \phi_{\text{Max}} \) has a small influence on \( \mu_e/\mu_f \): at \( \phi = 20\% \), the drag would vary by \(< \pm 1.5\% \) if \( \phi_{\text{Max}} \) is changed by \( \pm 10\% \) respectively.
Figure 4.2: (a) Same plot as in figure 4.1 but, instead as a function of Reynolds number $Re_{Bulk, Eilers}$ based on Eilers effective viscosity. (b) Percentage relative difference between measured friction factor and its predicted value for a suspension with higher effective viscosity expressed as a function of the particle size in inner units $d_p^+$. 

From a cursory glance, the good collapse of data on the single phase curve in figure 4.2 seems to suggest that an effective viscosity based description is practically sufficient to estimate the drag in a suspension, at least for the concentrations used in this study. However, noticeable differences (especially for $\phi = 20\%$) hints towards the role of change in particle size, a parameter that is usually missing from the effective viscosity formulation.

To better appreciate the trends, the previous data is now plotted separately for a few representative $Re_{2H}$ in figure 4.3. Instead of the absolute value of the friction factor, the percentage difference between the value for particle-laden flow $f$ and for single phase flow $f_{SP}$ is plotted (normalized by $f_{SP}$). The prediction based on the effective viscosity is indicated by the dashed lines and the measured values can lie both above (for low $Re_{2H}$, see figure 4.3a) or below (at high $Re_{Bulk}$, see figure 4.3d) these predictions. The solid lines, calculated from the particle-wall ($pw$) theory of Costa et al. (2016), seem to well represent the drag for $\phi \leq 10\%, d_p/D \leq 0.03, Re_{Bulk} \leq 25000$. Above these values, the drag is always lower from what is predicted by this theory. The derivation of the friction factor ($f_{pw}$) based on the particle-wall theory is involves extending the calculations for a channel flow in Costa et al. (2016) to a pipe flow.

Assuming that one of the many empirical correlation for turbulent single phase friction factor (in our case, we have used the explicit relationship by Duan et al. 2012) is applicable in the homogenous suspension region of the
Figure 4.3: Relative change in friction factor (due to addition of spheres) compared to single phase flow of Newtonian fluid at the same $Re_{Bulk}$. The $\star$ symbols in (a) corresponds to DNS simulations at similar experimental conditions (for two sizes and two volume fractions). The solid lines correspond to the predictions from the particle-wall layer theory (Costa et al. 2016). The dashed lines corresponds to the friction factor simply based on an increase in the effective suspension viscosity.

reduced-diameter pipe away from the particle-wall layer, one can find $f_{pw} =$
Figure 4.4: Results for DNS at $Re_{Bulk} = 10150$ for two particles sizes ($D/d_p \approx 10$ and 21) at both low ($\phi = 5\%$) and high ($\phi = 20\%$) volume fractions. (a) Particle concentration profile along the radial direction and (b) fractional contribution of stresses normalized by the value for single phase flow at the same $Re_{Bulk}$: $\sum \tau_V = $ viscous, $\sum \tau_T = $ turbulent and $\sum \tau_P = $ particle-induced contribution. The first $\#\#$ in $P\#\#Dd\#\#$ (on the x-axis of figure (b)) corresponds to the percent volume fraction and the second $\#\#$ corresponds to the particle size $D/d_p$. Courtesy of Mehdi Niazi Ardekani, KTH.

\[
(1 - 2\delta_{pw}/D)^{-1}(3.6\log_{10}(6.115/(Re_{Bulk}(1 - 2\delta_{pw}/D)(\nu_c/\nu)\sqrt{\pi/4})))^{-2}.
\]

Here, $\delta_{pw}$ is equal to the thickness of the particle-wall layer $\approx d_p$. Thus, for a limited range of parameters (mentioned above), the increase in drag can be attributed to the formation of a quasi-steady particle layer near the wall. With increasing particle size, the apparent departure from theory, which is shown to work quite well for a channel flow for a range of particle sizes and $Re_{Bulk}$, is most likely due to particle migration towards the core, thus violating the assumption of a uniform $\phi$ in the central region. Due to this migration, drag increases at a slower rate with increasing particle size (at a fixed $\phi$).

Migration is ascertained using numerical simulations as shown in figure 4.4a. The drag in simulations is in very good agreement with the experimentally measured value for the larger particle size $d_p/D \approx 0.1$, as shown in figure 4.3a. The agreement is not so good for the smaller particle size: $d_p/D \approx 0.05$. A separate simulation, with increased coefficient of friction, showed a marginally higher value of drag at the highest $\phi$ but, this was unable to explain the discrepancy.

A potential cause of the discrepancy could be the limited development length available in experiments ($L/D \approx 230$) and hence, a longer pipe would be needed to shed more light on this issue. Nevertheless, the peculiar non-monotonic change in drag is qualitatively observed in both experiments and
simulations: at low $\phi = 5\%$, the drag reduces with increasing particle size from $d_p/D \approx 0.05$ to 0.1; however, at higher $\phi = 20\%$, the drag increases with the same increase in particle size (see figure 4.3a). Migration and the non-monotonic change in pressure drop with particle size was initially observed in our work with turbulent square duct flow (Zade et al. 2018), which prompted the extension of such measurements to a round pipe flow.

The above non-monotonic behaviour can be quantitatively understood by calculating the average stress-contribution due to the viscous, turbulent and particle effects (see section 1.5). This stress distribution is shown in figure 4.4b. The viscous stresses remain very small, nearly equal to single phase flow and approximately constant for all particle-laden cases. At low $\phi = 5\%$, turbulent stresses increase for smaller particles and decrease for larger particles (compared to single phase flow). Initial increase in turbulent Reynolds shear stress for low $\phi$ can be due to increase in the intensity of ejection and sweep events (Pan & Banerjee 1997). Alternately, Shao et al. (2012) reasoned that particles disrupt the large coherent structures and thus, reduce the streamwise fluctuations. However, particles may also enhance wall-normal fluctuations, especially at low $\phi$, due to creation of small scale coherent structures at the particle-scale i.e. vortices shed by the particles. These opposing effects may cause a marginal increase in the Reynolds shear stress for smaller $\phi$. Such an increase was also observed by Picano et al. (2015), who investigated only one particle size.

For high $\phi = 20\%$, the turbulent stress decreases for both particle sizes, due to intense particle-fluid interaction that destroys the coherence of both large and small flow structures; the decrease being more prominent for the larger particles. Thus, it can be said that the initial increase in turbulent stresses with concentration saturates at some maxima at a certain threshold value of $\phi$, and this threshold is lower for larger particles. This decrease in turbulent stresses for larger particles is more than compensated by a significant increase in the particle-induced stresses, at $\phi = 20\%$. At $\phi = 5\%$, the particle-induced stresses were lower and nearly equal for both particle sizes. However, for $\phi = 20\%$, in the core region (see figure 4.4a), the particle concentration for large particles approaches the maximum packing fraction. Turbulence is greatly suppressed and particle-induced stress is the dominant momentum transport mechanism in this region (figure not shown). Smaller particles, on the other hand, display a more uniform concentration profile away from the wall, which explains the greater success of the particle-wall layer theory for that case (see figure 4.3a).

For increasing $Re_{Bulk}$, as shown in the panels of figure 4.3, the percentage change in drag successively declines. A recent numerical study by Yu et al. (2019) has also highlighted that under similar particle size and volume fractions the effect of particles on turbulence modulation is lower with an increase in $Re_{Bulk}$. Another evidence about the dwindling effect of particles at higher Reynolds number comes from the experimental study of Bakhuis et al. (2018), albeit in a Taylor-Couette flow set-up at significantly higher Reynolds number for $\phi$ up to 6\%.
Particle size in inner units gives an approximate idea about the size of particles relative to the smaller scales in turbulence. The relative change in drag with particle size (in inner units) for all cases is shown in figure 4.2b. For larger \( d_p^+ \geq 150 \) or, alternately, for a larger \( Re_{bulk} \) at a given particle size, the drag lies within \( \pm 5\% \) of the prediction based on a simple effective viscosity formulation. This hints that at larger \( d_p^+ \), the concentration distribution is more uniform and there is no need for a particle-wall layer compensation. Deviation from the above behaviour occurs, especially for \( \phi = 20\% \), for smaller \( d_p^+ \leq 150 \) or, alternately, for lower \( Re_{bulk} \). And these differences are most likely due to migration and consequent reduction of turbulence and increase of particle induced stresses. The exact cause behind the above observations is still a matter of discussion.

4.2. Oblate particles in a pipe

A similar study as above was undertaken with oblate disk like particles in the pipe. This was done with the aim to check whether oblate particles can induce drag reduction compared to single phase flow, as recently seen in the turbulent channel flow simulations of Ardekani et al. (2017). Two particle sizes with an aspect ratio (diameter to height) \( AR = 1/3 \) (corresponds to the same \( AR \) as in Ardekani et al. 2017) and 1/5 with a pipe to particle equivalent diameter \( D/d_{eq} = 9 \) and 6.5 respectively were investigated up to \( \phi = 19\% \). The equivalent diameter \( d_{eq} \) of the oblate particle corresponds to a sphere with the same volume.

The panels in figure 4.5 shows the measured value of drag as percent relative change compared to single phase flow, in a similar fashion to figure 4.3. A small drag reduction is seen at a low \( Re_{Bulk} = 4000 \) for \( \phi = 10\% \). Thereafter, the drag is higher for all \( Re_{Bulk} \) and it increases with \( \phi \). On comparing figures 4.3a (for spheres) and 4.5d (for oblates), both of which corresponds to nearly the same \( Re_{Bulk} \), an important observation is that the drag is substantially lower than for spherical particles for the same \( \phi \) and \( D/d_{eq} \). Smaller oblates seem to lower the drag more than larger oblates.

The rationale behind drag reduction observed in the turbulent channel flow of Ardekani et al. (2017) was the disappearance of the particle-wall layer. Moreover, oblates in the near-wall region rotate significantly slower than spheres near the wall and tend to statistically stay parallel to the wall, which leads to a decrease of the turbulent mixing. The curvature of the round pipe disrupts such an alignment. Furthermore, the alignment is more difficult for the larger oblates than for the smaller ones. This could, perhaps, explain the absence of drag reduction for the pipe as compared to the channel.
4.2. Oblate particles in a pipe

Figure 4.5: Relative change in friction factor (due to addition of oblate disc like particles) compared to single phase flow of Newtonian fluid at the same $Re_{Bulk}$. 
Chapter 5

Summary of the papers

In the following paragraphs, a short summary of the papers appended in the later part of the thesis is provided. These papers describe results for a square duct flow.

Paper 1

*Experimental investigation of turbulent suspensions of spherical particles in a square duct*

Understanding the effects of variation in particle size and concentration at different flow rates of turbulent flows is a pre-requisite if predictive abilities for such systems are to be developed. The presence of finite-size particles puts strenuous demands on numerical simulations to resolve the flow down to the smallest relevant scale in order to accurately describe the interaction between the phases. Such a numerical exercise becomes prohibitively expensive with increasing flow Reynolds number $Re_{Bulk}$. Hence, experiments are ideally suited to probe the above extreme regimes, provided measurements can be performed with acceptable spatiotemporal resolution.

With this thought in mind, in this work we have constructed an experimental facility that consists of a commonly observed flow geometry - a square duct. By suspending refractive-index-matched (RIM) spherical hydrogel particles in water, a Newtonian fluid, the combination of Particle Image Velocimetry (PIV) and Particle Tracking Velocimetry (PTV) is used for measuring the fluid and particle phase velocities under turbulent flow conditions at moderately dense concentrations.

Three particle sizes namely: $2H/d_p = 40, 16$ and $9$ ($2H$ being the duct full height and $d_p$ being the particle diameter) are each investigated at volume fractions of $\phi = 5, 10$ and $20\%$. For the variation of bulk flow Reynolds number $Re_{2H} \in (10000, 27000)$, this translates to a particle size in inner units varying from $d_p^+ \in (15, 165)$. The chosen particles are nearly neutrally-buoyant with a particle to fluid density ratio ($\rho_p/\rho_f$) of 1.0035 for the two largest sizes (which are also optically clear and thus permit PIV-PTV) and 1.01 for the smallest size (which are semi-transparent and therefore prevent PIV-PTV measurements). The set-up is satisfactorily validated against previously published results for
single phase flow statistics at a lower Reynolds number $Re_{2H} \approx 5600$, as well as for turbulent pressure drop over a range of $Re_{2H}$. Velocity and concentration statistics are reported for three spanwise planes, namely $z/H = 0$ (plane of the wall-bisector), $z/H = 0.4$ and $z/H = 0.8$. The side walls are located at $z/H = \pm 1$.

In the particle-laden cases, due to slight mismatch in density, gravitational settling effects are seen in the horizontal duct at lower $Re_{2H}$. This is qualitatively explained by calculating the ratio of the particle sedimenting velocity $U_T$ in quiescent conditions to the characteristic turbulent velocity $u_\tau$, i.e. the Rouse number $Ro = U_T/\kappa u_\tau$, $\kappa$ being the von Kármán constant. For the lower $Re_{2H} \approx 10000$, this corresponds to particles of all sizes preferentially residing in the bottom half of the duct, more in the central plane, $z/H = 0$, than closer to the side walls. The inhomogeneous particle distribution is reflected in the asymmetry of the mean streamwise velocity, which is faster in the top half of the duct due to fewer particles than the bottom half. At the highest $\phi = 20\%$, the flow in this particle rich bottom region is characterised by extreme reduction in turbulent fluctuations, similar to that in a porous bed that is slowly moving forward. Away from the particle rich region, the streamwise turbulence intensity and primary Reynolds shear stresses increases. The average drag (friction factor) is significantly larger than that of single-phase duct flow at this low $Re_{2H}$ and increases with $\phi$. For the lowest $Re_{Bulk} \approx 10000$, the pressure drop is fairly independent of the particle size if $\phi \leq 10\%$. Size dependence only appears at larger $\phi$.

The picture changes for the highest $Re_{2H} \approx 27000$, where particles are in almost full suspension and the drag is not significantly different than single phase flows. Despite the relatively smaller increase in drag compared to the earlier case at lower $Re_{Bulk}$, size dependence is now apparent at all $\phi$. The drag is generally higher than single phase flow but reduces monotonically with increasing particle size for $\phi$ up to 10\%. This is attributed to the reduction of turbulent stress contribution to the stress budget: larger particles more effectively break coherent eddies in the flow and hence, reduce the turbulent stresses. However, for $\phi = 20\%$, the drag first decreases and then increases with increasing particle size thus, showing a non-monotonic behaviour. This happens despite the larger reduction of turbulent stresses for larger particles, and hence, must be due to increased particle-induced stresses. Particles migrate towards the core due to shear-induced inertial effects which damps turbulence but increases particle stress in that region. An alternate mechanism is hypothesised for the drag increase for the smallest particles at $\phi = 20\%$: since the smaller particles are larger in number for a given $\phi$, in the near-wall region this could lead to more number of contact points and collisions with the wall. This may increase the friction even if the momentum of each particles decreases for reducing particle size. The fact that the observed drag can lie both above or below the predictions of an effective viscosity formulation, depending on the flow Reynolds number and particle size, has given insights into the complexity of
Summary of the papers

particle-fluid interactions. These results have also motivated simulations as well as experiments to investigate drag for multiple particle size and volume fractions in a turbulent pipe flow.

Paper 2

*Buoyant finite-size particles in turbulent duct flow*

The assumption of neutral-buoyancy allows the study of finite-size effects in the absence of gravity. However, in many applications, e.g. sediment transport, the role of gravity is important and particles may either continually slide and roll along the channel bottom, or undergo an intermittent saltation type of motion induced by lift and drag forces by surrounding fluid turbulence, as well as due to collisions with neighbouring particles.

In this work, we perform RIM PIV-PTV experiments with hydrogel particles ($\rho_p/\rho_f = 1.0035$) in the same set-up as in the study reported in Paper 1 at a smaller, yet turbulent, $Re_{2H} = 5600$. Two particle sizes: $2H/d_p = 14.5$ and 9 for multiple $\phi \in [1,5]\%$ are used, which on account of density difference and finite-size mostly travel along the bottom of the square duct. Interface resolved DNS simulations using the immersed boundary method are also performed for one case, $\phi = 1\%$ of smaller particles ($2H/d_p = 14.5$), to compare with experimentally measured particle distribution, velocity statistics and drag. Despite a very small average $\phi$, preferential accumulation of particles at the bottom of the duct raises the local average volume fraction to around 10% and thus, excellent agreement between experiments and simulations serves to cross validate both approaches. It partly demonstrates the suitability of hydrogel particles to be used as rigid spheres.

Results show that in the sedimenting regime, the average drag is very similar for both particle sizes for a given $\phi$, an observation that is coherent with our previous study at $Re_{2H} \approx 10000$ with sedimentation up to $\phi = 10\%$. However, the fluid velocity statistics are noticeably different in terms of the Reynolds shear stress, streamwise and wall-normal turbulent intensities. The changes are more pronounced for smaller particles at the same $\phi$, indicating the importance of particle number. Particle centred mean concentration and velocity distribution has revealed the drafting motion of a trailing particle in the wake of the reference particle, which can cause particles to cluster and move in a train-like manner along the bottom wall.

Paper 3

*Turbulence modulation by finite-size spherical particles in Newtonian and viscoelastic fluids*

Viscoelastic polymer additives, even in very small quantities, are known to considerably reduce turbulent drag and this phenomenon has been investigated in many studies over the last five decades. However, very little is know about the effect of finite-size particles in this viscoelastic turbulence regime.
This is one of the first studies that explores such a system. Accordingly, the influence of varying particle concentration $\phi \in [0, 20\%]$ of neutrally-buoyant particles of a fixed size, $2H/d_p = 10$, on drag and turbulence field is investigated in our square duct. The $Re_{Bulk} \approx 11000$, is kept nearly constant for both Newtonian (NF) and Viscoelastic (VEF) suspending fluid for consistent comparison. Effects of sedimentation are negligible due to the high flow rates in both types of fluids.

The Weissenberg number of the VEF flow is indirectly estimated to lie between 1-2 and, for single phase flow, the drag reduction is around 43% when compared to a NF flow (at the same $Re_{Bulk}$, is kept nearly constant for both Newtonian (NF) and Viscoelastic (VEF) suspending fluid for consistent comparison). Effects of sedimentation are negligible due to the high flow rates in both types of fluids.

The total pressure loss for particles in NF is significantly higher than the predictions based on the increased effective viscosity of the suspension, for $\phi \geq 25\%$. As with NF, particles are observed to preferentially migrate towards the core in VEF with increasing $\phi$. However, the migration towards the core is stronger and weaker towards the wall in VEF, for all $\phi$ considered. This has been attributed to the weaker turbulence in single phase VEF to start with. Migration towards the core further suppresses the Reynolds shear stresses but, also increases the particle-induced stresses. The resultant effect is that total drag rises for VEF with $\phi$, at a faster rate than in NF. Interestingly, if drag is compared at the same flow rate, as is often done in drag reduction studies, the drag reduction effect due to polymer additives is completely overshadowed by particles.

Furthermore, the streamwise and wall-normal fluid velocity fluctuations are reduced in a large portion of the channel for NF, whereas they may increase or decrease for VEF. The magnitude of the secondary flow velocity increases with $\phi$ for both types of fluids; more for VEF than NF.

**Paper 4**

*Finite-size spherical particles in a square duct flow of an elastoviscoplastic fluid: an experimental study*

In this work we extend our experimental campaign to study particle dynamics in a different kind of non-Newtonian fluid, with viscoplastic as well as elastic properties. Even after yielding, the viscosity of the fluid is significantly higher than other fluids used in our previous studies and hence, the single phase flow has a smaller Reynolds number.

Single phase flow of two elastoviscoplastic fluids, with different rheological constants: yield stress $\tau_y = 1.38$ (thicker) and 0.26 (thinner), is studied using PIV for multiple spanwise planes, to determine the extent of unyielded plug in the core region as well as the intensity of secondary flow that is driven by elasticity in the yielded region away from the core and the corners. With
increasing Reynolds number, the intensity and size of the secondary flow region increases and the fraction of the unyielded region decreases. The friction factor agrees well with the laminar Newtonian correlation \((f = 16/Re^*)\) if an equivalent Reynolds number \(Re^*\), accounting for the parameters of the Herschel-Bulkley model, is used. The \(Re^* \in (1, 200)\).

Flow field in the presence of spherical hydrogel particles, having a size of \(2H/d_p = 12\), for \(\phi = 5\) and 10\% is investigated in one of the fluids (thinner) at two spanwise planes: one at the centre and another closer to the wall. The most striking observation concerns the change in particle distribution: at low flow rates particles are largely focussed at the four corners of the duct, whereas at high flow rates they are dispersed between the centre and the corners. The first effect, at low flow rates is supposed to be due to the non-uniformity of the first normal stress difference distribution in the duct. The second effect is supposed to be due to the increasing intensity of secondary flow, that arises due to the second normal-stress difference as well as inertia. Shadowgraphs have also been used for complementary visualisation of the streamwise development of particle distribution. In terms of the effect on the fluid flow field, particles introduce velocity fluctuations that are not correlated in the streamwise and wall-normal directions. The size of the plug is also reduced compared to pure fluid case. Interestingly, an effective viscosity like relationship seems to collapse drag data for all particle-laden cases on the single phase flow curve, a feature that is also observed for particles in a laminar flow of Newtonian fluid.
Chapter 6

Conclusions and outlook

In this work, we have built an experimental set-up that can investigate the full flow features of both the particle (spherical) and fluid phases, in laminar and turbulent flows of Newtonian and non-Newtonian suspending fluid, up to moderately dense concentrations. The associated round pipe facility can be used for relatively simpler drag measurements with many more particle shapes and higher particle concentrations.

6.1. Main results

From the study of varying Reynolds number \( \text{Re}_{2H} \) for different sizes and concentration \( \phi \) of spherical particles, we have obtained important insights about the flow field in both the sedimenting as well as the fully suspended regime where the influence of gravity is negligible. In contrast to laminar flows where drag is only a function of the volume fraction of spherical particles, i.e. an effective viscosity formulation is sufficient, it is shown that in turbulent flows, the drag can be higher (at lower \( \text{Re}_{2H} \)) as well as lower (at higher \( \text{Re}_{2H} \)) than such predictions, and can display a complex dependence on particle size. Increase in particle size reduces turbulent fluctuations but, also increase particle-induced stresses in regions of high concentrations, i.e. towards the core and near the wall. This differential stress distribution is also reflected in the shape of the mean streamwise velocity profile that is more parabolic for smaller particles.

From the study of sedimenting particles at a low \( \text{Re}_{2H} = 5600 \), we have shown a reasonable agreement between the results of fully resolved DNS simulations as well our experiments thus partly validating both methods at least for small \( \phi \). Apart from this, the averaged particle centred statistics through PTV have helped us identify the relative particle concentration and velocity distribution in the neighbourhood of a reference particle in shear. Increased inertia is shown to lead to the formation of long train like particle clusters.

Particles have been shown to increase drag in a viscoelastic turbulent fluid flow. For a given \( \phi \), this increase is higher than the increase in a turbulent Newtonian fluid flow. As in the previous studies, the net increase or decrease depends on the relative contribution of turbulent stresses as well as particle-induced stresses. In turbulent viscoelastic fluid flow, the tendency of particles
to migrate towards the core is higher thus, leading to higher particle-induced stresses.

In laminar elastoviscoplastic fluid, with increasing Reynolds number, the size of the plug in the core reduces and the intensity of secondary flow due to the second normal stress difference increases. This coupled with increasing influence of inertia changes the particle distribution from being localised at the corners to being dispersed between the core and the corners. The drag is well predicted by a simple effective viscosity effect, applicable in the laminar flow regime.

The measurements and simulations in the round pipe have also indicated the robustness of the peculiar non-monotonic behaviour of drag as a function of particle size. The limits of the particle-wall theory is also defined. Larger particles are more prone to migrate towards the core and damp turbulent fluctuations than smaller particles. The cause for the approach of all drag curves to the single phase flow curve at higher Reynolds number needs more investigation. It has also been shown that oblate disk like particles that can cause a marginal reduction in drag in a turbulent channel flow, are not so effective in reducing drag in a round pipe flow.

6.2. Future

As should be apparent from this thesis, the multidimensionality of particle-laden flows presents a fertile turf to any investigator to perform both fundamental and applied research. Some of the observations in our experimental work have been explained whereas other still allow speculation. The most conclusive understanding will perhaps be obtained by using a combination of experiments (say, to make a quick parameter sweep) and fully resolved numerical simulations (to unearth the physical mechanism at a few strategic points in the vast parameter space). Simplified models can then be confidently developed.

Future studies should aim at obtaining more information using the existing data from concluded experiments. As early as the start of the project, it was thought of using stereoscopic PIV and, later, tomographic PIV to yield more information in a larger flow domain. Stereoscopic PIV should be straightforward even in the presence of particles. Instead of aligning the laser sheet along the streamwise direction as done in the present configuration, future studies should orient the laser sheet perpendicular to the streamwise direction. This would illuminate the whole cross-section and reveal the instantaneous particle distribution in the square section. In such a configuration PIV-PTV is well suited to measure the secondary flow and, using stereoscopic PIV, the streamwise component can also be extracted. This would require the use of a pulsed laser to accommodate the resulting fast out-of-plane streamwise motion, instead of the continuous laser used in the present work.

Time-resolved measurements coupled with the Taylor’s hypothesis of frozen turbulence can yield new information about the flow structures as it is done in Dennis & Nickels (2008). We have already developed a stable Lagrangian
particle tracking algorithm to detect and separate different individual particle trajectories. More work in this direction will certainly help in understanding the intermittency in particle motion for high concentrations.

The intensity of Rhodamine dye absorbed by particles is sensitive to change in temperature (Xu & Cai 2009). This property can potentially be used to study heat-transfer applications in the presence of particles.

In the last few months, we have modified the set-up to include a small flow section which can be completely filled with hydrogel particles and the flow of fluid through the interstitial pores can be investigated for varying flow rates; a study that can shed light on the flow physics through deformable porous media. It is thus, a unique facility that would hopefully be used to further the understanding of multiphase flows.

The papers originating from the present work are appended in the latter part of the thesis.
Acknowledgements

This thesis attempts at describing findings that are made in a rather large set-up over a period of slightly more than four years. Building a large set-up, just like writing a long and complex code, is a daunting task and the extent of challenges that it poses can be appreciated in a true measure by those who have been through this process themselves. That being said, it has been an enriching experience, full of important lessons, both from the perspective of science and personal development. Every once in a while one gets fortunate to avail skilful hands in the form of good colleagues, an insightful paragraph in a book which clears longstanding doubts, or inspiration from a hearty talk that motivates one to go on despite extended spells of disappointment. I have regularly been fortunate on all fronts. Finally, only time will tell whether this work was justified, whether it correctly answered a few questions and whether it provoked new ones. Till then, I repeat with John Henry Newman, ‘…one step enough for me.’

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The ambience of the Fluid Physics Laboratory, as any other workplace, has been reflective of the people passing its corridors. During my own short stay, I have been witness to many seniors leaving and new candidates arriving. It only seems like yesterday that I had Tomas Rosén as my room mate and seniors like Marco, Sohrab, Renzo, Julie, Sissy, Ramin and Elias occupying the neighbouring rooms. All of them have now graduated and are doing well. Thank you for your kind association. For those that are part of the present lot, I wish you all the best for the time ahead. A special thanks to Krishne (we’ll be neighbours now), Cecilia (you’re living your dream), the Monday morning meeting group (present - Calvin, Susumu, Jakob, Korneliya, Gusten, Ahmad and Rohan, past - Nitesh, Ugis), the not-so-regular Friday afternoon/Tuesday morning meeting group (present - Outi, Arash, Ali, Dhiya, Anthony, Emad, Nicoló, Kazan, Daulet, past - Ekaterina, Iman, Hamid), Henrik A, Michael L, Antonio S, Anders D, Santhosh, Yushi, Marcus, Sembian, Kentaro, Aidan, Johan, Manash, Simon, Federico, Julien, Fredrik J, Ashwin, Shyang Maw Lim, Niclas, Matteo, Giando and Prabal. I would also like to thank previous postdocs including Ninge, Shumpei, Takuya and Eda. Your company has always cheered me.

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Bibliography


Bec, J., Biferale, L., Cencini, M., Lanotte, A., Musacchio, S. & Toschi, F.


Kawata, T. & Obi, S. 2014 Velocity–pressure correlation measurement based on


Lecha, A. & Lecha, M. 1984 Drag reduction and heat transfer measurements with polyacrylamides on a model of a district heating system. In *Proc. 3rd Int. Conf. on Drag Reduction, Bristol*, pp. 2–5.


Nikuradse, J. 1930 Untersuchungen über turbulente Strömungen in nicht kreisförmi-


Seo, K. W., Kang, Y. J. & Lee, S. J. 2014 Lateral migration and focusing of


Vester, A. K., Örlü, R. & Alfredsson, P. H. 2016 Turbulent flows in curved


Xue, S.-C., Phan-Thien, N. & Tanner, R. 1995 Numerical study of secondary


Part II
Papers