Residual stresses in paperboard and the influence of drying conditions

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Abstract

The drying sequence in the manufacturing process for paperboard involves evaporation of water, primarily from within the fibres. The vapour is then transported out of the web by pressure or concentration gradients. As the moisture transport from the paper web to the ambient is quicker than the moisture transport within the fibre network to the surfaces of the web, moisture gradients develop through the thickness of the web. This work concerns effects on the mechanics of paper drying from the variation in moisture through the relatively thin structures of paper and paperboard.

Distributions of inplane residual stresses through paper materials in the unloaded state after drying are believed to be caused by the varying moisture through the thickness during drying. The distributions in general exhibit compressive stress near the board surfaces and tensile stress in the interior of the board. This may be modified after drying and is also affected by structural variation in the material between different plies of multi-ply paperboards.

The stress development during drying is important because it influences the resulting material properties of the paper and because it can lead to curl, which is a quality problem. The residual stresses themselves are an error source in simulation or evaluation of the mechanical behaviour of paper.

In this work, residual stress distributions in paperboard were determined experimentally, to clarify the mechanisms of residual stress build-up. An experimental method for such tests was also developed. Based on the experimental findings, the mechanics of paper drying was modelled and the stress build-up simulated. Simulation offers a way of studying how the properties of paper develop during drying of wet paper webs.

Keywords: paper, drying, mechanical properties, residual stress, curl
Preface

It has been an excellent couple of years. Paper is an interesting material and I have truly enjoyed getting acquainted with it.

I have had several advisors, which is both good and bad. Sören Östlund has given me lots of freedom and encouraged me to follow the many twists and turns. Leif Carlsson has communicated his opinions on most everything from across the Atlantic in Florida. Christer Fellers gave advice on the experimental work and has provided a well of literature on paper physics. Their efforts are deeply appreciated.

The paper physics group, together with STFI-Packforsk, provides a good atmosphere for discussing research. Petri Mäkelä also graciously contributed to one of the papers. Thank you to past and present colleagues at Solid Mechanics for providing a nice atmosphere in which to work and for playing floorball with me. The discussions with Johan Alfthan (of both of the above groups) on paper science have been very stimulating.

A good side of being a PhD-student is the outlook one obtains of what other PhD-students are doing. Perhaps the most exciting development in paper physics over the last few years has been the surfacing of papers by paper chemists of relevance to paper mechanics. Actors in order of appearance on my horizon have been: Annsoifie Torgnyndotter, Janne Laine, Malin Eriksson, Andrew Horwath, Magnus Gimåker and Jennie Forsström. As most of this work is not of immediate relevance to the present thesis, they are instead mentioned here: thank you for the inspiration!

The work was carried out at KTH Solid Mechanics and at STFI. The project was financed by FPIRC up to 2003 and by BiMaC since then. Tom Lindström, program director of both of these, has definitely had an influence both socially and professionally. And without FPIRC, I might not have met Erik Baggerud, whose project was obviously synergistic to this one.
This thesis consists of an introduction and the following six papers:

**Paper A**

**Paper B**

**Paper C**

**Paper D**

**Paper E**

**Paper F**
"Modelling the influence of drying conditions on the stress build-up during drying of paperboard" by M. Östlund, submitted to *Journal of Engineering Materials and Technology*. (presented at the 2005 ASME ‘IMECE’ conference in Orlando, Florida)
Other reports on this topic, by the same author(s), that are not part of the thesis:


“Experimental determination of residual stresses in paperboard”, Licentiate thesis no. 86, KTH Solid Mechanics, 100 44 Stockholm, Sweden (2003) *(papers B and C were featured in this thesis under different titles)*
Introduction

Sweden is a forested country and the forest industry is one of the country’s most important. Papers and paperboards for different purposes are major products in this industry. The context of this thesis is the manufacturing process for paper and board. The objective of the work is to increase the understanding of this process, to enable the manufacturing of enhanced paper. Although mechanical properties may be more important for packaging paper than for printing paper, the work should in principle apply for the manufacturing of all forms of paper (absorbents being a third major category).

Paper is made by dispersing wood fibres in water and then removing the water, whereby the fibres bond to one another without additional adhesives. The fibres are a few millimeters long and about 20 to 30 \( \mu \text{m} \) in the cross-direction, tube-shaped or ribbon-like depending on their treatment. Because of their length, the fibres will largely be oriented in the plane of a paper. While fibres in the trees or in wood fibre reinforced composites should be stiff, the same fibres should in papermaking be flexible in order to form a good network. To some extent the stiffness may be regained by applying load on the fibres as they are drying, once the network has developed.

Papermaking (see Figure 1) is typically divided into forming, pressing and drying. In forming, the fibre suspension is typically sprayed onto a wire to form the thin paper web, but the forming section also contains the initial dewatering, by gravity and by suction boxes. Pressing continues the dewatering in a series of nips where the web passes between a pair of cylinders. Drying implies that remaining water is evaporated by heat application. The moisture ratio (the weight of moisture normalized by the weight of the solids) is useful to characterize the dewatering. At forming, the moisture ratio is about 100, before pressing of the order of 4, while after pressing it
is only between 1.5 and 1. Drying sections can be over 100 meters long (half of the paper machine), and a characteristic drying time may be about one minute for paperboard, which is thicker paper. The mechanical properties of paper develop as the last water is removed, which is why this thesis is centred on the drying. (The earlier processes are certainly important in making property development possible.)

The cohesion of the paper web would be due to surface forces from the air/water interfaces at high moisture (forming), capillary forces at more moderate moisture ratios while chemical bonds between the fibres are assumed to account for the strength in dry papers [1]. It is actually not clear what type of bonds these may be (hydrogen bonds is a common speculation), but it is known that the capillary forces bringing fibres in close contact are necessary for bonds to develop. Freeze-dried paper does not develop any strength during drying.

**Figure 1:** Schematic of the manufacture of machine-made paper

In the most common type of drying section, the web is guided past steam-heated metal cylinders of 100°C temperature or more. For a symmetrical setup, the web will in general be held in contact with the cylinders by a felt or drying wire on top of the web, one for the top row of cylinders and one for the bottom row as the felt needs to be on opposite sides of the web. The water evaporated at the side facing the cylinder will typically also leave the web through this surface in the free draw after the cylinder. While inplane shrinkage can be restrained while on the cylinder, there is cross machine directional shrinkage in the free draws, in general leading to
variability across the web as well as affecting the average properties of the paper. There is also a single-felted version of cylinder drying, where the drying wire is between the cylinder and the web on alternate cylinders. These cylinders are sometimes vacuum rolls rather than steam-heated.

Drying rate can be increased by simultaneous pressing. This is utilized for example in Condebelt drying [2], where the web is pressed onto a heated steel belt. Here, dewatering is through the opposite, wet side of the web, and the vapour condenses on a cold steel belt on this side. The contact with the smooth hot belt during drying gives good printability to this side and the lack of shrinkage yields high inplane stiffness and strength, while the compaction leads to decreased bending stiffness. There is also a deviation from a flat shape of the dry paper known as curl. Impulse drying, where extreme temperatures and pressures causes water flashing to vapour as the pressure is released followed by water removal by the pressure of the steam, is not yet commercial [3]. Paper qualities where extensibility or fracture energy is important tend to go the other way and use through-air impingement and other non-contacting drying methods to dry the web without hindering the shrinkage at all.

The drying rate for paper is defined as the negative of the time derivative of the moisture ratio multiplied by the grammage (the grammage of paper is dry mass divided by surface area). It is fairly constant for most drying methods going into the drying. As the last of the free water leaves a surface, the drying rate would start to decrease, then stabilize again while there is still free water in the interior and then decrease toward zero as shown in Figure 2. For small moisture gradients, the stabilization (which may even show as a second plateau) would not be easily identifiable. In the industry, the temperature of the drying cylinders is usually increased along
the drying section to maintain a fairly constant drying rate all the way till the web is practically dry. A typical drying rate for such cylinder drying is about 4-8 g/m²s.

Figure 2: Development of the drying rate for drying at constant ambient temperature (from the simulations in Paper F)

The division into forming, pressing and drying is based on the equipment used for the different purposes. Regarding the development of paper properties, a more scientific division of the dewatering would be according to where in the structure the water resides, and what type of water it is. There would be a web saturation point early in the forming section (moisture ratio at about 20), when all the pores of the web are filled with water. This point is perhaps more commonly known as the dry line on a paper machine, as it would also be the point when a gas phase first penetrates the web.
Then there would be a fibre saturation point, when all the remaining water is inside the fibres (moisture ratio about 1). As water between the fibres can largely be removed by pressing, the fibre saturation point would be close to the start of the drying section. The simple centrifugation test to determine the water retention value (WRV) often yields similar values to the fibre saturation point (as determined by the solute exclusion technique). Forsström et al. [4] however indicates that the WRV is highly dependent on water bound to the surfaces of the fibres, which does not influence the fibre saturation point. WRV is supposed to be an estimate of the amount of water present in the paper after pressing and is also affected by fines that will take up water outside of the fibres [5].

The next point of interest is the hygroscopic point (moisture ratio of 0.3), with the significance that the remaining water in the fibres is ‘bound’. Bound water is here defined as water whose physical properties are different from free water, due to interaction with the cellulose. Properties affected include freezing temperature and thermal expansion. Mechanical properties of paper such as stiffness and the shrinkage at decreasing moisture should be related to dewatering of the fibres, but little is known about the relative roles of free and bound water in this respect.

There is of course water present also in dry paper, with a moisture ratio of about 0.09 at 23°C and 50% RH (relative humidity). The relation between RH in the air and moisture in the paper is given by a sorption isotherm, briefly mentioned in paper F. Paper expands less at moisture uptake than it contracted at the initial drying, due to irreversible closure of pores in the fibres and a decrease of water holding ability at the surfaces of the fibres. That different paper qualities are actually at different moisture ratios at 23°C and 50% RH causes problems for most empirical expressions for the sorption isotherm. That the removal of the different water fractions is sequential is an assumption, so it is possible that none of the
specific states described above is actually passed in the drying of paper [6].

The final paper is approximately orthotropic, where the anisotropy is just as much due to drying conditions as to fibre orientation from the forming. It is elastic up to a limit, with significant hardening (or at least deforming at the same load) beyond that limit. Paper is much weaker in the thickness direction than in the in-plane directions. The structure is porous, compressible and hygroscopic. The density is roughly 750 kg/m$^3$, which is half the density of cellulose.

This work concerns residual stresses in paper materials and the stress development that leads up to these. Residual stresses are defined as those that remain in a piece of material when the external forces are removed, implying a distribution of stress over a cross-section as the average stress on any cross-section is zero. Studied here are distributions of the stress components in the two in-plane directions through the thickness, as in Figure 3. Residual stresses are not to be confused with Kubát’s internal stress [7], which is a material property quantifying the time dependence of paper, nor with the drying stress [8], which is the average inplane stress resulting from external loading of the paper during drying.

![Figure 3: A possible residual stress profile](image)

Figure 3: A possible residual stress profile
As the stiffness of paper prior to drying is negligible compared to that of dry paper, the residual stresses should also be negligible prior to drying. There should further be negligible variation of elastic strain through the thickness (that would lead to stress once the stiffness develops) because the bonds between the fibres are not fully developed at the start of drying. After drying on the other hand, there are now more than a handful experimental determinations of residual stress in paperboard, many (but not the first) of which can be found in Papers B, C and D. The main reason to study residual stresses is because they offer clues as to how paper dries. (Asymmetric stress development leads to curl, which is a second good reason to understand the stresses in paper during drying.)

Something should also be mentioned here about different structural levels on which to study paper. Since paper is a fibrous material, loads would presumably be carried by parts of the fibres and not by the pores (“parts” because load is transferred between the fibres at the joints, so both ends of a fibre should be stress-free). Assuming as an example that one third of the cross-sectional area of the paper is fibre cross-sections, the average fibre stress would be three times the macroscopic stress in that direction (see Figure 4). Wood fibres are composites to a much higher extent than paper, so there would definitely be a distribution of stress between the different structural elements of a fibre as well. Throughout this work paper is treated as a continuum, which is adequate so long as the smallest element of the discretization is actually large enough to be representative.
Figure 4: Stress on different structural levels (compare Figure 3): the fibre stresses would be higher than the continuum stress because of smaller area.

There have been a few previous attempts to determine residual stresses on the macroscopic level in paperboard. A group led by Waterhouse in the United States did a pioneering series of investigations in the late 1980s. One article [9] was published at the time while the remaining work was reported in a more recent book [10]. They employed a layer removal method and, at least in some cases, evaluated the data with the Treuting-Read method (see Paper A, where it is also used). The method was fundamentally sound and some interesting results were generated. They did not comment on the many sources of error in the method that might have influenced the results. In 1999, a paper by Sunderland et al. [11] appeared. They used the principle of the layer removal method – to relieve the stress in part of the sheet – without actually removing any material. Instead, they used moisture to relax the stress. Moisture also swells the paper and the difference in curl response between two subsequent moisturizations was assumed to be related to the released stress. A problem is the difficulty in determining how far the moisture has travelled into the paper. There will not be a sharp line dividing the moisturized zone from the dry zone and the question of how much moisture is needed to achieve the stress release was not
addressed. Also, the moisture might remove the effects of restraints during the original drying, thereby causing (local) irreversible shrinkage [12].

Residual stresses in paper are believed to be analogous to cooling stresses in glass and polymers. When manufacturing conditions of these materials are such that the surface of a part solidifies before the interior, compressive stress is found at the surface and tensile stress in the interior, as shown in Figure 3. Temperature, governing the solidification of these materials, has much the same effect as moisture in paper. A decreasing temperature (moisture ratio) leads to thermal (moisture driven) contraction in the material. The time-shift in the contraction of the different layers of the material (from temperature or moisture gradients during solidification) would not lead to residual stresses in itself. However, as the surface layers contract, the material in the centre will just comply. When the interior contracts, it will do so only by compressing the adjoining solid material. This difference in the response of the neighbouring material during the contraction would lead to residual stresses.

A different, and possibly difficult to grasp, way of looking at this comes from the fact that the distribution of elastic strain corresponding to the residual stress must be balanced by other, inelastic forms of strain, for total strain to be constant. Compressive stress at the surfaces corresponds to more of this inelastic strain developing in the surface layers, supposedly because they dry first. The inelastic strains in question were specified for the modelling part of this work while the above actually has to hold, vague as it may be.

There are two primary areas where residual stresses are important. One is shape distortion. That is, the (lack of) symmetry of the residual stress profile influences the shape of the paper. Altering the residual stress-state – intentionally or unwittingly – will thus also change the shape of the paper. The other area of importance is residual stress as a source of error. The residual stresses are in general neglected, more
because of their not being known than because they are assumed small. This will cause the results of in-plane load-displacement calculations to be incorrect. Many tests of mechanical properties also assume for their evaluation that some certain type of stress-state is prevailing in the test piece. If the residual stresses are unknown, this assumption would in general not be valid. A simple example of residual stresses as a source of error is in the tensile test. A test of an elastic-perfectly plastic specimen with the elastic modulus $E=10$ GPa and the yield stress $\sigma_y=10$ MPa would look as in Figure 5 if the specimen had a parabolic residual stress distribution with 10 MPa compression at the surfaces. The determination of a yield stress from such test data would obviously be erroneous.

![Tensile test data](chart.png)

**Figure 5:** Tensile test data for an elastic-perfectly plastic specimen with the yield stress 10 MPa when residual stresses are non-zero

A group of researchers in the United States have proposed a third effect of the residual stresses [13,14]. They argued that the variation in stress causes additional
creep, thereby increasing the creep compliance (which should be a material property). Such an effect on the creep compliance has not been verified. Models using variation in stress as driving force for additional creep are however quite successful in explaining the mechano-sorptive effect, i.e. more creep at cyclic humidity than at constant RH [13].

For certain kinds of residual stress-state, the effects of the stresses are more tangible. Tensile stress at the surface would facilitate crack growth, which might influence the strength of the material. Compressive stress at the surface is dangerous because contact with water would modify the stress-state, leading to dimensional instability. Tensile stress at the surface is stable in that sense, because that is what the water contact would lead to. The desired residual stress-state is likely to be different for different qualities of paper. (Paper with zero residual stress, incidentally, would be just as susceptible to stress-state modification by water contact as paper with compressive stress at the surfaces.)

Papers E and F concern modelling and simulation of the stress development during drying of paperboard. The mechanics model is based on the assumption of additivity of different strains (which is based on the different deformations being caused by independent mechanisms). The development of all forms of strain except the elastic strain was explicitly modelled, while the total strain may be either prescribed or calculated from prescribed forces using force equilibrium. Types of strain include a hygroscopic strain that corresponds to the shrinkage of paper when the fibres de-swell. This strain should not vary through the thickness of the dry paper. The inelastic strains that balance the variation in elastic strain were due to time-dependent material behaviour and to the increase in stiffness during drying. An increase in stiffness corresponds to less contraction at unloading, i.e. a loss of elastic
strain and a corresponding build-up of inelastic strain. Experimental results for residual stress distributions in paperboard were well predicted.

While the objective of modelling was here primarily to study the stress development, a model such as in paper F opens a much bigger perspective. It could in principle be used to study all of what happens to the paper from the press section onwards or ultimately from forming of the web onwards, if desired. Supposing some understanding of how properties such as strength develop during drying, there would be no principle difficulty in including them in the simulations. A model of paper drying and its impact on the material would likely provide an important tool for predicting the properties of paper after drying. Such predictions may perhaps be done as in [15], based on empirical correlations and neglecting the development phase. It is however felt that studying the development of all properties through the manufacturing based on physics offers additional value. Whether it is necessary to consider through-thickness variation in the web or whether simpler 2D modelling is adequate will of course vary from case to case. Simulation offers a cost-effective way of studying changes to a given drying process or drying equipment compared to full scale testing as well as improving the understanding of how the wet web is transformed into paper.

While many properties of paper are primarily moisture-dependent, there is also an influence of applied load during drying. Drying the paper under tension increases in-plane stiffness and strength while it decreases strain to failure, hygroexpansivity, time-dependence of the material and out-of-plane strength. It also increases the final dimensions of the sheet. The reasons for the change in property in terms of the structure are only qualitatively known. The main directions in paper are known as MD (machine direction), CD (cross-machine direction) and ZD (thickness direction). At least stiffness is actually only changed in the direction of the tension and
not in the cross-direction [16]. The inplane elastic moduli were the only material properties whose development was included in the model for the mechanics of paper drying (they are required rather than optional to calculate stresses).

The build-up of stiffness in paper during drying is of course a field of its own and there is both previous research in the literature and much debate going on in the research community. Htun [8] argued that stiffness is more influenced (increased) by stress at higher moisture ratio than at the end of drying. Wahlström [17] showed by experiments that the stiffness correlated in a linear fashion with the total strain during drying. (The same, incidentally, holds for hygroexpansivity, even correlating with the absolute value of total strain for different pulps with different free shrinkage [18].) However, Htun (and de Ruvo) [19] had by then already shown that total strain during drying is not the structural cause of stiffness in the paper. They did this by varying the conditions for stress relaxation in restrained drying, coming up with dramatically higher stiffness for short drying time and low temperature. This indicates that stress relaxation during drying tends to decrease stiffness in the dry paper, while at constant total strain during drying.

Zhang et al. [20] demonstrated that it is not entirely easy to vary the amount of stress relaxation during drying of paper. They showed a constant stiffness for varying drying temperature (and varying stress development) at restrained drying, which indicates that for practical purposes the linear relation between strain during drying and stiffness may well hold also for different drying conditions. (Figure 6 in Paper C shows a case where it does not hold.) Unless the result of [19] is ever contradicted, the linear dependence of stiffness on total strain during drying is however limited to being a useful simple model and can not claim to be the structural reason for stiffness in paper. In papers E and F, the stiffness was assumed to be affected by the stress at each point in time during drying, and it was shown that all known
experimental results can be fulfilled with this type of a model, at least regarding stiffness averaged over the thickness. The reason for using such a model of the physical dependence was that the objective was to increase the understanding of the drying process, rather than to obtain good enough predictions.

The ambition was however to correctly predict also local elastic moduli through the thickness. Supposing that stress at each point in time during drying influences the stiffness and the stress varies through the thickness during drying, there should be every possibility to obtain a profile of elastic modulus through the thickness.

(Compare for example the larger inelastic strain in the surface layers when there is compressive residual stress in the surface layers mentioned on page 9: build-up of inelastic strain is one effect of stress during drying.) The experimental results of a stiffness distribution in Paper E were not conclusive and with Paper F it was shown that both significant and negligible stiffness distributions can be obtained with the same basic dependence of stiffness on the stress during drying. The conclusion would be that it is difficult to avoid structural variations through the thickness influencing the stiffness data. More research is needed.
Summary of the papers and division of work between the authors:

Paper A: A method for experimental determination of the through-thickness distribution of residual stress in paperboard is presented. It relies on removing layers from the board by surface grinding and determining the change in curvature as bending stiffness and, possibly, stress distribution is changed. The stresses in the original board are calculated from the curvatures using the Treutling-Read method. Geometrically non-linear effects at the large deformations taking place are avoided by performing the tests on narrow enough strips of the board. Other sources of error are investigated and commented upon. The most important are the cutting of the strips, the stress introduced by the grinding and the possibility of plastic deformation when the strips are flattened.

M. Östlund did the work, S. Östlund, Carlsson and Fellers provided guidance.

Paper B: Experimental results are presented for some laboratory-made paperboards. The degree of restraints during drying is shown not to influence the residual stresses in a significant way. A higher level of beating of the pulp, however, leads to higher residual stresses. All of the residual stress-states determined showed compressive stress at the surfaces. This does at least not contradict the hypothesis that the gradient in moisture ratio during drying is the cause of residual stresses on the macroscopic level in paper. Surprisingly, the drying gradient did not influence the total free shrinkage of paperboards. It was discussed why residual stresses in paperboard do not decrease with time like stress in a stress relaxation test does.

M. Östlund did the work, S. Östlund, Carlsson and Fellers provided guidance.
**Paper C:** Like in paper B, residual stress distributions for a number of laboratory-made paperboards are presented. This time drying conditions were varied. This was done to study the relation between moisture transport scenarios during drying and the residual stresses. That the drying gradient causes the residual stresses in paperboard was here shown beyond reasonable doubt. The different drying conditions influenced also the mechanical properties of the boards. Relaxation and the dependence of paper properties on drying stress are discussed in that context. Finally, the relation between residual stresses and curl of paper is discussed.

M. Östlund did the work, S. Östlund, Carlsson and Fellers provided guidance.

**Paper D:** As in B and C, experimentally determined residual stresses are presented, in this paper for boards made on an industrial paper machine. The influence of post-production operations like surface sizing, curl control and coating were shown, as well as providing an overview of what happens with regard to residual stresses in real papermaking. The main result was that operations involving rewetting of the board surfaces lead tensile stress to build-up in the surface layers. This implies that intentionally reversing the residual stresses after drying might be good for (inplane and out-of-plane) dimensional stability.

M. Östlund did the work, S. Östlund, Carlsson and Fellers provided guidance.
**Paper E:** The first of the modelling papers studies the effects of variables possibly varying through the thickness on the mechanics of paper drying. A material model for drying paper was presented and used to elucidate what a varying moisture ratio through a paper material might lead to. The first result was that moisture gradients during drying influence the development of shrinkage and stiffness, meaning that the local behaviour of paper cannot be determined from macroscopic tests. Secondly, it was shown that the build-up of stress leading up to the residual stresses can be simulated from a sound description of the physics. Lastly, it was shown that a varying stress history through the thickness may lead to a variation in material properties.

Petri Mäkelä, in the course of his work at STFI-Packforsk, made the majority of the experiments. M. Östlund made remaining experiments, the modelling, the simulations and wrote the major part of the paper. S. Östlund provided guidance.

**Paper F:** Here, an attempt was made of simulating all that happens in paper drying from basic input data such as temperature and humidity of the drying media and prescribed forces or strains. The drying model predicted that high temperature drying, although causing large moisture gradients at the start of drying, also removes the moisture gradients at a relatively higher moisture ratio, eliminating the effects on the mechanics. The effect of drying conditions on mechanical properties was also simulated and compared (favourably) to the existing experimental evidence. Also curl was calculated for asymmetric drying and a stress distribution for a multiply board with structural variation through the thickness was shown.

M. Östlund did the work. However, the paper relies heavily on the published works of Erik Baggerud.
Suggestions for future work

The model developed in paper F may be used to study a paper web being transported through a drying section, in order to observe the variability across the web and the effects of boundary conditions. Presumably, this requires the model to be implemented in a finite element code. It could have been attempted already after paper E as the final work of this thesis, but was not given priority.

The equations presented here governing paper shrinkage in all three dimensions as water is removed are entirely empirical, which is the biggest single weakness of the model. They should at the very least be based on a proper understanding of how paper shrinks, i.e. how de-swelling of fibres relate to paper shrinkage, and ideally be modelled explicitly in terms of shrinkage forces. This feature of the model is necessary to predict the density variations through paperboard that develops during drying. Of course it might also open the way for modelling of how fibre joints form and thereby how stiffness and strength develops in terms of structure of the web.

Finally, the modelling of material properties such as elastic moduli and time constants featured here are very much first attempts. Further research would be recommendable. The same applies to properties like permeability and diffusivity, though that research may not be done within this academic discipline. The understanding of the build-up of these properties should however benefit much from the above mentioned description of paper shrinkage, as the properties are primarily structurally controlled.
References


