Introduction of the Academic Factor Quality Minus Junk to a Commercial Factor Model and its Effect on the Explanatory Power

An OLS Regression on Stock Returns

MARIT ANNINK

REBECCA LARSSON
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Abstract

The ability to predict stock returns is an ability many wish to possess, and in an accurate way as possible. For many years there has been an interest in the field of factor models explaining the returns, with the aim to increase the explanatory power. This is however a complex business since the factors and their improvement of explanatory power need to be significant. Now and then, researchers come up with new significant factors that have a positive impact on models. AQR Capital Management is no exception to this, since they in 2013 presented the factor Quality Minus Junk, earning significant risk-adjusted returns. This bachelor thesis work within mathematical statistics and industrial engineering and management, aims to investigate whether or not the commercial multi-factor model used at the public pension fund Fjärde AP-fonden will be improved by adding the factor Quality Minus Junk, in the sense of explanatory power. The method used is mainly based on multiple linear regression and three three-year time periods are studied ranging from 2010 to 2018. The results from this thesis work show that the QMJ factor provides significant increases in explanatory power for one of three time periods, the most recent period 2016–2018. However, since the results are inconclusive further studies are needed in order to better understand how to interpret the results and whether or not to include the QMJ factor in the model.

Keywords

Sammanfattning


Nyckelord

Faktormodeller, Riskmodeller, Quality Minus Junk, Regressionsanalys, Kandidatexamsarbete, Tillämpad matematik, Fjärde AP-Fonden, Förklaringsgrad.
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Authors

Marit Annink, annink@kth.se
Rebecca Larsson, relarss@kth.se
Industrial Engineering and Management
KTH Royal Institute of Technology

Place for Project

Stockholm, Sweden
KTH Royal Institute of Technology and Fjärde AP-Fonden

Examiner

Jörgen Säve-Söderbergh
Department of Mathematics
KTH Royal Institute of Technology

Supervisors

Jimmy Olsson
Head of Division Mathematical Statistics
Department of Mathematics
KTH Royal Institute of Technology

Julia Liljegren
Doctoral Student
Department of Industrial Economics and Management
KTH Royal Institute of Technology

Nils Everling
Quantitative Analyst
Fjärde AP-Fonden
## Contents

1 Introduction ................................. 1
  1.1 Background .................................. 1
  1.2 Purpose ..................................... 2
  1.3 Problem Formulation ......................... 2
    1.3.1 Research Questions ....................... 2
  1.4 Knowledge Base .............................. 2
  1.5 Disposition of Report ....................... 3
  1.6 Scope ...................................... 3

2 Mathematical Theory .......................... 4
  2.1 Multiple Linear Regression ................... 4
  2.2 Ordinary Least Squares ....................... 4
  2.3 Explanatory Power ............................ 5
  2.4 Detection and Treatment of Outliers .......... 5
    2.4.1 Cook’s Distance ......................... 6
    2.4.2 DFBETAS ................................ 6
    2.4.3 DFFITS ................................ 6
  2.5 Hypothesis Testing ............................ 7
    2.5.1 Test for Significance of Regression ...... 7
    2.5.2 t-test .................................. 7
    2.5.3 F-test .................................. 7
  2.6 Multiple Testing .............................. 8
  2.7 Overfitting .................................. 8
  2.8 Spearman’s Rank Correlation Coefficient .... 8

3 Financial Theory .............................. 9
  3.1 Return ..................................... 9
  3.2 Risk-Adjusted Return ......................... 9
  3.3 Investment Strategies ....................... 9
  3.4 Risk ....................................... 9
  3.5 Market Beta ................................ 10
  3.6 MSCI Europe Index ........................... 10

4 Factor Model Theory ........................... 11
  4.1 Linear Factor Model .......................... 11
  4.2 Previous Work ............................... 11
  4.3 Commercial Risk Model ....................... 12
  4.4 Quality Minus Junk .......................... 13

5 General Method ............................... 16

6 Descriptive Method ............................ 16
  6.1 Assumptions ................................. 16
  6.2 Tools ..................................... 16
  6.3 Data Collection .............................. 16
    6.3.1 AP4 .................................. 16
    6.3.2 AQR Capital Management .................. 17
  6.4 Data Transformation ......................... 17
<table>
<thead>
<tr>
<th>Section</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.4.1</td>
<td>Transformation of Date Format</td>
<td>17</td>
</tr>
<tr>
<td>6.4.2</td>
<td>Transformation of Data Type Format</td>
<td>18</td>
</tr>
<tr>
<td>6.4.3</td>
<td>Transformation of Return Format</td>
<td>18</td>
</tr>
<tr>
<td>6.5</td>
<td>Weighting of Stocks</td>
<td>18</td>
</tr>
<tr>
<td>6.6</td>
<td>Deletion of Outliers</td>
<td>18</td>
</tr>
<tr>
<td>6.7</td>
<td>Time Period</td>
<td>19</td>
</tr>
<tr>
<td>6.8</td>
<td>Part 1: 12-factor Model</td>
<td>19</td>
</tr>
<tr>
<td>6.8.1</td>
<td>Regression</td>
<td>19</td>
</tr>
<tr>
<td>6.8.2</td>
<td>Regression Analysis</td>
<td>19</td>
</tr>
<tr>
<td>6.9</td>
<td>Part 2: Add QMJ to 12-factor Model</td>
<td>19</td>
</tr>
<tr>
<td>6.10</td>
<td>Part 3: 13-factor Model</td>
<td>20</td>
</tr>
<tr>
<td>6.10.1</td>
<td>Regression</td>
<td>20</td>
</tr>
<tr>
<td>6.10.2</td>
<td>Regression Analysis</td>
<td>20</td>
</tr>
<tr>
<td>6.10.3</td>
<td>Beta Analysis</td>
<td>20</td>
</tr>
<tr>
<td>6.10.4</td>
<td>Sector Analysis</td>
<td>20</td>
</tr>
<tr>
<td>6.11</td>
<td>Comparison</td>
<td>21</td>
</tr>
<tr>
<td>7</td>
<td>Results 22</td>
<td></td>
</tr>
<tr>
<td>7.1</td>
<td>Part 1: Without QMJ 22</td>
<td></td>
</tr>
<tr>
<td>7.1.1</td>
<td>Deletion of Outliers 22</td>
<td></td>
</tr>
<tr>
<td>7.1.2</td>
<td>Explanatory Power 22</td>
<td></td>
</tr>
<tr>
<td>7.2</td>
<td>Part 2 23</td>
<td></td>
</tr>
<tr>
<td>7.2.1</td>
<td>Deletion of Outliers 23</td>
<td></td>
</tr>
<tr>
<td>7.2.2</td>
<td>Explanatory Power 23</td>
<td></td>
</tr>
<tr>
<td>7.2.3</td>
<td>Stock Analysis 24</td>
<td></td>
</tr>
<tr>
<td>7.2.4</td>
<td>Factor Beta Significance for All Stocks 24</td>
<td></td>
</tr>
<tr>
<td>7.2.5</td>
<td>Weighted Portfolio Beta Significance 26</td>
<td></td>
</tr>
<tr>
<td>7.2.6</td>
<td>Factor Beta Percentage of Whole Model 27</td>
<td></td>
</tr>
<tr>
<td>7.2.7</td>
<td>Number of Stocks Per Sector 28</td>
<td></td>
</tr>
<tr>
<td>7.2.8</td>
<td>QMJ Factor Significance by Sector 28</td>
<td></td>
</tr>
<tr>
<td>7.2.9</td>
<td>Percentage of QMJ-Beta per Sector 29</td>
<td></td>
</tr>
<tr>
<td>7.2.10</td>
<td>Factor Correlations 29</td>
<td></td>
</tr>
<tr>
<td>7.3</td>
<td>Comparison 31</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>Discussion 32</td>
<td></td>
</tr>
<tr>
<td>8.1</td>
<td>Mathematical Discussion 32</td>
<td></td>
</tr>
<tr>
<td>8.1.1</td>
<td>Discussion of Explanatory Power 32</td>
<td></td>
</tr>
<tr>
<td>8.1.2</td>
<td>Discussion of Significance 33</td>
<td></td>
</tr>
<tr>
<td>8.1.3</td>
<td>Discussion of Weighted Portfolio Beta Significance 34</td>
<td></td>
</tr>
<tr>
<td>8.1.4</td>
<td>Discussion of the Sensitivity of the QMJ Factor 34</td>
<td></td>
</tr>
<tr>
<td>8.1.5</td>
<td>Beta Percentage Analysis 37</td>
<td></td>
</tr>
<tr>
<td>8.1.6</td>
<td>Sector Analysis 37</td>
<td></td>
</tr>
<tr>
<td>8.1.7</td>
<td>Analysis of Factor Betas in Combination With Sectors 38</td>
<td></td>
</tr>
<tr>
<td>8.2</td>
<td>CRM Framework 38</td>
<td></td>
</tr>
<tr>
<td>8.2.1</td>
<td>Large Total R-squared 38</td>
<td></td>
</tr>
<tr>
<td>8.2.2</td>
<td>Significant $t$-statistic of Factor Sensitivities 38</td>
<td></td>
</tr>
<tr>
<td>8.2.3</td>
<td>Low Correlation 39</td>
<td></td>
</tr>
<tr>
<td>8.2.4</td>
<td>Significance for Individual Stocks 39</td>
<td></td>
</tr>
<tr>
<td>8.2.5</td>
<td>Conclusion CRM Framework 39</td>
<td></td>
</tr>
<tr>
<td>8.3</td>
<td>Discussion of Method 39</td>
<td></td>
</tr>
<tr>
<td>8.4</td>
<td>Further Research 40</td>
<td></td>
</tr>
</tbody>
</table>
CONTENTS

8.5 Conclusion ............................................................................................................ 40

9 List of References .................................................................................................... 41

10 Appendices .............................................................................................................. 44
   10.1 Definition of Ratios .......................................................................................... 44
   10.2 Stock Names ...................................................................................................... 45
   10.3 Company-specific Information .......................................................................... 49
List of Tables

1. Explanatory power for each period ........................................... 23
2. Explanatory power for each period ........................................... 23
3. Stock analysis for each period ................................................ 24
4. Period 1 2010—2012: Percentage of significant stocks according to \( t \)-value for each factor ................................................................. 24
5. Period 2 2013—2015: Percentage of significant stocks according to \( t \)-value for each factor ................................................................. 25
6. Period 3 2016—2018: Percentage of significant stocks according to \( t \)-value for each factor ................................................................. 25
7. Period 1 2010—2012: Portfolio’s weighted \( t \)-value for each factor ................................................................. 26
8. Period 2 2013—2015: Portfolio’s weighted \( t \)-value for each factor ................................................................. 27
9. Period 3 2016—2018: Portfolio’s weighted \( t \)-value for each factor ................................................................. 27
11. Period 2 2013—2015: Beta percentage of whole model ................ 27
13. Number of stocks per sector ..................................................... 28
14. Period 1 2010—2012: Percentage of stocks with significant \( t \)-value per sector ..................................................... 28
15. Period 2 2013—2015: Percentage of stocks with significant \( t \)-value per sector ..................................................... 28
16. Period 3 2016—2018: Percentage of stocks with significant \( t \)-value per sector ..................................................... 29
17. Period 1 2010—2012: Percentage of the QMJ-beta relative to all factors for each sector 29
18. Period 2 2013—2015: Percentage of the QMJ-beta relative to all factors for each sector 29
19. Period 3 2016—2018: Percentage of the QMJ-beta relative to all factors for each sector 29
20. Comparison of explanatory powers before and after QMJ introduction 31

List of Figures

1. Cumulative returns of QMJ factors from long sample of domestic (U.S) stocks ........................................ 14
2. Cumulative returns of QMJ factors from broad sample of global stocks ........................................ 15
3. The outlier stock 15 plotted against stock 1, 2 and 3 ........................................ 22
4. Period 1 2010—2012: Comparison of beta significance ........................................ 25
5. Period 2 2013—2015: Comparison of beta significance ........................................ 25
6. Period 3 2016—2018: Comparison of beta significance ........................................ 26
7. Period 1 2010—2012: Spearman’s factor correlation ........................................ 30
8. Period 2 2013—2015: Spearman’s factor correlation ........................................ 30
10. GPRV Analyses for stocks in period 2 ............................................. 35
11. GPRV Analysis for companies in period 1 ............................................. 49
12. GPRV Analysis for companies in period 2 ............................................. 50
1 Introduction

1.1 Background

Within the field of *factor models*, research is conducted regarding their ability to predict future returns on investments. Factor models measure the extent to which a portfolio of stocks is influenced by a range of economic factors, e.g. oil price or an index [1]. During the 20th century, these models have developed into having included more factors and such factors with higher explanatory power. That search for higher explanatory power is still ongoing, since it is truly beneficial for an actor to be able to predict returns in an accurate way. This concern is mainly crucial for fund management firms, since their main businesses and goals are to optimize their investment processes in order to get the highest possible returns on their investments.

Public pension funds are no exception to this statement. By managing the national pension system’s capital buffer, their goal is to generate a high long-term return and maximum benefit for the current and future pensioners. One could therefore argue that fund managers’ abilities to predict returns on investments is even more crucial in the case of pension funds since their performance affects us all. Therefore, one interesting aspect would be to investigate if their models for predicting returns could develop further by introducing recent research within the field.

For a model, there are many ways of improving mathematically. By *regression analysis*, one finds several tools for how to achieve this goal. For instance, by adding factors, a model’s explanatory power would increase if the factors themselves have high enough *explanatory power*. This kind of measure indicates how much of the stock returns that can be explained by the factors used in the model and is therefore of high relevance in improving models [2].

The Swedish national pension fund *Fjärde AP-fonden (AP4)* uses a number of factor models in its investment processes. One of them, hereinafter referred to as a *Commercial Risk Model (CRM)*, is a global 12-factor model containing both macroeconomic and equity market factors and uses common statistical techniques to calculate stock sensitivities to these factors in order to help investors estimate both tracking error and sources of risk of their portfolios. By knowing which factors that most accurately indicate the movements of stock returns, investment strategies can be improved and adjusted for generation of higher returns – the ideal factors in a risk model are those that best explain the movement of the stock returns.

With precursors like *value*, *size* and *momentum*, one of the "latest" factors in the field of factor models, is the factor *Quality Minus Junk, QMJ* [3, 4]. The US-based hedge fund *AQR Capital Management* focuses, besides on capital management, on research within the field of factor models and one of their most recent and known research is this QMJ factor, which has been shown to earn significant returns across developed markets [4]. In this case *quality* refers to high quality and *junk* refers to low quality and the authors give a definition on how to measure quality. *Small* and *big* refer to the size of the stock. In short, the QMJ factor is formed as the buying of quality stocks (long position in quality stocks) and the selling of junk stocks (short position in junk stocks). The factor is then defined as the average return of two size-sorted quality portfolios minus the average return of two size-sorted junk portfolios.

Since all the pension funds’ mission is to generate a high long-term return, improvements of currently used models are welcomed. By introducing a factor into a model, one could potentially draw benefits from these attempts. By specifically using this commercial 12-factor model and to introduce the academic factor Quality Minus Junk, some improvement would potentially be
achieved and this is what this thesis will investigate. Without knowing the exact impact of this introduction, the results will be of interest for the rest of the Swedish pension system and in general regarding introductions of academic factors into established commercial factor models.

1.2 Purpose

The purpose of this thesis work is to, through regression analysis and financial analysis, assess the impact of the factor Quality Minus Junk on AP4's model. The results retrieved from this thesis may also be useful to the general knowledge about factor models and adding of factors and in particular about academic factors, such as Quality Minus Junk. It will be assessed how great the mathematical impact is and if this impact is significant to the model.

By conducting this exploratory study, the results, could potentially generate valuable insights for similar firms in the Swedish pension system. Hence, the impact of this study could potentially have greater effects than imagined at first glance.

1.3 Problem Formulation

In the attempt of improving AP4’s commercial risk model, the QMJ factor will be added to the model. More specifically the knowledge about the current 12-factor model will be assessed by regressing these factors of the model. The QMJ factor will be added to the model as an equity market factor. A new regression of the now 13 factors will be carried through. By regression analysis, this new model will be compared to the original one and different regression analysis tools will give an indication of the impact of the factor on the model in the mathematical sense.

1.3.1 Research Questions

The research question that will be answered in this thesis is:

- What is the impact of the QMJ factor on the currently used risk model for European stocks?

This research question can be divided into the following two sub-questions:

- What is the effect of the QMJ factor on the explanatory power of the model?
- Should the QMJ factor be included into the risk model?

1.4 Knowledge Base

In order to be able to measure the mathematical impact on the model, regression analysis will be used. Regression analysis is a statistical method that will be explained further in 2.1 Mathematical theory. The main part of the knowledge will be retrieved from the KTH course SF2930 Regression analysis along with the books Introduction to Linear Regression Analysis and An Introduction to Statistical Learning [2, 5].

Knowledge within the area of factor models, will be retrieved from literature. Regarding the QMJ factor, the research paper Quality Minus Junk by Clifford S. Asness, Andrea Frazzini and Lasse H. Pedersen will be used [4].
1 INTRODUCTION

1.5 Disposition of Report
This thesis will handle both technical aspects and aspects connected to the area of industrial management. In particular, this thesis will concern mathematical statistics in the form of multiple linear regression. Concerning the aspect of industrial management, a focus will lie on finance since factor models are a major part of the thesis. The mathematical theory and the financial theory will be presented separately. However, there will be no separation of these two aspects in regards of separate methods or such; they go hand in hand, and the discussion of the research questions will deal with them both.

1.6 Scope
This thesis work will investigate European stocks from the MSCI Europe Index, in the time period 2010–2018. The impact of one factor on the current model is going to be analysed, the QMJ factor. The primary methodology used for this exploratory study is multiple linear regression. Explanations on these decisions will be given throughout the thesis.
2 Mathematical Theory

2.1 Multiple Linear Regression

When a dependent response variable should be approximated from a number of independent variables, multiple linear regression is used. The model is set in the following way:

\[ y_i = \sum_{j=0}^{k} x_{ij} \beta_j + e_i, \quad i = 1, ..., n, \]

where \( y_i \) represents the response variable and the \( x_{ij} \) represents the regressor variables. The normally distributed error term is denoted by \( e_i \). The term \( \beta_0 \) is the intercept and \( \beta_1, \beta_2, ..., \beta_n \) represent the regressor coefficients for each of the regressor variables and these regressor coefficients will be estimated. There are \( n \) number of observations and \( k \) number of regressor variables [2, p. 68, 72].

Matrix notation can be used to display the model, then in the following way

\[ Y = X \beta + e, \]

where

\[
Y = \begin{pmatrix}
y_1 \\
y_2 \\
\vdots \\
y_n 
\end{pmatrix}, \quad X = \begin{pmatrix}
1 & x_{11} & \cdots & x_{1k} \\
1 & x_{21} & \cdots & x_{2k} \\
\vdots & \vdots & \ddots & \vdots \\
1 & x_{kn} & \cdots & x_{nk}
\end{pmatrix}, \quad \beta = \begin{pmatrix}
\beta_0 \\
\beta_1 \\
\vdots \\
\beta_n
\end{pmatrix}, \quad e = \begin{pmatrix}
e_1 \\
e_2 \\
\vdots \\
e_n
\end{pmatrix}.
\]

For the linear regression to be possible to perform and give fair results, the following five assumptions must be satisfied [2, p. 129]:

1. There is approximately a linear relationship between the response and the regressors.
2. \( E[e_i] = 0 \), the expected value of the error terms are equal to zero.
3. \( E[e_i^2] = \sigma^2 \), the variance of the error terms are constant.
4. The errors are uncorrelated.
5. The errors are normally distributed.

In order to estimate the regression coefficients \( \beta \), Ordinary Least Squares (OLS), is used. This method builds on minimizing the sum of the residuals \( \hat{e}^T \hat{e} = ||\hat{e}||^2 \) and the normal equation \( X^T \hat{e} = 0 \), where \( \hat{e} = Y - X \hat{\beta} \) and \( \hat{\beta} \) is the least-squares estimator [2, p. 70–73].

2.2 Ordinary Least Squares

The method of ordinary least squares, OLS, can be used to estimate the regression coefficients \( \beta_j \). It is assumed that the error term \( \varepsilon \) in the model has \( E[\varepsilon] = 0 \), \( Var[\varepsilon] = \sigma^2 \) and that the errors are uncorrelated. These estimates are found by minimizing the distance from the observations \( y_i \) to the fitted values \( \hat{y}_i \), also called residuals. The least-squares function is given by

\[ S(\beta_0, \beta_1, ..., \beta_k) = \sum_{i=1}^{n} e_i^2 = \sum_{i=1}^{n} (y_i - \beta_0 - \sum_{j=1}^{k} \beta_j x_{ij})^2 \]
and should be minimized with respect to $\beta_0, \beta_1, \ldots, \beta_k$. When using matrix notations the least-squares function is given by \cite[p. 70–73]{2}

$$S(\beta) = \sum_{i=1}^{n} \varepsilon_i^2 = \varepsilon^T \varepsilon = (y - X\beta)^T(y - X\beta)$$

and the least-squares estimator of $\beta$ is finally given by

$$\hat{\beta} = (X^T X)^{-1} X^T Y.$$  

### 2.3 Explanatory Power

The explanatory power, $R^2$ or $R$-squared, is a measure on how well the independent explanatory variables together explain the variance of the dependent response variable. Hence, the explanatory power is a tool in determining the regression model’s validity. Mathematically $R^2$ is defined as

$$R^2 = 1 - \frac{SS_{Res}}{SS_T},$$

where $SS_{Res}$ is the residual sum of squares, expressed as $SS_{Res} = \sum_{i=1}^{n}(y_i - \hat{y}_i)^2$, and $SS_T$ is the total sum of squares, expressed as $SS_T = \sum_{i=1}^{n}(y_i - \bar{y})^2$.

It is desirable to have a high value of $R^2$, since it minimizes the error term $\hat{e}$ and therefore implies that the estimated model of the response variable is improved. An $R^2$ value of 0.5 means that 50% of the model’s response can be explained by the explanatory variables.

Furthermore, there is an Adjusted $R^2$, $R^2_{Adj}$. This measure is adjusted so that unnecessary variables are not used in the model, by taking into account the degrees of freedom of these variables. If only $R^2$ is used, a larger number of variables will be preferred since the explanatory power will increase when more variables are used. However, the Adjusted $R^2$ will be lower if an independent variable is included in the model with a low explanatory power. Adjusted $R^2$ for a $p$-term equation is defined as the following \cite[p. 87–88]{2}

$$R^2_{Adj} = 1 - \frac{SS_{Res} / (n-p)}{SS_T / (n-1)},$$

where $n$ is the number of observations and $p$ is the number of parameters.

It is challenging to determine whether a certain $R^2$ value is good or not. In general, this depends on the application. In physics applications, the likelihood of near-1 $R^2$ values are greater than in the applications of e.g. psychology, in which $R^2$ values of 0.1 are more realistic \cite[p. 70]{5}. In the area of finance, $R^2$ values generally do not exceed 0.4 \cite{6}.

### 2.4 Detection and Treatment of Outliers

An outlier is an extreme observation considerably different from the majority of the data. Depending on its location in x space, it can have varying effects on the regression model. Outliers can be "bad" values, in the sense that they occur as a result of faulty measurement or analysis of the data. In this case, the outlier should be deleted from the data set since it does not provide the model with an accurate representation of reality. Outliers can also be unusual but perfectly plausible observations. In this case the outlier should not be deleted from the data set since it contributes accurately to the model and the model falls short in an estimation and prediction sense \cite[p. 152–153]{2}.  


For detecting outliers, many different statistical tests have been proposed. Outliers are often identified by unusually large residuals. However, there are many measures for looking at both the location of the point in $x$ space as well as the response variable $y$. Following, are such measures.

### 2.4.1 Cook’s Distance

Cook’s distance is a measure of the squared distance between the least-squares estimates based on all $n$ points $\hat{\beta}$ and the estimate obtained by deleting the $i$th point, $\hat{\beta}_{(i)}$. This measure is called a deletion diagnostic, since it measures the influence of the $i$th observation if it is removed from the sample. Cook’s distance measure is defined as the following:

$$D_i(X^TX, pMS_{Res}) = D_i = \frac{(\hat{\beta}_{(i)} - \hat{\beta})^T X^T X (\hat{\beta}_{(i)} - \hat{\beta})}{pMS_{Res}},$$

where $p$ is the number of parameters and $MS_{Res}$ is the residual mean square.

Large values of $D_i$ indicate considerable influence on the least-squares estimates $\hat{\beta}_{(i)}$.

The $D_i$ statistic may be rewritten as

$$D_i = \frac{r_i^2}{p} \frac{\text{Var}(\hat{\beta}_{ij})}{\text{Var}(e_i)},$$

where $h_{ii}$ are the diagonal elements of the hat matrix $H = X(X^T X)^{-1}X^T$ [2, p. 215–216].

### 2.4.2 DFBETAS

DFBETAS is another deletion diagnostic, which indicates how much the regression coefficient $\hat{\beta}_j$ changes, if the $i$th observation were deleted. This statistic is defined as:

$$DFBETAS_{j,i} = \frac{\hat{\beta}_{ij} - \tilde{\beta}_{ij}}{\sqrt{s_{ij}^2 C_{jj}}},$$

where $C_{jj}$ is the $j$th diagonal element of $(X^T X)^{-1}$ and $\tilde{\beta}_{ij}$ is the $j$th regression coefficient computed without use of the $i$th observation.

A large $DFBETAS_{j,i}$ indicates that observation $i$ has considerable influence on the $j$th regression coefficient [2, p. 217].

### 2.4.3 DFFITS

DFFITS is another deletion diagnostic, which indicates how much the fitted value $\hat{y}_i$ changes if observation $i$ is removed. So, the influence of the $i$th observation on the predicted or fitted value. This statistic is defined as:

$$DFFITS_i = \frac{\hat{y}_i - \tilde{y}_i}{\sqrt{s_{(i)}h_{ii}}},$$

where $h_{ii}$ are the diagonal elements of the hat matrix $H = X(X^T X)^{-1}X^T$.

Observations with corresponding values for which $|DFFITS_i| > 2\sqrt{p/n}$, demand further attention [2, 217–218].
2.5 Hypothesis Testing

Hypothesis testing is a commonly used method for analysing the significance of the independent variables. There are several tests used for this purpose.

Adding a variable to a regression model always causes the sum of squares for regression to increase and the residual sum of squares to decrease. A decision has to be made whether the increase in the regression sum of squares is sufficient to warrant using the additional regressor in the model. The addition of a regressor also increases the variance of the fitted value, so regressors that are of real value in explaining the response must only be included. Adding of an unimportant regressor may also increase the residual mean square, which may decrease the usefulness of the model. Tests for making sure that the added regressor significantly explains the response variable, are hence crucial [2, p. 88].

2.5.1 Test for Significance of Regression

The test for significance of regression is a test to determine if there is a linear relationship between the response $y$ and any of the regressor variables $x_1, x_2, ..., x_k$. This procedure is often thought of as an overall or global test of model adequacy. The hypotheses for testing the significance of any individual regression coefficient $\beta_j$, are

\[
H_0 : \beta_1 = \beta_2 = ... = \beta_k = 0 \\
H_1 : \beta_j \neq 0, \text{ for at least one } j
\]

where $H_0$ is called the null hypothesis. Rejection of the null hypothesis implies that at least one of the regressors $x_1, x_2, ..., x_k$ contributes significantly to the model [2, p. 84].

2.5.2 $t$-test

A so-called $t$ statistic can be used to test the hypothesis, where

\[
t_0 = \frac{\hat{\beta}_j}{\sqrt{\hat{\sigma}^2_{\beta_j}}}
\]

and follows a $t_{\alpha/2, n-k-1}$ distribution if the null hypothesis $H_0 : \beta_j = 0$ is true. That is, the null hypothesis $H_0$ is rejected if $|t_0| > t_{\alpha/2, n-k-1}$.

Since the regression coefficient $\hat{\beta}_j$ depends on all of the other regressor variables $x_{i,j}$ in the model, this $t$-test is really a marginal test. It tests the contribution of $x_j$ given the other regressors in the model [2, p. 88].

In the case of this thesis, the null-hypothesis is interpreted as a factor not contributing to the model. And more specifically, the QMJ factor not contributing to the explanation of the portfolios’ total return. One wants to be able to reject the null-hypothesis as often as possible in order to say that a factor is significant for the model. In order to reject the null-hypothesis the $t$-value should be greater than 2.

2.5.3 $F$-test

A method commonly used to analyse the independent variables is the $F$-test. From this method, a $p$-value can be calculated which shows the probability that the next observation yields an equally extreme value as the previously observed values. By having a chosen level of significance, normally 0.05, one can by using the $p$-value, determine if a variable should be kept or not in the model. A
null hypothesis is usually formulated, in which the coefficient $\beta_j$ for an independent variable equals to zero [2, p. 84–85].

### 2.6 Multiple Testing

The phenomenon of multiple testing occurs when a set of hypotheses are tested simultaneously and the potential danger is to get a significant result due to chance. If 13 hypotheses are to be tested at a significance level of 0.05, the probability of observing at least one result only due to chance is given by

$$P(\text{at least one significant result}) = 1 - P(\text{no significant results}) = (1 - 0.05)^{13} = 0.51$$

So, with 13 tests, there is a 51% chance of observing at least one significant result even if all tests are in fact not significant [7, p. 1].

### 2.7 Overfitting

The phenomenon of overfitting of data means that the model follows the errors, or noise, too closely. It is an undesirable situation since the fit obtained will not yield accurate estimates of the response on new observations that were not part of the original training data set. When a given method yields a small training mean square error, the data is said to be overfitted. This occurs since the procedure of statistical learning is working too hard to find patterns in the training data. There is a risk in picking up on patterns that are just caused by random chance rather than by true characteristics of the unknown function $f$ [5, p. 22, 47].

### 2.8 Spearman’s Rank Correlation Coefficient

Spearman’s rank correlation coefficient is a non-parametric measure of correlation; that is, statistics based on being either distribution-free or having specified distribution with parameters unspecified. The coefficient estimates how well the relationship is between two variables and can be explained by a monotone function. A perfect Spearman correlation of $\pm 1$ occurs when one of the two variables is a perfect monotone function of the other variable. The coefficient is calculated by:

$$\hat{\rho}_S = 1 - \frac{6S(d^2)}{n(n^2 - 1)}$$

where

$$S(d^2) = \sum_{i=1}^{n} [R(x_i) - R(y_i)]^2$$

$\{R(x_i)\}_1^n$ and $\{R(y_i)\}_1^n$ are observation ranks [8, p. 27–28].
3 Financial Theory

This part will serve as an introduction to the theory surrounding finance. Mainly this will be theory on a basic level and will explain main concepts within the area of finance that are relevant to this thesis work.

3.1 Return

The return is defined as the difference between the selling price and purchasing price of an asset plus any cash distributions, expressed in percentage of the buying price [1, p. 1125].

3.2 Risk-Adjusted Return

The risk-adjusted return shows how an asset has performed over and above a benchmark asset with the same risk. It measures how much return an investment has yielded relative to the risk the investment has beared over a time period [9].

3.3 Investment Strategies

Two commonly used concepts in finance are:

- Long market position
- Short market position

The long position can be referred to a positive investment in a security — the buying of the security, with the expectation that the security will rise in value.

The short position is referred to a negative investment in a security — the selling of the security today with the intention of repurchasing it later at a lower price. The expectation is thus that the security initially will decrease in value [1, p. 405].

3.4 Risk

Risk is a term that can have different meanings. One definition is that risk is the variance of the return, which is the expected square deviation from the mean. Risk can also be defined as the standard deviation of the return, which is the square root of the variance of the return. In finance, the standard deviation is also referred to as volatility. Risk in the financial sense, can be firm-specific (idiosyncratic) or systematic and could therefore also have different definitions. The firm-specific risk is uncorrelated and affects a particular security. The systematic risk is perfectly correlated and affects all securities. Other types of risk within finance can e.g be Value-at-Risk or Expected Shortfall.

There exists a historical trade-off between risk and return. Higher returns can be achieved only at higher levels of risk. Diversification can be applied in order to average out the independent (idiosyncratic) risk in a large portfolio. The systematic risk will however be considered and beared in exchange for earning higher returns [1, p. 354–356, 364–372].
3.5 Market Beta

The market beta measures the security’s sensitivity to the systematic risk. A security’s market beta is related to how sensitive its underlying revenues and cash flows are to general economic conditions.

Market beta is expressed as the expected percentage change in the excess return of a security of a 1% change in the excess return of the market, where the excess return is the return of the stock minus the risk-free interest rate. A market beta-value of 1 indicates that the price of the security is strongly correlated with the market — only has systematic risk, and a market beta-value of 0 indicates that there is no correlation with the market. A security with beta 2 carries twice as much systematic risk as an investment in the market portfolio. A beta that is negative, indicates that the return of the security moves inversely to the market [1, 375–379].

Throughout this thesis different types of beta are used with different meanings. When referring to market beta it will be referred to as market beta. When referring to other types of beta, other words will be used, e.g only beta.

3.6 MSCI Europe Index

The MSCI Europe Index captures large and mid market capitalization reflection across 15 Developed Market countries in Europe. The factors that drive risk and return are Value, Low size, Momentum, Quality, Yield and Low Volatility. Market capitalization is a company’s market value which is computed by outstanding number of shares times price of share. Market capitalization (market cap) is typically divided into three different categories; Large (>10 billion), Mid (>2 billion, <10 billion) and Small (<2 billion) [10].
4 Factor Model Theory

This part will bring to light the theory surrounding factor models. Firstly, a general introduction to factor models and their characteristics. Secondly, an explanation of the most well-known factor models and lastly an introduction to the commercial factor model and the Quality Minus Junk factor that this thesis project is focusing on.

A factor model measures the extent to which a portfolio of stocks is influenced by a range of economic factors, e.g. oil price or an index [1, p. 501–5016]. The term factor is in the mathematical sense represented by the regression variable that aims to explain the response variable — in this case the stock returns. Since a factor model can illustrate the risk of a portfolio, the factor model can be referred to as risk model.

4.1 Linear Factor Model

A linear factor model assumes that the rate of return of an asset $i$ is given by

$$ r_i = a_i + b_{1,i}f_1 + ... + b_{k,i}f_k + e_i $$

where the $f_j$ are factors, $a_i$ and $b_j$ are constants and $e_i$ represents the error term [11, p. 12–13].

The simplest case is when the model only considers one factor. That is, the rate of return is given by

$$ r_i = a_i + b_i f + e_i $$

4.2 Previous Work

Factor models have for a long time been a hot topic of research. Since the early beginning where the model only consisted of one single factor to today’s multi-factor models in different forms. The research often means that already existing models are extended with some new factor or some of the existing models’ factors are tweaked, with the aim of increasing the explanatory power of the model. However, this kind of research is not as easy as it sounds — the ability to predict stocks is puzzling even today, but if successful work is made within this field, many advantages can be leveraged, which explains its popularity.

During the early 1960’s, William Sharpe, John Lintner and others developed the today commonly used Capital Asset Pricing Model (CAPM) [12, 13]. This model allows for identifying the efficient portfolio of stocks without having any knowledge of the expected return of each security or the cost of capital of an investment. It is a 1-factor model containing a market factor (MKT).

An additional popular factor model is the so-called Fama-French three-factor model that was developed by Eugene Fama and Kenneth French as a criticism to CAPM with its one single explanatory variable [3]. This model is today a standard model for studies of asset returns. In addition to the market factor (MKT) in CAPM, the size factor Small Minus Big (SMB) and the value factor High Minus Low (HML) were now also considered for explaining asset returns. Later, the momentum factor Up Minus Down (UMD) was introduced by John Carhart [14].

In 2014, Fama and French made a comeback with the Fama-French five-factor model that now except for the previously mentioned factors in their three-factor model also included Robust Minus Weak (RMW) and Conservative Minus Aggressive (CMA) [15]. That search for new factors and
4 FACTOR MODEL THEORY

The development of factor models is constantly ongoing. Companies such as capital management firms today combine their businesses with research within the field to improve their own businesses whilst some companies are suppliers of these factor models and need good selling arguments for their potential clients.

Even though factor models are popular tools, their ability to explain asset returns is in reality modest. There is a relatively low $R^2$ connected to the explanation of asset returns. Financial science often overvalues the explanatory powers as very high. But general stock price movements are notoriously unpredictable and financial economists have developed the theory of efficient markets in order to explain why they should be unpredictable. Explanatory powers of modern factor models often reaches a maximum of 40% [6]. Hereinafter, this thesis work will refer to risk model.

4.3 Commercial Risk Model

One of AP4’s risk models, supplied by a third party, models returns and risk for stocks. In this thesis we refer to it as the Commercial Risk Model (CRM). The total return of a stock $i$ during period $t$ for this model is given by

$$r_{it} = a_i + \sum_j b_{ij}^{ME} F_{jt}^{ME} + \sum_j b_{ij}^{EM} F_{jt}^{EM} + b_S F_S^t + b_C F_C^t + e_{it}.$$

The total return of a portfolio $p$ during period $t$ is given by

$$r_{pt} = \sum_i w_i a_i + \sum_j (\sum_i w_i b_{ij}^{ME}) F_{jt}^{ME} + \sum_j (\sum_i w_i b_{ij}^{EM}) F_{jt}^{EM} +$$

$$+ (\sum_i w_i b_S^t) F_S^t + (\sum_i w_i b_C^t) F_C^t + \sum_i w_i e_{it}$$

where $ME =$ macroeconomic, $EM =$ equity market, $S =$ sector and $C =$ country.

The risk model has 12 factors; 7 macroeconomic factors and 5 equity market factors.

Macroeconomic factors:  
- Global Yield  
- Emerging Markets Bond Yield  
- Credit  
- Oil Price  
- Commodities Price  
- EUR/USD  
- JPY/USD

Equity market factors:  
- Global Market  
- Global Small-Cap Premium  
- Global Growth/Value Premium  
- Global Sector Factors  
- Country Factors

The explanatory power of this model is, on average, relatively high according to the writers. From 2012 to 2015 the model’s $R^2$ varied between 28% and 37%. It rises considerably when it is used for analysis of a portfolio – the more stocks in a portfolio the more the company-specific risk is diversified away. According to observations of this model, large cap companies exhibit a closer fit to the model with a significantly higher $R^2$. Small cap companies often have stock-specific risk that the model does not capture, resulting in lower $R^2$ values.
4.4 Quality Minus Junk

AQR Capital Management is a U.S.-based hedge fund that has written many papers on the topic of factor models, for example *The Devil in HML’s Detail* and *Betting Against Beta* [16, 17]. In 2013, Clifford S. Asness, Andrea Frazzini and Lasse H. Pedersen at AQR wrote a research paper on an approach in finding successful trading strategies by looking at the quality aspect of a stock. Their research paper investigates the performance of the so-called *Quality Minus Junk* factor, (QMJ), that has appeared to earn significant risk-adjusted returns in the U.S. and globally across 24 countries [4].

Quality in this case is defined as the characteristics that investors should be willing to pay a higher price for, everything else equal. Historically, high-quality stocks have generated high risk-adjusted returns while low-quality junk stocks have generated negative risk-adjusted returns. A QMJ portfolio that takes a long position in quality-stocks and shorts junk stocks, produces high risk-adjusted returns. The definition of quality contains the following quality characteristics, and their exact measures can be found in Appendix 10.1:

1. **Profitability**: More profitable companies should command a higher stock price.
   - Profitability is measured by Gross Profits Over Assets (GPOA), Return On Equity (ROE), Return On Assets (ROA), Cash Flow Over Assets (CFOA), Gross Margin (GMAR) and Fraction Of Earnings Composed Of Cash (ACC).
   - The profitability score is given by
     \[
     Profitability = z(z_{gpoa} + z_{roe} + z_{roa} + z_{cfoa} + z_{gmar} + z_{acc})
     \]

2. **Growth**: Investors should command a higher price for stocks with growing profits.
   - Growth is measured as the 5-year prior growth in profitability, averaged across over measures of profitability.
   - The growth score is given by
     \[
     Growth = z(z_{\Delta gpoa} + z_{\Delta roe} + z_{\Delta roa} + z_{\Delta cfoa} + z_{\Delta gmar} + z_{\Delta acc})
     \]

3. **Safety**: Investors should pay a higher price for a stock with a lower required return; that is, a safer stock.
   - Safe securities are defined as companies with Low Beta (BAB), Low Idiosyncratic Volatility (IVOL), Low Leverage (LEV), Low Bankruptcy Risk (O-score and Z-score) and Low ROE Volatility (EVOL).
   - The safety score is given by
     \[
     Safety = z(z_{bab} + z_{ivol} + z_{lev} + z_{o} + z_{z} + z_{evol})
     \]

4. **Payout**: More shareholder-friendly companies should command a higher stock price.
   - The payout score is defined by using Equity Net Issuance (EISS), Debt Net Issuance (DISS) and Total Net Payouts Over Profits (NPOP).
   - The payout score is given by
     \[
     Payout = z(z_{eiss} + z_{diss} + z_{npop})
     \]
**Quality**: That is, all stocks are rated by z-scores for each of the quality components. The final quality score is given by the following expression where all quality components are included:

\[
Quality = z(Profitability + Growth + Safety + Payout)
\]

At the end of each calendar month, stocks are assigned to two size-sorted portfolios based on their market capitalization. The QMJ factor is long (buys) the top 30% high-quality stocks and is short (sells) the bottom 30% junk stocks within the universe of large stocks and similarly within the universe of small stocks. The QMJ factor is the average return on the two high-quality portfolios minus the average return on the two low-quality (junk) portfolios as can be seen below.

\[
QMJ = \frac{1}{2}(Small\ Quality + Big\ Quality) - \frac{1}{2}(Small\ Junk + Big\ Junk)
\]

\[
= \frac{1}{2}(Small\ Quality - Small\ Junk) + \frac{1}{2}(Big\ Quality - Big\ Junk)
\]

\[
= \frac{1}{2}(QMJ\ in\ small\ stocks) + \frac{1}{2}(QMJ\ in\ big\ stocks)
\]

The average $R^2$ increases when all four quality components are included, reaching 40% in the U.S. and 31% in the global sample but still leaving a large part of the cross section of prices unexplained.

By studying the graphs in Figure 1 and Figure 2 from the QMJ research paper, it becomes clear that the QMJ factor has consistently generated positive excess returns and risk-adjusted returns over time [4].

![Figure 1: Cumulative returns of QMJ factors from long sample of domestic (U.S) stocks](image)

Figure 1: Cumulative returns of QMJ factors from long sample of domestic (U.S) stocks
When the QMJ factor has a large regression coefficient (or beta-value), this means that the stock returns are explained largely by this QMJ factor. Since the QMJ factor is expressed in returns from high-quality portfolios minus the returns from the low-quality portfolios, a stock that has a high so-called QMJ-beta represents a company with relatively high quality. A stock that has a low QMJ-beta should instead possess an average level of quality, since the returns from the high respective low-quality portfolio does not differ that much. A negative QMJ-beta means that the returns from the low-quality portfolio exceeds the returns of the high-quality portfolio and that there is no correlation between a high quality and high returns — but the other way around.
5 General Method

Necessary data on stock returns and factor data was collected. The data was then transformed in order to retrieve a coherent data set. The commercial 12-factor model was replicated and some important properties were extracted. The QMJ factor was then added to the model, enabled by transformations, and a new regression was made. A comparison between the two models’ properties could then be illustrated. The QMJ factor’s impact was then analyzed from different perspectives, such as a beta sensitivity analysis of the factor as well as a sector analysis.

6 Descriptive Method

In the following section, a more detailed method will be presented where descriptions are found on how to perform the different steps of this thesis work; from general assumptions and tools needed, explanations of data transformations and residualisation of the factors, to more specific steps on regressions, regression analyses and steps on how the QMJ factor was added to the model.

6.1 Assumptions

For this method, the following assumptions were made:

- The stocks would not change sector code over time.
- Only European stocks were to be analyzed, originating from the MSCI Europe Index.
- The weighting of the stocks were not to be changed on a daily basis.

6.2 Tools

For this method, the following programs were used:

- The statistical program $R$.
- Excel
- Python

6.3 Data Collection

Data was retrieved from two different sources; AP4 and from AQR Capital Management:

6.3.1 AP4

Data from AP4 included data on the macroeconomic factors, the equity market factors and the response variable as well as data concerning factor co-variance matrices.

Macroeconomic Factors:

- *Global Yield*: Weekly percentage change of an index of 10-year government bond yields. This data is expressed in %.
- *Emerging Markets Bond Yield*: Weekly percentage change of an index of emerging sovereign bond yields. This data is expressed in %.
6 DESCRIPTIVE METHOD

- **Credit**: Weekly percentage change of a Credit Default Swap (CDS) index. This factor reflects default risk in global corporate bonds. This data is expressed in %.

- **Oil Price**: Weekly percentage change of crude oil spot price in US dollars. This data is expressed in %.

- **Commodities Price**: Weekly percentage change of a commodities price index. This data is expressed in %.

- **EUR/USD and JPY/USD**: Measured as the weekly percentage changes in the exchange rates. This data is expressed in %.

**Equity Market Factors:**

- **Global Market**: Weekly total returns of a global equity index in US dollars.

- **Global Small-Cap Premium**: Weekly total return spread in US dollars between a small cap and large cap index, according to SMB [3].

- **Global Growth/Value Premium**: Weekly total return spread in US dollars between a growth and a value index, according to HML [3].

- **Global Sector Factors**: Weekly total return in US dollars of 10 global indices corresponding to GICS sectors.

- **Country Factors**: Weekly total returns in local currency of country equity where the stock belongs.

**Other Data:**

- **Response variable** \( y \) in form of daily returns of the 300 European stocks from the MSCI Europe Index. The specific stocks are shown in Appendix 10.2.

- Factor co-variance matrices.

All the data needed to make a regression on the 12-factor model was therefore in place.

### 6.3.2 AQR Capital Management

Data from AQR Capital Management included data on the QMJ factor [18]. This data included the returns from the QMJ data on 24 countries, as well as aggregated returns sorted on Global, Global ex. USA, Europe, North America and Pacific. Global data was chosen for the regression since the model in its entirety is global and the potential increase in explanatory power should not be due to region.

### 6.4 Data Transformation

The data retrieved from AP4 and AQR was not coherent. Transformations on the data were therefore necessary, in order to secure accurate results. Below, these types of transformations are presented.

#### 6.4.1 Transformation of Date Format

The data from AP4 had different time frames. The response \( y \) consisted of daily returns, and the factor data consisted of weekly data. The data from AQR regarding the QMJ factor was on the form of daily returns. A transformation from daily to weekly data was therefore made for the response data.
6.4.2 Transformation of Data Type Format

All data was not of the same data type and therefore needed to be converted into what in this case made sense, numerical format. The macroeconomic factor data was expressed in percent, hence converted to decimal form. Since the results retrieved in this method later would be compared to the results in the CRM report – the 12-factor model, the method needed to be the same. The CRM residualizes equity factors according to the process described below:

- **Global Market**: Perform an OLS regression of this variable on the seven macroeconomic variables. Then use the residuals of this regression as the final Global Market factor.

- **Global Small-Cap Premium**: Perform an OLS regression of this variable on the seven macroeconomic variables and the residualised global market factor. Then use the residuals of this regression as the final Global Small-Cap Premium factor.

- **Global Growth/Value Style Premium**: Perform an OLS regression of this variable on the seven macroeconomic variables and the two residualised equity market factors. Then use the residuals of this regression as the final residual style factor.

- **Global Sector**: Perform an OLS regression of this variable on the seven macroeconomic factors and the three residualised equity market factors listed above. The residuals of this regression make up the final Global Sector factor. There are ten sectors.

- **Local Market/Country**: Perform an OLS regression of this variable on the seven macroeconomic factors, the first three residualised equity market factors and all ten residualised sector factors. The residuals from this regression make up the final Local Market/Country factor.

6.4.3 Transformation of Return Format

The format of having the returns expressed in percent, was changed into expressing them in decimal form.

6.5 Weighting of Stocks

Every stock was weighted against the MSCI Europe Index. When using these stocks to form a hypothetical portfolio, the weights need to be used in order to compute properties of that portfolio. The weights of the stocks are dependent on time, i.e. the weights for the different stocks changes on daily basis. Data of the last weights that were available, were used. However, the weights that were given did not sum up to one due to loss of data when transforming from a daily to weekly format. The retrieved weights were therefore first re-weighted by dividing with the total weights to retrieve their real weights, and therefore summed up to 100%. Those weights were the ones being used later on in the regression analysis.

6.6 Deletion of Outliers

In order to find potential outliers, analyses of the stocks’ regressions individually could be made where the t-values and F-values were particularly taken into account. If these values were abnormal, the stocks were to be looked into more thoroughly. The measures used for this analysis were Cook’s Distance, DFBETAS and DFFITS. If these measures could not explain the abnormalities, the corresponding stocks were removed from the data set.
6.7 Time Period
For the regressions and regression analyses, three-year time periods were considered since this is
the default timeframe of the CRM. A three-year time period was also reasonable when considering
what time frame is essential to a stock’s return. It is known that in general stocks are strongly
dependent on happenings close in time and it was therefore not relevant to look at longer time
periods.

6.8 Part 1: 12-factor Model
In this section, the regression and regression analyses of the 12-factor model are presented.

6.8.1 Regression
In order to find the relationship between the response variable $y$ and the factors $x_1, ..., x_{12}$, an OLS
regression was made in R including the 12 original factors — the 7 macroeconomic factors and the
5 equity market factors and the response variable on the set of stocks.

6.8.2 Regression Analysis
In order to analyze the regression made for the 12-factor model, the following regression analysis
tools and conditions were used:

- Explanatory power (R-squared and Adjusted R-squared)
- $t$ statistic
- $F$ statistic
- Outlier diagnostics

Even though the explanatory power is of greatest interest, it is important to look at e.g. the $t$
statistics in order to know if the impact is significant or not. Both $t$- and $F$-statistics were derived
and outliers detected according to section 7.2.1. It was important to do this step before calcul-
ating the explanatory power, due to the potential impact of the outliers on the explanatory power.

The explanatory power was derived using the weighting method described in section 6.5. In order to
get the explanatory power, every weight was multiplied with the explanatory power and then later
summed up. An explanatory power for the whole portfolio was then obtained. Both R-squared
and Adjusted R-squared were computed.

6.9 Part 2: Add QMJ to 12-factor Model
In order for the adding of the QMJ factor to be made, the factor needed to be residualised according
to the process mentioned earlier in section 6.4.2. The QMJ factor was added before the Sector and
Country factors, as shown below:

- Global Market factor
- Global Small-Cap Premium factor
- Global Growth/Value Style Premium factor
6 DESCRiptive METHOD

• QMJ factor: Here the Global QMJ returns was added. An OLS regression was performed on the 7 macroeconomic factors and the 3 residualised equity market factors and the extraction of the residuals made up the new QMJ factor data.

• Global Sector factor

• Local Market/Country factor

6.10 Part 3: 13-factor Model

In this section, the regression and regression analyses of the 13-factor model are presented. It was in this section that the impact of the QMJ factor on the model was analysed, according to different measures.

6.10.1 Regression

In order to find the relationship between the response variable $y$ and the factors $x_1,\ldots,x_{13}$, a regression was made in $R$ including the 13 factors — the 7 macroeconomic factors and the 6 equity market factors.

6.10.2 Regression Analysis

In order to analyse the regression made for the 13-factor model, the following regression analysis tools and conditions were used:

• Explanatory power

• $t$ statistic

• $F$ statistic

• Factor beta (regression coefficient)

• Correlation

• Outlier diagnostics

6.10.3 Beta Analysis

In order to get a better understanding of how the QMJ impacts the model, the regression coefficient of the QMJ factor was analysed, or the beta-value of the QMJ factor. The stocks with the highest and lowest beta-values corresponding to the QMJ factor were also noted. These stocks were then to be analyzed more deeply according to some of the properties of the quality definition. Other beta analyses were also made in different forms.

6.10.4 Sector Analysis

The sectors to which the stocks belong according to the Global Industry Classification Standard (GICS) [19], were analysed with respect to the QMJ factor. In this way, an understanding could be gained regarding how the returns of different kinds of stocks and sectors can be explained by the QMJ factor.
6.11 Comparison

A comparison between the results of the explanatory power retrieved in Part 1 and 3 was made in order to understand how the QMJ factor had impacted the model in that sense. Since the explanatory power was the most important measure for this thesis, it was natural to compare them and see the clear differences between the 12- and 13-factor model.
7 Results

The results will be displayed in a Part 1 and a Part 2, similar to the those presented in the descriptive method. In each of the parts, results will be displayed for the three time periods; Period 1 (2010–2012), Period 2 (2013–2015) and Period 3 (2016–2018). Short comments will be made regarding these results and what they say.

7.1 Part 1: Without QMJ

This section covers relevant results that belong to the 12-factor model, that is; the model without the QMJ factor.

7.1.1 Deletion of Outliers

The same procedure that is done in section 7.2.1 is made in this case too, leaving the data set with 298 stocks. The outliers were also plotted against other stocks to investigate if there were any clear differences. Figure 3 shows the plots of stock 15, Heineken, against stock 1, 2 and 3. Plots a), b) and c) show that there is no clear difference between the data points.

![Graphs showing plots of stock 15 against stocks 1, 2, and 3.](image)

Figure 3: The outlier stock 15 plotted against stock 1, 2 and 3

7.1.2 Explanatory Power

The explanatory power of the three time periods are presented in Table 1, in the sense of R-squared and Adjusted R-squared.
Both R-squared and Adjusted R-squared are increased over time. In period 1, R-squared and Adjusted R-squared are the same and in period 2 and 3 they are different. The Adjusted R-squared is lower than R-squared.

### 7.2 Part 2

This section covers relevant results belonging to the 13-factor model that includes the QMJ factor.

#### 7.2.1 Deletion of Outliers

Two out of the 300 available stocks show abnormal results regarding their $t$-values and $F$-values. In addition to this, they both showed an explanatory power of 100%. Regarding the measures that can indicate outliers; Cook’s Distance, DFBETAS and DFFITS, these stocks shows no abnormality in comparison to the other 298 stocks. For all of the three periods, the stocks showing abnormal $t$-values and $F$-values are the following:

**Heineken:**
- $t$-value for the QMJ factor: $2.419731e+15$
- $F$-Value: $1.266902e+32$

**Iberdrola:**
- $t$-value for the QMJ factor: $4.029447e+14$
- $F$-value: $4.393272e+31$

Even if there exists no clear explanation to why these two stocks are abnormal, a question arises whether or not they are results of measuring error or if they simply are naturally abnormal stocks in the sense of returns. Since there exists no such information, the two stocks are removed from the data set which now contains 298 stocks.

#### 7.2.2 Explanatory Power

The explanatory power of the three time periods are presented in Table 2, in the sense of R-squared and Adjusted R-squared.

<table>
<thead>
<tr>
<th></th>
<th>Period 1</th>
<th>Period 2</th>
<th>Period 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>R-squared</td>
<td>0.2853423</td>
<td>0.3492593</td>
<td>0.3783207</td>
</tr>
<tr>
<td>Adjusted R-squared</td>
<td>0.2253711</td>
<td>0.2946517</td>
<td>0.3261518</td>
</tr>
</tbody>
</table>

Table 2: Explanatory power for each period

Both R-squared and Adjusted R-squared are appearing to increase over time. All three periods have a difference between their R-squared value and Adjusted R-squared value of approximately 5 percentage units.
7.2.3 Stock Analysis

In Table 3, stocks are presented for each time period representing the stocks with the highest QMJ-betas (both positive and negative valued) and the stocks with the lowest positive QMJ-beta. The values within the parentheses show these QMJ-betas.

<table>
<thead>
<tr>
<th></th>
<th>Period 1</th>
<th>Period 2</th>
<th>Period 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stock with highest QMJ-beta (positive)</td>
<td>Orion (+0.78)</td>
<td>RSA Insurance Group (+0.70)</td>
<td>Arcelormittal (+1.90)</td>
</tr>
<tr>
<td>Stock with highest QMJ-beta (negative)</td>
<td>Unicore (–0.79)</td>
<td>Tesco (–0.98)</td>
<td>G4S (–0.40)</td>
</tr>
<tr>
<td>Stock with lowest QMJ-beta</td>
<td>Telia (+0.0005)</td>
<td>Phillips (+0.0008)</td>
<td>Deutsche Post (+0.012)</td>
</tr>
</tbody>
</table>

Table 3: Stock analysis for each period

From this table, it becomes clear that none of the periods have stocks that reoccur more than once. Thus, there is no clear connection between the periods or the types of stocks that are more or less QMJ sensitive.

7.2.4 Factor Beta Significance for All Stocks

Tables 4, 5 and 6 in this section show the percentage of stocks within the universe with statistically significant $t$-values (the absolute value of $t$ is strictly larger than 2) for each factor. This is made for each of the three time periods. For each time period a circle diagram is also shown to visualize the difference in size of significance for each factor.

<table>
<thead>
<tr>
<th>Global Yield</th>
<th>Bond Yield</th>
<th>Credit</th>
<th>Oil</th>
<th>Commodities Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.738255</td>
<td>4.362416</td>
<td>3.691275</td>
<td>4.026846</td>
<td>1.342282</td>
</tr>
<tr>
<td>EUR/USD</td>
<td>JPY/USD</td>
<td>Market</td>
<td>Small-Cap</td>
<td>Value Premium</td>
</tr>
<tr>
<td>60.067114</td>
<td>47.651007</td>
<td>3.691275</td>
<td>2.348993</td>
<td>1.006711</td>
</tr>
<tr>
<td>QMJ</td>
<td>Sector</td>
<td>Country</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.348993</td>
<td>98.993289</td>
<td>16.778523</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4: Period 1 2010–2012: Percentage of significant stocks according to $t$-value for each factor
7 Results

Figure 4: Period 1 2010–2012: Comparison of beta significance

<table>
<thead>
<tr>
<th>Global Yield</th>
<th>Bond Yield</th>
<th>Credit</th>
<th>Oil</th>
<th>Commodities Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>EUR/USD</td>
<td>JPY/USD</td>
<td>Market</td>
<td>Small-Cap</td>
<td>Value Premium</td>
</tr>
<tr>
<td>10.067114</td>
<td>83.557047</td>
<td>82.550336</td>
<td>13.087248</td>
<td>12.751678</td>
</tr>
<tr>
<td>QMJ</td>
<td>Sector</td>
<td>Country</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.026846</td>
<td>64.42953</td>
<td>14.09396</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 5: Period 2 2013–2015: Percentage of significant stocks according to $t$-value for each factor

Figure 5: Period 2 2013–2015: Comparison of beta significance
By observing the circle diagrams and the tables, it becomes clear that the QMJ-beta significance is increasing over time. In comparison to the remaining factors, QMJ has one of the lower significance values in period 1 and 2 but in period 3 it is more skewed to the higher values. The results in period 1 and 2 are expected, due to the theory of multiple testing presented in section 2.6, which means that there are some falsely significant results.

### 7.2.5 Weighted Portfolio Beta Significance

By weighing the $t$-values for each of the individual stocks and summing them up, the portfolio’s factor significance can be determined. Tables 7, 8 and 9 show the portfolio’s weighted $t$-values for each factor and period. The QMJ factor is of particular interest.

<table>
<thead>
<tr>
<th>Global Yield</th>
<th>Bond Yield</th>
<th>Credit</th>
<th>Oil</th>
<th>Commodities Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.04523611</td>
<td>0.09372496</td>
<td>−0.19238210</td>
<td>−0.08657412</td>
<td>0.28927452</td>
</tr>
<tr>
<td>EUR/USD</td>
<td>JPY/USD</td>
<td>Market</td>
<td>Small-Cap</td>
<td>Value Premium</td>
</tr>
<tr>
<td>−2.28066122</td>
<td>1.86773251</td>
<td>−0.65356639</td>
<td>0.48130330</td>
<td>−0.3899167</td>
</tr>
<tr>
<td>QMJ</td>
<td>Sector</td>
<td>Country</td>
<td></td>
<td></td>
</tr>
<tr>
<td>−0.23162436</td>
<td>6.48882041</td>
<td>0.75860520</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 7: Period 1 2010–2012: Portfolio’s weighted $t$-value for each factor
7 RESULTS

<table>
<thead>
<tr>
<th>Global Yield</th>
<th>Bond Yield</th>
<th>Credit</th>
<th>Oil</th>
<th>Commodities Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.6293506</td>
<td>0.3404308</td>
<td>0.846752</td>
<td>1.3932847</td>
<td>0.6524828</td>
</tr>
<tr>
<td>EUR/USD</td>
<td>JPY/USD</td>
<td>Market</td>
<td>Small-Cap</td>
<td>Value Premium</td>
</tr>
<tr>
<td>-0.5314509</td>
<td>-3.0917335</td>
<td>3.0416333</td>
<td>0.6794308</td>
<td>0.3079302</td>
</tr>
<tr>
<td>QMJ</td>
<td>Sector</td>
<td>Country</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-0.5205979</td>
<td>3.1059512</td>
<td>0.6994109</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 8: Period 2 2013–2015: Portfolio’s weighted $t$-value for each factor

<table>
<thead>
<tr>
<th>Global Yield</th>
<th>Bond Yield</th>
<th>Credit</th>
<th>Oil</th>
<th>Commodities Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.74133691</td>
<td>0.74816359</td>
<td>-0.07426301</td>
<td>0.54949657</td>
<td>1.21568476</td>
</tr>
<tr>
<td>EUR/USD</td>
<td>JPY/USD</td>
<td>Market</td>
<td>Small-Cap</td>
<td>Value Premium</td>
</tr>
<tr>
<td>-2.03634884</td>
<td>-1.26602263</td>
<td>4.14257544</td>
<td>0.76702140</td>
<td>-0.24508264</td>
</tr>
<tr>
<td>QMJ</td>
<td>Sector</td>
<td>Country</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.01085317</td>
<td>3.05008768</td>
<td>0.6084467</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 9: Period 3 2016–2018: Portfolio’s weighted $t$-value for each factor

Since a factor is significant if the absolute value of the $t$-value is strictly larger than 2, the tables above show that the QMJ factor is only significant in period 3. Worth mentioning is however that the majority of the remaining factors are not significant either according to the significance condition.

7.2.6 Factor Beta Percentage of Whole Model

To determine the extent to which each factor influences the model, the percentage of each factor’s beta (i.e. regressor coefficient) of the whole model is shown in Tables 10, 11 and 12. The QMJ factor is of particular interest.

<table>
<thead>
<tr>
<th>Global Yield</th>
<th>Bond Yield</th>
<th>Credit</th>
<th>Oil</th>
<th>Commodities Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.5560774</td>
<td>0.113898</td>
<td>1.294736</td>
<td>2.132523</td>
<td>3.837018</td>
</tr>
<tr>
<td>EUR/USD</td>
<td>JPY/USD</td>
<td>Market</td>
<td>Small-Cap</td>
<td>Value Premium</td>
</tr>
<tr>
<td>20.264178</td>
<td>7.89973</td>
<td>6.016978</td>
<td>11.445315</td>
<td>0.732445</td>
</tr>
<tr>
<td>QMJ</td>
<td>Sector</td>
<td>Country</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.24751</td>
<td>30.912075</td>
<td>4.547518</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 10: Period 1 2010–2012: Beta percentage of whole model

<table>
<thead>
<tr>
<th>Global Yield</th>
<th>Bond Yield</th>
<th>Credit</th>
<th>Oil</th>
<th>Commodities Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.3250328</td>
<td>0.679103</td>
<td>6.218186</td>
<td>10.104108</td>
<td>4.563399</td>
</tr>
<tr>
<td>EUR/USD</td>
<td>JPY/USD</td>
<td>Market</td>
<td>Small-Cap</td>
<td>Value Premium</td>
</tr>
<tr>
<td>2.731091</td>
<td>7.062931</td>
<td>31.046847</td>
<td>12.161589</td>
<td>3.408724</td>
</tr>
<tr>
<td>QMJ</td>
<td>Sector</td>
<td>Country</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6.768995</td>
<td>11.293441</td>
<td>2.696553</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 11: Period 2 2013–2015: Beta percentage of whole model


Table 12: Period 3 2016–2018: Beta percentage of whole model

Over time the percentage of the QMJ-beta increases. In period 3, the QMJ-beta is covering 17% of the model. This is the second highest beta-percentage in the model during this time period.

7.2.7 Number of Stocks Per Sector

Table 13 shows the distribution of the 298 stocks in each of the 10 GICS-sectors.

Table 13: Number of stocks per sector

The majority of stocks are in sector 40: Informational Technology and in sector 20: Energy.

7.2.8 QMJ Factor Significance by Sector

Tables 14, 15 and 16 in this section show the percentage of stocks that are significant for the QMJ factor for each sector. That is, the distribution of QMJ factor significance across sectors for each three periods.

Table 14: Period 1 2010–2012: Percentage of stocks with significant $t$-value per sector

Table 15: Period 2 2013–2015: Percentage of stocks with significant $t$-value per sector
The percentage of stocks that are significant clearly increases over time. Clearly there is a great difference between the percentages in periods 1 and 2 with the ones in period 3.

### 7.2.9 Percentage of QMJ-Beta per Sector

Tables 17, 18 and 19 illustrate what the percentages of the QMJ-beta is in relation to all the other factors’ betas, for each of the ten sectors. This gives an indication to what extent the QMJ factor explains the stock returns in relation to the other factors in the different sectors.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Cons. Disc</td>
<td>4.053743</td>
<td>4.053743</td>
<td>4.053743</td>
</tr>
<tr>
<td>Cons. Stap.</td>
<td>7.242825</td>
<td>7.242825</td>
<td>7.242825</td>
</tr>
<tr>
<td>Energy</td>
<td>6.048371</td>
<td>6.048371</td>
<td>6.048371</td>
</tr>
<tr>
<td>Financials</td>
<td>5.782813</td>
<td>5.782813</td>
<td>5.782813</td>
</tr>
<tr>
<td>Health Care</td>
<td>6.405901</td>
<td>14.664318</td>
<td>14.664318</td>
</tr>
<tr>
<td>Industrial</td>
<td>5.30525</td>
<td>10.267301</td>
<td>10.267301</td>
</tr>
<tr>
<td>Info. Tech</td>
<td>8.809046</td>
<td>10.054339</td>
<td>10.054339</td>
</tr>
<tr>
<td>Materials</td>
<td>6.877741</td>
<td>11.795476</td>
<td>11.795476</td>
</tr>
<tr>
<td>Telecom</td>
<td>8.371406</td>
<td>12.660288</td>
<td>12.660288</td>
</tr>
<tr>
<td>Utilities</td>
<td>6.256312</td>
<td>20.015632</td>
<td>20.015632</td>
</tr>
</tbody>
</table>

The results in the tables cannot yield a conclusive result. The sectors in which the QMJ factor is more influential vary with each period. In period 3, it becomes clear that the highest percentage is generated in the sector Utilities, where the QMJ-beta stands for 20% of the full model’s beta.

### 7.2.10 Factor Correlations

Figures 7, 8 and 9 show Spearman’s correlation between the factors, in order to find potential pairwise factor correlations. The highest value that can be obtained is ±100 and the lowest correlation is 0.
Figure 7: Period 1 2010–2012: Spearman’s factor correlation

Figure 8: Period 2 2013–2015: Spearman’s factor correlation
For all three time periods, the QMJ factor correlations with the other 10 factors are very low. There is no clear difference between the time periods. The correlations were only checked between the first 10 factors due to simplicity reasons. A full inclusion of factors would mean a huge amount of correlation matrices.

### 7.3 Comparison

In Table 20, the explanatory powers are presented for each measure — R-squared and Adjusted R-squared — before and after the QMJ factor has been added to the model. The percentage changes resulting from adding the QMJ factor are also displayed.

<table>
<thead>
<tr>
<th></th>
<th>Period 1</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Years</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>R-squared</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Without QMJ</td>
<td>0.2853423</td>
<td>0.3492593</td>
<td>0.3783207</td>
<td></td>
</tr>
<tr>
<td>With QMJ</td>
<td>0.2878403</td>
<td>0.3522319</td>
<td>0.3885319</td>
<td></td>
</tr>
<tr>
<td>Percentage unit change when adding QMJ</td>
<td>0.2498</td>
<td>0.2973</td>
<td>1.0211</td>
<td></td>
</tr>
<tr>
<td>Percentage change when adding QMJ</td>
<td>0.8754</td>
<td>0.8511</td>
<td>2.6991</td>
<td></td>
</tr>
<tr>
<td><strong>Adjusted R-squared</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Without QMJ</td>
<td>0.2253711</td>
<td>0.2946517</td>
<td>0.3261518</td>
<td></td>
</tr>
<tr>
<td>With QMJ</td>
<td>0.2226426</td>
<td>0.2929292</td>
<td>0.325525</td>
<td></td>
</tr>
<tr>
<td>Percentage unit change when adding QMJ</td>
<td>-0.2729</td>
<td>-0.1723</td>
<td>0.6401</td>
<td></td>
</tr>
<tr>
<td>Percentage change when adding QMJ</td>
<td>-1.2106</td>
<td>-0.5845</td>
<td>1.9624</td>
<td></td>
</tr>
</tbody>
</table>

Table 20: Comparison of explanatory powers before and after QMJ introduction

The table shows that the introduction of QMJ increases the R-squared in Period 1 and Period 3 and the Adjusted R-squared only increases in Period 3. In Period 2, it instead decreases.
8 Discussion

The discussion will analyze and discuss the results that have been retrieved in this thesis and will also discuss the method that has been used. The discussion is divided into parts; a mathematical part discussing the mathematical aspects of the thesis, a part analyzing whether the QMJ factor should be included in the CRM, as well as a part discussing the model that has been used and the potential further research within the subject of this thesis. The aim of the discussion section is to be able to answer the research questions of this thesis and end up with a conclusion.

8.1 Mathematical Discussion

This mathematical discussion will deal with the analysis and discussion of the results from section 7, including the discussion of the explanatory power and the analyses made on the QMJ-betas. The analyses will be made for each of the three time periods. In general regarding the analyses, there needs to be an awareness about the impact that the results actually carry. Since the QMJ factor is not always significant, one needs to be careful with drawing to large of conclusions regarding the other results that are retrieved, e.g. the QMJ-betas and their potential values in investigating the overall impact of the factor. The term beta here refers to the regression coefficient.

8.1.1 Discussion of Explanatory Power

When discussing whether or not a certain factor is relevant to a model, explanatory power is an important measure. From 7.3. in Results, the explanatory powers of the models with and without the QMJ factor are visible for the three time periods. These are compared for each time period. In financial modelling it is common to use R-squared and not Adjusted R-squared. But when comparing models with different number of regressors, it is important to take the Adjusted R-squared into account. This is made, in order to check that the potential impact of QMJ is not due to chance. It is therefore concluded that both R-squared and Adjusted R-squared are of relevance but in this case the Adjusted R-squared is more of importance because of the different number of variables.

For period 1 (2010–2012), the adding of the QMJ factor yields a 0.9% increase in R-squared, translated into 0.2 percentage units. The Adjusted R-squared however decreases with 1.2%, translated into 0.27 percentage units. This means that the model slightly penalizes the QMJ-factor. The conclusion is that for period 1, the adding of the QMJ factor yields both a positive and negative impact on the model in the explanatory power sense, but the decrease is relatively small.

For period 2 (2013–2015), R-squared increases with 0.85% translated into 0.3 percentage units when the QMJ factor is added. In the sense of Adjusted R-squared, there is a decrease of 0.58%, translated into a difference of 0.17 percentage units. The conclusion for period 2, is that the QMJ factor penalizes the model to the degree that the Adjusted R-squared is lower with than without the factor. However, the difference is almost not noticeable and one could therefore conclude that the inclusion of the QMJ factor makes no difference.

For period 3 (2016–2018), the model with the QMJ factor has a higher explanatory power than the model without the factor, both in the sense of R-squared and Adjusted R-squared. A difference of 1.02 percentage units, translated into a 2.69% increase in R-squared and a difference of 0.64 percentage units, translated into an increase of 1.96% in Adjusted R-squared, is yielded. The conclusion for period 3 hence is that the QMJ factor improves the model.
The conclusion in the sense of explanatory power is that the model that includes the QMJ factor in two out of three time periods displays an increase in explanatory power. The second period displays a decrease in explanatory power that can be disregarded, while the third period shows larger increases when considering Adjusted R-squared. As earlier mentioned, factor models are generally able to explain less than 50% of stock returns, and one might question the practical relevance of these models. However, a change in explanatory power as the one illustrated in period 3, might prove good for the model and generally any increase in explanatory power is good for the model given that there exists no trade-off.

8.1.2 Discussion of Significance

As presented in the Theory section, the null-hypothesis is defined as the following:

\[ H_0 : \beta_1 = \beta_2 = \ldots = \beta_k = 0, \]

which means that the regressors are not significant for the explanation of the response. In the case of this thesis, that is interpreted as the QMJ factor not contributing to the explanation of the portfolios’ total return. One wants to be able to reject the null-hypothesis as often as possible in order to say that QMJ is significant for the model.

For each of the three periods the percentage of significant stocks according to \( t \)-value has been presented for each of the 13 factors – the macroeconomic factors and equity market factors including the QMJ factor. This is done in order to see how the QMJ factor acts relative to the other factors. It is easy to draw the conclusion that the null-hypothesis is rejected when receiving a small percentage value, and to still include the QMJ would be something bad. But when comparing to other factors one becomes more careful drawing that conclusion.

For period 1 (2010–2012), the null-hypothesis for the QMJ factor can be rejected in 2.34% of the cases. This result is in the same percentage range as the factor Small-Cap and is even better than the factors Commodities Price and Value Premium. For the factor Sector, close to 99% of the stocks are significant and about 60% for the factor EUR/USD and 48% for JPY/USD.

For period 2 (2013–2015), the null-hypothesis for the QMJ factor can be rejected in 4.02% of the cases. This number places the QMJ factor as the factor with the lowest percentage of stocks that are significant for the factors. The factor Credit has the next lowest percentage and Market and JPY/USD have the highest percentages with 82% and 83% respectively. The results for period 1 and 2 could be explained by the theory of multiple testing which is explained in section 2.6. If this would be the only condition for including a factor or not, QMJ would not be included. However, Small-Cap, Commodities Price and Value Premium would then also be excluded, and they are not.

For period 3 (2016–2018), the null-hypothesis for the QMJ factor can be rejected in 40.2% of the cases. This value is remarkably high, which shows that QMJ is significant to a large number of stocks. When comparing to other factors, most of those factors’ percentage \( t \)-values have been increasing and one can therefore say that it is expected that the QMJ factor also would increase. However, it has had a higher growth rate than most others factors. Hence, it indicates that QMJ has made a significant impact to the explanatory power.

The conclusion of this analysis is that the QMJ factor in two out of three cases has the lowest percentage of significant stocks according to the \( t \)-value. The QMJ factor hence has had more significance in the period closest to today, 2016 – 2018.
8.1.3 Discussion of Weighted Portfolio Beta Significance

From the results in section 7.2.5, it becomes clear that the QMJ factor, as earlier mentioned, only is significant for the weighted portfolio in period 3. Since this is the case, one needs to be cautious proceeding with the analyses of the other results obtained. One could argue that these results are not even valid, since the factor is not significant for the model. However, as argued in section 8.1.2, it is hard to draw any conclusions since the other factors also have inconclusive results and are still included in the model.

8.1.4 Discussion of the Sensitivity of the QMJ Factor

The regression analysis can identify which stocks for each of the three observed periods that are explained the most and the least by the QMJ factor. That is, the QMJ-betas for each period that are the highest (both positive and negative valued) and lowest. By analysing these three stocks for period 3 that possess these characteristics, a conclusion can be made whether or not these results from the regression analysis can be viewed as reliable. Period 3 was chosen since it has showed the most interesting results for the QMJ factor. The stocks are analysed from a quality-perspective in the sense that some of the measures of quality defined in the QMJ article are obtained for each of the individual stocks. The data for calculation of these measures have been obtained both from the companies' annual reports and the database Orbis [20]. Data on the market betas are retrieved from Yahoo Finance [21]. This approach allows for a general overview of the stocks and a natural comparison between the two stocks in each time period. The analysis will also include a so called GPRV analysis, which is a patented tool from Infront Analytics for assessing the relative value of listed stocks using fundamental analysis [22]. It will provide a graphical view of a stock’s attractiveness through 4 categories: growth, profitability, risk and value.

Together these analyses will form a conclusion about the stocks and their respective QMJ-betas. Below, the stocks are presented for period 3 and a following comparison of them based on the quality measures profitability, growth and safety. GPRV analyses for period 1 and 2 can be found in Appendix 10.3. Worth mentioning is that growth over three years has been considered and not the five year growth defined in the QMJ report. This growth aspect will therefore not carry the same importance as the rest of the quality measures in the analysis.
Period 3: 2016–2018: According to the results of the regression, in the period 2016 – 2018, the stock with highest positive QMJ-beta is ArcelorMittal (MT), the stock with highest negative QMJ-beta is G4S (GFS) and the stock with lowest positive QMJ-beta is Deutsche Post (DPW).

ArcelorMittal (MT): Luxembourg-based leading steel and mining company and supplier of quality steel products in markets such as automotive, construction, household appliances and packaging. Present in 60 countries [26].

G4S (GFS): British company acting as a leader in global, integrated security. Also a provider of logistics and cash solutions as well as risk consulting. Present in 125 countries [27].

Deutsche Post (DPW): Germany-based deliverer of mail and parcel in Germany and the world. An expert provider of dialogue marketing and press distribution services and corporate communications solutions [28].

Figure 10 above shows the GPRV analyses of these stocks and below are the different quality measures for the stocks displayed – that is, the profitability (GPOA, ROE, ROA, GMAR) for 2016, 2017 and 2018, the profitability growth 2016 – 2018 and the market beta (BAB).
Discussion

1. Profitability 2016:
   - GPOA: 8.5%
   - ROE: 5.4%
   - ROA: 2.3%
   - GMAR: 11.2%

2. Profitability 2017:
   - GPOA: 9.1%
   - ROE: 11.1%
   - ROA: 5.4%
   - GMAR: 11.4%

3. Profitability 2018:
   - GPOA: 9.9%
   - ROE: 12.1%
   - ROA: 5.8%
   - GMAR: 11.8%

4. Growth 2016–2018:
   - GPOA: 3.5%
   - ROE: 11.1%
   - ROA: 4.8%
   - GMAR: 4.7%

5. Safety:
   - BAB: 1.84 (0.92)

**Comparison of the Stocks:** When analysing the different ratios it becomes clearer why QMJ explains the stocks’ returns differently. ArcelorMittal has higher margins in almost every ratio besides the growth rates. ArcelorMittal is therefore more profitable and a stock with higher quality. When looking at the growth rates for Deutsche Post and comparing them to ArcelorMittal, the growth rates that are driven by revenue growth, are much higher for Deutsche Post. It seems likely that Deutsche Post had a revenue problem and turned the business around. When instead comparing the growth rates between 2017–2018, the GPOA-growth is 3.8% and the GMAR-growth is 2.4% which is either in line with ArcelorMittal or much lower. The returns on assets and returns on equity are much lower than the ones of ArcelorMittal and it is then concluded that Deutsche Post is expected to not have as high quality as ArcelorMittal. The company with the most negative QMJ-beta was G4S. When comparing it becomes clear that this should be expected. The company is suffering from a problem with their return on equity and return on assets and has almost no growth in the respect of profitability. This means that the stock has a lower quality and it is therefore expected that it should have a big QMJ-beta in terms of absolute value but with a negative correlation. When taking into account the market betas of the stocks, there are however
no clear contributions to explaining the results. But a decision is made to value the profitability and growth higher than the safety of the stocks.

As argued in section 4.4, a high QMJ-beta should reflect a high-quality stock and a low QMJ-beta should reflect a more average-leveled quality of the stock since the returns of the high- and low-quality portfolios do not greatly differ. The results confirm this hypothesis and hence the results can be viewed as reliable.

8.1.5 Beta Percentage Analysis

An approach in order to decide the importance of the QMJ factor for the overall model, is to analyse the beta-percentage of the factor. In this section, beta-percentage refers to the percentage of each factor’s beta to the whole model. Since the general beta-value indicates a change on the response variable \( y \) due to a unit change in the explanatory variable \( x \) (with all other variables kept constant), a higher QMJ-beta indicates a higher importance of it for the model. These results are visible in section 7.2.6.

For period 1 (2010–2012), the QMJ factor has a beta-percentage of 4.24751\%. This makes it the eighth highest beta of all the 13 factors. The factor has a beta-percentage higher than the ones of the factors Bond Yield, Credit, Oil, Commodities Price and Value Premium. For period 2 (2013–2015), the QMJ factor has a beta-percentage of 6.768995\%. This makes it the fifth highest value of all the 13 factors. The factor has a beta-percentage higher than the ones of the factors Global Yield, Bond Yield, Credit, Commodities Price, EUR/USD, Value Premium and Country. For period 3 (2016–2018), the QMJ factor has a beta-percentage of 16.660711\%. This makes it the second highest value of all the 13 factors. The factor has a beta-percentage higher than all the other factors except the Global Market factor.

The conclusion for this beta analysis is that the QMJ factor’s beta-percentage increases over time and is an important factor in period 3 for explaining stock returns. It increases from 4.24\% to 6.76\% to 16.66\% — that is, the response changes 16 times the change in the QMJ factor. As mentioned, the impact of these conclusions should not be overly amplified when evaluating its overall impact.

8.1.6 Sector Analysis

One additional approach in order to understand the QMJ factor is to study its significance per sector, for each of the three periods. This analysis is based on ten sectors from GICS. For each period and sector, the percentage of stocks with significant \( t \)-values was studied.

For period 1 (2010–2012), five out of the ten sectors have no stocks with significance for the QMJ factor. The other sectors have percentages in the range of 1.39\% – 8.0\%, where Consumer Staples has the highest value of 8\%. For period 2 (2013–2015), five out of the ten sectors again have no stocks with significance. However, there is now one sector that has changed — Consumer Staples now has no significant stocks while Utilities has 5.26\% significant stocks. The sectors with significant stocks have percentages in the range of 1.79\% – 9.72\%, where Information Technology has the highest value of 9.72\%. For period 3 (2016–2018), all of the ten sectors now have stocks with significance. The sectors with significant stocks have percentages in the range of 20.0\% – 94.74\%, where Utilities has the highest value of 94.74\%.

The conclusion is that the number of sectors with significant stocks according to \( t \)-value has increased over time as well as the percentage of significant stocks within each sector, which is not
surprising given the previous results. Health Care is a sector that has increased its percentage from 3.33% to 6.67% to 60% of significant stocks. Information Technology has in a similar manor increased from 1.39% to 9.72% to 28%.

8.1.7 Analysis of Factor Betas in Combination With Sectors

In addition to the previous analysis, it might also be interesting to analyze the QMJ-betas for each sector in order to understand which sectors QMJ is more sensitive in. In this section, beta-percentage refers to the percentage of each factor’s beta to the whole model.

For period 1 (2010–2012), the QMJ factor has the highest beta-percentage in the sector Consumer Discretionary where it reaches a value of 9.483%. The rest of the factors have values ranging from 5.330% to 7.962%. For period 2 (2013–2015), the QMJ factor has the highest beta-percentage in the sector Information Technology where it reaches a value of 8.809%. The rest of the factors have values ranging from 4.053% to 8.371%. For period 3 (2016–2018), the QMJ factor has the highest beta-percentage in the sector Utilities where it reaches a value of 20.015. The rest of the factors have values ranging from 4.053% to 14.664%.

The conclusion is that the QMJ factor has the general highest beta-percentage in period 3, a value of 20.015%, which represents the sector Utilities. For each period, there is a new sector that holds the highest beta-percentage for the QMJ factor. For each period, no sector reoccurs in having the highest beta-percentage; there are new sectors for each period and hence there exists no distinct pattern on the relationship between high beta-percentages and sectors.

8.2 CRM Framework

In order to determine if a factor should be included in the model, four market standard metrics have been selected to consider. These will be referred to as CRM framework and include:

- Large total R-squared.
- Significant t-statistics of the factor sensitivities (absolute value of t should not exceed 2).
- Low correlation among factors.
- Large fraction of individual stocks with statistically significant factor sensitivities.

8.2.1 Large Total R-squared

As stated earlier, the Adjusted R-squared is of more relevance than the simple R-squared. The Adjusted R-squared is only significantly increased in period 3. However, the other periods are not affected at all or the decrease is small enough to be neglected. Since the results are inconclusive, it is difficult to state a conclusion. Most likely QMJ would not damage the model to the extent that it should not be included in the model. However, to support this conclusion, further research is required.

8.2.2 Significant t-statistic of Factor Sensitivities

In order to investigate if the QMJ factor is significant for the portfolio, the results from section 7.2.5 are taken into account. The results show that the factor is only significant in period 3, not in period 1 and 2. Again the results are inconclusive and since the factor in the majority of cases is not significant, the conclusion can be drawn that the QMJ should not be added. However, when comparing to the other factors that are included in the 12-factor model, the majority of those
factors are not significant. This makes it difficult to conclude if the QMJ factor should be included or not. The significance in period 3 indicates that QMJ is of interest and further research needs to be conducted.

8.2.3 Low Correlation

To add the QMJ factor in the model it is important that the correlation between QMJ and the other factors is low. In section 7.2.10 the correlations between these factors are shown and there is almost no correlation. When only considering this criteria it is concluded that the QMJ factor could be included to the model without degrading it. One could look at other correlation measures such as multicollinearity but since QMJ is residualised, this is not of interest.

8.2.4 Significance for Individual Stocks

In section 7.2.4, the percentage of stocks that are significant is shown. The values 2.34% and 4.02% from period 1 and 2 seem low but when comparing to the other factors it does not have to be interpreted in that way. In period 1 there are many other factors with even lower percentages and are still included in the model. However, when comparing to the factors of period 2, the value of the QMJ factor is low. In period 3 it is as high as 40.27%, which is much higher than the majority of factors. This indicates a good level of significance. The conclusion drawn in section 8.2.2 is also applicable in this section; more research is needed.

8.2.5 Conclusion CRM Framework

From the analysis of the CRM Framework, it can be concluded that there is no clear recommendation about whether to include the QMJ factor or not. Since the results are ambiguous, one must be careful when expressing them. If all periods would have had the same results as the ones in period 3, the conclusion would be to include the QMJ factor. However, since this is not the case further research is needed, see section 8.4.

8.3 Discussion of Method

In order to understand if the method is good enough to use for reliable results, one must consider different phenomena that can occur and affect the model and its abilities to fit a model that can accurately predict future observations. Since the method of this thesis work includes making t-tests for each stock, the issue of multiple testing described in 4.1.6 could be a phenomena occurring. For this thesis work, this aspect of multiple testing has not been investigated, hence it could be something affecting the model.

One of the dangers of having too many variables in a model, is the overfitting of data. If the factor is highly significant for the model it should be added. However, adding too many factors may lead to an overfitted model which is a serious problem when using the model to predict future observations. However, in this case only one factor is added and it is assumed that the problem of overfitting is not present for the 13-factor model.

A condition for the modelling to work correctly was that the data needed to be correct. The data from AQR is difficult to analyse and is calculated through complex calculations. The assumption was made that the calculations were correct. Nevertheless, this is of course something that could have influenced the model and its quality.
Another important aspect was the programming part of the thesis work. In order to make sure that the programming was correct, the available co-variance matrices from the CRM was compared to the self-constructed model. The matrices were the same and it was then concluded that the programming was done correctly. However, there could still be unknown bugs that could have influenced the results.

8.4 Further Research

The field of factor models is, as already expressed in this thesis, a field which is a popular area of research, since the benefits of improved models can have great impact on the areas in which the factor models can be applied. This thesis work has just touched the surface on the introduction of new factors into existing models, and in specific the factor Quality Minus Junk into a 12-factor model consisting of macroeconomic factors and equity market factors. In this thesis, three time periods are studied, and the number of time periods could definitely be broadened in order to in a greater extent understand and explain the QMJ factor and in which sort of time periods it is of greater value. It could also be of value to try and include whole cycles in order to better understand how the factor acts in recessions or booms.

Another way to improve this thesis work, is to include more data. While 300 stocks perhaps was enough for the scope of this thesis, a wider range of stocks could definitely improve the model and its reliability. Not only including European stocks but also global stocks could improve the understanding of the QMJ factor.

8.5 Conclusion

The answer to the research question; What is the impact of the QMJ factor on the currently used risk model for European stocks?, can not with certainty be concluded at this stage. The answer to the sub-question What is the effect of the QMJ factor on the explanatory power of the model? is that the explanatory power increases in period 3 for the model. The QMJ factor influences the model differently during the studied time periods, and this is an important aspect that needs to be studied further before being able to answer this question with full certainty. The sub-question Should the QMJ factor be included into the risk model? does not either have an unambiguous answer. According to the CRM Framework, QMJ enhances the model, especially during period 3. Before relying on the analysis by the CRM Framework one should be careful to include the QMJ factor without conducting more research, see section 8.4. There, it is concluded that the scope is too narrow in the sense of small number and types of stocks and the whole business cycle is not investigated.
9 List of References


10 Appendices

10.1 Definition of Ratios

Profitability

\[ GPOA = \frac{\text{Revenue} - \text{CostOfGoodsSold}}{\text{TotalAssets}} = \frac{GP}{AT} \]

\[ ROE = \frac{\text{NetIncome}}{\text{BookEquity}} = \frac{IB}{BE} \]

\[ ROA = \frac{\text{NetIncome}}{\text{TotalAssets}} = \frac{IB}{AT} \]

\[ CFOA = \frac{\text{NetIncome} + \text{Depreciation} - \Delta \text{WorkingCapital} - \text{CapitalExpenditures}}{\text{TotalAssets}} = \frac{CF}{AT} \]

\[ GMAR = \frac{\text{Revenue} - \text{CostOfGoodsSold}}{\text{TotalSales}} = \frac{GP}{SALE} \]

\[ ACC = \frac{-\Delta \text{WorkingCapital} - \text{CapitalExpenditures}}{\text{TotalAssets}} = \frac{MWCPD}{AT} \]

Working Capital = Current Assets - Current Liabilities - CashShort Term Instruments + Short Term Debt + Income Taxes Payable

Book-Equity = Shareholders’ Equity – Preferred Stock

Shareholders’ Equity = Stockholders’ Equity or Sum of Common Equity and Preferred Stock

Growth

\[ \text{GrowthGPOA} = \frac{(GP_t - GP_{t-5})}{AT_{t-5}} \]

\[ \text{GrowthROE} = \frac{(IB_t - IB_{t-5})}{BE_{t-5}} \]

\[ \text{GrowthROA} = \frac{(IB_t - IB_{t-5})}{AT_{t-5}} \]

\[ \text{GrowthCFOA} = \frac{(CF_t - CF_{t-5})}{AT_{t-5}} \]

\[ \text{GrowthGMAR} = \frac{(GP_t - GP_{t-5})}{SALE_{t-5}} \]

\[ \text{GrowthACC} = \frac{(MWCPD_t - MWCPD_{t-5})}{AT_{t-5}} \]

Safety

\[ BAB = -\beta = -\text{Market Beta} \]

\[ IVOL = -\sigma^i = -\text{Stock’s Idiosyncratic Volatility} \]
\[ LEV = -\frac{(LongTermDebt + ShortTermDebt + MinorityInterest + PreferredStock)}{AT} \]

**Payout**

\[ EI\text{SS} = -\text{One-year Percent Change in Split-adjusted Number of Shares} = -\log \frac{\text{SHROUT}_{\text{ADJ}, t}}{\text{SHROUT}_{\text{ADJ}, t-1}} \]

\[ DI\text{SS} = -\text{One-year Percent Change in Total Debt} = -\log \frac{TOTD_{t}}{TOTD_{t-1}} \]

\[ NPOP = \frac{\text{Total Net Payout}}{\text{Profits}} \]

### 10.2 Stock Names

AP MOLLER MAERSK B  
GROUPE BRUXELLES LAMBERT  
ELECTROLUX B  
ERICSSON (LM) B  
ESSILORLUXOTTICA  
EURAZEO  
FIAT CHRYSLER AUTOMOBILE  
VIVENDI  
GECINA  
GLAXOSMITHKLINE  
COMPASS GROUP  
DIAGEO  
HAMMERSON  
HEINEKEN HOLDING  
HEINEKEN NV  
HENKEL VORZUG  
HENNES MAURITZ B  
ACCOR  
LAFARGEHOLCIM  
IBERDROLA  
UMICORE  
IMERYS  
INVESTOR B  
JOHNSON MATTHEY  
KERRY GROUP A  
KINGFISHER  
KONE B  
KBC GROUPE  
LAND SECURITIES GROUP  
LEGAL GENERAL GROUP  
LLOYDS BANKING GROUP  
L’OREAL  
LUFTHANSA  
LVMH MOET HENNESSY  
MAPFRE  
MARKS SPENCER GROUP  
MEDIOBANCA  
GEA GROUP  
WARTSILA B  
MICHELIN  
MUENCHENER RUECKVERSICHT  
ING GROEP  
ASSICURAZIONI GENERALI  
NESTLE  
Nokia Corp  
NORSK HYDRO  
UNITED UTILITIES GROUP  
NOVO NORDISK B  
ATLAS COPCO A  
ATLAS COPCO B  
OMV AG  
ORKLA  
PARGESA HOLDING INH  
PEARSON  
PERNOD RICARD  
KONINKLIJKE PHILIPS  
KERING  
ADECCO GROUP  
PORSCHE AUTOMOBIL VZG  
PRUDENTIAL  
ATLANTIA  
VODAFONE GROUP  
RECKITT BENCKISER GROUP  
RELX (GB)  
SWISS LIFE HOLDING  
UPM-KYMMENE  
REPSOL  
ROCHE HOLDING GENUSS  
ROLLS-ROYCE GROUP
ROYAL BANK OF SCOTLAND
AXA
RIO TINTO PLC (GB)
RWE STAMM
SAFRAN
SAINSURY (J)
SAINT-GOBAIN
SAMPO A
JULIUS BAER GROUP
NOVARTIS
SANOFI
SAP
ESSITY B
SCHINDLER HOLDING NAMEN
SCHINDLER HOLDING PART
SCHNEIDER ELECTRIC
SCHRODERS
UBS GROUP
SWISS RE
BALOISE HOLDING
SEVERN TRENT
VINCI
ROYAL DUTCH SHELL B
SIEMENS
LEONARDO
SKAND.ENSKILDA BANKEN A
SKANSKA B
SKF B
SEGO
SWATCH GROUP NAM
SMITH & NEPHEW
SMITHS GROUP
SOCIETE GENERALE
SODEXO
SOLVAY
STANDARD CHARTERED
TELECOM ITALIA
TELECOM ITALIA RNC
RSA INSURANCE GROUP
SGS
SVENSKA HANDELSBK A
INTESA SANPAOLO
BBVA
TELEFONICA
TESCO
THALES
THYSSEN KRUPP
TOTAL
BANCO SANTANDER
UCB (GROUPE)

UNIBAIL-RODAMCO-WE
UNILEVER PLC (GB)
METSO CORP
E. ON
VOLKSWAGEN STAMM
VOLKSWAGEN VORZUG
VOLVO B
WHITBREAD
FERGUSON
WOLTERS KLUWER
WPP
BARCLAYS
ZURICH INSURANCE GROUP
AEGON
BASF
INTERCONTINENTAL HOTELS
SSE
BAYER
ABB LTD
SANDVIK
BEIERSDORF
HSBC HOLDINGS (GB)
ASTRAZENECA
SWATCH GROUP INH
SECURITAS B
FIN RICHEMONT NAMEN A
BIC
AGEAS
UNILEVER NV (NL) CERT
BMW STAMM
BNP PARIBAS
KONINKLIJKE KPN
DNB
3I GROUP
ASSOCIATED BRITISH FOODS
ATOS
SCOR
ASSA ABLOY B
RENAULT
BOUVYGES
STMICROELECTRONICS
BAE SYSTEMS
BRITISH LAND COBP
BT GROUP
JERONIMO MARTINS SGPS
DANONE
BUNZL
AHOLD DELHAIZE
CAPGEMINI
CARLSBERG B
AIR LIQUIDE
CARREFOUR
CASINO
NATURGY ENERGY GROUP
ASML HLDG
SWEDBANK
ARCELORMITTAL A
VOESTALPINE
MERCK KGAA STAMM
NORDEA BANK
ADIDAS
ENI
NATIONAL GRID
RANDSTAD NV
TELE2 B
SWEDISH MATCH
DASSAULT SYSTEMES
HENKEL STAMM
METRO STAMM (NEW)
DEUTSCHE TELEKOM
ORION-YHTYMÄE B
FRESENIUS MEDICAL CARE
CENTRICA
EDP ENERGIAS DE PORTUGAL
RYANAIR HOLDINGS
BHP GROUP (GB)
ORANGE
ERSTE GROUP BANK
IMPERIAL BRANDS
ALSTOM
SES A-FDR
BRITISH AMERICAN TOBACCO
SWISSCOM
FORTUM CORP
STORA ENSO R
AIRBUS
LONZA GROUP
ENEL
PUBLICIS GROUPE
WILLIAM DEMANT HOLDING
MORRISON WM SUPERMARKETS
UNITED INTERNET
GIVAUDAN
TELIA CO
ELISA A
VEOLIA ENVIRONNEMENT
CARNIVAL PLC (GB)
NOKIAN RENKAAT
GEBERIT
SONOVA HOLDING
QIAGEN
NOVOZYMES B
DEUTSCHE POST
ANHEUSER-BUSCH INBEV
TELENO
DEUTSCHE BOERSE
DSV DE SAMMENSLUT VOGN
G4S
INTL AIRLINES GROUP
INDITEX
EQUINOR
JC DECAUX INTERNATIONAL
FRAPORT
LONDON STOCK EXCHANGE
BANCO SABADEL
RED ELECTRICA CORP
KLEPIERRE
SNAM
CREDIT AGRICOLE
LINDT SPRUENGLI NAMEN
KUEHNE NAGEL INTL
HANNOVER RUECK
CNP ASSURANCES
BURBERRY GROUP
AKZO NOBEL
YARA INTERNATIONAL
PROXIMUSTERNA
ANGLO AMERICAN
ILIAD
FERROVIAL
ADMIRAL GROUP
NESTE CORPORATION
RAFFEISEN BANK INTL
ROYAL DUTCH SHELL A
ENGIE
EDF
COLOPLAST B
COLRUYT
TRYG
ANTOFAGASTA
EUTELSAT COMMUNICATIONS
HUSQVARNA B  
ADP  
AVIVA  
COMMERZBANK  
COVIVIO  
STANDARD LIFE ABERDEEN  
GRIFOLS  
EXPERIAN  
LEGRAND  
MILLICOM INTL CELL SDR  
PRYSMIAN  
CAIXABANK  
ICADE  
BUREAU VERITAS SA  
SUEZ  

GALP ENERGIA SGPS B  
FRESNILLO PLC  
TENARIS (IT)  
NATIXIS  
EXOR  
UNICREDIT  
CRH  
CREDIT SUISSE  
DAIMLER  
DANSKE BANK  
DEUTSCHE BANK  
ALLIANZ  
KONINKLIJKE DSM
10.3 Company-specific Information

Figure 11: GPRV Analysis for companies in period 1
Figure 12: GPRV Analysis for companies in period 2

(a) RSA Insurance Group [32]  
(b) Tesco [33]  
(c) Philips [34]