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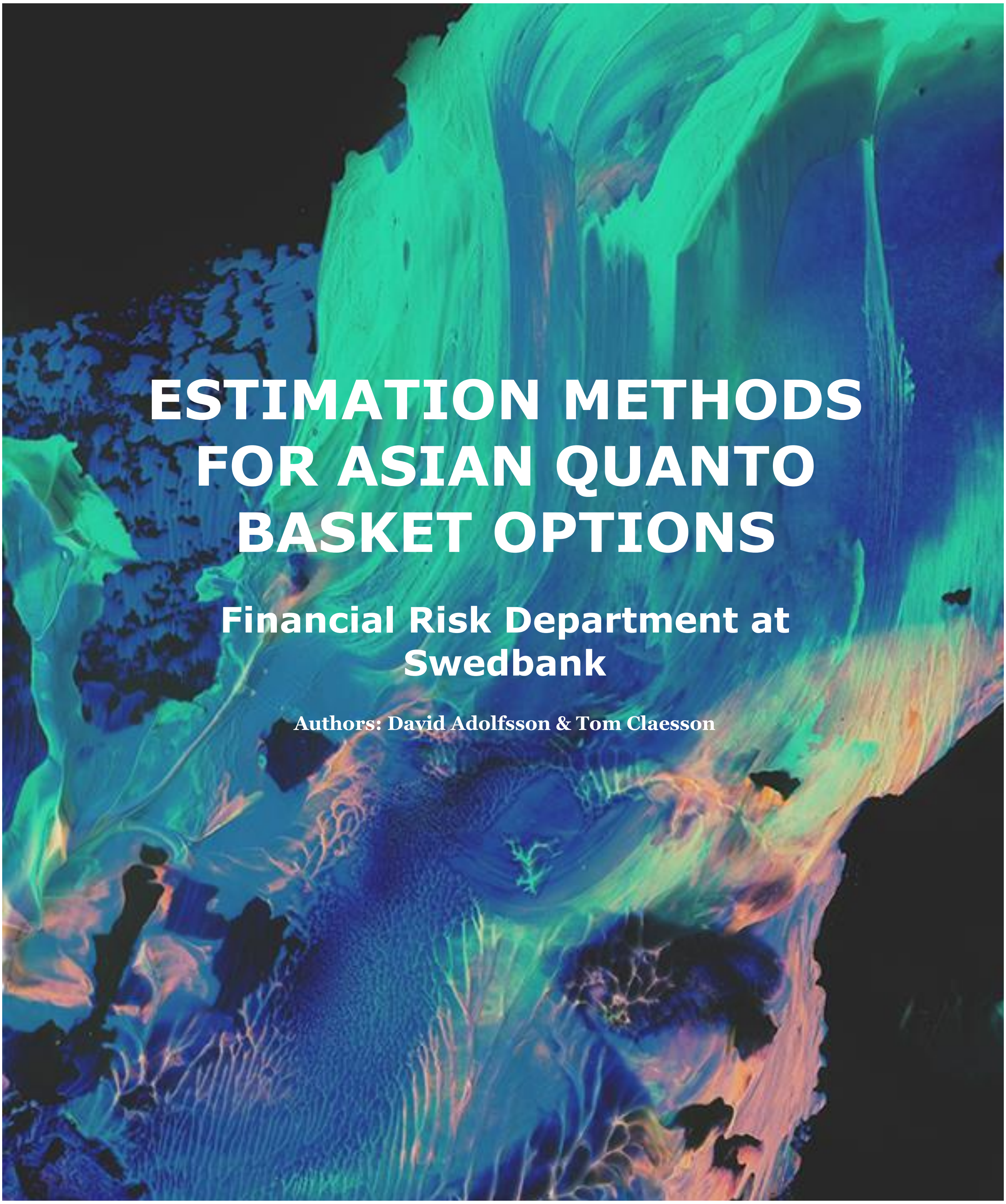

ESTIMATION METHODS FOR ASIAN QUANTO BASKET OPTIONS

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Abstract

All financial institutions that provide options to counterparties will in most cases get involved with Monte Carlo simulations. Options with a payoff function that depends on asset's value at different time points over its lifespan are so called path dependent options. This path dependency implicates that there exists no parametric solution and the price must hence be estimated, it is here Monte Carlo methods come into the picture. The problem though with this fundamental option pricing method is the computational time. Prices fluctuate continuously on the open market with respect to different risk factors and since it's impossible to re-evaluate the option for all shifts due to its computing intensive nature, estimations of the option price must be used. Estimating the price from known points will of course never produce the same result as a full re-evaluation but an estimation method that produces reliable results and greatly reduces computing time is desirable. This thesis will evaluate different approaches and try to minimize the estimation error with respect to a certain number of risk factors.

This is the background for our master thesis at Swedbank. The goal is to create multiple estimation methods and compare them to Swedbank's current estimation model. By doing this we could potentially provide Swedbank with improvement ideas regarding some of its option products and risk measurements. This thesis is primarily based on two estimation methods that estimate option prices with respect to two variable risk factors, the value of the underlying assets and volatility. The first method is a grid that uses a second order Taylor expansion and the sensitivities delta, gamma and vega. The other method uses a grid of pre-simulated option prices for different shifts in risk factors. The interpolation technique that is used in this method is called *Piecewise Cubic Hermite* interpolation. The methods (or referred to as approaches in the report) are implemented to handle a relative change of 50 percent in the underlying asset's index value, which is the first risk factor. Concerning the second risk factor, volatility, both methods estimate prices for a 50 percent relative downward change and an upward change of 400 percent from the initial volatility. Should there emerge even more extreme market conditions both methods use linear extrapolation to estimate a new option price.

Key words: Monte Carlo simulation, Estimation, Option, Taylor expansion, Sensitivities, Price interpolation, Grid.

Sammanfattning

Alla finansiella institutioner som handlar optioner kommer att stöta på Monte Carlo simulering. Optioner som beror av de underliggande tillgångarnas värde under flera tidsperioder sägs ha "vandringsberoende" vilket gör att det är inte bara en tidsperiod som avgör priset på optionen. När flera olika tidsperioder påverkar priset på optionen finns det inte någon sluten formel, vilket gör att Monte Carlo metoder ofta används. Monte Carlo simuleringar är extremt datorintensivt vilket leder till att prissättningen av optioner är ett tidskrävande moment. Riskfaktorer fluktuerar ständigt på den öppna marknaden vilket gör att institutioner måste uppdatera priset på optioner på daglig basis. Eftersom Monte Carlo simuleringar är så pass tidskrävande är det orimligt att genomföra en ny omvärdering för alla skiften i riskfaktorerna, vilket blir problematiskt. För att lösa detta kan estimering av priset på optioner göras med hjälp av olika estimeringsmetoder, vilket är huvudsyftet med denna rapport. En estimeringsmetod kommer aldrig att generera lika bra resultat som en full omvärdering med hjälp av Monte Carlo men målet är att reducera värderingstiden och minimera felprissättning. Målet är att implementera och analysera olika estimeringsmetoder för att på så sätt ge underlag till förbättringsmöjligheter till den befintliga estimeringsmetoden hos Swedbank.

Denna uppsats baseras på två estimeringsmetoder. Den första metoden använder andra ordningens Taylorutveckling där känsligheterna delta, gamma och vega uppskattas i flera punkter över estimeringsintervallet. Dessa används sedan för att estimera det nya priset på optionen vid skiften i de olika riskfaktorerna. Den andra metoden använder interpoleringstekniken *Piecewise Cubic Hermite* interpolering mellan för-simulerade Monte Carlo optionspriser utplacerade över estimeringsintervallet för att estimera det nya optionspriset. Bägge metoderna tar hänsyn till skiften i de underliggande tillgångnas gemensamma indexvärde och dess volatilitet. Metoderna klarar av att estimera ett pris på optioner där det sker ett skifte på $+-50$ procent i värdet på de underliggande tillgångarna och ett volatilitetsskifte inom intervallet -50 till 400 procent. Skulle det visa sig att det uppstår ännu mer extrema skiften på marknaden så extrapolerar metoderna fram optionspriser linjärt.

Nyckelord: Monte Carlo simulering, Estimering, Option, Taylorutveckling, Känsligheter, Prisinterpolering, Rutnät.

Contents

1 Introduction

1.1 Description of problem

Predicting the price of a stock in the future is impossible and this creates a problem when pricing options. The probability distribution of a finite sum of log-normally distributed and mutually correlated stochastic variables is unknown, hence no exact explicit pricing formula exists. This implies a mathematical problem with pricing non linear options and their combinations. Since a full re-evaluation using Monte Carlo methods is unreasonable for all shifts in the different risk factors due to its computing intensive nature, an attempt to estimate the price from a known point is often done. This can be done in a multiple different ways, a few of which will be explained and compared in this thesis. The main question that it will answer is,

- How can different estimation methods reduce the estimation error of the price of Asian Quanto Basket options when there is a change in the underlying assets' value and or volatility?

1.2 Swedbank

Swedbank's roots go as deep as 1820 when Sweden's first savings bank was founded in Gothenburg by Eduard Ludenorff. In the first decades of the 19th century the establishment of savings banks grew rapidly and by 1870 there were over 300 savings banks in Sweden. In 1997 a merger between Sparbanken Sverige and Föreningsbanken created FöreningsSparbanken that had over five million customers. FöreningsSparbanken AB used the name Swebank in international context and 2006 the bank changed name to Swedbank AB (Swedbank, 2019a).

Today Swedbank has over 7.3 million private and 0.6 million corporate customers mainly located in Sweden and in the Baltic states. Swedbank's goal has since the start been to help as many people and businesses as possible to achieve a solid financial sustainability. The core values of Swedbank are open, simple and caring (Swedbank, 2019b).

1.3 Background of problem

Quoted prices on active markets determine the fair value of options but in some cases there are no such market prices available. In those cases Swedbank will use widely accepted valuation models to determine the price, such as discounted future cash flows (Swedbank, 2018). One method of forecasting the future cashflow is by using Monte Carlo simulations and then, by discounting these, a fair value for the option can be determined. Monte Carlo simulations are widely used among all financial institutions and is the backbone for evaluating exotic options. The crucial problem with Monte Carlo simulations is the computational time and time to market which is the time it takes for the issuer to bring the product available on the market for the customers.

This issue is relevant to Swedbank because of current and future regulations on banks. They must be able to price the products and have models for assessing risk to be able to determine the amount of capital they have to hold to cover potential losses. With a more precise model this capital requirement can be reduced which can lead to a higher return on capital. We choose to attempt to estimate a price for an Asian quanto basket option because Swedbank has its largest exposure to these types of options. Hence improvement ideas for this area would be most relevant to Swedbank.
Using Monte Carlo methods to price all Swedbank's options is computing-intensive. When the underlying asset's and its volatility changes in value so does the option value. This change in price for the option can be calculated in a number of different ways, some of which are doing the exact price simulation all over again. Since the stock market fluctuates it is hard to determine when to do this and how often. An approximate way of doing this is to calculate the option's sensitivities, the greeks. These can reflect the change in price for an option when there is a change

in the underlying asset's value and or volatility. The sensitivities *delta*, *gamma* and *vega* and how these can be used to estimate a new price of the option will one of the central approaches in this thesis. The *delta* and *gamma* is the first and second order derivative with respect to the value of the underlying asset's and *vega* is the first order derivative with respect to the asset's volatility. The other central approximation method uses fair valued option prices, by using interpolating techniques a estimation of the option price can be done when there is a change in the different risk factors.

1.4 Goal

The goal with the master thesis is to implement and analyze different estimation methods for the price of options and compare them to the current method used at Swedbank. To compare the methods we will look at number of Monte Carlo re-evaluation required to build the model and its precision. The ultimate goal is to improve the risk measurement for options.

1.5 Purpose

Swedbank is implementing an upgraded system for modelling, calculation and monitoring of market and counterparty risk. The initiative is done in project form where improvements to the current estimation model also are in scope in order to align and prepare for future upcoming regulations. Within this domain the project has identified the risk measurement of options as a possible area.

1.6 Limitations

There are several risk factors that have an impact on price of options and shifts in these occur frequently. This thesis will focusing on estimating option prices based on shifts in the two most impactful risk factors. These are the value of the underlying assets and their respective volatility. This implicating that risk factors such as interest rate, foreign exchange rate and time are constant during the pricing process. Another limitation that have been done is that we are only testing our methods on a Asian Quanto Basket option.

1.7 Approach and Outline

In this thesis, different estimation methods for measuring risk for options will be implemented and analyzed using the software Matlab 2017a. The first approach is first order Taylor expansion, *delta*-approximation. The second is *delta-gamma*-approximation, which is a second order Taylor expansion. The third is *delta-gamma-vega*-approximation, which also take the volatility into consideration. The fourth approach is a grid with fair value option prices. The final approach is *delta-cross-gamma* which is Swedbank's current approximation method to evaluate some type of options. Approach 3 will be developed further to the *Delta-Gamma-Vega grid* later in the thesis and approach number 4 is later referred to as *Price-interpolation grid*. Approach 3 and 4 will be compared to Swedbank's internal approach, which we call approach 5. Important to notice is that we have not implemented approach 5. Approach 3-5 will be compared to each other evaluated both on accuracy and computational complexity, in terms of Monte Carlo re-evaluations. Results from approach 1 and 2 will not be analyzed in this thesis due to the fact that they only have the ability to estimate the option price for shifts in one risk factor. These were used to evaluate if a grid solution could improve an estimation model. They will also however be used to help the reader get a better understanding of approach 3.

The thesis is structured as follows: Chapter 2 will include all relevant theory for the reader to get a good understanding of the topic. Such as what is an option, Monte Carlo simulation, Brownian Motion, interpolation techniques and what the greek sensitivities are. Chapter 3 will guide the reader through the methods and model implementation, focusing on the most central approaches (3 and 4), which are mentioned above. Chapter 4 will show the main results and Chapter 5 will cover the conclusions from the results. Chapter 6 include a discussion and further development suggestions of the thesis.

2 Theory

This chapter will guide the reader through all the necessary theory about the pricing problem.

2.1 Options

European option
A European option gives the owner the right but not the obligation to buy (call option) or sell (put option) an underlying asset at a fixed date in the future known as the exercise date at a predetermined price called the strike price. What separates options from each other is time to maturity, strike price, payoff function and how and when they can be exercised. The most common type of option is the European option.

Definition 1.1 A European call option on the underlying asset S with strike price K and exercise date $\mathbf{x}$ follows the following properties.

- The holder of the contract has, exactly at the time $\mathbf{x}$, the right to buy the underlying asset S for K SEK from the underwriter of the option.

(Bjork 1999, 77)

The European option can unlike most other options be valuated through a parametric pricing formula, the Black-Scholes (1973) formula

$$\text{call}_t = S_t \Phi(d_1) - K e^{-r(T-t)} \Phi(d_2)$$

where

$$d_1 = \frac{\log(S_t/K) + \left(r + \sigma^2/2\right)(T-t)}{\sigma\sqrt{T-t}}$$

$$d_2 = \frac{\log(S_t/K) + \left(r - \sigma^2/2\right)(T-t)}{\sigma\sqrt{T-t}} = d_1 - \sigma\sqrt{T-t}.$$

Where S_t is the price of the underlying asset at time t, K is the strike price, r is the annual risk-free interest rate, T-t is the time left to expiration measured in years, σ is the underlying annual volatility and Φ is the cumulative distribution function of a standard normal distribution with mean of zero and standard deviation of 1. Under the standard Black-Scholes assumptions, the underlying asset has a continuous identical independent distribution of normal returns. Price or value of an option is affected by many underlying risk factors, for instance volatility, strike price, value of the underlying asset's or foreign exchange rates. The market price of an option is commonly referred to as spot price or net present value of the option (Daníelsson 2011, 115-116).

Important to highlight is that the buyer of the option has the **option** but not obligation to exercise it. The strike price is for a call option in the previous definition, the price you can pay for an asset S at the exercise date. If the asset's price is above the strike price, the rational decision is to exercise the option. If the asset price however is below the strike price you would rather buy the asset for less than the strike price in the open market. This is evident when looking at the payoff function for a simple European call, $(S-K)^+$. This shows that if the asset's price is below the strike price at the exercise date it would not be beneficial to buy the asset for K. An option that is not at it's maturity date and below the strike price is however not worthless and will still have a value. This is called intrinsic value or often referred to as time value and is the difference between the options market price and maximum of the payoff function. An options lifespan, the time between $t \to T$, can vary greatly from hours to decades. An "at the money"-option is when the value of the underlying assets is equal to the strike price.

Quanto option
A quanto option, also known as a quantity-adjusting option is a cross-currency financial instrument where the underlying asset is denominated in a currency that is different compared to the currency that the option is denominated in (Investopedia, 2018a).

Asian option
An Asian option is instead of only looking at the underlying asset's price at the maturity date the arithmetic or geometric average of the asset's price over a pre-determined time period or time points. This average is then used to determine the payoff as per the function

$$(\overline{S} - K)^+. \tag{1}$$

Where $\overline{S}$ is the average for S in the different time points and K is the strike price. The arithmetic mean for Asian option has the following equation

$$\overline{S} = \frac{1}{m} \sum_{j=1}^{m} S_j. \tag{2}$$

Where S_j is the asset's value at the different pre determined time points (Glasserman 2003, 8).

Basket option
A basket option is much like the simple European option, but instead of only looking at one asset the payoff is determined by at least two underlying assets

$$Payoff = \text{MAX}\left[\sum_{i=1}^{n} w_i \cdot (\frac{S_{iT}}{S_{it}} - K)\ , 0\right]. \tag{3}$$

Where S_{it} is the stock price for asset i at time t and S_{iT} is the stock price for the same asset at time T. The individual weight, w_i, is the weight for each asset and describes how much of the total capital that is allocated in asset S_{it} at time t. This type of payoff function will return the basket options return in percent. This can then be multiplied by a pre-determined nominal value decided between the buyer and seller of the option, say 100 Swedish kronor. This means that if the payoff function in Equation (3) returns 0.1, the payoff in Swedish kronor will be 10 SEK (Glasserman 2003, 105).

Asian Basket option
Then there are options called Asian-Baskets. The name of the option contains information of how the option is structured. In this case it is an Asian option, meaning the exercise price will be an average of some sort. The payoff function will be dependent on two or more assets since it is a basket option and the payoff will be a combination of Equation (2) and (3)

$$Payoff = \text{MAX}\left[\sum_{i=1}^{n} w_i \cdot (\frac{\overline{S}_{iT}}{S_{it}} - K)\ , 0\right]. \tag{4}$$

Asian Quanto Basket option
The option that will be evaluated in this thesis is a so called Asian Quanto Basket option. The payoff function will depend on two or more assets since it is a basket option and the payoff will be a combination of (2)-(4) but now also depending on the exchange rate, X_T, between the payout currency and the currency of the underlying assets at time t. The following payoff function fall out:

$$Payoff = \text{MAX}\left[\sum_{i=1}^{n} w_i \cdot (\frac{\overline{S}_{iT}}{S_{it}} X_T - K \cdot X_t)\ , 0\right]. \tag{5}$$

Path dependent options
An Asian option is path dependent since its payoff function depends on the stocks value at different time points over the life of the option. This means that the payoff is not only determined by the underlying asset's prices at the exercises date but over multiply dates. The number of dates and length between them is option specific and can greatly vary. Path dependency is a problem when pricing the option because there exists no parametric solution and the options path must be simulated. This can by done by using the random walk theory with Geometric Brownian motion (Chandra, Mukherjee and Sengupta 2015).

2.2 Brownian Motion

To be able to price an Asian quanto basket option we must try to simulate the assets future price movements and for that Geometric Brownian Motion is commonly used. To understand and be able to use this process it is vital to understand the theory behind one dimensional Brownian Motion (BM) and random walk theory. This is the basis for a Monte Carlo simulation and is what generates the different outcomes (paths) seen in Figure 1. By analyzing these paths the future expected cash flow can be estimated and used to determine the fair value option price.

2.2.1 One dimension

A stochastic process $\{W(t), 0 \leq t \leq T\}$, is a one dimensional Brownian motion on [0,T] characterized by the following properties:

(i) $W(0) = 0$;

(ii) With a probability of one W(t) is continuous on $[0, T]$;

(iii) $\{W(t_1) - W(t_0), W(t_2) - W(t_1), \ldots, W(t_k) - W(t_{k-1})\}$ are all independent increments for any k and any t in $0 \leq t_0 < t_1 < \cdots < t_k \leq T$;

(iv) The process has Gaussian increments, $W(t) - W(s) \sim N(0, t-s)$ for any $0 \leq s < t \leq T$.

A process $X(t)$ is a Brownian motion with constants μ and $\sigma > 0$ where μ is the drift and σ^2 the diffusion coefficient if

$$\frac{X(t) - \mu t}{\sigma}$$

is a standard Brownian motion. By using,

$$X(t) = \mu t + \sigma W(t)$$

construction of X from a standard Brownian motion W is complete.

By the properties defined for a Brownian motion, $X(t) \sim N(\mu t, \sigma^2 t)$. The stochastic differential equation

$$dX(t) = \mu dt + \sigma dW(t)$$

is solved by X. If the drift μ and diffusion coefficient σ is time dependent but deterministic the Brownian motion should be defined by the following stochastic differential equation

$$dX(t) = \mu(t)dt + \sigma(t)dW(t).$$

Solving the stochastic differential equation we get,

$$X(t) = X(0) + \int_0^t \mu(s)ds + \int_0^t \sigma(s)dW(s)$$

where $X(0)$ is a arbitrary constant. The process X has independent increments and continuous sample paths. The mean of each increment $X(t) - X(s)$ is normally distributed with,

$$\mathrm{E}[X(t) - X(s)] = \int_s^t \mu(u)du$$

and variance,

$$\mathrm{Var}[X(t) - X(s)] = \mathrm{Var}\left[\int_s^t \sigma(u)dW(u)\right] = \int_s^t \sigma^2(u)du \tag{6}$$

(Glasserman 2003, 79-80).

2.2.2 Random Walk Construction

If $Z_1, ..., Z_n$ are independent standard normal stochastic variables, $t_0 = 0$ and $W(0) = 0$ for a standard Brownian motion the following values $W(t+1)$ can be created using:

$$W(t_{i+1}) = W(t_i) + \sqrt{t_{i+1} - t_i} Z_{i+1}, \quad i = 0, \ldots, n-1. \tag{7}$$

For $X \sim \mathrm{BM}(\mu, \sigma^2)$ with constant μ and σ and given $X(0)$, set

$$X(t_{i+1}) = X(t_i) + \mu(t_{i+1} - t_i) + \sigma\sqrt{t_{i+1} - t_i} Z_{i+1}, \quad i = 0, \ldots, n-1 \tag{8}$$

(Glasserman 2003, 81).

2.2.3 Geometric Brownian motion

A geometric brownian motion is a Brownian motion with initial value $\log(S(0))$, meaning that it's an exponential brownian motion. The major advantage with a geometric Brownian motion compared to the regular Brownian motion is that it can not produce negative values which is a desirable feature when simulating assets such as stocks. This is because an exponential function only takes positive values.
The percentage change when using a geometric Brownian motion is,

$$\frac{S(t_2) - S(t_1)}{S(t_1)}, \frac{S(t_3) - S(t_2)}{S(t_2)}, \ldots, \frac{S(t_n) - S(t_{n-1})}{S(t_{n-1})}. \tag{9}$$

These returns are also independent for $t_1 < t_2 < ... < t_n$ since they use relative change in comparison to Brownian motions absolute change.
The basic properties for a geometric Brownian motion is as follows. If X satisfies

$$dX(t) = \mu dt + \sigma dW(t) \tag{10}$$

where W is a standard Brownian motion it follows that $X \sim \mathrm{BM}\left(\mu, \sigma^2\right)$. Using Ito's formula for $S(t) = S(0)\exp(X(t)) \equiv f(X(t))$ produces the following equation,

$$\begin{aligned} dS(t) &= f'(X(t))dX(t) + \frac{1}{2}\sigma^2 f''(X(t))dt \\ &= S(0)\exp(X(t))[\mu dt + \sigma dW(t)] + \frac{1}{2}\sigma^2 S(0)\exp(X(t))dt \\ &= S(t)\left(\mu + \frac{1}{2}\sigma^2\right)dt + S(t)\sigma dW(t). \end{aligned} \tag{11}$$

Different from a regular Brownian motion the geometric Brownian motion process is typically defined as an SDE of the form

$$\frac{dS(t)}{S(t)} = \mu dt + \sigma dW(t). \tag{12}$$

Inconsistencies when comparing the models (11) and (12) are apparent and is because "μ can be interpreted differently". The variable μ is the drift factor for the Brownian motion in Equation (11) that was then exponentiated to define $S(t)$ and is the drift for $logS(t)$. On the other hand in Equation (12) the drift factor for $S(t)$ is $\mu S(t)$ which implies,

$$d\log S(t) = \left(\mu - \frac{1}{2}\sigma^2\right)dt + \sigma dW(t). \tag{13}$$

To verify this Ito's formula can be used.

To indicate that a process S is on the type of (12) we will use the notation $S \sim \mathrm{GBM}\left(\mu, \sigma^2\right)$. Even though μ is not the drift factor for either $S(t)$ or $logS(t)$ it will still be referred to as this. The variable σ in (12) is referred to as the volatility parameter of $S(t)$; the diffusion coefficient of $S(t)$ is $\sigma^2 S^2(t)$.

If S $\sim \mathrm{GBM}\left(\mu, \sigma^2\right)$ and S has the initial value of $S(0)$ the solution to (13) is given by,

$$S(t) = S(0)\exp\left(\left[\mu - \frac{1}{2}\sigma^2\right]t + \sigma W(t)\right). \tag{14}$$

If $u < t$ the more general equation is,

$$S(t) = S(u)\exp\left(\left[\mu - \frac{1}{2}\sigma^2\right](t-u) + \sigma(W(t) - W(u))\right). \tag{15}$$

This proves the previously stated independence of returns in (9). Using the fact that the increments from W are independent and normally distributed a simple recursive procedure for simulating values of S at $0 = t_0 < t_1 < ... < t_n$ is created.

$$S\left(t_{i+1}\right) = S\left(t_i\right)\exp\left(\left[\mu - \frac{1}{2}\sigma^2\right]\left(t_{i+1} - t_i\right) + \sigma\sqrt{t_{i+1} - t_i}Z_{i+1}\right)\ i = 0, 1, \ldots, n-1 \tag{16}$$

where $Z_1, Z_2, ..., Z_n$ are standard normal distributed random variables
(Glasserman 2004, 93-94).

A quanto option has a similar way of generating geometric brownian paths as an ordinary option. But instead of only depending on the development on the underlying asset's it also depends on the relationship between a foreign exchange rate and the settled currency of an option. This results in a similar equation as (16) but now with the following expression:

$$S\left(t_{i+1}\right) = S\left(t_i\right)\exp\left(\left[r_f - \delta_f - \rho\sigma_x\sigma_s - \frac{1}{2}\sigma_s^2\right]\left(t_{i+1} - t_i\right) + \sigma_s\sqrt{t_{i+1} - t_i}Z_{i+1}\right)\ i = 0, 1, \ldots, n-1 \tag{17}$$

where $Z_1, Z_2, ..., Z_n$ are standard normal distributed random variables.

r_f=Risk free foreign interest rate.
δ_f=Continuous dividend yield.
ρ=Correlation between foreign currency and issued currency.
σ_x=Volatility of foreign exchange rate.
σ_s=Volatility of underlying S.
$t_{i+1} - t_i$=Timestep, which is constant for all i.

To get the complete derivation of (17), the reader is referred to (Kwok and Wong 2000, 258-259). Basically, it is just repeating step (11-16) but instead of using (10):

$$dX(t) = \mu dt + \sigma dW(t)$$

as process, the following dynamics can be used:

$$dX(t) = (r_f - \delta_f - \rho\sigma_x\sigma_s)dt + \sigma_s dW(t).$$

Where the drift factor is the risk free rate in a domestic currency world (Kwok and Wong 2000, 259). By using the dynamics above we also consider the correlation between two different countries currencies and their individual risk free rates. In our case we will study the United States and Sweden, since the basket is denominated in US dollars and payout occur in Swedish SEK.

2.3 Monte Carlo Simulation

Monte Carlo methods are widely used in the real world and not to mention the financial industry. By using i.d.d random variables we can simulate future cash flows for stocks. Starting with the initial value of the stock today and simulating different scenarios into the future. By increasing the number of simulations or commonly referred to as paths, value of the future cash flow will converge to the fair value, as stated in *Law of large numbers* below. This method is fundamental for pricing options and the discounted future cash flow will respond to the fair value of an option (Glasserman 2003, 1).

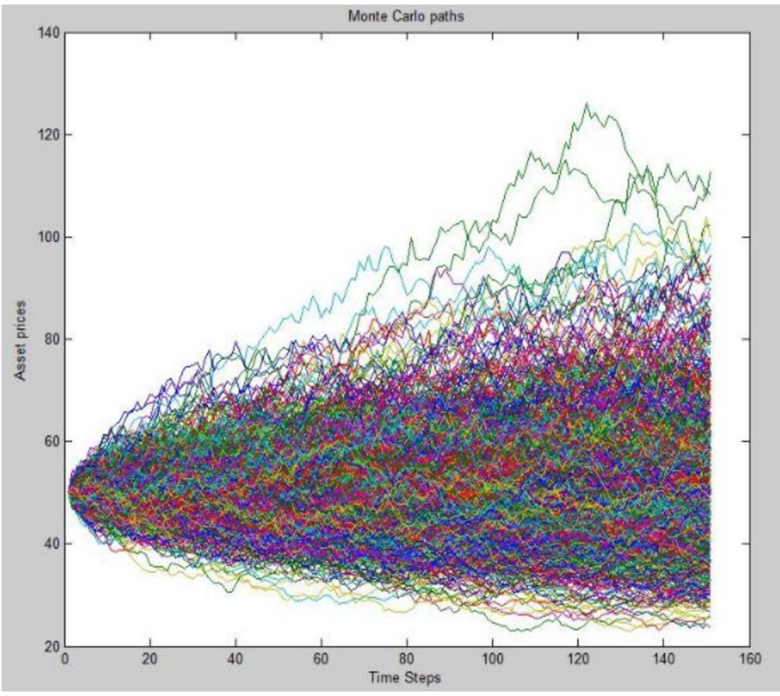

Figure 1: Visual illustration of simulated Monte Carlo paths (Towardsdatascience 2018).

2.3.1 Law of large numbers

The law of large numbers is a mathematical theorem about the theory of probabilities. The theorem says that the arithmetic mean of a big number of independent observations of a stochastic variable is close to the expected value of the variable. Hence the law of large numbers is sometime called *law of averages*. This assumption is crucial to a Monte Carlo simulation and is why Geometric Brownian motion is used to create a large amount of paths. When analyzed, these create different option prices but when the average of them are calculated we get the fair value for the option as per this law. To put it pure mathematically it looks like the following,

Theorem- Law of large numbers of discrete stochastic variables
Let $X_1, X_2, ..., X_n$ be an independent trials process, with finite expected value $\mu = E(X_j)$ and finite variance $\sigma^2 = V(X_j)$ for $j = 1, ..., n$.
Let $S_n = X_1 + X_2 + \cdots + X_n$. Then for any $\epsilon > 0$,

$$P\left(\left|\frac{S_n}{n} - \mu\right| \geq \epsilon\right) \to 0$$

as $n \to \infty$. Equivalently,

$$P\left(\left|\frac{S_n}{n} - \mu\right| < \epsilon\right) \to 1$$

as $n \to \infty$
(Grinstead and Snell 2003, 307).

2.4 The greek sensitivities

The greek sensitivities will be used in approach 3 and is what gives the model the ability to estimate the option price for different shifts in the two risk factors. This will be better explained in Section 2.4.2 when the theory behind Taylor polynomials is explained. In the following section we will describe how these greek sensitivities are estimated.
The sensitivities that are used in this thesis are *delta*, *gamma* and *vega*. A simple explanation of *delta* is that it estimates how much the price of the option changes for a 1 SEK change in asset price. A *delta* of 0.65 for instance will result in an increase of 0.65 SEK in the price of the option when the underlying asset increases by 1 SEK. Depending on the payoff function for the option, *delta* can be either positive or negative. What determines the *delta* is how the payoff function is structured but also where the asset price is with respect to the strike price. *Gamma* describes the change in *delta* for a 1 point move in the underlying asset price. It defines the convexity for the option value relative to the underlying asset. *Gamma* is the second derivative of the value of a option with respect to the underlying assets' prices (Daníelsson 2011, 116-117).

Vega is like *delta*, the first derivative but with respect to volatility instead of the underlying asset's prices. *Vega* describes the change in the price of an option for a one percent change in implied volatility (Investopedia, 2018b).

There is two different broad categories for estimating the sensitivities, methods involving at least two values of the parameter of differentiation and methods that do not. The first category is called finite-difference approximations and is in theory easier to implement and is why it's used in this thesis. The negative part of this method is that it produces biased estimates and therefore requires balancing the bias and variance. The other category, pathwise methods, produces unbiased estimates since it differentiates a probability density rather than an outcome.

2.4.1 Finite-Difference Approximation

Assume a model with a parameter θ ranging over some interval of the real line. Further on, suppose that for each value of θ there is a process for generating a stochastic variable $Y(\theta)$, which is the output for the model at parameter θ. Let

$$\alpha(\theta) = \mathrm{E}[Y(\theta)].$$

The problem of estimating the derivative breaks down to finding a way to estimate $\alpha'(\theta)$. In the case of option pricing $Y(\theta)$ is the discounted payoff of the option, $\alpha(\theta)$ is option's price, and θ is any underlying risk factor that could have an impact of the option's price. When θ is the initial price of an underlying asset, then $\alpha'(\theta)$ is the option's *delta*. In the same way the *gamma* can be calculated but now using the second order derivative of the option's initial price with respect to the underlying asset, put mathematically $\alpha''(\theta)$. When the underlying risk factor θ is volatility, $\alpha'(\theta)$ is called *vega*.

Approach for derivative estimation

To estimate the derivative $\alpha'(\theta)$ we simulate independent replications $Y_1(\theta), \ldots, Y_n(\theta)$ of the model at parameter θ and n additional replications $Y_1(\theta+h), \ldots, Y_n(\theta+h)$ at $\theta+h$, for some $h > 0$. Then by taking the average over each set of replications, $\overline{Y}_n(\theta)$ and $\overline{Y}_n(\theta+h)$ are received and the following forward-difference estimator can be defined:

$$\hat{\Delta}_F \equiv \hat{\Delta}_F(n,h) = \frac{\overline{Y}_n(\theta+h) - \overline{Y}_n(\theta)}{h}. \tag{18}$$

Then by adding a replication $\theta - h$, two similar replications of $\theta - h$ are defined and further on the central-difference estimator can be defined by

$$\hat{\Delta}_C \equiv \hat{\Delta}_C(n,h) = \frac{\overline{Y}_n(\theta+h) - \overline{Y}_n(\theta-h)}{2h}. \tag{19}$$

The central estimator is more computational demanding in that sense that two additional points, $\theta+h$ and $\theta-h$, need to be computed for estimating $\alpha(\theta)$ and $\alpha'(\theta)$. As seen, the forward-difference estimator only requires one additional point, $\theta+h$.
Now illustrating the forward-difference estimator compared to the central-difference estimator in a simple example in Figure 2. Let's assume a Black-Scholes option with the following attributes; volatility=0.3, interest rate=5 percent, strike price=100 and 0.04 years (about two weeks) until expiration. A comparison is done at the tangent line with underlying asset price of 95, using forward-difference from prices at 95 and 100 and a central difference with prices of 90 and 100. With respect to the tangent line at 95 one can see that the slope of the central difference is clearly closer than the slope of the forward difference.

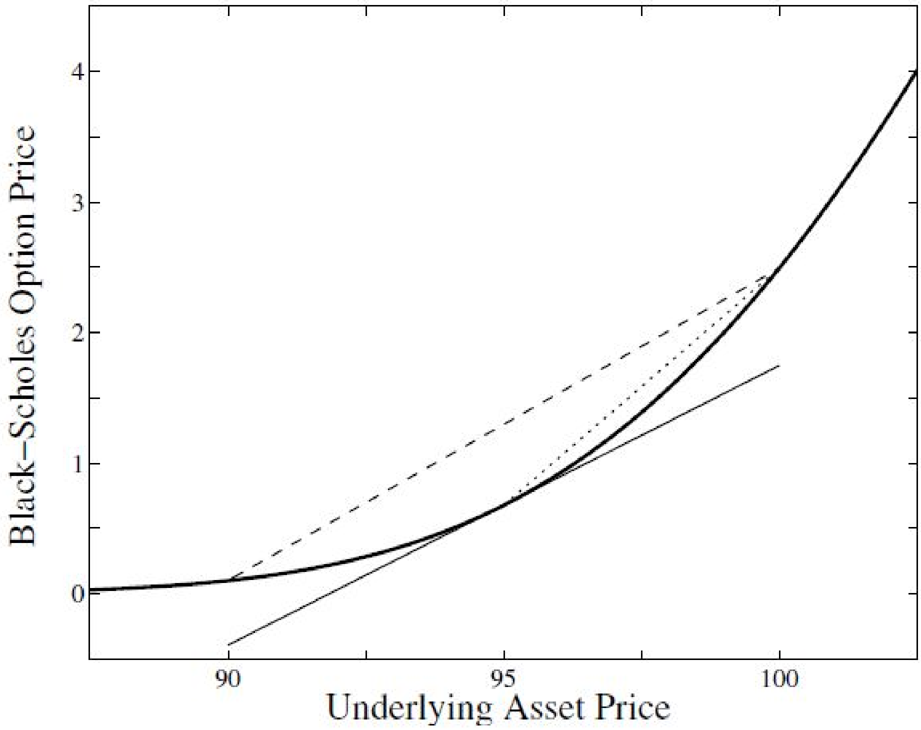

Figure 2: Comparison of forward-difference approximation (dotted) and central-difference approximation (dashed) with exact tangent to the Black-Scholes formula (Glasserman 2004, 380).

When using deterministic algorithms to estimate the derivatives there is a problem with using very small values of h since in applications of Monte Carlo, specifically the variability of the estimates, prevents the user from doing that. Due to this, the user shall use a "sufficient" large value of h to get as accurate estimations of derivatives as possible but this leads to possible round-off errors. So when implementing it is advisable to be aware of the possible round-off error but it is rarely the main issue to accurately estimate the derivatives using simulation.

Second order derivative
The central-difference estimator has the following form:

$$\frac{\overline{Y}_n(\theta + h) - 2\overline{Y}_n(\theta) + \overline{Y}_n(\theta - h)}{h^2} \tag{20}$$

(Glasserman 2003, 384-385).

2.4.2 Taylor polynomials

When the greek sensitivities *delta*, *gamma* and *vega* are estimated they can be used in a Taylor approximation to estimate the option price for shifts in the different risk factors.*Delta*, *gamma* are in fact the first and second order derivative of the option price with respect to the value of the underlying asset and *vega* is the first order derivative with respect to volatility. In words the new option price can be estimated with the following equation,
New option price = old option price + *delta* * (new stock price at time t+1 – starting stock price at time t) + 0.5 * *gamma* * (new stock price at time t+1 - starting stock price)^2+*vega* * (new volatility at time t+1 – starting volatility at time t).

The more theoretical explanation of Taylor polynomials is that it's used to best describe the behaviour of a function around a specific point. For a function $f(x)$ about $x = a$ the 1st order Taylor polynomial will be

$$P_1(x) = f(a) + f'(a)(x - a) \tag{21}$$

and that function best describes the behaviour of f near a compared to any other polynomial of degree 1, since P_1 and f both have the same derivative and value at a, $P_1(a) = f(a)$ and $P_1'(a) = f'(a)$. Since its a first order Taylor polynomial the approximation will be linear.

To better approximate $f(x)$ higher degree polynomials can be used as long as f is differentiable to that degree. For a function f that is twice differentiable near a the polynomial

$$P_2(x) = f(a) + f'(a)(x - a) + \frac{f''(a)}{2}(x - a)^2 \tag{22}$$

satisfies $P_2(a) = f(a), P_2'(a) = f'(a)$, and $P_2''(a) = f''(a)$ and can best approximate the behaviour of f around a compared to any other polynomial of degree max 2. For the general case where $f^{(n)}(x)$ exists the polynomial will be

$$\begin{aligned} P_n(x) = f(a) + \tfrac{f'(a)}{1!}(x - a) + \tfrac{f''(a)}{2!}(x - a)^2 \\ + \tfrac{f'''(a)}{3!}(x - a)^3 + \cdots + \tfrac{f^{(n)}(a)}{n!}(x - a)^n \end{aligned} \tag{23}$$

and be the best approximation for f around $x = a$. P_n is the nth order Taylor polynomial for f (Adams and Essex 2013, 271).

2.5 Interpolation techniques

A few interpolation techniques will be tested and the reason why interpolation is needed is because it will implicate a better approximation among the different approaches.
Suppose the value of a quantity y is uniquely determined by the value of some other quantity x. Since the exact dependence of $y = f(x)$ is unknown there is an interest of finding that dependence. To obtain this dependence, variables are measured in different situations. These different situations implies the value $y = f(x)$ of the unknown function $f(x)$ for several values $x_1, ..., x_n$. Based on this, a prediction of the value $f(x)$ is desirable for all other values x. When x is between the lowest and highest value of $x_{i=1,...,n}$, it is called *interpolation*. When x is lower than the lowest value and higher than the highest value of $x_{i=1,...,n}$, it is called *extrapolation* (Pownuk and Kreinovich, 2017).

2.5.1 Linear interpolation

One of the most common interpolation techniques is *linear interpolation* and is based on the assumption that the function $f(x)$ is linear on the interval $[x_1, x_2]$. This leads to the following formula for $f(x)$:

$$f(x) = \frac{x - x_1}{x_2 - x_1} \cdot f(x_2) + \frac{x_2 - x}{x_2 - x_1} \cdot f(x_1). \tag{24}$$

The main argument of using this type of interpolation is because of its simplicity, and in many practical situations linear interpolation works quite well. The easiest functions to compute are also linear ones. But in computational science not to mention financial situations, often very complex computations are necessary which does not include linearity (Pownuk and Kreinovich, 2017).

2.5.2 Piecewise Cubic Hermite interpolation

The most effective interpolation techniques are based on piecewise cubic polynomials. Hermite interpolation uses the values of the function and the first derivatives at the nodes, the interpolated values. The interpolation functions are local cubics, thereby the name *Piecewise Cubic Hermite interpolation.*

Let $P(x_k) = y_k$, $k = 1, ..., n$ be the interpolating polynomial.

Let $h_k := x_{k+1} - x_k$ be the length of the kth subinterval.

Then

$$\delta_k = \frac{y_{k+1} - y_k}{h_k}.$$

Let $d_k := P'(x_k)$.

Note: If $P(x)$, the interpolant, is piecewise linear, then d_k is not defined since $d_k = \delta_{k-1}$ on the left of x_k, but on the right of x_k, $d_k = \delta_k$. In this case it´s undefined since $\delta_{k-1} \neq \delta_k$.

For higher-order interpolants, cubics for instance, it is possible to force the interpolant to be smooth at x_k, which sometimes is referred to as breakpoints. By forcing the derivative at the end of one piecewise cubic to match the derivative at the next piecewise cubic, smoothness is received.

Suppose $P(x) = c_1x^3 + c_2x^2 + c_3x + c_4$ where $P(0) = 0, P(1) = 3, P'(0) = 1$ and $P'(1) = 2$.

This gives following equations:

$$\begin{aligned} c_4 &= 0 \\ c_1 + c_2 + c_3 + c_4 &= 3 \\ c_3 &= 1 \\ 3c_1 + 2c_2 + c_3 &= 2. \end{aligned}$$

Solving the equations give the following:

$$c_4 = 0, c_3 = 1, c_2 = 5, c_1 = -3.$$

The cubic polynomial, $P(x) = -3x^3 + 5x^2 + x$, matches with the two data points and two derivatives that are specified. Since the specified interval were at [0,1] it is simplified. Now consider the following cubic polynomial on the interval $x_k \leq x \leq x_{k+1}$ expressed in terms of local variables $s = x - x_k$ and $h = h_k$:

$$P(x) = \frac{3hs^2 - 2s^3}{h^3} y_{k+1} + \frac{h^3 - 3hs^2 + 2s^3}{h^3} y_k + \frac{s^2(s-h)}{h^2} d_{k+1} + \frac{s(s-h)^2}{h^2} d_k. \tag{25}$$

This is a cubic polynomial in s (and hence in x) that satisfies all 4 interpolation conditions: Two derivative values (possible unknown) and two on function values.

$$\begin{aligned} P(x_k) &= y_k, \quad P(x_{k+1}) = y_{k+1} \\ P'(x_k) &= d_k, \quad P'(x_{k+1}) = d_{k+1}. \end{aligned}$$

Interpolants for derivatives are known as Hermite. By knowing both function values and first order derivative at a set of points, Piecewise Cubic Hermite migth be used to fit data (Bickley, 1968).

2.6 The Black-Scholes model

This famous model is quickly described to support the theory behind estimation of volatility discussed in Section 2.7.

This is a special case and let us consider a financial market consisting of only two assets. The first asset is the stock price S and the second one is a so called risk free asset with price process B. Then assume we have a market with the following dynamics:

$$\begin{aligned} dB(t) &= rB(t)dt \\ dS(t) &= S(t)\alpha(t, S(t))dt + S(t)\sigma(t, S(t))dW(t) \end{aligned} \tag{26}$$

where r, α and σ are deterministic constants and W is a Wiener process.

Then we can specialize the dynamics above to the case of the Black-Scholes model,

$$\begin{aligned} dB(t) &= rB(t)dt \\ dS(t) &= \alpha S(t)d(t) + \sigma S(t)dW(t). \end{aligned} \tag{27}$$

Reader is guided to Bjork (1998, 76-90) for all details and deeper understanding.

2.7 Historical volatility

Volatility is one of the most significant parameter when pricing an option but also one of the hardest to estimate. The historical volatility is commonly used as an indication of what the volatility for the assets is. Volatility is described as a statistical measure of the dispersion of returns for an asset or market index. Measurement of volatility can either be done by using the standard deviation or variance between returns from that same asset or market index. Riskier assets are generally connected to higher volatility. In stock market volatility is often associated with big swings in either directions, the market is called volatile if the stock rises and falls more than one basis point over a sustained period of time. The Volatility Index (VIX) expresses the market volatility. VIX was created to measure the 30-day expected volatility of the U.S. stock market derived from spot prices of S&P 500 call and put options. What many investors do not know when buying options is that they are paying a greater amount of money for the option if the implied volatility is higher (Investopedia, 2018c).

An example; assume that a quantitative investor wants to value a plain vanilla, European call, with one year to expiration date. The volatility is not constant over time and the future volatility is unknown, so approximation is needed. By using historical volatility, for instance two years back, the investor can begin to value the option. It's common that the lifetime of the option and calibration length is the same, meaning that the calibration length of volatility in this case is two year back of assets' prices.

Estimation of volatility, σ, is done by using standard Black-Scholes GBM model (27) under the objective measure P. Where P is the observed market given by (26), S has a log-normal distribution and by observing historical prices one may define

$$\xi_1, \ldots, \xi_n$$

by

$$\xi_i = \ln\left(\frac{S(t_i)}{S(t_{i-1})}\right).$$

Where $\xi_1, \ldots, \xi_n$ are i.d.d random variables with

$$E[\xi_i] = \left(\alpha - \frac{1}{2}\sigma^2\right)\Delta t$$

and

$$\mathrm{Var}[\xi_i] = \sigma^2 \Delta t.$$

From a statistical point of view, the estimate of σ is given by

$$\sigma^* = \frac{S_\xi}{\sqrt{\Delta t}}$$

where the sample variance, S_ξ^2, is given by

$$S_\xi^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(\xi_i - \overline{\xi}\right)^2$$

$$\overline{\xi} = \frac{1}{n}\sum_{i=1}^{n}\xi_i \tag{28}$$

(Bjork 1999, 93).

2.8 Risk free interest rate

The risk free interest also called risk free rate of return is the return on investments with no risk. No risk means that there is a zero chance for a default and the investor will get his or hers money back guaranteed plus a small return, the risk free return. Since there are no assets of this type, the risk free interest rate does not exist. It can only be approximated and to do that investments with a very low risk are used to benchmark the return. The three month U.S treasury bill is for example often used by US based investors because the probability that the US will default is practically zero.

This return or interest rate will then in theory be the absolute minimum an investor will be expecting for an investment. This is because if it is lower than this the investor will be accepting a higher risk for the same or lower expected return which no rational investor would be willing to do. This rate can be used to discount cash flows in risk neutral environments. It is also used in models like the Black-Scholes model for pricing European options (Investopedia, 2018d).

3 Method and model implementation

3.1 Methods

To reduce the computing time approaches 1-4 discussed in this chapter will simulate the stocks in the basket as an index. This means that instead of creating an individual path or future estimated value for each individual asset they are group up together. By then estimating the parameters needed for a Monte Carlo simulation for the entire group as one, prediction for the future index value can be done. Approach 5 (Swedbank's own method) instead estimates a future path or value with individual Monte Carlo paths. By saying simulating individually we mean that each individual index component (stock) has its own path with assumed correlation between the components in the Monte Carlo setup. Simulating our methods as an index will greatly reduce computing time but still generate reliable results. The potential side effects from this will be discussed later in this thesis.

Approach 3 and 4 will also benefit from a grid based solution with multiply pre-evaluated points scattered over the estimation interval. For approach 4 these points will only include a fair option price estimated with Monte Carlo method as described in Section 2.3. The points in Approach 3 will on the other hand include both a fair value option price and the greek sensitivities described in Section 2.4. Approach 5 will in contrast only have one point with a pre-evaluated fair value option price and the greek sensitivities. Meaning that approach 5 has a similar setup as approach 3 but without a "grid-effect".

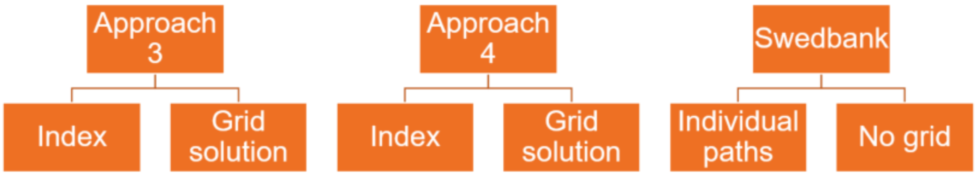

Figure 3: A simple overview of the key aspects for each approach. Swedbanks own model is referred to as approach 5 in this thesis.

To evaluate the results from the different methods a fair value grid will be created using Monte Carlo simulations and will be referred to as *reference points*. This will then be used as an answer key and considered as the true value of an option and what the methods will try to estimate.

In the following sections where we discuss the methods we will refer to points where we have estimated a fair value option price to as *price point*. Points where we have both a fair value option price and estimated sensitivities will be referred to as *simulated values*.

The underlying assets are ten American stocks. The portfolio is equally weighted and the return is depending on both the share development, relationship between US dollar and SEK and the participation rate, which determines the leverage. Leverage means that the final payoff is calculated using Equation (5) and multiplied by the participation rate. The underlying assets of the basket are listed below.

1. Kellogg Co (K UN Equity)
2. Kimberly-Clark Corp (KMB UN Equity)
3. Coca-Cola Co/The (KO UN Equity)
4. Mcdonald'S Corp (MCD UN Equity)

5. Pepsico Inc (PEP US Equity)
6. Procter Gamble Co/The (PG UN Equity)
7. Southern Co/The (SO UN Equity)
8. AtT Inc (T UN Equity)
9. United Parcel Service-Cl B (UPS UN Equity)
10. Verizon Communications Inc (VZ UN Equity)

3.1.1 Approach 1-3

Approach 1 will use the Greek sensitivity *delta* and Equation (21), the first order Taylor expansion, to estimate a new option price when there is a change in the value of the underlying assets. A *delta* grid is implemented in approach 1b and is done by simulating the option price and *delta* for different shifts in the value of the underlying assets. These *simulated values* are then used to estimate the new price when there is a shift in the value of the underlying assets. This is done by interpolating between the *simulated values* from the two closest points. The method is illustrated in Figure 4 where the two linear lines are the *delta* approximations (approach 1) from the two *simulated values* and the third curved line is the fair value for the option for different values of underlying assets. If there was no grid the estimated price would be the value obtained in the red point, but due to the grid-solutions weighing between the two *simulated values* we receive the value in the green point.

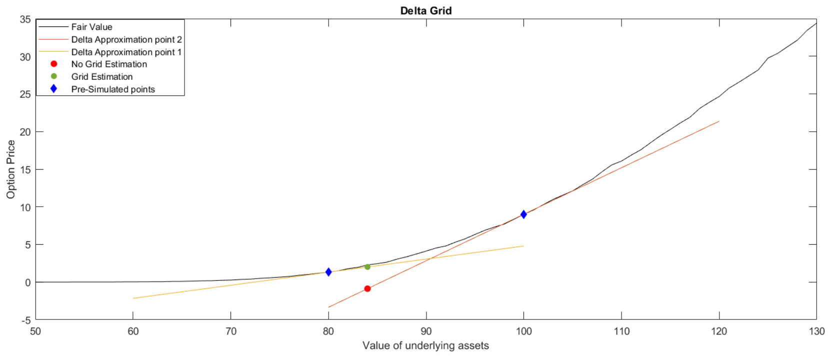

Figure 4: Illustration of approach 1, No Grid Estimation (red) and approach 1b, Grid Estimation (green) for an Asian option with strike price 100.

Approach 2 uses the same concept as approach 1b but with the added factor of the second order derivative, *gamma* which is described in Equation (22) showing second order Taylor expansion.

Approach 3 is a more complex approach using three sensitivities *delta*, *gamma* and *vega* and therefore we move away from a single risk factor, value of the underlying assets, and also consider volatility. This model uses the same concept for shifts in the value of underlying assets as approach 2. The volatility sensitivity *vega* is used much like *delta* described in approach 1b. It also benefits from a grid solution that includes *simulated points* like Figure 5 shows. The exact placement of these *simulated points* can be seen in the appendix, Table 14. Using the Greek sensitivities from closest points and then weighting between them a more accurate estimation can be made over a larger interval.

3.1.2 Approach 4

Approach 4 is a more simple grid based solution with *price points*, meaning pre-evaluated points for different shifts in volatility and the value of the underlying assets. Interpolation between these option prices is then used to estimate a new option price for different shifts in the two risk factors. Figure 5 below illustrates the intuition behind the 4th approach. The blue points represent fair value option prices for different shifts in volatility and underlying assets. All the option prices together represent an n x m-matrix. Important to notice here is that when implementing this model the distance between the option prices are not symmetric as the figure might show.

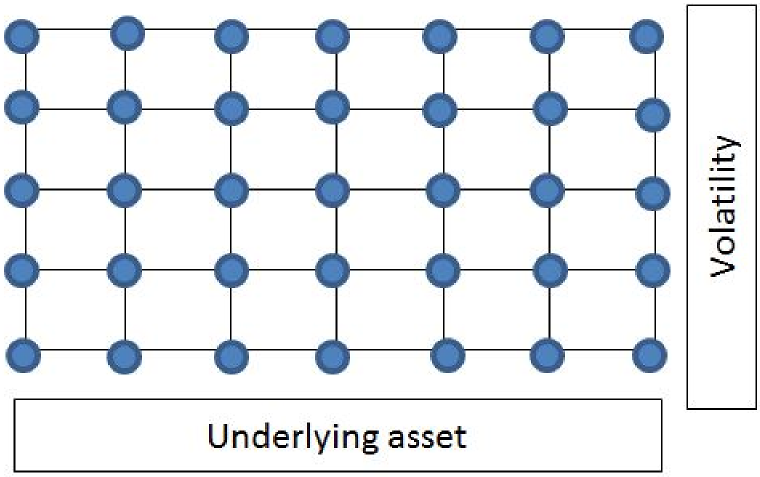

Figure 5: The intuition behind a grid solution, blue dots represents *price points.*

Each point is connected to two lines, one includes the points with the same value of the underlying assets and the other one for points with the same volatility. These lines are created by *Piecewise Cubic Hermite* interpolation with one of the risk factors locked. These two lines are then used for each of the four points to generate four estimated values. Weighting between these points is then used to estimate a new option price.

3.1.3 Approach 5 - Swedbank's internal estimation model

Approach 5 is Swedbank's current estimation model and is used for certain types of options and is not implemented by us. The estimation of the option price is done with respect to the individual asset's in contrast to approach 3 and 4 that estimates the option as an index.
This approach estimates the value of a multivariate function of risk factors based on second order Taylor polynomials. So this approach can import several risk factors but now focusing on value of underlying assets and volatility. Approach 5 ignores the higher infinitesimal items when calculating. Given a pricing function $f(\vec{x})$: $R^n \to R$, and given the initial value of this function at point $\vec{x}'\,(t_{\text{sim}} + \Delta t)$, Swedbanks internal model estimates the function value at point $\vec{x}''\,(t_{\text{sim}} + \Delta t)$ by using Taylor expansion:

$$f\left(\vec{x}''\right) = f\left(\overrightarrow{x'}\right) + f'\left(\overrightarrow{x'}\right) \cdot \Delta\vec{x} + \frac{1}{2} \cdot f''\left(\overrightarrow{x'}\right) \cdot \Delta\vec{x}^2 + \frac{1}{2} \cdot \sum_{i \neq j} f_{ij}^n\left(\overrightarrow{x'}\right) \cdot \Delta x_i \Delta x_j$$

where, $\vec{x}'\,(t_{\text{sim}} + \Delta t) = (x_1', x_2', \ldots, x_n')$ denotes a vector of n risk factors at time $(t_{\text{sim}} + \Delta t)$ without shifts and similarly $\vec{x}''\,(t_{\text{sim}} + \Delta t) = (x_1'', x_2'', \ldots, x_n'')$ denotes a vector of n risk factors at time $(t_{\text{sim}} + \Delta t)$ with different shifts i in the specified risk factors.

Note: Notional at time t_i as seen at time is equal to t_{sim}.

Also $\overrightarrow{\Delta x} = (\Delta x_1, \Delta x_2, \ldots, \Delta x_n)$ denotes a vector of risk factor value change for each risk factor i with $\Delta x_i = \frac{x_i''}{x_i'} - 1$ when the shift is relative.

$f'(\vec{x})$ and $f''(\vec{x})$ are first respectively second order derivatives with respect to the argument of $\overrightarrow{x}$. Then we obtain:

$$f'\left(\vec{x}'\right) = \left(\frac{\partial f(\vec{x})}{\partial x_1}\bigg|_{x_1 = x_1'}, \frac{\partial f(\vec{x})}{\partial x_2}\bigg|_{x_2 = x_2'}, \ldots, \frac{\partial f(\vec{x})}{\partial x_n}\bigg|_{x_n = x_n'} \right)$$

$$f''\left(\vec{x}'\right) = \left(\frac{\partial^2 f(\vec{x})}{\partial x_1^2}\bigg|_{x_1 = x_1'}, \frac{\partial^2 f(\vec{x})}{\partial x_2^2}\bigg|_{x_2 = x_2'}, \ldots, \frac{\partial^2 f(\vec{x})}{\partial x_n^2}\bigg|_{x_n = x_n'} \right)$$

$f_{ij}''(\vec{x}')$ are the cross gamma derivatives

$$f_{ij}''\left(\vec{x}'\right) = \frac{\partial^2 f(\vec{x})}{\partial x_i \partial x_j}\bigg|_{x_i = x_i', x_j = x_j'}$$

and

$$\Delta \vec{x}^2 = \left(\Delta x_1^2, \Delta x_2^2, \ldots, \Delta x_n^2\right).$$

3.2 Model implementations

A description of the approaches and their implementation will be described in this section. This will be done for the most central approaches 3-4. However approach 1-2 uses the same concept as approach 3 but with less complexity.

3.2.1 Evaluation method

All approaches will be based on Monte Carlo simulations with geometric brownian motion (GBM) to evaluate a fair value option price.

1. Using Equation (17) to simulate the paths

$$S\left(t_{i+1}\right) = S\left(t_i\right) \exp\left(\left[r_f - \delta_f - \rho\sigma_x\sigma_s - \frac{1}{2}\sigma_s^2 \right] \left(t_{i+1} - t_i\right) + \sigma_s \sqrt{t_{i+1} - t_i} Z_{i+1} \right) i = 0, 1, \ldots, n-1.$$

2. Using the path simulated in the previous step we read the values at the option-specified dates. The mean value, $\overline{S}$, is then calculated for these points with Equation (2).
3. Using $(\overline{S} - K)^+$ we get the option's payoff at time T.
4. This process is then repeated 20 000 times and the option's payoff is stored in a vector, Payoff.
5. The fair value of the option will be received due to the law of large numbers.
6. Discount Payoff vector.

3.2.2 Simulating the Greek sensitivities

To implement approach 3 referred to as *Delta-Gamma-Vega grid* the Greek sensitivities, using theory from 2.4.1, and fair value option prices must be estimated. Where these *simulated values* are located on the grid will be discussed in detail in Chapter 6.

1. Simulate option prices ($Price$) using Monte Carlo for different shifts in underlying risk factors θ using Equation (17) in relation to the strike price, as in step 3 in 3.2.1.

2. Simulate dependent option prices (same Z) but now involving an small change, h, in the portfolio value. Option price for the portfolio with a positive h is $Price_H$ and for the option with a negative h $Price_L$.

3. By taking the difference between $Price_H$ and $Price$ and dividing by h, forward *delta* sensitivities are estimated, see Equation (18).

4. By adding $Price_H$ and $Price_L$ together and subtracting by $Price$ multiplied by 2 then dividing the whole expression by h^2, central *gamma* sensitivities are estimated. This according to the Equation (20).

5. Repeat step 2 for a small shift in volatility instead of the underlying assets.

6. By taking the simulated option prices with increased volatility, subtracting the option price ($Price$) with no shift and dividing by h, forward *vega* sensitivities are estimated, see Equation (18).

3.2.3 Delta-Gamma-Vega Grid - Approach 3

1. A matrix with the Greek sensitivities and option prices for different shifts is constructed using all the steps in 3.2.2. Where these *simulated values* exactly are located will be discussed in Chapter 6.

2. First the shift in both volatility and underlying assets has to be determined.

3. Based on the shift in the underlying assets a vega matrix is created by interpolating or extrapolating between the pre-simulated points for *vega*.

4. Adjust for the change in value of the underlying assets as follows
   i. Determine in between what *simulated values* the shift in the value of the underlying assets is located.
   ii. Use second order Taylor expansion (22) and the *delta gamma* values from both points to get 2 estimated option prices.
   iii. Inverse interpolation is used between these so that the final value is weighted towards the closest point.
   iv. If the point is outside the grid the boundary point is only used and no interpolation is needed.
   v. Returns the estimated value for the option with respect to the change in the underlying asset's value

5. Now we have to adjust the options price for the shift in volatility.
   i. Determine in between what *simulated values* the shift in the volatility is located.
   ii. Use first order Taylor expansion (21) and the vega values from both points to get 2 estimated option prices.
   iii. Inverse interpolation is used between these so that the final value is weighted towards the closest point.
   iv. If the point is outside the grid the boundary point is only used and no interpolation is needed.

v. The Vega adjustment will return the change in price a volatility change has in absolute terms. The new estimated option price subtracted by the starting option price.

6. To get the final estimated option price with respect to both the change in value of the underlying assets and volatility we add the values from step 4 and 5.

3.2.4 Price Interpolation Grid - Approach 4

1. A n x m-matrix with different shifts is simulated with Monte Carlo methods, using Equation (17) in relation to the strike price. Where these *price points* exactly are located will be discussed in Chapter 6.

2. To get to a desired point the shift in volatility and the underlying assets is determined.

3. A search in both x and y direction determines within which 4 (2 x 2) points this shift is located.

4. Price interpolation

   i. Estimate four new values, one from each *price point* using the values from the *Piecewise Cubic Hermite* interpolation

   ii. Weighting between these 4 points (new 2 x 2- matrix) is then used by taking the distance from nearby points risk factors to the targets. This will return the estimated new price for the option.

5. If the point is outside the grid linear extrapolation will be used between the closest points to determine the price.

6. Target option price is received.

3.2.5 Reference points

To evaluate our approaches and Swedbank's current solution a theoretical price-surface is created consisting of separate and simultaneous shifts in the options underlying risk factors θ, for volatility and the value of the underlying assets. This solution is created by very computing intensive Monte Carlo simulations described earlier and has to be done for every separate shift. This price surface will be the backbone for the whole project since it will act as a reference point that we will compare the models ability to estimate a new price to. The price surface will be the best representation of the fair value of the option for a combination of the different risk factors θ and will be covering a shift from -50 to 50 percent change in the value of the underlying assets and a volatility shift of -50 to 400 percent change. The step length for value of the underlying assets will be a 1 percent (0.01) change from the initial portfolio value. Step length for the volatility will be a 10 percent (0.1) change from the initial volatility. This will create a surface consisting of $101 * 36 = 3636$ full re-evaluations using Monte Carlo simulation.

To summarize, we are using Equation (17) in relation to the strike price and applies different shifts in value of the underlying assets (portfolio value), S, and its volatility σ_s. This gives us the fair value option surface found in appendix Figure 13.

4 Results

When simulating the option as an index compared with individual paths there will be a slight difference in spot price. This effect is shown in Table 1 and is based on our evaluated Asian Quanto Basket option. The percentage shift is calculated by simulating the basket as an index and then comparing the prices generated by simulating them with individual paths. Here using 4 different combinations of shifts when simulating individually where each combination sum up to 1.0, which gives the corresponding value as simulating as an index. These 4 different combinations are showed and specified in column 2 below.

Table 1: Simulating Monte Carlo paths as an index versus simulating individually.

Index / Individual	**Relative shift in underlying asset's value**	**Price**	**% change**
Index	1.0	6.4616	0
Individual	1.0 1.0 1.0 1.0 1.0 1.0 1.0 1.0 1.0 1.0	6.6306	2.62
Individual	0.5 0.5 0.5 0.5 0.5 1.5 1.5 1.5 1.5 1.5	6.7796	4.92
Individual	0.3 0.3 0.3 0.3 0.3 1.7 1.7 1.7 1.7 1.7	6.8491	6.00
Individual	0.5 0.6 0.7 0.8 0.9 1.1 1.2 1.3 1.4 1.5	6.7057	3.78

Since Approach 5, *delta-cross-gamma*, is evaluated based on individual shifts in the risk factors there will be an initial price difference compared to approach 3 and 4. Thus, to reduce the effects by the initial price difference between the models they will be compared on the relative change from their respective spot price instead of absolute change. Since approach 1-2 only uses the single risk factor, value of the underlying assets, they can't be used in a fair comparison. Hence they will not be a part of the results and the focus will be on approach 3-5.

To compare the different approaches Swedbank produced 75 option prices (approach 5) based on different shifts in the two risk factors. By applying the same shifts for all approaches in relation to the fair value reference points, comparable surfaces can be created. The full surface created by all reference points can be found in Appendix Figure 13. Figure 6 shows the surface for the reference points that will be used to evaluate the three different approaches and will be seen as the true value the models attempt to replicate.

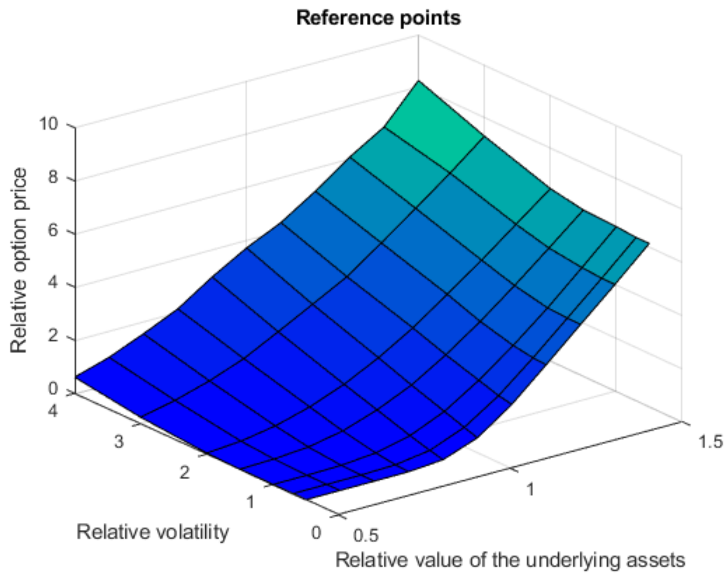

Figure 6: A surface of 75 fair value option prices created by multiple Monte Carlo re-evaluations.

To clarify, a relative option price change of 8 represents an option price $8 * spotprice$. In that case we have an increase of 800 percent from the initial spot price.
Two types of figures will be presented in the following section, one is each approaches relative option-surfaces against the references points. The other one is the total difference between the approaches and the reference points, the error. Here positive values are overestimations and negative values underestimations of the option price. The yellow surface in the error plots represent zero error and is therefor a theoretical optimum. The exact value for all the different approaches and shifts can be found in Table 3 in appendix.

4.1 Approach 3

Starting with approach 3, the *Delta-Gamma-Vega grid*, we can see that it has a tendency to underestimate the option price. This is evident in Figure 7 where the surface for the reference points is above the estimated values from approach 3. The approach handles small shift reasonably well but it struggles to estimate the more extreme shifts in the underlying assets and volatility. Figure 8 shows a clear mismatch when there is a -50 percent shift in the value of the underlying assets and a volatility increase of 400 percent.

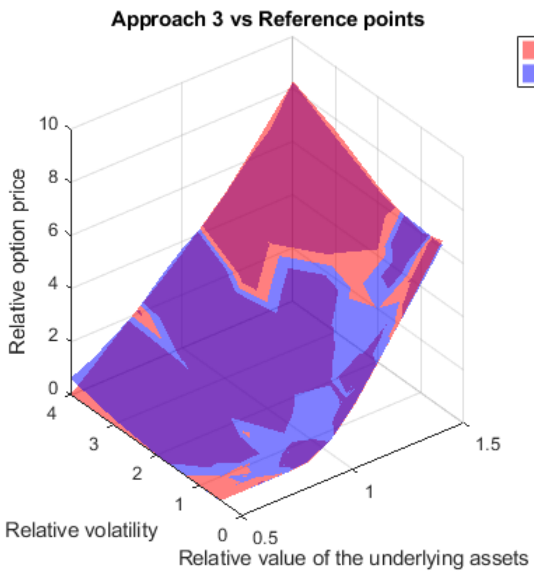

Figure 7: Surface estimated by approach 3 against the reference points.

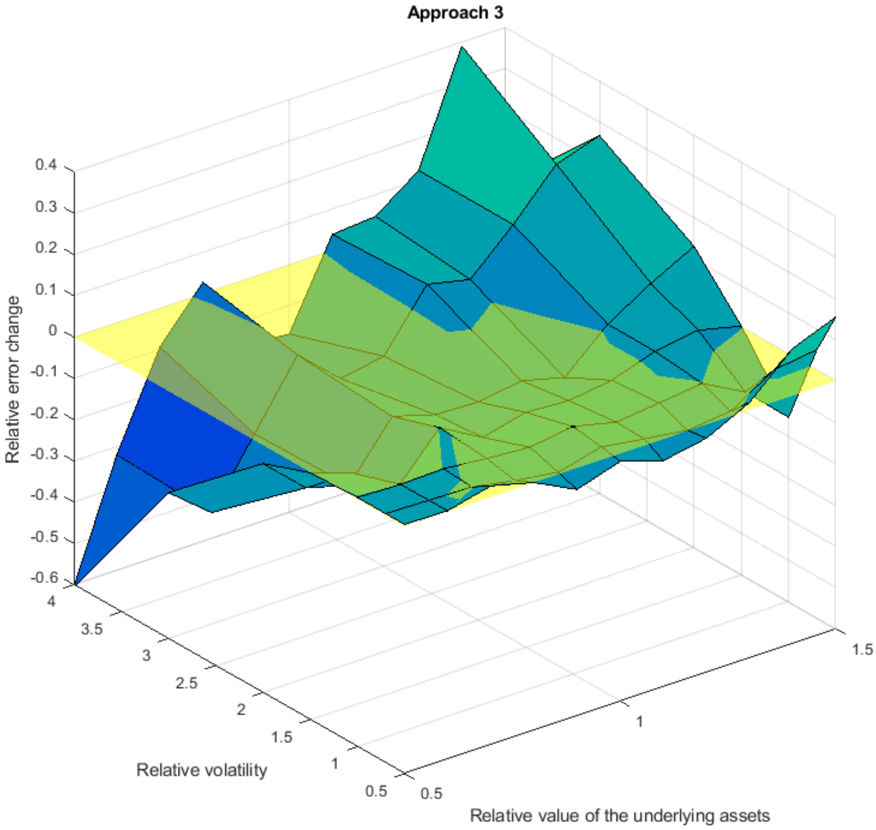

Figure 8: The difference between approach 3 and the reference points, the error.

4.2 Approach 4

Approach 4, the *Price interpolation grid*, shows no clear sign of either overestimating or underestimating the option price as is most apparent in Figure 9. The model even seem to be able to handle the more extreme shifts, see Figure 10, that approach 3 struggled with.

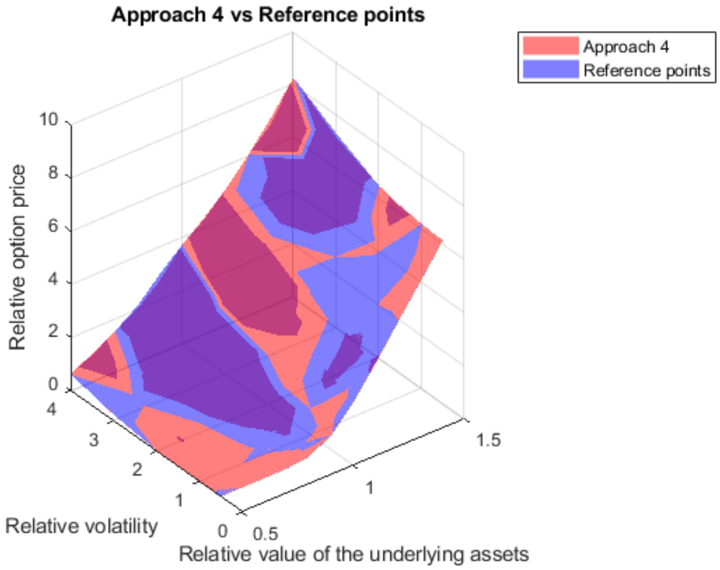

Figure 9: Surface estimated by approach 4 against the reference points.

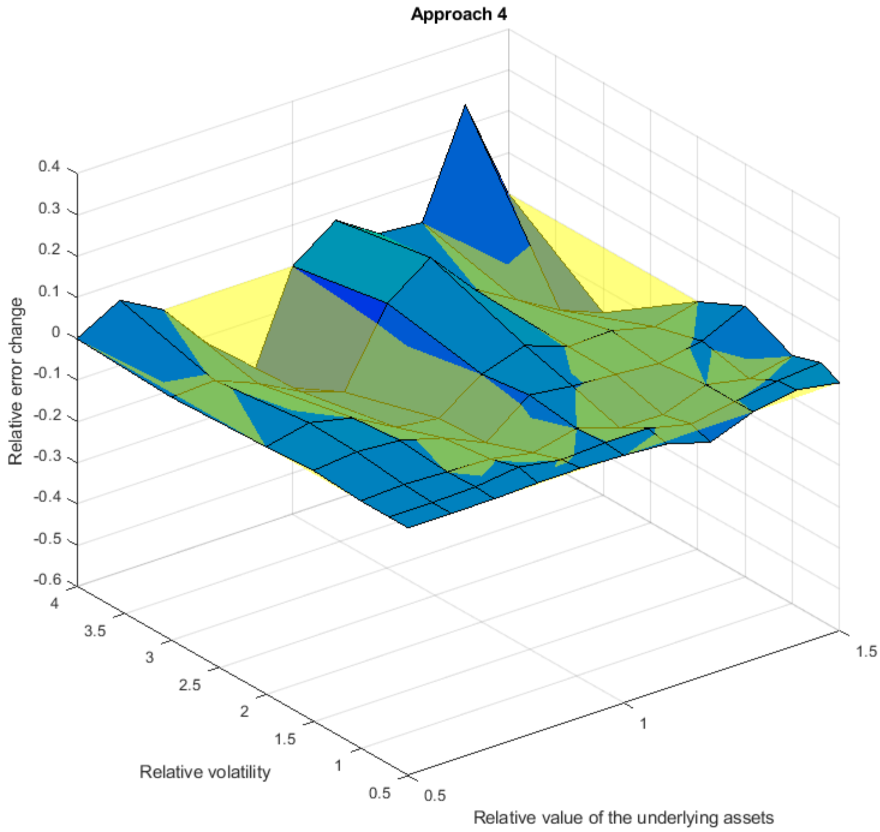

Figure 10: The difference between approach 4 and the reference points, the error.

4.3 Approach 5

Approach 5, the *delta-cross-gamma* model struggles to estimate the option price for even relativly small changes in the risk factors. This is evident in both Figure 11 and 12 where the two surfaces diverge quickly as the size of the shifts increase. At the most extreme points, the estimated error is more than 200 percent of the spotprice which can be seen in Figure 12.

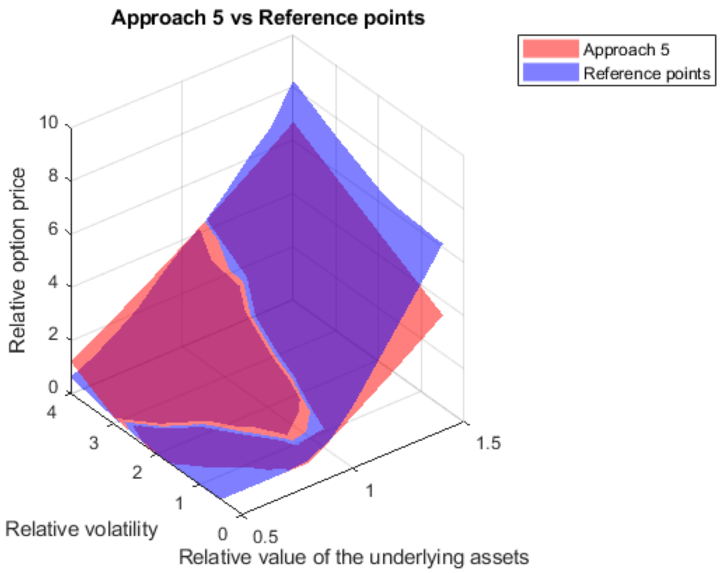

Figure 11: Surface estimated by approach 5 against the reference points.

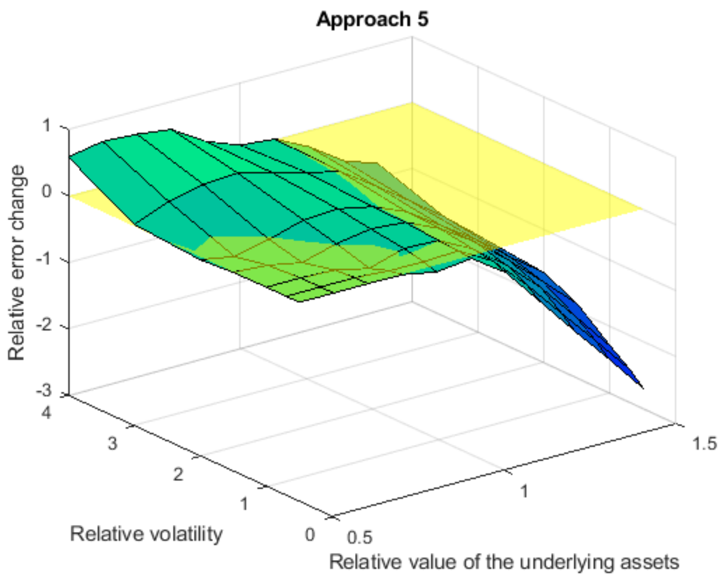

Figure 12: The difference between approach 5 and the reference points, the error.

4.4 Computational complexity

Each model requires a different amount of re-evaluations to be built up and calculating these is the easiest way of comparing the models computational complexity. Table 2 shows the total number of re-evaluations for each of three approaches. Since approach 5 estimates the sensitivities for each one of the individual stocks the amount of re-evaluations increase as the number of stocks in the basket increase. For the other two approaches the number of assets in the basket (index) does not matter and the number of re-evaluations will be constant. Since approach 3-4 estimates the option price and or its sensitivities for the basket as an index they can more accurately be compared.

Table 2: Number of Monte Carlo re-evaluations required to build up each of the three models.

Method	Approach 3	Approach 4	Approach 5
Number of evaluations	42	28	96

5 Conclusion

When studying the figures presented in Chapter 4 it's evident that approach 5, Swedbanks's current model struggles as the size of the shifts increase. It is only accurate for a small interval around the starting value of the risk factors. This can partly be explained by the difference in simulating the basket option as an index versus individual assets. However, that is only a small part of the problem. It uses the Greek sensitivities, but only from the starting point (see Figure 16 in appendix). Since these are only reliable in a relativly small interval the error will increase as the size of the shift increases. Approach 3 uses the same principle but with the added benefit from a grid solution (see Figure 14 in appendix), hence enabling it to be accurate over a much larger interval.

The difference between approach 3 and 4 is smaller but when closely studied approach 3 handles the small shifts marginally better (see Table 3 in appendix) but it struggles with the extreme shifts. Therefore approach 4 (see Figure 15 in appendix) is considered as the better of the two since the methods are supposed to be able to cover the entire interval. This could be improved by adding more *simulated values* to the grid-solution for approach 3 but would lead to a more computing intensive solution. Approach 4's number one spot is also strengthened by the fact that it requires the least amount of re-evaluations to be built up, see Table 2. It is hence not only the most accurate when looking at the interval as a whole but the least computing intensive of the three approaches. For all models the estimation time to build up each model is negligible.

6 Discussion

The size of the grids are decided by expert opinions from employees at Swedbank, in terms of the max shifts for the value of the underlying assets. A daily shift for a solid American portfolio that is greater than 50 percent is highly improbable. For the volatility boundaries we studied the historical volatility of similar indexes. Figure 17 in Appendix 8.5, shows the annual volatility estimations for OMX 30, a Swedish index. All observations are above 5 percent and the majority is under 40 percent with the exception of a few outliers. Testing the approaches on a grid that covers these boundaries will prove that they can handle most shifts that occur in the open market. Approach 4, which is the price interpolation is in our model based on $7 * 4 = 28$ number of Monte Carlo re-evaluations. Meaning that the matrix is based on seven different shifts in the underlying index and four different shift in the index volatility. Experimenting where the *price points* should be located have also been done. We have seen that the effect of shifts in the underlying assets produced the biggest error and hence decided to put more emphasis to handle shifts of that kind. This implicates a solution with more *price points* for shifts in the underlying assets compared to volatility.

Where the *price point* have been scattered on each axis is quite symmetric, see Figure 15 again. By choosing to scatter out the *price points* with a symmetric strategy with respect to both risk factors the solution becomes more general. Our generated reference points, recall Figure 6, implicates that we always know what a correct value is, based on that certain point in time. Because of our symmetric strategy, the focus has been on covering the whole interval both in terms of shifts in the underlying assets and volatility. By focusing on smaller intervals the solution could be improved but on the other hand it would lose the bigger picture. There will always be a trade off between the size of the grid and accuracy, given the same amount of re-evaluations.

Due to the fact that estimating the Greeks for different shift require more full re-evaluations we had to choose the grid-points a bit more scarcely. Figure 14 in appendix shows where these *simulated values* are placed on the grid and the total number of full re-evaluations are shown in Table 2 and totals 42. These can most certainly be placed more optimally but as mentioned earlier it depends on where the focus is, hence the priority of this project was not to produce the optimal grid for a specific option but a general one that fits most.

In Methods (3.1) one can see that shifts in our approaches, 3 and 4 are made on the basket as an index. The reason for why this simplification has been done, when it evidently does not yield the same results as Table 1 shows is due to the complexity of the problem. Let's consider the option that we have been focusing on, a basket with ten underlying assets. If we would use the same method as in approach 4 but now creating a grid that uses the stocks as separate risk factors we will quickly encounter a problem. Instead of shifting the value of the underlying assets as an index we would now try to shift each individual asset with respect to the same risk factors and replicating the same relative shifts that have been done in our current solution. So, we are about to replicate seven relative shifts among each of the ten underlying stocks and four among their individual volatilites. To replicate the relative shifts among the underlying asset we get the following equation,

$$a^b = c$$

a=Number of shifts of risk factor.
b=Number of underlying assets.
c=Total combinations.
which gives the following combinations of fair value option prices based on all shifts in the value of the underlying assets;

$7^{10} = 282475249$.

For the aware reader we can see that at this point in time we have not even taken the volatility into consideration hence we start to realize that this cannot be implemented... Even for a smaller grid solution the computing time required the grid quickly outweighs the benefit in simulating the stocks as individuals instead of an index.

However simulating as an index could be favourable with only having one underlying asset, which would result in a similar solution to the one used in both approach 3 and 4.

6.1 Further development

Further development can be done in the form of including more risk factors. This expands dimensions to the pricing problem and makes it even more realistic. Interesting risk factors to include in the pricing problem would be time dependency, interest rate or exchange rate. By including those risk factors the grid solutions would be expanded and the computational time would in turn then also be increased. Another development area would be testing the approaches on a completely different option type. This would bring more generality into the approaches.

A smarter grid could also be implemented that decides where to optimally place the *price point* for for approach 4 and *simulated values* for approach 3. This for each specific product with respect to the underlying risk factors. Here machine learning could be used so that the system could learn to see patterns for each option specific product and its underlying assets. It should then be able to place the pre-simulated values more efficiently based on a desired size and accuracy of the grid.

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8 Appendices

8.1 Reference points

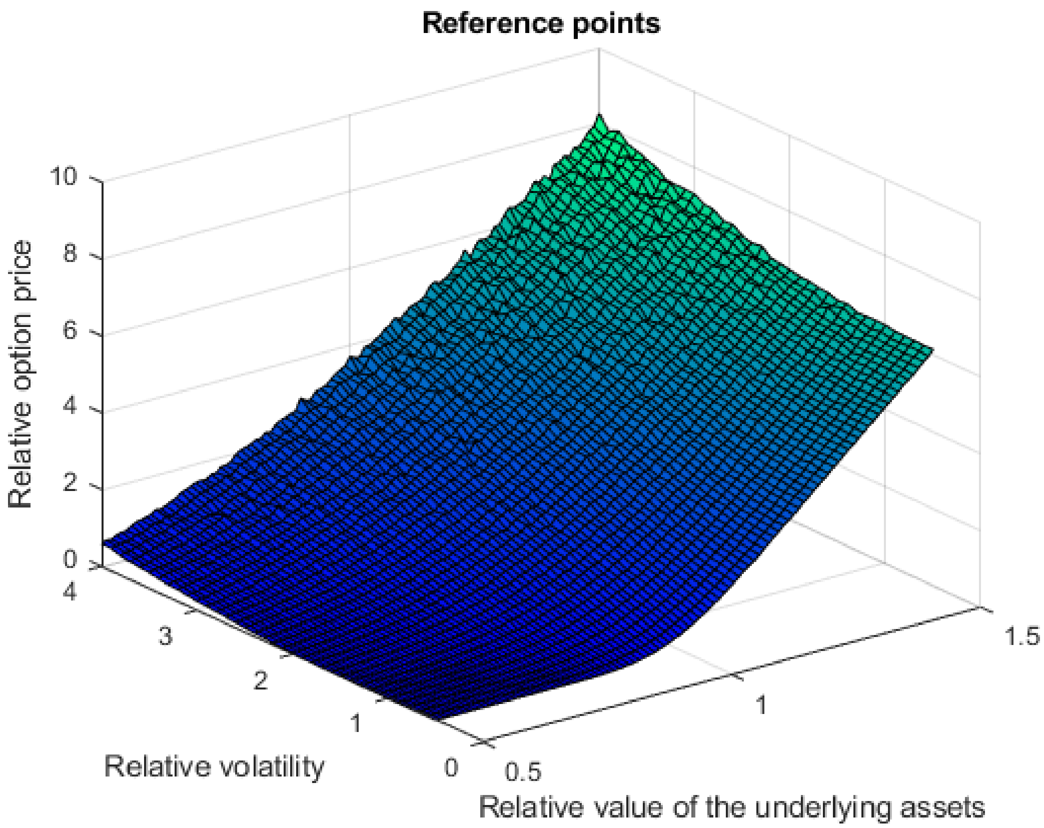


Figure 13: Surface of all 3636 reference points for SWEO713D.

8.2 Relative shifts used in Approach 3

- / -	0.5 / 0.8	0.5 / 1.0	0.5 / 1.2	- / -
1.0 / 0.6	1.0 / 0.8	**1.0 / 1.0**	1.0 / 1.2	1.0 / 1.4
- / -	1.5 / 0.8	1.5 / 1.0	1.5 / 1.2	- / -
- / -	2.5 / 0.8	2.5 / 1.0	2.5 / 1.2	- / -
- / -	4.0 / 0.8	4.0 / 1.0	4.0 / 1.2	- / -

Volatility

Underlying assets

Figure 14: Simulated values representing all relative shifts in the two risk factors. Spot price is found in **1.0/1.0**.

8.3 Relative shifts used in Approach 4

0.5 / 0.5	0.5 / 0.7	0.5 / 0.85	0.5 / 1.0	0.5 / 1.15	0.5 / 1.3	0.5 / 1.5
1.0 / 0.5	1.0 / 0.7	1.0 / 0.85	**1.0 / 1.0**	1.0 / 1.15	1.0 / 1.3	1.0 / 1.5
2.0 / 0.5	2.0 / 0.7	2.0 / 0.85	2.0 / 1.0	2.0 / 1.15	2.0 / 1.3	2.0 / 1.5
4.0 / 0.5	4.0 / 0.7	4.0 / 0.85	4.0 / 1.0	4.0 / 1.15	4.0 / 1.3	4.0 / 1.5

Volatility

Underlying assets

Figure 15: Reference points representing all relative shifts in the two risk factors. Spot price is found in **1.0/1.0**.

8.4 Relative shifts used in Approach 5

- / -	- / -	- / -	- / -	- / -
- / -	- / -	**1.0 / 1.0**	- / -	- / -
- / -	- / -	- / -	- / -	- / -
- / -	- / -	- / -	- / -	- / -
- / -	- / -	- / -	- / -	- / -

Volatility

Underlying assets

Figure 16: Simulated values representing all relative shifts in the two risk factors. Spot price is found in **1.0/1.0**.

8.5 Historical volatility

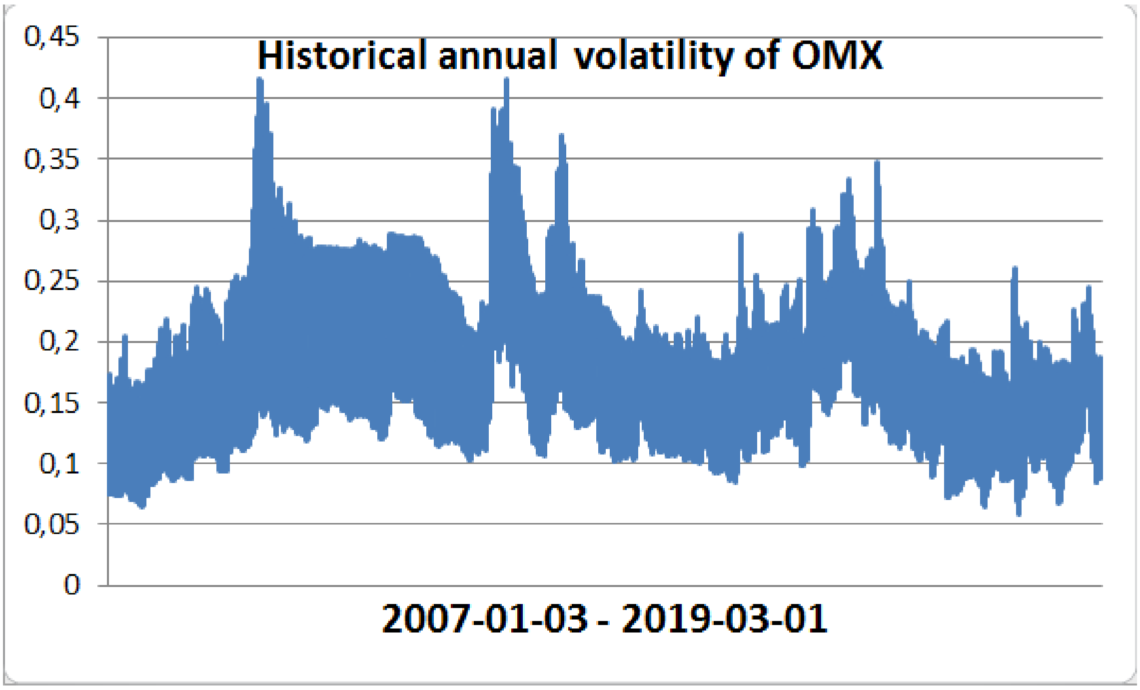
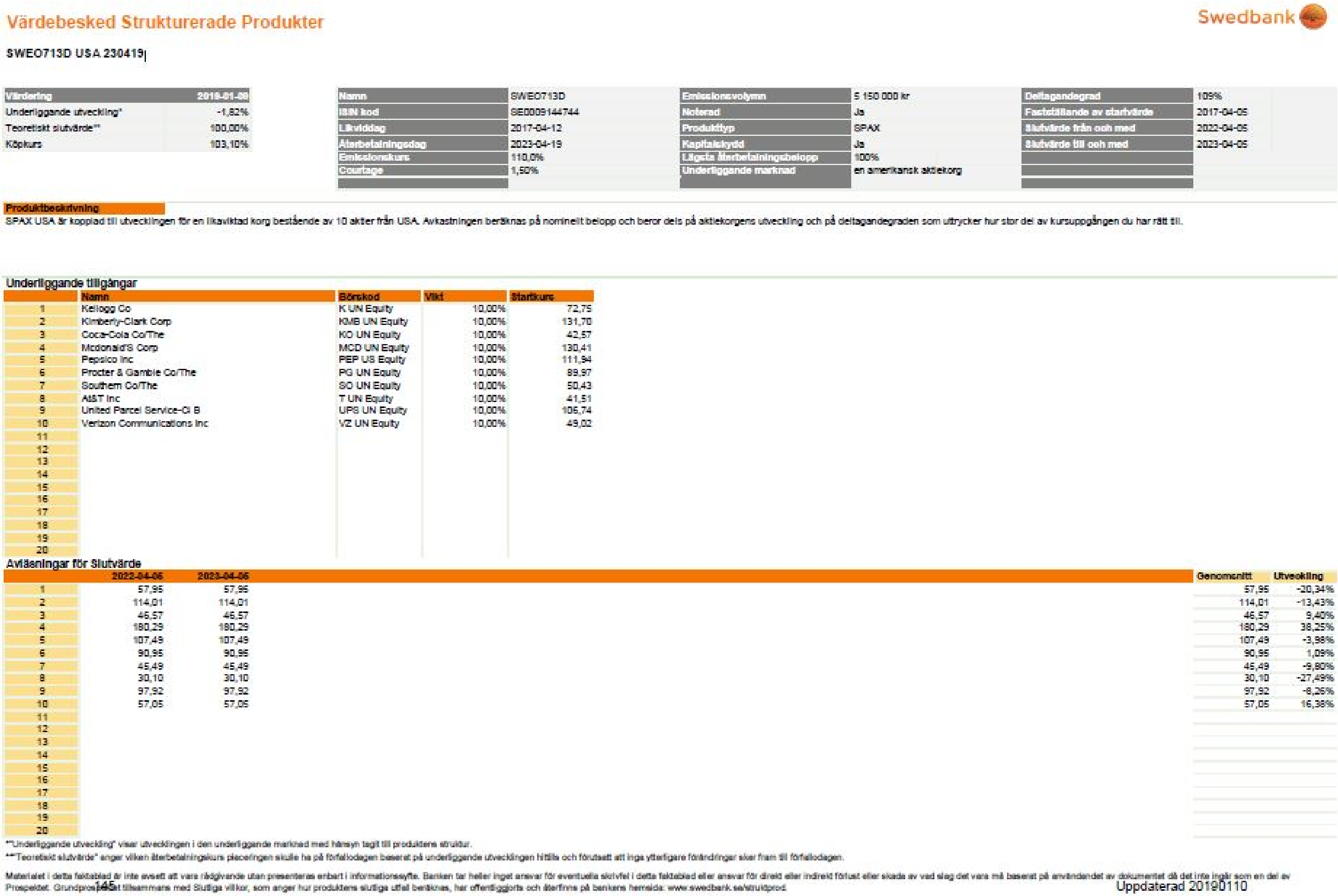


Figure 17: Historical shifts in volatility on annual basis since 2007-01-03 until 2019-03-01

8.6 Evaluated option



Värdebesked Strukturerade Produkter

Swedbank

SWEO713D USA 230419

Värdering	2019-01-09
Underliggande utveckling*	-1,82%
Teoretiskt slutvärde**	100,00%
Köpkurs	103,10%

Namn	SWEO713D	Emissionsvolymn	5 150 000 kr	Deltagandegrad	109%
ISIN kod	SE0009144744	Noterad	Ja	Fastställande av startvärde	2017-04-05
Likviddag	2017-04-12	Produkttyp	SPAX	Slutvärde från och med	2022-04-05
Återbetalningsdag	2023-04-19	Kapitalskydd	Ja	Slutvärde till och med	2023-04-05
Emissionskurs	110,0%	Lägsta återbetalningsbelopp	100%		
Courtage	1,50%	Underliggande marknad	en amerikansk aktiekorg		

Produktbeskrivning

SPAX USA är kopplad till utvecklingen för en likaviktad korg bestående av 10 aktier från USA. Avkastningen beräknas på nominellt belopp och beror dels på aktiekorgens utveckling och på deltagandegraden som uttrycker hur stor del av kursuppgången du har rätt till.

Underliggande tillgångar

	Namn	Börskod	Vikt	Startkurs
1	Kellogg Co	K UN Equity	10,00%	72,75
2	Kimberly-Clark Corp	KMB UN Equity	10,00%	131,70
3	Coca-Cola Co/The	KO UN Equity	10,00%	42,57
4	Mcdonald'S Corp	MCD UN Equity	10,00%	130,41
5	Pepsico Inc	PEP US Equity	10,00%	111,94
6	Procter & Gamble Co/The	PG UN Equity	10,00%	89,97
7	Southern Co/The	SO UN Equity	10,00%	50,43
8	AT&T Inc	T UN Equity	10,00%	41,51
9	United Parcel Service-Cl B	UPS UN Equity	10,00%	106,74
10	Verizon Communications Inc	VZ UN Equity	10,00%	49,02
11				
12				
13				
14				
15				
16				
17				
18				
19				
20				

Avläsningar för Slutvärde

	2022-04-06	2023-04-06	Genomsnitt	Utveckling
1	57,95	57,95	57,95	-20,34%
2	114,01	114,01	114,01	-13,43%
3	46,57	46,57	46,57	9,40%
4	180,29	180,29	180,29	38,25%
5	107,49	107,49	107,49	-3,98%
6	90,95	90,95	90,95	1,09%
7	45,49	45,49	45,49	-9,80%
8	30,10	30,10	30,10	-27,49%
9	97,92	97,92	97,92	-8,26%
10	57,05	57,05	57,05	16,38%
11				
12				
13				
14				
15				
16				
17				
18				
19				
20				

*"Underliggande utveckling" visar utvecklingen i den underliggande marknad med hänsyn tagit till produktens struktur.

**"Teoretiskt slutvärde" anger vilken återbetalningskurs placeringen skulle ha på förfallodagen baserat på underliggande utvecklingen hittills och förutsatt att inga ytterligare förändringar sker fram till förfallodagen.

Materialet i detta faktablad är inte avsett att vara rådgivande utan presenteras enbart i informationssyfte. Banken tar heller inget ansvar för eventuella skrivfel i detta faktablad eller ansvar för direkt eller indirekt förlust eller skada av vad slag det vara må baserat på användandet av dokumentet då det inte ingår som en del av Prospektet. Grundprospektet tillsammans med Slutliga villkor, som anger hur produktens slutliga utfall beräknas, har offentliggjorts och återfinns på bankens hemsida: www.swedbank.se/struktprod

145

Uppdaterad 20190110

Figure 18: Evaluated product SWEO713D, Swedbank

8.7 Approaches versus reference point

Table 3: Relative option prices for approaches 3-5 and the relative option price for the same shifts taken from reference points, known as the true value.

Underlying asset's shifts	Volatility shift	Approach 3	Approach 4	Approach 5	Reference point
0,5	1	0,003872048	5,03E-06	0	5,03E-06
0,5	0,5	0,003872048	0	0	0
0,5	0,7	0,003872048	9,01E-07	0	0
0,5	1,5	3,87E-03	0,013492332	0	0,005526
0,5	2	0	0,044899167	0	0,044899
0,5	3	0	0,235129768	0,274607794	0,246035
0,5	4	0,003872048	6,10E-01	1,193820505	6,10E-01
0,6	1	0,001963798	0,006215516	0	0,001612
0,6	0,5	0,001963798	1,43E-06	0	0
0,6	0,7	0,001189315	5,53E-05	0	1,51E-05
0,6	1,5	0,001963798	0,058214425	0	0,040083
0,6	2	0	0,161669785	0	0,138812
0,6	3	0,199717493	0,524142021	0,762744554	0,529365
0,6	4	0,684522303	1,063461835	1,68326097	1,005379
0,7	1	0,027226421	2,39E-02	0	0,02393
0,7	0,5	0,027226421	6,03E-06	0	6,03E-06
0,7	0,7	4,38E-05	0,004713121	0	0,002021
0,7	1,5	0,027226421	0,150698249	0	0,140123
0,7	2	0,194501727	0,362460085	0,342707598	0,36246
0,7	3	0,739677033	0,866613573	1,26448584	0,914093
0,7	4	1,445191125	1,539811917	2,18630596	1,539812
0,8	1	0,132766755	1,36E-01	0	1,40E-01
0,8	0,5	0,004735052	0,008390631	0	0,004814
0,8	0,7	0,015317503	0,05237587	0	0,03664
0,8	1,5	0,395317443	0,374785831	0,395224432	0,393073
0,8	2	0,690387588	0,668276352	0,856749699	0,726357
0,8	3	1,380949578	1,296474054	1,779831645	1,399089
0,8	4	2,153370902	2,011063004	2,70295547	2,126101
0,9	1	0,425069781	0,415794254	0,460052558	0,439888
0,9	0,5	0,052723934	0,108994668	0	0,104665
0,9	0,7	0,190873459	0,225807109	0,182757595	0,227455

0,9	1,5	0,779688349	0,77098127	0,922219206	0,836306
0,9	2	1,160134159	1,155653776	1,384396325	1,227098
0,9	3	1,949963126	1,921558316	2,308781971	2,060477
1	0,5	0,558640018	0,577748754	0,53719197	0,577749
1	0,7	0,731021419	0,745241995	0,722313923	0,740879
1	1,5	1,386846117	1,433255811	1,462818501	1,418021
1	2	1,804354287	1,875842873	1,925647472	1,875843
1	3	2,65077019	2,783104012	2,851336822	2,742222
1	4	3,550282486	3,7184207	3,77706805	3,718421
0,9	4	2,81719224	2,726857668	3,2332095	2,977827
1,1	1	1,795684808	1,785884047	1,553551967	1,820093
1,1	0,5	1,429937752	1,474919368	1,09009208	1,482323
1,1	0,7	1,558429998	1,588022911	1,275474781	1,584321
1,1	1,5	2,169241499	2,208906325	2,017022315	2,207065
1,1	2	2,591970881	2,682029288	2,480503139	2,647247
1,1	3	3,47744343	3,546096898	3,407496193	3,429707
1,1	4	4,361062383	4,399264602	4,334531126	4,32331
1,2	1	2,766915109	2,74277604	2,12070845	2,780002
1,2	0,5	2,59391243	2,58823058	1,656596715	2,627004
1,2	0,7	2,616354183	2,635788855	1,842240154	2,656463
1,2	1,5	3,086497918	3,104865967	2,584830655	3,110099
1,2	2	3,480708742	3,559435233	3,048963331	3,56295
1,2	3	4,373505229	4,355338118	3,977260085	4,349509
1,2	4	5,198833711	5,16131494	4,905598722	5,152352
1,3	1	3,839016566	3,840090359	2,701469462	3,84009
1,3	0,5	3,807290991	3,788776887	2,236705876	3,788777
1,3	0,7	3,771440428	3,798962278	2,422610052	3,80033
1,3	1,5	4,123188506	4,098472331	3,166243515	4,112051
1,3	2	4,505134407	4,493321515	3,631028043	4,493322
1,3	3	5,422348022	5,244281127	4,560628501	5,300194
1,3	4	6,208622323	6,086217813	5,490270843	6,086218
1,4	1	4,950054783	4,986102748	3,29583499	5,006848
1,4	0,5	5,059606313	4,963844524	2,830419551	4,945593
1,4	0,7	4,986286908	4,964521922	3,01658447	4,950762

1,4	1,5	5,198815853	5,176880229	3,7612609	5,171378
1,4	2	5,568496832	5,467517483	4,226697275	5,493135
1,4	3	6,510127575	6,160256336	5,157601438	6,276465
1,4	4	7,257347696	7,119209259	6,088547484	6,868589
1,5	1	6,04352066	6,197066743	3,903805043	6,197067
1,5	0,5	6,294349294	6,13895676	3,437737757	6,138957
1,5	0,7	6,183561047	6,156088967	3,624163414	6,133633
1,5	1,5	6,256870861	6,309059432	4,369882805	6,254632
1,5	2	6,614286917	6,484304453	4,835971032	6,484304
1,5	3	7,580334788	7,154844868	5,7681789	7,312834
1,5	4	8,288500728	8,26683635	6,700428645	8,266836