Designing radiation protection for a linear accelerator using Monte Carlo-simulations

Jonatan Lindahl

June 14, 2019
Master’s Thesis in Engineering Physics & Medical Physics, 30 ECTS  
Designing radiation protection for a linear accelerator using Monte Carlo-simulations  
Jonatan Lindahl (lindahl.jonatan@outlook.com)  
Supervisor: Anders Garpebring (anders.garpebring@umu.se)  
Examiner: Heikki Tölli (heikki.tolli@umu.se)  
Department of radiation sciences  
Umeå University  
SE-901 87 Umeå, Sweden  
© 2019
Abstract

The department of Radiation Sciences at Umeå University has obtained an old linear accelerator, intended for educational purposes. The goal of this thesis was to find proper reinforced radiation protection in an intended bunker (a room with thick concrete walls), to ensure that the radiation outside the bunker falls within acceptable levels. The main method was with the use of Monte Carlo-simulations.

To properly simulate the accelerator, knowledge of the energy distribution of emitted radiation was needed. For this, a novel method for spectra determination, using several depth dose measurements including off-axis, was developed. A method that shows promising results in finding the spectra when measurements outside the primary beam are included. The found energy spectrum was then used to simulate the accelerator in the intended bunker. The resulting dose distribution was visualized together with 3D CAD-images of the bunker, to easily see in which locations outside the bunker where the dose was high.

An important finding was that some changes are required to ensure that the public does not receive too high doses of radiation on a public outdoor-area that is located above the bunker. Otherwise, the accelerator is only allowed to be run 1.8 hours per year. A workaround to this problem could be to just plant a thorn bush, covering the dangerous area of radius 3 m. After such a measure has been taken, which is assumed in the following results, the focus moves to the radiation that leaks into the accelerator’s intended control room, which is located right outside the bunker’s entrance door.

The results show that the accelerator is only allowed to be run for a maximum of 6.1 or 3.3 hours per year (depending on the placement of the accelerator in the room), without a specific extra reinforced radiation protection consisting mainly of lead bricks. With the specific extra protection added, the accelerator is allowed to be run 44 or 54 hours per year instead, showing a distinct improvement. However, the dose rate to the control room was still quite high, 13.7 µGy h$^{-1}$ or 11.2 µGy h$^{-1}$, compared to the average dose received by someone living in Sweden, which is 0.27 µGy h$^{-1}$. Therefore, further measures are recommended. This is however a worst case scenario, since the leakage spectrum from the accelerator itself was simulated as having the same energy spectrum as the primary beam at 0.1% of the intensity, which is the maximum leakage dose according to the specifications for the accelerator. This is probably an overestimation of the intensity. Also, the energy spectra of the leakage is probably of lower energy than the primary beam in at least some directions. Implementing more knowledge of the leak spectra in future work, should therefore result in more allowed run hours for the accelerator.
Contents

1 Introduction 1

2 Theory 2
  2.1 Spectra determination by transmission measurements ................. 2
  2.2 Spectra determination by dose transmission measurements .......... 3
    2.2.1 A novel method for spectra determination including the buildup
    region and off-axis depth doses ...................................... 3
  2.3 Buildup and off-axis .............................................. 4
  2.4 Tikhonov regularization ........................................... 6
    2.4.1 The L-curve ..................................................... 8
  2.5 Beam geometry and inverse square law ................................ 8
    2.5.1 Inverse square law ............................................ 8
    2.5.2 Straight down approximation ................................... 9
    2.5.3 Cone beam fraction ............................................ 10
  2.6 Monte Carlo-simulations .......................................... 11
    2.6.1 Pencil beam .................................................... 14
    2.6.2 Dose normalization ........................................... 14
  2.7 Uncertainty ....................................................... 15

3 Method 16
  3.1 Spectra determination ............................................ 16
    3.1.1 Simulations .................................................... 16
    3.1.2 Measurements .................................................. 17
  3.2 Reinforced radiation protection .................................. 20
    3.2.1 Design .......................................................... 20
    3.2.2 Simulations .................................................... 22
    3.2.3 Dose normalization ............................................ 24
    3.2.4 Analysis of the resulting dose ................................ 24
  3.3 Hardware & software .............................................. 25

4 Results 26
  4.1 Spectra determination ............................................ 26
    4.1.1 Simulations .................................................... 26
    4.1.2 Measurements .................................................. 27
    4.1.3 Calculating the spectra ...................................... 28
  4.2 Reinforced radiation protection .................................. 34
    4.2.1 Dose normalization ........................................... 34
    4.2.2 Simulations .................................................... 34
    4.2.3 Analysis of the resulting dose ................................ 39
5 Discussion and future work

5.1 Spectra determination ........................................... 42
5.2 Reinforced radiation protection ................................. 43
   5.2.1 Conclusion .................................................. 45

Appendix A Spectra determination ................................. A1
   A.1 Figures .......................................................... A1
   A.2 Data ............................................................. A3

Appendix B Reinforced radiation protection ....................... B1
   B.1 Outlier .......................................................... B1
   B.2 Dose back and right ......................................... B2
      B.2.1 Case Right ................................................ B2
      B.2.2 Case Back ................................................ B3
   B.3 Simulation amount ............................................. B4
1 Introduction

The department of Radiation Sciences at Umeå University has a goal to get access to a linear accelerator for educational- and research-purposes. When an accelerator from the radiation treatment at the University Hospital of Umeå (NUS) was replaced, it was taken over by the department for this purpose.

A previous risk analysis performed by students showed that the intended location, an existing bunker for an old cobalt-60 radiotherapy machine, for the accelerator did not have enough radiation protection. This needs to be strengthened and investigated further so that the leakage radiation outside the bunker is guaranteed to fall within acceptable levels.

This will be investigated in this master thesis where the goal is to find appropriate radiation protection for the intended bunker. The purpose of this work is to give a suggestion on reinforced radiation protection that might allow for the accelerator to be installed, and used for education. A brilliant opportunity for the department!

By constructing models of the bunker and accelerator, Monte Carlo-simulations could be performed to determine the level of radiation. This was mainly done using GATE\(^1\), a simulation platform built upon a radiation transport simulation toolkit developed at CERN\(^2\) known as Geant\(^3\). To be able to build a good simulation, knowledge of the energy distribution that radiates out from the radio therapy head in all directions is needed. The approximate energy spectrum in the primary direction is known. However, the energy spectra of the leakage radiation in other directions are unknown. A novel method for determining the mega voltage spectra by different depth dose\(^4\) measurements was developed. Finally, proposals for enhanced radiation protection to the intended linear accelerator in the planned location were produced, together with resulting dose distributions outside the bunker.

In short, the goals for this project were:

- Develop a novel method to find the spectrum of a high energy linear accelerator
- Suggest how to reinforce the radiation protection for the intended linear accelerator
- Analyse the resulting dose outside the bunker, with and without the protection

\(^1\)http://www.opengatecollaboration.org/
\(^2\)https://home.cern/
\(^3\)https://geant4.web.cern.ch/
\(^4\)Dose is the energy deposited by radiation in a mass, it has the special unit Gy [J/kg].
2 Theory

To accurately simulate the beam of a linear accelerator, knowledge of what energy spectra the accelerator radiates in all directions is needed. The measurement of low intensity x-rays in the energy range for diagnostic use ($\approx 100$ kV) can be done by an ordinary scintillation detector of rather small size, but for higher energies a large crystal of at least 25 cm may be needed [3]. Scintillation detectors work by the use of a crystal that transform ionising radiation to visible light, a process called scintillation, with an intensity proportional to the energy of the incident x-ray. The light can then be measured by a photo multiplier-tube, and thus one obtain a measure of the x-ray energy. Another problem that makes the ordinary scintillator unfit for the measurement of a megavoltage (MV) beam generated from a linear accelerator is that the dead times would be too large. The scintillation processes in the crystal are slow, about 230 ns for an ordinary NaI(Tl) scintillator [4]. Thus, it can not handle the high intensity of the radiation pulses from the accelerator [15]. Hence, another method was developed for this purpose in section 2.2.1. In addition to knowledge of the spectra, it is important to understand the beam geometry, such that a proper amount of photons are simulated at different angles, section 2.5.3.

For an accurate measurement at different locations in the beam, the spread of the beam need to be accounted for. Both that the intensity decreases with the distance, section 2.5.1, and that the beam angle changes depending on the distance away from the centre axis of the beam, section 2.5.2.

A short description of monte carlo is given in section 2.6, together with the useful concept of the pencil beam for efficiency. It is also important to be able to normalize the simulation result such that it represent some real life parameter, section 2.6.2. Lastly, the theory for the uncertainty estimation is presented in section 2.7.

2.1 Spectra determination by transmission measurements

One method to determine the spectra is to measure several transmissions

$$t_{m,\text{meas}} = \frac{I_m}{I_0}, \quad (1)$$

where $I_0$ and $I_m$ are the measured intensity before and after the passage through an attenuator, respectively. The index $m$ refer to different thicknesses of attenuator material. The attenuator can be any material that absorbs x-rays. The transmission through a material can be estimated using a discrete spectrum of $N$ energy bins, by

$$t_{m,\text{calc}}(\mathbf{c}) = \sum_{n=1}^{N} c_n \cdot e^{-\mu_n d_m}, \quad (2)$$

where $c_n$ is the weight of energy $E_n$ in the energy spectrum defined by the vector $\mathbf{c}$, $d_m$ is the thickness of the material and $\mu_n$ is the linear attenuation coefficient in that material for an energy $E_n$. 

2
The energy spectrum can then be found, according to Manciu et al. [14], from the transmission measurements of the poly energetic beam by the use of an optimisation tool to minimise the penalty function

\[ P_T(c) = \frac{1}{M} \sum_{m=1}^{M} \sqrt{(t_{m,\text{calc}}(c) - t_{m,\text{meas}})^2} + \frac{\alpha}{N} \sum_{n=2}^{N} (c_n - c_{n-1})^2, \]  

(3)

by varying the spectrum \( c \). In eq. (3) \( M \) is the number of measurement points, \( N \) is the number of energy bins in the spectrum, and \( \alpha \) is an arbitrary constant to control the smoothness of the spectrum. The reason for dividing the first term with \( t_{m,\text{meas}} \) is to more evenly account for the fact that a lower dose is measured at a larger depth because of the attenuation. In theory this will give a unique solution (eq. (3) is convex). But, the weak energy dependence [10, 3] for the linear attenuation coefficient, \( \mu \), makes the system very unstable. In other words, several different spectra distributions \( c \) can quite accurately fit to the measured transmissions. Yet another problem is that the pure intensity will also be difficult to measure because of the high intensity of the beam pulses [15].

### 2.2 Spectra determination by dose transmission measurements

A more available option is to measure the dose that is transmitted through a medium, instead of the pure intensity, with one of the many dose detectors that are designed for this kind of x-ray intensity and energy. However, the dose is not directly proportional to the intensity and therefore the theory gets a bit more complicated. One solution is to adjust eq. (2) accordingly, on a form suggested by Krmar et al. [13], and obtain

\[ t_{m,\text{calc}}(\Phi) = \frac{1}{D_0} \sum_{n=1}^{N} \Phi_n \cdot E_n \cdot \frac{\mu_{en}(E_n)}{\rho} \cdot e^{-\mu_n d_m}, \]  

(4)

where \( \mu_{en} \) is the energy absorption coefficient for air, \( D_0 \) the dose in air without the attenuator, \( \rho \) the density of air and \( \Phi_n \) the photon fluence at energy \( E_n \), i.e. \( \Phi \) also describes the photon spectrum and is thus a scaled version of \( c \). Eq. (4) is a simplified model of the real case and does not account for scattered radiation from the rest of the attenuation medium. Both eq. (2) and (4) also suffer from the flaw that they can only modulate depth dose from the dose maximum and deeper, i.e. nothing in the buildup region (section 2.3). To avoid a lot of those problems a novel method is used, using Monte Carlo-simulated 3D-dose distributions, which is presented in section 2.2.1 below.

#### 2.2.1 A novel method for spectra determination including the buildup region and off-axis depth doses

The intuitive method uses a table of dose distribution data in a water phantom from Monte Carlo-simulations of several mono-energetic beams. In this way the calculated
dose at a point $r_m$ simply becomes

$$D_{m,\text{calc}}(c) = \sum_{n=1}^{N} c_n \cdot D_n(r_m),$$

(5)

where $D_n(r_m)$ is the dose from the simulation of a mono-energetic beam of energy $E_n$ at position $r_m$, which is a point inside the simulated water phantom. Now, all that is left is to design a new penalty function that can compare a point $D_{m,\text{calc}}$ from eq. (5) for some spectrum $c$ with a measured dose $D_{m,\text{meas}}$ at the same point in space. A penalty function of the same form as previously in eq. (3), becomes

$$P_D(c) = \frac{1}{M} \sum_{m=1}^{M} \sqrt{(D_{m,\text{calc}}(c) - D_{m,\text{meas}})^2} + \frac{\alpha}{N} \sum_{n=2}^{N} (c_n - c_{n-1})^2.$$  

(6)

The goal is to minimize the new penalty function, just as before, by optimising the spectrum $c$. This method open up for the use of off-axis\(^5\) dose measurements as well as the inclusion of the buildup region, since it is all included in the simulated data. This extra data contain more energy dependence than just the usual depth dose curve in the middle of the field, see section 2.3. Thus the system becomes more stable, in theory.

2.3 Buildup and off-axis

When a photon beam enters a denser material the dose starts low and increases to a dose maximum, as can be seen in fig. 2.1, this is called the buildup-region.

\[ \text{Figure 2.1} \quad \text{– Monte Carlo-simulated depth doses for eight mono energetic beams through a water phantom, all normalized to their dose maximum.} \]

\(^5\text{A line that do not coincide with the beam axis, which is the centre axis of the beam.}\)
This is a result of escaping electrons, meaning that an ionised electron near the surface can escape the medium and deposit its energy far away in the material of lower density due to its longer electron range there. Hence, the dose maximum occur at a depth roughly the same as the electron range in that material for an electron of the same energy as the incident photon. At that depth an escaping electron is, ideally, replaced by another, as illustrated in fig. 2.2. This phenomena is called \textit{charged particle equilibrium} (CPE) because an escaping charged particle (electron) is replaced by another and an equilibrium follows. At larger depths than the dose maximum the dose decreases again due to attenuation of the beam in the medium. There is so called \textit{transient charged particle equilibrium} (TCPE) in the area with decreasing dose [4].

\begin{figure}[h]
  \centering
  \includegraphics[width=0.5\textwidth]{fig2.2.png}
  \caption{For a homogenous medium, CPE exist in \textit{dV} if the distance between the boundaries of the small volume \textit{dV} and the big volume \textit{V} is seperated by at least the electron range. The ionised electrons should be produced uniformly. Thus, all the electrons that escape \textit{dV} will be replaced by another from \textit{V}. Image adapted from Attix [4].}
\end{figure}

The Compton effect is the dominant interaction from x-rays ranging between around 20 keV to 30 MeV [4] in low Z-materials such as water and is illustrated in fig. 2.3.

\begin{figure}[h]
  \centering
  \includegraphics[width=0.5\textwidth]{fig2.3.png}
  \caption{Compton scattering for an incident photon of energy $h\nu$, where $h$ is Planck’s constant and $\nu$ is the frequency, hitting an electron escaping at an angle $\theta$ and the photon of reduced energy $h\nu'$ continues at an angle $\theta'$.}
\end{figure}
The Compton scattering angle can be derived from the conservation of momenta and energy, and results in

\[
\cos(\theta) = \left( 1 + \frac{h\nu}{m_0c_0^2} \tan \left( \frac{\theta'}{2} \right) \right),
\]

(7)

where \( m_0 \) is the rest mass of the electron, \( c_0 \) is the speed of light and the rest according to the annotations from fig. 2.3. It can be seen from eq. (7) that the two scattering angles depend on the incident photon energy \( (h\nu) \) and thus the scattered radiation in a media has an energy dependence. Hence, if one were to measure the dose just outside the field, one would find an energy dependent dose distribution from the scattered radiation.

### 2.4 Tikhonov regularization

The penalty function, eq. (6), can be minimized using \textit{Tikhonov regularization}, which is a method to solve an ill conditioned system

\[
Ax = b,
\]

(8)

where \( A \) is a known matrix, \( b \) a known vector and \( x \) the vector of variables to be solved for. The method employ a regularization using a suitable \textit{Tikhonov matrix} \( \Gamma \). To solve the system in eq. (8), one can minimize the problem

\[
||Ax - b||_2^2 + ||\Gamma x||_2^2
\]

(9)

which have the regularization added as a second term. The solution is then [7]

\[
\tilde{x} = (A^T A + \Gamma^T \Gamma)^{-1} A^T b,
\]

(10)

where \( A^T \) stands for the transpose of the matrix \( A \). Eq. (9) corresponds to a penalty function of the form

\[
P_{Tikhonov}(c) = \sum_{m=1}^{M} (D_{m,calc}(c) - D_{m,meas})^2 + \alpha \sum_{n=2}^{N} (c_n - c_{n-1})^2,
\]

(11)

if one use the following:

\[
A = \begin{bmatrix}
D_1(r_1) & \cdots & D_N(r_1) \\
\vdots & \ddots & \vdots \\
D_1(r_M) & \cdots & D_N(r_M)
\end{bmatrix},
\]

(12)

which is the table of doses for the \( N \) different mono energies and \( M \) positions of interest. Further,

\[
b = \begin{bmatrix}
D_{1,meas} & \cdots & D_{M,meas}
\end{bmatrix}^T,
\]

(13)
is the column vector of measured points, matching the points in the matrix $A$. The spectrum to solve for is $x = c$, and finally

$$
\Gamma = \alpha \cdot \begin{bmatrix}
1 & 0 & 0 & \ldots & 0 \\
-1 & 1 & 0 & \ldots & 0 \\
0 & -1 & 1 & \ldots & 0 \\
\vdots & \ddots & \ddots & \ddots & \vdots \\
0 & \ldots & 0 & -1 & 1
\end{bmatrix}.
$$

(14)

The normalization constants $M$ and $N$, in eq. (6), can be included in the arbitrary constant $\alpha$. Now, the only thing missing in eq. (11) from eq. (6) is to divide the first term with the measured dose. This can be done by introducing a weighting matrix

$$
W = \text{diag}(b)^{-1},
$$

(15)

where $\text{diag}(b)$ is a diagonal matrix with the elements $b_i$ on the diagonal. The minimization problem then becomes

$$
||\sqrt{W} \cdot (Ac - b)||_2^2 + ||\Gamma c||_2^2,
$$

(16)

which have the solution

$$
\tilde{c} = (A^TWA + \Gamma^T\Gamma)^{-1}A^TWb.
$$

(17)

One advantage of using this method instead of an optimizer, aside from the fact that it is way faster, is that no initial guess is needed and thus the result becomes unbiased. Which is good since we want to find an unknown spectrum. However, for a better solution one can use the boundary conditions that the lowest and highest energy bins of the spectra should be equal to zero, if the first bin correspond to a zero-energy and the highest bin is of higher energy than the electrons generated in the linear accelerator. Because, energy can not be created from nothing, no photons can exist of higher energy than the electrons hitting the target. This can be implemented simply by adding two more equations by expanding $W$, $A$ and $b$ such that the new system to solve becomes

$$
\begin{bmatrix}
W & \ldots & 0 & 0 \\
\vdots & \ddots & \ddots & \vdots \\
0 & \ldots & w_1 & 0 \\
0 & \ldots & 0 & w_2
\end{bmatrix} \cdot \begin{bmatrix}
A \\
1 & 0 & \ldots & 0 \\
0 & 0 & \ldots & 1
\end{bmatrix} \cdot c = \begin{bmatrix}
b \\
0 \\
0 \\
0
\end{bmatrix},
$$

(18)

where $w_1$ and $w_2$ is arbitrary constants that sets the weights of the new boundary conditions. That is, they set the importance of meeting the criteria of the boundary conditions.

---

6Except the missing square root of the first term, which however should not affect the solution. The only change is that the value of $\alpha$ may differ. In other words, it solves the same problem.
2.4.1 The L-curve

To find a suitable value for the regularization parameter $\alpha$ in eq. (14), one can plot the solution norm

$$||\alpha^{-1} \cdot \Gamma c||_2,$$

as a function of the residual norm

$$||\sqrt{W} \cdot (Ac - b)||_2,$$

don logarithmic scales [7]. The solution norm is the square root of the second term in eq. (16) divided by $\alpha$, since it is $\alpha$ we want to find, and the residual norm is the square root of the first term. This should create an "L-shaped" curve, hence the name, for which proper values of $\alpha$ should correspond to points close to the corner in the L-shaped curve. At that point, the idea is that $\alpha$ is large enough to make the solution norm small, without making the residual norm too large. This is no exact method but it gives a good first guess of the constant.

2.5 Beam geometry and inverse square law

2.5.1 Inverse square law

For a point source, which is a rather good approximation for the beam from a typical accelerator (typical focus is only a few mm in size) [22], the beam will spread out in all directions. This means that the intensity, photons per unit area, will decrease as

$$I = I_0 \cdot \left(\frac{r_0}{r}\right)^2,$$

where $I$ is the intensity a distance $r$ from a source of intensity $I_0$ at a distance $r_0$. This relation is called the inverse square law and is visualised in fig. 2.4. This means that if one were to double the distance from a source, the intensity decreases to only a quarter. This implies that if one want to measure at a larger distance, a lot more photons are required initially to maintain the signal and obtain the same signal-to-noise ratio (SNR).

**Figure 2.4** – A visualisation of the inverse square law. $S$ is the source, the red lines represent individual photons, $A$ is a unit area and $r$ is a distance from the source [8].
2.5.2 Straight down approximation

For off-axis measurements, that is, for measurements away from the centre axis of the beam, the beam direction changes from straight down to have an angle, as illustrated in fig. 2.5. This must be taken into account if a depth dose is going to be measured off-axis.

Figure 2.5 – A 2D-representation of the spread of the beam from a point source. SSD is the distance between the source and the water, $\theta$ is the angle to the desired measurement location marked as "Start" at an off-axis distance $L$ at the water surface. The corresponding off-axis distance at a depth $d$ is marked $L'$ and the new depth with $d'$.

For an off-axis distance $L$ at the surface that is a distance SSD\(^7\) from the source, the dose depth measurement should follow the line (or rather the 3D-vector) that ends at an off-axis distance

$$L' = L \cdot \left(1 + \frac{d}{SSD}\right),$$

for a depth $d$. Note, that for ordinary measurements along the centre axis ($L = 0$), eq. (22) still stands since then also $L' = 0$ and thus the measurement is straight down in the water with $\theta = 0$ as expected. Also,

$$\lim_{SSD \to \infty} L \cdot \left(1 + \frac{d}{SSD}\right) = L,$$

meaning that for a large distance SSD (SSD $\gg d$), $L' \approx L$ and one could choose to measure straight down as an approximation instead of using eq. (22). It can be useful to do so if it is hard to determine the distance, SSD, to the source.

When measuring the depth dose, one can choose to obtain the dose at coordinates $(x, y, z) = (\text{crossline}, \text{inline}, \text{depth})$ with origo at the surface in the middle of the beam. Crossline is the direction from left to right as seen from the front toward the accelerator when the beam points straight down at the floor. Inline is the direction from and toward

\(^7\)Source Surface Distance - the distance from the source to the surface.
the gantry\textsuperscript{8}. Depth is the direction straight down along the centre axis of the beam. Then, the depth dose can be normalised for the spread, at the surface, by solving for $I_0$ in eq. (21) with

$$r = \sqrt{x^2 + y^2 + (z + \text{SSD})^2},$$

and $r_0 = \text{SSD}$. The off-axis depth dose can be corrected to be as it would have been if the beam pointed straight down by changing the inline/crossline coordinates to the ones at the point "Start" in fig. 2.5 and with a new calculated depth

$$d' = \sqrt{(x - L_x)^2 + (y - L_y)^2 + z^2},$$

where $L_x$ and $L_y$ is the start off-axis distance in the crossline- and inline-direction, respectively. This will make the measured data match the one from the generated pencil beams, described in section 2.6.1.

2.5.3 Cone beam fraction

When no collimator jaws (lead blocks that collimates the beam) is mounted on a linear accelerator the beam cross section usually is a circular disc that is sufficiently large to cover a square for the intended maximum field size. For the case represented in fig. 2.6, where the diameter of the beam is the same as the diagonal of the square field, the angle

$$\theta = \tan^{-1}\left(\frac{d}{2 \cdot \text{SAD}}\right) = \tan^{-1}\left(\frac{\text{FS}}{\sqrt{2} \cdot \text{SAD}}\right),$$

by simple geometry, with the annotations from the figure.

\textbf{Figure 2.6} – The cross section of a green beam at a Source Axial Distance (SAD) from the source, making a cone of an angle $\theta$. A square field of field size FS is shown in red. The diagonal, $d$, of the square field is also the diameter of the cone. The so called isocenter is usually located at an SAD of 1 m, and is where the field size is defined.

\textsuperscript{8}The machine that holds the linear accelerator and can rotate it around the patient.
It can be useful to calculate how large the primary beam is in comparison to all other directions, which is the "leakage beam", and it can be done by comparing the areas that they cover. The surface area for a sphere with radius $r$, that is, the area that is irradiated from an isotropic point source at a distance $r$ from the source, is given by

$$A_{sphere} = 4\pi r^2. \quad (27)$$

The surface area of the transparent cap in fig. 2.7 is given by [21]

$$A_{cap} = 2\pi r^2(1 - \cos \theta). \quad (28)$$

The surface area of the rest of the sphere is then given by

$$A_{rest} = A_{sphere} - A_{cap}, \quad (29)$$

where $A_{sphere}$ and $A_{cap}$ come from eq. (27) and (28), respectively. The ratio between the surface area of the transparent cap, $A_{cap}$, and the rest of the sphere, $A_{rest}$, simplifies to

$$\frac{A_{rest}}{A_{cap}} = \frac{A_{sphere}}{A_{cap}} - 1 = \frac{4\pi r^2}{2\pi r^2(1 - \cos \theta)} - 1 = \frac{2}{1 - \cos \theta} - 1. \quad (30)$$

Figure 2.7 – An illustration of the spherical cap, the cap is the transparent part. $r$ is the radius of the sphere and $\theta$ an angle.

### 2.6 Monte Carlo-simulations

Monte Carlo-simulation is a randomized method to solve problems numerically. A mathematical example could be to evaluate the value of $\pi$ by randomly throw a ball in a boxed circle of radius one (area = $\pi$). Then the ratio between the number of hits inside the
circle (hits a radius less than one from the centre of the circle) and the total number of throws should correspond to the ratio between the area of the box and the circle, which is $\pi/4$, then simply multiply by four for an estimate of $\pi$ (a good animation can be found at Wikipedia\textsuperscript{9}).

The physics behind the transport of charged and uncharged particles is complex and the equations that describe the transport are difficult to solve analytically. See fig. 2.8 for an example of some of the interactions as a result from one single photon. The radiation transport is often complex enough for infinite homogeneous media, but even more complexity arises when realistic geometries are used. That is why a numerical method is appropriate. Monte Carlo gets increasingly advantageous with the complexity of the problem [3].

![Figure 2.8](image_url) – A simplified diagram showing some of the interactions from one single incident photon, $\gamma_0$, incoming from the left. The diagram is not to scale and all interactions are not included. A total of 12 individual photons ($\gamma_i$), 14 individual electrons ($e_i$) and 3 positrons ($e^+_i$), are shown. P.P. stands for pair production, P.E. for photoelectric effect, Brems. for Bremsstrahlung and Comp. for Compton scattering, which are all different interaction types. In-flight annihilation and annihilation are also both interaction types.

In Monte Carlo-simulations the particles are typically simulated one at a time. The path length before an interaction and what interaction that will occur, are both chosen randomly from probability distributions. After that interaction, the particle either dies\textsuperscript{10}.

\textsuperscript{9}https://en.wikipedia.org/wiki/Monte_Carlo_method

\textsuperscript{10}Is not simulated further, gets deleted.
because it deposited all of its energy\footnote{Rather when the particle has less than a set threshold value of energy left.} in the interaction, or it continues for the next \textit{step}. In the next step the path length and interaction type is once again chosen randomly from probability distributions, but now potentially with a new incident particle type, energy, position and direction. This continues until the particle dies or leaves the defined simulation area, outside of which the particle is simply deleted. For each interaction the dose from the interaction can be calculated and stored. This means that a simulation with too few photons can get an unlikely dose distribution. In other words, a very poor estimate with huge errors. But as the number of photons are increased, so is the reliability of the simulated dose distribution.

Because of the need for a huge number of calculations, the simulations are performed on computers. Only a few years ago, Monte Carlo-simulations were exclusively available to large research centers. Today, an ordinary desktop computer is sufficient for smaller simulations and thus, a powerful tool has become readily available. That, together with the fact that the many interactions of radiation with matter are all governed by probability distributions of discrete stochastic processes, which makes it perfect for Monte Carlo-simulations, are probably the reasons why the most data currently available today in radiation dosimetry are derived using Monte Carlo-simulations \cite{3}. The random numbers generated by a computer can never be truly random. The criteria for the random numbers generated is that they must be uncorrelated numbers and have long repetition time, that is, a long time before the number series repeats itself. These criteria are often met sufficiently well in the so called pseudo-random numbers used, i.e. they are random enough and allow for good results.

The foundation for a simulation in the software used in this work, GATE, is to define the source, geometry, physics and detectors. Other details might be the choice of which pseudo-random engine to use and whether to visualize the simulation or not. The visualization is used with advantage when defining the simulation, and to test that the simulation run as it is supposed to. The source is defined by particle type, energy, distribution and shape. Also, the number of initial particles to be simulated from the source is set, which set the length of the simulation. The geometry is all defined inside a box that is appropriately called the \textit{world}. Each geometry is assigned a physical material. Ideally one would like to simulate every single physical event at an infinitesimal scale, but a too small scale would be to compute-intensive, i.e. take too much time, and it is even impossible to use an infinitely small scale. Instead the physics is chosen from a library of physics lists, where one get to choose an appropriate list for the intended work. It is also possible to tweak the physics settings for the advanced user. The dose detector in the simulation environment is called a dose actor and can be attached to any geometry. The dose actor covers the entire geometry of choice and the user selects the desired resolution. The resolution sets the voxel size. A voxel represents some finite 3D volume in space, usually rectangular, and is assigned a single value, e.g. the dose. That is, a voxel is a pixel with depth. The absorbed dose is calculated as the sum of all energy
depositions in a voxel, divided by its mass. Typically this is in the units of MeV kg$^{-1}$ but is usually converted to the SI unit of the dose, J kg$^{-1}$, simply by the use of the relation $1 \text{MeV} = 1.60217646 \times 10^{-13} \text{J}$ [6].

2.6.1 Pencil beam

Instead of simulating a wide beam, that needs a lot of photons to cover the large area, one can simulate a narrow beam that needs less initial photons to cover the same number of photons per unit area and thus obtain the same SNR. This narrow beam is called a pencil beam, because of its slim shape. From this pencil beam, a wide beam can be produced simply by adding the resulting 3D-dose distribution together side-by-side with each other to create a larger beam, as illustrated in fig. 2.9.

![Pencil beam diagram](image)

**(a)**

**Figure 2.9** – The dose distribution from a small square pencil beam the size of one pixel, pointing into the page, is shown in (a). The small squares are pixels and the larger dotted green squares represent the desired five-by-five beam, that is composed of 25 narrow beams, resulting in (b).

2.6.2 Dose normalization

The amount of radiation dose delivered by the linear accelerator is typically measured by ionisation chambers in the primary beam and is simply called *monitor units* (MU). The machine is usually calibrated such that 100 MU gives 1 Gy under certain reference conditions [9]. Such reference conditions could be the dose at 10 cm depth in a water phantom at 90 cm SSD. The intensity of the simulated beam is set by a user defined number of initial photons. To get a connection between the number of simulated photons and the delivered dose from the real accelerator, one could simulate the reference
conditions and measure the dose from a specific amount of photons. Then, the amount of photons corresponding to 100 MU (or 1 Gy under reference conditions) is given by

\[ N_{100MU} = \frac{N_{MC}}{D_{MC,ref}}, \]  

(31)

for \( N_{MC} \) simulated photons that give the dose \( D_{MC,ref} \). To scale a simulation of \( N_i \) photons to correspond to an accelerator that delivers 100 MU, one can use a scale factor

\[ S_i = \frac{N_{100MU}}{N_i}. \]  

(32)

Then, the dose is given by

\[ D_i = S_i \cdot D_{MC,i}, \]  

(33)

where \( D_{MC,i} \) is the dose in a voxel from a simulation of \( N_i \) photons, and \( D_i \) the dose scaled to a 100 MU run of the accelerator. It can be useful to scale the dose to the dose rate

\[ \dot{D} = D_i \cdot \frac{\text{MU}_{\text{acc.}}}{100}, \]  

(34)

where \( D_i \) comes from eq. (33) and \( \text{MU}_{\text{acc.}} \) is how many MU the accelerator delivers per time unit.

### 2.7 Uncertainty

The simulations can each be seen as an independent measurement and thus the standard deviation can be calculated as

\[ \sigma = \sqrt{\frac{1}{N-1} \sum_{i=1}^{N} (x_i - \bar{x})^2}, \]  

(35)

where \( N \) is the number of measurements (or simulations), \( x_i \) is a value in a voxel \( x \) for the simulation \( i \) and

\[ \bar{x} = \frac{1}{N} \sum_{i=1}^{N} x_i, \]  

(36)

that is, the mean in the voxel \( x \) of all simulations. The standard deviation of the mean is given by

\[ \sigma_{\bar{x}} = \frac{1}{\sqrt{N}} \sigma, \]  

(37)

for an independent random variable \( x \). The standard deviations, \( \sigma_X \) and \( \sigma_Y \), from two independent variables \( X \) and \( Y \) respectively, can be added together as

\[ \sigma_{X+Y} = \sqrt{\sigma_X^2 + \sigma_Y^2}. \]  

(38)

The uncertainty can be neatly expressed as the so called coefficient of variation,

\[ CV = \frac{\sigma}{\bar{x}}, \]  

(39)

which gives the uncertainty as a fraction of the value in question.
3 Method

To accurately simulate a linear accelerator, one need to find what energy spectra that spreads out from the machine in all directions. The energy spectrum in the primary beam of an existing accelerator at the radio therapy department at NUS was found using the novel method developed in section 2.2.1, by the use of depth dose measurements, including off-axis measurements, and corresponding Monte Carlo-simulated depth doses.

Knowledge about the spectra was then used to simulate the accelerator in the intended bunker. To do so, the room was sketched and areas of dose measurements were defined. Extra radiation protection was designed and simulated as well, such that a comparison could be done with and without the extra protection. The resulting dose distribution outside the bunker was normalized to a dose rate that would result from a real life run of the accelerator, and analysed by heat maps to identify hot spots. Finally, a more thorough analysis of the most interesting areas was performed.

3.1 Spectra determination

3.1.1 Simulations

Several monoenergetic pencil beams with a cross section of \(1 \times 1 \text{mm}^2\) were simulated in GATE with physics list "EmStandardOpt3" [1], hitting a \(50 \times 50 \times 50 \text{cm}^3\) water phantom. The dose actor was defined as a smaller volume of \(101 \times 101 \times 500 \text{mm}^3\), and was located in the middle of the larger phantom, placed with the longer dimension in the beam direction, as can be seen in fig. 3.1.

![Figure 3.1](image-url) – A screenshot from a simulation of the pencil beam for 100 initial photons of 0.25 MeV each. The green lines are the trajectories of the photons, which all start off at the position marked Beam.
The dose distribution matrix generated from the dose actor would become impractically large\textsuperscript{12} for the desired voxel size of $1 \times 1 \times 1 \text{mm}^3$, if the entire water phantom in fig. 3.1 was used as the dose actor. That is why the dose actor was chosen as a smaller volume. The dose distribution from each of the monoenergetic pencils, ranging from 0.25 MeV with 0.25 incremental steps up to 8 MeV, was then used to each generate a $5 \times 5 \text{cm}^2$ beam, simply by adding them together as described in section 2.6.1. Then each energy was combined to get a 4D energy table of 3D-dose distributions. The table could then be used to find the matrix $A$ in eq. (18), simply by interpolating for the desired energies and spatial locations. A zero energy of zero dose was also added to the table of practical reasons. One extra pencil was simulated with a Varian 6 MV spectrum (table A.1 in appendix A.2) from the publication by Sheikh-Bagheri et al.\textsuperscript{18}, and used to make another $5 \times 5 \text{cm}^2$ beam. This was done to be able to compare the simulated dose with a measured one. All simulations were performed with the "MersenneTwister" random engine, one of three choices in GATE.

3.1.2 Measurements

The measurements used for the determination of the energy spectrum were performed on an existing linear accelerator, a Varian Clinac iX, at the radio therapy department in Umeå. The setup consisted of a large water filled phantom, named BluePhantom (IBA Dosimetry), and can be seen in fig. 3.2. The BluePhantom includes a motorised detector mount that, together with the included software, allow to pre-define at which coordinates in the water tank the measurements should be performed. Then, one simply turn on the beam and the measure protocol of all the pre-defined points will be measured automatically. The measured dose data is stored together with corresponding coordinates.

A square field size of $\text{FS} = 5 \times 5 \text{cm}^2$ was used and the photon energy set to 6 MV. The BluePhantom was placed such that the beam pointed straight down through the middle with the water surface a distance $\text{SSD} = 90 \text{cm}$ from the source. In practice, this was accomplished by adjusting the height of the tank until the surface was 10 cm higher than the green laser, seen in fig. 3.2, which marks the isocenter that is 1 m from the source. Three different distances, $L_1 = 0$, $L_2 = 30 \text{mm}$ and $L_3 = 50 \text{mm}$, from the centre axis of the beam was chosen for the depth dose measurements. The vectors that the depth dose should follow down in the phantom were calculated as described in section 2.5.2, resulting in $L'_1 = 0$, $L'_2 = 39.5 \text{mm}$ and $L'_3 = 65.8 \text{mm}$, with a measurement depth of $d = 285 \text{mm}$. The actual locations in the BluePhantom that were measured are visualised as the blue dots in fig. 3.3. The detectors used were a calibrated photon diode (PFD\textsuperscript{13}G)\textsuperscript{13} for the dose measurements and a compact air ionization chamber (CC13)\textsuperscript{14} present in a corner of the primary field as a reference to compensate for potential dose rate variations of the beam.

\textsuperscript{12}The software usually crashes with too big matrices and it would be heavy to analyse the big matrices.

\textsuperscript{13}IBA dosimetry semi-conductor shielded detector with high spatial precision.

\textsuperscript{14}Standard chamber for clinical use, IBA dosimetry.
A crossline dose profile was also measured at 10 cm depth for a validation of the pencil beam approach. This was simply a measurement from left to right at a constant depth in the BluePhantom. This was used to compare the shape of the dose profile from the generated $5 \times 5 \text{cm}^2$ beam with the real measured one.

Figure 3.2 – The measurement setup used on the Varian Clinac iX together with the BluePhantom water tank. The green lasers, that marks the isocenter, are turned on as well.
Figure 3.3 – The size of the simulated dose distributions, $101 \times 101 \times 500 \text{mm}^3$, shown in blue. A straight box of size $5 \times 5 \times 50 \text{cm}^3$ is shown in grey as a reference for a theoretical beam with no spread. The green represents the actual $5 \times 5 \text{cm}^2$ beam, which spreads as if it was generated from a point source at SSD = 90 cm. The actual measurement locations are shown in blue and the corresponding re-calculated points in red.
3.2 Reinforced radiation protection

3.2.1 Design

The entire bunker was sketched in 3D-CAD (Autodesk Fusion 360). The measurements were taken from blue prints of the room and supplemented with some actual physical measurements in the bunker. Each new material needed to be saved as a separate .STL\textsuperscript{15} format file before it could be imported to GATE, due to limitations in the GATE software that only allow one material per file. Two different cases, with different orientation of the linear accelerator, was investigated with and without extra radiation protection. Both cases can be seen together in fig. 3.4. In the same figure, some extra radiation protection by the entrance door, is shown enlarged. This extra protection was chosen since one can actually see into the bunker through a small gap when the entrance door is closed.

![Diagram of the bunker with extra concrete blocks by the entrance door shown enlarged. Beams in green, the red circles around the beams are just a marking for 1 m distance from the source.](image)

Figure 3.4 – Showing both cases in the bunker at once. The extra concrete blocks by the entrance door are shown enlarged. Beams in green, the red circles around the beams are just a marking for 1 m distance from the source.

The walls are mainly 70 cm thick and surrounded by the ground, with one thicker wall of 2 m facing toward a cyclotron bunker. The floor is 30 cm thick and the roof (not shown in the figures) is 1 m thick with a bit of ground on top, followed by tiles on a public outdoor-area. The "water walls" are fictional for the purpose of the simulation and is 1 m thick. A radiation protection reinforcement consisting of 21 lead bricks, each brick of size $5 \times 10 \times 20$ cm\textsuperscript{3}, is shown enlarged for each case in fig. 3.5 and 3.6, respectively. The extra lead were added mainly to prevent too much radiation to the front wall and the

\textsuperscript{15}A file format that saves the surfaces as triangles.
entrance door. Lead is a good choice because of its high density and high atomic number (Z), which enable a compact radiation protection that do not take up too much space. Another advantage is that it efficiently absorbs high energy photons of more than 4 MeV due to pair production\(^{16}\) [3]. In turn, it lowers the penetration ability of the radiation, and thus, less photons escape the bunker.

**Case Right** The engineers had already started to put the accelerator together in the bunker and the preferred position would probably be similar to the one named case Right, that can be seen in fig. 3.5.

![Figure 3.5](image)

**Figure 3.5** – Case Right has the beam pointing at the right wall. The source is positioned 1000 mm up from the floor, 3070 mm from the back wall and 4045 mm from the left wall in the room. Enlarged is the extra lead (blue). The thicker centre part of the extra lead is centered at the source position in the left right position and with the bottom edge \(\approx 92\) mm below the source. Some of the fictional water walls are displayed as well.

\(^{16}\)Pair production is an interaction of a high energy photon with a nucleus, creating an electron and a positron, which both have far less penetration ability than the photon [4].
**Case Back** From a radiation protection point of view, case Back, as seen in fig. 3.5, is probably a better choice. This is because the source is better covered by the extra concrete wall, 50 cm thick, which extends out into the room, between the door and the accelerator position.

![Case Back Diagram](image)

**Figure 3.6** – Case Back has the beam pointing at the back wall. The source is positioned 1000 mm up from the floor, 4514 mm from the back wall and 5585 mm from the left wall in the room. Enlarged is the extra lead (blue), consisting of the same lead bricks as in fig. 3.5, and is all centered at the source position in the left right position, with the bottom edge 120 mm below the source. Some of the fictional water walls are displayed as well.

### 3.2.2 Simulations

The simulations were performed in GATE with the same physics list and random engine, "EmStandardOpt3" and "MersenneTwister", as for the pencil beams in section 3.1.1. The source was simulated as a point source. The primary beam was a cone that cover a field size of $FS = 40 \times 40 \text{cm}^2$ at a distance $SAD = 100\text{cm}$ from the source, which is where the field size is usually defined. That is, the cone with an angle $\theta = 15.793^\circ$ from eq. (26). The spectrum for the primary beam was taken from the work by Sheikh-Bagheri et al. [18] and is a Monte Carlo-simulated spectrum for a Siemens 6 MV linear accelerator, which is the same brand and energy as for the intended accelerator. The spectra was simulated in histogram mode, meaning that the spectra was simulated by energies uniformly distributed in the energy bins from the last energy up to the next. The spectrum used is shown in fig. 3.7.
Figure 3.7 – The primary spectrum of a Siemens 6MV linear accelerator [18]. The spectrum can also be found in appendix A.2, table A.1. The fraction, $c$, of each energy bin is shown on the vertical axis. The sum of all $c$ equals unity.

The leakage radiation was simulated as monoenergies starting from 0.25 MeV with 0.25 incremental steps up to 7 MeV, followed by 7.5 MeV and 8 MeV, that was spread from the point source in all directions other than the directions covered by the primary beam. Two of the highest energies, 7.25 MeV and 7.75 MeV, were skipped simply because it is expected to be few photons of the highest energies in the spectrum and the simulation time is long for high energy photons. The dose distributions from the monoenergetic simulations were then combined for an arbitrary spectrum afterwards. This was done by simply adding the voxel values from each simulation, multiplied by a corresponding weighting factor of how abundant that specific energy was in the spectrum. This opens up for a lot of flexibility and is one of the great benefits in this kind of simulation. Unfortunately the intended linear accelerator could not be started yet and thus no measurement could be performed on that exact accelerator. However, since the found spectrum for $\alpha = 2.6$ in fig. 4.9 is close to the theoretical spectrum for Siemens 6 MV as well, this is what was used as the leakage spectrum in the simulations. The spectrum was found by the method in section 3.1, from measurements on a Varian 6 MV linear accelerator. The amount of photons simulated as leakage compared with the primary beam were scaled to correspond about 0.1% dose in a point 1 m behind the accelerator, compared to the dose 1 m from the source in the primary beam. This is namely the maximum specified leakage according to Siemens for the specific accelerator used (Siemens Oncor 6 MV) [19]. The geometry for the leakage beam is about 52 times as big as the primary beam according to eq. (30). Thus, the leakage spectrum was scaled by a factor of $52 \cdot 0.1 \% = 5.2 \%$. The body of the linear accelerator was not simulated, it is only visualised in the figures.
to understand how it would affect the layout of the bunker. Same thing goes for the cobalt-60 therapy machine. Both of which would probably attenuate even more of the radiation.

**The water walls** The purpose of the water walls was to represent what dose a human, whom is basically made out of water, would get if standing just outside the bunker. The walls are each 1 m thick and made out of liquid water. The entire walls were set as dose actors with an voxel size close to $10 \times 10 \times 2 \text{cm}^3$, where the finest resolution was in the thickness-direction.

**Materials** The walls, floor and roof of the bunker are made out of ordinary concrete and were simulated as such, with the elemental composition of concrete from NIST [5]. The small storage room for radioactive sources (fig. 3.4) had walls made out of 4 mm lead in the simulations. This is because the door to the storage is classed as 4 mm lead equivalent and thus the walls should be at least as thick. The extra concrete blocks, fig. 3.4, are made out of so called *high density concrete* and were simulated as ConcreteBA from NIST [5]. The actual composition was unknown, but should however have been something similar. The door was simulated composed of the following five layers: 1 mm steel, 5 mm lead, 70 mm boron plastic, 5 mm lead and 1 mm steel. The boron plastic was simulated as plastic (polyethylene) from NIST [5], modified by the addition of 5% boron (B) by weight.

### 3.2.3 Dose normalization

The same simulation setup as used previously for the pencil beam, fig. 3.1 in section 3.1.1, was used to determine the reference dose $D_{MC,ref}$ for $N_{MC}$ photons at 10 cm depth in the water phantom, with the pencil beam replaced by the same primary beam as for the main simulations, described in section 3.2.2, at SSD = 90 cm.

### 3.2.4 Analysis of the resulting dose

The analysis of the resulting dose distribution outside of the bunker was performed by 2-dimensional (floor plane) heat maps of the mean values of all the dose deposited from top to bottom in the water walls. This made it easy to see where potential hot spots might be. The most interesting areas, for example high dose somewhere outside the entrance door, was investigated more thoroughly. An analyse of the position of the voxel of maximum dose in the water walls was also investigated further. The corresponding uncertainty was calculated by eq. (39) and aided to better understand if the dose in a voxel was high because of some rare large dose deposition (high CV), or seemed to be a result from likely events (low CV).
3.3 Hardware & software

A total of six computers were used simultaneously for the simulations, they had the following specifications:

- One computer with Windows 10, AMD Ryzen 5 1600 hexa-core 3.2 GHz CPU, SSD-HDD and 32 GB RAM 2133 MHz
- Two computers with Ubuntu 16.04.02, Intel Xeon E5-1620 v3 quad-core 3.5 GHz CPU, SSD-HDD and 32 GB RAM 2133 MHz
- Two computers with Windows 7, Intel Core i7-6700 quad-core 3.4 GHz CPU, SSD-HDD and 24 GB RAM 2133 MHz
- One computer with Windows 10, Intel Core i7-6700 quad-core 3.4 GHz CPU, SSD-HDD and 24 GB RAM 2133 MHz

The simulation software, GATE, needed to run on Linux-based systems. Therefore, a so called virtual box that makes it possible to run a virtual computer as a usual program, was used on the Windows-based systems. More specifically Oracle VM\textsuperscript{17} was the virtual box program of choice here. It was used together with vGATE 8.0\textsuperscript{18}, which is a mountable virtual computer running Linux, that have the GATE-software pre-installed.

Fun fact: in this project, simulations have been performed totalling at least 31453 core hours from 4389 number of individual simulations and a total of $4.5830e11$ primary photons!

\textsuperscript{17}https://www.virtualbox.org/
\textsuperscript{18}http://www.opengatecollaboration.org/vGATE80
4 Results

A comparison between simulated and measured data is shown. The found spectra estimated from measurements including the off-axis is presented as well as the spectra found when only including the centre depth dose. The leakage spectrum used for the simulation of the linear accelerator in the bunker is shown, followed by the resulting dose distributions outside the bunker. Both with and without extra radiation protection, consisting of lead bricks and some high density concrete bricks. The result from the dose reference simulation, for the normalization of the simulated dose distribution, is shown as well. Finally, a more thorough analyse of the most interesting areas is performed.

4.1 Spectra determination

4.1.1 Simulations

The centre depth doses in water from the generated pencil beams from all the 32 monoenergies are shown in fig. 4.1, normalized to the dose maximum of the highest energy. Every monoenergetic pencil was generated from five individual simulations of 100e6 photons each. Thus, the total number of photons simulated for the 4D energy and dose distribution table was $16 \times 10^9$, and took about 623 core hours. Some of the energies can also be seen in an earlier figure, used to describe the build-up in the theory section 2.3, fig. 2.1.

![Figure 4.1](image_url)

Figure 4.1 – The centre depth doses from 32 monoenergetic $5 \times 5 \text{cm}^2$ beams generated from the simulations of $1 \times 1 \text{mm}^2$ pencils, together with a zero energy of zero dose.

The time simulated on a single core of a processor.
4.1.2 Measurements

A comparison between the measured depth dose and a simulated one is shown in fig. 4.2. The simulated pencil beam was based on the theoretical spectrum of a Varian 6 MV accelerator, the same brand as the machine the measurements were performed on, and was put together to a $5 \times 5 \text{cm}^2$ beam. The result from the profile dose measurement is shown in fig. 4.3 together with the same dose profile from the simulated dose distribution.

![Figure 4.2](image.png)

**Figure 4.2** – The depth dose from a simulated single $1 \times 1 \text{mm}^2$ beam plotted together with the center depth dose from a $5 \times 5 \text{cm}^2$ beam generated from the single beam, and a measured centre depth dose of the same field size. All normalized to 100 mm depth.
4.1.3 Calculating the spectra

Since the simulated 4D table comes from straight pencils, the simulated beams lack the cone shaped spread, i.e. they are straight non-diverging beams. Thus, to match the simulated table data, the measured data needs to be corrected to as if it was measured from such a straight beam without the spread. This was done by the inverse square law and re-calculating the coordinates, as described in section 2.5.2. The locations of the re-calculated points are visualised as red dots in fig. 3.3. All the measurement points were put together in a single vector \( \mathbf{b} \), and data for \( \mathbf{A} \) interpolated for the desired energies for which the spectrum is going to be found, and then interpolated for the actual coordinates of the measured points in \( \mathbf{b} \), from the 4D energy and dose distribution table, that was found in section 4.1.1.

The spectrum, \( \mathbf{c} \), could then be found simply by the use of \( \mathbf{A} \) and \( \mathbf{b} \) in eq. (17). However, the solution can falsely give negative values of the spectrum, which of course makes no sense, since there cannot be a negative number of photons. Hence, the function "lsqnonneg" in Matlab, an optimization tool, was used to minimize the problem in eq. (16). This optimizer has a constraint such that the solution is non-negative, that is, \( \mathbf{c} \geq \mathbf{0} \). But, for proper values of \( \alpha \) the resulting spectra was all positive or near positive, see fig. A.2 in appendix A.1. The resulting L-curve was produced as described in section 2.4.1 and is shown in fig. 4.4. After experimenting with different regularization parameters for different input data, it seems like values of \( \alpha \) just before the "corner" in the L-curve
are the most appropriate, i.e. they produce the best results.

**Figure 4.4** – The L-curve from 20 different $\alpha$, ranging from 0.1 to 100, equally spaced on a logarithmic scale. The tickets represent $\alpha$, not all $\alpha$ are presented with numbers.

A spectrum was estimated for the energies starting from 0.25 with 0.25 incremental steps up to 6 MeV to make a good comparison with the theoretical spectrum for the exact same energies, and is shown in fig. 4.5 for three different values of the regularization parameter $\alpha$. The value of $\alpha$ that produced the best looking spectrum was found to be $\alpha = 2.6$, and was thus chosen to be the proper regularization parameter. The weighting constants were set to $w_1 = 1000$ and $w_2 = 10$, after experimenting with different values. The large value of $w_1$ is justified because it corresponds to a zero-energy. Also, the high value does not affect the higher energies in the spectra much at all. The estimated dose, that is, $A \cdot c$, is shown together with the measured points, $b$, in fig. 4.6.
Figure 4.5 – The estimated spectra for three different values on the regularization parameter $\alpha$, together with the theoretical Varian 6 MV spectrum.

Figure 4.6 – The measured dose shown together with the resulting estimated dose for $\alpha = 2.6$. The measurement points are divided into three sections corresponding to three different lines of re-calculated points in fig. 3.3. Where section 1 is from the central depth dose and section 2 and 3 from the off-axis measurements of 30 mm and 50 mm, respectively.
Using the ordinary centre depth dose only, gives the spectra in fig. 4.7, for the same regularization parameters as in fig. 4.5. The resulting depth dose, for $\alpha = 2.6$, is shown in fig. 4.8. The corresponding L-curve can be found in appendix A.1, fig. A.1. The resulting spectra got even worse for smaller $\alpha$.

**Figure 4.7** – The estimated spectra for three different values on the regularization parameter $\alpha$, together with the theoretical Varian 6 MV spectrum, here using only the ordinary centre depth dose as input, neglecting any off-axis measurements.
Another spectrum was also generated for a wider range of energies, namely energies starting from 0.25 MeV with 0.25 incremental steps up to 7 MeV, followed by 7.5 MeV and 8 MeV, shown in fig. 4.9. The wider range of energies better justify a large weighting constant $w_2$, since the energy goes beyond what the accelerator should produce. The estimated spectrum for $\alpha = 2.6$ is the spectrum that was used as the leakage spectrum in the following simulations of the bunker, resulting in the dose distributions found in section 4.2.
Figure 4.9 – The final estimated spectrum for $\alpha = 2.6$, shown together with the theoretical Varian 6 MV spectrum (on which the measurements were performed) and for Siemens 6 MV (brand and energy of the linac that is going to be used in the intended bunker). The data of this graph can be found in appendix A.2, table A.1.
4.2 Reinforced radiation protection

4.2.1 Dose normalization

The dose normalization simulation, described in section 3.2.3, took about 158 core hours and resulted in the reference dose $D_{MC,\text{ref}} \approx 2.6 \times 10^{-5}$ Gy as can be seen from the mean ROI value in fig. 4.10a. This was a result from 100 individual simulations totalling $N_{MC} = 10 \times 10^9$ photons. Thus, from eq. (31), the number of photons that correspond to an 100 MU run of the accelerator (1 Gy in the reference conditions mentioned in section 2.6.2), resulted in $N_{100\text{MU}} \approx 3.8 \times 10^{14}$. The CV seen in fig. 4.10b, calculated from eq. (39), indicates a typical uncertainty of about 0.5% for the same ROI as the reference dose.

![Figure 4.10](image1)

**Figure 4.10** – Slices at 10 cm depth in a $50 \times 50 \times 50$ cm$^3$ water phantom, showing the dose in (a) together with corresponding CV in (b). The slice is 1 mm thick. A ROI is shown in purple and the ROI statistics are shown as well.

4.2.2 Simulations

The simulated dose distributions in the water walls were transformed to heat maps showing the mean dose, section 3.2.4, and are shown together with CAD images of the bunker. Some slices of extra interest are shown as well. Case Right is presented first, both without and with the extra radiation protection, in two different angles in fig. 4.11 - 4.14. The same thing for case Back in fig. 4.15 - 4.18. All the heat maps are normalized by eq. (34), with $\bar{\text{MU}}_{\text{acc}} = 300 \cdot 60 = 18000$ MU/h. Meaning that the dose maps show the resulting dose rate from running the accelerator at a constant dose rate of 300 MU/min, which was the maximum dose rate of the accelerator when it was in clinical use. Observe the logarithmic colour scale, where the black corresponds to doses low enough to run the accelerator around the clock, because the total dose during a one year period would still be less than the limit of 0.1 mGy [20]. The amount of simulated photons, simulation
times and the number of simulations can be found in table B.1 in appendix B.3.

**Case Right** Heat maps for the mean dose outside the bunker for case Right, together with absolute dose shown in slices of higher interest, see fig. 4.11 - 4.14.

**Figure 4.11** – Case Right without the extra radiation protection.

**Figure 4.12** – Case Right with the extra radiation protection.
**Figure 4.13** – Case Right without the extra radiation protection.

**Figure 4.14** – Case Right with the extra radiation protection.
Case Back  Heat maps for the mean dose outside the bunker for case Back, together with absolute dose shown in slices of higher interest, see fig. 4.15 - 4.18.

Figure 4.15 – Case Back without the extra radiation protection.

Figure 4.16 – Case Back with the extra radiation protection.
Figure 4.17 – Case Back without the extra radiation protection.

Figure 4.18 – Case Back with the extra radiation protection.
4.2.3 Analysis of the resulting dose

An analysis of an important slice outside the entrance door for both cases, followed by analysis of the dose in the slice covering the roof directly outside of the bunker, where there is a public outdoor-area. The axes show the length of the sides, from the centre of the slices. The maximum dose in any voxel for each water wall is shown in table 1. The dose rate is normalized with eq. (34), with $\dot{\text{MU}}_{\text{acc.}} = 18000 \text{ MU/h}$, i.e. in the same way as was done in section 4.2.2.

Table 1 – The maximum voxel value of the dose, shown in Gy, from each of the six water walls for both case Right and case Back. Each value is also presented with its CV. All values rounded to three significant digits. Normalized to a 100 MU run by eq. (33).

<table>
<thead>
<tr>
<th>Case</th>
<th>Front wall</th>
<th>Left wall</th>
<th>Right wall</th>
<th>Back wall</th>
<th>Top wall</th>
<th>Bottom wall</th>
</tr>
</thead>
<tbody>
<tr>
<td>Right</td>
<td>$7.59 \times 10^{-8}$</td>
<td>$1.98 \times 10^{-9}$</td>
<td>$1.08 \times 10^{-3}$</td>
<td>$1.18 \times 10^{-6}$</td>
<td>$3.03 \times 10^{-7}$</td>
<td>$1.42 \times 10^{-4}$</td>
</tr>
<tr>
<td>CV</td>
<td>$6.82 \times 10^{-2}$</td>
<td>$7.41 \times 10^{-1}$</td>
<td>$3.80 \times 10^{-3}$</td>
<td>$1.57 \times 10^{-2}$</td>
<td>$2.90 \times 10^{-2}$</td>
<td>$1.70 \times 10^{-3}$</td>
</tr>
<tr>
<td>Back</td>
<td>$6.22 \times 10^{-8}$</td>
<td>$8.66 \times 10^{-9}$</td>
<td>$4.42 \times 10^{-6}$</td>
<td>$4.86 \times 10^{-4}$</td>
<td>$3.13 \times 10^{-7}$</td>
<td>$1.41 \times 10^{-4}$</td>
</tr>
<tr>
<td>CV</td>
<td>$9.37 \times 10^{-1}$</td>
<td>$2.93 \times 10^{-1}$</td>
<td>$1.23 \times 10^{-2}$</td>
<td>$1.07 \times 10^{-2}$</td>
<td>$4.02 \times 10^{-2}$</td>
<td>$2.40 \times 10^{-3}$</td>
</tr>
</tbody>
</table>

Table 2 shows the maximum time the accelerator can be run at 300 MU/min before reaching the annual dose limit of 0.1 mSv\(^{20}\) [20]. This is calculated simply by how many hours of the maximum dose rate that would result in 0.1 mSv. The table also shows the maximum amount of hours with the higher annual dose limit of 6 mSv, that applies to a controlled area for authorised category A or B personnel only [20]. The "top wall" is a water wall directly above the roof, where the public outdoor-area is.

Table 2 – The maximum amount of hours to run the accelerator at 300 MU/min per one year. The less strict dose limit for authorised personnel is shown in parentheses.

<table>
<thead>
<tr>
<th>Case</th>
<th>Front wall</th>
<th>$\dot{D}_{\text{max}}^{\text{Front}}$ [Gy h(^{-1})]</th>
<th>Top wall</th>
<th>$\dot{D}_{\text{max}}^{\text{Top}}$ [Gy h(^{-1})]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Right shielded</td>
<td>7.3 (44)</td>
<td>$1.37 \times 10^{-5}$</td>
<td>1.8</td>
<td>$5.46 \times 10^{-5}$</td>
</tr>
<tr>
<td>Right unshielded</td>
<td>1.0 (6.1)</td>
<td>$9.87 \times 10^{-5}$</td>
<td>1.8</td>
<td>$5.56 \times 10^{-5}$</td>
</tr>
<tr>
<td>Back shielded</td>
<td>9 (54)</td>
<td>$1.12 \times 10^{-5}$</td>
<td>1.8</td>
<td>$5.63 \times 10^{-5}$</td>
</tr>
<tr>
<td>Back unshielded</td>
<td>0.5 (3.3)</td>
<td>$1.82 \times 10^{-4}$</td>
<td>1.7</td>
<td>$5.77 \times 10^{-5}$</td>
</tr>
</tbody>
</table>

\(^{20}\)Sv is short for Sievert and is the special unit for equivalent dose and effective dose. It is basically the absorbed dose corrected by factors that has to do with how harmful the radiation type is. For photons the conversion factor to the SI unit of the absorbed dose is just unity, meaning that, $1 \text{ Sv} = 1 \text{ Gy}$ [11].
Outside the entrance door  A more thorough analysis of an important slice outside the entrance door is shown, including the dose and its CV. First shown for case Right in fig. 4.19 and then case Back in fig. 4.20, both with the reinforced shielding. The positions of the slices can be seen in fig. 4.12 and 4.16, for case Right and Back, respectively. The entrance door is located to the right in fig. 4.19 and 4.20, and extends from 130 cm above the lower edge of the slice and is 231 cm high. For case Back, a slice other than the slice containing the maximum voxel value is chosen because of a rare event, see appendix B.1.

Figure 4.19 – Slices showing dose in (a) and corresponding CV in (b) for case Right.

Figure 4.20 – Slices showing dose in (a) and corresponding CV in (b) for case Back.
Outside the roof  A presentation of a slice directly above the roof, where there is a public outdoor-area. However, the simulation only includes the concrete roof. In real life there is a bit of gravel, soil and concrete tiles before reaching the people that might be there. Shown first is case Right in fig. 4.21 and then case Back in fig. 4.22, both with the reinforced shielding.

![Figure 4.21](image1)

**Figure 4.21** – Slices showing dose in (a) and corresponding CV in (b) for case Right.

![Figure 4.22](image2)

**Figure 4.22** – Slices showing dose in (a) and corresponding CV in (b) for case Back.
5 Discussion and future work

The main purpose of this work was to find a reinforced radiation protection for a bunker, such that the radiation dose outside the bunker would fall within acceptable levels when operating a linear accelerator in there. This would allow for the accelerator to be installed, and used for education. A brilliant opportunity for the department!

5.1 Spectra determination

The first obstacle was to find the energy spectra of the accelerator, which was needed to properly simulate the accelerator. It was harder than expected and turned out to become a fairly large part of this work, resulting in a new method for spectra determination, a method with great potential. However, this method was not the main objective, so it was not polished and could not be used to find a good leakage spectra. That is why the leakage spectrum was simulated as having the same energy distribution as the spectrum estimated from measurements of a primary beam.

The method could be improved further for determination of the primary beam spectrum by simulating the entire radiotherapy head, including the collimators, flattening filter, etc. This would include the contribution to the depth dose from so called head scatter [9] and other factors that may affect the measurements. It would however take a long time to simulate because an enormous amount of photons would be required since it is not possible to use a pencil beam in that kind of simulation. But, there would be other benefits as well, so it might be worth the effort. One would namely get around the straight down approximation and inverse square law, section 2.5. Also, the scattered radiation in the phantom would probably be more accurately simulated because it has a certain range in the phantom. Meaning that for a real life beam that diverges, the scattered radiation from a beam line away from the beam centre axis would have an increasing distance to the beam centre axis with depth in the phantom (see the illustration in fig. 2.5 in section 2.5.2). This would imply that the dose from the pencil beam, that is non-diverging, would overestimate the contribution from scattered radiation at large depths for the centre depth dose. However, the opposite is seen in fig. 4.2. It can be due to the fact that head scatter, collimator leakage etc. are not included in the simulations, which would all increase the centre depth dose at larger depths [9]. Another reason could be that a theoretical spectrum was used to simulate the pencil in fig. 4.2, a spectrum that might overestimate the low energy photons, which would also explain why the depth dose deviates a bit from the measured.

A suggestion for the improvement of determination of the leakage spectrum would be to simulate the exact dimensions and compositions of the measuring phantom, including the walls of the tank, and then simulate the entire room for different monoenergies. In this way all scattered radiation in the room would be able to reach the inside of the water phantom in the same way as in real life. This would, in theory, create an accurate energy table from which the spectrum, of whatever energies that radiates from the accelerator,
could be found. But, this would require a huge amount of photons, though it is still not impossible to do.

The dose profile at 100 mm depth from the measurement and the simulated pencil beam agrees well with each other in fig. 4.3. The slight curvature of the dose profile inside the 5 cm wide beam seems to coincide really well. The measured and simulated profiles deviate a bit at the edges, and should thus not be used for finding the spectra. It would also be hard to precisely follow the edge of the beam in such a measurement. The dose outside the beam seems to have the same shape but it still differs a bit in amplitude, which might be due to the detector energy respond. The detector is calibrated in the primary beam and may response falsely for the lower energies of the scattered radiation. The dose profile together with the comparison of depth doses, validates the pencil beam method used, where the narrow pencil beams were simply added together side-by-side.

The resulting estimated spectra seems to benefit from the inclusion of some off axis measurements, as can be seen comparing fig. 4.5 and 4.7, which demonstrates the potential of the novel method. The method is also practical to use since all measurement points can be obtained from a single phantom in a stationary position. If one has access to the same, or similar, phantom as used in section 3.1.2 it gets even easier. With this setup all the measurement points can be predefined and is then fast to measure from a single, automatic, measurement protocol.

Unfortunately the L-curve does not present an obvious choice of the regularization parameter $\alpha$. But it is still a great help to find proper values of $\alpha$ to choose from. Then one can compare different spectra from a few different $\alpha$ and choose the most likely by inspection, as long as one keeps an eye on the estimated and measured data points, for example fig. 4.6, so they do not start to deviate too much.

5.2 Reinforced radiation protection

Monte Carlo-simulations is a great tool to identify eventual hot spots in designing a bunker for a linear accelerator. This may enable a design that does not need as thick barriers as recommended by analytic methods, for example the method in a report by NCRP [17]. Because one can focus on the hot areas of high dose rather than using simplified equations that force the use of extra thick barriers, to ensure that it weighs up the uncertainty from the simplifications. Also, simulations are perfect for the curious mind, since one can choose to explore things with a large freedom, even things practically impossible in real life. Such as the simulation of individual monoenergies from a linear accelerator, and see how they affect the radiation dose outside the bunker. Either from the dose distribution of each energy, or by combining them to an arbitrary spectra. In this work, a lot of simulations that are not a part of the end results have been performed. A bit of trial-and-error was used to obtain the end results. This is also one of the strong aspects of simulations - design what you want to try and let the computer do all the work. If you are not satisfied with the result, optimize by making changes and re-run.
The dose normalization in section 4.2.1 showed that as many as roughly $3.8 \times 10^{14}$ photons are needed for 1 Gy under the reference conditions, or in other words, for 100 MU. This is a big number, and would take a little over 700 years to simulate on a typical quad-core processor. But fortunately one can often manage with less photons, and then scale the data afterwards. It is the statistics that matters. In radiation protection one often speak of tens of percent in uncertainty. Hence, the uncertainties of only a couple of percent, seen from the simulations in section 4.2.3, are more than acceptable. Keep in mind that only a tiny fraction of 100 MU is simulated (see table B.1 in appendix B.3), and still such strong statistics can be found even outside the concrete walls of the bunker!

A surprising and important finding was the high dose to the public outdoor-area above the bunker roof. This limits the run time of the accelerator to only 1.8 hours per year! But, for instance, if a thorn bush or a large statue is placed such that it covers an area about 3 m in radius (see fig. 4.21 and 4.22 for the dose maps), the limitation instead becomes the dose to the intended control room, which is right outside the bunker’s entrance door.

The leakage was simulated as a worst case scenario with the spectrum of a primary beam and the intensity set to the maximum specified dose of 0.1% [19]. The leakage spectra is probably less intense and of lower energies in at least some of the angles [12]. In future work, more knowledge of the leak spectra should be implemented. This would probably lower the dose outside the bunker, allowing for more run hours of the accelerator.

Above the roof to the right of the bunker, there is a PET-MRI facility. The first part of the room that is hit by the radiation from intended accelerator is a technical room where there usually are no people. Also, the doses are probably low, as can be seen in the upper 1 m in fig. B.2a and B.4a in appendix B.2. The last meter is the portion of the water wall that sticks up beyond the roof of the bunker, which is located right beneath the ground level, on which the PET-MRI facility stands on. There is also concrete walls and other material before reaching the inside of the room. Thus, the dose inside the PET-MRI room is probably low enough to be neglected. If case Right is chosen and one wants to further decrease the dose to the PET-MRI, one can simply reduce the maximum field size, which is larger than a $40 \times 40$ cm$^2$ field at a distance of 100 cm from the source in the simulations.

The simulation model used should be validated in future work. An attempt using TLD was performed during this work, only to realise that the build-up is too large and the spectra in the room varies quite a lot, so no conclusion could be made. The difference in the energy spectra makes the build-up different and thus unreliable data was measured by the small TLD, even though they have a good energy response [16]. A suggestion of

21A combined Positron Emission Tomography scanner and a Magnetic Resonance Imaging camera.
22Thermoluminescent dosimeter, in other words, a dose measuring device.
how to validate the model would be with the use of TLD inside PMMA-cubes of thickness 5 cm. In this way one would probably avoid build-up problems and the same PMMA-cubes can be simulated in a couple of measurement locations in the same simulated room. Then one can simply compare the absolute doses and see if they agree.

5.2.1 Conclusion

The most important findings from this work can be found in table 2. The table says that for authorised personnel of category A or B, the accelerator is allowed to be run a maximum of 44 or 54 hours\(^{23}\) per year for case Right and Back, respectively. That is quite an improvement from 6.1 and 3.3 hours, without the reinforced radiation protection.

However, the dose rate to the control room is still quite high, 13.7 µGy h\(^{-1}\) for case Right, or 11.2 µGy h\(^{-1}\) for case Back. As a comparison, the average dose received by someone living in Sweden is 2.4 mSv per year \([2]\) (including the average contribution of 0.9 mSv per year from medical diagnostics), which translates to 0.27 µGy h\(^{-1}\) for photons. Thus, further measures of reinforced radiation protection are recommended. One such measure to consider is to remotely control the accelerator, which removes the need for someone to be in the control room located just outside the entrance door.

From a radiation protection point of view, case Back is the preferred case, just as suspected in section 3.2.1. On the other hand, case Right is not much worse and might be the better choice in practice, since the accelerator better fits the layout in the room, and is easier to install in that location. Another advantage is that it does not radiate directly at the back wall, behind which there is going to be a corridor built. However, there is going to be a bit of ground between the bunker and the corridor, that is probably enough protection even for case Back. But there is tough regularization on the dose limit (0.1 mSv per year) since the corridor is a public place where people may be.

My personal suggestion would be to plant a thorn bush in the public outdoor-area above the roof, centered at the source position, and to use case Right, with the alteration to move the source a bit closer to the right wall. It would not hurt to make the concrete wall (that extends out in the room between the accelerator and the entrance door) even longer, and maybe thicker, which should neither be too expensive, nor alter the room layout too much. This might be enough to lower the dose in the control room to more reasonable levels, which should make the installation and use of the accelerator feasible.

\(^{23}\)Neglecting the dose to the public outdoor-area above the roof because of the workarounds, using a thorn bush or statue, as mentioned earlier. This brings the focus to the control room outside the entrance door instead.
References


Appendix A  Spectra determination

A.1  Figures

Figure A.1 – The L-curve from 20 different $\alpha$, ranging from 0.1 to 100, equally spaced on a logarithmic scale. The tickets represent $\alpha$, not all $\alpha$ are presented with numbers. This L-curve is from the estimation of the spectra using the depth dose only, see fig. 4.7.
Figure A.2 – The resulting spectra from the same input data as for fig. 4.5 in section 4.1.3, using the solution from eq. (17) instead of the optimization tool, "lsqnonneg". For \( \alpha = 2.6 \) the spectrum is all positive.
### A.2 Data

Table A.1 – Varian 6 MV and Siemens 6MV spectra from the work by Sheikh-Bagheri et al. [18] and Result is the estimated spectrum from the result of measuring on a Varian Clinac iX 6 MV linear accelerator (and is also the spectrum used as the leakage in the simulations).

<table>
<thead>
<tr>
<th>E [MeV]</th>
<th>Siemens 6 MV</th>
<th>Varian 6 MV</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>2.45E-02</td>
<td>2.05E-02</td>
<td>3.95E-02</td>
</tr>
<tr>
<td>0.5</td>
<td>1.24E-01</td>
<td>1.21E-01</td>
<td>6.97E-02</td>
</tr>
<tr>
<td>0.75</td>
<td>1.22E-01</td>
<td>1.26E-01</td>
<td>8.76E-02</td>
</tr>
<tr>
<td>1</td>
<td>1.02E-01</td>
<td>1.09E-01</td>
<td>9.51E-02</td>
</tr>
<tr>
<td>1.25</td>
<td>8.65E-02</td>
<td>9.36E-02</td>
<td>9.48E-02</td>
</tr>
<tr>
<td>1.5</td>
<td>7.37E-02</td>
<td>8.02E-02</td>
<td>8.93E-02</td>
</tr>
<tr>
<td>1.75</td>
<td>6.33E-02</td>
<td>6.95E-02</td>
<td>8.06E-02</td>
</tr>
<tr>
<td>2</td>
<td>5.49E-02</td>
<td>5.98E-02</td>
<td>7.04E-02</td>
</tr>
<tr>
<td>2.25</td>
<td>4.73E-02</td>
<td>5.13E-02</td>
<td>6.03E-02</td>
</tr>
<tr>
<td>2.5</td>
<td>4.13E-02</td>
<td>4.40E-02</td>
<td>5.10E-02</td>
</tr>
<tr>
<td>2.75</td>
<td>3.62E-02</td>
<td>3.79E-02</td>
<td>4.29E-02</td>
</tr>
<tr>
<td>3</td>
<td>3.19E-02</td>
<td>3.33E-02</td>
<td>3.62E-02</td>
</tr>
<tr>
<td>3.25</td>
<td>2.79E-02</td>
<td>2.86E-02</td>
<td>3.07E-02</td>
</tr>
<tr>
<td>3.5</td>
<td>2.50E-02</td>
<td>2.50E-02</td>
<td>2.64E-02</td>
</tr>
<tr>
<td>3.75</td>
<td>2.21E-02</td>
<td>2.16E-02</td>
<td>2.29E-02</td>
</tr>
<tr>
<td>4</td>
<td>1.95E-02</td>
<td>1.83E-02</td>
<td>1.99E-02</td>
</tr>
<tr>
<td>4.25</td>
<td>1.71E-02</td>
<td>1.59E-02</td>
<td>1.74E-02</td>
</tr>
<tr>
<td>4.5</td>
<td>1.52E-02</td>
<td>1.32E-02</td>
<td>1.51E-02</td>
</tr>
<tr>
<td>4.75</td>
<td>1.32E-02</td>
<td>1.09E-02</td>
<td>1.29E-02</td>
</tr>
<tr>
<td>5</td>
<td>1.16E-02</td>
<td>8.67E-03</td>
<td>1.08E-02</td>
</tr>
<tr>
<td>5.25</td>
<td>9.93E-03</td>
<td>6.28E-03</td>
<td>8.68E-03</td>
</tr>
<tr>
<td>5.5</td>
<td>8.48E-03</td>
<td>3.92E-03</td>
<td>6.69E-03</td>
</tr>
<tr>
<td>5.75</td>
<td>6.97E-03</td>
<td>1.34E-03</td>
<td>4.84E-03</td>
</tr>
<tr>
<td>6</td>
<td>5.60E-03</td>
<td>4.16E-05</td>
<td>3.19E-03</td>
</tr>
<tr>
<td>6.25</td>
<td>4.12E-03</td>
<td>-</td>
<td>1.88E-03</td>
</tr>
<tr>
<td>6.5</td>
<td>2.73E-03</td>
<td>-</td>
<td>9.10E-04</td>
</tr>
<tr>
<td>6.75</td>
<td>1.54E-03</td>
<td>-</td>
<td>3.09E-04</td>
</tr>
<tr>
<td>7</td>
<td>7.55E-04</td>
<td>-</td>
<td>7.28E-05</td>
</tr>
<tr>
<td>7.25</td>
<td>3.09E-04</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>7.5</td>
<td>8.30E-05</td>
<td>-</td>
<td>0.00E+00</td>
</tr>
<tr>
<td>7.75</td>
<td>1.70E-05</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>8</td>
<td>2.47E-06</td>
<td>-</td>
<td>0.00E+00</td>
</tr>
</tbody>
</table>
Appendix B  Reinforced radiation protection

B.1 Outlier

Case Back (shielded) had a deviating dose value in a voxel from some rare event that did not seem to fit in with the rest of the data in the slice from the front water wall, fig. B.1. The slice was located about 1 m further to the right than the slice outside the entrance door, shown in fig. 4.16, which had the maximum dose when neglecting this single deviating voxel value of $1.12 \times 10^{-5}$ Gy h$^{-1}$ (CV = 0.937).

![Diagram](image.png)

**Figure B.1** – Outside the front wall for case Back (shielded). Slices showing dose in (a), and corresponding CV in (b).
B.2 Dose back and right

The dose in the slices of interest in right and back water walls for case Right and Back, with the reinforced radiation protection. Corresponding CV is shown as well.

B.2.1 Case Right

The dose in the slices of interest from the right and the back water walls in fig. 4.12 and 4.14 are shown in fig. B.2 and B.3, respectively.

**Figure B.2** – Outside the right wall for case Right (shielded). Slices showing dose in (a), and corresponding CV in (b).

**Figure B.3** – Outside the back wall for case Right (shielded). Slices showing dose in (a), and corresponding CV in (b).
B.2.2 Case Back

The dose in the slices of interest from the right and the back water walls in fig. 4.16 and 4.18 are shown in fig. B.4 and B.5, respectively.

![Figure B.4](image1)

Figure B.4 – Outside the right wall for case Back (shielded). Slices showing dose in (a), and corresponding CV in (b).

![Figure B.5](image2)

Figure B.5 – Outside the back wall for case Back (shielded). Slices showing dose in (a), and corresponding CV in (b).
### B.3 Simulation amount

**Table B.1** – Information on the simulations of the bunker. Prim for primary photons, Leak for leakage photons and Both is their sum.

<table>
<thead>
<tr>
<th>Simulation</th>
<th>Photons</th>
<th>Core hours</th>
<th>No. of sim.</th>
</tr>
</thead>
<tbody>
<tr>
<td>CaseRight__Prim</td>
<td>$1.00 \times 10^{10}$</td>
<td>488</td>
<td>100</td>
</tr>
<tr>
<td>CaseRight__Leak</td>
<td>$6.00 \times 10^{10}$</td>
<td>$4.11 \times 10^3$</td>
<td>600</td>
</tr>
<tr>
<td>CaseRight__Both</td>
<td>$7.00 \times 10^{10}$</td>
<td>$4.60 \times 10^3$</td>
<td>700</td>
</tr>
<tr>
<td>CaseRight__Shield__Prim</td>
<td>$3.00 \times 10^{10}$</td>
<td>$1.37 \times 10^3$</td>
<td>300</td>
</tr>
<tr>
<td>CaseRight__Shield__Leak</td>
<td>$6.00 \times 10^{10}$</td>
<td>$3.37 \times 10^3$</td>
<td>600</td>
</tr>
<tr>
<td>CaseRight__Shield__Both</td>
<td>$9.00 \times 10^{10}$</td>
<td>$4.74 \times 10^3$</td>
<td>900</td>
</tr>
<tr>
<td>CaseBack__Prim</td>
<td>$5.00 \times 10^9$</td>
<td>375</td>
<td>50</td>
</tr>
<tr>
<td>CaseBack__Leak</td>
<td>$3.00 \times 10^{10}$</td>
<td>$1.83 \times 10^3$</td>
<td>300</td>
</tr>
<tr>
<td>CaseBack__Both</td>
<td>$3.50 \times 10^{10}$</td>
<td>$2.20 \times 10^3$</td>
<td>350</td>
</tr>
<tr>
<td>CaseBack__Shield__Prim</td>
<td>$1.00 \times 10^{10}$</td>
<td>631</td>
<td>100</td>
</tr>
<tr>
<td>CaseBack__Shield__Leak</td>
<td>$3.00 \times 10^{10}$</td>
<td>$2.47 \times 10^3$</td>
<td>300</td>
</tr>
<tr>
<td>CaseBack__Shield__Both</td>
<td>$4.00 \times 10^{10}$</td>
<td>$3.11 \times 10^3$</td>
<td>400</td>
</tr>
</tbody>
</table>