This thesis contributes to research and practice within the field of special education in mathematics with more knowledge about, and an understanding of, students’ meaning(s) of inclusion in mathematics. The results show that research studies on inclusion in mathematics education use the term inclusion to reflect both an ideology and a way of teaching, although these two uses are most often treated separately and independently of each other. The results also show how students’ meaning(s) of inclusion can be described by three overarching discourses: the Discourse of the mathematics classroom setting, the Discourse of assessment, and that of accessibility in mathematics education. Within these Discourses, smaller discourses make issues of meanings of inclusion for the students visible in terms of testing, grades, tasks, the importance of the teacher, being valued or not valued, the like or dislike of mathematics, the classroom organisation, teaching approaches and being in a small group.
The meaning(s) of inclusion in mathematics in student talk

Inclusion as a topic when students talk about learning and teaching in mathematics
THE MEANING(S) OF INCLUSION
IN MATHEMATICS IN STUDENT TALK

Inclusion as a topic when students talk about learning and teaching in mathematics

HELENA ROOS

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Abstract


This thesis contributes to research and practice within the field of special education in mathematics with more knowledge about, and an understanding of, students’ meaning(s) of inclusion in mathematics education. Three research questions guide the study: What meaning(s) is/are ascribed, and how is inclusion used, in mathematics education research? What meaning(s) do the students ascribe to inclusion in mathematics learning and teaching? And what frames students’ meaning(s) of inclusion in mathematics learning and teaching?

The first part of this study began with a systematic literature review on the notion of inclusion in mathematics education research, and the search resulted in 1,296 research studies. Of these, 76 studies were retained after the criteria for time span and peer-reviewed research were applied and 19 duplicates had been removed.

The second part of the study involves a case study of three students and their meaning(s) of inclusion in mathematics education. The selected school was a lower secondary school in an urban area of Sweden. The school had set out to work inclusively, meaning their aims were to include all students in the ordinary classroom teaching in every subject and to incorporate special education into the ordinary teaching with no fixed special education groups. Three students were chosen for this part of the study: one in Grade 7 and two in Grade 8. Edward, one of the students in Grade 8, was chosen because he was thought to be a student in access to mathematics education. The other two students were chosen because they were thought to be struggling to gain access to mathematics education: Veronica in Grade 7 and Ronaldo in Grade 8 (the same class as Edward).

In this study, the object of the study is the meaning(s) of inclusion in student talk. This study is an instrumental and collective case (Stake, 1995), as it involves several students’ meaning(s) aimed at developing a more general understanding of inclusion in mathematics education. The case is also an information-rich case (Patton, 2002), with contributions from students in mathematics education at an inclusive school. Applying Flyvbjerg’s (2006; 2011) notions, one can also call this kind of selection “information-oriented”, and the case is an extreme one – a choice made in order to get “a best case scenario”. An extreme case is a case used to “obtain information on unusual cases
which can be especially problematic or especially good in a more closely defined sense” (Flyvbjerg, 2011, p. 307).

The data in this study consists of both observations and interviews conducted during the spring semester 2016. The observations took place in a Grade 7 and Grade 8 classroom at the same school where the interviewed students were enrolled. At least one mathematics lesson each month for each class was observed, and student interviews followed each observation. The observations were used to provide a context for the interviews and to support the analysis.

In this study, discourse analysis (DA) as described by Gee (2014a; 2014b) was chosen as both the theoretical frame and as an analytical tool because of its explanatory view on discourse, with description foregrounded. With the help of DA, this study describes both the meaning(s) and the use of the notion of inclusion in mathematics education research. It also describes students’ meaning(s) of inclusion in mathematics education as well as framing issues in student talk of inclusion in mathematics education.

From Gee’s point of view, DA encompasses all forms of interaction, both spoken and written, and he provides a toolkit for analysing such interaction by posing questions to the text. Gee distinguishes two theoretical notions, big and small discourses, henceforth referred to as Discourse (D) and discourse (d). Discourse represents a wider context, both social and political, and is constructed upon ways of saying, doing, and being: “If you put language, action, interaction, values, beliefs, symbols, objects, tools, and places together in such a way that other recognize you as a particular type of who (identity) engaged in a particular type of what (activity), here and now, then you have pulled of a Discourse” (Gee, 2014 a, p. 52, Gee’s italics). When looking at discourse (with a small d), it focuses on language in use – the “stretches of language” we can see in the conversations we investigate (Gee, 2014a, 2014b), meaning the relations between words and sentences and how these relations visualize the themes within the conversations. These small discourses can inform on how the language is used, what typical words and themes are visible, and how the speakers or writers design the language. According to Gee (2015), big Discourse sets a larger context for the analysis of small discourse.

The results of the first part of the study answer to the research question, What meaning(s) is ascribed, and how is inclusion used in mathematics education research? They show that research on inclusion in mathematics education use the term inclusion when both referring to an ideology and a way of teaching, although these two uses are usually treated separately and independently of each other.
The results of the second part of the study answer to the following research questions: What meaning(s) do the students ascribe to inclusion in mathematics learning and teaching? And what frames students’ meaning(s) of inclusion in mathematics learning and teaching? These questions show how meaning(s) of inclusion in student talk can be described by three overarching Discourses: the Discourse of mathematics classroom setting, of assessment, and of accessibility in mathematics education. Within these Discourses, smaller discourses make issues of meanings of inclusion for the students visible in terms of: testing, grades, tasks, the importance of the teacher, (not) being valued, the dislike of mathematics, the classroom organization, and being in a small group.

This study shows the complexities and challenges of teaching mathematics, all while simultaneously handling students’ diversity and promoting the mathematical development of each student. To enhance students’ participation and access demands that the teacher knows her or his students, is flexible, has a pedagogical stance and tactfulness, and is knowledgeable in mathematics and mathematics education. It also demands that the teacher is able to take a critical stance and resist the prevailing discourse of assessment that can sometimes overshadow the mathematics education, and in a sense, almost become mathematics for the students.

Furthermore, this study also shows how complex and challenging it is to be a mathematics student: they are required to relate to, understand, and participate in many Discourses existing at the same time in a single mathematics classroom. These Discourses interrelate and are embedded in power relations between students and teachers and institutions. This demands that the students are alert and able to use various symbols and objects as well as recognize patterns, and then act accordingly. Hence, to be able to fully participate, you have to be able to talk the talk and walk the walk (Gee, 2014a). This means that not only do you have to use the language correctly, but also you have to act properly at the right time and place.
Acknowledgements

When people tell me they think I must be very smart to be doing research, I always reply, “It’s not that you have to be that smart, but damn, you have to be stubborn to pursue your research goal!” And given that stubbornness runs in the family, I am so very grateful to my mother, my father, and my grandmothers Anna-Lisa and Helfrid, who have all passed on their stubbornness and persistence to me. I am also so very grateful to my parents for their support in every part of my life. And although my grandmothers are not alive, I know they are watching over me and are so proud of me: Tänk farmor, att den lilla däkan gjorde det, ja herregud i himlen mormor!

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be your mother and for your support, always. Tobias – what can I say? You are my everything, and when the going gets tough, you are always by my side – my security, my love. Thank you for all the times you said, “Go, do your research – I will take care of this”. Thank you from the bottom of my heart for having my back through thick and thin.
Prologue

Throughout this research process I have worn, and still wear, three bracelets on my wrist. I never take them off. They are a constant reminder of me being me – not what anyone else wants me to be, not what society expects me to be. Just me. They serve as a reminder of the power and trust inside me. The first one has written on it, *Hej Livet, Nu köö vi!* (*Hey life, let’s go!*). This reminds me of my inner joy, my love for exploring, finding new things and new roads to travel both physically and mentally. The second bracelet reads, *She believed she could, so she did.* This reminds me of the saying, *When the going gets tough, the tough get going.* Nobody shall set my limits. I am the only one to do that, and if I believe I can, then I can. Like the famous quote from the movie *Dirty Dancing*, “NOBODY puts baby in a corner!” On the third bracelet is written the word *Breathe*. This reminds me that everything shall pass and everything will be just fine – just take deep breaths.

In relation to this study, there were many times when I had to look long and hard at my bracelets, finding strength in their messages. Sometimes, when I felt lost and confused, I reminded myself that this is a new research path, so the only way to walk down the path is my own. I have to follow and trust my own thoughts. Sometimes, I felt as if someone, or something, was trying to set my limits. When that happened, the little stubborn girl inside of me told me to stand in my own truth, believe in myself and not let anyone other than myself set my limitations and my borders. Borders set by others are there to be crossed. And when I felt despair and thought that this will never work out, I had been reminded to just take deep breaths and let my shoulders drop from my ears. I would think, *What feels like an impossibility right now will probably end in possibility sooner or later if I just let it go.* With that stated, letting go of work doesn’t write a PhD thesis. So what has to be let go of is the feeling of not being able to do something. Then it’s just a matter of believing in the good your research will do, and then write. Just write. And that is what I did. I wrote a thesis about something that really engages me – every student’s right to have opportunities to be included in the teaching and learning of mathematics in primary school.
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Thesis overview

This thesis is a compilation of five articles. Surrounding the articles, a deeper description of the entire study is made, starting with the Introduction with the aim and research questions. Thereafter, the background is presented, which concerns the area of inclusion in mathematics and special educational needs in mathematics. Thereafter, a prior licentiate study (Roos, 2015) with the same overarching aim to investigate inclusion in mathematics education is summarized, but here, it is made with teacher talk in focus. Following that is a description of the theoretical framing of the study and the methodology connected to the theoretical framing. Thereafter, the articles written within this study are presented (see list of appended articles following this section). The last part of this thesis presents a summary of the results and a general discussion of the main findings of the study of inclusion in mathematics education. Conclusions, implications and suggestions for further research in the area are also presented here.
List of appended articles


Roos, H. (2019c). I just don’t like math, or I think it is interesting, but difficult … Mathematics classroom setting influencing inclusion. Paper in the proceedings to CERME11, 2019.

Roos, H. (submitted). Same, same, but different – consistency and diversity in participation in an inclusive mathematics classroom.

Articles I–IV are re-published articles with permission from the journals and proceedings.
INTRODUCTION

The overarching aim of this study is to contribute to knowledge about students’ meaning(s) of inclusion in mathematics education. This is a highly important area, as both research and practice struggle to find ways to increase inclusion in mathematics education.

A notion frequently used in relation to, or instead of, participation in the education for every student is inclusion, or related notions such as inclusive pedagogy. Often, inclusion is used to describe an ideological stance or a way of working in mathematics (Roos, 2019a) in order to provide “a meaningful education for all” (Florian, Black-Hawkins & Rouse, 2017, p. 14). “For all” implies the focus of inclusive education is not only on low attainers and their deficiencies but also on issues of diversity in order to avoid marginalization (Florian, Black-Hawkins & Rouse, 2017; Alderton & Gifford, 2018). However, at the same time, the notion for all affords a gaze on all students’ learning, raising contradictions regarding who is seen, heard and supported. This has been intensely debated in research (e.g. Popkewitz, 2004). Inclusive settings and working inclusively can be defined as ways of accommodating all learning differences among students within a classroom and creating opportunities for every student to participate in the education (Barton, 1997). Inclusive settings can also be discussed on a societal level, as in, talking about including every student from a socio-political view (e.g. Strahler-Pohl & Pais, 2013). In this study, every student means, in the spirit of Popkewitz (2004), not departing from the differences of the individual and trying to accommodate all students into what is understood as “normal” for similarly aged students regarding learning mathematics (which implicitly suggests “all” are divided into those who do understand the mathematics and those who do not), but rather departing from the opportunities of every student. Therefore, this study derives from three interrelated areas: the inclusive mathematics classroom, special educational needs in mathematics, and inclusion in mathematics education. These three areas will be briefly presented in the following part of the introduction in order to set the scene.
Inclusive mathematics classrooms include a diverse mix of students in regard to gender, cultural and social backgrounds, and ability. These diversity issues are also frequently discussed in relation to access and equity in mathematics education research (Bishop & Forgasz, 2007). Often when research discusses inclusion in education, excluded groups of students are mentioned (e.g. Nilholm, 2007; Janhukainen, 2011). Excluded groups are often referred to as the grouping of students with disabilities, in terms of diagnosis, into special schools or special groups. This implies that little attention has been paid to what happens in a regular mathematics classroom foregrounding inclusion (Roos, 2015). Thus, the focus of this study is to investigate inclusion in mathematics education with a focus on diversity in access to mathematics within a regular mathematics classroom. Moreover, a gap has been identified in research on inclusion regarding inclusion foregrounding students (Scherer et al. 2016; Roos, 2019). Consequently, this study aims at listening to students and observe how they talk about being included in mathematics education. More specifically, this listening is systematically explored in order to generate knowledge about students’ meaning(s) of inclusion in mathematics education. However, this does not refer to just any student in the aim to uncover the meaning(s) of every student, but rather those who are seen by the teachers as students in special educational needs in mathematics (SEM).

But what is special educational needs in mathematics (SEM)? To answer that, we need to go beyond the inclusive mathematics classroom and the particular culture of school. SEM exists because of normalization processes in our society that regulate what is normal to attain in mathematics at a certain age or at a certain time. If the student does not fit into that “normal” frame in mathematics learning, she or he is labelled as being in SEM. Magne (2006) defines SEM as the need for something else other than what is usually offered in the mathematics education to enhance the learning. Accordingly, one direction in SEM is towards mathematics difficulties, with students struggling to gain access to mathematics. The reason for the struggles and the mathematics difficulties can be explained in a range of ways involving multiple explanations. Many aspects have an influence on mathematics
difficulties, and we need to consider and monitor all of them both in research and practice in order to find effective ways “of identifying, remedying and preventing mathematics difficulties” (Gifford & Rockcliffe, 2012 p. 5).

Another direction in SEM is towards mathematics facility, where students are in access to mathematics, and most often, easily master the mathematics offered in the education. This is because, according to Magnes’ (2006) definition of SEM, where although the students master the content, they still need special education because something other than the offered mathematics education is needed to enhance learning. This combination of facility and needs makes the notion of inclusion both challenging and interesting, as it has to work in two directions – one direction towards mathematics difficulties and one direction towards mathematics facilities. Hence, the directions of SEM display a diversity in regard to access to mathematics. In this thesis, mathematics education is seen as a frame factor for the mathematics presented in the education. To be able to gain access to mathematics, you need to have access to the mathematics education. Additionally, if mathematical content is not presented in the education, it limits access to mathematics.

Following from the above, SEM is seen in this study as something that may occur regardless of whether a student is a high or a low achiever. This level of achievement can be for a short or a long period of time, and it can be in general or in specific areas of mathematics. The phrase, “student in need of SEM”, implies a student is in need of something other than what is usually offered in order to appropriately develop her or his mathematical knowledge (Bagger & Roos, 2015) and enhance learning. In need is also used to highlight the fact that SEM is not seen as something static and eternal, but rather the student is in need at a certain time and in a certain situation, but may not be in need in another time or another situation. Hence, in SEM suggests a situated standpoint. When discussing inclusive education, often the SEM students in mind are low achievers who struggle to gain access to the content. Nevertheless, as stated, even a high achiever who is in access to mathematics can be in SEM, as she or he may need specific solutions in order to have optimal opportunities to participate and to be included. It
is common for schools to describe themselves as inclusive; however, what they mean by that is often cloaked, and the question that arises is whether it leads to possibilities for every student to participate in mathematics education.

This study is situated in the setting of inclusion in mathematics education; thus the study has a special educational starting point regarding mathematics learning and teaching. Special education is a major field within the educational research paradigm and is of interest both in terms of research and practice. The field of special educational needs (SEN) seeks to identify what needs in the education have to be met in order to empower all students and create ideal learning opportunities. Special educational needs in mathematics (SEM) is a minor field within this larger context. This field is influenced by theories and research from different research paradigms, mathematics education, the educational and the psychological research paradigm, among others.

The research presented in this thesis is a development from a prior study (Roos, 2015) investigating teacher talk regarding inclusion in mathematics education. That study contributed with insights on what inclusion in mathematics education can be found in teacher talk and insights on influencing factors on the inclusion process in mathematics education, focusing on teaching and organization. The results from this prior study are further elaborated in the forthcoming chapter, Background. Although that study was an important piece in the inclusive mathematics education puzzle, it felt like an (perhaps even more) important piece was missing, namely, what about the student talk of inclusion? What is the meaning(s) of inclusion in mathematics education for them? And what influencing issues of the process of inclusion is visible in their talk of mathematics learning and teaching? Hence, this study aims to add that missing piece. This will lead to further insights and knowledge in the field of inclusion in mathematics education. Consequently, this study is situated in two different, yet overlapping research paradigms – mathematics education and special education, specifically within the field of inclusion (Figure 1).
This study takes a social view on learning mathematics, which implies that meaning, thinking and reasoning are seen as interaction and participation in social activities (Lerman, 2000). Humans almost always consider symbols (such as notions, words, things, etc.) in the world in terms of meaning and handle symbols by ascribing them meaning in social practices (Gee, 2014a), even mathematics. Fairclough (2003) describes meaning-making as occurring within social practices. This socially produced ascribing of meaning can also be understood as a moment of exploration, engagement and negotiation. Wenger (1998) refers to the social process of meaning making as negotiation of meaning, stressing the process of experiencing the world and a meaningful engagement in the world: “Meaning arises when any symbol (can be a word, image, or thing) ‘stands for’ (is associated with) something else than itself.” (Gee, 2014a, p. 230). In this thesis, this association is understood as what happens when people in social practice negotiate, explore and ascribe meaning into phenomena, things, et cetera. This implies that the meaning-making is within social practice, and the meaning is not in the heads of persons but rather in the social practice between persons, as stated by Gee (Gee, 2014a). Gee’s theories and understanding of meaning have been used in order to capture students’ meaning(s) of inclusion, as they harmonises well with the overall purpose of contributing to an understanding of students’ meaning(s) of inclusion in mathematics education. Gee’s theories suggest that meaning is not static but rather situated in time and space. Applied to the context of this study, this means that I come to the study
from an understanding that, for instance, the meaning of mathematics has developed over the years and implies different things and usages depending on, for instance, whether you are in a lower secondary classroom, at a master’s program at a university, or at a car factory. Hence, meaning is ascribed as “the result of social interactions, negotiations, contestations, and agreements among people. It is inherently variable and social” (Gee, 2012, p. 21). This suggests meanings are rooted in the negotiation between different social practices (Gee, 2012).

Historically, mathematics have developed over thousands of years by meaning-making in social practices; hence, you could say that mathematics itself is a social construct (Hacking, 1999). If one assumes that learning mathematics is about meaning-making in a social practice, it is not too far-fetched to say that if one is not included in mathematics lesson at school (i.e. physically there, but not participating in the mathematical practice), then they are not making meaning in mathematics. If you are included, you are making meaning in the mathematics education. Hence, to be included, and the meaning of inclusion as participation in the meaning-making that happens in the social practice of mathematics education is of importance. Notably, it is not just “any” participation but a participation in the mathematical practice of learning and teaching. In this study, inclusion is defined and thought upon as processes of participation in the mathematics education, where participation is seen “a process of taking part and also to the relations with others that reflect this process” (Wenger, 1998, p. 55). More specifically, in this study, processes of participation, hence inclusion, are about taking part in the mathematics education and meaning-making with peers and teachers in learning situations in mathematics.

To conclude, in this study, participation is variable and about taking part in the mathematics education and relating to other peers and teachers in learning situations in mathematics. Inclusion is regarded as processes of participation in the mathematics education. In relation to inclusion, SEM is about finding ways to meet a diversity in access to mathematics in the education in order to enhance processes of participation. Here, access is
closely connected to participation and inclusion, because when opportunities to take part and relate to others in learning situations in mathematics are enhanced, then it is likely that opportunities to gain access to mathematics will also be enhanced.

As stated, the overarching aim of this study is to investigate the meaning(s) of inclusion in mathematics education in student talk of inclusion. An important notion here is meaning. In this study, \textit{meaning} is understood as a dynamic process rooted in negotiation between social practices in the space between persons, influenced by time and space.
Aim and research questions

The aim of this study is to contribute to research and practice within the field of special education in mathematics with knowledge about, and an understanding of, students’ meaning(s) of inclusion in mathematics education.

Three research questions guide the study:

What meaning(s) is/are ascribed, and how is inclusion used, in mathematics education research?

What meaning(s) do the students ascribe to inclusion in mathematics learning and teaching?

What frames students’ meaning(s) of inclusion in mathematics learning and teaching?

It is not surprising that these questions interrelate, and in this study, the two research questions regarding students’ meaning(s) are considered as two sides of coin.
BACKGROUND

This study focuses on meanings of inclusion in primary school mathematics education in student talk. Although every student in the classroom is important, the focus of this study is on the student in special educational needs in mathematics (SEM). Hence, this chapter will present and discuss SEM and the relation between SEM and inclusion in mathematics education. Also, the chapter presents research on learning mathematics with students foregrounded. In addition, the previous study of inclusion in mathematics education with focus on teachers talk of inclusion (Roos, 2015) is described and serves as a starting point for this study of students’ meaning(s) of inclusion. The chapter ends with a reflection on the change from the teacher perspective to the student perspective on inclusion.

Special educational needs in mathematics

According to a Swedish government proposal from the late 1980s, special education can be interpreted as “activities for students that fall outside the natural variability of diversity” (Proposition 1988/89: 4 p. 80). This implies there is a natural variability in our classroom, but it does not say how to define it and what criteria to use when doing so. Consequently, it is left to the school and the teachers to define the natural variability and act accordingly to provide special education to those who fall outside this variability. Connecting the subject of mathematics to special education needs, the variability and diversity concerns knowledge in mathematics. Hence, SEM becomes relative and socially constructed, depending on who or what defines natural diversity among students. Consequently, SEM is connected to issues of power. That becomes particularly visible when it comes to who has the right to make definitions, establish criteria and decide who has the right to get special education (Swedish Research Council, 2007). The interpretation is always situated in culture and time, meaning the interpretation and use of the term special needs itself “depend[s] ultimately on value judgements about what is important or desirable in human life and not just on empirical fact” (Wilson, 2002, p. 61).
SEM is discussed by teachers and schools in practice but unfortunately not as much among scholars. It is also a term that is hard to define and has different definitions depending on from what epistemological field it derives (Bagger & Roos, 2015). Also, notions used to describe the “special” differ depending on the epistemological field. From which field the research derives can be seen in the use of terms and definitions used (when there are any). When discussing SEM, these fields can be psychological, social or pedagogical. A couple notions used among scholars are, for example, children with mathematics difficulties (e.g. Dowker, 2009; Gifford & Rockliffe, 2012) and low attainers (e.g. Alderton & Gifford, 2018). Both definitions of SEM derive from the educational field. Other notions used are dyscalculia (e.g. Kaufmann, 2008; Butterworth, Varma & Laurillard, 2011) and mathematics anxiety (e.g. Hannula, 2012), which derive from the field of psychology. SEM students (Magne, 2006) is also used, deriving from a social and pedagogical field. Bagger and Roos (2015) suggest the term students in special educational needs in mathematics, which is adapted from Magne (2006) and used in this study. The reason for using this term is that this study takes off from a social, relational and pedagogical perspective on mathematics learning and participation, focusing how teaching and learning activities affect students’ learning in mathematics. Drawing on Silfver, Sjöberg and Bagger (2013), the need is something that may occur regardless of whether the student is a high or low achiever, whether it is required for a shorter or longer period in time, and whether it is in general or more specific areas in mathematics. Hence, the student is in SEM because it signals that it is not a monolithic deficiency within the student but rather something dynamic that the student can get in and out of (Bagger & Roos, 2015). To conclude, SEM is a conceptual notion describing a specific need in the mathematics education in order to meet diversity, though, depending on the institutional environment in where it is situated. This implies SEM is understood differently depending on the variation of national and school settings.

As stated in the introduction, research on inclusive education most often focuses on SEM students who are seen as low achievers that struggle to gain access to a mathematics education. However, a SEM student can
achieve high and have access to mathematics, as she or he might need specific solutions in the education in order to have optimal opportunities to participate. Hence, if inclusive education aims at meeting every student, then it needs to consider every student.

Looking at research within the SEM field, it not only investigates casual explanations of SEM, hence meanings, but also issues of norms, power, empowerment and to “include the excluded” (e.g. Pais, 2014; Healy & Powell, 2013). In addition, teaching and learning for students in terms of struggle or in terms of access to a mathematics education is of interest in this research (e.g. Lewis & Fisher, 2016; Leikin, 2011). This is further elaborated in the following sections.

Struggling to gain access to mathematics education
One direction in SEM concerns mathematics difficulties, with students struggling to gain access to mathematics. The reasons for the struggles and the mathematics difficulties can be explained in a range of ways. There are multiple explanations for the difficulties: cognitive, certain types of impairment (such as for instance visual impairment), and educational, and these create barriers for students’ participation in mathematics education. Accordingly, there are a diversity of aspects influencing mathematics difficulties, and all of them need to be considered in order to find effective ways “of identifying, remedying and preventing mathematics difficulties” (Gifford & Rockliffe, 2012 p. 5).

Mathematical learning disability (MLD) is a notion used to refer to mathematics difficulties stemming from a cognitive, biological origin (Lewis & Fischer, 2016; Lewis, 2014). Hence, the explanation for the struggle in mathematics derives from the field of psychology, where the classification of MLD often is made by mathematics achievement scores below the 25th percentile (Lewis & Fisher, 2016). However, sometimes MLD also is an abbreviation for mathematical learning difficulties (instead of disability) (Scherer et al. 2016); in this case, the definition moves towards environmental and societal explanations.
The notion of dyscalculia is often connected to MLD focusing on disability, and researchers sometimes (e.g. Mazzocco et al. 2013) consider these two notions identical. *Dyscalculia* is a term that refers to a specific learning disability in arithmetic (Shalev, 2000; Butterworth, 2011; Gifford, 2008) that is the result of a cognitive difficulty. It has been used more frequently in research and practice during the last decade, for instance, in Denmark, where the Danish Ministry of Education decided to create a dyscalculia test, which is currently in the making (Lindeskov & Lindhardt, 2015). However, there is confusion about its definition and the way to test for it, and hence, the actual rate of prevalence. Also, the notion limits possible causes and excludes environmental and societal explanations. That makes dyscalculia a debated notion, and researchers (mostly in the pedagogical field) have chosen to avoid using this notion (Gifford, 2008).

Low achievement (LA) is also a notion sometimes used when talking about mathematics difficulties and is sometimes compared to MLD (e.g. Mazzocco et al. 2013). Some researchers argue that low achievement is a social construct, "not a fact but a human interpretation of relations between the individual and the environment” (Magne 2003, p. 9), while others explain the notion of low achievement by the presence of cognitive disorders or discrepancy to IQ (Scherer, Beswick, DeBlois, Healy & Moser Opitz, 2016). Thus, this notion derives both from the psychological and pedagogical fields, depending on the explanation. Important to take into consideration is that there is no consensus on the operational definition of, and difference between, MLD and LA (Lewis, 2013). This implies that the methods and tests that determine whether mathematics difficulties derive from environmental or cognitive explanations are not reliable (Lewis & Fisher, 2016; Lewis, 2013). As Magne (2003) suggests, if one focuses on the environment in relation to the achievement of the individual student, then student’s relation to “regular” achievement in mathematics becomes important to identify. One way to identify “regular” achievement is to look at the national curricular goals in mathematics to see the expected mathematical knowledge for a certain grade. Therefore, given that curricular goals, and the interpretations of those goals, differ between countries, schools and teachers, the definition of what a low achiever is also differs.
Following Magnes’ (2003) definition, moving from cognitive explanations towards environmental explanations, these explanations can be described from different perspectives. One way to group the environmental explanations is by social, cultural and political explanations. This grouping also reflects issues addressed in research concerning challenges to meet diversity in mathematics education (Abreu, Gorgorió & Björklund, 2018). To make even more issues visible within social explanations, we find the categories of class (e.g. Gutiérrez, 2007; 2008) and gender (e.g. Mendick, 2005; Weist, 2011; Forgasz & Rivera, 2012). In addition, within cultural explanations, we find language and ethnicity (e.g. Meaney, Trinick & Fairhall, 2013; Barewell et al. 2016). The political aspect of mathematics education can be regarded not only as overarching the others but also as deeply affecting the educational systems at hand, suggesting that educational explanations for students who struggle to gain access is at the classroom and teacher levels, as well as the political level (e.g. Valero & Zevenbergen, 2004). It is Gutiérrez (2013) who refers to a sociopolitical turn in mathematics education that highlights identity and power at play. By using this socio-political perspective, formerly unknown explanations affecting learning and teaching in mathematics can be highlighted and addressed. Hence, socio-political explanations of access to mathematics can offer another dimension in regard to challenges to meet the diversity in mathematics education.

In this study, students struggling to gain access implies a struggle for opportunities to participate in the mathematics education, and thus, through the mathematics education, enhanced access to mathematics.

In access to mathematics education

Another direction in SEM is towards the mathematics facility, when students are in access to mathematics education and usually master the mathematics presented in the education easily. Notions used in research describing this are gifted students in mathematics (e.g. Oktaç, Fuentes & Rodriguez Andrade, 2011; Wistedt & Raman, 2011) and mathematically talented students (e.g. Shayshon, Gal, Tesler & Ko, 2014). Solomon (2009) refers to Pimm (1987), who uses the notion of “full
participation”, which can be seen as another way of describing students in access. Here, “full participation in mathematics constitutes being able to use not just individual technical terms but also phrases and modes of argument—being a mathematician involves speaking like a mathematician” (Solomon, 2009, p. 11). Following Magnes’ (2006) definition of SEM, although the students may master the content, they still need special education because something else than the offered mathematics education is needed in order to enhance learning.

Research on students in access to mathematics in inclusive classrooms highlights the importance of the teacher “to reach all students in mathematics classrooms” (Freiman, 2011, p. 161) to enhance learning. It is also important that the teacher provide social and intellectual stimulation to all the students, so that everybody has the possibility to “attain their maximum potential” (Oktaç, Fuentes & Rodriguez Andrade, 2011, p. 362). One way to stimulate students in access is highlighted by Diezmann and Watters (2001;2002), who discusses the importance of challenging tasks and getting opportunities to discuss these tasks. Similarly, Wistedt and Raman (2011) stress the need for sufficient stimulation and challenges for the students to meet their potential, focusing on students in access in order to raise the quality of the mathematics education and promote equity. This perspective on equity in relation to students in access is also seen in a study focusing on gifted students and their right to have opportunities to learn mathematics (Leikin, 2011). Here, Leikin (2011) poses the question, “What type of ability grouping is the most effective for mathematically gifted students?” This can be a rather provoking question in relation to the inclusive field, where ability grouping is strongly criticized (e.g. Boaler, William & Brown, 2000). In relation to this, Solomon (2009) points out that, although students choose to study mathematics at the undergraduate level (which can be seen as a form of ability grouping) and are perceived as “good at mathematics”, they nevertheless express identities of exclusion rather than the expected identity of participation. Hence, this seems like the norm when one is a “gifted student” and the comparison and grouping of ability cloaks issues of underlying values and assumptions, and processes of participation. If the needs are regarded as situated (Bagger & Roos, 2015), then a student who more
often is in access to mathematics education might struggle to gain access and participate in mathematics education in certain situations, such as the test situation (Bagger, 2017a).

In this study, students in access to mathematics means access to the mathematics worked with in the classroom but with a need for opportunities for full participation in relation to a mathematical content (suggesting the need for other mathematics than what is presented in the education), and with a need to relate to others, and in turn, enhanced access to mathematics.

To conclude, on one hand, there seems to be a focus on labelling students who struggle for access by using terms like MLD, LA and dyscalculia. Yet, on the other hand, when discussing students who have access to mathematics education, there seems to be an emphasis on the importance of meeting the students’ potential. Instead of focusing on different labels and definitions in research, would it not be better to focus on meeting every students’ potential in mathematics (not only the students in access)? Additionally, across and within countries, schools handle SEM differently (both students who struggle for access and students in access). One way to handle this is to work inclusively in different ways (Persson & Persson, 2012). But what does it mean to work inclusively in mathematics education? This will be further elaborated in the next section entitled Access in mathematics in relation to inclusion and participation.

Access in mathematics education in relation to inclusion and participation

Inclusive practices and inclusive settings can be defined as ways of accommodating all differences among students within the classroom, creating opportunities for every student to participate in the education (Barton, 1997). This implies that an inclusive mathematics classroom holds a diversity of students. Within this diversity, there are most likely students in special educational needs in mathematics (SEM). Thus, teaching mathematics in these inclusive classrooms is complex, involving issues such as differences in learning trajectories and equity (Scherer et al. 2016). In relation to equity, research highlights the
importance of taking diversity as a point of departure in inclusive classrooms (e.g. Sullivan, 2015; Roos, 2019a), which implies that taking diversity into account is positive and closely related to improving access to mathematics education (Nasir & Cobb, 2007). When talking about inclusion and access to mathematics, research in mathematics education make social, ethnic, cultural and gender issues visible (Athew, Graven, Secada & Varlero, 2011; Bishop. & Forgasz, 2007; Forgasz & Rivera, 2012; Solomon, 2009). The research associates, or replaces, the notion of inclusion with the notion of equity, indicating a strong connection to socio-political issues, and thereby, foregrounds justice in mathematics education (Roos, 2019a). This can be exemplified with how Pais and Valero (2011) and Valero (2012) discuss processes of inclusion and exclusion in society at large in mathematics education by using the notion of equity, suggesting that, to move towards equity, we need to recognize and address these processes of inclusion and exclusion. Thus, social justice (Pais, 2014) is one reason for discussing inclusion from a socio-political stance.

A key aspect of teaching and learning in mathematics is the teacher and the teacher’s awareness of students’ prerequisites (Anthony & Walshaw, 2007; Anthony, 2013). If the teaching is student-centred, there is evidence that the students will be more positive towards mathematics (Noyes, 2012). This is an important aspect of teaching mathematics in inclusive classrooms, as studies show that students’ negative perception of the subject influences engagement (Lewis, 2013; Murray, 2011; Andersson, Valero, Meaney, 2015), and thereby, access. However, as Andersson et al. (2015) point out, this dislike of mathematics is not always static and stable in nature; it can depend on available contexts. The teachers’ choice of tasks, the teachers’ way of engaging students, and the teacher’s awareness of the students and sensitivity towards the students are also important issues in relation to students’ interest in mathematics as a subject (Sullivan, Zevenbergen & Mousley, 2003), as it affects their participation and access. Consequently, the awareness of the teacher in regard to the students’ diversity, the tasks and the social interaction is of the utmost importance for teaching in inclusive mathematics classrooms. Similarly, Secher-Schmidt (2016) argues that we need to consider both the way we teach and test mathematics, and
the support available to the individual student in order to develop inclusive practices in school. Scherer et al. (2016) discuss how to build an inclusive mathematics education and teach by promoting participation as well as evolving teaching practices and intervention strategies. This is accomplished by focusing on the learning of every student in the classroom, and the learning situations allow for the meeting of these differences. Hence, the differences in the inclusive classroom are seen as a resource for participation, and in the long run, a key for access to the mathematics education. This is supported by Gervasoni et al. (2012), who found that when teachers directed attention to learning opportunities taking diversity as a point of departure, instead of deficits, it increased their pedagogical actions. Another important aspect highlighted in the research about inclusive classrooms is the sense of belonging for the students (Rose & Shevlin, 2017). According to Gervasoni et al. (2012), creating the experience of belonging and feeling valued is a key challenge for all school communities and teachers. This suggests that the sense of not belonging is a learning obstacle and a form of social exclusion that perpetuates and reproduces social patterns of (dis)advantage (Civil & Planas, 2004), thus influencing students’ inclusion in a negative way. Alderton and Gifford (2018) describe this from a discourse perspective, highlighting how students who are identified as low attainers are constructed in social practices imbued with power relations. Morgan (2005) pinpoints these social power relations in relation to inclusion and exclusion from a language perspective when critiquing how the national guidance for mathematics support in the UK makes assumptions about definitions and concepts in mathematics. These assumptions reinforce differences in access to mathematics education for students considered as high or low achievers. Hence, how the society, school and teacher provide a feeling of belonging in both governing documents and in the classroom is important.

In the appended article “Inclusion in mathematics education: an ideology, a way of teaching, or both?” (Roos, 2019a), a deeper analysis with descriptions of ascribed meaning(s) and how inclusion is used in mathematics education research is detailed.
Inclusion in mathematics education – foregrounding students

When looking at research on inclusion in mathematics education foregrounding students, it is noticeable how there are fewer studies than those that focus on the teacher or educational perspectives (Roos, 2019a). However, there are studies, like for example, Kleve and Penne (2016), who investigate students’ stories of inclusion in terms of “stories about participation” (p. 42) (or the lack of them) in (mathematical) discourses. The authors use the terms “insiders” and “outsiders” to describe students’ participation in mathematics education and highlight the importance of disciplinary understanding, insight and the meta-awareness of mathematics as a discipline in order to be an “insider” and participate, which is crucial for learning mathematics (Kleve & Penne, 2016). Also, Solomon (2009) foregrounds students by looking at students’ developing identities of inclusion in mathematics. Similar to Kleve and Penne (2016), she uses the term “outsiders” to describe why many learners of mathematics feel alienated by the world of mathematics. In her investigation of identity in mathematics, discourses of ability, competition, performance and comparison are seen. All these discourses are perpetuated by high-stakes testing, which in turn, seems to have a great impact on what happens in the mathematics classroom.

More research foregrounding students can be found when interpreting inclusion as processes of participation, although they do not explicitly use the notion of inclusion; for example, Anthony, Kaur, Ohtani and Clarke (2013) stress the importance of attending to students’ voices when investigating effective pedagogy and learning outcomes in mathematics. At the same time they affirm classroom practices as culturally situated in order to enhance understandings of mathematics classrooms around the world. Here, in the culturally situated classroom, the role of social, emotive and motivational factors in students’ learning processes are important. Motivational factors are also seen by Murray (2011), who examined secondary school students’ perspectives on declining participation. Here, the reasons for declining participation included the finding that mathematics was seen as boring, difficult and not well taught. The solution suggested by the students was that the
education needed to make mathematics more enjoyable and relevant in addition to showing the importance of mathematics. Hence, the teacher and the teaching are significant. When investigating what a good mathematics teacher means from the student point of view, Anthony (2013) highlights that the attribute of “goodness”, when describing a good teacher, was influenced by the diverse socio-political reality of the students. However, all the students talked about the importance of caring teachers and of teachers who explain things well. Additionally, an important issue was the co-construction of a unique learning community. Hence, “mathematics knowledge is created in the spaces and activities that the classroom community shares within a web of economic, social and cultural difference” (p. 223). Even McDonough and Sullivan (2014) highlight the teacher and the awareness of the teacher in relation to students. McDonough and Sullivan (2014) suggest that the teacher gather data on their students’ view of mathematics in order to become more informed about the students’ knowledge and disposition, which in turn, will improve their teaching. Also, they suggest both researchers and teachers gather more than one type of data and use multiple semi-structured interviews for an in-depth exploration of students’ meanings of mathematics (McDonough & Sullivan, 2014) in order to make visible any issues that may be hidden. Solomon (2009) proposes that “the way in which central practices are hidden from many students, causing them to remain on the margins, lacking the means of ownership” is the major issue regarding exclusion of students in mathematics education (p. 163). Her suggestion to overcome this and promote inclusion is to make the hidden practices visible and explicit for students and to reflect upon the teacher–student relationship in order to be aware of the power and authority embedded in the social community of practice.

It is important to stress that, although the above text is an attempt to describe research on students’ meanings regarding inclusion in mathematics education and mathematics in a kind of homogenic way, the individual students’ meaning is always most important so as not to marginalize and create stereotypes. It is also important to consider that the meanings presented in research may not represent the diversity of students’ meanings (Cremin, Mason & Busher, 2011). Hence, there is
always a need to take the individual student into account and not make any common assumptions.

The previous study on inclusion in mathematics education – a summary

The study presented in this PhD thesis, which focuses on students’ meaning(s) of inclusion in mathematics education, is preceded by a previous study that also focuses on inclusion in mathematics education but with teachers’ meaning foregrounded (Roos, 2015). This chapter is a summary of the results of the study. The aim of the previous study was to contribute to research and practice in mathematics and special education with more knowledge about, and an understanding of, how all students can be included in the mathematics education in primary school from a teacher perspective. The research questions focused on were:

- What can inclusion in mathematics be in primary school, and what influences the process of inclusion in mathematics?

- What, from an inclusive perspective, appears to be important in the learning and teaching of mathematics?

The study was an investigation of inclusion in mathematics education with teachers’ meaning foregrounded. The central person of this study was a remedial teacher in mathematics, Barbara. Barbara was chosen for the study based on her broad experience of teaching mathematics to SEM students and her recognized skills according to colleagues, the principal, and in the municipality, where she had been working with mathematics in an overarching team concerning special needs. Barbara worked at Oakdale Primary School, a large primary school with three classes in each grade, from preschool class (6-year-old students) up to Grade 6 (12-year-old students). The choice of Barbara at Oakdale Primary School warranted that inclusion in mathematics education would become visible. Thus, inclusion in mathematics education at Oakdale Primary School may be seen as a critical case (Flyvbjerg, 2006).
However, although Barbara was the central person in the study, *inclusion in mathematics education* was the object of study.

The empirical data consisted of interviews, observations and documentary sources. All these different data were collected in a selected, intermittent way (Jeffrey & Troman, 2004). The analysis was partly made during the data collection, enabling the research to move from a wide scope to a narrower focus, back and forth in a process. This way of looking at the data is a part of a hermeneutics approach – looking at the whole and the parts of a process. A static-dynamic analysis (Aspers, 2007) was conducted to find key words, make codes and create categories. In this coding and categorization, a theoretical framework was applied.

The theoretical framework was part of the social learning theory of Wenger (1998) on communities of practice. In this theory, learning is seen as a process of social participation. One unit of analysis in this theory is identity, another is communities of practice (COP), which is an informal community where people involved in the same social setting form the practice (Wenger, 1998). In order to identify different communities of practice, the concepts of mutual engagement, joint enterprise and shared repertoire were used. These “three dimensions” (p. 72) are the source of a community of practice according to Wenger (1998). In the study, only the part of Wenger’s (1998) theory constructing COP was used. This could be limiting but also an advantage, as it allowed a focus on, and to delve deeply into, the data. Communities of practice offered a way to structure the data. In addition to COP, Asp-Onsjö’s (2006) view on inclusion as social, spatial and didactical was used. Here, didactical inclusion made it possible to connect to the teaching and learning of the subject of mathematics. This study also uses the notion of *reification* to discover negotiations of meaning, and hence, participation in the communities of practice. This notion is used by Palmér (2013) to describe negotiation. Wenger’s concept *negotiation of shared meaning* is used as a tool describing the interplay in and between different communities of practice. *Boundary objects* is a notion also used when referring to items used to negotiate shared meaning. Accordingly, a participatory perspective was adopted, which reflects that inclusion is
not limited to being physically present in the classroom – one can be included in the mathematical practice anywhere, as this form of inclusion has no physical boundaries.

Four communities of practice were identified at the site based on the data, community of mathematics classroom, community of special education needs in mathematics, community of mathematics at Oakdale Primary School, and community of student health. These practices overlapped and influenced each other, forming a constellation of interconnections. In these communities, the process of inclusion was visible in both similarities and differences and inclusion in mathematics came to the fore. Also, in relation to the object of study inclusion in mathematics, three different cases were identified: the case of the principal of Oakdale Primary School, the case of Barbara at Oakdale Primary School, and the case of mathematics teachers at Oakdale Primary School.

The results showed that despite differences between the communities of practice regarding inclusion in mathematics education, there were also similarities. In both the community of the mathematics classroom and the community of special education needs in mathematics, Barbara talked about in or out of the classroom, and about both students and herself as a remedial teacher being in or out of the classroom. When talking about students, in or out of the classroom was an issue of social inclusion for Barbara. In the community of special education needs in mathematics, both the principal and the mathematics teachers talked about in or out of the classroom. Hence, in and out of the classroom is a reification of inclusion in mathematics in both the community of mathematics classroom and the community of special education needs in mathematics, but the data implied this reification was twofold, as it concerned both the students and/or the remedial teacher.

Teacher knowledge was visible in the community of the mathematics classroom, the community of mathematics at Oakdale Primary School, and the community of special education needs in mathematics. Barbara talked about teacher knowledge in the community of the mathematics classroom and the case with mathematics teachers mentioned it in the community of mathematics at Oakdale Primary School. The principal
spoke about *teacher knowledge* in the community of special education needs in mathematics, the community of mathematics at Oakdale Primary School, and the community of mathematics classroom. In relation to teacher knowledge, the *development of mathematics education* was highlighted. This was seen in the community of mathematics at Oakdale Primary School in the cases of Barbara and the principal. This could be related to the case of the mathematics teacher talk about *teaching mathematics* in the community of special education needs in mathematics. *Teacher knowledge* was a reification in the communities of mathematics classrooms, the community of mathematics at Oakdale Primary School, and in the community of special education needs in mathematics. The members of these communities of practice negotiated about what kind of teacher knowledge is necessary to be able to support the SEM students. The notion of *development of mathematics education* is a reification in the community of mathematics at Oakdale Primary School.

*Representations* were visible in the community of special education needs in mathematics in the cases of Barbara and mathematics teachers. Barbara also mentioned representations in the community of mathematics classroom. Hence, representations were reifications in the community of special education needs in mathematics.

*Time* and *time and support* were mentioned in the case of mathematics teachers both in the community of special education needs in mathematics and the community of the mathematics classroom, although there were differences in how they spoke about it. In the community of special education needs in mathematics, they talked about giving enough time for the SEM students to learn mathematics and to also make time for them, which seems to be a prerequisite for didactical inclusion. In the community of the mathematics classroom, they talked about having time and extra support in the classroom when teaching mathematics, which is in regard to having all students in the classroom – spatial inclusion.

*Courses* were mentioned by all three cases in the community of special education needs in mathematics. Courses are talked about as a means for including all students in the mathematics taught – didactical
inclusion. But it was also talked about as spatially being out of the classroom. Courses were also mentioned in the community of mathematics at Oakdale Primary School, in terms of spatially excluding students from the classroom. At the beginning of the study, courses were seldom mentioned, but over time, they were mentioned more by Barbara, implying that courses were a notion Barbara used as a boundary object. Hence, Barbara was a broker between the communities of special education needs in mathematics and mathematics at Oakdale Primary School.

*Mapping knowledge* was highlighted by the case of Barbara in the community of student health as well as that of mathematics at Oakdale Primary School. It seemed that mapping knowledge in mathematics was a part of the didactical inclusion. To be able to support the SEM students, the teachers and the remedial teacher needed to know the students’ level of knowledge in mathematics. The materials used to map knowledge in mathematics *DIAMANT* and “Förstå och använda tal” (in English “Use and understand numbers”) (McIntosh, 2008) can be seen as boundary objects between the communities of student health and mathematics at Oakdale Primary School.

To summarize and highlight each COP in the community of special education needs in mathematics, courses and in and out of the classroom were visible in all three cases. In two of the three cases (Barbara and the mathematics teachers) Representations were visible. In the case of Barbara, student participation, stigmatization, intensive teaching, changing roles in the classroom, tasks and strategies, recognising similarities, connection with content, and preparing and immersing were visible. In the case of mathematics teachers, listening to students, being responsive, time, and teaching mathematics were visible. In the case of the principal, teacher knowledge was visible.

In the community of mathematics classrooms, there were no similarities between the cases regarding inclusion in mathematics. In the case of Barbara, participation in classroom teaching, flexible solutions, connection of content, approaches and material, didactical discussions, teacher knowledge, representations and flexible solutions were visible. In the case of
mathematics teachers, *time and support* in the classroom, being able to reach and *challenge the SEM students* and *self-esteem and self-confidence* were visible. In the case of the principal, *teacher knowledge* was visible.

In the **community of mathematics at Oakdale Primary School**, the *development of mathematics education* was visible in two of the cases (those of Barbara and the principal) and *cooperation* was also visible in two of the cases (the case of Barbara and the case of mathematics teachers). In the case of Barbara, *working alone, cooperation and discussions, mandate, courses* and *mapping knowledge* were visible. In the case of mathematics teachers and the case of the principal, *teacher knowledge* was visible. *Mathematical discussions* were seen in the case of the principal. Both Barbara and the principal highlighted the important role of discussions but in different ways. The principal highlighted mathematical discussions on an overall level, while Barbara highlighted discussions together with cooperation with other teachers.

In the **community of student health**, there were no similarities between the cases regarding inclusion in mathematics education. *Mapping knowledge* and *preventive work* was visible in the case of Barbara. *Individual action plans* were visible in the case of the principal. It is worth noting that no members from the case of mathematics teachers participated in this community.

Steaming from the study of the process of inclusion in mathematics education at Oakdale Primary School, three notions were created as a set of principles to demonstrate forms of inclusion in mathematics education at Oakdale Primary School: dynamic inclusion, participating inclusion and content inclusion. These notions stem from a meta reflection on the results. *Dynamic inclusion* suggests a focus on how the mathematics (special) education is arranged in terms of different ways of giving and receiving support, in or out of the classroom, from the mathematics teacher or the remedial teacher, or in the form of a short course. *Participating inclusion* highlights the importance of listening and being vigilant to the SEM students regarding their participation in the teaching. *Content inclusion* highlights how the participation is influenced by the way the content was presented and worked with, depending on
the needs and wishes of the SEM student and the specific mathematical content. These three forms of inclusion suggest that a mathematics teacher and remedial teacher in mathematics need to be aware of all forms of inclusion. These three forms interact with each other, and although there are no clear borders between them, they nevertheless influence each other.

The results show different aspects in the teaching of mathematics regarding students in SEM in the different COPs. One of these aspects is how to support SEM students in mathematics from an inclusive perspective. Two notions are created to describe these aspects: content flow and recognition of similarities. Content flow describes teachers’ awareness of making connections between different teaching and learning situations, and the awareness of making connections in the mathematical content. A part of content flow was the importance of considering situated knowledge in the teaching of mathematics through preparation, immersing and repeating in the mathematics education along with an awareness of mathematical tasks and mathematical representations. The recognition of similarities focuses on how and what to teach in mathematics, and as a teacher, to help students recognize similarities in mathematics between different teaching and learning situations, thus enhancing the inclusion process in mathematics education. These notions are further explained in Roos (2017).

From an inclusive perspective, several aspects in mathematics education in the previous study appeared to be important considering inclusion from the teacher perspective. However, it is important to stress that this was a case study that does not claim the results are generalizable but rather highlights aspects at this particular school. With that in mind, one can nevertheless identify similarities (and differences) with other schools and situations. One aspect was how the organization creates space for the development of mathematics education in terms of time for cooperation and discussions among teachers. Both Gregory (2006) and Cobb et al. (2013) highlight the importance of a connection between the organization and the practice, and the results of the previous study pointed to the same conclusion. Another important aspect was the need to have a well-functioning team working with SEM at the school in order
to develop the teaching of SEM and work with preventions for SEM. This was something the special teacher and the teachers lacked at the investigated school. Yet another important aspect was the level of knowledge of the remedial and the mathematics teachers regarding the learning and teaching of mathematics. Finally, the most important aspect was listening to the students’ meaning to be able to find out how to include the individual student. This aspect was key, and it led me to the next research endeavour, investigating students’ meaning of inclusion in mathematics education.

Transitioning from teachers’ to students’ meanings of inclusion

In the shift of focus from teachers’ to students’ meanings of inclusion, some issues were reconsidered and adjusted according to the new aim and the new research questions. The aim and the research questions of the new project immediately follow.

The aim of this study is to contribute to research and practice within the field of special education in mathematics with more knowledge about, and an understanding of, students’ meaning(s) of inclusion in mathematics education.

Three research questions guide the study:

What meaning(s) is/are ascribed, and how is inclusion used, in mathematics education research?

What meaning(s) do the students ascribe to inclusion in mathematics learning and teaching?

What frames students’ meaning(s) of inclusion in mathematics learning and teaching?

The first adjustment concerns the choice of school for empirical collection. In the first study, with a focus on teachers’ meaning of inclusion in mathematics education, the investigated school was chosen
because of the special teacher in mathematics and her teacher experience and recognized skills. Therefore, the choice of school was a consequence of choosing the teacher. The school was a public primary school with students who were between 6–12 years old. In the new empirical investigation, a school setting out to work inclusively was selected. This implies that the school aims to include all students in the ordinary classroom teaching in every subject, incorporating the special education into the ordinary teaching, with no fixed special education groups, as stated by the school. The reason for this selection is that students participating in an inclusive setting were sought. The school is a public upper primary school, with students who are between 13–16 years old, which seems to be a natural step from lower primary to upper primary in the research process. Also, finding an upper primary could provide benefits, as the students are a bit older and can perhaps more easily reflect on their meaning(s). The selected school is considered a typical Swedish public upper primary school, apart from the school’s choice to work inclusively.

Another adjustment is the change of theoretical perspective. As described, the previous study adopted social learning theory perspectives focusing on communities of practice (Wenger, 1998). This social perspective helped me investigate both the individual teacher as well as groups of teachers at the investigated school. Together with the conceptual framework focusing on inclusion (Asp-Onsjö, 2006), issues of inclusion in mathematics education from a teacher perspective were identified. These theoretical perspectives were helpful in this identification, and thus inclusion in mathematics education focusing on teachers’ meanings could be described. However, the change from teachers’ to students’ meanings of inclusion came with the need for another theoretical lens. The findings of the previous study brought issues to the fore regarding the identification of influencing aspects on inclusion, which could not be grasped with COP and the conceptual framework of Asp-Onsjö (2006). These issues concerned how to understand social interactions in relation to inclusion in mathematics education and how to describe them in detail. Consequently, a shift of theoretical perspective was needed to understand these issues. Therefore, another theoretical perspective is used in the present study.
investigating students’ perspective on inclusion in mathematics education. This theoretical perspective is Discourse Analysis (DA) and will be described in the next chapter, Theoretical Framing.
Summary background

In this chapter, SEM and its relation to inclusion in mathematics education was discussed. It was shown that the term “inclusion” is hard to define and has different definitions depending on from which epistemological field it derives. In this thesis, the term “students in special educational needs in mathematics” is used because it departs from a social, relational and pedagogical perspective on mathematics learning and participation. Also, one’s needs are viewed upon as something that may occur regardless of whether the student is a high or low achiever, whether it is required for a shorter or longer period of time, and whether it is in general or more specific areas in mathematics. The student as being in SEM reflects that the student can get in and out of the specific needs rather than signifying a monolithic deficiency within the student (Bagger & Roos, 2015). When investigating research regarding students struggling to gain access, as well as students in access, an interpretation can be made that students in both cases are in need of special education in mathematics education, and thus, there are reasons to study them both. Therefore, both students struggling to gain access and students in access are in regarded as SEM students in this study.

Research foregrounding students’ meanings of learning mathematics in inclusive classrooms shows how processes of participation in mathematics education are influenced by external factors. The factors are about making mathematics education enjoyable for students and motivating them. This can be accomplished by teachers showing a caring attitude and being aware of the students, explaining the content well, and making “hidden” mathematics visible. These factors can be regarded as culturally situated and different depending on the individual student(s).

In the description of the previous study regarding inclusion in mathematics education foregrounding teachers’ meanings (Roos, 2015), issues of participation in the form of dynamic inclusion, participating inclusion, and content inclusion became visible. These notions stemmed from a meta reflection on the results. Another issue stemming from the
meta reflection was the importance of supporting students in mathematics education between different situations with similar content – to create a content flow. This can be achieved by helping students to recognize similarities. Then, the mathematics education can focus on how and what to teach in mathematics and create the content flow by preparing, immersing and repeating along with making the students aware of mathematical tasks and mathematical representations.

The chapter ends with a reflection on the transition from teacher to student meanings of inclusion. Here, in the shift of focus, theoretical and methodological issues were reconsidered and adjusted according to the new aim and new research questions. These issues concerned what school to investigate (an inclusive school) and the change of theoretical perspective from COP to DA. DA as a theoretical perspective will be further discussed in the next chapter, Theoretical Framing.
THEORETICAL FRAMING

This chapter presents the theoretical perspective of this research. As previously mentioned, this study takes a social view on learning when investigating inclusion in mathematics education. This view considers knowledge in mathematics as well as learning to be social interactions. Hence, investigating social interaction in mathematics education becomes an important piece in the puzzle of understanding mathematics learning. In the study, social interaction is seen from a participatory perspective, which means the process of participating and interpersonal meetings is central. Therefore, the whole process of social interaction(s) and what surrounds these in interpersonal meetings is in focus. To investigate students’ meaning(s) of inclusion in mathematics education through social interactions, a theoretical frame explaining social interaction is needed. In this research, the theoretical and methodological frame of discourse analysis (DA) has been chosen in order to explain social interaction and describe inclusion in mathematics education foregrounding students’ meaning(s) of inclusion. However, given that inclusion is considered as a process of participation that is continuously ongoing, this theoretical and methodological frame provides a snapshot of the process – it may not be the same in other situations, other cultures or in other times.

The first part of this chapter introduces DA and the different approaches of DA used in research. Thereafter, the choice of approach of DA in this study as well as the underlying assumptions of this particular approach and reflections on the use of this specific perspective of DA are presented.

Discourse Analysis (DA)

Given that discourse is a central in every instance of DA (Bayley, Cameron & Lucas, 2013), it is important to understand the notion of discourse. Discourse is understood and used in a variety of ways within research, and it can even be the case that notions other than discourse
are used in DA as the analytical instance, for example, Foucault (1994) uses “statements” as the analytical category in DA (Bayley et al. 2013). When using discourse in a broader sense, it “cover[s] all forms of spoken interaction, formal and informal, and written texts of all kinds” (Potter & Wetherell, 1987, p. 7). Then, discourse(s) can be recognized in social interactions between humans with the help of the language available in the social process. Because of different corresponding views in the use of discourse, Trappes-Lomax (2004) suggests there is nothing to gain with one single definition of discourse. Instead, he provides a set of definitions: 1) “the linguistic, cognitive and social processes whereby meanings are expressed and intentions interpreted in human interaction”, 2) “the historically and culturally embedded sets of conventions which constitute and regulate such processes”, 3) “a particular event in which such processes are instantiated”, and 4) the product of such an event, especially in the form of visible text, whether originally spoken and subsequently transcribed or originally written” (p. 136). In this study, discourse is understood as historically and culturally embedded sets of conventions which constitute and regulate linguistic, cognitive and social processes.

DA is a theoretical and methodological approach that has been developed over the years to counteract to the constructivist perspective and to try to go beyond attitudes and behaviour to be able to see the social (Potter & Wetherell, 1987). When applying DA, different applications and approaches are available because views on discourse can be slightly different, and thus, their application will be slightly different. Some research applies DA as an analytical tool, while other research applies it as a theory, and yet other research applies it as both. In this study, DA is applied as both an analytical tool and as a theory. The different approaches concern how to use discourse analysis as a methodological tool, and they also concern what type of DA is applied. Although DA has several approaches available; all of them concern the study of language in use and examining patterns of language beyond its use in sentences (Trappes-Lomax, 2004). That is, the focus is on meaning of language in interaction and to uncover how things work in the social building of the society (Gee, 2014a). By extension, from a DA perspective, when we create language, we are active reproducers of
culture (Gee, 2014a). Hence, common to all of the approaches in DA is the focus on interpersonal interactions between people – the language – how we communicate and can actually see, hear, and read. Also, common to all of the approaches of DA is study beyond text; that is, the tool of DA helps to construe discourses at different levels in society. Hence, to construe discourses implies, by analysing the use of a certain type of language in a certain type of situation, you can say something about the social world. When going beyond the language, this something becomes visible in construed discourses.

Approaches in DA

As previously stated, there are different approaches in DA, and they can be divided into different categories (Trappes-Lomax, 2004): rules and principles, context and culture, functions and structures, and power and politics. It is important to stress is that, although these categories interrelate, there are no clear-cut borders between them. Nevertheless, they are useful for making the directions of the application of DA visible.

Considering the category of rules and principles, this approach “seek[s] to understand the means by which language users […] make sense” (Trappes-Lomax, 2004, p. 136). These means can be various factors, such as contextual or the utterances of others. Approaches of DA within this category are, for example, conversation analysis and speech act theory (Trappes-Lomax, 2004), which focus on the rules and principles of language.

The category, context and cultures, refers to approaches within DA that focus on situational and cultural differences through language (Trappes-Lomax, 2004). Discursive Psychology is one of the approaches within this category. These approaches focus on reframing questions about cognition. Potter and Wetherell (1987), who are regarded as “the founders” of DA, use DA as a way to reach beyond attitudes and behaviours. At the time when Potter and Wetherell wrote about DA, this was a new way of looking at the world. Discursive psychology, as an approach, is used to reframe questions about cognition and rewrite them into a discourse perspective, with the aim to find out what people know by investigating the text (Barwell, 2013). In discursive psychology, DA
can be used not only as an analytical tool but also a theory when the goal is to make the underlying thoughts in discourse visible. Within mathematics education, scholars have used discursive psychology to investigate, for example, mathematics teacher knowledge (Barwell, 2013) and discursive practices in mathematics teacher education (Skog, 2014). Another approach within context and culture is presented by Gee (2014a), who offers a way to use DA as both a theory and a method with a focus on description. Big and small discourses are used as overarching theoretical notions, (D)iscourse and (d)iscourse, where big Discourse focuses on a wider context that is social and political, and small discourse focuses on language in use, as in, what we can see in the conversations or stories we investigate.

In the category, functions and structures, the linguistic view in terms of text and grammar in language is in focus (Trappes-Lomax, 2004). Here, system functional linguistics, SFL (Halliday, 2007) is one way of looking at discourse. In SFL, the focus is the positioning and organization of ideas and how these ideas relate to one another in texts. This is made through a detailed analysis of the text, where text is examined from a semantic point of view. One example of SFL in mathematics education is Morgan (2006), who uses Halliday’s linguistics in the analysis of students’ solutions to mathematics tasks. The focus is on social semiotics, where Morgan (2006) uses SFL as an analytical tool to identify activities, relations and the function of the text. A Scandinavian example of the use of SFL in mathematics education is with Ebbelind and Segerby (2015) who use SFL to gain insight in the operationalization of different concepts of mathematical literacy in texts.

Power and politics is another category in DA, where the research has an interest in power and political issues. Critical discourse analysis, CDA (Fairclough, 2010), is a well-used theory and analytical tool within this category. Fairclough (2010) uses CDA to analyse language and its involvement in the workings of capitalist societies. Hence, the critique in CDA is the focus on identifying preventions and limitations in society. One example of the use of CDA in mathematics education research is with Herbel-Eisenmann and Wagner (2007), who use CDA to uncover the social positioning of mathematics textbooks experienced by
students. Here, the CDA perspective is used to highlight how the text positions the students in relation to teachers and other students. Another use of discourse seen within the category of *power and politics* is the use of discourse by the French philosopher and social theorist Michel Foucault. When using Foucault’s notion of discourse, power becomes relational. The discourse is formed by power relations within and between individuals and institutions (Foucault & Hurley, 1997). Hence, the definition of discourse is related to knowledge, truth and power regulating what is possible in a social situation – who is permitted to do what and when (Foucault & Hurley, 1997). Foucault’s notion of discourse does not provide any analytical tools, but rather it is more of a philosophical view on social life, bringing awareness to power relations, positions and governmentality in society. In mathematics education, there are several scholars who draw from Foucault. One example is Meaney (2004), who draws on Foucault’s ideas of power and control when investigating positions for a project in an indigenous community that was developing a mathematics curriculum. In the Scandinavian context, Bagger (2015) draws on Foucault’s thoughts about power, positioning, disciplining and governing when investigating national tests in mathematics.

This multi-faceted division indicates a broad and diverse use of DA, which can be interpreted as an asset because DA can be used in different ways in different areas. As Gee (2014, p. 1) states, “No one theory is universally right or universally applicable. Each theory offers tools which work better for some kinds of data than they do for others”. Accordingly, Gee (2014) has a fairly pragmatic view of the use of theory as a set of tools in relation to data. However, Ryve (2011) critiques the diverse use of discourse in the field of mathematics education, claiming a tension in the use of discourse. According to Ryve (2011), this tension is visible in the different uses and nonrelated research in mathematics education using discourse which, from his point of view, undermines further sophisticated development of theoretical approaches in this field. Hence, the multi-faceted division regarding the use of DA can be interpreted as both an asset and a tension.
Looking at the different categories in relation to a critical approach, one might say that all DA is critical (although there is a specific approach called critical discourse analysis, CDA) given that “all language is political and all language is a part of the way we build and sustain our world, cultures, and institutions” (Gee, 2014a, p. 10). Although, some approaches in DA are more descriptive in nature, with the goal to understand how language works in order to understand the world. In the more critical approaches of DA (like CDA), the goal is not only to offer explanations but also to intervene and highlight controversies of the world (Gee, 2014a).

Many more scholars use these approaches as well as other approaches of DA. Nonetheless, as stated, they all have in common an interest in investigating the social in society through what we actually see, hear and read to make the invisible in society visible. However, as shown, DA can look and be used very differently. The choice of approach and use of DA depends ultimately on the aim and research question. This brings us to the choice of approach in this study, which is described in the next section.

DA in this study

Which DA approach should be chosen for this study of inclusion in mathematics education? One possible way to investigate what frames inclusion in mathematics education is to investigate how students talk about the ways in which they are included in mathematics taught in different situations. This can be done by identifying the ways in which students talk about, act, and produce items in school mathematics, which also falls is in line with the aim of this study – to contribute to research and practice within the field of special education in mathematics with more knowledge about, and an understanding of, students’ meaning(s) of inclusion in mathematics education. Although issues of inclusion automatically bring to mind issues of power, this is not what is foregrounded in the aim of this study. The aim stresses a need to understand inclusion in students’ meaning(s) of inclusion in mathematics education and what frames inclusion in mathematics education. This is specified in the research questions, What meaning(s) do the students ascribe to inclusion in mathematics learning and
teaching, and what frames students’ meaning(s) of inclusion in mathematics learning and teaching? Hence, the aim and the research questions would benefit from a theory and methodology that can describe discourses framing inclusion. That is why an approach of DA from the category *context and cultures* is applied in this study, with a focus on situational and cultural differences. DA as described by Gee (2014a 2014b) was chosen because of its explanatory view on discourse, with description foregrounded. However, in the background, issues related to critical and political aspects are present. With help from these descriptions, I describe how students ascribe meaning(s) to inclusion in mathematics and what frames inclusion in mathematics.

From Gee’s standpoint, DA encompasses all forms of interaction, both spoken and written, and he provides a toolkit for analysing such interactions. As mentioned, Gee distinguishes between two theoretical notions, big and small discourses, henceforth referred to as *Discourse* (*D*) and *discourse* (*d*). Discourse represents a wider context, both social and political, and is constructed upon ways of saying, doing, and being. Discourse is socially accepted ways of using language and other expressions of acting and thinking (Gee, 2012): “If you put language, action, interaction, values, beliefs, symbols, objects, tools, and places together in such a way that other recognize you as a particular type of who (identity) engaged in a particular type of what (activity), here and now, then you have pulled of a Discourse” (Gee, 2014 a, p. 52). Hence, recognition is crucial in Discourse, for instance, being recognized as an ambitious, lazy or struggling mathematics student. However, Discourse and recognition are reflexively related (Gee, 2014a), that is, they create each other and no one knows what came first, like the hen and the egg. Discourse always encompasses language plus “other stuff” (Gee, 2014a, p. 52), including actions, interactions, values, symbols, objects, tools and places. Discourses are simultaneously embedded in various social institutions involving various sorts of properties and objects. For example, a Discourse can be school mathematics, where the recognition lies in the mathematics classroom (the place), items such as a textbook and a ruler (tools), the mathematical symbols on the blackboard and in the textbook and the way a mathematics lesson is acted out (actions, interactions and values). When language and the other stuff are
combined in a way that makes them recognizable, the result is Discourse, and the persons engaged in that Discourse are recognized as a particular type, in this case, mathematics students and/or mathematics teachers.

Even though description is foregrounded in Gee’s approach to Discourse, Discourses are intrinsically ideological because they involve a set of values and the distribution of social goods, as in, who is “normal” (p. 159), and who isn’t? (Gee, 2012). Discourses are resistant to internal criticism, as talking about critical opinions undermines the Discourse and those who hold critical opinions are defined as outsiders of the Discourse. In a Discourse, positions are defined in relation to other, ultimately opposing Discourses, and marginalize concepts and values central to other Discourses. Thus, Discourses are intimately related to the hieratical structure of society (Gee, 2012). According to Gee (2014a), Discourses have no clear boundaries; on the contrary, Discourses are constantly developing and changing, as they are affected by the society in time and space. For instance, a Discourse can split into two or more Discourses, and two or more Discourses can fuse into one. This makes Discourses uncountable and constantly on “the move”. New Discourses emerge, and old Discourses die all the time. This constant ongoing change of Discourse happens because of a Discourse’s position in relation to other Discourses (Gee, 2014a). According to Gee (2014a), Discourses do not need to be “grand” or large scale because Discourses are out in the society as a synchronization of people – a kind of dance by people in time and history. This synchronized dance recognizes you as a particular sort who doing a particular something. “Being able to understand a Discourse is being able to recognize such ‘dances’” (Gee, 2012, p. 152). Hence, a Discourse can also be small-scaled. According to Gee (2012), all people early in life learn a specific way of being a person. This is described as a primary Discourse which gives an initial sense of who I am and sets the foundations of my everyday language. Discourses established later in life in a wider community than in the primary Discourse(s) are called secondary Discourses. These are established in institutions, for instance, in schools (Gee, 2012). Hence, in this study, only secondary Discourses will be used, as the focus is on students meaning(s) of inclusion in mathematics education.
According to Gee (2015), big Discourses set the context for the analysis of small discourses. This brings us to the notion (d)iscourse (with lowercase d), which focuses on language in use – the “stretches of language” we can see in conversations or stories that we investigate (Gee, 2014a, 2014b), meaning the relations between words and sentences, and how these relations visualize the themes within the conversations. These small discourses investigate how the language is used, what typical words and themes are visible, and how the speakers or writers design the language. Small discourse studies the flow of language in use, and the connections across this flow of language in use (Gee, 2015). In one of Gee’s earlier works (2012), the notion of cultural models is used as a synonym to the notion of figured worlds. In a later work (2014a), Gee only uses the notion of figured worlds, which will be used from now on in this thesis. Gee (2012) describes these figured worlds as informal theories that people construct from experiences in the world. In his later work (2014a), he describes figured worlds as typical stories for people in social groups. These typical stories contain images, metaphors and narratives constructed socially and culturally – the form of a simplified world capturing what is taken for granted by a group of people. According to Gee (2014a), the figured worlds are partly in our heads and partly in the space between people, in the talk, and in the society within, for example, the media. We build these models of the world to be able to understand and act in society. And, because society changes, and what is typical in a situation changes, the figured worlds change along with it; hence, they are dynamic and situated (Gee, 2014a): “Figured worlds give us yet another tool for discourse analysis” (Gee, 2014a, p. 90), leading one to ask what typical stories the words in the text invite the readers or listeners to assume (Gee, 2014a). Accordingly, figured worlds are a part of the small discourse, although interrelated with big Discourse because figured worlds are part of “a set of related social practices” (Gee, 2012, p. 104).

To summarise, Discourse is a socially accepted ways of using language, and acting. Small discourse is about the patterns, figured worlds and flow in the language produced within the context of Discourse. Discourse “analysis embeds little “d” discourse analysis into the ways
Underlying assumptions of DA according to Gee

According to Gee, discourse(s) can be recognized in social interactions between humans with the help of the language available in the social process. This implies that, by using language in a social process, you also do things and are things; hence, there are connections between saying, doing and being, and between information, actions and identity (Gee, 2014a). Here, recognition plays an important role in a discourse (Gee, 2014a) because we recognize and can be recognized through our actions and utterances. Hence, language, both in its written and spoken forms, is a central function in any discourse. Gee’s way of viewing the notion of Discourse has underlying assumptions in other social theorists’ ways of describing social and historical processes of learning and identity. Gee’s notion of Discourse is meant to cover a breath of aspects of notions within this field (Gee, 2014a). Moreover, Foucault’s notion of discourse is related to Discourse, according to Gee (2014a). Foucault describes discourses as representations of knowledge, truth and power, governing what is possible to talk about, who can do what and when (Foucault & Hurley, 1997). The discourse is created by humans, and between the individual and the agreed system of representations of the collective. This agreement of representations is closely related to recognition, which Gee refers to when identifying Discourse. Also, Foucault’s ideas about governing relate to Gee’s Discourse in that they are constructed on ways of saying, doing and being, and acted out with actions, interactions and values.

Discourse is connected to the actual context (Foucault, 1969/2002). Thus, to make sense of discourse, the particular context where the language is used is important to recognize (Gee, 2014a: Potter & Wetherell, 1987). Conventionally, context is used as “that which surrounds” (Cole, 1996), but when connected to discourse, the language and the interpersonal interactions in the surroundings are also involved in context. Accordingly, context is related to the setting in which language is produced, including the people, their body language and their shared knowledge. Shared cultural knowledge lies within the context, making
context something that is already there in terms of the physical surroundings and also something that is construed by the way people use language (Gee, 2014a). This way of considering context is related to Halliday’s (Halliday, 2007) definition of context in language education, where context is described as the verbal environment and is divided into context of situation and context of culture. Context of situation refers to the language as text – the type of text, which refers to the genre of the text. Context of culture refers to a reflection of the speakers/writers’ system where language is seen as a system. Skott (2009) has a slightly different interpretation of context and refers to contexts as “shifting intersubjectively established and continually (re-)generated settings” (p. 31). Here, the focus is on the participating individuals and their involvement; hence, the shifting and regenerating of settings depend ultimately on the individuals participating in practices.

Gee (2014 a) refers to the notions “forms of life” and “language games” by Wittgenstein (1953) when describing DA. By forms of life, it is implied that “what has to be accepted, the given, is – so one could say – forms of life” (Wittgenstein, 1953, p. 226e). These forms of life are formed by language and culture, where language is imbued in culture and vice versa. According to Wittgenstein, language is built up with a range of “language games”. These language games are influenced by context, activity and culture and are perceived as “systems of communication” (Wittgenstein, 1953, p. vii). Thus, we can identify a resemblance between language games and Gee’s small discourse as well as a resemblance between forms of life and Gee’s Discourse.

Another notion referred to by Gee is that of “practices”. A practice can be referred to as a social contextual doing (Skott, 2009); hence, it is interrelated with context. All language is ascribed meaning from a practice, suggesting that no language stands on its own, all language is influenced by social and contextual doings. Gee (2014a) describes practice as implicit or explicit rules or conventions governing a situation determining who has acted appropriately or normally. Thus, practice is “a socially recognised and institutionally or culturally supported endeavour” (Gee, 2014a, p. 231). One example of this is how the language is used together with other actions to enact an activity, for
instance, how to speak and act when taking a mathematics test. Here, participants in the practice knows how to speak and act and what to do when taking the test. Foucault (1994, p. 11) uses the term “discursive practices” and describes this as “characterized by the demarcation of a field of objects ... by the setting of norms ... of descriptions that govern exclusions and selections”. Hence, according to Foucault, discursive practices are instances that govern a setting with norms and objects. Even Fairclough (2010, p. 95) uses the term “discursive practice”, referring to it as “the production, distribution and consumption of a text”, which has a dialectical relationship to the social practice. Hence, the discursive practice from Fairclough’s perspective is embedded in the social practice, where the social practice not only covers language but also other dimensions such as institutional and organizational. One example of a discursive practice can be how to behave, act, speak and listen during a mathematics lesson. This discursive practice is embedded in the social practice of school. Gee also refers to how the social theorist and philosopher Bourdieu (1990) uses “practice”. Accordingly, Bourdieu has a theory of practice, which is considered as a “Grand Theory” implying it is an abstract and normative theory which is generic and can be applied to different fields of research. Here, practice is an overarching notion seen “as the result of social structures in a particular field (structure; macro) where certain rules apply and also of one’s habitus (agency; micro), i.e. the embodied history that is manifested in our system of thinking, feeling, perceiving and behaving” (Walter, 2014, p. 15). Likewise, the notion of practice occurs in Communities of Practice by Lave and Wenger (1991), which Gee also makes reference to. Lave and Wenger (1991, p. 98) describe communities of practice as “a set of relations among person, activity, and world, over time and in relation with other tangential and overlapping communities of practice”. Both in Bourdieu’s practice and in Lave and Wenger’s communities of practice, we can see the resemblance to Gee’s theory of Discourse in the attempt to theorize and categorize social relations and activities in a society by recognising sets of relations. Similarly, these sets of relations overlap and relate.

Language is a central empirical instance in all DA, even in DA from Gee’s perspective, where language is ways of saying, doing and being
that have a certain function in a certain situation (Gee, 2014a). Hence, the way Gee views language origins from a functional view on language, and Halliday is often referred to by Gee. Halliday is a linguist who created a system for analysing text called Systemic Functional Linguistics (see the section Different approaches in DA). Halliday refers to language as functional text, meaning language is doing “some job in some context” (Halliday & Hasan, 1989, p. 10). Hence, the function of the language in each and every situation is in focus. A notion used by Gee to describe the complexity of language in use, and how the language incorporates words and phrases from another text, as in, a kind of borrowing, is *intertextuality*. Here, Gee refers to Fairclough’s definition of intertextuality as the property texts have when they are rich with quotations from other texts. Accordingly, intertextuality deals with how the producer of text in a certain situation borrows text from other situations. These situations depend on how the producer of the text views the world and acts in the world. Another term for these other situations is “figured worlds”. As mentioned, Gee describes this as a picture of a simplified world that captures what is taken to be typical or normal. Here Holland et al.’s (1998, p. 52) description of figured worlds is used as Gee’s example: “A socially and culturally constructed realm of interpretation in which particular characters and actors are recognized, significance is assigned to certain acts, and by a set of agents who engage in a limited range of meaningful acts or changes of state as moved by a specific set of forces.”

These underlying assumptions of DA by Gee imply that Discourse aims to cover a lot: situated identities; ways of performing and recognizing characteristic identities and activities; ways of coordinating and being coordinated by other people, things, tools technologies, symbol systems, places and times; and characteristic ways of acting-interacting-feeling-emoting-valuing-gesturing-posturing-dressing-thinking-believing-knowing-speaking-listening-reading and writing (Gee, 2014a, p. 58).

Reflections on the use of DA according to Gee

In this study, Discourse is understood as historically and culturally embedded sets of conventions that constitute and regulate linguistic, cognitive and social processes. Accordingly, Discourse involves
interpersonal interactions between humans, where recognition is central through language. The language is spoken and written texts in a practice, with practice defined here as socially recognized and institutionally and culturally supported doings. In this study, the practice is mathematics education in school. Text is understood as the language produced in this practice, framed by the context and the culture. In this study, the culture is perceived as embedded in the context, and context is defined as the intersubjective setting in which language is produced. Accordingly, context is something that is already there in terms of the physical surroundings and also something that is construed by the way people use language. As stated, DA according to Gee (2014a, 2014b) is applied in this study because of its explanatory power to describe, in this case, students’ meaning(s) of inclusion in mathematics education.

Despite the critique of how Gee presents Discourse (Sfard & Prusak, 2005, Chronaki, 2013) as a kind of monolithic phenomenon of socially accepted associations, I do not align my position with how Gee (2014a) describes Discourses, or identity work, as either monolithic or neutral when describing meanings and framing factors of inclusion in mathematics education and how they relate. In this study, I regard the different discourses or Discourses as not entirely separate, but rather as often overlapping in a dialectic relationship, meaning they relate and influence each other. The Discourse implies a particular space between humans where recognition and belonging is central through language. Moreover, I do not consider discourse(s) or Discourse(s) to be neutral but rather consider them as socially accepted associations guided by practice and context, and also, the other way around, where practice and context are guided by discourse and Discourse.

In this study, secondary Discourses are in focus and used as small-scale Discourses in relation to the focus of the study – students’ meaning(s) of inclusion in mathematics education. This implies the Discourses are about ways of being, doing and talking in the practice of mathematics education in school. The discourses study the flow of language in use, and the connections across this flow of language in use. This implies the discourses study how the language is used by the students, what typical
words, themes and figured worlds are visible and how the speakers or writers design the language. The figured worlds are seen as part of the discourses as well as communicators between the Discourses and discourses. By using words and themes and figured worlds to construe discourses, Discourses can also be construed. Then, the key distinction between discourse and Discourse is that discourse is seen at text level describing issues tightly connected to the students, and Discourse is seen on a larger level, describing framing issues for the students. In this study, with the help of discourses, I expect to find students’ meaning(s) of inclusion in mathematics education, and with the help of Discourses, I expect to be able to find what frames inclusion in mathematics. Thus, discourses and Discourses work together and interrelate. In this study of inclusion in mathematics education, I see the relation between Discourse and discourse as an enduring dialogic process in time and space, creating and recreating the students’ meaning(s) of inclusion in mathematics education. Here, Discourses set a larger context for the analysis of discourses (Fig. 2).

Also, given that big and small discourses are a form of DA, a critical and a political aspect is present in the analysis.

![Figure 2](image-url)  
*Figure 2.* The dialogic process between discourse and Discourse regarding the meaning of inclusion in mathematics.
To operate D(d)iscourse in the study, a toolkit presented by Gee is used as a methodological instrument. This toolkit consists of questions to be asked to the text based on tools of inquiry. The use of the toolkit in this study will be presented in the next chapter, Methodology.

Theoretical framing – summary

To be able to investigate inclusion in mathematics education through social interactions, a theoretical frame in order to explain social interaction is needed. In this study, the theoretical and methodological frame of discourse analysis (DA) has been chosen in order to explain social interaction and frame students’ meanings of inclusion in mathematics education. However, given that inclusion and participation is considered a continuously ongoing process, this theoretical and methodological frame provides only a snapshot of the process.

DA is a theoretical and methodological approach that has been developed over the years as counteract to the constructivist perspective to try to go beyond attitudes and behaviour to be able to see the social (Potter & Wetherell, 1987). Different applications and approaches are found within DA; thus, given that views on discourse are different, their application will also be slightly different. Some research applies DA as an analytical tool, while other research applies it as a theory, and yet other research applies it as both. Although the field of DA has several approaches available, all of them concern the study of language in use and examining patterns of language beyond its use in sentences (Trappes-Lomax, 2004). That is, the focus is on language meaning in interaction and to uncover how things work in the social building of the society (Gee, 2014a). Also, common to all the approaches of DA is study that goes beyond text, that is, the tool of DA helps to construe discourses at different levels in society.

To explain the different approaches of DA, a division can be made into different fields (Trappes-Lomax, 2004): Rules and principles, context and culture, functions and structures, and power and politics. These fields are interrelated, and there are no clear-cut borders between them.
Considering the field of *Rules and principles*, this field “seek[s] to understand the means by which language users [...] make sense” (Trappes-Lomax, 2004, p. 136). These means can be various factors, such as contextual factors or the utterances of others. The field, *context and cultures*, refers to approaches within DA with a focus on situational and cultural differences through language (Trappes-Lomax, 2004). In the field, *functions and structures*, the linguistic view in terms of text and grammar in language is in focus. *Power and politics* is another field in DA, where the research has an interest in power and political issues.

For this study, an approach of DA from the field of *context and cultures*, with a focus on situational and cultural differences, was chosen – more specifically, DA as applied by Gee (2014a, 2014b) was chosen, because of its explanatory view on discourse, with description foregrounded. However, in the background, issues involving critical and political aspects are present. With help from this description, I describe how students express the meaning(s) of inclusion in mathematics education and what frames inclusion in mathematics education. DA encompasses all forms of interaction, both spoken and written, and Gee provides a toolkit for analysing such interactions. Gee distinguishes two theoretical notions, big and small discourses – *Discourse (D)* and *discourse (d)*. Discourse represents a wider context, both social and political, and it is constructed upon ways of saying, doing, and being: “If you put language, action, interaction, values, beliefs, symbols, objects, tools, and places together in such a way that other recognize you as a particular type of who (identity) engaged in a particular type of what (activity), here and now, then you have pulled of a Discourse” (Gee, 2014 a, p. 52). When looking at discourse (with a small d) from Gee’s standpoint, the focus is on language in use: the “stretches of language” we can see in conversations we investigate (Gee, 2014a, 2014b), meaning the relations between words and sentences, and how these relations visualize the themes within the conversations. These small discourses can inform on how the language is used, what typical words and themes are visible, and how the speakers or writers design the language. According to Gee (2015), Discourse sets a larger context for the analysis of discourse.
Underlying assumptions of DA as applied by Gee imply that Discourse aims to cover much ground: situated identities; ways of performing and recognizing characteristic identities and activities; ways of coordinating and being coordinated by other people, things, tools, technologies, symbol systems, places and times; and characteristic ways of acting-interacting-feeling-emoting-valuing-gesturing-posturing-dressing-thinking-believing-knowing-speaking-listening-reading and writing (Gee, 2014a, p. 58). When Gee describes discourse analysis as a tool, he makes the underlying assumptions operationalizable.

Discourse and discourse (Gee, 2014a) are used in this study to describe and explain students’ meaning of inclusion in mathematics education by analysing language. When analysing the language of the students (and the teachers), the relations between words and sentences in their text and how these relate visualize themes and figured worlds within the conversations. From these themes, discourses can be construed. Furthermore, Discourses can be construed when adding considerations of the context and practice in the research.

However, it is important to be mindful when using Gee’s notions of Discourse and discourse, and not see them as clear-cut categories. A critique of Gee’s way of using discourse within mathematics education research concerns this issue and the way Gee describes Discourse and identity (e.g. Sfard & Prusak, 2005, Chronaki, 2013). The critique is in regard to how Gee presents Discourse as a kind of monolithic phenomenon; however, I see Discourses, or identity work, as neither monolithic, nor neutral but rather as a way of describing meanings of inclusion in mathematics education and how they interrelate. In this study, I regard the different discourses or Discourses as not entirely separate, but rather as often overlapping in a dialectic relationship, meaning they relate and influence each other. Thereby, small discourses and big Discourses work together and interrelate. In this study of inclusion in mathematics education, I see the relation between discourse and Discourse as an enduring dialogic process in time and space, creating and recreating the story of inclusion in mathematics education.
METHODOLOGY

In this study, students’ meaning(s) of inclusion in mathematics education is framed and described. This is made by studying student talk and social actions in the context of mathematics education in lower secondary school.

This chapter starts off with a discussion on case studies. Then, a description of the setting of the study in terms of the Swedish educational system, choice of school and participants follows. Next is a description of the data constructed in the study followed by a description of the data analysis process and an empirical example showing the analytical process. The chapter ends with quality considerations, including ethics when researching students’ meaning(s) of inclusion.

Case studies

A case study attempts to capture something of special interest in detail (Patton, 2002) and involves an intensive study of an individual unit (Flyvbjerg, 2011). To make the choice of a particular individual unit of study and to set its boundaries is a key factor in conducting a case study (Flyvbjerg, 2011). Ragin (1992a) calls this process “casing”, and it is a research tactic used to find boundaries for the case and to “wash[es] the empirical units of its specificity” (p. 220) to make only certain features relevant. According to Flyvbjerg (2011), case studies stress developmental factors and a relation to the environment. This implies that cases occur and evolve over place and time and therefore are connected to their specific environment. Hence, the individual unit of study is demarcated by the environment, which in turn creates the boundaries for the individual unit of study. Case studies are also intense, with depth that involves developments over time, details, richness and variance. In this study, the casing involves students in an inclusive school setting. However, not just any students but rather students regarded as being in SEM by the teachers.
Stake (1995) distinguishes between three different kinds of case studies: intrinsic, instrumental, and collective. *Intrinsic case studies* aim at learning about a specific case in depth. In *instrumental case studies*, a particular case is used to learn and develop a more general understanding of factors that go beyond the case itself. And *collective case studies* look at several cases within the same case study. What is a case then? According to Ragin (1992b), a case can be theoretical, empirical or both and is bound to an object or a process. The case reflects a subject of special interest that one seeks to understand (Stake, 1995) and relate to the aim of the research. A detailed case study brings valuable knowledge to a study that goes in depth about what is being investigated and is important for a nuanced view of reality (Flyvbjerg, 2011).

When looking at the unit of analysis in a case study, more than one object can be studied (Ragin, 1992a; Patton, 2002), yet still be within the overall case. What constitutes this overall case ultimately depends upon the research questions and the way the study is conducted. This is highlighted by Ragin (1992b), who states that researchers would probably not know what their cases are until the research is completed. Also, Ragin (1992b, p. 6) stresses the importance of repeatedly asking the question, “What is this a case of?” during the research process.

In this study, the object of study is the meaning(s) of inclusion in student talk. To be able to understand students’ meaning(s) of inclusion in mathematics education, there was a need to study student talk. This reflects how the study is an instrumental and collective case (Stake, 1995), as it is a case with several students’ meaning(s) aiming at develop a more general understanding of inclusion. To be able to grasp student talk, the investigation required a school context concerning students participating in mathematics education. Also, given that the focus of the object of the study is students’ meanings of inclusion, this school ought to be working inclusively (meaning it aims at include all students in the ordinary classroom teaching in every subject). Consequently, a school working inclusively was contacted in the selection of SEM students. Patton (2002) describes this kind of choice as an information-rich case for in-depth study. From such an in-depth study, one can learn about issues that are central to the purpose of the study, also called
“purposeful sampling”. The information-rich case (Patton, 2002) in this study is thus that of students in mathematics education at an inclusive school. Using Flyvbjerg’s (2006; 2011) notions, one can also call this kind of selection “information oriented”, and the case is an extreme one – a choice made in order to get “a best case scenario”. An extreme case is a case to “obtain information on unusual cases which can be especially problematic or especially good in a more closely defined sense” (Flyvbjerg, 2011, p. 307). This extreme case can help to understand the limits of theories and be helpful in developing new concepts. One can also say that this case is somewhat critical. By “critical”, Flyvbjerg (2011) refers to cases that achieve information by permitting logical deductions like “[I]f this is (not) valid for this case, then it applies to all (no) cases”. (Flyvbjerg, 2011, p. 307). Hence, searching for critical cases implies a search for a “most likely” or a “least likely” case. When looking for extreme and critical cases in this investigation, the extreme case is used to obtain information about students’ meaning(s) of inclusion. This case is expected to be an especially good case because, over the past several years, the school have been explicitly working inclusively (for a thorough description of this inclusivity approach, see section “The school”). Further, the case is expected to be especially good, as the students selected for the study at this school are considered as SEM students and not randomly selected students (for a thorough description of the selection of student, see the section “Participating students – extreme cases”). If students’ meaning(s) of inclusion is not explicit in this case, namely, SEM students at an inclusive school, then where would it be?

Sometimes sceptical voices are raised about the value of conducting case studies, which Flyvbjerg (2011) argues are connected to misunderstandings about case studies. One such misunderstanding is that “general and theoretical knowledge is more valuable than concrete case knowledge”( p. 302). However, Flyvbjerg argues that we certainly can learn a lot from case studies, perhaps even more than from other types of studies. This because of the in-depth and concrete environment-dependent knowledge that is produced in case studies. According to Flyvbjerg, case studies are not about proving something but about learning something about our social world. Another misunderstanding
brought to the fore by Flyvbjerg (2011) is the opinion that if the study is not generalizable, then the study does not contribute to research development. Flyvbjerg claims that case studies can indeed be generalizable, depending on the case and how it is chosen, and concludes that you can often generalize a single case because the force of example and transferability are underestimated. Consequently, if the case is chosen with care and is thoroughly described, there is a force in the example of the case which also makes it possible to compare with other situations and reflect upon the similarities and differences.

In this study, as previously stated, the choice of individual unit of study is the meaning(s) of inclusion in student talk and the case is SEM students at a school working inclusively. To complete the “casing” and to underpin the force of the example and the possibility of transferability, there is a need to further describe the school setting, the mathematics lessons, and the selected students. These descriptions will set the boundaries for the case. Hence, in the following sections, the setting of the study will be further elaborated and described.

The setting of the study
The setting of the study refers to the school and the mathematics education at this school as well as the choice of students as information-rich and extreme/critical cases. To get a sense of the school culture this school, and thus this study, is embedded in, the section starts with an overall description of the Swedish educational system.

The Swedish educational system
In Sweden, students start compulsory school the year they become 7 years old, and the compulsory school ends the year they become 16 years old. Before compulsory school, most children have attended one year of preschool. Until 2018, preschool was voluntary, but now it is obligatory. Lower primary school is divided into Grades 1–3 (students 7–9 years old), and upper primary school is divided into Grades 4–6 (students 10–12 years old). Thereafter, most often the students change school to enter lower secondary school, which is Grades 7–9 (students 13–16 years old). It is most common for the primary and secondary
school to be a public school ("comprehensive school" in the UK), but there are also private school alternatives. After compulsory school, most students enter a national programme at an upper secondary school. Most often these programmes are for three years and end with an upper secondary degree. For all levels of the educational setting, there are no school fees in Sweden.

The teaching of mathematics starts at the preschool class level. Most often the mathematics teacher in preschool class and primary school is a general teacher who covers other subjects as well. In lower secondary school, the teachers who teach mathematics are usually specialized in mathematics. Many schools have at least one special teacher in mathematics development (henceforth referred to as "special teacher"), which is a teacher who is specialized in SEM (the special teacher has 1.5 years additional training in SEM at an advanced level, which is studied after the teaching degree and in general at least, three years of working in the profession).

In the Swedish mathematics curriculum, it is stated that "teaching should aim at helping students develop knowledge of mathematics and its use in everyday life and different subject areas" (Swedish National Agency for Education, 2018, p. 55). The teaching in mathematics should give students the opportunity to develop five abilities (Swedish National Agency for Education, 2018):

- Formulate and solve problems using mathematics and also assess selected strategies and methods, use and analyse mathematical concepts and their interrelationships, choose and use appropriate mathematical methods to perform calculations and solve routine tasks, apply and follow mathematical reasoning, and use mathematical forms of expression to discuss, reason and give an account of questions, calculations and conclusions. (Swedish National Agency for Education, 2018, p. 56)

These five abilities are meant to be thought of in each area of the so-called core content. For both primary school and lower secondary school, the core content is divided into the following main sections:
The school
A public lower secondary school in an urban area in Sweden was chosen for this study. The school has approximately 550 students and 5 classes in each grade from Grade 7 (13 year olds) to Grade 9 (16 year olds). The catchment area is both urban and suburban, with detached houses and apartments, and there is a cultural as well as a social diversity. This school has set out to work inclusively, meaning its aim is to include all students in the ordinary classroom teaching in every subject and to incorporate special education into the ordinary teaching with no fixed special educational groups: “Inclusion is a core issue for us, everybody is welcome in the classroom. […] The support will primarily take place within the classrooms by co-teaching between the teachers and special teachers” (from the school’s website, own translation). As a first step in the selection of the school, the principal was contacted over the phone. I explained the aim of the study, and the principal explained the idea of inclusion at the school. He also thought it was a good idea to study inclusion and was interested in contributing to the study. The initiative for the inclusive approach at the school came from the special teachers at the school who were unsatisfied with the special education and support for students in need. The special teachers felt that when working with special educational groups, they stigmatized students rather than enhanced their learning. They talked to the principal, who was interested. After one semester of preparing with study visits to other schools that work inclusively and having collegial discussions, the approach was implemented over a period of two years. This resulted in the cancellation of all fixed special educational groups, and every student was present in the ordinary classroom, where two teachers were present most of the time. This was usually with one special teacher and one regular teacher. When this study started, the school had been working inclusively for five years. This way of working is not typical for Swedish schools, although it has become more common in recent years, as studies have shown good results when working inclusively (e.g. Persson & Persson, 2012). As previously mentioned, the focus of the
object of study is students’ meanings of inclusion. Hence, the selection of this school was made because it worked inclusively and that meant I would get an information-rich case. This case would help me investigate central issues of inclusion in mathematics education – a purposeful sampling for investigating students’ meaning of inclusion.

The process of informing the school
After selecting the school, emails were sent to a teacher at the school with information about the study, and a date was decided for when I would come and meet the teachers and present the study for all the mathematics teachers at the school. On 21 October, a meeting was held with the mathematics teachers at the school where I presented the study. Nine mathematics teachers and the principal were present. The teachers discussed the selection and were keen on the idea to select both students struggling to gain access and students in access. In this study, I wanted to focus on students in SEM, but as I did not want to impose my interpretation of SEM on anyone, the teachers were asked to reflect on SEM and suggest SEM students. They reflected on this at the meeting and found two criteria: students are in SEM if they struggle to reach the accepted knowledge requirements according to the curriculum, or if the teachers find that they have difficulties challenging the students who are in mathematics facility and able to reach the highest knowledge requirement. Together, we agreed on two classes, Grades 7 and 8, and to choose 2–4 students in each class. Thereafter, the principal and the teachers gave them consent forms to fill in if they wanted to participate in the study (Appendices 5–6). I collected the written consent both by email and by hand. Before entering the field on 11 December, I visited the chosen classes and introduced the project to the students. Here, I choose to describe the project as being about inclusion, and not particularly about SEM, out of ethical considerations. The students

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1 As previously stated, in this study students struggling to gain access implies struggles for gaining opportunities to participate in the mathematics education, and by that, enhanced access to mathematics. Students in access to mathematics suggests an access to the mathematics worked with in the classroom but with a need to get opportunities for full participation in relation to the mathematical content (implying a need for other mathematics than what is presented in the education), and relate to others, and by that enhanced access to mathematics.
received written information about the project, which was included in the letter of consent (Appendices 1–4).

Participating students – extreme cases
As described above, the selection of students was made after suggestions from the mathematics teachers and the special teacher. They suggested students they perceived as being in some kind of SEM and were either students struggling to gain access or students with access to mathematics. Although, they were mindful in their choice, for instance, they did not choose students who had many social struggles or many struggles in many subjects. If the students and parents consented (see Appendices 1–4), the students were offered to take part in the study.

Six students were chosen to be followed: three in Grade 7 and three in Grade 8. One student in Grade 7 and one in Grade 8 were chosen because they were perceived as students in access to mathematics, Billy (Grade 7) and Edward (Grade 8). The teachers said that these students needed something else than what was usually offered in their mathematics education in order to enhance learning, but the teachers struggled to know what that was. The other students were chosen because they were perceived as struggling to gain access to mathematics, Veronica and Slatan in Grade 7, and Jeff and Ronaldo in Grade 8. The students chose their own pseudonyms. The selection did not take gender into consideration, so the fact that there is only one girl is by random chance.

In this thesis, the focus is on Veronica in Grade 7 (13 years old), Ronaldo in Grade 8 (14 years old) and Edward (14 years old) in the same class as Ronaldo. This choice of focus is because Ronaldo and Edward were taking part in the same mathematics education in the same classroom but were perceived as either in struggle to gain access or in access. Veronica was chosen because she was in a different class. Her meaning was interesting to investigate further and to compare with Ronaldo and Edward because she was in another mathematics class. Due to the time-consuming process of analysing these three cases in depth, the other three students are not included in the results.
Veronica articulates, “Math is pretty hard” and “I don’t like maths”. The mathematics teachers describe her as struggling to gain access to mathematics, and she is just above the threshold for a passing grade. Ronaldo talks about himself as a student with learning difficulties: “I have difficulties within all subjects, and it’s like concentration and all that.” He says that he forgets: “I don’t remember, I have to repeat a lot”, referring to his work with mathematics at the lessons. The mathematics teachers describe him as struggling to gain access to mathematics and just above the threshold for a passing grade. Edward talks of himself as a student that thinks mathematics is really easy and does not need much support. He says he does not struggle at all, as mathematics works “automatically” for him and he “already knows” most of the content in mathematics presented during the lessons. The mathematics teachers perceive him as having excessive access to mathematics, and he has the highest grade in mathematics.

Reflection on the selection of participating students
In the selection of SEM students in this study, the teachers discussed the students they thought were struggling to gain access and students in access. The definition of SEM was not something that was imposed by me, but nevertheless, the teachers seemed to have an idea of what it means to be a “typical” SEM student. This suggests there is a collective meaning of a SEM student in access and a SEM student in struggle to gain access at the school. This meaning is imposed on the students by a school mathematics discourse; hence, the idea of being in need is socially constructed.

Participating teachers
Although the focus of this study is on students’ meanings of inclusion, the fact that the students take part in mathematics education also means that the teachers are important despite being in the background. The teachers who took part in this study are the special teacher, Karen (who has a degree as a Special Education Teacher in Mathematics); the Grade 7 Mathematics teacher, Oliver (who has a degree as a Mathematics teacher for lower secondary school); the Grade 8 Mathematics teacher, Tess (who has a degree as a Mathematics teacher for lower secondary school); and the special teacher in Reading and Writing, Christie (who
has a degree as a Special Education Teacher in Reading and Writing). The principal at the school was also interviewed. The interviews with the teachers and the principal are not considered a part of the case but rather as providing additional information to understand the environment.

The data

In this study, the research questions aim at describing inclusion in mathematics education, both with a focus on research in the area and a focus on students’ meaning of inclusion. Two different sets of data have been used to answer all three research questions. The first set of data contains research literature aiming to answer Research Question 1, which focuses on inclusion in research (also found in Article 1). The second set of data contains interviews and observations aiming to answer Research Questions 2 and 3, focusing on students’ meaning of inclusion (also found in Roos, 2019b; Roos, 2019c). This will be further elaborated in the following paragraphs.

Data for Research Question 1

To answer Research Question 1, What meaning(s) is ascribed, and how is inclusion used in mathematics education research? I needed to investigate research literature, and hence, collect relevant literature. This collection was carried out by a systematic search of literature. In the search, various criteria were used for selecting studies to review. One criterion was that the studies connected inclusion to mathematics education and school. Also, only studies in English or in the Scandinavian languages were selected, as I am fluent in those languages. The date of publication was another criterion – to find research on the recent state of inclusion in mathematics education, the time span was set to 2010–2016. Finally, a quality criterion was used, and therefore, only peer-reviewed research was included.

Five databases were searched: ERIC, Web of Science, PsycINFO, SwePub, and the Mathematics Education Database. The databases were selected because their coverage of pedagogical and educational research both nationally and internationally. The search string contained the
words inclus* or inklu* and education* or school or undervis* or skol* or utbildn*, and then math* or matemat* was added. Thereafter, the criteria for timespan and peer-reviewed research were applied. To limit the search in PsycINFO, another criterion was added to search for research concerning 6–17 year olds (school age). In Web of Science, the criterion research within education and special education was added in order to limit the result. In SwePub, the search string (inclus*) (education) (year) and (math*) was used because of the difference in structure of the database. The search resulted in 1,296 research studies. Of these, 76 studies were retained after the aforementioned criteria were applied and 19 duplicates had been removed. These studies comprised 52 journal articles, 18 book chapters, 4 conference papers and 2 theses. Table 1 displays the distribution of the research selected by year.

<table>
<thead>
<tr>
<th></th>
<th></th>
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<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>12</td>
<td>9</td>
<td>12</td>
<td>9</td>
<td>23</td>
<td>7</td>
<td>76</td>
</tr>
</tbody>
</table>

*Table 1. Distribution of the research material by year.*

Data for Research Questions 2 and 3

To answer Research Questions 2 and 3 – What meaning(s) do the students ascribe to inclusion in mathematics learning and teaching? And what frames students’ meaning(s) of inclusion in mathematics learning and teaching? – the meaning the student ascribes to inclusion in the classroom needed to be understood. Arising from this was the question of what type of data can show the students meaning(s)? To generate data capturing the meanings, I chose to observe classrooms and interview students not only once but several times so that I could follow them, interview them, and make observations in order to obtain in-depth data. Also, the longer perspective could serve as a surety for capturing the students meaning(s), not only once, but over time. When spending time in the field interviewing the students and participating in their mathematics education at least twice a month (approximately 45–60 minutes each lesson), it was clear that I needed to talk to the teachers.
too. During the semester, the involved teachers and the principal were interviewed. Both the observations and interviews were conducted during spring semester 2016 (two observations were made the autumn semester 2015 when the project was introduced to the students in each class).

**Observations in the study**

Classroom observations were made to understand the environment and also to be able to pose situated questions in the interviews. The observations were in a Grade 7 and a Grade 8 classroom at the school. Observation notes were made and photos were taken of the blackboard, for example. At least one mathematics lesson each month for each class was observed, and student interviews followed each observation. The observations were collected to be used in the analysis as a part of the collected data as well as to provide a context for the interviews. In total, 21 lessons were observed, totalling approximately 17 h (1030 min) (Table 2).

<table>
<thead>
<tr>
<th>Grade 7</th>
<th>Grade 8</th>
<th>Extra math – voluntary in Grades 8 and 9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Date</td>
<td>Length</td>
<td>Date</td>
</tr>
<tr>
<td>11/12/2015</td>
<td>50 min</td>
<td>11/12/2015</td>
</tr>
<tr>
<td>22/1/2016</td>
<td>50 min</td>
<td>22/1/2016</td>
</tr>
<tr>
<td>4/2/2016</td>
<td>50 min</td>
<td>3/2/2016</td>
</tr>
<tr>
<td>12/2/2016</td>
<td>50 min</td>
<td>12/2/2016</td>
</tr>
<tr>
<td>22/3/016</td>
<td>60 min</td>
<td>16/3/2016</td>
</tr>
<tr>
<td>5/4/2016</td>
<td>60 min</td>
<td>4/4/2016</td>
</tr>
<tr>
<td>12/4/2016</td>
<td>60 min</td>
<td>15/4/2016</td>
</tr>
<tr>
<td>27/4/2016</td>
<td>50 min</td>
<td>26/4/2016</td>
</tr>
<tr>
<td>24/5/2016</td>
<td>60 min</td>
<td>25/5/2016</td>
</tr>
<tr>
<td>3/6/2016</td>
<td>50 min</td>
<td>3/6/2016</td>
</tr>
</tbody>
</table>

*Table 2. Overview of observations.*

An example of a mathematics lesson

As part of the “casing” that enables the reader to get a sense of the mathematics lessons at the school, in this section, I will describe the
physical surroundings of the school and one mathematics lesson. At this lower secondary school, the students have lockers in the corridor where they can store their books and other personal items. The classrooms are locked, and there is no specific material in the classrooms. The students have one mathematics textbook and one notebook. The textbook is a loan from the school for the school year, and it is to be returned to the school before the summer break. Specific material is carried by the teacher to each classroom (such as rulers, calculators, protractors, etc.). The teachers work in teams, and they have a desk and a computer in a room where the teams are placed. They also keep specific materials in the room, which are brought to the classrooms in a cart. Following is an example of how the events in the observed classroom unfolded.

15 April 2016. Lesson with Grade 8 (the class Ronaldo and Edward attend)

The students come in to the classroom and sit down at their assigned seats. There are high tables and high chairs, with two students at each table. The tables are placed in three rows in the room. At the front is a blackboard, and a desk and chair for the teacher. On the blackboard, it says “11.40–12.30” in the left corner (which indicates the time of the lesson) and “equations, count – new stuff” is written below the time. This indicates the content of the lesson. When the students sit down and are quiet, the teacher, Tess, says, “Hello. Today I am going to repeat a little about how to do equations with you, and add some new stuff so that you are prepared for Monday when we have a substitute teacher. Let’s take out our notebooks”. The special teacher (Karen) comes into the room and stands quietly at the back. Then Tess writes on the blackboard: “4X-10=50”. Tess says, “I want you to write this down in your notebook and solve it.” When the students work in their notebooks, Karen walks around to help the students. After a little while, Tess encourages one student to present a solution on the blackboard. When that is done (correctly), Tess says, “Equations always end with that
you show X=”. After this, the students get a new task, work on it by themselves, and then another student presents a solution on the blackboard. The same procedure is repeated one more time. Tess says, “Equations always end with showing X equals” and writes “X=” on the blackboard. Then Tess presents “What is new now, X on both sides of the equal sign”. She uses a web-based digital teaching resource (Skolplus) to explain how to solve tasks with X on both sides. This is shown by a projection. Karen helps Tess with the explanations by writing on the blackboard while Tess explains orally. Then the students are invited to work with the tasks shown in the program. First they are asked to write the tasks presented by the program in their notebooks; then they are asked to work on them based on the thinking of a balancing scale, with the representation presented in the program. Karen walks around in the classroom and talks to students as they work in the notebooks. After this, there are seven minutes left where the students work with equations tasks in the textbook. Karen helps some students who need help (for instance, Ronaldo). The students indicate that they need help by raising a hand. When the time is out, the students take their textbooks and notebooks and walk out of the classroom. Tess stays to collect her things and briefly talks to Karen about the lesson, then both leave the classroom.

Naturally, there were differences between the observed lessons, but in general, the structure of each lesson was similar to the one above. One difference is that, after the first presentation on the blackboard, the special teacher would leave with one to four students to go into a small room next to the classroom. This is not shown in the narrative above because during this particular class the special teacher stayed in the classroom for the entire lesson. Another exception from this structure was seen in two of the lessons observed. In one lesson, there were elaborations with geometrical objects instead of working with tasks on the blackboard or in the textbook. In the other lesson, the teacher and
the special teacher divided the students into two groups. The special teacher had one group of students, which were the students who choose to repeat basic notions in geometry, and the teacher stayed with the students who wanted to do more advanced tasks in the classroom.

Interviews in the study

The interviews took place in a small room near the mathematics classroom, which was familiar to the students. The interviews were audio recorded. To be able to grasp students’ meaning(s) of inclusion in mathematics, the interviews included questions about the situations and content of the mathematics education from that particular week and what meaning(s) these situations had for the students in terms of being included in the mathematics education. Before the first and last interview, the students filled in a questionnaire (Appendices 7–8) about the mathematics education in terms of learning and teaching, for example, When telling the teacher how you have solved a task, how do you feel? The questionnaire was inspired by questions from a self-assessment questionnaire regarding mathematics from a Swedish national test (Swedish National Agency for education, 2014, p. 34). The inspiration was because the design of the questions does not focus on the actual mathematical knowledge of the students, but rather on the students reflections on their knowledge and learning situations in mathematics. The questions were modified to fit the aim of this study, focusing on students’ meanings of inclusion in mathematics education (see Appendices 7–8). The questions in the first and the last interview were based on the questionnaire. The other interviews were semi-structured based on observations, for example, How did you perceive the task introduced on the blackboard? What tasks did you do? How did that go? Was something easy/hard? When do you learn mathematics best? (Appendices 9–10). Also, every interview ended with the question, Is there anything else you would like to tell me about the lesson, the teaching, or the learning? This question was posed to open up for covering anything we had not discussed in the interview. Both the questionnaire and the questions asked in the interviews aimed at finding students’ meaning(s) of inclusion in mathematics and when they really mean that they have learned mathematics, hence, aiming towards processes of participation and the meaning(s) of inclusion in student
talk. The interviews presented here are only those used and analysed in this thesis. The interviews varied in length and lasted between 7–31 minutes. The students in Grade 8 were interviewed six times each, and the student in Grade 7 was interviewed five times (she was sick one time). To summarize, there were 17 student interviews, which lasted approximately 8 h (458 min). An overview of the student interviews is shown in Table 3 below.

<table>
<thead>
<tr>
<th>Name</th>
<th>Edward</th>
<th>Ronaldo</th>
<th>Veronica</th>
</tr>
</thead>
<tbody>
<tr>
<td>nr</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>3/2–16 31 min</td>
<td>3/2–16 30 min</td>
<td>2/3–16 20 min</td>
</tr>
<tr>
<td>2</td>
<td>2/3–16 15 min</td>
<td>2/3–16 14 min</td>
<td>6/4–16 11 min</td>
</tr>
<tr>
<td>3</td>
<td>6/3–16, 10 min</td>
<td>16/3–16 7 min</td>
<td>22/4–16 14 min</td>
</tr>
<tr>
<td>4</td>
<td>22/4–16 22 min</td>
<td>6/4–16 12 min</td>
<td>4/5–16 10 min</td>
</tr>
<tr>
<td>5</td>
<td>24/5–16 16 min</td>
<td>20/4–16 12 min</td>
<td>3/6–16 11 min</td>
</tr>
<tr>
<td>6</td>
<td>31/5–16 14 min</td>
<td>31/5–16 10 min</td>
<td></td>
</tr>
</tbody>
</table>

Table 3. Overview of student interviews.

To be able to grasp the context of the mathematics education and underlying values and thoughts about inclusive education at the school, the mathematics teachers in the observed classrooms and the special teachers involved in the education as well as the principal were interviewed. In total, five teacher interviews were conducted during the semester when the empirical investigation took place, lasting approximately 2.5 h (143 m). An overview of the teacher interviews is shown in Table 4 on page 72.
Referring on the role of an interviewer

The purpose of interviewing is to enter another person’s way of making meaning in the world, and it begins with the assumption that the meanings of others are meaningful and can give us information and knowledge (Patton, 2002). In DA, interviews are a means to identify and explore the participants’ practices (Potter, 1996): “An interview can be a particularly effective way of getting at the range of interpretative repertoires that a participant has available as well as some of the uses to which those repertoires are put” (Potter, 1996, p. 15). However, from a discursive perspective, interviews do not occur in a complete vacuum but in interaction with the interviewer. In the interaction, the way the interviewer poses questions and acts in the interview situation influences the participants. This can be somewhat challenging, as the question-answer format might lead the participant to reflect on certain topics (Potter, 1996). In this study, by being mindful when posing questions, I tried not to impose my own categories and constructions on the students. Also, this was one of the reasons for repeated interviews. By having several interview sessions with the students over time, I could reflect on the posing of the questions in each interview. Moreover, this meant that the students got a chance to get to know me and the interview situation and thus became increasingly more secure and willing to answer and elaborate on questions without me leading to specific topics.

<table>
<thead>
<tr>
<th>Name</th>
<th>Karen (Special Education teacher in Mathematics)</th>
<th>Christie (Special Education teacher in Reading and Writing)</th>
<th>Tess (Mathematics teacher, Grade 8)</th>
<th>Oliver (Mathematics teacher, Grade 7)</th>
<th>Principal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Duration</td>
<td>3/2–16</td>
<td>6/4–16, 10 min</td>
<td>13/5–16, 35 min</td>
<td>19/5–16, 33 min</td>
<td>3/6–16, 41 min</td>
</tr>
</tbody>
</table>

*Table 4. Overview of teacher interviews.*
Data analysis

Studying student talk regarding their participation in mathematics education and identifying aspects of learning and teaching in the stories is important when looking for ascribed meanings and framing aspects on inclusion in mathematics. In this study, Discourse Analysis (DA) is used as a theory (see the chapter, Theoretical Framing) and as an analytical tool to accomplish this task. By analysing the language and its use in specific situations, something can be understood about the social world. When going beyond the text in this study, construed discourses of what frames and constitutes students’ meaning(s) of inclusion in mathematics education give us access to the particular social world of the student(s).

As previously described in the chapter, Theoretical Framing, this study uses Gee’s (2014a, 2014b) notions of big and small discourses as theoretical concepts of DA. Here, big Discourse(s) describes social and political contexts and small discourse(s) focuses on language in use (both spoken and written) and what stretches of languages are visible in the investigated stories (Gee, 2014a). “Stretches of language” is Gee’s notion to describe small conversations that are evident in the investigated stories.

To analyse different forms of spoken and written language, Gee (2014b) provides 28 tools that highlight the communication by posing questions to the text. These questions open up for investigation of the text and what is beyond the text in terms of d(D)iscourse(s). Some of the tools are linguistic, hence, close to the text and the context of the text, while other tools give access to an interpretive level and are closer to the big picture of what is happening in the social world. Thus, the tools allow for the investigating of the text on two different levels, but they are at the same time communicating to connect these levels. Gee (2014b) divides the tools into four main focus categories: language and context; saying, doing and designing; building things in the world; and theoretical tools (Appendix 11).
The data analysis process
The following is a narrative of how the analysis process was conducted in this study. Important to mention is that, although the narrative tells the story in a straight timeline, the entire analysis involved a constant back-and-forth process.

The process of analysing the texts (the student interviews and observations) started off with reading one of the interviews as a whole transcribed interview. Then I started to search for excerpts in the interview relating to meaning(s) of learning and teaching mathematics. When an excerpt of text was identified as relating to meaning(s) of learning and teaching mathematics, it was selected by colour marking. As a result, excerpts where the students talk about, for example, a field trip or reasons why they were tired was not further focused upon.

In the third phase, the 28 tools Gee (Appendix 11) were applied to this text by posing the questions provided by Gee (2014b). It soon became clear that not all the tools were useful in relation to this type of study and set of text. That is why, after trying this process on two interviews, a selection of tools was made. An example of an unselected tool is, for example, the big conversation tool, because its purpose is to highlight debate and discussions between or among Discourses, which does not fit with the aim of the study to understand students’ meaning(s) of inclusion in mathematics education.

I divided the applied tools into two sets of related tools, linguistic and interpretative (Table 5). The linguistic tools are used to investigate the texts and the context of the texts in detail. The interpretative tools then provide interpretations of what is going on – the big picture of what is happening in the student talk and how student talk can indicate students’ meaning of inclusion. The division was also guided by the four main focus categories Gee uses to explain the tools: language and context; saying, doing and designing; building things in the world; and theoretical tools. Simultaneously with the analysis of the interviews, some of the tools required the inclusion of observations, for example, the frame tool (with the question, Can you find out anything additional about the context?) and the big Discourse tool (with the question, What Discourse is this a
language part of?). The relation between the linguistic tools and the interpretative tools and the four categories are displayed in Table 5. Here, the correspondence between the linguistic and the interpretive tool is shown in the way they have been written in the table. Consequently, during the analysis, there was a back-and-forth process between looking into detail linguistically and doing an interpretation. The analysis can be understood as an ongoing communication between the linguistic and interpretative levels; an example of this can be found with *deictic expressions*. If you take a word like “it”, for example, what does it refer to? What do I need to fill in to understand what “it” is? And how does “it” tie in with the context? Moving between the linguistic level and the interpretative level made students’ meanings of inclusion in the language and context visible in terms of what the SEM students were saying, doing and designing as well as how they were building things in relation to inclusion in mathematics. When applying the theoretical tools, what the students were saying, doing, designing and building in terms of meaning(s) of inclusion in mathematics could be construed into D(d)iscourses.
<table>
<thead>
<tr>
<th>LINGUISTIC TOOLS</th>
<th>INTERPRETATIVE TOOLS</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Language and context tools</strong></td>
<td></td>
</tr>
<tr>
<td><em>Diexis tool</em> – How are deictics being used to tie what is said to context?</td>
<td><em>Fill in tool</em> – What needs to be filled in to achieve clarity? What is not being said explicitly but is assumed to be known?</td>
</tr>
<tr>
<td><em>Making strange tool</em> – What would someone find strange if that person did not share the knowledge and assumptions? Hence, what is taken for granted by the students?</td>
<td></td>
</tr>
<tr>
<td><em>Subject tool</em> – Why has the student chosen the particular subject of the conversation? How does the student organize information in terms of subject and predicates?</td>
<td></td>
</tr>
<tr>
<td><em>Intonation tool</em> – How does the students’ intonation contour contribute to the meaning of utterances? (in the transcripts, emphasized words are in bold).</td>
<td><em>Frame tool</em> – Can I find out any more about the context, and if so, does this change the analysis?</td>
</tr>
<tr>
<td><strong>Saying, doing, and designing tools</strong></td>
<td></td>
</tr>
<tr>
<td><em>Vocabulary tool</em> – what sort of words are being used, and how does the distribution of words function to mark the communication in terms of style?</td>
<td><em>Doing and not just saying tool</em> – What is the student who is talking trying to do? (can be several things).</td>
</tr>
<tr>
<td><em>Why this way and not that way tool</em> – Why does the student build and design grammar in this way and not in some other way?</td>
<td></td>
</tr>
<tr>
<td><em>Stanza tool</em> – How do stanzas cluster into larger blocks of information?</td>
<td><em>Topic and theme tool</em> – What is the topic and theme for each clause? What theme is a set of clauses? When the theme was not the topic and deviated from the usual choice, why was it chosen?</td>
</tr>
<tr>
<td>Building things in the world tools</td>
<td>Theoretical tools</td>
</tr>
<tr>
<td>-----------------------------------</td>
<td>-------------------</td>
</tr>
<tr>
<td><em>Significance building tool</em> – How are words and grammatical devices used to build up or lessen significance for certain things and not others?</td>
<td><em>Context is reflexive tool</em> – How is what the student is saying helping to create or shape relevant context? How is what the speaker is saying helping to reproduce context?</td>
</tr>
<tr>
<td><em>Intertextuality tool</em> – How are words and grammatical devices used to quote, refer to, or allude to other text or other styles of social language?</td>
<td></td>
</tr>
<tr>
<td><em>System and knowledge building tool</em> – How are the words and grammar being used to privilege or de-privilege specific sign systems (e.g. everyday or scientific mathematical concepts) or different ways of knowing and believing?</td>
<td></td>
</tr>
<tr>
<td><em>Topic flow or topic chaining tool</em> – What are the topics of all the main clauses, and how are these topics linked to each other (or not) to create a chain?</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Theoretical tools</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><em>Intertextuality tool</em> – How are words and grammatical devices used to quote, refer to or allude to other text or other styles of social language?</td>
<td><em>Situated meaning tool</em> – What situated meaning does the communication have?</td>
</tr>
<tr>
<td><em>Figured world tool</em> – What typical figured worlds are the words and communication assuming and inviting listeners to assume?</td>
<td></td>
</tr>
<tr>
<td><em>Big “D” Discourse tool</em> – What Discourse is this language a part of? What sort of actions, interactions, values, beliefs, and objects, tools, technologies and environments are associated with this sort of language within a particular discourse?</td>
<td></td>
</tr>
</tbody>
</table>

*Table 5. List of linguistic and interpretative tools used in the analysis.*
When answers on the questions from the tools were found in the texts, the passages from the text were placed into a table with the selected tools displayed in Table 5. Each interview had a table, and a set of data could be placed into several places in the table because the answer to the questions asked to the text could be within the same stanza from the interview. Table 6 (on page 80) is an example with data from an interview with Ronaldo on 16 March 2016. The original table had all the tools used; this is a shortened version meant solely to present the outline of the table.

<table>
<thead>
<tr>
<th>Tool</th>
<th>Questions</th>
<th>Clause</th>
</tr>
</thead>
<tbody>
<tr>
<td><em>Diexis tool</em></td>
<td>What does it tie to, connect to?</td>
<td>I was supposed to have an extra going-through, it is just as well, so I don’t forget that too.</td>
</tr>
<tr>
<td><em>Fill in tool</em></td>
<td>What needs to be filled in?</td>
<td>I don’t forget that too.</td>
</tr>
<tr>
<td><em>Subject tool</em></td>
<td>Why this subject?</td>
<td>But then, when you are in a smaller group, you dare to say more than I dare.</td>
</tr>
<tr>
<td><em>Intonation tool</em></td>
<td>What is intonated?</td>
<td><strong>Than</strong> I dare</td>
</tr>
<tr>
<td><em>Doing not just saying tool</em></td>
<td>What is it trying to do?</td>
<td>Good, then I dare to say stuff too. It feels like I am developing more then.</td>
</tr>
<tr>
<td><em>Vocabulary tool</em></td>
<td>What words are used?</td>
<td>Dare, sure, forget</td>
</tr>
<tr>
<td><em>Topic and theme tool</em></td>
<td>What is the topic and theme?</td>
<td>When you are in a smaller group, you dare to say more. Good, then I dare to say stuff too. It feels like I am developing more then.</td>
</tr>
<tr>
<td><em>Topic flow or topic chaining tool</em></td>
<td>How are the topics linked?</td>
<td>Small group – dare feels good</td>
</tr>
</tbody>
</table>

**Table 6. Example of analytical table for each interview.**

In the fourth phase, the research questions were related to the extracted data and placed into tables. What can what the students are talking about in relation to the answer to the tools say about the meaning(s) they ascribe to inclusion in mathematics learning and teaching, and what
frames and influences students meaning(s) of inclusion in mathematics learning and teaching? One example of the analysis process in this phase was the question, What meaning(s) is contributed by intonation? Words that are stressed in relation to the context are construed as something appearing to be a meaning ascribed for the students in relation to their learning and teaching. Another example is the way that stanzas are clustered in relation to the topics in the stanzas. Does the same topic appear in several stanzas, and are the topics in the stanzas linked? If so, these topics are construed as being a meaning for the students in relation to their learning and teaching. Taking the example in Table 6 above, when Ronaldo intonates then I dare, and uses the word “dare” to talk about being in a small group, the small group is construed as appearing to be a meaning ascribed by Ronaldo to inclusion in mathematics education.

In the fifth phase of the analysis process, a comparison in and between interviews was made for each student. Did the same topic and theme reoccur in the stanzas, and were there topic chains in, or/and between interviews? Why? Were the same words used? Hence, stretches of language and discourses for each student were construed. In the students’ answers, it was observed that nearly the same answers would reappear over time; hence, a type of saturation was reached.

In the sixth phase of the analytical process, a comparison was made between the students. When students addressed the same aspects, different themes emerged, or, to use Gee’s (2011) terminology, different “stretches” of language appeared. This was made by writing down the three students’ discourses from each interview on a whiteboard. Then differences and similarities could be seen, and mutual discourses could be identified and scrutinized.

The seventh phase of the analytical process construed Discourses by how the students constructed the stretches of language, and if such stretches could be seen in classroom observations and photos, then how? This phase required a look back at the interviews with the teachers and the principal. How did they describe the mathematics education in comparison to the discourses construed by the students?
As stated in the beginning of this narrative, even though the narrative tells a story of a straight line of work, the entire analysis process was a constant back-and-forth way of working. Especially when looking into the relationship between discourse and Discourse, this was made by first construing discourse(s), and then, using the The Big “D” discourse tool to construe Discourses. This was made in a continuous back-and-forth process, acknowledging time and space, creating and recreating the students’ meaning(s) of inclusion in mathematics education, where the Discourse(s) set a larger context for the analysis of discourses.

Exemplifying the analytical process

The tools that were used in the analysis and the adopted questions for each tool are presented below as an example of the analytical process. The example is an excerpt from an interview with Veronica, who replies to a question about being in the classroom or being outside the classroom with the special teacher. A version of this exemplification is also included in the appended Article V.

Usually, me and, I think it is two more (students), join Karen [the special teacher] sometimes, so before and after a test, we like look through the test and stuff. […] Why does it feel nice to be outside the classroom? […] I don’t know… it’s less people. It’s like just three or four persons. […] It’s like you get an extra occasion [with explanations]. If you don’t get it the first time, you can get one more time. (Veronica)

*The diexis tool* – How are diectics, connecting words like “it”, “this”, and “that”, used to tie what is said to the context? In the above example, Veronica uses “it” to tie the small group to the number of students in the group and to what happens in the small group.

*The fill in tool* – The questions, What needs to be filled in to achieve clarity? And what is not being said explicitly, but assumed to be known? are used in the above example to fill in what Veronica means by an “extra occasion” and also why she wants it, namely, being given the
opportunity to have something explained one more time in a small group.

The subject tool – The question, Why has the student chosen the particular subject of the conversation? is used to see how the student organizes information in terms of subject and predicates. In the excerpt, Veronica places the subject “me” first, and then two more students who join Karen, which implies that Veronica considers herself important in this group.

The intonation tool – The question, How does the students’ intonation contour contribute to the meaning? is used to determine if the students are emphasizing something. In the above example, Veronica hesitates when trying to describe why it is nice to be outside the classroom.

The doing and not just saying tool – This tool is used to go beyond the text by asking, What is the student trying to do? Veronica says, “you can get one more time”; Hence, going beyond the text, she explains that she wants to have one more occasion with the teacher explaining outside the classroom in order to get access to the mathematics.

The topic and theme tool – This tool asks, What is the topic and theme for each part? When the theme is not the topic and has deviated from the first choice, why was it chosen? The topic for this clause is about being outside the classroom in a small group and why Veronica wants to be there. She did not introduce the subject to be outside the classroom, but she chooses to bring up the subject of the number of students and getting an extra occasion in the small group. You can also see a connection to another topic, tests.

The topic chaining tool – This tool asks how the topics are linked to each other to create (or not) a chain creating an overall topic of coherent sense? In Veronica’s case, the topic of being in a small group is reoccurring and a chain is seen in and between interviews.

The vocabulary tool – What types of words are being used, and how does the distribution of words function to mark the communication, in terms
of style? Here, the word “nice” reoccurs in Veronica’s texts when talking about being in the small group and getting support by the special teacher.

*The context is reflexive tool* – This tool asks, What does the student say that helps to reproduce context? What social groups, institutions, or cultures support and normalize the practices? In Veronica’s case, she is reproducing the norm of being in a small group and the social group of students in special educational needs in mathematics.

*The identity building tool* – The questions, What socially recognizable identity or identities is the student trying to enact or get others to recognize? And how is the student positioning others? are used to identify both how Veronica positions herself and others. In this excerpt, it is visible that Veronica is positioning herself and two others as being in need of more explanations.

*The situated meaning tool* – What situated meaning does the communication have? In this excerpt, the situated meaning is about why the small group is something Veronica needs to be able to access the mathematics.

*The figured world tool* – What typical figured worlds are the words and communication assuming and inviting listeners to assume? Veronica talks about being outside the classroom as something assumed to be known.

*The Big “D” discourse tool* – This tool asks what Discourse this language is a part of and what sort of values, beliefs, objects, tools, and environments are associated with this sort of language within this particular Discourse. In the above excerpt, the environment of being outside the classroom and the values of Veronica herself being in need of an extra occasion are visible. To be able to look at the environment, objects, and tools, the observations need to be examined.

In the previous example, the first ten tools point at what Veronica is saying, doing, designing and building in terms of meaning(s) of
inclusion. When using the theoretical tools (situated meaning, figured world, and big Discourse tool) together with the other tools, there is an indication of a discourse of being in a small group. To be able to construe a big Discourse, the observations need to be taken into consideration by looking at objects, tools, and the environment as well as other texts from Veronica and the other students. In this example, the observation notes show that a few students (often Veronica) sometimes join the special teacher in a small room next to the classroom.

Quality considerations researching students’ meaning(s)

Given that this study is a qualitative case study, it is unfruitful to apply quality criterion from the quantitative field rooted in a positivist paradigm, such as validity and reliability. However, there are other quality criteria important to take into consideration when doing a qualitative study (Adler & Lerman, 2003; Goodchild; 2011; Niss, 2010; Tracy, 2010). The quality criteria discussed in research mainly concern three interrelated areas: 1) Ethical considerations, which are also argued to be an overarching issue to consider in all parts of research (Goodchild, 2011; Adler & Lerman, 2003); 2) The design of the research, including a worthy topic, rich rigour in terms of sufficient use of theoretical constructs, data collection, and analysis process (Tracy, 2010); and 3) The reporting of the research with a communicative quality (Niss, 2010) and a significant contribution (Tracy, 2010). These three overarching criterion areas are discussed below in relation to this PhD study.

Ethical considerations

When referring to research ethics, you can talk about research ethics and the researchers’ ethics (Swedish Research Council, 2017, p. 12). These two types of ethics deal with different issues in the research. Research ethics are often used when discussing ethical considerations regarding the informants of the project. In contrast, the researchers’ ethics deal with the researchers’ own responsibility towards the research and the research community (Swedish Research Council, 2017). You could say that the researcher’s own ethical responsibility forms the basis for all
research ethics, and the researcher herself or himself bears the ultimate responsibility for good quality and morally acceptable research (Swedish Research Council, 2017). This can be compared with the notions of *external and internal ethics* from Floyd and Arthur (2012), where external ethical issues are obvious and visible aspects, for example written consents, while internal ethical issues relate to ethical and moral dilemmas of the researcher herself or himself in relation to the research conducted. Tracy (2010) also discusses this and uses the terms “procedural ethics” and “relational ethics”. Here, “procedural” refers to external ethics and “relational” refers to internal ethics, such as researcher’s self-consciousness. Consequently, external ethical issues fall more in line with research ethics, and internal ethical issues are more in line with researchers’ ethics.

Ethics are present all through the research process, from creating the research question to writing the results (Goodchild, 2011). In this study of students’ meaning of inclusion in mathematics education with a focus on students in special educational needs in mathematics, both internal and external issues need to be taken into consideration. When writing the research questions, what meaning(s) do the students ascribe to inclusion in mathematics learning and teaching? And what frames students’ meaning(s) of inclusion in mathematics learning and teaching? Ethical considerations were present, and many questions arose: How will I get close to the students? How can I do research with the students, and how can they get a feeling of being a part of the research without feeling exposed? How can I grasp a process in the field and write it down without offending the students or the teachers and the school?

Before entering the field, written consent from the guardians as well as information about the project and consent from the students are external issues of importance. Given that students’ meanings are in focus in this project, I find it important that the students as well as the guardians give their consent (Appendices 1–4). This is why the consents were translated into English, Spanish and Arabic. It is important to also obtain consent from the school and the teachers involved (Appendices 5–6). In the written consent, it is important to make clear that the participant always has a choice regarding involvement in the research and to explaining the
risks (Alderson & Morrow, 2011). The participants must understand that participation in the research is voluntary, and the permission the students and their guardians give can be withdrawn at any time. This is about respecting a child’s choices in education (Alderson & Morrow, 2011).

Ethics is especially important when going into the field. Here, the researcher needs to be aware of how to handle relations with informants, both professional and personal, as well as how to handle inside knowledge, conflicting roles, and anonymity (Floyd & Arthur, 2012). It is particularly important in this study, as it concerns students in special educational needs in mathematics who can be regarded as particularly vulnerable. I have met students, teachers, and the principal, and I have held roles not only as a researcher but also as an expert in SEM, in addition to my role as a kind of mathematics teacher. My awareness of my different roles in the field work is important, as I have come to possess inside knowledge about the students that the teachers did not know. This presented a conflict between my different roles because what the students told me in confidence was of great interest to the teachers. What could I tell them without compromising the students or the study? I ended up being very selective about what I told the teachers. I only told the teachers this type of information when a student said that he wanted me to tell the teachers. This happened on one occasion when Billy, a student in Grade 7, told me he would like to be given more advanced tasks, both on the test and in the education. He wanted me to tell the teachers about it, and therefore, I did.

In terms of the Swedish Research Council’s (Swedish Research Council, 2008) view on ethics, they have four main requirements regarding research ethics for an individual’s protection: information, approval, confidentiality, and appliance. These four requirements are examples of external ethical considerations. If strictly looking at what the law says concerning research ethics, there is a Swedish law from 2004 called the Ethical Review Law, which stipulates that all research concerning human beings shall be ethically reviewed. To do this review, ethical review boards have been set up. This law will be applied if the research concerns sensitive personal data, if the research involves physical
intervention or aimed at physical or psychological stress, or there is a clear risk of harm to the subject (Swedish Research Council, 2017). However, this involves interpretation; for example, what does it mean that a person is at risk of psychological stress? Even if the research does not acquire an ethical review according to the law, the researcher should apply the principles of research ethics and the research codex. The research codex is a collection of guidelines for researchers that are not legal documents but moral guidelines for human responsibility in relation to research (see www.codex.vr.se). In relation to this, Goodchild (2011) talks about being mindful of the impact on participants by conducting an ethical risk assessment reflecting on the impact of the wellbeing of the participants. This particular study concerns meanings of students in school who are in special educational needs in mathematics, and this required me as a researcher to be part of their mathematics education to some degree. My presence as I talked and interviewed them could be potentially psychologically stressful for them. Therefore, before entering the field, an ethical review at the local ethical review board, Sydost, was conducted. From there, I received an advisory ethical review which pointed out important issues when entering the field, such as making sure to inform the student and the guardians about both their participation and their right to end the participation at any time. Also, one should be mindful in the selection of students and practise confidentiality. It did not recommend a central ethical review.

It was also important to consider the power relations between me as an adult researcher and the student as a child (Christensen, 2004). Here, my role as a researcher was special and needed to be considered in order to get close to the students, meaning I had to gain their trust as a researcher. In doing so, it was important to consider the students’ social agency and active participation in research to be able to hear the meanings of the students (Christensen, 2004). It was also important in this process to take into consideration the fine line between the rights of the students to be heard and the rights of the students to be protected in research (Alderson & Morrow, 2011).
From an internal ethical perspective, it is also important that I consider how I present the students in this thesis and be vigilant about the formulations; the informants are our companions in the research and deserve gentle treatment (Walford, 2008). I also considered this when writing the results. Hence, in this study, I do not focus on failures and deficiencies but rather on opportunities for development. The focus is not on the individual student per se but on students’ meaning(s) of inclusion.

To conclude, as a researcher, I have an ethical responsibility, which implies much more than merely following the rules (Athew et al. 2011). As Tracy (2010) writes, ethics constitutes the end goal of qualitative quality research.

The design of the research

When designing research, the choice of topic is important. According to Tracy (2010), a worthy topic is a topic that is relevant, timely, significant, interesting or evocative. The topic of this study was chosen partly because of the prior study on inclusion in mathematics focusing on teachers’ meanings and partly because of the lack of research on inclusion in mathematics education foregrounding students’ meaning (Roos, 2019a). Accordingly, it is relevant because it targets a research gap and is somewhat evocative because when referencing inclusion in educational research, a subject such as mathematics is often not included. Instead, overarching issues of participation in education are often reflected on, such as human rights, equity, and the labelling of students. Also, a topic should be closely connected to the research questions, which in turn, must be clear and precise (Niss, 2010). In this study, the topic – students’ meaning of inclusion in mathematics education – is framed by the following research questions: What meaning(s) is ascribed, and how is inclusion used, in mathematics education research? What meaning(s) do the students ascribe to inclusion in mathematics learning and teaching? And what frames students’ meaning(s) of inclusion in mathematics learning and teaching? Hence, the research questions aim at describing inclusion in mathematics education, both in terms of meaning(s) of inclusion in
mathematics education research and students’ meaning(s) of inclusion in mathematics education.

Another quality criterion regarding the design of the study is rationality when identifying the reasons for what we do in terms of theoretical constructs and data collection (Goodchild, 2011). Tracy (2010) calls this a “rich rigor” in research. Regarding theoretical choices, a good theory “should help researchers understand what is going on in the classroom, it is essential for framing the research, it is essential in interpreting the evidence, it is essential in the development of knowledge” (Goodchild, 2011, p. 14). In this study, the choice of theoretical constructs depends on the aim and research questions as well as the form of the empirical sample. Given that the aim was to understand students’ meaning(s) of inclusion in mathematics education and the empirical data mainly consists of student interviews, this meant a theoretical approach for the text was needed. Also a theory was needed that could describe student views on inclusion in mathematics education. Regarding data generation, it is important to pose the questions, Are there enough data? Is the environment of the study appropriate in relation to the aim? And did the researcher spend enough time to gather interesting and significant data (Tracy, 2010)? In this study, the environment is an inclusive mathematics education; hence, it aims at framing an appropriate environment in relation to the aim of the study. The data collection was planned to continue until I was able to get the entire picture of students’ meaning(s) of inclusion. Over time, it was seen that nearly the same answers reappeared, even though different questions were posed, hence, a type of saturation was reached. For example, in several of the interviews, a student kept returning to the lack of being given a challenge in the mathematics education. It was talked about in relation to tests, to tasks, when “going through”, etc. Realizing that saturation was likely to have been reached ended the data collection. Although a type of saturation was reached, there still may be aspects of the students’ meanings of the mathematics education that were not addressed. It can be that the “wrong” questions were posed, there were perhaps things they did not want to tell me as a researcher, or it may simply be that certain things did not come up in the discussions.
The reporting of the research

The reporting of the results in this study started with writing a number of articles. The first empirical article aimed at testing the analytical approach for interpreting the data. In the interpretation, it is important to be honest and open when reporting all issues. Also, one should be mindful of the style one uses when reporting (Goodchild, 2011). In the first empirical article (Roos, 2018), a first attempt at interpreting the data by using DA was made. Here, I was vigilant. I listened carefully to the interviews and observations used in the analysis and read them thoroughly several times. Also, when writing the article, I was careful not to write anything negative in the interpretations and to be as objective as possible in the descriptions, even though the result points out a critical issue for the students’ participation in mathematics education, namely, assessment. Then, in Roos (2019b) and Roos (2019c), I used the same thoughts about being vigilant in the interpretations and writings as in Roos (2018). I was careful about how I reported on the findings and the descriptions of the students and what they said in “thick descriptions”; hence, I considered the participants. This is also a quality criterion highlighted by both Goodchild (2011) and Tracy (2010). Another issue in relation to thick descriptions in qualitative studies is generalization. The results of qualitative research cannot be generalized as results in quantitative research can be. However, providing thick descriptions makes it possible for the reader to make transfers (Lincoln & Guba, 1985), which can be seen as a type of generalization: “It is argued that if the author gives full and detailed descriptions of the particular context studied, readers can make informed decisions about the applicability of the findings to their own or other situations” (Walford, 2008, p. 17). Flyvbjerg (2006) argues that you can generalize on the basis on a single case if the selection of the case is strategic in relation to the research questions. Then, the case study may be central to scientific development because “the force of example” is underestimated (p. 228).

Other criteria when writing about the research are resonance, contribution and meaningful coherence (Tracy, 2010). Here, resonance stands for “research’s ability to meaningfully reverberate and affect an audience” (p. 844). This is a hard task and impossible for me to comment
on, even though I kept this in mind when I wrote the articles and it is the overarching text of this thesis. Given that one part of this study is a case study, I have tried to be as descriptive as possible, providing thick descriptions that should enable the reader to make transfers between the research and their own situation.

Regarding the contribution of this thesis – for whom is it written? Who is the audience? (a quality criterion, according to Adler and Lerman, 2003) This study is written for both researchers and practitioners. For researchers, the aim is to provide a sustainable development on the notion of inclusion in mathematics education research and a study foregrounding the meaning(s) of students. For practitioners, the aim is to provide insights on how different aspects of the education, both at a societal level as well as at the school level, affect students’ participation in mathematics education. Does it extend knowledge in the field of mathematics education and inclusion? I would argue – yes. It sheds light on the meaning(s) of students, which are seldom highlighted. It also sheds light on the use of the notion of inclusion, both in research and in practice. Regarding its use in practice, it problematizes and provides a multifaced picture of students’ meanings of inclusion. Can this study improve practice? I argue that it can. Not directly, but rather indirectly, as it gives insight into critical aspects that frame and influence inclusion in mathematics teaching and learning. Also, it contributes to our understanding of what meaning(s) students ascribe to inclusion in mathematics learning and teaching; hence, it adds to our understanding of social life in this time, space and culture.

Concluding the discussion on quality on reporting the research in this study, I argue it is meaningfully coherent, meaning the study achieves its stated purpose, and uses methods and representation practices that lend themselves well to the adopted theories (Tracy, 2010); in this case, those within the research fields of mathematics and special education. Also, the intention in the writing is to thoughtfully interconnect the results of this research to prior research in the area.
Summary methodology

The first part of this study involves a systematic literature review on the notion of inclusion in mathematics education research. Five databases were searched with the words inclus* or inklu* and education* or school or undervis* or skol* or utbildn*, and then math* or matemat* was added. Thereafter, the criteria for timespan and peer-reviewed research were applied. The search resulted in 1,296 research studies. Of these, 76 studies were retained after the aforementioned criteria were applied and 19 duplicates had been removed.

The second part of this study examines a case study of three students and their meaning(s) of inclusion in mathematics education. The selected school is a lower secondary school in an urban area in Sweden. The school had set out to work inclusively, meaning their aim was to include all students in the ordinary classroom teaching in every subject and to incorporate the special education into the ordinary teaching, with no fixed special educational groups. The cases are seen as information-rich cases (Patton, 2002), as the students participate in an inclusive mathematics education. This type of selection is considered information-oriented (Flyvbjerg, 2006).

Three students were chosen: one in Grade 7 and two in Grade 8. The student in Grade 8, Edward, was chosen because he was perceived as student in access to mathematics education. The other two students were chosen because they were perceived as struggling to gain access to mathematics education: Veronica in Grade 7 and Ronaldo in Grade 8 (the same class as Edward). The data in this study consists of both observations and interviews construed during the spring semester 2016. The observations took place in a Grade 7 and a Grade 8 classroom at the school where the interviewed students participated. At least one mathematics lesson each month for each class was observed, and student interviews followed each observation. The observations were used to provide a context for the interviews as well as to support the analysis.

In the study, Discourse Analysis (DA) is used as a theory and as an analytical tool. Analysing language and its use in specific situations can...
tell us more about the social world. To analyse different forms of spoken and written language, Gee (2014b) provides tools that highlight the communication by posing questions to the text. These questions open up for investigation of the text and what is beyond the text in terms of discourse(s). Some of the tools are linguistic, hence, close to the text and the context of the text, while other tools give access to the interpretive level, which is closer to the big picture of what is happening in the social world. In this study investigating students’ stories of their own participation, the tools have been adapted according to the aim, and the questions asked to the text are connected to each tool.

Quality criteria of qualitative research have been taken into consideration (Adler & Lerman, 2003; Goodchild; 2011; Niss, 2010 Tracy, 2010). The quality criteria discussed in research mainly concern three interrelated areas: ethical considerations, the design of the research, and the reporting of the research. In this study, ethical considerations concern the participants, being in the field, and writing the thesis. Given that the students are in focus in this project, they, as well as the guardians, were required to give their consent (Appendices 1–4). This is why the consent form was translated into English, Spanish, and Arabic. It was also considered important to obtain consent from the school and the teachers involved (Appendices 5–6). The participants needed to understand that participation in the research is voluntary, and the permission of the students and their guardians can be withdrawn at any time. This is about respecting a child’s choices in education (Alderson & Morrow, 2011). Ethics is especially important when going into the field. In this study, it is even more of an issue because it concerns students in special educational needs in mathematics. I have met with the students, the teachers and the principal, and I have had different roles. My awareness of my different roles in the field work is of importance, as I have come to possess inside knowledge about the students that the teachers did not know.

Regarding the design of the study, the choice of topic is important. According to Tracy (2010), a worthy topic is a topic that is relevant, timely, significant, interesting or evocative. The topic of this study was chosen partly because of the prior study studying inclusion in
mathematics education focusing on teachers meanings, and partly because of the lack of research on inclusion in mathematics education foregrounding students’ meaning(s) (Roos, 2019a). Also, the design of the study – it’s rationale in identifying the reasons for what we do, in terms of theoretical constructs and data collection – is important (Goodchild, 2011). In this study, the choice of theoretical constructs depended on the aim and research questions as well as the form of the empirical sample. This means that, as the aim was to understand students’ meaning of inclusion in mathematics education and the data mainly consisted of student interviews, a theoretical approach in order to understand the text was needed. Also needed was a theory that could describe student views on inclusion in mathematics education. Regarding data collection, in this study, the environment is an inclusive mathematics education; hence, it is a purposeful sample aimed at framing an appropriate environment.

Regarding the reporting of the research, it is important to be vigilant about the style that is used when reporting (Goodchild, 2011). In this study, I was vigilant when writing the results; moreover, I read through and listened to the interviews and observations used in the interpretation several times. Additionally, I was careful not to write anything negative in the interpretations, even though the results highlighted critical issues for the students’ participation in mathematics education. Moreover, the participants were considered in how they were presented and what quotations were chosen.

Concerning the contribution of this thesis, this study was written with both research and practice in mind. For research, the aim is to highlight the use of inclusion in mathematics education research and to provide with a sustainable development on the notion of inclusion in mathematics education as well as foregrounding students’ meanings of inclusion in mathematics education. For practice, the aim is to provide with insights on how different aspects in the education, both at a societal level as well as school level, frame students’ meaning(s) of inclusion in mathematics education. It also problematizes, and provides a multifaced picture of, what students’ meaning(s) of inclusion can be, which in turn, can inform and improve practice.
BACKGROUND AND CONTENT OF THE ARTICLES IN THIS THESIS

In this chapter, a short description of the background, purpose, and context of the appended articles in this PhD thesis is given as well as a short description of their content. Additionally, a reflection on the outcome of the articles and their possible contributions illustrates how each article has allowed me to progress in the research process. Through presenting how and why the articles have been produced, my research process as well as some decision-making factors in this PhD project are made visible.

Given that the articles originate from the same study, there are overlapping issues of theory and methodology in them. Accordingly, a short summary of the theory used in all the articles is provided below (for a deeper description, see the chapter, Theoretical Framing). Thereafter, a short reflection on the connection to the aim and research questions of the thesis is made for each article and the connections between the articles are discussed. Although the articles originate from the same study, they were written in different phases of the PhD project; thus, there are some differences in the wording and definitions, which is discussed at the end of this section.

Theoretical framing and methodology – summary

Discourse analysis (DA) has been used as theory and a tool in all the articles. In Article I, it is used to frame definitions and roles of inclusion in research studies in the field of mathematics education. In Articles II–V it is used to frame students’ meaning(s) of inclusion in mathematics education. DA as described by Gee (2014a, 2014b) has been used, with big and small discourses (Discourse with capital D and discourse with lowercase d) as theoretical concepts. Here, Discourse(s) refers to social and political context(s) embedding numerous social institutions simultaneously. (D)iscourses are language plus “other stuff” (Gee, 2014a, p. 52), such as actions, interactions, values, beliefs, symbols,
objects, tools, and places. Small d discourse focuses on language in use (both spoken and written), as in, what small conversations are visible in the investigated stories (Gee, 2014a).

Gee (2014a) is drawn upon to analyse different kinds of texts – the text in articles and text in the form of transcriptions of interviews and observation notes. These texts were analysed with the help of the tools of inquiry provided by Gee (2014a), which are both linguistic and interpretative. While examining the texts, I asked the questions of each tool to learn more about the text being examined. Moving between the linguistic level and the interpretative level made students’ meanings of inclusion in the language and context visible. Also, what the SEM students were saying, doing, and designing as well as how they were building text in relation to inclusion in mathematics were made visible. When applying the theoretical tools, anything that is made visible could be construed into D(d)iscourses.

In Articles II–V, the investigation took place at a municipal lower secondary school (Grades 7–9) in Sweden. The school has roughly 550 students, and it set out to apply inclusive work. The goal of applying inclusive work reflects how the school does not typically apply special education in small groups, all teaching occurs in the regular classroom, and there are almost always two teachers in each lesson; however, one special education teacher in mathematics and one regular mathematics teacher teach the mathematics lessons. The school is an urban one, albeit on the outskirts of a city, and has a varied catchment area of residents who live in both apartment blocks and detached houses. Therefore, the students participating in the study have differing socio-economic backgrounds.

Two classes (Grade 7 and Grade 8) selected by the mathematics teachers at the school were visited, and observations were made. The classes were selected according to how long the students had participated in inclusive classrooms at the school. The school and how it works with inclusive settings was rather new in Grade 7, whereas in Grade 8, the students had already been engaged in how the school worked with inclusive issues for at least a year, and they planned to continue this for
another year. Another criterion was that the classes should be able to handle having a researcher in the classroom. The teachers identified students in need of SEM in the two classes based on the definition presented in the introduction; their selection included both students struggling with access to mathematics education and students who needed additional learning challenges. In the articles, texts from three of the selected students were analysed in depth. The students are Veronica in Grade 7, and Ronaldo and Edward in Grade 8. Veronica says, “Maths is pretty hard”, and further stating, “I don’t like maths”. The mathematics teachers talk about her as a student struggling to gain access to mathematics, and she is just above the threshold for a passing grade. Ronaldo describes himself as a student with learning difficulties: “I have difficulties within all subjects, and it’s like [to do with] concentration and all that.” He also says that he forgets things: “I don’t remember, I have to repeat a lot.” The mathematics teachers talk about him as a student who struggles to gain access to mathematics education, and he also is just above the threshold for a passing grade. In contrast, Edward describes himself as a person who thinks mathematics is really easy and does not need much help at all. He does not have to make any effort – mathematics works “automatically” for him, and he “already knows” most of what they are doing in maths class. The mathematics teachers talk about him as a student with access to mathematics education, and he has the highest grade possible in mathematics. These students are described in depth in the section, “Participating students – cases”.

Article I – Inclusion in mathematics education: An ideology, a way of teaching, or both?

Background and prerequisites of Article I

When I entered the research field of inclusion in mathematics, it soon became clear to me that it is a diverse field in terms of its theoretical, epistemological and methodological approaches, which follow a psychological, didactical or pedagogical research paradigm. Sometimes these perspectives seemed to be interrelated, and at other times, in opposition, which even prevented me from understanding the
prevailing view on inclusion in mathematics education. In order to go further, there was a need to gain a deeper understanding of what inclusion can be in mathematics education research and be able to situate my study in this field theoretically, methodologically, and empirically. Thus, a systematic literature review was made to ensure this thesis did not end up as just another empirical study contributing to the diversity without any deeper understanding in terms of inclusion in mathematics education.

During the process of collecting and analysing research regarding these issues, a need emerged to dig deeper into the underlying meanings, values and assumptions of inclusion in mathematics education. How can inclusion be defined and used, and how did I want to define and use inclusion in this study? Hence, the systematic literature review required a set of theoretical lenses that could help me understand the underlying meanings in the reviewed articles. The literature review thus became not only a review but also a study with a theoretical framing. I had been curious about the theory and methodology of discourse analysis (DA), as it seemed to lend itself to the empirical investigation of this research. And after reading more about it, it seemed well suited for investigating the underlying values in research articles, book chapters, conference papers and theses as well. Accordingly, DA as described by Gee (2014a) was used to analyse the texts in the systematic literature review. The article was accepted and published online in the international journal, Educational Studies in Mathematics, on 25 October 2018 (Roos, 2019a).

Content of Article I

The systematic literature review focuses on the definitions and roles of inclusion in mathematics education research. The aim is to highlight the current meaning(s) of the notion to help promote the sustainable development of the notion of inclusion in mathematics education.

Discourse analysis was used to analyse 76 studies published between 2010 and 2016. The studies comprised 52 journal articles, 18 book chapters, 4 conference papers, and 2 theses. The discourse analysis procedure began by posing questions to the texts. The questions were chosen from those developed by Gee (2014b) and adapted to suit the
systematic literature review. The questions were chosen to illuminate the meanings, operationalizations and Discourses of inclusion in mathematics education research. The adaptations were made to highlight the context of this study – mathematics education and research on inclusion – using words in the questions like “inclusion” and “scholar”, for example, “How are the words and grammatical devices used to build a viewpoint on inclusion?” and “What are the scholars saying about the subjects?”

The results show that the term inclusion is used both as an ideology and as a way of teaching, and these two uses are most often treated separately and independently of each other. The results also show a division between meanings ascribed to inclusion and operationalizing inclusion. “Meanings ascribed” refers to research that discusses issues and meanings of inclusion, whereas “operationalizing inclusion” refers to how to work with inclusion. It was found that meanings and the operationalization of inclusion were discussed at two levels – on a societal level as a form of ideology and on a classroom level as a form of teaching. Usually, these texts would not discuss both levels; however, there were exceptions where a few of the investigated studies contained both levels. Another result was that only one of the studies foregrounded students in relation to inclusion and mathematics.

When inclusion was treated as an ideology, values were articulated; however, when treated as a way of teaching, interventions were brought to the fore. When the notion of inclusion was used as an ideology, the most extensive discourse concerned meanings ascribed to inclusion and dealt with equity in mathematics education. The other visible discourses concerning meanings ascribed using inclusion as an ideology were mathematics (not) for all and including (or excluding) SEM students. When operationalizing inclusion in mathematics education on an ideological level, the discourse of inclusion in relation to exclusion, the discourse of valuing diversity and the discourse of equity in mathematics education were visible.

When inclusion is used as a way of teaching, the most extensive discourse relates to the operationalization of inclusion and concerns
teaching interventions for mathematical engagement. The other visible discourses regarding the operationalization of inclusion is the discourse of teaching for maximising opportunities in mathematics for all and the discourse valuing diversity. A total of 45 out of 76 studies discussed the operationalization of inclusion in mathematics in terms of how to work in school and in classrooms, using words like “teaching”, “classrooms”, “lessons” and “instructions”. However, 21 of the 45 studies that discussed the operationalization of inclusion did not discuss the notion of inclusion per se but rather took its meaning for granted. There were only nine studies found with meanings ascribed to inclusion regarding a way of teaching. These studies encompass the discourse of participation and the discourse of inclusion in relation to exclusion.

To summarize, when inclusion is used as an ideology, its meanings and values are articulated. These values are a vision of inclusion on an overarching level. However, if the vision is not connected to operationalizing inclusion, it may not have any real impact in the classroom. On the other hand, if inclusion is only used to refer to a way of teaching, it is hard to justify why we need to work with inclusion in mathematics education. Instead, to promote the sustainable development of inclusion, the two uses need to be connected. In the future, inclusion should be thought of more in terms of the nuances of a colour: it originates from the same core but with variations expressing its various aspects that connect and interrelate with its ideological and operational aspects. Based on the systematic literature review, if the sustainable development of inclusion in mathematics education is to be promoted, scholars need to connect and interrelate the operationalization and meanings of inclusion, both in society and in the mathematics classroom, as well as investigate students’ meanings of inclusion in research.

Reflections on Article I

Article I frames meanings of the notion of inclusion in research in terms of D(discourses), and by that, it contributes with insights in mathematics education research by explaining the current use of the notion of inclusion. It also contributes to the background of this study. Accordingly, it responds to Research Question 1 in the overall PhD
study: What meaning(s) is ascribed, and how is inclusion used in mathematics education research? Article I provides a set of theoretical and methodological lenses in the form of DA tested out on written text. Furthermore, it makes a research gap visible, concerning the lack of bringing students’ meanings to the fore in research regarding inclusion in mathematics education. This brings us to Articles II, III, IV and V, all of which aim at describing students’ meanings of inclusion in mathematics education.

**Article II – The influence of assessment on students’ experiences of mathematics**

**Background and prerequisites of Article II**

In the initial phase of analysing the interviews with the students, I noticed that the students talked a lot about assessment in different ways without me having asked about it. The observation notes also show that assessments were discussed in the classrooms. Hence, assessment seemed to somehow frame the students’ meaning(s) of inclusion in mathematics teaching and learning. This observation laid the ground for the first empirical article of this PhD thesis, which investigates the influence of assessment by analysing the first interviews with the two students in Grade 8 (Edward and Ronaldo). Aside from investigating the influence of assessment to students’ participation, this analysis was a first attempt to apply DA as described by Gee (2014a, 2014b) to the interviews and observations generated in this project. This resulted in a paper, which was presented at the 22nd MAVI Conference and later published in a Springer book with selected papers from this conference (Roos, 2018).

**Content of Article II**

The analysed text presented in this paper focuses on two students, Edward and Ronaldo. The analysis found stretches of language that address assessment several times, although assessment was not an expressed focus of the interview questions. The topics of discussion found in the students’ text were tests, writing on tests and one’s feelings when taking tests. Grades were also visible in the student talk, where
both the topics of grades and tests were visible in the observation notes. These stretches about assessment indicate a Discourse of assessment in mathematics in which the students position themselves as being assessed in mathematics, which clearly influences their meanings of inclusion mathematics education. Interestingly, the students are categorized by their teachers as either in access or struggling to gain access to mathematics education, yet both are greatly influenced, albeit differently, by the Discourse of assessment in mathematics.

To summarize, it seems as though assessment does not provide relevant input to support the SEM students in this study in developing their mathematical knowledge, as the Swedish National Agency (2018) prescribes, in order to increase the achievement of student goals. On the contrary, assessment seems to spark other meanings for the students, of which inclusion serves as the frame: in Ronaldo’s case, this is evidenced through his expressed feelings about mathematics and how he talks about himself as a low performer whereas, in Edward’s case, it is evidenced through the burden he expresses about having to present his knowledge. In this sense, it seems that assessment is even an obstacle for Edward, a student in access, in developing his knowledge of mathematics.

Reflections on Article II

Article II makes one Discourse influencing students’ participation in mathematics education visible, the Discourse of assessment. Also, it showed that DA was applicable for this type of empirical data, as it was useful construing framing discourses. It also helped me find answers in the text by posing questions to the text. The article connects to Article I by showing an empirical example of students’ meaning(s) of inclusion in mathematics education. The acknowledgement of assessment as a framing factor in mathematics education is not new in research, but it adds new insight to the research field in how it frames students participation and, in turn, inclusion in mathematics education.

The article responds to parts of Research Question 2 in the overall PhD project: What frames students’ meaning(s) of inclusion in mathematics learning and teaching? However, given that only a small part of the
empirical material was investigated for this article, further studies are needed in order to fully respond to Research Question 2. This brings us to Articles III, IV and V.

**Article III – Challenges at the border of normality: Students in special educational needs in an inclusive mathematics classroom**

**Background and prerequisites of Article III**

After finishing the empirical data collection, much time was spent transcribing the interviews and going through the observation notes. When typing, I continuously conducted the analysis with the help of DA, with a focus on the research question, What frames students’ meaning(s) of inclusion in mathematics learning and teaching? In addition to assessment, another issue was stressed in the students’ stories of participation, namely, how the mathematics education challenged the students, and how it framed their participation. This analysis ended up in construing the *Discourse of accessibility in mathematics education*. I wrote an article with the aim to highlight the challenges of students to access mathematics education. This article was accepted for, presented at, and published in the proceedings of the Mathematics Education and Society 10th International Conference (MES) in 2019.

**Content of Article III**

This article (as well as the other articles) departs from the intersection of research in mathematics education and special education. At this intersection, issues of diversity, equity, participation and normalization in terms of mathematics (not) for all become visible and are often framed by the notion of inclusion (Roos, 2019a). In this paper, the focus is on text from the same two students featured in Article II.

The results of this study show how the two students perceived being in special educational needs in mathematics (SEM), either as students in access to mathematics education or as students in a struggle to gain access, are challenged when participating in mathematics education.
Distinguishing between (d)iscourse as stories in texts and (D)iscourses as social and political recognizable units, the result shows the same, yet different, discourses that emphasize certain challenges: tasks, the importance of the teacher, to be (un)valued, and math is boring – all indicating a Discourse of accessibility in mathematics education.

The discourse of task is construed by the fact that both Edward and Ronaldo talk about how tasks affect their participation. For Edward, it is the theme of the task and the mathematical challenge of the task that are important. He needs to be engaged and have a challenge in the task in order to be an active participant. Ronaldo expresses insecurity about how to approach tasks. He also describes how hard it is for him to comprehend problem-solving tasks involving text.

The discourse of the importance of the teacher is construed by the fact that both Edward and Ronaldo talk about the importance of the mathematics teacher and how the teacher can enhance or diminish their participation in mathematics education. This can be facilitated by teachers listening to students, adjusting the pace when giving instructions, being mindful about the level of the presented mathematical content, and adjusting the teaching according to the needs of the students.

The discourse of being (un)valued is construed by the fact that both students talk about being valued as SEM students. In Edward’s case, it is about how he perceives himself as sometimes not valued, which can be seen in his disappointment at not being appreciated in the classroom. In Ronaldo’s case, it is about him wanting to be valued as a SEM student and receive special education in a smaller group sometimes.

The discourse of math is boring is construed by the fact that both Edward and Ronaldo talk about mathematics as something being boring. Edward has negative comments about the subject and describes mathematics as “the most boring subject, because when we are going to maths class, then it feels like you just dig yourself down into the sand”. Ronaldo expresses boredom in relation to listening for a long time and boredom because of the lack of variation: “It gets so bloody tedious, or like, boring like hell in the end. So vary things.” Taken together, these
discourses paint a picture of challenges for accessibility for the SEM students on the boarder of normality, which is manifested in Ronaldo and Edwards texts. Hence, a Discourse of accessibility frames the students’ view on participation in mathematics education. The accessibility is challenged in two ways, the students are challenged in their participation since they do not fit into the ‘normal’ education, and mathematics education is challenged to meet every student’s needs to promote equity.

Reflections on Article III

Article III makes visible yet another Discourse influencing students’ participation in mathematics education, the Discourse of accessibility. The article connects with Article II in the way that it adds another piece to the inclusive puzzle. Also, it implicitly connects issues of access to issues of assessment, for instance, tasks. This article opened up for critical aspects for the SEM students teaching and learning in terms of participation, hence critical aspects for inclusion in mathematics education. That made me reflect about the meaning of normality in relation to mathematics education, both what is normal to know at a certain age, but also what is normal in terms of what and how to teach. That’s why the title of the article is “challenges at the boarder of normality”. In my data, I saw that these students somehow were on the border on what was expected as normal to know in their grade. To meet these two students’ needs, the teachers had to provide alternative approaches from what is usually taught at this school in this classroom in order to include these students on the border. Also, reflecting on the result, the teacher seems to be an important actor for promoting inclusion in mathematics education. It is not only in how the teacher teaches and organizes the classroom and the mathematics education per se but also in how the teacher relates and connects with the students, human to human. There seems to be something in the student talk about the teacher. It is hard to explain with words, but it has to do with how they are appreciated as humans and met with a pedagogical stance in a tactful way. This article provides with insights about what challenges students on the border of normality; hence, it contributes with knowledge about important aspects of access for SEM students for both practice and research.
This article partly responds to Research Question 3 in the overall PhD project: What frames students’ meaning(s) of inclusion in mathematics learning and teaching? However, in the analysis, yet other stretches of language were visible in the students’ stories, and these stretches of language lead to Article IV.

**Article IV – I just don’t like math, or I think it is interesting, but difficult … Mathematics classroom setting influencing inclusion**

**Background and prerequisites of Article IV**
Almost in the same breath as construing the Discourse of accessibility in mathematics education, another Discourse was construed – the Discourse of mathematics education setting. In the analysis process, these Discourses overlap and interrelate (as well as overlap and interrelate with the Discourse of assessment); hence, they were not easily separated. However, to be able to describe students’ meaning(s) of inclusion in mathematics teaching and learning, issues concerning the setting of the mathematics classroom needed to be highlighted in depth, as they were salient in the language stretches of all the interviewed students. Consequently, an article was written with the aim to highlight the setup of the mathematics classroom and its relation to students’ meanings of inclusion. This article was accepted and presented in Thematic Working Group 25 “Inclusive Mathematics Education – Challenges for Students with Special Needs” at the 11th Congress of the European Society for Research in Mathematics Education Conference (CERME) 2019. Here, in addition to Edward and Ronaldo’s stories, stories from Veronica, a girl in Grade 7, were added to the analysis. This data was added in order to highlight both a different grade, a different classroom, a different gender, and to see if there were any significant differences between the students’ stories.

**Content of Article IV**
The results show that students’ participation in mathematics education is framed by how the mathematics education is set up.
The discourse of classroom organization involves the use of the textbook, discussions and working with peers, “going-through”\textsuperscript{2} and teaching approaches. Here, students talk about how different approaches in the teaching, such as oral explanations, various ways of working and taking notes, playing math games and doing practical mathematics, influence the students’ participation in a positive way. The discourse of being in a small group is mainly construed by the talk of the students struggling to get access in mathematics education, Ronaldo and Veronica. Both Veronica and Ronaldo sometimes share their feelings when they talk about being in a small group. Words like “it feels good”, “it’s like you get an extra occasion”, “I dare to say stuff too – it feels like I am developing more” and “I concentrate better, and it is peaceful and quiet” indicate positive feelings. Edward talks about the small group as well, but in terms of not joining because “I don’t think I would get something out of it, I don’t.” All these small discourses contributed to the construction of the Discourse of mathematics classroom setting.

The result shows that, even though the same issues regarding organization frame the three students’ meanings of inclusion in mathematics teaching and learning, there is a diversity within the issues, which raises a critical question: Is the inclusive classroom setting really inclusive in terms of participation and access to mathematics?

Reflections on Article IV

Article IV makes yet another Discourse influencing students’ participation in mathematics education visible, the Discourse of mathematics classroom setting. This article connects with Articles II and III in the way that it adds another piece to the inclusive puzzle. When reflecting on the result, the importance of the setting of the mathematics education is important, but how diverse should it be in order to meet the diversity of the students? Also, included in the setting are aspects of

\textsuperscript{2} In Swedish mathematics education, it is common for the lesson to start or end with a “going-through” (in Swedish, genomgång). Andrews and Nosrati (2018) point out three instances of what can be considered “going-through”: when the teachers inform the students of what to work with, when presenting new models, and when demonstrating solutions to problems the students find difficult.
classroom culture in terms of norms and power. This made me reflect on the complexity of mathematics teaching and the importance of taking diversity as a point of departure in mathematics education. This article contributes to the research field with knowledge about the complexity of setting up the mathematics classroom in order to include every student from the meaning(s) of students. It also contributes to practice by emphasizing the importance of being mindful regarding classroom organization, teaching approaches, and aspects related to being in a small group.

Similar to Articles II and III, this article responds to Research Question 3 in the overall PhD study: What frames students’ meaning(s) of inclusion in mathematics learning and teaching? Until now, the analysis had made it possible to construe three interrelated Discourses influencing inclusion: The Discourse of assessment; the Discourse of accessibility; and the Discourse of mathematics classroom setting. It was also seen there were both differences and similarities in the students’ stories about participation in mathematics education and in the D(d)iscourses. This reflection of similarities and differences leads into Article V.

**Article V – Same, same, but different: Consistency and diversity in participation in an inclusive mathematics classroom**

**Background and prerequisites of Article V**

Each of the empirical articles (Articles II, III and IV) thoroughly describe one Discourse each, like doing a puzzle by adding one piece at a time. Now it was time to try to take all the pieces of the inclusion puzzle into consideration and put them together in an effort to describe the whole picture of framing factors on students’ meaning(s) of inclusion in mathematics learning and teaching. This is in order to see the whole picture of the puzzle, and also, given that all the D(d)iscourses relate and sometimes overlap, it made sense to describe them together. Hence, Article V strives at merging the results of Articles II, III and IV to show all the construed D(d)iscourses and their interconnections. Empirical data from all the interviews with Edward, Ronaldo, and Veronica were
analysed; and together with all the observations, it became possible to see similarities and differences in their stories of participation in the mathematics classroom. This article was submitted to a journal in November 2018 and was resubmitted in April 2019.

Content of Article V

This article reports on three construed Discourses that influence and describe students’ meaning(s) of inclusion in mathematics education in the form of participation in mathematics education: the Discourse of assessment, the Discourse of mathematics classroom setting, and the Discourse of accessibility in mathematics education. These construed Discourses overlap and display a multifaceted and complex picture of participation and what frames this participation. Interrelations between Discourses are visible in the topic of students’ texts. And although the same topic reoccurs, different aspects are stressed – like a coin, a topic can have (at least) two sides. Consequently, although there are many similarities between the students, there are also differences.

In this study, it is evident that Edward, a student in access to mathematics with a top grade, regards mathematics as something highly boring; in fact, it is his least-preferred subject. One reason for this can be explained by his talk of being unvalued and that the education does not really fit in with his needs. Perhaps this is because he is seen as a SEM student, but in reality, given that he passes with the highest grade, the teachers do not really treat him as a SEM student.

Neither Veronica nor Ronaldo were keen on participating in the classroom because this caused them to feel uncomfortable. For them, participation in the small group was more secure. This can be seen as an expression of stigmatization, where the label “special needs student” creates obstacles in their participation. This is similar to the result by Civil and Planas (2004), which found that special needs students identified with certain forms of participation.

To conclude, participation in mathematics education in an inclusive classroom is multifaceted and complex. The discourses can be seen as gatekeepers framing students’ participation, and thereby their access to
mathematics education. Additionally, the students’ access to mathematics education and mathematics is influenced by the participation in mathematics education, which makes participation and access inevitably interconnected, united by the idea of inclusion. Hence, inclusion is not easily defined or accomplished but rather it is an ongoing process of participation. However, in this process of participation, ideological statements are not the only decisive elements for inclusion, as individual, group and subject-specific issues must also be taken into consideration.

Reflections on Article V
Similar to Articles II–IV, Article V responds to Research Question 2 in the overall PhD project – What frames students’ meaning(s) of inclusion in mathematics learning and teaching? The article can be regarded as a summary of the results of the empirical part regarding students’ meaning of inclusion of this PhD study. The main contribution of this article is the illumination of all three construed Discourses together, not in any chronological order or hierarchy but rather as simultaneously existing and interconnected D(d)iscourses. Hence, Article V merges the content of Articles II, III and IV. I considered this article important to write in order to be able to reflect on the three simultaneously existing and interrelating Discourses at the same time. This makes visible the complexity of students’ meaning(s) of inclusion in mathematics education as well as the interconnection between access to mathematics (education) and participation. As a former teacher and a researcher, another reflection is the students’ ambivalence in relation to small groups. Is it that schools end up labelling the students when they join small groups, or is it necessary in order to provide them with a good mathematics education? This article contributes with insights to the research field, specifically, how complex inclusive mathematics teaching and learning is and that there are several influencing factors of inclusion that coexist.

Connecting Article V back to the results of Article I, one can say that in Article V, inclusion is used both as an ideology and as a way of teaching. Although the focus of Article V is on meanings ascribed to the notion of inclusion and is more connected to a way of teaching than to an
ideological perspective on inclusion in mathematics education, the ideological perspective is visible in the choice of school and in the endeavour of visualizing framing issues for inclusion.

Discussion of the article process

As previously described, this thesis includes a compilation of five articles which were produced in a process over time. During this process, theoretical and methodological approaches have been refined and developed. Also, each of the articles were written for a particular audience, depending on the aims and scopes of the particular journal or conference. Reflecting on the trajectory of the overall study, there are some shifts related to the theoretical and methodological aspects of the articles. One such shift is the use of the notion, ethnography. Article II was written simultaneously with Article I, and was actually, to some extent, finished before Article I. It is also the first article that uses the tools of Gee on the data from the interviews. The use of the notion of ethnography in Article II was a leftover from the previous licentiate study of meanings of inclusion in mathematics education from a teacher perspective. In that study, ethnography was used, and when writing Article II, the theoretical foundation for this thesis was still in development. In the other articles, the notion of ethnography is not used. Another notion that occurs in only one article is the notion of experience, which is used in Article II. However, it not used further in this PhD study, as it has connotations to a cognitive perspective. This was not intended in Article II, and thus was changed in the coming papers and articles. Another such issue is how the notion of SEM is described. In Article II, it is described as an “educational initiative that may occur if a student is a high or low achiever” (Roos, 2018, p. 1). However, in the later work and in this thesis, the definition of SEM has slightly changed and is about finding ways to meet a diversity in access to mathematics in the education in order to enhance processes of participation. Yet another issue is the way Gee’s (2014a 2014b) theory is explained and used in the articles. During the process of data analysis, I grew more and more acquainted with both the theory and the analytical process. This made the description of the theory and method of analysis slightly different between the papers. This may result in some differences in how
the analysis and theory are reflected upon. Therefore, the text in this overarching description of the thesis reflects the stage the theory and analysis were at when this study was finalized.
SUMMARY RESULTS

This chapter offers a summary of the contribution of the appended articles in this PhD study in relation to the overarching aim of contributing to research and practice within the field of special education in mathematics with more knowledge about, and an understanding of, students’ meaning(s) of inclusion in mathematics education. This was carried out by a meta-analysis of how the D(d)iscourses frame students’ inclusion in mathematics education. The meta-analysis of the D(d)iscourses is guided by the theoretical approach of DA as described by Gee, which suggest the Discourses identified in each article, in relation to the discourses, are used to identify issues of inclusion in mathematics education from a student perspective. Hence, the purpose of doing a meta-analysis of the findings of all the articles written within this study is to synthesize the results of this study. After this meta-analysis, a summary of the results is shown in relation to the three overall research questions in this project: What meaning(s) is ascribed, and how is inclusion used, in mathematics education research? What meaning(s) do the students ascribe to inclusion in mathematics learning and teaching? And what frames students’ meaning(s) of inclusion in mathematics learning and teaching?

For an overall picture of construed D(d)iscourses and what these D(d)iscourses mean in terms of inclusion in mathematics education, the following questions were formulated and asked to each of the articles appended in this thesis:

1. What Discourse(s) is/are construed?
2. What discourses are construed?
3. What has been found, in terms of inclusion in mathematics education, in relation to the result of the D(d)iscourses?
4. How can what has been found be translated into influencing issues to be considered in the mathematics education to enhance inclusion?
The meta-analysis made it possible to synthesize the results from all the articles, which can be seen in Table 7. Here, the three themes, *Discourse(s), discourse(s)* and *influencing factors for inclusion* serve as headings for each category in the table. Although the articles and categories are separated in Table 7, there are interrelations between the D(d)iscourse(s) presented in each article. Also, Article V aims at describing all three Discourses construed in the case study answering to Research Questions 2 and 3. After Table 7, a description of the meta-reflection of the result in the study is displayed by answering the three research questions:

What meaning(s) is ascribed, and how is inclusion used, in mathematics education research?

What meaning(s) do the students ascribe to inclusion in mathematics learning and teaching?

What frames students’ meaning(s) of inclusion in mathematics learning and teaching?

<table>
<thead>
<tr>
<th>Article</th>
<th>Discourse(s)</th>
<th>discourse(s)</th>
<th>Influencing factors for inclusion in mathematics education</th>
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<tbody>
<tr>
<td>I</td>
<td>Discourse of inclusion in mathematics in society</td>
<td>(meanings ascribed) equity mathematics (not) for all including (or excluding) SEM students (operationalization) inclusion in relation to exclusion valuing diversity equity</td>
<td>How equity is valued (or unvalued) in mathematics education. Making mathematics education accessible for (not) all students. How the prevailing view on SEM students influences mathematics education. Revealing exclusive factors in mathematics education to enhance inclusion. How valuing diversity can enhance access and participation in mathematics education. The importance of creating opportunities for students to participate in mathematics education, and succeed in participation between peers, and with the content. A reflection about what favours the students, inclusive or special interventions. How interventions can achieve meaningful participation in mathematics classrooms.</td>
</tr>
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</table>

<p>| Discourse of inclusion in mathematics in the classroom | (meanings ascribed) Participation Inclusion in relation to exclusion (operationalization) Teaching interventions for mathematical engagement Teaching for maximizing opportunities in mathematics for all | | |</p>
<table>
<thead>
<tr>
<th>II</th>
<th>Discourse of assessment</th>
<th>How the mathematics education by assessment limits participation, and thereby, limits access to mathematics (education).</th>
</tr>
</thead>
<tbody>
<tr>
<td>III</td>
<td>Discourse of accessibility in mathematics education</td>
<td>The (un)challenge in tasks influencing participation and access. The pedagogical stance and tactfulness of the teacher enhancing or diminishing students’ participation. How the mathematics education values students is of importance for students’ participation. The meaning of mathematics as something boring challenges students’ participation.</td>
</tr>
<tr>
<td>IV</td>
<td>Discourse of mathematics education setting</td>
<td>Classroom organization How the organization, in terms of the textbook, discussions and “going-through” frames students’ participation and how variation in teaching approaches increases students’ participation. How being in a small group enhances or diminishes students’ participation in mathematics education; also, how the label of “SEM student” may affect participation and access.</td>
</tr>
<tr>
<td>V</td>
<td>Discourse of assessment</td>
<td>Testing Grades The construction of tests and the demands on the students, in terms of explaining, influences both participation and access. Grading influences what students perceive as mathematics and thereby limits their participation and access.</td>
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<td></td>
<td>Discourse of accessibility in mathematics education</td>
<td>Tasks The importance of the teacher (Not) being valued Dislike</td>
</tr>
<tr>
<td></td>
<td>Discourse of mathematics education setting</td>
<td>Classroom organization Being in a small group</td>
</tr>
</tbody>
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Table 7. The meta-analysis of the result in Articles I–V.
Inclusion in mathematics education in society and in the classroom

It can be seen from the results of Article I that research in mathematics education has diverse ascribed meanings of inclusion. Also, the use of inclusion in mathematics education research is diverse (see “Article I – Inclusion in mathematics education: An ideology, a way of teaching, or both?”). Stemming from that result are issues for access and participation on two levels, the societal level and the mathematics classroom level. Looking at the societal level, it is important to consider is how equity is valued (or unvalued) in mathematics education. Also, on this societal level, it is important to reflect on making mathematics education accessible for all students, or at least reflect on the question, If it is not for all, then why? By posing this question, the prevailing view of SEM students influencing mathematics education becomes visible and factors in mathematics education excluding students can be revealed and inclusion can be enhanced. Another way of enhancing access and participation can be by valuing diversity in mathematics education on an overarching level.

Looking into research on inclusion at the classroom level, the research highlights the importance of creating opportunities for students to participate in the mathematics education and succeed in participation between peers, with content meant to enhance access and participation. Also highlighted in research was that there is a need in mathematics education to reflect on what favours the particular students, inclusive or special interventions, and in what way. Hence, the focus was on not only how interventions can achieve meaningful participation in mathematics classrooms, but also how the teaching practices can maximize opportunities for participation in mathematics education. One way to do that in mathematics teaching is by addressing diversity and ensuring that the needs of all students are addressed. This can be made by changing the perspective of the education rather than trying to fit students into mathematics education – the aim should be that the mathematics education fits the diversity of the students.
Similarities and differences in meaning(s) of inclusion in mathematics education

When examining the construed discourses in the student interviews, the meaning(s) students ascribe to inclusion in mathematics teaching and learning were made visible. The discourses for the three students in this study were the same, yet different. The similarities can be found in what the students talked about, and the differences lie in how the students talked about “the what”.

An issue discussed in almost every interview was tasks and how the tasks influence the students’ participation in mathematics education – hence, the discourse of tasks. Here, the students talked about mathematics tasks in both different and similar ways, for instance, Veronica and Ronaldo spoke of the tasks in similar ways. Their participation seemed to be hindered by being asked to complete word problems. Also, both described equations as difficult, although Veronica had a strategy for solving equations correctly. Another similarity was seen in their uncertainty when choosing a method for solving tasks. They both expressed being unsure of how to approach word problems and how to choose a suitable solution. Hence, a meaning of inclusion for these students relates to being able to encounter and understand word problems and equations. The repetition of how to solve a task was also a way of enhancing participation for Veronica and Ronaldo, yet it hindered Edward; instead, what was important for Edward’s participation was the context of the tasks and how well the presentation of the tasks could challenge him. Consequently, the meaning of inclusion in relation to tasks for these students are to be able to gain access to the mathematical content within the tasks. For both Veronica and Ronaldo, they require guidance in both how to read word problems and in how to approach them. For Edward, the tasks need to challenge, engage and be interesting.

Another discourse construed from the students’ text was the importance of the mathematics teacher, who could enhance or diminish their participation in mathematics education. Here, the teacher’s awareness of what mathematics to present and how to do so was important for
Edward’s participation. For Ronaldo, the pace and method of teaching were crucial for his participation, as was obtaining targeted support from a special teacher. This was also important for Veronica’s participation, as was the teacher’s awareness of how and when to explain something. Veronica needed the teacher to talk a lot and be around in the classroom, asking her how she is doing when she works independently. Consequently, a meaning of inclusion from the student is teachers’ awareness in relation to the mathematics content as well as the particular student’s needs.

A discourse related to the importance of the teacher was the discourse of (not) being valued. Here, the meaning of inclusion was seen in how the students were appreciated and seen by the teachers. In both Veronica’s and Ronaldo’s cases, their participation in mathematics education was influenced by how they were valued as SEM students and how much attention was put into adapting the education to their specific needs. In Edward’s case, his participation was influenced by not being valued as a SEM student and not seen in the classroom.

The participation of Veronica, Edward and Ronaldo was challenged by the fact that they disliked their mathematics education or found it boring, hence, a discourse of dislike was construed. The dislike was partly explained by how the education was set up. Consequently, one meaning of inclusion from the three students was to find mathematics education enjoyable and appealing in its setup. The setup could be changed by addressing identified critical aspects in the classroom organization. Critical aspects of the classroom organization were often identified in the student talk, and a discourse of classroom organization was construed. In this discourse, it was shown that the students’ participation was framed by the use of the textbook, discussions, the “going-through” approach as well as other teaching approaches. This made a meaning of inclusion visible through being able to access and understand all parts of the classroom organization. One way to enhance understanding and open up for access is with different teaching approaches, which was seen in the text from all three students. Also how to work in mathematics in terms of experiments, math games, “practical stuff”, being outside doing math, and variations of these, is a way of enhancing inclusion and hence
a meaning of inclusion for the students. In relation to variation, to be able to join a small group sometimes was also visible in the student talk and the discourse of being in a small group was construed. Here, Veronica and Ronaldo shared feelings of security when talking about being in a small group outside the classroom. They also addressed that being in a small group allowed them to have the content presented in the classroom repeated to them, and it also gave the possibility to ask the special teacher to explain something more thoroughly. Hence, being in a small group means the mathematical content can be explained in more detail and from a special educational point of view by providing additional representations in the explanations. Here, the mathematics that had not been understood by the students might be revealed. Consequently, one meaning of inclusion involves a feeling of security and repetition in the mathematics education setting in order to “find” the mathematics.

Framing Discourses on students’ meaning(s) of inclusion

The construed Discourses in this study, the Discourse of mathematics classroom setting, the discourse of assessment, and the discourse of accessibility in mathematics education, can all be regarded as framing Discourses on students’ meanings of inclusion in mathematics teaching and learning. The small discourses are close to the text and thus describe the meaning(s) students ascribe to inclusion in mathematics teaching and learning. The big Discourses are construed from the discourses and can thus be seen as an interpretation of factors framing and influencing students’ meaning(s) of inclusion in mathematics learning and teaching. A short summary of the framings and influencing factors in each Discourse are described below.

Discourse of mathematics education setting

This discourse involves how the organization of the mathematics education, in terms of textbook use, discussions, and “going-through”, frames the students’ meaning of inclusion. However, it does not only frame inclusion but also pinpoints these issues as framing factors for inclusion in mathematics education. In contrast, it can be seen how a
variation in teaching approaches increases students’ participation and access, thus enhancing inclusion. Additionally, this discourse can make visible how being in a small group can enhance or diminish students’ participation in mathematics education, making yet another framing factor visible.

**Discourse of assessment**

This discourse involves how the participation for the students in mathematics education is limited by assessment, and thereby their access to mathematics (education) becomes limited. The testing and the demands on the students in terms of explaining in writing on tests also strongly frames both participation and access. Grading frames what students perceive as mathematics and who they are in relation to mathematics. Thus, grading limits students’ participation and access in mathematics education.

**Discourse of accessibility in mathematics education**

This discourse involves how the (un)challenge in tasks influence participation and access for students and thereby frames students’ meanings of inclusion as well as limits or enhances their access to mathematics. Moreover, it shows how the pedagogical stance and tactfulness of the teacher can enhance or diminish students’ participation and thereby their access to mathematics. Hence, the teachers’ awareness of both the tasks and every student works to frame the students’ meaning of inclusion. This relates to how the mathematics education values the students, which also frames inclusion in mathematics education. Another framing factor of students’ meaning of inclusion is the idea of mathematics as something boring, which contributes to limiting access to mathematics education.

To conclude, these overarching Discourses can not only be seen as framing but also, to some extent, as limiting factors for inclusion in mathematics education for the students in this study. If relating the Discourses construed in Articles II–V to Article I, one can say that, in Articles II–V, inclusion is seen as both an ideology and as a way of teaching. The focus of the articles is on meanings ascribed to the notion of inclusion and connected to a way of teaching. Though, the ideological
perspective is visible in the way the investigated school views inclusion in mathematics education, namely, that every student shall participate in the classroom, and the results should be discussed and related to an ideological perspective. Also, the ideological perspective is visible in the choice of school, and in the endeavour of visualizing framing issues for inclusion, which suggests that the school is trying to find a way to include every student.
DISCUSSION

This study began with a systematic literature review of how the notion of inclusion in mathematics research is defined and used in research. From that, an investigation of students’ meaning(s) of inclusion in mathematics education followed. In this chapter, the results from these two parts will first be discussed. Then, a comparison is made with the results from the licentiate thesis in which teachers’ meaning(s) of inclusion is investigated (Roos, 2015). Together with the results from the licentiate thesis, the results from this study will provide an even more versatile elaboration on inclusion in mathematics education. Thereafter, a discussion of the methodological and theoretical approach in this study is made. The chapter ends with a discussion on the main contributions and conclusions of this research as well as its implications for practice and research.

Research on the notion of inclusion

In Article I, we saw how, when inclusion is used as an ideology in mathematics education research, its meanings and values are articulated. Here, thoughts and visions of inclusion in mathematics are on an overarching societal level, with words like “equity” and “social justice” at the fore. On this societal level, the operationalization of inclusion is not often discussed, meaning it is not made explicit how to do inclusion. Consequently, although a vision is visualized on a societal level, this vision may not have any real impact in mathematics classrooms or on students’ inclusion in mathematics education. In Article I, we also see that a usual way to use inclusion is when referring to a way of teaching without reflecting on any ideological aspects. However, when doing this, it becomes hard to justify the need to work inclusively in mathematics education at all. One conclusion from Article I is that, if one wants to promote a sustainable development of inclusion, ideology and ways of teaching need to be connected. If they are connected, the notion of inclusion can become more sustainable, and hopefully, the research of inclusion in mathematics education can
become more coherent and thus more influential. With that stated, I do not mean to imply that the notion of inclusion has to be used and interpreted in the same way, but rather I suggest thinking of inclusion as more like the nuances of a colour: it originates from the same core, but with variations expressing its various aspects that connect and interrelate with its ideological and operational aspects. In relation to this, Article I shows that the meaning(s) and operationalization(s) of the notion of inclusion in research depend on the theory used, which in turn, depend on the current research paradigm. However, quite often, the research lacks any definition of inclusion. Many scholars use the notion without discussing it, leaving it for the reader to do the interpretation. In this case, one could say that not even one nuance of the colour of inclusion is provided. When comparing this result to other similar studies in the area (Lewis & Fisher, 2016; Secher Schmidt 2015, 2016), these scholars put the undefined notion of inclusion in the background and opt to foreground special educational needs and mathematics instead. Hence, the result in Article I adds to prior research by foregrounding the notion of inclusion and its use in research. Another striking finding in Article I is the lack of students’ meaning in research on inclusion in mathematics education. Thus, a need is identified – the need to take students into consideration in research on inclusion. This need leads to the next step when investigating students’ meaning(s) of inclusion in mathematics education.

Students’ meaning(s) of inclusion

In this study, the choice of individual unit of study is the meaning(s) of inclusion in student talk, and the case is SEM students at a school working inclusively. Here, inclusion is defined as processes of participation. However, not just any participation, but the participation of SEM students in mathematics education. In this study, processes of participation in mathematics education and what frames these processes is described by three interrelated Discourses: the Discourse of mathematics classroom setting, the Discourse of assessment, and the Discourse of accessibility in mathematics education.
The Discourse of mathematics classroom setting describes how the organization of the mathematics education frames the students’ meaning(s) of inclusion. However, the Discourse of mathematics classroom setting does not only frame inclusion, but also it pinpoints issues that limit access to mathematics education and thereby access to mathematics. Issues discussed by the students that may limit participation in mathematics education are the textbook, the going-throughs, and the discussions. Regarding the textbook, how it is organized and how it provides access to knowledge as well as how to write in the notebook are important factors to consider in the mathematics education, and this is also seen internationally (Fan & Miao, 2013). However, these limitations can also be seen as opportunities. Perhaps by using the textbook as one way among several to do mathematics in the mathematics classroom, students’ participation in mathematics would be increased. Hence, putting variation in teaching approaches to the fore in the planning and implementation of the mathematics education in the classroom could increase students’ participation. Also, having different going-throughs at different levels in the mathematics classroom may increase students’ participation in mathematics. Additionally, reflecting more carefully on how to pair students during discussions and using challenging tasks in the discussions in relation to the students’ access to mathematics education, like Diezmann and Watters (2001; 2002) suggest, may increase students’ participation in mathematics education. In the long run, this might increase the students’ access to mathematics. This implies that, to enhance inclusion in the Discourse of mathematics classroom setting, diversity among students need to be taken into consideration and a diverse teaching is needed, as pointed out by Sullivan (2015). Another issue visible in the Discourse of mathematics classroom setting is how small groups outside classrooms may enhance or diminish SEM students’ participation in mathematics education. Thus, being in a small group outside the classroom is not always an example of exclusion and stigmatization, but can instead be a prerequisite for inclusion. Hence, the result of this study shows a different finding than that of Boaler, William and Brown (2000), who found stigmatization in relation to the grouping of students. Hence, inclusion and exclusion are not really about physical positions but rather about students’ feelings of
participation in mathematics education. As a former teacher and now a researcher, I experience an ambivalence in relation to small groups. Are schools (and research) labelling SEM students when teaching mathematics in small groups outside the classroom or are small groups necessary in order to provide an inclusive mathematics education where every student get access to mathematics education? If looking at inclusion from an ideological perspective, small groups may label students and limit their participation in mathematics education, like Civil and Planas (2004) identified. However, based on students’ meaning of inclusion in this study, both Veronica and Ronaldo appreciated being in a small group, sometimes. Hence, it seems like being in a small group can both limit and enhance participation in mathematics education. It limits participation in mathematics education by not being able to participate in the regular classroom education, but it enhances participation in mathematics education by having the possibility to gain access to mathematics by repetitions and opportunities to discuss and ask the special teacher for more thorough explanations. Perhaps, it is the agency of every student that is of main importance for inclusion. Both Ronaldo and Veronica emphasize the possibility to be in a small group, not an external dictated decision to be in a small group. Them being free to choose and that the small group is an offer for them to take – or not – may increase their participation in mathematics education. At the investigated school, the small group is a dynamic phenomenon, it occurs when the teachers or students think it is necessary, although not at every mathematics lesson. However, the small groups are only used for students struggling to gain access to mathematics education. Then an interesting question not yet investigated arises: Would a student in access to mathematics education benefit from such participation? Would Edward’s participation and access to mathematics increase if he had the possibility to join a small group sometimes based on his needs, with “questioning, discovery, rigor and creativity” like Raman and Wiestedt (2011, p. 348) suggest?

The Discourse of assessment describes how assessment limits the students’ participation in mathematics education and how this influences their access to mathematics and the way they think about mathematics as well as what the students identify as being mathematics.
For the three students, the prevailing summative assessment in the form of tests is limiting their participation in mathematics education. Also, for Edward and Ronaldo, the grading is what they perceive as being mathematics, not the actual mathematical activity. According to the Swedish curriculum,

Mathematics is by its nature a creative, reflective, problem-solving activity that is closely linked to social, technological and digital development. Knowledge in mathematics gives people the preconditions to make informed decisions in the many choices faced in everyday life and increases opportunities to participate in decision-making processes in society. (The Swedish National Agency for education, 2018, p. 55)

However, this is not how the students talk about mathematics. Rather, they talk about mathematics as bringing a very specific way of producing answers on tests and about getting good grades. This might be governed by another part in the same curriculum where the knowledge requirements are described. Here, how to “account for and discuss approaches” (p. 63) and how to “reason about their approaches and make proposals on alternative approaches” (p. 64) for the students in this study seem to imply a static procedure of justifying lines of thought. In relation to this, a question arises: If the assessment used in the mathematics education are not only written tests but also formative assessments, would the student talk about mathematics and their meaning(s) of inclusion in mathematics education change?

Interestingly, Veronica did not refer to grades at all during the study. Is this just a coincidence, or is it because she is in Grade 7 and Ronaldo and Edward are in grade 8, where grading becomes more explicit? Or, are there gender differences? These questions merit further investigation, as research stresses gender differences in mathematics education (e.g. Mendick, 2005). Overall, the prevailing culture of assessment in Swedish schools as a measure of quality (Lundahl, 2014) is highlighted by how the Discourse of assessment framed and limited students’ participation in mathematics education. It is reasonable to assume that this culture of assessment is governed by political statements and agendas. This
governing is seen in other research on assessment in mathematics, and how the ways of assessing students to some extent counteract equity and social justice and even exclude SEM students (Bagger, 2017b). This can be explained by how institutional Discourses govern individuals in what to say, do, and how to act, interact and value (Bagger, 2017b); hence, they are a form of institutional power. If the institutional power governing assessment would be redistributed and issues of assessment would be re-negotiated, then the Discourse of assessment would maybe change. Perhaps then, the issues reflected on by the students in this study in terms of how assessment limits their participation in mathematics education and their access to mathematics, would fade. As a result, their inclusion in mathematics education may be enhanced.

In the Discourse of accessibility, it can be seen how the (un)challenge in tasks frames participation in mathematics education and access to mathematics for the three students. Thus, the (un)challenge in tasks frames their meaning(s) of inclusion in mathematics education as well as limits their access to mathematics. Would Ronaldo’s and Veronica’s participation in mathematics education increase if they were given reading comprehension exercises in relation to mathematical texts? This might be one possible way to enhance inclusion, as they both struggle with word problems. Research (Österholm, 2006) has shown that reading mathematical texts with symbols differs from reading other types of text. Hence, there may be a need for education in reading comprehension for mathematical texts that include symbols. Perhaps such an educational strategy could increase the student(s) participation in mathematics education and their access to mathematics. Further, would Veronica’s and Ronald’s participation in mathematics education increase if they gained access to specific strategies when solving equations, given that this is an issue for both of them? In Edwards’ case, would his participation in mathematics education increase if he encountered more challenging tasks, including challenging discussions of these tasks? The importance of challenging tasks for students in access has been shown in prior studies (e.g. Diezmann & Watters, 2002). In addition, if Edward was given more challenging tasks, he may gain enhanced access to mathematics and be more motivated to have mathematical discussions with peers, as it has been shown that there is
a relationship between the complexity of tasks and the extent and type of discussions that engage the students in access (Diezmann & Watters, 2001).

The Discourse of accessibility also shows how the teacher can enhance or diminish students’ participation and thereby their access to mathematics education. For example, for Ronaldo, it is important that the teacher recognizes him as a SEM student and goes at a slow pace, which would enable him to follow. For Veronica, it is important to be “checked upon” and recognized as a SEM student during each lesson. For Edward, it is important to be recognized by the teacher in the classroom, both as a student in access to mathematics education and as a person. This implies teaching mathematics requires more than mathematical, didactical and pedagogical skills. It requires a competence to see the individual students’ prerequisites as a student in mathematics and as a person, and be able to put all these pieces together to see the “whole student in mathematics”. Hence, teacher’s awareness of the students and sensitivity towards the students are important to enhance the participation of students, which is a finding from previous research (Sullivan, Zevenbergen & Mousley, 2003; Anthony, 2013). Ljungblad (2016) calls this the “pedagogical stance” and the “tactfulness of the teacher”.

Another framing factor on students’ meaning(s) of inclusion is their dislike of mathematics, which challenged Veronica’s, Edward’s, and Ronaldo’s participation. It has been shown in other studies that boredom creates decreased participation (Murray, 2011). The dislike is partly explained by how the education is set up and partly explained by the fact that mathematics seemed to be hidden from the students in some way. That is, the procedures (remembering a specific way of doing, etc.) and the format to present the line of thought in mathematics seemed to cloak what mathematics is. As Solomon (2009) highlights, one way to promote participation for the students is to make the hidden practices visible and explicit for them and the teacher to reflect upon the student and teacher relationship with power relations embedded. If mathematics education in the classrooms of the study would have a more varied setting and the mathematics teachers reflected about the
hidden practices and teacher–student relationships, maybe the participation of the student, as well as inclusion, would be enhanced. One way to accomplish this may be to identify important issues for inclusion in the classroom organization. One such important issue is seen in the discourse of classroom organization. Here, using different mathematical experiments, math games, and practical stuff is seen in the student talk. In addition, it was observed that the students talked about varying these approaches to make mathematics education more engaging.

To conclude, these overarching Discourses can be seen not only as framing but also, to some extent, as limiting factors for inclusion in mathematics education for the students in this study. Although the Discourses and discourses for the three students in this study are the same, they are nevertheless different. It is like gazing through a faceted crystal (where the crystal is the D(d)iscourse): The crystal is the same, but depending on the angle from which one looks, different objects and colours come into focus. Hence, it is always about the standpoint taken and about the individual student. Nevertheless, the D(d)iscourses in this study do tell us something about the students’ meaning(s) of inclusion in mathematics education not only on an individual level but also on a more general level. As Flyvbjerg (2011) concludes, a single case can often be generalized because the force of example and transferability are underestimated. On a general level, one conclusion is that assessment frames and limits inclusion in mathematics education. Another conclusion is that how the setting of the mathematics education is planned and how the school and the teachers reflect and work in this setting frames the students’ meaning of inclusion in mathematics education. Also, how teachers and the organization at a school reflect upon the use of small groups as a way of enabling students to gain access to mathematics frames the students’ meaning of inclusion in mathematics education. Other framing issues of students’ meaning(s) of inclusion on a general level are how the teacher explains and values the SEM students and what tasks are used in mathematics education, in terms of challenge and access to mathematics.
A final conclusion to be made from this study on a general level is if mathematics education can move away from mathematics being identified as a boring subject, then accessibility to mathematics education and, in turn, to mathematics would perhaps increase. When comparing this result with other research on inclusion in mathematics education foregrounding students (e.g. Kleve & Penne, 2016; Solomon, 2009), where being an “insider” and developing identities as a mathematics student is in focus, this study builds on these results. It does so by trying to frame students’ meaning(s) of inclusion in mathematics education to highlight possibilities to become an insider and what is crucial for developing an identity as an insider.

Student vs teacher meaning(s) of inclusion

When comparing the results in this study of students’ meaning(s) of inclusion in mathematics education with the results in the licentiate study of teachers’ meaning(s) of inclusion in mathematics education (Roos, 2015), what can be seen in terms of similarities and differences?

In the licentiate study of teachers meaning(s) of inclusion in mathematics education in primary school, a set of three principles emerge that explain the process of inclusion at the investigated school. The first principle is dynamic inclusion. Dynamic inclusion refers to courses of intensive teaching during a short period, both in and out of the classroom (and with both students and the special teacher), listening to students and changing roles as teachers. The second principle is content inclusion. Content inclusion refers to representations, tasks, strategies and generalizations, didactical discussions, teacher knowledge, recognition of similarities, connection of content, and to reach and challenge the SEM students. The third principle is participating inclusion. Participating inclusion involves being responsive, listening to students, encouraging students’ participation, and developing self-esteem and self-confidence.
Similarities

Several things are present in both studies on inclusion in mathematics education. The first concerns where to be, and why and when receiving the special education. The students talk about sometimes being in a small group to get some peace and quiet and an extra opportunity for explanations. The teachers talk about a dynamic way of providing special education, sometimes being in the classroom and sometimes in a small group to be able to go deep into explanations according to the students’ specific needs. Another visible similarity is the talk of tasks. Here, the students express how tasks limit their participation, either in terms of the challenge of reading word problems and of equations, or the un-challenge. The teachers talk about tasks with several different representations inviting the students to participate in mathematics education and the problem of finding tasks in the classroom that invite every student, hence, meeting diversity in the classroom. In relation to tasks, the teachers talk about strategies when approaching tasks; this is also an issue for both Veronica and Ronaldo, especially when they talk about word problems and equations. The teachers also talk about how to reach and challenge the SEM students in the classroom and in the education. This is also visible in the student talk. Edward talks about not being challenged, and Ronaldo and Veronica point out the importance of receiving special mathematics education, in terms of attention that is given to their specific needs. This relates to another similarity, the teachers talk about being responsive to students in order to promote student participation in mathematics education, and the students talk about the importance of being valued by the teachers and the importance of the teachers regarding them as SEM students. Another similarity is when the teachers talk about teacher knowledge, the students talk about the importance of being met at the right level, which can be understood as a form of teacher knowledge. Being met at the right level for Edward is to be challenged in mathematical severity, for Ronaldo it is to be given explanations at a slow pace, and for Veronica, it is being given oral explanations.
Differences

The teachers talk about the importance of teacher discussions to enhance teacher knowledge by collegial learning. Also, they point out issues of recognizing similarities and the importance of connecting the content between situations for the students. This is not visible in the student talk. Another thing made visible in the teacher talk is students’ self-esteem and students’ self-confidence in relation to learning in mathematics. This is not really visible in the student talk; instead, they talk about mathematics as something boring in relation to learning. Hence, there seems to be an interpretation by the teachers that students’ self-esteem has an impact on learning mathematics, but the students do not express this in relation to learning; instead, they relate learning in mathematics to how mathematics education is accessible. In other words, the teachers have a tendency to place issues of learning within the individual student, and the students have a tendency to put issues of learning outside in the learning environment. If one relates this to two special educational perspectives – a relational perspective, in which the heritage of the problem is described as located in the environment, and a categorical perspective, where the heritage of the problem is placed inside of the student (Persson, 2008) – one might say that the teachers have a tendency to place the reason for not participating in mathematics education with the students, while the students themselves place the reason for not participating in mathematics education as being a result of the environment.

A striking difference is the impact of assessment, which is seen in students’ meaning(s) of inclusion in mathematics education. This is not seen at all in teachers’ meaning(s) of inclusion. Maybe the fact that the study of teachers’ meaning(s) of inclusion is made in primary school, where grading is not relevant until Grade 6 is a factor in this difference. Although, based on the empirical material available, it is not possible to say if assessment is seen as an influencing factor in teachers’ meaning(s) of inclusion in mathematics education in lower secondary school. However, given that it is such a decisive influencing Discourse on students’ meaning(s) of inclusion in mathematics education, this is something worth investigating further. Another difference can be seen in how the setting of the mathematics classroom had an impact on students’
meaning(s) of inclusion in mathematics education. This is not visible in the teachers’ talk. They do not talk about specific setups and teaching approaches and the need to vary them, as the students do; instead, they talk about content-specific issues, such as the use of different representations.

It is important to take into consideration that the two studies compared were conducted at different schools and involve different grades, students, and teachers. Therefore, explicit conclusions cannot be drawn from this comparison. However, interesting issues arise in the similarities and differences, that can inform research and practice regarding inclusion in mathematics education, for instance, the role that assessment plays in relation to inclusion in mathematics education and the role of the mathematics classroom setting for inclusion.

Reflecting on the similarities and differences shown above, there might be consequences for students’ access and participation in mathematics education depending on the degree of consensus on meaning(s) of inclusion in mathematics education in the particular learning situation of the particular student. If there are many similarities regarding meanings of inclusion from students and teachers, this may lead to enhanced participation in mathematics and increased access to mathematics. If there are many differences, this may lead to declining participation in mathematics education and decreased access to mathematics. Here, inclusion can be seen as a dynamism striving for full participation in mathematics education and ideal access to mathematics. However, it is important to point out that there may be institutional and political discourses in the teachers’ and student talk, which to some extent, is also a part of the meaning(s) of inclusion.

Theoretical and methodological concerns

In the first part of the present study, there is a systematic literature review on the notion of inclusion in mathematics education research. Here, the timespan in the search could have been expanded to also include the history of the notion of inclusion. However, as the aim was to grasp the recent state of the notion and the result ended up with 76
studies, the search had served its purpose. However, it is important to recognize the limitation of the scope using “inclusion” as a word in the search string. This limitation means that research conducted in the spirit of inclusion but that uses a different term is not visible, and by that, not part of the study. This limitation can be a reason for the tendency in interventionist studies to focus on low achievement and disability rather than ethnicity, gender, language and class. Interventionist studies that focus on the latter issues do not generally use the notion of inclusion.

In Article I, DA as described by Gee (2014a, 2014b) is used. The analytical tools provided by Gee are adjusted according to the research questions, small discourses are construed, and from them, overarching big Discourses. Thus, DA is not only an analytical tool but also a tool for sorting and visualizing issues of inclusion in research by providing a form of structure. In this way, DA contributed on both a theoretical and methodological level to construe discourses and Discourses, and by that, provided insights into different foci present in research regarding inclusion in mathematics education today. In this study, Gee’s (2014a, 2014b) approach on DA is used in a different way than it usually has been used. Gee (2014b) suggests that a discourse analysis starts with looking at the big picture, at Discourse(s). But, he also writes that “each reader may well find their own favoured order” (p. 2) and “there is no necessary order to the tools” when applying them to data in written and spoken form and observations (p. 195). I contacted James Paul Gee by email and asked if it is possible to apply the tools only to written text, which he said is doable (email conversation 5 October 2017). Hence, the way Gee’s discourse analysis is used on written text, recognizing small discourses first, and then big Discourses, can be seen as a development and a contribution on how to use DA and do systematic literature reviews. In the analysis, I found it prosperous to first apply the tools helping to construe the discourses and then apply the tools helping to construe the Discourses. This order worked for this specific data and for me as a researcher, as suggested by Gee (2014b), I found my favoured order.

In the second part of the study, a case study of three students and their meaning(s) of inclusion in mathematics education was conducted. When
doing research, there is always a tension between theoretical considerations and an ideal way of doing things as well as the practicalities. One such tension can be seen in the selection of SEM students. In this study, I want to focus on students in SEM, but given that I do not want to impose my interpretation of SEM on anyone, the teachers were asked to reflect on SEM and suggest SEM students. Therefore, at the meeting with all the teachers before entering the field, they reflected on this and found two criterions – students are in SEM if they struggle to reach the accepted knowledge requirements according to the curriculum, or if the teachers find that they have difficulties in challenging the students who are in the mathematics facility reaching the highest knowledge requirement. Thereafter, the selection of SEM students was made after suggestions from the mathematics teachers and the special teacher. From an ethical perspective, it is necessary to let the teachers with their background information suggest students, so as not to expose vulnerable students. This make teachers in the selection process “gatekeepers”. In relation to the selection, the teachers base their selection on a collective meaning of what a SEM student in access to mathematics education is, and what a SEM student in struggle to get access is. This meaning is then imposed upon the students by the school mathematics discourse. Hence, the idea of being “in need” is socially constructed. If being in need in mathematics had been constructed differently or by other teachers, the selection of students in this study might had been different.

In this case study, there is a claim made that the object of study is the case of meaning(s) of inclusion in mathematics in student talk. Hence, this is a case with more than one student in the overall case. These students are considered as extreme and critical cases because they are in SEM and are cases within the overall case. The extreme case provides with important information about the meaning(s) of inclusion for SEM students. The critical case helps us to reflect about the case, as what is seen as valid for this case, may apply to all cases (Flyvbjerg, 2011). Another claim of this study is that this is an information-rich case in order to get a “a best case scenario”. This because the choice of the school was for an inclusive school. Consequently, if students’ meaning(s) of inclusion are not explicit in this case, then where are they made explicit?
Given the choice of an inclusive school setting and the choice of SEM students meaning(s) of inclusion in this setting, this study claims there is a force in this example and possibilities of transferability, as suggested by Flyvbjerg (2011).

According to Gee (2014b), the validity of a DA is never “once and for all” (p. 195), but rather it is always open for further discussion. However, there are four elements highlighted by Gee constituting validity for a DA: convergence, agreement, coverage, and linguistic details. I will discuss below in what ways the DA used in this study on students’ texts meets these criteria. The convergence involves how the analysis offers compatible and convincing answers to many of the tools’ questions. In this analysis, given that the students have been interviewed during an entire semester (at least five times each) and given that the tools of inquiry are applied to all texts, a broad range of both compatible and convincing answers can be found and an in-depth perspective of students’ meaning(s) of inclusion in mathematics education can be reached. However, using a list of tools can be challenging, as some tools may not be well suited to the actual set of data, and also, some tools perhaps are not well suited to the research questions. This is why, after the first use of the tools of inquiry – a selection of tools useful for this particular study – both research questions and data set is made (this is described in the section, ”The data analysis process”). However, this can be seen as a weakness in the convergence. Agreement implies that the answers to the tools are more convincing when more members of the construed Discourses agree that the analysis shows the function of the language. It also implies how other discourse analysts and researchers support the conclusion. In this research, I did not discuss the Discourses with the interviewed students who are members of the construed Discourses, which can be seen as a weakness in validity. However, as the study on inclusion in research in mathematics education (Roos, 2019a) and parts of the study of students’ meaning of inclusion in mathematics education (Roos, 2018; Roos; 2019b; Roos 2019c) have been published, the way DA is applied in this study can be considered as accepted by other researchers. This can be seen as high validity. Coverage is when the analysis is considered more valid the more it can be applied to related data. In this study, because I have analysed 17 different
interviews from three different students with 21 observations, it can be seen as having high coverage. Finally linguistic details imply that, the more the analysis is tied to the linguistic structure, the higher the validity. In this study, parts of the analysis are tightly connected to the linguistic structure in the application of the linguistic tools. By using the linguistic tools, specific words, subjects, stanzas and deictics used by the students are seen as the specific language of the D(d)iscourses. Also, the many interviews conducted during the entire semester provided with data to an in-depth study. For instance, issues appeared in the students’ text after two or three interviews that maybe would not have appeared if the students had not had the time to get to know me and trust me as a researcher. The reason for this may be that the students became comfortable with talking about their own learning of mathematics with me. For example, in the first interview, some of the students are cautious and hesitant, but in the second interview, they are more open in their answers. By the third interview, they are really engaged in the discussion.

To conclude, the way DA is used in this study is a methodological contribution. DA has a critical and political view, and although it is not foregrounded, it nevertheless exists like a backdrop. What is foregrounded is the descriptive part of DA, in this case, how students’ meaning(s) of inclusion in mathematics education can be described.

**Main contributions, conclusions and implications**

A contribution of this study is the analysis of how inclusion is used in mathematics education research, as an ideology and as a way of teaching, and how these two uses are most often treated separately and independently of each other. From this result, scholars can shift the positions of future research on inclusion in mathematics education and thus try to treat inclusion both as an ideology and a way of teaching at the same time. Or at least they can be aware of the different uses and be careful to define the use and the meaning of inclusion in the specific research.
A conceptual contribution of this study is the interpretation of the notion of SEM, which was expanded to include both students in struggle to gain access and students in access to mathematics education. In relation to this, the notion of inclusion becomes both challenging and interesting, as it required working in two directions to cover the diversity of access to mathematics. Another conceptual contribution in regard to the notion of access is the relationship between participation, inclusion, and access. Here, the students’ participation in mathematics education is framed by the D(d)iscourses and thereby their access to mathematics is framed by the extent of their participation in mathematics education. However, their access to mathematics is in turn framed by their participation in mathematics education, which makes participation and access inevitably interconnected and joined by the idea of inclusion. Thus, the idea of inclusion as processes of participation aims at not only moving towards full participation in mathematics education but also moving towards full access to mathematics. This relationship between participation, inclusion, and access, work in both a positive and a negative direction regarding full participation and enhanced access for the individual student. Also, this relationship is relative to the individual student. Ronaldo’s and Veronica’s participation and access in the discourse of going-through is one example. Ronaldo participates in the regular mathematics education, a form of physical inclusion, but given that his participation in mathematics education is hindered by the number of going-throughs he requires and his need for structure and slow instructions, his access to mathematics is hindered. Veronica also participates in the regular mathematics education, but her participation is enhanced by getting many going-throughs and instructions from the mathematics teacher; therefore, her access to mathematics increases. Hence, by aiming at working inclusively, the education strives towards full access and full participation. Though, to be able to work towards full access and full participation, the education must be mindful and reconsider the individual students’ needs in the mathematics education in order to really know how to include the students. This suggests that there is no one-size-fits-all solution regarding inclusion in mathematics education, but rather inclusion depends on the students and teachers involved in the process.
From a methodological perspective, the main contribution is how DA as described by Gee can first be adjusted and then used in similar ways in both a data set containing research texts and in a data set containing interviews and observations. From a theoretical perspective, the main contribution is how focusing on small-scale Discourses in relation to a social phenomenon such as inclusion can provide insights that are not only on an overall political level but also on an overall descriptive level.

This study also contributes to both research and practice in how students’ meaning(s) of inclusion can be described by the three overarching Discourses construed in this research: the Discourse of mathematics classroom setting, of assessment, and of accessibility in mathematics education. This description can be related to previous research on inclusion in mathematics education (e.g. Solomon, 2009; Kleve & Penne, 2016) and to the situation in Swedish schools regarding the inclusion of SEM students (e.g. Persson & Persson, 2012). By doing this, issues that might be hidden in both research and practice concerning inclusion in mathematics education can be made visible. Then, perhaps not only issues found in this specific research will be illuminated, but also other issues situated in the specific time and space where inclusion in mathematics education is investigated will surface in relation to the three Discourses in this study.

For practice, insights are provided on how different aspects of mathematics education, both at a societal level as well as the school level, influence students’ meaning(s) of inclusion in mathematics education. It also problematizes and provides a multifaced picture of what students’ meaning of inclusion in mathematics education can be. This can inform and improve practice regarding the view on assessment and the setting of the mathematics education and issues of accessibility in the specific practice. Here, the small discourses make issues of what frames inclusion for the SEM student visible in terms of testing, grading, tasks, the importance of the teacher, (not) being valued, the dislike of mathematics, the classroom organization, and being in a small group. These discourses also suggest how to work at an individual level, both in the mathematics classroom and at the organizational level at the school in order to include every student in primary school mathematics.
education. For example, the result of this study shows an approach to the small group outside the classroom as one way to enhance inclusion in mathematics education.

Reflecting on the results of this study, what can be concluded? One thing, which is also indicated in the title of Article V, is that the meaning(s) of inclusion in mathematics education in student talk is the same, yet very different when listening to individual SEM students. Because of the differences of students in mathematics classrooms, the education in these classrooms needs to be able to meet these differences; thus, diversity among students demands diversity in mathematics education. This diversity in the education needs to take into consideration the issues of both participation in mathematics education and access to mathematics. This adds to previous research about valuing diversity (Brown, 2015; Krainer, 2015; Sullivan, 2015) and taking diversity as a point of departure in the mathematics education, and it also raises awareness of issues of participation and access.

Another conclusion is that inclusion is not equivalent with every student being in the same classroom. Although the investigated school set out to work inclusively from an ideological point of view, with all students in the same classroom, some D(d)iscourses limit students’ participation in mathematics education, and thus the possibility to be in a small group outside the classroom is expressed as positive for the students. Hence, one implication is that we in Sweden need to move from an almost monolithic view of inclusive classrooms as a physical room where every student is always present physically to a more dynamic view on inclusion, which is more situated and related to the students and their prerequisites. This implies that education needs to move from implicitly trying to fit the students into what is considered “normal” of all students towards departing from the opportunities of every student. With this stated, there is a need to be mindful and careful so as not to label students in SEM and not create stigma, as highlighted by Civil and Planas (2004). However, it is also important to be aware that stigma can be present and be created inside an inclusive mathematics classroom as well. Here, the individual students’ agency is of importance to be able to, with the help from teachers and special teachers, decide when and
how to receive special education. This would be interesting to further investigate, namely, how to work inclusively in mathematics education to enhance participation and access not only with both students’ and teachers’ meanings foregrounded but also from a more sociopolitical view on inclusion that takes research on inclusion in mathematics on the societal level into consideration. Is it possible to join these three viewpoints to enhance participation in mathematics education and access to mathematics?

Another conclusion is that a student in access to mathematics education also can be in special educational needs. This implies a need to acknowledge these SEM students and offer special mathematics education in order to enhance their participation in mathematics education and access to mathematics. This is an important path to further investigate, namely, how to support students in access to enhance their mathematics learning from a special educational perspective, especially as most studies that have been conducted have a cognitive focus (e.g. Oktaç, Fuentes & Rodriguez Andrade, 2011; Shayshon, Gal, Tesler & Ko, 2014).

If one takes a critical perspective on the results of this study, how might inclusion in mathematics education be different? How might changes in practice change D(d)iscourses enhancing students participation in mathematics education and access to mathematics by working inclusively? Considering the three Discourses, there are issues visible in the D(d)iscourses that can be changed. For example, when looking at the organization of the mathematics education, changing it from being governed by a textbook to a more flexible way of organizing, with more variations in how to do mathematics. More variations would perhaps change the D(d)iscourses and thereby perhaps the students’ meanings of inclusion in mathematics would change as well. Another critical issue is the social construct of the SEM student. If the construction of a SEM student changed and diversity would be taken as a point of departure in the mathematics education, maybe the D(d)iscourses would change, and as a result, the way school and the society look upon special needs would change.
Looking back at this investigation, it is striking how complex and challenging teaching mathematics is. Teachers are expected to be able to handle students’ diversity and promote all students’ mathematical development. To enhance students’ participation in mathematics education and access to mathematics demands that the teacher knows her or his students, is flexible, has a pedagogical stance and tactfulness, and is knowledgeable in mathematics and mathematics education. Also, it demands that the teacher is able to take a critical stance and resist the prevailing discourse of assessment that can sometimes overshadow the mathematics education, and in a sense, almost become mathematics for the students.

It is also striking how complex and challenging it is being a student in mathematics. They are expected to relate to, understand, and participate in all the Discourses existing at the same time in a single mathematics classroom. These Discourses interrelate and are embedded in power relations between students and teachers and institutions. This demands that the students be alert, be able to use various symbols and objects, recognize patterns, and act accordingly. Hence, to be able to fully participate in mathematics education, you have to talk the talk and walk the walk (Gee, 2014a), meaning you not only have to use the language right, but also you have to act right at the right time and the right place to be fully included in the mathematics education and thereby gain access to mathematics.
Further reading

In this study of inclusion in mathematics education, several interesting issues arose. However, not all of them fit within the aim and research questions of this particular study. The following articles, presented chronologically, fall on the outskirts of this study but are nevertheless worth mentioning.


This research process reminds me of the words of the Swedish artist Lars Winnerbäck when he sings, “I am carving in stone. Slowly, I think I begin to see a silhouette. Some arms and some legs, I am working into the stone, until I see a figure” (own translation). In pursuing this research endeavour, in finding that figure deep inside the stone, one thing has been the most important – the freedom in the research process. As hard as it has been not being part of a research project or a research school, having to create my own field to investigate and govern my own process has been vital for me. For that reason, I will end this thesis with one of my own poems. I dedicate it to every person who has trusted me in this research endeavour. I am forever grateful that I have had the freedom to do my thing. I hope this research freedom will continue forever despite the cold political winds blowing in these times.

Free

Lock me up and restrain me – and I will explode.
Make limitations and obstructions and I will be sad and broken.
Give me green meadows and unlimited space of air.
Give me wild forests and an open sea.
Give me wind in my face every day.
Let my thoughts run free up, up into the sky – to the moon and above.
Let my soul fly free like a feather in the wind – creating its own magic dance.
Let me be free – then everything – just everything will be just … fine.

Slutet gott, allting gott.

Växjö, March 2019
Syftet med denna studie är att bidra till forskning och praktik inom fältet specialpedagogisk matematikdidaktik med mer kunskap om, och en förståelse för hur varje elev kan bli inkluderad i matematikundervisningen ur ett elevperspektiv. Forskningsfrågorna i studien var: Vilken eller vilka mening(ar) tillskrivs inkludering i matematikdidaktisk forskning? Vilken eller vilka mening(ar) tillskriver elever inkludering i lärande och undervisning i matematik? Vad inramar elevers mening(ar) om inkludering i lärande och undervisning i matematik?

organisatoriska åtgärder för att stödja samarbete och diskussioner; att ha väl fungerande team som arbetar med förebyggande åtgärder i matematikundervisningen, det vill säga att utnyttja den lärarkunskap som finns i organisationen om och i matematikdidaktiska frågor, samt sist, men inte minst; att lyssna på elevernas röst. Från denna studie syntes alltså ett behov av att lyssna på elevernas röst i undervisningen. Men hur lyssnar vi på elevernas röst i forskningen? Denna fråga levde vidare och togs med in i doktorandstudien. Således blev det övergripande målet i doktorandstudien att undersöka elevers mening(ar) av inkludering i matematikundervisningen.

Doktorandstudien inleddes med att undersöka vilken, eller vilka, mening(ar) som tillskrivs begreppet inkludering i matematikdidaktisk forskning. Detta för att undersöka hur begreppet används samt att se hur elevernas röst kommer fram i forskning. För att undersöka detta gjordes en systematisk litteraturstudie med fokus på att identifiera mening(ar) av begreppet inkludering i forskning och en artikel som analyserade resultatet av detta skrevs (Roos, 2019a). Resultatet visade att det finns en mångfald av sätt att använda begreppet inkludering i matematikdidaktisk forskning, och det finns en mångfald av tillskrivna meningar för begreppet i forskningen (se artikel 1 - Inclusion in mathematics education: an ideology, a way of teaching, or both?). Resultatet visade även att frågor kring access och deltagande i matematikundervisningen var centrala i forskningen. På en samhällelig nivå visualiserades detta genom forskning som betonande viten av reflektion i jämlighetsfrågor och hur jämlighet värderas (eller inte) i matematikundervisning. På samhällsnivå synliggjordes även reflektioner kring att göra matematiken tillgänglig för alla elever, eller reflektioner kring varför inte matematik är eller kan vara tillgänglig för alla elever. Genom att denna reflektion blir den rådande synen på elever i särskilda utbildningsbehov i matematik (SUM) synlig och faktorer i matematikundervisningen som exkluderar elever kan synliggöras. Genom att synliggöra dessa faktorer kan inkludering i matematikundervisning öka, om man adresserar dessa faktorer i undervisningen. Ett annat sätt att öka access och deltagande i matematikundervisningen kan vara genom att värdera mångfald i matematikundervisningen på en övergripande nivå.
Resultatet av den systematiska litteraturstudien visade även att mycket av den forskning som fokuserar på inkludering i matematik på klassrumsnivå belyser vikten av att skapa möjligheter för elever att delta i matematikundervisningen, lyckas i samarbetet med kamrater samt i lyckas i relation till det matematiska innehållet för att öka access och deltagande. Vikten av att i undervisningen reflektera kring vad som är viktigt för individen, om det är att vara i klassrummet, eller att delta i interventioner i specialundervisning och på vilket sätt. Således betonas hur interventioner kan hjälpa till att uppnå meningsfullt deltagande i matematikklassrummet, med större möjligheter för deltagande i matematikundervisningen. Ett sätt att göra det är att byta perspektiv i undervisningen, och istället för att försöka få elever att passa in i den befintliga matematikundervisningen, skapa undervisningen utefter den mångfald av elever som finns i klassrummet. Slående var att det fanns endast några få forskningsstudier som undersökte inkludering med fokus på elevers mening.

Efter den systematiska litteraturstudien genomfördes en studie med fokus på att undersöka elevers mening av inkludering i matematik, just för att tillföra elevperspektivet till forskning om inkludering i matematikundervisning. I denna studie är inkludering definerat som processer av deltagande. Dock, inte vilket deltagande som helst, utan deltagande i matematik klassrummet av elever i SUM. Detta medförde att elever som var i någon form av SUM, samt hade kännedom om inkluderande matematikundervisning eftersöktes. Således valdes en högstadieskola som utger sig för att arbeta inkluderande i all undervisning. Med detta menar de att de har som mål att inte ha några särskilda undervisningsgrupper, utan målet är att alla elever ska delta i den undervisning som bedrivs i de vanliga klassrummen. Elever som gick i årskurs 7 respektive 8 valdes ut i samspråk med de undervisande lärarna och speciallärare i matematikutveckling. Sex elever följdes under en termin. I denna avhandling har tre elever fokuserats, en flicka, Veronica, i årskurs 7, och två pojkar, Ronaldo och Edward, i årskurs 8. Edward valdes på grund av att han uppleves av lärarna som en elev med god access till matematikundervisningen. De andra två eleverna
valdes på grund utav att de upplevdes kämpa med att få access till matematikundervisningen.


Gee har 28 undersökningsverktyg som belyser kommunikation genom att ställa frågor till texten. Dessa frågor öppnar upp för undersökning av texten och vad som finns bakom texten i termer av d(D)iskurs(er). Några av verktygen är lingvistiska och ligger nära texten och textens sammanhang, medan andra verktyg ger tillgång till tolkning av den större bilden och det större sammanhangen avseende vad som sker i den sociala världen. I denna studie av elevers tal om sitt eget deltagande i matematikundervisningen, har dessa verktyg använts i relation till denna studies syfte och forskningsfrågor.
Resultatet av denna analys presenteras i forskningsartiklarna II-IV. Resultatet kan kortfattat beskrivas med hjälp av tre interrelaterade Diskurser: Diskursen matematikundervisningens uppbyggnad, bedömnings-Diskursen och tillgänglighets-Diskursen. Dessa Diskurser beskriver elevers mening(ar) av inkludering i matematikundervisning, och vad som ramar in och påverkar denna/dessa mening(ar).

**Diskursen matematikundervisningens uppbyggnad** visar hur organisationen av matematikundervisningen gällande användning av läroböcker, diskussioner med klasskamrater och genomgångar ramar in och påverkar elevers mening(ar) av inkludering matematikundervisning. Emellertid visar denna Diskurs inte endast vad som ramar in, men visar även begränsande faktorer för elevernas access till matematik. Diskursen visar även hur att vara i en liten undervisnings-grupp kan förstärka eller minska elevernas deltagande i matematikundervisningen, vilket visar på att den lilla undervisnings-gruppen både ramar in och påverkar inkludering i matematikundervisningen. Ur detta resultat föds en kritisk fråga; hur påverkas access och deltagande i matematikundervisning när man är klassad som en SUM-elev?

**Bedömnings-Diskursen** visar hur elevers deltagande i matematikundervisningen är begränsad av bedömning i termer av betyg och prov. Hur test konstrueras, och de krav testen har på elevers skriftliga förklaringar och svar påverkar starkt SUM-elevernas deltagande och därmed deras access till matematik. Betygen påverkar även vad elever upplever vara matematik, och därmed konstituerar betygen en begränsande faktor för elevers deltagande och access i matematikundervisning.

**Tillgänglighets-Diskursen** visar hur utmaningar, eller brist på utmaningar, i matematikuppgifter påverkar elevernas deltagande och access. Diskursen visar även att ibland begränsar matematikuppgifterna elevernas deltagande i matematikundervisning, och därmed deras inkludering. I denna Diskurs blir det även synligt hur lärarens pedagogiska takt och hållning kan öka eller minska elevers deltagande,
och därmed deras access till matematiken. Följaktligen påverkar lärarens medvetenhet elevers mening av inkludering. Detta relaterar till hur matematik-undervisningen värderar elever, något som också påverkar elevers mening(ar) av inkludering i matematik. En annan begränsande faktor som återfinns i denna tillgänglighets-Diskurs är uppfattningen av matematik som ett tråkigt ämne. Detta blir en begränsning för access till matematik.

För att sammanfatta, de tre övergripande Diskurserna kan ses som inramande, men också till viss utsträckning begränsande för inkludering i matematikundervisningen ur ett elevperspektiv. Även om D(d)iskurserna för de tre eleverna i denna studie var samma, var de ändå olika. Det är som att titta igenom en kristall (där kristallen är D(d)iskursen), kristallen är densamma, men beroende på från vilket håll man tittar, kommer olika objekt och olika färger i fokus. Således, det handlar alltid om vilken utgångspunkt som tas, och om den individuella eleven.

D(d)iskurserna i denna studie säger något om elevernas mening av inkludering i matematikundervisning, inte bara på en individnivå, men också på en mer generell nivå. På en mer generell nivå kan sägas att bedömning i matematik påverkar och begränsar elevers syn på inkludering. Vi kan också konkludera att hur matematikundervisningen är uppbyggd, och hur skolan och lärarna reflekterar kring, och arbetar med denna uppbyggnad av matematikundervisningen påverkar elevernas mening av inkludering. Även hur lärare och organisation på en skola reflekterar kring och använder små grupper i undervisningen som ett sätt att öka elevernas access till matematiken påverkar elevernas mening av inkludering.

En annan slutsats som kan dras av resultaten av denna studie på en mer övergripande nivå handlar om tillgänglighet till matematik-undervisningen. Hur och vilka uppgifter som används i matematikundervisningen samt hur läraren förklarar och värderar SUM-eleverna påverkar tillgängligheten till matematiken för eleverna. Ytterligare en reflektion kring resultaten är hur vi kan gå ifrån synen på matematik
som ett tråkigt ämne, och därmed öka tillgängligheten i matematikundervisningen för eleverna.

För att konkludera, det finns ingen ”one-size-fits-all” lösning på inkludering i matematikundervisning, det beror alltid på de elever och lärare som är involverade i lärande och undervisningsprocessen. Följaktligen, en mångfald av elever kräver en mångfald i matematikundervisningen.
REFERENCES


Lindenskov, L., & Lindhardt, B. (2015). Developing test materials for developmental dyscalculia for Danish pupils in grade 4: the political background and the research based design. The 8th Nordic research conference on special needs education in mathematics.


Straehler-Pohl, H., & Pais, A. (2013). To participate or not to participate? That is not the question! In B. Ubuz, Ç. Haser, & M. A. Mariotti (Eds.), *CERME 8: Congress of the European Society for Research in Mathematics Education* (pp. 1794–1803). Turkey: Congress of the European Society for Research in Mathematics Education.


Swedish National Agency (2014). *Teacher information, test A Mathematics, year 6*.


APPENDIX

List of Appendices

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Appendix 1 – Letter of Approval to Students and Guardians in English

Information for the students

I am a PhD student in mathematics education at Linnaeus University, and for a period of 6 months I will be conducting a study about mathematics at your school. As part of my study, I will be observing students in special education needs in mathematics. I will be talking to and interviewing the students and recording audio and video. The study will be conducted in collaboration with the regular and special needs teachers at your school but will not entail any change in teaching practices.

All the information collected will be confidential, and everyone involved will be de-identified. The collected material will only be used for research purposes, which includes publication of articles, conference input, and a thesis. Participation is totally voluntary, and you decide the level of your involvement. If you decide to take part, you can change your mind and leave the project at any time.

The researcher will inform you orally about the study and you will only be interviewed if you wish to be. If you wish to take part in the study, please put your signature at the bottom of page 2 of this information letter.

Please contact me if you have any questions.

Best regards,

Helena Roos, PhD student and lecturer in mathematics education, Linnaeus University in Växjö, Helena.Roos@lnu.se 0470-708834

Participation in research project Inclusion in Mathematics
Yes I would like to take part in the research project, spring term 2016.

____________________________________________________________
Student’s signature

____________________________________________________________
Name in block capitals
Class: ___________________________________________
Information for guardians

At Linnaeus University, there is currently a research project about inclusion in mathematics in school. Its purpose is to develop an understanding of and knowledge about ways to include students in the mathematics education.

As part of this project, a study will be conducted in the spring term 2016 in which students with special education needs in mathematics will be observed 1–3 days a week. The study will include both regular mathematics teaching and special needs teaching. As part of the study, audio and video will be recorded in the classroom, and observations of students and student groups in mathematics classes and special needs education in mathematics will be conducted. We will also be talking to and interviewing students and teachers. This means that your child could be interviewed. The study will be conducted in collaboration with regular and special needs teachers at your school but will not entail any change in teaching practices. The principal has given his approval to the study. All the information that is collected will be treated confidentially. Collected material will be used for research purposes only, which includes the publication of articles, conference input, and a thesis. Participation is totally voluntary, and students who take part can change their mind and leave the project at any time. We would therefore like to ask for your consent for your child to take part in the study. If you give your consent, please sign the attached reply form and return it to the class teacher within a week.

Please contact me if you have any questions.

Best regards,
Helena Roos
PhD student and lecturer in mathematical didactics, Linnaeus University in Växjö, Helena.Roos@lnu.se +46 (0)470-708834

Supervisor: Hanna Palmer, Linnaeus University, Hanna.Palmer@lnu.se, +46 (0)470 708641
Participation in research project Inclusion in Mathematics

We give our consent for our child to take part in the research project in spring term 2016.

Name of student__________________________________________

Class: ______________________________________________________

_________________________________________________________
Parent signature

_________________________________________________________
Name in block capital
Information till elev och föräldrar.

Jag är doktorand i matematikdidaktik vid Linnéuniversitetet som under 6 månader kommer att genomföra en undersökning gällande särskilda utbildningsbehov i matematik. I min undersökning kommer jag att göra ljud och videoupptagningar och observationer av elever och elevgrupper under matematiklektioner samt samtala och intervjuar elever. Genomförandet av undersökningen kommer att ske i samarbete med lärare och speciallärare på din/Ditt barns skola.

Eleverna kommer att informeras om studien muntligt av forskaren och enbart de elever som vill kommer att intervjuas. För att eleverna ska få vara med i studien måste dock du/ni ge ert samtycke. Vänligen bekräfta om du/ni gör detta eller inte genom att återlämna sida två av detta informationsbrev till klassläraren.

Skolan, läraren och elever kommer att behandlas konfidentiellt i studien, det vill säga all information som samlas in kommer att behandlas med tillförsikt och alla medverkande kommer att avidentifieras. Insamlat material kommer enbart användas i forskningssammanhang vilket innebär publicering av artiklar, konferensbidrag samt i en avhandling. Det är helt frivilligt att vara med och du/ni väljer själv hur du/ni vill göra. Om du väljer att vara med kan du när som helst ångra dig och avbryta om du inte vill vara med längre.

Undersökningarna kommer att äga rum under våren 2016.

Jag hoppas att du som elev vill delta och att ni som föräldrar ger er tillåtelse att ert barn får delta i undersökningen.

Kontakta gärna mig om ni har några frågor.
Mvh
Helena Roos
doktorand och universitetsadjunkt i matematikdidaktik,
Linnéuniversitet i Växjö, Helena.Roos@lnu.se 0470-708834
Deltagande i forskningsprojekt i matematik.

☐ Vi/jag godkänner medverkan i forskningsprojektet vt 2016.

☐ Vi/jag vill inte medverka i forskningsprojektet.

____________________________________________________________
Elevens underskrift

____________________________________________________________
Namnförtydligande

____________________________________________________________
Målsmans Underskrift

____________________________________________________________
Namnförtydligande
Información para el alumno.

Soy doctoranda en educación matemática en la Universidad Linné y durante seis meses voy a realizar una investigación sobre matemáticas en tu escuela. En mi investigación voy a realizar un seguimiento de los alumnos que tienen una necesidad especial de formación en matemáticas. Voy a hablar y entrevistar a esos alumnos y haré grabaciones sonoras y de vídeo. La realización de la investigación va a tener lugar en colaboración con los maestros y los maestros especiales en tu escuela, pero eso no va a significar ningún cambio en la enseñanza.

Toda la información que se recopile va a ser confidencial y todos los que participen van a hacerse anónimos. El material recopilado va a ser utilizado solo en contextos de investigación, lo cual significa la publicación de artículos, subvenciones de conferencias y una tesis. La participación es absolutamente voluntaria y tú eliges cómo quieres realizarla. Si eliges participar, puedes arrepentirte e interrumpir tu participación en cualquier momento.

La investigadora va a informarte de manera oral acerca del estudio y solo vas a ser entrevistado si tú quieres. Si quieres participar en el estudio, firma la página dos de esta carta informativa.

Contáctame si tienes alguna duda.

Saludos cordiales,
Helena Roos
doctoranda y ayudante de cátedra en educación matemática, Universidad Linné en Växjö, Helena.Roos@lnu.se 0470-708834
Participación en el proyecto de investigación Inclusión en las matemáticas

Quiero participar en el proyecto de investigación, semestre de primavera 2016.

____________________________________________________________
Firma del alumno

____________________________________________________________
Nombre en letras mayúsculas

Clase: _________________________________________
Información para los titulares de la custodia.

En la Universidad Linné se realiza un proyecto de investigación acerca de la inclusión de las matemáticas en la escuela de enseñanza primaria, con el objeto de desarrollar la comprensión y los conocimientos acerca de cómo pueden ser incluidos los alumnos en la enseñanza de las matemáticas.

Durante el semestre de primavera 2016 va a tener lugar una investigación en el marco de este proyecto, donde se realizará un seguimiento de los alumnos que tengan necesidad especial de formación en matemáticas durante 1 a 3 días por semana. El estudio va a incluir tanto la enseñanza ordinaria de las matemáticas como la enseñanza especial. En la investigación se van a hacer grabaciones sonoras y de vídeos en las aulas, observaciones de los alumnos y grupos de alumnos durante las lecciones de matemáticas, y en la enseñanza especial de las matemáticas. En el proyecto también vamos a conversar y entrevistar a alumnos y maestros. Esto puede implicar que su hijo sea entrevistado. La realización de la investigación va a tener lugar en colaboración con los maestros y los maestros especiales de la escuela de su hijo, pero eso no va a significar ningún cambio en la enseñanza. El director ha dado su aprobación a la investigación. Toda la información que se recopile va a ser procesada de manera confidencial. El material recopilado va a ser utilizado solo en contextos de investigación, lo cual significa la publicación de artículos, subvenciones de conferencias y una tesis. La participación es absolutamente voluntaria y, si decide participar, puede arrepentirse e interrumpir esa participación en cualquier momento. Queremos preguntarle si permite que su hijo participe en la investigación. Si aprueba la participación de su hijo, firme el talón de respuesta adjunto y entréguelo al maestro en el plazo de una semana.

Contácteme si tiene alguna duda.

Saludos cordiales,
Helena Roos, doctoranda y ayudante de cátedra en educación matemática, Universidad Linné en Växjö, Helena.Roos@lnu.se 0470-708834
Participación en el proyecto de investigación Inclusión en las matemáticas

Aprobamos la participación de nuestro hijo en el proyecto de investigación, semestre de primavera 2016.

Nombre del alumno:

Clase: ________________________________

Firma del progenitor o tutor

Nombre en letras mayúsculas
Appendix 4 – Letter of Approval to Students and Guardians in Arabic

للمعلومات معلومات

ليه جامعة في الدكتوراه شهادة على الحصول الرياضيات تدريس أسلوب مجال في أدرس أنا Linnéuniversitetet ين الخاصية المدرسة في الرياضيات عن بتحقيق أشهر 6 خلال ستقوم التي ، مقابلتهم و التلاميذ مع التحدث ساقوم الرياضيات تعليم إلى خاصة حاجة في تلاميذ التحقيق خلال سايح في المتخصصين الأساتذة و الأساتذة مع بالتعاون التحقيق سيتم فيديو تصوير و صوتي بتسجيل القيام و التعليم في تغيير أي ذلك يعني لن لكن و ببك الخاصة المدرسة.

لا تعمل لن كما محددون غير سيئلون المشاركون كل و سرية ستظل جميعها سيتم التي المعلومات كل كذلك و المؤتمرات خلال ، المقالات نشر في يعني مما أني، البحوث نطاق في إلا المجموعة المعلومات المشاركة اختارت إذا .لا أم المشاركة بنفسك اختارت أنت و اختياري أمر المشاركة دكتوراه طروحة في ذلك تريد كنت إن المشاركة عن التوقف و أليك تغيير وقت أي في بإمكانك.

كنت إذا ذلك عن موافقة كنت إن إما مقابلتك تم لن و الدراسة عن شفهيا بإعلامك الباحث سيقوم الإخبارية الرسالة هذه من الثانية الصفحة في التوقع فيجوب الدراسة هذه في المشاركة على موافق.

أسرتي أي لديك إذا بي بالاتصال بك أهلا

التحيات أطيب مع

فكشو في لينيه جامعة الرياضيات تدريس أسلوب في جامعتي مدرسة و دكتوراه طالبة بروس هيلينا
Helena.Roos@lnu.se 708834-0470

الرياضيات في الاشتمال عن علمي بحث مشروع في المشاركة

الربيع الفصل في العلمي البحث مشروع في المشاركة أريد

________________________
التلميذ توقيع

________________________
بالكامل الاسم

:الصف

الأمور لأولياء معلومات
The department of mathematics at Linnaeus University is currently implementing a research project on mathematics in primary school, with the aim of developing knowledge and information about how to include students in mathematics education during the fourth quarter of the 2016 academic year. The study will be conducted in fields of this project, where the students who have special needs in mathematics education, about 1-3 days a week. The study will be conducted in both regular and special education. We will record audio and visual in the classroom in addition to student and student groups during mathematics lessons and special mathematics education. We will also conduct interviews and conversations with students and teachers throughout the project. It means that we will interview your child. The study will be conducted in cooperation with teachers and teachers in the special school of your child, but it does not mean any change in education. The director of the school has approved this study. All data presented here will be treated in a confidential manner. Data will not be used except in a research context, i.e. in publication, presentation, dissertation. Participation is voluntary and the child can change his opinion at any time and stop participating. We would like to ask if you agree to participate your child in this study? If you agree, we ask you to sign the enclosed form and give it to the teacher within a week.

Translated by
Språkservice
Reference: MDH63
Malmö, Sweden:
www.sprakservice.se
I am a PhD student in mathematics education at Linnaeus University, and for a period of 6 months, I will be conducting a study about mathematics at your school. As part of my study, I will be observing students in special education needs in mathematics. I will be talking to and interviewing the students and recording audio and video.

All the information collected will be confidential, and everyone involved will be de-identified. The collected material will only be used for research purposes, which includes publication of articles, conference input and a thesis.

If you wish to take part in the study, please put your signature at the bottom of page 2 of this information letter.

Please contact me if you have any questions.

Best regards,
Helena Roos
PhD student and lecturer in mathematics education, Linnaeus University in Växjö, Helena.Roos@lnu.se 0470-708834

Participation in research project Inclusion in Mathematics

Yes I would like to take part in the research project, spring term 2016.

____________________________________________________________
signature

____________________________________________________________
Name in block capitals
Appendix 6 – Letter of Approval to Teachers in Swedish

Hej!

Jag är doktorand matematikdidaktik vid Linnéuniversitetet som under sex månaders tid kommer att genomföra en undersökning på din skola gällande inkluderande undervisning i matematik ur ett specialpedagogiskt perspektiv.

I min undersökning kommer jag att göra ljud och videoupptagningar samt observationer av elever och elevgrupper under matematiklektioner och vid specialundervisning i matematik. Jag kommer även att samtala och intervjua elever. All information som samlas in kommer att behandlas konfidentiellt och alla medverkande kommer att avidentifieras.

Resultatet kommer att sammanställas i en avhandling som så småningom kommer att publiceras.

Undersökningarna kommer att äga rum under våren 2016.

Jag hoppas att du är villig att delta i forskningsprojektet.

Mvh
Helena Roos
doktorand och universitetsadjunkt i matematikdidaktik,
Linnéuniversitet i Växjö, Helena.Roos@lnu.se, 0470-708834

Jag medverkar gärna i forskningsprojektet

-----------------------------------------------------------------------------------------------------------------------------
Underskrift
-----------------------------------------------------------------------------------------------------------------------------
Namnförtydligande

Appendix 7 – Self-Assessment Form in English
**SELF-ASSESSMENT**

Name: 

Grade: 

How do you feel in the following situations?

<table>
<thead>
<tr>
<th>When:</th>
<th>SURE</th>
<th>PRETTY SURE</th>
<th>UNSURE</th>
<th>VERY UNSURE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Answer a question you think you know from the teacher during going-through</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tell the teacher how you solved a task</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tell a peer how you solved a task</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Show the teacher how you solved a task in writing in your notebook</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Discuss solutions of tasks in mathematics with your peers</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Use mental arithmetic</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Choose a method to solve a mathematical task</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Use a calculator</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Determine if an answer is reasonable</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Get help from the special teacher in mathematics</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Go out of the classroom with the special teacher</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Describe what is meant by ( \frac{1}{8} )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Describe what is meant by mean value</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Describe what is meant by perimeter</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
# SJÄLVSKATTNING

**Namn:**  
**Klass:**  
Fundera på hur du känner i följande situationer:

<table>
<thead>
<tr>
<th>När du ska:</th>
<th>SAKER</th>
<th>GANNSKA SÄKER</th>
<th>OSÄKER</th>
<th>MYCKET OSÄKER</th>
</tr>
</thead>
<tbody>
<tr>
<td>svara på en fråga du tror att du kan från läraren under en genomgång i helklass</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Berätta för läraren hur du har löst en uppgift</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Berätta för en kompis hur du har löst en uppgift</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>visa läraren hur du löst en uppgift skriftligt i din bok</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Diskutera lösningar av matematikuppgifter med klasskompisar</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>använda huvudräkning</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>välja metod för att lösa en matematikuppgift</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>använda miniräknare</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>avgöra om ett svar är rimligt</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>få hjälp av specialläraren</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gå ut ur klassrummet med specialläraren</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>beskriva vad som menas med 1/8</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Beskriva vad som menas med medelvärde</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Beskriva vad som menas med omkrets</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Appendix 9 – Interview Guide in English

The open interview guide for the individual student interviews.

1. How was your last mathematics lesson?
2. Was something good?
3. Why was it good? (Depending on the answers, I might need to follow up and ask about what was bad in the mathematics lesson).
4. Did you understand what you did in the mathematics lesson? If not, what did you not understand? Do you know why you didn’t understand it?
5. Specific questions about the content of the last mathematics lesson. Look at pictures taken from the blackboard, describe what they were doing, and talk about the work using tasks from the textbook.
Appendix 10 – Interview Guide in Swedish


4. **Upplevelse av läraren** – När din mattelärare förklarade på lektionen, var det lätt att förstå? Vad gjorde din/a lärare på lektionen? Hade de (matte respektive speciallärare) olika uppdrag? Var det någon lärare som hjälpte dig mer?
Appendix 11 – Gee’s 28 tools of Inquiry Adapted to the Study

Language and context tools

1. **Diexis tool** – How are deictics being used to tie what is said to context?
2. **Fill in tool** – What needs to be filled in to achieve clarity? What is not being said explicitly but is assumed to be known?
3. **Making strange tool** – What would someone find strange if that person did not share the knowledge and assumptions? Hence, what is taken for granted by the students?
4. **Subject tool** – Why has the student chosen the particular subject of the conversation? How does the student organize information in terms of subject and predicates?
5. **Intonation tool** – How does the students’ intonation contour contribute to the meaning of utterances? (in the transcripts, emphasized words are in bold).
6. **Frame tool** – Can I find out any more about the context, and if so, does this change the analysis?

Saying, doing, and designing tools

7. **Doing and not just saying tool** – What is the student who is talking trying to do? (can be several things).
8. **Vocabulary tool** – What sort of words are being used, and how does the distribution of words function to mark the communication in terms of style?
9. **Why this way and not that way tool** – Why does the student build and design grammar in this way and not in some other way?
10. **Integration tool** – What was included and what was left out in terms of optional arguments? When were clauses turned into phrases? What perspectives are being communicated by the way in which information is packed into clauses?
11. **Topic and theme tool** – What is the topic and theme for each clause? What theme is a set of clauses? When the theme is not the topic, and had deviated from the usual choice, why was it chosen?
12. *Stanza tool* – How do stanzas cluster into larger blocks of information?

**Building things in the world**

13. *Context is reflexive tool* – How is what the student saying helping to create or shape relevant context? How is what the speaker saying helping to reproduce context?

14. *Significance building tool* – How are words and grammatical devices being used to build up or lessen significance for certain things and not others?

15. *Activities building tool* – What activity or activities is this communication building or enacting? What social groups, institutions, or cultures support and norm the activities?

16. *Identity building tool* – What socially recognizable identity or identities is the speaker trying to enact or get others to recognize? How is the speaker positioning others?

17. *Relationship building tool* – How are words and grammatical devices used to build and sustain or change relationships of various sorts among the speaker, other people, social groups, cultures and/or institutions?

18. *Politics building tool* – How are words and grammatical devices used to build what counts as a social good and how social goods are or should be distributed in society?

19. *Connection building tool* – How are words and grammatical devices used in the communication to connect or disconnect things?

20. *Cohesion tool* – what is the student trying to communicate or achieve by using cohesive devices in the way he or she does?

21. *System and knowledge building tool* – How are the words and grammar being used to privilege or de privilege specific sign systems (e.g. every day or scientific mathematical concepts) or different ways of knowing and believing?

22. *Topic flow or topic chaining tool* – What are the topics of all main clauses and how are these topics linked to each other (or not) to create a chain?
Theoretical tools

23. *Situated meaning tool* – What situated meaning has the communication?

24. *Social language tool* – How are words and grammatical devices used to signal and enact a given social language?

25. *Intertextuality tool* – How are words and grammatical devices used to quote, refer to or allude to other text or other styles of social language?

26. *Figured world tool* – What typical figured worlds are the word and communication assuming and inviting listeners to assume?

27. *Big “D” Discourse tool* – What Discourse is this language a part of? What sort of actions, interactions, values, beliefs, and objects, tools, technologies and environments are associated with this sort of language within a particular discourse?

28. *Big C Conversation tool* – What issues, debates and claims is the communication claiming to assume? Can the communication be seen as carrying out a historical or widely known debate or discussion between or among Discourses? Which Discourses?
Appendix 12 – Mattevisan

MATTEVISAN

Melodi: John Brown’s Body

Term plus term blir summa när man räknar addition
Term minus term blir differens i subtraktion
Faktor gånger faktor blir produkt det vet du väl
I vår multiplikationstabell

Refräng: Matte matte det är toppen, man blir pigg i huvudknoppen, räknesätten är de fyra och jag lär mig alla nu.

Täljaren på taket och så nämnaren längst ner
när man räknar division en kvot som svar man ser.
Alla dessa uttryck ska man kunna utantill
om man matte klara vill.

Refräng: Matte, matte det är toppen, man blir pigg i huvudknoppen,
räknesätten är de fyra och jag lär mig alla nu.

THE END