DC-link and Machine Design Considerations for Resonant Controllers Adopted in Automotive PMSM Drives

Oskar Wallmark and Mojgan Nikouei

1 Department of Electric Power and Energy Systems, KTH Royal Institute of Technology, Teknikringen 33, Stockholm, Sweden

Abstract: This paper presents an analysis of the harmonic content on the dc-link (battery) side in electric drives with permanent-magnet synchronous machines (PMSMs) adopting fractional-slot concentrated windings (FSCWs) which can have a significant harmonic content. Analytical expression for predicting the ripple content in the dc-link (battery) current utilizing data from a two-dimensional finite element (2D-FEM) based model is presented and found to be in good agreement with corresponding experimental results. The FEM model is also used to demonstrate that the resulting harmonic content in the phase currents when operating at speeds well above the bandwidth of a conventional proportional+integral (PI) type controller can result in a substantial increase in magnet losses. Finally, a previously reported, experimentally evaluated model of an automotive traction battery and associated battery cable conventional proportional+integral (PI) type controller can result in a substantial increase in magnet losses. Finally, a previously reported, experimentally evaluated model of an automotive traction battery and associated battery cable is combined with an implemented model of the FSCW-PMSM (utilizing data from the 2D-FEM model), converter and control to predict the resulting harmonic content in the battery current. The presented methods can be utilized when analyzing the impact of harmonics on both the ac and dc side of an automotive electric drive system.

1 Introduction

While a high stator-slot fill factor and axially short end windings are benefits of fractional-slot concentrated windings (FSCWs), the increase in harmonic content (compared to conventional distributed windings) represent a well-known challenge with this type of winding [1]. With a conventional proportional+integral (PI) type current controller (implemented in the rotor-fixed dq-reference frame), at high rotor speeds, the frequencies of these harmonics can be significantly higher than the achievable current-control bandwidth. This results in a harmonic content in the phase currents causing additional losses and torque ripple.

Since the frequencies of the introduced harmonics are known, the addition of resonant part(s) in the current controller can eliminate the impact of the harmonics, resulting in sinusoidal currents. During the last decade or so, the properties of resonant-type controllers in electric power applications (single- and multi-phase systems) and how they can be tuned have been investigated in the literature, see, e.g., [2–4]. However, to the knowledge of the authors, two issues that become important in an automotive traction application have not been addressed fully in the literature. First, while sinusoidal ac currents can be realized with a resonant-type current control, the harmonic content in the electric machine results in harmonic content in the applied phase voltages, which results in a harmonic content in the dc link and, in turn, in the battery current. Second, this increase in harmonic content (and losses) on the dc (battery) side must be put in contrast to the additional machine losses arising if the phase currents have a significant harmonic content (e.g., without the use of a resonant-type current control). Of particular importance is the location of the increased machine losses.

If the loss increase is significant in the rotor, this can lead to excessive magnet temperatures with a potential demagnetization as a result. With the above issues quantified, a good base is formed to determine whether modifications to the machine design (commonly, skewing the rotor or reducing the axial segmentation of the permanent-magnet segments) are required to mitigate the harmonic content.

The above issues are addressed in this paper as follows. First, a condensed review (closed-loop dynamics, phase margin and implementation) of proportional-integral-resonant (PIR) type current controllers implemented in the dq-reference frame is presented in Section 2. This section is tutorial in nature with the exception of Section 2.3, which details the achievable phase margins in the case of a limited switching frequency of the power electronic converter. In Section 3, the losses in a prototype 30-kW PMSM developed for an automotive traction application are computed using a finite-element method (FEM) model evaluated at two representative operating points. The FEM simulations are carried out with perfectly sinusoidal voltage supply (representing the case when the harmonics are well above the bandwidth of the closed control loop) and with perfectly sinusoidal phase currents (representing the case with a resonant-type current control) and the two cases are compared. In Section 4, a model for predicting the harmonic dc-link (battery) current is presented. The model in Section 4 is then compared to corresponding experimental results with good agreement in Section 5.1. The experimentally verified model is then combined with the experimentally verified model of an automotive traction battery and battery presented in [5] to predict the harmonic content in the battery current and its associated losses. Finally, concluding remarks are given in Section 6.
2 Closed-Loop Dynamics, Phase Margin, and Digital Implementation

2.1 Open-Loop Dynamics

In the rotor-fixed dq-reference frame and with the resonant part added, the current controller part for $i_d$ (the controller part for $i_q$ is similar and not reported here) in [6] can be expressed as

$$v_d = F_d \left( k_{id} - i_d \right) - \omega_r L_{iq} - R_{ad} i_d$$
$$F_d = k_{pd} + \frac{k_{id}}{s} + \frac{k_{pd}}{s + (h \omega_r)^2}$$

where $s$ is the Laplace variable, $\omega_r$, the electrical rotor speed, and $-\omega_r L_{iq} R_{ad}$ represent decoupling and active damping, respectively. Assuming no parameter errors, the transfer function $G_k$ from $v_d$ to $i_d$ becomes $G_k = \frac{1}{(s L_d + R_s + R_d)}$. The open-loop transfer function $G_k$ becomes

$$G_k = \frac{k_{pd}}{s + \alpha_c} \left( 1 + \frac{\omega_c}{s} + \frac{\alpha_c s}{s^2 + (h \omega_r)^2} \right).$$

2.2 Closed-Loop Dynamics

Neglecting the time delay ($T_d \to 0$), the closed-loop transfer function $G_c$ becomes

$$G_c = \frac{G_k}{1 + G_k} = \frac{\text{num}}{\text{den}}$$
$$\text{num} = \alpha_c s^2 (s + \alpha_c + \alpha_r) + \alpha_c (h \omega_r)^2 (s + \alpha_c)$$
$$\text{den} = s^2 + 2 \alpha_c s \left( s^2 + \alpha_c (\alpha_c + \alpha_r) \right) + (h \omega_r)^2 (s + \alpha_c)^2.$$

Assuming $\alpha_c \ll \alpha_c, G_c$ becomes

$$G_c \approx \frac{\alpha_c}{s + \alpha_c}.$$

Hence, except around the resonant frequency $\omega \approx h \omega_r$, the closed-loop dynamics approximates a first-order system with the bandwidth $\alpha_c$.

2.3 Phase Margins

To consider the impact of the time delay $T_d = 1.5/f_s$, the phase margin $\phi_m$ is now studied. With the rule-of-thumb $\alpha_c \leq 2\pi f_s/10$ [9] and with $\alpha_c = 0$, it can be shown that $\phi_m$ becomes $\phi_m = 36^\circ$ if $\alpha_c = 2\pi f_s/10$. With $\alpha_c = 0$, for a harmonic of angular frequency $h \omega_r$ to be damped sufficiently, the condition $h \omega_r < \alpha_c$ is reasonable which, with $\alpha_c = 2\pi f_s/10$ and $h = 6$, yields $\omega_r < 2\pi f_s/(10h) \approx 0.105 f_s$. For $f_s = 50$ kHz, this corresponds to an upper shaft speed limit of around 12.5 krpm for a PMSM with eight poles ($p = 8$).

With $\alpha_c = 2\pi f_s/10$ and $\alpha_r = \alpha_c/10$, the phase margin of (3) is a function of $h \omega_r$ and is depicted in Fig. 1 (a) for $h = 6$. From the figure, it can be seen that obtaining a phase margin $\phi_m \geq 20^\circ$, $\omega_r \leq 0.12 f_s$ which is only a minor improvement compared to without the resonant part of the controller. Hence, for a sixth-order harmonic ($h = 6$) to be significantly damped, PI control is sufficient for speeds below $\omega_r < 2\pi f_s/(10h)$. In a practical application, however, due to quantization noise and limitations in terms of current and voltage sensor bandwidths, it can be motivated to increase $f_s$ while keeping the bandwidth $\alpha_c$ fixed. Fig. 1 (b) shows the phase margin when $\alpha_c$ is fixed to $\alpha_c = 2\pi \cdot 1000$ rad/s while $f_s$ is varied. As seen, at $f_s = 50$ kHz, for the margin $\phi_m \geq 30^\circ$, $\omega_r \leq 2100$ rad/s which corresponds to a shaft speed of around 5000 rpm for a PMSM with eight poles.

![Fig. 1: Phase margin as function of $f_s$ and $\omega_c$: (a) $\alpha_c = 2\pi f_s/10$ and $\alpha_r = \alpha_c/10$; (b) $\alpha_c = 2\pi \cdot 1000$ rad/s and $\alpha_r = \alpha_c/10 = 2\pi \cdot 100$ rad/s.](image)

**Remark:** PIR controllers can also be used to reduce the resulting torque ripple which, by necessity, results in harmonics in the phase currents. Such approaches are presented in [8, 10].

2.4 Discrete-time implementation

Discretizing using the direct form II transposed (DFII) structure with $I_d$ and $I_q$ as integrator states and $\omega_r$ as the rotor speed (measured in electrical rad/s), the controller can be expressed as:

```
//Resonators
b0=ar*kpd/(h*wr)*sin(h*wr*Ts);
alpha1=2*(1-cos(h*wr*Ts));
yd=-b0*Iq+xiq;
ydref=kpd*(idref-iq)-Rad*id;
vdref=kpd*(idref-iq)-Rad*id
```

$I_d$ and $I_q$ are the control signals, and $y_d$ is the current control output.
Table 1: 80 Nm @ 4000 rpm.

<table>
<thead>
<tr>
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<tbody>
<tr>
<td>Stator winding</td>
<td>368 W</td>
<td>522 W</td>
<td>-31.5%</td>
</tr>
<tr>
<td>Stator lam.</td>
<td>389 W</td>
<td>404 W</td>
<td>3.8%</td>
</tr>
<tr>
<td>Rotor lam.</td>
<td>66 W</td>
<td>66 W</td>
<td>0%</td>
</tr>
<tr>
<td>PMs</td>
<td>64 W</td>
<td>54 W</td>
<td>-16%</td>
</tr>
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</table>

which represents a special case of the resonant filter analyzed in [4]. Note that due to the division with \( \omega_r \) when \( b_0 \) is computed, the resonant parts of the controller \( (y_d \text{ and } y_q) \) should only be added to \( v_{dref} \) and \( v_{qref} \) above a selected low-speed region. Voltage saturation and integrator anti-windup should preferably be added (see, e.g., [6]).

3 Current harmonics and its impact on machine losses

Commonly, when machine losses are computed using FEM, perfectly sinusoidal phase currents are assumed. When the frequency of the harmonics in the machine are well above the bandwidth of the closed current control loop, the controller is essentially blinded for this high-frequency content. This results in close to sinusoidal applied voltages which, in turn, results in a harmonic content in the phase currents. Hence, it is of interest to compare how the resulting losses and their distribution are changed for these two situations. In the appendix, brief details of a 30-kW FSCW-PMSM (described more in detail in [11]) are reported along with details of the corresponding FEM model.

Fig. 2 shows sample phase voltages and phase currents when the FEM model is fed with perfectly sinusoidal voltages and currents, respectively. The corresponding losses in the stator and rotor laminations with sinusoidal voltages is marginal only. Second, the largest increase in losses in the stator and rotor laminations with speed, field weakening operation) are reported in Table 1 and Table 2, respectively. From the reported losses, some interesting details should be highlighted. First, the predicted relative increase of losses occurs in the permanent magnets, and this effect is pronounced at high speeds. Hence, it is of interest to compare how the resulting losses and their distribution are changed for these two situations. In the appendix, brief details of a 30-kW FSCW-PMSM (described more in detail in [11]) are reported along with details of the corresponding FEM model.

Fig. 2 shows sample phase voltages and phase currents when the FEM model is fed with perfectly sinusoidal voltages and currents, respectively. The corresponding losses in the different parts of the machine for operating points corresponding to 80 Nm at 4000 rpm (corresponding to rated operation) and 35 Nm at 8000 rpm (corresponding to a high-speed, field weakening operation) are reported in Table 1 and Table 2, respectively. From the reported losses, some interesting details should be highlighted. First, the predicted increase in losses in the stator and rotor laminations with sinusoidal voltages is marginal only. Second, the largest predicted relative increase of losses occurs in the permanent magnets, and this effect is pronounced at high speeds. From these results, it is clear that investigations similar to the above should be carried out when design means like skewing the rotor (which reduces the torque density) and reducing the axial length of the permanent-magnet segments (complicating the magnet mounting process) are considered.

Table 2: 35 Nm @ 8000 rpm.

<table>
<thead>
<tr>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Stator winding</td>
<td>151 W</td>
<td>139 W</td>
<td>-8.6%</td>
</tr>
<tr>
<td>Stator lam.</td>
<td>403 W</td>
<td>404 W</td>
<td>0.25%</td>
</tr>
<tr>
<td>Rotor lam.</td>
<td>41 W</td>
<td>41 W</td>
<td>0%</td>
</tr>
<tr>
<td>PMs</td>
<td>106 W</td>
<td>70 W</td>
<td>-34%</td>
</tr>
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</table>

4 DC-link interaction

If a PIR current controller is used and (close to) perfectly sinusoidal phase currents are realized, the increase in harmonic content on the dc link (and battery current) must be considered. This effect can be analyzed as follows. First, as is well known, the current–flux dynamics of a PMSM in the rotor-fixed \( dq \)-reference frame can be expressed as

\[
v_d = R_s i_d + \frac{d\psi_d}{dt} \omega_r \psi_q \tag{8}
\]

\[
v_q = R_s i_q + \frac{d\psi_q}{dt} + \omega_r v_d \tag{9}
\]

where \( R_s \) is the stator resistance per phase. Assuming perfectly sinusoidal currents, we have \( i_d = i_d^* \) and \( i_q = i_q^* \), and the flux linkages \( \psi_d \) and \( \psi_q \) can be approximated as

\[
\psi_d \approx \psi_d^* + \psi_{dth} \cos (h \omega_r t + \phi_{dth}) \tag{10}
\]

\[
\psi_q \approx \psi_q^* + \psi_{qth} \sin (h \omega_r t + \phi_{qth}) \tag{11}
\]

where the asterisk (*) denotes the specific operating point and it is assumed that the harmonic content in \( \psi_d \) and \( \psi_q \) is dominated by a single harmonic (of order \( h \)). Inserting (10) and (11) into (8) and (9), it is evident that \( v_d = v_d^* + \Delta v_d \) and \( v_q = v_q^* + \Delta v_q \) where \( \Delta v_d \) and \( \Delta v_q \) can be expressed

\[
\Delta v_d = R_s i_d + \frac{d\psi_d}{dt} \omega_r \phi_{dth} \tag{12}
\]

\[
\Delta v_q = R_s i_q + \frac{d\psi_q}{dt} + \omega_r v_d \phi_{qth} \tag{13}
\]
as

\[
\Delta v_d = \frac{d\psi_d}{dt} - \omega_r \psi_q = -h \omega_r \psi_{dh}^* \sin(h \omega_r t + \varphi_{dh}^* - \varphi_{dh}) - \omega_r \psi_{qh}^* \sin(h \omega_r t + \varphi_{qh})
\]

\[
\Delta v_q = \frac{d\psi_q}{dt} + \omega_r \psi_d = h \omega_r \psi_{dh}^* \cos(h \omega_r t + \varphi_{dh}^*) + \omega_r \psi_{qh}^* \cos(h \omega_r t + \varphi_{dh}^*).
\]

The power \( P \) can now be expressed as \( P = P^* + \Delta P \) where \( \Delta P \) is found as

\[
\Delta P = \frac{3}{2} \left( \Delta v_{d\text{ref}}^* + \Delta v_{q\text{ref}}^* \right).
\]

Now, consider the model of the electric drive and the dc-link depicted in Fig. 3. As demonstrated in [12], the dc-link voltage dynamics can be linearized as

\[
C_{dc} \frac{d\Delta v_k}{dt} = \Delta i_b - \frac{\Delta P}{E_b}.
\]

Noting that \( \Delta v_k = -(R_b + s L_b) \Delta i_b \), (15) can be solved for \( \Delta i_b \) resulting in

\[
\Delta i_b = \frac{\Delta P}{E_b (C_{dc} L_b s^2 + R_b C_{dc} s + 1)}.
\]

By inserting (14) into (16), the amplitude of the oscillation of \( i_b \) (of angular frequency \( h \omega_r \)) can be expressed as

\[
\text{max}(\Delta i_b) = \frac{3 \max(\Delta v_{d\text{ref}}^* + \Delta v_{q\text{ref}}^*)}{2 E_b (C_{dc} L_b h^2 \omega_r^2 + R_b C_{dc} h \omega_r + 1)}.
\]

By identifying the magnitude (\( \psi_{dh}^* \) and \( \psi_{qh}^* \)) and phase

\[
\varphi_{dh}^* \quad \text{and} \quad \varphi_{qh}^*
\]

of the dominant flux-linkage harmonics using FEM, (17) can be used to determine the harmonic content in the battery current.

5 Experimental verification and prediction of harmonic battery current in a traction application

5.1 Experimental verification

The above described approach to predict the harmonic content on the dc link is experimentally verified as described below. The FSCW-PMSM described in the appendix is connected to a speed controlled servo drive (see Fig. 4 for a picture of the laboratory setup). The converter, switching at 20 kHz, is fed from the armature of a dc generator and the resulting dc-link voltage is controlled to \( E_b = 80 \) V by controlling the field current of the dc generator. The dc-link voltage control is very slow and its dynamics can be neglected. By charging the 100 \( \mu \)F dc-link capacitor (i.e., \( C_{dc} = 100 \mu \)F), the effective resistance and inductance of the dc-link are identified to \( R_b \approx 610 \) m\( \Omega \) and \( L_b \approx 4.2 \) mH, respectively.

In the experimental results reported in Fig. 5, the FSCW-PMSM is operated at \( \omega_r = 75.4 \) rad/s with \( i_{d\text{ref}} = 0 \) A and \( i_{q\text{ref}} = -50 \) A (generation operation is considered so that dc-link instabilities caused by the interaction between the constant-power load inductive source can be avoided [12]). The bandwidth of the closed current control loop is set to \( \alpha_c = 220 \) rad/s and the machine is operated with and without the resonant part of the current controller added. The resonant part is resonating at the angular frequency \( 6 \omega_r \) with the gain \( \alpha_c = \alpha_c / 10 = 22 \) rad/s. As can be seen in Fig. 5 (c), the harmonic content in the phase current is effectively reduced with the resonant part added. Fig. 5 (a)–(b) show that the harmonic content on the dc link is substantial, both with and without the resonant part added.
Corresponding simulation results when the FSCW-PMSM is operated at $\omega_r = 75.4$ rad/s with $i_d^{\text{ref}} = 0$ A and $i_q^{\text{ref}} = -50$ A. Blue curves: with resonant link added in the current controller; red curves: without resonant link added.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig5.png}
\caption{Experimental results when the FSCW-PMSM is operated at $\omega_r = 75.4$ rad/s with $i_d^{\text{ref}} = 0$ A and $i_q^{\text{ref}} = -50$ A. Blue curves: with resonant link added in the current controller; red curves: without resonant link added.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig6.png}
\caption{Corresponding simulation results when the FSCW-PMSM is operated at $\omega_r = 75.4$ rad/s with $i_d^{\text{ref}} = 0$ A and $i_q^{\text{ref}} = -50$ A. Blue curves: with resonant link added in the current controller; red curves: without resonant link added.}
\end{figure}

5.2 Battery Current in a Traction Application

The values for the dc link are now selected to $E_b = 450$ V, $R_b = 50$ m$\Omega$, and $L_b = 2$ $\mu$H, which is in agreement with the experimentally verified high-frequency model (valid for frequencies above around 1 kHz) of an automotive traction battery and associated battery cable presented in [5]. The capacitance is selected to 25 $\mu$F, which is reasonable in an automotive application with a silicon-carbide based converter switching at 100 kHz (resulting in a peak-to-peak battery current ripple due to the switching of the converter of around 5–10%). The bandwidth of the closed current control loop is set to $\alpha_v = 2.2$ kHz and the when the resonant part of the current controller added, the resonant part is resonating at the angular frequency $\omega_0$, with the gain $\alpha_v = \alpha_c/10 = 220$ rad/s. At the rated point of operation, (4000 rpm and 80 Nm) with operation with maximum-torque-per-ampere (MTPA), $i_d^{\text{ref}} = -64.3$ A and $i_q^{\text{ref}} = 120.1$ A. From the FEM model (identifying both sixth and twelfth harmonic order), this yields $\psi_{d6} = 3.6$ mVs, $\psi_{q6} = 6.4$ mVs, $\varphi_{d6} = -2.15$ rad, $\varphi_{q6} = 0.32$ rad, $\psi_{d12} = 0.5$ mVs, $\psi_{q12} = 0.4$ mVs, $\varphi_{d12} = -2.32$ rad, and $\varphi_{q12} = 1.98$ rad.

As can be seen in Fig. 7 (a), the predicted harmonic content in the battery current is substantial. The predicted peak-to-peak battery current ripple using (17) and taking only the sixth-order harmonics into account becomes 36.7 A, which is in good agreement with the simulation. Although recent research [13, 14] indicates that the lifetime of lithium batteries is not affected directly by the ripple content, the associated increase in the cell temperature can indirectly...
reduce the battery’s lifetime. Also, the dc-link voltage variation can be problematic for additional loads connected to the dc-link.

Fig. 7: Simulation results when the FSCW-PMSM is operated at \( \omega_r = 1675.5 \text{ rad/s} \) (4000 rpm) and 80 Nm with \( i_{\text{ref}}^q = -64.3 \text{ A} \) and \( i_{\text{ref}}^d = 120.1 \text{ A} \). Blue curves: with resonant link added in the current controller; red curves: without resonant link added.

6 Concluding remarks

This paper has addressed the impact of harmonics on the dc-link (battery) side for automotive electric drives of FSCW-PMSM type controlled using PIR current controllers to produce (close to) sinusoidal phase currents. Apart from a brief analysis and review of the controller type, results from FEM-based simulations showed that the resulting harmonic content in the phase currents when operating at speeds well above the bandwidth of a conventional PI controller can result in a substantial increase in magnet losses, an aspect that should be kept in mind by the machine designer when considering design mitigation means to reduce the harmonic content of the machine. A simple, analytical expression for predicting the ripple content in the dc-link current was presented which utilizes data from a corresponding, two-dimensional FEM model. The model was then experimentally verified and a good agreement between the measured and predicted dc-current was obtained. A previously reported, experimentally verified high-frequency model of an automotive traction battery and associated battery cable was then used to predict the resulting ripple on the dc-link voltage and battery current. A substantial harmonic content in the battery current was predicted. The above suggested approach can be adopted by the machine and electric drive system designer(s) when analyzing the impact of harmonics on both the ac and dc side of an automotive electric drive system.

7 Acknowledgments

This work has been supported in part by the Swedish Electromobility Centre (SEC) and STAndUP for Energy strategic research framework.

8 References

9 Appendices

9.1 PMSM Specifications

Key specifications of the FSCW-PMSM detailed in [11] are reported in Table 3.

<table>
<thead>
<tr>
<th>Specification</th>
<th>Value</th>
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<tr>
<td>No. of poles (p)</td>
<td>8</td>
</tr>
<tr>
<td>No. of stator slots</td>
<td>12</td>
</tr>
<tr>
<td>No. of turns/slot</td>
<td>16</td>
</tr>
<tr>
<td>Steel quality</td>
<td>M250-35A</td>
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<td>Active length (L_a)</td>
<td>130 mm</td>
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<tr>
<td>Air-gap length</td>
<td>0.75 mm</td>
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<tr>
<td>Outer stator radius</td>
<td>105.8 mm</td>
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<tr>
<td>Rotor radius</td>
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<td>Magnet axial length (l_mag)</td>
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<td>Magnet width (w_mag)</td>
<td>15.3 mm</td>
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<tr>
<td>Magnet type</td>
<td>Vacodym 854 AP</td>
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<tr>
<td>Magnet conductivity (σ_mag)</td>
<td>694 kS/m</td>
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</table>

9.2 FEM Model Details

The adopted 2D-FEM model is implemented using Comsol Multiphysics∗ and is illustrated in Fig. 8.

![Fig. 8: FEM model: (a) Mesh; (b) Sample magnetic field lines; (c) Sample instant current density; (d) Average lamination loss density.](image)

The BH-curve for the M250-35A steel is obtained from [15] and the data is extrapolated using the simultaneous exponential extrapolation method described in [16]. The permanent magnets have an assigned electrical conductivity of 694 kS/m [17]. To compute the eddy currents in the permanent-magnet segments, the numerical approach in [18] is followed meaning that the boundary condition that the surface integral of the (axial component of the) current density in each permanent-magnet segment should be zero is implemented. This forces the eddy currents to form paths similar to those found when solving a full 3D model. To account for the axial segmentation of the permanent-magnet segments, the average eddy-current losses in the permanent-magnet segments \( P_{mag} \) are computed as [18]

\[
P_{mag} = \frac{l_{mag}^2}{l_{mag}^2 + w_{mag}^2} \frac{1}{T_{per}} \int_0^{T_{per}} \left( \frac{L_a}{\sigma_{mag}} \int_S J_z^2 dS \right) dt
\]

where the surface \( S \) is the cross-sectional surface of all permanent-magnet segments and \( T_{per} \) is the period time (or a multiple thereof) of the induced permanent-magnet eddy currents (one electrical period in this case). The average loss density in triangular mesh element \( i' \) (belonging to the stator or the rotor lamination) is computed as

\[
\rho_{loss,i'} = \sum_{\nu=1}^{\nu_{max}} k_{eddy} \omega \left( |B_{x,i'},\nu|^2 + |B_{y,i'},\nu|^2 \right) + \sum_{\nu=1}^{\nu_{max}} k_{hyst} \omega \left( |B_{x,i'},\nu|^2 + |B_{y,i'},\nu|^2 \right)
\]

where the (frequency-dependent) loss factors \( k_{eddy} \) and \( k_{hyst} \) are identified using the loss data provided in [15] and \( \nu_{max} \) is the highest harmonic order considered (corresponding to 2.5 kHz in this case).

*Comsol Multiphysics is a registered trademark of Comsol AB, Stockholm, Sweden.