EXAMPLES FOR DESIGN OF FOUNDATIONS

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EXAMPLES FOR DESIGN OF FOUNDATIONS

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Example 1

Design a strip footing founded on a firm sand according to

a) Swedish Building Code (SBN 75)
b) the deformation method
c) ultimate bearing capacity formulas

The load on the slab \( Q = 650 \text{ kN/m} \).

The sand has \( \rho = 1.8 \text{ t/m}^3 \), \( \rho' = 1.1 \text{ t/m}^3 \) and \( \phi' = 36^\circ \).

The ground water level is 2 m below ground surface, i.e. in the same level as the slab.

Design also for a maximum total settlement according to Canadian manual (1978). Weight sounding \( M_w = 30 \text{ ht/0.2 m} \).

Solution:

a) SBN 75

Allowable soil pressure:

\[
\sigma = b n (1 - \frac{b}{31}) (1 - \tan^2 \alpha)
\]
For strip footing $\frac{b}{31} = 0$, and a vertical load $\alpha = 0^\circ$.
The ground water level is in the same level of footing $h = 0$.
The depth of footing under the ground surface $d = 2.0 \text{ m}$, and for firm sand layer, from Table 23:5332:

$n = 0.22 \text{ MPa/m}$

and $\sigma_{\text{max}} = 0.5 \text{ MPa}$

Substituting the value $n$ in 23:5332,

$\sigma = 0.22 b \text{ MPa}$

$\sigma_{\text{act}} = \frac{Q}{b} = \frac{0.650}{b} \text{ MPa}$

As $\sigma = \sigma_{\text{act}}$

$0.22 b = \frac{0.65}{b}$

then $b = \frac{0.65/0.22}{1} = 1.72 \text{ m}$

($b = 1.7 \text{ m}$)

and $\sigma = 0.22 \times 1.7 = 0.37 \text{ MPa} < 0.5 \text{ MPa}$.

Comment: the foundation is designed with safety factor equal 3 according to SBN 75.

b) The deformation method

According to Maslov's method, the allowable soil pressure $\sigma_m$ is calculated by the formula:

$\sigma_m = \frac{\pi \rho \rho' (b \tan \phi' + d \frac{\rho}{\rho'} + c \cot \phi'/g \rho') + g \rho d}{\cot \phi' - \frac{\pi}{2} + \phi'}$

substituting the values:

$\sigma_m = \frac{\pi \cdot 10 \cdot 1.1 (b \tan 36^\circ + 2 \frac{1.8}{1.1} + 0)}{\cot 36^\circ - \frac{\pi}{2} \frac{36 \cdot \pi}{180}} + 10 \cdot 1.8 \cdot 2$
After transformation

\[ \sigma_m = 57.9 \, b + 297 \]

as \( \sigma_{act} = \frac{650}{b} \)

and \( \sigma_m = \sigma_{act} \) then \( b = 1.67 \, m \) \( (b=1.70 \, m) \).

Comment: The allowable soil pressure \( \sigma_m \), is the pressure that don't develop the plastic zone below the depth \( z_{max} \), where \( z_{max} = b \tan \phi' \) (see Fig 1.2).

c) By Canadian manual 1978

The allowable bearing pressure is calculated by:

\[ q_a = \frac{1}{FS} \left( cN_C + \rho gd Nq + \frac{1}{2} \rho'gb N_Y \right) \]

For \( \phi' = 36^\circ \) from Fig 3.4: \( Nq = N_Y = 32.67 \) \( \text{ (after Hansen)} \)

As \( q_q = \sigma_{act} = \frac{650}{b} \) and assumed \( FS = 3 \) then,

\[ \frac{650}{b} = \frac{1}{3} (1.8 \cdot 10 \cdot 2 \cdot 32.67 + \frac{1}{2} 1.1 \cdot 10b \cdot 32.67) \]

After transformation

\[ b^2 + 6.54b - 650 = 0 \]

\[ b = 1.37 \, m \text{ or } b = 1.40 \, m \]
therefore:
\[ \sigma_{act} = \frac{650}{b} = 464 \text{ kPa} \]

(Comment: \( \sigma_{act} = 0.46 \text{ MPa} < 0.5 \text{ MPa (SBN 75)} \)).

d) By DIN 4017

The ultimate bearing capacity is calculated by

\[ q_u = (\rho_1 g d N_q v_g + \rho_2 g b N_Y v_Y) \]

As strip foundation \( v_Y = v_q = 1 \).

For \( \phi = 36^\circ \):
\[ N_q = 39.48 \]
\[ N_Y = 25.36 \]

Assumed \( FS = 3 \), therefore \( q_a = \frac{q_u}{3} \).

Substituting the values:
\[ \frac{650}{b} = \frac{1}{3}(1.8 \cdot 10 \cdot 2 \cdot 39.48 \cdot 1 + 1.1 \cdot 10 \cdot 25.36 \cdot 1) \]

After transformation:
\[ b^2 + 2.53b - 6.86 = 0 \]
\[ b = 1.64 \text{ m} \] \( b=1.60 \text{ m} \)

therefore
\[ \sigma_{act} = \frac{650}{1.6} = 406 \text{ kPa} \]

(Comment: \( \sigma_{act} = 0.406 \text{ MPa} < 0.5 \text{ MPa (SBN 75)} \)).
e) Hungarian foundation specification (MSZ 15004)

Assume the reduction factor to be $\alpha_1 = 0.7$, $\alpha_2 = 0.7$, $\alpha_3 = 0.6$

$$\alpha = \alpha_1 \cdot \alpha_2 \cdot \alpha_3 = 0.29$$

And the allowable pressure is:

$$q_a = a q_u$$

The ultimate bearing capacity of the soil is calculated by:

$$q_u = \rho'gb\gamma + \rho gd q$$

for $\phi = 36^\circ$: $\gamma = 40.2$ and $q = 37.8$

Substituting the values:

$$q_u = 1.1 \cdot 10 \cdot b \cdot 40.2 + 1.8 \cdot 10 \cdot 2 \cdot 37.8$$

$$q_a = \frac{650}{b} = 0.29(422b + 1360.8)$$

After transformation:

$$b^2 + 3.23 - 5.31 = 0$$

$$b = 1.2 \text{ m.}$$

$$\sigma_{act} = \frac{650}{1.2} = 542 \text{ kPa}$$

$$= 0.54 \text{ MPa} > 0.5 \text{ MPa (SBN 75)}$$

Comment: The $\alpha$ is $1/FS$, as the value $\gamma$ is high according to c) and d), so the footing width is only 1.2 m.
f) by Handbook Foundation Engineering (Kezdi)

The allowable bearing pressure is obtained from the formula:

\[ q_a = \frac{1}{FS} (cN_c + \rho d Nq + \frac{1}{2} \rho'gbN\gamma) \]

as \( q_a = q_{act} = \frac{650}{b} \), \( c = 0 \) and \( FS = 3.0 \)

for \( \phi' = 36^\circ \), bearing capacity factors (Table 3.1):

\[ Nq = 37.75, \ N\gamma = 56.31 \]

Substituting the values:

\[ \frac{650}{b} = \frac{1}{3} (10 \cdot 1.8 \cdot 2 \cdot 37.75 + \frac{1}{2} 1.1 \cdot 10 \cdot 56.31, b) \]

\[ b^2 + 4.39b - 6.3 = 0 \]

\[ b = 1.14 \text{ m} \ (b=1.20 \text{ m}) \]

Comments: The values of \( N_c \) are almost the same for d), e) and f) but \( N\gamma \) has a big variation. By different solutions of ultimate bearing capacity, the variation of \( b \) is from 1.2 m to 1.60 m, this is due to the variation of the \( N\gamma \) value. The value of \( b = 1.2 \text{ m} \) obtained by e) and f) will give a pressure not permitted according to SBN 75.

g) Design for a total maximum settlement

According to the Canadian manual for structures in sand the allowable maximum total settlement is 3.8 cm. The settlement may be estimated using the relationship:

\[ s = \frac{b \ \eta_{net}}{E_s} f_c \]

where \( E_s = \) modulus of elasticity of the soil
\( f_c = \) settlement coefficient, as given in Fig. 5.3 (Kany)
\( q_n = \) net design bearing pressure
Example 2

A friction pile 30 x 30 of precast concrete is driven to 12 m depth in firm sand. Calculate the bearing capacity from

a) static point of view
b) dynamic point of view
c) SW-bored pile instructions.

The pile driving hammer has a free fall of 0.5 m. Allowable remaining settlement for the 6000 kN pile is 0.7 mm according to the Swedish Pile codes.

The sand has $\rho = 1.8 \text{ t/m}^3$, $\rho' = 1.1 \text{ t/m}^3$ and $\phi' = 34^\circ$. The ground water level is 2 m below the ground surface. Young's modulus is $3 \times 10^7 \text{ kPa}$.

Also calculate the bearing capacity of the pile, considering the effect of critical depth.

Example 2
The value of \( q_n \) is equal to \( q-pgd \), where \( q = \frac{650}{b} \), therefore:

\[
q_n = \frac{650}{b} - pgd = \frac{650}{b} - 1.8 \cdot 10^2 = \frac{650}{b} - 36
\]

For strip footing \( \frac{L}{b} = \alpha \), and assumed the compressible layer \( z = 2B \), by Fig. 5.3, \( f_c = 1.07 \). The result of weight sounding is given \( M_w = 30 \text{ kN/m} \), which means the cone static resistance \( q_C = 10 \text{ MPa} \) (\( q_C = 0.33 M_w \), Sw Bored instructions). The modulus of elasticity may be determined by the relationship:

\[
E_s = (1.5 \div 2) q_c
\]

(Canadian manual, SAA code)

if assumed

\[
E_s = 1.7 q_c,
\]

\[
E_s = 1.7 \cdot 10 \text{ MPa} = 17 \text{ MPa} = 170 \cdot 10^2 \text{ kPa}
\]

Substituting the values:

\[
3.8 \cdot 10^{-2} = \frac{b(\frac{650}{b} - 36)}{170 \cdot 10^2} 1.07
\]

After solution:

\[
b = 1.37 \text{ m} \quad (b=1.40 \text{ m})
\]

Comment: the method require a approximation of the value of the modulus of elasticity \( E_s \), therefore it is useful only for preliminary design purposes.
FIG 5.3 (After Kany)
CURVES FOR CALCULATION OF SETTLEMENT AT THE CRITICAL POINT, C

\[ s = \frac{q'B}{E_s} \]

WHERE \( q' = q - \gamma D_f \)

\[ L/B = 1.0 \quad 1.5 \quad 2.0 \quad 3.0 \quad 5.0 \]
a) Statically:

The bearing capacity of the pile: \( P = P_f + P_p \)

\( P_f \) = skin friction: For one pile element \( dz \) (Fig 2.2), equilibrium gives:

\[
\frac{dP}{dz} = K_s \sigma_v \theta \tan \delta dz
\]

\[
P_f = \int_0^L K_s \sigma_v \theta \tan \delta dz = P_H \tan \delta.
\]

**Fig. 2.2**

where \( K_s = \frac{\sigma_v}{\sigma_h} \), \( \theta \) = perimeter of pile and \( \tan \delta \) is the friction between the pile and the soil.

**Fig. 2.3** shows the distribution of horizontal pressure, then the resulting horizontal load \( P_H \) is:

\[
P_H = K_s \left( \frac{1}{2} \rho \rho z_1^2 + \rho \rho z_1 + \rho \rho z_2 + \rho \rho z_2^2 \right)
\]

After Broms, the angle of friction at the sand/concrete pile contact

\[
\delta = \frac{3}{4} \phi = \frac{3}{4} \times 34^\circ = 25.5^\circ
\]

and \( K_s = 2 \) for firm sand

substituting the values:

\[
= 2 \times 4 \times 0.3 \left( \frac{1}{2} \times 10 \times 1.8 \times 2 + 2 \times 10 \times 1.8 + 10 \times 1 \times 1.8 \times 8 \right) \tan 25.5^\circ
\]

\[
P_f = 1090 \text{ kN}
\]
The point resistance is calculated by Caquot and Kerisel:
\[ P_p = A_p \rho g L \tan \phi \]

where:
- \( A_p \) = cross-sectional area of the pile tip
- \( \rho g L \) = effective overburden pressure at the pile tip.

Substituting the values:
\[ P_p = 0.3 \times 10 \times \left( \frac{2 \times 1.8 + 10 \times 1.1}{12} \right) \times e^{\tan \phi} \]
\[ P_p = 1470 \text{ kN} \]

Then the bearing capacity of the pile is
\[ P = P_f + P_p = 1090 + 1470 = 2560 \text{ kN} \]

The Canadian Manual

Point resistance: Assume that \( L < D_{\text{critical}} \) therefore:
\[ P_p = (\rho g L N_q) A_p \]

where:
- \( \rho g \) = effective unit weight of the soil
- \( N_q \) = the bearing capacity factor for pile as derived from Brezantsev (1961)

For \( \phi = 34^\circ \), \( N_q \approx 75 \)

Substituting the values:
\[ P_p = 0.3 \times (1.8 \times 10 \times 2 + 1.1 \times 10 \times 10) \times 75 \]
\[ P_p = 985 \text{ kN} \]

Calculation of the skin friction follows the same procedure that was presented, then:
\[ P = 1090 + 985 = 2075 \text{ kN} \]
b) Dynamically

The bearing capacity of the pile is calculated by the following pile driving formula: (SBN 75)

\[ P_u = 0.8 n h \frac{Q_r (1-0.1 \frac{Q_p}{Q_r})}{(e+c)/2} \]

- \( n \) = correction factor for the drop hammer (1 for a free falling hammer)
- \( Q_r \) = weight of the hammer (at least 3 tons for a concrete pile). Here \( Q_r = 4 \) tons.
- \( Q_p \) = total weight of the pile
  \[ Q_p = 2.4 \times 0.3^2 \times 12 = 2.59 \text{ tons} \]
- \( h \) = height of drop of hammer: 0.5 m
- \( e \) = remaining penetration: 0.7 mm = 0.7x10^{-3} m
- \( c \) = rebound of the soil

The rebound of the soil for a concrete pile can be assumed.

\[ c = \frac{P_u l_e}{A_p E_p} + \frac{P_u l_f}{A_f E_f} \]

\[ l_e = \frac{Q_r}{q_p}, \text{ where } q_p \text{ is the weight of the pile per metre, here} \]
\[ l_e = \frac{4}{2.4 \times 0.3^2} = 18.7 \text{ m} \]

\( l_f \) = length of the follower. Here \( l_f = 0 \)

\( E_p \) = modulus of elasticity (Young's modulus):
\[ 3 \times 10^7 \text{ kPa} \]

Substituting the values in the expression of \( c \):

\[ c = \frac{P_u \times 18.7}{0.3^2 \times 3 \times 10^7} = 6.9 \times 10^{-6} P_u \]

Substituting the values in the pile driving formula:

\[ P_u = 0.8 \times 1 \times 0.5 \times 4(1-0.2 \times 2.59/4.0)/(0.7+0.0069 P_u) \]
\[ \Rightarrow 0.0035 P_u^2 + 0.7 P_u - 14960 = 0 \]
\[ P_u = 1980 \text{ kN} \]
c) by Sw. Bored Pile Instructions with $N_s = \phi' = 34^\circ$

For diagram 2.1421a, with $\frac{L}{D} = \frac{12}{0.3} = 40$, one obtains
the value $N_q = \frac{P_{sf}}{\sigma_v'} = 42$ for a bored pile.

where $P_{sf}$ = the failure pressure at the bored pile point
$\sigma_v'$ = effective overburden pressure at the pile
   point level.
$N_q$ = the bearing capacity factor.

For driven piles the bearing capacity is 3 times that
of bored piles, so $N_q = 3 \times 42 = 146$ for a driven pile.

$\sigma_v' = 2 \times 1.8 \times 10 + 10 \times 1.1 \times 10 = 146$ kPa

The failure pressure of the driven pile:

$P_{sf} = 146 \times 146 = 18,396$ kPa

According to the diagram 2.14212

$p_m/\sigma_v' = 12$ for bored piles

Experience gives $p_m/\sigma_v' = 3 \times 12 = 36$ for driven piles

The average overburden pressure

$\sigma_v = \frac{0 + 146}{2} = 73$ kPa

then

$p_m = 36 \times 73 = 2628$ kPa

As $p_m$ is the permissible shaft stress with the safety
factor of 2, then the failure skin friction is assumed:

$P_{mf} = 2628 \times 2 = 5256$ kPa

and the bearing capacity of the pile:

$P = A_p (P_{sf} + P_{mf})$
$= 0.3^2 (18396 + 5256) = 2128$ kN
The maximum shaft resistance can be calculated by the formula:

\[ P_{mf} = K_s \bar{\sigma}_v \frac{L_1}{D} \left[ 1 - \left(1 - \frac{\sigma'_u}{\bar{\sigma}_v} \right) \left(1 - \frac{10}{n}\right)^2 \right] \tan \rho \]

- \( K_s \) = earth pressure coefficient = 1.5
- \( \bar{\sigma}_v \) = mean value of \( \sigma'_v \) = 73 kPa
- \( \sigma'_u \) = 0
- \( L_1 \) = 12 m
- \( D \) = 0.3 m
- \( \delta \) = 25°5
- \( n = \frac{L_1}{D} = 40 \)

Substituting the values:

\[ P_{mf} = 1.5 \times 73 \times 4 \times 40 \left[ 1 - \left(1 - \frac{10}{40}\right)^2 \right] 0.48 \]

\[ P_{mf} = 3700 \text{ kPa} \]

and the bearing capacity of the pile

\[ P = A_p (P_{sf} + P_{mf}) = 0.3^2 (18396 + 3700) = 1989 \]

This is almost the same value as obtained from the diagram 2.14212 and use of the safety factor of 2.0.

Comment: The bearing capacity of the pile is calculated by different methods, and is 2000 ÷ 2500 kN. If a safety factor of 3 is assumed, the allowable load is 660 kN. The Sw. Bored Pile Instructions consider critical depth.
d) Calculation of the bearing capacity of the pile by the Australian Code (SAA, 1978).

The unit skin friction and point resistance reach limiting values at a pile depth of about 8 to 20 times the pile diameter depending on the relative density. The value of critical depth $D_c$, the bearing capacity factor $N_q$, and the factor of correction for different types of pile and relative density are presented in Table A. 1.1.2.

In this case as $\phi' = 34^\circ$, the sand is medium dense and

$$\frac{D_c}{d} = 8, \quad F = 1.0 \quad \text{and} \quad N_q = 100.$$  

where $d$ is the diameter of the pile. For equivalent area, $d = 0.34 \text{ m}$ and $D_c = 8 \times 0.34 = 2.72 \text{ m}$.

The unit skin friction at critical depth is:

$$P_{mf} = F \times \sigma'_v = 1(1.8 \times 10 \times 2 + 1.1 \times 10 \times 0.72) = 44 \text{ kPa}$$

The unit point resistance:

$$P_{sf} = \sigma' v N_q = 44 \times 100 = 4400 \text{ kPa}.$$  

The bearing capacity of the pile is calculated by:

$$Q = Q_f + Q_p = \frac{P_{mf}}{2 \theta} D_c + P_{mf} \theta(L-D_c) + A_p P_{sf}$$

Substituting the values:

$$Q = \frac{44}{2} \times 4 \times 0.3 \times 2.72 + 44 \times 0.3 \times 4(12-2.72) + 0.3^2 \times 4400$$

$$Q = 958 \text{ kN}$$

Comment: Considering the effect of critical depth, the bearing capacity is reduced to almost 50% of the values obtained by a), b) and c).
Example 3

Calculate the primary consolidation settlement in the clay in case a) to f) for one layer solution. The soil consist of 10.0 m clay on sand. The clay is normally consolidated, \( p = 1.6 \, \text{t/m}^3 \), and the compression modulus number \( m = 10.0 \). The ground water table is 1.0 m below the ground surface.

a) Loading over a large area
b) Lowering of the ground water table by 1.0 m
c) Lowering of the pore pressure in the bottom of the clay layer
d) Building 10 x 10 m\(^2\) on the ground surface
e) Building 10 x 10 m\(^2\) founded 2 m below the ground surface
f) Building on piles.

Solution:

\[
\delta = h \frac{1}{m} \ln \left( \frac{p + \Delta \sigma'}{\sigma_0'} \right)
\]

if OCR = 1.
where $\sigma'_o$ is the effective overburden pressure and $\Delta\sigma'$ is the additional pressure due to the loading. Effective pressure at the middle of the layer =

$$\sigma'_o = 1.6 \times 10 \times 1.0 + 0.6 \times 10 \times 4 = 40 \text{ kPa}$$

As $q = \Delta\sigma' = 10 \text{ kPa}$

Then

$$\delta = 1000 \times \frac{1}{10} \ln \frac{40+10}{40} = 22 \text{ cm}$$

b) Lowering of ground water table by 1.0 m

Effective overburden pressure at 5.5 m below the ground surface

$$\sigma'_o = 1.6 \times 10 \times 10 + 0.6 \times 10 \times 4.5 = 43 \text{ kPa}$$

Additional pressure due to lowering of ground water table.

$$\Delta\sigma' = (1.6-0.6) \times 10 \times 1.0 = 10 \text{ kPa}$$

The settlement

$$\delta = 900 \times \frac{1}{10} \ln \frac{43+10}{43} = 19 \text{ cm}$$
c) Lowering of pore pressure in the bottom of the clay layer.

![Diagram](Image)

Fig. 3.3

Effective pressure at 9.0 m under ground surface.

\[ \sigma'_o = 1.6 \times 10 \times 1 + 0.6 \times 10 \times 8.0 = 64 \text{ kPa}. \]

Additional pressure

\[ \Delta \sigma' = 80 - 50 \text{ kPa} = 30 \text{ kPa} \]

The settlement

\[ \delta = 200 \times \frac{7}{10} \ln \frac{64+30}{64} = 7.7 \text{ cm} \]

d) Building 10 x 10 m\(^2\) on the ground surface.
Effective pressure at 5 m under the ground surface

\[ \sigma'_o = 40 \text{ kPa} \] (after a))

\[ \Delta\sigma' = \frac{Q}{(b+z)(1+z)} = \frac{4000}{(10+5)(10+5)} = 18 \text{ kPa} \]

\[ \delta \approx 1000 \ln \left( \frac{40+18}{40} \right) = 37 \text{ cm} \]

e) Building 10 x 10 m² founded 2 m below the ground surface.

![Fig. 3.5](image)

Calculate the pressure at 6.0 m below the ground surface

\[ \sigma'_o = 1.6 \times 10 \times 1.0 + 0.6 \times 10 \times 5.0 = 46 \text{ kPa} \]

Additional pressure

\[ \Delta\sigma' = \frac{4000-1.6\times10\times2.0\times10\times10}{(10+4)(10+4)} = 4 \text{ kPa} \]

\[ \delta = \frac{1}{800} \times \frac{1}{10} \ln \frac{46+4}{46} = 6.7 \text{ cm} \]
f) Building on piles

Assume that the load is transferred to the soil at the depth of 2/3 Lp, then the theoretical footing is located 4.0 m below the ground surface. The effective pressure at 7.0 m below the ground surface:

$$\sigma'_o = 1.6 \times 10 \times 1 + 0.6 \times 10 \times 6 = 52 \text{ kPa}$$

The additional pressure 3 m below the theoretical footing:

$$\Delta \sigma' = \frac{4000}{(10+3)(10+3)} = 24 \text{ kPa}$$

$$\delta = 600 \times \frac{1}{10} \ln \frac{52+24}{52} = 23 \text{ cm}$$
Example 4

A quadratic pile group is loaded by 450 kN, Fig. 4.1. How large settlement will occur after one year? The clay is normally consolidated, $\rho = 1.6 \text{t/m}^3$, $m = 10$, $\beta = 0$ and $c_v = 6.8 \times 10^{-8} \text{m}^2/\text{s}$.

Solution

The submerged density of the sand layer $= \rho' = \rho_d - \frac{\rho_d}{\rho_s} \rho_w = 1.8 - \frac{1.8}{2.65} \times 1.0 = 1.12 \text{t/m}^3$
The load carried by the pile group is assumed to be transferred to the soil through a theoretical footing located 0.3-0.4 times the pile length up from the pile point. The load is assumed to spread within frustum of a pyramid with the side 2:1 and to cause uniform additional vertical pressure at lower levels, the pressure at any level being equal to the load carried by the group divided by the cross-sectional area of the pyramid at that level. The level of a theoretical footing: \((0.3 \pm 0.4)L = 5\) m. (at level \(-0.6\) m).

The effective overburden pressure at any depth:
\[
\sigma'_o = (1.8 \times 1.0 + 1.12 \times 1.10 + 0.6 \times 9.0) \times 10 + 0.6 \times z \times 10 = 83 + 6z
\]

where \(z\) is from the theoretical footing (see Fig. 4.1).

The uniform additional vertical pressure:
\[
\Delta \sigma' = \frac{Q}{(B+z)^2} = \frac{450}{(2+z)^2}
\]

The settlement is calculated by:
\[
\delta_i = \Delta h \frac{1}{m} \ln \frac{\sigma'_o + \Delta \sigma'}{\sigma'_o}
\]

The clay layer is divided by sublayers \((2+2+6+6)\) m, and the settlement of every layer is calculated.

<table>
<thead>
<tr>
<th>No. of layer</th>
<th>h (m)</th>
<th>Middle level (m)</th>
<th>z (m)</th>
<th>(\sigma'_o) (kPa)</th>
<th>(\Delta \sigma') (kPa)</th>
<th>(\delta_i)</th>
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<td>13.0</td>
<td>161</td>
<td>2</td>
<td>0.7</td>
</tr>
</tbody>
</table>

\[\Sigma \delta_i = 15.7 \text{ cm} \approx (16 \text{ cm})\]
The total settlement of the clay layer is 16 cm. Assume that the consolidation is of double drainage, \(2h = 16 \text{ m}\), and \(h = 8.0 \text{ m}\).

The time factor is defined by:
\[
T_v = \frac{C_v t}{h^2} = \frac{6.8 \times 10^{-8} \times 3.15 \times 10^6}{8^2} = 0.33
\]

Therefore the degree of consolidation \(U = 0.20\).

The settlement after one year:
\[
\delta = 0.2 \times 16 \text{ cm} = 3.2 \text{ cm}.
\]

Example 5

A road embankment shall be founded on clay. The clay is overconsolidated by 30 kPa in the upper 14 m. Underneath is 6 m normally consolidated clay. Under the clay is permeable sand. Pore pressure measurements in the clay show hydrostatic pressure down to the level -14 m. The pore pressure at the level -16 and -18 is 140 kPa, and at the level -20 the pressure is 130 kPa. The ground water table is 1.0 m below the ground surface. Calculate the primary consolidation settlement without and with 12 m timber piles. Note: 2 layer solution, 2:1 method for stress. Settlement below preconsolidation pressure is neglected.
The total pressure, pore pressure and effective pressure are presented in the following table.

<table>
<thead>
<tr>
<th>Level</th>
<th>Total pressure</th>
<th>Pore pressure</th>
<th>Effective pressure</th>
</tr>
</thead>
<tbody>
<tr>
<td>+0.0</td>
<td>$\sigma_0 = 0$</td>
<td>$U_0 = 0$</td>
<td>$\sigma'_0 = 0$</td>
</tr>
<tr>
<td>-1.0</td>
<td>$\sigma_0 = 1.6 \times 1.0 \times 10.0 = 16$ kPa</td>
<td>$U_0 = 0$</td>
<td>$\sigma'_0 = 16$ kPa</td>
</tr>
<tr>
<td>-14.0</td>
<td>$\sigma_0 = 1.6 \times 14 \times 10.0 = 224$ kPa</td>
<td>$U_0 = 130$ kPa</td>
<td>$\sigma'_0 = 94$ kPa</td>
</tr>
<tr>
<td>-15.5</td>
<td>$\sigma_0 = 224 + 1.5 \times 1.5 \times 10 = 246.5$ kPa</td>
<td>$U_0 = 137.5$ kPa</td>
<td>$\sigma'_0 = 109$ kPa</td>
</tr>
<tr>
<td>-16.0</td>
<td>$\sigma_0 = 246 + 5 \times 0.5 \times 1.5 \times 10 = 254$ kPa</td>
<td>$U_0 = 140$ kPa</td>
<td>$\sigma'_0 = 114.0$ kPa</td>
</tr>
<tr>
<td>-18.0</td>
<td>$\sigma_0 = 254 + 2 \times 1.5 \times 10 = 284$ kPa</td>
<td>$U_0 = 140$ kPa</td>
<td>$\sigma'_0 = 144$ kPa</td>
</tr>
<tr>
<td>-18.5</td>
<td>$\sigma_0 = 284 + 0.5 \times 1.5 \times 10 = 291.5$ kPa</td>
<td>$U_0 = 137.5$ kPa</td>
<td>$\sigma'_0 = 154$ kPa</td>
</tr>
<tr>
<td>-20.0</td>
<td>$\sigma_0 = 291.5 + 1.5 \times 1.5 \times 10 = 314$ kPa</td>
<td>$U_0 = 130.0$ kPa</td>
<td>$\sigma'_0 = 184$ kPa</td>
</tr>
</tbody>
</table>
Additional load from the embankment:

- Without piles:
  \[ \Delta \sigma_1' = \frac{Q}{(b+z)x1} = \frac{1.9 \times 1.5 \times 10 \times 12}{12+z} = \frac{342}{12+z} \text{ kPa} \]

  where \( z \) is the distance from the ground surface.

- With piles:
  \[ \Delta \sigma_2' = \frac{342}{12+z'} \text{ kPa} \]

  where \( z' \) is the distance from the theoretical footing.

<table>
<thead>
<tr>
<th>Level (m)</th>
<th>( \Delta \sigma_1' )</th>
<th>( \sigma_0' + \Delta \sigma_1' )</th>
<th>( \Delta \sigma_2' )</th>
<th>( \sigma_0' + \Delta \sigma_2' )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-8</td>
<td>88</td>
<td>75</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-10</td>
<td>101</td>
<td>96</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-14</td>
<td>141</td>
<td>107</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-15.5</td>
<td>151</td>
<td>121</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-18.5</td>
<td>181</td>
<td>165</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-20.0</td>
<td>201</td>
<td>195</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>29</td>
<td></td>
<td>87</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>25</td>
<td></td>
<td>105</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>19</td>
<td></td>
<td>113</td>
<td></td>
</tr>
<tr>
<td>7.5</td>
<td>17</td>
<td></td>
<td>126</td>
<td></td>
</tr>
<tr>
<td>10.5</td>
<td>15</td>
<td></td>
<td>169</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>15</td>
<td></td>
<td>199</td>
<td></td>
</tr>
</tbody>
</table>

The distribution of pressure is presented in Fig. 5.2.

The settlement is calculated by:

- Without piles:
  \[ \delta = h \frac{1}{m} \ln \left( \frac{\sigma_0' + \Delta \sigma_1'}{\sigma_0'} \right) \]

- With piles:
  \[ \delta = 300 \frac{1}{12} \ln \frac{126}{109} + 300 \frac{1}{12} \ln \frac{169}{109} = 6.20 \text{ cm} \]
Comment: As the upper clay layer is overconsolidated and the additional pressure is less than the overconsolidation pressure, the settlement can be neglected in this layer. The use of piles in this case produce more settlements than without piles, due to the distribution of effective overburden pressure. Pile foundations do not always reduce settlements in comparison with shallow footing.
Example 6

Calculate the bearing capacity of a pile group of 16 driven concrete piles in clay. The pile group is square with c/c 1.0 m between the piles (see Fig. 6.1). The piles are square 0.3x0.3 m. The pile tip and the pile end is at the level +1.0 m and -16.0 m. Soil properties according to Fig. 6.2.

Solution:
\( \tau_{fu} \) from level \(-2.0\) m to \(-16\) m is \( \approx 22\) kPa.

\( \tau_{fu} \) at tip leven \( \approx 33\) kPa.

1. Ultimate bearing capacity of a single pile:

\[
P = P_s + P_p
\]

\[
P_s = \tau \times \theta \times L
\]

\( \theta \) = perimeter of the pile

\( \tau = \tau_{fu} \times 0.8 \) \((0.8:\) reduction factor due to concrete pile\)

\[
P_s = 22 \times 0.8 \times 4 \times 0.3 \times 14.0 = 295\ kN
\]

\[
P_p = 9 \times \tau_{fu} A_p = 9 \times 33 \times 0.3^2 = 27\ kN.
\]

Comment: The value of \( P_p \) is \( \approx 10\% P_s \).

\[
P_T = 295 + 27 = 320\ kN
\]

Bearing capacity of the pile group

\[
P = n \times P_T = 16 \times 320 = 5120\ kN.
\]

2. Calculate the bearing capacity of the pile group as a theoretical footing.

Dimension of the pile group 3 x 3 m. \((B=3\) m\)

\[
P_T = P_s + P_p
\]

\[
P_s = \tau_{fu} \times 4 \times B \times L = 22 \times 4 \times 3 \times 14 = 3700\ kN
\]

\[
P_p = 9 \times \tau_{fu} B^2 = 9 \times 33 \times 3^2 = 2670\ kN
\]

\[
P_T = 3700 + 2670 = 6370\ kN
\]

\( \therefore \) Bearing capacity of the pile group is 5120 kN.
Example 7

A silo building, 9 x 9 m, weighing 1100 tons will be founded 1 m below the ground surface. The soil is clay to 80 m, \( p = 1.6 \text{ t/m}^3 \), \( \tau_{fu} = (12+1.0z)\text{kPa} \) where \( z \) = the depth in metre below ground surface. Available timber pile: 18 m \( \phi 12.5 \) cm top

12 m \( \phi 20 \) cm in top

The increase in diameter is 10 mm/m. Decide foundation method, slab size, number, length and placing of piles if piles are used. Safety factor of 3 of bearing capacity. The slab must not be greater than 11 x 11 m. The ground water table is 1.0 m below the ground surface. \( \sigma_{\text{timber}} = 4500 \text{ kPa} \). The Swedish pile code must be considered.

Solution given:

![Diagram](image)

- Pile No.1 = 18 m; \( \phi_1 = 12.5, \phi_2 = 30.1 \text{ cm} \)
- Pile No.2 = 12 m; \( \phi_1 = 20, \phi_2 = 32 \text{ cm} \)
- \( \sigma_{\text{timber}} = 4500 \text{ kN} \)
- \( F_c = 3 \)

Fig. 7.1
1. Calculate the bearing capacity of the foundation without pile. The bearing capacity of a foundation in cohesive soil is evaluated by

\[ q_b = cN_cK_c + qN_qK_q \quad (\phi=0) \]

where

- \( N_c = 5.7 \) (Terzaghi)
- \( N_q = 1.0 \) (Terzaghi)
- \( k_c = k_q = (1+0.35 \frac{D}{B})(1+0.2 \frac{B}{L}) \)
- \( D = 1.0; \ B = L = 10.0 \ m \)
- \( k_c = k_q = (1+0.35 \frac{1.0}{9})(1+0.2 \frac{9.0}{9.0}) = 1.24 \)

The medium value of cohesion between \( D \) and \( (D+ \frac{B\sqrt{2}}{2}) \):

\[ c = 12 + 1 + \frac{1}{2}(0+4.5\sqrt{2}) = 16 \ kPa \]

Substituting the values:

- \( q_b = 16 \times 5.7 \times 1.24 + 10 \times 1.6 \times 1.0 \times 1.0 \times 1.24 \)
- \( q_b = 113 \ + \ 20 = 133 \ kPa \)
- \( q_{act} = \frac{11000}{9.0 \times 9.0} = 135.8 \)

The use of pile is necessary.
2. Calculate foundation with pile No. 1.

According to the Swedish Pile Code (Table 23:611) spacing center to center: 5 D

\[ c/c = 5 \times 0.305 = 1.5 \text{ m} \]

Bearing capacity of a single pile:

\[
P_{br} = \tau_s a_s
\]

\[
P_{br} = \tau_0 L \left( \frac{D_1+D_2}{2} \right) + \frac{axL^2}{6}(D_1+2D_2)
\]

where \( a = \) increase in shear strength

Substituting the values:

\[
P_{br} = 13 \times 18 \times 3.14 \left( \frac{0.305+0.125}{2} \right) + 1.0 \times 18^2 \times 3.14 \left( 0.305+2 \times 0.205 \right) = 252 \text{ kN}
\]

Allowable load for a single pile \( P_a = \frac{252}{3} = 84 \text{ kN} \).

Number of piles \( \frac{Q}{P_a} = \frac{11000}{84} = 131 \) piles

As the minimum spacing (c/c) is 1.5 m, the dimension of the pile cap is minimum:

\[ B = 1.5 \times \sqrt{131} = 17.2 \text{ m} \]

The maximum allowable dimension is 11 x 11 m, therefore the use of pile No. 1 only is not permitted.
3. The compound of two timber piles (Fig. 7.4).

The driving of the pile 12 m in the soil produce a disturbance zone and reduce the cohesion between the soil and the pile. According to the Swedish Pile Code the cohesion from ground surface is $f_L$ (here 3.0 m) is equal to 0. The cohesion below this level is $\frac{1}{3} \tau_{fu}$ for the upper pile. The ultimate bearing capacity of a single pile:

$$P_{br} = \tau_s A_s = (12+13)x18\pi x \frac{0.125+0.35}{2} + \frac{10x18^2\pi}{6}(0.305+2x0.125) + \frac{1}{3}(12+4)9\pi \frac{0.2+3x0.01+0.2+12x0.01}{2} + \frac{1}{3} \times \frac{1.0x9^2}{6} \pi (0.23+2x0.32) = 452 \text{ kN}.$$

Allowable load: $p_a = \frac{452}{3} = 151 \text{ kN}$

The diameter of the upper pile is 20 cm, then

$$\text{Area} = \pi x \frac{20^2}{4} = 314 \text{ cm}^2$$

As the maximum allowable pressure of a pile is 4500 kN/m$^2$. The allowable load: $p_a = 4500 x 314 x 10^{-4} = 141 \text{ kN}$.

Therefore:

The number of piles $= \frac{11000}{141} = 78$ piles.

Take 81 piles placed 9 x 9.

The minimum spacing $c/c = 6D$

$c/c = 6 x 20 = 120 \text{ cm}.$
The dimension of the cap:

\[ B = 9 \times 1.20 + 10.8 \text{ m} \quad (10.8 \times 10.8) \]

The bearing capacity of a pile group is greater than for the tip only.

\[ p^{\text{tip}} = 9 \tau_{fu}xBxL = 9(12+31)\times10.8^2 = 45000 \text{ kN} \]

and \( 45000 > 3\times1100 \text{ kN} \).

Conclusion: Pile cap 11 x 11 m, 81 piles (9x9). The pile is compound by pile 12 m and 18 m.
Example 8

Calculate the settlement of a building founded on floating piles in clay. The pile group is 15 x 20 m and the piles are 18 m long, carrying a load of 28 MN, including the weight of the pile cap. The cap is founded 1 m below the ground surface.

The dry crust has a density of 1.9 t/m³. The clay is normally consolidated. The ground water table is 3.0 m below the ground surface. The clay deposit is 40 m deep and is underlaid by a non-cohesive soil.

The clay has a density of \( \rho = 1.6 \text{ t/m}^3 \). The compression modulus number \( m = 11.5 \), and the stress exponent \( \beta = 0 \). (3 layers and 2:1 method shall be used.)

Fig. 8.1
Solution:

The level of the theoretical footing is -13 m ($\frac{1}{3}$ length of pile from pile tip). The clay below this level can be subdivided into 3 layers.

<table>
<thead>
<tr>
<th>Layer No</th>
<th>interval</th>
<th>thickness (m)</th>
<th>medium level</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-13 to -20</td>
<td>7</td>
<td>-16.5</td>
</tr>
<tr>
<td>2</td>
<td>-20 to -29</td>
<td>9</td>
<td>-24.5</td>
</tr>
<tr>
<td>3</td>
<td>-29 to -40</td>
<td>11</td>
<td>-34.5</td>
</tr>
</tbody>
</table>

The settlement of every layer is calculated by:

$$s = h \frac{1}{m} \ln \frac{\sigma' + \Delta \sigma'}{\sigma'}$$

Settlement of layer No. 1:

The additional pressure at level -16.5 due to the weight of the building (2:1 method)

$$\Delta \sigma_1 = \frac{Q}{(B+2)(L+z)}$$

$$\Delta \sigma_1 = \frac{28000}{(15+3.5)(20+3.5)} = 64.5 \text{ kPa}$$

due to the excavation for foundation:

$$\Delta \sigma_1 = \frac{x g x d x B x L}{(15+15.5)(20+15.5)} = 5.3 \text{ kPa}$$

The original effective pressure:

$$\sigma'_o = \sigma_o - u$$

$$\sigma_o = 1 \times 10 \times 1.9 + 15.5 \times 1.0 \times 1.6 = 267 \text{ kPa}$$

$$u = 13.5 \times 1.0 \times 10 = 135$$

$$\sigma'_o = 132 \text{ kPa}$$

Substituting the values:

$$s_1 = 700 \times \frac{1}{11.5} \ln \frac{135+64.5-5.3}{132} = 22.6 \text{ cm}$$
Settlement of layer No. 2

Effective pressure at level -24.5 m:

\[ \sigma'_o = 132 + 8 \times 0.6 \times 10 = 180 \text{ kPa} \]

Additional effective pressure:

\[ \Delta \sigma_1' = \frac{28000}{(15+11.5)(20+11.5)} = 33.6 \text{ kPa} \]

\[ \Delta \sigma_2' = \frac{5700}{(15+23.5)(20+23.5)} = 3.4 \text{ kPa} \]

\[ s_2 = 900 \frac{1}{11.5} \ln \frac{180+33.6-3.4}{180} = 12.3 \text{ cm} \]

Settlement layer No. 3

\[ \sigma'_o = 180 + 10 \times 10 \times 0.6 = 240 \text{ kPa} \]

\[ \Delta \sigma_1' = \frac{28000}{(15+21.5)(20+21.5)} = 18.5 \text{ kPa} \]

\[ \Delta \sigma_2' = \frac{5700}{(15+33.5)(20+33.5)} = 2.2 \text{ kPa} \]

\[ s_3 = 1100 \times \frac{1}{11.5} \ln \frac{240+18.5-2.2}{240} = 6.3 \text{ cm} \]

Total settlement:

\[ s_T = s_1 + s_2 + s_3 = 22.6 + 12.3 + 6.3 = 41.2 \text{ cm} \]
Example 9

A circular concrete pile is driven into loose clay and will be used as a tension pile. The pile is 26 m long with its top 8 m below the ground surface. The diameter is 0.34 m. Shear strength according to Fig. 9.1. Calculate the bearing capacity, the displacement at a safety factor of 2.5 and the displacement at failure.

Data: $G_s = 150 \ \tau_{fu}$, $E_p = 3.6 \times 10^7$ kPa

Solution:

Calculation of the initial displacement modulus $K_s$. According to the data for soil (Fig. 9.1)

$$\tau_{fu} = 47 \text{ kPa}$$

As Torstensson (1973) showed that the shear strength determined with the field vane test varied with time to failure according to

$$\frac{\tau_{cr}}{\tau_o} = 1.21 \left(\frac{t}{t_o}\right)^{-0.053}$$

where

- $\tau_{cr}$ = critical shear strength
- $t_o$ = time to failure in a standard test (1 min)

Thus, if the time to failure is about 3 hours, the vane shear strength should be multiplied by a factor of 0.9 to obtain $\tau_{cr}$.

$$\tau_{cr} = 47 \times 0.9 = 42.30 \text{ kPa}$$

Investigations also showed that the shaft resistance one month after installation was 0.9 times the undrained strength determined by the field vane test with the same time to failure, therefore:

$$\tau_{shaft} = 42.30 \times 0.9 = 38 \text{ kPa}$$
Shear modulus of the clay:

\[ G_s = 150 \times \tau_{cr} = 150 \times 42.30 = 6.3 \times 10^3 \text{ kPa} \]

\[ \frac{L}{d} = \frac{26.0}{0.34} = 76.5 \]

\[ \frac{E_p}{G_s} = \frac{3.6 \times 10^7}{6.3 \times 10^3} = 5.7 \times 10^3 \]

(According to Fig. 9.2)

then

\[ K_s = 0.27 \frac{G_s}{d} = 0.27 \frac{6.3 \times 10^3}{0.34} = 5.0 \times 10^3 \text{ kN/m}^3 \]

The bearing capacity of pile \( (P_f) \) can be calculated as:

\[ P_f = \tau_{shaft} \times \Theta \times L \]

\( \Theta = \) perimeter of the pile

\[ P_f = 38 \times \pi \times 0.34 \times 26 = 1055 \text{ kN} \]

The displacement of the pile at a load equal to \( \frac{1}{2.5} \) the bearing capacity \( \delta_f/2.5 \) will then be:

\[ \delta_f/2.5 = \frac{\tau_{shaft}}{K_s} = \frac{P}{\Theta \times L} = \frac{1}{K_s} = \frac{P}{\Theta \times L K_s} \]

\[ \delta_f/2.5 = \frac{1055}{2.5 \times 0.34 \times 26 \times 5 \times 10^3} = \frac{1055}{347.15 \times 10^3} = 3.04 \times 10^{-3} \text{ m} \]

According to the Fig. 9.3, the displacement at failure \( \delta_f \) will be

\[ \frac{\tau}{C_a} = \frac{1}{2.5} = 0.4 \Rightarrow \frac{\delta}{\delta_f} = 0.19 \Rightarrow \delta_f = \frac{\delta}{0.19} = \frac{3.04 \times 10^{-3}}{0.19} = 16 \times 10^{-3} \text{ m} \]
Fig. 2.1: Data for the pile and soil used in the example 9
Example 10

A pile group of 9 friction concrete piles is loaded by the point load \( Q = 2500 \text{ kN} \), acting 0.25 m from the center, see Fig. 10.1. Calculate required stop driving criteria of the most loaded piles. The piles are driven through a 4 m compressible layer and into firm non-cohesive soil. An overload of 20 kPa will be placed outside the pile cap, and will cause negative skin friction. The friction is supposed to be \( \frac{2}{3} \) of the shear strength of the clay. By the piling a free falling hammer will fall 40 cm. The density of the pile is 2.4 \( \text{t/m}^3 \). The dead weight of the pile cap can be neglected. The codes must be followed. Safety factor is 3. Young's modulus of the piles \( E = 3.10^7 \text{ kPa} \).

Solution
The transformation of the load $Q$ to the center of the pile cap will produce a moment and the equilibrium equation can be written:

$$Q \times 0.25 = R_1 \times 1.5 + R_2 \times 1.5 \quad \text{(See Fig. 4C.2)}$$

As $R_1 = R_2$ (the couple of load) therefore the total additional compression load is

$$R = \frac{Q \times 0.25}{3}$$

The additional load due to negative skin friction:

$$P_{nsf} = \tau \times \theta \times L$$

$$P_{nsf} = \frac{2}{3} \times 10 \times 4 \times 0.3 = 32 \text{ kN}$$

The total load for the most loaded pile is:

$$P_{act} = 247 + 32 = 380 \text{ kN}$$

$380 \text{ kN} < 450 \text{ kN}$, where $450 \text{ kN}$ is the maximum allowable load for concrete piles

The ultimate bearing capacity $P_u$ is calculated by (4.5.5 SBN).

$$P_u = 0.8 \times h \times Q_r \left(1 - 0.1 \frac{Q_p}{Q_r}\right) / (e+c/s)$$

where

- $P_u = 3 P_{act}$ \quad \text{(FS=3.0)}$
- $h = 0.4 \text{ m}$
- $Q_r = \text{weight of hammer, assumed equal 30 kN}$
- $Q_p = \text{weight of pile}$
- $Q_p = \frac{C}{4} \times L \times A = 2.4 \times 10 \times 10 \times 0.3 \times 0.3 = 21.6 \text{ kN}$
- $P_{le} = \frac{P_u}{P_{le}}$
- $c = \frac{A}{P} + 0$ \quad \text{(no follower)}$
- $1e = \frac{Q_r}{Q_p} = \frac{30}{2.4 \times 1.0 \times 10.0 \times 0.3} = 13.9 \text{ m}$
- $c = \frac{3 \times 280 \times 13.9}{0.3 \times 3.10^7} = 0.0059 \text{ m}$
a) original system of load

b) Equivalent system of load

Fig 10.2
Substituting the values in the formula of $P_u$

$$3 \times 380 = 0.8 \times 30 \times (\frac{0.1 \times 21.6}{30}) / (e + 0.0059/2)$$

After the solution:

- $e = 0.0048$ m
- $e \approx 5$ mm

Conclusion:
The design load of 380 kN requires less than 5 mm of remaining penetration for the last blow.
Example 11

Calculate the maximum horizontal load that the pile group in Fig. 11.1 can take. The piles have a diameter of 0.3 m, length of 5 m and a failure moment of 50 kNm. The piles are flexibly coupled to the cap. The soil is non-cohesive with $\phi' = 36^\circ$ and $\rho = 1.8 \text{ t/m}^3$. The ground water table is deep. The lateral resistance can be calculated according to Broms' method. Soil pressure onto the pile cap is neglected.

Solution:

![Diagram of pile group and ground levels](image)

Fig. 11.1

The failure of a laterally loaded pile takes place either when the maximum bending moment in the loaded pile reaches the ultimate resistance or when the lateral earth pressure reaches the ultimate lateral resistance of the soil along the total length of the pile.

a) Suppose that the limit condition is the failure moment of the pile:

The failure takes place when the maximum bending moment exceeds the yield resistance of the pile section and a plastic hinge forms at the section of maximum bending moment. Suppose that this point is situated at the depth $t$, (see Fig. 11.2).
The soil reaction at this depth can be calculated by:

\[ \sigma_p = 3K_p \gamma y t \]

where

- \( K_p = \tan^2 (45^\circ + \phi) \)
- \( K_p = \tan^2 (45^\circ + \frac{36^\circ}{2}) = 3.85 \)
- \( b \) = diameter of the pile
- \( t \) = depth of the plastic hinge

Substituting the values

\[ \sigma_p = 3 \times 3.85 \times 10 \times 1.8 \times 0.3 \times t \]
\[ \sigma_p = 62.5 \text{ t kN/m} \]

Take the resultant moment at the depth \( t \):

\[ 62.5t \times \frac{t}{2} \times \frac{2}{3}t - 50 = 0 \]
\[ t = 1.34 \text{ m} \]
The projection of horizontal forces:

\[ 62.5t \times \frac{t}{2} - P_{\text{top}} = 0 \]

\[ P_{\text{top}} = \frac{62.5 \times 1.34^2}{2} = 56 \text{ kN} \]

b) Calculate the force \( P_{\text{top}} \) considering the limit condition is the lateral reaction of the soil:

The soil reaction at the tip of the pile:

\[ \sigma_p = 3 \frac{K_p \gamma LD}{L} = 3 \times 3.85 \times 1.8 \times 10 \times 5 \times 0.3 \]

\[ \sigma_p = 312 \text{ kN/m} \]

The diagram of the force and soil reaction is shown in Fig. 11.3. Moment equilibrium at the pile tip gives:

\[ P_{\text{top}} \times L = \frac{1}{2} \times L \times 312 \times \frac{L}{3} \]

\[ P_{\text{top}} \times 5 = \frac{1}{2} \times 5 \times 312 \times \frac{5}{3} \quad P_{\text{top}} = 260 \text{ kN} \]

The limit condition is the a) failure moment of the pile. Therefore \( P_{\text{top}} = 56 \text{ kN} \). As the group consists of 4 piles, then:

\[ H = 4P = 4 \times 56 = 224 \text{ kN} \]

c) Check that piles can take the actual tension force.

Calculate the vertical tension force. The horizontal force \( H \) produces the moment \( M = H \times 1 \) at the bottom of the pile cap. This moment will be taken by a couple of vertical loads, see Fig. 11.3.

The equation will be written:

\[ V = \frac{H \times 1}{3 \times 2} = 38 \text{ kN} \]

where 3 is the spacing of the pile and 2 is the number of the pile in every row of piles.
The weight of one pile = \( \frac{\pi x 0.3^2}{4} \times 5 \times 2.4 \times 10 = 8.5 \text{ kN} \)

The weight of the cap pile = \( 5 \times 5 \times 1 \times 2.4 \times 10 = 600 \text{ kN} \)

Then, the vertical load on every pile due to the weight of the cap pile is 150 kN. The horizontal force \( P_{\text{top}} \) also makes contribution to resist the tension force and may be calculated by:

\[
P_f = P_{\text{top}} \tan \delta
\]

where

\[
\delta = \frac{3\phi}{4} = 27^\circ
\]

then \( p_f = 56 \times \tan 27^\circ = 28.5 \text{ kN} \).

As the resistance force is greater than the tension force, the tension piles will not be withdrawn.
Example 12.

Calculate the safety factor to failure of the quadratic footing in Fig. 12.1. The footing is loaded by the vertical load \( V = 200 \) kN and the horizontal load \( H = 60 \) kN. The soil is clay with a density of \( \rho = 1.6 \) t/m\(^3\) and shear strength \( \tau_{fu} = 26 \) kPa. The ground water table is 0.7 m below the footing.

Solution

Ultimate bearing capacity is determined by the formula:

\[
q_u = c N_c N_d i_c + q_f D N_q \gamma_d i_q + q_d B \gamma_y d_y i_y
\]

For clay \( \phi = 0 \) and \( N_c = 5.7; N_q = 1; \) and \( \gamma_y = 0 \).

Shape factors:

\[
\gamma_q = 1 + 0.2 \frac{B}{L} = 1 + 0.2 \times 1 = 1.2
\]

\[
\gamma_c = 1 + 0.2 \frac{B}{L} = 1 + 0.2 \times 1 = 1.2
\]

Effect of the overburden (depth factor)

\[
d_q = d_c = 1 + 0.35 \frac{D}{b} = 1 + 0.35 \times \frac{1}{2} = 1.18
\]

Inclination factors:

\[
i_q = \left( 1 - \frac{0.7 \times H}{V} \right)^3 = \left( 1 - \frac{0.7 \times 60}{200} \right)^3 = 0.43
\]

\[
i_c = \frac{1}{2} + \frac{1}{2} \sqrt{\frac{H}{B.E.L.}} = \frac{1}{2} + \frac{1}{2} \sqrt{\frac{60}{2.2.26}} = 0.8
\]
Substituting the values:

\[ q_f = 26 \cdot 5.7 \cdot 1.2 \cdot 1.18 \cdot 0.83 + 10 \cdot 1.6 \cdot 1.0 \cdot 1.2 \cdot 1.18 \cdot 0.49 = \]

\[ = 174 + 11 = 185 \text{ kPa} \]

\[ q_{act} = \frac{V}{A} = \frac{200}{2 \cdot 2} = 50 \text{ kPa} \]

\[ F = \frac{q_f}{q_{act}} = \frac{185}{50} = 3.7 \]

The safety factor is equal to 3.7.
Example 13.

Calculate the permissible load for the rectangular footing in Fig. 13.1. The soil is clay with $\rho = 1.7 \text{ t/m}^3$ and $\tau_{fu} = 45 \text{ kPa}$. The ground water table is 0.7 m below the ground surface. The foundation depth is 1.5 m. The safety factor is chosen to $FS = 3$.

Substituting the values:

\[
q_f = 45 \cdot 5.7 \cdot 1.15 \cdot 1.48 \cdot 1 + 10 \cdot 1.7 \cdot 1.5 \cdot 1.15 \cdot 1.48 \cdot 1
\]

$q_f = 480 \text{ kPa}$

$Q_f = q_f \cdot A = 480 \cdot 1.1 \cdot 1.5 = 792 \text{ kN}$

$Q_{allow} = \frac{Q_f}{3} = \frac{792}{3} = 264 \text{ kN}$
Example 14.

A strip footing with width 2.2 m will be vertically loaded by 70 kN/m and horizontally loaded by a triangular load, see Fig. 14.1. The soil is sand with $\phi' = 32^\circ$, $\rho_d = 1.85$ t/m$^3$ and $\rho' = 1.15$ t/m$^3$. The ground water table is in the level of the ground surface. Calculate the safety factor.

\[ H = \frac{1}{2} \times 20 \times 1.8 = 18 \text{ kN/m} \]

The horizontal force $H$ produces a moment at the bottom of the footing and is equal to:

\[ M = H z = 18 \times 0.6 = 10.80 \text{ kNm} \]

The excentricity:

\[ e = \frac{M}{V} = \frac{10.80}{70} = 0.15 \text{ m} \]

(see Fig. 14.2)

The effective width:

\[ B = B_o - 2e = 2.2 - 0.15 = 1.90 \text{ m} \]

The ultimate bearing capacity:

\[ q_f = \frac{q \rho' B_L}{g N_f \nu_f d_f i_f} \]

shape factor $\nu_f = 1$, $0.4 B_L / L = 1$ because $L \gg \infty$
depth factor $d_f = 1$

inclination factor $i_f = \left(1 - \frac{H}{V}\right)^3 = \left(1 - \frac{18}{70}\right)^3 = 0.41$
Fig. 14.2
Suppose that $F = 1$: $N_\rho = 8.20$ and $\phi' = 23^\circ$.

As the real value of $\phi'$ is $32^\circ$, therefore the safety factor expressed in terms of the shear strength of the soil ($\tan \phi$) is:

$$F = \frac{\tan 32^\circ}{\tan 23^\circ} = 1.47$$

On the other hand the safety factor can be expressed by terms of load. In this case:

$$F = \frac{N_\rho(32^\circ)}{N_\rho(23^\circ)} = \frac{28}{8.2} = 3.4$$

The difference in the two safety factors depends on the function of $N_\rho = f(\phi)$ and $\tan \phi$. 

\[ q_f = 10.15 \cdot 4.90 N_\rho \cdot 4.1 \cdot 0.44 = 4.48 N_\rho \]

\[ Q_f = q_f BL = 4.48 N_\rho \cdot 9.1 = 8.51 N_\rho \]

\[ A_s = V = 70 \therefore 70. F = Q_f = 8.51 N_\rho \]

\[ F = \frac{8.51 N_\rho}{70} \]
Example 15.

A long retaining wall of concrete with a height of 3.0 m and a width of 2 m is founded on concrete piles 0.3 x 0.3 m. The piles are stiffly coupled to the wall, which is supposed to be completely smooth. The acting load is supposed to be taken by the piles. The ground water table is in the ground surface. Behind the wall, water is pumped away. The clay has $\rho = 1.6 \text{ t/m}^3$, and $\tau_{fu} = 20 \text{ kPa}$.

The required safety factor $FS = 1.3$ is applied on the shear strength of the clay and the failure moment of the piles $M_{\text{failure}} = 65 \text{ kN m}$. The density of the concrete is $2.4 \text{ t/m}^3$. Calculate the length of the piles and the spacing between the piles.

Solution

given $\tau_{fu} = 20 \text{ kPa}, FS = 3: \tau_{allow} = \frac{20}{1.3} = 15 \text{ kPa}$

$M_{\text{failure}} = 65 \text{ kN m}, FS = 3: M_{\text{allow}} = \frac{65}{1.3} = 50 \text{ kN m}$

1. Determination of the load acting on the pile group:
   1.1 Vertical load
   Vertical load due to the weight of the wall:
   
   $A_{pg} = 2 \cdot 3 \cdot 10 \cdot 2.4 = 144 \text{ kN/m}$

   surcharge load $= qB = 10 \cdot 2 = 20 \text{ kN/m}$

   Total vertical load = 164 kN/m.

   1.2 Horizontal load due to earth pressure:
   
   $pa = K_a q + K_a \rho gz - 2c \cdot K_{ac}$

   $A_s \phi = 0 \quad K_a = 1$

   $K_{ac} = \sqrt{1 + \frac{2}{3} \frac{r}{r}} \quad \text{or} \quad \sqrt{1 + \frac{2}{3} \frac{r}{r}}$
Clay

\( \rho = 1.6 \text{ t/m}^3 \)

\( q = 20 \text{ kPa} \)

\( \phi = 0.3 \text{ m} \)

\( M_{\text{failure}} = 65 \text{ kN} \cdot \text{m} \)

Fig. 15.4
where $r$ is the factor due to the friction between the soil and the structure, and $a$ is the adhesion between the soil and the wall, suppose $r = a = 0$ in this case and the active pressure is evaluated at $z = 0$ 
$p_a = 10 - 2 \cdot 15 = \text{20 kPa}$
$z = 3$ 
$p_a = 10 + 10 \cdot 1.6 \cdot 3 - 30 = \text{28 kPa}$

As the effect of active earth pressure is small in this case, it is not considered in the analysis.

**Horizontal force due to the water pressure:**

$$p_w = g \cdot p_w \cdot z$$

$z = 0$ 
$p_w = 0$

$z = 3$ 
$p_w = 15 \cdot 3 = 30 \text{ kPa}$

Horizontal force due to $p_w = 15 \cdot 3 = \text{45 kPa}$

**Horizontal force acting on the wall:** 
$5 \cdot 3 = \text{15 kN/m}$

**Total horizontal force:** 
$15 + 45 = \text{60 kN/m}$

**1.3 Moment:** the moment due to the horizontal force

$$M = p_w \cdot 1 + p_h \cdot 1.5 = 45 \cdot 1 + 15 \cdot 1.5 = \text{67.5 kNm/m}$$

2. **Diagram of the force acting on the structure**

Fig. 15.2 shows the total load acting on the structure and on every pile. Note that by the effect of the moment, a couple of forces are produced at the right and the left row of the wall.

3. **Determination of the length and the spacing of the piles to support the vertical load of the structure:**

As $0.8 \cdot 1 \cdot \theta = V$, where $\theta$ is the perimeter of the pile and $l$ is the length of the pile

then 
$$l_e = \frac{130}{0.8 \cdot 15 \cdot 0.3 \cdot 4}$$

$$l_r = 9.1 \text{ m/m}$$

and 
$$l_f = \frac{34}{0.8 \cdot 15 \cdot 0.3 \cdot 4} = 2.4 \text{ m/m}$$

$l_r', l_f$ are the lengths of the pile for every meter of the wall.
Fig. 45.2. System of loads
4. Determination of the length of the piles to support the horizontal load of structure (Broms's method, 1964)

The load, the deflection, the soil reaction and the bending moment are shown in Fig. 15.3 where the intensity of the soil pressure is calculated by

\[ p = 2 \cdot T_B = 9 \cdot 15 \cdot 0.3 = 40.5 \text{ kN/m} \]

Note: According to Broms, it has been assumed that the lateral soil reaction is equal to zero to a depth of 1.5 D and equal to 9 \( T_D \) below this depth.

At failure, the restraining moment at the head of the pile is equal to the ultimate moment resistance of the pile section. The maximum moment occurs at the level where the total shear force in the pile is equal to zero at a depth 1 (see Fig. 15.4 a).

Equilibrium equation of moment at the head of the pile:

\[ 40.5 \left( l - 0.45 \right) \left( \frac{l - 0.45}{2} + 0.45 \right) = 2 \cdot 50 \]

\[ l^2 - 0.45^2 = \frac{2 \cdot 2 \cdot 50}{40.5} = 4.94 \]

\[ l^2 = 5.40 \quad \therefore \quad l = 2.26 \]

The part of the pile with the length \( \Delta l \) (located below the point of maximum bending moment) can be evaluated by taking the moment with the tip pile (see Fig. 15.4 b).

\[ 50 = -40.5 \cdot \frac{\Delta l^2}{8} + 40.5 \cdot \frac{\Delta l}{2} \left( \frac{\Delta l}{2} + \frac{\Delta l}{4} \right) \]

\[ 3 \frac{\Delta l^2}{8} - \frac{\Delta l^2}{8} = 50 \]

\[ \Delta l = 4.94 \quad \therefore \quad \Delta l = 2.20 \text{ m} \]

Minimum length of the pile \( l + \Delta l \)

\[ 2.26 + 2.20 \approx 4.5 \text{ m} \]

The allowable horizontal load

\[ H_{allow} = 40.5(1-0.45) = 40.5(2.3-0.43) = 75 \text{ kN/pile} \]
Fig. 45.3

Deflection, soil reaction, bending moment

Fig. 45.4

M = 50 kNm

\[ \Delta l / \Delta l' \]

\[ \Delta l / \Delta l' \]
5. The spacing between the piles:

The length of the pile is 4.5 m, so the spacing of the pile at the left row:

\[ c = \frac{4.5}{2.4} = 1.9 \text{ m} \]

at the right row:

\[ c = \frac{4.5}{9.1} = 0.5 \text{ m} \]

As the Building Code requires that \( c/c = 5-6 \text{ d or 1.7 m} \) in this case, it is assumed that \( c/c = 1.7 \text{ m} \), the length of the pile at the right row equal to:

\[ l_r = 9.1 \times 1.7 = 15.5 \text{ m} \]

Conclusion

At the left row \( l = 4.5 \text{ m}, c/c = 1.9 \text{ m} \)
At the right row \( l = 15.5 \text{ m}, c/c = 1.7 \)

6. Comments

The length of the pile can be determined by the following procedure

a) Determination of \( l = 1.5D + f \) (the depth of maximum positive moment) \( M_{\text{max}}^{\text{pos}} = p(1.5D+0.5f)-M_{\text{max}}^{\text{neg}} \)

where \( M_{\text{max}}^{\text{pos}} \) = maximum moment occurring at a depth \((1.5D+f)\)
equal to the ultimate moment resistance of the pile section
\( M_{\text{max}}^{\text{neg}} \) = the restraining moment at the head of the pile
\( D \) = diameter of the pile
\( p \) = maximum horizontal load at the head of the pile, and can be calculated by \( p = 9 \tau D f \).

At failure the restraining moment is equal to \( M_{\text{max}}^{\text{pos}} \) and

\[ 2M = 9 \tau D f (1.5D+0.5f) \]

and after the transformation

\[ D \tau f^2 + 3D^2 \tau f - M = 0 \]
b) The part of the pile $\Delta l$ below the point of maximum moment:

$$M_{\text{max}}^{\text{pos}} = 2.24 D \tau \Delta l^2$$

And the length of the pile is equal to

$$L = 1.5 D + f + \Delta l$$