A median voter model of health insurance with ex post moral hazard*

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Abstract

One of the main features of health insurance is moral hazard, as defined by Pauly (1968); people face incentives for excess utilization of medical care since they do not pay the full marginal cost for provision. To mitigate the moral hazard problem, a coinsurance can be included in the insurance contract.

We analyze under what conditions there is a conflict between individuals on what coinsurance rate should be set with public health insurance, and we establish conditions for a median-voter equilibrium. Then we allow the public insurance to be supplemented with private insurance, and we establish conditions under which public provision will lead to larger aggregate spending than private provision does.

Keywords: health insurance; moral hazard; public provision; median voter

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1 Introduction

No matter what system for the provision of health care—private or public—a country has opted for, the consumer pays only a small part of the total cost out-of-pocket at the occasion of consumption. While insurance premiums pay for the bulk of the cost in a private system, tax receipts are used if provision is public. But irrespective of how health care is financed, we have to deal with the fact that once people have fallen ill they face incentives to consume more than optimal health care, since they do not have to pay the full marginal cost for the care they utilize. This is in the health economics literature referred to as moral hazard (Pauly, 1968). Or sometimes as ex post moral hazard (Zweifel and Breyer, 1997) to stress the fact that it is something arising after the bad state has occurred—as opposed to ordinary moral hazard which is a change in behavior before the actual accident.

The problem of ex post moral hazard has attracted a lot of attention in conjunction with private health insurance. And in Feldstein (1973) and Feldman and Dowd (1991) it is shown that it is not just a problem of theoretical interest, but also of substantial empirical relevance.

The usual way of mitigating moral hazard is to require patients to pay some part of the costs out-of-pocket, i.e. to include a coinsurance in the insurance contract. The larger the part paid out-of-pocket (the higher the coinsurance rate) the less excess utilization of medical care. On the other hand, the higher the coinsurance rate the less risk reduction. So, there is an inherent conflict between reducing excess utilization and reducing risk when deciding on the coinsurance rate.1 The optimal coinsurance rate makes an ideal trade-off between minimizing deadweight losses and reducing risk.

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1It is in the interest of the buyer of insurance to reduce overconsumption since the premium will depend on the expected costs for the buyer’s medical care.
Not everybody will want the same coinsurance rate since people differ in how they want to strike the balance. With private health insurance the market can offer buyers different contracts, so that people preferring a lower coinsurance have to pay higher premiums. This is generally not the case when health insurance is fully tax-funded. Then people cannot choose how much to pay and get a coinsurance in accordance with their contribution, but instead one contract applies to everyone.

It is quite different to have a uniform coinsurance rate determined in a political process, than having different rates varying in accordance with one’s preferences. It will, for instance, have different consequences for efficiency and distribution. Our first objective is to provide insights on what factors cause individuals to have different preferences over policy alternatives: Under what conditions is there a conflict in society on what coinsurance rate should be set? A second objective is to give conditions for the existence of a median voter equilibrium. Having done that, we compare the median voter equilibrium coinsurance rate with the rates people choose with private actuarial insurance. Further, we allow the public insurance to be supplemented with private insurance. Then we answer two questions: Who will buy the extra coverage? And how does the coinsurance rates people now face compare with the rates chosen with pure private provision?

To realize the importance of what coinsurance is set, one should keep in mind that what rate is set will affect how much health care people will consume. And in the end this will determine the aggregate level of health care expenditures in the economy. It is therefore interesting to analyze what system, public or private, render the highest aggregate spending on medical services. This is also done in the paper.

An objection to the claim that coinsurance rates determine demand could
be that many types of medical services are rationed, so that people cannot choose to consume as much as they want to. But some types of care are easier to ration than others. Care that is labor intensive can be rationed by restricting the supply of doctors and nurses creating waiting lists for surgery, for instance (see Besley et al, 1999, for an analysis of the importance of waiting lists as devices for rationing). But waiting lists is a rationing device not available for all types of care. Pharmaceutical drugs, for instance, only requires a prescription to be filled out by a physician—not a very time consuming procedure—and then the patient can treat himself at home. An indication of the problem with rationing pharmaceutical drugs is that Medicare does not cover ambulatory drugs for the fear of moral hazard (Schweitzer, 1997).

So, although rationing is an important aspect of health care we do believe that there are enough instances where rationing does not take place—or is so incomplete—to merit analysis where care is not rationed.

This paper is most closely related to two separate research lines. One concerns positive analysis of public provision of private goods in general (Usher, 1977, Epple and Romano, 1994). In these papers it is assumed that the good is provided uniformly to everybody. Wilson and Katz (1983), however, ana-

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2This is probably more common when provision is public rather than private. In fact, the combination of letting richer people pay a larger share of the health care bill by tax funding while at the same time restricting everybody to consume more or less the same amount, is probably an important political rationale to why public provision is so common (see Besley and Gouveia, 1994, for a discussion).

3A further indication of the problems with rationing drugs is the dramatic increase in expenditures on drugs that as taken place in most OECD countries during the last decade, eating into health care budgets. That rationing is harder with drugs than other types of care and demand have to be restricted in some other way, is also indicated by the fact that coinsurance rates are usually much higher for drugs than other types of care.
lyzes what should characterize a non-rationed good that a political majority finds beneficial to subsidize. Further, they analyze what the level of subsidization chosen by the majority depends upon. One conclusion is that goods with large compensated price elasticities are bad targets, because subsidies lead to too much wasteful consumption, i.e. large deadweight losses. The two examples of goods they mention as actually being subsidized are education and health care.

Deciding on a level for subsidization or setting a coinsurance rate could seem like two sides of the same coin. But then we forget that even without public intervention there will be “subsidization” of health care at the time of consumption. So, the situation to compare the outcome under public provision with is not one of zero deadweight losses since there will be deadweight losses even with private provision.

Another way of putting this is that the provision of health care should be analyzed in expected utility terms and what the government provides should be considered as insurance. This is done in the second line of research this paper is closely related to: the literature on public provision of health insurance. Breyer (1991, 2000) does this from a normative perspective, while Gouveia (1997) does it from a positive perspective in a voting model. Gouveia, however, abstract from moral hazard, and everybody consumes the same amount of health care.

The rest of the paper is organized as follows; in Section 2 the theoretical model is introduced and some results for publicly provided health insurance is presented. Further the conditions for a political equilibrium are discussed. In Section 3 we compare public with private insurance, and in Section 4 we allow the public insurance to be supplemented with private insurance. In Section 5 we analyze whether aggregate medical care utilization will be
larger with public or private provision of health insurance. Finally, Section 6 concludes and summarizes.

2 Tax financed health insurance

2.1 The Model

In this section we analyze publicly provided health care insurance. In the economy there are \( n \) individuals who differ only in endowed income, \( y_i \). For the individuals there are two possible states of the world: ill, \( I \), and well, \( W \). They all face probability \( p \) of becoming ill, and therefore they are well with probability \( (1 - p) \). So \( p \) is exogenous and constant between individuals. The individuals can consume two types of goods, \( c \), non-health goods, and \( m \), a composite health good. The prices of both goods are normalized to one.

In case of good health utility depends only on general consumption \( U^W(c) \). An individual that has fallen ill gets utility both from general consumption and from medical care, and preferences are given by the separable utility function \( U^I(c, m) \) such that \( U'_c, U'_m > 0, U''_{cc}, U''_{mm} < 0 \), and \( U''_{cm} = U''_{mc} = 0 \). By consuming medical care one’s utility thus increases but full health is never recovered.

The government provides health insurance that reimburses the individual for part of his medical expenses. The patient’s out-of-pocket payment will equal \( \beta m_i \), where \( \beta \) is the coinsurance rate. The health insurance is financed from a proportional income tax. The total tax payment for the individual is \( T^i = ty_i \), where \( t \) is the tax rate and \( y_i \) is endowed income for individual \( i \). There are no other public expenditures, so tax receipts are used solely for providing health insurance.

We assume that the government balances the budget in expected terms,
so tax revenues should equal the government’s expected costs for medical care:

\[ \sum_i T^i = t \sum_i y_i = p(1 - \beta) \sum_i m_i. \]  

(1)

The tax rate \( t \) needed to raise enough revenue can now be expressed in terms of mean income, \( \bar{y} \), and mean medical care expenditures, \( \bar{m} \):

\[ t = p(1 - \beta)\bar{m}\frac{1}{\bar{y}}. \]  

(2)

And individual \( i \)'s tax payment will hence be:

\[ T^i = p(1 - \beta)\bar{m}\frac{y_i}{\bar{y}}. \]  

(3)

It is instructive to compare the tax payments for different individuals with the premiums they would have to pay if health insurance was instead purchased on the market. The actuarially fair insurance premium, \( q_i \), equals the expected claim:

\[ q_i = p(1 - \beta)m_i. \]  

(4)

All individuals with \( m_i > \bar{m}y_i/\bar{y} \) will have their health insurance subsidized if it is publicly provided and financed from a proportional tax on income. Who will be subsidized depends on the income elasticity of medical care. If the income elasticity equals one, everybody spends the same share of income on medical care, so that the ratio \( m_i/y_i \) is constant. From this follows that \( m_i = \bar{m}y_i/\bar{y} \) and, hence, everybody’s tax payments equals their actuarial insurance premium. If the income elasticity is less (larger) than one individuals with below (above) mean income are subsidized.

The objective in this subsection is to study how the desired coinsurance rate varies between individuals. Each individual has to make two decisions. In the first stage, they decide which rate of coinsurance they prefer, and in the second stage they decide what amount of medical care to consume in case of
illness. When making their decision in stage one, they will consider how much medical care they will utilize in stage two depending on the coinsurance rate. So before we are able to derive an expression defining the optimal coinsurance, we have to derive an expression for optimal expenditure on medical care given \( \beta \).

The budget constraint facing the ill individual is

\[
y_i = c_i + \beta m_i + T_i.
\]

Substituting the budget constraint for \( c_i \) in the utility function, we can write the maximization problem as

\[
\max_m U^I(y_i - \beta m_i - T_i, m_i).
\]

The first order condition for this problem is

\[
\frac{U^I_y}{U^I_m} = \beta,
\]

which implicitly defines \( m \) as a function of \( \beta \) and \( y_i \) (as well as \( t \), but we suppress \( t \) in the following). By differentiating (7) with respect to \( m, \beta \) and \( y \) it can be shown that \( dm/d\beta < 0 \) and \( dm/dy > 0 \).

To determine the optimal coinsurance, we maximize expected utility with respect to \( \beta \)—taking the dependency of \( m_i \) and \( \overline{m} \) on \( \beta \) into account. The tax payment \( T_i = p(1 - \beta)\overline{m} \frac{y_i}{\overline{y}} \) is now endogenous, so the budget constraint when ill is

\[
y_i = c^I_i + \beta m(\beta, y_i) + p(1 - \beta)\overline{m}(\beta, \overline{y}) \frac{y_i}{\overline{y}}
\]

and when well

\[
y_i = c^W_i + p(1 - \beta)\overline{m}(\beta, \overline{y}) \frac{y_i}{\overline{y}}.
\]
Substituting this into the utility functions we get the following maximization problem for each individual to solve:

$$\max_{\beta} EV_i = pV^I(\,y_i - \beta m(\beta, y_i) - p(1 - \beta)\overline{m}(\beta, \overline{y})\frac{y_i}{\overline{y}}, m(\beta, y_i)) + (10)$$

$$(1 - p)V^W(\,y_i - p(1 - \beta)\overline{m}(\beta, \overline{y})\frac{y_i}{\overline{y}})$$

where $V^I$ and $V^W$ are the indirect utility functions (utility given that $m$ is chosen optimally) when ill and well, respectively. The first order condition for the problem is

$$\frac{V^W}{V^I} = \frac{p m_i}{1 - p} \left( \frac{m_i}{pm - p(1 - \beta)\overline{m} \frac{y_i}{\overline{y}}} - 1 \right).$$

Expression (11) defines the optimal $\beta$ implicitly.

Our main interest in this section is to analyze under what conditions there is a conflict between individuals on the (uniform) coinsurance rate in the publicly provided health insurance. To see how the optimal coinsurance rate varies between individuals we differentiate the first order condition with respect to $\beta$ and $y$ and obtain (for details see Appendix A.1)

$$\frac{d\beta^*}{dy} = \frac{1}{V^I} \left\{ \frac{\Omega_i}{c_i^j} \left[ \rho_i^I \left( \frac{C^W}{m_i} - \beta \eta \right) - \rho_i^W \left( \frac{C^W}{m_i} - \beta \right) \right] + 1 - \eta \right\} \frac{S.O.C.}{(12)}$$

where $\eta$ is the income elasticity, $\rho_i^j = -\frac{V_i}{V^j} \frac{\partial c_i^j}{\partial \beta}$, $j = I, W$ is the relative risk aversion, and $\Omega_i = m_i - p\overline{m} \frac{y_i}{\overline{y}} + p (1 - \beta)\overline{m} \frac{y_i}{\overline{y}} > 0$ (we show that $\Omega > 0$ in Appendix A.1). The second order condition, $S.O.C < 0$, is assumed to be satisfied.

The sign of $d\beta^*/dy$ is ambiguous. It will depend on the size of the income elasticity and how relative risk aversion changes with income. Noteworthy, however, is that it does not depend on the individual’s price elasticity. The
price elasticity is an important factor in setting the optimal coinsurance, but with tax financed health insurance everybody base their decision on the same price elasticity—how the average demand for medical care responds to price changes. So even if the price elasticity differs between individuals, this will not imply a conflict on preferred coinsurance.

Analyzing how income elasticity and risk aversion affect the preferred coinsurance, we have the following result:

**Proposition 1** If people are constantly relatively risk averse (CRRA) and the income elasticity is equal to one, everybody wants the same coinsurance when insurance is publicly provided and financed by a proportional tax.

Proof: If $\rho^I = \rho^W$ (CRRA) expression (12) collapses to

$$
\frac{d\beta^*}{dy} = \frac{1}{V_{cI}} - \frac{(1 - \eta) \left( \frac{\rho_{mI} \rho_{yI}}{y_{cI}} + 1 \right)}{S.O.C.},
$$

where it can be seen that if $\eta = 1$ and it follows that $d\beta^*/dy = 0$.

It can also be seen that the optimal coinsurance will decrease (increase) with income if the income elasticity is smaller (larger) than one. The intuition for this result is the following: The income elasticity affects $\beta^*$ in two ways. First, the income elasticity will determine how the tax-price for health insurance varies between individuals, as discussed earlier. If $\eta < 1$, low income earners are subsidized given the proportional income tax. The lower the income, the larger the subsidy. Therefore, poorer people will want more health insurance, i.e. lower coinsurance. If $\eta > 1$ the reverse is true. When $\eta = 1$ nobody is subsidized and there is no tax-price argument for different persons wanting different coinsurance.

Second, even if there was no subsidization—and everybody paid their actuarial premium regardless of the size of the income elasticity—Proposition
1 still holds. If $\eta < 1$, the higher the income the lower will the share of income spent on medical care be, and the lower relative risk will the individual be exposed to. Hence, richer people will be exposed to lower risk relative to their income. And this will drive up their preferred coinsurance. Again, if $\eta > 1$ the argument works the other way round. When $\eta = 1$ everybody regardless of income face the same relative risk, so the risk exposure is no source of conflict regarding what coinsurance should be set.

In short, both effects work in the same direction if either $\eta < 1$ or $\eta > 1$, and neither effect is present if $\eta = 1$.

To see how the other crucial parameter, the relative risk aversion, affects the preferred coinsurance rate, assume the income elasticity is equal to one. Then we find that equation (12) may be written as

$$\frac{d\beta^*}{dy} = -\frac{1}{V_c^d} \frac{\Omega_w}{S.O.C.} \frac{\rho^I - \rho^w}{\rho^I}$$

We find that

- with increasing relative risk aversion, $\rho^W_i > \rho^I_i$, the optimal coinsurance will be lower the higher the income.

- with decreasing relative risk aversion, $\rho^W_i < \rho^I_i$, the optimal coinsurance will be higher the higher the income.

This result is quite intuitive and should need no discussion.

### 2.2 Political Equilibrium

So far we have looked at peoples’ optimal coinsurance rates. Now we will look at the equilibrium coinsurance rates—essentially whether a median-voter equilibrium exists.
We assume that decisions are taken by majority vote. Median-voter equilibria apply to policy issues where disagreement between voters is likely to be one-dimensional. Our problem is really two-dimensional, with both the tax rate $t$ and the coinsurance $\beta$ subject to choice. However, the problem will reduce to one dimension through the government budget constraint and we can therefore treat $\beta$ as the only choice-variable.

We will rely on the single-crossing condition introduced by Gans and Smart (1996) to check the existence of a median voter equilibrium. If underlying preferences are defined over a two-dimensional real choice-variable but attention can be restricted to one dimension, single-crossing can be checked with the Spence-Mirrlees condition that marginal rates of substitution should be ordered. In our application this amounts to checking if individuals’ indifference curves cross only once in $(\beta, t)$-space.

To obtain an expression for the indifference curves we differentiate expected utility with respect to $\beta$ and $t$ (and taking into account that $m$ is a function of $\beta$ and $t$):

$$
\frac{dt}{d\beta} = -\frac{pV_c^{it}}{pV_c^{it} + (1-p)V_c^{it}m_y}{m_y}.
$$

If the slope of the indifference curve can be said to either decrease or increase monotonically with $y$, the single crossing condition will be satisfied. To start with the second term, $m_i/y_i$, whether this will increase or decrease with $y$ depends solely on the income elasticity. If $\eta < 1$ it decreases and if $\eta > 1$ it increases. To see how the first term, involving the marginal utilities, changes with $y$ we take the derivative with respect to $y$. It turns out that the sign of the derivative will have the same sign as the following expression (see Appendix 2)

$$
\rho_i^W = \rho_i^I \frac{1 - t - \eta\beta m_y}{1 - t - \beta \frac{m}{y_i}}.
$$

11
Then we have the following result:

**Proposition 2** Sufficient conditions for the existence of a median-voter equilibrium are either

- \( \eta \leq 1 \) together with CRRA or DRRA, or
- \( \eta \geq 1 \) together with CRRA or IRRA.

As long as \( \eta \leq 1 \) (and assuming both numerator and denominator to be positive) and people are CRRA or DRRA, i.e. \( \rho^W \leq \rho^I \), the expression will be negative, and the first term of (15) will decrease with \( y \). At the same time \( \eta \leq 1 \) guarantees that the second term of (15) is non-increasing in \( y \). So then we know that \( dt/d\beta \) is monotone in \( y \) and single-crossing holds. (When \( \eta = 1 \) and \( \rho^W = \rho^I \), \( dt/d\beta \) is constant between individuals and indifference curves do not cross at all.)

On the other hand, if \( \eta \geq 1 \) and people are CRRA or IRRA, i.e. \( \rho^W \geq \rho^I \), the expression will be positive, and the first term of (15) will increase with \( y \). At the same time \( \eta \geq 1 \) guarantees that the second term is non-decreasing in \( y \). So again we know that the \( dt/d\beta \) is monotone in \( y \) and single-crossing holds.

Proposition 2 gives conditions for the existence of median voter equilibrium. What characterizes the equilibrium? By construction all individuals will have identical insurance contracts with the same coinsurance rate, \( \beta \). This means that some individuals typically will get more insurance than they desire, while others get less. The only person always getting the exact right amount is the median voter. For the special cases we looked at in (13) and (14) the median voter will be the individual with median income.\(^4\) This

\(^4\)The income distribution is assumed to be skewed to the right, so that the median voter will have less than mean income.
follows from the monotonicity in voters’ type. But we cannot from this con-
clude that the median voter always will be identical to the median income
earer. In the general case (12) the derivative cannot be signed, and it can
well be that it is positive over one range of incomes and negative over an-
other range. In which case the median voter will not necessarily earn median
income.

3 Comparison with private health insurance

To put the results obtained so far into perspective we will here point out
in what respects the characteristics of tax financed health insurance differs
from privately purchased health insurance concerning the choice of coinsur-
ance rate. We will assume that everybody when buying health insurance on
the market will pay their actuarially fair premium. This presupposes that
contracts can be written conditional on income: high income earners will
have to pay larger premiums than low income earners because high income
earners will demand more medical services. People maximize expected utility
with respect tot $\beta$ given the same budget constraints as in the case of public
insurance, substituting the premium in (3) for the tax in (4):

$$\max_{\beta} EV_i = pV^I(y_i - \beta m(\beta, y_i) - p(1 - \beta)m(\beta, y_i), m(\beta, y_i)) + \sum (17)$$

$$(1 - p)V^W(y_i - p(1 - \beta)m(\beta, y_i)).$$

The first order condition for $\beta^{*M}$, the optimal coinsurance with private in-
surance ($M$ = private case), is:

$$\frac{\frac{V^W_i}{V^I_i}} = \frac{p}{1 - p} \left( \frac{m_i}{pm_i - p(1 - \beta)\frac{\partial m_i}{\partial \beta}} - 1 \right).$$

An obvious difference between public and private provision is that every-
bodies get their preferred coinsurance with private but not with public health
insurance. Then there are differences that affect individuals’ optimal coinsurance rates. First, the “price” of insurance coverage is different: tax payments do not in general equal the actuarial premium. A second difference affecting optimal coinsurance rates concerns price elasticities. With private insurance people base their decision on what coinsurance to prefer on their own price elasticity. With public insurance people base their decision on how mean expenditure on medical care responds to price changes. If price elasticities are constant over individuals this will of course not matter.

The first of these two differences means that the income elasticity only matter in one way for the choice of optimal coinsurance in private insurance, and not in two ways as with public insurance (as discussed in relation to expression (13)). The “price” of insurance coverage does not change with the income elasticity. This means that if \( \eta < 1 \), everything else equal, individuals with below mean income (and a tax price below the actuarial premium) has a lower \( \beta^* \) with public than with private insurance. For individuals with above mean income (and a tax price above the actuarial premium) it is the other way round.

The second of these differences means that there is an extra element in private insurance, that causes \( \beta^{*M} \)’s to differ. The larger the (absolute value of) individual’s price elasticity the larger his \( \beta^{*M} \).

What does these differences imply for the sign of \( d\beta^{*M}/dy \)? In particular, are an income elasticity equal to one, \( \eta = 1 \), and CRRA sufficient conditions for everybody to have the same \( \beta^{*M} \), i.e. does Proposition 1 hold also for private insurance? At first sight it appears that it does not hold; that we also have to assume that price elasticities are constant over individuals. But the fact is that assuming \( \eta = 1 \) and CRRA will imply that price elasticities are constant between individuals.
To see this, differentiate the first order condition for optimal $m$ with respect to $m$, $\beta$, and $y$ and you arrive at the following relationship between income- and price-elasticity, and relative risk aversion:

$$\varepsilon_i = -\eta \left( \beta \frac{m_i}{y_i} - \frac{1}{\rho_i} \frac{c_i}{y_i} \right),$$

(19)

where $\varepsilon_i$ is the price elasticity of individual $i$. This expression tells us that we cannot vary the three parameters freely. Making assumptions for two of them will—most often—imply something particular for the third. For instance, assuming $\eta = 1$ and CRRA implies constant price elasticities.\(^5\) (The relations between income elasticity, price elasticity and relative risk aversion can be seen in Table 1.) We then have the following result.

**Proposition 3** If people are constantly relatively risk averse (CRRA) and the income elasticity is equal to one, everybody choose the same coinsurance rate if premiums are actuarially fair.

Under these conditions private insurance is identical to public insurance.

The relations in Table 1 implies that, except under the conditions in Proposition 3, the sign of $d\beta^*M/dy$ is ambiguous in all cases. This is because the three parameters pull $\beta^*M$ in different directions. Consider, for instance, the likely case of $\eta < 1$, CRRA and a price elasticity which decreases with income. The fact that the income elasticity is less than one makes richer individuals prefer a higher coinsurance rate than individuals less well off. On the other hand, rich individuals have lower price elasticities than poor individuals, which make the rich prefer a lower coinsurance rate than the

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\(^5\)To see why, notice that expression (19) should hold for every possible $\beta$, not just optimal $\beta'$s. Assume that everybody faces the same $\beta$. Also assume that $\eta = 1$. Then $m_i/y_i$ and $c_i/y_i$ are constant over individuals, and in order for the price elasticity to vary between individuals, $\rho$ has to vary.
Table 1: Relation between income elasticity, relative risk aversion and price elasticity

| Income elasticity $\eta$ | Relative risk aversion $\rho$ | How the price elasticity (absolute value of) changes with income, $|\varepsilon|$ |
|--------------------------|-------------------------------|--------------------------------------------------|
| CRRA                     | 1                             | Constant                                         |
| DRRA                     | Decreasing                    |                                                  |
| IRRA                     | Increasing                    |                                                  |
| CRRA                     | $<1$                          | Decreasing                                       |
| DRRA                     | Decreasing                    |                                                  |
| IRRA                     | Constant, Increasing or Decreasing |                                                |
| CRRA                     | $>1$                          | Increasing                                       |
| DRRA                     | Constant, Increasing or Decreasing |                                              |
| IRRA                     | Constant, Increasing or Decreasing |                                              |

poor. The net effect is uncertain. Similar reasoning can be applied for other combinations of the three parameters.

4 Supplementary private insurance

In the cases of public insurance we looked at in Section 2, expressions (13) and (14), individuals with incomes larger or smaller than the median would prefer a coinsurance rate different, from the one set by the median voter. The fact that nearly half the population get less insurance coverage than they desire leaves room for a private insurance that will meet the demand for larger insurance coverage. We will now extend the analysis in Section 2 and allow for a supplementary insurance which covers some part of the individuals’ out-of-pocket costs.\(^6\)

In this section we shall analyze how much extra coverage—how much lower coinsurance rate—will different individuals demand. In particular, we

\(^6\)Examples of such supplementary insurance are the so called “Medigap” plans available in the USA, that cover deductibles and coinsurances required by Medicare (Schweitzer, 1997).
want to answer the question: Will the coinsurance rate chosen when supplementary insurance is purchased, be higher or lower than the one chosen when insurance is purely privately provided? The answer has important implications for whether aggregate utilization of medical care is larger with public provision (with private supplementary insurance) or with pure private provision. We shall restrict attention to the case when the income elasticity is less than one, $\eta < 1$, and people are characterized by constant relative risk aversion, CRRA.\footnote{The assumption of $\eta < 1$ finds strong support in the literature, see for example (Getzen 2000).}

When buying supplementary insurance the individual takes the coverage provided by the government, $\beta^P$ ($P =$ public), as given, and then decides on how much he wants to reduce this coinsurance. We denote the reduction of the public coinsurance rate $\alpha$, so the coinsurance rate faced by the individual when having bought supplementary insurance is $\beta^S = \beta^P - \alpha$ ($S =$ supplementary). Assuming perfect competition on the private insurance market, the actuarially fair premium for the extra coverage is

$$q^S_i = p\alpha m_i = p(\beta^P - \beta^S)m_i. \quad (20)$$

When deciding on degree of supplementary coverage, $\beta^S$, the individual faces a problem similar to equations (10) and (17) in the previous sections. He maximizes the following expression:

$$\max_{\beta^S} EV_i = pV^I \left( y_i - \beta^S m(y_i, \beta^S) - p(\beta^P - \beta^S)m(y_i, \beta^S) - T_i, m(y_i, \beta^S) \right)$$

$$+ (1 - p)V^W \left( y_i - p(\beta^P - \beta^S)m(y_i, \beta^S) - T_i \right), \quad (21)$$

the second step being the same as in equation (6). The first order condition
is
\[
\frac{V_c^{W}}{V_c^{I}} = \frac{p}{1-p} \left( \frac{m_i}{pm_i + p(\beta^P - \beta^S) \frac{\partial m}{\partial \beta}} - 1 \right).
\] (22)

The only difference between expression (22) and (18) is that $\beta^P - \beta^S$ in (22) replaces $1 - \beta^M$ in (18). The difference reflects the fact that the marginal cost for extra coverage is lower when buying supplementary rather than pure private insurance.\(^8\) It arises from the fact that the lower $\beta$ induces increased medical care utilization, and this imposes higher costs on the insurer. With pure private insurance this extra cost will be fully reflected in a higher premium. While with supplementary insurance, some part of the extra costs falls on the government, so the premium for the extra coverage does not fully reflect the increase in costs. As an effect the presence of supplementary insurance will increase the government’s costs for providing health insurance.

The fact that the marginal cost for additional coverage is lower with supplementary insurance compared to pure private insurance, has implications for the choice of optimal coinsurance, $\beta^*$. Comparing (22) and (18) one can see that $V_c^{W}/V_c^{I}$ is closer to 1 in (22). This suggests that the difference between $c^W$ and $c^I$ is larger with pure private insurance, and hence it also suggests that $\beta^S < \beta^M$.

This supposition can be confirmed more formally, so we have the following result:

**Proposition 4** If $\eta < 1$ and CRRA, then $\beta^*S < \beta^*M$.

Noting that pure private insurance can be considered a special case of supplementary insurance (with $\beta^P = 1$, i.e. no public coverage), the result is proved by using expression (22) to show that $\frac{d\beta^*S}{d\beta^M} > 0$. (See Figure 1

\(^8\)It is $pm_i - p(\beta^P - \beta^S) \frac{\partial m}{\partial \beta}$ with supplementary insurance, versus $pm_i - p(1 - \beta^M) \frac{\partial m}{\partial \beta}$ with pure private insurance.
for an illustration on how co-insurances rates varies with income in different regimes.)

Will there be a change in optimal public co-insurance rates, $\beta^P$, when supplementary insurance is available? Yes. Individuals paying taxes higher than the actuarial premium will now vote for $\beta^P = 1$, since they now can buy insurance coverage at a lower cost on the market. When $\eta < 1$ and CRRA this will be everybody with $y_i > \overline{y}$. Individual’s with $y_i < \overline{y}$, on the other hand, have no reason to change their voting behavior since public provision still is cheapest.

Will the political equilibrium change when supplementary insurance is available? No. Although $\beta^P$ is not monotone in voters’ types any longer—there is a discontinuity at $\overline{y}$—the individuals are still ranked in the same order in terms of $\beta^p$. So the median voter still earns median income. And just as before everybody with $y_i > y_m$ want a higher coinsurance than the median voter, while everybody with $y_i < y_m$ want a lower coinsurance than the median voter.

5 Is aggregate expenditure larger with private or public insurance?

The total effect on aggregate spending for medical care when switching from pure private to public provision (with or without supplementary private insurance) can be separated in two effects. The first is that, given coinsurance rates, the premiums paid for private insurance does not equal tax payments for most persons, and hence net incomes (income net of premium/tax payment) differ. Tax-funding redistributes net income. Call this the income effect. The second effect is that people’s coinsurance rates change: we move
away from a situation where everybody can choose their own rate to a situation where the median voter’s rate applies to everyone (if supplementary insurance is available people can however choose a lower rate). Call this the price effect.

Just as in the previous section we will here restrict attention to the case when the income elasticity is less than one, \( \eta < 1 \), and people are characterized by constant relative risk aversion, CRRA. It follows that the price elasticity (in absolute value) must be decreasing in income (see Table 1).

Then, the income effect works unambiguously in the direction of increasing aggregate spending on medical services when switching from private to public provision. The reason being that the proportional tax in combination with an income elasticity of less than one, redistributes net income from the rich to the poor. And—also due to the income elasticity being less than one—poor people spend a larger fraction of their income on medical services than richer people do.\(^9\)

To isolate the price effect we want to see what happens with aggregate spending when coinsurance rates change while incomes net of premiums/tax payments remain the same. An individual will then increase his spending on medical care if the coinsurance rate he faces with public provision is lower than the rate he faced with private provision. The extent of the increase depends both on how much the coinsurance rates change, and how large the individual’s response to these changes are. The aggregate price effect in the economy, depends on how many individuals face lower versus higher coinsurance rates with public compared to private provision, as well as on

\(^9\)The higher spending induced by redistribution of net income when switching from private to public provision, does not mean that deadweight losses increase. Income effects do not create deadweight losses.
how much more or how much less utilization of care those changes induce.

It is hard to see how changed coinsurances affects aggregate spending when comparing pure private with pure public provision. This is because we cannot say—without making further and strong assumptions—who faces a lower and who faces a higher coinsurance rate after switching from pure private to pure public, let alone how much more or less care they utilize.

Comparing pure private insurance to public insurance added with supplementary private insurance is easier. The reason being that, with reasonable assumptions, everybody faces a lower coinsurance in the latter case. We know that \( \beta^S < \beta^M \) for everybody with \( y_i < y_m \) (Proposition 4). Everybody with \( y_i > y_m \), on the other hand, faces the same coinsurance rate, \( \beta^P_m \), which is the one preferred by the median income earner. This must be lower than \( \beta^M \) if \( \frac{d\beta^M}{dy} > 0 \).\(^{10}\)

Then we have the following result:

**Proposition 5** A sufficient condition for larger aggregate spending on medical care with public insurance (with supplementary private insurance) than with pure private insurance, is that optimal pure private coinsurance rates increase with income, i.e. \( \frac{d\beta^M}{dy} > 0 \).

The reason is that now the income and the price effect will work in the same direction: expanding demand when we switch from private to public insurance.

\(^{10}\)That \( \frac{d\beta^M}{dy} > 0 \) is not known for sure when \( \eta < 1 \) and CRRA (see discussion in Section 4).
6 Summary and conclusions

If rationing is possible there are very good reasons for a majority of the population to prefer public provision of health insurance. Although high income earners pay substantially larger taxes than low income earners everybody get to consume the same amount (quality) of medical care. Public provision is thus under these conditions a very efficient redistributive device.  

In this paper we have analyzed the situation when rationing is not possible and the problem with ex post moral hazard arises. The model does not contain voting over whether provision of health care should be public or private, but rather what coinsurance rate should be set given public provision. This way we focused on how the demand for medical services and risk reduction differs for an individual, depending on how insurance is provided.

We find that if the income elasticity is equal to one and individuals are constantly relatively risk averse, there is no conflict between individuals on what coinsurance rate should be set. This is also true if health insurance is privately provided. However, making the more reasonable assumption of an income elasticity of less than one, there is a conflict: the optimal coinsurance rate increases with income.

The way we model the political process we know that the median voter will be certain to benefit from public provision. Maintaining the assumption of an income elasticity of less than one and constant relative risk aversion, low income earners will be subsidized. But whether they will benefit from public provision depends on if the gains of subsidization outweighs the disadvantage of getting an inappropriate amount of risk reduction. For high income earn-

\[11^1\text{It has been shown that even when an optimal income tax is available, public provision of private goods can enhance efficiency, see Blomquist and Christiansen (1995) and Boadway and Marchand (1995).}\]
ers, on the other hand, there are no gains from public provision. The fact that half the population get higher coinsurance than preferred, leaves room for private supplementary insurance—reducing the actual coinsurance rate. With our assumptions we find that it will be low-income earners that will purchase supplementary private insurance. We have also shown that under very reasonable conditions aggregate spending on medical services will be larger with public insurance (added with supplementary private insurance) compared with pure private insurance.

Finally, we want to point out some limitations of our study and indicate what could be done in the future. First, it is probably too strong an assumption that no rationing at all takes place. Even for the type of services where rationing is hard, e.g. pharmaceuticals, the no-rationing assumption is quite strong. Second, in our model everybody faces the same probability of becoming sick. A more realistic model would account for the differences in probability of falling ill we know exist, and for the negative correlation between health and wealth (Smith, 1999).
Appendix

A.1 Publicly provided health insurance

To study how optimal coinsurance, $\beta$, varies with income, we differentiate the first order condition for optimal $\beta$ with respect to $\beta$ and $y$, this yields

$$0 = (1-p)V_{ic}^{iv} \left[ 1-p(1-\beta)\frac{m}{y} \right] \left[ \frac{y_i}{y} \left( \frac{m}{y} - (1-\beta)\frac{\partial m}{\partial \beta} \right) \right] dy$$  \hspace{1cm} (23)

$$+ (1-p)V_{ic}^{iv} \left[ \frac{p}{y} m \frac{1}{y} - p(1-\beta)\frac{\partial m}{\partial \beta} \frac{1}{y} \right] dy$$

$$-pV_{ic}^{ii} \left[ 1-\beta \frac{\partial m_i}{\partial y} - p(1-\beta)\frac{m}{y} \right] \left[ m_i + p\frac{y_i}{y} \left( (1-\beta)\frac{\partial m}{\partial \beta} - \frac{m}{y} \right) \right] dy$$

$$-pV_{ic}^{ii} \left[ \frac{\partial m_i}{\partial y} + p\frac{1}{y} \left( (1-\beta)\frac{\partial m}{\partial \beta} - \frac{m}{y} \right) \right] dy + (S.O.C) \frac{d\beta}{dy}$$

$$= Ady + (S.O.C) \frac{d\beta}{dy},$$

and $\frac{d\beta}{dy} = -\frac{A}{s.o.c}$. We assume the second order condition to be fulfilled, so in the following we concentrate on the sign of $A$.

Multiplying all terms in $A$ by $\frac{V_i}{V_{ic}}$ and inserting the first order condition for optimal $\beta$, we can divide the expression into two parts, one containing the indirect utility function, $G$, and one that does not, $F$. Simplify $F$:

$$\frac{F_i}{V_{ic}^{ii}} = p \left[ \frac{m_i}{y_i} - p(1-\beta)\frac{\partial m_i}{\partial y_i} \right] \frac{m_i + p\frac{y_i}{y} \left( (1-\beta)\frac{\partial m}{\partial \beta} - \frac{m}{y} \right)}{\frac{m_i}{y_i} - p(1-\beta)\frac{\partial m_i}{\partial y_i}}$$  \hspace{1cm} (24)

$$-p\frac{\partial m_i}{\partial y} + p \left[ \frac{m_i}{y_i} - p(1-\beta)\frac{\partial m_i}{\partial y_i} \right] \frac{1}{\frac{m_i}{y_i} - p(1-\beta)\frac{\partial m_i}{\partial y_i} \frac{y_i}{y} - p\frac{\partial m_i}{\partial y}}$$

$$= p \left[ \frac{m_i}{y_i} - \frac{\partial m_i}{\partial y_i} \right]$$

Simplify $G$: 
\[
\frac{G_i}{V_{cW}^{iW}} = (1 - p)pW_{cc}^{iW} \left[ 1 - p(1 - \beta) \frac{\bar{m} - (1 - \beta) \frac{\partial m}{\partial \beta}}{\bar{m}} \right] \phi \frac{\bar{m} - (1 - \beta) \frac{\partial m}{\partial \beta}}{\bar{m}} \times
\]
\[
\frac{p}{1 - p} \frac{m_i - p \frac{\eta_j}{y}}{p \frac{\eta_j}{y}} \frac{\partial m}{\partial y} \frac{\bar{m} - (1 - \beta) \frac{\partial m}{\partial \beta}}{\bar{m}} \frac{V_{cc}^{iW}}{V_{cW}^{iW}} \left[ 1 - \beta \frac{\partial m_i}{\partial y} - p(1 - \beta) \frac{\eta_j}{y} \right] \frac{m_i - p \frac{\eta_j}{y} \left( \frac{\bar{m} - (1 - \beta) \frac{\partial m}{\partial \beta}}{\bar{m}} \right)}{\Omega}
\]
\[
= p \Omega_i \left[ \Phi_i \left( \frac{V_{cc}^{iW}}{V_{cW}^{iW}} - \frac{V_{cc}^{iI}}{V_{cW}^{iI}} \right) + \beta \frac{\partial m_i}{\partial y} \frac{V_{cc}^{iI}}{V_{cW}^{iI}} \right]. \tag{25}
\]

Denoting the income elasticity \( \frac{\partial m}{\partial y} \frac{\eta_j}{m_i} = \eta \), and thus \( \frac{\partial m}{\partial y} \frac{\eta_j}{y} = \eta \frac{\eta_j}{y} \) and multiplying by \( \frac{c_i}{c_j} \), defining \( \rho^j = -\frac{V_{cc}^{iI}}{V_{cW}^{iI}} c_j, j = I, W \) as the relative risk aversion and then adding \( F \) and \( G \) together we have:

\[
\frac{G_i + F_i}{V_{cW}^{iI}} = p \frac{m_i}{y_i} \left\{ \Omega_i \left[ \rho_i \left( \frac{c_i}{m_i} - \beta \eta \right) - \rho_i \left( \frac{c_i}{m_i} - \beta \right) \right] + 1 - \eta \right\}. \tag{26}
\]

And thus we have:

\[
\frac{d \beta^*}{d y} = \frac{1}{V_{cW}^{iI}} \frac{p \frac{m_i}{y_i} \left\{ \Omega_i \left[ \rho_i \left( \frac{c_i}{m_i} - \beta \eta \right) - \rho_i \left( \frac{c_i}{m_i} - \beta \right) \right] + 1 - \eta \right\}}{S.O.C.}, \tag{27}
\]

which is expression (12) in Section 2.

To sign this expression we need to know the sign of \( \Omega_i = m_i + p \frac{\eta_j}{y} \left[ (1 - \beta) \frac{\partial m}{\partial y} - \bar{m} \right] = -\frac{\partial c_i}{\partial y} - \beta \frac{\partial m}{\partial y} \). Since we know that \( \frac{\partial m}{\partial y} < 0 \) signing \( \Omega \) amounts to signing \( \frac{\partial c_i}{\partial y} \).

Using the first order condition for \( \beta \), we can write

\[
0 < \frac{V_{cW}^{iW}}{V_{cW}^{iI}} = \frac{p \frac{m_i - p \frac{\eta_j}{y}}{y} \left[ \frac{\bar{m} - (1 - \beta) \frac{\partial m}{\partial \beta}}{\bar{m}} \right]}{1 - p \frac{p \frac{\eta_j}{y}}{y} \left[ \frac{\bar{m} - (1 - \beta) \frac{\partial m}{\partial \beta}}{\bar{m}} \right]} < 1 \tag{28}
\]
since $V^I_c > V^W_c$ with a positive $\beta$. This equation may also be written as

$$0 < \frac{p}{1-p} \left[ \frac{m_i}{p \bar{y} (\bar{m} - (1-\beta) \frac{\partial m}{\partial \beta})} - 1 \right] < 1,$$

from which it follows

$$\frac{m_i}{p \bar{y} (\bar{m} - (1-\beta) \frac{\partial m}{\partial \beta})} > 1.$$  \hspace{1cm} (30)

Now, assume that $\frac{\partial c^I}{\partial \beta} < 0$:

$$-\frac{\partial c^I}{\partial \beta} = m_i + \beta \frac{\partial m_i}{\partial \beta} - \bar{m} \frac{y_i}{\bar{y}} + p(1-\beta) \frac{\partial \bar{m}}{\partial \beta} \frac{y_i}{\bar{y}} > 0$$

$$\Rightarrow m_i - \bar{m} \frac{y_i}{\bar{y}} + p(1-\beta) \frac{\partial \bar{m}}{\partial \beta} \frac{y_i}{\bar{y}} > 0$$

$$m_i > \bar{m} \frac{y_i}{\bar{y}} - p(1-\beta) \frac{\partial \bar{m}}{\partial \beta} \frac{y_i}{\bar{y}},$$

and we know this holds from (30).

In short, $V^I_c > V^W_c$ implies $\Omega > 0$.

### A.2 The Political Equilibrium

$$EU_i = pV^I(y_i - \beta m_i - ty_i, m_i) + (1-p)V^W(y_i - ty_i)$$  \hspace{1cm} (32)

Differentiate with respect to the coinsurance rate, $\beta$, and the tax rate, $t$ gives:

$$\left[ pV^I_c (-m_i - \beta \frac{\partial m_i}{\partial \beta}) + pV^W_m \right] d\beta + \left[ pV^I_c (\beta \frac{\partial m_i}{\partial t} - y_i) + pV^W_m \frac{\partial m_i}{\partial t} - (1-p) V^W_c y_i \right] dt = 0,$$

which—by the first order condition for optimal $m$—simplifies to

$$\frac{dt}{d\beta} = -\frac{pV^I_c}{pV^I_c + (1-p)V^W_c} \frac{m_i}{y_i}.$$  \hspace{1cm} (34)
Differentiating with respect to $y$ we have

\[
\frac{dt}{d\beta} = \frac{p(1-p)V_{e}^{W}V_{e}^{I}(1-t - \beta\frac{\partial m_i}{\partial y}) - p(1-p)V_{e}^{W}V_{e}^{I}(1-t)}{(pV_{e}^{I} + (1-p)V_{e}^{W})^2} \\
= \frac{p(1-p)V_{e}^{W}V_{e}^{I} \left[ \frac{\rho_i^W 1 - t}{C_{e}^W} - \frac{\rho_i^I 1 - \beta\frac{2m_i}{W} - t}{C_{e}^I} \right]}{(pV_{e}^{I} + (1-p)V_{e}^{W})^2} \\
= \frac{p(1-p)V_{e}^{W}V_{e}^{I} \left[ \frac{\rho_i^W - \rho_i^I 1 - \beta\frac{2m_i}{W} - t}{C_{e}^I} \right]}{(pV_{e}^{I} + (1-p)V_{e}^{W})^2}.
\]

(35)

### A.3 Private Health Insurance

To study how a change in income affects the desired coinsurance rate, $\beta$, with private insurance we differentiate the first order condition for $\beta$, expression (18), with respect to $\beta$ and $y$ (index $i$ is here dropped)

\[
0 = (1-p)V_{e}^{W} \left[ 1 - p(1-\beta) \frac{\partial m_i}{\partial y} \right] \left[ p m_i - p(1-\beta) \frac{\partial m_i}{\partial \beta} \right] dy \\
+ (1-p)V_{e}^{I} \left[ p \frac{\partial m_i}{\partial y} - p(1-\beta) \frac{\partial^2 m_i}{\partial \beta \partial y} \right] dy \\
- pV_{e}^{I} \left[ 1 - \beta \frac{\partial m_i}{\partial y} - p(1-\beta) \frac{\partial m_i}{\partial \beta} \right] \left[ m_i(1-p) + p(1-\beta) \frac{\partial m_i}{\partial \beta} \right] dy \\
- pV_{e}^{I} \left[ \frac{\partial m_i}{\partial y}(1-p) + p(1-\beta) \frac{\partial^2 m_i}{\partial \beta \partial y} \right] dy + (S.O.C) d\beta \\
= Bdy + (S.O.C) d\beta
\]

and $\frac{d\beta}{dy} = -\frac{B}{S.O.C}$. We assume the second order condition to be fulfilled, so in the following we concentrate on the sign of $B$.

Multiplying by $\frac{V_{e}^{I}}{V_{e}^{W}}$ and inserting the first order condition for $\beta$ yields

\[
\frac{B_i}{V_{e}^{I}} = (1-p) V_{e}^{W} \left[ 1 - p(1-\beta) \frac{\partial m_i}{\partial y} \right] \left[ p m_i - p(1-\beta) \frac{\partial m_i}{\partial \beta} \right] \times \\
\frac{1 - p}{1 - p} \frac{m_i - pm_i + p(1-\beta) \frac{\partial m_i}{\partial \beta}}{pm_i - p(1-\beta) \frac{\partial m_i}{\partial \beta}}
\]

27
\[-p \frac{V^{t \tau}}{V^{t \tau}} \left[ 1 - \beta \frac{\partial m_i}{\partial y} - p(1 - \beta) \frac{\partial m_i}{\partial y} \right] \left[ m_i (1 - p) + p (1 - \beta) \frac{\partial m_i}{\partial \beta} \right] \]
\[+ p \left[ \frac{\partial m_i}{\partial y} - p (1 - \beta) \frac{\partial^2 m_i}{\partial \beta \partial y} \right] \frac{m_i - pm_i + p(1 - \beta) \frac{\partial m_i}{\partial \beta}}{pm_i - p(1 - \beta) \frac{\partial m_i}{\partial \beta}} \]
\[-p \left[ (1 - p) \frac{\partial m_i}{\partial y} + p (1 - \beta) \frac{\partial^2 m_i}{\partial \beta \partial y} \right].\]

We will use the following relations:

\[ \frac{\partial m_i}{\partial y} m_i = \eta \Rightarrow \frac{\partial m_i}{\partial y} = \eta \frac{m_i}{y_i} \]

(38)

\[ \frac{\partial m_i}{\partial \beta} m_i = \varepsilon_i \Rightarrow \frac{\partial m_i}{\partial \beta} = \varepsilon_i \frac{m_i}{\beta}. \]

(39)

If we allow the price elasticity to vary between individuals we have

\[ \frac{\partial^2 m_i}{\partial \beta \partial y} \]

\[ = \varepsilon_i \frac{\partial m_i}{\partial y} + \frac{\partial \varepsilon_i}{\partial y} \frac{m_i}{\beta} \]

\[ = \frac{m_i}{\beta} \left( \varepsilon_i \frac{\eta}{y_i} + \frac{\partial \varepsilon_i}{\partial y} \right) \]

(40)

(41)

Substituting equation(41) in equation(37), and dividing the expression into two, one containing the indirect utility function, \(G\), and one part containing the terms that do not, \(F\), we have:

\[ F_i = p \left[ \frac{\partial m_i}{\partial y} - p (1 - \beta) \frac{m_i}{\beta} \left( \varepsilon_i \frac{\eta}{y_i} + \frac{\partial \varepsilon_i}{\partial y} \right) \right] \frac{m_i - pm_i + (1 - \beta) \frac{\partial m_i}{\partial \beta}}{pm_i - p (1 - \beta) \frac{\partial m_i}{\partial \beta}} \]

\[ - p \left[ (1 - p) \frac{\partial m_i}{\partial y} + p (1 - \beta) \frac{m_i}{\beta} \left( \varepsilon_i \frac{\eta}{y_i} + \frac{\partial \varepsilon_i}{\partial y} \right) \right] \]

\[ = \frac{p (1 - \beta) \frac{\partial \varepsilon_i}{\partial y} m_i}{1 - (1 - \beta) \frac{\beta}{\beta}}. \]

(42)

(43)

This expression will be equal to zero whenever the price elasticity is constant between individuals, i.e. \( \frac{\partial \varepsilon_i}{\partial y} = 0 \).
Here we see that if $\eta = 1$ and we have CRRA, $G$ will equal zero.

Adding the two parts we have

$$
\frac{d\beta}{dy} = \frac{1}{V^W_i} \left[ \Theta \left( \rho^I - \rho^W \right) - \eta \frac{\beta m_i}{y_i} \left( \rho^I + \frac{V^W_i}{V^W_c} \left( \frac{y_i - p(1 - \beta)\eta m_i}{\eta} \right) \right) \right] + \frac{\partial \theta_{\eta y}}{\partial \beta_{y}} \frac{\eta y_i}{y_i} S.O.C
$$

(45)

### A.4 Price elasticity, income elasticity and risk aversion

From the first order condition for medical care we have $\frac{V^W_m}{U^i_c} = \beta$. Differentiating this with respect to $m$ and $y$ we have

$$
\frac{\partial m_i}{\partial y} = \frac{U^i_{cc} \beta}{U^i_{cc} \beta^2 + U^i_{mm}}
$$

(46)

and

$$
\frac{\partial m_i}{\partial \beta} = \frac{U^i_{cc} \beta m_i - U^i_c}{U^i_{cc} \beta^2 + U^i_{mm}}
$$

(47)
and as before we use the definitions of the income and price elasticities and get

\[
\frac{\partial m_i}{\partial \beta} = -\eta \left( \frac{m_i}{y_i} \beta - \frac{U_i}{U_{cc} y_i} \right) = \varepsilon_i. \tag{48}
\]

Then the price elasticity may also be written as

\[
\varepsilon_i = -\eta \frac{m_i}{y_i} \beta + \eta \frac{U_i}{U_{cc} y_i} c_i
\]

\[
= -\eta \frac{m_i}{y_i} \beta - \eta \frac{c_i}{\rho_i y_i}
\]  

where \( \rho = -U_{cc}/U_c \) is relative risk aversion.

### A.5 Supplementary private insurance

**Proof of Proposition (4):** We look at the derivative \( \frac{da}{d\beta_P} \), and from the relation \( \frac{da}{d\beta} = -\frac{da}{d\beta_P} \) we will sign the derivative \( \frac{da}{d\beta_P} \). The derivative \( \frac{da}{d\beta_P} \) is obtained by differentiating (22) with respect to \( \beta_P \) gives and \( \alpha \) (where index \( i \) is dropped).

\[
\begin{align*}
\left\{ pV_{ci} \left[ \frac{\partial m_i}{\partial \beta_P} - p \frac{\partial m_i}{\partial \beta_P} - p\alpha \frac{\partial^2 m_i}{\partial \beta^2 P} \right] \right\} d\beta_P + \\
\left\{ pV_{ci} \left[ -m_i - \beta_P \frac{\partial m_i}{\partial \beta_P} - p\alpha \frac{\partial m_i}{\partial \beta_P} + p\frac{y_i}{\bar{y}} - p(1 - \beta_P) \frac{\partial m}{\partial \beta_P} \right] \right\} \times \\
\left\{ m_i - p m_i - p\alpha \frac{\partial m_i}{\partial \beta_S} \right\} d\beta_P + \\
\left\{ (1 - p)V_{ci} \left[ -p \frac{\partial m_i}{\partial \beta_P} - p\alpha \frac{\partial^2 m_i}{\partial \beta^2 P} \right] \right\} d\beta_P + \\
\left\{ pV_{ci} \left[ -p\alpha \frac{\partial m_i}{\partial \beta_P} + p\frac{m_i}{\bar{y}} - p(1 - \beta_P) \frac{\partial m}{\partial \beta_P} \right] \right\} \times \\
\left\{ -p m_i - p\alpha \frac{\partial m_i}{\partial \beta_S} \right\} d\beta_P + (S.O.C) \, d\alpha
\end{align*}
\]

\[
= 0 \tag{50}
\]

After using the facts that \( \frac{\partial m}{\partial \beta_P} = \frac{\partial m}{\partial \beta_S} \), \( \frac{\partial^2 m}{\partial \beta^2} = \varepsilon \frac{m (\varepsilon - 1)}{\beta^2} \), assuming CRRA,
and some manipulations we arrive at the following expression

\[
\frac{d\alpha}{d\beta^p} = -\frac{1}{V_e} \left( \frac{\phi V^W}{c^I V^W_e} \frac{m\alpha}{\beta(\beta + \varepsilon\alpha)} + \frac{D}{S.O.C} \left[ c^I \Omega \beta m + c^W \left( m + \beta^P \frac{\partial m}{\partial \beta} \right) \right] \right) < 0. \tag{51}
\]

\(C < 0\) since \(\varepsilon < 0\), and \(\beta + \varepsilon\alpha\) since \(\alpha < \beta\) and \(|\varepsilon| < 1\).

\(D < 0\) since \(\frac{\phi V^W}{c^I V^W_e} < 0\) and also \(c^I \Omega \beta m + c^W \left( m + \beta^P \frac{\partial m}{\partial \beta} \right) < 0\). Concerning the last term, although it is the case that \(c^I < c^W\) and \(\Omega < m + \beta^P \frac{\partial m}{\partial \beta}\) (which we know from the fact that \(\frac{\partial c^I}{\partial \beta^p} = \Omega - \left( m + \beta^P \frac{\partial m}{\partial \beta} \right) < 0\)) the fact that the first term is multiplied with \(\beta m\) will make sure this term dominates.
References


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Figure 1: Optimal and actual coinsurance rates when the income elasticity is less than one, $\eta < 1$, and people are characterized by constant relative risk aversion, CRRA, and after having assumed $\frac{d\beta^{M^*}}{dy} > 0$. $y_m$ — median income, $\beta^{public^*}$ — optimal coinsurance rates public case, $\beta^{public}$ — actual coinsurance rate public case, $\beta^{private^*}$ — optimal (and actual) coinsurance rates private case, $\beta^{supplementary^*}$ — optimal (and actual) coinsurance rates for the case with public and supplementary private insurance (everybody to the right of $y_m$ faces $\beta^{public}$).