A STUDY ON THE DCC-GARCH MODEL’S FORECASTING ABILITY WITH VALUE-AT-RISK APPLICATIONS ON THE SCANDINAVIAN FOREIGN EXCHANGE MARKET

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Abstract
This thesis has treated the subject of DCC-GARCH model’s forecasting ability and Value-at-Risk applications on the Scandinavian foreign exchange market. The estimated models were based on daily opening foreign exchange spot rates in the period of 2004-2013, which captured the information in the financial crisis of 2008 and Eurozone crisis in the early 2010s. The forecasts were performed on a one-day rolling window in 2014. The results show that the DCC-GARCH model accurately predicted the fluctuation in the conditional correlation, although not with the correct magnitude. Furthermore, the DCC-GARCH model shows good Value-at-Risk forecasting performance for different portfolios containing the Scandinavian currencies.

Key words: Multivariate GARCH, Conditional Correlations, Forecasting, Time-varying covariance matrices, Exchange rate returns, Variance-Covariance matrix.
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1. INTRODUCTION

Predicting the volatility is important for most actors involved in the financial markets. It is used over a wide range of areas such as estimating the market risk, risk management in portfolio management and for pricing financial derivatives. The volatility of the financial markets exhibits patterns that differ from other time series. The most distinguished features are the heteroscedasticity and the phenomena of volatility clustering, hence the normal time series models suffer the risk of not providing accurate forecasts. The lack of adequate models for approaching heteroscedastic time series lead to the development of the Autoregressive Conditional Heteroscedastic model (ARCH) by Engle in 1982, presented in *Autoregressive conditional heteroscedasticity with estimates of the variance of UK inflation*. This model was further developed to a Generalized ARCH (GARCH) by Bollerslev, presented in his paper *Generalized Autoregressive Conditional Heteroscedasticity* published in 1986.

A further feature is the volatility covariation of different financial assets. To be able to account for the effect of covariation, Multivariate GARCH models (MGARCH) were developed. Unlike the ARCH and GARCH models, the MGARCH is a generic name that represents several models with different approaches to estimating the covariation between different assets. The first MGARCH models developed were the Vector Error Correction (VEC-GARCH) and BEKK models, and since, further models have been developed e.g. Constant Conditional Correlation-GARCH (CCC-GARCH) presented by Bollerslev (1990). This thesis makes use of the Dynamic Conditional Correlation-GARCH (DCC-GARCH) presented by Engle and Sheppard (2001), considering that it is a generalization of the complex CCC-GARCH which gives a computational advantage when estimating large covariance matrices.

When considering the foreign exchange market, the topic of their volatility has been concerned in multiple areas, and is a key factor in political, macroeconomic and financial decisions. Ever since the phasing out of the Bretton Woods system, the movements of the exchange rates has been of higher importance, and hence the ability of prediction. This is because of during the Bretton Woods system, the currencies of the member countries of the International Monetary Fund (IMF) had fixed exchange rates against the dollar. The
importance of this was even more illuminated after the financial crisis in 2008 and the Eurozone crisis.

Foreign exchange is often part of many portfolios containing financial assets. To be able to adequately manage the risk, it is important for portfolio managers to accurately account for the covariation between different assets. This makes the use of Multivariate GARCH models appealing, due to the ability of forecasting covariation between financial assets.

Previously, there has been a great amount of research conducted on the topic of volatility clustering and time series modelling. The topic of modeling the conditional correlation and covariation of different currencies have been studied before, in e.g. Bollerslev (1990) and Kerney and Patton (2000). However, a majority of the studies conducted have focused on the more commonly traded currencies, opening up for studies on the application of the model on many other currencies. The DCC-GARCH model, which is used in this thesis, has been studied before, see e.g. Billio, Gobbo and Coperin (2005) and Orskaug (2009). However, earlier studies have mainly focused on other financial assets like equity and commodities. Hence, a study on the topic of modelling conditional correlation and covariance with the DCC-GARCH, especially for less traded currencies, could bring a new perspective on the use of DCC-GARCH model.

This report focuses on the forecasting performance of the conditional correlations by the DCC-GARCH model, as well as investigating how well the DCC-GARCH forecasts can be applied on Value-at-Risk calculations. The estimations and forecasts is constructed by data from the Scandinavian foreign exchange spot rates against the dollar (USD/SEK, USD/NOK, USD/DKK) with the logarithmic returns being the measure of volatility.

The following section presents the background and features of the foreign exchange market and financial markets in general. Section three presents the theory of the models used in this report. Section four presents the empirical results of the estimation of the models. Section five evaluates the forecasting performance of the DCC-GARCH model. Section six contains the Value-at-Risk forecasting and backtesting. Section seven holds the conclusions of the estimations and forecasts of the conditional correlation and Value-at-Risk. Lastly, a list of the references used in this report as well as the appendices is found.
2. ECONOMIC THEORY

In this chapter the essential concepts of the exchange rate market and its major factors of impact are presented, followed by a brief discussion about important economic terms when considering the movements of exchange rates. Furthermore, a discussion about the global financial crisis of 2008 and Eurozone crisis of the 2010’s and their effect on the exchange rate market follows. Lastly, there is a brief discussion of Denmark’s participation in the ERM II (European Union 2018).

2.1 USE OF VOLATILITY PREDICTION

The volatility of different assets is crucial in numerous financial areas, such as pricing financial derivatives and estimating the market risk and the value of an asset. This is important for example when calculating the risk for and hedging a portfolio. Being able to correctly predict the volatilities could give investors an important edge on the market.

2.2 EXCHANGE RATES AND FOREIGN EXCHANGE MARKETS

Exchange rates is a quotient given by the price of a nation's currency divided by the price of another nation’s currency. The standard expression, known as direct quoting and used in this report, is the price of a unit of a foreign currency expressed in terms of units of the domestic currency. As an example, if USD/SEK is traded at 9.1 it means that 1.0 USD is bought for 9.1 SEK. In this thesis the US dollar is used as the base currency while the other currency is used as the quote currency.

Exchange rates can be of two types; spot or forward. Spot exchange rates is the current exchange rate and forward exchange rates are set and traded directly, but the actual payment and delivery of the currency is done at a later date and time decided at the time of the deal. In this thesis, the spot opening rates are used when calculating the returns.

The foreign exchange (FX) market is an example of what is called an over-the-counter market (OTC-market). This means that every transaction goes directly from seller to buyer, unlike for example the stock market where every trade is supervised by an exchange. The foreign exchange market is in terms of trading volume by far the largest market in the world (Record 2003).
2.3 WHAT AFFECTS THE EXCHANGE RATES

The foreign exchange market never closes and is to a great extent affected by monetary decisions and policies as well as political decisions on national and international level. The result of this is that the exchange rates are affected by a different set of parameters than many other financial assets. Three important factors when estimating the movement of exchange rates are the Gross Domestic Product (GDP), a country’s key interest rate and the inflation. These three measures and their impact on the exchange rates and the volatility are discussed more thoroughly in the following sections.

2.3.1 GROSS DOMESTIC PRODUCT

The Gross Domestic Product is composed by a country’s private consumption expenditure, business investments, governmental spending and net export during one year. The GDP might be considered as the most widely used economic indicator of a country’s well-being, and therefore it has a great impact on the exchange rates. Generally speaking, a growth in GDP will lead to a rise in the value of a country’s currency. The GDP is also used as an indicator for inflation changes and is used in many interest rate decisions carried out by the responsible institution, both of which are discussed below.

2.3.2 KEY INTEREST RATES

The key interest rate is normally decided by the central bank of the given country, and is revised a number of times per year. The interest rate is one of, if not the most widely used tool to govern an economy, where the key interest rate is a tool for the central bank to maintain a desired level of inflation. A higher key interest rate will make it more expensive for people to borrow money, generally generating a falling inflation, while lowering the interest rate would make it cheaper to borrow money, normally causing the inflation to rise. The relationship between the interest rates and inflation plays a big role in the pricing of currency and therefore also the volatility of the exchange rate.

It can be seen that markets that offer a higher yield on the risk free return, is also is traded at a higher price, i.e. if the central bank decides to raise the interest rate, more investors will turn to that market, the demand for the domestic currency will increase and consequently it will be traded at a higher level. This is shown in the theory of Interest Rate Parity, given by:
$$S_t = \frac{(1 + i_a)}{(1 + i_b)} \times E_t(S_{t+k})$$

where $S_t$ is the current spot rate at time $t$, $i_a$ and $i_b$ are the nominal interest rates of two countries and $E_t(S_{t+k})$ is the expected future spot exchange rate at time $t + k$ (Feenstra, Taylor 2008).

2.3.3 INFLATION

Inflation can be described as how much the general price level in a country changes during a set period of time and is affected by the levels of supply and demand for a country’s goods. A higher demand will contribute to the inflation-level rising while a lower demand will do the opposite. A higher inflation basically means that you need more money to buy the same amount of goods, i.e. the currency is worth less, and has depreciated. As discussed in section 2.3.2, inflation is indirectly closely linked to the interest rates, due to the effect on supply and demand.

2.4 VOLATILITY CLUSTERING OF FINANCIAL ASSETS

The concept of volatility clustering is the trend that large changes in asset prices cluster together. Large changes are likely to be followed by large changes, and likewise are small changes expected to be followed by small changes. This phenomena of serial dependency is known as volatility clustering, and is observed when there are periods, or clusters, of high volatility and periods of low volatility. In practice, this implies that the impact of a large price movement leads to a setting of high volatility on the market, tending to persist for a period of time, leading to a period of high volatility (Ruey 2005).

2.5 FINANCIAL CRISIS 2008

The global financial crisis of 2008 had its origin in the mortgage market in the United States in the early 2000s (Jones 2009). In the year of 2001, the US experienced a mild recession of the economy. In times of terrorist attacks and the dot-com-bubble not far in mind, the Federal Reserve were afraid of a deeper recession and wanted to stimulate the economy. The Federal Reserve lowered the key interest rate, the Federal Funds Rate, from 6.5% to 1.75% over a period of 18 months (Figure 12, Appendix). People who earlier had not been able to be
granted a loan for a house were now granted loans and investing in real estate. Large financial institutions packed up thousands of mortgages and sold shares to investors. The rising price level of the house market made real estate investments and mortgage securities a safe bet. Investors pushed the lenders to create more mortgages, which resulted in that people with worse and worse credit rating got granted loans. The mortgages and securities where then packaged and sold to different investors, which lead to an end result of an overleveraged and overvalued housing market.

The bubble started to burst in 2007 which lead to that a series of large banks and lenders declared bankruptcy. The Federal Reserve initiated a series of reductions of the key interest rate in an attempt to milden the struck of the crisis, and was followed by central banks all over the world (FiU 2008/09:24).

2.6 EUROZONE CRISIS

The Eurozone crisis starting in late 2009 and culminating in 2013 may be considered as a sequel to the financial crisis in 2008. In short, the reasons for the crisis may be derived to the initial faults of the monetary union (Lane 2012). As an example, countries as Greece and Italy with large budget deficit were as a part of the monetary union able to loan at the same interest rate as steady economies like Germany. Cheap credits given by the union lead to an overleveraged housing market in several euro countries, which might have been one of the reasons for the crash in 2009. In the aftermath of the global financial crisis, several countries in the Eurozone had debts that became too hard to manage, and with the falling credit ratings new loans became hard to get. This lead to substantial peaks in the interest rates of the affected countries.

2.7 THE DANISH KRONE AND THE ERM II

An important note to the discussions is that the Danish krona (DKK) is pegged to the Euro (EUR) via the ERM II. This means that USD/DKK follows the USD/EUR in a range of plus/minus 2.25% (European Union 2018). Much of the discussion when evaluating the USD/DKK therefore concerns the actions by the Eurozone and the European Central Bank. The reason for not choosing to look at the USD/EUR, is that the aim of the report is to evaluate the actions on the Scandinavian markets. Even though the results would basically be the same, this method makes it easier for the reader to get the full picture.
3. STATISTICAL THEORY

In this section the statistical concepts, tests and models that are being used in the thesis are presented. The presentation of the different concepts follows a chronological order i.e. in the order they are used in the thesis. The chapter handles both univariate and multivariate models evaluation, GARCH and MGARCH models, and the measures used to evaluate the forecasting. Furthermore, this section explains the theory behind Value-at-Risk, as well as the forecasting and backtesting procedure.

3.1 NORMALITY TESTS

Financial time series data tend to suffer from volatility clustering and have a heavy tailed distribution (Ruey 2005). The following sections consists of univariate as well as multivariate tests of normality, to confirm that the data exhibits the characteristics of financial time series.

3.1.1 JARQUE-BERA TEST OF NORMALITY

To test whether the univariate distributions of the logarithmic returns are normally distributed or not, a Jarque-Bera test of normality is to be performed (Cryer, Chan 2008). The test examines whether there are any excess skewness or kurtosis significantly different from zero in the sample. The test statistic and hypotheses are defined below;

\[ JB = \frac{n - 1 + 1}{6} \left( S^2 + \frac{1}{4} (K - 3)^2 \right) \]

Where \( S = \text{skewness} = \frac{\frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x})^3}{\left( \frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x})^2 \right)^{3/2}} \) and \( K = \text{excess kurtosis} = \frac{\frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x})^4}{\left( \frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x})^2 \right)^2} \).

\[ H_0 : S = K = 0 \]
\[ H_a : S \neq 0 \text{ or } K \neq 0 \]

The null hypothesis of the Jarque-Bera test is that the excess kurtosis equals the skewness which equals to zero. Why the excess kurtosis is used is because of that the normal distribution has a kurtosis of three. The kurtosis is a measure of how probable the extreme cases of a probability distribution is. Loosely explained, it can be stated as the “thickness” of the tails of the distribution. The skewness on the other hand is a measure of the symmetry of
the probability distribution. A positive skewness indicates that it has a long tail to the right, and a negative skewness has correspondingly a long left tail. The null hypothesis is thus that the logarithmic returns is normally distributed, versus the alternative that they are not.

3.1.2 MULTIVARIATE TEST OF NORMALITY

Since the aim of the thesis is to model with a multivariate approach, a test of multivariate normality is to be conducted. There exists a set of different tests for this, but this thesis makes use of the Henze-Zirkler’s Test of Multivariate Normality (Baringhaus, Henze 1988). The null hypothesis of the test is that the data are multivariate normally distributed, versus the alternative that the data are not. Below, the test statistic and hypotheses are specified.

\[
H_0: \text{The data are multivariate normally distributed} \\
\text{versus} \\
H_a: \text{The data are not multivariate normally distributed}
\]

\[
HZ_u = n \left( 4I_E + T_{n,u}I_{E^c} \right)
\]

Where \(I_E, I_{E^c}\) are indicator functions with \(E\) which is the sample covariance matrix.

\[
T_u(d) = \frac{1}{n^2} \sum_{j=1}^{n} \sum_{k=1}^{n} \exp \left( -\frac{u^2}{2} |Y_j - Y_k|^2 \right) \\
- 2(1 + u^2)^{\frac{d}{2}} \frac{1}{n} \sum_{j=1}^{n} \exp \left( -\frac{u^2}{2(1 + u^2)} |Y_j|^2 \right) + (1 + 2u^2)^{\frac{d}{2}}
\]

\[
u = 2^{-\frac{1}{d}} \left\{ \left( 2d + 1 \right)n / 4 \right\}^{1/(d+4)}
\]

And \(d\) denotes the number of dimensions and \(Y_n\) is the observations from a \(d\)-variate distribution.
3.2 ARCH MODELS

To be able to accurately capture the conditional variance of financial data, Engle (1982) introduced the autoregressive conditional heteroscedasticity (ARCH) model. The conditional variance refers to that the variance of today is conditioned on past values of the variance. As discussed in section 2.4, the volatility of financial time series tends to cluster in cycles of high volatility as well as cycles of low volatility and that it varies over time. The conditional variance of the returns is not constant, and is denoted \( \sigma^2_{t|t-1} \), which indicates that it is dependent on its past lagged values through time.

Even though the ARCH model captures the varying volatility of financial time series, it has its limitations and a discussion of generalizations including the univariate as well as the multivariate case is following. Furthermore, the ARCH model assumes, as with the standard autoregressive process, that the variance of the error term depends on its lagged values, i.e. the previous conditional variances of the error term.

Formally, the ARCH process is a regression model with the conditional variance as the outcome variable and the past lags of the squared returns as the covariates.

\[
r_t = \mu + \epsilon_t, \quad t = 1, \ldots, T
\]

where \( \mu \) is the mean of the process and \( \epsilon_t \) is the residual. The residual \( \epsilon_t \) can be expressed as \( \epsilon_t = z_t \sigma_t \), where \( z_t \sim N(0,1) \).

\[
\sigma^2_t = \alpha_0 + \alpha_1 \epsilon^2_{t-1} + \cdots + \alpha_q \epsilon^2_{t-q}
\]

This specifies a stochastic process for the residuals of the ARCH process, which functions as a predictor for the size of the upcoming residuals. Since \( z \) is presumed to have zero mean and unit variance, that is that the variance as well as standard deviation equals to one, the conditional variance of \( r \) equals the variance of \( \sigma^2_{t|t-1} \).

A weakness of the ARCH model is that requires a high order of the stochastic process when capturing the conditional variance of a time series. This leads to a large number of parameters to be estimated. Furthermore, the ARCH model assumes that positive and negative shocks
has the same effect on the volatility of a time series. As shown by Paul (2007), it is common that the volatility financial asset prices respond differently to positive and negative shocks. The weakness regarding the number of parameters to be estimated is solved in the generalized ARCH models (GARCH), which is discussed in the following chapter.

3.3 GARCH MODELS

Bollerslev (1986) and Taylor (1986) independently worked on and introduced the Generalized ARCH model (GARCH) where a limited number of the lags of the conditional variance is replacing an infinite number of lagged squared returns. A generally accepted notation of the GARCH is GARCH \((p, q)\), where \(p\) stands for the number of lag variances to include, and \(q\) stands for the number of lags of the squared returns to include. Hence, a GARCH \((0, q)\) is simply an ARCH \((q)\).

The major weakness of the ARCH model is that in practice, a large number of lags were needed. Research has shown that the GARCH \((1,1)\) has proven to be sufficient to estimate the volatility (Brooks, 2014). Another weakness of the ARCH- and GARCH model is that they assume that all parameter \(\alpha\) is non negative. If this assumption is not fulfilled, the model might predict negative volatility which in practice is impossible. Several models have been developed using the GARCH \((p,q)\) as a foundation to correct for this problem, for example Brooks (2014). However, they are not discussed further since only the standard GARCH is considered in this thesis. Below, the general GARCH \((p,q)\) model is presented.

\[
\begin{align*}
  r_t &= \mu_t + a_t \\
  a_t &= h_t^{1/2} z_t \\
  h_t &= \alpha_0 + \alpha_1 a_{t-1}^2 + \cdots + \alpha_q a_{t-q}^2 + \beta_1 h_{t-1} + \cdots + \beta_p h_{t-p}
\end{align*}
\]
Notation, in line with Orskaug (2009)

\( r_t \): log return of an asset at time \( t \)
\( \mu_t \): expected value of the log return of an asset at time \( t \)
\( a_t \): mean corrected return of an asset at time \( t \)
\( h_t \): the conditional variance at time \( t \)
\( z_t \): a sequence of iid N (0,1) random variables
\( \alpha_0, \alpha_1, ..., \alpha_q \): parameters of the model
\( \beta_0, \beta_1, ..., \beta_q \): parameters of the model
\( p, q \): orders of the GARCH model

The conditional variance in the GARCH model is dependent on the squared returns as shown above, and hence varies over time. Since the values are squared, the model does not distinguish positive and negative shocks, only the magnitude of the shock.

3.4 MULTIVARIATE GARCH MODELS

The univariate GARCH model can be generalized to Multivariate GARCH models (MGARCH). The MGARCH models accounts for and estimates the interaction effect between the volatility of different assets. Being able to observe and estimate the interaction effect and account for it in the model would generate more information about the volatility covariation of assets. The MGARCH model is generally given by:

\[
  r_t = H_t^{1/2} \eta_t
\]

where the \( r_t \) is as a vector of log-returns of \( N \) assets and \( \eta_t \) is an iid vector error process such that \( E(\eta_t \eta_t^T) = I \), hence the only element that needs to be specified, is the matrix process of \( H_t \). Suggested by Silvennoinen and Teräsvirta (2008), there are four different approaches to specifying the matrix process.
3.4.1 MODELS OF THE CONDITIONAL COVARIANCE MATRIX

Models of the conditional covariance matrix are generalizations of the univariate GARCH model. Examples of models that are using this approach is the VEC model suggested by Bollerslev, Engle and Woolridge (1988) and the BEKK model, presented in Engle and Kroner (1995).

3.4.2 FACTOR MODELS

Factor models originate from economic theory and were first presented by Engle, Ng and Rothschild (1990). The concept builds on the idea of $H_t$ being generated by a number of underlying factors that are conditionally heteroscedastic, that possess a GARCH structure and are potentially correlated, which constructs the structure of the conditional covariance matrix.

3.4.3 MODELS OF CONDITIONAL VARIANCES AND CORRELATIONS

Models of conditional variances and correlations are built on the concept of modelling the conditional standard deviations and correlations by decomposing the conditional covariance matrix. The first of these models were presented as the Constant Conditional Correlation model (CCC) by Bollerslev in 1990, and has been further developed since. The models of conditional variances and correlations are further discussed in section 3.5.

3.4.4 NONPARAMETRIC AND SEMIPARAMETRIC APPROACHES

The advantages of the non-parametric approaches are that they do not impose a certain structure in the data, which otherwise could be misspecified, but suffers other problems that non-parametric approaches usually do, e.g. not being able to estimate dynamic correlations or covariances. Further readings about the different multivariate GARCH models and their applications can be found in Silvennoinen and Teräsvirta (2008).

This paper solely focuses on models of conditional variances and correlations and evaluate whether or not a time-varying covariance matrix is necessary, i.e. if the data are imposing a dynamic covariance structure.
3.5 DCC-GARCH MODELS

The Dynamic Conditional Correlation GARCH model (DCC-GARCH) was first introduced by Engle and Sheppard (2001) and can be viewed as an extension of the CCC-GARCH model presented by Bollerslev (1990). The difference between the CCC-GARCH and DCC-GARCH is that the DCC-GARCH allows the correlation structure to be dynamic and vary over time. The DCC-GARCH model is easier to compute than many other complex MGARCH models, where the most distinguished advantage is that the number of parameters that are estimated in the correlation process are independent on the number of series that are to be estimated, which renders in a large computational advantage when estimating large covariance matrices (Engle 2002).

3.5.1 TEST FOR DYNAMIC CORRELATIONS

To confirm whether the data contains dynamic correlations or not, a test of non-constant correlation has to be performed. The result of the test gives an indication of what type of model to be estimated and used throughout the rest of the thesis. This thesis makes use of the test proposed by Engle and Sheppard in *Correlation Multivariate GARCH: Theoretical and Empirical properties of Dynamic Conditional* (2001), with the following hypotheses

\[ H_0: R_t = \bar{R} \]

\[ H_a: \text{vech}(R_t) = \text{vech}(\bar{R}) + \beta_1 \text{vech}(R_{t-1}) + \cdots + \beta_p \text{vech}(R_{t-p}) \]

The test is based on an auxiliary regression of the product of the univariate GARCH processes’ standardized residuals regressed on a constant and the lagged products. \( R_t \) is the covariance matrix which under the null is iid with variance matrix given by the identity function. \( \text{vech} \) uses the elements above the diagonal of the covariance matrix. The test statistic can then be expressed as

\[
\frac{\delta' X' X \delta}{\hat{\delta}^2} \sim \chi^2_{(s+1)}
\]

where \( X \) is the regressor matrix and \( \hat{\delta} \) is the estimate regression parameters. The test statistic is asymptotically \( \chi^2_{(s+1)} \) where \( s \) is the lag of the product. If the null hypothesis of constant correlation is rejected, a model that allows dynamic correlation is needed.
### 3.5.2. THE MODEL

In line with Engle (2002), the DCC-GARCH can be presented as follows:

\[
H_t = D_t R_t D_t
\]

Where \( H_t \) represents the conditional covariance matrix and \( R_t \) the conditional correlation matrix. \( D_t \) is generally viewed as univariate GARCH models, but are not restricted to being univariate GARCH models. The model could also include functions of other variables.

\[
D_t = \text{diag}\{\sqrt{h_{i,t}}\}
\]

The elements of \( D_t \) are written as univariate GARCH models:

\[
h_{i,t} = \omega_i + \sum_{p=1}^{P} \alpha_{ip} \rho_{it-p}^2 + \sum_{q=1}^{Q} \beta_{iq} h_{it-q}
\]

where the normal restrictions for univariate GARCH models are imposed. Those restrictions include the restriction of non-negativity of the variance, as well as stationary of the model.

\( R_t \) can be derived as:

\[
R_t = Q_t^{-1} Q_t^{-1} Q_t^{-1}
\]

where

\[
Q_t = \left( 1 - \sum_{m=1}^{M} \alpha_m - \sum_{n=1}^{N} \beta_n \right) Q + \sum_{m=1}^{M} \alpha_m (\epsilon_{t-m} \epsilon_{t-m}') + \sum_{n=1}^{N} \beta_n Q_{t-n}
\]

and

\[
Q_t^* = \begin{bmatrix} \sqrt{q_{11}} & 0 & 0 \\ 0 & \sqrt{q_{22}} & 0 \\ 0 & 0 & \sqrt{q_{33}} \end{bmatrix}
\]
and $\bar{Q}$ is the unconditional covariance of standardized residuals from the first stage of estimation.

Another crucial assumption for the DCC-GARCH model is that the conditional correlation matrix ($R_t$) holds the properties of a covariance matrix, that is, being positive definite. As proven by Engle and Sheppard in Correlation Multivariate GARCH: Theoretical and Empirical properties of Dynamic Conditional (2001), we see that if $Q_t$ is positive definite, it implies that $R_t$ is positive definite. Furthermore, the parameters $\alpha$ and $\beta$ are scalars and must individually be larger than zero but $\alpha + \beta < 1$. This condition stands for the univariate GARCH model, but it is applied to the multivariate case in the DCC-GARCH (Orskaug 2009).
3.5.3 ESTIMATION OF THE MODEL

When estimating the DCC-GARCH model, the model is estimated in a two-step procedure.

As seen in Silvennoinen and Teräsvirta (2008), the first step of estimation is that the residuals of the univariate GARCH models are estimated by the quasi-likelihood function:

\[
QL_1(\phi | r_t) = -\frac{1}{2} \sum_{t=1}^{T} (k \log(2\pi) + \log(|I_k|) + 2 \log(|D_t|) + r_t D_t^{-1} I_k D_t^{-1} r_t)
\]

\[
= -\frac{1}{2} \sum_{t=1}^{T} (k \log(2\pi) + 2 \log(|D_t|) + r_t D_t^{-2} r_t)
\]

\[
= -\frac{1}{2} \sum_{t=1}^{T} \left( k \log(2\pi) + \sum_{n=1}^{k} \left( \log(h_{it}) + \frac{y_{it}^2}{h_{it}} \right) \right)
\]

When the first stage is estimated, the parameters of the dynamic correlation are estimated conditioned on the parameters estimated in the first step, which generates a second likelihood function. Through this likelihood function the parameters of the dynamic correlation is then calculated.

3.6 MODEL EVALUATION

3.6.1 LJUNG-BOX TEST

Before deciding what model to use, it is important to evaluate if there is autocorrelation left in the residuals of the estimated model. If all autocorrelations in the residuals are jointly zero, it means that the null hypothesis of random errors cannot be rejected. The test’s hypotheses are defined as,

\[ H_0: \hat{p}_1 = \cdots = \hat{p}_k = 0, k = 1, \ldots, h \]

versus

\[ H_a: At \ at least \ one \ \hat{p}_k \neq 0, k = 1, \ldots, h \]

\[
Q = n(n + 2) \sum_{k=1}^{n} \frac{\hat{p}_k^2}{n - k}
\]
where \( n \) is the number of observations, \( k \) denotes the given lag and \( h \) is the number of lags being tested. If the statistic \( Q > \chi^2_{1-\alpha,h} \), the null of no autocorrelation in the residuals is rejected.

### 3.6.2 CHOOSING THE BEST MODEL

When deciding what model and order to use, there is a few ways one can evaluate which specified model to use. The purpose for the use of this in this thesis is to evaluate which model specification that minimizes the loss of information, hence the model that contains the most information of those tested. This is done to narrow down the number of models to estimate and later on use when forecasting. The first approach to evaluating the model is, is the Akaike Information Criterion (AIC) which aims to maximize the conditional likelihood function of an ARCH-model with the order \((p,q)\) and corrects it for asymptotic bias (Burnham et al. 2004)

\[
AIC = -2 \log(L) + 2k
\]

The AIC is a relative measure to compare different models and should not be interpreted by their value alone, since they are arbitrary constants and strongly affected by sample size (Burnham et al. 2004).

Another way of choosing the superior model is to use the Bayesian Information Criterion (BIC), which was derived by Schwarz (1978) and is given by the formula:

\[
BIC = -2 \log(L) + K \log(n).
\]

As with the AIC, one computes and compares the BIC-value of each individual model, and then selects the model with the lowest value. The BIC penalizes more complex models more heavily, hence one should not choose the larger model if the two information criterions contradict each other. There have been concern presented about the use of the BIC, in Durham et al. (2004), especially in situations where the sample size is small and when estimating its target model. Although, this is not further discussed in this paper.
In this thesis, the AIC- and BIC-scores for seven models with different DCC lags are evaluated and the best three are used further.

3.7 FORECASTING

The forecasting of the conditional correlations is carried out over the period of 2014-01-01 - 2014-12-31, where the predicted values are compared to a proxy for the actual correlations. The forecast for the conditional correlations and covariances are estimated for one day ahead. Rolling forecasting on a one-day basis is used, meaning that the model used for forecasting is refitted each day. The forecast window is made up of 260 days. When estimating the actual correlations, no intraday data are available, which means that other procedures have to be used. When estimating the actual correlations, a 10-day moving average of the correlations on the daily logarithmic returns are used to estimate the real correlations.

3.7.1 FORECASTING THE COVARIANCE MATRIX

When estimating the DCC-model one step ahead in time, the following formula is applied.

\[ H_{t+1} = D_{t+1}R_{t+1}D_{t+1} \]

From this formula it can be seen that the elements of the DCC-model, \( D_t \) and \( R_t \) needs to be forecasted individually.

The forecast for the \( D_t \) is:

\[ E[D_{t+1}|F_t] = diag \left( E[h_{1,t+k}|F_t], ..., E[h_{n,t+k}|F_t] \right) \]

Where \( F_t \) is the information given at time \( t \).
As shown by Orskaug (2009), finding that the elements of the conditional correlation matrix are not themselves forecasts causes trouble when trying to find unbiased forecasts. In Orskaug (2009), two approaches to computing the forecasts of the conditional correlation matrix are presented. Findings show that the second method presented has better bias properties for most correlation matrices, with support from the empirical studies of Engle and Sheppard (2001). Here it is assumed that $\tilde{Q} \approx \tilde{R}$ and that $E[R_{t+l}|F_t] \approx E[Q_{t+l}|F_t]$. Resulting in $E[R_{t+l}|F_t] = (1 - a - b)\tilde{Q} + a\varepsilon_t\varepsilon_t^T + bQ_t$. This enables the computation of the forecast for the covariance matrix by $R_{t+k} = \tilde{D}_{t+k}R_{t+k}\tilde{D}_{t+k}$.

### 3.8 FORECASTING EVALUATION

One way of measuring the forecasting performance of a model is to compare how well the model can predict the outcome out of sample, i.e. how well the predicted correlations correspond to the actual correlations estimated by the chosen proxy. In this section, two of the most common measures for evaluating forecasting performance are reviewed as well as their strengths and weaknesses. These measures are used to evaluate how well the DCC-GARCH model predicts the volatility for different currency pairs.

#### 3.8.1 MAE

The first measure is the Mean Absolute Error (Matsuura, Wilmott 2005) which is the measure of how close the absolute values of the predicted correlations are to the actual correlations, given by the following formula:

$$MAE = \frac{\sum_{t=1}^{T} |y_t - y_t'|}{T}$$

#### 3.8.2 RMSE

The second measure used is the Root Mean Square Error (RMSE) which also is a measure of the distance between the predicted correlations and actual correlations, but with another approach than the MAE (Matsuura, Wilmott 2005). Instead of comparing the absolute values, the rooted squares are being compared.

$$RMSE = \sqrt{\frac{\sum_{t=1}^{T} (y_t - y_t')^2}{T}}$$
As discussed by Matsuura and Willmott (2005), the RMSE presents a weakness in its sensitivity to outliers, due to squaring the errors. Contrary the RMSE gives a computational advantage over the MAE in different mathematical models and calculating model error sensitivity, with the strength of not using the absolute values.

3.9 APPLICATION OF VALUE AT RISK

The basic concept of Value-At-Risk (VaR) is to measure and quantify the risk of large losses in a specific asset or a portfolio in a set period of time. Hence, it is a measure of the value of an investment given the risk of the investment, the value at risk. Value-at-Risk calculations can be applied when one wants to know the maximum loss in value of an asset or portfolio in a set period of time, with a given confidence level. As an example, with a 95% confidence level, what is the largest possible loss in the value of a portfolio the following trading day? This can be explained by the following formula (Cao et al. 2010).

\[ \text{VAR}_p (1 - \alpha) = -\sigma_p * q_p (\alpha) \]

Where \( \alpha \) = confidence level of the Value-at-Risk, \( \sigma_p \) = the standard deviation of the portfolio, and \( q \) = the standardized distribution.

3.9.1 FORECASTING THE VALUE-AT-RISK

When estimating the Value-at-Risk for a portfolio, there are three commonly used approaches; Historical simulation, Parametric approaches and Monte Carlo simulation. In this thesis a parametric approach to forecast the Value-at-Risk is used. The parametric approach uses the Variance-Covariance matrix from which the portfolio variance can be estimated (Markowitz 1952)

\[ \sigma_p^2 = \sum_{i=1}^{N} \omega_i^2 \sigma_i^2 + 2 \sum_{i=1}^{N} \sum_{j=1}^{N} \sigma_i \sigma_j \gamma_{ij} \]

Where \( \sigma_p^2 \) = the portfolio variance, \( \omega_i \) = weight of asset \( i \), \( \sigma_i \) = standard deviation of asset \( i \), \( \sigma_j \) = standard deviation of asset \( j \) and \( \gamma_{ij} \) = the covariance between asset \( i \) and asset \( j \).

The Value-at-Risk is then calculated by multiplying the portfolio standard deviation with the average mean of the portfolio. This approach is flexible, easy to handle and is used by many
practitioners. In contradiction to the historical simulation this enables one to analyze how the Value-at-Risk calculations is effected in different situations by being able to add specific market scenarios (Wiener 1997).

The mean of the portfolio is then to be viewed as the $\mu$ and the portfolio variance is viewed as $\sigma$ in a normal distribution. This is one of the drawbacks of this approach since it assumes a normal distribution, which the market returns are not. These parameters are then used to estimate the distribution for the portfolio. From this distribution, the lower 1% or 5% quantile is then drawn and the given value is the lower Value-at-Risk limit (Wiener 1997). In this thesis, the variance-covariance matrix is refitted each day in line with section 3.7.2., which leads to a new fitted distribution each day.

3.9.2 BACKTESTING THE VALUE-AT-RISK

To test the performance of the Value-at-Risk forecasts one can count the violations in relation to the expected number of violations. A violation is if the actual loss is larger than the predicted Value-at-Risk value. This can then be used as a measure of the performance of the Value-at-Risk forecast. The expected number of violations ($q$) can be written as

$$q = (1 - \alpha) \times t$$

where $\alpha = confidence\ level$ and $t = number\ of\ observations$.

The violation ratio ($VR$) can then be calculated by

$$VR = \frac{z}{q}$$

where $z = actual\ number\ of\ violations$

If the actual violations correspond perfectly to the expected violations, the result would be a violation ratio equal to 1, indicating that the model predict the losses appropriately.

If the violation ratio is larger than 1, it indicates that the model underestimates the risk. Correspondingly, a violation ratio smaller than 1 indicate that the model overestimates the risk.

To test whether the actual violations differ from the predicted violations, a Kupiec test is performed. The Kupiec test is a proportion of failures-test, which examines whether or not
the number of Value-at-Risk exceedances is in line with the chosen confidence level. The test is based on a likelihood ratio of the binomial distribution presented below (Kupiec 1995).

\[ f(x) = \binom{N}{x} p^x (1 - p)^{N-x} \]

\[ LR = -2 \ln[(1 - p)^{N-x} p^x] + 2 \ln\left[\left(1 - \frac{x}{N}\right)^{N-x} \left(\frac{x}{N}\right)^x\right] \]

The null hypothesis of the test is that the number of Value-at-Risk violations correspond to the chosen confidence level. If the null hypothesis is rejected, the models’ predicted Value-at-Risk is violated too often for it to be seen as an accurate prediction method.

The forecasting and backtesting are carried out on two equally weighted portfolios. One consisting of USD/SEK and USD/NOK denoted as Portfolio A, and the other consisting of USD/SEK, USD/NOK and USD/DKK denoted as Portfolio B.
4. EMPIRICAL ANALYSIS

This section presents the results of the data evaluation and model evaluation, as well as the estimations of the correlation and covariance matrices. This is followed by a discussion rooted in economic theory, aiming to build a foundation for answering the main issues of the thesis.

4.1 THE DATA

The analysis and estimations performed in the report is done in R, partly by own computations by the authors, but also with help of the existing packages rmgarch and rugarch (Ghalanos 2018). The data are obtained from Thomson Reuters EIKON database (2018) and consist of the time series of the daily opening spot rates for the USD/SEK, USD/NOK and USD/DKK. The time series for the estimations is in the period 2004-01-01 - 2013-12-31. In Appendix the inflation level, interest rates and GDP of each country during the period 2004-2014 are presented in Figure 11, 12 and 13 to 14 respectively. This data are obtained from OECD web page (OECD 2018).

The movement of the Scandinavian exchange rates are similar, which can be seen in Figure 1, where for example all three currencies depreciated during the financial crisis of 2008 as well as appreciated during the boom in the years ahead of the crisis.

![Figure 1. Spot rates for the USD/SEK USD/NOK USD/DKK.](image-url)
Considering the logarithmic returns, the pattern is similar for all three exchange rates. The volatility is relatively low until the third quarter of 2008, when the financial crisis hit, and the volatility increased by a great amount as seen by the large spikes. The effects of the financial crisis are shown in a continuous higher level of volatility in the years to follow, although gradually decreasing towards the level of volatility before the financial crisis. Another factor might be the Eurozone crisis that began in 2010, which contributed to a higher volatility in the Scandinavian exchange rates.

Figure 2: Logarithmic returns of USD/SEK

Figure 3: Logarithmic returns of USD/NOK
As shown in Table 1, the mean and standard deviation of the different exchange rates are similar to a large extent. The USD/SEK and USD/NOK shows some lower minimum and maximum values than the Danish krone, which might be explained by the fact that the Danish krone is pegged to the Euro, i.e. the USD/DKK shows the same movement as the USD/EUR. Furthermore, all three exchange rates exhibit signs of non-normality in form of a positive skewness as well as excess kurtosis.

**Table 1: Descriptive statistics of the logarithmic returns of the exchange rates**

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>St. Dev.</th>
<th>Min</th>
<th>Max</th>
<th>Skewness</th>
<th>Kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td>USD/SEK</td>
<td>0.0000</td>
<td>0.009</td>
<td>-0.049</td>
<td>0.048</td>
<td>0.133</td>
<td>5.562</td>
</tr>
<tr>
<td>USD/NOK</td>
<td>0.0000</td>
<td>0.008</td>
<td>-0.054</td>
<td>0.048</td>
<td>0.246</td>
<td>6.089</td>
</tr>
<tr>
<td>USD/DKK</td>
<td>0.0000</td>
<td>0.007</td>
<td>-0.038</td>
<td>0.034</td>
<td>0.039</td>
<td>4.814</td>
</tr>
</tbody>
</table>

Figure 4: Logarithmic returns of USD/DKK
Figure 5: Squared returns of USD/SEK

Figure 6: Squared returns of USD/NOK

Figure 7: Squared returns of USD/DKK
As discussed in section 2.2 volatility clustering is a common phenomenon in financial time series. By visually examining the squared returns of the exchange rates, it appears to be that they suffer from volatility clustering. To quantify this, descriptive statistics for the average squared returns of the three series for periods of 2004Q1-2008Q2, 2008Q3-2009Q2 and 2009Q3-2013Q4 are found below.

Table 2: Descriptive statistics of three different time periods average squared returns

<table>
<thead>
<tr>
<th>Period</th>
<th>Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>Period 1</td>
<td>0.000039</td>
</tr>
<tr>
<td>Period 2</td>
<td>0.000178</td>
</tr>
<tr>
<td>Period 3</td>
<td>0.000060</td>
</tr>
</tbody>
</table>

As seen in Table 2 the mean of the squared returns of period 2 is five and three times higher than the mean of the squared returns of the period before and the period after the financial crisis respectively which indicates that the time series suffer from volatility clustering. This along with the skewness and excess kurtosis shown in Table 1 further indicates that these time series exhibit the normal characteristics of financial time series.

4.1.1 DATA INSPECTION

Before beginning to construct the GARCH and MGARCH, the data is examined to confirm that the logarithmic returns of the exchange rates are non-normal which calls for the use of models that can handle heteroscedastic data.

4.1.1.2 RESULTS OF NORMALITY TESTS

The univariate series are tested to confirm that the individual exchange rate series are non-normal.

Table 3: Jarque-Bera scores of the exchange rates

<table>
<thead>
<tr>
<th></th>
<th>$\chi^2$</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>USD/SEK</td>
<td>721</td>
<td>0.000</td>
</tr>
<tr>
<td>USD/NOK</td>
<td>1062</td>
<td>0.000</td>
</tr>
<tr>
<td>USD/DKK</td>
<td>358</td>
<td>0.000</td>
</tr>
</tbody>
</table>
In all cases, the test statistic concludes that the null hypothesis is rejected on a five percent level. Thus, the Jarque-Bera test gives support to that the univariate samples of the logarithmic returns are not normally distributed which is expected due to the characteristics of financial time series.

To further investigate the non-normality in the sample, a test for multivariate normality is performed.

<table>
<thead>
<tr>
<th>Table 4: Henze-Zirkler score</th>
</tr>
</thead>
<tbody>
<tr>
<td>HZ</td>
</tr>
<tr>
<td>MVAR</td>
</tr>
</tbody>
</table>

The test indicates that the data are not multivariate normally distributed and hence the assumed conditional distribution used further on is the multivariate Student’s-t distribution, which allows fat tails. In the next section, the multivariate Student’s-t distribution is derived.

4.1.2 MULTIVARIATE STUDENT’S-T DISTRIBUTION

As discussed in Orskaug (2009) there are several different approaches for generalizing the univariate Student’s-T distribution to the multivariate representation. The optimization relies on the conditional distribution. The most commonly used generalization is the one considered in this thesis, in line with Orskaug (2009). The joint density of the standardized errors $z_t$ is

$$f(z_t|\nu) = \prod_{t=1}^{T} \frac{\Gamma \left( \frac{\nu + n}{2} \right)}{\Gamma \left( \frac{\nu}{2} \right) [\pi (\nu - 2)]^{n/2}} \left[ 1 + \frac{z_t^T z_t}{\nu - 2} \right]^{-\frac{n+\nu}{2}}$$
4.1.3 TEST OF DYNAMIC CORRELATIONS

To evaluate whether or not there is a dynamic structure in the correlations, a test of constant correlation, presented in section 3.5.1, is performed with the following output.

<table>
<thead>
<tr>
<th>Table 5: Test of dynamic correlations</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\chi^2$</td>
</tr>
<tr>
<td>----------</td>
</tr>
<tr>
<td>9.7125</td>
</tr>
</tbody>
</table>

The null of constant correlation is rejected in favor of the alternative, which implies that there is a dynamic structure present in the correlations. Hence, a DCC-GARCH model is estimated to handle the dynamic structure in the correlations.

4.2 MODEL RESULTS

The following section includes a presentation of the AIC and BIC scores, with methodology presented in section 3.6.2, for different orders of the DCC-GARCH.

4.2.1 CHOOSING THE BEST MODEL AIC/BIC

<table>
<thead>
<tr>
<th>Table 6: AIC and BIC scores for different DCC models</th>
</tr>
</thead>
<tbody>
<tr>
<td>AIC</td>
</tr>
<tr>
<td>-----</td>
</tr>
<tr>
<td>DCC (1,1)</td>
</tr>
<tr>
<td>DCC (1,2)</td>
</tr>
<tr>
<td>DCC (2,1)</td>
</tr>
<tr>
<td>DCC (2,2)</td>
</tr>
<tr>
<td>DCC (2,3)</td>
</tr>
<tr>
<td>DCC (3,2)</td>
</tr>
<tr>
<td>DCC (3,3)</td>
</tr>
</tbody>
</table>

In Table 6, it is shown that the model with the lowest BIC score is the DCC-GARCH (1,1). The GARCH order in all of the models is the GARCH (1,1) since it is standard and acknowledged as a generally good model in financial time series. Discussed in section 3.6.2, the BIC penalizes more complex models more heavily, which can be seen in Table 6. The
model choice is based on the AIC and BIC scores. As seen in Table 6, the AIC and BIC is rising with the added orders. The chosen models are therefore the DCC-GARCH (1,1), DCC-GARCH (1,2) and the DCC-GARCH (2,1). As the table shows, the DCC-GARCH (1,1) has the lowest BIC value but the other two are included in the estimation process for the possibility of comparison.

4.2.2 DCC ESTIMATION RESULTS

The fit of the DCC-GARCH (1,1) model is presented below, with the estimated coefficients.

Table 7: DCC-GARCH (1,1) parameter estimates.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimated Coef</th>
<th>St. Err.</th>
<th>t-value</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(c_{11})</td>
<td>-0.0002</td>
<td>0.0001</td>
<td>-1.1082</td>
<td>0.2678</td>
</tr>
<tr>
<td>(\alpha_{11})</td>
<td>0.0382</td>
<td>0.0119</td>
<td>3.2244</td>
<td>0.0013</td>
</tr>
<tr>
<td>(\beta_{11})</td>
<td>0.9567</td>
<td>0.0108</td>
<td>88.9845</td>
<td>0.0000</td>
</tr>
<tr>
<td>(c_{22})</td>
<td>-0.0002</td>
<td>0.0001</td>
<td>-1.3823</td>
<td>0.1669</td>
</tr>
<tr>
<td>(\alpha_{22})</td>
<td>0.0316</td>
<td>0.0044</td>
<td>7.2506</td>
<td>0.0000</td>
</tr>
<tr>
<td>(\beta_{22})</td>
<td>0.9641</td>
<td>0.0032</td>
<td>298.2285</td>
<td>0.0000</td>
</tr>
<tr>
<td>(c_{33})</td>
<td>-0.0002</td>
<td>0.0001</td>
<td>-1.4933</td>
<td>0.1354</td>
</tr>
<tr>
<td>(\alpha_{33})</td>
<td>0.0350</td>
<td>0.0044</td>
<td>7.8901</td>
<td>0.0000</td>
</tr>
<tr>
<td>(\beta_{33})</td>
<td>0.9623</td>
<td>0.0033</td>
<td>297.4652</td>
<td>0.0000</td>
</tr>
<tr>
<td>(\alpha)</td>
<td>0.0190</td>
<td>0.0041</td>
<td>4.6580</td>
<td>0.0000</td>
</tr>
<tr>
<td>(\beta)</td>
<td>0.9760</td>
<td>0.0062</td>
<td>158.5197</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

For the DCC-GARCH (1,1), all parameters except the means are significant on a 5% level. It can therefore be concluded that the DCC-GARCH (1,1) accurately captures both the univariate ARCH and GARCH structures of the time series as well as the interaction between the different assets.

Furthermore, the DCC-GARCH (1,1) in Table 7, it can be seen that all the individual GARCH series fulfill the criteria that \(\alpha + \beta < 1\). The DCC parameters also follow the same criteria. It can be seen that the scalars \(\alpha_{11} + \beta_{11} = 0.9949\), \(\alpha_{22} + \beta_{22} = 0.9777\), \(\alpha_{33} + \beta_{33} = 0.9973\) as well as the joint parameters \(\alpha + \beta = 0.9950\). The individual parameters are all larger than zero at the same time as the sum is less than 1, which ensures positive unconditional variance.
Looking at the DCC-GARCH (1,2) and DCC-GARCH (2,1) parameter estimates in Table 8 and 9, it can be seen that the GARCH parameters are practically the same as for the DCC-GARCH (1,1). Some interesting information can be found in the DCC parameters, shown last in each table. When comparing them it can be seen in Table 9 that the $\alpha_2$ term for the DCC-GARCH (2,1) has a value close to zero, with the other parameters being close to identical to the DCC-GARCH (1,1).

Considering the parameter estimates for the DCC-GARCH (1,2) in Table 8, it is noted that no DCC-parameter estimates are significant. However, in contradiction to the DCC-GARCH (2,1) it can be seen that the DCC-parameter estimates differ, which implies a different
structure. Therefore, evaluating the DCC-GARCH (1,2) further might be of interest if the parameters are to be significant, while continuing with the DCC-GARCH (2,1) is not. Since the DCC-GARCH (1,1) is the only model with all significant parameters, it is also the only model used to estimate the conditional correlations and covariances as well as forecasting the conditional correlations and Value-at-Risk.

4.3 MODEL EVALUATION

To confirm that the model has accurately captured the information in the time series, a test for autocorrelation in the residuals is performed. The residuals of the estimated model should be merely white noise.

4.3.1 LJUNG-BOX TEST

As derived in section 3.6.1, a Ljung-Box test is performed on the residuals of the estimated univariate GARCH (1,1) models. The result of the test indicate that the residuals contains no autocorrelation, meaning that the model does an adequate job of explaining the variance over time.

<table>
<thead>
<tr>
<th></th>
<th>( \chi^2 )</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>USD/SEK</td>
<td>28.337</td>
<td>0.1017</td>
</tr>
<tr>
<td>USD/NOK</td>
<td>24.893</td>
<td>0.2056</td>
</tr>
<tr>
<td>USD/DKK</td>
<td>23.894</td>
<td>0.2470</td>
</tr>
</tbody>
</table>

As shown in Table 10 all three univariate time series used in the DCC model, have a p-values larger than 0.05 with 20 degrees of freedom and does therefore not exhibit any significant signs of autocorrelation in the residuals.

4.4 ESTIMATION DISCUSSION

As seen by the estimated conditional correlations of the markets in Figure 15-17 (Appendix), the correlations between the exchange rates are high, drifting around 0.8, and falling to levels as low as 0.6. The high correlations between the SEK, NOK and DKK are likely a result of the similarities of the countries when it comes to economic and political decisions. The
countries have had similar interest rates, inflation and GDP during the given time period. When evaluating the correlations between the Danish krone with the Swedish krona and the Norwegian krone, the correlations are high here as well, even though the Danish krone is pegged to the Euro. A potential explanation to this is the close ties Sweden and Norway have with the countries in the Eurozone. This due to Sweden being a part of the European Union and Norway being part of the European Economic Area (EEA). The results may not be very surprising, but still bring valuable information when e.g. hedging a portfolio consisting of currencies.

A drop in the conditional correlation is displayed 2006Q3-2007Q2. What lead to this drop in correlations cannot be said with certainty, but one possible explanation for the lower correlations between the SEK and the NOK can be the difference in inflation rates. However, the true nature of this drop is something that needs to be studied further.

When evaluating and analyzing the estimated conditional covariances and correlations there are also a couple of macroeconomic events and factors to be considered as possible sources of explanation for structural breaks in the pattern. The global financial crisis of 2008 and the Eurozone crisis around 2010 are both possible sources. The effects of the financial crisis in 2008 are clearly illustrated in both the conditional covariances and correlation. As seen in Figure 18, Figure 19 and Figure 20, the covariance shows clear spikes during the financial crisis in 2008, which can be seen as the main factor for the increased covariance. However, there are no clear signs of a change in the correlations, which an increase in the univariate series’ volatility might be an explaining factor for.

During the period of the financial crisis, the central banks of the United States, Sweden, Denmark and Norway all established a series of reductions in the key interest rate. As an example, the Swedish central bank cut the country’s key interest rate during the period of February 2008 to September 2009 from 4.75% to 0.25%. (Figure 12, Appendix). The Federal Reserve System (FED) started cutting their interest rate earlier than the others (Figure 12, Appendix). Following the interest rate parity, the Swedish, Norwegian and Danish currencies all depreciated when the central banks began to lower the interest rates. The interest rates are also a potential explanation for the behavior of the exchange rates during the financial crisis when it comes to the extent of depreciation and covariance. The results clearly show that the SEK and NOK had a larger depreciation as well as a higher covariance, than what they had
against the DKK. The European Central Bank made interest rate cuts, smaller than those of the Swedish and Norwegian Central Banks, which lead to an appreciation of the DKK against the SEK and NOK as shown in (Figure 12, Appendix; Figure 1)

During the years of the Eurozone crisis, similar patterns for the different exchange rates can be observed in the conditional correlations as well as the conditional covariances. A cluster of higher conditional covariance can be observed between 2010 and 2012, while the conditional correlation does not show any tendencies of a clear pattern. This indicates that the separate exchange rates have higher volatility during this period of time, which might be due to the instability in European economies during these years. Considering the conditional correlation for the three series, they seem to have similar behavior in the second half of 2012. The correlation substantially drops for all series, where SEK vs DKK has the largest drop from about 0.8 to 0.5. One factor behind the drop in correlation during these years may be that as stated earlier, the DKK is pegged to the Euro, and therefore behaves different.
5. FORECASTING

The forecasting, as discussed in section 3.7 is carried out on the period 2014-01-01 - 2014-12-31 where a one-day rolling forecast window, with a refitted model every day is applied. The forecasting performance the DCC-GARCH is evaluated. As shown in Figure 8, Figure 9 and Figure 10, the forecasts by the DCC-GARCH model are all stable and does not seem to fluctuate a lot.

To evaluate the forecasting performance of the DCC-GARCH (1,1) the predicted correlations is compared towards the actual returns based on the proxy derived in section 3.7. They are both visually examined and measured by evaluation measures.

5.1 FORECASTS

In line with the previous estimations for the period 2004-2013, the predicted correlations for all three currency pairs continue to be high, although dropping slightly compared to the earlier stages of the series, ranging around 0.7. What can be seen is that the predicted correlations of the DCC-GARCH models is very stable and not especially volatile.

5.2 FORECAST EVALUATION

![Figure 8: Predicted and Realized correlations SEK/NOK DCC (1,1)](image-url)

Figure 8: Predicted and Realized correlations SEK/NOK DCC (1,1)
When visually examining the forecasts, the forecasts seem to follow the same pattern as the true correlations, accurately predicting increasing and decreasing correlations. However, the DCC-GARCH model does not accurately capture the magnitude of the spikes in correlation. This follows what is shown in the previous section, the stability of the predictions of the DCC-GARCH model.
When examining the RMSE and MAE for the different predicted correlations, it is seen that all three seem to range around 0.25 for the RMSE and 0.185 for the MAE. However, it would have to be compared to additional models, to evaluate how well the model performs.

When comparing the performance of the DCC-GARCH model for the different currencies, it is seen that during 2014 the models did a better job in predicting the correlation between the SEK and the NOK.

All of these results must be taken with a great amount of caution since the realized correlations is based on a moving average rather than realized correlations based on intraday data which reduces the extent of conclusions that can be made from the result.
6. VALUE-AT-RISK

6.1 VALUE-AT-RISK FORECASTING

As previously discussed the Value-at-Risk is forecasted at a one-day horizon on a 95% and 99% confidence level, and is forecasted for 260 days. The predicted Value-at-Risk values show that the predicted daily loss is higher at the end of the year. The forecasted Value-at-Risk is performed with the DCC-GARCH (1,1). Forecasting and backtesting of the predicted Value-at-Risk is carried out on two different equally weighted portfolios.

**Portfolio A:** Swedish Krona and Norwegian Krone

**Portfolio B:** Swedish Krona, Norwegian Krone and Danish Krone

When looking at the forecasted Value-at-Risk, shown in Figure 21-24 (Appendix), it shows that the Value-at-Risk forecasted by the DCC-GARCH model is not especially volatile. Furthermore, the DCC-GARCH model accurately predicts the magnitude of the returns, i.e. when the returns get more volatile, the DCC-GARCH model predicts a lower boundary for the Value-at-Risk.

It is also evident that the DCC-GARCH model predicts a lower Value-at-Risk boundary for portfolio A compared to portfolio B, which is a result of the fact that Portfolio A is less diverse and therefore has a higher portfolio variance (Figure 21 to 24, Appendix).
6.2 VALUE-AT-RISK BACKTESTING

As derived in section 3.9.2, the Violation Ratios and Kupiec test are examined to evaluate the Value-at-Risk performance by the DCC-GARCH model. Above, the violation ratios and Kupiec test on a 1% and 5% level is found.

When evaluating the Kupiec tests at the 1% Value-at-Risk level, it is evident that the forecasted Value-at-Risk is in line with the actual returns of the portfolios. The Kupiec test shows significant results for the model, with p-values larger than 0.05. This indicates that the DCC-GARCH model is adequate for calculating the Value-at-Risk at a 1% level.

When evaluating the Kupiec tests it can be seen that at the 5% level, the Value-at-Risk for both Portfolio A and Portfolio B accurately predict the risk, having p-values higher than 0.05. Although, with violation ratios of 0.6154 and 0.5385 and p-values relatively close to 0.05, there are some tendencies that the DCC-GARCH model overestimates the risk for both Portfolios.

<table>
<thead>
<tr>
<th>Table 12: Violation Ratios and Kupiec Tests for Portfolio A</th>
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<tbody>
<tr>
<td>Violation Ratio</td>
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<td>Kupiec Test</td>
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<td>p-value</td>
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<table>
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<th>Table 13: Violation Ratios and Kupiec Tests for Portfolio B</th>
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</thead>
<tbody>
<tr>
<td>Violation Ratio</td>
</tr>
<tr>
<td>Kupiec Test</td>
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<td>p-value</td>
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7. CONCLUSION

The aim of this thesis was to evaluate to what extent the DCC-GARCH model can be used to predict the conditional correlations between the Scandinavian currencies quoted against the dollar. The report did also examine how well the DCC-GARCH model performs when being used for Value-at-Risk calculations for different equally weighted portfolios.

From the results in the report, it has shown that DCC-GARCH model shows tendencies of being able to estimate changes in conditional correlations for Scandinavian currencies against the dollar when being calculated on a one-day rolling forecast. It is also evident that even though the DCC-GARCH model can predict the movement of the correlation, it keeps underestimating the magnitude of the change in conditional correlation. Furthermore, for the forecasting period, it seems that the model does an equally good job of predicting the different correlations for different currency pairs. Although, an important note is that many results have a potential of becoming more accurate if they were to be based on high frequent data. This means that one has to be cautious when drawing conclusions from the results presented in this thesis.

When evaluating the Value-at-Risk performance of the DCC-GARCH model, it has been shown that the DCC-GARCH (1,1) model fulfills a purpose when forecasting the Value-at-Risk. Although the DCC-GARCH model had Kupiec test p-values higher than 0.05, the DCC-GARCH model showed tendencies of overestimating the risk at the 5% Value-at-Risk level. However, the DCC-GARCH model showed significant Value-at-Risk forecasts for both portfolios on a 1%, and 5% Value-at-Risk level, at the 5% significance level for the Kupiec test. The DCC-GARCH models’ computational advantages compared to other multivariate GARCH models, as well as showing significant Value-at-Risk results, are reasons to why practitioners might consider using the DCC-GARCH model in real life scenarios.
8. FURTHER STUDIES

When it comes to further studies concerning the multivariate DCC-GARCH models, there are a few areas that would be interesting. The first obvious improvement to this thesis would be to use intraday data to estimate the realized correlations. The possibility to use intraday data for the realized correlation would mean that the proxy for correlations would come closer to the actual correlation, which would result in a more accurate evaluation of the forecasts. Though, one thing to take into account when using high frequency data are the computational complexity in multivariate models, which could limit the use of the data depending on how often the model would have to be refitted.

A second area of interest would be to keep studying other smaller, more volatile currencies. Many studies have been conducted on the major Foreign Exchange quotas, but not to the same extent on the less commonly traded currencies. Those studies would have the potential to broaden the perspective on the use and performance of DCC-GARCH models. In addition to this, this thesis only addressed the DCC-GARCH model. More models and their performance could be examined, like the models that directly model the conditional covariance matrix e.g. VEC and BEKK models.

Another potential extension to this study would be to more thoroughly examine how the DCC-GARCH model performs during calm and less calm periods. In this thesis the financial crisis of 2008 and the Eurozone crisis were discussed as well as their effects on the conditional correlations. However, this thesis did not compare the forecasting performance of the DCC-models during times of different volatility.

Lastly, to get a deeper understanding of the Value-at-Risk applications, one could consider forecasting and evaluating other types of portfolios, with different weights or assets. Another potential extension is to compare the DCC-GARCH model to other uni- and multivariate GARCH models.
REFERENCES


Matsuura, Kenji and Willmott, Cort J. 2005, *Advantages of the mean absolute error (MAE) over the root mean square error (RMSE) in assessing average model performance*.


ELECTRONIC REFERENCES


APPENDIX

Figure 11: Inflation

Figure 12: Interest rates
Figure 13: GDP Denmark, Norway, Sweden

Figure 14: GDP United States, Eurozone
Figure 15: Conditional correlations SEK/NOK DCC (1,1)

Figure 16: Conditional correlations SEK/DKK DCC (1,1)

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Figure 20: Conditional Covariance NOK/DKK DCC (1,1)
Figure 21: 1% Value-at-Risk for Portfolio A

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Figure 24: 5% Value-at-Risk for Portfolio B