GNSS Radio Occultation (GNSS-RO) is an opportunistic Earth sensing technique where GNSS signals passing through the atmosphere are received in low Earth orbit and processed to extract meteorological parameters. As signals are received along an orbit, the measured Doppler shift is transformed to a bending angle profile (commonly referred to as bending angle retrieval), which, in turn, is inverted to a refractivity profile. Thanks to its high vertical resolution and SI traceability, GNSS-RO is an important complement to other Earth sensing endeavors. In the lower troposphere, GNSS-RO measurements often get degraded and biased due to sharp refractive gradients and other complex structures. The main objective of this thesis is to explore contemporary retrieval methods such as phase matching and full spectrum inversion to improve their performance in these conditions. To avoid the bias caused by the standard inversion, we attempt to derive additional information from the amplitude output of the examined retrieval operators. While simulations indicate that such information could be found, it is not immediately straightforward how to achieve this with real measurements. The approach chosen is to examine reflected signal components and their effect on the amplitude output.
GNSS Radio Occultation Inversion Methods and Reflection Observations in the Lower Troposphere

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Licentiate Dissertation in Systems Engineering

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Abstract

GNSS Radio Occultation (GNSS-RO) is an opportunistic Earth sensing technique where GNSS signals passing through the atmosphere are received in low Earth orbit and processed to extract meteorological parameters. As signals are received along an orbit, the measured Doppler shift is transformed to a bending angle profile (commonly referred to as bending angle retrieval), which, in turn, is inverted to a refractivity profile. Thanks to its high vertical resolution and SI traceability, GNSS-RO is an important complement to other Earth sensing endeavors. In the lower troposphere, GNSS-RO measurements often get degraded and biased due to sharp refractive gradients and other complex structures. The main objective of this thesis is to explore contemporary retrieval methods such as phase matching and full spectrum inversion to improve their performance in these conditions. To avoid the bias caused by the standard inversion, we attempt to derive additional information from the amplitude output of the examined retrieval operators. While simulations indicate that such information could be found, it is not immediately straightforward how to achieve this with real measurements. The approach chosen is to examine reflected signal components and their effect on the amplitude output.
Preface

This thesis is divided into two parts. The first part describes the topic of GNSS Radio Occultation as well as the various software that was implemented to enable the included studies. The second part consists of the included scientific publications.
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Part I

Introduction
1 Motivation and overview

Weather forecasting has always been of great interest to society. Its applications range from day-to-day planning of activities for individuals, to optimizing grocery supply chains, to warning vulnerable citizens of dangers related to extreme weather events. With the onset of global warming, extreme weather forecasting and climate research grow more relevant than ever. The ability to accurately measure properties of the atmosphere is crucial for weather forecasting as well as for researchers to better understand the changing climate. With the use of satellites, remote sensing has enabled mankind to boost this ability significantly, since collection of data is no longer limited to easily accessible geographic sites. There are currently a wide range of sensors in orbit, observing the Earth both passively and actively to provide an accurate representation of the atmosphere. GNSS Radio Occultation (GNSS-RO) is one of them, and is a remote sensing technique that relies on signals (primarily used for navigation) from GNSS satellites. The GNSS signals are intercepted after having propagated through the Earth’s atmosphere, and inverted into vertical profiles of various atmospheric parameters. These vertical profiles are similar to what radiosonde measurements produce, with the benefit that they can be performed over oceans and other areas of the Earth that are difficult to reach. Approximately 2000 RO measurements are made every day and assimilated into weather models, and there are additional missions planned to increase the global coverage to take further advantage of the many different GNSS systems in orbit. The high vertical resolution and potential SI traceability of the measurements provide a very useful complement to other Earth weather sensing systems, which makes GNSS-RO one of the most important techniques for improving weather forecasting.

The objective of this thesis is to improve this inversion process for measurements in the lower troposphere. In RO, several different methods are used for inversion. In this work we mainly investigate one of them, called Phase Matching (PM)[1]. We focus on the amplitude of the output of the PM operator in an attempt to extract additional information about conditions at the Earth’s surface, such as the refractive properties of the
atmosphere. Furthermore we show that reflected signal components are admitted by the PM operator as well as the related Full Spectrum Inversion (FSI) operator, which can, potentially, contribute additional information about the lower troposphere.

The thesis is structured in two parts; the first part introduces the field of GNSS-RO, and the second part consists of the scientific publications made in this thesis work. The first part is organized as follows: Section 2 gives an overview of the history and applications of GNSS-RO as well as some of the current challenges in the fields. Section 3 discusses surface reflections as they relate to GNSS-RO. Section 4 introduces the implemented simulation environment, the Wave Optics Propagator (WOP). Section 5 gives an overview of the different ways to process data obtained from GNSS-RO measurements. Section 6 discusses the contributions of the included publications and how they relate to the main objective. Section 7 presents suggestions on how to move on with the findings presented in this thesis.

2 The GNSS radio occultation technique

GNSS-RO is an opportunistic technique, in which the main idea is to receive GNSS signals in low-Earth orbit (LEO) after they propagate through the Earth’s atmosphere. From the point of view of a GNSS satellite, an occultation event can be described as when the receiver either rises from or sets behind the Earth. While the receiver is not completely shadowed, the atmosphere will act as a lens on the signal. The fundamental aspect of the technique relies on the assumption of geometrical optics, i.e., considering the signal (an electromagnetic wave) as a manifold of rays, and the assumption of a spherically symmetric atmosphere in which these rays bend. Figure 2.1 describes the geometry of an occultation, observed from the side. This view is commonly referred to as the occultation plane. The bending angle and the impact parameter are denoted as $\alpha$ and $a$, respectively. The length of the vectors describing the positions of the GNSS and LEO satellites are $R_{GNSS}$ and $R_{LEO}$, respectively, and the angle between them is referred to as the separation angle, $\theta$. The angles between a signal
Figure 2.1: A schematic of the geometry in a RO event. The figure is not to scale.

ray and the GNSS and LEO vectors, respectively, are denoted as $\varphi_{GNSS}$ and $\varphi_{LEO}$. Thus, the received signal is used to measure the bending angle of the signal rays, which in turn can be analytically inverted to information describing the refractive properties of a vertical cross section of the atmosphere. The way this inversion is done is by means of the inverse Abel transform.

Employing Bouguer’s rule [2], a ray propagating through a spherically symmetric medium can be assigned a kind of angular momentum, or impact parameter, which is conserved:

$$rn(r) \sin \varphi = a = \text{const}, \quad (2.1)$$
where \( r \) is the distance from the center of curvature of the atmosphere \(^1\), \( n(r) \) is the refractive index, \( \varphi \) is the angle between the ray’s direction and a vector pointing to the center of curvature, and \( a \) is the conserved impact parameter. This impact parameter is conserved for all points on the ray, and is assumed to uniquely describe a ray, meaning that two different rays will not have the same impact parameter. At the ray trajectory’s lowest point \( r_t \), the tangent altitude, \( \varphi \) becomes a right angle and (2.1) is reduced to

\[
 r_t n(r_t) = a, \tag{2.2} 
\]

which yields a relation between impact parameter, height, and refractive index at the tangent point. It is useful to define another common measure of altitude related to the tangent point: the straight line tangent altitude (SLTA). The SLTA is the height of an imagined straight line between transmitter and receiver, measured at \( r_t \). As such, a zero SLTA means that the line of sight between satellites is tangential to the Earth’s surface, and a negative SLTA means that the line of sight is below the Earth’s surface. The assumption of geometrical optics allows us to analytically describe the bending angle \( \alpha \) of a ray, using the Abel transform. Assuming the ray passes through a spherically symmetric body whose refractive properties are described by \( n(r) \), the bending angle is defined as

\[
 \alpha(a) = -2a \int_{r_t}^{\infty} \frac{1}{n} \frac{1}{\sqrt{r^2 n(r)^2 - a^2}} \, dr. \tag{2.3} 
\]

Likewise, given a bending angle profile \( \alpha(a) \), the inverse Abel transform inverts it to refractive index:

\[
 n(a_1) = \exp \left( \frac{1}{\pi} \int_{a_1}^{\infty} \frac{\alpha(a)}{\sqrt{a^2 - a_1^2}} \, da \right), \tag{2.4} 
\]

\(^1\)the Earth and its atmosphere can be considered an ellipsoid. One defines an occultation plane, a radius of curvature, and a center of curvature, providing the closest match to the curvature of the ellipsoid for set of positions of the GNSS and LEO during an occultation.
which can then be mapped to $n(r)$ as (2.2) yields

$$r_1 = \frac{a_1}{n(a_1)}. \quad (2.5)$$

It is important to note that when discussing the atmosphere, it is convenient to talk about refractivity, $N$, rather than refractive index, $n$. The relation between them is as follows:

$$N = (n - 1)10^6. \quad (2.6)$$

In this thesis the term refractivity will exclusively refer to this definition. However, there are other ways to define it. Figure 2.2 shows a refractivity profile and its corresponding bending angle profile. It is convenient to subtract the Earth’s radius of curvature at $r_t$ from the impact parameter to get impact height on the $y$ axis.

As a received signal is processed to produce a bending angle profile, (2.3) and (2.4) are of central importance to RO, in that they define the relationship between ray paths and the refractivity of the atmosphere. Given a refractivity profile and background data, it is possible to estimate temperature, pressure, vapor pressure and electron density using the following approximation:

$$N \approx 77.6 \frac{P}{T} + 3.73 \cdot 10^5 \frac{P_w}{T^2} - 4.03 \cdot 10^7 \frac{n_e}{f^2}, \quad (2.7)$$

where $T$ is temperature in Kelvin, $P$ and $P_w$ are total pressure and vapor pressure in mbar, $n_e$ is free electron density in electrons per cubic meter, and $f$ is the signal frequency in Hz. In the neutral atmosphere, $n_e$ is negligible.

A notable limitation of the RO technique is the assumption of spherical symmetry. The application of Bouguer’s rule - and subsequently the Abel transform - enforces this assumption. In reality, the Earth and its atmosphere are not spherical, nor is the atmosphere spherically symmetric. This means that the bending angle profile is actually more of an averaged profile for a vertical cross section of the atmosphere, and refractive properties in
Figure 2.2: Example of a refractivity profile (left) and its corresponding bending angle profile (right). Compared to height, impact height will be shifted slightly upwards, due to the definition of impact parameter.
any horizontal direction can disturb the measurements. Additionally, when
the refractive gradient gets so sharp that the curvature of a ray matches
or exceeds the curvature of the Earth, super-refraction (SR) occurs. This
means that no ray will have its tangent point in a SR layer of the at-
mosphere, and we experience a corresponding gap in the received signal
(illustrated in Fig. 2.3. The inverse Abel transform cannot uniquely map a
bending angle profile with SR to just one refractivity profile, as illustrated
by Fig. 2.4.

Another complication comes from the ionosphere, which introduces a
bias in bending angle retrievals. The standard way of correcting this bias
is with a linear combination of the bending angles retrieved from the two
carrier frequencies of the GNSS transmitter [3]. This is effective in most
cases, although there will still be some residual ionospheric error [3]. Ad-
ditionally, occasional fluctuations in the ionosphere can greatly affect the
quality of the signal.

2.1 History

RO was initially conceived to examine the atmospheres of other planets
in our solar system, dating all the way back to the 1960s. In these cases,
a spacecraft is equipped with a radio transmitter that transmits a sig-
nal as it passes close to a celestial body. All known planets in our solar
system, as well as several other celestial bodies, have been examined in
this way. Once GPS satellites were in orbit around Earth, opportunisti-
cally using them as RO transmitters made the technique economically
achievable to apply to the Earth’s atmosphere, and the GPS/MET proof-
of-concept mission launched in 1995. The mission was considered a success,
and since then there have been several missions that have provided data
to operational weather forecast centers as well as to the research commu-
nity. Among the notable missions are CHAMP, GRACE, FORMOSAT-
3/COSMIC, METOP-A, METOP-B, and the recently launched METOP-
C. A more complete history of GNSS-RO is given by [4].
Figure 2.3: Comparison of a signal in a case with no SR (top left) and a signal with SR (bottom left), and their respective bending angle profiles (top right and bottom right). There is a gap in the SR signal amplitude corresponding to the atmospheric layer between the top of the SR layer and the Earth surface. The SR layer has caused the signals passing through this layer to go much deeper in SLTA, and appear in reverse order, i.e. deeper SLTA corresponds to heights increasing towards the SR layer top.
Figure 2.4: Example of a refractivity profile (left) containing SR and its corresponding bending angle profile (right). In theory, the spike around 2.6 km goes to infinity. The dashed refractivity profile is the result of inverting the bending angle profile with the inverse Abel transform. It can be seen that the refractivity from the inverted bending angle is biased.
2.2 Applications

One of the main applications for GNSS-RO is numerical weather prediction (NWP). The bending angle and refractivity profiles provide a useful complement to other techniques. Due to the geometry of an occultation, measurements offer a high vertical resolution compared to systems that scan the Earth "from above". Data from GNSS-RO contribute to approximately 10% of the overall forecast error reduction in the forecasts made by the European Centre for Medium-ranged Weather Forecasts (ECMWF) [5]. As the methods of processing RO data improve, reanalyses become appropriate, where the improved data is assimilated into weather models to assess the performance of different missions as well as the weather models themselves [6]. In addition to medium-range forecasting, several studies have shown how GNSS-RO data can improve severe weather forecasting [7, 8, 9].

Additionally, RO data is used to investigate specific phenomena and parameters in the atmosphere. Several studies show the impact of RO in determining the height of the planetary boundary layer [10, 11, 12]. As the record of RO data becomes longer and longer, it becomes more relevant to apply in climate research [13].

GNSS-RO is also useful for profiling the ionosphere, for example, in finding and tracking irregularities [14]. It has been suggested that gravity waves noticeable in the ionosphere can predict the wave height of tsunamis [15]. Furthermore, there are studies attempting to find anomalies in the ionosphere before earthquakes [16, 17].

2.3 Current problems

The quality of GNSS-RO measurements is still poor in the lower troposphere, especially in the subtropics. This is in part a consequence of the instrument not being able to track the signal. However, even intact signals may have poor quality. This can be partly explained by two previously mentioned problems: SR and horizontal gradients. The presence of SR regions in the atmosphere inevitably leads to some degradation, or loss, of
the signal [18]. To address this, Xie et al. proposed a parametric reconstruction of the lowest part of the refractivity profile, which, however, still requires certain a priori information [19, 20]. Furthermore, the existence of horizontal refractive gradients can cause several different rays to have the same impact parameter. These horizontal gradients cause significant bending angle errors [21].

Ionospheric effects on the signal can also introduce errors. Not only do ionospheric perturbations cause a residual ionospheric error after the standard correction, there are even cases where ionospheric conditions combined with strong gradients magnify errors in bending angle measurements [22].

Somewhat related to - or inspired by - GNSS-RO is the emerging technique LEO-LEO RO [23]. The idea is to launch both transmitter and receiver, to able to control the wavelength of the signal. As the cost of launching small satellites has decreased significantly in the last decades, this kind of mission is now feasible.

3 Surface reflections

While most signal rays received in LEO pass through the atmosphere without obstacle, some rays are reflected in the ocean. This phenomenon has been known for a long time [24, 25, 26], and despite the fact that the reflected components contain meteorological information [27] they are still not assimilated in NWP models. A modified Abel transform for reflected bending angles was recently proposed by [28], but has not, at the time of writing, been evaluated in assimilations.

The grazing reflections described above should not be confused with a related technique, GNSS reflectometry (GNSS-R) [29]. In contrast to GNSS-RO, GNSS-R instead receives signals that are reflected at a sharp angle in the ocean, providing information about, e.g., altimetry and ocean winds.
4 Wave optics propagator

There are two common ways of simulating the signal propagation in a RO event: ray-tracing (based on geometrical optics) and wave optics propagation (also referred to as physical optics). Both have their advantages and disadvantages. A ray tracer enables examination of specific phenomena, such as the effects of spherical inhomogeneity in any direction (i.e., conditions where the atmosphere is not spherically symmetric), scattering, polarization, etc. However, it is confined to examining the effects on single rays, and can not easily represent the entire signal as it would look in the LEO orbit. WOP, on the other hand, considers a two-dimensional geometry where it is easy to sample the field along any arbitrary orbit. Effects such as bending due to radial gradients as well as gradients along the propagation direction are incorporated. At the expense of not being able to examine individual rays, it is instead capable of capturing the multipath phenomenon. The simulation of a GNSS signal in the WOP is done in three stages. Firstly, a cylindrical wave is propagated from the transmitter to the beginning of the atmosphere, assuming vacuum. Secondly, the wave is propagated through the atmosphere by using the multiple phase screen (MPS) technique, also known as the Fourier Split-Step technique[30]. Finally, as the wave reaches vacuum again, it is propagated to orbit by computing a diffraction integral. It is possible - and in some cases even preferable - to use the MPS technique all the way to orbit, though this significantly increases the run-time of the simulation.

At BTH we have implemented a WOP based on the work of Rasch [31], capable of simulating occultation events with one-dimensional and two-dimensional refractivity profiles. This provides an opportunity to validate and examine the various retrieval algorithms investigated in the thesis. In the following, a description of the WOP is presented, specifically the MPS technique and the propagation to orbit, described in further detail in [31].

The WOP geometry is two-dimensional with the origin in the center of a circular Earth, and considers the atmosphere inside a box, as shown in Fig. 4.1. This box is positioned in a way that puts a significant part of the Earth inside. The GNSS transmitter is considered to be stationary,
Figure 4.1: A typical box in WOP, within which the Earth and its atmosphere are considered. The dashed line represents the LEO orbit.

approximately 26000 km away from Earth, at a vertical position equal to the middle of the box’s side. The LEO transmitter’s orbit segment has a constant radius of approximately 800 km more than the radius of the Earth, which is approximately 6371 km. The wave’s principal direction is along the z axis, and the phase screens are aligned along the y axis. The dimensions of the box itself depend on the desired resolution of the simulation as well as on the region of interest in the atmosphere.

4.1 Multiple Phase Screens

The fundamental idea in the WOP is to propagate a cylindrical wave through the atmosphere by considering the Helmholtz equation:

$$\nabla^2 \psi + k_0^2 n^2 \psi = 0,$$

where $k_0$ is the wave number in vacuum, $n$ is the refractive index of the medium, and $\psi$ is the electrical field. By assuming that the wave travels predominantly in the $z$-direction and letting $u$ denote the simplified field,
the scalar version of (4.1) can be reduced to

$$\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} + 2ik_0 \frac{\partial u}{\partial z} + k_0^2(n^2 - 1)u = 0, \quad (4.2)$$

which in the frequency domain yields

$$-\omega^2 \tilde{u} + \frac{\partial^2 \tilde{u}}{\partial z^2} + 2ik_0 \frac{\partial \tilde{u}}{\partial z} + k_0^2(n^2 - 1)\tilde{u} = 0, \quad (4.3)$$

where $u$ and $\tilde{u}$ describe the wave in space and a spatial frequency domain, respectively. The MPS technique relies on considering the medium as a set of vertical screens. Between each screen, the wave is propagated under the assumption that $n$ is constant in $z$. The change in $n$ is applied as a phase delay every time the wave reaches a new phase screen.

The propagation between phase screens is done in the frequency domain, where $\tilde{u}(\omega, z) = \mathcal{F}\{u(y, z)\}$ and $\mathcal{F}$ denotes the Fourier transform. Assuming a constant $n$, this finally yields

$$u(y, z + \Delta z) = \exp\left(ik_0(n - 1)\Delta z\right)$$

$$\cdot \mathcal{F}^{-1}\left\{\exp\left(i k_0 \left(\sqrt{1 - \frac{\omega^2}{k_0^2}} - 1\right)\Delta z\right) \mathcal{F}\{u(y, z)\}\right\}. \quad (4.4)$$

At this point we can use (4.4) to propagate the field in small steps of $\Delta z$. The first exponential applies the phase delay due to the atmosphere’s refractive index, and the second factor propagates the wave in free space. Figure 4.2 demonstrates the amplitude of the GNSS signal as it passes through an atmosphere.

To minimize the noise from discontinuities while using the Fourier transform, Rasch[31] suggests applying a tapering window to the edges of the phase screens as well as to the segments that lie inside the Earth. However, setting the field amplitude to zero inside the Earth with no tapering will result in a behavior similar to surface reflection.
4.2 Propagation to orbit using a diffraction integral

Using the MPS technique to propagate the wave all the way to the LEO is possible, but computationally costly and not always necessary. If we assume a neutral atmosphere (without an ionosphere), there will be a large region between the atmosphere and LEO where no refraction occurs. Thus, we can solve the Helmholtz equation using a diffractive integral on the last
screen:

\[ u(y_0, z_0) = \sqrt{\frac{k_0}{2\pi}} \int_S u(y) \frac{z_0}{\sqrt{z_0^2 + (y - y_0)^2}} \exp(ik_0\sqrt{z_0^2 + (y - y_0)^2} - i\frac{\pi}{4}) \frac{1}{(z_0^2 + (y - y_0)^2)^{1/4}} dy, \]

where \( y_0 \) and \( z_0 \) are the coordinates of the LEO satellite at a point in orbit, and \( S \) is the screen. For simulations where an ionosphere is considered at high altitudes, this solution is insufficient. However, for the purposes of this thesis, there is no need to consider an ionosphere. At this point, \( u \) gives the amplitude and total phase of the field at any point in orbit. The total phase can be interpreted as a ray’s optical path length in meters. Subtracting the distance between transmitter and receiver results in excess phase, which is the quantity commonly used by data centers.

5 Processing of RO data

The processing of GNSS-RO signals can be roughly divided into three steps. At the lowest level (by data centers typically referred to as level 0), binary data is parsed to reconstruct the excess phase and amplitude measurements of a signal. Additionally, it is important to extract the geometric parameters of the system, such as position, velocity and radius of curvature of the Earth. The output of this processing step is then a data product commonly referred to as level 1, containing phase, amplitude and ephemeris data, such as orbit coordinates (this is also the output of the WOP). In the following step of processing, the bending angle profile is retrieved, and then inverted to refractivity using Abel inversion. A refractivity profile along with the various atmospheric parameters yielded from (2.7) constitute level 2 data products. Averaging these profiles into gridded monthly means yields the final level 3 data products. This section describes in further details various ways of bending angle retrieval, i.e., the first processing step on level 1. There are two main categories of retrieval used by data centers - geometrical optics (GO) and wave optics (WO). Furthermore, related to WO there are time-frequency methods of analysis that yield two-dimensional spectra instead of one-dimensional profiles.
5.1 Geometrical optics

As the name implies, GO retrieval relies on the geometry of the occultation and Bouguer’s rule. The geometry yields that

\[ \alpha = \varphi_{\text{GNSS}} + \varphi_{\text{LEO}} + \theta - \pi, \tag{5.1} \]

and Bouguer’s rule as \( n \) approaches unity becomes

\[ r_{\text{GNSS}} \sin(\varphi_{\text{GNSS}}) = r_{\text{LEO}} \sin(\varphi_{\text{LEO}}) = a. \tag{5.2} \]

Finally, the frequency shift (Doppler) \( f_d \) relates to the system thus,

\[ f_d = \frac{f}{c} (v_{\text{GNSS}} \cdot \hat{e}_{\text{GNSS}} + v_{\text{LEO}} \cdot \hat{e}_{\text{LEO}}), \tag{5.3} \]

where \( f \) is the carrier frequency, \( c \) is the speed of light, \( v \) are the respective velocities, and \( \hat{e} \) are the directions of the rays at transmitter and receiver. With this, the bending angle can be derived iteratively. In the WOP, the GNSS transmitter is considered to be stationary, and the LEO is considered to be circular, which allows us to use a much simpler equation without considering the Doppler:

\[ \frac{d\psi}{ds} = -\sin \varphi_{\text{LEO}}, \tag{5.4} \]

where \( \psi \) is the optical path length between GNSS and LEO in meters (given by the excess phase of the signal and the distance between the satellites), and \( ds \) is an infinitesimally small orbit segment.

This method of retrieval has a resolution that is limited by the size of the ray’s Fresnel zone, which can range between approximately 0.3 km and 1.4 km, depending on refractivity and frequency. Figure 5.1 shows a model bending angle profile corresponding to a refractivity profile with a local peak at a height of 5 km that is too narrow for GO to handle. The refractivity profile is defined as

\[ N(h) = N_0 \exp \left(-\frac{h}{H}\right) \left(1 + 0.01 \exp \left(-\frac{(h-B)^2}{W^2}\right)\right), \tag{5.5} \]
where \( N_0 \) is the refractivity at the surface, \( H \) is the scale height, \( B \) is the height at which the bump is centered in meters (in this case 5000), and \( W \) is the distance from the peak of the bump to its bottom (in this case 100 m). At sufficiently large refractive gradients, there are multipath phenomena that cannot be resolved due to GO retrieval’s fundamental assumption that ray paths do not overlap with each other. This manifests itself in the method completely breaking down, producing an invalid result. In addition to sharp enough gradients, multipath also occurs in cases of surface reflection, meaning that GO retrieval is incapable of resolving such scenarios. To address both these issues, there is a class of operators that examine the signal’s frequency content, classified as WO methods, described in the following chapter. However, for high altitudes, GO retrieval is sufficient, since there is no significant multipath. Additionally, the small-scale structures that would be resolved by the increased resolution of WO methods tend to be smaller than, and thus masked by, the noise floor.

### 5.2 Wave optics

There are several different WO operators, and they all rely on the fundamental mechanism of stationary phase, which is why they are sometimes referred to as Fourier integral operators (FIOs). Chief among them are full spectrum inversion (FSI), phase matching (PM) and the canonical transform (CT) \cite{1}\cite{32}\cite{33}. This thesis is mainly concerned with PM and FSI. As they are described in more detail in Paper 3, this section will only give a brief overview. The core idea is to transform the RO signal from the time domain to a frequency domain. The bending angle is tied to the derivative of the phase of the transformed function, and the impact parameter is found in different ways depending on the specific operator.

For FSI, the RO signal is transformed using a standard Fourier transform, and the frequencies are then mapped to an impact parameter using geometrical optics. The advantage of using the Fourier transform is the computational speed gained by applying the Fast Fourier transform (FFT). However, FSI requires a nearly circular orbit.

PM, instead, considers a spectrum of impact parameters as its kernel,
Figure 5.1: Refractivity profile (left) and bending angle profiles from GO and the Abel transform (right). The peak in bending angle cannot be resolved by GO.
resulting in a direct mapping to impact parameter space. This forfeits the benefit of the FFT while instead allowing any type of receiving trajectory. For this reason PM is used with airborne RO.

The third operator, CT, relies on back-propagation, i.e., the field as received in LEO is propagated backwards to a straight line, which enables a Fourier transform. It has later on been developed, and a combination of CT and FSI has resulted in a second version, CT2 [34].

The WO methods have a resolution of approximately 60 m, which is important in the lower parts of the atmosphere, where changes in refractivity can be quite dramatic. Figure 5.2 shows that PM can easily resolve the bump that GO could not. Additionally, they all yield an amplitude as a sort of "by-product", this by-product constituting the focus of this thesis.

5.3 Time-frequency analysis

By employing, e.g., the short-time Fourier transform (STFT) with a sliding window, it is possible to analyze RO signals in time and frequency simultaneously. Several other time-frequency analysis methods (also called radio-holographic methods) have been employed, most notably the multiple signal classification (MUSIC) algorithm [35, 26], and the Wigner distribution function[36]. These methods are commonly used to identify occurrences of multipath phenomena (including reflection) as well as other artifacts in the signal. Up front, the algorithms yield a spectrum in a time-frequency domain, though it is possible to map this to an impact parameter-bending angle domain. Figure 5.3 illustrates this mapping.

6 Contributions

The objective of this thesis is to investigate ways in which extraction of atmospheric information from GNSS-RO signals can be improved in the lower troposphere. The topic is of interest because horizontal gradients as well as SR degrade the signal. The inspiring idea for the work for Paper 1 was a parametric reconstruction of the refractivity in the lower troposphere, proposed by [19]. One of the parameters needed for the reconstruction is
Figure 5.2: Refractivity profile (left) and bending angle profiles from PM and the Abel transform (right). The peak in bending angle is resolved by PM.

the impact parameter at the surface of the Earth. Paper 1 attempts to identify this surface impact parameter by examining the amplitude that PM yields. It was found that in simulated cases this information can be extracted by locating the impact parameter where the amplitude turns to zero. However, these cases are idealized and do not consider loss of tracking above the lowest point, non-circular orbits, or horizontal gradients.

Paper 2 began as an attempt to apply the method in Paper 1 on real data. That was problematic for two main reasons: (1) PM amplitudes
Figure 5.3: Radio hologram of a RO measurement (left) mapped to impact parameter-bending angle domain (right). The tail in the bottom left-hand side of the left plot is a reflection signature.

in real measurements do not go to zero as quickly as in ideal scenarios due to stronger noise, and (2) PM amplitudes go to zero whenever the instrument loses tracking, which can occur at altitudes above the Earth’s surface. Instead amplitude spikes were noticed around the surface in some measurements, even when the tracking was lost at a higher altitude. These spikes were shown to be caused by reflected parts of the signal. They can be identified in measurements with deep tracking as well, by truncating the signal appropriately. The reflection spikes typically occur at an impact parameter slightly below the one corresponding to the surface.

Paper 3 uses the method of identifying reflection spikes presented in Paper 2, but using FSI instead of PM. The reduced computational complexity is valuable for further studies of this phenomenon.

7 Future work

In this thesis I show that information about the lower troposphere can, likely, be found in the amplitude of signals processed with FIO operators. Additionally, the results of Papers 2 and 3 indicate that the reflected com-
ponents of the signals can be of interest. As truncation of the signal may not be the most appropriate method, future work will consist of further investigations of surface reflections, FIO amplitudes, and the information that can be derived thereof. It is of great interest to further investigate whether retrieval of reflected bending angles can contribute to finding the surface impact parameter. Furthermore, the use of various radio-holographic methods can provide additional tools for understanding and characterizing parameters and errors.

Bibliography


Part II

Publications
Paper 1
DETERMINING THE REFRACTIVITY AT THE BOTTOM OF THE ATMOSPHERE USING RADIO OCCULTATION

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ABSTRACT

High accuracy of impact height is important to get reliable Radio Occultation (RO) measurements of the atmosphere refractivity. We have made an investigation on how accurately we can measure the impact height at ground level using wave optics simulations, realistic refractivity profiles, a realistic simulator for an advanced RO instrument including noise, and using phase matching for the inversion. The idea of the investigation is to increase the measurement accuracy of impact height at low altitudes and to give reliable measurements even in cases of super-refractive layers. We present statistics on the accuracy and precision of the determination of the impact height at ground, as well as the resulting accuracy and precision in the measured refractivity.

Index Terms— radio occultation, GPS, marine boundary layer

1. INTRODUCTION

The radio occultation (RO) technique is a method for sounding the Earth’s atmosphere by inverting global navigation satellite system (GNSS) signals that pass through it without hitting the ground. By applying the inverse Abel transform the bending angles of the optical rays can be found. From the found bending angles, it is possible to retrieve the atmosphere’s refractive index, which yields valuable information about humidity, temperature and pressure at a high vertical resolution[1]. This data is mainly used in numerical weather prediction and climate research. The existence of super-refractive (SR) layers in the atmosphere makes RO measurements unreliable, as the inverted refractivity becomes negatively biased due to atmospheric ducting[2]. An approach to this problem is proposed by Xie et al. in [3]. The idea is to reconstruct SR profiles with the help of two constraints: the height corresponding to the upper boundary of the SR layer, and the impact parameter corresponding to the Earth’s surface (which we refer to as $a_{\text{min}}$). Xie’s reconstruction is promising, however it is only evaluated on cases where these constraints were known beforehand. Therefore our paper specifically addresses the issue of the latter constraint, i.e. how to find an estimate for $a_{\text{min}}$.

A common method to invert RO signals is phase matching (PM)[4]. The signal is integrated over time and transformed into a complex function of the impact parameter. The bending angle is derived by taking the derivative of the phase of this function. The significance of the its amplitude, however, is less clear. It can be used to determine the lower cut-off impact parameter of the data, and possibly to detect the presence of a SR layer.

In this paper, we investigate the amplitude of the PM with a focus on the relation between its decline and $a_{\text{min}}$. To do this we perform simulated occultations using realistic atmosphere data[5], and present results both for ideal conditions and for the case when instrument noise is added.

2. REFRACTIVITY AND IMPACT PARAMETER AT GROUND

Assuming spherical symmetry, the path of an optical ray passing through the atmosphere satisfies Bouger’s rule[6]. This rule defines a constant $a$, called impact parameter for each ray:

$$ a = r n(r) \sin(\phi) $$

(1)

where $r$ denotes the magnitude of a vector going from the center of curvature of the Earth to the position of the ray, $n(r)$ denotes the atmosphere’s refractive index at the ray’s position, and $\phi$ denotes the angle between this radius vector and the ray’s direction vector. At a ray’s lowest point $r_t$ (the tangent altitude), $\phi$ equals $\frac{\pi}{2}$ radians and (1) evaluates to:

$$ a = r_t n(r_t) $$

(2)

In the case of an atmosphere with a SR layer, the inverse Abel transform introduces a bias in the refractivity profile, as shown in the left panel of Fig. 1. Below the upper boundary of the SR layer (around 1.2 km) the true (solid line) and biased (dashed line) refractivities differ from each other. While the true refractivity has its lowest point at $h = 0$, the biased curve terminates some 100 meters above ground. It is known that the impact height that corresponds to the lowest point on both
these curves must be equal to the impact height at the lowest point on the bending angle curve (right) \( (a_{\text{min}}) \). Since we know that the tangent point radius at this point is equal to the radius of curvature of the Earth, \( r_c \), we can use (2) to directly calculate the refractive index at ground as \( n(r_c) = \frac{a_{\text{min}}}{r_c} \), and the refractivity is found through

\[
N = (n - 1) \cdot 10^6.
\]  

From accurate knowledge of \( a_{\text{min}} \) we can therefore calculate \( N(r_c) \) even in the case of SR. However, the bending angle curve in Fig. 1 is too ideal compared to a retrieved bending angle. It is created from a forward Abel transform of a refractive index profile, and takes no account of the wave properties of the signal. The lowest point in a bending angle diagram generated by PM on a real (or realistically simulated) signal has no well determined \( a_{\text{min}} \). Thus, in order to use this method for calculating the lowest refractivity we need to develop a method to correctly determine \( a_{\text{min}} \), and in order to do that we need to make a close analysis of the structure of the simulated signal data close to ground.

\[
\begin{array}{c}
\text{Fig. 1. Example of super-refractivity. The inverted refractivity profile (left) suffers from a bias. Both refractivity profiles yield the same bending angle (right).}
\end{array}
\]

\[
\begin{array}{c}
\text{N} = (n - 1) \cdot 10^6.
\end{array}
\]  

From accurate knowledge of \( a_{\text{min}} \) we can therefore calculate \( N(r_c) \) even in the case of SR. However, the bending angle curve in Fig. 1 is too ideal compared to a retrieved bending angle. It is created from a forward Abel transform of a refractive index profile, and takes no account of the wave properties of the signal. The lowest point in a bending angle diagram generated by PM on a real (or realistically simulated) signal has no well determined \( a_{\text{min}} \). Thus, in order to use this method for calculating the lowest refractivity we need to develop a method to correctly determine \( a_{\text{min}} \), and in order to do that we need to make a close analysis of the structure of the simulated signal data close to ground.

\[
\begin{array}{c}
\text{3. PHASE MATCHING}
\end{array}
\]

For a signal \( u(t) \) received in Low Earth Orbit (LEO), Jensen et al. define the PM operation as follows[4]:

\[
U(a) = \int_{r_{\text{min}}}^{r_{\text{max}}} u(t) \exp \left( -ik_0 s(t, a) \right) dt
\]  

where \( s(t, a) \) is the optical path length of a model ray path. The function \( s(t, a) \) can be defined for any \( a \), and as such a range is chosen to include the Earth’s surface by a large margin. The phase of \( U(a) \) is then differentiated with respect to \( a \) to yield the bending angle \( \alpha(a) \). Consequently, \( \alpha(a) \) is defined for values that do not make physical sense, and we look into \([U(a)]\) to determine the “cutoff point”, \( a_{\text{min}} \). We define \( \hat{U}(a) \) as the normalized \( U(a) \) with respect to its values at high altitudes. The \([U]\) function has a very clear structure; all instances will have a sharp decline as the transmitter is shadowed by the Earth at low impact heights. Since \( a_{\text{min}} \) has to be located above this decline, we define a halfway point \( a_{1/2} \) such that \([U(a_{1/2})] = \frac{1}{2} \). An example of a typical \([U]\) function can be seen in Fig. 2, with \( a_{\text{min}} \) and \( a_{1/2} \) highlighted. From these quantities we also define \( \Delta a = a_{\text{min}} - a_{1/2} \).

\[
\begin{array}{c}
\text{Fig. 2.} \quad [\hat{U}(a)] \text{ for case 1 from the reference dataset[5]. The impact parameter corresponding to the Earth’s surface is located just above the sharp decrease of amplitude.}
\end{array}
\]

\[
\begin{array}{c}
\text{4. SIMULATIONS}
\end{array}
\]

In order to simulate a wave passing through the atmosphere we have implemented a wave optics propagator (WOP)[7], inspired specifically by the work of Rasch[8]. The WOP solves the Helmholtz equation iteratively along an axis in a two-dimensional space. This technique has been called the multiple phase-screen technique[9]. We model the Earth by multiplying the wave amplitude with a tapering window if \( r > r_c \):

\[
w(r) = e^{-\frac{(r-r_c)^2}{L_T^2}}
\]  

where \( r \) denotes the distance from the center of curvature, and \( r_c \) denotes the Earth’s radius of curvature. We use a tapering length \( L_T = 100 m \). The purpose of this window is to prevent artifacts that occur if the wave is not continuous. As input the WOP needs a one-dimensional atmosphere profile of refractivity, which is then assumed to be a spherically symmetrical atmosphere of the Earth. The output of the WOP is the amplitude and phase of the signal received in LEO, along with the corresponding geometry of transmitter, receiver and Earth.

We use the 55 reference cases defined by Healy[3] as input to the WOP. These cases are divided into 4 categories, where each category is increasingly “difficult” to invert. Category 4 cases have such a large refractivity gradient that they are SR, and ducting occurs. After the WOP we perform PM on the simulated signals and note their respective \( \Delta a \) values.
5. NOISE AND FILTERING

In order to make the signals more realistic we add noise to the WOP output. As the signal used in the simulation was of L1 frequency, and the amplitude was normalized to 1 at high altitude, an appropriate (complex-valued) Gaussian noise was added with $\sigma = 0.0537$, which corresponds to a signal-to-noise ratio of 25 dB at high altitude. As the sample rate of the signal is 100 Hz, this represents a signal-to-noise power density of 45 dBHz, which is a pessimistic value in comparison to the 55 dBHz expected for current and future RO instruments[10]. To filter the noise, we employed a sliding window in which we use linear regression to fit a polynomial of the second degree to each point in the noisy amplitude function. For this study, we use a window length of 170 m to remove all jaggedness from the decline of $|\hat{U}|$. In the left panel of Fig. 3 is an example of a noisy profile, and to the right, how it looks after filtering.

![Fig. 3. $|\hat{U}|$ function with noise (left), and filtered using a sliding polynomial fitting window (right).](image)

6. RESULTS

In Fig. 4 and Fig. 5 we show the $\Delta a$ values determined with ideal conditions and noisy conditions. We present the mean ($\mu$) and standard deviation ($\sigma$) of the distributions as well. The super-refractive (category 4) profiles in the dataset are highlighted with circles. Figures 6 and 7 show the resulting errors in refractivity if $N$ is calculated using (2) and (3), with $a_{min}$ approximated using $a_{1/2}$ and the mean of $\Delta a$, i.e.

$$N_{est} = 10^6 \left( \frac{a_{1/2} + \mu(\Delta a)}{r_c} - 1 \right)$$

(6)

The errors are relative and are calculated by $N_{error} = \frac{N_{min} - N_{est}}{N_{min}}$.

It is clear that the SR cases are clustered around $\pm 20^\circ$, which would correspond to the moist, subtropical regions where these ducting type of profiles are quite common. It is also clear that there appears to be no particular difference in the accuracy of the estimates for the SR cases as compared to the entire set, but they seem to be more precise. There is no particular reason as to why this would be the case, and it is most likely caused by the low number of profiles used in the analysis. The small difference between the results of noise-free and noisy signals shows that the method is robust to realistic instrument noise.

![Fig. 4. The spread of $\Delta a$ values from noise-free signals by latitude, with category 4 cases highlighted.](image)

![Fig. 5. The spread of $\Delta a$ values from noisy signals by latitude, with category 4 cases highlighted.](image)

7. DISCUSSION AND CONCLUSION

The results in this paper indicate that refractivity at the bottom of the atmosphere can potentially be determined from the amplitude of the PM output. However, there are several limitations and challenges that need to be addressed before this method is ready for real occultation data.

The proposed method relies on a certain type of boundary condition for the Earth. The one used in these simulations is not chosen because it is accurate in a physical sense. Further simulations with more accurate boundary conditions might be needed.

The surface of the Earth is not entirely smooth, and as such finding an accurate value for $a_{min}$ in cases of varying
The spread of relative $N$ errors from noise-free signals by latitude, with category 4 cases highlighted.

Fig. 6.

The spread of relative $N$ errors from noisy signals by latitude, with category 4 cases highlighted.

Fig. 7.

topography seems unrealistic. Regions where the terrain is flat (e.g. the oceans) seem more suitable.

We used our proposed method to find $N_{\text{min}}$ on the same data where we calculated the mean $a_{\text{min}}$. This biases the results, and independent data is needed to make a proper validation. This can be achieved using in-orbit RO measurements and corresponding refractivity data, which is our planned next step for further investigations.

To conclude, we show that a simulated RO signal transformed by the PM integral contains some information about the lowest possible impact height. The fact that SR cases do not exhibit any form of bias implies that these results have the potential to aid in determining one of the constraints needed for the reconstruction method proposed by Xie et al. Furthermore our results indicate that the performance of our method is not significantly affected by noise. How this approach could be applied to real RO data is a question which requires in-depth analysis. We hope to be able to address this question in our future research.

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8. REFERENCES


Analysis of reflections in GNSS radio occultation measurements using the phase matching amplitude

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Abstract. It is well-known that in the presence of super-refractive layers in the lower-tropospheric inversion of GNSS radio occultation (RO) measurements using the Abel transform yields biased refractivity profiles. As such it is problematic to reconstruct the true refractivity from the RO signal. Additional information about this lower region of the atmosphere might be embedded in reflected parts of the signal. To retrieve the bending angle, the phase matching operator can be used. This operator produces a complex function of the impact parameter, and from its phase we can calculate the bending angle. Instead of looking at the phase, in this paper we focus on the function’s amplitude. The results in this paper show that the signatures of surface reflections in GNSS RO measurements can be significantly enhanced when using the phase matching method by processing only an appropriately selected segment of the received signal. This signature enhancement is demonstrated by simulations and confirmed with 10 hand-picked MetOp-A occultations with reflected components. To validate that these events show signs of reflections, radio holographic images are generated. Our results suggest that the phase matching amplitude carries information that can improve the interpretation of radio occultation measurements in the lower troposphere.

1 Introduction

GNSS radio occultation (RO) is a technique used for sounding the Earth’s atmosphere. Assuming spherical symmetry of the atmosphere, the bending angles of GNSS signals passing through the atmosphere can be found and assimilated into numerical weather prediction systems. The bending angle measurements contain valuable information due to their relation to the atmosphere’s refractivity, which can yield information about humidity, temperature and pressure (e.g., Kursinski et al., 2000; Yunck et al., 2000). The geometry of the transmitter, Earth and receiver, as well as the short wavelength of the signals, results in a measurement with high vertical resolution. In many RO events, the instrument in orbit receives reflected components of the signal as well as direct ones. Boniface et al. (2011) have shown that reflected signals contain meteorological information. A method to detect these reflected components was suggested by Hocke et al. (1999) and has later been used on real data and shown to work (e.g., Beyerle et al., 2002; Pavelyev et al., 2002). This method uses a radio hologram generated by subtracting a ray-traced reference field from the received signal. An effort to flag occultation events where reflections are present is described by Cardellach and Oliveras (2016), based on a supervised learning approach classifying such radio holographic images. It has since then been employed by the Radio Occultation Meteorology Satellite Application Facility (ROM SAF) to flag millions of occultation events. Cardellach and Oliveras also investigate whether knowledge of these reflections can improve the quality of RO data, but Healy (2015) concludes that a binary reflection flag is probably inappropriate for assimilation purposes. Gorbunov (2016) proposes a technique based on the canonical transform to retrieve bending angle profiles of reflected rays and achieves a good agreement with the ROM SAF database.

When processing a RO signal, we use the phase matching (PM) operator (Jensen et al., 2004), which outputs a complex...
Figure 1. Using a sliding window for the signal yields PM amplitudes that correspond to the reflection model. Panel (a) shows a case with weak refractive gradients; panel (b) shows a case with strong refractive gradients.

Figure 2. \(|U|\) for a simulated signal that is propagated to a shallow orbit and one that is propagated to a deeper orbit (blue). For illustration the truncated signal is not tapered, producing dramatic oscillations around 4 km.

function of impact parameter whose phase is proportional to the signal’s bending angle. Although the amplitude of this function may contain valuable information as well, it has not been appropriately investigated.

In this paper, we compare the PM amplitudes of simulated measurements to real measurements made by the GRAS instrument aboard MetOp-A. We demonstrate signatures corresponding to surface reflections and truncate the received signal to distinguish them from the much stronger signatures of the direct components. We justify this truncation using simulated data and a simple geometric model for which reflected components are expected to appear in the signal, and we provide radio holographic images as a means of validation. Finally we discuss the difference in structure between simulated and real signals, as well as the potential future uses of the PM amplitude. In the Appendix we show that the PM operator admits reflected signal components.

2 Phase matching

Jensen et al. define PM as an operator that transforms a complex signal of time \(u(t)\) to impact parameter space:

\[
U(a) = \frac{1}{k_0} \int_{t_{\min}}^{t_{\max}} u(t) \exp \left(-ik_0S(t,a)\right) dt.
\]  (1)

Here, \(k_0\) is the wavenumber in vacuum associated with the carrier frequency of the GNSS transmitter, \(S(t,a)\) is the optical path length of a model ray path, and \(a\) is impact parameter. The derivative of \(\arg(U(a))\) with respect to \(a\) is proportional to the bending angle. As this integral is defined for any impact parameter \(a\), it is important to determine at which \(a\) the function does not contain relevant information anymore. The lowest \(a\) (which we call \(a_{\min}\) in this paper) corresponds to a ray that is tangential to the Earth’s surface. To make sure that all information in the signal is mapped to impact parameter space, we compute the \(U\) for impact parameter values going all the way to the Earth’s surface. This also ensures that we include reflected rays in the \(U\) function. It is not obvious that the PM method should work for reflected rays, as the geometrical model ray path is constructed for a direct ray, but in the Appendix it is shown that the standard PM technique admits reflected rays as well, provided the Earth surface is smooth.
Figure 3. An event classified as category 1. SLTA_{min} is at approximately −73 km, and the signal is truncated at SLTA = −50 km. The reflection spike is located at an impact height of 1919 m, and \( a_{min} \) is located at an impact height of 2017 m.

Figure 4. An event classified as category 1. SLTA_{min} is at approximately −73 km, and the signal is truncated at SLTA = −60 km. The reflection spike is 1881 m, and \( a_{min} \) is 1992 m.

3 A model for reflected rays

In Fig. 1 we use data from two real MetOp-A measurements to demonstrate that some of the features we see in the amplitude for the complex function \( U \) are caused by reflections. By overlaying the predicted straight line tangent altitude (SLTA) for the direct rays (blue line) and reflected rays (red line) as a function of impact height the reflection is illustrated. To generate these SLTA plots we use the co-located ECMWF refractivity profiles. The black lines show the amplitude of the \( U \) function when passing segments of the signal to the PM operator using a 10 km sliding window. The relationship between reflected bending angle and impact parameter is well-known (see, e.g., Pavelyev et al., 2011; Gorbunov, 2016) and can be described as

\[
\alpha(a) = -2a \int_{R_U(R_E)}^{\infty} \frac{1}{\sqrt{a^2 + n^2(r)}} \frac{d\ln n}{dr} dr - \pi + 2 \arcsin\left(\frac{a}{R_E n(R_E)}\right),
\]

(2)

where \( \alpha \) is the bending angle, \( R_E \) the Earth radius of curvature, and \( n \) the refractive index as a function of radius. The bending angle of a direct ray is given by the same expression without the last two terms. The integral can be evaluated numerically using a number of techniques, and we employ the method described in Rasch (2014). The SLTA for fixed values of the LEO and GNSS orbital radii is given by

\[
SLTA = \frac{r_L r_G \sin \theta}{\sqrt{r_L^2 + r_G^2 - 2r_L r_G \cos \theta}} - R_E.
\]

(3)
where $r_L$ and $r_G$ are the LEO and GNSS orbit radii, and $\theta$ is the separation angle between the satellites, given by

$$\theta = \pi + \alpha - \arcsin\left(\frac{a}{r_L}\right) - \arcsin\left(\frac{a}{r_G}\right).$$  \hspace{1cm} (4)

4 MetOp-A data

The data from occultation events are collected from the COSMIC Data Analysis and Archive Center (CDAAC) web interface, specifically day 2007.274 with the metopa2016 designation, indicating reprocessed measurements from MetOp-A. The signal amplitude, excess phase and orbit data needed for PM are all found in the atmPhs files. In these files, the orbit coordinates are given with the Earth’s center of mass as the point of origin. To fulfill the assumption of local spherical symmetry, we translate the coordinates so that they instead consider the center of curvature of the Earth at the occultation point. This is done by collecting center of curvature data from the corresponding atmPrf files. As the atmPrf files contain bending angle and impact height values, these were used to control the accuracy of the PM implementation.

For simulating a GNSS signal as it passes through the atmosphere, a wave optics propagator (WOP) is used with the multiple phase screen technique (see, e.g., Benzon et al., 2003; Benzon and Gorbunov, 2012; Rasch, 2014). The WOP uses a simpler, two-dimensional geometry where the GNSS transmitter is stationary. The three-dimensional geometry data in the atmPhs files are thus projected onto a plane where the LEO orbit is defined by the separation angle. Both transmitter and receiver are assigned a constant radius, and the Earth is assigned the radius of curvature for the particular event. The separation angles can be modified to simulate an occultation event tracked to a lower SLTA than its...
Figure 7. An event classified as category 1. SLTA\textsubscript{min} is at approximately \(-72\) km, and the signal is truncated at SLTA = \(-60\) km. The reflection spike is located at an impact height of 1940 m, and \(a\)\textsubscript{min} is 1993 m.

Figure 8. An event classified as category 3. SLTA\textsubscript{min} is at approximately \(-87\) km, and the signal is truncated at SLTA = \(-70\) km. The reflection spike is located at an impact height of 2075 m, and \(a\)\textsubscript{min} is 2204 m.

corresponding measurement. For an atmosphere, the high-resolution, co-located refractivity profile from ECMWF also provided by CDAAC, is used (echPrf files). These profiles do not go all the way to the ground – the last bit is extrapolated linearly. To simulate surface reflections, we set the electromagnetic field to zero on all parts of the phase screens that lie inside the Earth. Although it is not clear whether this method has a solid physical basis it appears to give quite accurate results and is routinely used in WOP simulations (Gorbunov, 2016; Levy, 2000).

As a frame of reference, radio holographic images are produced by constructing a “beating function” from the excess phase, signal amplitude and a reference field and using the short-time fast Fourier transform with a sliding window, similar to Boniface et al. (2011).

5 Surface reflections

In the lowest part of an occultation (SLTA around \(-80\) km, impact height around 2.1 km), where the signal becomes shadowed by the Earth, the magnitude of \(U\) also decreases. The lowest direct ray becomes diffracted by the Earth’s surface and gradually decreases in magnitude over a region corresponding to the Fresnel zone size, which is seen clearly in simulated data and frequently less clearly in measured data. This transition occurs over a few hundred meters (Kursinski et al., 2000), around SLTA\textsubscript{min}, and \(a\)\textsubscript{min}, whose values are determined by the Earth radius and the refractive index at ground. Quite often when the signal is lost at an SLTA value that is substantially higher than SLTA\textsubscript{min} we see a spike in \(|U|\) around \(a\)\textsubscript{min}. This spike is most easily explained as a reflection. If tracking of the signal goes all the way to the surface this spike overlaps with the direct signal and is obfuscated. In Fig. 1 we see that the reflected signal and the di-
rect signal coincide around $a_{\min}$ (approximately at 2.1 km). For panel a this occurs at the deepest SLTA (around $-90$ km) and for panel b this again occurs around $-90$ km, but the deepest part of the signal goes to $-150$ km and occurs at an impact height around 2.8 km, which corresponds to a region of strong refractivity gradients. The value for SLTA where the direct and reflected signals join is called SLTA$_{\min}$. By cutting the signal well above SLTA$_{\min}$ at an SLTA around $-50$ km, it is clear from Fig. 1 that we would only keep the upper parts of the reflected and direct signals and completely remove the signal parts containing the diffraction signature around $a_{\min}$ and the deep signals caused by strong refractivity gradients. In Fig. 2 we can see the same thing. The abrupt decrease in $|U|$ around an impact height of 2.1 km corresponds to the point where the reflected and direct signals join, and the direct signal becomes diffracted. By cutting the signal at $-65$ km the lower parts of the reflected and direct signals are removed, and the reflection signature becomes clearly separated from the direct signal. The reflection spike appears a small distance below $a_{\min}$. This small distance depends on where the signal is truncated, which is illustrated by Fig. 1.

The nature of the PM operator is such that sharp discontinuities in the signal introduce significant noise in the $|U|$ function. To avoid these, we taper the signals using a one-sided Tukey window (Bloomfield, 2004). In the case of the sliding window used in Fig. 1, we use a two-sided window. The noise generated by omitting any sort of tapering is demonstrated in Fig. 2, where the simulated signal ends abruptly.

6 Results

We present 10 cases where PM is performed on real signals alongside their simulated counterparts based on ECMWF’s co-located refractivity profiles. These cases are classified in
categories 1 through 4 based on the sharpest gradient above 100 m, in the same manner as the reference dataset from ECMWF (Healy, 2012), where category 4 has the sharpest gradients. Overall, the structure of $|U|$ is similar to that of a step function, both in simulations and using real data. By truncating the signals at an appropriate SLTA we can distinguish previously hidden reflection spikes in $|U|$. For this study we pick the altitude of truncation qualitatively. Figures 3 through 12 show the received signal (first from the left), $|U|$ for a simulated signal (second from the left), $|U|$ for the received signal (third from the left) and a radio hologram generated from the signal (fourth from the left). The first, second and third plots from the left are color coded so that the blue plots describe the complete signals, and the red plots describe the truncated signals. All the truncated signals have been tapered using the aforementioned Tukey window to avoid introducing additional noise.

Simulations on cases classified as category 3 and 4 – shown in Figs. 8 through 12 – give rise to a sharply negative spike at an impact height corresponding to the sharp gradient in the refractivity profile. This structure cannot be found in the real data. Moreover, the real data show a high level of noise that is not found in simulated data.

We note that there are peculiar oscillating structures in the real data, particularly in Fig. 4 at 6 km, Fig. 7 at 4 and 6 km, Fig. 8 at 7.5 km, Fig. 11 at 5 and 9 km and Fig. 12 at 5 and 9 km. These oscillating structures are not found in the simulated data.

Furthermore we note that in all cases but Fig. 11, the reflection spike appears some distance below the $a_{\text{min}}$ calculated from the ECMWF profiles.

While these 10 hand-picked cases all clearly contain reflected components, there are several measurements in which the reflected parts cannot be distinguished. This is typically either because the measurement was not deep enough or be-
cause $|U|$ was too noisy. When the measurements are sufficiently deep, and the noise level of $|U|$ is low, there are very clear reflection spikes. This is corroborated by the radio holographic images, which also show very clear reflection patterns in those cases. The bright yellow trail around 0 Hz is the direct signal, whereas the more faint trail going off to negative frequencies (and appearing again at positive frequencies due to aliasing) is the reflected signal. The signal has been divided everywhere by the average signal amplitude in the window. This is done to make the reflection and the direct signal for low SLTA stand out against the strong signature of the direct signal for high SLTA. This also often causes the hologram to give the appearance of containing strong broadband noise for low SLTA, but it is actually the amplification of weak noise.

7 Conclusions

The results in this paper show that the signatures of surface reflections in GNSS RO measurements can be significantly enhanced when using the PM method by processing only an appropriately selected segment of the received signal. We can then identify reflection signatures even in cases where they are normally obscured by the direct signal’s influence on the PM amplitude. This signature enhancement is demonstrated by simulations and confirmed with MetOp-A measurements. For further validation of the reflections, radio holographic images are provided for comparison. For the cases presented, clear and sharp reflection spikes in the PM amplitude are corroborated with clear reflection patterns in their corresponding radio holograms. Weaker or less distinct spikes have more noise in their corresponding radio holograms.

The events containing reflected signals presented here are hand-picked to illustrate that the approach can be useful for identifying reflections in real measurements. There are still questions that can only be answered by larger volumes of data, e.g., if the PM amplitude can possibly be used to retrieve information related to the reflected signal. Furthermore, we observe that the reflection spikes vary greatly in shape. Additionally, simulated reflection spikes always occur very close to the surface impact parameter, but in real measurements this is not always the case. The reason for these variabilities needs to be investigated. At this point we have only analyzed events over water, since we expect those reflections to be much clearer and numerous compared to events over land.

When comparing the PM amplitudes of real and simulated signals, some interesting discrepancies were found. First, the characteristic dip in $|U|$ associated with a region of sharp gradient in the refractivity could not be identified in the real data, being completely drowned in noise or simply not present. Second, the levels of noise in the $|U|$ function tend to increase close to the Earth surface. As no noise was added to the simulations, it is not clear whether this is due to instrument noise or atmospheric disturbances. Third, peculiar oscillatory features at quite distinct heights were seen in the real data but not at all in the simulated. These features are not understood at present, but it is likely that they have to do with horizontal gradients in the refractive index and with aliasing of the reflected signal onto the direct signal due to low sampling rate. What horizontal gradients do to the PM amplitude has not been investigated, but it is likely that it will decrease the amplitude and cause additional noise, perhaps resulting in these peculiar oscillations. Regarding the aliasing of reflected signals, it is quite clearly seen in some of the radio holograms that the reflected signal is still strong when it has been aliased and again approaches the direct signal in frequency. This may cause a degradation of the PM method (the integral over the stationary phase point) at the specific impact height that belongs to the direct signal at the point where the direct and reflected aliased signal match in frequency.

Data availability. The data from occultation events are collected from the COSMIC Data Analysis and Archive Center (CDAAC) web interface, found at the URL http://cdaac-www.cosmic.ucar.edu/cdaac/index.html.
Appendix A: Phase matching for reflected rays

It is not obvious that the PM method should work for reflected rays without modifications, but we will show that it does, under the assumption of reflections taking place on a perfectly smooth surface. First we will review the method used for PM of direct (non-reflected) rays, and then we will show that the result is the same for reflected rays. For the full details of the PM method the reader is referred to Jensen et al. (2004).

A1 Direct rays

Under the assumption of a spherically symmetric atmosphere we can use Bouger’s rule

\[ rn(r) \sin \phi = a, \quad (A1) \]

where \( r \) is the distance from the Earth center of curvature, \( n \) the refractive index, \( \phi \) the angle the ray makes with the radial vector and \( a \) the impact parameter. A ray is emitted from the GNSS satellite (at \( r_G \)) with angle \( \phi_G \), being smaller than \( \pi \). The ray makes its closest approach to the Earth when \( \phi = \pi/2 \). The ray then exits the atmosphere and is received at the LEO satellite (at \( r_L \)) with the angle \( \phi_L \), being larger than \( \pi \). The total bending of the ray (measured positive towards the Earth) is given by the bending angle

\[ \alpha(a) = -2a \int_a^{\infty} \frac{1}{\sqrt{r^2n(r)^2-a^2}} \frac{d \ln n}{dr} dr. \quad (A2) \]

The optical path length for the ray is given by the integral over the refractive index along the path of the ray

\[ S = \int n(r) ds, \quad (A3) \]

where the term \( ds \) is an infinitesimal length along the ray. Under the spherical symmetry assumption the integral can be recast in a very attractive form, i.e.,

\[
S(t,a) = \sqrt{r_L(t)^2-a^2} + \sqrt{r_G(t)^2-a^2} \\
-2a \int_a^{\infty} \frac{1}{\sqrt{r^2n(r)^2-a^2}} \frac{d \ln n}{dr} dr \\
-2 \int_a^{\infty} \sqrt{r^2n(r)^2-a^2} \frac{d \ln n}{dr} dr. \quad (A4)
\]

The last term is connected to the bending angle in the following way:

\[
\int_a^{\infty} \alpha(a')da' = -2 \int_a^{\infty} \sqrt{r^2n(r)^2-a^2} \frac{d \ln n}{dr} dr. \quad (A5)
\]

which can be verified by taking the derivative with respect to \( a \) on both sides. Using also the definition for the bending angle (Eq. A2) we can write

\[
S(t,a) = \sqrt{r_L(t)^2-a^2} + \sqrt{r_G(t)^2-a^2} + \alpha(a)a \\
+ \int_a^{\infty} \alpha(a')da'. \quad (A6)
\]

The impact parameter is generally connected to a certain point in time, and certain values for \( r_L \) and \( r_G \), in a complicated way. Whatever this connection may be, the angles in the system must fulfill

\[
\theta(t) + \phi_G + \phi_L - \pi = \alpha, \quad (A7)
\]

where \( \theta \) is the separation angle between the satellites. We can rewrite this using Bouger’s rule

\[
\theta(t) + \arcsin(a/r_G(t)) + \arcsin(a/r_G(t)) = \alpha. \quad (A8)
\]

For every value of \( a \) there will be a corresponding value for \( t \). In that sense one could write the optical path length as a function of \( t \) only, i.e.,

\[
S(t) = \sqrt{r_L(t)^2-a(t)^2} + \sqrt{r_G(t)^2-a(t)^2} + \alpha(a(t))a(t) \\
+ \int_a^{\infty} \alpha(a')da'. \quad (A9)
\]

In the phase matching method we perform an integral for each value of a given impact parameter \( a_g \) where we wish to find the bending angle. The signal is given by

\[
u(t) = |u(t)| \exp(i k S(t)). \quad (A10)\]

where \( k \) is the wavenumber, and \( i = \sqrt{-1} \). We subtract a geometrical model for the ray and form an integral as

\[
U(a_g) = \int_{t_{\text{min}}}^{t_{\text{max}}} |u(t)| \exp(i k (S(t) - S_g(t,a_g))) dt, \quad (A11)\]

where \( t_{\text{min}} \) and \( t_{\text{max}} \) are the start and stop times of the signal, and the geometrical rays is given by

\[
S_g(t,a_g) = \sqrt{r_L(t)^2-a_g^2} + \sqrt{r_G(t)^2-a_g^2} + a_g \alpha(a_g) = \\
= \sqrt{r_L(t)^2-a_g^2} + \sqrt{r_G(t)^2-a_g^2} + a_g \left( \theta(t)\right) \\
+ \arcsin(a_g/r_G(t)) \\
+ \arcsin(a_g/r_G(t)) - \pi. \quad (A12)\]

and the terms in the brackets are the bending angle from Eq. (A8). The integral will get its main contribution from the
point where there is a stationary phase point, characterized by
\[
\frac{d}{dt} \left( S(t) - S_g(t, a_g) \right) = 0. \tag{A13}
\]

The time derivative of \( S \) is given by
\[
\frac{dS}{dt} = \frac{1}{r_l(t)} \frac{d}{dt} \sqrt{r_l(t)^2 - a(t)^2}
+ \frac{1}{r_G(t)} \frac{d}{dt} \sqrt{r_G(t)^2 - a(t)^2 + a(t) \frac{d\theta}{dt}}. \tag{A14}
\]

Likewise, the time derivative of \( S_g \) is
\[
\frac{dS_g}{dt} = \frac{1}{r_l(t)} \frac{d}{dt} \sqrt{r_l(t)^2 - a_g^2}
+ \frac{1}{r_G(t)} \frac{d}{dt} \sqrt{r_G(t)^2 - a_g^2 + a_g \frac{d\theta}{dt}}. \tag{A15}
\]

Hence, the stationary phase point occurs where \( a(t_g) = a_g \). At that point the difference in optical path length becomes
\[
S(t_g) - S_g(t_g, a_g) = \int_{a_g}^{\infty} \alpha(a') da', \tag{A16}
\]

and the phase matching integral is given by
\[
U(a_g) = C(t_g) \exp \left( ik \int_{a_g}^{\infty} \alpha(a') da' \right), \tag{A17}
\]

where \( C(t_g) \) is an amplitude factor depending on the signal amplitude and phase in the region around the stationary phase point. The bending angle as a function of impact parameter is thus found by taking the derivative of the phase of the function \( U \) with respect to \( a_g \), i.e.,
\[
\alpha(a_g) = -\int \frac{dU(a_g)}{U(a_g)} \tag{A18}
\]

### A2 Reflected rays

For rays suffering reflection the Bouger’s rule still applies, but the ray never reaches the point where \( \phi = \pi/2 \). Instead the ray is reflected at the point where \( r = R_E \), where \( R_E \) is the Earth’s radius of curvature. Using the definition \( R_E n(r_l) = a_E \) we find the angle the ray makes with the radial vector at reflection to be
\[
\phi_E = \arcsin \left( \frac{a}{a_E} \right). \tag{A19}
\]

Here we naturally assume that \( a < a_E \); otherwise the ray would never reach the surface and be reflected. Since we assume the surface to be completely smooth, the radial vector is parallel to the surface normal, and since the incidence angle with respect to the surface normal is equal to the reflected ray angle with respect to the surface normal, we find that the ray angle after reflection is
\[
\phi_E = \pi - \phi_E. \tag{A20}
\]

We conclude that the ray suffers a negative bending of \( \pi - 2\phi_E \) radians due to the reflection. The total bending angle for a reflected ray is therefore given by
\[
\alpha(a) = -2a \int_{a_E}^{\infty} \frac{1}{\sqrt{r^2 n(r)^2 - a^2}} \frac{d\ln n}{dr} dr - \pi + 2\phi_E. \tag{A21}
\]

The integral for the optical path length becomes more complicated (although the derivation is straightforward):
\[
S(t) = \sqrt{r_l(t)^2 - a(t)^2} - \sqrt{a_E^2 - a(t)^2} + \sqrt{r_G(t)^2 - a(t)^2}
- \sqrt{a_E^2 - a(t)^2} - 2at \int_{a_E}^{\infty} \frac{1}{\sqrt{r^2 n(r)^2 - a^2}} \frac{d\ln n}{dr} dr
- 2 \int \sqrt{r^2 n(r)^2 - a^2} \frac{d\ln n}{dr} dr. \tag{A22}
\]

We can rewrite slightly using Eqs. (A21) and (A19):
\[
S(t) = \sqrt{r_l(t)^2 - a(t)^2} - \sqrt{a_E^2 - a(t)^2} + \sqrt{r_G(t)^2 - a(t)^2}
- \sqrt{a_E^2 - a(t)^2} + 2at \int_{a_E}^{\infty} \frac{\alpha(a)}{a_E} \frac{d\ln n}{dr} dr. \tag{A23}
\]

The derivative with respect to time yields
\[
\frac{dS}{dt} = \frac{r_l(t) \dot{r}_l(t) - a(t) \dot{a}(t)}{\sqrt{r_l(t)^2 - a(t)^2}} + \frac{\dot{r}_G(t) \dot{r}_G(t) - a(t) \dot{a}(t)}{\sqrt{r_G(t)^2 - a(t)^2}}
+ \dot{a}(t) \int_{a_E}^{\infty} \frac{1}{\sqrt{r^2 n(r)^2 - a^2}} \frac{d\ln n}{dr} dr, \tag{A24}
\]

where dot signifies derivative with respect to time. The last term can be rewritten using Eq. (A21):
\[ \frac{dS}{dt} = \frac{r_L(t)\hat{r}_L(t) - a(t)\hat{a}(t)}{\sqrt{r_L(t)^2 - a(t)^2}} + \frac{r_G(t)\hat{r}_G(t) - a(t)\hat{a}(t)}{\sqrt{r_G(t)^2 - a(t)^2}} + \hat{a}(t) \left[ \alpha(t) + \pi - 2\arcsin \left( \frac{a(t)}{a_E} \right) \right] + a(t)\dot{\alpha}(t) + \hat{\alpha}(t) \left[ -\alpha(t) - \pi + 2\arcsin \left( \frac{a(t)}{a_E} \right) \right]. \] (A25)

The expression simplifies to

\[ \frac{dS}{dt} = \frac{r_L(t)\hat{r}_L(t) - a(t)\hat{a}(t)}{\sqrt{r_L(t)^2 - a(t)^2}} + \frac{r_G(t)\hat{r}_G(t) - a(t)\hat{a}(t)}{\sqrt{r_G(t)^2 - a(t)^2}} + a(t)\dot{\alpha}(t). \] (A26)

The geometrical conditions in Eq. (A8) are still valid, i.e.,

\[ \begin{align*}
\alpha(t) &= \theta(t) + \arcsin(a(t)/r_L(t)) \\
&+ \arcsin(a(t)/r_G(t)) - \pi.
\end{align*} \] (A27)

Taking the derivative with respect to time yields

\[ \dot{\alpha}(t) = \dot{\theta}(t) + \frac{\hat{a}(t) - a(t)\hat{\alpha}(t)/r_L(t)}{\sqrt{r_L(t)^2 - a(t)^2}} + \frac{\hat{a}(t) - a(t)\hat{\alpha}(t)/r_G(t)}{\sqrt{r_G(t)^2 - a(t)^2}}. \] (A28)

Inserting this expression into Eq. (A26) we get

\[ \frac{dS}{dr} = \frac{r_L(t)\hat{r}_L(t)}{r_L(t)^2 - a(t)^2} + \frac{r_G(t)\hat{r}_G(t)}{r_G(t)^2 - a(t)^2} + a(t)\hat{\theta}(t), \] (A29)

which is the very same expression as Eq. (A14). Hence, the stationary phase point again occurs where \( a(t_g) = a_g \). At this point we have

\[ \begin{align*}
S(t_g) - S_g(t_g, a_g) &= -2\sqrt{a_E^2 - a_g^2} + a_g\pi \\
&- 2a_g \arcsin \left( \frac{a_g}{a_E} \right) - 2 \int_{a_g}^\infty \frac{\sqrt{n(r)^2 - a_g^2} \frac{d\ln n}{dr}}{a_g} dr. \] (A30)

This is the term that appears in the phase of the phase matching function \( U(a_g) \). Taking the derivative with respect to \( a_g \) leads to

\[ \frac{d}{da_g} \left( S(t_g) - S_g(t_g, a_g) \right) = 2a_g \int_{a_g}^\infty \frac{1}{\sqrt{r^2 n(r)^2 - a_g^2}} \frac{d\ln n}{dr} dr + \pi - 2\arcsin \left( \frac{a_g}{a_E} \right), \] (A31)

which is the negative of the bending angle for a reflected ray as given in Eq. (A21). Consequently the phase matching method works in the exact same way for direct and reflected rays. It should be stressed that these derivations are only valid when the Earth surface can be considered smooth. When the surface is not smooth the incoming ray will change impact parameter upon reflection. Due to this the expression for the optical path length becomes a function of the old and new impact parameter, and the simple geometrical ray model used in the phase matching method cannot lead to a stationary phase point. This is basically a case of multipath in the impact parameter domain. It may be argued though that this is of little consequence for real measurements since the occultation measuring instrument will not record signals that deviate too strongly from direct rays, as they quite rapidly become heavily Doppler shifted with increasing reflection angle. For this reason reflected signals will only be seen at impact parameters that are very close to the value at the Earth surface. These rays are of grazing incidence, and under such circumstances the surface may always be considered as flat (Beckmann and Spizzichino, 1963).
Author contributions. JR, TS, AC and MIP designed the study and TS performed the simulations and processing. TS prepared the manuscript with contributions from all co-authors. JR performed the calculations for the reflection model and wrote the Appendix.

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References


Paper 3
Comparing reflection signatures in radio occultation measurements using the full spectrum inversion and phase matching methods

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ABSTRACT

Global Navigation Satellite System radio occultation (GNSS-RO) is an important technique used to sound the Earth’s atmosphere and provide data products to numerical weather prediction (NWP) systems as well as to climate research. It provides a high vertical resolution and SI-traceability that are both valuable complements to other Earth observation systems. In addition to direct components refracted in the atmosphere, many received RO signals contain reflected components thanks to the specular and relatively smooth characteristics of the ocean. These reflected components can interfere the retrieval of the direct part of the signal, and can also contain meteorological information of their own, e.g., information about the refractivity at the Earth’s surface. While the conventional method to detect such reflections is by using radio-holographic methods, it has been shown that it is possible to see reflections using wave optics inversion, specifically while inspecting the amplitude of the output of phase matching (PM). The primary objective of this paper is to analyze the appearance of these reflections in the amplitude output from another wave optics algorithm, namely the much faster full spectrum inversion (FSI). PM and FSI are closely related algorithms - they both use the method of stationary phase to derive the bending angle from a measured signal. We apply our own implementation of FSI to the same GNSS-RO measurements that PM was previously applied to and show that the amplitudes of the outputs again indicate reflection in the surface of the ocean. Our results show that the amplitudes output from the FSI and PM algorithms are practically identical and that the reflection signatures thus appear equally well.

Keywords: radio occultation, wave optics, reflections, full spectrum inversion

1. INTRODUCTION

GNSS radio occultation (GNSS-RO) is a technique for sounding the Earth’s atmosphere. As GNSS signal rays pass through without reaching the Earth’s surface, they are intercepted by low-Earth orbit (LEO) satellites. Due to the refractive properties of the atmosphere, the rays will bend as they are propagated through. For each point in the receiver’s LEO, the bending angles of the rays can be computed as a function of their conserved impact parameter. Measuring their Doppler shift yields (under the assumption of spherical symmetry) vertical bending angle profiles, which in turn contains information about the refractive properties of the atmosphere. Knowing the chemical composition of the air subsequently allows for derivation of additional parameters such as humidity, temperature and pressure.\textsuperscript{1} These measurements are of great value to numerical weather prediction (NWP) applications; the complementary geometry and the SI-traceable nature of the measurements make them useful for calibration and validation of other Earth-observing systems.

When the Earth’s surface is reflective and smooth enough (i.e., over oceans), the receiver in LEO will record some reflected rays in addition to the direct ones. The identification and retrieval of reflected signal components can potentially contribute additional meteorological information, as Ref. 2 shows by means of a ray tracing comparison. At this point, however, it is unclear exactly what information in the signal is useful. Studying a data set from the Radio Occultation Meteorology Satellite Application Facility (ROM SAF), Ref. 3 concludes

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that a binary flag indicating the presence or absence of reflection is not sufficient for assimilation into NWP systems. Nevertheless, detection of events containing reflections remains an important task, to further study the phenomenon.

The most common way of identifying reflections is by time-frequency analysis, as they will appear as a Doppler delay in the spectrogram corresponding to a signal. Based on this approach a method of classifying spectrograms containing reflections using a support vector machine is presented in Ref. 7. The use of artificial neural networks to improve both the classification performance and the computational efficiency of detection is proposed in Ref. 8. Reference 9 shows that reflections can also be identified and retrieved using a Fourier integral operator (FIO) (originally intended for the retrieval of direct bending angles), specifically by a modification of the canonical transform. Reference 10 employs a different FIO, phase matching (PM), and shows that by properly truncating the measured signal, a reflection signature will appear in the amplitude of the operator’s output, corresponding to an impact parameter near the Earth’s surface. This surface impact parameter is an important data point when trying to parametrically reconstruct super-refractive refractivity profiles. Reference 11 explores the correlations between the occurrence of reflections and several meteorological parameters, and suggests that reflected bending angle profiles could be assimilated into NWP models. The forward operator for reflected bending angles proposed in this study also depends on the surface impact parameter.

In this paper we revisit the idea of truncating RO signals and processing them with an FIO. The goal is to confirm that the Full Spectrum Inversion (FSI) operator admits reflected rays in the same way as the PM operator does. To accomplish this we replicate a part of the results presented in Ref. 11 using the FSI operator and compare them to the results yielded from PM. The paper is organized as follows. Section 2 describes the implementation details and differences between FSI and PM. In Sect. 3, we outline how the signals are processed and analyzed to match the previous study. Section 4 describes the results and in Sect. 5 we give the conclusion.

2. IMPLEMENTATION

Throughout the description of the FSI and PM operators, we define a received GNSS-RO signal $V(t)$ as

$$V(t) = A(t) \exp(ik\varphi(t)),$$

where $A(t)$ is the real-valued amplitude, $i$ is the imaginary number, $k$ is the wave number related to the carrier frequency, and $\varphi(t)$ is the phase, or optical path length of the signal. Additional relevant parameters include the following ephemeris data: The distance from the Earth’s center of curvature to transmitter and receiver $r_G(t)$ and $r_L(t)$ respectively, and the angle of separation $\theta(t)$ between the radius vectors. Applying either of these operators $F$ to a signal will map it to impact parameter space:

$$F\{V(t)\} = \alpha(a),$$

where $\alpha$ denotes bending angle and $a$ denotes impact parameter. Subtracting the local radius of curvature of the Earth $r_E$ from $a$ yields impact height.

We will not give a detailed account about the theoretical justifications for how these operators yield bending angle measurements. For this the reader is referred to Ref. 15 and Ref. 12, respectively. Instead we present summaries of the implemented algorithms, to give an idea of what computations are made.

Both FSI and PM require significant upsampling of the signal. Typically, an RO signal has a frequency of 50 or 100 Hz, but for these operators to be work it needs to be interpolated to approximately 2000 Hz. Additionally, a central step in both methods is the differentiation of a noisy phase. Since differentiation is sensitive to noise, some form of filtering is required to reach satisfactory results. In our implementations, we perform a polynomial least squares fit with a sliding window that can be analytically differentiated. The window lengths are not chosen with any sort of quality control in mind.
2.1 Full Spectrum Inversion

The main idea of FSI is to perform a Discrete Fourier Transform (DFT) to move the signal into the frequency domain, since a ray’s corresponding frequency ($\omega$) is proportional to its impact parameter. Differentiating the phase with regards to $\omega$ results in the arrival time of each frequency component, which is used to sample the ephemeris data and compute the bending angle for each impact parameter. Supposing that the radial velocities of transmitter and receiver are constant, the algorithm is straightforward, as can be seen in Alg. 1.

Algorithm 1 Full spectrum inversion assuming constant radial velocities

1: Subtract a linear phase from $V(t)$ based on the center frequency.
2: Upsample $V(t)$.
3: Let $\hat{V}(\omega) = \int_{t_0}^{t_n} V(t) \exp(-i\omega t) dt$.
4: Add the center frequency to $\omega$.
5: Let $u(\omega) = \angle \hat{V}(\omega)$.
6: Let $u_p(\omega)$ be a local polynomial fit of $u(\omega)$.
7: $t(\omega) = -\frac{du_p}{d\omega}$.
8: $a = \frac{\omega}{k \frac{df}{dt}}$.
9: $\alpha = \theta(\omega) + \arcsin\left(\frac{a}{r_{ck}(\omega)}\right) + \arcsin\left(\frac{a}{r_{LG}(\omega)}\right) - \pi$.

However, in a real measurement there will be variations in the radial velocities. To address this, the signal is resampled with regards to $\theta$ and a corrective phase term $\varphi_m$ is subtracted from the signal to remove the interference of radial variations. This phase term is derived from a model impact parameter, and for this we use the impact parameter yielded from the geometrical optics method applied on a smoothed signal phase. Ultimately the modified algorithm shown in Alg. 2 operates in a “pseudo frequency” domain, and includes some preprocessing of the signal as well as the iterative solution of another equation to find $a$.

The term ”FSI amplitude” refers to $|\hat{V}_s(a)|$, i.e. the amplitude of $\hat{V}_s$ in impact parameter space. Since FSI is makes use of the standard Fourier transform, the Fast Fourier Transform (FFT) can be employed to significantly reduce the computational complexity from $O(n^2)$ to $O(n \log n)$. The other operations are all made in approximately linear time.

2.2 Phase Matching

The core concept of PM is to construct a set of test rays where $S_t(t,a)$ describes their optical path length, and use as the kernel of the FIO. Since the test rays are defined in impact parameter space, so will the PM output $\hat{V}$ be. Thus, there is no intermediate step to map frequency to impact parameter. Finally the derivative of the phase of $\hat{V}$ is proportional to the bending angle. Compared to FSI, an implementation of PM will arguably look much more straightforward, as shown in Alg. 3.

Similar to the FSI case, the term ”PM amplitude” refers to $|\hat{V}(a)|$. Since PM does not make use of a standard Fourier transform, it does not benefit from the FFT. Thus, for every point in the impact parameter domain, every point in the time domain needs to be considered for the integral. This results in a computational complexity of $O(n^2)$.

Although the methods are said to be virtually equivalent in literature, some small-scale differences in bending angle can occur. However, these differences occur at heights where the corresponding amplitude is low, indicating that the differences amount to noise rather than anything else. Figure 1 shows a measurement with a quite stable amplitude and thus quite small differences in bending angle, while Fig. 2 shows a measurement where the amplitude is noisier, and subsequently so are the differences in bending angle.
Algorithm 2 Full spectrum inversion for real measurements

1: Calculate a model impact parameter $a_m(\theta)$.  
2: Resample $V(t)$ with regards to $\theta$.  
3: Let $\phi_m(\theta) = \int_0^\theta \left( \frac{d r_L}{d \theta'} \sqrt{1 - \left( \frac{a_m}{r_L} \right)^2} + \frac{d r_G}{d \theta'} \sqrt{1 - \left( \frac{a_m}{r_G} \right)^2} \right) d \theta'$.  
4: Let $V_s(\theta) = V(\theta) \exp \left( -i k \phi_m(\theta) \right)$.  
5: Subtract a linear phase from $V_s(\theta)$ based on the center frequency.  
6: Upsample $V_s(\theta)$.  
7: Let $\hat{V}_s(\omega) = \int_{\theta_{\text{start}}}^{\theta_{\text{end}}} V_s(\theta) \exp(-i \omega \theta) d\theta$.  
8: Add the center frequency to $\omega$.  
9: Let $u_P(\omega)$ be a local polynomial fit of $u(\omega)$.  
10: $\alpha(a) = \theta(\omega) + \arcsin \left( \frac{a}{r_L(\omega)} \right) + \arcsin \left( \frac{a}{r_G(\omega)} \right) - \pi$.  

Algorithm 3 Phase matching

1: Upsample $V(t)$.  
2: Define $a$ as an independent variable with a desired resolution.  
3: Let $\phi_L = \arcsin \left( \frac{a}{r_L(t)} \right)$ and $\phi_G = \arcsin \left( \frac{a}{r_G(t)} \right)$.  
4: Let $\alpha(t) = \theta(t) + \phi_L + \phi_G - \pi$.  
5: $S(t,a) = \sqrt{r_L(t)^2 - a^2} + \sqrt{r_G(t)^2 - a^2} + \alpha(t) a$.  
6: $\hat{V}(a) = \int_{t_{\text{start}}}^{t_{\text{end}}} V(t) \exp \left( -i k S(t,a) \right) dt$.  
7: Let $u(a) = \angle \hat{V}(a)$.  
8: Let $u_P(a)$ be a local polynomial fit of $u(a)$.  
9: $\alpha(a) = -\frac{1}{k} \frac{du_P}{da}$. 
Figure 1. An example of a retrieved bending angle (left) and its corresponding amplitude (right) of a measurement with a weak refractive gradient.

Figure 2. An example of a retrieved bending angle (left) and its corresponding amplitude (right) of a measurement with a strong refractive gradient.
Figure 3. An event classified into category 1. $SLTA_{\min}$ is at approximately $-73$ km, and the signal is truncated at $SLTA = -50$ km. The reflection spike is located at an impact height of 1919 m, and $a_{\min}$ is located at an impact height of 2017 m.

3. EXPERIMENTAL SETUP

The goal of this paper is to reproduce the results from Ref. 11, using FSI instead of PM. Thus, the same data and parameters will be used. RO measurements were downloaded from the COSMIC Data Analysis and Archive Center (CDAAC), specifically from the Metop-A mission on October 1st, 2007. Ten events over the ocean have been selected, such that their sharpest refractive gradient varies to cover a span of typical, realistic cases. The events have been classified into categories based on the maximum gradient of their co-located refractivity profile. These categories are defined for a reference dataset of refractivity profiles, where category 1 has the smallest gradients and category 4 the largest (super-refractive) ones. For each measurement, a suitable truncation point was set at hand-picked straight line tangent altitudes (SLTA) such that a reflection spike will be clearly visible. Using the same inputs for both PM and FSI, the signals are processed and the amplitudes of the outputs are plotted for comparison. No filtering is used for the amplitudes.

4. RESULTS

Figures 3 through 12 show FSI amplitudes on top of PM amplitudes. $SLTA_{\min}$ refers to the SLTA where direct and reflected rays join, which is right at the Earth’s surface. The surface impact parameter $a_{\min}$ is received from ECMWF’s co-located refractivity profiles. These profiles stop a small distance above the surface, and this last bit is linearly extrapolated. It can clearly be seen that the respective amplitudes both give rise to the reflection signature, as the structures match very closely.

In all cases there is an offset in impact height between the reflection spike and $a_{\min}$. The size of this offset depends on at which altitude the signal is truncated; for the reflection spike to occur at $a_{\min}$, the signal would need to be truncated precisely at $SLTA_{\min}$.

The level of noise is more dramatic in measurements corresponding with refractivity profiles of higher categories. Figure 5 and Fig. 6 in particular are examples of amplitudes with low levels of noise and distinct reflection spikes, while measurements from higher categories like Fig. 11 and Fig. 12 typically show more dramatic noise levels and more smeared out reflection spikes. Additionally, the occurrence of wildly oscillating structures that cannot just be attributed to noise is more common in the higher categories.

5. CONCLUSION

The goal of this study was to confirm that reflection spikes can be found in the FSI amplitude just as well as in the PM amplitude. An implementation of FSI was made, and ten RO signals with varying levels of noise were
Figure 4. An event classified into category 1. $SLT A_{min}$ is at approximately $-73$ km, and the signal is truncated at $SLT A = -60$ km. The reflection spike is 1881 m, and $a_{min}$ is located at an impact height of 1992 m.

Figure 5. An event classified into category 1. $SLT A_{min}$ is at approximately $-86$ km, and the signal is truncated at $SLT A = -60$ km. The reflection spike is located at an impact height of 1948 m, and $a_{min}$ is located at an impact height of 2008 m.
Figure 6. An event classified into category 1. $SLTA_{min}$ is at approximately $-65$ km, and the signal is truncated at $SLTA = -50$ km. The reflection spike is located at an impact height of 1849 m, and $a_{min}$ is located at an impact height of 1888 m.

Figure 7. An event classified into category 1. $SLTA_{min}$ is at approximately $-72$ km, and the signal is truncated at $SLTA = -60$ km. The reflection spike is located at an impact height of 1940 m, and $a_{min}$ is located at an impact height of 1993 m.
Figure 8. An event classified into category 3. $SLT_{A_{min}}$ is at approximately −87 km, and the signal is truncated at $SLT_{A} = −70$ km. The reflection spike is located at an impact height of 2075 m, and $a_{min}$ is located at an impact height of 2204 m.

Figure 9. An event classified into category 3. $SLT_{A_{min}}$ is at approximately −84 km, and the signal is truncated at $SLT_{A} = −50$ km. The reflection spike is located at an impact height of 2057 m, and $a_{min}$ is located at an impact height of 2182 m.
Figure 10. An event classified into category 3. $SLA_{min}$ is at approximately $-85$ km, and the signal is truncated at $SLA = -50$ km. The reflection spike is located at an impact height of 1928 m, and $a_{min}$ is located at an impact height of 2200 m.

Figure 11. An event classified into category 3. $SLA_{min}$ is at approximately $-99$ km, and the signal is truncated at $SLA = -50$ km. The reflection spike is located at an impact height of 2363 m, and $a_{min}$ is located at an impact height of 2301 m.
processed using a manual truncation. The results presented show that reflection spikes are apparent in the FSI amplitude. For cases with low refractive gradients, the spikes are often quite distinct, while sharper gradients tend to result in less clear signs of reflection. This matches well with the behavior of the PM amplitude. While the noise characteristics are not exactly the same, the structures of the respective amplitudes are very similar, including reflection signatures. There is no indication that one operator is able to more clearly depict an event containing reflection than the other. This is an expected result - both FSI and PM are FIOs, meaning they both rely on stationary phase approximations to process signals.

FSI allows for faster processing than PM, and it may be that useful information can be gained from the amplitude curves by varying the value for the truncation SLTA. In this way one can make the upper edge of the reflection spike move around, and by fitting a suitable model of the spike one can possibly retrieve the SLTA as a function of impact parameter for the reflected ray. This in turn could allow for the retrieval of such things as the value for $a_{\text{min}}$ and $SLTA_{\text{min}}$.

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ABSTRACT

GNSS Radio Occultation (GNSS-RO) is an opportunistic Earth sensing technique where GNSS signals passing through the atmosphere are received in low Earth orbit and processed to extract meteorological parameters. As signals are received along an orbit, the measured Doppler shift is transformed to a bending angle profile (commonly referred to as bending angle retrieval), which, in turn, is inverted to a refractivity profile. Thanks to its high vertical resolution and SI traceability, GNSS-RO is an important complement to other Earth sensing endeavors. In the lower troposphere, GNSS-RO measurements often get degraded and biased due to sharp refractive gradients and other complex structures. The main objective of this thesis is to explore contemporary retrieval methods such as phase matching and full spectrum inversion to improve their performance in these conditions. To avoid the bias caused by the standard inversion, we attempt to derive additional information from the amplitude output of the examined retrieval operators. While simulations indicate that such information could be found, it is not immediately straightforward how to achieve this with real measurements. The approach chosen is to examine reflected signal components and their effect on the amplitude output.