Mutual Coupling Effects on Direction of Arrival Estimation

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Introduction: The demand for mobile communication has been growing almost exponentially for the last years and continues to increase as it becomes easier to use, is more widely available and offers a greater variety of services. In order to fulfill the increasing demands on capacity and coverage *smart* or *intelligent* antennas have been suggested. These antennas increase the spectral efficiency of a wireless system by using arrays of antenna elements to shape RF signals in particular directions. Generally, smart antennas can be divided into two categories: *switched multi-beam* and *adaptive*.

Adaptive antennas utilize sophisticated signal processing algorithms to continuously distinguish between desired signals, multipath and interfering signals. In some scenarios, directions of arrival (DOAs) are estimated as well. This makes it possible to smoothly track users with main lobes and inteferers with nulls and thereby constantly maximizing the SINR (signal-to-interferens and noise ratio).

DOA can be an important parameter for an adaptive antenna system to estimate, for example, in high mobility wireless communication systems. In this paper, we address the plausibility of assuming the array to be ideal (which is often the case in array signal processing theory) when the received signals are actually collected with a physical array consisting of standard folded dipoles. In reality, the antenna elements will couple electromagneticly to each other affecting the characteristic impedances and the element patterns. This *mutual-coupling* effect is well known to antenna designers and has in *array signal processing* often been dealt with by positioning *dummy* elements at the boundaries of the array to create a homogenous electromagnetic environment for the inner elements. However, the question still remains whether or not these inner elements can be assumed to be ideal.

In this study we first measured the element patterns (or steering vectors) of a twelve element antenna array (built by Allgon) to get an idea of the magnitude of the departure from ideality of a practical antenna array. These results were then used in array signal processing simulations. The performance of two DOA estimation algorithms, *MUSIC* [1] and *ESPRIT* [2], was studied and compared.

Theory: The following is a short overview of some array signal processing theory of interest for this study. More detailed analysis can be found in references 1, 2 and 3.

The output, $\mathbf{x}(t)$, from an array of m elements can be described as:

$$\mathbf{x}(t) = \mathbf{A}(\theta)\mathbf{s}(t) + \mathbf{n}(t) \qquad (1),$$

where $\mathbf{A}$ is the collection of array steering vectors ($\mathbf{a}(\theta)$, the element factors), $\mathbf{s}(t)$ is the complex representation of d narrowband *non-coherent* signals, and, $\mathbf{n}(t)$ represents the noise.

The signal parameters of interest are of spatial nature. Therefore, we form the output *spatial covariance* matrix defined as:

$$\mathbf{R} = \mathbf{x}(t)\mathbf{x}^H(t) = \mathbf{A}E\{\mathbf{s}(t)\mathbf{s}^H(t)\}\mathbf{A}^H + E\{\mathbf{n}(t)\mathbf{n}^H(t)\} = \mathbf{A}(\theta)\mathbf{S}\mathbf{A}^H(\theta) + \sigma^2\mathbf{I} \qquad (2).$$

Here $(*)^H$ denotes Hermetian conjugate, E(*) is the expectation operator, σ^2 the noise power and $\mathbf{I}$ is the identity matrix. Through eigendecomposition it is possible to rewrite the covariance matrix as:

$$\mathbf{R} = \mathbf{U}_s\Lambda_s\mathbf{U}_s + \sigma^2\mathbf{U}_n\mathbf{U}_n^H \qquad (3)$$

where the columns of $\mathbf{U}_s$ span the signal subspace and the columns of $\mathbf{U}_n$ span the noise subspace. $\Lambda_s = \mathrm{diag}\{\lambda_1,\ldots,\lambda_d\}$ is a diagonal matrix of real and positive eigenvalues which correspond to *d* signal eigenvectors. The true DOAs, corresponding to array steering vectors, span a space containing the signal subspace:

$$S = \mathrm{span}\{\mathbf{U}_s\} = \mathrm{span}\{\mathbf{A}(\theta)\mathbf{S}\} \subseteq \mathrm{span}\{\mathbf{A}(\theta)\}. \qquad (4)$$

This relationship is the basis of all signal subspace techniques such as MUSIC and ESPRIT, the two algorithms studied in this paper.

MUSIC

MUSIC is a spectral-based algorithmic solution to the DOA estimation problem. The estimates of vectors that approximately span the noise subspace are obtained by choosing the eigenvectors corresponding to the *m-d* smallest eigenvalues of $\hat{\mathbf{R}}_N$ (the sampled covariance matrix measured at the array). Schmidt [1] proposed the following algorithm as one possible measure of the *closeness* of an element of the array manifold, *A* (the collection of all array steering vectors), with $\hat{S}$, the estimated signal subspace:

$$P_M(\theta) = \frac{\mathbf{a}^H(\theta)\mathbf{a}(\theta)}{\left|\mathbf{a}^H(\theta)\hat{\mathbf{U}}_n\right|^2} = \frac{\mathbf{a}^H(\theta)\mathbf{a}(\theta)}{\mathbf{a}^H(\theta)\hat{\mathbf{U}}_n\hat{\mathbf{U}}_n^H\mathbf{a}(\theta)} \qquad (5).$$

This so-called MUSIC *pseudo spectrum* is searched for peaks over the parameter range of interest. The location of the *d* largest peaks of P_M are the estimates of the DOAs.

ESPRIT

ESPRIT [2] is a parametric based algorithm that gives a solution to the DOA problem without knowing the array manifold. In this case the array is assumed to be composed of two identical subarrays where every sensor in one of the subarrays has a *doublet* in the other.

The two subarrays are assumed to contain m/2 elements each, and be displaced from each other by a known displacement vector **D**. The total array output is modeled as:

$$\mathbf{x}(t) = \begin{bmatrix}\overline{\mathbf{A}}\\ \overline{\mathbf{A}}\Phi\end{bmatrix}\mathbf{s}(t) + \begin{bmatrix}\mathbf{n}_1(t)\\ \mathbf{n}_2(t)\end{bmatrix} = \begin{bmatrix}\mathbf{A}_1\\ \mathbf{A}_2\end{bmatrix}\mathbf{s}(t) + \begin{bmatrix}\mathbf{n}_1(t)\\ \mathbf{n}_2(t)\end{bmatrix} \qquad (6),$$

where the *m/2* × *d* matrix $\overline{\mathbf{A}}$ contains the array manifold vectors common to the two subarrays. The matrix Φ is a diagonal *d* × *d* matrix of the phase delays between the doublet sensors for the *d* wavefronts, and is given by

$$\Phi = \mathrm{diag}\{e^{j\omega\tau_1},\ldots,e^{j\omega\tau_d}\} \qquad (7)$$

where τ_k is the time delay in propagation of the k^{th} emitter signal between the two subarrays (ω is the center frequency). As the DOAs are related to the time delay by:

$$\theta_k = -\arcsin\left\{\frac{c\tau_k}{|\mathbf{D}|}\right\} \tag{8},$$

the DOA estimation problem can be reduced to that of finding $\mathbf{\Phi}$.

From equations 2 and 3 one may find that there exists a full rank $d \times d$ matrix $\mathbf{T}$ such that $\mathbf{U}_s = \mathbf{AT}$ [3]. This means that using equation 6 we can separate $\mathbf{U}_s$ into $\mathbf{U}_1 = \mathbf{A}_1\,\mathbf{T}$ and $\mathbf{U}_2 = \mathbf{A}_2\,\mathbf{T}$.

Defining $\Psi = \mathbf{T}^{-1}\mathbf{\Phi}\mathbf{T}$ we get

$$\mathbf{U}_2 = \mathbf{U}_1\Psi \tag{9}.$$

The DOA estimates can now be obtained by applying equation 8 to the eigenvalues of Ψ. In ESPRIT this is being done by eigendecompositioning of $\hat{\mathbf{R}}_N$, forming $\hat{\mathbf{U}}_1$ and $\hat{\mathbf{U}}_2$ from the d principal eigenvectors and solving for Ψ in either a Least-Square (LS) or a Total-Least-Square (TLS) sense. An eigendecomposition of Ψ then provides the eigenvalues, their arguments being related to the DOAs through equation 8.

The antenna array: The antenna studied in the simulations was designed by Allgon System AB and contains twelve antenna elements. The inner eight are used for array signal processing whereas the outer four elements were terminated in 50 ohms. The objective was to create identical individual element patterns, the assumption most often made in array signal processing theory. The gain and phase responses of the array were measured and the measurements combined in the *calibrated* array steering vectors, $\mathbf{a}_c(\theta)$. These calibrated steering vectors were then used in the simulations. The outer right element was used as phase reference.

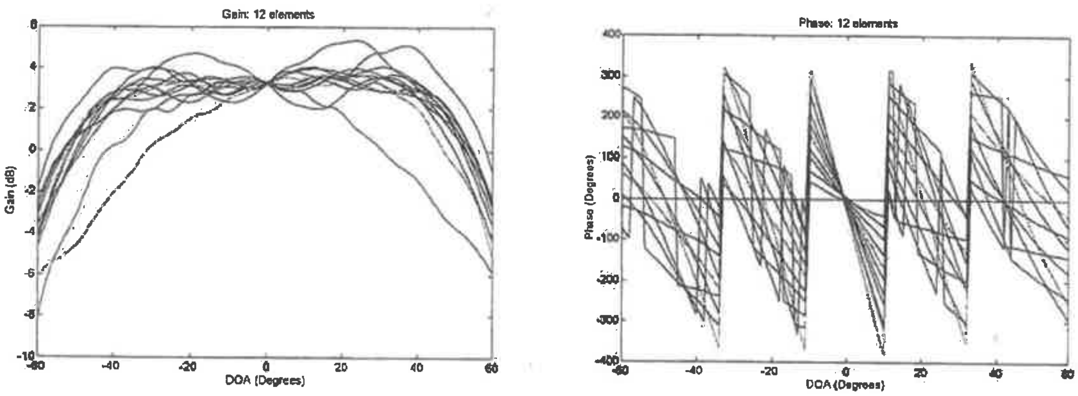

Figure 1. The gain and phase responses of all 12 elements in the array. The inner eight are used for array signal processing whereas the outer four act as dummies.

As can be seen in Figure 1, the gain and phase response for each element differs quite dramatically. The inner eight seem to be fairly similar but can hardly be considered as ideal omni-directional sensors.

Simulations: The purpose of the simulations was to establish whether or not it is plausible to assume ideal arrays when the received signal data is collected with the array described above. The following questions were considered:

* How accurately can a DOA be resolved with the array?
* How closely can two sources be spaced and still be resolvable?
* How many sources can be resolved at the same time?

The simulations were carried out using MATLAB and a 486 PC. The signals are assumed to be *narrow-banded* and the noise to be *spatially white*. The SNR is set to 3dB (note that in communication applications, the SNRs are generally substantially greater than 3dB for successful link establishment) and the number of samples to 100. The situation is assumed to be planar (azimuth without elevation) and the parameter range is reduced to $\pm 60°$ (typical base station antenna application).

The signal waveforms were assumed to be sinusoidal but their amplitude and phase were randomly generated for each element and sample.

For both MUSIC and ESPRIT, the sampled covariance matrix is constructed with calibrated array steering vectors. Eigendecomposition of $\hat{\mathbf{R}}_N$ then gives $\hat{\mathbf{U}}_s$, $\hat{\mathbf{U}}_n$, $\hat{\mathbf{U}}_1$ and $\hat{\mathbf{U}}_2$. In MUSIC the spectrum is created using $\hat{\mathbf{U}}_n$ and ideal steering vectors (omni-directional elements). The spectrum is then searched for peaks. Maximum overlap ESPRIT [4] is used, and Ψ_{TLS} is created in a TLS sense using $\hat{\mathbf{U}}_1$ and $\hat{\mathbf{U}}_2$. The estimated DOAs thus obtained can be used to assess the impact of the assumption of ideality on the accuracy of DOA estimation using the chosen methods.

As measures of estimation accuracy the "Root-Mean-Square-Error" (RMSE) was used.

$$\mathrm{RMSE} = \sqrt{\tfrac{1}{N}\sum_{i=1}^{N}(\hat{\theta}-\theta)^2} \qquad (10),$$

where N is the number of repetitions, $\hat{\theta}$ the estimated DOA and θ the true DOA.

Results: For one DOA at broadside, the RMSE values for both MUSIC and ESPRIT are less than 2.0°. However, the estimated RMSE values increase with angle and the highest values are achieved at the boundaries (the array used is not symmetric, *cf.*, figure 1). At 60° the resolutions are 5.8° and 5.5° for MUSIC and ESPRIT respectively.

As is the case for one DOA, two sources are more difficult to resolve at the boundaries (see for example Figure 2) than at broadside. Whereas MUSIC was not able to resolve two sources spaced 10° apart at angles larger than 50°, ESPRIT was still able to do so.

For both MUSIC and ESPRIT, maximum 5 sources could be resolved with the array. This is two less than theoretical for MUSIC and one less than theoretical for ESPRIT. In general, ESPRIT was found to outperform MUSIC. The reason for this is that MUSIC requires all steering vectors used for creating the spectrum to be similar to those collecting the data. In ESPRIT the only necessary requirement is that the *doublets* are fairly similar.

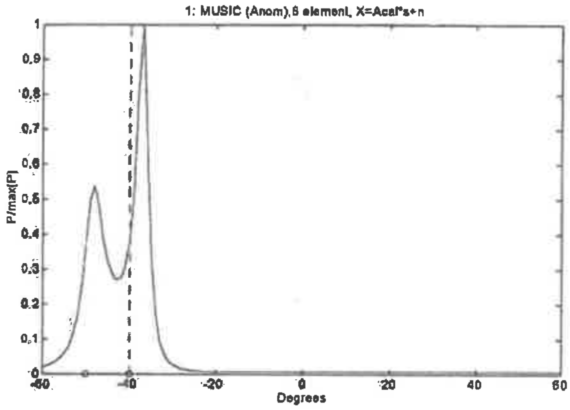

Figure 2. The Music spectrum for two DOAs at -50° and -40°.

Discussions and Conclusions: The effects of mutual coupling on the performance of antenna arrays are well known to antenna designers but, perhaps, less so to signal processing engineers. We have therefore studied the effects of unmodelled errors in array manifold vectors on two high-resolution DOA estimation algorithms, MUSIC and *ESPRIT*

The scenario chosen for simulation might be thought to be unrealistic. While certainly a difficult scenario, this situation could arise, for example, when adaptive arrays are employed to increase the range of coverage from a particular base station, and a mobile unit is at the edge of this increased coverage area. While the individual element SNR is 3dB, after appropriate signal processing, the output SNR could be substantially increased to something on the order of 11-12dB. This is in fact the objective when employing adaptive antenna arrays to such systems.

There are of course other factors, such as noise, that may affect the antenna array errors. One may also argue that in mobile communications accurate DOAs is not the main goal and that what is really of interest in these applications are the actual signal waveform estimates. Still our study shows that mutual coupling effects will indeed affect the DOA estimation when performed with a practical antenna array consisting of standard dipoles, indicating that this also could be a severe, and neglected, problem in other adaptive antenna applications.

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