Robust and Agile Attitude Control for Triple CubeSat Eye-Sat

BENJAMIN CHARBAUT
Ce que l'on conçoit bien s'énonce clairement,
Et les mots pour le dire arrivent aisément.

[...]

Travaillez à loisir, quelque ordre qui vous presse,
Et ne vous piquez point d'une folle vitesse :
Un style si rapide, et qui court en rimant,
Marque moins trop d'esprit que peu de jugement.

J'aime mieux un ruisseau qui, sur la molle arène,
Dans un pré plein de fleurs lentement se promène,
Qu'un torrent débordé qui, d'un cours orageux,
Roule, plein de gravier, sur un terrain fangeux.

Hâtez-vous lentement, et, sans perdre courage,
Vingt fois sur le métier remettez votre ouvrage :
Polissez-le sans cesse et le repolissez ;
Ajoutez quelquefois, et souvent effacez.

Nicolas BOILEAU (1636-1711)
Foreword

"Don’t ever start working before you finish your Thesis!" insisted Ulf Ringertz, professor and teacher of Flight Mechanics at KTH. This work is only a Master Thesis, but I wish I had taken his advice more seriously! One year after leaving CNES at the end of my final year-internship and starting working for Airbus, I am finally on the brink of completing my studies. One year of trying to work in the mornings, in the evenings, or on week-ends. One year that taught me how important the work-life balance is.

Although he kept me busy for all this time, I am deeply grateful to Frédérick Viaud for having been an awesome supervisor. "Careful, you might not be able to finish all this work", he said at the very beginning. But the work was stimulating on its own, and Frédérick’s constant availability, alacrity and creativity made it even more challenging, and playful. Thanks for a great time!

I would also like to express my gratitude to Gunnar Tibert, my supervisor at KTH, for understanding the reasons of this one-year-delay, and for making this final year project even possible by conciliating Swedish and French views on confidentiality.

I consider myself lucky to have benefited from an ideal work environment at CNES, for which I would like to thank Stéphane Berrivin and the AOCS service, and the permanent Eye-Sat team, Antoine, Fabien, Nicolas and Christophe. Special thanks to Alain Gabori-
AUD, the originator of this whole nanosatellite adventure, for his valuable mentoring.

This project was also about experiments, which were only possible thanks to Pierre-Emmanuel Martinez, Laurent Rivière, Simon Debois and Yann Le Huédé, both for the preparation and realisation of wheel, magnetometer and magnetorquer tests.

Let us not forget that Eye-Sat is a student-driven project! I really appreciated the start-up spirit my fellow interns and I developed throughout these months, and the momentum that came with it, both in work and in more casual fields... Je coinche!

Now that this work is close to achievement, a special thought for my fellow KTH student Sarra Fakhfakh, without whom I would have never heard of the Eye-Sat project in the first place.

Last but not least, I would like to thank my closest relatives and Nicole, for their daily support.
Abstract

Eye-Sat is a student-designed 3U-CubeSat, to be launched to a sun-synchronous orbit from where it will map the zodiacal light, a faint glare caused by the reflection of Sun on interplanetary dust. Such mission requires an accurate 3-axis attitude control, for which Eye-Sat is equipped with reactions wheels, magnetorquers, magnetometers and a star tracker. The star tracker can only be used for inertial pointing, which confines its use to shooting phases. A solution based on the remaining 3 equipment is proposed for the other mission phases, providing 3-axis pointing with high agility, for ground station tracking, at the cost of a slightly degraded accuracy. The magnetometers and magnetorquers work in closed-loop, while manoeuvres are performed in open-loop by the reaction wheels, which also ensure gyroscopic stabilisation of the spacecraft. Since this design relies on only one sensor, efforts have been put into making it robust to the imperfections of the magnetometers. Robustness to potential changes in the mission or the design has also been taken into consideration. Performance assessments carried out on a preliminary tuning have demonstrated the capacity of this magnetic-based mode to recover 3-axis pointing when exiting the survival mode, to provide a 3-axis pointing accuracy better than 8° in the worst case, and to sustain slews up to 0.87° s⁻¹ in download.
**Sammanfattning**

Eye-Sat är en 3U-CubeSat utformad och byggd av studenter. Den ska placeras i en solsynkron omloppsbana där den kommer att kartlägga zodiakljuset, en svag bländning producerad när solens ljus reflekterar på interplanetärt damm. Detta rymduppdrag kräver en precis reglering kring tre axlar och därför är Eye-Sat utrustad med fyra reaktionshjul, magnetspolar för kraftmomentgenerering, magnetometrar och en stjärnsensor. Stjärnsensorn kan endast användas för inertial attitydreglering, vilket begränsar användningen till fotograferingsfasen. En strategi baserad på de återstående regleringsdonen och sensorerna föreslås för de andra rymduppdragsfaserna, vilken ger treaxlig pekning för markstationsspårning, men med något sämre noggrannhet. Magnetometrarna och magnetspolararna arbetar i sluten reglering, medan manövreringen genomförs i öppen reglering med reaktionshjulen, vilka också säkerställer gyroskopisk stabilisering av rymdfarkosten. Eftersom denna utformning är beroende av endast en sensor är det kritiskt att göra den robust mot mätfel hos magnetometern. Robusthet mot potentiella framtida förändringar i utformningen har också beaktats. Preständabedömningar som gjorts vid en preliminär inställning har demonterat att den magnetiska regleringen kan återställa treaxlig pekning när man lämnar den säkra regleringmoden. En treaxlig pekningsnoggrannhet bättre än 8 grader i värsta fall och vinkelhastigheter upp till 0.87 grader/s i nedladdningsfasen.
Résumé

Eye-Sat est un CubeSat 3U réalisé par des étudiants. En orbite solaire-synchrone, il réalisera une cartographie de la lumière zodiacale, un halo diffus produit par la réflexion des rayons du Soleil sur les poussières interplanétaires. Une telle mission nécessite un contrôle d’attitude 3 axes performant, pour lequel Eye-Sat est équipé de roues à réaction, de magnétocoupleurs, de mangéomètres et d’un senseur stellaire. Le senseur stellaire ne pouvant être utilisé qu’en pointage inertielle, son usage est cantonné aux prises de vue. Pour les autres phases de la mission, une solution basée exclusivement sur les 3 autres équipements est proposée. Elle assure un pointage 3 axes avec l’agilité requise pour le suivi de la station sol, au prix d’une précision de pointage légèrement dégradée. Les mangéomètres et les magnétocoupleurs fonctionnent en boucle fermée, tandis que les manœuvres sont réalisées en boucle ouverte par les roues. Enfin, les roues assurent la stabilisation gyroscopique du satellite. Le contrôle d’attitude reposant sur un unique senseur, il était important de le rendre robuste aux imperfections des mangéomètres. La robustesse aux éventuelles modifications de la mission ou du design est également assurée. Les simulations réalisées avec des réglages préliminaires démontrent la capacité de ce mode magnétique à recouvrer un pointage 3 axes en sortie de mode survie, à assurer une erreur de pointage 3 axes meilleure que 8° dans le pire cas, et à réaliser des rotations jusqu’à 0.87° s⁻¹ en suivi de station sol.
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Notations

- Vectors are denoted by bold letters. In figures however, an overhead arrow will be used. When necessary, a subscript indicates the reference frame in which they are expressed, as in \( \mathbf{v}|_F \).

- Matrices are written between square brackets, as in \([M]\).

- The direction cosine matrix from a frame \( \mathcal{N} \) to a frame \( \mathcal{F} \), also called transfer matrix from \( \mathcal{N} \) to \( \mathcal{F} \), is denoted by \([\mathcal{F}/\mathcal{N}]\), so that \( \mathbf{v}|_F = [\mathcal{F}/\mathcal{N}] \mathbf{v}|_N \). Its rows are the base vectors of frame \( \mathcal{F} \) expressed in the base of frame \( \mathcal{N} \). By analogy, the corresponding quaternion is denoted by \( Q_{\mathcal{F}/\mathcal{N}} \).

- \( \Omega_{\mathcal{B}/\mathcal{F}} \) is the rotation vector of frame \( \mathcal{B} \) with respect to frame \( \mathcal{F} \).

- \([\ddot{\mathbf{x}}]\), with \( \mathbf{x} \) a 3-dimensional vector, is the matrix such that \( \ddot{\mathbf{x}} \mathbf{v} = \mathbf{x} \times \mathbf{v} \) for any 3-dimensional vector \( \mathbf{v} \). The coefficients of this matrix depend on the reference frame in which it is expressed.

- The derivative of a vector \( \mathbf{v} \) with respect to time in the frame \( \mathcal{N} \) is denoted by \( \frac{d\mathbf{v}}{dt}|_\mathcal{N} \).

- Time derivatives of scalar quantities are denoted by a dot, as in \( \dot{\theta} \).
Glossary

AOCS  Attitude and Orbit Control System  
CAD  Computer-Aided Design  
COTS  Commercial Off-The-Shelf  
CSKB  CubeSat-Kit Bus  
GNSS  Global Navigation Satellite System  
IGRF  International Geomagnetic Reference Field  
ITRF  International Terrestrial reference Frame  
LEOP  Launch and Early Operations Phase  
MAG  Magnetometers  
MAS  Acquisition and Survival Mode  
MFV  End-of-life Mode  
MGT  Coarse Transition Mode  
MLT  Launch Mode  
MNO  Normal Mode  
MTB  Magnetorquers Board  
PD  Proportional-Derivative  
P-POD  Poly-Picosatellite Orbital Deployer
<table>
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<th>Acronym</th>
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<tr>
<td>RWS</td>
<td>Reaction Wheels</td>
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<tr>
<td>TAI</td>
<td>International Atomic Time</td>
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<tr>
<td>TC</td>
<td>Telecommand</td>
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<tr>
<td>TM</td>
<td>Telemetry</td>
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<tr>
<td>TT</td>
<td>Terrestrial Time</td>
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<tr>
<td>UTC</td>
<td>Coordinated Universal Time</td>
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Chapter 1

Introduction

1.1 About CNES

CNES (Centre National d’Études Spatiales, National Centre of Space Studies) is France’s space agency, founded in 1961. With a yearly budget of €2.33 billion in 2017, the second per capita in the world for a civil space agency [1], it is in charge of proposing and executing France’s space policy, by supporting research and industrial application and leading space innovation.

CNES is also in charge of international space cooperation. It represents France within ESA (European Space Agency), of which it is the first contributor with a budget of €833 million [1] out of a €5.75 billion total in 2017 [2]. CNES is also responsible for collaboration with non-European space actors through joint programmes.

The activities of CNES span over five domains [3]:

- Launchers, guaranteeing autonomous access to space with Ariane, in collaboration with Arianespace and ESA;

- Science, with collaborations to both national and European scientific missions. CNES
was a major contributor to the design of the comet-lander Philae launched in 2004, pioneered exoplanet detection with CoRoT (Convection, Rotation and planetary Transit) from 2006 to 2014, and launched MICROSCOPE (Micro-Satellite à trainée Compensée pour l’Observation du Principe d’Équivalence, Drag-Compensated Micro-Satellite for the Observation of the Equivalence Principle) in 2016, a mission which tests the universality of free fall\(^1\);

- Earth Observation, starring the SPOT programme (Satellites Pour l’Observation de la Terre, Satellites for Earth Observation) for mapping, vegetation monitoring and estimation of the impacts of natural disasters, or more recently, the MicroCarb mission which will map the carbon dioxide sources and sinks starting in 2020;

- Telecommunications. Although this business is mainly private, CNES supports and validates technologies that industry will implement, the main effort being on high bandwidth services in remote areas. Aside classical telecommunications, CNES is in charge of the LEOP (Launch and Early Orbit Phase) of Galileo satellites, the European GNSS (Global Navigation Satellite System), and manages Argos, a satellite-based search and rescue system, which is capable of locating distress calls all over the globe using the Doppler effect;

- Defence. CNES works jointly with the DGA (Direction Générale de l’Armement, General Directorate of Armaments, in charge of procurement, research and development for the French military) on the elaboration of space systems for military applications, including very high resolution Earth Observation, military telecommunications, and electronic intelligence.

The workforce of CNES is some 2400-strong, out of which 75% are engineers and executives, and 36% are women [5]. It is spread over 4 sites:

---

\(^1\)The equivalence principle states that the inertial and the gravitational mass are equal. MICROSCOPE will test this assertion at a relative precision of \(10^{-15}\). Previous observations demonstrated the validity of the equivalence principle at a relative precision of \(10^{-13}\) [4].
• Headquarters in Paris-Les Halles, with approximately 200 people;

• DLA (Direction des Lanceurs, Launchers Directorate), Paris-Daumesnil, with approximately 200 people. It is in charge of launchers research and development;

• CSG (Centre Spatial Guyanais, Guiana Space Centre), in Kourou, French Guiana, with approximately 300 people. It is Europe’s spaceport, from where Ariane 5, Soyuz ST and Vega rockets are launched;

• CST (Centre Spatial Toulousain, Toulouse Space Centre), in Toulouse, France. With approximately 1700 people, it is the largest site, in charge of orbital systems.

1.2 About nanosatellites

The small satellite designation usually applies to satellites weighing less than 500 kg. This satellite class breaks down into several categories, depending on their mass [6]:

• Minisatellites, from 100 kg to 500 kg;

• Microsatellites, from 10 kg to 100 kg;

• Nanosatellites, from 1 kg to 10 kg;

• Picosatellites, from 0.1 kg to 1 kg;

• Femtosatellites, below 0.1 kg.

Although nanosatellites were launched during the early days of space exploration, between 1957 and 1962, none was launched between 1963 and 1996 [7]. Nanosatellites made their return to space during the last two decades, offering space solutions for education, science and technological demonstration at a reduced cost. More recently, industrial applications, especially under the form of low-cost satellite constellations, also take advantage of nanosatellite
solutions.

The relative cheapness of nanosatellite missions is both explained by a massive use of COTS (Commercial Off-The-Shelf) components, and by standardisation impelled in 1999 by Bob Twiggs and Jordi Puig-Suari of Stanford University and California Polytechnic State University, who co-invented the CubeSat standard [8].

A CubeSat is composed of elementary cube-shaped units, or U, with exterior dimensions $10 \times 10 \times 11 \text{cm}^3$, and a maximum mass of 1.33 kg, although deviations are possible depending on the mission [9]. Units can be combined up to 27U, the most common sizes being 1U, 2U, 3U, 6U, 12U and 18U.

The structure contains 4 rods, arranged in a standardised pattern, along which CubeSat-compatible equipment boards can be slid for assembly. CubeSat equipment are usually stacked together and connected using a single CSKB (CubeSat Kit Bus) 104-pin connector, which both provides electrical supply and ensures data transfer.

![Figure 1.1: A 3U-CubeSat next to its P-POD. Source: Wikimedia.](image)

Due to the lack of a nanosatellite-dedicated launch vehicle as of today, CubeSats are often launched \textit{piggyback} with a larger satellite, or even brought on-board a cargo to the Interna-
tional Space Station from where it is eventually released. Nevertheless, ultra-light launchers currently under development may offer tailor-made launch solutions to CubeSats in the near future. Rocket Lab, a space company from the USA and New Zealand, entered the test phase of its Electron rocket earlier this year, 2017. With its 17 m height and 1.2 m diameter, it is capable of delivering a 150 kg payload to a 500 km Sun-Synchronous Orbit\(^2\) [10].

The standardisation of nanosatellites also addresses launcher separation, with various on-orbit deployers for CubeSats. The P-POD (Poly-Picosatellite Orbital Deployer), shown in Figure 1.1, is the most common of them, and was developed jointly with the CubeSat standard. It can accommodate a total of 3U.

A review of all nanosatellite missions up to 2010 was performed in [7]. Although it does not solely include CubeSats and considers both modern and early nanosatellites\(^3\), it sheds light on the AOCS on-board these satellites, with only 40% of them having an active attitude control, and 20% having no attitude control at all. Most of the time, the AOCS only aimed at reducing the angular rates of the satellite, while 15% of the missions used it to point instruments and 4% to point solar arrays\(^4\). Tracking a ground station was only performed by 2% of nanosatellite missions\(^5\).

Reference [11] focuses specifically on CubeSat missions up to 2012, studying a total of 112 satellites. Education was the main purpose of these CubeSat missions (37%), followed by technological demonstration (34%), science (23%) and communications (6%). Universities were the main CubeSat developers, with 69% of CubeSat missions, the remaining 31% being developed by industry. The most common form factor was 1U, totalling 61% of all missions,

\(^2\)A Sun-Synchronous Orbit is an orbit whose plane has a constant orientation relative to the Sun throughout the year.

\(^3\)Nanosatellite missions from the 1950s and 1960s amount to 17, out of a 94 missions total.

\(^4\)CubeSats are often covered with solar cells on all faces.

\(^5\)Due to the fast-evolving trend of CubeSats, developed in the following paragraphs, these figures are likely to be obsolete.
although there was a clear discrepancy between universities and industry, the latter preferring 3U-CubeSats and higher. The discrepancy was also clear when it came to mission relevance, with only one industry-built beepsat\textsuperscript{6} out of a 41 missions total, which means only 41% of university-built CubeSats were designed to perform actual missions. Finally, [11] highlights the vast number of failures, amounting to 41%, with 10% of the missions suffering launch failure. Once again, the statistical difference between universities and industry is clear, with a success rate of only 45% for academics, versus 77% for industry.

As pointed out in reference [11], the world of CubeSats is evolving fast: at the time this article was written, less and less beepsats were launched, leaving room for actual missions, and while only 112 CubeSats had been launched up until then, 80 were manifested for launch in 2013. According to the Nanosatellite & CubeSat Database [12], a total of 861 nanosatellites have been launched as of November 18, 2017, out of which 796 are CubeSats, and 535 are currently in orbit. A total of 361 nanosatellites manifested for the sole year 2017, and 419 are announced for 2018. This surge raises concerns about the increased collision risk, since most nanosatellites do not feature propulsion.

The market is now dominated by private companies, with 437 satellites in orbit, versus 293 for universities. CubeSats offer unmatched opportunities for low-cost satellite constellations, both for Earth observation and communication. Let us cite for instance the EU-funded QB50 network, for which 2U and 3U-CubeSats are designed by partner universities and institutes to study the lower thermosphere and atmospheric re-entry, of which 36 were launched in 2017 [13]. On February 14, 2017, an Indian PSLV rocket released a record 103 CubeSats into orbit [14], out of which 88 belonged to Planet, a US-based imaging company. With a total of 149 3U-CubeSats within its constellation, it aims at imaging the totality of Earth’s surface every day [15].

\textsuperscript{6}A beepsat is a satellite whose only mission is to send a signal back to Earth to prove it has reached orbit, the most famous example being Sputnik.
The increased accessibility of nanosatellite missions, both technically and economically, has opened a whole new field of space applications for a wide range of actors, from students and academics to industrials. On some occasions, a CubeSat even became the first satellite of its home country! It was for instance the case for ESTCube-1 of Estonia in 2013 [16]. The market is however heavily dominated by North America, with 59% of nanosatellites being US-owned, while Europe’s share amounts to 25% [12].

1.3 About JANUS and Eye-Sat

The Janus programme was started in 2012 by CNES, in partnership with a dozen French universities [17], with the objective of promoting space activities for students through the development of CubeSats. In addition to its educative purpose, the essence of Janus is to test new technologies and perform science in orbit, in collaboration with industrials and research institutes.

Eye-Sat is the pilot project of Janus [18]. Initiated in 2012, it aims at producing a state-of-the-art CubeSat with student teams, using the support of CNES experts and facilities. It will perform an astronomy mission, and on-orbit demonstration of CNES technological innovations.

1.4 Scope of this work

At the time of this work, Eye-Sat was in its C/D development phase. The A/B development phase of the AOCS was performed in 2014 [19]. Previous activities in the C/D phase featured the development and validation of a survival mode, in 2016 [20]. The rest of the work focused on the normal mode, in which the mission is carried out, with the design of an attitude estimator and the guidance functions.
However, uncertainties on the performance of the attitude-related equipment and the unavailability of the star tracker in a variety of cases motivated a change in the AOCS architecture, and triggered the need for a magnetic-based normal mode, which constitutes the topic of this work. This mode will be entered directly when exiting the survival mode, and will be used for all mission phases except shooting, for which the availability of the star tracker is guaranteed.

The magnetic-based normal mode must achieve the following goals:

- It has to rely only on magnetometers for attitude determination, and on magnetorquers and reaction wheels for actuation.

- It has to be robust, to provide a fast convergence of the pointing with stringent entry conditions.

- It has to ensure 3-axis pointing with reasonable accuracy, in order to prepare for the transition to a stiffer control for the shooting phases.

- It has to be agile, in order to perform slew manoeuvres in a limited amount of time, and track a ground station.

In order to meet these goals, this work adopts a bottom-up approach, from the equipment to the design of the controller.

- Chapter 2 gives elements of mission analysis;

- The performance of the attitude-related equipment is assessed in chapter 3. The in-flight usage of the equipment is discussed, based on test results;

- Chapter 4 introduces the dynamics of the satellite, and translates it into equations;
• Chapter 5 is dedicated to the design of the attitude determination and control. Support functions for navigation and guidance are briefly introduced;

• A stability analysis is performed in chapter 6;

• A preliminary tuning of the control laws is proposed in chapter 7, taking into consideration criteria established in chapter 6;

• Finally, the performance of the magnetic-based normal mode is evaluated in chapter 8 on reference cases.
Chapter 2

Mission analysis

2.1 Mission definition

Eye-Sat is a 3U-CubeSat, whose aim is to conduct the first satellite mission dedicated to the mapping of the intensity and polarisation of the zodiacal light.

The zodiacal light is a faint glow caused by the scattering of sunlight by interplanetary dust, whose spectrum extends in the visible and near-infrared wavelengths. It is a significant contributor to the night sky brightness, and even the first one throughout the infrared [21]. Therefore, characterising it is crucial for the quality of astronomical observations in these wavelengths.

The zodiacal light is stronger near the Sun and the ecliptic, and in the direction of the gegenschein, i.e. opposite to the Sun. However, its distribution is asymmetric, and displays an annual variation.

Eye-Sat’s mission is designed to last one year in order to cover the annual variation of the zodiacal light. The areas of interest will be imaged repeatedly throughout the year, in
wavelengths – blue, green, red and near-infrared – and with 3 different polarisations [22].

The outreach of the mission will be to shoot a deep panoramic view of the Milky Way in the four wavelengths used for imaging the zodiacal light.

2.2 Description of rotations

The relative orientation, or attitude, of two frames can be described by means of Euler rotation sequences. In this work, the 3-2-1 set\(^1\) of Euler angles will be used.

![Diagram of Euler angles sequence](image)

Figure 2.1: The 3-2-1 Euler angles sequence.

In figure 2.1, the \(\{\psi, \theta, \phi\}\)\(^2\) sequence describes the orientation of \(\mathcal{F}\) relative to \(\mathcal{N}\). The coordinates of a vector \(\mathbf{v}\) in either frame are linked by \(\mathbf{v}|_{\mathcal{F}} = [\mathcal{F}/\mathcal{N}] \mathbf{v}|_{\mathcal{N}}\), where \([\mathcal{F}/\mathcal{N}]\) is the direction cosine matrix that describes the orientation of \(\mathcal{F}\) relative to \(\mathcal{N}\).

\[
[\mathcal{F}/\mathcal{N}] = \begin{bmatrix}
c\theta c\psi & c\theta s\psi & -s\theta \\
c\phi s\theta c\psi - c\psi s\phi & s\phi s\theta s\psi + c\phi c\psi & s\phi c\theta \\
c\phi s\theta c\psi + s\psi & c\phi s\theta s\psi - s\phi c\psi & c\phi c\theta
\end{bmatrix}
\]  

(2.1)

c and s are abbreviations standing for \(\cos\) and \(\sin\). The rows of \([\mathcal{F}/\mathcal{N}]\) are the coordi-

\(^1\)3-2-1 means that the rotations are performed sequentially around the third, then the second, and finally the first axis. The order in which the rotations are performed is crucial, since rotations in space are not commutative. The 3-2-1 set of angles is sometimes called Cardan angles.

\(^2\)The angles are respectively designated by yaw, pitch and roll.
nates of the base vectors of $\mathcal{F}$ expressed in frame $\mathcal{N}$, and its columns are the base vectors of $\mathcal{N}$ expressed in frame $\mathcal{F}$. Consequently, $[\mathcal{F}/\mathcal{N}]^\top = [\mathcal{N}/\mathcal{F}]$. Direction cosine matrices are orthogonal matrices: $[\mathcal{F}/\mathcal{N}]^\top = [\mathcal{F}/\mathcal{N}]^{-1}$ for any frame $\mathcal{N}$ and any frame $\mathcal{F}$.

For any frames $\mathcal{B}$, $\mathcal{F}$ and $\mathcal{N}$, direction cosine matrices are composed as follows\(^3\):

$$[\mathcal{B}/\mathcal{F}] [\mathcal{F}/\mathcal{N}] = [\mathcal{B}/\mathcal{N}]. \quad (2.2)$$

The rotation of frame $\mathcal{F}$ with respect to frame $\mathcal{N}$ is described by the vector $\mathbf{\Omega}_{\mathcal{F}/\mathcal{N}}$. The reader is referred to [23, pp. 93–94] for the demonstration.

$$\mathbf{\Omega}_{\mathcal{F}/\mathcal{N}} = \begin{bmatrix} \dot{\phi} - \sin(\theta)\dot{\psi} \\ \cos(\varphi)\dot{\theta} + \sin(\varphi)\cos(\theta)\dot{\psi} \\ -\sin(\varphi)\dot{\theta} + \cos(\varphi)\cos(\theta)\dot{\psi} \end{bmatrix}_{\mathcal{F}}. \quad (2.3)$$

The Euler angles are a minimal set, \textit{id est} 3 coordinates are used to describe rotations around the 3 axes, which any smaller set cannot achieve. The downside to being a minimal set is the existence of singularities: any minimal set has a geometric singularity that results in a kinematic singularity\(^4\). For the 3-2-1 Euler angles sequence, this singularity occurs when $\theta = \pm 90^\circ$, for which the description of attitude becomes non-unique. To illustrate this, let

\(^3\)This directly results from the composition of rotations.
\(^4\)A geometric singularity is a singularity in the description of the orientation, while a kinematic singularity occurs while differentiating this orientation.
us consider the direction cosine matrix (2.1) with $\theta = 90^\circ$:

\[
\begin{bmatrix}
F/N
\end{bmatrix} =
\begin{bmatrix}
0 & 0 & -1 \\
s\varphi c\psi - c\varphi s\psi & s\varphi s\psi + c\varphi c\psi & 0 \\
c\varphi c\psi + s\varphi s\psi & c\varphi s\psi - s\varphi c\psi & 0 \\
0 & 0 & -1 \\
\sin (\varphi - \psi) & \cos (\varphi - \psi) & 0 \\
\cos (\varphi - \psi) & -\sin (\varphi - \psi) & 0
\end{bmatrix}
\]

(2.4)

\[
= 
\begin{bmatrix}
0 & 0 & -1 \\
\sin (\varphi - \psi - \psi) & \cos (\varphi - \psi - \psi) & 0 \\
\cos (\varphi - \psi - \psi) & -\sin (\varphi - \psi - \psi) & 0
\end{bmatrix}
\]

(2.5)

The values of $\varphi$ and $\psi$ do not matter individually, as the direction cosine matrix only depends on their difference, $\varphi - \psi$. Therefore, an infinity of couples $\{\varphi, \psi\}$ can be used to describe the same orientation.

To resolve the natural singularities of minimal sets, one has to introduce an additional coordinate\(^5\). If chosen correctly, the new set is redundant but does not feature any singularity. Direction cosine matrices, for instance, are highly redundant, since they comprise 9 coordinates. Let us introduce $[C]$, a generic direction cosine matrix.

\[
[C] =
\begin{bmatrix}
C_{1,1} & C_{1,2} & C_{1,3} \\
C_{2,1} & C_{2,2} & C_{2,3} \\
C_{3,1} & C_{3,2} & C_{3,3}
\end{bmatrix}
\]

(2.6)

Since direction cosine matrices are orthogonal, they describe rotations. From any direction cosine matrix (2.6), it is possible to extract a unitary rotation axis $e = [e_1, e_2, e_3]^T$\(^6\) and a

---

\(^5\)For $n \geq 3$ coordinates, only 3 are free degrees of freedom. The others are linked by orthogonality constraints.

\(^6\)The subscript indicating the frame in which the coordinates of the rotation axis are expressed can be omitted, since by definition, $[C]e = e$, so for any couple of frames $\{N, F\}$, $e|_F = [F/N]e|_N = e|_N$. 

14
principal rotation angle $\Theta$ [23, p. 98].

\[
\Theta = \cos^{-1} \left( \frac{C_{1,1} + C_{2,2} + C_{3,3} - 1}{2} \right) \tag{2.7}
\]

\[
e = \frac{1}{2 \sin \Theta} \begin{bmatrix} C_{2,3} - C_{3,2} \\ C_{3,1} - C_{1,3} \\ C_{1,2} - C_{2,1} \end{bmatrix}. \tag{2.8}
\]

The couple $\{e, \Theta\}$ already constitutes a 4-coordinate non-singular set describing the attitude, but it will be used here to define the corresponding attitude quaternion $Q$.

\[
Q = \begin{bmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \end{bmatrix} = \begin{bmatrix} \cos \frac{\Theta}{2} \\ e \sin \frac{\Theta}{2} \end{bmatrix}. \tag{2.9}
\]

$q_0$ is called the scalar part of the quaternion, while $[q_1, q_2, q_3]^T$ is called the vector part.

By analogy with the notation $[\mathcal{F}/\mathcal{N}]$ for the direction cosine matrix, the quaternion describing the orientation of the $F$ frame with respect to the $N$ frame is denoted by $Q_{\mathcal{F}/\mathcal{N}}$.

Let us define the quaternion product, which shall be denoted by the symbol $\otimes$.

\[
Q' \otimes Q = \begin{bmatrix} q_0' & -q_1' & -q_2' & -q_3' \\ q_1' & q_0' & q_3' & -q_2' \\ q_2' & -q_3' & q_0' & q_1' \\ q_3' & q_2' & -q_1' & q_0' \end{bmatrix} \begin{bmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \end{bmatrix}. \tag{2.10}
\]

This definition of the quaternion product preserves the composition of rotations, that is to say $Q_{B/F} \otimes Q_{F/N} = Q_{B/N}$ any frames $B$, $F$ and $N$, analogously to (2.2).
The conjugate of a quaternion $Q = [q_0, q_1, q_2, q_3]^\top$ is denoted $Q^\star$, and its coordinates are $[q_0, -q_1, -q_2, -q_3]^\top$. It describes the reciprocal rotation: $Q_{F/N}^\star = Q_{N/F}$.

Without too much mathematical detail about quaternions, a few properties are listed hereafter:

- $Q$ and $-Q$ describe the same attitude\(^7\);
- $||Q||_2 = \sqrt{q_0^2 + q_1^2 + q_2^2 + q_3^2} = 1$;
- $Q \otimes Q^\star = [1, 0, 0, 0]^\top$.

\section*{2.3 Reference systems}

Reference systems are used to describe the position or orientation of objects in space at a given date. A complete reference system is thus composed of a time scale and origin, a reference frame, and a spatial origin.

In what follows, the time scale will be UTC\(^8\). CNES Julian Days\(^9\), which are based on UTC, will also be used. Reference frames used in this work are presented in subsections 2.3.1 to 2.3.6 below. Time and space origins are defined only when necessary.

\subsection*{2.3.1 Inertial frame}

The inertial frame selected for this work is EME2000, which is the Earth’s Mean Equator and Equinox frame taken at the epoch J2000, i.e. on January 1\(^{st}\), 2000 at 0:00 Terrestrial

\(^7\)The "−" sign results in reversing the rotation axis and rotating of the opposite angle $−\Theta$, which results in the same rotation.

\(^8\)Coordinated Universal Time.

\(^9\)Number of days since the 1\(^{st}\) of January, 1950 0:00 UTC.
The origin of the inertial frame is the centre of mass of the Earth. Its $x$-axis points in the direction of the vernal equinox\(^{11}\) at epoch J2000, its $z$-axis is aligned with Earth’s mean rotation axis at epoch J2000, and its $y$-axis completes the trihedron in a direct manner, as shown in Figure 2.2.

The inertial frame will be denoted by the letter $\mathcal{I}$ hereafter.

### 2.3.2 Terrestrial frame

The terrestrial frame of reference is the ITRF (International Terrestrial Reference Frame). It is linked to Earth’s rotation, and comes with successive realisations to match the evolutions of Earth’s axis of rotation and angular rate. The latest realisation was issued in 2014.

---

\(^{10}\)The Terrestrial Time (TT) is 32.184 s ahead of the International Atomic Time (TAI) by definition. The TAI is itself 32 s ahead of the UTC as of November 2017. This difference is increased each time a leap second is introduced in the UTC to compensate for the slowing of Earth’s rotation [24].

\(^{11}\)The direction of the Sun from Earth at the vernal equinox, that is around the 21st of March.
The origin of this frame is Earth’s centre of mass. Its $z$-axis is aligned with Earth’s mean rotation axis, its $x$-axis lies in the mean equatorial plane and points through the International Reference Meridian. The $y$-axis completes the trihedron in a direct manner. The terrestrial frame will be denoted by the letter $\mathcal{R}$ hereafter.

The $z$-axes of $\mathcal{R}$ and $\mathcal{I}$ differ due to the precession and nutation of Earth’s axis of rotation, to which $\mathcal{R}$ is adjusted. The $x$ and $y$-axes of $\mathcal{R}$ rotate with respect to $\mathcal{I}$, as they follow the rotation of the Earth. The axes of $\mathcal{R}$ are shown in Figure 2.2 together with those of $\mathcal{I}$.

### 2.3.3 Local orbital frame

The origin of the local orbital frame is the centre of mass of the satellite. Its $z$-axis points opposite to the local direction of Earth’s centre of mass. Its $x$-axis points in the direction of the orbit normal $n_{\text{orb}}^{12}$. The $y$-axis completes the trihedron in a direct manner. It lies in the orbital plane, and points as close as possible to the direction opposite to the velocity vector of the satellite$^{13}$. The local orbital frame will be denoted by the letter $\mathcal{O}$ hereafter.

![Local Orbital Frame Diagram](image)

**Figure 2.3:** The local orbital frame. The orbit is dashed, arrows indicate the direction of the rotation.

$^{12}$The orbit normal is orthogonal to the orbital plane and points in the direction of the rotation vector of the on-orbit movement.

$^{13}$The classical definition of the axes has been changed to facilitate the description of relative orientations with the mission frames.
2.3.4 Local magnetic frame

The local magnetic frame is linked to the local magnetic field $\mathbf{B}$, and is shown in Figure 2.4. Its $y$-axis is collinear to $\mathbf{B}$ but points in the opposite direction. The $x$-axis is orthogonal to the local magnetic plane$^{14}$, and points in the direction of the orbit normal $\mathbf{n}_{\text{orb}}$ (see subsection 2.3.3). $z$ completes the trihedron in a direct manner. The orbit and the magnetic lines are drawn in the same plane in Figure 2.4. This is not strictly true, but will hold as a first order approximation. The magnetic frame will be denoted by the letter $\mathcal{M}$ hereafter.

![Figure 2.4: The local magnetic frame. The orbit is dashed, arrows indicate the direction of the rotation. The magnetic lines are plain, arrows indicate the direction of positive magnetic field.](image)

2.3.5 Target frame

The target frame coincides with the satellite frame (see subsection 2.3.6) when the satellite is perfectly pointed according to the guidance law$^{15}$. The target frame will be denoted by the letter $\mathcal{T}$ hereafter. The orientation of frame $\mathcal{T}$ with respect to frame $\mathcal{I}$ will be described by the target quaternion $\mathbf{Q}_\mathcal{T}$, and the rotation by $\mathbf{\Omega}_\mathcal{T} = \begin{bmatrix} \omega_{\mathcal{T},1} & \omega_{\mathcal{T},2} & \omega_{\mathcal{T},3} \end{bmatrix}_\mathcal{T}$.

$^{14}$The plane containing the local magnetic field line.

$^{15}$See section 5.4 for a description of the guidance law.
### 2.3.6 Satellite frame

The satellite frame is linked to the satellite. Its axes are described in section 2.6. Its originates at the centre of the \(-x\)-face of the satellite, as shown in Figure 2.9. The satellite frame will be denoted by the letter \(S\) hereafter. The orientation of frame \(S\) with respect to frame \(T\) will be described by the error direction cosine matrix \([S/T]\) or the error quaternion \(Q_{S/T}\), and the rotation by \(\Omega_{S/T}\). In what follows, the notation \(\varphi, \theta, \psi\) for Euler angles shall only be used to describes the orientation of \(S\) with respect to \(T\).

\[
[S/T] = \begin{bmatrix}
c\theta c\psi & c\theta s\psi & -s\theta \\
sc\varphi c\psi - c\varphi s\psi & s\varphi s\theta s\psi + c\varphi c\psi & s\varphi c\theta \\
c\varphi s\theta c\psi + s\varphi s\psi & c\varphi s\theta s\psi - s\varphi c\psi & c\varphi c\theta 
\end{bmatrix}
\]  

(2.11)

\[
\Omega_{S/T} = \begin{bmatrix}
\dot{\varphi} - \sin(\theta)\dot{\psi} \\
\cos(\varphi)\dot{\theta} + \sin(\varphi)\cos(\theta)\dot{\psi} \\
-\sin(\varphi)\dot{\theta} + \cos(\varphi)\cos(\theta)\dot{\psi}
\end{bmatrix}
\]

(2.12)

Under the hypothesis that \(\varphi, \theta, \psi \ll 1\), and assuming that the derivatives \(\dot{\varphi}, \dot{\theta}\) and \(\dot{\psi}\) are of the same order of magnitude as \(\varphi, \theta\) and \(\psi\), the first order approximation of \([S/T]\) and \(\Omega_{S/T}\) is:

\[
[S/T] = \begin{bmatrix}
1 & \psi & -\theta \\
-\psi & 1 & \varphi \\
\theta & -\varphi & 1
\end{bmatrix}
\]  

(2.13)

\[
\Omega_{S/T} = \begin{bmatrix}
\varphi \\
\dot{\theta} \\
\dot{\psi}
\end{bmatrix}
\]

(2.14)

Let us introduce the error vector\(^{16}\) \(\epsilon = [\varphi, \theta, \psi]_S^T\). At first order, the error quaternion

\(^{16}\)One must keep in mind that this is not a proper vector, but rather a notation. In particular, expressing
reads \( Q_{S/T} = [1, \epsilon] \). In what follows, \( Q_{S/T} \) and \( \Omega_{S/T} \) will alternatively be denoted by \( \delta Q \) and \( \delta \omega \). For applications in following chapters, the target angular velocity \( \Omega_T \) needs to be expressed in the satellite frame \( S \): \( \Omega_T|_S = [S/T] \Omega_T|_T \). At first order in \( \epsilon \), one gets:

\[
\Omega_T = \begin{bmatrix}
\omega_{T,1} + \psi \omega_{T,2} - \theta \omega_{T,3} \\
-\psi \omega_{T,1} + \omega_{T,2} + \varphi \omega_{T,3} \\
\theta \omega_{T,1} - \varphi \omega_{T,2} + \omega_{T,3}
\end{bmatrix}_{S}.
\]  
(2.15)

2.4 Orbital characteristics

Eye-Sat’s orbit is an ellipse\(^{17}\), with Earth located at one of its foci. It can be described by the six Keplerian orbital elements, which are shown in Figure 2.5:

- Right ascension of the ascending node \( \Omega_0 \), the angle made by the point at which the orbit crosses the equator from South to North with the \( x \)-axis of the inertial frame \( \mathcal{I} \)\(^{18}\);
- Inclination \( i_0 \) of the orbital plane with respect to the equatorial plane;
- Argument of the perigee \( \omega_p \), measured from the ascending node;
- Semi-major axis \( a_0 \) of the ellipse;
- Eccentricity \( e_0 \) of the ellipse\(^{19}\);
- True anomaly \( \nu \), that is the angular position of the satellite on the orbit measured from the perigee.

Additional quantities will be used, such as the on-orbit velocity of the satellite \( v_0 \), its orbital angular rate \( \omega_0 \), and its orbital period \( T_0 \).

---

\(^{17}\)Under the Keplerian approximation.
\(^{18}\)See section 2.3.
\(^{19}\)\(0 \leq e_0 < 1\) for an ellipse, \(e_0 = 1\) for a parabola, and \(e_0 > 1\) for an hyperbola.
Since Earth is not a perfect sphere, harmonic models have been developed to describe its shape. The first order term of that decomposition, $J_2$, accounts for the presence of a bulge at the equator, which exerts a torque on the orbit and makes the ascending node drift. This phenomenon is called the \textit{regression of nodes} [25, p. 95], and the drift is given by:

$$\dot{\Omega}_0 = -\frac{3\sqrt{\mu_\oplus J_2 R_\oplus^2}}{2a_0^{7/2} (1 - e_0^2)^2} \cos i_0.$$ \hspace{1cm} (2.16)

$\mu_\oplus$ is Earth’s gravitational constant, and $R_\oplus$ is Earth’s equatorial radius.

Eye-Sat’s orbit was chosen to be a sun-synchronous orbit, which means that the regression rate $\dot{\Omega}_0$ is equal to the angular rate corresponding to the motion of the Earth around the Sun. The Earth circles the Sun in one sidereal year $T_{SY}$, that is 365.256363004 mean solar days of 86 400 s each [26]: the ascending node must circle the Earth within one sidereal year as well. Since the rotation vector of the Earth about the Sun points towards the celestial North, $\dot{\Omega}_0$ must be a positive quantity.

$$\dot{\Omega}_0 = \frac{2\pi}{T_{SY}}.$$ \hspace{1cm} (2.17)

![Orbital elements. The orbit is dashed.](image_url)
The inclination $i_0$ is therefore obtained by equating equations (2.16) and (2.17):

$$i_0 = \cos^{-1}\left(\frac{-4\pi a_0^{7/2} (1 - e_0^2)^2}{3T_{SY} \sqrt{\mu / \mu_\oplus} J_2 R_\oplus^2}\right).$$  \hspace{1cm} (2.18)

Sun-synchronous orbits have high inclinations, making them quasi-polar. Eye-Sat’s orbit was chosen to be a 6–18 sun-synchronous orbit [22], which means the ascending node corresponds to 6:00 local time. This is particularly convenient, since Eye-Sat’s orbit will always be facing the Sun, and will avoid eclipse for most of the year. The pointing of its solar panels is therefore facilitated.

Eye-Sat’s nominal altitude $h_0^{20}$ was chosen to be 690 km, which ensures that it will de-orbit naturally in less than 25 years, as imposed by the French legislation\(^{21}\). This results in a semi-major axis $a_0$ of 7,068,136.3 m. The nominal Keplerian orbital elements are specified in the first column of Table 2.1.

However, since Eye-Sat will be launched *piggyback*, it is not guaranteed that this orbit will be the one used for the mission. The ascending node might be changed to 18:00 local time, and the altitude might differ from its nominal value 690 km. Going higher should not be considered, in order to comply with the French legislation, but the possibility of being launched to a lower altitude must be treated as a plausible scenario.

\(^{20}a_0 = R_\oplus + h_0.\)

\(^{21}\)The Law on Space Operations (LOS) was passed in 2008, and is partially applied since 2010. It will be fully applied from 2020 onwards. Its prerogatives against the proliferation of space debris impose that every spacecraft sent to Low-Earth Orbit (LEO) must de-orbit within 25 years, which is in accordance with ESA’s guidelines.
<table>
<thead>
<tr>
<th>$h_0$ (km)</th>
<th>690</th>
<th>500</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_0$ (m)</td>
<td>7068136.3</td>
<td>6878136.3</td>
</tr>
<tr>
<td>$\Omega_0$ (local time)</td>
<td>6:00</td>
<td>6:00</td>
</tr>
<tr>
<td>$i_0$ (°)</td>
<td>98.1501</td>
<td>97.4043</td>
</tr>
<tr>
<td>$\omega_p$ (rad)</td>
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<td>0</td>
</tr>
<tr>
<td>$e_0$</td>
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<td>0.001</td>
</tr>
<tr>
<td>$\omega_0$ (rad s$^{-1}$)</td>
<td>$1.062 \times 10^{-3}$</td>
<td>$1.107 \times 10^{-3}$</td>
</tr>
<tr>
<td>$T_0$ (s)</td>
<td>5913.8</td>
<td>5677.0</td>
</tr>
</tbody>
</table>

Table 2.1: Keplerian orbital elements, with additional elements.

Being launched to a lower altitude would imply stronger constraints on the attitude control\(^{22}\). In what follows, it will be assumed that the altitude can be somewhere between 500 and 690 km. The orbital elements for the lowest altitude are given in the second column of Table 2.1.

### 2.5 Environment

#### 2.5.1 Solar illumination

Eye-Sat’s orbit is sun-synchronous, which means the ascending and descending nodes correspond to fixed local times, respectively chosen as 6:00 and 18:00. This means that ground trace of Eye-Sat will roughly travel Earth’s terminator, and that its solar panels will enjoy a constant illumination on the orbit\(^{23}\), and throughout the year, or so would it be if Eye-Sat’s orbit were strictly polar and Earth’s spin axis were not tilted.

Figure 2.6 shows the non-intuitive motion of the orbital plane with respect to the direction of the Sun throughout the year, with $i_\oplus$ denoting Earth’s axial tilt, $M_\oplus$ its mean anomaly

\(^{22}\)See section 4.3 for a description of disturbances.

\(^{23}\)No eclipse occurs when travelling along the terminator.
about the Sun, measured from the Summer solstice\textsuperscript{24}, and $\alpha_{\text{sun}}$ the Sun incidence on the orbital plane. $i_\oplus$ is equal to 23.43695\degree \textsuperscript{[27]}, and the value of $i_0$ is taken from Table 2.1, assuming an altitude of 690 km.

Figure 2.6: Relative positions of the Sun, the Earth and Eye-Sat’s orbit throughout the year.

Under the assumption of Earth’s orbit being circular, $\alpha_{\text{sun}}$ is given by the following relation:

$$
\alpha_{\text{sun}} = \cos^{-1}\left(\sin i_0 \left(\cos^2 M_\oplus \cos i_\oplus + \sin^2 M_\oplus\right) - \cos i_0 \cos M_\oplus \sin i_\oplus\right). \quad (2.19)
$$

Figure 2.7: Sun incidence on the orbital plane.

\textsuperscript{24}The mean anomaly would normally be measured from perihelion, which happens at the beginning of January. It differs from the true anomaly in its definition — the rate of the mean anomaly is constant throughout the orbit for an elliptical orbit — but they are strictly equal for a circular orbit.
The evolution of $\alpha_{\text{sun}}$ throughout the year is plotted in Figure 2.7. Surprisingly, the minimum incidence of $1.55^\circ$ is neither obtained at the equinoxes nor at the Summer solstice, for which it amounts to $15.29^\circ$. The maximum is however reached at the Winter solstice, with an incidence of $31.58^\circ$. The dashed line represents the incidence above which eclipses occur on the orbit, which happens about 84 days a year. Eclipses last 20 minutes at most [22].

Both results on the sun incidence and the occurrence of eclipses are very close to those from [22], based on celestial mechanics simulations. These figures are more severe for a 500km altitude.

### 2.5.2 Magnetic field

The best-fit dipole for Earth’s magnetic field has a magnetic moment $^{25} M_\oplus = 7.79 \times 10^{22}$ A m$^2$ [28]. Following the notations introduced in Figure 2.8, the magnetic field generated by a dipole is:

$$ B = \frac{\mu_0 M}{4\pi r^3} (2 \cos \beta r + \sin \beta \beta). \quad (2.20) $$

![Figure 2.8: Magnetic dipole.](image)

---

$^{25}$This value dates back to 2000. According to the trend presented in reference [28], the value of the dipole should be around $7.7 \times 10^{22}$ A m$^2$ in 2017.
Assuming this equivalent dipole is located at the centre of the Earth, the maximum and minimum intensities of the magnetic field it generates at the altitude of Eye-Sat’s orbit are respectively $\mu_0 M_\oplus/2\pi a_0^3$ and $\mu_0 M_\oplus/4\pi a_0^3$. The values for circular orbits at altitudes of 690 km and 500 km are gathered in Table 2.2.

<table>
<thead>
<tr>
<th>$h_0$ (km)</th>
<th>690</th>
<th>500</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_{\oplus,\text{min}}$ (µT)</td>
<td>20.06</td>
<td>23.94</td>
</tr>
<tr>
<td>$B_{\oplus,\text{max}}$ (µT)</td>
<td>44.12</td>
<td>47.88</td>
</tr>
</tbody>
</table>

Table 2.2: Minimum and maximum magnetic field strength on Eye-Sat’s orbit.

Of course, the dipolar field is only a zero-order approximation of Earth’s magnetic field, which displays geographical and temporal variations of direction and intensity. Reference [20] mentions an intensity of the field between 20 and 47 µT at an altitude of 700 km, which is consistent with the approximate results from Table 2.2. Therefore, the intensity of the magnetic field is assumed to be between 20 and 50 µT.

### 2.6 The satellite

Eye-Sat is a 3U-CubeSat, with a mass of approximately 4.2 kg.\(^{26}\)

\(^{26}\)With a slight deviation from the 1.33 kg per U mentioned in section 1.2.
The satellite body, shown in yellow in Figure 2.9, has exterior dimensions of approximately $34 \times 10 \times 10\text{cm}^3$. Each of the four solar panels, shown in blue in Figure 2.9, is approximately 34 cm long and 10 cm large.

Figure 2.9 features the axes of the satellite frame $\mathcal{S}$. Attitude-related equipment are listed below, with their orientation with respect to the satellite axes.

**Solar panels**

The solar cells are pointed in the $-x$-direction. Each panel bears 6 solar cells, each generating approximately 1 W power under normal Sun illumination at a distance of 1 AU. The 4 panels total 24 W.

**Instrument**

Eye-Sat’s instrument is an optical telescope, named IRIS (*Instrument Réalisé pour l’Imagerie Spatiale*, Instrument Realised for Space Imaging), which features coloured and polarised filters for the analysis of the zodiacal light. Its line of sight is aligned with the $+x$-direction.

**S-band antennae**

Two S-band — 2 GHz — patch-antennae are located on the $+z$ and $-z$-faces of the satellite body. Together, they provided an omnidirectional radiation pattern\(^{27}\) used for TM/TC.

**X-band antenna**

One X-band — 8 GHz — patch-antenna is installed on the $-z$-face of the satellite body. It is a directional antenna, providing high data rates for payload telemetry download\(^{28}\).

---

\(^{27}\)Although some sectors may be affected by the presence of the solar panels.

\(^{28}\)See appendix A.1 for details about the radiation pattern and the impact on pointing.
2.7 Mission phases

Eye-Sat’s mission will feature the following phases: launch, de-tumbling, survival, standby, manoeuvre, shooting, downloading and end-of-life. All these phases correspond to different attitude control modes\textsuperscript{29}.

Launch

During launch, all satellite systems are off, and batteries are charged. When the satellite is deployed from the launcher, a switch turns its systems on.

De-tumbling

When the satellite is released from its launching pod, it might be rotating at rather high rates\textsuperscript{30}. The rotation rate is reduced before the solar panels open to secure electrical supply.

Survival

When the solar panels are open, the satellite finds itself in survival phase. The aim of this phase is to stabilise the satellite so that the solar panels face the Sun. The satellite body rotates to produce a gyroscopic stabilisation. As little equipment as possible is used, and the pointing error can be large.

In the event of fault detection during the mission, the satellite switches back to survival mode.

\textsuperscript{29}Detailed in chapter 5.
\textsuperscript{30}Up to $10^9 \text{s}^{-1}$ per axis [20].
Standby

During standby, the satellite charges its batteries, and is ready to switch to the following phases of shooting or downloading. The satellite is 3-axes stabilised.

Manoeuvre

Switching to shooting or downloading from standby, and back, requires a manoeuvre. The satellite is 3-axes stabilised.

Shooting

The satellite is 3-axes stabilised and performs an inertial pointing. When the pointing is good, the camera is switched on to take pictures.

Download

The satellite is 3-axes stabilised. The omnidirectional S-band antennae send TM to the S-band receiver on ground, and receive TC. The directional X-band antenna is pointed towards the X-band receiver on ground, and sends payload data. Detail about station pointing is provided in appendix A.

End of life

At the end of its mission, the satellite is passivated. Batteries are emptied and power lines are physically cut, preventing explosion of the satellite and subsequent creation of debris. The satellite will then de-orbit due to atmospheric drag within 25 years.
Chapter 3

Equipment

Eye-Sat has four attitude control-related devices. Sensing is performed by magnetometers and a star tracker, while actuation is ensured by magnetorquers and reaction wheels.

This setting features two natural couples, as sensors and actuators work in pair. The star tracker provides direct access to the attitude and angular rate, which can be used in a closed-loop by the reaction wheels.

Magnetometers, however, do not yield 3-axis attitude knowledge, since they only measure the magnetic field $\mathbf{B}$: a rotation around this vector is invisible to them. Therefore, they only deliver 2-axis information.

Magnetorquers have the same limitation. They generate a magnetic moment $\mathbf{M_{MTB}}$, which produces a torque $\mathbf{T_{MTB}}$ by interacting with the ambient magnetic field:

$$\mathbf{T_{MTB}} = \mathbf{M_{MTB}} \times \mathbf{B} \quad (3.1)$$

Due to the properties of the cross product\(^1\), no torque can be generated along $\mathbf{B}$, which is also the direction about which a rotation cannot be seen by the magnetometers: the non-observable axis is also the non-commandable axis for the magnetometers-magnetorquers.

\(^1\) $\mathbf{v_1} \times \mathbf{v_2}$ is orthogonal to both $\mathbf{v_1}$ and $\mathbf{v_2}$. 

33
couple. Furthermore, magnetometer measurements can be used in the loop without actually estimating the attitude. This shortcut is exploited in chapter 5.

This part introduces the sensors and actuators, their characteristics, as well as the in-flight usage for each of them.

### 3.1 Magnetometers

#### 3.1.1 Characteristics

Eye-Sat is equipped with two 3-axis magnetometers, used in cold redundancy. They provide a measurement of the ambient magnetic field $B_{\text{MAG}}$, which is affected by misalignments, scaling factors, bias and noise. The two latter errors can be both intrinsic and extrinsic, as the satellite and its components have a parasitic effect on magnetic measurements. The following model can be used to describe the behaviour of the magnetometers:

$$B_{\text{MAG}} = [A_{\text{MAG}}] B + \delta B_{\text{MAG}} + n_{\text{MAG}},$$

(3.2)

- $[A_{\text{MAG}}]$ is the configuration matrix, bearing the scaling factors on its diagonal, and the misalignment terms on its non-diagonal elements. This matrix evolves over time, mainly under the effect of thermal cycling;

- $\delta B_{\text{MAG}}$ is the bias. Like the configuration matrix, it varies over time, mainly due to thermal cycling;

- $n_{\text{MAG}}$ is the measurement noise.

The values of $[A_{\text{MAG}}]$, $\delta B$, and the standard deviation of $n_{\text{MAG}}$ for the nominal and redundant sets were determined from tests performed in the non-magnetic chamber of CNES. This
facility consists of a building isolated from magnetic perturbation\textsuperscript{2}, in which air coils are used to actively compensate Earth’s magnetic field. The norm of the resulting magnetic field is inferior to $50\,\text{nT}$ for a test volume of 2 cubic metres\textsuperscript{3}. In addition to compensating for Earth’s magnetic field, the coils can be used to generate fields up to $70\,\mu\text{T}$ in norm, in any direction [29].

For each magnetometer set, magnetic fields from $5\,\mu\text{T}$ to $50\,\mu\text{T}$\textsuperscript{4} were generated along each of the 6 signed satellite axes. For each signed axis, 4 magnitudes were tested, resulting in 24 configurations for each magnetometers set. For each of these configurations, 20 measurements were performed, yielding a total of 480 measurements per set. Measurements were performed in parallel by a finely calibrated reference magnetometer.

For each magnetometer set, the following post-treatments were performed on the raw measurements:

1. For each configuration, the mean value obtained from the 20 measurements is calculated. The difference between the raw measurements and the filtered one gives 20 values of the noise vector $n_{\text{MAG}}$.

2. The standard deviation of $n_{\text{MAG}}$ on each axis is denoted by $\sigma(n_{\text{MAG}})$, and calculated from the 480 measured noise vectors.

3. Finally, the values of $[A_{\text{MAG}}]$ and $\delta B_{\text{MAG}}$ are computed via a least-square regression performed on the 24 filtered values, compared to the reference measurements.

The characteristics of the nominal and redundant sets of magnetometers are summarised in Table 3.1, under the form of typical values determined from test results.

\textsuperscript{2}It is isolated from other facilities, built from non-magnetic materials, and most electrical devices are located outside the building.

\textsuperscript{3}In practice, the resulting field as measured by the reference magnetometer was below $1\,\text{nT}$ for the duration of the test.

\textsuperscript{4}In accordance with the magnitude of the magnetic field on Eye-Sat’s orbit, as exposed in subsection 2.5.2.
Table 3.1: Characteristics of the magnetometers.

<table>
<thead>
<tr>
<th></th>
<th>Nominal</th>
<th>Redundant</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resolution (nT)</td>
<td>73</td>
<td>7.5</td>
</tr>
<tr>
<td>Scaling factors</td>
<td>$1 \pm 0.10$</td>
<td>$1 \pm 0.25$</td>
</tr>
<tr>
<td>Misalignment terms</td>
<td>0.01</td>
<td>0.10</td>
</tr>
<tr>
<td>$\delta B_{MAG}$ (µT)</td>
<td>7</td>
<td>6</td>
</tr>
<tr>
<td>$\sigma (n_{MAG})$ (nT)</td>
<td>55</td>
<td>900</td>
</tr>
</tbody>
</table>

Scaling factors and biases are rather large for both magnetometers. Misalignments and noise, however, are much larger for the redundant set, by one order of magnitude. The noise level is really low for the nominal set, as the standard deviation is smaller than the resolution, whereas for the redundant set, it even compares to the norm of the measured field. Filtering is therefore crucial for the use of such noisy measurements. These typical values show that magnetometer measurements are not exploitable without prior calibration.

### 3.1.2 In-flight usage

The configuration matrix and the bias vector have to be estimated prior to flight, and/or in flight. The noise level has to be dealt with by filtering.

The processing of the magnetometer measurement first consists in a calibration, to invert equation (3.2):

$$B_{\text{meas}} = [A_{MAG}]^{-1} (B_{MAG} - \delta B_{MAG}).$$

(3.3)

At this stage, the measurement is still noisy. The second stage consists in applying low-pass filtering to lower the noise levels.
The resulting measurement is not a perfect image of \( \mathbf{B} \), of course. Residual scaling factors, misalignments and biases persist, for the knowledge of \([A_{MAG}]\) and \(\delta B_{MAG}\) cannot be exact.

For Eye-Sat, they will be evaluated on-ground, with the fully-assembled satellite. However, since these parameters will certainly evolve over the mission, estimating them on-board would be an interesting way of improvement. This opportunity is discussed in subsection 5.6.8.

### 3.2 Star tracker

The star tracker is located next to the instrument, with its line of sight in the \(+x\)-direction. By imaging the sky and comparing the stars to an on-board catalogue, it estimates the attitude quaternion \(Q_{S/T}\).

The star tracker will be used in closed-loop with the reaction wheels. As announced in section 1.4, this control loop is beyond the scope of the present work, and no further detail will be given on the star tracker and its in-flight usage.

### 3.3 Magnetorquers

#### 3.3.1 Characteristics

Eye-Sat’s MTB (MagneTorquer Board) consists of two ferromagnetic-core coils oriented along the \(y\) and \(z\)-axes of the satellite, and one air-core coil along the \(x\)-axis.
When supplied with a current $I$, a coil generates a magnetic moment $\mathcal{M}$:

$$\mathcal{M} = \kappa N S I$$

(3.4)

$S$ is the section area of the coil, $N$ is the number of windings, and $\kappa$ the magnetic gain. $\kappa$ is equal to 1 for air-core coils, but is strictly superior to 1 for ferromagnetic-core coils.

The following model is used to describe the relation between the moment commanded to the magnetorquers $\mathcal{M}_{\text{com}}$ and the moment they actually generate $\mathcal{M}_{\text{MTB}}$:

$$\mathcal{M}_{\text{MTB}} = [A_{\text{MTB}}] \mathcal{M}_{\text{com}}.$$  

(3.5)

$[A_{\text{MTB}}]$ is the coupling matrix. It bears scaling factors on its diagonal. Its non-diagonal elements account for both misalignments and mutual induction between the coils.

The determination of $[A_{\text{MTB}}]$ was performed in the non-magnetic chamber of CNES, described in subsection 3.1.1. Earth’s magnetic field was compensated so that the resulting magnetic field is close to being null in the test zone\(^5\).

The MTB was commanded with magnetic moments. A finely calibrated reference magnetometer was used to measure the magnetic field generated by the MTB at a fixed distance of 50 cm from the geometrical centre of the MTB, in the 6 principle directions of the equipment. The equivalent dipole moment was then estimated from these 6 measurements.

The tests were performed in two steps:

1. Each one of the 3 coils was commanded with a magnetic moment of $\pm 0.1 \text{ A m}^2$ and

\(^5\)As already mentioned in subsection 3.1.1, it was measured as being below 1 nT for the duration of the test.
±0.2 A m². The results from this first phase were used to determine the capacity of the MTB to reach the announced capacity of ±0.2 A m² per axis. They were checked against a thermal dependency model to estimate the temperature range in which the nominal level of performance can be met.

2. Each coil was commanded with a magnetic moment in \{-0.2 \pm 0.2\} A m². All 27 combinations of commands for the 3 coils were tested. The 27 measurements of magnetic moment were used to determine \([A_{MTB}]\) with a least-squares regression.

In addition, the characteristic time of the coils were determined from their measured electrical resistance, and an estimation of their inductance.

The characteristics of Eye-Sat’s MTB are summarised hereunder, under the form of typical values determined from test results.

- Maximum amplitude \(M_{\text{max}} = 0.2\ A\ m^2\);
- Characteristic time: 3 ms;
- Scaling factors: 1 ± 0.20;
- Misalignment/mutual induction terms: 0.02.

The misalignment and mutual induction terms are rather small, but may still induce a few degrees of angular error. The scaling factors are large, and need to be corrected.

3.3.2 In-flight usage

The coupling matrix must be accurately estimated on-ground. This estimation must be performed with the assembled satellite, since the structure and other components may influence the coupling matrix. The estimated matrix is then used for calibration of the commanded moment.
Given the dimensions of Eye-Sat, the magnetometers and the MTB can only be some centimetres away from each other. The norm of the magnetic field generated by a dipole moment of 0.2 A m\(^2\) at a distance of 10 cm is approximately 40 µT in the axis of the dipole, 20 µT perpendicularly to it, according to equation (2.20). These values are of the same order of magnitude as the measured magnetic field of Earth, and would impede any measurement.

Therefore, magnetorquers must be switched off when performing magnetometer measurements. It is mentioned in subsection 3.3.1 that the characteristic time of the coils is typically 3 ms, which means 99\% of the dipole generated by the coils has vanished after 15 ms.

To take into account the necessity to switch off the magnetorquers and wait for their tranquillisation, a technique called time-sharing is used. It is presented in Figure 3.1.

![Figure 3.1: Time-sharing for MAG and MTB.](image)

The magnetorquers are only actuated for a ratio \(\alpha_{MTB}\) of the AOCS time step \(T_{AOCS}\). This must be taken into consideration when computing the command to be sent to the magnetorquers.

The AOCS control loop issues a magnetic moment command \(\mathcal{M}_{AOCS}\). The integral of the magnetic moment over one AOCS time step must be preserved, which is why the target magnetic moment to be commanded is:

\[
\mathcal{M}_{TS} = \frac{\mathcal{M}_{AOCS}}{\alpha_{MTB}}. \tag{3.6}
\]
Since the magnetorquers have a limited actuation capacity per axis $M_{\text{max}}$, $M_{\text{TS}}$ must be saturated. However, the saturation shall preserve the direction of action:

$$M_{\text{sat}} = \frac{M_{\text{TS}}}{\max \left(1, \frac{M_{\text{TS},1}}{M_{\text{max}}}, \frac{M_{\text{TS},2}}{M_{\text{max}}}, \frac{M_{\text{TS},3}}{M_{\text{max}}} \right)} \quad (3.7)$$

Finally, the saturated magnetic moment must be calibrated, to correct for the coupling:

$$M_{\text{com}} = [A_{\text{MTB}}]^{-1} M_{\text{sat}}. \quad (3.8)$$

### 3.4 Reaction wheels

#### 3.4.1 Characteristics

Eye-Sat is equipped with 4 reaction wheels, both for redundancy and for enhanced performance. Indeed, the fourth wheel provides a degree of freedom when computing the command to send to each wheel, and increases the actuation capacity.

The characteristics of the reaction wheels were determined from tests performed on the engineering model. The wheel was laid flat on a non-skid insulated surface, and submitted to steps of various levels to test the accuracy of the response, and the ability to operate in the specified angular speed range. It was then submitted to ramps of various slopes through the whole speed domain, to assess the maximum torque capacity. The characteristics of the wheels are presented hereunder, under the form of typical values derived from tests.

- Moment of inertia: $1.5 \times 10^{-6}$ kg m$^2$;
- Minimum speed: 500 rpm;
- Maximum speed: 10 000 rpm;
- Maximum torque: $4 \times 10^{-5}$ N m.
The minimum and maximum speed were determined as the range for which the behaviour of the reaction wheel was nominal. Below, the friction and stiction torques\(^6\) make the control of the speed chaotic, and above, the torque capacity is affected. The maximum torque indicated here corresponds to the value of the torque at high speeds, which can be achieved on the whole domain. Higher torques are achievable at low speeds only.

The wheels are arranged in a pyramidal pattern, shown in Figure 3.2. This configuration was chosen to be symmetric, in order not to discriminate any satellite axis. The direction vectors of the reaction wheels \(\mathbf{u}_{\text{RW},i}\) have the same projection on all satellite axes.

This configuration is characterised by the wheel direction matrix \([M_{\text{RWS}}]\), whose columns are the direction vectors of the reaction wheels expressed in the satellite frame \(S\):

\[
[M_{\text{RWS}}] = \begin{bmatrix}
\mathbf{u}_{\text{RW},1}\big|_S & \mathbf{u}_{\text{RW},2}\big|_S & \mathbf{u}_{\text{RW},3}\big|_S & \mathbf{u}_{\text{RW},4}\big|_S
\end{bmatrix}
\]

\[
= \frac{1}{\sqrt{3}} \begin{bmatrix}
1 & 1 & 1 & 1 \\
1 & -1 & -1 & 1 \\
1 & 1 & -1 & -1
\end{bmatrix}.
\]

3.4.2 In-flight usage

The characteristics of the wheels account for the existence of a deadzone below 500 rpm in absolute value, in which the wheel cannot be controlled accurately. This deadzone must

---

\(^6\)The stiction torque is the static friction which has to be overcome for the wheel to start spinning.
therefore be avoided at all times, which will be done by using the degree of freedom provided by the fourth wheel. The reaction wheels will be commanded in speed.

**Use of the kernel**

The wheel direction matrix \([M_{\text{RWS}}]\) can be seen as the transfer matrix from the 4D-wheel frame to the 3D-satellite frame \(\mathcal{S}\). Let us denote by \(h_{\text{RWS}}\) the vector of individual wheel angular momentums along their rotation axes, and \(H_{\text{RWS}}\) the angular momentum generated by the wheel array in the \(\mathcal{S}\) frame:

\[
h_{\text{RWS}} = \begin{bmatrix} h_1 & h_2 & h_3 & h_4 \end{bmatrix}^\top
\]

\[
H_{\text{RWS}} = \begin{bmatrix} H_1 & H_2 & H_3 \end{bmatrix}^\top.
\]

The relation between these two vectors is:

\[
H_{\text{RWS}} = [M_{\text{RWS}}] h_{\text{RWS}}.
\]

(3.13)

However, it is the inverse transformation one needs to perform, since the attitude control law issues the vector \(H_{\text{RWS}}\) to be commanded to the wheel array, but the angular momentum commands to be sent to each wheel individually are the components of \(h_{\text{RWS}}\). The goal is to choose a vector \(h_{\text{RWS}}\) that satisfies equation (3.13).

Since \([M_{\text{RWS}}]\) is a matrix of rank 3, this problem has one degree of freedom, which corresponds to the kernel of \([M_{\text{RWS}}]\):

\[
\text{ker} [M_{\text{RWS}}] = \mathbb{R} k_{\text{RWS}}.
\]

(3.14)
\( k_{\text{RWS}} \) is a generator of the kernel:

\[
\begin{bmatrix}
1 & -1 & 1 & -1
\end{bmatrix}^T.
\] (3.15)

Therefore, if \( h_{\text{RWS}} \) satisfies equation (3.13), then:

\[
H_{\text{RWS}} = [M_{\text{RWS}}] (h_{\text{RWS}} + \lambda k_{\text{RWS}}), \forall \lambda \in \mathbb{R}.
\] (3.16)

This means the component of the command sent to the wheels \( h_{\text{RWS}} \) along \( \ker [M_{\text{RWS}}] \) can be chosen freely.

In order to calculate a candidate \( h_{\text{RWS}} \), let us introduce the pseudo inverse of \( [M_{\text{RWS}}] \), denoted by \( [M_{\text{RWS}}^\star] \):

\[
[M_{\text{RWS}}^\star] = [M_{\text{RWS}}]^T ([M_{\text{RWS}}] [M_{\text{RWS}}]^T)^{-1}.
\] (3.17)

The direct consequence from the definition of \( [M_{\text{RWS}}^\star] \) in equation (3.17) is:

\[
[M_{\text{RWS}}] [M_{\text{RWS}}^\star] = [I_{3 \times 3}].
\] (3.18)

Therefore, \( h_{\text{RWS}}^\star = [M_{\text{RWS}}^\star] H_{\text{RWS}} \) satisfies equation (3.13).

This solution is orthogonal to the kernel. Since \( [M_{\text{RWS}}^\star] [M_{\text{RWS}}] h_{\text{RWS}}^\star = h_{\text{RWS}}^\star \), and since the kernel of \( [M_{\text{RWS}}] \) is included in that of \( [M_{\text{RWS}}^\star] [M_{\text{RWS}}] \), then \( h_{\text{RWS}}^\star : k_{\text{RWS}} = 0 \). The kernel component of \( h_{\text{RWS}} = h_{\text{RWS}}^\star + \lambda k_{\text{RWS}} \) can thus be chosen independently. For Eye-Sat, it will be used to avoid the dead-zone around zero velocity.

\([M_{\text{RWS}}]\) can thus be seen as the projection from the 4-dimensional space of the individual wheel momenta to the 3-dimensional space of the wheel array momentum, along the null-
The actuation envelope materialises the maximum actuation available in all directions. In 3D, it is thus a polyhedron. All actuation vectors contained within this polyhedron are achievable, and all vectors out of its envelope are out of reach.

For the 4-wheel configuration introduced in subsection 3.4.1, it is chosen to constrain the wheels to one sign domain — \([h_{\text{min}}, h_{\text{max}}]\) or \([-h_{\text{max}}, -h_{\text{min}}]\) — to avoid zero-crossing. \(h_{\text{min}}\) and \(h_{\text{max}}\) respectively correspond to the minimum and maximum wheel speed introduced in subsection 3.4.1:

\[
\begin{align*}
h_{\text{min}} &= 8.10 \times 10^{-5} \text{ N m s} \\
h_{\text{max}} &= 1.46 \times 10^{-3} \text{ N m s}
\end{align*}
\]

The signs attributed to wheels correspond to the sign of the components of \(\mathbf{k}_{\text{RWS}}\): wheels 1 and 3 are within \([h_{\text{min}}, h_{\text{max}}]\), wheels 2 and 4 are within \([-h_{\text{max}}, -h_{\text{min}}]\).

According to reference [30], the vertices of the actuation envelope are obtained when all wheels are saturated: all at \(\pm h_{\text{min}}\) or \(\pm h_{\text{max}}\). This actually applies to the 4D-envelope in the frame of wheels, which is a hypercube with \(2^4 = 16\) vertices.

The 3D-envelope is obtained by projecting this hypercube using \([M_{\text{RWS}}]\). Since the projection is performed along the kernel vector \(\mathbf{k}_{\text{RWS}}\), the vertices which are collinear to \(\mathbf{k}_{\text{RWS}}\) will be projected onto the null vector, leaving only 14 vertices in 3D.

The actuation envelope of Eye-Sat’s wheel array for the angular momentum is shown in

\[
\begin{bmatrix}
h_{\text{min}} & -h_{\text{min}} & h_{\text{min}} & -h_{\text{min}} \\
h_{\text{max}} & -h_{\text{max}} & h_{\text{max}} & -h_{\text{max}}
\end{bmatrix}^\top
\]
Figure 3.3. Blue crosses indicate vertices, and centres of edges and faces. The weakest directions of actuation are towards the centres of the faces, the strongest are towards the vertices. It is called a symmetric envelope because it presents the same extension on all its vertices. The smallest actuation capacity is the radius of the sphere inscribed in the envelope, which is $H_{\text{rad}} = 1.26 \times 10^{-3} \text{ N m s}$.

Using the same method, the smallest torque capacity is $T_{\text{rad}} = 6.53 \times 10^{-5} \text{ N m}$. The torque envelope is homothetic to the angular momentum envelope shown in Figure 3.3.
Command strategy

The wheels are commanded in speed, and a null-space action along $k_{RWS}$ results in all the
wheel speeds increasing or decreasing by the same amount, in absolute value. It is chosen to
use the degree of freedom to set the angular momentum of the slowest wheel in absolute value
to $h_{\text{min}}$. The sign is determined by the sign domain assigned to the corresponding wheel.
The speed of all the wheels is minimised in the process.

Ensuring that the slowest wheel spins at $\pm 500 \text{ rpm}$ minimises the power consumption$^8$. Furthermore, the higher the speed, the higher the friction torques: minimising the wheel speed also protects it from degradation.

The commanded wheel angular momenta are calculated as follows, from a commanded
array momentum $H_{RWS}$:

1. The solution from the pseudo-inverse is computed:

$$h_{RWS}^\ast = [M_{RWS}^\ast] H_{RWS}.$$  

It is orthogonal to the kernel vector $k_{RWS}$.

2. The angular momentum of the slowest wheel is computed:

$$h_{RWS,\text{min}} = \min_i (h_{RWS,i}^\ast \text{sign} (k_{RWS,i})).$$

3. The angular momentum of the slowest wheel is set to $h_{\text{min}}$ in absolute value, by sub-
tracting $h_{RWS,\text{min}}$ and adding $h_{\text{min}}$ to all the wheels:

$$h_{RWS} = h_{RWS}^\ast + (h_{\text{min}} - h_{RWS,\text{min}}) \text{sign} (k_{RWS}).$$

---

$^8$The electrical power consumed by the wheel, neglecting the consumption of the electronics, is $T\omega$, with $T$ the torque and $\omega$ the angular rate of the wheel.
Chapter 4

Satellite dynamics

4.1 Satellite properties

Eye-Sat is constituted of a central body and four deployable solar panels. Although these large arrays may be prone to oscillations, as described in subsection 4.3.2, the satellite shall be treated as a non-deformable rigid body, and it will be shown in subsection 6.2.2 that the control loop damps these oscillations.

For rotational motions, the satellite can be described by its matrix of inertia $[I_{\text{sat}}]$, expressed in the satellite frame $S$ at Eye-Sat centre of mass. The reader is referred to Figure 2.9 for the definition of its axes.

Since Eye-Sat has an axial symmetry about its $x$-axis, the matrix of inertia reduces to a diagonal matrix\(^1\), its moments of inertia about $y$ and $z$ being equal. The CAD model of

\(^1\)In reality, the order of magnitude of the non-diagonal terms is $1 \times 10^{-3}$ kg m\(^2\) at most. They are neglected in the present analysis.
Eye-Sat gives $I_1 = 4.5 \times 10^{-2}$ kg m$^2$ and $I_2 = 6.8 \times 10^{-2}$ kg m$^2$.

$$
[I_{\text{sat}}] = \begin{bmatrix}
I_1 & 0 & 0 \\
0 & I_2 & 0 \\
0 & 0 & I_2
\end{bmatrix}.
$$

(4.1)

The rotation of the satellite relative to an arbitrary frame $\mathcal{F}$ is described by the rotation vector $\Omega_{S/F}$. The dynamic of the spacecraft is described by its angular momentum $H_{S/F}$.

$$
H_{S/F} = [I_{\text{sat}}] \Omega_{S/F}
$$

(4.2)

The total angular momentum of the satellite relative to the inertial frame $\mathcal{I}$ shall be denoted $H_{\text{tot}}$. It is composed of the angular momentum of the rigid body, $H_{\text{sat}}$, and that of the reaction wheels $H_{\text{RWS}}$, where, using notations introduced in subsections 2.3.5 and 2.3.6:

$$
H_{\text{sat}} = [I_{\text{sat}}] \Omega_{S/I} \\
= [I_{\text{sat}}] (\Omega_{T} + \delta \omega).
$$

(4.3)

### 4.2 Control torques

Eye-Sat’s attitude control-related actuators, introduced in chapter 3, actively generate torques to control the orientation of the spacecraft.

#### 4.2.1 Magnetorquers

Magnetorquers generate a magnetic moment $\mathcal{M}_{\text{MTB}}$, which produces a torque $T_{\text{MTB}}$ by interacting with the ambient magnetic field $B$.

$$
T_{\text{MTB}} = \mathcal{M}_{\text{MTB}} \times B
$$

(4.4)
$T_{\text{MTB}}$ is an external torque, generated by interaction with the environment. It thus affects the total angular momentum of the satellite. $T_{\text{MTB}}$ tends to align the magnetic moment with the direction of the magnetic field.

From the minimum norm of the magnetic field on the orbit, determined in subsection 2.5.2, and the magnetic moment generated by the MTB in the weakest direction\(^2\), from subsection 3.3.1, one gets that the minimum magnetic torque capacity is $4.0 \times 10^{-6}$ N m at 690 km, and $4.8 \times 10^{-6}$ N m at 500 km.

### 4.2.2 Reaction wheels

Eye-Sat is equipped with 4 reaction wheels, which were introduced in section 3.4. The dynamic of each one of them is characterised by its angular momentum $h_i$ about its axis of rotation. The angular momentums are gathered in $h_{\text{RWS}}$:

$$h_{\text{RWS}} = \begin{bmatrix} h_1 & h_2 & h_3 & h_4 \end{bmatrix}^\top.$$  \hspace{1cm} (4.5)

The angular momentum of the 4-wheel array is $H_{\text{RWS}}$:

$$H_{\text{RWS}} = [M_{\text{RWS}}] h_{\text{RWS}}$$  \hspace{1cm} (4.6)

$$= \begin{bmatrix} H_1 & H_2 & H_3 \end{bmatrix}^\top_s.$$  \hspace{1cm} (4.7)

The variation of this angular momentum generates a torque $T_{\text{RWS}}$ which is applied to the satellite:

$$T_{\text{RWS}} = - \left( \frac{dH_{\text{RWS}}}{dt} \right)_s.$$  \hspace{1cm} (4.8)

\(^2\)However, the time-sharing ratio $\alpha_{\text{MTB}}$ defined in subsection 3.3.2 must be applied to this capacity.
The minus sign comes from the reciprocity of actions. In the absence of external torques, the angular momentum of the total satellite system, composed of the body plus the reaction wheels, is constant. Therefore, accelerating the wheels with a torque \( \left( \frac{dH_{\text{RW}}}{dt} \right)_S \) results in applying an opposite torque \( T_{\text{RWS}} = - \left( \frac{dH_{\text{RW}}}{dt} \right)_S \) to the satellite body. Contrary to \( T_{\text{MTB}} \), \( T_{\text{RWS}} \) is an internal torque: it does not affect the total angular momentum of the satellite.

The torque capacity in the weakest direction is \( 6.5 \times 10^{-5} \text{N m} \), according to subsection 3.4.2.

### 4.3 Disturbances

Eye-Sat is submitted to disturbance torques, some external — they change the total angular momentum \( H_{\text{tot}} \) — and some internal — they only cause an exchange of angular momentum between parts of the spacecraft. External disturbance torques shall be denoted by \( T_d \). The main disturbances are introduced hereunder.

#### 4.3.1 External disturbances

**Magnetic torque**

Once assembled, a satellite can retain a residual magnetic moment in the absence of magnetic actuation, or even acquire such residual moment in operation\(^3\). The residual magnetic moment \( M_{\text{res}} \) for Eye-Sat is assumed to be 0.01 A m\(^2\) per axis, which results in a norm of 0.017 A m\(^2\) [20]. The resulting disturbance torque is given by the following relation:

\[
T_{d,\text{mag}} = M_{\text{res}} \times B_\oplus. \tag{4.9}
\]

\(^3\)In particular, reaction wheels are commonly known to acquire a magnetic charge over a mission due to their rotation.
The maximum magnitude of the magnetic field on Eye-Sat’s orbit is $44.12\,\mu T$ at an altitude of 690 km, and $47.88\,\mu T$ at 500 km, according to subsection 2.5.2. The maximum norm of $T_{d,\text{mag}}$ is thus $T_{d,\text{mag},\text{max}} = 7.5 \times 10^{-7}\,\text{N m}$ at 690 km, and $T_{d,\text{mag},\text{max}} = 8.1 \times 10^{-7}\,\text{N m}$ at 500 km.

**Aerodynamic torque**

The residual atmosphere on orbit generates drag, which, combined with the geometry of Eye-Sat, produces a disturbance torque $T_{d,\text{aero}}$. The magnitude of the drag is proportional to the density of the atmosphere$^4$, which is itself affected by the solar activity$^5$.

Eye-Sat will be launched in 2018 or later, which means it will operate close to a solar minimum$^6$. Assuming a medium solar activity, the magnitude of the aerodynamic disturbance torque on Eye-Sat at an altitude of 700 km is $T_{d,\text{aero}} = 6 \times 10^{-8}\,\text{N m}$ [20]. The same value is assumed to hold for an altitude of 690 km.

The MSISE-90 model for the upper atmosphere [32] taken for a mean solar activity yields an atmospheric density of approximately $4 \times 10^{-14}\,\text{kg/m}^3$ at an altitude of 700 km, versus $7 \times 10^{-13}\,\text{kg/m}^3$ at 500 km. The magnitude of the aerodynamic disturbance torque on Eye-Sat at an altitude of 500 km is thus approximately $T_{d,\text{aero}} = 1 \times 10^{-6}\,\text{N m}$.

**Radiation pressure torque**

Eye-Sat receives radiation mainly from the Sun, directly and by reflection on Earth, and from Earth’s albedo. The radiation pressure generated on Eye-Sat translates into a force and a torque applied at its centre of mass. The magnitude of this torque is approximately

---

$^4$Under the assumption of a continuous medium, which is far from being the case but will be used here only to demonstrate a trend. In any case, an increase of the density induces an increase of the drag.

$^5$A higher solar activity results in a higher density at a given altitude.

$^6$The current solar cycle began in 2008, and will end in 2019. The solar maximum was reached between 2012 and 2015 [31].
\[ T_{d, \text{rad}} = 1 \times 10^{-9} \text{N m at an altitude of 700 km} \ [20]. \]

The effects of the reflection and the albedo are proportional to \(1/(R_\oplus + h_0)^2\), thus decreasing with the altitude\(^7\). Changing it from 700 to 500 km results in a 6% increase. The effects of the altitude on the solar pressure are assumed negligible. Therefore, the order of magnitude mentioned above holds for the whole range of altitude considered for Eye-Sat’s mission.

**Gravity gradient torque**

The intensity gravity field decreasing with the altitude, different parts of Eye-Sat experience different gravitational accelerations. The resulting torque is the gravity gradient torque, whose expression at first order is \([23, \text{p. 190}]\):

\[
T_{d, gg} = \frac{3\mu_\oplus}{(R_\oplus + h_0)^3} \mathbf{r}_\oplus \times ([I_{\text{sat}}] \mathbf{r}_\oplus). \tag{4.10}
\]

Expressed in Eye-Sat’s frame \(S\), the gravity gradient torque reads:

\[
T_{d, gg} = \frac{3\mu_\oplus}{(R_\oplus + h_0)^3} \begin{bmatrix} 0 \\ r_{\oplus,1} r_{\oplus,3} (I_1 - I_2) \\ r_{\oplus,1} r_{\oplus,2} (I_2 - I_1) \end{bmatrix}_S, \tag{4.11}
\]

where \(\mathbf{r}_\oplus = \begin{bmatrix} r_{\oplus,1} & r_{\oplus,2} & r_{\oplus,3} \end{bmatrix}_S\).

The magnitude of the gravity gradient torque is thus \(3\mu_\oplus (I_2 - I_1) / (R_\oplus + h_0)^3\), which yields \(T_{d, gg} = 8 \times 10^{-8} \text{N m at an altitude of 690 km}\), and \(T_{d, gg} = 9 \times 10^{-8} \text{N m at an altitude of 500 km}\).\(^{7}\)  

\(^{7}\)This relation is derived from the propagation of a spherical wave.
Synthesis of external disturbance torques

Table 4.1 summarises the disturbance torques to which Eye-Sat is submitted at altitudes of 690 km and 500 km. At 690 km, the magnetic torque $T_{d, \text{mag}}$ is the main contributor, whereas at 500 km, the contributions of $T_{d, \text{mag}}$ and $T_{d, \text{aero}}$ are of the same order of magnitude. The resulting disturbance at 500 km is approximately twice as big as at 690 km.

<table>
<thead>
<tr>
<th>$h_0$ (km)</th>
<th>690</th>
<th>500</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_{d, \text{mag}}$ (Nm)</td>
<td>$7.5 \times 10^{-7}$</td>
<td>$8.1 \times 10^{-7}$</td>
</tr>
<tr>
<td>$T_{d, \text{aero}}$ (Nm)</td>
<td>$6 \times 10^{-8}$</td>
<td>$1 \times 10^{-6}$</td>
</tr>
<tr>
<td>$T_{d, \text{rad}}$ (Nm)</td>
<td>$1 \times 10^{-9}$</td>
<td>$1 \times 10^{-9}$</td>
</tr>
<tr>
<td>$T_{d, \text{gg}}$ (Nm)</td>
<td>$7.8 \times 10^{-8}$</td>
<td>$8.5 \times 10^{-8}$</td>
</tr>
<tr>
<td>$T_d$ (Nm)</td>
<td>$1 \times 10^{-6}$</td>
<td>$2 \times 10^{-6}$</td>
</tr>
</tbody>
</table>

Table 4.1: Synthesis of external disturbance torques.

In the scope of this study, the reaction wheels are not used in a closed-loop. The mitigation of external torques thus relies on the MTB, whose worst-case torque capacity was determined in subsection 4.2.1 as $4.0 \times 10^{-6}$ Nm at 690 km, and $4.8 \times 10^{-6}$ Nm at 500 km. In both cases, they dominate the disturbance torques by a factor 2 to 4.

One must bear in mind that these values are derived from worst-case individual contributions.

4.3.2 Internal disturbances

Wheels micro-vibrations, misalignments and scaling factors

Operating the reaction wheels comes with disturbances, of three main types:

- micro-vibrations, caused by vibrations of the ball-bearings and internal unbalance;
- torque scaling factors, resulting in the torque achieved by a wheel being equal to the commanded one, times a factor different from 1;

- wheel misalignments, resulting in the realised action not being strictly collinear with the commanded one.

Micro-vibrations mainly affect pointing stability. This is a matter of importance for the shooting phase of the mission, for which reaction wheels are used in a closed-loop, but not for the scope of this study.

Torque scaling factors are disregarded, since wheels are not commanded in torque in the scope of this study. Such scaling factors do not exist for a speed command\textsuperscript{8}.

Wheel misalignments directly impact the direction of the embarked angular momentum, and therefore participate in the pointing budget.

**Flexible modes**

Although the spacecraft is assumed rigid in the scope of the dynamical study reported in this work, its solar panels are not strictly rigid. It is reported in [19] that the lowest-frequency flexible modes of the solar panels was around 4 Hz.

Since this disturbance is internal, it can only be triggered by momentum exchange between the rigid body and the appendage. Therefore, it must be made sure that such exchange is damped by the attitude control, by properly tuning its cut-off frequency.

\textsuperscript{8}The speed command is achieved by comparing the measured speed to the target one. Most of the time, speed sensors are tachometers, which do not suffer from scaling factors.
4.4 Dynamical system

The external torques applied to Eye-Sat are the disturbance torques, gathered in $T_d$, and the control torque produced by the magnetorquers $T_{MTB}$. The conservation of the angular momentum of the satellite in the inertial frame of reference $I$ reads:

$$\left(\frac{dH_{tot}}{dt}\right)_I = T_{MTB} + T_d. \quad (4.12)$$

Once again, $T_{RWS}$ does not appear in equation (4.12), for it is an internal torque, included in the left-hand term.

The derivation in the satellite frame of reference $S$ generates a gyroscopic term:

$$\left(\frac{dH_{tot}}{dt}\right)_S = \left(\frac{dH_{tot}}{dt}\right)_I - \Omega_{S/I} \times H_{tot}. \quad (4.13)$$

Therefore:

$$\left(\frac{dH_{tot}}{dt}\right)_S = -\Omega_{S/I} \times H_{tot} + T_{MTB} + T_d. \quad (4.14)$$

Assuming the matrix of inertia is a constant quantity, the expanded version of the equation reads:

$$[I_{sat}] \left(\frac{d(\Omega_T + \delta\omega)}{dt}\right)_S + \left(\frac{dH_{RWS}}{dt}\right)_S = - (\Omega_T + \delta\omega) \times ([I_{sat}] (\Omega_T + \delta\omega) + H_{RWS})$$

$$+ T_{MTB} + T_d. \quad (4.15)$$

Rearranging terms and using the shorthand $T_{RWS} = - \left(\frac{dH_{RWS}}{dt}\right)_S$, one gets:

$$[I_{sat}] \left(\frac{d\delta\omega}{dt}\right)_S = - [I_{sat}] \left(\frac{d\Omega_T}{dt}\right)_S - (\Omega_T + \delta\omega) \times ([I_{sat}] (\Omega_T + \delta\omega) + H_{RWS})$$

$$+ T_{RWS} + T_{MTB} + T_d. \quad (4.16)$$
4.5 Detailed contributions

The terms of equation (4.16) are detailed hereunder, at first order in \( \epsilon \) and \( \delta \omega \). The components of \( T_{MTB} \) will not be detailed here, but an approximation is given in subsection 6.1.2 for a geocentric pointing, corresponding to the *standby* mission phase.

\[
[I_{sat}] \left( \frac{d\delta \omega}{dt} \right)_S = \begin{bmatrix} I_1 \dot{\psi} \\ I_2 \dot{\theta} \\ I_2 \ddot{\psi} \end{bmatrix}_S \tag{4.17}
\]

\[
- [I_{sat}] \left( \frac{d\Omega_T}{dt} \right)_S = \begin{bmatrix} I_1 \left( \omega_{T,1} + \psi \omega_{T,2} - \theta \omega_{T,3} + \dot{\psi} \omega_{T,2} - \dot{\theta} \omega_{T,3} \right) \\ I_2 \left( -\psi \omega_{T,1} + \omega_{T,2} + \varphi \omega_{T,3} - \dot{\psi} \omega_{T,1} + \dot{\varphi} \omega_{T,3} \right) \\ I_2 \left( \theta \omega_{T,1} - \varphi \omega_{T,2} + \omega_{T,3} + \dot{\theta} \omega_{T,1} - \dot{\varphi} \omega_{T,2} \right) \end{bmatrix}_S \tag{4.18}
\]

\[
- \Omega_T \times ([I_{sat}] \Omega_T) = \begin{bmatrix} 0 \\ (I_2 - I_1) \left( \omega_{T,1} \omega_{T,3} + \theta \left( \omega_{T,2}^2 - \omega_{T,3}^2 \right) - \varphi \omega_{T,1} \omega_{T,2} + \psi \omega_{T,2} \right) \\ (I_1 - I_2) \left( \omega_{T,1} \omega_{T,2} + \psi \left( \omega_{T,2}^2 - \omega_{T,1}^2 \right) + \varphi \omega_{T,1} \omega_{T,3} - \theta \omega_{T,2} \right) \end{bmatrix}_S \tag{4.19}
\]

\[
- \Omega_T \times ([I_{sat}] \delta \omega) = \begin{bmatrix} I_2 \dot{\theta} \omega_{T,3} - I_2 \dot{\psi} \omega_{T,2} \\ I_2 \ddot{\psi} \omega_{T,3} - I_1 \dot{\varphi} \omega_{T,3} \\ I_1 \dot{\psi} \omega_{T,2} - I_2 \dot{\theta} \omega_{T,1} \end{bmatrix}_S \tag{4.20}
\]
\[-\Omega_T \times \mathbf{H}_{RWS} = \begin{bmatrix}
H_2 (\theta \omega_{T,1} - \varphi \omega_{T,2} + \omega_{T,3}) - H_3 (-\psi \omega_{T,1} + \omega_{T,2} + \varphi \omega_{T,3}) \\
H_3 (\omega_{T,1} + \psi \omega_{T,2} - \theta \omega_{T,3}) - H_1 (\theta \omega_{T,1} - \varphi \omega_{T,2} + \omega_{T,3}) \\
H_1 (-\psi \omega_{T,1} + \omega_{T,2} + \varphi \omega_{T,3}) - H_2 (\theta \omega_{T,1} + \psi \omega_{T,2} - \theta \omega_{T,3})
\end{bmatrix}_S \]

(4.21)

\[-\delta \omega \times ([I_{sat}] \Omega_T) = \begin{bmatrix}
I_2 \dot{\psi} \omega_{T,2} - I_2 \dot{\omega}_{T,3} \\
I_2 \dot{\varphi} \omega_{T,3} - I_1 \dot{\psi} \omega_{T,1} \\
I_1 \dot{\theta} \omega_{T,1} - I_2 \dot{\varphi} \omega_{T,2}
\end{bmatrix}_S \]

(4.22)

\[-\delta \omega \times ([I_{sat}] \delta \omega) = \begin{bmatrix}
0 \\
0 \\
0
\end{bmatrix}_S \]

(4.23)

\[-\delta \omega \times \mathbf{H}_{RWS} = \begin{bmatrix}
\dot{\psi} \mathbf{H}_2 - \dot{\theta} \mathbf{H}_3 \\
\dot{\varphi} \mathbf{H}_3 - \dot{\psi} \mathbf{H}_1 \\
\dot{\theta} \mathbf{H}_1 - \dot{\varphi} \mathbf{H}_2
\end{bmatrix}_S \]

(4.24)

\[\mathbf{T}_{RWS} = -\begin{bmatrix}
\dot{\mathbf{H}}_1 \\
\dot{\mathbf{H}}_2 \\
\dot{\mathbf{H}}_3
\end{bmatrix}_S \]

(4.25)

\[\mathbf{T}_d = \begin{bmatrix}
T_{d,1} \\
T_{d,2} \\
T_{d,3}
\end{bmatrix}_S \]

(4.26)

All components of equation (4.23) are second-order quantities.
Chapter 5

Attitude control

5.1 Architecture

The preliminary design of Eye-Sat’s AOCS was performed in [19]. It initially comprised 5 modes, 3 functional (MAS, MGT, MNO) and 2 non-functional (MLT, MFV). The diagram of modes is shown in Figure 5.1, and modes are introduced hereunder.

- **MLT (Mode Lancement, Launch Mode)** in the mode which is active until the satellite is deployed from the launcher.

- **MAS (Mode Acquisition Survie, Acquisition and Survival Mode)** is the most robust mode. It is based on few equipments and a simple conception to prevent failures. Its
purpose is to secure electrical supply by pointing the solar arrays towards the Sun, as it is activated after launch and after MGT or MNO failure. Estimation and actuation are respectively performed by magnetometers and magnetorquers. Since magnetic estimation and actuation only allow for 2-axis attitude control, the satellite body is spun, which provides gyroscopic stiffness. The detailed design of MAS was performed in [20].

- MGT (Mode Grossier de Transition, Coarse Transition Mode) switches the star tracker on, and performs acquisition of stellar-based attitude measurements. The spin introduced in MAS must be stopped, and reaction wheels are spun to provide gyroscopic stabilisation. In-loop estimation and actuation are magnetic-based. Once the acquisition is successful, transition to MNO is allowed.

- MNO (Mode Normal, Normal Mode) is the mode in which the mission is performed. It is the mode for which performance needs are maximal, with Eye-Sat being 3-axis pointed. Estimation is based on star tracker measurements, and actuation is performed by the reaction wheels. The magnetorquers are used for the off-loading of the wheels.

- MFV (Mode Fin de Vie, End-of-life Mode) is the final mode. It is activated for disposal of the satellite: AOCS in non-active, batteries are drained and power circuits are passivated.

A change in configuration moved the star tracker in alignment with the instrument, along the satellite’s x-axis. The advantage of this change is that in shooting, since scenes are chosen for neither Earth, the Moon nor the Sun to be in the instrument’s field of view, the star tracker will not be dazzled by any of them, and will be able to provide attitude estimation. However, the star tracker would almost always be dazzled by the Earth while tracking the X-band ground station. This triggered the need for a MNO based on magnetic estimation. Since star tracker measurements will be available in shooting, this particular mission phase will still use the star tracker for estimation. In case of a star tracker failure, the possibility to perform shooting with magnetic estimation is left, as a degraded use case.
With a magnetic-based MNO, MGT is no longer needed. Transition from MAS to MNO can be performed directly. The new AOCS architecture is shown in Figure 5.2, with 4 modes, 2 functional and 2 non-functional.

![2-mode AOCS architecture](image1)

MLT, MAS and MFV are unchanged in this new architecture. MNO comprises 4 sub-modes, as shown in Figure 5.3. *Standby*, *download* and *shooting* are used for performing the corresponding mission phases.

![MNO sub-modes architecture](image2)

The control is shown in Figure 5.4 under a schematic form. The *navigation* block is described in section 5.3. The *guidance* block is described in section 5.4. The estimation is composed of the *filtering*, *calibration* and *MAG* blocks, described in subsection 5.6.2. The command of MTB is described in subsection 5.6.3 to 5.6.5. The wheel command is presented in subsections 5.6.6 and 5.6.7.
5.2 Context

The upcoming sections of this chapter are dedicated to the design of the magnetic-based MNO, capable of 3-axis attitude control and sufficient agility to perform X-band downloading and manoeuvres.

Standby is the least stringent sub-mode in terms of pointing performance. Its main purpose is to charge the batteries, which requires that the solar panels receive sufficient illumination. Given the power needs during this mission phase, a $30^\circ$ half-cone pointing precision is required. It must however dissipate the exceeding kinetic energy when entering MNO from MAS. In MAS, the satellite body is spun about the $x$ axis, at an angular rate of $22\omega_0$, which is approximately $1.4^\circ\text{s}^{-1}$ at an altitude of 500 km. This high rotation rate must be taken into consideration for the design of the estimator.

The manoeuvre sub-mode is used to transit from standby to download or shooting. In order not to interfere with the mission planning, the manoeuvring time must be limited to a
reasonable fraction of an orbit. The most stringent manoeuvre is from \textit{standby} to \textit{shooting}, since some shooting scenes induce a 90°-slew about either $y$ or $z$, which are associated to the greater inertia $I_2$. Transitions from \textit{standby} to \textit{download} may induce similar angular displacements, but around the $x$-axis, of lower inertia $I_1$. Finally, Eye-Sat must be able to perform a 90°-slew about either $y$ or $z$ within less than 1000 s\textsuperscript{1}.

The \textit{download} sub-mode is a key design driver, for kinematics are imposed by the on-orbit motion, and target angular velocities can be as high as 0.87° s$^{-1}$ about the $x$-axis, as shown in appendix A.4. In order to achieve reasonably high data rates, the pointing performance is required to be at least 14° half-cone, according to appendix A.3.3.

Since actuation is based on reaction wheels in \textit{shooting}, no specific requirement is derived for this work.

The following requirements shall thus be fulfilled by the attitude control system:

- 30° half-cone pointing precision about the $x$-axis in \textit{standby};
- 14° half-cone pointing precision about the $z$-axis in \textit{download};
- slewing capacity of 90° in 1000 s around either the $y$ or the $z$-axis;
- ability to achieve 1° s$^{-1}$ angular rate about the $x$-axis in \textit{download};
- ability to perform magnetic measurements at angular rates up to 1.4° s$^{-1}$.

No pointing stability requirement has been derived. Pointing stability would only be critical for \textit{shooting}.

\textsuperscript{1}The orbital period is approximately 6000 s within the considered altitude range.
Pointing performance requirements are global targets, which include errors from control, guidance, navigation, as well as equipment misalignments and calibration errors. Therefore, an allocation of 70% of these requirements is taken for the control, which shall achieve 20° half-cone pointing precision about the $x$-axis in standby and 10° half-cone pointing precision about the $z$-axis in download, all other errors being null.

5.3 Navigation

Eye-Sat’s navigation functions consist of an orbit propagator, a Sun ephemeris calculator, and a function deriving the position of the satellite in the terrestrial frame $R$. Estimation functions are located in the functional block associated to control.

The need for such navigation functions is dictated by two factors:

• On-board guidance, which requires knowledge of the orbital characteristics to model the target magnetic field;

• Autonomous download, which requires knowing the relative position of the spacecraft and the X-band antenna, and the Sun direction to correctly point the solar panels during this phase.

The on-board propagator is RK4+J6, meaning Earth’s gravity potential is modelled up to the 6th zonal harmonic, and integration is performed using Runge-Kutta’s 4th order method. Since the on-board propagator drifts, it must be re-initialised based on observations and a more accurate, ground-based propagator. The re-initialisation must take place at least every 3 days in order to meet the mission needs [22].
5.4 Guidance

The guidance profile is computed on board, with the ground only providing the timeline of the transitions between sub-modes, with the exception of shooting, for which a quaternion must also be issued for each scene to be imaged.

The on-board guidance function issues at each flight software time step:

- the target quaternion $Q_T = Q_{T/I}$, describing the target attitude of the spacecraft with respect to the inertial frame;

- the target angular rate $\Omega_T = \Omega_{T/I}$ with respect to the inertial frame;

- the target magnetic field $B_0$, which corresponds to the modelled magnetic field in the target reference frame $T$. In other words, it corresponds to the magnetic field the perfectly well-pointing satellite would experience. The magnetic field is computed from the IGRF model in its latest realisation\(^2\), up to the 8\(^{th}\) order.

The definition of $Q_T$ and $\Omega_T$ is detailed hereunder for each sub-mode.

5.4.1 Standby

The satellite follows a geocentric pointing profile, with its $z$-axis pointing opposite to the direction of the centre of the Earth, and its $x$-axis pointing in the direction of $n_{\text{orb}}$. Therefore, the target frame $T$ and the local orbital frame $O$ are equal.

\(^2\text{International Geomagnetic Reference Field. One realisation covers a 5-year period, the latest one to date being the 12\(^{th}\) generation, covering 2015–2020.}\)
The target direction cosine matrix is:

\[
[T/I] = \begin{bmatrix}
  n_{\text{orb}}|_I^T \\
  (r_{\oplus}|_I \times n_{\text{orb}}|_I)|_I^T \\
  r_{\oplus}|_I^T
\end{bmatrix}
\]  

(5.1)

Neglecting the precession of the orbital plane, the target angular rate is equal to the orbital rate:

\[
\Omega_T = \begin{bmatrix}
  \omega_0 \\
  0 \\
  0
\end{bmatrix}.
\]  

(5.2)

5.4.2 Download

The $-z$-axis points towards the X-band ground receiver. The $-x$-face, which bears the solar cells, is pointed as close as possible to the opposite of the orbit normal $n_{\text{orb}}$, whose direction remains close to that of the Sun, as shown in subsection 2.5.1.

The target direction cosine matrix is:

\[
[T/I] = \begin{bmatrix}
  \left( \frac{n_{\text{orb}}|_I - (r_X \cdot n_{\text{orb}}) r_X|_I}{\|n_{\text{orb}} - (r_X \cdot n_{\text{orb}}) r_X\|} \right)^T \\
  \left( r_X|_I \times \frac{n_{\text{orb}}|_I - (r_X \cdot n_{\text{orb}}) r_X|_I}{\|n_{\text{orb}} - (r_X \cdot n_{\text{orb}}) r_X\|} \right)^T \\
  r_X|_I^T
\end{bmatrix},
\]  

(5.3)

where $r_X$ is the direction vector pointing from the ground X-band antenna towards the satellite.
5.4.3 Shooting

In shooting, the guidance quaternion $Q_T$ is constant and defined by the ground. The target angular rate $\Omega_T$ is null.

5.4.4 Manoeuvre

The manoeuvre sub-mode consists in rallying a target attitude from the current attitude. The gap is bridged with a bang-bang sequence, composed of an acceleration phase at constant torque, a steady rate phase, and a deceleration phase at constant torque. The rallying profile is calculated on-board, based on momentum and torque allocations.

Figure 5.5 shows the principle of a bang-bang profile for a manoeuvre between two inertial pointings, in 1D.

Figure 5.5: Bang-bang guidance profile. The angular target is dashed red.
5.5 Philosophy of the control

Due to the star tracker being unavailable during some *download* sequences, the MNO has to rely on the 3 remaining AOCS-related equipment: magnetometers, magnetorquers, and reaction wheels. As stated in the introduction to chapter 3, the magnetometers and the magnetorquers form a *natural couple*: as shown in section 5.6, the magnetometer measurements can be used to compute the command of the magnetorquers without actually estimating the attitude in the process.

Magnetometers and magnetorquers shall thus be used in closed-loop, thereby providing 2-axis attitude control\(^3\). To perform 3-axis attitude control, one has to use the reaction wheels to perform gyroscopic stabilisation, generating a gyroscopic stiffness along the non-magnetically commandable axis. This is achieved by commanding the wheels with an angular momentum of constant norm in a direction orthogonal to the magnetic field, further referred to as the *embarked* angular momentum \(H_{\text{emb}}\). The embarked angular momentum has to be rotated during manoeuvres so that it always points in the same inertial direction\(^4\), in order to avoid generating parasitic gyroscopic torques that would dramatically impact the agility of the satellite. The rotation of the embarked angular momentum is shown in Figure 5.6.

The magnetic control torque is too soft an action to allow performing agile manoeuvres. In addition to the magnetic closed-loop and the embarked angular momentum, one has to command the reaction wheels in open-loop to follow the guidance profile.

---

\(^3\)The axis collinear to \(B\) is neither observable nor commandable.

\(^4\)This would strictly speaking be true only if the satellite followed exactly the target attitude at all times.
In a nutshell, here are the ingredients to the magnetic-based MNO:

- An open-loop reaction wheel command to follow the guidance profile;
- A magnetic closed-loop to correct residual pointing errors about the guidance profile;
- An embarked angular momentum to provide passive third-axis stabilisation.

5.6 Detailed design

This section explores the conception of the magnetic-based MNO. Subsections 5.6.1 to 5.6.5 present the magnetic control, subsection 5.6.6 details the choice of the embarked angular momentum, and subsection 5.6.7 deals with the open-loop wheel command. Subsection 5.6.8 proposes a way-forward for improving the magnetic control by performing Kalman-filtering on the measured magnetic field.

5.6.1 Target magnetic field

The target magnetic field $B_0$ generated by the guidance function corresponds to the magnetic field that perfect magnetometers would measure if the magnetic field model were exact and
the satellite perfectly followed the guidance profile.

The unitary version of the target magnetic field is denoted by $b_0$, and the unitary magnetic field is denoted by $b$. Figure 5.7 displays $b$ and $b_0$ for a poorly-pointing and a well-pointing satellite.

$\vec{b} \cdot \vec{b}_0 = \vec{b}_0$

Figure 5.7: Target magnetic field for a poorly-pointing satellite (left), and a well-pointing satellite (right).

$B_0$ is linked to the satellite reference frame $S$. The relation between $B_0$ and $B$ is detailed hereafter, under the assumption that the magnetic field model is perfect:

$$ B_0 = \begin{bmatrix} B_{0,1} \\ B_{0,2} \\ B_{0,3} \end{bmatrix}_S $$

$$ B = \begin{bmatrix} B_{0,1} \\ B_{0,2} \\ B_{0,3} \end{bmatrix}_T $$

In other words:

$$ B_0|_S = [T/S] B|_S. $$
At first order in $\epsilon$:

$$\left[ T/S \right] = \begin{bmatrix}
1 & -\psi & \theta \\
\psi & 1 & -\varphi \\
-\theta & \varphi & 1
\end{bmatrix} = \left[ I_{3 \times 3} \right] + \left[ \epsilon \right].$$

(5.7)

This yields the following expression, valid for both non-unitary and unitary versions of the magnetic field:

$$B_\theta = B + \left[ \epsilon \right] B.$$  

(5.8)

### 5.6.2 Estimation

As stated in subsection 3.1.1, the magnetic field measured by the magnetometers is:

$$B_{\text{MAG}} = [A_{\text{MAG}}] B + \delta B_{\text{MAG}} + n_{\text{MAG}},$$

(5.9)

where $[A_{\text{MAG}}]$ is the configuration matrix, bearing the scaling factors and misalignment terms, $\delta B_{\text{MAG}}$ is the bias and $n_{\text{MAG}}$ is the measurement noise.

The measurement is corrected for the known part of the bias, scaling factors and misalignments, and transposed to the satellite frame $S$ as exposed in subsection 3.1.2. The calibrated magnetic measurement $B_{\text{meas}}$ still suffers from residual misalignments and scaling factors — which shall be disregarded here — and residual bias $\delta B_{\text{meas}}$. The noise is not affected in the calibration process.

$$B_{\text{meas}} = B + \delta B_{\text{meas}} + n_{\text{MAG}}.$$  

(5.10)
The derivative of $B_{\text{meas}}$ is used in the control-loop, described in subsection 5.6.3. The derivative can be approximated by the Euler difference:

$$\frac{dB_{\text{meas}}}{dt}(t) = \frac{B_{\text{meas}}(t) - B_{\text{meas}}(t - \Delta t)}{\Delta t},$$

(5.11)

with $\Delta t$ the AOCS time step. If the standard deviation of components of $B_{\text{meas}}$ is $\sigma(B)$, then the standard deviation of the components of its derivative is $2\sigma(B)/\Delta t$: the noise level is increased by the derivation process.

In Laplace’s domain, the transfer function of the pure derivative is:

$$H_d(p) = p,$$

(5.12)

where $p$ is Laplace’s variable. The asymptotic Bode magnitude plot for this transfer function is shown in Figure 5.8.

![20 log $|H_d(p)|$ vs $\omega$](image)

**Figure 5.8:** Asymptotic Bode magnitude plot for transfer function $H_d$.

The Bode magnitude plot clearly illustrates that the higher the frequency, the higher the transmitted noise. Therefore, the magnetic measurement has to be filtered before being used in the attitude control loop. Since measurement noise is a high-frequency perturbation, a low-pass filter is chosen. In Laplace’s domain, the transfer function of the first-order low-pass
The filter is:

\[ H_1(p) = \frac{1}{1 + \tau p}, \]  

(5.13)

where \( \tau \) is the time constant of the filter. The cut-off frequency \( \omega_c \) of this filter is \( 1/\tau \). The filter is unitary for frequencies below \( \omega_c \), and acts as an integrator on signals of higher frequencies, which it regularises\(^5\). The asymptotic Bode magnitude plot for \( H_1 \) is shown in Figure 5.9.

\[ 20 \log |H_1(p)| \]

\( \omega_c \)

\( \omega \)

(a)

\[ 20 \log |H(p)| \]

\( H_1 \cdot H_d(p) \)

\( H_d(p) \)

\( \omega_c \)

\( \omega \)

(b)

Figure 5.9: Asymptotic Bode magnitude plot for transfer functions \( H_1 \) (a) and \( H_d \cdot H_1 \) (b).

However, combining this first-order low-pass filter with the derivation annihilates the benefits of regularisation at high frequencies, since the resulting transfer function is:

\[ H_1(p) \cdot H_d(p) = \frac{p}{1 + \tau p}. \]  

(5.14)

This transfer function behaves as a differentiator at frequencies below \( \omega_c \), but has a unitary gain for high frequencies, meaning the measurement noise is simply fed to the control loop. The asymptotic Bode magnitude plot for \( H_d \cdot H_1 \) is shown in Figure 5.9.

\(^5\)Only asymptotic behaviours are described here.
A second-order low-pass filter is proposed:

\[
H_2(p) = \frac{1}{(1 + \tau p)^2}. \tag{5.15}
\]

The cut-off frequency remains \(1/\tau\), but the global transfer function becomes:

\[
H_2(p) \cdot H_d(p) = \frac{p}{(1 + \tau p)^2}. \tag{5.16}
\]

The asymptotic Bode magnitude plot for \(H_d \cdot H_2\) is shown in Figure 5.10. The resulting filter behaves as a differentiator for low-frequency signals, and an integrator for frequencies higher than \(\omega_c\). The tuning of the filter consists in choosing the correct value for \(\omega_c = 1/\tau\), so that measurement noise is attenuated, but the slow-varying \(B\) field is unaltered.

![Asymptotic Bode magnitude plot](image)

Figure 5.10: Asymptotic Bode magnitude plot for transfer functions \(H_2\) (a) and \(H_d \cdot H_2\) (b).

Low-pass filtering induces a phase difference. In order for the measured and the target magnetic fields to display the same delay, filtering is applied to both.

Low pass filters of order \(k > 2\), with a transfer function \(H_k(p) = 1/ (1 + \tau p)^k\) would even further attenuate the noise levels, but at the cost of an increased phase delay.
5.6.3 PD controller

Calibrated and filtered magnetometer measurements will be fed into a pseudo proportional-derivative controller. The classical proportional and derivative terms will be approximated at first order in $\epsilon$ and $\delta\omega$, and due to magnetic properties discussed in subsection 5.5, they will only apply on 2 axes.

In a PD controller, the proportional term acts as a spring, and keeps the satellite pointed after the transitory phase. The derivative term acts as a damper: it dissipates the exceeding kinetic energy, and has a determining role in the transitory phase.

Commanding the magnetorquers with the magnetic moment corresponding to the desired torque is not a straightforward process, and can introduce errors, as exposed in section 5.6.4. Instead, the properties of the magnetic measurement and actuation will be exploited.

The magnetic moment commanded to the magnetorquers, $\mathcal{M}_{MTB}$, is the sum of a proportional term $\mathcal{M}_K$ and a derivative term $\mathcal{M}_D$.

**Proportional term**

The proportional term in an ideal 3-axis PD-controller would be $T_K = -K\epsilon$, with $K$ the proportional gain. However, it is the magnetic moment $\mathcal{M}_K$ one has to command here.

At first order in $\epsilon$, equation (5.8) gave a relation between $b$ and $b_0$: $b_0 = b + [\bar{\epsilon}]b$, so $(b_0 - b) \times b = (\epsilon \cdot b) b - \epsilon$. This last quantity is equal to $-\epsilon$, minus its component along $b$.

So, by commanding $\mathcal{M}_K = K (b_0 - b) / B_0$ to the magnetorquers, and assuming $B_0 = B^6$,

---

Meaning the magnetic field model gives the exact norm of the magnetic field. This hypothesis is discussed in subsection 5.6.8.
one would get the desired proportional term:

$$T_K = -K\epsilon + K(\epsilon \cdot b)b$$  \hspace{1cm} (5.17)

The division by $B_0$ in the definition of $\mathcal{M}_K$ allows for the resulting torque to be independent of the norm of the local magnetic field, which varies as the satellite orbits the Earth. A division by $B_0$ rather than $B$ was chosen to avoid influence of the residual measurement bias. See subsection 5.6.4 for discussions.

The expression of $\mathcal{M}_K$ is based on the difference $b_0 - b$. Since $b \times b$ is null, it is useless to command the second term of the difference. It might actually bring additional error, since $b$ is only known through $B_{\text{meas}}$, which is biased. Since this bias would directly translate into a pointing error, one should instead command:

$$\mathcal{M}_K = K\frac{b_0}{B_0}.$$  \hspace{1cm} (5.18)

The error in this command only comes from the inaccuracy of the magnetic model and the guidance profile.

The proportional term of a classical PD controller is only approximated under the assumption of small angular errors. Nevertheless, the spring effect is true for any angle. Figure 5.11 shows that the torque generated by commanding $\mathcal{M}_K$ collinear to $b_0$ tends to bring $b$ collinear to $b_0$ in the satellite frame.
Derivative term

The derivative term in an ideal 3-axis PD controller would be $T_D = -D\delta\omega$, with $D$ the proportional gain.

From the definition of $b_0$ given in equation (5.6), and assuming the magnetic field model is perfect, one gets:

$$
\left(\frac{db_0}{dt}\right)_S = \left(\frac{db}{dt}\right)_T = \left(\frac{db}{dt}\right)_S + \left[\delta\omega\right]b.
$$

(5.19)

$(\frac{db_0}{dt})_S$ and $(\frac{db}{dt})_S$ shall further be respectively denoted by $\dot{b}_0$ and $\dot{b}$.

$$(\dot{b}_0 - \dot{b}) \times b = (\delta\omega) b - \delta\omega,$$

which is the projection of $\delta\omega$ on the plane orthogonal to $b$.

The derivative term to be commanded to the magnetorquers is thus:

$$
\mathcal{M}_D = \frac{\dot{b}_0 - \dot{b}}{B_0}.
$$

(5.20)

Once again, dividing by $B_0$ makes the action independent from the norm of the local magnetic field.
The desired derivative torque is obtained:

\[ T_D = -D\delta\omega + D (\delta\omega \cdot \mathbf{b}) \mathbf{b}. \]  

(5.21)

It is worth noting that this expression is a valid approximation of the derivative term, without assumptions on the magnitude of \( \delta\omega \).

### 5.6.4 Bias annihilation

On-board knowledge of the magnetic field comes from magnetometers, whose measurements are biased and noisy. The noise level can be reduce with low-pass filtering, but bias estimation is a more complex topic. Mathematically discarding it would thus be interesting.

The magnetic measurement is \( \mathbf{B}_{\text{meas}} = \mathbf{B} + \delta\mathbf{B}_{\text{meas}} \), where the noise has been omitted. By definition, and assuming that the variation of the bias is much slower than that of \( \mathbf{B}_{\text{meas}} \), its derivative can be neglected:

\[
\frac{dB_{\text{meas}}}{dt} = \lim_{dt \to 0} \frac{B_{\text{meas}}(t + dt) - B_{\text{meas}}(t)}{dt}
= \lim_{dt \to 0} \frac{B(t + dt) + \delta B_{\text{meas}} - B(t) - \delta B_{\text{meas}}}{dt}
= \frac{dB}{dt}.
\]

(5.22)

Therefore, differentiation annihilates the measurement bias.

The unitary form of the magnetic measurement is \( \mathbf{b}_{\text{meas}} = (\mathbf{B} + \delta\mathbf{B}_{\text{meas}})/\|\mathbf{B} + \delta\mathbf{B}_{\text{meas}}\| \). Differentiating it does not cancel the bias, since the denominator has a non-null derivative.

Of the magnetic moments \( \mathcal{M}_K \) and \( \mathcal{M}_D \) defined respectively in equations (5.18) and

---

7The time-variation of the bias is mainly driven by thermal cycling, whereas the variation of the magnetic field is driven by the rotational and orbital motions of the satellite.
only $M_D$ is bias-dependent, through $\dot{b}$. Noting that $(\frac{dB}{dt})_S = \dot{B}b + B\dot{b}$, and knowing from equation (5.22) that $(\frac{dB_{\text{meas}}}{dt})_S = (\frac{dB}{dt})_S$, one gets:

$$\left(\frac{dB_{\text{meas}}}{dt}\right)_S \times b = B\left(\dot{b} \times b\right). \tag{5.23}$$

Assuming that $B_0 = B$, the derivative torque expressed in equation (5.21) can be obtained by alternatively commanding:

$$M_D = \frac{D}{B_0}\left(\dot{b}_0 - \frac{\dot{B}_{\text{meas}}}{B_0}\right), \tag{5.24}$$

where $\dot{B}_{\text{meas}} = (\frac{dB_{\text{meas}}}{dt})_S$. The moment commanded to magnetorquers is now independent from the magnetometer bias.

### 5.6.5 Phases

Magnetorquers have a limited actuation capacity $M_{\text{max}}$ per axis (see subsection 3.3.1), so saturation has to be dealt with. The choice of the gains $K$ and $D$ must thus take saturation into account, so that it is either avoided, or used to optimise the command. Since there is a division by $B_0$ in both $M_K$ and $M_D$, saturation is more likely to happen when $B_0$ is small.

The purpose of the proportional term is to keep the satellite well-pointed, while that of the derivative term is to dissipate the exceeding kinetic energy. $D$ is thus the key parameter for the transitory phase, for instance when switching from MAS to MNO, while $K$ is crucial to post-transitory phase performance. In order to optimise performance in both cases, 2 sets of gains $\{K, D\}$ are chosen, corresponding to the 2 phases of the magnetic control: the acquisition phase, which corresponds to the rate reduction phase, and the converged phase, with finer pointing.

---

$^8B = ||B||$. 

81
The transition between the 2 phases is performed based on an estimation of the off-pointing $\Theta$ and time. An angular threshold $\Theta_{\text{thr}}$ and 2 time thresholds $t_{\text{acq/conv}}$ and $t_{\text{conv/acq}}$ are defined. Transition is performed based on the following conditions:

- Transition from acquisition phase to converged phase if $\Theta < \Theta_{\text{thr}}$ during $t_{\text{conv/acq}}$.
- Transition from converged phase to acquisition phase if $\Theta > \Theta_{\text{thr}}$ during $t_{\text{acq/conv}}$.

The off-pointing $\Theta$ is determined using the magnetometer measurements and the target magnetic field:

$$\Theta_{\text{mag}} = \cos^{-1}(b_{\text{meas}} \cdot b_0).$$

(5.25)

It thus only contains a 2-axis information.

Time thresholds $t_{\text{conv/acq}}$ and $t_{\text{acq/conv}}$ must be chosen such that transition to the converged phase is allowed only if the satellite has a small off-pointing for a prolonged time, and fall-back to the acquisition phase is not triggered by an erroneous or isolated measurement.

### 5.6.6 Embarked angular momentum

As presented in section 5.5, since the magnetic loop can only provide 2-axis attitude control, Eye-Sat must have an embarked angular momentum orthogonal to $B$, so that a gyroscopic stiffness torque is generated in the direction of the magnetic field. In practice, $H_{\text{emb}}$ does not have to be strictly orthogonal to $B$, but $\|H_{\text{emb}} \cdot B\|$ has to be as small as possible.

Under the approximation made in subsection 2.3.4, the orbit normal $n_{\text{orb}}$ is orthogonal to $B$. Its rotation rate in the inertial frame is about $1^\circ$ per day because of the regression of nodes. $n_{\text{orb}}$ is therefore a quasi-inertial direction. The following expression is a good
candidate for $H_{\text{emb}}$:

$$H_{\text{emb}} = H_{\text{emb}} n_{\text{orb}, T},$$  \hspace{1cm} (5.26)

where $n_{\text{orb}, T} |_S = n_{\text{orb}} |_T$. This choice allows for taking advantage of the rotation of the satellite around the Earth in standby, where $\Omega_T$ is aligned with $n_{\text{orb}}$. Equation (4.16) features the gyroscopic term $-\Omega_T \times H_{\text{RWS}}$. $H_{\text{emb}}$ being a component of $H_{\text{RWS}}$, it generates the gyroscopic term $-\Omega_T \times H_{\text{emb}}$, which tends to align the embarked angular momentum with the target angular velocity, as shown in Figure 5.12.

![Figure 5.12: Gyroscopic torque generated by the embarked angular momentum.](image)

The expression for $H_{\text{emb}}$ given in equation 5.26 will thus be used. In addition, setting it collinear to $n_{\text{orb}, T}$ ensures that the embarked angular momentum will not rotate with the satellite, and will thus remain pointed in the same inertial direction — if guidance and pointing errors are neglected.

### 5.6.7 Open-loop wheel control

The magnetic loop cannot perform slew manoeuvres, because the torques it generates are weak and one axis is non-commandable. On the other hand, reaction wheels can provide torques stronger by one or two orders of magnitude.

Manoeuvres shall thus be performed by the reaction wheels in open-loop. The guidance function, introduced in section 5.4, delivers the target angular rate $\Omega_T$. The opposite of the
target angular momentum of the satellite will be commanded to the wheels:

\[ H_{\text{man}} = -[I_{\text{sat}}] \Omega_T. \]  

(5.27)

Finally, the angular momentum commanded to the wheels will be:

\[ H_{\text{RWS}} = H_{\text{emb}} + H_{\text{man}}. \]  

(5.28)

5.6.8 Kalman filtering of magnetic measurements

The assumption that \( B = B_0 \) is used on several occasions in this section. However, this hypothesis might directly affect the pointing performance if it is not verified, like it may be the case in the event of a solar storm.

In order to prevent such deterioration, a valuable improvement of the estimation would consist in implementing an extended Kalman filter to enhance the magnetic field measurements, using the on-board magnetic field model for the prediction phase and ensuring that the norm of the measured field matches that of the modelled one in converged state.

In addition to guaranteeing that the assumption on norms is correct, such filter would allow for in-flight estimation of the magnetometer bias.

This function has finally been implemented in the flight software following the present work.
Chapter 6

Stability analysis

This chapter explores the stability of the control law designed in chapter 5. Main difficulties in this analysis are the non-commandability of one of the axes with magnetic actuation, and the presence of the wheel momentum, which generates gyroscopic torques.

The analysis will be limited to linearisable cases, thus assuming no saturation occurs\(^1\). In addition, it is assumed that the realisation of commands is perfect. Stability will only be evaluated for the geocentric pointing, corresponding to the \textit{standby} sub-mode. It will be assumed that the results of this analysis apply in \textit{download} and \textit{manoeuvre} sub-modes.

In section 6.1, a state-space representation approach is used to analytically derive stability criteria. These criteria are taken into consideration when tuning the control laws in chapter 7. It is then checked that the chosen tuning is stable in section 6.2, and a frequency analysis is performed.

\(^1\)This condition is fulfilled by the \textit{acquisition} phase of the magnetic control, whose purpose is to avoid saturation, as explained in section 7.2
6.1 Stability conditions

6.1.1 Assumptions

In this section, the following assumptions will be made:

- Expressions in $\epsilon$ and $\delta\omega$ will be limited to first-order development.
- It is assumed that the local magnetic field is contained within the orbital plane\(^2\).

6.1.2 Magnetic torque

At first order, the direction of the magnetic field $B$ is seen in the target frame $T$ as a sinusoidal signal with angular frequency $\omega_0$:

$$B = B(t)b$$

$$= B(t) \begin{bmatrix} 0 \\ -\cos(\omega_0 t) \\ -\sin(\omega_0 t) \end{bmatrix}_T.$$

Equation (6.1)

At first order in $\epsilon$, its expression in the satellite frame $S$ is:

$$B = B(t) \begin{bmatrix} -\psi\cos(\omega_0 t) + \theta\sin(\omega_0 t) \\ -\cos(\omega_0 t) - \varphi\sin(\omega_0 t) \\ \varphi\cos(\omega_0 t) - \sin(\omega_0 t) \end{bmatrix}_S.$$

Equation (6.2)

\(^2\)See subsection 2.3.4.
In the satellite frame $S$, the first-order target magnetic field is:

$$
\begin{align*}
B_0 &= B_0(t) b_0 \\
&= B_0(t) \begin{bmatrix}
0 \\
-\cos(\omega_0 t) \\
-\sin(\omega_0 t)
\end{bmatrix}.
\end{align*} \tag{6.3}
$$

Further on in this section, $\cos(\omega_0 t)$ and $\sin(\omega_0 t)$ will be respectively denoted by $c$ and $s$.

Expressing the magnetic moment commanded to the magnetorquers, designed in subsections 5.6.3 and 5.6.4, requires expressing $\dot{b}_0$ and $\dot{B}_{\text{meas}}$:

$$
\dot{b}_0 = \begin{bmatrix}
0 \\
\omega_0 s \\
-\omega_0 c
\end{bmatrix}_S \tag{6.4}
$$

$$
\dot{B}_{\text{meas}} = \left( \frac{dB}{dt} \right)_S + B(t) \begin{bmatrix}
-\dot{\psi}c + \dot{\theta}s \\
-\dot{\phi}s \\
\dot{\varphi}c - \dot{s}
\end{bmatrix}_S + \omega_0 \begin{bmatrix}
\theta c + \psi s \\
-\varphi c + s \\
-c - \varphi s
\end{bmatrix}_S. \tag{6.5}
$$

Therefore, $\mathcal{M}_K$ and $\mathcal{M}_D$ become:

$$
\mathcal{M}_K = \frac{K}{B_0} \begin{bmatrix}
0 \\
-c \\
-s
\end{bmatrix}_S \tag{6.6}
$$

$$
\mathcal{M}_D = \frac{D}{B_0} \begin{bmatrix}
0 \\
\omega_0 s \\
-\omega_0 c
\end{bmatrix}_S - B_0 \begin{bmatrix}
-\psi c + \theta s \\
-c - \varphi s \\
\varphi c - s
\end{bmatrix}_S - B_0 \begin{bmatrix}
-\dot{\psi}c + \dot{\theta}s \\
-\dot{\phi}s \\
\dot{\varphi}c
\end{bmatrix}_S + \omega_0 \begin{bmatrix}
\theta c + \psi s \\
-\varphi c + s \\
-c - \varphi s
\end{bmatrix}_S. \tag{6.7}
$$
Assuming that the target magnetic field is perfectly modelled, \( B_0 = B \). One must also keep in mind that \( \mathbf{b} = \begin{bmatrix} -\psi c + \theta s & -c - \varphi s & \varphi c - s \end{bmatrix}^T_s \).

The expression of the magnetic command torques, at first order in \( \epsilon \) and \( \delta \omega \), is therefore:

\[
\mathcal{M}_K \times \mathbf{B} = K \begin{bmatrix} -\varphi \\ (\psi c - \theta s) s \\ (-\psi c + \theta s) c \end{bmatrix}^S_s
\]

\[
\mathcal{M}_D \times \mathbf{B} = D \begin{bmatrix} -\dot{\varphi} \\ (\dot{\psi}c - \dot{\theta}s) s \\ (-\dot{\psi}c + \dot{\theta}s) c \end{bmatrix}^S_s + \omega_0 \begin{bmatrix} 0 \\ (-\theta c - \psi s) s \\ (\theta c + \psi s) c \end{bmatrix}^S_s
\]  

(6.8)

(6.9)

### 6.1.3 Substitution of variables

Let us introduce the following substitution:

\[
\begin{cases}
\alpha = \varphi \\
\beta = \theta c + \psi s \\
\gamma = \psi c - \theta s.
\end{cases}
\]

(6.10)

These quantities are the coordinates of \( \epsilon \) in the local magnetic reference frame \( \mathcal{M} \):

\[
\begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = [\mathcal{M}/\mathcal{T}] \epsilon,
\]

(6.11)

where:

\[
[\mathcal{M}/\mathcal{T}] = \begin{bmatrix}
1 & 0 & 0 \\
0 & c & s \\
0 & -s & c
\end{bmatrix}.
\]

(6.12)
However, $\alpha$, $\beta$ and $\gamma$ are not the angle errors as measured in the local magnetic frame.

The first and second derivatives of this new set of variables are:

\[
\begin{align*}
\dot{\alpha} &= \dot{\phi} \\
\dot{\beta} &= \dot{\theta}_c + \dot{\psi}_s + \omega_0 \gamma \\
\dot{\gamma} &= \dot{\psi}_c - \dot{\theta}_s - \omega_0 \beta \\
\ddot{\alpha} &= \ddot{\phi} \\
\ddot{\beta} &= \ddot{\theta}_c + \ddot{\psi}_s + 2 \omega_0 \dot{\gamma} + \omega_0^2 \beta \\
\ddot{\gamma} &= \ddot{\psi}_c - \ddot{\theta}_s - 2 \omega_0 \dot{\beta} + \omega_0^2 \gamma.
\end{align*}
\] (6.13) (6.14)

### 6.1.4 State-space representation

Since it is assumed that the satellite is in a geocentric-pointing *standby* sub-mode, with $\omega_0 > 0$:

\[
\begin{align*}
\Omega_T &= \begin{bmatrix} \omega_0 \\ 0 \\ 0 \end{bmatrix}_T \\
H_{\text{RWS}} &= \begin{bmatrix} H_{\text{RWS}} \\ 0 \\ 0 \end{bmatrix}_S.
\end{align*}
\] (6.15) (6.16)
Equation 4.16 reduces to the following system:

\[
\begin{align*}
I_1 \ddot{\phi} &= -K \phi - D \dot{\phi} + T_{d,x}^x \quad (1) \\
I_2 \ddot{\theta} &= [(I_2 - I_1) \omega_0 - H_{RWS}] \omega_0 \theta + [(2I_2 - I_1) \omega_0 - H_{RWS}] \dot{\psi} + K \sigma_\gamma + D \dot{\sigma}_\gamma + T_{d,y}^y \quad (2) \\
I_2 \ddot{\psi} &= [(I_2 - I_1) \omega_0 - H_{RWS}] \omega_0 \psi - [(2I_2 - I_1) \omega_0 - H_{RWS}] \dot{\theta} - Kc_\gamma - Dc_\dot{\gamma} + T_{d,z}^z \quad (3)
\end{align*}
\]

The first line of this system is decoupled from the other two. Denoting \( T_{d,1} \) by \( T_{d,a} \), it can be re-written using the new set of variables defined in equation (6.10):

\[
I_1 \ddot{\alpha} = -K \alpha - D \dot{\alpha} + T_{d,a}. \quad (6.18)
\]

The other two lines form a coupled system. The new set of variable can be used by forming linear combinations of the two last lines, denoted by (2) and (3):

\[
\begin{align*}
(2)c + (3)s \\
(3)c - (2)s.
\end{align*}
\]

The coupled system thus reads, after substitution:

\[
\begin{align*}
I_2 \left( \ddot{\beta} - 2 \omega_0 \dot{\gamma} - \omega_0^2 \beta \right) &= [(I_2 - I_1) \omega_0 - H_{RWS}] \omega_0 \beta + [(2I_2 - I_1) \omega_0 - H_{RWS}] (\dot{\gamma} + \omega_0 \beta) + T_{d,\beta}^\beta \\
I_2 \left( \ddot{\gamma} + 2 \omega_0 \dot{\beta} - \omega_0^2 \gamma \right) &= [(I_2 - I_1) \omega_0 - H_{RWS}] \omega_0 \gamma - [(2I_2 - I_1) \omega_0 - H_{RWS}] (\dot{\beta} - \omega_0 \gamma) - K \gamma - D \dot{\gamma} + T_{d,\gamma}. \quad (6.19)
\end{align*}
\]
where $T_{d,\beta} = cT_{d,\beta} + sT_{d,3}$ and $T_{d,\gamma} = cT_{d,\gamma} - sT_{d,2}$. After simplifications, it can finally be re-written as:

$$
\begin{align*}
I_2 \ddot{\beta} &= \Delta_1 \omega_0 \beta + \Delta_2 \dot{\gamma} + T_{d,\beta} \\
I_2 \ddot{\gamma} &= \Delta_1 \omega_0 \gamma - \Delta_2 \dot{\beta} - K \dot{\gamma} - D \ddot{\gamma} + T_{d,\gamma},
\end{align*}
$$

(6.20)

where $\Delta_1 = (4I_2 - 2I_1) \omega_0 - 2H_{RWS}$ and $\Delta_2 = (4I_2 - I_1) \omega_0 - H_{RWS}$.

The state vector $\Xi$ is defined as:

$$
\Xi = \begin{bmatrix} \beta \\ \gamma \\ \dot{\beta} \\ \dot{\gamma} \end{bmatrix}.
$$

(6.21)

The system can be written in matrix form:

$$
\dot{\Xi} = [A] \Xi + [C],
$$

(6.22)

where:

$$
[A] = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{\Delta_1 \omega_0}{I_2} & 0 & 0 & \frac{\Delta_2}{I_2} \\ 0 & \frac{\Delta_1 \omega_0 - K}{I_2} & - \frac{\Delta_2}{I_2} & - \frac{D}{I_2} \end{bmatrix},
$$

(6.23)

$$
[C] = \frac{1}{I_2} \begin{bmatrix} 0 \\ 0 \\ T_{d,\beta} \\ T_{d,\gamma} \end{bmatrix}.
$$

(6.24)
6.1.5 Stability criteria

If disturbance torques are neglected, the necessary and sufficient conditions for the stability of the decoupled axis are:

\[
\begin{aligned}
K & > 0 \\
D & > 0.
\end{aligned}
\]  
(6.25)

The necessary and sufficient condition for the stability of the coupled system is the strict negativity of the real parts of the eigenvalues of \([A]\). A necessary condition is that all the coefficients of the characteristic polynomial of \([A]\) strictly have the same sign. The characteristic polynomial is:

\[
\chi_{[A]}(\lambda) = \lambda^4 + \frac{D}{I_2} \lambda^3 + \frac{(K - 2\Delta_1 \omega_0) I_2}{I_2^2} \lambda^2 - \frac{D \Delta_1 \omega_0}{I_2} \lambda + \frac{\Delta_1 \omega_0 (\Delta_1 \omega_0 - K)}{I_2}.
\]  
(6.26)

The necessary condition induces:

\[
\begin{aligned}
D & > 0 \\
\Delta_1 & < 0.
\end{aligned}
\]  
(6.27)

The second condition can also read \(H_{\text{RWS}} > (2I_2 - I_1) \omega_0\). \(K > 0\) is not needed for the stability of the coupled system, but it is dictated by the condition on the decoupled axis. It is a sufficient condition for the coefficients of \(\chi_{[A]}\) to all have the same sign.

The 3 conditions on \(K\), \(D\) and \(H_{\text{RWS}}\) are necessary conditions for stability, but there is no guarantee that they are sufficient.
6.2 Stability of the tuning

6.2.1 Stability of the tuning

The magnitude of the embarked angular momentum $H_{emb}$ is tuned to $8 \times 10^{-4}$ N m s in section 7.5. The gains $K$ and $D$ are respectively set to $4 \times 10^{-6}$ N m rad$^{-1}$ and $2.2 \times 10^{-3}$ N m s rad$^{-1}$ for acquisition in section 7.2.

The necessary conditions expressed in equations (6.25) and (6.27) are met. For the decoupled $x$ axis, these necessary conditions are sufficient, and the tuning guarantees the stability. For the coupled $y$ and $z$ axes, it must be checked that the real parts of the roots of $\chi_{[A]} (\lambda)$ are strictly negative, which is the necessary and sufficient condition for stability. These roots are determined numerically, and the condition is satisfied:

$$\begin{align*}
\lambda_1 &= -0.0270 \\
\lambda_2 &= -0.0037 \\
\lambda_3 &= -0.0008 + j0.0042 \\
\lambda_4 &= -0.0008 - j0.0042.
\end{align*}$$

(6.28)

Therefore, the tuning guarantees the stability of the system.

However, the stability of the chosen values does not give insight on the margins on each of the parameters that may influence stability. By how much can the altitude, $H_{emb}$, $K$, $D$, $I_1$ or $I_2$ vary from their reference values after the tuning performed in section 7.2?

Figure 6.1 displays the locus of the 4 roots of $\chi_{[A]} (\lambda)$ when varying these 6 parameters. Tunable parameters — $H_{emb}$, $K$ and $D$ — are varied by ±50%. The altitude explores the range within which the mission can take place — between 500 km and 690 km according to section 2.4. Finally, the moments of inertia $I_1$ and $I_2$, which are less likely to change much, are
varied by approximately ±10% about their reference values of respectively $4.5 \times 10^{-2} \text{kg} \cdot \text{m}^2$ and $6.8 \times 10^{-2} \text{kg} \cdot \text{m}^2$. The ranges within which the variation is performed are gathered in Table 6.1. For each variable, the interval consists of 101 equally distributed points.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Reference value</th>
<th>Minimal value</th>
<th>Maximal value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Altitude (km)</td>
<td>500</td>
<td>500</td>
<td>690</td>
</tr>
<tr>
<td>$H_{emb}$ (N m s)</td>
<td>$8 \times 10^{-4}$</td>
<td>$4 \times 10^{-4}$</td>
<td>$12 \times 10^{-4}$</td>
</tr>
<tr>
<td>$K$ (N m rad$^{-1}$)</td>
<td>$4 \times 10^{-6}$</td>
<td>$2 \times 10^{-6}$</td>
<td>$6 \times 10^{-6}$</td>
</tr>
<tr>
<td>$D$ (N m s rad$^{-1}$)</td>
<td>$2.2 \times 10^{-3}$</td>
<td>$1.1 \times 10^{-3}$</td>
<td>$3.3 \times 10^{-3}$</td>
</tr>
<tr>
<td>$I_1$ (kg m$^2$)</td>
<td>$4.5 \times 10^{-2}$</td>
<td>$4.0 \times 10^{-2}$</td>
<td>$5.0 \times 10^{-2}$</td>
</tr>
<tr>
<td>$I_2$ (kg m$^2$)</td>
<td>$6.8 \times 10^{-2}$</td>
<td>$6.0 \times 10^{-2}$</td>
<td>$8.0 \times 10^{-2}$</td>
</tr>
</tbody>
</table>

Table 6.1: Parameter ranges for the sensitivity study of the stability.

The first observation is that within the chosen ranges for all parameters, the real part of the roots remain strictly negative. The real roots are never brought very close to the imaginary axis. The only risk may come from the complex conjugate roots, which are very sensitive to the variation of $H_{emb}$. Decreasing this parameter would dramatically impact the stability of the system. The only other parameter that has a notable effect on the stability is the derivative gain $D$, whose diminution brings the roots closer to the imaginary axis.

This analysis confirms the stability for a reasonable range of tunings and changes in Eye-Sat’s configuration. However, the root-locus plots were drawn by varying only one parameter at a time. Conclusions should thus be used with care.
6.2.2 Frequency analysis

In this subsection, the frequency response of Eye-Sat to disturbances in *standby* sub-mode is studied, for the *acquisition* tuning determined in section 7.2. The main goal of this analysis is to verify the rejection of the flexible modes at 4 Hz introduced in section 4.3.2, which corresponds to approximately 27 rad s\(^{-1}\).
The full system of dynamical equations, (6.18) and (6.20), is repeated hereunder:

\[
\begin{align*}
I_1 \ddot{\alpha} &= -K\alpha - D\dot{\alpha} + T_{d,\alpha} \\
I_2 \ddot{\beta} &= \Delta_1 \omega_0 \beta + \Delta_2 \dot{\gamma} + T_{d,\beta} \\
I_2 \ddot{\gamma} &= \Delta_1 \omega_0 \gamma - \Delta_2 \dot{\beta} - K\gamma - D\dot{\gamma} + T_{d,\gamma},
\end{align*}
\]

(6.29)

A Laplace transform is applied to this system, with \( p \) as the Laplace variable. For commodity, initial conditions are omitted and the same notations are used for quantities in the Laplace domain and in the time-domain.

\[
\begin{align*}
I_1 \alpha p^2 &= -K\alpha - D\alpha p + T_{d,\alpha} \\
I_2 \beta p^2 &= \Delta_1 \omega_0 \beta + \Delta_2 \gamma p + T_{d,\beta} \\
I_2 \gamma p^2 &= \Delta_1 \omega_0 \gamma - \Delta_2 \beta p - K\gamma - D\gamma p + T_{d,\gamma},
\end{align*}
\]

(6.30)

The associated transfer functions \( \alpha/T_{d,\alpha}, \beta/T_{d,\beta}, \beta/T_{d,\gamma}, \gamma/T_{d,\beta} \) and \( \gamma/T_{d,\gamma} \) are gathered in Table 6.2. Notations used in this table are defined in equation (6.31).

\[
\begin{align*}
F_1 (p) &= I_1 p^2 + Dp + K \\
F_2 (p) &= I_2 p^2 + Dp + K - \Delta_1 \omega_0 \\
F_3 (p) &= I_2 p^2 - \Delta_1 \omega_0 \\
F_4 (p) &= F_2 (p) F_3 (p) + \Delta_2^2 p^2.
\end{align*}
\]

(6.31)

\[
\begin{array}{ccc}
& T_{d,\alpha} & T_{d,\beta} & T_{d,\gamma} \\
\hline
\alpha & \frac{1}{F_1 (p)} & 0 & 0 \\
\beta & 0 & \frac{F_2 (p)}{F_4 (p)} & \frac{\Delta_2 p}{F_4 (p)} \\
\gamma & 0 & -\frac{\Delta_2 p}{F_4 (p)} & \frac{F_3 (p)}{F_4 (p)}
\end{array}
\]

Table 6.2: Transfer functions in \textit{standby} sub-mode.
The associated transfer functions are plotted in Figures 6.2, 6.3, 6.4, 6.5 and 6.6. Since their inputs and outputs do not have the same dimension, the outputs are normalised by a division by $K$. Therefore, amplitudes displayed on these graphs must be handled with care, and the study will focus on resonances and rejections.

The $\alpha$ axis behaves as a classical second-order low-pass filter, with natural frequency $\sqrt{K/I_1}$ and damping ratio $D/(2\sqrt{KI_2})$. Its Bode plot, displayed in Figure 6.2, does not feature any zero or any resonance. The cut-off frequency is around $2 \times 10^{-3}$ rad s$^{-1}$, thereby neatly rejecting the flexible modes.

![Figure 6.2: Bode plot of $\alpha/T_{d,\alpha}$.](image)

The study of the coupled $y$ and $z$ axes is a bit more complicated to apprehend. $\beta/T_{d,\gamma}$ and $\gamma/T_{d,\beta}$ only differ by their sign, so the cross-influence of disturbance torques is equal on the $\beta$ and $\gamma$ axes. $\beta/T_{d,\beta}$ and $\gamma/T_{d,\gamma}$ also share the same form, excepted for the influence of the magnetic control.

Neglecting $\Delta_p^2 p^2$ in the denominator of $\beta/T_{d,\beta}$, a term which is linked to the gyroscopic
stiffness and is thus crucial for stability, one could simplify $F_2(p)$ and wipe the influence of the magnetic control in $\beta/T_{d,\beta}$. Therefore, the $\beta$ axis is only linked to the magnetic control through gyroscopic coupling with the $\gamma$ axis.

The Bode diagram of $\beta/T_{d,\beta}$ is shown in Figure 6.3. It displays a resonance at $8.7 \times 10^{-4}$ rad s$^{-1}$, resonance which has disappeared at the orbital rate $\omega_0$, which constitutes the fundamental frequency of the on-orbit disturbances. Flexible modes at 27 rad s$^{-1}$ are well rejected.

![Bode plot of $\beta/T_{d,\beta}$](image)

Figure 6.3: Bode plot of $\beta/T_{d,\beta}$.

The Bode plots of the crossed transfer functions $\beta/T_{d,\gamma}$ and $\gamma/T_{d,\beta}$, shown respectively in Figures 6.4 and 6.5, are strictly equal. They also display a resonance at $8.7 \times 10^{-4}$ rad s$^{-1}$, and flexible modes are clearly rejected.
Figure 6.4: Bode plot of $\beta/T_{d,\gamma}$.

Figure 6.5: Bode plot of $\gamma/T_{d,\beta}$.

$\gamma/T_{d,\gamma}$ features a double real zero, at the angular frequency $\sqrt{-\Delta_1\omega_0/I_2}$, that is $4.8 \times 10^{-3}$ rad s$^{-1}$. Disturbances at this frequency are rejected by the closed-loop, due to the influence of the embarked angular momentum. A resonance at $8.7 \times 10^{-4}$ rad s$^{-1}$ is also visible in Figure 6.6.
Flexible modes are clearly rejected on this axis as well.

Figure 6.6: Bode plot of $\gamma/T_{d,\gamma}$.

The frequency analysis confirms that the magnetic closed-loop damps the disturbance caused by the first oscillatory mode on the solar panels.
Chapter 7

Tuning

7.1 Estimation

It has been chosen in subsection 5.6.2 to filter the magnetic measurements with a low-pass filter of order \( k \geq 2 \), with a transfer function of the following form:

\[
H_k(p) = \frac{1}{(1 + \tau p)^k}.
\]  

(7.1)

This transfer function can be approximated at first order in \( \tau p \ll 1 \) by:

\[
H_k(p) \approx 1 - k \tau p.
\]  

(7.2)

Rewriting the Laplace variable \( p \) as \( j \omega \), where \( j^2 = -1 \) and \( \omega \) is the angular frequency, the argument can be approximated by \( k \tau \omega \). The phase difference thus increases linearly with the angular frequency when \( \tau p \ll 1 \). The delay for such signals is \( k \tau \) at first order.

Therefore, increasing the order of the filter introduces a delay on the measured magnetic field, which is proportional to the order of the filter. The second-order low-pass filter thus arises as a good trade-off between noise reduction and low delay.
The largest rotation rates Eye-Sat will face in MNO are when the satellite enters MNO from MAS, as stated in section 5.2. In MAS, the target angular rate is $22\omega_0$, which is equal to $2.44 \times 10^{-2} \text{rad s}^{-1}$ at an altitude of 500 km. In order to properly estimate the attitude with the magnetometers during that phase, the cut-off frequency of the low-pass filter has to be at least one decade higher, i.e. $\omega_c \geq 2.44 \times 10^{-1} \text{rad s}^{-1}$. This condition translates into $\tau \leq 4.1 \text{s}$. Raising the cut-off frequency reduces the delay, but at the cost of an increased noise level.

Taking some margin with respect to this requirement, it is arbitrarily chosen to limit the delay to 5 s, which corresponds to $\tau = 2.5 \text{s}$. The cut-off-frequency is then:

$$\omega_c = 4 \times 10^{-1} \text{rad s}^{-1}. \quad (7.3)$$

### 7.2 Acquisition phase

#### 7.2.1 Analytical tuning

Subsection 5.6.5 introduced the principle for tuning the gains for the acquisition phase: during this phase, the derivative term dissipates the exceeding kinetic energy.

The magnetorquers are capable of generating a magnetic moment $\mathcal{M}_{\text{max}} = 0.2 \text{A m}^2$ along each satellite axis\(^1\). The capacity must be shared between the proportional and the derivative action. Since the latter one has the preponderant role in acquisition, the preliminary criterion would be that the proportional action never saturates the actuator.

The proportional part of the magnetic moment is $\mathcal{M}_K = K b_0 / B_0$, whose norm is simply

---

\(^1\)See subsection 3.3.1.
It thus depends on the norm of the magnetic field. According to section 2.5, the minimum norm of the magnetic field at 500 km is approximately $2 \times 10^{-5}$ T. The condition on $K$ in *acquisition* is thus $K_{\text{eq}} \approx 4 \times 10^{-6}$ N m rad$^{-1}$. By analogy with a classical PD-controller, the damping coefficient $\xi$ along a given axis is defined as:

$$\xi = \frac{D}{2\sqrt{K I}}, \quad (7.4)$$

with $I$ the moment of inertia of the considered axis. The critical damping regime\(^2\) is obtained for $\xi = 1$. The regime giving the fastest convergence below 5% of static error is obtained for $\xi = 1/\sqrt{2}$, at the price of a slight overshoot.

Choosing $\xi = 1/\sqrt{2}$ yields:

$$D = \sqrt{2KI}. \quad (7.5)$$

Since $D$ and $K$ are scalar quantities, the tuning is common to all the axes of the satellite. To avoid having $\xi < 1/\sqrt{2}$ on any axis, which would damage the performance by generating large oscillations, the tuning has to be performed on the axis bearing the largest moment of inertia. Therefore, $D = \sqrt{2KI_2}$, which yields $D = 7.4 \times 10^{-4}$ N m s rad$^{-1}$ for $K = 4 \times 10^{-6}$ N m rad$^{-1}$.

### 7.2.2 Numerical tuning

The numerical tuning is chosen based on a 12 000 s-reference test case, with the control phase set to *acquisition*, with transition to *converged* inhibited. The initial conditions are:

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---

\(^2\)Defined as the regime giving fastest convergence without overshooting. $\xi > 1$ corresponds to an overdamped regime, with a slower return to equilibrium than with the critical regime, and $0 < \xi < 1$ corresponds to an under-damped regime. Such regime is characterised by oscillations.
• $a_0 = 6\,878\,136.3\,\text{m}$
• $e_0 = 0.00001$
• $\omega_p = 0\,\text{rad}$
• $i_0 = 1.700\,057\,4\,\text{rad}$
• $\Omega_0 = 3.150\,537\,2\,\text{rad}$
• $M = 0\,\text{rad}$
• $Q_{S/I} = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}$
• $\Omega_{S/I} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}\,\text{rad}\,\text{s}^{-1}$
• $\Omega_{\text{RWS}} = \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}\,\text{rpm}$.

Preliminary tests on wide ranges of parameters have established the ranges of interest for the gains, which are shown in Table 7.1. The embarked angular momentum $H_{\text{emb}}$ is set to $8 \times 10^{-4}\,\text{N}\,\text{m}\,\text{s}$.

<table>
<thead>
<tr>
<th></th>
<th>Minimum</th>
<th>Maximum</th>
<th>Step</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K,\text{(N m rad}^{-1})$</td>
<td>$3.5 \times 10^{-6}$</td>
<td>$4.5 \times 10^{-6}$</td>
<td>$0.5 \times 10^{-6}$</td>
</tr>
<tr>
<td>$D,\text{(N m s rad}^{-1})$</td>
<td>$1.8 \times 10^{-3}$</td>
<td>$2.4 \times 10^{-3}$</td>
<td>$0.2 \times 10^{-3}$</td>
</tr>
</tbody>
</table>

Table 7.1: Ranges explored for the tuning of acquisition gains.

The figures of merit are:

• the convergence time to a 3-axis pointing error below $10^\circ$, assuming perfect guidance and sensor calibration;

• the maximum 3-axis pointing error in converged regime;

• the maximum pointing error around each axis in converged regime.
Considering the low number of cases and tunings studied, the preliminary tuning is picked by hand among the configurations displaying good results in all 3 areas. The final choice is:

\[
\begin{align*}
K &= 4 \times 10^{-6} \text{ N m rad}^{-1} \\
D &= 2.2 \times 10^{-3} \text{ N m s rad}^{-1}.
\end{align*}
\] (7.6)

The associated performance indicators are:

- Convergence time to 10°: 3340 s
- Maximum 3-axis pointing error: 6.0°
- Maximum pointing error around $x$: 3.5°
- Maximum pointing error around $y$: 5.3°
- Maximum pointing error around $z$: 2.7°.

$K$ is actually chosen at the limit defined in subsection 7.2.1, and $D$ is much larger than expected, yielding a damping coefficient $\xi_1 = 2.59$ on the $x$ axis, and $\xi_2 = 2.11$ on the $y$ and $z$ axes.

### 7.3 Converged phase

#### 7.3.1 Analytical tuning

As stated in subsection 5.6.5, the converged phases aims at maintaining a fine pointing. By analogy with a classical PD controller, the static error around one axis, in presence of a disturbance torque $T_d$, is:

\[
\Theta = \frac{T_d}{K}.
\] (7.7)
Therefore, a large $K$ is needed. In order for the proportional action to be maximal at every position on orbit, it has to be made sure that it always saturates the magnetorquers.

$\mathcal{M}_K$ is inversely proportional to the norm of the magnetic field, whose maximum at 500 km altitude is $5 \times 10^{-5}$ T. The magnetorquers can generate $\sqrt{3} \mathcal{M}_{\text{max}} = 0.35 \text{ A m}^2$ in their strongest direction. To always saturate the magnetorquers with the proportional term, the following condition has to be met: $K \geq 1.7 \times 10^{-5} \text{ N m rad}^{-1}$.

Because of saturation, the damping coefficient does not have a significant meaning in this case, and the tuning of $D$ is left to the numerical case.

### 7.3.2 Numerical tuning

The numerical tuning is chosen based on a 12 000 s-reference test case, with the control phase set to *converged*, with transition to *acquisition* inhibited. The initial conditions are taken from a converged *acquisition* phase, with the tuning parameters determined in subsection 7.2.2.

- $a_0 = 6.892948.0 \text{ m}$
- $e_0 = 0.0017$
- $\omega_p = 0.5960 \text{ rad}$
- $i_0 = 1.6999083 \text{ rad}$
- $\Omega_0 = 3.1595810 \text{ rad}$
- $M = 0.0012 \text{ rad}$
- $Q_{s/x} = \begin{bmatrix} -0.4723 & 0.5352 & 0.4661 & -0.5228 \end{bmatrix}$
\[
\begin{align*}
\mathbf{\Omega}_{S/I} &= \begin{bmatrix} 1.1542 & 0.1434 & 0.1626 \end{bmatrix} 10^{-3} \text{rad s}^{-1} \\
\mathbf{\Omega}_{RWS} &= \begin{bmatrix} 1001.1 & -5010.0 & 1000.0 & -5014.6 \end{bmatrix} \text{rpm}
\end{align*}
\]

Preliminary tests on wide ranges of parameters have established the ranges of interest for gains which are shown in Table 7.2. The embarked angular momentum \( H_{\text{emb}} \) is set to \( 8 \times 10^{-4} \text{N m s} \).

<table>
<thead>
<tr>
<th></th>
<th>Minimum</th>
<th>Maximum</th>
<th>Step</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K ) (N m rad(^{-1})</td>
<td>7.0 \times 10^{-6}</td>
<td>9.0 \times 10^{-6}</td>
<td>1.0 \times 10^{-6}</td>
</tr>
<tr>
<td>( D ) (N m s rad(^{-1})</td>
<td>4.0 \times 10^{-3}</td>
<td>6.0 \times 10^{-3}</td>
<td>0.5 \times 10^{-3}</td>
</tr>
</tbody>
</table>

Table 7.2: Ranges explored for the tuning of \textit{converged} gains.

The figures of merit are:

- the maximum 3-axis pointing error in converged regime;
- the quadratic 3-axis pointing error in converged regime;
- the quadratic pointing error around each axis in converged regime.

Considering the low number of cases and tunings studied, the preliminary tuning is picked by hand among the configurations displaying good results in all 3 areas. The final choice is:

\[
\begin{align*}
K &= 8 \times 10^{-6} \text{N m rad}^{-1} \\
D &= 4.5 \times 10^{-3} \text{N m s rad}^{-1}.
\end{align*}
\]

(7.8)

\( K \) is actually chosen at a much lower value than what was foreseen in subsection 7.3.1. The associated performance indicators are:

- Maximum 3-axis pointing error: 4.8°
- Quadratic 3-axis pointing error: 3.0°
• Quadratic pointing error around $x$: $1.1^\circ$

• Quadratic pointing error around $y$: $2.1^\circ$

• Quadratic pointing error around $z$: $1.8^\circ$.

### 7.4 Phase transition

The following transition conditions were introduced in subsection 5.6.5 when describing the need for an *acquisition* and a *converged* phase:

- Transition from *acquisition* phase to *converged* phase if $\Theta < \Theta_{\text{thr}}$ during $t_{\text{conv/acq}}$.
- Transition from *converged* phase to *acquisition* phase if $\Theta > \Theta_{\text{thr}}$ during $t_{\text{acq/conv}}$.

#### 7.4.1 Angular threshold

The off-pointing is estimated from the magnetometer measurements and the modelled magnetic field as $\Theta = \cos^{-1}(b_{\text{meas}} \cdot b_0)$.

Estimating the off-pointing with this formula comes with two error sources: the error in modelling the direction of the magnetic field with $b_0$, which is a guidance error, and that in measuring the direction of the magnetic field as $b_{\text{meas}}$, which is a measurement bias.

The tests reported in subsection 3.1.1 indicate that the measurement bias can be as high as $7\,\mu\text{T}$ per axis prior to calibration, and the norm of the magnetic field at an altitude of $690\,\text{km}$ can be as low as $20\,\mu\text{T}$. The angular error realised in evaluating $\Theta$ with $\Theta_{\text{mag}}$ is denoted by $\Theta_{\text{err}}$. The measured magnetic field is $B_{\text{meas}} = B + \delta B$, as shown in Figure 7.1.

\[\text{See section 2.5.}\]
The worst-case in estimating $\Theta$ with $\Theta_{\text{mag}}$ is when $B \cdot \delta B_\parallel < 0$. Then:

$$\Theta_{\text{err}} = \tan^{-1}\left(\frac{\|\delta B_{\perp}\|}{\|B\| - \|\delta B_\parallel\|}\right).$$

With a 7µT bias on each axis and $B = 20$ µT, $\Theta_{\text{err}}$ amounts to 37.3°.

Keeping an allocation of 10° for the AOCS error\(^4\) and adding a 30% margin, it is chosen to tune $\Theta_{\text{thr}}$ to 50°.

The tuning of this angular threshold can be adjusted if more information is available about the possibility to calibrate the measurement bias in flight, and the magnitude of the residual bias.

### 7.4.2 Time threshold

There is a risk that measurements $\Theta_{\text{mag}}$ punctually exceed $\Theta_{\text{thr}}$ because of measurement noise. To avoid triggering unwanted fall-back from converged to acquisition, $t_{\text{acq/conv}}$ must be used as a filtering duration. It is tuned to 2 s, which corresponds to 8 AOCS time steps\(^5\).

---

\(^4\)This is a 2-axis error. The value is in accordance with the 10° half-cone pointing error requirement in download mentioned in section 5.2.

\(^5\)See section 8.1 for a description of the software implementation.
\( t_{\text{conv/acq}} \), however, is used to ensure that acquisition is really finished before switching to converged gains. Since the threshold \( \Theta_{\text{thr}} \) is large, there is an uncertainty about how well the satellite is pointing. Therefore, \( t_{\text{conv/acq}} \) is tuned to 2000 s, which is in accordance with the characteristic time of the transient phase observed in subsection 8.2.1.

### 7.5 Constant wheel momentum

#### 7.5.1 Analytical tuning

The choice of the magnitude of the embarked angular momentum is performed on two competing criteria:

- A large norm for the embarked angular momentum enhances the stability of the satellite.
- A small norm for the embarked angular momentum allows for more agility, since more wheel momentum can be dedicated to manoeuvres.

It has been shown in subsection 3.4.2 that the norm of the angular momentum envelope of the reaction wheels configuration in the worst-case direction is \( H_{\text{max}} = 1.26 \times 10^{-3} \text{ N m s} \), which has to be shared into two allocations, one for the embarked momentum, the other one for manoeuvres.

Subsection 6.1.5 established a stability criterion based on the wheel momentum along the orbit normal:

\[
H_{\text{RWS}} > (2I_2 - I_1) \omega_0. \tag{7.10}
\]

The worst case would come from the two components of \( H_{\text{RWS}}, \ H_{\text{emb}} \) and \( H_{\text{man}} \), pointing
in opposite directions, yielding the following condition:

\[ H_{\text{emb}} > \frac{H_{\text{max}} + (2I_2 - I_1) \omega_0}{2}. \]  \hspace{1cm} (7.11)

An altitude of 500 km, which is the most stringent case within our interval of study, produces an orbital angular frequency \( \omega_0 = 1.107 \times 10^{-3} \text{ rad s}^{-1} \). The condition for stability is then \( H_{\text{emb}} > 5.75 \times 10^{-4} \text{ N m s} \).

### 7.5.2 Numerical tuning

The values explored for \( H_{\text{emb}} \) are \{6, 7, 8\} \( 10^{-4} \text{ N m s} \). Going further would dramatically affect the slewing capacity of the spacecraft.

The tuning value is chosen based on the scenario described in subsection 7.2.2 for the tuning of the acquisition gains. Figure 7.2 compares the 3-axis pointing convergence for \( H_{\text{emb}} = 7 \times 10^{-4} \text{ N m s} \) and \( H_{\text{emb}} = 8 \times 10^{-4} \text{ N m s} \). The performance for \( H_{\text{emb}} = 6 \times 10^{-4} \text{ N m s} \) is close to instability, and is not shown on this graph.

The convergence is much better for \( H_{\text{emb}} = 8 \times 10^{-4} \text{ N m s} \). This setting leaves an allocation of \( 2.5 \times 10^{-4} \text{ N m s} \) available for manoeuvres, which allows the satellite to perform a 90° slew around either its \( y \) or \( z \)-axis in 7 minutes, which is well within the requirement of performing this slew in less than 1000 s expressed in section 5.2. Therefore, \( H_{\text{emb}} \) is set to \( 8 \times 10^{-4} \text{ N m s} \).
Figure 7.2: Influence of the embarked momentum on the convergence in *acquisition*.

### 7.6 Summary

The results of this tuning campaign are gathered hereunder. They are only preliminary, and must be refined with a much larger variety of cases. The tuning values shown here can be used as a good starting point for such extensive campaign.

- $K_{acq} = 4 \times 10^{-6}$ N m rad$^{-1}$
- $D_{acq} = 2.2 \times 10^{-3}$ N m s rad$^{-1}$
- $K_{conv} = 8 \times 10^{-6}$ N m rad$^{-1}$
- $D_{conv} = 4.5 \times 10^{-3}$ N m s rad$^{-1}$
- $H_{emb} = 8 \times 10^{-4}$ N m s
- $\Theta_{thr} = 50^\circ$
- $t_{conv/acq} = 2000$ s
- $t_{acq/conv} = 2$ s.
Chapter 8

Simulation results

8.1 Simulation environment

8.1.1 Software implementation

The AOCS software is developed in the Matlab/Simulink environment. Each function corresponds to a Simulink library. These functions are translated into an automatically-generated C code for implementation in the flight software.

The AOCS software has a dedicated partition in the flight software. The flight software is executed at a frequency of 4 Hz. However, this frequency might be brought down to 1 Hz due to the latency time of some equipment.

8.1.2 OCEANS

The simulations are performed using Simulink and OCEANS (Outil de Conception et d’Études pour les Analyses SCAO, Conception and Studies Tool for AOCS Analyses), a Simulink environment developed by CNES. It essentially provides standardised tools for the planning, parameterisation and post-processing of test campaigns.
8.1.3 Environment models

Magnetic field

The magnetic field is modelled using the IGRF in its 2015 realisation, including derivatives. The field is modelled up to the 13th order.

Atmosphere

The atmosphere is modelled using the MSIS86\textsuperscript{1} model. A medium geomagnetic activity is assumed, with a geomagnetic index of 5. The solar flux is set to a medium value of $120 \times 10^{-22} \text{ W Hz}^{-1} \text{ m}^{-2}$.

Other models

The gravity, albedo and solar flux are also modelled, without any specific settings or choices being made.

8.1.4 Equipment models

Magnetometers

The magnetometer model includes the effect of misalignments, scaling factors, biases and noise.

However, misalignments, scaling factors and biases are corrected by a perfect sensor calibration in simulation cases, unless stated otherwise.

Magnetorquers

The magnetometer model includes the effect of misalignments and scaling factors.

\textsuperscript{1}Mass Spectrometer and Incoherent Scatter, in its 1986 version.
However, misalignments and scaling factors are corrected by a perfect actuator calibration in simulation cases, unless stated otherwise.

**Reaction wheels**

The reaction wheel model includes a response time of 0.5 s. The allowable torque is bounded. A speed increase or decrease is achieved with a ramp at maximal torque. No wheel noise is included.

### 8.2 Acquisition phase performance

#### 8.2.1 Rate reduction

The performance of the *acquisition* control phase in rate reduction is evaluated by switching to MNO after 18 000 s in MAS, which allowed it to converge. The initial conditions are gathered hereunder. Transition to the *converged* control phase is inhibited, and the active sub-mode is *standby*. The orbital period is 5677.0 s.

- Julian Day: 24827 00:00:00 (December 22, 2017)
- \( a_0 = 6 878 136.3 \) m
- \( e_0 = 0.00001 \)
- \( \omega_p = 0 \) rad
- \( \dot{i}_0 = 1.700 \) 057 4 rad
- \( \Omega_0 = 3.150 \) 537 2 rad
- \( M = 0 \) rad
- \( \Omega_{S/I} = \begin{bmatrix} 2.31 \times 10^{-2} & 2.71 \times 10^{-4} & -2.17 \times 10^{-4} \end{bmatrix} \) rad s\(^{-1} \)
- $\Omega_{\text{RWS}} = \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}$ rpm.

Figure 8.1 displays the 3-axis pointing error, and Figure 8.2 shows the 1-axis pointing errors. The 3-axis pointing error converges below $10^\circ$ is about 1600 s. Oscillations disappear after approximately 2000 s.

Figure 8.1: 3-axis pointing error for the acquisition control phase in rate reduction.

Figure 8.2: 1-axis pointing errors for the acquisition control phase in rate reduction.
Angular rates are displayed in Figure 8.3. The high initial rates are damped very quickly. Figure 8.4 focuses on the first 500 s of the simulation, showing a ramp on the x axis. This behaviour has to be put in relation with the ramp of the wheels, shown in Figure 8.5. Wheels 1 and 3 take about 8 s to reach their commanded speed, while wheels 2 and 4 take approximately 40 s. This latter time scale corresponds to the duration of the ramp visible in Figure 8.4. These ramps introduce a transient on the y and z axes, which is then damped by the closed-loop control.

The rapidity of the convergence is partly explained by the tuning of the embarked momentum $H_{emb}$ to $8 \times 10^{-4}$ N m s. In standby, the embarked momentum points in the $+x$-direction. At MAS exit, the angular momentum of the satellite is mainly along this axis, with a norm of approximately $1.0 \times 10^{-3}$ N m s. Turning the wheels on directly transfers 80% of that momentum to the wheel array, with 20% left to be damped by the magnetic closed-loop.

Figure 8.3: Angular rates for the acquisition control phase in rate reduction.
Figure 8.4: Angular rates for the acquisition control phase in rate reduction (detail).

Figure 8.5: Wheel speeds for the acquisition control phase in rate reduction.
8.2.2 Standby

The performance of the acquisition control phase in converged standby sub-mode is evaluated over 18 000 s. Transition to the converged control phase is inhibited. The initial conditions are gathered hereunder. They correspond to a converged acquisition phase, so that the control is converged for the duration of the simulation. The orbital period is 5677.0 s.

- $a_0 = 6 892 948.0$ m
- $e_0 = 0.0017$
- $\omega_p = 0.5960$ rad
- $i_0 = 1.699 908 3$ rad
- $\Omega_0 = 3.159 581 0$ rad
- $M = 0.00120$ rad.

3-axis and 1-axis pointing errors are respectively shown in Figure 8.6 and Figure 8.7. The 3-axis pointing error oscillates at the double of the orbital frequency $\omega_0$. So do the 1-axis pointing errors around the $x$ and $z$ axes, whereas the pointing error around the $y$ axis oscillates at the orbital frequency $\omega_0$. These oscillations are correlated with the rotation of the magnetic field, shown in Figure 8.8. For instance, around 7000 s, the magnetic field is aligned with the satellite $z$ axis, which coincides with a maximum of the off-pointing around the $z$ axis, which is not commandable.
Figure 8.6: 3-axis pointing error for the *acquisition* control phase in *standby*.

Figure 8.7: 1-axis pointing errors for the *acquisition* control phase in *standby*.
Figure 8.8: Magnetic field for the *acquisition* control phase in *standby*.

Figure 8.9 shows the evolution of the angular rates, and compares them to the target ones. The actual angular rates oscillate about the target ones, and maximum differences around each axis are well correlated with the maxima of the magnetic field along the same axis, which corresponds to the parts of the orbit when this axis is not commandable.

In addition, the angular rates shown in Figure 8.9 are noisy, which is linked to the actuation of the magnetorquers, shown in Figure 8.10. The noise in the actuation comes from the derivative term of the control, with noise being transmitted from the magnetic measurements. It can be checked in this figure that with this tuning, almost no saturation occurs over an orbit.
The performance of the acquisition control phase in standby is evaluated on the last orbit of this test case, that is between 12 309 s and 18 000 s. Key figures are gathered hereunder.

- Maximum 3-axis pointing error: 6.01°
• Maximum 1-axis pointing error around $x$: 3.29°
• Maximum 1-axis pointing error around $y$: 5.18°
• Maximum 1-axis pointing error around $z$: 3.83°
• Quadratic 3-axis pointing error: 4.35°
• Quadratic 1-axis pointing error around $x$: 1.65°
• Quadratic 1-axis pointing error around $y$: 3.40°
• Quadratic 1-axis pointing error around $z$: 2.15°.

8.2.3 Download

The performance of the acquisition control phase in converged download sub-mode is evaluated over 18 000 s. Transition to the converged control phase is inhibited. The initial conditions are gathered hereunder. They correspond to a converged acquisition phase. The control sub-mode is standby for most of the simulation. It is only switched to download when a visibility occurs. A manoeuvre is performed at transitions. The orbital period is 5677.0 s.

• $a_0 = 6\,892\,948.0$ m
• $e_0 = 0.0017$
• $\omega_p = 0.5864$ rad
• $i_0 = 1.699\,902\,5$ rad
• $\Omega_0 = 3.162\,965\,6$ rad
• $M = 0.001\,19$ rad.
This scenario features 2 station visibilities. The configuration of these visibilities are shown in Figure 8.11. The first visibility implies a high elevation, and lasts for 421 s. The second one is close to the horizon, and lasts for 255 s.

Figure 8.11: Ground station visibilities for first (a) and second (b) visibilities. The green cross indicates the location of the ground station. The full red line indicates the trajectory of the spacecraft on its orbit. The dashed red line is Eye-Sat’s trace, the projection of the trajectory on the surface of the Earth. The green circle indicates the visibility cap of the station, that is the area in which the trace of the satellite must be for visibility to occur.

For the first visibility, the target sub-mode is switched from standby to download at \( t = 1000 \) s. It is switched back to standby at \( t = 2500 \) s. For the second one, switches are commanded at \( t = 6800 \) s and \( t = 8000 \) s.

Figure 8.12 shows the 3-axis pointing error for the duration of the simulation, compared to the pointing error for the exact same time range, but in standby sub-mode the whole time, in order to evaluate the pointing error caused by the tracking of the ground station. For both visibilities, the pointing error is first lowered by the tracking motion, before increasing at the end of the sequence. Upon switching back to standby, an extra off-pointing is visible. It is dissipated in less than one orbit, which means it is possible to perform downloads on two consecutive orbits.
Figure 8.12: 3-axis pointing error for the *acquisition* control phase in *download*, compared to the pointing error in *standby*.

Figure 8.13 shows the pointing error for each axis. The frequency of oscillations is increased when tracking the station.

Figure 8.13: 1-axis pointing errors for the *acquisition* control phase in *download*.

Figure 8.14 shows the half-cone pointing error about \( z \), which indicates how well the ground antenna is pointed by Eye-Sat. It is better than 5\(^{\circ}\) for the first visibility, and better
than 8° for the second one.

Figure 8.14: Half-cone pointing error about z for the acquisition control phase in download.

It is worth noting that the peaks seen around 1000 s, 2500 s and 7000 s are not physical, but correspond to a punctual discontinuity in the guidance function when switching between standby and manoeuvre.

Figure 8.15 displays the angular rate and the target one. The bang-bang guidance in manoeuvre is clearly visible. Figure 8.16 shows the spin rate of the wheels. Allocations are respected, wheels are not saturated. It is worth noting that the high rotation rates when tracking the ground station are achieved by unloading the reaction wheels, whose speed decreases in the first half of the visibility, before retrieving their original rate.
Figure 8.15: Angular rates for the acquisition control phase in download.

Figure 8.16: Wheel speeds for the acquisition control phase in download.

These results confirm the capacity of the satellite to perform downloads on two consecutive orbits. In addition, it indicates that the acquisition control phase already fulfills the requirement of a pointing error inferior to $10^\circ$ half-cone about $z$ during download.
8.3 Converged phase performance

8.3.1 Standby

The scenario is the one described in subsection 8.2.2, for comparison of performance in acquisition and in converged control phase.

Figure 8.17 shows the 3-axis pointing error, and Figure 8.18 shows the 1-axis pointing errors. The converged control phase enhances the pointing on all axes.

Figure 8.17: 3-axis pointing error for the converged control phase in standby.
Figure 8.18: 1-axis pointing errors for the \textit{converged} control phase in \textit{standby}.

Figure 8.19 compares the magnetic moment of the magnetorquers in \textit{acquisition} and \textit{converged} control phases. As planned, there is almost always one axis saturated in the \textit{converged} phase, to ensure a maximal stiffness.

Figure 8.19: Magnetorquers magnetic moment for the \textit{converged} control phase in \textit{standby}.
The performance on the last orbit are gathered in Table 8.1, and compared to the ones obtained in acquisition. The gain is particularly high in terms of quadratic error, with a 31% drop for the 3-axis pointing error.

Both the acquisition and the converged control phases satisfy the 15° pointing requirement in standby.

### 8.3.2 Download

The scenario is the one described in subsection 8.2.3, for comparison of performance in acquisition and in converged control phase.

Figure 8.20 compares the 3-axis pointing error for the converged and acquisition control phases. Even though the pointing may punctually be better for the acquisition tuning, it is generally enhanced by the converged tuning, including during the visibility. Figure 8.21 confirms the gain axis by axis.

<table>
<thead>
<tr>
<th></th>
<th>Acquisition (°)</th>
<th>Converged (°)</th>
<th>Relative difference (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum 3-axis pointing error</td>
<td>6.01</td>
<td>4.76</td>
<td>-21</td>
</tr>
<tr>
<td>Maximum 1-axis pointing error around x</td>
<td>3.29</td>
<td>2.83</td>
<td>-14</td>
</tr>
<tr>
<td>Maximum 1-axis pointing error around y</td>
<td>5.18</td>
<td>3.35</td>
<td>-35</td>
</tr>
<tr>
<td>Maximum 1-axis pointing error around z</td>
<td>3.83</td>
<td>3.29</td>
<td>-14</td>
</tr>
<tr>
<td>Quadratic 3-axis pointing error</td>
<td>4.35</td>
<td>2.99</td>
<td>-31</td>
</tr>
<tr>
<td>Quadratic 1-axis pointing error around x</td>
<td>1.65</td>
<td>1.14</td>
<td>-31</td>
</tr>
<tr>
<td>Quadratic 1-axis pointing error around y</td>
<td>3.40</td>
<td>2.07</td>
<td>-39</td>
</tr>
<tr>
<td>Quadratic 1-axis pointing error around z</td>
<td>2.15</td>
<td>1.81</td>
<td>-16</td>
</tr>
</tbody>
</table>

Table 8.1: Performance of the converged control phase in standby and comparison with acquisition.
Figure 8.22 shows that the half-cone pointing precision is enhanced when tracking the ground station, with a 1 to $2^\circ$ improvement for both visibilities.

Figure 8.20: 3-axis pointing error for the converged control phase in download.

Figure 8.21: 1-axis pointing errors for the converged control phase in download.
The converged control phase improves the pointing performance in download. The requirement of a $10^\circ$ half-cone precision about $z$ for the control, which was already fulfilled by the acquisition tuning, is met by the converged tuning.

8.4 Additional results

8.4.1 Flight software frequency

In this subsection, the impact of lowering the frequency of the flight software from 4 to 1 Hz on the pointing performance is evaluated, both in standby and in download sub-modes, with the converged tuning.

Standby

The scenario is the one described in subsection 8.2.2, so that the results can be compared directly.

Figure 8.23 shows the 3-axis pointing error for both flight software frequencies. Surprisingly, the slower frequency produces slightly better performance, but the difference is
not really marked. This slight enhancement is confirmed in Figure 8.24 for the axis-by-axis off-pointing.

Figure 8.23: 3-axis pointing error for the converged control phase in standby, with a software frequency of 1 Hz.

Figure 8.24: 1-axis pointing errors for the converged control phase in standby, with a software frequency of 1 Hz.

The performance on the last orbit of this scenario is summarised in Table 8.2. The 1 Hz
control is better in every category, by a few per cent. This improvement may be linked to the actuation of the magnetorquers, which is less noisy.

<table>
<thead>
<tr>
<th></th>
<th>4 Hz (°)</th>
<th>1 Hz (°)</th>
<th>Relative difference (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum 3-axis pointing error</td>
<td>4.76</td>
<td>4.53</td>
<td>−4.8</td>
</tr>
<tr>
<td>Maximum 1-axis pointing error around $x$</td>
<td>2.83</td>
<td>2.71</td>
<td>−4.2</td>
</tr>
<tr>
<td>Maximum 1-axis pointing error around $y$</td>
<td>3.35</td>
<td>3.12</td>
<td>−6.9</td>
</tr>
<tr>
<td>Maximum 1-axis pointing error around $z$</td>
<td>3.29</td>
<td>3.24</td>
<td>−1.5</td>
</tr>
<tr>
<td>Quadratic 3-axis pointing error</td>
<td>2.99</td>
<td>2.92</td>
<td>−2.3</td>
</tr>
<tr>
<td>Quadratic 1-axis pointing error around $x$</td>
<td>1.14</td>
<td>1.07</td>
<td>−6.1</td>
</tr>
<tr>
<td>Quadratic 1-axis pointing error around $y$</td>
<td>2.07</td>
<td>2.02</td>
<td>−2.4</td>
</tr>
<tr>
<td>Quadratic 1-axis pointing error around $z$</td>
<td>1.81</td>
<td>1.81</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 8.2: Performance of the converged control phase in standby with a software frequency of 1 Hz, compared with 4 Hz.

Download

The scenario is the one described in subsection 8.2.3, so that the results can be compared directly.

Figure 8.25 shows the 3-axis pointing error, Figure 8.26 the axis-by-axis pointing errors, and Figure 8.27 the half-cone pointing error about $z$. This time, the plots are quite different for 1 Hz and for 4 Hz, probably due to the high dynamics of the tracking, to which a 4 Hz control can be more reactive.

A deterioration of 1 to 2° can be observed in Figure 8.27 due to the lower frequency for the first visibility, whereas the performance are similar for the second one. The major difference between the two frequencies is with the damping of the errors introduced during the tracking, which is much more efficient at 1 Hz and can be observed on all 3 plots. One
orbit after the second visibility, a peak in the pointing error can be seen at 4 Hz, whereas such peak is absent at 1 Hz.

Figure 8.25: 3-axis pointing error for the converged control phase in download, with a software frequency of 1 Hz.

Figure 8.26: 1-axis pointing errors for the converged control phase in download, with a software frequency of 1 Hz.
Figure 8.27: Half-cone pointing error about $z$ for the converged control phase in download, with a software frequency of 1 Hz.

Lowering the frequency of the flight software to 1 Hz does not endanger the pointing performance of the spacecraft, which are still compliant with the requirements formulated in section 5.2.

It was expected that according to the Nyquist-Shannon theorem, lowering the frequency would not endanger the controller whose cut-off frequency is located around $2 \times 10^{-3} \text{ rad s}^{-1}$. This is of no danger for the magnetic measurements either, since the maximum angular rate when exiting the MAS is $1.4^\circ \text{s}^{-1}$.

Switching to 1 Hz has also the advantage of easing the integration of the MNO in the flight software, since the MAS already runs at 1 Hz. The flight software can therefore have the same sequencing for both modes.

For these reasons, it has finally been chosen to switch the frequency from 4 to 1 Hz.
8.4.2 Magnetometer bias

In this subsection, the influence of a non-compensated magnetometer bias is studied. It is stated in subsection 5.6.4 that the magnetometer bias is annihilated mathematically, which will be checked with this example. The scenario is the one described in subsection 8.2.2. The active sub-mode is \textit{standby}, and the \textit{converged} gains are used.

A bias of $\pm 7 \mu T$ is introduced on each axis of the magnetometer, and no correction is performed.

Figures 8.28 and 8.29 show that the pointing error curves overlap perfectly after a convergence time. The initial difference is imputable to the convergence time of the second-order low-pass filter applied to the magnetic measurement. This confirms that the control is robust to magnetometer bias.

![Figure 8.28: 3-axis pointing error for the converged control phase in standby, with an uncorrected magnetometer bias.](image)

Figure 8.28: 3-axis pointing error for the \textit{converged} control phase in \textit{standby}, with an uncorrected magnetometer bias.
Figure 8.29: 1-axis pointing errors for the converged control phase in standby, with an uncorrected magnetometer bias.
Chapter 9

Conclusions

The in-flight usage of the attitude-related equipment was derived from test results in chapter 3, and served as a basis for the design of the attitude determination and control in chapter 5.

The preliminary tuning of the control laws proposed in chapter 7 was proven stable analytically in chapter 6 for the geocentric pointing, and numerically in chapter 8 for a variety of cases.

The performance estimated in chapter 8 cover the requirements regarding pointing convergence, pointing precision and agility, with consequent margins. Only half of the pointing error budget was allocated to the attitude control, with an equal allocation for other error sources, such as navigation, guidance, and equipment flaws. The robustness of the attitude control to lowering the flight software frequency or a non-corrected magnetometer bias was also demonstrated.

Future work will focus on performing a fine tuning of the attitude control laws proposed in this work, and a thorough performance assessment. In addition, the stellar-based part of
the normal mode is still to be designed, and will be used for the *shooting* mission phase. The transitions between the magnetic-based control presented in this work and the stellar-based control will be a key topic, since the stellar-based control will have to absorb the residual pointing error left by the magnetic-based control before pictures are taken. The performance in convergence will be of critical importance for the operability of the satellite.

This work provided me with a thorough technical introduction to attitude control and guidance, with both software and hardware aspects. Furthermore, being in a small team enabled many direct interactions with other subsystems, such as mechanics, flight software or thermal control, which helped me build a better understanding of the spacecraft system as a whole, and created a very dynamic and pleasant work environment. Working on Eye-Sat at CNES was a highly valuable experience: one could not hope for a better start into a satellite engineering career.
Appendix A

Ground station download

This chapter is dedicated to an in-depth analysis of the download mission phase, introduced in section 2.7. Section A.1 provides a general introduction of the download problem. Section A.2 introduces the tool used to study the download phase. Section A.3 gives insight on the station visibilities and their duration. Section A.4 derives kinematic and dynamical results from the geometrical study of the trajectory.

A.1 Position of the problem

During the shooting mission phase, the payload generates data which have to be downloaded to the ground for processing. This data amounts to a daily 14.3 Gbits, on average\textsuperscript{1}.

The payload telemetry is downloaded via the directional X-band antenna, located on the $-z$ face of Eye-Sat. The ground receiver is located in Toulouse, at ENAC\textsuperscript{2}. In the WGS84\textsuperscript{3}, it is located at 43.5639°N of latitude, 1.4816°E of longitude and an altitude of 204m.

The link between the X-band patch antenna and the ground antenna is unidirectional.

\textsuperscript{1}Value coming from undocumented project discussions.
\textsuperscript{2}École Nationale de l’Aviation Civile, the French National School of Civil Aviation.
\textsuperscript{3}World Geodetic System, in its 1984 realisation.
The uplink is achieved using the S-band antennae only, which communicate with a dedicated ground station.

The X-band patch antenna is directional, which means that its radiation pattern is highly anisotropic, with the signal being strongest along the axis of the antenna and depleting quickly when moving away from it. The strength of the signal directly impacts the data rate. Figure A.1 shows the achievable data rate as a function of the pointing error, along the worst-case direction\(^4\).

![Figure A.1: Achievable data rate as a function of the pointing error of the satellite.](image)

The link is also affected by the atmosphere, and obstacles that may lie between the satellite and the ground antenna. An elevation mask \(\mu_X\) is used to account for these effects: it is assumed there is no ground-satellite visibility if the elevation of the satellite \(\epsilon_X\) is inferior to \(\mu_X\). The elevation of the satellite is the angular distance from the satellite to the local horizon, as seen from the ground station. The elevation mask is set to 10°.

\(^4\)The radiation pattern is disturbed by the satellite itself, and in particular its solar panels.
The number and duration of the visibilities are fully determined by the on-orbit motion of the satellite, with a high impact from the altitude: a higher altitude provides more and longer visibilities. The detailed analysis is performed in section A.3.

Since a good pointing accuracy is needed to achieve high data rates, the satellite has to actively track the ground station. The guidance kinematics are therefore imposed by the relative motion of the satellite and the ground station. This translates into imposed dynamics, and it must be made sure that the actuators can cope with it. The detailed analysis is performed in section A.4.

A.2 Method

Analyses performed in sections A.3 and A.4 rely on a Matlab-based tool developed in the scope of this Master thesis work, in order to study the download mission phase.

This tool consists in a purely geometrical simulator of the satellite motion in the terrestrial frame $\mathcal{R}^5$, taking into account the rotation of the Earth. It is not an orbit propagator, and it does not include disturbances of any kind. The following assumptions are made:

- Earth is assumed to be a sphere of radius $R_\oplus$;
- the orbit is assumed circular, and is travelled at a constant velocity;
- the precession of the orbital plane under the effect of the $J_2$ term is disregarded. Its influence over one orbit is negligible, amounting to a drift of about $0.07^\circ$ in longitude every orbit.

The user inputs are the following:

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$^5$See section 2.3
$^6$See section 2.4.
• the semi-major axis $a_0$. The inclination $i_0$ is deduced, to ensure the orbit is sun-synchronous;
• the longitude of the ascending node at the beginning of the simulation;
• the number of orbits to simulate;
• the number of points per orbit;
• the position of the ground station in frame $R$;
• the elevation mask $\mu_X$.

The orbits are discretised into equally-spaced points. For each of them, the position of the satellite in frame $R$ is computed. The results include the following, for simulated point:

• the position of the satellite in frame $R$;
• the position of the satellite relative to the ground station in frame $R$;
• the position of the satellite as azimuth, elevation and distance, from the ground station. This defines how the ground stations sees the satellite;
• the on-ground trace of the satellite, i.e. the projection of the trajectory onto Earth’s surface, in frame $R$;
• the orbit normal $n_{\text{orb}}$ in frame $R$.

These outputs are post-processed to determine the presence and duration of visibilities. The tools also computes the visibility sector of the ground station, i.e. the part of the orbital sphere within which the satellite must lie for a visibility to happen. This visibility sector can alternatively be defined as the part of Earth’s surface within which the trace of the satellite
must lie.

Additionally, the download guidance\textsuperscript{7} quaternion is computed at each time step, and differentiated to yield the target angular rate $\Omega_T$. Finally, $\Omega_T$ is multiplied with the matrix of inertia $[I_{sat}]$ to yield the target angular momentum of the satellite. The value of $[I_{sat}]$ is given in section 4.1.

Finally, the tools returns a collection of plots, including the visualisation of the orbit, its trace, the ground stations and its visibility sector. Figure A.2 provides an example of such a plot, for altitudes of 500 km and 690 km. The green cross and the green circle respectively show the position of the ground station and its visibility circle projected on the ground, with $\mu_X = 10^\circ$. The red line and the red dashes respectively show the trajectory and the on-ground trace of the satellite. The visibility occurs when the dashed line enters the green circle.

Figure A.2 shows that the visibility sector is clearly larger at 690 km, as expected, meaning that more visibilities will occur, and that these visibilities will be longer.

\textsuperscript{7}See section 5.4.
Both trajectories presented in Figure A.2 cross the visibility centre at its centre, thereby providing the longest visibilities possible and making the satellite fly at the Zenith of the ground station. Visibilities can be much shorter, and bring the satellite to a much lower elevation, depending on how the trajectory crosses the visibility sector.

A.3 Visibilities

A.3.1 Visibility sector

The visibility sector is the portion of the orbital sphere that can be seen from the ground station, taking into consideration the elevation mask $\mu_X$. Under the assumption of a spherical Earth and a circular orbit, this visibility sector is a spherical cap, characterised by its half-cone angle to the centre of the Earth $\Gamma_X$, as shown in Figure A.3.

The satellite is seen by the ground station at an azimuth $\alpha_X^8$, an elevation $\epsilon_X$ and a distance $d_X$. The distance to the edge of the spherical cap corresponding to the visibility sector is denoted by $d_{\mu}$. Since the orbit is assumed circular, the radius of the orbit is $a_0$.

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8Not shown in Figure A.3
Figure A.3: Visibility from the ground antenna. The orbit is dashed, with the visibility sector in bold red.

The expression of $d_\mu$ is obtained by using the equation of the orbital circle, which yields a second-degree equation in $d_\mu$:

\[
(d_\mu \cos \mu_X)^2 + (R_\oplus + d_\mu \sin \mu_X)^2 = a_0^2.
\]  

(A.1)

\(d_\mu\) is obtained by taking the positive root of equation (A.1):

\[
d_\mu = -R_\oplus \sin \mu_X + \sqrt{(R_\oplus \sin \mu_X)^2 + a_0^2 - R_\oplus^2}.
\]  

(A.2)

Finally, $\Gamma_X$ can be determined from equation (A.3).

\[
\cos (\Gamma_X) = \frac{R_\oplus + \sin (\mu_X) d_\mu}{a_0}.
\]  

(A.3)

The maximum linear distance travelled on the spherical visibility cap is denoted by $\odot_X$.

\[
\odot_X = 2\Gamma_X a_0.
\]  

(A.4)
Neglecting the component of the velocity of the satellite in frame $\mathcal{R}$ which is due to Earth’s rotation\(^9\), an approximation of the longest possible visibility is provided in Table A.1, along with the half-cone angle of the spherical cap and the longest linear distance through it.

<table>
<thead>
<tr>
<th>$h_0$ (km)</th>
<th>$\Gamma_X$ (°)</th>
<th>$\Omega_X$ (km)</th>
<th>Maximum visibility (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>500</td>
<td>14.05</td>
<td>3372.4</td>
<td>443</td>
</tr>
<tr>
<td>690</td>
<td>17.29</td>
<td>4266.7</td>
<td>568</td>
</tr>
</tbody>
</table>

Table A.1: Dimensions of the visibility sector.

The dimension of the visibility sector, both the half-cone angle and the maximum linear distance, is rather accurate. The assumption on the velocity leading to the estimate of the duration of the visibility is rough, but provides a rule-of-thumb value, which will be confirmed in subsection A.3.2.

### A.3.2 Visibility distribution

This subsection focuses on the occurrence and duration of visibilities. It is assumed that the statistical distribution of the orbits in terms of latitude at the ascending node is uniform. This is the case in practice since the altitude of the sun-synchronous orbit was not chosen to ensure the reproducibility of the orbits over time.

The tool described in section A.2 was run for a collection of single orbits, with their longitude at ascending node spaced by $0.1^\circ$, both at 500 km and 690 km. The total visibility time over each orbit is shown in Figure A.4.

\(^9\)It is approximately $363.4 \text{ m s}^{-1}$ (respectively $373.5 \text{ m s}^{-1}$) at the latitude of the X-band ground station, compared to $7.61 \times 10^3 \text{ m s}^{-1}$ (respectively $7.51 \times 10^3 \text{ m s}^{-1}$) for the component due to the on-orbit motion at an altitude of 500 km (respectively 690 km), which amount to less than 5% error.
Figure A.4: Occurrence and duration of visibilities.

Two longitude sectors provide visibility at an altitude of 500 km, one from $-7.6^\circ$ to $33.2^\circ$, corresponding to the crossing of the visibility sector from the equator towards the North Pole, and one ranging from $161.7^\circ$ to $-157.6^\circ$, when descending from the North Pole towards the equator. The first case corresponds to flying over the station in the morning, the second one in the evening, due to the orbit being sun-synchronous 6-18. The sum of the two longitude sectors is $81.6^\circ$ of longitude, meaning that visibility occurs for $22.7\%$ of orbits at an altitude of 500 km.

At an altitude of 690 km, the two longitude sectors range from $-12.0^\circ$ to $38.5^\circ$, and from $156.8^\circ$ to $-152.7^\circ$, respectively, totalling $100.9^\circ$ of longitude. Visibilities occur for $28.0\%$ of orbits.

The maximum duration of visibilities is 444.7 s at 500 km, versus 568.4 s at 690 km, which corroborates the estimates given in Table A.1 with a very good accuracy.
Short visibilities are rather scarce. The mean duration of visibilities is 349.1 s at 500 km, with a median at 384.8 s. At 690 km, the mean is 445.4 s and the median is 491.2 s.

A.3.3 Daily visibility

The satellite completes 15.2 orbits a day at an altitude of 500 km, and 14.6 at 690 km. Taking into consideration the precession of the orbital plane, the ascending node drifts westwards by 23.59° of longitude at every orbit at 500 km, 24.57° at 690 km.

Since the longitude slots which give visibilities are 40.8° wide each at 500 km, and 50.5° each at 690 km, it is guaranteed that at least 2 visibilities occur every day at 500 km, and even 4 at 690 km\(^{10}\).

Figure A.5 shows the total visibility time over 15 consecutive orbits, indexed by the longitude at ascending node of the first orbit. Visibilities lasting less than 120 s are disregarded.

\(^{10}\)As Pascal GUELFI, my mathematics teacher in Mathématiques Spéciales always said: "You cannot walk over a 40.8°-long tile by doing 23.59°-long steps". With a 50.5°-long tile and 24.57°-long, you even step in twice!
The longitude is discretised in steps of 0.1°. Considering 15 orbits is conservative for the sizing-case, at 500 km.

At 500 km, 2 to 4 acceptable visibilities occur daily, with a mean duration of 1178 s. The worst-case day provides 841.5 s of visibility.

At 690 km, 3 to 5 acceptable visibilities occur daily, with a mean duration of 1865 s. The worst-case day provides 1547.5 s of visibility.

The altitude of 500 km is clearly the scaling case. Considering that 14.3 Gbits of data are collected every day, downloading them in 1178 s requires a data rate of 12.5 Mbits/s. According to Figure A.1, this data rate can be achieved with a pointing accuracy better than 14°. As stated in section 5.2, 4° are provisioned for the navigation and guidance errors, so the control error shall be less than 10° half-cone. If this specification is met, it is therefore possible to download the data generated by the payload at 500 km, on average. On days when the visibility is not sufficient, it must therefore be possible to store the data on-board, to download it the following day.

Finally, it is worth noting that downloading the data requires to perform up to 4 download sequences a day, which means 2 consecutive orbits can include download phases: this has to be taken into consideration for the power budget, since the download phase has a high power consumption, and the solar arrays have a poor illumination.

A.4 Kinematic and dynamic analysis

The maximum rotation rate during the download phase is reached when Eye-Sat flies right above the ground station, i.e. when crossing the centre of the visibility sector, as shown in
Figure A.2. Neglecting the component of the velocity due to Earth’s rotation and assuming a plane movement, the angular rate of Eye-Sat at the zenith of the ground station is 
\[ \omega_0 \left( R_\oplus + h_0 \right) / h_0, \] 
which amounts to \( 1.52 \times 10^{-2} \text{ rad s}^{-1} \) at 500 km, that is \( 0.87^\circ \text{ s}^{-1} \).

The kinematics of the satellite are constrained by the pointing of the X-band patch antenna, which must be directed towards the ground station. In addition, the solar panels have to be pointed as close to \(-n_{\text{orb}}\) as the first constraint allows them to, as stated in subsection 5.4.2. These constraints induce 3-axis kinematics on Eye-Sat.

The guidance profile is enforced by the reaction wheels in open-loop, as designed in section 5.5. Kinematics about the 3 satellite axes are pondered by the inertias to yield angular momenta, and it must be made sure that the reaction wheels can cope with such demand.

In the simple case described above, the rotation is about Eye-Sat’s \( x \)-axis, which bears the smallest inertia, and amounts to \( 6.85 \times 10^{-4} \text{ N m s} \). The angular momentum commanded to the wheels is established in subsections 5.6.6 and 5.6.7, and reads:

\[ H_{\text{RWS}} = H_{\text{emb}} n_{\text{orb, } T} - [I_{\text{sat}}] \Omega_T. \]  

In the Zenithal case described above, the \( x \) axis is aligned to \( n_{\text{orb}} \), which makes \( \Omega_T \) collinear to \( n_{\text{orb}} \) and of same sign as well. Therefore, the rotation about \( x \) is simply a transfer of the embarked momentum \( H_{\text{emb}} = 8 \times 10^{-4} \text{ N m s} \) to the satellite body, and the capacity of the wheels is not exceeded.

What happens when considering an arbitrary visibility? A transverse rate component is generally present, with a higher inertia, and it is unclear whether the wheel capacity is sufficient. The tool introduced in section A.2 is used to sweep all possible visibilities, and check the maximum wheel rotation rate. The analysis is performed on the same set of orbits as in section A.3, both at 500 km and 690 km.
Figure A.6: Maximum angular momentum of the wheel array in *download*.

The maximum norm of the angular momentum of the wheel array when following the guidance profile over one orbit is shown in Figure A.6. It must be kept in mind that visibilities only happen around 0° and 180° of longitude at the ascending node, and that the minima around −90° and 90° do not correspond to orbits with visibilities. The dashed line corresponds to the wheel capacity in the worst direction, as established in subsection 3.4.1. It is clear that the wheel capacity is not reached at 690 km, but it seems that this limit may be exceeded at 500 km, depending on the direction of the angular momentum in its envelope.

The curve displays a peculiar pattern around the longitude giving a Zenithal *download*, with two peaks flanking a local minimum. At 500 km, the peaks correspond to 2.3° and 21.7° in longitude, with the minimum at 13.0°. Corresponding trajectories are shown in Figure A.7.

The local minimum corresponds to the zenithal visibility, as expected, for there is no rate on transverse axes at the zenith. When moving away in longitude from this zenithal visibility, transverse rates appear, and produce an increase of the momentum, up until the side peaks. Past them, the rates are reduced due to the distance to the station, which balances the transfer of the rate to the transverse axes.
Figure A.7: Trajectories of extremal wheel angular momentum at 500 km, at longitudes 2.3° (a), 13.0° (b) and 21.7° (c).
The rate of the fastest wheel over one orbit following the *download* guidance profile is shown in Figure A.8. The reader is reminded that the wheel speed is always superior to 500 rpm, by construction, and that the maximum rate must not exceed 10000 rpm. The analysis confirms that this limit is not reached when following the *download* guidance\(^\text{11}\).

\(^{11}\)Lower altitudes should however not be attempted!
Bibliography


