Characterisation of Time-dependent Statistical Failure of Fibre Networks
Applications for Light-weight Structural Composites

Amanda Mattsson

Main supervisor: Tetsu Uesaka
Co-supervisor: Christina Dahlström

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Faculty of Science, Technology and Media
Mid Sweden University, SE-851 70 Sundsvall, Sweden
Phone: +46 (0)10 142 80 00
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ABSTRACT

The future of a sustainable society requires that materials not only be renewable, but also leave as small a carbon footprint in the environment as possible. One such product is light-weight composite material for transportation packages. Cellulose fibres have been and will continue to be ideal for this purpose.

The strength design of light-weight composites is becoming increasingly important. The challenge is to neither over- nor under-design, but instead to target the right strength under realistic loading conditions. The question then is: What is right strength? Under realistic loading conditions (e.g., fatigue, random loading, and creep), materials fail differently from what one expects from tests of static strength: materials often fail at much lower stresses than are measured in these tests, the failure is time-dependent, and time to failure is highly variable.

Therefore, to answer the above question, we have set up the following objectives: (1) theoretically formulate time-dependent statistical failure (TSF), and examine the validity of the model; (2) define material parameters describing the multi-faceted strength characteristics based on this formulation; (3) develop an experimental method to determine the material parameters; (4) investigate the impacts of fibre properties and network structures; and finally (5) characterise containerboard (the fibre material used in corrugated boxes) samples in terms of the new material parameters. The results for these five objectives are presented below, one by one.

(1) A general formulation of TSF, originally proposed by Coleman [1] for fibre failures, has been used. We have found that this model is indeed valid, even at the fibre network level, with only two restrictions: the existence of a lower bound on weakest-link scaling and an approximate nature of the Weibull distribution.

(2) Accordingly, we have defined three material parameters that characterise different aspects of material strength: short-term strength, durability/brittleness, and reliability. We call these parameters the new strength metrics.

(3) Although the newly defined material parameters are
Abstract

most comprehensive, it takes up to several months to determine them by using creep tests. We have developed a new method, using constant loading rate (CLR) tests, that not only gives values comparable to those from creep tests, but also requires only about one day, allowing a drastic reduction in the testing time.

(4) Monte-Carlo simulations of lattice networks have been performed to determine the basic relationships between fibre properties and network failures. The brittleness of an individual fibre (or a breaking element) influenced both brittleness and reliability of the fibre network, the higher the brittleness, the lower the reliability. Reliability, on the other hand, exhibited more intricate relationships with fibre properties and network structures. Several important analytical relationships have been derived.

(5) Finally, using the CLR tests, we have characterised commercial containerboards in terms of the new strength metrics. Containerboard, as a cellulose fibre network, is quite comparable to typical stiff polymer-based fibre composites (e.g., glass fibres and aramid fibres). However, the reliability and durability/brittleness of containerboard varied considerably within the operating windows, suggesting ample opportunities to fine-tune these properties even using current papermaking practices.

The fact that the multi-faceted nature of strength can be expressed by three parameters is remarkable, and the implications are profound for how materials are designed and new materials developed. It is the author’s hope that this thesis will be of some use when it comes to redefining materials for a sustainable society, particularly the renewable alternative – cellulose fibres.
SAMMANFATTNING


För att kunna svara på den ovanstående frågan har vi satt upp följande målsättningar: (1) teoretiskt formulera tidsberoende, statistiska brott och utvärdera modellens validitet, (2) definiera materialparametrar som beskriver de mångfacetterade styrkeegenskaper som är baserade på formuleringen, (3) utveckla en experimentell metod för att bestämma materialparametrarna, (4) undersöka effekterna av fiberegnskaper och nätwerksstrukturer, och slutligen (5) karaktärisera prover av containerboard (det papper som används för bland annat wellpapå) baserat på de nya materialparametrarna. Resultaten för dessa målsättningar presenteras nedan, en efter en.


(2) Vi har med hjälp av ovanstående formulering definierat tre materialparametrar som karaktäriserar de olika aspekterna på materialets styrka: hållfasthet vid snabb belastning, uthål-
lighet/sprödhet och tillförlitlighet. Dessa parametrar utgör de nya mätten på styrka.

(3) Det tar upp till flera månader att bestämma dessa materialparametrar genom att utföra krypprov. Vi har utvecklat en metod med konstant belastningshastighet (CLR) som inte bara ger jämförbara resultat med krypproven, utan även drastiskt minskar testtiden till runt en dag.


Det faktum att den mångfacetterade karaktären av styrka kan uttryckas med tre materialparametrar är anmärkningsvärt. Det innebär att det är möjligt att påverka det sätt som materialet designas och hur nya material kan utvecklas. Förhoppningen är att denna avhandling kommer att vara till nytta för att omdefiniera material i framtiden för ett hållbart samhälle, särskilt det förnyelsebara alternativet – cellulosafibrer.
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LIST OF PAPERS

This thesis is based on the following papers, herein referred to by their Roman numerals:

**Paper I**

*Time-dependent statistical failure of fiber networks*
Amanda Mattsson and Tetsu Uesaka

**Paper II**

*Time-dependent breakdown of fiber networks: Uncertainty of lifetime*
Amanda Mattsson and Tetsu Uesaka
*Physical Review E*, 95(5), 2017, 053005-1–053005-10

**Paper III**

*Characterisation of time-dependent, statistical failure of cellulose fibre networks*
Amanda Mattsson and Tetsu Uesaka
*Cellulose*, 25(5), 2018, 2817–2828

**Paper IV**

*New strength metrics for containerboards: Influences of basic papermaking factors*
Amanda Mattsson and Tetsu Uesaka
*Accepted for publication in Nordic Pulp & Paper Research Journal*, 2018

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Related papers

The following publications by the author are not included in this thesis.

**Time-dependent, statistical failure of paperboard in compression**
Amanda Mattsson and Tetsu Uesaka
CONTRIBUTIONS TO THE PAPERS

The author’s contributions to the papers included in this thesis are as follows:

**Paper I**
Principal author: simulations, analysis, manuscript preparation

**Paper II**
Principal author: simulations, analysis, manuscript preparation

**Paper III**
Principal author: experimental work, analysis, manuscript preparation

**Paper IV**
Principal author: experimental work, analysis, manuscript preparation
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1.1 General remarks

Currently, transportation fuel is one of the biggest contributors to carbon dioxide emissions. Therefore, improving fuel efficiency and simply reducing the weight of anything involved in transportation are two of the most crucial targets in research and development. Light-weight solutions are necessary for both transportation vehicles and the packaging used for transporting goods.

Light-weight composites have been developed over the years to enhance performance at a given weight. The first generation of light-weight composites were fibre-reinforced plastics that enabled significant weight reduction while maintaining strength comparable to traditional metals. More recent light-weight composites further pushed the limits by introducing honey-comb and lattice structures to achieve extremely light-weights while maintaining stiffness and strength. The underlying ideas for these structures are inspired by the structures used in nature; Fig. 1.1 shows an example of the extremely light structure of a deep-sea sponge. This is known as a glass sponge, which has a unique lattice structure with diagonal supports, making it very light and stiff. The lattice’s load-carrying elements possess a graded structure in its radial direction, giving an unusually high fracture toughness (e.g., [2]). Today, these types of light-weight structures have a large number of applications in automobiles, bullet trains, passenger- and cargo aircrafts, and military-purpose drones.

Perhaps the most widely used light-weight composites are corrugated boards and boxes made of cellulose fibres, for example, corrugated containerboards (Fig. 1.2a), boards (Fig. 1.2b), and box (Fig. 1.2c). These are considered the first generation of light-weight structural composites. These materials established their dominant position as transport packaging many years ago because of their high performance, light-weight, and low cost, and will continue to be used as Internet commerce continues to expand.
Chapter 1. Introduction

Figure 1.1: A deep-sea sponge with an extremely light structure. These are known as glass sponges that create structures with impressive properties. [3]

Although the use of light-weight composites has been expanding, a new design challenge has also emerged. For example, the demand for light-weight severely limits the use of a "relaxed" safety factor, i.e., arbitrarily raising the factor to compensate for uncertainties related to end use and strength design. In other words, the material must be designed to have the right strength under realistic loading conditions, and it should be neither over- nor under-designed. This poses an important question regarding the design of this promising class of materials: What is the right strength under realistic conditions?

1.2 Problem definition

Under realistic loading conditions (e.g., impulse loading, vibration, fatigue, random loading, and creep), materials fail very differently from what one expects from static strength, which is routinely measured in testing labs. For example, under creep and fatigue conditions, materials are known to fail at much lower stresses than indicated by their static strength. At the same time, the failure is time-dependent,
1.3 Background

Material strength is traditionally evaluated in terms of the critical stress at which the material fails under monotonically increasing

meaning that the time to failure is highly variable (i.e., exhibiting great statistical variation), as discussed later. The fundamental question therefore concerns how to systematically attack such complex problems of time-dependent statistical failure (TSF) under varying loading conditions.

1.3 Background

Figure 1.2: (a) Containerboards of cellulose fibres in corrugated shapes, (b) corrugated boards of cellulose fibres, and (c) a corrugated box of cellulose fibres.
displacement (or load) conditions, and this is called the material’s static strength. Common static strength tests for cellulose materials used in packaging applications are short-span compression tests (SCTs), edgewise compression tests (ECTs), and box compression tests (BCTs).

However, it is known that materials often fail even at much lower stress levels if they are subjected to stresses for prolonged periods, as in creep and fatigue conditions. It has also been shown that failures do not necessarily happen at peak loads, as illustrated by the loading history shown in Fig. 1.3. Material strength is generally time-dependent or loading-history dependent. In this regard, an interesting question is: Does stronger material necessarily perform better in end uses – i.e., is stronger necessarily better? These questions may sound absurd to those pursuing strength increase as a primary target in new material development or product quality improvement.

Figure 1.3: An example of a loading history in which the failure does not happen when it is expected to.

Figure 1.4 shows an example of creep failures of corrugated boxes in compression, as tested by Nyman [4]. (The figure is based on Nyman’s PhD work, and the data were received as his courtesy.) Prior to creep tests, ordinary compression tests (box compression test, BCT) were performed for Box A and Box B, both with the same size and configuration. BCT results showed that Box B was stronger (2.92 kN) than Box A (2.44 kN). However, creep strength, as measured by time to collapse (i.e., lifetime), showed the opposite, that Box A had a longer lifetime than did Box B (Fig. 1.4a). In other words, Box A was
1.3. Background

more durable than Box B, even though its "ordinary" strength was lower. This poses a question of the relevance of typical static strength to longer-term strength properties.

Another important aspect of long-term performance is variability. Each data point in Fig. 1.4a is, in fact, an average of 10 measurements of creep lifetime. The scatter is plotted at each load level in Fig. 1.4b. The variation in lifetimes among the boxes was extremely large, and the coefficient of variation (COV, i.e., standard deviation divided by the mean value) ranged from 34 % to 77 % at the different load levels, although the measurements were made under nominally constant environmental conditions. Though the large scatter of creep lifetime data was recognised even in the early literature on the creep failure of container boxes [5–9], the area has remained largely unexamined. This is because the strong loading-history dependence makes testing difficult and time consuming, and the data are so scattered that results are often confusing and difficult to analyse.

From the above example, it would be easy to conclude that the scatters are due to defects introduced in the converting process, rather than to the materials (i.e., containerboards) themselves. However, when examining the materials, we found that containerboards do exhibit huge variation in lifetimes, as illustrated by examples of typical creep failure curves in Fig. 1.5a and by the corresponding frequency
distribution in Figure 1.5b. The coefficient of variation (COV) exceeded 100%, and this large variation in creep lifetime is rather universal, having also been found in many other materials [10–12]. The static strength values, on the other hand, only showed a COV of 4–10% (Figure 1.5b), even though the same containerboard was tested as in the creep failure tests. This big difference between the lifetime variability and static strength variability has been observed by many other researchers [10–16].

The great uncertainty concerning lifetime poses an enormous challenge to both design practice and research. First, it is a source of over-design (i.e., implementing an overly high safety factor) of boxes, which are subjected to creep or long-term loading. Second, the great variation and highly skewed distribution towards a shorter lifetime makes the average lifetime almost meaningless. Third, the extremely long tail of the lifetime distribution makes creep failure tests difficult, because, even though the mean lifetime is a few days, data indicate that longer lifetimes could easily exceed one month, meaning that testing would require a prohibitively long time. Therefore, the volume of research into time-dependent statistical failure (TSF) is disproportionately small, despite the fact that this is the most common form of failure.

![Creep Compression Curves and Failures of a Containerboard](image1.png)

**Figure 1.5**: (a) Typical creep compression curves and failures of a containerboard; (b) the frequency distribution of its lifetime and static strength. The lifetime and strength values are normalised by their means.

TSF has traditionally been dealt with in the areas of fatigue strength and creep strength (e.g., [17] and [18]) for many years. Be-
1.3. Background

cause of the complexity of its underlying mechanisms, approaches to time-dependent failure are largely empirical and phenomenological, though several important empirical laws have been found (e.g., [19–21]). These laws are widely used in organising fatigue and creep strength data and in providing insights for the later development of damage evolution models (e.g., [22]). The basic limitation of these approaches is that they are mainly deterministic, so there is no systematic treatment of the huge scatter inherent in fatigue and creep strength. In the area of cellulosic materials, the subject has been actively investigated as the creep failure of container boxes under constant or cyclic humidity conditions [5–9, 23, 24]. In particular, since cyclic humidity conditions accelerate creep deformation, as compared with deformation under equilibrium humidity conditions, special attention is paid to the area of accelerated creep of cellulosic materials.

Although the enormous scatter of creep lifetime data was recognised in the early literature, the systematic treatment of TSF is still in its infancy in the case of cellulosic materials (for a review, see, e.g., [25]).

The first rigorous treatment of TSF is probably Coleman’s model [1, 26, 27]. This model considers the evolution of damage given an arbitrary loading history including monotonic loading, creep, and fatigue, and incorporates a probabilistic failure criterion. It is therefore ideal for describing TSF. It is based on the three postulates: (1) weakest-link scaling (WLS), (2) an algebraic form of the probabilistic failure criterion, and (3) damage evolution rules (in exponential or power-law forms). Although Coleman’s model was intended for an individual fibre (i.e., a chain of breakable elements), Phoenix et al. extended Coleman’s approach to a system of fibre bundles (i.e., a parallel arrangement of breakable elements) as a model for uniaxially reinforced fibre-polymer composites [10, 28–31]. Later, Christensen (e.g., [32, 33]) considered the time-dependent extension of a single crack, and obtained the lifetime distribution (for creep) by assuming that (1) the crack growth rate is a power-law function of stress, (2) the crack grows unsteadily at a critical stress intensity, and (3) the strength measured in a far field follows the Weibull distribution. Based on this, they arrived at the following relationship between static strength variability and creep lifetime variability, i.e., the Weibull exponents of
Chapter 1. Introduction

lifetime \( (m_{\text{creep}}) \) and strength \( (m_{\text{strength}}) \) \( (\beta \) is also called the reliability parameter, as will be described shortly): 

\[
m_{\text{creep}} = \beta \quad \text{and} \quad m_{\text{strength}} = \beta (\rho + i) \quad (1.1)
\]

where \( \rho \) is a stress-related exponent for damage (or crack) growth, \( i = 1 \) for Coleman’s model, and \( i = 0 \) for Christensen’s model. (For brittle and quasi-brittle cases, such as \( \rho >> 1 \), both approaches essentially give the same result.) According to this relationship, the seemingly small variability (i.e., high Weibull exponent) of static strength is due to the high stress-sensitivity of damage growth (i.e., high \( \rho \)) in brittle or quasi brittle materials. Although the approaches taken by Coleman and Christensen are phenomenological, they have provided a powerful tool for organising hopelessly scattered creep lifetime data. In addition, many of the postulates included in their formulations are also touching upon some of the key topics of statistical physics: disorders and damage growth, size scaling, the validity of the Weibull distribution, and the onset of avalanche failure.

1.4 Scope

To summarise the literature, Coleman’s formulation was found to be the most attractive for applying to a wide variety of failures under realistic loading conditions. Its only question is that it was defined for an individual fibre, and has not been validated for applications to fibre network systems. Second, the literature contains no practical test to determine the characteristics of time-dependent statistical failure (TSF). Current creep testing is too time consuming to perform, so it is imperative to develop an experimental method that is practical while also allowing the theoretically sound characterisation of materials. Third, in the area of TSF, there is no explicit relationship between fibre properties, network structure, and network properties in both theoretical and experimental senses. This is also important when it comes to applications using cellulosic fibres in light-weight structures.

Within this scope, we have set the following research objectives:

1. Theoretically formulate TSF and examine the validity of the model.
1.4. Scope

2. Define material parameters that describe the multi-faceted strength characteristics based on the formulation.

3. Develop an experimental method to determine the material parameters.

4. Investigate the impacts of fibre properties and network structures.

5. Characterise containerboard samples in terms of the new material parameters.
2.1 Coleman’s formulation of time-dependent statistical failure

As mentioned earlier, Coleman’s formulation of time-dependent statistical failure (TSF) is based on three postulates. The first one, weakest-link scaling (WLS), is generally stated as:

\[ 1 - F_l(t) = [1 - F_s(t)]^M \]  \hspace{1cm} (2.1)

where \( F_l(t) \) and \( F_s(t) \) are the cumulative distribution functions of lifetime, \( t \), for an element (fibre) of size \( l \) and a system (network) of size \( s \) respectively, and \( M = l/s \). The second postulate is a breakdown rule that defines how damage, \( \Omega(t) \), evolves as a function of time and force. Taking a power law form, we have:

\[ \frac{d\Omega}{dt} = \kappa(f(t)) = cf(t)^\rho \]  \hspace{1cm} (2.2)

where \( f(t) \) is the force (or stress) history, and \( c \) and \( \rho \) are constants; \( \rho \) determines the force sensitivity of damage growth. The third postulate is a probabilistic failure criterion:

\[ F_s(t) = \Psi(\Omega(t)) \]  \hspace{1cm} (2.3)

where the function \( \Psi(\Omega) \) is a single-valued, positive, and monotonically increasing function of \( \Omega \). The function may take any form as long as it satisfies this basic condition, but Coleman chose a power-law form with the exponent \( \beta \).

Using these three postulates, one obtains the following expression for the cumulative distribution function of lifetime, \( t_B \):

\[ F(t_B) = 1 - \exp \left\{ - \left[ \int_{0}^{t_B} \left( \frac{f(t)}{S_c} \right)^\rho \, dt \right]^\beta \right\} \]  \hspace{1cm} (2.4)
where $f(t)$ is load at time $t$, and $S_c$, $\rho$, and $\beta$ are material parameters, as will be described shortly. (An extensive explanation of these postulates, as well as the derivations leading to this cumulative distribution function of lifetime, can be found in Paper I [34].) This formula was originally derived by Coleman [1], later generalised by Phoenix [10], and also re-derived by Curtin and Sher based on a damage-evolution model [22]. The most important feature of this model is that it can take into account any loading history, such as creep, constant loading rate, fatigue, and random loading, and determine lifetime distributions for each history.

As will be shown later, the applicability of this formula was tested for two different loading cases: creep loading and constant loading rate. The distributions (derived by Coleman) for these two special cases are presented below. More detailed descriptions can be found in Paper III [35].

For creep loading, i.e., $f(t) = f_0 = constant$, the lifetime distribution is given by:

$$F(t_B) = 1 - \exp\left\{-\left(\frac{f_0}{S_c}\right)^{\rho\beta} t_B^\beta\right\}$$

(2.5)

From the above expression, one can determine various statistical quantities which are practically important. One such parameter is the median, i.e., $F(t_{B,\text{median}}) = 1/2$:

$$t_{B,\text{median}} = (\ln 2)^{1/\beta}\left(\frac{f_0}{S_c}\right)^{-\rho}$$

(2.6)

Although the mean of lifetime is often reported, it is difficult to accurately determine the mean from the experimental data of lifetime, because of the extremely long tail of the lifetime distributions. The median, on the other hand, is an easy parameter to determine for a modestly large number of tests (e.g., more than 10).

For a constant loading rate (CLR) test, $f(t) = \alpha t$, where $\alpha$ is the loading rate, we can determine the distribution of strength, $f_B (= \alpha t_B)$, by:
2.2. Physical meanings of the material parameters $S_c$, $\rho$, and $\beta$

\[
G(f_B) = 1 - \exp \left\{ -\left( \frac{1}{(\rho + 1)S^\rho} \right)^\beta f_B^{\beta(\rho+1)} \right\} \tag{2.7}
\]

In the same way, the mean strength, $f_{B,\text{mean}}$, is given by:

\[
f_{B,\text{mean}} = \left\{ (\rho + 1)S^\rho \right\}^{\frac{1}{\rho + 1}} \Gamma \left( 1 + \frac{1}{\beta(\rho + 1)} \right) \tag{2.8}
\]

where $\Gamma$ is a gamma function.

2.2 Physical meanings of the material parameters $S_c$, $\rho$, and $\beta$

As seen in these expressions (Eqs. 2.5 and 2.7), the statistical failure responses in two different loading cases or two different timescales (i.e., creep and static loading tests) are completely determined by the material parameters, i.e., $S_c$, $\rho$, and $\beta$. These parameters have special meanings, as described below.

The parameter $S_c$, characteristic strength, essentially represents short-term strength, i.e., it is the creep load at which approximately 63% of the samples fail within one time unit (second) of creep. (See Eq. 2.5; note that $1 - \exp(1) = 0.63$.)

The parameter $\rho$ has a dual meaning: brittleness and durability. In Coleman’s formulation [1], $\rho$ is defined in the damage evolution law:

\[
\frac{d\Omega}{dt} \propto cf(t)^\rho \tag{2.9}
\]

where $\Omega$ is a damage parameter. At a given force $f(t)$, the higher the $\rho$, the higher the rate of damage growth, i.e., the network becomes more brittle. Another meaning comes from the molecular interpretation given by Phoenix [14]:

\[
\rho = \frac{U_0}{kT} \tag{2.10}
\]

where $U_0$ is a potential barrier to the thermal fluctuations of atoms and $kT$ is its thermal energy, with $k$ being the Boltzmann constant and $T$ the absolute temperature. As the potential barrier becomes
comparatively higher than the thermal energy (i.e., the higher the $\rho$), fewer atoms cross the barrier, so the system becomes more stable (or more durable). Conversely, if the temperature increases, then $\rho$ decreases and the material becomes more ductile (or less brittle).

The parameter $\beta$ is the Weibull exponent of lifetime distribution (Eq. 2.5): a higher value means less variation of lifetime, i.e., less uncertainty and thus more reliable. From the material characterisation viewpoint, it is therefore the material property representing the reliability of long-term strength (i.e., creep lifetime).

Among these parameters, characteristic strength, $S_c$, is the closest to a routinely measured property (static strength). However, both durability/brittleness, $\rho$, and reliability, $\beta$, aspects, are largely overlooked when characterising the strength properties of cellulosic materials.

Once all these material parameters are determined and a loading history is given, then one can calculate the probability that the fibre or network fails up to a certain time, $t_B$, from Eq. 2.4.
3.1 Numerical approach: Monte-Carlo simulations of statistical failure of fibre networks
(Papers I and II)

3.1.1 Model geometry and boundary conditions

A two-dimensional network with a central-force triangular lattice has been used in validating Coleman’s formulation and investigating the impacts of fibre properties and network structures on the material parameters (Fig. 3.1). This model has a long history in the literature in the context of statistical failure (e.g., [36–38]). The particular advantage of this model is that, because of its simplicity, one can perform a large number of failure simulations time efficiently. Nevertheless, even though its geometry is highly simplified, the model retains the essential network mechanics (i.e., long-range correlation) and rich statistical mechanics, equivalent to a full-scale fibre network model [39]. Unlike the fibre bundle model (FBM), this model does not require the definition of special load-sharing rules. (The FBM is a simplified mode, where fibres are aligned in the loading direction and fail one at a time, to track complex failure combinatorics. Once a fibre fails, either the neighbouring fibres (local-load sharing rule) or all the remaining fibres (equal-load sharing rule) carry the load, and then the failure process continues until all fibres fail.) Instead, load sharing is determined by force equilibrium. Another important point is that the coordination number of this network is 6, greater than 4 (i.e., the isostaticity point in 2D, at which the degree of freedom and the number of constraints are equal [40–43]). Therefore, the initial structure of this network model is rigid, unlike some of the percolation network models (e.g., [44]). With this model, disorders can be introduced into the element (fibre) properties (e.g., stiffness and characteristic strength of fibre) and into the network geometry (e.g., Fig. 3.2), as will be described later.
Chapter 3. Approaches

Constant forces, either tensile or compressive, were applied at the top boundary to simulate creep, and a traction-free boundary condition was applied at the sides. Periodic boundary conditions, however, were not used in this study to avoid introducing artificial length scales into the failure phenomena of the network. The simulations were performed 1000 times for each size in Paper I [34]. In Paper II [45], 1000 repetitions were used for each of the four load levels, since several load levels are required for the three material parameters to be determined for the various investigations. This number of repeats was earlier found to be sufficient to detect non-Weibull behaviour in a finite size range [30, 46].

![Figure 3.1: Schematic of the original structure of the fibre network tested.](image)

3.1.2 Stochastic failure model of fibre

The fibres in the network are assumed to break according to the time-dependent failure model, Eq. 2.5. The parameter $\beta$ (i.e., the Weibull exponent or reliability parameter) for the individual fibres, $\beta_f$, is set to 1 in all simulations. The significance of this is that the probability of failure per unit time is given by:
3.1.2. Stochastic failure model of fibre

Figure 3.2: An example on a disordered structure of the fibre network tested.

\[ h(t) = \frac{F'_f(t)}{1 - F_f(t)} = \left( \frac{T(t)}{T_c} \right) \rho_f \] (3.1)

That is, the probability of failure is determined only by the force at the current time and not by the force in the past, i.e., it is a memory-less, or Markov, process (e.g., [47]). This memory-less element (fibre) failure rate was chosen to investigate the origins of the system’s (network’s) memory ($\beta$).

The simulations start by applying a dead load, and the forces acting on individual fibres are calculated. Based on these forces, random numbers are generated according to Eq. 2.5, and the lifetime values of individual fibres are calculated. The fibre with the shortest lifetime is chosen to break, and the modulus of this fibre is set to zero (or a small number). Then, the forces in each fibre are recalculated for the new state of mechanical equilibrium. The lifetime values of surviving fibres are updated using the following formula:

\[ t_{B2} - t_1 = \left( \frac{T(0)}{T(t_1)} \right)^\rho (t_{B1} - t_1) \] (3.2)
where \( t_1 \) is the time of the first fibre failure in the network and \( t_{B1} \) and \( t_{B2} \) are the first and second estimates of the lifetime of a surviving fibre, respectively. \( T(0) \) and \( T(t_1) \) are the forces of the surviving fibres before and after the first fibre failure, respectively. The minimum updated lifetime of all surviving fibres is then identified; this determines the second fibre failure. This process continues until avalanche failure occurs. This algorithm more faithfully reproduces fibre breaking processes than does the earlier method for triangular lattice models (e.g., [22, 48, 49]) and is in the same spirit as those used for fibre bundle models (FBMs) (e.g., [50]). More detailed information on the model and its settings can be found in Papers I and II [34, 45].

### 3.1.3 Determination of avalanche failure

Network failure, which recalls an avalanche, is defined by the following two conditions: the ratio of the current to initial creep rate exceeds a certain value, \( r \), and this high creep rate \( (r) \) is repeated for a certain number of consecutive time steps, \( n \). Within the parameter space tested, \( r = 100 \) and \( n = 5 \) were found to consistently and acceptably detect the initiation of avalanche failure.

### 3.2 Experimental determination of the material parameters \( S_c, \rho, \) and \( \beta \)

(Papers III and IV)

#### 3.2.1 Materials

Apart from the computer simulations, the material parameters \( S_c, \rho, \) and \( \beta \) were also determined experimentally, using the formation by Coleman.

Various containerboard samples (i.e., liner and fluting used in corrugated board) of different qualities and basis weights were tested in Papers III and IV [35, 51]. The samples were collected from board mills in Sweden, Germany, Austria, and the Czech Republic. These specimens were subjected to both compression creep tests and constant loading rate (CLR) compression tests, as described below. Information
3.2.2 Creep tests

A series of compression creep tests was performed under different loads to determine the material parameters according to the procedure described in the next section. The time to failure was determined when the creep rate surpassed a certain threshold value. Typical creep failure curves are shown in Fig. 3.5a. A long-span compression tester (LCT) with finger supports was used, which was built at Rise Bioeconomy (formerly Swedish Forest Products Research Laboratories, STFI) (Fig. 3.3). (Testing procedures are detailed in [52].) Applied loads were varied at 3–4 levels, and at each load level, 50–100 samples were tested.

Figure 3.3: Creep (constant load) equipment.
Chapter 3. Approaches

3.2.3 Constant loading rate (CLR) tests

Constant loading rate (CLR) tests were performed at different loading rates to determine the material parameters (described in the next section). The samples were compressed at a given CLR until failure occurred. The failure point was determined by setting a criterion for the relative decrease in the tangent modulus of the stress-strain curve. Some examples of typical force-displacement curves (for the same material as used in the creep tests in Fig. 3.5a) are shown in Fig. 3.5b. The equipment used in compressing the containerboard specimens was originally designed at the Forest Products Laboratory in Madison, Wisconsin, USA [53], but was rebuilt to fit an MTS machine (see Fig. 3.4). Loading rates were varied at four levels, and at least 20 tests were performed at each loading rate. For more details about the testing, see Paper III [35].

Figure 3.4: Constant loading rate (CLR) equipment.
3.3. Numerical procedure for determining the material parameters $S_c$, $\rho$, and $\beta$

The procedure for determining the three material parameters, characteristic strength, $S_c$, durability/brittleness, $\rho$, and reliability, $\beta$, can be illustrated by transforming the original equations for creep, i.e., Eq. 2.5 and Eq. 2.7 for CLR test into their Weibull formats as follows:

$$
\log(-\log(1 - F(t_B))) = \log(t_B)\beta + \log(f_0)\rho\beta - \log(S_c)\rho\beta
$$

(3.3)

$$
\log(-\log(1 - G(f_B))) = \log(f_B)\beta(\rho + 1) - \log(S_c)\beta\rho - \log(\rho + 1)\beta - \log(\alpha)\beta
$$

(3.4)

In the case of creep (Eq. 3.3), plotting $\log(-\log(1 - F(t_B)))$ and $\log(t_B)$ gives the slope as $\beta$, and, by changing the applied load, $\log(f_0)$, we obtain $S_c$ and $\rho$ from the intercepts. In the case of CLR tests, plotting $\log(-\log(1 - G(f_B)))$ against $\log(f_B)$ gives the slope as $\beta(\rho + 1)$, and from the intercepts, we can obtain $S_c$ and $\rho$ by changing the loading rate ($\alpha$). In the actual determination, we directly solve the original equations (Eqs. 2.5 and 2.7) using a numerical solver for nonlinear equations, i.e., `fitnlm` [54] in the MATLAB environment. The starting
Chapter 3. Approaches

values (i.e., initial guesses) for this nonlinear fitting were obtained from the Weibull plots based on Eqs. 3.3 and 3.4. This procedure was also used for the data from the Monte-Carlo simulations.
Chapter 4

RESULTS AND DISCUSSION

4.1 Monte-Carlo simulations of creep failure of fibre networks (Papers I and II)

4.1.1 Time-dependent failures under creep loading

It would be instructive first to see how fibre networks break as a function of time, without being bothered by the complex combinatorics of the failure process. This would give us important insights into the essence of statistical failures, and into the meanings of various material parameters defined earlier.

Creep failure simulations have been performed for triangular lattices, as paradigms of fibre networks (see the original structure in Fig. 3.1). Constant forces were applied on the top boundary until the structure failed (see the section "Approaches" for more details). Typical creep curves obtained from the simulations are given in Fig. 4.1, where the effects of different set values of the durability/brittleness parameter of fibre, $\rho_f$ (the durability/brittleness parameter of the network is indicated as $\rho$), are shown: (a) $\rho_f = 5$, (b) $\rho_f = 10$, and (c) $\rho_f = 20$. Each marked point corresponds to an individual fibre break. As expected, with increasing $\rho_f$, the network becomes more brittle, and fewer fibre breaks are required before avalanche-type failure occurs. The range of the $\rho_f$ values used here already exceeds the threshold (i.e., $\rho_f = 2$) that was defined by Curtin for the tough-brittle failure transition [22]. Since these creep curves represent a sequence of elastic failures of fibres, their shapes are rather erratic, unlike the typical creep curves for viscoelastic bodies.

The corresponding damage evolutions are shown in Fig. 4.2, indicating which fibres in the network structure failed at the time of the avalanche failure. With increasing values of $\rho_f$, the network becomes more brittle and fewer fibres break before the whole network
fails. For elastic continuums, it is known that there is a critical crack length at which unstable crack growth initiates. Indeed, such critical cracks are clearly present in networks with higher values of \( \rho_f \). In Paper II [45], the size of these clusters with respect to the set value of \( \rho_f \) was discussed and analysed further in the case of both regular (Fig. 3.1) and disordered (Fig. 3.2) structures.

### 4.1.2 Validation of Coleman’s model in fibre networks

As mentioned earlier, Coleman’s approach consists of three postulates: (1) weakest-link scaling, (2) an algebraic form of the probabilistic failure criterion, and (3) a power-law (or exponential) form of damage evolution. To justify using Coleman’s model in the case of a fibre network, instead of fibre, we need to examine these postulates as they apply to a fibre network. We first examined the third postulate,
4.1.2. Validation of Coleman’s model in fibre networks

Figure 4.2: Damage evolutions, corresponding to the creep curves shown in Fig. 4.1, at three different values of the durability/brittleness parameter for fibre, $\rho_f$. Each point represents an individual fibre break. (a) $\rho_f = 5$, (b) $\rho_f = 10$, and (c) $\rho_f = 20$.

both analytically (Papers I and II [34, 45]) and experimentally [52], determining that it indeed holds at the fibre network level. This was again confirmed experimentally in Paper III [35], and is also shown in the section "Development of experimental characterisation methods for containerboards (Papers III and IV)". However, postulates (1) and (2) are still in question. We examined these postulates by looking at the size dependence of creep lifetime distributions. (Note that postulate (2) means that creep lifetime follows the Weibull distribution.) We will now examine the first and second postulates.

The length and width of the network were increased and the distributions were plotted in the Weibull format (Fig. 4.3). Each
distribution curve was shifted by $ln(N)$ to reveal the emergence of weakest-link scaling, since, if weakest-link scaling holds, all the shifted curves should form a single curve (or line) in the Weibull plot. As seen in Fig. 4.3, the shifted distribution curves gradually collapse into a single curve (called a characteristic distribution), implying that weakest-link scaling emerges as the network size increases. However, the collapsed curve is slightly curved, not straight, indicating that the underlying distribution is not the Weibull distribution. We found that the resulting distribution is the double-exponential distribution, called Duxbury-Leath-Beale (DLB) distribution. This distribution had initially been found for static strength, first in random fuse models [55] and later in fibre bundle models (FBMs) [56]. However, it is important to note that, in a typical experimentally accessible probability range (e.g., 0.01–0.99), the non-linearity (i.e., non-Weibull nature) of the distribution is subtle, making it very difficult to detect in a statistically significant manner. In other words, the Weibull distribution approximately holds for the lifetime distribution of this fibre network. This was also confirmed by a number of experimental observations reported in the literature (e.g., [14, 28, 32]).

Figure 4.3: Lifetime distributions for different sizes of the fibre network plotted in the Weibull format. $N$ is the total number of fibres and $t_B$ is the lifetime of the networks. The broken line is an estimate of the characteristic distribution function. The total number of repetitions is 1000 for each network size.
4.2. Impacts of fibre properties and network structures on the material parameters $S_c$, $\rho$, and $\beta$ (Papers I and II)

In conclusion, Coleman’s formulation [1], approximately holds, not only for the individual fibre but also for the fibre network. There are only two conditions. First, there is a lower bound on weakest-link scaling, meaning that in small networks, weakest-link scaling does not hold. Second, the Weibull distribution is an approximation, so if we extend the distribution beyond the experimentally obtained range, there will be over-estimation in the lower and upper tails. This further implies that the distribution depends on the size of the network, so the material parameters also vary with the network size, as seen in Fig. 4.4 (and discussed in detail in Paper III [35]). In these graphs, typical error bars for each data point are of the same size as the plotted symbols.

The parameter $S_c$, characteristic strength, decreases very slowly with increasing network size, $N$, as shown by its logarithmic dependence (Fig. 4.4a). The parameter $\rho$, the durability/brittleness parameter, maintains a value corresponding to the set value for the individual fibres, $\rho_f (\approx 10)$, and is independent of the network size (Fig. 4.4b). In other words, the load dependence is preserved when moving up in the structural hierarchy from fibre to network. The parameter $\beta$, reliability or Weibull shape parameter, continues to increase with increasing network size, $N$, but relatively slowly, with a logarithmic dependence (Fig. 4.4c). Interestingly, this logarithmic dependence was predicted by an analytical model of damage evolution formulated by Curtin [22]. The increase in the reliability parameter ($\beta$) with network size has previously been reported experimentally (in terms of the static strength of concrete [57] and paper materials [46]) as well as numerically in FBMst incorporating local load sharing.

4.2 Impacts of fibre properties and network structures on the material parameters $S_c$, $\rho$, and $\beta$
(Papers I and II)

The impacts of fibre properties and network structures on the statistical failure of fibre networks are particularly difficult to investigate using experimental means. Monte-Carlo simulations, however, are ideal for systematically exploring the parameter space. This section reports on the introduction of different changes into the fibre network to
investigate their effects on the material parameters of the network. The changes were compared with the default, i.e., the original structure (Fig. 3.1), and if applicable, also with one other. This thesis presents the highlighted effects of some of these changes; the full results are presented in Papers I and II [34, 45].

4.2.1 Disordered structure

The original structure (Fig. 3.1) was distorted to see how doing so affected the material parameters. Each node was displaced by a small amount which is controlled by a Gaussian random variable with zero mean and a varying standard deviation (e.g., Fig. 3.2).

Figure 4.5 shows the material parameter results for two different sets of values of fibre durability/brittleness, $\rho_f = 10$ (quasi-brittle) and $\rho_f = 50$ (brittle). First, with increasing standard deviation, the characteristic strength, $S_c$, decreased almost linearly. In particular, at
4.2.2 Fibre durability/brittleness parameter, $\rho_f$

$\rho_f = 50$, $S_c$ decreased more rapidly than at $\rho_f = 10$. Apparently, characteristic strength is more sensitive to structural disorder in a more brittle than less brittle network. The durability/brittleness parameter for the network, $\rho$, remained constant at values corresponding to those of the fibres (i.e., $\rho_f = 10$ and 50). In other words, the durability/brittleness of the network, $\rho$, is entirely controlled by that of the fibres, $\rho_f$, and not by the structural disorder. The reliability parameter for the network, $\beta$ (the reliability parameter for the individual fibres is $\beta_f$), decreased with the degree of distortion, standard deviation. More brittle fibres ($\rho_f = 50$) consistently exhibited lower $\beta$ values. Interestingly, $\beta$ declined below unity, which is the value of $\beta_f$. An important note is that in all fibre bundle models (FBMs) proposed for time-dependent failure (e.g., [28, 30, 31]), $\beta \geq \beta_f$. Such a result is reasonable because the load-sharing structure of FBMs tends to suppress the variability of lifetime at the network level, as seen in the analytical solution for the equal-load sharing case (e.g., [30]). Experimental data, however, commonly have less-than-unity values of $\beta$ [10–15, 52]. The values for fibres and filaments (e.g., glass, carbon, and graphite fibre) typically tend to be much less than unity, whereas those for fibre composites (which are load-sharing structures) and fibre networks tend to be higher, sometimes being greater than unity. The results obtained here, therefore have important implications for long-term material uncertainty and structure.

4.2.2 Fibre durability/brittleness parameter, $\rho_f$

The durability/brittleness parameter for individual fibres, $\rho_f$, was varied to see how doing so affects the material parameters of the network, i.e., $S_c$, $\rho$, and $\beta$ (Fig. 4.6). All the fibres in the network structure were the same, and varied from a typical quasi-static range value (e.g., $\rho_f = 10$) to a value in the super-brittle range (e.g., $\rho_f = 200$, corresponding to graphite fibre [15]). With increasing $\rho_f$, the characteristic strength, $S_c$, sharply increases and plateaus in the high brittleness range. The parameter $\rho$ equals the comparable parameter for individual fibres, $\rho_f$, as predicted earlier. The parameter $\beta$ decreases sharply with increasing $\rho_f$ and plateaus in the high brittleness range. These results are qualitatively consistent with experimental observations.
Chapter 4. Results and discussion

Figure 4.5: Effects of the structural distortion at two values of the durability/brittleness parameter for fibre, \( \rho_f \), i.e., \( \rho_f = 10 \) and \( \rho_f = 50 \), on (a) dimensionless characteristic strength, \( S_c/T_c \) (\( T_c \) is the set characteristic strength of the fibres), (b) the durability/brittleness parameter, \( \rho \), and (c) the reliability parameter, \( \beta \).

presented in the literature: higher brittleness of the component fibres (\( \rho_f \)) is often associated with higher (short-term) strength (\( S_c \)) but poor long-term reliability (\( \beta \)) (or increased uncertainty, e.g., [11, 15]).

4.2.3 Effects of different disorders on the reliability parameter, \( \beta \)

A comparison of the effects of three different types of distortions on the reliability parameter, \( \beta \), is shown in Fig. 4.7. These distortions are: (1) strength, introduced by varying the characteristic strength of the fibre, \( T_c \) (Eq. 7 in Paper II [45]); (2) stiffness, generated by varying the cross-sectional area of the fibres; and (3) structure, generated as described in the previous section "Disordered structure". Since the distributions of strength and stiffness were created from the uniform distributions, standard deviations were recalculated to be compared with the structural disorder. The reliability parameter, \( \beta \), was com-
4.3 Development of experimental characterisation methods for containerboards (Papers III and IV)

Figure 4.6: Effects of the durability/brittleness parameter for fibre, $\rho_f$, on (a) dimensionless characteristic strength, $S_c/T_c$ ($T_c$ is the set characteristic strength of the fibres), (b) the durability/brittleness parameter, $\rho$, and (c) the reliability parameter, $\beta$.

pared as the ratio to the initial value of the "undistorted structure". The strength disorder has a rather modest impact on $\beta$, whereas the stiffness and structural disorders have strong negative impact on the parameter. A natural speculation from this result is that the network variability ($\beta$) may be more influenced by stress distributions within the structure than by "threshold" strength distributions.

4.3 Development of experimental characterisation methods for containerboards (Papers III and IV)

Since we have demonstrated that Coleman’s model for individual fibres is also applicable to the fibre network and that the three material parameters characterise different aspects of time-dependent statistical failure (TSF), we will proceed to develop an experimental method to determine these parameters.
Chapter 4. Results and discussion

A series of experiments for containerboards was performed using two different loading cases, i.e., creep (constant load) and constant loading rate (CLR) (see more in the section "Approaches"). These represent two different loading cases, as shown by their cumulative distributions (Eqs. 2.5 and 2.7).

4.3.1 Creep lifetime and strength distributions: Comparisons

The typical frequency distribution of creep lifetime obtained at a given applied load is shown in Fig. 4.8a. It is characterised by a large number of premature failures (i.e., short lifetimes) and, at the same time, by a persistent tail in the very long lifetime range. Figure 4.8b shows the cumulative distribution function of the same lifetime data as in Fig. 4.8a, $F(t_B)$, plotted in the Weibull format. The creep lifetime data approximately follow the Weibull distribution, as was observed earlier [52] and as predicted by the current model. The highly skewed distribution of the lifetime is an indication of a very low value of the reliability parameter (the Weibull shape parameter), $\beta$, which is often less than unity.

Another important test for the model validation concerns the damage evolution law, Eq. 2.9 (related to Coleman’s third postulate).
4.3.1. Creep lifetime and strength distributions: Comparisons

Figure 4.8: (a) A typical frequency distribution of creep lifetime, measured at an applied load of 56.0 N. The lifetime data are made dimensionless by dividing them by the median lifetime. Containerboard samples with a basis weight of 140 g/m². (b) Cumulative distribution function of the same lifetime data as in (a) plotted in the Weibull format. \( t_B \) is the lifetime of the containerboards.

If the damage evolution law is in a power-law form, then the model predicts that (1) increasing creep loads horizontally shifts the cumulative distributions in the Weibull format to the left, and (2) when plotted in a log-log format, the mean (or median) lifetime vs. applied load relationships should be linear. Figure 4.9a examines the first condition. The individual curves indeed shift horizontally to the left with increasing applied load. To examine the second condition, the median lifetime values are plotted against applied load in a log-log format in Fig. 4.9b. The relationship is linear, as predicted by Eq. 2.6, and such linearity was observed consistently in all other sample sets. (The reason for using median, instead of mean, values is that it is difficult to precisely determine the mean lifetime in creep experiments, because of the presence of extremely long-live specimens. Particularly at low levels of applied load, not all samples fail within the set experimental time period.)

Figure 4.10a shows the frequency distribution of strength obtained by CLR tests at the same loading rate. The data are typically centred around the mean with a very small scatter, unlike the lifetime distributions. The corresponding data are plotted in the Weibull format in Fig. 4.10b, again showing the Weibull distribution (i.e., a straight line in the Weibull plot).

Although both the lifetime distributions and strength distributions
belong to the same Weibull distribution family, the difference between these distributions resides in their shape parameter (or Weibull exponent), $m$. Coleman [1] and more recently Christensen et al. [58] derived the relationship between the Weibull exponents for creep lifetime, $m_{\text{creep}}$, and for strength, $m_{\text{strength}}$ (Eq. 2.1). Since $\rho$ is in the order of 20–60 in the case of cellulosic materials, it is understandable that $m_{\text{strength}}$ is always much higher than $m_{\text{creep}}$. In other words, strength distributions always have much smaller scatters than do creep lifetime distributions.

Figure 4.10: (a) A typical frequency distribution of strength, measured at a loading rate of 1 $N/s$. The strength data are made dimensionless by dividing them by their mean value. Containerboard samples with a basis weight of 140 $g/m^2$. (b) Cumulative distribution function of the same strength data as in (a) plotted in the Weibull format. $f_B$ is the strength of the containerboards.
4.3.2. Characterisation of containerboards with the material parameters $S_c$, $\rho$, and $\beta$

In the case of CLR tests, the model predicts that the strength distributions plotted in the Weibull format will be horizontally shifted to the right with increasing loading rates. Figure 4.11a indeed exhibits such a trend. It also predicts that taking the mean strength values from Fig. 4.11a and plotting them against the corresponding loading rates will yield a linear relationship in the log-log plot. The result (Fig. 4.11b) exhibits precisely such a relationship.

In summary, the model originally proposed by Coleman describes very well the statistical failure responses of cellulosic materials subjected to entirely different loading histories, creep tests, and CLR tests.

![Figure 4.11: (a) Cumulative distribution function of strength plotted in the Weibull format for four different loading rates (1–500 N/s). $f_B$ is the strength of the containerboards. Containerboard samples with a basis weight of 140 g/m². (b) Mean strength vs. loading rate for the data presented in (a).]

4.3.2 Characterisation of containerboards with the material parameters $S_c$, $\rho$, and $\beta$

The three material parameters, characteristic strength, $S_c$, the durability/brittleness parameter, $\rho$, and the reliability parameter, $\beta$, were determined by performing both creep tests and CLR tests on a series of commercial containerboard samples. (The procedure for determining these parameters can be found in the section "Approaches".) Both these test methods were performed to determine whether they gave comparable results. Figure 4.12 shows the results for a series of commercial containerboard samples. In these tests, samples for creep tests
and CLR tests were taken in different time periods, independently, from the same containerboard. Therefore, we were unable to perform "controlled randomisation", i.e., taking specimens, randomising them, and then splitting the specimens into two tests. As a result, the computed parameters unfortunately varied greatly. However, examining individual cases, there was no systematic difference in the results between the creep and CLR tests, suggesting that both tests provide comparable results. These results have an important implication. Creep tests are difficult to perform in terms of controlling the test environment, loading conditions, and individual equipment variability. In addition, creep testing takes an uncertain length of time, and it is difficult to increase the number of samples. However, CLR tests are quick and all other conditions can be well controlled. Therefore, the results suggest that CLR tests may be an ideal method to determine these three material parameters in a practical way.

Figure 4.12: Material parameters determined by creep and CLR compression tests. (a) Specific characteristic strength, $S_c/(\text{basis weight } \times \text{width})$, (b) the durability/brittleness parameter, $\rho$, and (c) the reliability parameter (or Weibull shape parameter), $\beta$. 
4.3.2. Characterisation of containerboards with the material parameters $S_c$, $\rho$, and $\beta$

With the CLR method, a large number of containerboard samples was collected and characterised by determining the material parameters (Paper IV [51]). Basic papermaking variables were varied to see how doing so affected the material parameters. These factors were basis weight, CD (cross machine direction) position of the paper machine, and the MD (machine direction) /CD relationship. In addition, different containerboard grades (i.e., fibre types) with approximately the same basis weights were compared. Figure 4.13 is a plot of the reliability parameter, $\beta$, against the durability/brittleness parameter, $\rho$, for some of these samples. There is a tendency that with increasing $\rho$, $\beta$ decreases, except for one outlier. In other words, highly brittle, though durable, materials tend to exhibit large variability in long-term performance (e.g., creep lifetime). This relationship was actually predicted in Fig. 4.6 from our Monte-Carlo simulation studies (Papers I and II [34, 45]). The basic mechanism is that a more brittle material (i.e., higher $\beta$) tends to be more sensitive to the presence of defects and to defect distributions, so the effect of material non-uniformity is

![Figure 4.13: Comparison of $\beta$ and $\rho$ between different collected containerboard samples.](image-url)

Figure 4.13: Comparison of $\beta$ and $\rho$ between different collected containerboard samples. Fluting is the corrugated material used in the middle of a corrugated board construction, and liner is the flat material used on both sides. Kraftliner is a liner with a high amount of virgin fibres, White top is a liner where the top side is bleached, Testliner is a liner made of recycled fibres, Fluting is made of semi-chemical pulp, and Recycled fluting is a fluting made of recycled fibres.
amplified, leading to more variability (i.e., lower $\beta$). Although higher $\rho$ is important in achieving higher durability, maintaining reliability ($\beta$) is also important. In this sense, it is interesting to see that the outlier sample exhibited both higher $\beta$ and higher $\rho$. In particular, achieving $\beta > 1$ is very significant for reliability, because at $\beta = 1$, a highly skewed distribution of lifetime becomes an exponential distribution, implying that premature failures (i.e., failure near zero lifetime) decrease drastically (i.e., theoretically from infinite to finite).

We also compared the material parameters, $\beta$ and $\rho$, obtained from our containerboard samples with those for other types of fibre materials. Such data are not widely available, but a limited number of data have been reported in the literature on fibre-based materials used for advanced composites [10, 11, 13–15] (see Fig. 4.14). There is, again, a clear trend that the higher the $\rho$ value (brittleness), the lower the $\beta$ (reliability). Interestingly, the cellulosic fibre networks used for containerboards exhibited rather high $\beta$ values (indicating high reliability) with modest $\rho$ values. These values are comparable to those for other types of fibre materials, such as Kevlar-epoxy and glass-

![Figure 4.14: Comparison of $\beta$ and $\rho$ between containerboard samples and typical fibre-based materials used for light-weight structural composites. Fluting is the corrugated material used in the middle of a corrugated board construction, and liner is the flat material used on both sides. Kraftliner is a liner with a high amount of virgin fibres and Fluting is made of semi-chemical pulp.](image)
4.3.2. Characterisation of containerboards with the material parameters $S_c$, $\rho$, and $\beta$ epoxy composites. By enhancing their performance further, these cellulose-based fibre networks may become even more competitive and favourable materials for structural composite applications.
CONCLUSIONS

Inspired by the fundamental question of whether stronger is necessarily better, we became interested in time-dependent statistical failure (TSF), initiating this study to find a new way of characterising the multifaceted "strength" of materials. Using the formulation developed by Coleman and others, we have identified three "facets" of strength: (1) characteristic strength, $S_c$, (2) a durability/brittleness parameter, $\rho$, and (3) a reliability parameter, $\beta$. These material parameters capture the time-dependent nature of the material, and describe the variability of both the short- and long-term properties.

Although Coleman's formulation is known as a rigorous approach, it has never previously been tested for a broad class of materials, particularly fibre network systems. By performing Monte-Carlo simulations of statistical failure in lattice fibre networks, we examined the postulates of the theory. The conclusion is that the theory is indeed applicable to network systems, with two minor modifications, namely: (1) for weakest-link scaling to appear, a certain minimum system (network) size is required; and (2) the underlying characteristic distribution of lifetime is not the exact Weibull distribution but has a double-exponential form. However, the Weibull distribution still persists within a typical, experimentally accessible probability range (e.g., 0.01–0.99) and is therefore a good approximation. Monte-Carlo simulations have also given us numerous insights into and a good understanding of how fibre properties translate into network failure properties through a disordered network structure. First, the reliability, $\beta$, of the network is proportional to the reliability of the individual fibres, $\beta_f$, and the proportional constant is highly dependent on the disordered structure and the stiffness non-uniformity (or more generally, stress uniformity) within the network. Second, the durability/brittleness, $\rho$, of the network is equal to the durability/brittleness of the individual fibres (or breaking elements), $\rho_f$, suggesting that durability/brittleness is primarily determined by the fibre properties. An interesting interaction was also found between $\beta$ and $\rho_f$, in that...
increasing $\rho_f$ decreases $\beta$, i.e., more brittle systems (networks) tend to suffer more variation in creep lifetime (i.e., long-term performance).

After validating the formulation of Coleman and others, we started developing an experimental method, i.e., constant loading rate (CLR) testing, for determining the material parameters by comparing the CLR results with the results of creep tests. These two tests differ in their measurement timescales, but the distributions are governed by the same set of material parameters. The results indicated that the two methods provided comparable values for the material parameters, and no systematic deviations were observed. An implication is that it is now possible to perform the much quicker CLR tests, instead of traditional creep tests, to determine the material parameters. This means a drastic reduction in testing time, which also enables an increase in the number of tested samples to raise statistical confidence in the results.

As the first application of the newly developed method, we performed a series of CLR tests on commercial containerboard samples with varying papermaking conditions and furnishes, and the results were compared with those for fibre-reinforced plastic composites. Containerboards are characterised as a material with relatively low durability/brittleness, but comparatively high reliability. These properties are closer to those of typical stiff polymer materials. Throughout the material spectrum, there is a clear trend towards "the higher the brittleness, the lower the reliability", as predicted by our simulations. For example, carbon fibres have very high durability/brittleness, but very low reliability.

The fact that the multi-faceted nature of strength can be expressed by only three parameters is remarkable. The implications are profound for the way materials will be designed and new materials developed. It is the author’s hope that this thesis will be of some use when it comes to redefining and developing materials for a sustainable society, particularly the renewable alternative – cellulose fibres.
ACRONYMS

BCT  box compression test
CLR  constant loading rate
COV  coefficient of variation
ECT  edgewise compression test
FBM  fibre bundle model
LCT  long-span compression test
SCT  short-span compression test
TSF  time-dependent statistical failure
WLS  weakest-link scaling
BIBLIOGRAPHY


